Control-Surface Structural Nonlinearities in Aeroelasticity: A State of the Art Review

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This paper presents a review and assessment of more than six decades of research in aeroelastic stability, including control-surface nonlinearities. The work covers free play, friction, cubic stiffness, and an actuator in a failure mode. The basic modeling of each nonlinearity is introduced to provide insights for those new in this field. A detailed description of findings is included that shows the two and three degrees of freedom typical section model has been widely investigated. For this type of system, it is noted that usually a nonlinearity in the pitch degree of freedom generates more complex limit-cycle oscillations than a nonlinearity in the control-surface rotation. A brief introduction of the methods most commonly employed in the theoretical and computational analyses is presented, including their pros and cons, as reported in the literature. This paper explores challenges yet to be overcome and opportunities for further studies.

Nomenclature

A	=	amplitude of harmonic motion
b_{eq}	=	linear equivalent damping coefficient
d	=	damping coefficient
f	=	Coulomb friction coefficient
k	=	control-surface stiffness
k_{eq}	=	linear equivalent stiffness
k_3	=	control-surface nonlinear stiffness
r	=	lever arm of control-surface actuator
u	=	structural displacement
α	=	parameter of nonlinear viscous damping force
β	=	control-surface rotation
δ	=	free-play region of control surface
τ	=	elastic torque
ω	=	frequency

I. Introduction

CONCENTRATED nonlinearities generally occur in combined forms on real aircraft structures. Woolston et al. [1] in 1955 were one of the first researchers to document the effect of structural nonlinearities on aeroelastic stability. They conducted a very interesting analysis using an analog computer at NACA. Their results gave rise to the important report "technical note 3539," which introduced the main effects of free play, hysteresis, and cubic springs acting on control surfaces. They noted that these nonlinearities usually contribute to a decrease in flutter speed, with the exception for the cubic hard spring. On the other hand, this early study also indicated that the potentially catastrophic flutter may become less violent or self-limiting under the influence of these nonlinear effects. Previous efforts have been made before the work of Woolston et al. However, they probably were unpublished or classified as confidential, such as the wind-tunnel test campaign conducted in 1953 [2].

Two years later, Woolston et al. [3] published a second and complementary investigation extending their analysis to a three degrees of freedom (DOF) system, instead of the two DOF considered previously. They employed the famous theoretical flutter model published around 15 years earlier by Theodorsen and Garrick [4]. Particularly for fixed free-play amplitude in the control-surface rotation, Woolston et al. [3] noted that the nonlinearity increased the stable range of airspeed in comparison with the linear flutter boundary computed by Theodorsen and Garrick [4]. This benefit decreased for large control-surface initial amplitudes, but the stable flight envelope remained larger than the linear one. Note that it was a relevant observation because their previous work, which considered just two DOF, showed theoretically and experimentally that free play in pitch reduced the stable flight envelope [1]. They also confirmed that for a soft spring, flutter oscillations were highly divergent, whereas a cubic hard spring generates stable limit-cycle oscillations (LCOs).

A few years later, Laurenson and Trn [5] clarified the effect of structural nonlinearities on a missile control-surface system. They noted that free play, preload, and friction were typical nonlinearities acting on those devices, and that they affect the performance characteristics besides changing the flutter boundaries for those systems. They highlighted the importance of understanding the influences of these nonlinear effects due to the potential for reducing costs and system weight and increasing efficiency. They employed a describing function (DF)-based technique to perform their analysis, similarly to that done by Woolston et al. [1]. The general conclusion from their work was that these nonlinearities decreased the effective stiffness of the system and because of this tend to cause the same effect as a reduction in stiffness in the linear system.

Breitbach [6] considered a locally lumped nonlinearity in the control mechanisms or in the connecting parts between wing and external stores. Their work found that control-surface free play can generate LCOs, and although they are not explosive, they may cause a long-term failure due to material fatigue. Similar LCO was previously observed by Haidl [7]. Both authors noted a reduction in the equivalent control-surface resonance frequency due to the free-play effect. They performed ground vibration tests considering different aileron trailing-edge amplitudes.

Although these early works described the main effects of structural nonlinearities on the aeroelastic stability, the relevance of these

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nonlinearities still attracts important attention from research centers and aerospace companies. In this context, this paper presents a literature overview from these nearly 65 years of investigations since the study published by Woolston et al. [1]. This work discusses fundamental aspects of modeling and presents a chronological description of important research in this field. This paper also summarizes the main characteristics of the methods often employed to investigate the nonlinear phenomena involved. Recommendations for future studies are presented as final remarks. The reader can note that the review of prior literature per se appears later in this paper after a discussion of the state of the art based upon the best current knowledge. This is because progress on this topic (and many other topics) has traced a circuitous path, and, from the perspective available today, not all of the earlier literature is equally relevant to this review of the current state of the art. By summarizing the present state of the art earlier in this paper, this will enable the reader to better appreciate prior contributions to our present knowledge in the light of today. Thus, in this present review of literature, the prior literature is discussed in the context of the current state of the art.

II. Certification Criteria and Current Practice in Industry

Control-surface actuators usually are complex devices connected to hydraulic systems. The actuating mechanism is based on transforming energy in the form of fluid pressure into mechanical force to generate the control-surface rotation. The final operating system (i.e., actuator and hydraulic circuit) presents nonlinear behavior, which is commonly neglected when performing ground vibration tests because a "control-surface rotation mode" is identified. (Note that, in practice, the term *vibration mode* in the sense of linear modal analysis is limited to linear systems. However, it is an acceptable practice for some nonlinear systems by considering small amplitude of motion.)

control-surface actuating mechanisms usually present geometric free play because it is inherent to the manufacturing and assembling processes. It is the most typical control-surface nonlinearity among those ones considered by aeronautical certification agencies and companies. The Military Specification MIL-A-8870C(AS) states that tests need to be performed to demonstrate that the free play for control surfaces, tabs, and other applicable surfaces is within specified limits [8]. For example, this document requires that a typical aileron may have a total free play not greater than 0.13 deg.

