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True damping and frequency prediction for aeroelastic systems: The PP method

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Abstract

This paper presents a numerical scheme for stability analysis of the aeroelastic systems in the Laplace domain. The proposed technique, which is called the PP method, is proposed for when the aerodynamic model is represented in the Laplace domain and includes complicated transcendental expressions in terms of the Laplace variable. This method utilizes a matrix iterative procedure to find the eigenvalues of the system and generalizes the other methods such as the P and PK methods for prediction of the flutter conditions. The major advantage of this technique over the other approximate methods is true prediction of subcritical damping and frequency values of the aeroelastic modes. To examine the present technique for stability analysis, some typical examples are used which illustrate its applicability and advantages.

Keywords: Aeroelastic stability; Laplace domain; Iterative procedure

1. Introduction

Aeroelasticity problems are generally classified in two categories: aeroelastic instability prediction and aeroelastic response calculations. The aeroelastic equations of motion can be treated in either the time or frequency domains. The stability analysis of the aeroelastic systems is usually performed in the frequency domain, while the dynamic response calculations are done in the time domain. It is mainly because in the frequency domain, only the frequency characteristics of the system are considered, while the time domain analysis usually encounters numerical integration difficulties. Furthermore, most of the aerodynamic theories developed in the frequency domain are limited to harmonic motion. ‘‘Another reason for the popularity of the frequency domain method is the powerful power spectral description of random loads such as gust loads, landing loads (over randomly rough surfaces), etc’’ (Dowell et al., 1996).

The general form of the governing equation of an aeroelastic system can be written in the Laplace domain as

$$\left[\frac{U^2}{b^2}\mathbf{M}p^2 + \frac{U}{b}\mathbf{C}p + \mathbf{K} + q_\infty \mathbf{Q}(p)\right]\mathbf{q} = \mathbf{0}, \tag{1}$$

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where U is the free-stream velocity, $q_\infty = \rho U^2/2$ is the flow dynamic pressure, ρ is the air density, and b is the reference length that is usually half of the reference chord. The matrices $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the generalized mass, damping and stiffness matrices of the structure, respectively, and $\mathbf{Q}(p)$ is a matrix describing the generalized aerodynamic force, $\mathbf{f}_a$, as follows:

$$\mathbf{f}_a = q_\infty \mathbf{Q}(p)\mathbf{q}. \tag{2}$$

The nondimensional Laplace transform parameter, p, can be expressed as

$$p = \frac{b}{U}(g + \mathrm{i}\omega), \tag{3}$$

where g and ω are the damping and circular frequency, respectively. The reduced frequency k and the rate of decay γ in transient motion are defined as

$$k = \frac{b\omega}{U}, \quad \gamma = \frac{g}{\omega}. \tag{4}$$

The instability condition of the aeroelastic system can be determined using the roots of the determinant of Eq. (1). Some numerical schemes have been devised to obtain the roots of the flutter determinant. If the matrix $\mathbf{Q}(p)$ is representable in the following polynomial form:

$$\mathbf{Q}(p) = \sum_{n=0}^{N} \mathbf{Q}_n p^n, \tag{5}$$

where $\mathbf{Q}_n$'s are invariant matrices with respect to p, one can write Eq. (1) in the form of a simple eigenvalue problem which can be solved using standard eigenproblem algorithms. This approach is known as the P method. However, most of the unsteady aerodynamic theories take on complicated forms and contain transcendental terms with respect to the complex variable p and thus cannot be used with the P method.

Edwards et al. (1979) employed the Laplace inversion integral to separate the aerodynamic loads into rational and nonrational parts and showed that only the former are involved in the aeroelastic stability of the wing. They also evaluated the approximate methods, starting from simple harmonic unsteady theory, with their exact results for the aeroelastic stability analysis.

The K method is another well-known technique to find the aeroelastic instability condition using Eq. (1). The basic idea in this method is to add enough artificial structural damping to the system to achieve simple harmonic motion at every reduced frequency k. Assuming $p = \mathrm{i}k$, Eq. (1) is altered as follows:

$$[-\mathbf{M}\omega^2 + \mathbf{C}\mathrm{i}\omega + \mathbf{K}(1 + \mathrm{i}\zeta) + q_\infty \mathbf{Q}(\mathrm{i}k)]\mathbf{q} = \mathbf{0}, \tag{6}$$

where ζ is the artificial structural damping. For a given value of k, Eq. (6) can be written in the form of a standard eigenvalue problem, which can be solved to find the corresponding U and ζ. The intersection of the curve of ζ versus U with the U axis ($\zeta = 0$) indicates the critical air speed. The obtained values of damping for each k can only be interpreted as the required damping (of the specific form) to achieve a simple harmonic motion (Bisplinghoff et al., 1996; Hodges and Pierce, 2002). Eq. (6) is equivalent to Eq. (1) only at the instability condition where $\zeta = 0$. Otherwise, the eigenvalues obtained from Eq. (6) do not correspond to Eq. (1). It is also worth noticing that the K method may sometimes lead to incorrect results in flutter calculation (Patil et al., 2004).

