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Optimization of Wing Structures for Flutter: An Interval Approach

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This paper presents an uncertainty based automated optimum design of airplane wing structures with multiple design constraints. The maximization of flutter velocity of supersonic aircraft wing structures is considered as the objective. The wing is assumed to fly through a specific flight condition and the restrictions imposed upon the behavior of the structure involve limitations on taxiing stress, gust stress, landing stress and the range in which the natural frequencies are allowed to fall along with strength requirements. The design parameters of the aircraft wing are assumed to be uncertain and are described by a range of values. Second order piston theory is used to predict the aerodynamic load distribution and the dynamic aeroelastic characteristics of the wing. The procedure is illustrated with two examples. One is a symmetric double wedge airfoil, based on a beam type of analysis, and the other is a supersonic airplane wing, based on a finite element analysis. An interval analysis - based nonlinear programming techniques (penalty function approach and sequential quadratic programming), is used for the optimum solution of two aircraft wings.

1. Design Criteria

The optimum design methods with deterministic parameters have been very well developed. In the present work, the problem of designing a supersonic wing structure with uncertain data is considered. The probability-based structural optimization method can be applied to problems in which design parameters are of a random nature. In design problems there exists a vast amount of uncertain data which can not be modeled using probability principles. Such problems can be solved using a fuzzy approach assuming fuzzy information for the geometry, strength (resistances) as well as applied loads. In this paper the problem of optimum structural design in a fuzzy environment is considered. The fuzzy set is divided into finite subsets by using discrete values of membership function. Each fuzzy subset represents the range of imprecision corresponding to a specified α value. Thus, an interval can be used to describe a fuzzy subset. In this work, the flutter velocity of an aircraft wing is maximized subject to constraints. The maximum stress, wing tip deflection, root angle of attack, the natural frequencies of the wing structure, the stresses induced in the wing structure due to taxiing, gust and landing loads are suitably constrained. The design criteria to be satisfied, in the specified flight condition, are the following:

1. The first p natural frequencies of the structure ω_i are to be excluded from certain bands by restricting them to lie between lower and upper bounds:

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$$
\left.\begin{aligned}
g_i\left(\vec{X}\right) &= \left(\omega_i^l / \omega_i\right) - 1 \le 0 \qquad \left(i = 1,2,\cdots,p\right) \\
g_{p+i}\left(\vec{X}\right) &= \left(\omega_i / \omega_i^u\right) - 1 \le 0 \qquad \left(i = 1,2,\cdots,p\right)
\end{aligned}\right\} \tag{1}
$$

2. The elastic deflection of the wing tip δ under a specified steady-state flight condition must not exceed a certain prescribed limit $\left(\delta^u\right)$:

$$g_{2p+1}\left(\vec{X}\right) = \left(\delta / \delta^u\right) - 1 \le 0 \tag{2}$$

3. The principal stress induced at any point of the wing σ_s during a prescribed steady-state flight condition must not exceed the yield point of the material $\left(\sigma_s^u\right)$:

$$g_{2p+2}\left(\vec{X}\right) = \left(\sigma_s / \sigma_s^u\right) - 1 \le 0 \tag{3}$$

4. The root angle of attack of the wing α_o under a given steady-state flight condition is to be less than a specified maximum value α_o^u:

$$g_{2p+3}\left(\vec{X}\right) = \left(\alpha_o / \alpha_o^u\right) - 1 \le 0 \tag{4}$$

5. The maximum principal stresses developed at any point of the wing during a specified gust condition $\left(\sigma_g\right)$ are restricted to be less than the permissible value σ_g^u:

$$g_{2p+4}\left(\vec{X}\right) = \left(\sigma_g / \sigma_g^u\right) - 1 \le 0 \tag{5}$$

6. The maximum principal stresses developed at any point of the wing during landing $\left(\sigma_l\right)$ are restricted to be less than the permissible value σ_l^u:

$$g_{2p+5}\left(\vec{X}\right) = \left(\sigma_l / \sigma_l^u\right) - 1 \le 0 \tag{6}$$

7. The maximum principal stresses developed at any point of the wing during taxiing $\left(\sigma_t\right)$ are restricted to be less than the permissible value σ_t^u:

$$g_{2p+6}\left(\vec{X}\right) = \left(\sigma_t / \sigma_t^u\right) - 1 \le 0 \tag{7}$$

Theses criteria are prescribed to avoid failure of the structure due to lack of strength, stiffness and due to mild harmonic forcing that might cause resonance.