Aerospace companies commonly perform free-play measurements during aircraft development, before or while conducting ground vibration modal tests [9]. Because of the very small maximum allowable free-play limits, a typical test procedure involves the use of highly controlled and precise devices [10,11]. The free-play measurements generally capture a hysteretic effect, which allows one to obtain the free play as the total width inside the hysteresis diagram constructed by using the hinge moment and the control-surface rotation [9,12]. The results are carefully documented in technical reports later evaluated by aeronautical certification agencies. The technical reports need to show that the certification requirements are properly addressed [13].

Because the control-surface free play decreases the effective actuating stiffness [3], aerospace companies perform linear flutter analysis for a range of values of stiffnesses to cover the possibilities for this equivalent stiffness (ES). (The ES due to the free play was demonstrated analytically by Hartog [14].) On the other hand, computational and analytical tools to evaluate the nonlinear effects due to free play are also used in industry. In this sense, recently, the Advisory Circular (AC) 25.629-18 commented on the analytical investigations involving control surfaces, "… Freeplay effects should be incorporated to account for any influence of in-service wear on flutter margins … " [15]. This same AC additionally states that, "… The combined freeplay of the damper and supporting elements between the control surface and fixed surfaces should be considered … " if flutter passive dampers are employed. However, in practice, due to the complexity of evaluating the whole impact of nonlinear phenomena on aircraft behavior, the allowable free play is still required to be limited.

III. Fundamentals of Modeling Control-Surface Nonlinearities

For a typical actuation system, the control-surface rotation β is obtained in terms of the difference of displacements described by two DOF u_i and u_j due to the flexibility of mounting system, in which control surfaces are attached. If the actuation device involves linear displacements, as usually is verified when the generation of movement at the actuator piston rod is caused by the compression of hydraulic fluid in a chamber, there is a nonlinearity associated with the lever arm r that exists to generate the control-surface rotation around the rotation axis. For small angles, β can be obtained approximately by $(u_j - u_i)/r$. However, the attachment mechanism may introduce that geometric nonlinearity.

Typical control-surface nonlinearities depend on the control-surface angular displacement (i.e., the rotation angle β) and its velocity $\dot{\beta}$, as described as follows:

1) Free play: Control-surface free play is usually described by a linear piecewise model describing the elastic torque $\tau \equiv \tau_\delta$ on the control-surface hinge line with a dead zone. It may appear with preload, which generates asymmetry, as noted by Lee et al. [16]:

$$\frac{\tau_\delta}{k} = \begin{cases} \beta + \delta_L & \text{if } \beta < -\delta_L \\ 0 & \text{if } -\delta_L \le \beta \le \delta_R \\ \beta - \delta_R & \text{if } \beta > \delta_R \end{cases} \tag{1}$$

where δ_L and δ_R are, respectively, the free play for negative and positive angle β of control-surface rotation.

2) Friction: Coulomb friction is one of the most typical friction effects associated to the control-surface rotation [16–18]. The incremental torque due to the friction τ_f is given by

$$\frac{\tau_f}{f} = \begin{cases} 1 & \text{if } \dot{\beta} > 0 \\ 0 & \text{if } \dot{\beta} = 0 \\ -1 & \text{if } \dot{\beta} < 0 \end{cases} \tag{2}$$

where f is the friction coefficient. However, Lee et al. [16] noted that, in practice, f is not constant over $\dot{\beta}$, as observed in the Stribeck friction effect [19,20].
Although friction forces exhibit a dissipative nature, it is important to highlight that they are usually neglected in aeroelastic stability analysis because friction properties can reduce substantially over time. Note that a conservative analysis is performed when friction forces are not included, similarly when neglecting the structural damping.

3) Nonlinear stiffness: The cubic effect is most commonly found in control-surface restoring moments. The total moment $\tau \equiv \tau_3$ is given by

$$\frac{\tau_3}{k} = \beta + \frac{k_3}{k}\beta^3 \tag{3}$$

and often $k_3 > 0$ (i.e., a hardening spring) [16,21].

4) Nonlinear damping forces: The connections of control-surface actuators in hydraulic systems can generate nonlinear effects typical of nonlinear viscous dampers. The additional nonlinear torque τ_d due to the damping effect is proportional to the control-surface velocity, as follows [22]:

$$\frac{\tau_d}{d} = \mathrm{sign}(\dot{\beta})|\dot{\beta}|^\alpha \tag{4}$$

where sign() is the signal function, || is the modulus function, and d is the damping coefficient. The parameter $\alpha = 2$ describes the quadratic damping, which is reported in the literature as a typical effect in actuation devices [23]. Additionally, hysteretic behavior can be observed, especially when considering actuators in failure mode, as reported by Huet et al. [24].

5) Actuator in failure mode: According to the literature, control-surface actuators typically exhibit friction and quadratic damping

[23]. For the linear flutter analysis, these devices are usually described by linear characteristics (i.e., a constant stiffness k), and, for some cases, also including a linear damping coefficient b. The values of k and b used in numerical simulations can be compared with the actuator characteristics obtained from the mechanical impedance test [25,26]. However, if these actuators are working in a failure mode, they exhibit nonlinear behavior such that the linear ES (k_{eq}) and damping coefficients (b_{eq}) depend on the amplitude and frequency of motion, that is, they are not constant as considered in the linear analysis. Huet et al. [24] present qualitative curves illustrating k_{eq} and b_{eq} over frequency ω for different amplitudes A of harmonic motion. The nonlinear torque τ_{fm} in failure mode is approximated by a linear equivalent τ_{eq} for each pair (ω, A), as follows:

$$\tau_{\mathrm{fm}}(\omega, A) \approx \tau_{\mathrm{eq}} = k_{\mathrm{eq}}(\omega, A)\beta + b_{\mathrm{eq}}(\omega, A)\dot{\beta} \tag{5}$$

In this context, recently, Rodrigues et al. [27] introduced a frequency-domain-based approach to investigate aeroelastic stability considering these linear ES and damping properties. However, if the mathematical model to describe the actuator nonlinear behavior is known, different methods can be employed, and Sec. V provides a brief description for several of them.

IV. Assessment of the Prior Literature in the Context of Current Knowledge

This section presents a description of several studies involving control-surface nonlinearities in aeroelastic systems. The main goal is to provide a brief but insightful discussion of the state of the art that includes citations to the prior literature. This topic has attracted more attention from the aeronautics community, as shown in Table 1, through the number of papers (NOPs) published over the years. The typical section airfoil with two or three DOF is the most often system considered by the researchers.

