NONLINEAR FLUTTER ANALYSIS BY APPLICATION OF A HARMONIC BALANCE AND A CONTINUATION METHOD

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Abstract: The application of a *continuation method* to the solution of a system of nonlinear flutter equation is presented. The continuation method use predictor-corrector techniques to follow solution curves of nonlinear equations represented by functions of a number of variables and parameters. This approach was successfully demonstrated for flutter stability and parameter studies of linear systems. In this paper the locally parameterized continuation process is applied to systems with interacting structural nonlinearities.
For practical analysis of complete aircraft a modal reduction is proposed, which is based on normal modes of the structure with all concentrated nonlinearities replaced by linear springs. Concentrated structural nonlinearities can be modeled by a harmonically equivalent stiffness matrix depending on the magnitudes of the complex generalized coordinates. In the solution process the stiffness matrix is modified by proper values resulting from *harmonic balance* of nonlinearities. Available results of calculation based on center-manifold reduction for an airplane with several nonlinearities in the control system and the 2D wing tests agree with those of the proposed continuation method which can serve as a powerful tool for the investigation of complex nonlinear flutter behavior.

1 INTRODUCTION

Static tests and ground vibration tests carried out on aircraft prototypes confirm existence of both distributed and concentrated structural nonlinearities. Nonlinearities of this type are indicated in static tests on the *force-deflection* diagrams or in ground vibration tests on the *frequency-amplitude* diagrams. Examples of such nonlinearities in real gliders can be found in [1,2]. That results in additional complications in analysis because flutter of a non linear system may be of the limited amplitude but on the other hand a non linear system which is stable with respect to small disturbances may be unstable with respect to large ones. The nonlinearities in flutter calculations can be handled either by direct integration of equations of motion in the time-domain or by iterative search in frequency-domain [2,3]. In both methods finite amplitude limit-cycle solution are the targets. For systems with concentrated structural nonlinearities the harmonic linearization method is applied and equivalent linear system, conserving mean energy value during one period of oscillation, is obtained for a given constant-amplitude harmonic motion. In this case the flutter problem is described by the system of nonlinear algebraic equations.
The finite amplitude limit-cycle oscillations (LCO) for systems with single nonlinearity can be found causing no serious difficulties. However, for greater number of nonlinearities iterative methods are necessary, because the amplitudes of aircraft deflections at concentration points (nonlinear springs) are not known prior to calculations but their ratios are determined by the

resulting flutter mode. Earlier works [2,3] confirm the efficiency of iterative process for systems having two nonlinearities.
A class of *continuation methods* [4] is one of recent developments in numerical analysis is. Continuation method was successfully applied to flutter calculation of systems without structural nonlinearities [5]. Application of a continuation method to the flutter solution for systems with many structural nonlinearities is presented on examples of a sailplane with nonlinear control stiffness of flaps, ailerons and elevator [6] and the LCO investigation on the wing tested in the wind tunnel [7,8].
This paper shows comparison of results obtained by continuation method with those obtained by the method based on center-manifold reduction [10] and wind tunnel tests [8]. Compatibility of results is satisfactory.

2 NONLINEAR FLUTTER EQUATIONS – FORMULATION IN TERMS OF CONTINUATION METHOD

When the structure of aircraft is nonlinear (for example result from backlash, solid friction, application of nonlinear correctors in control system etc..) then the stiffness matrix $[K]$ in flutter equation (1)

$$\left(-\omega^2[M]+[K]\right)\{q\}=\rho\omega^2\left[A(k)\right]\{q\} \tag{1}$$

where: $\{q\}$ - coordinates vector,
$[M]$ - mass matrix,
$[K]$ - (complex) stiffness matrix,
$\left[A(k)\right]$ - (complex) aerodynamics matrix, the elements of which are functions of the reduced frequency $k=\omega b/V$
ω - circular frequency,
b - reference length,
V - free-stream velocity,
ρ - free-stream density.