2. Interval Optimization Problem

The optimization of two example wings is considered for illustration. The first example deals with the determination of the thickness (T) and chord length (C) of the hollow symmetric double-wedge airfoil as shown in Figure 1. The maximization of flutter speed while driving the airfoil through a specified flight condition is treated as objective. Thus the optimization problem becomes: Find $\vec{X} = \{x_1, x_2\}^T \equiv \{T, C\}^T$ which optimizes the flutter Mach number $f_1(\vec{X})$ subject to the constraints specified in Eqs. (4) to (12). In the second example (Figure 2), the finite element method is used for structural idealization. The overall dimensions of this wing correspond approximately to those of the Concord wing. The constant strain triangular membrane elements are used to represent the wing cover skin, triangular shear elements are used for ribs and spars and pin-jointed bar elements are used for the axial load carrying capacity of ribs and spars. The thicknesses of the skin, ribs and spars and the cross-sectional areas of the pin-jointed bars are treated as design variables. Thus the optimization problem can be stated as: Find $\vec{X} = \{x_1, x_2, \cdots, x_6\}^{\mathrm{T}}$ which optimizes $f_1(\vec{X})$ subject to the constraints stated in Eqs. (1) to (7) with $f_1(\vec{X})$, the flutter Mach number, calculated using piston theory for supersonic flight conditions [2].

In the interval-approach, all the system parameters, including the design variables and the objective function, are treated as interval variables. Hence we need to apply interval arithmetic to every step of the calculations. During actual programming, we need to adjust the order in which different interval parameters are considered in any specific equation. This is because, when the program executes the equation using interval parameters, the new order will not only minimize the computational time but also lead to a reduced interval range for the result. In addition, the truncation approach[3], is used based on a comparison between the ranges of the input parameters and the ranges of the computed responses. The purpose of truncation is to make reasonable modifications to the output range before using it in the next interval operation. The truncation approach can give reasonable predictions for the solution even when the widths of the input parameters or the interval ranges of other influencing parameters are quite large.

In some computational steps, using interval arithmetic may not only seem to be redundant, but might lead to an invalid result which does not follow the physics of the equation. If these invalid operations are used in the computation, the final solution will be incorrect. To avoid such problems, a combinatorial approach is used instead of the interval operation in order to comply with the physical logic. Thus it is necessary to understand the physical meaning of each equation before implementing the interval analysis.

3. Numerical Results and Discussion

The multivariable constrained optimization problems are solved using nonlinear programming techniques (penalty function approach and sequential quadratic programming). In the first example, two design variables are considered. In the second example, six design variables are used in the optimization with the first four corresponding to the thickness of the skin in four different regions, the fifth one denoting the thickness of the rib and spar webs and the sixth one representing the cross-sectional areas of the pin-jointed bars. The objective is maximization of flutter velocity. A clamped boundary condition is specified along the roots of the wings. The design data for the first and the second example wings are given in Tables 1 and 2, respectively.

For the second example wing, only the top half of the wing is idealized for the finite element analysis. Since the load vector in the steady-state design condition depends on the root angle of attack as well as the displacement state, an iterative process is used to determine the correct values of the root angle of attack and nodal displacements, which provide the specified gross lift. The stresses induced in the finite elements are determined from known nodal displacements. The natural frequencies and mode shapes of the wing are found from the eigenvalue analysis and by reducing its order to one-third of that of the corresponding static problem. With system equivalent reduction expansion process (SEREP) technique[5], only the transverse displacement degrees of freedom are retained.

