

Distributing Three-Dimensional Aerodynamic Load to Fem Nodes

Sun Weimin^a

Nanjing Research Institute of Electronics and Technology, Nanjing 210039, China

Abstract. Aiming at the requirement of the load conversion in aeronautics and astronautics static strength analysis, a three-dimensional aerodynamic load equivalent distribution method based on least squares is proposed. Based on the principle of the closest distance, the corresponding relationship between the aerodynamic node and the finite element node is established to ensure that each finite element node has its corresponding aerodynamic node. Under the requirement of minimizing the variance of load obtained by each node allocation, the least squares algorithm is used to obtain the results of the load conversion. Through an engineering example of aerodynamic distribution, it is verified that the method presented in this paper not only has high accuracy but also can ensure the uniformity of load distribution before and after load distribution.

1 Introduction

In the static strength analysis of aeronautics and astronautics field, it is generally necessary to transfer the aerodynamic load to FEM (finite element method) nodes. The principle is that the equivalent force is more reasonable under the premise of ensuring the resultant forces and the resultant moments. Mapping to the target finite element model node to ensure that the aerodynamic load is correctly applied to the structural finite element model is a very tedious but important task [1].

Transferring the aerodynamic load to FEM (finite element method) nodes was first studied in the field of aeroelastic analysis. The typical treatment method is based on spline interpolation theory and virtual work principle. The aerodynamic loads are transformed into finite element nodal loads by spline interpolation matrix. MSC.Nastran aeroelastic manual introduces several types of interpolation algorithms based on beam-slice mathematical analysis theory, such as line spline interpolation, surface spline interpolation and user-defined display interpolation function [2]. Based on the principle of surface spline interpolation, Zhang [3] used thin-plate spline and bending thin plate interpolation to calculate the aerodynamic loads on the main wing of an aircraft. Gao et al. [4] developed a method for aerodynamic load conversion based on spring-cantilever model with minimal strain energy criterion. The method based on spline interpolation is often applicable to the case of small number of load interpolation points. When the number of load points increases to more than 10 and the load changes dramatically, the error of load interpolation results will increase dramatically [3].

For the above reasons, the domestic aviation field adopts converting algorithms such as three-point method,

four-point method and multi-point method (also known as three-point, four-point and multi-point row). The principle of the algorithm is based on the static equilibrium condition (four-point row and multi-point row also use the principle of minimum strain energy), which distributes one aerodynamic load to three, four or more finite element model nodes respectively. Chen et al. [6] introduced the specific process of three-point and multi-point methods in detail, and verified the feasibility and validity of the method with load distribution of Eagle 500 aircraft. Zhang [3] and others introduced the specific process of three-point method and four-point method, and verified the feasibility of the method by the load distribution examples of leading edge slot, trailing edge flaps and nozzles of a large transport aircraft. Wang [7] introduced the implementation process of the multi-point method in detail, and distributed the aerodynamic load of a plane's flat tail equivalently. Lei Li et al. [5] analyzed and compared three load conversion methods, i.e. three-point method, four-point method and multi-point method, by using relevant load conversion examples.

In this paper, based on the actual engineering requirement of aeronautical structure static strength simulation analysis, in view of the basic principle of multi-point method and considering the continuity of aerodynamic load distribution on the structure, this paper presents an efficient three-dimensional aerodynamic load equivalent distribution method, and then validates the effectiveness of the proposed method through a load conversion example.

^a Corresponding author: flygys@sohu.com

2 The basic principle of multi-point method

The multi-point load equivalent method can also be called energy minimization method. The basic idea is to allocate the load according to the principle of minimum strain energy of the system. The finite element nodes near the aerodynamic load nodes are allocated more loads, and conversely less loads [5,6]. The algorithm of multi-point load equivalent method is based on the minimum strain energy of the system after load conversion and satisfies the static equilibrium condition. The schematic diagram is shown in Figure 1. The aerodynamic load P_O on the aerodynamic load node O is to be distributed to n finite element nodes ($n > 3$).