The main fundamentals of nonlinear dynamics involving control-surface structural nonlinearities were documented in 1954 [2], 1955 [1], and 1957 [3], and the effort was concentrated in the NACA and the U.S. Air Force. The relevant works of Hoffman and Spielberg [2] and Woolston et al. [3] revealed that flutter dynamics could be significantly changed by nonlinearities. In 1959, Shen [28] reexamined the works of Woolston et al. [3] by considering an analytical approach, as done previously by Shen and Hsu [29]. However, Woolston and Andrews [31] noted that Shen and Hsu [29] considered dissimilar quantities to compare their works, but both of them presented correct results in terms of flutter boundary. On the other hand, Shen [28] used the ideas proposed by Kryloff and Bogoliuboff [30] (in their so-called harmonic linearization method) to conclude that nonlinearity of the hysteresis type described by an equivalent structural damping coefficient is also a function of the amplitude, such as known previously for the free play. Shen's works introduced new strategies to employ DFs to investigate the effects of control-surface nonlinearities. Almost 20 years later, researchers from the Naval Air Systems Command (USA), in 1977, and from the AGARD, in 1978, published two reports that definitely marked the resumption of research on this topic around the world [5,6].

Table 1 Number of papers (NOPs) published over the years

Years	NOP
1950–1960	8
1961–1970	0
1971–1980	4
1981–1990	10
1991–2000	18
2001–2010	37
2011–2020	50

A. First Systems and Methods

Theodorsen and Garrick's [4] works were definitely decisive for the first studies involving structural nonlinearities in control surfaces. The modeling of the flutter phenomenon additionally provides the analytical dynamics of a typical section airfoil, besides the method to identify the aeroelastic instability. However, eigenvalue extraction, combined with DFs, was the basis for the methods employed in several works. Lee [32], for example, developed an iterative procedure to evaluate the flutter phenomenon using the DF approach. The author considered local nonlinear stiffness for different systems and computed the linearized representation by assuming previous amplitude of motion. In particular, the models of twin control surfaces considering distributed springs and the tactical missile wing pair represented important advances in terms of system representation for that time. Because of this, the work also evaluated the influence of the number of modes used to truncate the system of equations when the generalized coordinate system is used. An eigenvalue extraction is employed to evaluate the stability of the linearized system considering multiple nonlinearities, such as free play, hysteresis, and bilinear stiffness. The author's conclusions emphasized that DF provided accurate results. On the other hand, it required an iterative procedure to consider large dynamic systems with multiple DOF and nonlinearities.

The challenges involving the use of DF-based method motivated researchers to consider different strategies. Laurenson et al. [33] focused on investigating numerical methods to integrate the system of equations in the time domain. The authors investigated a control surface with a loose hinge and/or joint slippage in the surface support structure. They considered two types of nonlinearities separately (i.e., free play and preload). Preload corresponds to an asymmetric free play occurring for non-null elastic torque. The system was described by two DOF, and steady-state aerodynamics was considered. The authors compared numerical results obtained by using the methods trapezoidal, Runge–Kutta (RK), Shanks, and Adams–Moulton. They noted that similar results can be obtained with any of these methods; however, they recommended the RK method as the most attractive approach based on computational efficiency. In particular to the preload nonlinearity, their work demonstrates that higher harmonics have important influence on predicting the system responses. Although they did not explicitly state, the authors indicated that DFs are more interesting when only free play is considered, and they have a limited applicability potential when considering preload and nonlinear stiffness.

Three years later, Lee and Tron [34] showed that under low preload, a DF could provide interesting results if the nonlinear stiffness is particularly bilinear. They also achieved good results if each nonlinearity was considered separately from each other. The authors studied the aeroelastic stability of the CF-18 wing fold, including outboard leading-edge flap considering a bilinear spring with a preload and free play. For the bilinear stiffness, their results demonstrated that LCO can occur with considerable increase in flutter speed above that for the nominal linear condition. On the other hand, positive aileron angles are more effective to alleviate the LCOs than negative angles when free play is considered. The DF method exhibited its (probably) main characteristics (i.e., simplicity of use and huge potential of applicability).

B. First Studies Investigating Chaotic Behavior

The years before the early 1990s marked a growing effort to understand chaotic behavior in aeroelastic systems. The chaotic oscillations were related to the flutter phenomenon considering beam-like structures, as reported by Dowell [35–37]. Dowell's works were considered by Zhao and Yang [38], who considered a cubic spring in the pitch DOF to investigate an airfoil. The authors consider the critical flutter motion as simple harmonic, as done previously by Yang and Zhou [39], and they named the approach as equivalent linearization (EL) method. They considered the two DOF typical section with nonlinearities in pitch and noted chaotic motion for a narrow airspeed range higher than the linear divergent speed. The authors concluded that a buckling panel submerged in supersonic flow is more subject to chaos than an airfoil in incompressible flow.

A report published in 1989 documented 3 years of a research program focused on investigating the effect of discrete nonlinearities in rigid control surfaces [40]. The authors carried out numerical simulations and experimental wind-tunnel tests by considering a DOF typical section with nonlinear springs. Although their results presented different types of motions, including LCO, the authors noted a narrow relation between initial conditions and chaotic responses. This program was part of an important effort by the U.S. Air Force Office of Scientific Research, which some years before was focused on free play without noting chaotic motions [41,42].

Asymmetric bilinear stiffness and hardening cubic stiffness in pitch were investigated by Price et al. [21]. The authors employed the dual-input DF technique to predict LCOs and compared with the finite difference time marching solution [43]. Their results showed LCOs at velocities well below the flutter boundary, in agreement with the results of McIntosh et al. [44], and they confirmed their previous observations that chaotic motion is strongly dependent on the initial conditions and the system parameters [45]. Price et al. [21] proved that the cubic nonlinearity for the two DOF typical section airfoil generates chaos, instead of the discontinuous nature of a bilinear restoring moment.