depends on the solution vector $\{q\}$. Strictly speaking matrix $[K]$ depends on amplitudes of aircraft vibration. Critical flutter conditions are in this case function of the vibration amplitudes $\{\beta\}$ of nonlinear springs, which are new parameters (new unknowns) in equations formulated in terms of continuation method:

$$\left(\omega^2\left([M]+\rho\left[A(k)\right]\right)-(1+ig)\left[K(\beta)\right]\right)\{q\}=0 \text{ - flutter equation;} \tag{2}$$

$\{q\}^H\{q\}-E=0$ - amplitude normalization;

$\mathrm{Im}(q_i)=0$ - phase normalization;

$k-\dfrac{\omega b}{V}=0$ - reduced frequency;

$$\{\beta\} - \left|[\beta^{NLin}]\{q\}\right| = 0 \text{ - deformation of nonlinear springs;} \tag{3}$$

where: $[M]$ - mass matrix,

$[A(k)]$ - aerodynamic matrix,

$[K\{\beta\}]$ - nonlinear stiffness matrix,

$[\beta^{NLin}]$ - rectangular matrix, which contains in *l* rows deformations of nonlinear springs corresponding to *n* particular vibration modes used in calculation,

ω - circular frequency,

g - artificial damping,

b - reference length,

V - free-stream velocity,

ρ - free-stream density,

E - vibration amplitude level.

Nonlinear stiffness matrix is defined by the equation

$$[K\{\beta\}] = [K_0] + [\beta^{NLin}]^T([K_{eqv}(\beta)] - [K^L])[\beta^{NLin}] \tag{4}$$

where: $[K_0]$ - generalized stiffness matrix,

$[K_{eqv}(\beta)]$ - diagonal matrix of harmonically equivalent stiffnesses,

$[K^L]$ - diagonal matrix of linear stiffnesses.

Continuation methods are a class of predictor corrector technique for the solution of systems of non linear algebraic equations which are functions of several parameters, defined over a specified range. Typically there is one more unknown than equations. One of unknowns is chosen as the continuation parameter, and is fixed during the corrector phase in order to give the same number of equations as unknowns. For calculation economy it is important to pick the largest possible predictor step while maintaining convergence in the corrector phase.

The system of 2*n*+3+*l* real equations (2,3) has 2*n*+5+*l* real unknowns $\{q\}$, ω, g, k, V, E and $\{\beta\}$. It means that in continuation process the one of parameters (the one of unknowns) must be fixed. For example if $g = 0$ during calculations, then continuation solution gives the critical flutter conditions (limit cycle solution) V_F, ω_F, $\{q_F\}$, $\{\beta_F\}$ dependent on vibration amplitude level E (*nonlinear flutter*). If $E = \varepsilon$ (ε - small value) is fixed in calculation, then the continuation method gives classical linear solution (*linear flutter*), which can be a starting point to nonlinear calculations.

The PITCON package of FORTRAN subroutines [4] was used to solve the system of equations (2,3). PITCON was designed at University of Pittsburgh to be a general-purpose package for the solution of undetermined system of non linear equations in which the number of equations is one less than the number of unknowns.

3 HARMONIC BALANCE

Previously described methods are based on the assumption that the aircraft motion is time-harmonic. Consequently, all nonlinearities of the structure must be *harmonically equivalent*. The effective tool in this case is the linearization method based on the principle of *harmonic balance* [11,12] (*harmonic linearization*), which provides equality of the amplitude of linearized system and the amplitude of the fundamental harmonic component of nonlinear system at $\overline{\beta} = \beta\cos\omega t$. For the nonlinear *force-deflection* function the real characteristic $M(\beta)$ (for example: *hinge moment-control surface deflection*) is replaced by the equivalent one

$$M(t) = \mathrm{Re}\left(\beta\left(C + iG\right)e^{i\omega t}\right) \tag{5}$$

defined for the time-harmonic motion $\overline{\beta} = \beta e^{i\omega t}$. Real control surface deflections are:

$$\beta_r = \mathrm{Re}(\overline{\beta}) = \beta\cos\omega t \tag{6}$$

Equivalent stiffness and damping coefficients, C and G, respectively are obtained by balancing the fundamental harmonic term of the Fourier series expansion of $M(\beta_r)$ during a single oscillation period. Harmonically equivalent stiffness

$$C(\beta) = \frac{1}{\pi\beta}\int_0^{2\pi} M(\beta\cos\omega t)\cdot\cos\omega t\cdot d(\omega t) \tag{7}$$

and damping

$$G(\beta) = \frac{1}{\pi\beta}\int_0^{2\pi} M(\beta\cos\omega t)\cdot\sin\omega t\cdot d(\omega t) \tag{8}$$

are the functions of the amplitude β in harmonic oscillation. Equivalent characteristic (*describing function*)

$$M(\beta_r) = \beta(C\cos\omega t - G\sin\omega t) \tag{9}$$

gives, after transformation, the following equation:

$$M(\beta_r) = M^C(\beta_r) + M^G(\beta_r) \tag{10}$$

where: $M^C(\beta_r) = C(\beta)\beta_r$ - linear function of β_r,

$M^G(\beta_r) = -G(\beta)\sqrt{\beta^2 - \beta_r^2}$ - hysteresis function of β_r.

The nonlinear force-deflection function is replaced after linearization by an equivalent one with elliptical hysteresis. Energy dissipated during one period of oscillation remains unchanged for the equivalent characteristics

$$E_d = \oint M(\beta_r) \cdot d\beta_r = \pi G \beta^2 \tag{11}$$

The (complex) matrix of *harmonically equivalent* stiffnesses in equation (4) $\left[K_{eqv}(\beta)\right]$ is diagonal

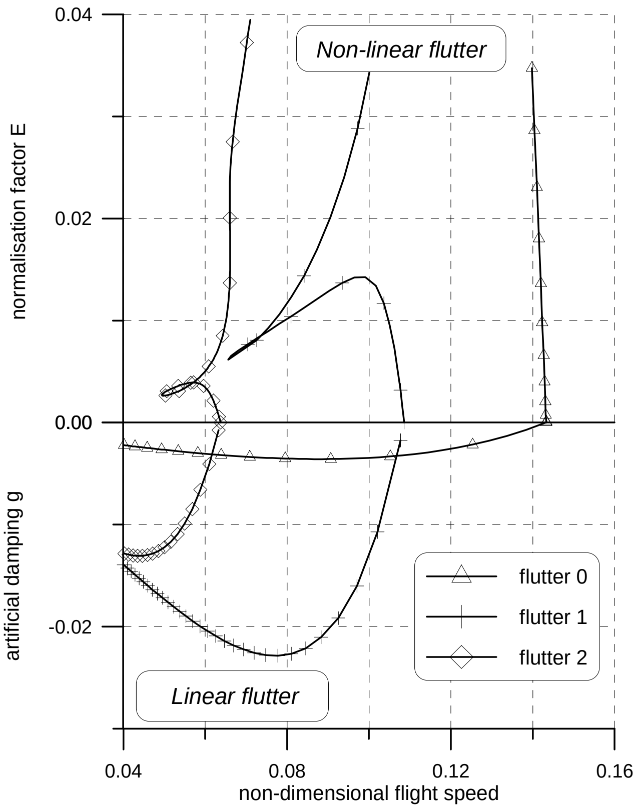
$$\left[K_{eqv}(\beta)\right] = \left[C(\beta)\right] + i\left[G(\beta)\right] \tag{12}$$

The conditions necessary for the application of *harmonic balance* are discussed in detail by Popow & Platow [11] and Shen [12]. The integrals (7,8) can by calculated analytically only for simple $M(\beta_r)$ functions. Typical example of analytical *describing functions* are presented in [1]. For the force-deflection characteristics obtained from tests, the only way to calculate integrals (7,8) is the use of numerical method. Efficient algorithm was elaborate [13] and used [14] to calculate *describing functions* for the force-deflection characteristics obtained from tests [1]. Practical tests conducted on aircraft, generally allow for estimation of nonlinearities such as: back-lash, solid friction, spring correctors and deflection stops. If nonlinearities are also found to be dependent upon velocity of the control deflection then the tests are more complex and therefore rarely performed on prototypes of aircraft.