The flutter speed is calculated by using the first four natural modes as generalized coordinates and equations are solved by using a double iterative scheme[2]. Piston theory[6] is used for the computation of the root angle attack, aerodynamic loads producing tip deflection, steady state stress and flutter velocity of wing structure. The rigid body mode and the first bending mode are used as generalized coordinates in the landing, taxing and gust stress analyses. The number of degrees of freedom considered in the finite-element analysis and the number of modes considered in the dynamic analysis are kept low for computational convenience.

The optimization results obtained with interval analysis are compared with those obtained using deterministic and probabilistic analyses for comparable data. Among the behavior constraints the taxiing stress constraint and gust stress constraint were found to be active. Figure 3 shows the maximization of flutter velocity with the number of iterations for optimizations based on deterministic, probabilistic and interval analyses. Comparing the results obtained using the three different approaches leads to the following conclusions: The optimum value of the objective function is higher in probabilistic optimization compared to the deterministic one. The results obtained with interval optimization are found to be in reasonable agreement with those obtained using the deterministic and probabilistic approaches.

4. Conclusion

A procedure is described for the automated optimum design of wing structures for maximum flutter velocity with consideration of stresses developed due to landing, gust and taxiing loads along with static and dynamic constraints. The procedure is demonstrated by considering the design of two example wings: one based on simple beam-type of analysis and the other based on finite element analysis. The optimization results obtained with interval analysis are compared with those obtained using deterministic and probabilistic analyses. The interval analysis-based results are found to be in good agreement with those obtained using deterministic and probabilistic analyses. The procedure outlined is expected to be useful during the preliminary design stages of an airplane structure. The interval analysis is expected to be more realistic and, hence, can be used in the optimum design of airplane wing structures.

5. References

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Table 1: Design data for example wing 1

Material properties (titanium) of wing:	
Young's modulus,	15.95×10^6 psi
Shear modulus	6.38×10^6 psi
Material density	8.7243845 lb-sec^2/ft^4
Additional masses :	
Fuselage weight	279.25466 lb
Engines weight	139.62733 lb
Fuel weight	210.75 lb
Static load condition :	
Altitude	35,000 ft
Air pressure	498 lbs/ft^2
Speed of sound	973.3 ft/sec
Air density	7.36×10^{-4} lbs-sec^2/ft^4
Steady state flight Mach number	2.0
Pull up acceleration	2.0 g
Flight duration	1 hr
Semispan	29.52756 ft
Solidity ratio of the cross section	0.075
Landing load condition:	
Weight of unsprung mass m_u	719.36 lb
Inclination of the shock strut (θ)	0.0
Landing gear location	5.904 ft from the root of wing
Lift factor	1.01

Duration of transient analysis	0.3 s
Tire force-deflection relation: (α, β) Sink velocity (V_v) Horizontal velocity at contact (V_l)	 $(1.25\times10^6$, 1.22) 118.1103 in/sec 147.6378 in/sec
Gust load condition: **Discrete gust(cosine type):** Maximum vertical velocity Length of gust of Forward velocity of flight Altitude **Power spectral approach:** Scale of turbulence	 9.84252 ft/sec 10 chords 508 ft/sec 24993.6 ft 2500 ft
Taxiing load condition: Velocity of airplane during taxiing (v) Number of modes considered(N) Weight of main (nose) landing gear $M_1 g$ $(M_2 g)$ Damping coefficient (λ_j) Time increment (Δt) Spring stiffness of tire of main (nose) Landing gear $k_{t1}(k_{t2})$ Static value of nonlinear spring force in Main (nose) landing gear $\overline{F}_{s1}$ $(\overline{F}_{s2})$ Limiting coulomb friction of main (nose) Landing gear F_1 (F_2) Distance between nose and main landing gears	 2 ft/sec 5 719.36 lb, 359.68 lb 0.025 0.002 sec 7.1377 lb/in (0.9993 lb/in) 6519 lb (786.87 lb) 505.8453 lb (303.5072 lb) 20 ft.