Hassig (1971) suggested an approximate true damping solution of Eq. (1) by a determinant iteration procedure that is known as PK method. Hassig presented a detailed description of the superiority of this method over the K method and showed that it generalizes the P method for stability analysis. In the PK method, the aerodynamic matrix $\mathbf{Q}(p)$ is approximated by $\mathbf{Q}(\mathrm{i}k)$ assuming low rate of decay oscillations, resulting in

$$\left[\frac{U^2}{b^2}\mathbf{M}p^2 + \frac{U}{b}\mathbf{C}p + \mathbf{K} + q_\infty \mathbf{Q}(\mathrm{i}k)\right]\mathbf{q} = \mathbf{0}. \tag{7}$$

Rodden et al. (1979) modified the PK method by adding an extra aerodynamic damping matrix into Eq. (7) as

$$\left[\frac{U^2}{b^2}\mathbf{M}p^2 + \frac{U}{b}\left(\mathbf{C} + \frac{\rho U b}{2}\frac{\mathbf{Q}^{\mathrm{I}}}{k}\right)p + (\mathbf{K} + q_\infty \mathbf{Q}^{\mathrm{R}})\right]\mathbf{q} = \mathbf{0}, \tag{8}$$

where $\mathbf{Q}^{\mathrm{R}}$ and $\mathbf{Q}^{\mathrm{I}}$ are the real and imaginary parts of $\mathbf{Q}(\mathrm{i}k)$. At a known airspeed, a matrix iterative procedure called "reduced frequency lining up process" is employed to find the corresponding eigenvalue of each aeroelastic mode. The procedure starts with an initial guess for k to calculate the matrix $\mathbf{Q}(\mathrm{i}k)$ and then iterates to find the eigenvalue which satisfies Eq. (8) (Rodden and Johnson, 1994).

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Chen (2000) proved that the added aerodynamic damping matrix in Eq. (8) is valid only at low reduced frequency or linearly varying $\mathbf{Q}(ik)$ and introduced the damping perturbation method for determination of the flutter condition. This method uses a reduced frequency sweeping technique for the extraction of the eigenvalues and is called the g-method. The major advantages of this method over the PK method are the better approximation of true damping at subcritical flight conditions and refining the aerodynamic load matrices for convergent oscillatory motion.

In the present work, a new numerical scheme is proposed for calculation of the eigenvalues of the aeroelastic equations in the Laplace domain. This numerical technique, termed the PP method, is an extension of the P method for when the aerodynamic model involves transcendental terms with respect to the Laplace variable and cannot be used with the P method directly.

2. The proposed algorithm

The generalized aerodynamic force matrix $\mathbf{Q}(p)$ can be written as

$$\mathbf{Q}(p) = \mathbf{Q}_p^{\mathrm{R}} + i\mathbf{Q}_p^{\mathrm{I}} = \frac{\mathbf{Q}_p^{\mathrm{I}}}{k} p + \mathbf{Q}_p^{\mathrm{R}} - \gamma \mathbf{Q}_p^{\mathrm{I}}, \tag{9}$$

where $\mathbf{Q}_p^{\mathrm{R}}$ and $\mathbf{Q}_p^{\mathrm{I}}$ are the real and imaginary parts of $\mathbf{Q}(p)$. Note that in the vicinity of very small values of k, as occurs near the divergence instability condition, the imaginary part of the generalized aerodynamic force matrix $\mathbf{Q}_p^{\mathrm{I}}$ goes to zero and thus we can replace the term $\mathbf{Q}_p^{\mathrm{I}}/k$ in Eq. (9) as

$$\left.\frac{\mathbf{Q}_p^I}{k}\right|_{k\to 0} = \left.\frac{\partial \mathbf{Q}_p^I}{\partial k}\right|_{p=g}. \tag{10}$$

Substituting Eq. (9) into Eq. (1) yields

$$\left[\frac{U^2}{b^2}\mathbf{M}p^2 + \frac{U}{b}\left(\mathbf{C} + \frac{\rho U b}{2}\frac{\mathbf{Q}_p^{\mathrm{I}}}{k}\right)p + (\mathbf{K} + q_\infty(\mathbf{Q}_p^{\mathrm{R}} - \gamma\mathbf{Q}_p^{\mathrm{I}}))\right]\mathbf{q} = \mathbf{0}, \tag{11}$$

that is a system of equations with real coefficient matrices. Defining the following new matrices:

$$\overline{\mathbf{M}} = \frac{U^2}{b^2}\mathbf{M}, \quad \overline{\mathbf{K}} = \mathbf{K} + q_\infty(\mathbf{Q}_p^{\mathrm{R}} - \gamma\mathbf{Q}_p^{\mathrm{I}}), \quad \overline{\mathbf{C}} = \frac{U}{b}\mathbf{C} + q_\infty\frac{\mathbf{Q}_p^{\mathrm{I}}}{k}, \tag{12,13,14}$$

one can state Eq. (11) in the state-space form

$$[\mathbf{A}(p) - p\mathbf{I}]\tilde{\mathbf{q}} = \mathbf{0}, \tag{15}$$

where $\tilde{\mathbf{q}}$ and $\mathbf{A}(\mathbf{p})$ are defined as follows:

$$\tilde{\mathbf{q}} = \begin{bmatrix}\mathbf{q}\\ \dot{\mathbf{q}}\end{bmatrix}, \quad \mathbf{A}(p) = \begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\overline{\mathbf{M}}^{-1}\overline{\mathbf{K}} & -\overline{\mathbf{M}}^{-1}\overline{\mathbf{C}}\end{bmatrix}. \tag{16}$$

