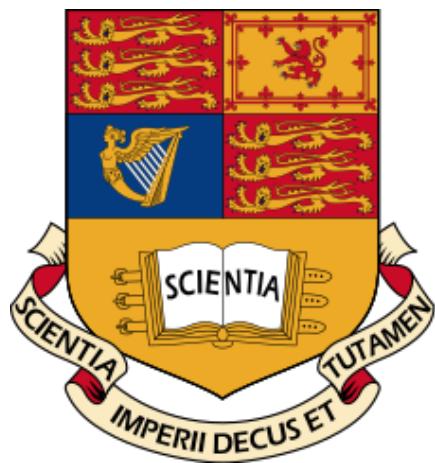


Imperial College London

Department of Electrical and Electronic Engineering

Final Year Project Report 2021



Project Title: State-decomposition techniques for sample efficient Deep Reinforcement Learning

Student: Edoardo David Santi

CID: 01373388

Course: MEng Electrical & Electronic Engineering

Project Supervisor: Prof. Kin K. Leung

Second Marker: Dr Tania Stathaki

Acknowledgments

I would like to express my gratitude to my supervisor Prof. Kin K. Leung for the continuous support and availability throughout the duration of this project and my excitement about the prospect of working with him as my PhD supervisor in the future. I would also like to express my gratitude towards my parents for their continuous support and encouragement throughout my studies.

Abstract

Your abstract

Contents

1	Introduction	2
1.1	Motivation	2
1.2	Project Definition	2
1.3	Structure of the report	2
2	Background	3
2.1	Reinforcement learning	3
2.2	Neural networks	6
2.3	Q-learning and Deep Q-learning	6
2.4	Problem of large state-spaces	7
3	State-Decomposition method	9
3.1	State-Decomposition method	9
3.1.1	Decomposing state trans. matrix	11
4	Design and testing of the proposed decomposition technique	13
4.1	Tools	13
4.1.1	Obtaining the state trans. matrix	14
4.2	The need for a suitable environment	14
4.3	Implementation of the DQN agents	17
4.4	Benchmark DQN agent	17
4.5	Ensuring fair comparison	18
5	Results	19
5.1	Evaluation method	19
5.2	Effect of different state representations on the performance	19
5.3	First approach	20
5.4	Initial results with Taxi2 and original state decomposition algorithm	20
5.5	Effect of varying the neural network's size	20
5.6	A simple approach to decomposition: 'DeltaSwitch'	21
5.7	Effect of using reduced encoding	24
5.8	Samples plots	25
5.9	Effect of wall / no wall	25
5.10	Effect of decomposing by destination or by position	26
5.11	A different approach based on transfer learning	27
5.12	Techniques to learn the 'switch'	30
5.13	Can have separate subsection for things that didn't work	32
6	Applicability of the proposed technique	34
7	Conclusions and Further Work	37
7.1	Extensions	37
7.1.1	Applications	39

A Code

43

Chapter 1

Introduction

1.1 Motivation

Reinforcement learning (RL) has been a very active research area in the last few years, achieving impressive feats in a wide range of applications. Some advantages of reinforcement learning algorithms are great flexibility, the capability of solving both very low and high-level problems [26] and not requiring a model of the environment in which they are used. RL has famously been used to solve games, such DeepMind's AlphaGo [24], a program that in 2016 beat the top-ranked Go player in the world. Other successful applications are in healthcare [34], robotics [2], autonomous driving [7] and natural language processing [21]. My project supervisor Prof. K. K. Leung has also been working on applications of RL in software-defined networks (SDNs), such as for the control of the placement of services [35] and the synchronisation of controllers in distributed SDNs [36].

Dealing with very large systems is one of the main problems encountered in the application of reinforcement learning in practical environments. RL is based on learning through interaction with the environment. As the size of the system increases, so does the complexity of the problem and it becomes increasingly difficult to learn how to interact with the environment, making the learning process longer. In practical scenarios, it might not be feasible to train an RL agent for long enough for it to be a feasible solution, as this might incur excessive costs or, in the case that the behaviour of the environment changes over time, the algorithm might not be able to learn fast enough to adapt to the changing environment.

1.2 Project Definition

The goal of this project is to develop the state-decomposition method proposed by my supervisor Prof. K. K. Leung. This aims to alleviate the problem of large state spaces of the Markov Decision Processes (MDPs) that underlay many practical environments. This new method leverages the characteristic of many control problems of being composed of many almost independent sub-problems that only seldom interact with each other. The method first identifies the sub-problems, then it trains separate RL agents for each of them. These are later combined to form a global agent that controls the whole system and takes into account their interactions.

While the high-level methodology is defined, the specifics of the algorithms are not. This project involves conducting research on the development of the state-decomposition method and to test it and benchmark it against mainstream algorithms to determine if it achieves improved performance.

1.3 Structure of the report

Chapter 2

Background

2.1 Reinforcement learning

"The reinforcement learning problem is meant to be a straightforward framing of the problem of learning from interaction to achieve a goal" [26].