Table 2: Interval optimization results for Double wedge airfoil (*active constraint)

	Initial Design	Bounds Lower	Bounds Upper	Optimum Design
Design variables,				
x_1, ft	[1.599 1.601]	[0.70 0.50]	[2.461 3.0]	[1.5271 1.5849]
x_2, ft	[18.741 18.759]	[1.969 2.5]	[19.685 21.0]	[18.6840 18.7799]
Behavior constraints				
Wing Tip deflection, δ, ft	[0.6579 0.6772]		5.937007	[0.6631 0.7002]
Root angle of attack, α_0 rad(deg)	[0.03891 0.0392] (2.2294 2.2459)		0.05 (2.8648)	[0.03747 0.0421] (2.1469 2.4121)
First natural frequency, ω_1, rad/sec	[29.9205 29.9307]	12.5664	45.1328	[29.1901 29.9984]
Second natural frequency, ω_2, rad/sec	[82.0919 82.1309]	10.2656	150.7968	[83.7824 85.4739]
Steady state stress, σ_s (lb/ft^2)	[2.9377e6 2.9596e6]		3.575575e7	[2.9647e7 3.1329e7]
Taxiing stress, σ_t (lb/ft^2)	[7.3200e4 7.4123e4] *		8.2213e4	[6.9830e4 7.2718e4]*
Landing stress, σ_L (lb/ft^2)	[9.890e6 1.547e7]		1.7731e7	[1.4792e7 1.5979e7]*
Gust stress, σ_g (lb/ft^2)	[4.302e5 4.662e5] *		4.665e5	[4.0929e5 4.4484e5]*

$f_1(x)$ = Flutter Mach number (M_F) = $[9.6284 \quad 9.5326]$

Table 3: Design data for example wing 2

Material Properties :	Material	Aluminum
	Young’s Modulus	10×10^6 psi
	Poisson’ Ratio	0.333
	Density	0.10 lb/cu. in
Details of the weight:	Planform area	1,474.5 sq. ft
	Engines	12,500 lb
	Fuselage and Payload	72,000 lb
	Fuel	92,500 lb
	Initial gross weight	192,500 lb
Flight conditions data:		
Altitude		25,000 ft
Pull-up acceleration		3.75 g
Flight Mach number		1.89
Pressure of air		786.33 lb/sq. ft
Density of air		0.001066 lb-sec^2/ft^4
Taxiing Analysis data:		
$N = 5$, $\Delta t = 0.002$ sec, λ_j =0.025, M_{1g} = 4992.0 lb, M_{2g} = 342.0 lb, M_{3g} = 386464 lb, $M_4 = 0.645 \times 10^8$ lb-in-s^2, k_{t1} = 96500.0 lb/in, k_{t2} = 13500 lb/in, $F_1^{'}$ = 1000 lb, $F_2^{'}$ = 600 lb, F_{s1} = 33000.0 lb, F_{s2} = 3500.0 lb.		
Gust Analysis:		
Discrete gust, cosine type;	maximum vertical velocity = 4.998 ft/sec	
Length of gust = 5 chords,	forward velocity of flight = 508 ft/sec	
Altitude = 24993.6 ft		
For spectrum approach;	scale of turbulence = 2500 ft	
Landing analysis data:		
Weight of unsprung mass below shock strut = 2300lb, landing gear location: X_l =80$^{//}$, Y_l =287.43$^{//}$, duration of transient analysis = 0.3 sec, tire force-deflection relation: α =560360, β =1.21, inclination of strut = 0° , sink velocity (V_v) = 45 in/sec.		

