# Swim Blater and Archimedes' Principle

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Fish has several methods for buoyancy in water, this methods can be classified as static lift or non-static lift. The difference between this two is the mechanisms fish use to move vertically in water. In non-static lift fish use their fins to generate drag forces which make them move up or down in water. In non-static lift fish do not use mechanical aspects to move, instead they use their density as a parameter to modify the balance between its weight and the buoyant force described by archimedes' principle. In this research we consider the static lift method use by fish having swim bladders. Using swim bladder fish may change its density to be greater than, equal to or lower than water's density so they may move vertically.

## I. INTRODUCTION

Experience shows an imersed object in water may have motion vertically: it can float up, remain at its initial position or sink down. In the next sections we cover this phenomenology **(II)** and explain it in terms of archimedes' principle **(III)**; then using a simplified model, we show how this behavior is related to the object's density compared with the density of water. Finally **(IV)** we apply this model to fish having swim bladder which use in a very efficient way this phenomenology as a static lift method for buoyancy.

## II. PHENOMENOLOGY

When an object is dropped in a swimming pool, there are three possibilities :

1. The object floats in the pool surface.
2. The object dips into the water and stops at some point without reaching the bottom.
3. The object dips until reaching the pool's bottom.

When the object is dropped, it gains kinetic energy before dipping into the pool. In the first case, the motion stops and the object begins to float until reaching the surface; then because of Newton's second law: **there is a force** pulling up the object which is greater than the force of gravity and is balanced when the object is partially inmersed. In the second case when the energy is dissipated because of water friction forces, the object stops and gets stuck at the reached depth; then according to Newton's second law, **there is a force** with the same magnitude and opposed to the force of gravity. Finally, in case three, when the object dips into the water it begins to slow its motion (until it is stopped by the bottom), then again because of Newton's second law : **there is a force** which partially balances the force of gravity.

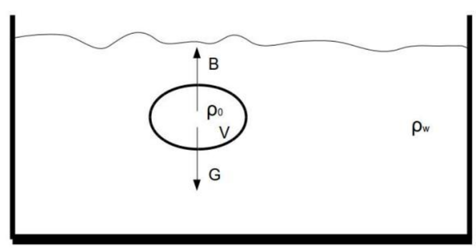
The unknown force is the buoyancy force which was first described by Archimedes of Syracuse about 200 B.C. Archimedes captured this phenomena in the well known Archimedes' principle which is described in the next section.

## III. ARCHIMEDES' PRINCIPLE

Archimedes' principle can be found in several general physics textbooks.The definition given in [2] is considered and it states:

*"A body wholly or partially immersed in a fluid will be buoyed up by a force equal to the weight of the fluid that the body displaces"*

Let's consider an impermeable body of volume $V$ and and mass $m$ with an uniform and constant mass density $\rho_0$ inmersed in a fluid of density $\rho_\omega$ as shown bellow.




Acording to Archimedes' principle and Newton's second law if the force buoying up the body is denoted by $B$ and the weight is $G$, then the the dynamics of this body is described by the next equation,

$$B - G = ma, \qquad (3.1)$$

the phenomenology described in the last session is fully contained in this formula. Indeed, first if force of gravity is smaller than the force of buoyancy then acceleration is positive and the body is pulled up. The force of gravity may be equal to the force of buoyancy and then the body remains at rest, inmersed. Finally if the force of gravity is bigger than the force of buoyancy then the body begins to dip down until reaching the botton.

Actually there is more information in equation (3.1). Archimedes principle for the buoyancy force $B$ and the properties of this body lead to,

$$\begin{aligned} m &= \rho_0 V \\ B &= \rho_\omega V g \\ G &= \rho_0 V g, \end{aligned} \tag{3.2}$$

doing some algebra on equation (3.1), relations (3.2) lead to a formula for accelerarion which depends only on densities; this formula is given by,

$$a = \left( \frac{\rho_\omega - \rho_0}{\rho_0} \right) g. \tag{3.3}$$

Taking a look to this relation, the physics behind it is again the same discussed previously, just with the most accurate result that motion depends on densities. Indeed, first if body's density is smaller than fluid's density then acceleration is positive and the body is pulled up until it is partially inmersed in the fluid. Densities may be equal, then forces are balanced and the body remains at rest, inmersed. Finally if body's density is bigger than fluid's density then the body weight is bigger than buoyancy force and the body goes down until it reaches the botton.