For any aeroelastic mode at a known airspeed, the iteration begins with an initial estimation of p and thus the generalized aerodynamic matrix and subsequently the matrix $\mathbf{A}(p)$ are calculated. The initial guess may play an important role in the convergence of the iteration procedure to the important aeroelastic modes and eigenvalues. Assume that we are interested to find the nth aeroelastic eigenvalue branch. At the low airspeeds the eigenvalue of the nth aeroelastic mode is very close to the natural frequency (ω_n) of the corresponding structural vibration mode and thus a good initial guess is $p = \mathrm{i}b\omega_n/U$. Since all the coefficient matrices in Eq. (11) are real, one can find the complex conjugate pairs of eigenvalues, from Eq. (15) as follows:

$$p_{m,n}^r = g_{m,n}^r \pm \mathrm{i}k_{m,n}^r, \tag{17}$$

where $p_{m,n}^r$ is the mth eigenvalue obtained at the rth iteration. Also the subscript n in $p_{m,n}^r$ shows the aeroelastic mode number that is under investigation. Considering only positive k in the iteration sequence, the eigenvalues are ordered by the reduced frequency ($k_{1,n}^r < k_{2,n}^r < \cdots$). The iterative procedure is formulated as follows:

$$[\mathbf{A}(p_{n,n}^{r-1}) - p_{,n}^r\mathbf{I}]\tilde{\mathbf{q}} = \mathbf{0}, \tag{18}$$

where $p_{,n}^r$ denotes the eigenvalues set at the current iteration. The convergence criterion is satisfied when

$$|p_{n,n}^r - p_{n,n}^{r-1}| \le \varepsilon, \tag{19}$$

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where ε is a small number, which is determined according to the desired error tolerance. The last iteration result can be used for initial estimation of p for another aeroelastic mode, namely

$$p^{0}_{n+1,n+1} = p^{r_c}_{n+1,n}, \tag{20}$$

where the superscript r_c, points to the calculated eigenvalues at the last iteration. Furthermore, the obtained eigenvalues at the current airspeed can be used as a very good initial guess of p for the next airspeed value. For a set of airspeed values, this procedure can be used to find the plots of damping and frequency of each aeroelastic mode versus airspeed.

It should be emphasized that to use the PP method, an unsteady aerodynamic code capable of evaluating the generalized aerodynamic matrix $\mathbf{Q}(p)$, in terms of the complex variable p is required. It is while the K and PK methods use $\mathbf{Q}(\mathrm{i}k)$ and most of aerodynamic codes are developed based on the simple harmonic motion. In the recent years some aerodynamic codes have been developed that are capable of computing $\mathbf{Q}(p)$ (Esfahanian and Behbahani-nejad, 2002; Soltani et al., 2004). Furthermore, there are some numerical techniques which can be used to calculate the generalized aerodynamic matrix using aerodynamic codes that are based on the harmonic motion formulation. For example, assuming an analytical expression for $\mathbf{Q}(\mathrm{i}k)$, the generalized aerodynamic matrix can be approximated using the damping perturbation method (Chen, 2000) as

$$\mathbf{Q}(p) \approx \mathbf{Q}(\mathrm{i}k) + g\mathbf{Q}'(\mathrm{i}k), \tag{21}$$

where

$$\mathbf{Q}'(\mathrm{i}k) = \frac{\mathrm{d}\mathbf{Q}(\mathrm{i}k)}{\mathrm{d}(\mathrm{i}k)}. \tag{22}$$

The matrix $\mathbf{Q}'(\mathrm{i}k)$ can be computed from $\mathbf{Q}(\mathrm{i}k)$ using finite difference schemes and for a better approximation, higher order derivatives can be used in Eq. (21). Therefore, even harmonic aerodynamic codes can be easily modified to implement the damping perturbation method for the calculation of the matrix $\mathbf{Q}(p)$.

3. Results and discussion

To examine the proposed technique, three typical examples are used for aeroelastic instability analysis. The first example is a 2-D typical wing section model in incompressible flow. The second example is the Goland wing and the other example is a 2-D panel in supersonic flow. The PP method is employed along with the P and PK methods and the results are compared in terms of accuracy and computational performance. The performed studies demonstrate that the present PP method can be successfully used with the aerodynamics models in the Laplace domain.

3.1. Example 1: 2-D typical wing section

The typical section, as shown in Fig. 1, is a simple spring-restrained model which represents a large aspect ratio wing where the translational and torsional springs reflect the wing structural bending and torsional stiffness, respectively. The structural mass and stiffness matrices can be written as

$$\mathbf{M} = \begin{bmatrix} m & S_\alpha \\ S_\alpha & I_\alpha \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} K_h & 0 \\ 0 & K_\alpha \end{bmatrix}, \tag{23}$$

where m, S_α and I_α are the section mass, static moment and the moment of inertia about the elastic axis, and K_h and K_α are the stiffness coefficients for plunging and pitching motion, respectively. The aerodynamic force vector in the Laplace domain can be written as

$$\mathbf{f}_a = \begin{bmatrix} \tilde{L} \\ -\tilde{M} \end{bmatrix}, \tag{24}$$

where $\tilde{L}$ and $\tilde{M}$ are the Laplace transform of the lift force and pitching moment about the elastic axis. Using the unsteady aerodynamic theory for a 2-D flat plate in incompressible potential flow, the aerodynamic matrix $\mathbf{Q}(p)$ can be written as follows (Bisplinghoff and Ashley, 1972):

$$\mathbf{Q}(p) = 2\pi([\mathbf{M}_1 + \bar{\phi}\mathbf{M}_2]p^2 + [\mathbf{C}_1 + \bar{\phi}\mathbf{C}_2]p), \tag{25}$$