4 EXAMPLES

4.1 Flutter limit cycle calculation of a sailplane with nonlinear control systems

Nine symmetric modes (seven vibration modes and two rigid body modes) of sailplane, used in [14] to testing the iteration process, were applied to LCO calculation by a continuation method [6]. Nonlinearities of outer flap controls, aileron controls and elevator controls are of *hard springs* type, generating cubic restoring forces. Inner flap control is keep linear in tests. Starting values to non-linear calculation (linear V-g solutions) were obtain by a continuation procedure applied to equations (2) for $\varepsilon = 10^{-8}$ (lower part of the graph). Target points ($g = 0$) in linear calculation are starting points to non-linear continuation process, in which constant unknown is changing from $E = \varepsilon$ to $g = 0$ (upper part of the graph).



4.2 Flutter limit cycle calculation of two sailplanes with nonlinear control systems

Examples assume nonlinearities concentrated in the points of connection between control surfaces and control systems of an aircraft, producing nonlinear restoring moments when the control surfaces rotate about hinge. It is assumed that each restoring moment $M(\beta)$ (10) is a cubic function of the local rotation angle β

$$M(\beta) = K(\beta + c\beta^3) \tag{13}$$

where K is a standard linear spring constant and c describes the strength of nonlinearity. To compare the results obtained by the continuation method with those obtained by the method based on center-manifold reduction [9,10] sample calculations of limit cycle amplitude and frequency were made for aileron and flap flutter of two gliders. All springs connected control surfaces to control systems were assumed to produce hardening cubic nonlinearities. The number of physical degrees of freedom used to calculate natural modes were nearly to 200. Six modal coordinates were taken into account including two or three rigid modes [10]. Both gliders had one nonlinear spring between aileron and control system with c=50 (13). As a final result of calculations the limit cycle amplitudes $\{\beta\}$ (3) (in radians) are plotted against the nondimensional flutter velocity V/Vo.

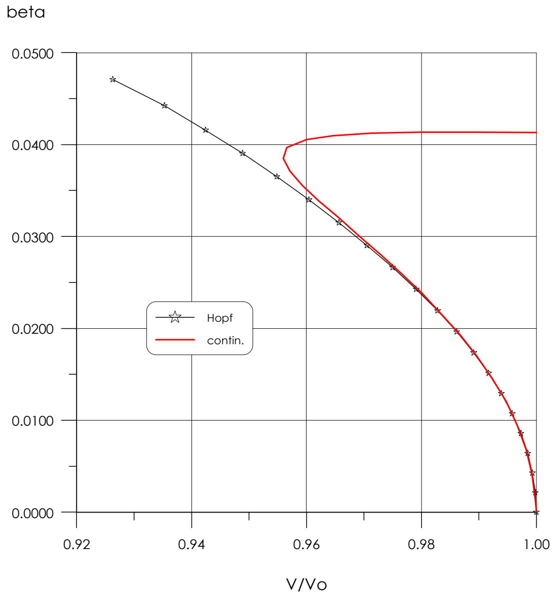


Figure 1: Flap limit cycle amplitude (symmetric flutter)