Assuming that there is a virtual beam element with length L_i and bending stiffness EI between the aerodynamic load node O and the finite element node i , the strain energy of the free end of the beam element, i.e. the finite element node i , when distributed to the load is as follows:

$$U_i = \frac{P_i^2 L_i^3}{6EI} \quad (1)$$

Then the strain energy of the entire system (n virtual beam elements) is:

$$U = \sum_{i=1}^n U_i \quad (2)$$

The load allocated to the finite element nodes also needs to satisfy the static equilibrium conditions, that is:

$$\begin{cases} \sum_{i=1}^n P_i = P_O \\ \sum_{i=1}^n P_i \bar{x}_i = 0 \\ \sum_{i=1}^n P_i \bar{z}_i = 0 \end{cases} \quad (3)$$

where $\bar{x}_i = x_i - x_O$, $\bar{z}_i = z_i - z_O$ is the coordinate of the finite element node relative to the aerodynamic load node and is a known quantity. Equation (3) above contains three equations, and the number of unknowns is more than three. The Lagrangian function can be established by introducing the principle of minimum strain energy and using the Lagrangian multiplier method:

$$F(\bar{\lambda}, \bar{\lambda}_x, \bar{\lambda}_z) = \sum_{i=1}^n \left(\frac{P_i^2 L_i^3}{6EI} - \bar{\lambda} P_i - \bar{\lambda}_x P_i \bar{x}_i - \bar{\lambda}_z P_i \bar{z}_i \right) \quad (4)$$

where $\bar{\lambda}$, $\bar{\lambda}_x$, $\bar{\lambda}_z$ is a Lagrange multiplier.

In order to minimize $F(\bar{\lambda}, \bar{\lambda}_x, \bar{\lambda}_z)$, let $\partial F(\bar{\lambda}, \bar{\lambda}_x, \bar{\lambda}_z) / \partial P_i = 0$, then obtain:

$$P_i L_i^3 = \lambda + \lambda_x \bar{x}_i + \lambda_z \bar{z}_i \quad (5)$$

where $\lambda = 3\bar{\lambda}EI$, $\lambda_x = 3\bar{\lambda}_x EI$, $\lambda_z = 3\bar{\lambda}_z EI$.

Substitute Eq. (5) into Eq. (3), we can obtain:

$$\begin{bmatrix} \sum_{i=1}^n L_i^3 & \sum_{i=1}^n \bar{x}_i L_i^3 & \sum_{i=1}^n \bar{z}_i L_i^3 \\ \sum_{i=1}^n \bar{x}_i L_i^3 & \sum_{i=1}^n \bar{x}_i^2 L_i^3 & \sum_{i=1}^n \bar{x}_i \bar{z}_i L_i^3 \\ \sum_{i=1}^n \bar{z}_i L_i^3 & \sum_{i=1}^n \bar{x}_i \bar{z}_i L_i^3 & \sum_{i=1}^n \bar{z}_i^2 L_i^3 \end{bmatrix} \begin{Bmatrix} \lambda \\ \lambda_x \\ \lambda_z \end{Bmatrix} = \begin{Bmatrix} P_O \\ 0 \\ 0 \end{Bmatrix} \quad (6)$$

It can be obtained by solving Eq. (6) and substituting into Eq. (5) to obtain all finite element node loads.

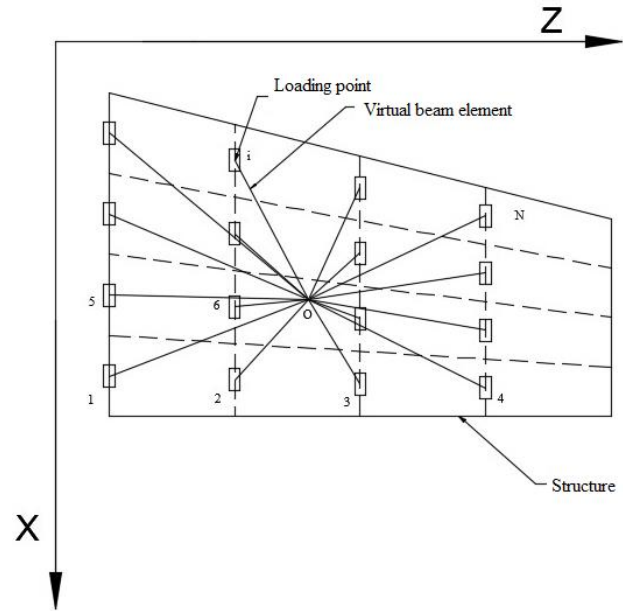


Figure 1. Schematic diagram of load equivalent calculation method for multi-point method.