Optimization data:

For design variables: lower bound = 0.04, upper bound = 0.5; upper bounds: taxing stress (σ_t) = 4.0e4 lb/in^2, gust stress (σ_g) = 5.1e4 lb/in^2, landing stress (σ_L)= 6.2e4 lb/in^2, wing tip deflection (δ) = 150 in, steady state stress (σ_s) = 6.5e4 lb/in^2, root angle of attack(α_o) =12 deg; lower and upper bounds: bending frequency = 3.0 rad/sec and 10.0 rad/sec; torsional frequency =10.0 rad/sec and 25 rad/sec; lower bound: flutter mach number = 1.89.

Table 4: Interval optimization results for wing (Example 2) (*active constraint)

	Initial Design	Bounds Lower	Bounds Upper	Optimum design
Design variables				
x_1, in	[0.1498 0.1501]	0.04	0.5	[0.1999 0.2204]
x_2, in	[0.1498 0.1501]	0.04	0.5	[0.0926 0.1086]
x_3, in	[0.1498 0.1501]	0.04	0.5	[0.0835 0.0969]
x_4, in	[0.1498 0.1501]	0.04	0.5	[0.0600 0.0754]
x_5, in^2	[0.2498 0.2501]	0.04	0.5	[0.2384 0.2443]
x_6, in^2	[0.2498 0.2501]	0.04	0.5	[0.2509 0.2565]
Behavior constraints				
Wing Tip deflection, δ, in	[63.7075 64.2415]		150	[54.6260 68.2301]
Root angle of attack, α_0 rad(deg)	(0.1624 0.1625) [9.3046 9.3108]		(0.2094) 12	(0.1488 0.1492) [8.5286 8.5485]
First natural frequency, ω_1, rad/sec	[7.4455 7.4696]	3.0	10.0	[4.2038 4.9123]
Second natural frequency,		10	25	[16.9910 17.3398]

ω_2 , rad/sec	[22.3271 22.3514]*		
Steady state stress, σ_s (lb/in^2)	[2.7089e4 2.7634e4]	6.50e4	[4.9511e4 5.0009e4]
Landing stress, σ_L (lb/in^2)	[5.1485e4 5.2134e4]	6.2e4	[5.7851e4 5.9898e4]*
Gust stress, σ_g (lb/in^2)	[1.8352e4 1.9123e4]	5.1e4	[4.7430e4 5.0139e4]*
Taxiing stress, σ_T (lb/in^2)	[2.8109e4 2.8465e4]	4.0e4	[3.3911e4 3.4804e4]*

$f_1(x)$= Flutter Mach number (M_F) = [7.0566 6.2390]