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where $\bar{\phi}$ is the Laplace transform of the Wagner function, which describes the circulatory lift built-up after a sudden change of angle of attack, and is given by (Bisplinghoff and Ashley, 1972)

$$\bar{\phi} = \frac{K_1(p)}{p[K_0(p) + K_1(p)]}, \tag{26}$$

where K_n denotes the modified Bessel function of the second kind and order n.

The rational function approximation method is another aerodynamic model which represents the unsteady aerodynamic loads in terms of several aerodynamic lags (Dowell et al., 1996). This approximation method for two aerodynamic lags can be obtained using Eq. (25) with the following approximation for $\bar{\phi}$:

$$\bar{\phi} \simeq \frac{1}{p} - \frac{0.335}{p+0.3} - \frac{0.165}{p+0.0455}. \tag{27}$$

The finite-state, induced-flow theory of Peters et al. (1995) is another useful unsteady aerodynamics model for a typical section. This model gives a very good approximation of the unsteady aerodynamic loads in state-space form and thus it can be easily used with the P method for aeroelastic stability analysis.

Theodorsen (1935) unsteady aerodynamic theory is a convenient model for harmonic oscillations of a typical section and can be used as an approximate model for the case of motions with low rate of decay. Theodorsen's function is expressed with Hankel functions of the second kind in terms of the reduced frequency and thus is used with the K or PK methods.

Table 1 shows the structural parameters of the typical section for two example cases. To examine the proposed algorithm, the aeroelastic stability of both examples is investigated using the PK method with Theodorsen aerodynamics as well as the P method with Peters' finite-state model with six induced inflow states. Furthermore, the PP method is employed with the Wagner model and the rational function approximation method in the Laplace domain.

Figs. 2 and 3 show the results for the stability analysis for Case 1 and Figs. 4 and 5 illustrate the same results for Case 2. The results show that for Case 1, the PK results are very close to the P and PP results, as long as the damping is

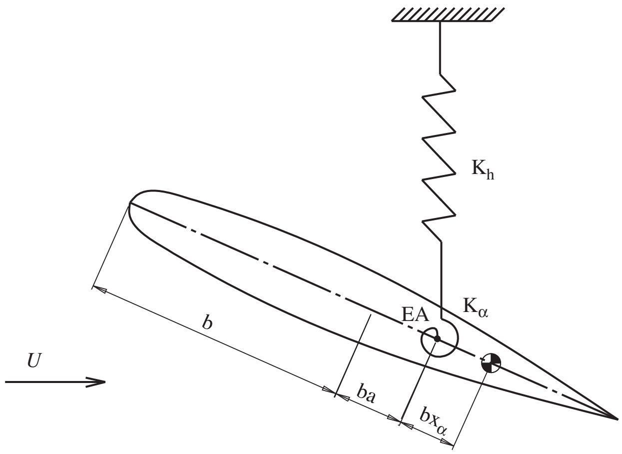

Fig. 1. 2-D typical section in incompressible flow.

Table 1
Dimensionless parameters of the typical section for two example cases.

Parameter	Description	Case 1	Case 2
$\sigma = \omega_\theta/\omega_h$	Uncoupled bending–torsion frequency ratio	0.4	0.2
a	Elastic axis location parameter	−0.2	−0.2
x_α	Dimensionless static unbalance	0.1	0.3
$\mu = m/\pi\rho b^2$	Mass ratio	20	10
$r_\alpha^2 = I_\alpha/mb^2$	Square of dimensionless radius of gyration	0.24	0.1

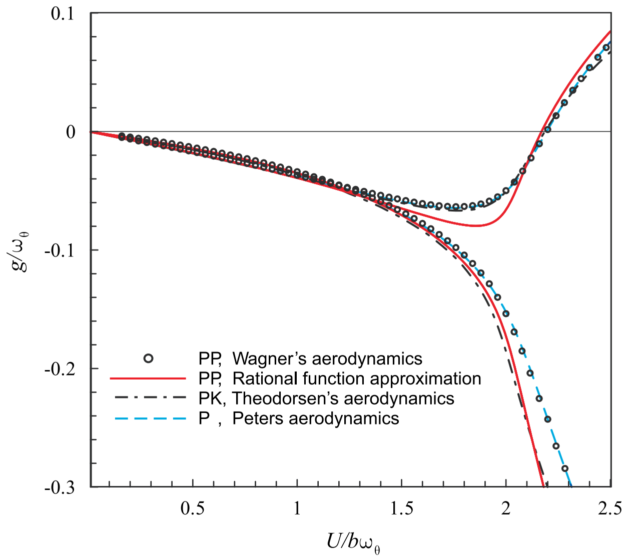

Fig. 2. Damping variation with airspeed for subsonic typical section (Case 1).