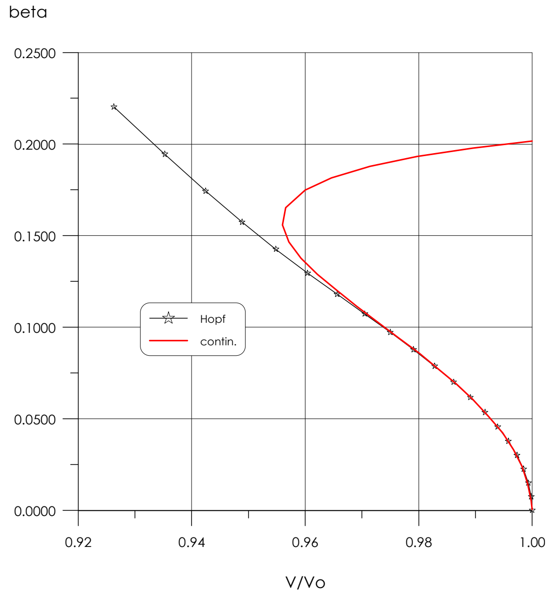


Figure 2: Aileron limit cycle amplitude (symmetric flutter)

The first glider revealed symmetric and also antisymmetric flap-aileron flutter at velocities Vo=187 km/h and Vo=178 km/h, respectively. Results of calculations are shown on Fig. 1-4.

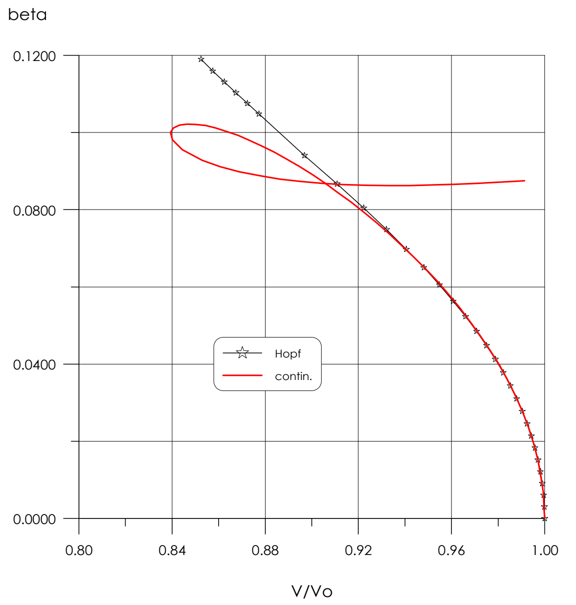


Figure 3: Flap limit cycle amplitude (antisymmetric flutter)

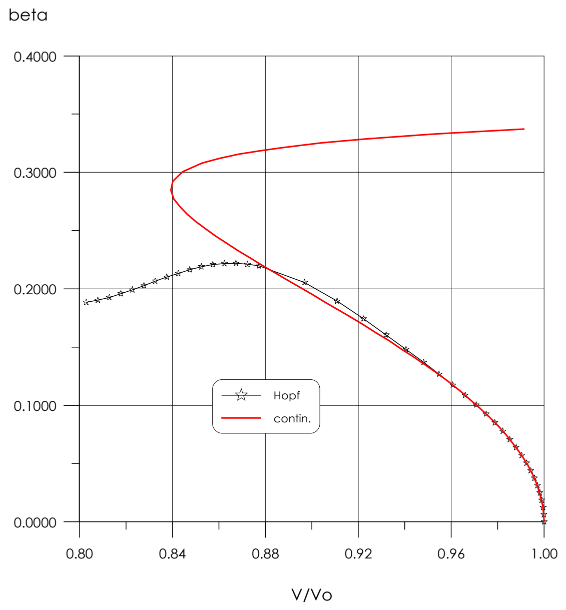


Figure 4: Aileron limit cycle amplitude (antisymmetric flutter)

Antisymmetric flutter at Vo=225 km/h occurred for the second glider. Result of calculations is presented in Fig. 5,6.