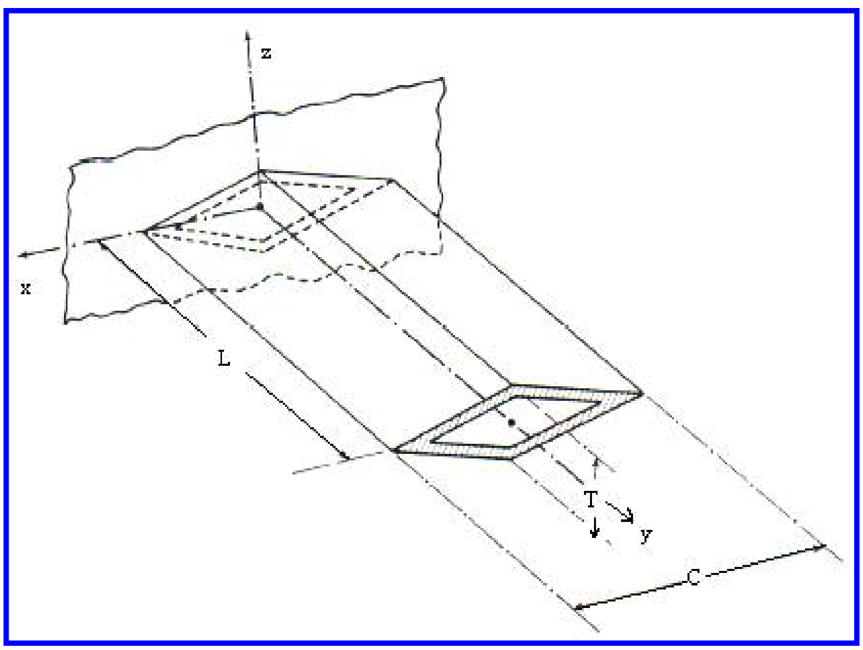

Figure 1: **Symmetric Double-wedge airfoil (Example 1)**

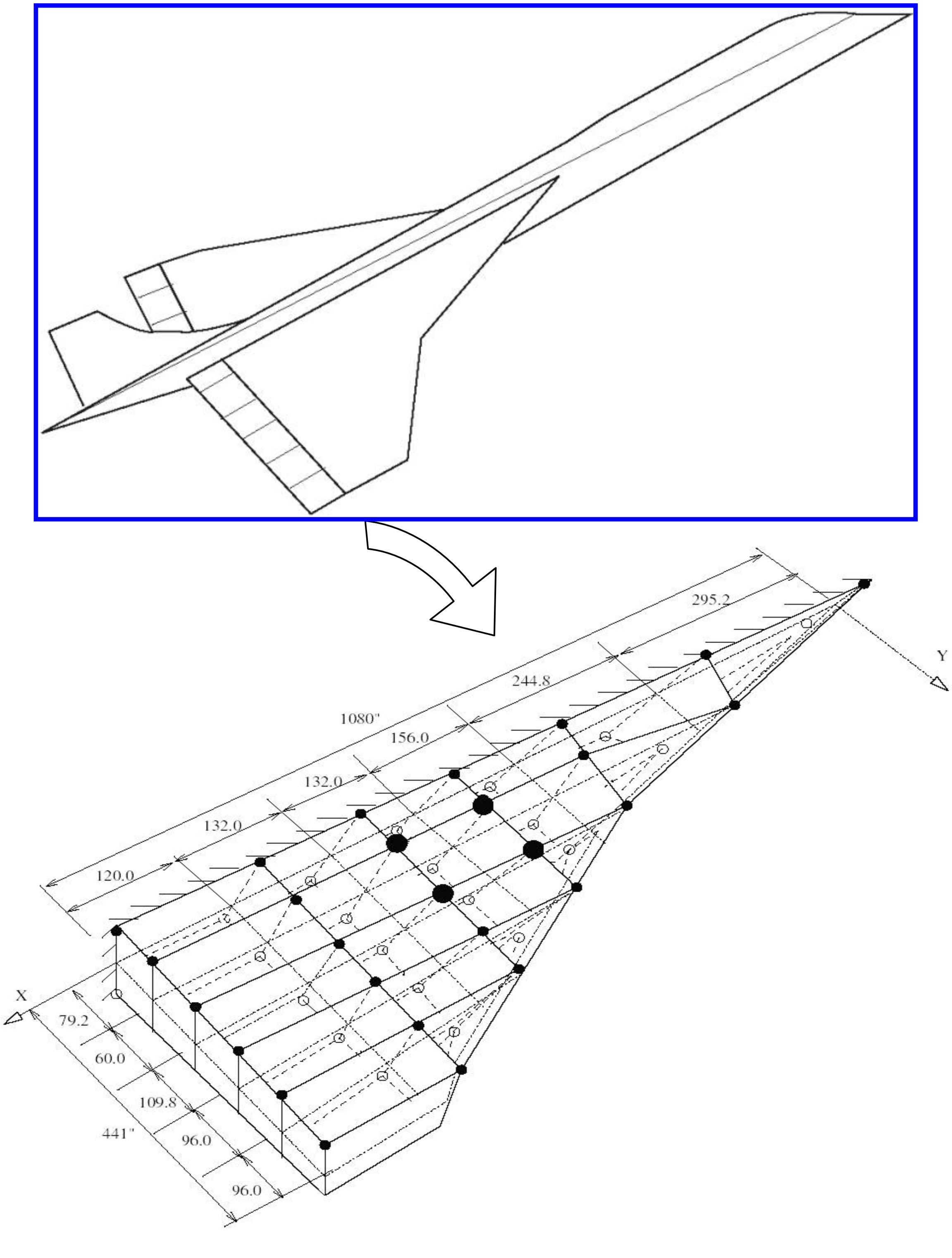

Figure 2: **Supersonic transport