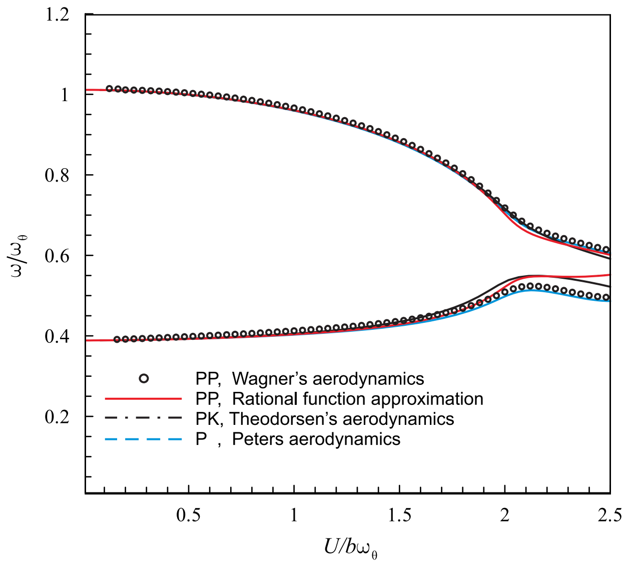

Fig. 3. Frequency variation with airspeed for subsonic typical section (Case 1).

small, as expected. However for Case 2, there is a considerable difference between the subcritical damping values of the highly damped aeroelastic mode that are obtained using the PK method when compared with the P and PP methods.

The results of the PP method with the Wagner aerodynamics model match the P method with the Peters finite state model. The PP results with the rational function approximation compare well with the reference P method and the small inconsistency between the two solutions is indeed due the used aerodynamics model.

3.2. Example 2: slender subsonic wing

In order to examine the application of the PP method for a more applied aeroelastic problem, the aeroelastic stability of the Goland wing is considered. It is a uniform wing made of a beam structure with the geometric and structural properties given in Table 2.

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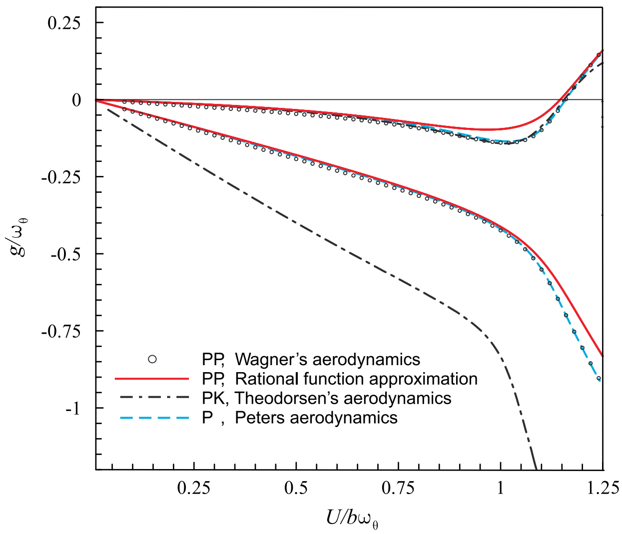

Fig. 4. Damping variation with airspeed for subsonic typical section (Case 2).

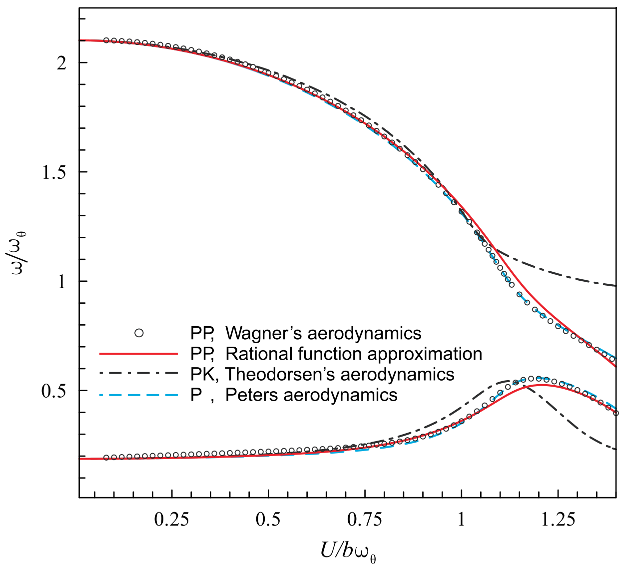

Fig. 5. Frequency variation with airspeed for subsonic typical section (Case 2).

Table 2
The geometric and structural properties of the Goland wing.