C. System Identification and Control

In the late 1990s, different authors numerically and experimentally investigated LCO due to different types of nonlinearities. Conner et al. [46], for example, investigated computationally and experimentally a three-DOF typical section with free play. The authors adapted the state-space aeroelastic model developed by Edwards et al. [47] and employed Hénon's technique to integrate the system of equations more accurately. Virgin et al. [48] presented complementary discussions on Conner et al.'s work and noted that some system responses during LCO were sensitive to small changes in the flow velocity, making a challenge for experimental observations in wind-tunnel tests. For this type of data, Trickey et al. [49] demonstrated the utility of employing a system identification approach to generate a linear approximation and develop techniques for online monitoring. O'Neil and Strganac [50] investigated softening and hardening spring nonlinearities on a two-DOF aeroelastic apparatus. Their results are consistent with the observations of Woolston et al. [3] that indicate transitions from stable to unstable responses. The study predicted regions of oscillation with decreasing amplitude transiting to LCO by increasing airspeed.

D. Different Types of Harmonic Balance Methods

Different variations of the first-order harmonic balance method (HBM) have been proposed from the beginning of the 2000s. However, DF-based analysis is still often considered by different researchers, as presented in this section. Tang et al. [51], for example, investigated the sensitivity to initial conditions for the onset of LCO in a typical section with control-surface free play. Their study confirmed the conclusions of Conner et al. [46] that the bifurcation velocities where LCOs occur are independent of the free-play amplitude. The authors also employed the HBM to compute the limit-cycle amplitude of all the DOF, including the linear DOF. They reduced the numbers of unknown parameters and obtained a linear algebraic system (instead of the typical nonlinear) by using the results of limit-cycle amplitude and frequency of the nonlinear DOF predicted by the DF combined with eigenvalue analysis.

Lee et al. [16] studied chaotic behavior considering free play, cubic stiffness, and hysteresis nonlinearities. They determined the amplitudes and frequencies of a supercritical post Hopf-type bifurcation using DF and integrating the system of equations to evaluate the influence of initial conditions. Lee et al. [52] considered a cubic nonlinear restoring torque to investigate soft and hard spring effects on the two-DOF airfoil flutter boundary. The authors identified the Hopf bifurcation point by evaluating the eigenvalues of the associated Jacobian matrix. Their results showed that hard springs in pitch generate oscillations with self-limited amplitude after the linear flutter boundary, which is independent of the initial angular displacement. On the other hand, soft springs can induce flutter below the linear flutter speed. These results were noted previously by Woolston et al. [1], and Lee and Desrochers [53] verified different harmonic components over airspeed, depending on the system parameters, like control-surface preload and mass ratio.

Bae et al. [54] investigated an aeroelastic system, including free play and bilinear spring in a control surface. Three different types of LCOs are evaluated in the time and frequency domains. They employed a DF to compute an ES and show that the ES of a bilinear spring is larger than that of free play. Liu and Dowell [55] showed a secondary bifurcation after the primary Hopf bifurcation in a two-DOF aeroelastic system with a hard spring in the pitch DOF. They employed the HBM to predict LCO amplitudes.

Lee et al. [56] extended the HBM to consider higher harmonics and introduced analytical expression to compute the LCO frequencies for a two-DOF airfoil considering cubic nonlinearity in the restoring forces. Their strategy does not employ iterative procedures to obtain frequencies and amplitude of motion. By including terms at least up to the ninth harmonic, the approach captures the secondary bifurcation identified by Liu and Dowell [55]. Baldelli et al. [57] employed a DF-based technique to determine the stability boundary of a nonlinear feedback system. They considered an uncertain structural control-surface stiffness and free play, and they identified two unstable LCO branches with frequencies near, respectively, the plunge and pitch natural frequencies.

Liu and Dowell [58] used the high-order HBM (HOHBM) to consider higher harmonics in the two-dimensional airfoil with control-surface free play experimented by Conner et al. [46]. They presented the complete procedure to solve the large nonlinear system containing two augmented variables and the limit-cycle harmonic amplitudes and fundamental frequency as unknowns. The authors also exposed the expressions for the first and third harmonic coefficients of the linear equivalent restoring force particularly due to free play. They investigated a subcritical bifurcation evaluating the influence of initial conditions and a hysteresis phenomenon.

Tang and Dowell [59] noted that free play can also induce LCO in wings with stores. They tested a delta-wing model with an external store in the wind tunnel. Their results demonstrated that the wingtip LCO amplitude is strongly dependent on the store pitch motion and initial conditions. Liu et al. [60] developed a harmonic-balance-based technique to investigate the frequency and amplitude of LCO in a two-DOF airfoil with cubic restoring force. Their high-dimensional HBM is less accurate than the classical HBM when the same number of harmonics is considered. However, they highlighted that the proposed approach is easier to implement computationally than the HBM when the number of high harmonics is large.

Liu et al. [61] developed the incremental HBM (IHBM) to study an airfoil with hysteresis nonlinearity in the pitching DOF. The authors used Floquet theory to assess the stability of the LCOs. Their approach allows one to track both stable and unstable limit cycles. This IHBM was also used by Chen et al. [62] to capture LCO considering cubic and uncertain springs to describe the pitch and plunge stiffnesses. Another variation of HBM was proposed previously by Chen and Liu [63] to study the LCO for this type of nonlinearities. Irani et al. [64] used a third-order HBM to investigate the LCO in a three-DOF airfoil with cubic restoring moments. They noted that the bifurcation diagram is dependent on the position of the airfoil elastic center, and that divergent oscillations may be observed when softening stiffness. Chen et al. [65] also investigated a similar system and proposed an EL technique to predict the LCO amplitudes. The authors commented that an IHBM can be considered to describe the high harmonics of motion.

E. More Recent Strategies to Investigate Control-Surface Nonlinearities

Vio and Cooper [66] used normal form [67] to investigate an aeroelastic system with bilinear stiffness. Because this method is typically restricted to continuous nonlinearities, the authors employed a polynomial approximation to represent the bilinear stiffness. They also used HBM to predict LCO, but the normal form method presented results in good agreement with numerical integration and more consistent than the HBM. Jones et al. [68] identified the LCO for piecewise linear systems at low-speed incompressible and transonic flows. They employed continuous functions to represent discrete nonlinearities. To investigate the transonic flow, the eigensystem realization algorithm was used to obtain reduced-order models (ROMs) from pulse responses of the linearized Euler simulations, as developed by Silva [69]. According to the author, the use of ROMs generates accurate results without the need for a costly computational fluid dynamics analysis.

