

1. A commodity q is traded at price p in a competitive market with price-taking consumers and firms.

There are 200 identical consumers each with income $Y = 80$. Each consumer has a utility function over numeraire consumption c and commodity q given by:

$$u(c, q) = c + 24q - 2q^2$$

There are M identical firms each with cost function given by:

$$c(q) = 32 + \frac{1}{2}q^2$$

The number of firms is fixed in the short run, but in the long run firms can freely enter or exit the market. Thus, the number of firms is flexible in the long run.

- a. Write down the consumer's first order condition.
 - b. Write down the firm's first order condition.
 - c. Write down the market equilibrium condition.
 - d. Write down the zero profit condition.
 - e. Solve for the long run equilibrium price, quantity, and number of firms.
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2. A commodity q is traded at price p in a competitive market with price-taking consumers and firms.

There are 1,280 identical consumers each with income $Y = 100$. Each consumer has a utility function over numeraire consumption c and commodity q given by:

$$u(c, q) = c + 20q - \frac{1}{2}q^2$$

There are 320 identical firms each with cost function given by:

$$c(q) = 32 + 10q + \frac{1}{2}q^2$$

- a. Calculate the equilibrium price and quantity.
- b. Calculate the total surplus (total utility + total profit) in equilibrium.
- c. Is the market in long run equilibrium?

A new technology lowers the cost of production so that the new cost function is:

$$c(q) = 32 + 5q + \frac{1}{2}q^2$$

- d. Calculate the new short run equilibrium price and quantity.
 - e. Calculate the total surplus in the new short run equilibrium.
 - f. Calculate the new long run equilibrium price, quantity, and number of firms.
 - g. Calculate the total surplus in the new long run equilibrium.
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3. A commodity q is traded at price p in a competitive market with price-taking consumers and firms.

There are N identical consumers each with income $Y = 50$. Each consumer has a utility function over numeraire consumption c and commodity q given by:

$$u(c, q) = c + 12q - \frac{1}{2}q^2$$

There are M identical firms each with cost function given by:

$$c(q) = 2 + \delta q + \frac{1}{2}q^2$$

The number of firms is fixed in the short run, but in the long run firms can freely enter or exit the market. Thus, the number of firms is flexible in the long run.

- a. Prove that the long run equilibrium price is equal to:

$$p = \delta + 2$$

- b. Prove that in the long run equilibrium, the total output of the commodity is

$$Q = N(10 - \delta)$$

and that the output per consumer is:

$$q_d = 10 - \delta$$

- c. Prove that in the long run equilibrium, the utility of each consumer is:

$$u = 50 + \frac{1}{2}(10 - \delta)^2$$

- d. Real GDP is a measure of real output per person. Using this model as an example, explain how:

- i. Population growth (an increase in N)
- ii. Productivity growth (a decrease in δ)

affect real GDP, real GDP per capita, and consumer well-being.