3 A three-dimensional aerodynamic load equivalent distribution method

For the three-dimensional load distribution, the multi-point method needs to project aerodynamic loads and finite element models into three coordinate axes and their vertical coordinate planes, respectively, and different node matching strategies may lead to abrupt changes in aerodynamic distribution (i.e., no loads are distributed on some finite element nodes). Based on this, this section refers to the basic principle of multi-point method, and fully considers the continuity of aerodynamic load distribution on the structure, proposes a three-dimensional aerodynamic load equivalent distribution method based on least square method.

3.1 Matching strategy between aerodynamic load nodes and finite element nodes

Based on the principle of closest distance, the corresponding relationship between source load and unit (and its nodes) is determined. A corresponding unit is assigned to each source load point, and then several finite

element nodes corresponding to each source load point are obtained. Note: When the finite element structure mesh is too dense, if only the method of "source load point-single finite element unit" is adopted, the distributed load will be concentrated in a very small area, and most of the nodes in other areas have no load. In view of this situation, the corresponding relationship of all "source load-finite element node" can be established by the aforementioned method at first, and then the finite element nodes which are not assigned to any source load point are extracted. Then the nearest source load points with these finite element nodes are obtained based on the nearest distance principle. Finally, a complete "source load-finite element node" corresponding relationship can be established (that is, there is at least one source load point for any finite element node).

3.2 Load distribution method based on "local resultant forces or resultant moment balance"

By using the theorem on moment of resultant force, a load is distributed to a set of parallel force systems passing through the target node. There are only three equations established here. When a source data point is divided into more than three target nodes (which usually belongs to this situation), an underdetermined linear equation system (the number of unknowns is more than the number of equations) will be established, which generally has infinite solutions, but according to distributing load from single point to multiple points, the requirement of non-inverse sign of load (that is, a load should be decomposed into multiple co-directional loads) should be satisfied. The least square method theory for solving undetermined equations is adopted, and the minimum variance of load obtained by each node distribution (i.e., the load is continuous and the sudden change is small) should be satisfied at the same time, and then the load distribution results can be obtained.

When the source load point is not in the area surrounded by the target load point, the direct use of moment balance method will lead to the negative sign of the load at the target load point. To overcome this shortcoming, there are two solutions: 1) refine the grid so that all source load points can fall into the structural grid; 2) when the first method is not adopted, load can be evenly distributed to several nodes near the source load point. Generally, the requirement of load distribution for moment error can be satisfied. If not, relevant optimization methods can be further used. The load on the finite element nodes is fine-tuned to satisfy the resultant moment balance before and after load sharing.

According to the above analysis, proper algorithm can be developed to obtain the three-dimensional aerodynamic load equivalent distribution.

3.3 Flow chart of the three-dimensional aerodynamic load equivalent distribution method

According to the above analysis, it can be determined that the flow of the three-dimensional aerodynamic load

equivalent distribution method proposed in this paper is as shown in Fig. 2 .

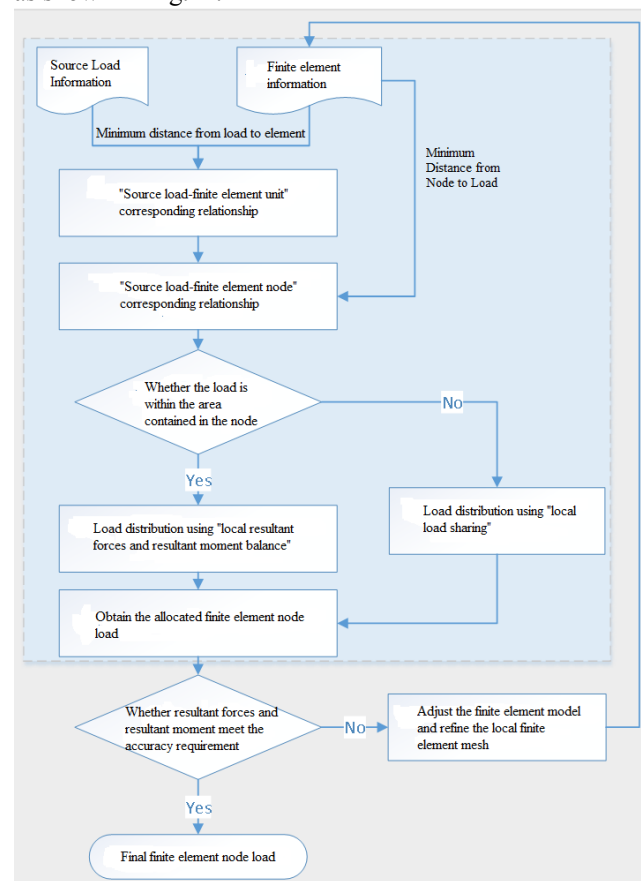


Figure 2. Algorithm flow chart

4 Example verification

4.1 Model description

To validate the present method, a aero structure is introduced with some source load points (aerodynamic load points) that are not in the area surrounded by the target load points (finite element nodes).