Chung et al. [70] employed the perturbation incremental method (PIM) to study a hysteresis structural nonlinearity in a two-DOF system. The authors noted that the method is suitable to detect complex aeroelastic responses. They presented a comparison of the dynamics due to hysteresis and free play, highlighting that the chaotic intervals are narrow in flow velocity for both nonlinearities, and that the first nonlinearity may generate different regions of LCO in comparison with free play. Arévalo and García-Fogeda [71] investigated a two-DOF slender wing–body configuration, including free play in the wing control mechanism. They identified different types of LCOs over flight speeds. Chaotic motions were verified below the linear flutter boundary and then change to LCO when the angle of attack is greater than a particular value. Vasconcellos et al. [72] assessed a different mathematical representation for a free-play-type nonlinearity. They noted that a hyperbolic tangent function can alternatively be used to describe the discontinuous free-play representation. Theoretical predictions of LCO are compared with Conner et al.'s [46] experimental results to demonstrate the feasibility of the approach.

Abdelkefi et al. [73] considered cubic springs for all three DOF in an airfoil. They employed quasi-steady approximation to derive the normal form of the Hopf bifurcation to characterize the system. The authors noted that this type of approximation implies comparable results with unsteady aerodynamics, and the pitch and plunge motions are periodic, but different amplitude of motion is verified for the flap DOF at airspeed higher than the linear flutter speed. In [74], the authors considered a polynomial expansion to employ the normal form of the Hopf bifurcation to characterize the type of instability in a two-DOF aeroelastic system. Emory and Patil [75] used nonlinear normal modes to predict the LCO in an aeroelastic system with cubic nonlinearity in the pitch DOF. They noted that with a proper selection of master coordinates, their semi-analytic method offers valuable insight into the mechanism of LCO.

Li et al. [76] investigated the cause of chaos in a two-DOF airfoil with cubic and free-play nonlinearities in pitch. They noted that chaotic behavior disappears if the elastic axis shifts from the midchord to the leading edge. Their results demonstrated that mass ratio has a small effect on the occurrence of chaos and that structural preload can eliminate the chaos. On the other hand, Price et al. [21,77] showed that chaotic motion can exist for this type of nonlinear system if a small structural preload is considered.

Chen and Liu [78] proposed the use of more Precise Integration Method (PIM) to investigate an airfoil with an external store. They included a free play in pitch and considered the nonlinear aeroelastic system as three linear subsystems due to its piecewise feature. The approach employs a predictor–corrector algorithm (PCA) to identify the switching points from one to another subsystem. According to the authors, the technique is a convenient alternative to capture LCO, in comparison with the classical RK method, because it does not require one to reduce the time step. Cui et al. [79] employed the PIM combined with the PCA to investigate the hysteresis effect in the pitch restoring torque to describe friction and free-play nonlinearities.

Dimitriadis [80] introduced numerical continuation with a detailed formulation to obtain the bifurcation diagram of an airfoil with control-surface free play without using software typically employed as AUTO. For a particular air velocity, Newton/Raphson-based methods are employed to obtain the convergence of some initial guess of LCO amplitude and frequency. The continuation allows to cover the velocity range where the guess for the next air velocity is well valued based on the converged values of the previous parameter. This approach improves the convergence for each parameter. This method is also used by Dimitriadis [81] to solve the large nonlinear algebraic system obtained by the HOHBM [58]. Finally, Dimitriadis [82] presented the complete bifurcation analysis of control-surface free play, where the result of the EL and the classical DF at the Hopf point is used as initial guess to employ the numerical continuation. The author showed that even for the typical section, the single free play can lead the system to a rich bifurcation behavior, usually not identified by linearized solutions.

Kholodar [83] presented a complementary physical explanation of the bifurcation airspeeds and frequencies of LCO for the system with free play for both zero and nonzero aerodynamic preload on the control surface. The work showed that a certain amount of the preload is destabilizing and can lead to higher-frequency oscillations because the preload increases the equivalent rotational stiffness of the control surface. Pereira et al. [84] noted that a hardening nonlinearity is dominant to determine the appearance and amplitude of LCO in comparison with free play. On the other hand, free play leads to the appearance of subcritical behavior and increases the region of dangerous LCO. They investigated a two-DOF airfoil considering these nonlinearities in pitch. Verstraelen et al. [85] used the EL technique to predict the LCO in a two-DOF airfoil with free play in pitch. They classified the range of amplitudes by three different domains {i.e., outside [left (or lower) and right (or upper) sides of free play] and inside free play}. Their numerical and experimental results demonstrated that these may exist two and three domain cycles.

Padmanabhan et al. [86] analyzed free play, Coulomb friction, and nonlinear stiffness and damping forces for two different aeroelastic systems considering the moving surface with torsional nonlinearities at the root actuator and a wing with a store on a flexible and nonlinear attachment. They showed that variations in effective damping associated to the nonlinear damping have a smaller role in LCO than the accompanying variation in effective stiffness when free play is considered. Frumusa and Vaccaro [87] presented data of free-play measurements in a horizontal tail of the M-346 trainer aircraft. They performed a linear equivalent modal analysis to verify the free-play amplitude. Their results demonstrated that this aircraft is able to withstand free play greater than those prescribed by MIL specifications, without incurring any dangerous aeroelastic instability.

Jinwu et al. [88] presented a review of advances in nonlinear aeroelasticity of aircraft. They cited Lee et al. [16], who cited Lee and Tron [34], to show that a typical free play obtained from experimental tests of aircraft is asymmetric and involves preload and hysteresis. Tang and Dowell [89] presented an overview of their experimental wind-tunnel tests performed to evaluate typical aeroelastic with structural nonlinearities. They provided insights into nonlinear phenomena and the fundamental physical aspects. Al-Mashhadani et al. [90] investigated a wing–flap–tab typical section, including free play in the tab. They noted LCO frequency jumps from low to high frequency. The authors found some differences between experimental and theoretical LCO predictions and more accurate comparisons for the linear condition than for LCO per se. The theoretical LCO presented different phases, increasing over time, as typically obtained when a classical integration method is employed to integrate numerically the system of equations instead of using Hénon's technique, as done by Conner et al. [91]. This technique was also used by Dai et al. [92] and Wayhs-Lopes et al. [93] to compare it to classical numerical integration. The first work employed the technique for free play as the single nonlinearity, whereas the second work shows how to apply the technique for multiple nonsmooth nonlinearities as free play and Coulomb friction. The authors show that Hénon's technique employed for both nonlinearities improves the results obtained when applied only for free play.