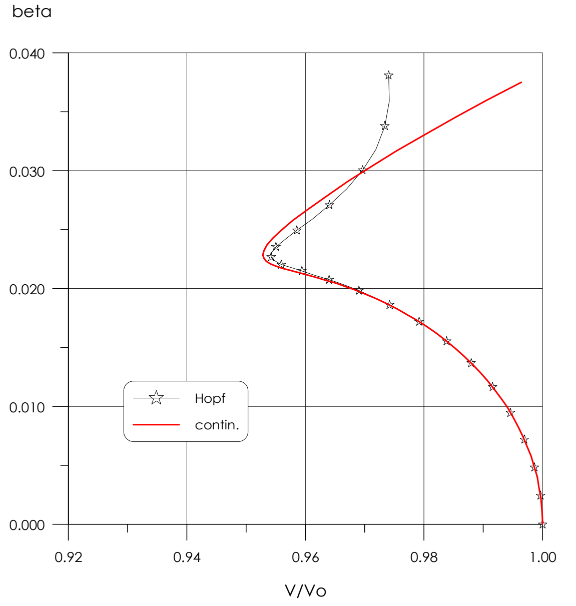


Figure 5: Flap limit cycle amplitude (antisymmetric flutter)

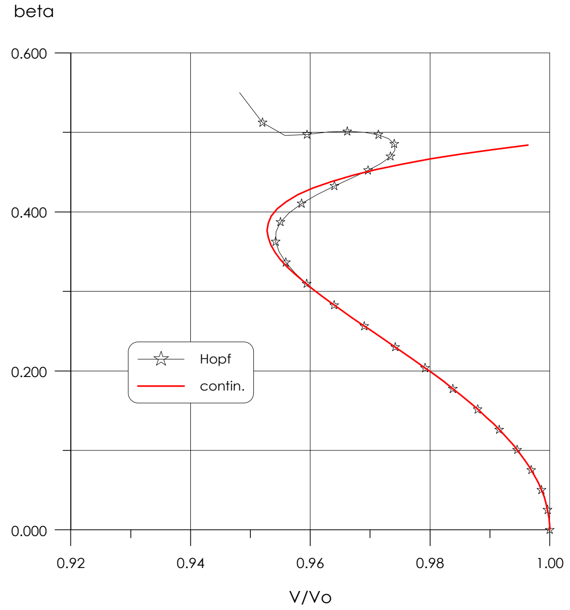


Figure 6: Aileron limit cycle amplitude (antisymmetric flutter)

Symbol **Hopf** in all figures denotes the amplitude of limit cycle oscillations calculated by the method based on center-manifold reduction [10]. There is a very good agreement between results of the present method and the method based on center-manifold reduction in a range of a few percent below linear flutter velocity Vo. However, beyond this interval a qualitative discrepancy of the results of both method are observed. It was impossible to establish the real behavior of gliders limit cycle oscillations because neither flight tests nor direct numerical integration of the nonlinear flutter equation were performed. Nevertheless, it is important that the limit cycle oscillations are detected below linear flutter velocity despite the fact that their amplitude is uncertain. These oscillations can be initiated by sufficiently high disturbance, magnitude of which is known from the present results of calculations and is given by the unstable branch of amplitude curves (the part of plots between the linear flutter velocity V/V0=1 and the turning point in Fig. 1-6).

4.3 Wind Tunnel tests

The Low Turbulence Wind Tunnel at Institute of Aviation has been chosen to prove the functionality of the new LCO model with computer control of a spring system [8]. This tunnel has a close rectangular test section with size 0.5x0.65 m. Maximum flow velocity is 85 m/s and can be controlled in whole range with great accuracy (dynamic pressure accuracy up to 0.1 mm H_2O). It has high stability of flow velocity, which is necessary in the LCO measurements, when even minor changes in velocity can cause changes in amplitudes of flutter modes and transient effects. It is particularly important in the most interesting cases for velocities close to the critical flutter speed or in range of LCO modes. The dimensions and kinematical scheme, of flutter model have been set to perform the tests in the most suitable conditions for validation.

