Econ 310 Week 3 Homework
Due: Tuesday 9/17

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1. A commodity q is traded at price p in a competitive market with price-taking consumers and firms.

   There are 120 identical consumers each with income $Y = 100$. Each consumer has a utility function over numeraire consumption c and commodity q given by:

   $$u(c, q) = c + 18q - \frac{1}{2}q^2$$

   There are 24 identical firms each with cost function given by:

   $$c(q) = \frac{1}{10}q^2$$

   a. Write down the consumer's first order condition.

   b. Write down the firm's first order condition.

   c. Write down the market equilibrium condition.

   d. Solve for the equilibrium price and total quantity.

   e. Calculate total consumer utility and total firm profit in equilibrium.

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2. Take the setup from problem 1. Suppose an ad valorem tax rate of 20% is placed on producers.

   a. Find the equilibrium price and total quantity traded under the tax.

   b. Calculate the tax revenue raised by the tax.

   c. Calculate the deadweight loss caused by the tax.

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3. Take the setup from problem 2. Suppose that instead of using an ad-valorem tax, a lump sum tax is levied on producers that generates the same tax revenue.

   a. What amount of tax must be charged to each producer in order to raise the same revenue from problem 2?

   b. Find the equilibrium price, total quantity, and total surplus under the lump sum tax. (Hint: Based on the lump sum principle, you shouldn't have to solve any new equations.)

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4. A commodity q is traded at price p in a competitive market with price-taking consumers and firms.

   There are 400 identical consumers each with income $Y = 100$. Each consumer has a utility function over numeraire consumption c and commodity q given by:

$$u(c, q) = c + 4q - \frac{1}{2}q^2$$

   There are 50 identical firms each with cost function given by:

$$c(q) = \frac{1}{2}q^2$$

   An ad-valorem tax rate of t is levied on the producers.

   a. Prove that the equilibrium price is equal to:

$$p = \frac{32}{9 - t}$$

   b. Prove that the equilibrium quantity is equal to:

$$Q = 1,600\left(\frac{1 - t}{9 - t}\right)$$

   c. Use Excel or any other software to plot tax revenue as t goes from 0% to 100%. How would you describe the shape of this curve? At approximately what tax rate is the tax revenue maximized? This pattern is known as the Laffer Curve.