Recently, Garrigues [94] presented a review of practices at Dassault Aviation and highlighted that the first harmonic linearization is used to evaluate the linear equivalent flutter speed through the *pk* method when free play or servo-actuator nonlinearities are considered in the military and business jet aircraft development. Jafari et al. [95] verified that LCO starts at low speed when free play is considered, in comparison with the nominal flutter airspeed. The authors noted that the LCO amplitude for a particular initial condition is strongly dependent on the free-play amplitude.

V. Current Theoretical/Computational Methods

There are several methods for solving nonlinear systems in both the time and frequency domains. Their formulations involve various numerical and analytical approaches, including iterative solutions. The classical approach is to use time-domain integration methods, which allow one to obtain different types of motion, such as decay responses, LCOs, chaotic motions, and others. Time marching methods, as the classical fourth-order RK, can be applied directly for continuous nonlinearities. On the other hand, discrete nonlinearities, such as free play, require combined approaches to identify the switching points more accurately during the integration process. In particular, for piecewise linear systems, analytical solutions of ordinary differential equations (ODEs) involving matrix exponential notation are the basis for the precise integration method combined with the PCA. The point transformation (PT) method is also an alternative, which allows one to use large time steps in comparison with classical methods, such as RK. However, these methods can be time consuming, and thus, the approaches written in the frequency domain can be an interesting alternative.

Methods written in the frequency domain usually involve algorithms that require less computational time, but they can be limited to periodic solutions. The HBM is one of the most commonly used methods particularly when considering a single harmonic, which considers a linear equivalent term representing the nonlinear function through the Fourier series. Multiple harmonics can also be considered in HOHBM to capture high-order harmonic responses. However, this increases the complexity of mathematical expressions to compute the Fourier series.

Bifurcation diagrams can be obtained by using iterative methods, as the numerical continuation and the PIM, which allow one to avoid time integration in over the total time interval for each bifurcation parameter. These methods commonly allow one to obtain an incremental solution for a different bifurcation parameter if a solution for another parameter value is previously computed. They usually are less time consuming in comparison with classical integration methods. On the other hand, the normal form and center manifold methods do not require iterative process. However, they can involve system dimensional reduction, which can eliminate more or less significant nonlinear terms. In addition, their accuracy can be limited to a neighborhood of critical points.

This section summarizes the main pros and cons for the most typical methods employed to study the effects of control-surface nonlinearities on system responses. A brief description of each method is presented, and, additionally, it is linked to relevant references that explore the method in more detail. These papers provide a deep understanding of various aspects involved in the formulation of these methods. However, the present paper may be helpful as an introduction to several methods. It is expected that an expert in nonlinear dynamics will be familiar with most, if not all, of the methods described. On the other hand, the content of this section is probably most useful to aerospace engineers who are not experts in nonlinear dynamics.

A. Classic Time Integration Methods

Time integration methods can be employed to integrate the nonlinear ODE. Commonly, the second-order ODEs are rewritten using the state-space realization (a first-order ODE) by defining the state vector with velocities and displacements at time t_{i+1} based on their values known at time t_i. Examples of these methods are Newmark, finite difference of Houbolt [96], RK, and others. Based on the literature, the method most often used in this field is the fourth-order RK, but each aeroelastic system may require a specific method to obtain more accurate results. The main characteristics of this strategy of solution comprise that time integration methods are commonly straightforward approaches of easy computational implementation [92]. They are convenient for coupling with flight control laws especially if the state-space representation is used [97]. On the other hand, time integration methods may produce numerical errors for discontinuous nonlinearities [91,98]. Because of this, Bayly and Virgin [99] combined a classic method with interpolations technique; Lin and Cheng [100,101] used bisection method, and Conner et al. [91] and Dai et al. [92] applied Hénon's technique to identify the switching points from inside to outside the free-play region. They are typically time consuming in terms of computational cost [91,102–105] and low efficiency to evaluate the effect of system parameters because requires a large number of computer runs [103,105].

B. Hénon's Technique-Based Methods

The main idea of this approach is to compute the state vector accurately to identify the switching points between inside and outside the free-play region (i.e., the free-play boundary), changing the integration independent variable from time to position. This strategy avoids numerical errors mainly in phase over time. References [91,92] define this approach in the context of aeroelastic systems, whereas [46,49, 72] compare numerical results with experimental data. The method is used to design controllers for LCO suppression [106,107], and Vasconcellos et al. [108] discuss the grazing effect. The method is also employed to investigate applications involving transonic flow [109,110]. Hénon's technique is suitable for any piecewise linear nonlinearity (discontinuous and discrete), as demonstrated in [91,93,111]. Wayhs-Lopes et al. [93] clarify that it is easily implemented computationally using the matrix notation, and it allows one to capture the exact switching point, adding only one integration step for each free-play border crossing, which is a substantial advantage in comparison with those methods based on interpolation or bisection [91]. The method captures the full spectrum of motion (decaying, LCO, nonperiodic motion, chaos, and divergent flutter). The benefits are evident mainly in chaos and chaotic transient behavior [72,91,92]. On the other hand, the method may be time consuming to carry out a complete analysis of stability because it requires a large number of numerical-integration runs [68,72].