The NACA0012 profile has been chosen for the uniform wing cross section. It is probably the most frequently used profile for basic investigation of non-stationary aerodynamics. The aerodynamic surface span of the wing is 0.5 m and the chord 0.2 m. It is assumed that measurement range of velocity is from 15 m/s to 40 m/s and flutter speed for linear portion of stiffness is about 20 m/s and the frequency range from 5 to 10 Hz, what results in the non-dimensional frequency factor *k* from 0.15 to 0.3.

The two degrees of freedom flutter model (Fig. 7,8) was designed to obtain possible low mass and was built of carbon composite excluding joints. The principal goals of the experiments were to prove the effectiveness of the new system of tuning the nonlinear stiffness characteristics of plunge K_h and pitch degrees of freedom K_α.

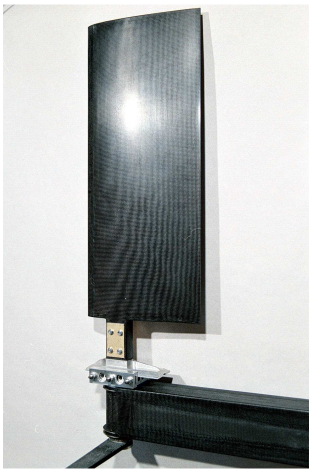


Figure 7: The view of the LCO model

Figure 8: The overall view of the wind tunnel test section

Great flexibility of possible characteristics both with hardening and softening non-linear rule has been obtained by application of the electro-dynamic exciters as the stiffness generators. The exciters were activated by the computer controlled system which produces the programmed displacement-force relation in real time. Using this concept any functional dependence between translation (or rotation) and the exciter force can be easily generated. Also the negative damping forces can be hypothetically produced.

Calculation results compared with the wind tunnel tests (cubic hardening nonlinearity in plunge) are shown on Fig. 9, 10. Test results are indicated by ▲ and +. Calculation results are indicated by solid lines.

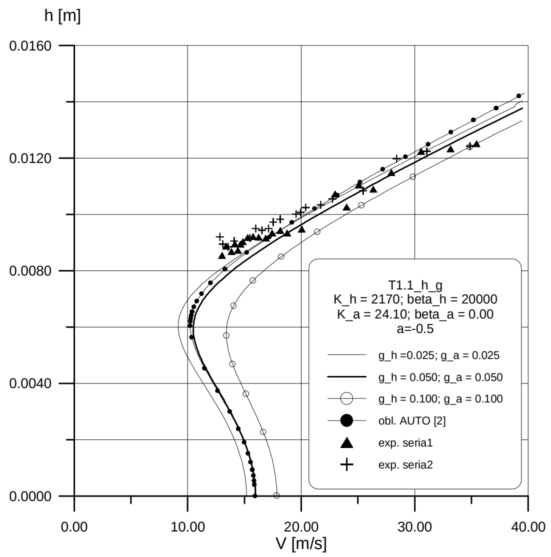


Figure 9: LCO - plunge

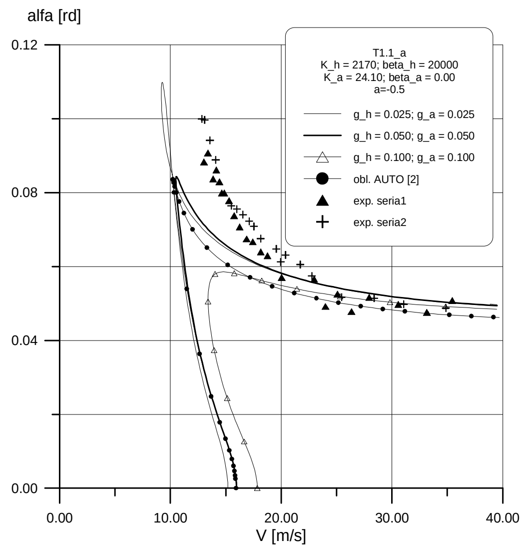


Figure 10: LCO - pitch

Examples of the time history and the phase shift are shown on Fig. 11 and Fig. 12.