4.2 Load distribution results

Surface spline interpolation method [2] and the method proposed in this paper are used to distribute the aerodynamic forces on the above structures. The results are shown in Fig. 3 and Table 1, respectively.

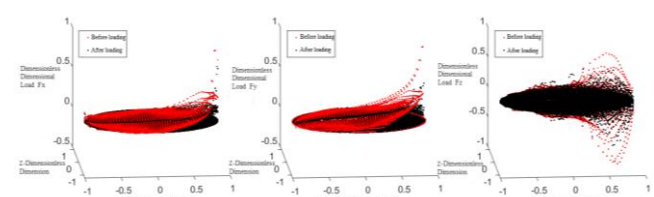


Figure 3. A comparison diagram of load distribution before and after the method presented in this paper.

Table 1. Surface spline interpolation method [2] and comparison of presented load distribution errors.

Resultant moment of force	Surface spline interpolation method	Method of this paper
Fx	0.00%	-0.23%
Fy	0.00%	0.00%
Fz	0.00%	-0.19%
Mx	-0.02%	-0.06%
My	0.00%	0.04%
Mz	-0.02%	-0.01%

As can be seen from Table 1, the error of load distribution by surface spline interpolation method is very small, while the error of this method is slightly larger (but it is also far less than the error requirement of engineering field). However, it can be found that the load distribution after load distribution by surface spline interpolation method is totally different from the original distribution, and there are a lot of negative loads. However, the load distribution obtained by this method is exactly the same as the original distribution, and there is no negative load. This shows that the method can not only meet the requirements of load distribution accuracy, but also obtain the same load distribution form as the original load distribution form.

5 Conclusion

Aiming at the load conversion requirements in aeronautics and astronautics static strength analysis, the advantages and disadvantages of many existing load distribution methods, such as surface spline interpolation, three-point method, four-point method and multi-point method, are analyzed. Based on local resultant forces and resultant moment balance, a three-dimensional aerodynamic load equivalent distribution method based on least squares is proposed. Based on the principle of closest distance, the corresponding relationship between aerodynamic nodes and finite element nodes is

constructed to ensure that each structural node in the finite element model has corresponding aerodynamic nodes. Under the requirement of minimum variance of load obtained by each node allocation (i.e., continuous load and small sudden change), the least square algorithm is used to obtain the load distribution results. Through an engineering example of aerodynamic distribution of a structure and comparing with the results of load distribution by surface spline interpolation method, it is proved that the method presented in this paper not only has high accuracy, but also can ensure the uniformity of load distribution before and after load distribution.

References

1. Zhang Jiangang, Sun Renjun, Tang Changhong. The method of transferring aerodynamic load to FEM nodes for large airplane [J]. Mechanics in Engineering, 2017, 39(1): 25-29 (In Chinese)
2. MSC.Software Corporation. MSC.Nastran Aeroelastic Analysis User's Guide [M]. MSC.Software Corporation, 2004.
3. Zhang Jiangang. Study on the interpolation method to calculate the aircraft wing aerodynamic press distribution on FEM nodes [J]. Aeronautical Science & Technology, 2017, 28(12): 14-18.(In Chinese)
4. Gao Shangjun, Yu Zhefeng, Wang Hai. Conversion of aerodynamic load based on spring-cantilever model with minimal strain energy criterion [J]. Aircraft Design, 2013, (6): 1-4(In Chinese)
5. Lei Li, Han Qing, Zhong Xiaoping. Comparative Analysis of Conversion Distribution Methods for Wing Structural Node Loads [J]. Advances in Aeronautical Science and Engineering, 2014, 5(3): 383-389 (In Chinese)
6. Chen Quanli, Xiong Jianqi. Application of the load equivariant method on aircraft static strength test. Aircraft Engineering, 2005(1): 63-65 (In Chinese)
7. Wang Zhuanli. FEM node load calculation of wing structure [J]. Hongdu Science and Technology, 2007(1): 7-14 (In Chinese)