Parameter	Description	Value
L	Wing span	6.09 m
b	Semi-reference-chord	0.9144 m
a	Elastic axis location parameter	-0.2
x_α	Dimensionless static unbalance	0.33
r_α^2	Square of dimensionless radius of gyration	0.25
m	Mass per length	35.719 kg/m
EI	Span-wise bending stiffness	9.77×10^6 N m^2
GJ	Span-wise torsion stiffness	0.988×10^6 N m^2

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Using the modal analysis technique and the Lagrange equations along with the virtual work principle the structural matrices are derived as follows (Bisplinghoff et al., 1996):

$$\mathbf{M} = m\begin{bmatrix} [\mathbf{A}_{m,n}] & bx_\alpha[\mathbf{S}_{m,n}] \\ bx_\alpha[\mathbf{S}_{m,n}] & b^2 r_\alpha^2[\mathbf{I}_{m,n}] \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} GJ[\mathbf{B}_{m,n}] & \mathbf{0} \\ \mathbf{0} & EI[\mathbf{T}_{m,n}] \end{bmatrix}, \tag{28}$$

where

$$\mathbf{A}_{m,n} = \int_0^L w_m(x)\theta_n(x)\,\mathrm{d}x, \quad \mathbf{T}_{m,n} = \int_0^L w''_m(x)w''_n(x)\,\mathrm{d}x, \quad \mathbf{B}_{m,n} = \int_0^L \theta''_m(x)\theta''_n(x)\,\mathrm{d}x, \tag{29}$$

in which $w_m(x)$ and $\theta_m(x)$ stand for the bending and torsion mode shapes of the beam, respectively. The generalized aerodynamic force corresponding to the bending and torsion modes are respectively given by

$$\mathbf{f}_{a_{(w_m)}} = \int_0^L w_m(x)\tilde{L}\,\mathrm{d}x, \quad \mathbf{f}_{a_{(\theta_m)}} = \int_0^L \theta_m(x)\tilde{M}\,\mathrm{d}x. \tag{30, 31}$$

Using Eqs. (30) and (31), the generalized aerodynamic matrix $\mathbf{Q}(p)$ can be written in the form of Eq. (25).

Figs. 6 and 7 show the results for the aeroelastic stability analysis of the Goland wing using two bending modes and two torsion modes. The results show the variation of aeroelastic damping and frequency versus the air speed at sea level. The results are obtained using the PK method with Theodorsen's aerodynamic model as well as the PP method with Wagner's and rational function approximation aerodynamics models. Figs. 6 and 7 show that the subcritical damping and frequency that are obtained using the PP method are approximately the same as those of the PK method for the third and fourth aeroelastic modes. However the difference between the results for the first and second modes is remarkable.

3.3. Example 3: 2-D plate in supersonic flow

Fig. 8 shows a simply supported 2-D flat plate in supersonic flow. Assuming that the plate is thin and free from in-plane stress, and using the modal analysis technique, the diagonal structural mass $\mathbf{M}$ and stiffness $\mathbf{K}$ matrices can be written as (Soltani et al., 2003)

$$\mathbf{M}_{n,n} = \frac{\rho_s h}{2}, \quad \mathbf{K}_{n,n} = \frac{Eh^3}{24(1-v^2)}\left(\frac{\pi n}{c}\right)^4, \tag{32}$$

where h is the plate thickness, ρ_s is the material density, v is the Poisson ratio of the material, and E is Young's modulus of elasticity. Based on the unsteady potential flow theory, the generalized aerodynamic matrix can be expressed

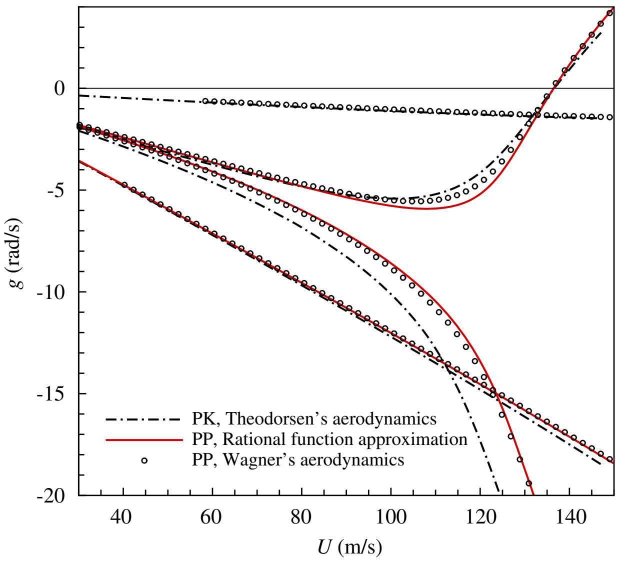

Fig. 6. Damping variation with airspeed for the Goland wing.

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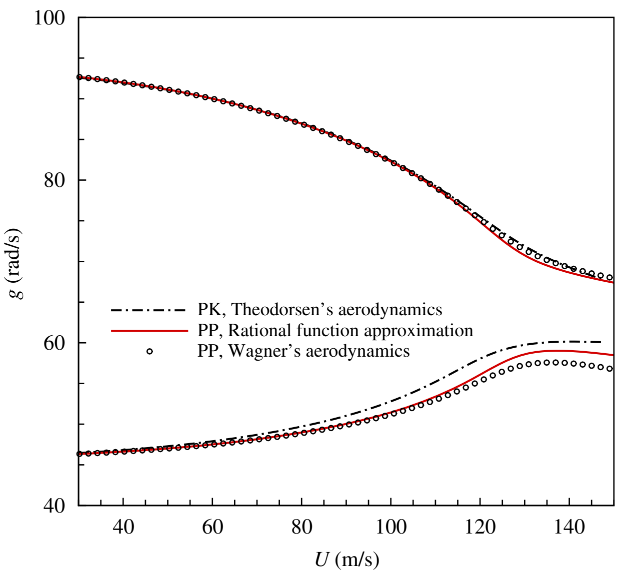

Fig. 7. Frequency variation with airspeed for the Goland wing.