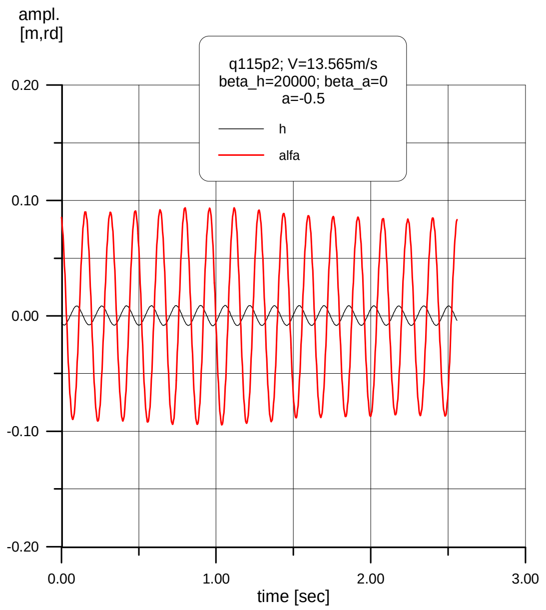


Figure 11: Plunge and pitch amplitudes (exp.)

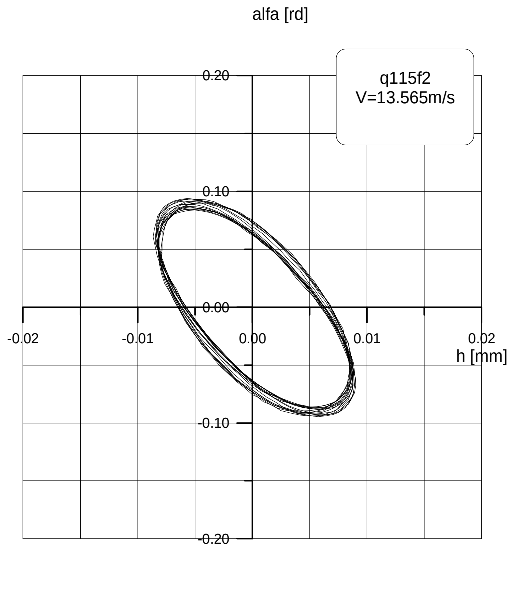


Figure 12: LCO (wind tunnel experiment)

The graphs were built based on the measured vibration amplitudes during the stable LCO for each wind velocity. During the test, vibration were measured for more than 40 velocities. Comparison of LCO obtain by calculations with experiment is good for plunge and acceptable for pitch.

5 CONCLUDING REMARKS

Continuation process has been successfully applied to flutter calculation of sailplane with structural nonlinearities. Results show that continuation method can be a powerful tool for analysis of complicated nonlinear flutter behavior of aircraft.

For practical analysis of complete aircraft a modal reduction is proposed, which is based on normal modes of the structure with all concentrated nonlinearities replaced by linear springs. In the solution process the stiffness matrix is modified by proper values, depending on the magnitudes of the complex generalized coordinates, resulting from harmonic balance. Solution, obtained for linear flutter analysis, was a starting point to nonlinear calculations by the continuation method. Especially the continuation process can be used to solve linear flutter by fixing in calculation infinitesimal values of vibration amplitudes. Starting point in this case can be find easily.

There is a very good agreement between results of the present method and the method based on center-manifold reduction in a range of a few percent below linear flutter velocity. However, beyond this interval a qualitative discrepancy of the results of both method are observed. It is important that the limit cycle oscillations are detected below linear flutter velocity. These oscillations can be initiated by sufficiently high disturbance, magnitude of which is known from the present results of calculations and is given by the unstable branch of amplitude curves.

Calculation performed for 2D model (with only plunge nonlinear spring) based on the harmonic balance and the continuation method show acceptable agreement with wind tunnel experiment.

Wind tunnel tests (with controlled nonlinear stiffness) are expensive and difficult to perform.

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