wing; Finite element idealization (Example 2)**

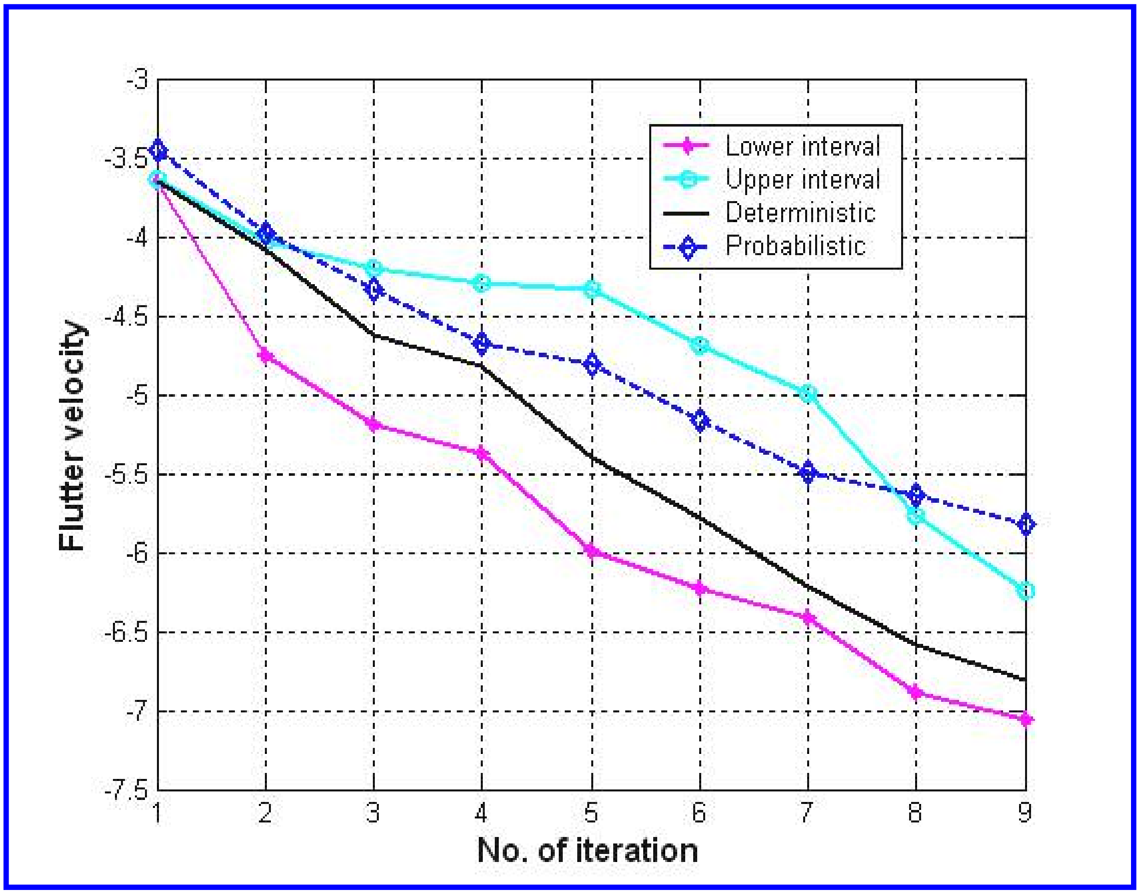

Figure 3: **Convergence of flutter velocity for example wing 2 with number of iterations.**