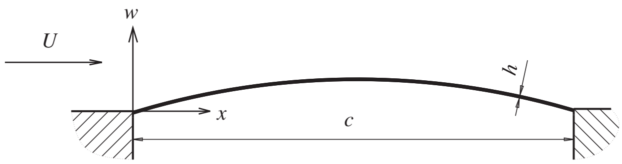

Fig. 8. 2-D panel in supersonic flow.

as (Soltani et al., 2003)

$$\mathbf{Q}(p) = \mathbf{M}_a p^2 + \mathbf{C}_a p + \mathbf{K}_a, \tag{33}$$

in which $\mathbf{M}_a$, $\mathbf{C}_a$ and $\mathbf{K}_a$ are the aerodynamic mass, damping and stiffness matrices, respectively, and can be written as

$$\mathbf{M}_{a_{m,n}} = \frac{1}{2c\beta^5} R_{m,n}, \tag{34}$$

$$\mathbf{C}_{a_{m,n}} = \begin{cases} \dfrac{1}{c\beta}\dfrac{M^2-2}{M^2-1}, & m = n, \\ 0, & m \neq n, \end{cases} \tag{35}$$

$$\mathbf{K}_{a_{m,n}} = \begin{cases} 0, & m = n, \\ \dfrac{2mn(1-(-1)^{m+n})}{c\beta(m^2-n^2)}, & m \neq n, \end{cases} \tag{36}$$

in which M is the Mach number and $\beta = \sqrt{M^2-1}$, and

$$R_{m,n} = \begin{cases} \int_0^1 f(\bar{\xi}) \dfrac{n\sin(m\pi\bar{\xi}) + (-1)^{m+n} m \sin(n\pi\bar{\xi})}{\pi(n^2-m^2)} \mathrm{d}\bar{\xi}, & m \neq n, \\ \int_0^1 f(\bar{\xi}) \left(\dfrac{\cos(m\pi\bar{\xi})}{2} + \dfrac{\sin(m\pi\bar{\xi})}{2m\pi} - \dfrac{\bar{\xi}\cos(m\pi\bar{\xi})}{2} \right) \mathrm{d}\bar{\xi}, & m = n, \end{cases} \tag{37}$$

where

$$f(\bar{\xi}) = \exp\left(\frac{-M^2\bar{\xi}}{\beta^2}p\right)\left[\left(\frac{M^2}{2}+1\right)K_0\left(\frac{M\bar{\xi}}{\beta^2}p\right) - 2MK_1\left(\frac{M\bar{\xi}}{\beta^2}p\right) - \left(\frac{M^2}{2}\right)K_2\left(\frac{M\bar{\xi}}{\beta^2}p\right)\right]. \tag{38}$$

By eliminating the aerodynamic mass matrix from Eq. (33) one can obtain the quasi-steady aerodynamics model that is widely used for supersonic panel flutter analysis and is called the piston theory. The quasi-steady aerodynamics model represents the aerodynamic loads as polynomial expressions in terms of p. Thus, for the aeroelastic stability analysis, this approximation is required so that one can use the P method, while the complete unsteady aerodynamics model requires an iterative procedure such as the proposed PP method.

Consider a flat plate with geometric and material properties given in Table 3.

The sound speed is 340 m/s and the air density is 1 kg/m^3. Using five structural modes, the aeroelastic stability of the plate is investigated using both the P and PP methods. Fig. 9 shows the damping values of the aeroelastic modes versus the Mach number and Fig. 10 shows the root-locus of the aeroelastic system. The graphs show slight difference between the results of the P and PP methods for the low Mach numbers but for the high Mach numbers the results match well. This is due to the influence of the aerodynamics mass matrix quickly vanishing as the Mach number increases.

3.4. Computational performance

The computational performance of the proposed PP method can be addressed in comparison with the PK method, since both are iterative in nature. To examine the computational costs of using the PP method over the PK method, the average number of evaluations of the aerodynamic matrix per velocity point is considered as the comparison index.

For this purpose, numerous test cases with random parameters and 50 velocity points are chosen from each of the previous three examples. The results show that the PP method can predict true damping values in subcritical conditions, while the average number of aerodynamic matrix evaluations of the PP method is approximately only 3–17% higher than the PK method (see Table 4). This increase is attributed to calculations with the complex numbers.

Table 3
The geometric and material properties of the flat plate.

Parameter	Description	Value
h	Thickness	2 mm
c	Chord	300 mm
ρ_s	Density	2800 kg/m^3
E	Young's modulus	70 GPa
ν	Poisson ratio	0.3

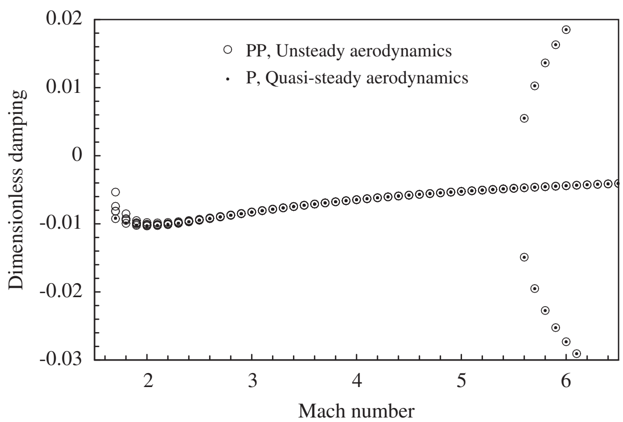

Fig. 9. Damping variation with Mach number for subsonic typical section for supersonic panel.