C. Continuous-Based Approaches for Discrete Nonlinearities

Continuous functions are employed to approximate the elastic restoring moment when free play is considered. Because of discontinuity of free play, tangent hyperbolic (TH) and rational polynomial (RP) are successfully applied in [72,76] to approximate the nonlinear restoring torque. The authors compared their results with experimental data and show that this strategy allows one to obtain more accurate results when integrating the system of equations (i.e., for the time-domain solution). Jones et al. [68] used TH combined with numerical continuation, and Dai et al. [92] compared the results obtained by using the TH with Hénon's technique. The main characteristics of this approach include that it is useful especially when a continuous representation of the nonlinearities is required. It may reduce the computational time to carry out numerical simulations, when compared with Hénon's technique and the PT method [72]. The use of TH allows one to capture instability and different nonlinear effects, including chaotic and transition behaviors [72]. Its use also allows one to achieve the subcritical bifurcation [111], and the use of RP allows one to accurately capture some types of LCO [76]. On the other hand, the use of continuous approach may require a substantial effort to obtain a representative mathematical function for each particular discrete nonlinearity. In particular, RP fails to predict chaotic motion and chaotic transients [72,92], whereas the switching points (from inside to outside the free-play region) are not computed, and because of this the bifurcation point is not exact [98].

D. Point Transformation Method

This time-domain method aims to avoid small time steps when integrating the system of equations. It directly calculates the traveling time to the switching points, which allows one to properly consider each corresponding subdomain. The method is based on the analytical solution of linear ODEs obtained through the matrix exponential notation; note that it is implicitly required that the nonlinear systems be piecewise linear. By starting with a set of initial conditions, the traveling time is computed by using each dynamic matrix and the appropriate state vector, obtaining the exact switching point [98]. Liu et al. [98,112] investigated free play and hysteresis, respectively, and Jones et al. [68] and Roberts et al. [113] employed it to find an initial steady solution to start the numerical continuation. The PT method may produce precise results [72]; capable of finding the exact switching point [98]; may be used to investigate the influence of initial conditions [58,114]; and captures different types of motion as periodic, period doubling, transient, and chaos [98,115]. On the other hand, it typically is limited to piecewise linear systems [104], it is time consuming [72,111], and it is not suitable to evaluate the effect of system parameters because it cannot predict unstable periodic motions [98].

E. Precise Integration Method

The conceptual idea for this method is similar to Hénon's technique. It employs a PCA to find the switching points. However, instead of using a numerical-integration time scheme, it considers the analytical solution of linear ODEs involving the matrix exponential notation (similarly to the PT method). The conventional PIM approximates the exponential matrix through the Taylor series expansion. Chen and Liu [78], Tian et al. [105], and Cui et al. [116] investigated a two-DOF system with free play, and Tian et al. [105] considered multiple free play and employed the Padé approximation instead of the Taylor series. References [117–121] present interesting studies in other fields (not aeroelasticity). The precise integration method allows the use of a larger time step than classical numerical-integration methods [78,117], and it may use the full computer precision to locate accurately the switching points [105]. However, the PIM may require several computational runs to find the switching points through the PCA, whereas Hénon's technique requires only one.

F. Harmonic Balance Method

The general idea is to write the system response through the sum of harmonic terms in time. Although HBM expresses a general approach, in terms of the number of harmonics, this name is usually used by authors when they consider only the first one. (In these cases, authors often employ the name "DF method.") Otherwise, the approach is commonly referred to as HOHBM, which is presented separately from this one in the present paper. Gelb and Vander Velde [43] introduce definitions (not for aeroelasticity), and several authors [28,39,122] define the approach for aeroelastic applications. Verstraelen et al. [85] and Gordon et al. [103] investigated three-DOF systems and compared with experimental results; several authors [109,123,124] considered free play in two-DOF systems, when Tang et al. [51] and Dimitriadis [82] included the control-surface rotation as a third DOF. Eller [18] and Padmanabhan et al. [86] investigated an aeroelastic system, including free play and friction; several authors [21,54,125] considered bilinear stiffness, whereas Liu and Zhao [126] and Padmanabhan and Dowell [127] included cubic stiffnesses. The main advantages of this method include that it is suitable to predict directly both stable and unstable LCOs (amplitude and frequency), including the cases with multiple limit cycles at each single flow speed [103,128]. The method is compatible with classic analytical stability analysis, such as those based on eigenvalue extractions [46], and it is not time consuming when compared with classic time integration and PT methods [104,105]. However, the method cannot predict high-order harmonics easily [21,51,104], and it cannot capture the effects of initial conditions [21,46,128]. It is not suitable to capture nonperiodic motion, including chaotic and transient behaviors [46,58,105,125]; it requires a specific analytical effort to calculate the DF of each nonlinearity before actually developing the numerical model [46]. In the case of multiple nonlinearities, it inevitably involves some iterative processes, which reduces its simplicity [58].

G. High-Order Harmonic Balance Method

The system response is written in terms of a sum of sine and cosine functions to approximate the nonlinear function by the Fourier series. Tamura et al. [129] introduced definitions considering the Duffing oscillator, and Liu and Dowell [58] compared with experimental results obtained by Conner et al. [46] for the three-DOF system with free play. These authors found results similar to Tang et al. [51], who performed a DF-based analysis. Dimitriadis [81] compared different strategies based on the HBM. Although it captures high-order harmonics responses, the method is not suitable to capture nonperiodic motion, including chaotic and transient behaviors [58].

H. High-Dimensional Harmonic Balance

This approach first arises as a variation of the HBM and aims to transform variables from the frequency to time domain by using the discrete Fourier transform. Dai et al. [130] noted that high-dimensional harmonic balance (HDHB) is mathematically equivalent to the time-domain collocation method. The HDHB method is successfully employed considering [60,131], and Dimitriadis [81] considered it to include free play in a generic transport aircraft. The method avoids the Fourier series, which involves tedious algebraic symbolic manipulations due to high harmonic coefficients [60], and the number of harmonics included in analysis does not influence the ease of deriving and implementing the HDHB system into a computer algorithm [131]. It requires $2n$ harmonics to result in a similar accuracy to that of HOHB with n harmonics [60,131] and it may produce physical aliasing [130], which are disadvantages to consider the method for practical applications.