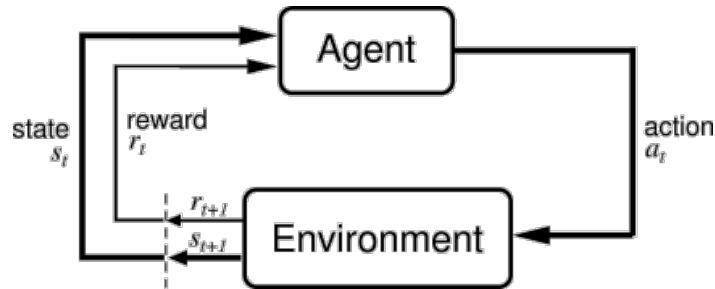


Figure 2.1: Interaction of the agent with the environment during time step t . Figure taken from [26]

The reinforcement learning *agent* continuously interacts with the *environment*. This interaction can be modelled as a sequence of time steps $t = 0, 1, 2, 3, \dots$ ¹. At each step t , the environment is represented by its *state* $S_t \in \mathcal{S}$, where \mathcal{S} is the set of all the possible states. Taking the *state* as input, the agent outputs an *action* $a_t \in \mathcal{A}$, where \mathcal{A} is the set of all the possible actions. In the next time step $t + 1$, the environment transitions to a new *state*² S_{t+1} and emits a *reward* $R_{t+1} \in \mathcal{R} \subset \mathbb{R}$, which the agent uses as feedback information to "learn" how to interact with the environment. Each time step can then be represented by the $\langle S_t, A_t, R_{t+1}, S_{t+1} \rangle$ tuple.

The agent implements a mapping from the possible states to the probabilities of choosing the different actions. This mapping is called *policy* and at time step t , it adopts symbol π_t . The probability of choosing action $A_t = a$ given the current state $S_t = s$ is denoted by $\pi_t(a|s)$. The goal of reinforcement learning is to learn the optimal policy, which is the policy that maximises the rewards, through repeated interaction with the environment. In general, we aim to maximise a function of the rewards at all time steps rather than the immediate reward, so the optimal policy doesn't necessarily maximise immediate rewards. The simplest case is to maximise

$$G_t = R_{t+1} + R_{t+2} + R_{t+3} + \dots + R_T \quad (2.1)$$

where T is a final time step. This approach can be used when the agent-environment interaction has an end, in which case a sequence of interactions from the initial time to time T can be called

¹The interaction with the environment can also be modelled in continuous time, however this is outside the scope of this project.

²Note that it is possible that $S_{t+1} = S_t$

an *episode*. In other cases, the interactions can continue indefinitely, thus this reward function is not feasible as $T = \infty$ and G_t could be infinite. To fix this, we apply a *discount rate* γ , with $|\gamma| < 1$, to future rewards:

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \quad (2.2)$$

This allows G_t to have a finite value in continuing tasks, which are those tasks that don't have finite time episodes.

In general, the dynamics of the environment could depend on the whole history of states, rewards and actions over an episode. However, we aim to have an agent which can choose the optimal action at each time step based only on the current state. Thus the knowledge of the current state and action should give the maximum possible amount of information about the dynamics of the environment. This is true if knowledge of the history of states, rewards and actions in the previous time steps does not provide any additional information about the dynamics of the environment than knowing the current state. States that follow this property are said to follow the *Markov property*. More formally, if the states follow this property

$$\Pr\{R_{t+1} = r, S_{t+1} = s' | S_0, A_0, R_1, \dots, S_{t-1}, A_{t-1}, R_t, S_t, A_t\} \quad (2.3)$$

$$= \Pr\{R_{t+1} = r, S_{t+1} = s' | S_t, A_t\} \quad (2.4)$$

$$= p(s', r | s, a) \quad (2.5)$$

This is the equation that governs Markov Decision Processes (MDPs) and hence a reinforcement learning task formulated with states that follow the Markov property is an MDP.

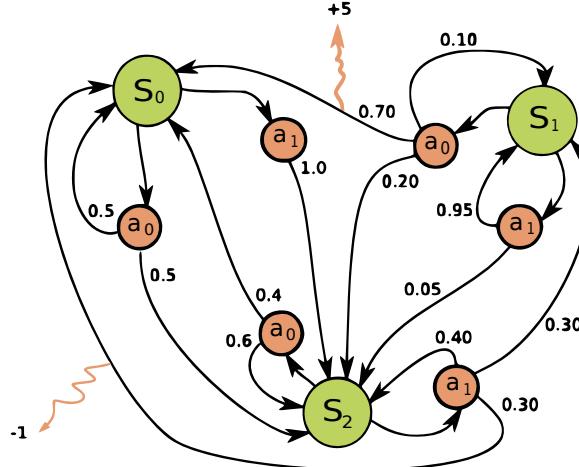


Figure 2.2: Graph representation of a Markov Decision Process. Note the rewards (zigzag arrows) that are released in certain transitions. [3]