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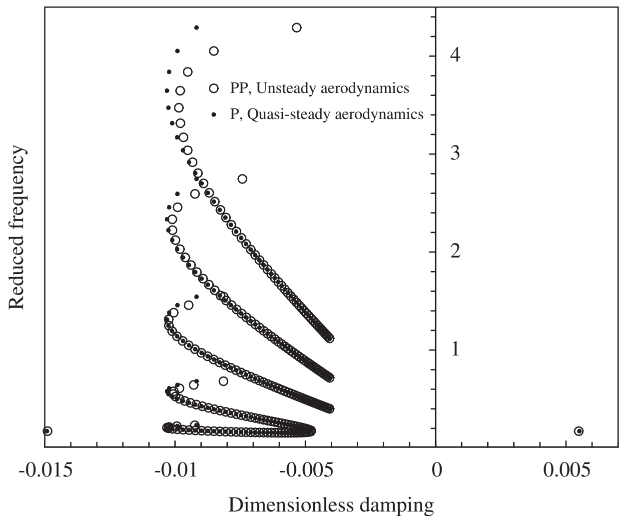

Fig. 10. Plot of root-locus for supersonic panel.

Table 4
The percentage of increase in the number of evaluation of the aerodynamic matrix of the PP method in comparison with the PK method.

Test case	Number of examples	Increment (%)
2-D typical wing section	100	6–15
Slender subsonic wing	50	8–17
2-D plate in supersonic flow	75	3–11

4. Conclusion

A new approach was presented to accurately evaluate aeroelastic system eigenvalues when the aerodynamic theory is in the Laplace domain. For this purpose, a matrix iterative procedure was introduced to determine the subcritical damping and frequency values of the aeroelastic modes. The proposed method was used to determine the aeroelastic stability margins of some examples in the subsonic and supersonic flow regimes. The new method provided the subcritical damping and frequency values of the aeroelastic modes that are in good agreement with the P method. Also the obtained aeroelastic instability conditions absolutely agree with the results of the other classical methods. The computational performance of the present algorithm is similar to the PK method and its major advantage over the PK method is the increased accuracy of the subcritical damping and frequency of the aeroelastic modes.

References

Bisplinghoff, R.L., Ashley, H., 1972. Principles of Aeroelasticity. Dover, New York.

Bisplinghoff, R.L., Ashley, H., Halfman, R.L., 1996. Aeroelasticity. Courier Dover Publications.

Chen, P.C., 2000. Damping perturbation method for flutter solution: the g-method. AIAA Journal 38, 1519–1524.

Dowell, E.H., Crawley, E.F., Curtiss, H.C., Peters, D.A., Scanlan, R.H., Sisto, F., 1996. A Modern Course in Aeroelasticity. Kluwer Academic Publishers, Netherlands.

Edwards, J.W., Ashley, H., Breakwell, J.V., 1979. Unsteady aerodynamic modeling for arbitrary motions. AIAA Journal 17, 365–374.

Esfahanian, V., Behbahani-nejad, M., 2002. Reduced order modeling of unsteady flows about complex configurations using the boundary element method. ASME Journal of Fluids Engineering 124 (4), 988–993.

Hassig, H.J., 1971. An approximate true damping solution of the flutter equation by determinant iteration. Journal of Aircraft 8, 885–889.

Hodges, D.H., Pierce, G.A., 2002. Introduction to Structural Dynamics and Aeroelasticity. Cambridge University Press, Cambridge.

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Patil, M.J., Yurkovich, R.N., Hodges, D.H., 2004. Incorrectness of the k method for flutter calculations. Journal of Aircraft 41 (2), 402–405.

Peters, D.A., Karunamoorthy, S., Cao, W.M., 1995. Finite state induced flow models; part I: two-dimensional thin airfoil. Journal of Aircraft 32 (2), 313–322.

Rodden, W.P., Harder, W.P., Bellinger E.D., 1979. Aeroelastic addition to NASTRAN aeroelastic analysis. NASA CR 3094.

Rodden, W.P., Johnson, E.H., 1994. User's guide of MSC/NASTRAN aeroelastic analysis. MSC/NASTRAN V68.

Soltani, N., Esfahanian, V., Haddadpour, H., Behbahani-Nejad, M., 2004. Unsteady supersonic aerodynamics based on BEM including thickness effects in aeroelastic analysis. Journal of Fluids and Structures 19, 801–813.

Soltani, N., Esfahanian, V., Haddadpour, H., 2003. Analytical prediction of panel flutter using unsteady potential flow. Journal of Aircraft 40, 805–807.

Theodorsen, T., 1935. General theory of aerodynamic instability and the mechanism of flutter. NACA TR 496.

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