I. Numerical Continuation

The method allows one to obtain bifurcation diagrams without integrating the system of equations. The approach depends on a given steady-state solution for a known value of the bifurcation parameter to compute the solution for an incremental value of that parameter. Several authors [132–135] introduced the formulation (not for aeroelasticity). Jones et al. [68] and Roberts et al. [113] investigated three-DOF piecewise linear systems, and Dimitriadis [80] considered free play and cubic stiffness. Dimitriadis [81] employed it combined with HOHB, and Dimitriadis [82] and Gordon et al. [103] employed it considering the HBM to obtain a preliminary solution. Gordon et al. [103] showed experimental results, Meyer [136,137] considered multiple nonlinearities, and Shukla and Patil [138] presented an optimal control design. Numerical continuation methods can completely characterize the whole spectrum of motion [80], and it is convenient to evaluate the effects of system parameters, including the adaptability to consider multiple parameter variations [113]. This approach involves relatively low computational cost [104], and it predicts the bifurcation points [102]. On the other hand, it is limited to construct bifurcation diagrams [104]. It is not easily suitable for high-DOF systems, although it is suitable for continuous nonlinearities [139]. Jones et al. [68] employed it for the following two different approaches involving piecewise linear systems: 1) combining with the PT method and 2) approximating discrete nonlinearities by continuous functions.

J. Incremental Harmonic Balance Method

The IHBM is derived from the HBM. It was developed as an alternative to solving mathematically complex equations [140]. It considers an incremental part achieved by using the Newton–Raphson algorithm. The fundamentals of this approach are presented in [141–145]. In particular, for aeroelasticity, Raghothama and Narayanan [146] and Ming et al. [147] investigated the dynamic behavior, including cubic stiffnesses, and Dimitriadis [81] and Ni et al. [140] considered free play. Liu et al. [61] considered hysteresis, and Liu et al. [148] investigated an external store. The method can properly consider strong, including multiple, nonlinearities [144,147]; it may track both stable and unstable LCOs [61] and captures the effect of changes in system parameters [147]. On the other hand, it is limited to periodic motions, as noted by Liu et al. [61].

K. Perturbation Incremental Method

It is a semi-analytical method that combines features from the perturbation and incremental methods [114]. The main definition of this approach is presented by Chan et al. [149]. Chung et al. [70,114] considered a two-DOF system with free play and hysteresis, respectively. The method is suitable for any piecewise linear system [114] and not time consuming in comparison with time integration [105]; it may track both stable and unstable LCOs, and it captures the effect of changing in system parameters [114]. However, the method is limited to weak nonlinearities [144]; it does not capture the effects of initial conditions [114], and it cannot accurately detect complex motions, as demonstrated by Tian et al. [105].

L. Normal Form

Normal form consists in reducing the nonlinear system of equations written previously using the state-space notation to its normal form via changing variables. Its linear part is diagonalized via standard linear transformation (using eigenvectors extracted from the linearized system matrix). The normal form is applied on the diagonalized system such that some nonlinearities are removed from the equation [150]. References [151–153] present very interesting works combining with the averaging method. Beyn et al. [133] and Zhang and Huseyin [154] compared it with the IHBM, and Vio and Cooper [66] considered a piecewise linear system. References [73,74,155,156] introduce the idea of bifurcations. It allows one to obtain a simpler equation to carry out the stability analysis; in principle, it reduces time marching and iterative processes [153] and has a relatively low computational cost [104]. On the other hand, its original formulation is limited to continuous nonlinearities [152]. However, Vio and Cooper [66] noted that piecewise linear nonlinearities may be approximated by continuous function before successfully applying the normal form method, limited to provide satisfactory results in the proximity of critical (Hopf bifurcation) points [138].

M. Center Manifold

It is usually applied before applying the normal form. It consists in reducing the system dimension, keeping the states related to pure imaginary eigenvalues. The center manifold theorem allows one to perform the nonlinear stability analysis similarly to the stability of linearized systems [102]. The approach is properly introduced by Verhulst [157]. Liu et al. [128] considered cubic stiffness, and Vio and Cooper [66] combined it with the normal form. Lee et al. [16] investigated the method when considering chaotic behavior. The approach presents relatively low computational cost [104], and it allows one to rewrite high-dimensional systems in low one [16]. On the other hand, it is limited to provide satisfactory results in the neighborhood of critical points [126,128,138], and it cannot capture effects of initial conditions [98].

There are several additional methods for nonlinear analysis in the literature. In particular, for aeroservoelasticity considering control-surface nonlinearities, it is possible to cite cell mapping [158], stochastic approaches [159], fictitious mass method [160,161], dealiasing HBM [162], Poincaré mapping-based analysis [163,164], and others.

VI. Conclusions

This review and assessment of the state of the art reveals that a substantial and relevant effort has been made to study the nonlinear dynamics of the two- and three-DOF typical sections, including control-surface nonlinearities. The findings provide important insight into the implications of control-surface nonlinearities in real aircraft. On the other hand, the wide use of this system model keeps the technology readiness level (as defined by Mankins [168]) compatible with basic research. Thus, there is an important challenge to undertake testing of more complex systems in representative operating condition.

The published works demonstrate that free play in pitch motion commonly implies a more complex dynamic behavior in comparison with free play in the control-surface rotation. As a general rule, Dowell and Tang [17] noted that there are two more important consequences of nonlinear effects in aeroelastic systems. The first one are steady-state oscillations due to the attenuation of exponentially growing motion predicted through linear analysis. The second consequence is that nonlinearities may make a stable system unstable under large disturbances and/or depending on the initial conditions. In practice, both cases require careful analysis from engineering teams when designing an aircraft, and this is the main reason that aeronautical certification agencies do not readily admit the presence of LCOs in flight.

Besides manned aviation, the recent popularization of unmanned aerial vehicles and the initiatives to use them for cargo transportation and military applications expand the class of aerial vehicles that can give importance to free play in maintenance tasks. Also, the research community may have an additional challenge to assess the effect of these nonlinearities in more realistic environment, as was done by Kholodar and Dickinson [170] for a jet regional aircraft. The potential presence of LCOs may require passive and active control strategies, which are also good opportunities for further research in this field. Finally, with more than one control-surface free play acting simultaneously on an aircraft, this is a topic not fully explored in the context of nonlinear aeroservoelasticity.

Based on this literature review, the typical section is the model more usually investigated. It is a well-known system mainly because Farmer designed the pitch and plunge apparatus [165,166], a nd NASA conducted experimental test campaigns in its transonic dynamic tunnel for developing active control technologies [167,168]. However, Kholodar and Dickinson [170] demonstrate that more complex systems can also be successfully investigated to improve the state of the art in this field.

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