Densities must be handle with care; thats why impermeability was requested since if the body allows the fluid to go inside without changing its shape, its density in average will be bigger. Uniform and constant density avoids the cases in which there is one or more cavities inside the body, in this case the buoyancy does not change but this fact inplies $m \neq \rho_0 V$.

What will happen if instead the object in the picture above we consider fish, indeed, fish!!.

## IV. FISH AND THEIR SWIM BLADDER

When immersed in water, fish experience the downward pull of gravity and the upward push of buoyancy. Fish have developed key adaptations allowing them to move up and down in the water. There are several approaches to do this [1], in this section we will consider fish with swim bladder as a mechanism for buoyancy only. Those fish have the ability modify their own weight making themselves lighter to float up or heavier to sink down. They accomplish this by changing the amount of gas in an inner bladder without changing their volume. This bladder is known as **swim bladder**. Let's consider a simple model as shown in the picture bellow where fish have a fixed volume $V$ and a density $\rho_p$ equiped with a swim bladder which can expand or compress changing its mass;

In this model fish have a non constant density since its mass changes depending on the gas input/output in their swim bladder. Then the mass of fish can be written as the sum of a mass $m_0$, related to the mass of its body without taking into account the gas inside their swim bladder, and the mass $m_b$ inside their swim bladder. Mass $m_0$ can be regarded as fixed since it doesn't change in the scale of time the mass $m_b$ varies, then for the density of fish we have:

$$\begin{aligned} \rho_p &= \frac{m_0 \pm m_b}{V} \\ &= \frac{m_0}{V} \pm \frac{m_b}{V} \\ \rho_p &= \rho_0 \pm \rho_b. \end{aligned} \tag{4.1}$$

Equation (4.1) describes the mechanism allowing fish to modify their density in terms of the swim bladder, and then as we saw previously, this process might result in fish floating up, being at rest at a given depth or sinking down without additional efforts. We can go even further if we in addition consider equation (3.3),

$$\begin{aligned} a &= \left( \frac{\rho_\omega}{\rho_p} - 1 \right) g, \\ &= \left( \frac{\rho_\omega}{\rho_0 \pm \rho_b} - 1 \right) g, \\ &= \left( \frac{1}{\frac{\rho_0}{\rho_\omega} \pm \frac{\rho_b}{\rho_\omega}} - 1 \right) g. \end{aligned} \tag{4.2}$$

Usually the density $\rho_0$ of fish is similar to the density of water [1], then $\rho_\omega \simeq \rho_0$ and equation (4.2) can be rewritten as follows:

$$a = \left( \frac{1}{1 \pm \frac{\rho_b}{\rho_0}} - 1 \right) g. \tag{4.3}$$

This last equation can be reduced if we use the next formula:

$$\frac{1}{1 \pm x} = 1 - (\pm x), \quad \text{for } 0 < x << 1 \;, \tag{4.4}$$

since:

$$\frac{\rho_b}{\rho_0} = \frac{m_b}{m_0} << 1.$$

Then equation (4.3) is equivalent to:

$$a = -\left(\pm\frac{\rho_b}{\rho_0}g\right). \tag{4.5}$$

The model we are considering show us the phenomelogy discussed in section **(II)** is a direct consequence of the mass amount in the swim bladder, which is essentially what happens in reality [1].

## V. CONCLUSIONS

Many fish have as a mechanism for buoyancy the mass input/output in their swim baldder. Adding mass from a critical value give this kind of fish the ability to sink down, mantaining mass in this critical value keep them floating at a fixed depth and lowering the mass from this critical value is equivalent to add a negative mass amount which makes fish become less dense than water so they begin to float up; this is a static lift method since fish don't have to use their fins or drag forces from currents to generate vertical motion in water. Then buoyancy for fish with swim bladder can be modelled with just one parameter as shown in section **(IV)**, namely, $\rho_b$ which is measured from a critical value $\rho_0$ as reference so it can be possitive, zero or negative meaning this that fish are denser than water, have the same density of water and are less dense than water, respectively.

[1] Quentin Bone and Richard Moore. *Biology of Fishes*. Taylor & Francis, 2008.

[2] J. Walker D. Halliday, R. Resnick. *Fundamentals of Physics*. John Wiley & Sons, Inc, 1993.