Most reinforcement learning algorithms involve the estimation of the *value functions*, which are functions of states or state-action pairs. A value function represents how good it is to be in a certain state or to perform a certain action in a certain state, in terms of the expected return G_t and given that we are following a certain policy. The action-value function is defined as

$$q_{\pi}(s, a) = \mathbb{E}_{\pi}[G_t | S_t = s, A_t = a] = \mathbb{E}_{\pi} \left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} | S_t = s, A_t = a \right] \quad (2.6)$$

Similarly, the state-value function is given by

$$v_{\pi}(s) = \mathbb{E}_{\pi}[G_t | S_t = s] \quad (2.7)$$

$$= \mathbb{E}_\pi \left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} | S_t = s \right] \quad (2.8)$$

$$= \sum_a \pi(a|s) \mathbb{E}_\pi \left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} | S_t = s, A_t = a \right] \quad (2.9)$$

$$= \sum_a \pi(a|s) q_\pi(s, a) \quad (2.10)$$

It is useful to specify the value functions in recursive format, known as the Bellman equations. This gives:

$$v_\pi(s) = \mathbb{E}_\pi[G_t | S_t = s] \quad (2.11)$$

$$= \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1} | S_t = s] \quad (2.12)$$

$$= \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a)[r + v_\pi(s')] \quad (2.13)$$

and

$$q_\pi(s, a) = \mathbb{E}_\pi[G_t | S_t = s, A_t = a] \quad (2.14)$$

$$= \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1} | S_t = s, A_t = a] \quad (2.15)$$

$$= \sum_{s',r} p(s',r|s,a)[r + v_\pi(s')] \quad (2.16)$$

For finite MDPs it can be proved that there always exists an *optimal policy* that maximises the state-values of all states the action-values of all state-action pairs, giving the optimal state-values $v_*(s)$ and $q_*(s, a)$:

$$v_*(s) = \max_\pi v_\pi(s), \quad \forall s \in \mathcal{S} \quad (2.17)$$

$$q_*(s, a) = \max_\pi q_\pi(s, a), \quad \forall s \in \mathcal{S}, \quad \forall a \in \mathcal{A} \quad (2.18)$$

Given full knowledge of the dynamics of the system³ and a fixed policy, we are able to evaluate the optimal state-values and action-values using methods based on dynamic programming [9]. One such method called *iterative policy evaluation*, which adopts the update step based on the Bellman equations given below, can be shown to converge to the state-values:

$$v_{k+1}(s) = \sum_{a \in \mathcal{A}} \pi(a|s) \left[R(s, a) + \gamma \sum_{s' \in \mathcal{S}} P(s, s', a) v_k(s') \right] \quad (2.19)$$

Policy iteration is one possible method of evaluating the state values when the optimal policy is not given, which alternates steps of policy evaluation and updates of the policy. The policy is updated so that in every state the greedy action, meaning the one with the highest action-value, is always picked. This process of solving the MDP when given the model of the environment is known as *planning*.

However, in many applications the model of the environment is unknown. Given a certain policy, it is possible in this case to approximate the state and action values by sampling the interaction with the environment. Monte Carlo methods interact with the environment by following the given policy and approximate each state-value using the returns obtained in each episode after the first or each visit to the state. The update that occurs every time the state occurs for the first time in the episode (or every time, depending on the version of the algorithm) is

$$V(S_t) \leftarrow V(S_t) + \alpha[G_t - V(S_t)] \quad (2.20)$$

³The probabilities of state-transitions and rewards given the current state-action pair

The disadvantage is that state-values cannot be updated until the end of an episode. Temporal Differences methods are another class of methods that solve this problem. Instead of relying on returns (G_t), they approximate them as the sum of the reward and the discounted state-value of the next visited state. The simplest TD algorithm's update step is given by

$$V(S_t) \leftarrow V(S_t) + \alpha(R_{t+1} + \gamma V(S_{t+1}) - V(S_t)) \quad (2.21)$$

There are many applications of reinforcement learning in which state-space and action-spaces are continuous. In that case, the MDP is not finite as there is an infinite number of possible states and actions. This will not be discussed as it is outside of the scope of this project, which only deals with discrete state and action spaces.

2.2 Neural networks

Should I give a brief introduction to neural networks

2.3 Q-learning and Deep Q-learning

SHOULD ADD A BIT OF INFORMATION ABOUT NEURAL NETWORKS

Many RL algorithms deal with the problem of solving an MDP without being given the optimal policy and the dynamics of the system. Q-learning [32] is one of the most noteworthy among these. It is based on Temporal Differences learning as the action-value updates follow a very similar formula. The basic value update in Q-learning is given by:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, a)] \quad (2.22)$$

Differently from the previously described TD algorithm, this operates on action-values instead of state-values. In Q-learning, the value update is done using the term $\max_a Q(S_{t+1}, a)$, which is the action-value of the greedy-action in the next visited state. This is not necessarily the action taken in the following step when visiting S_{t+1} , thus this is an *off-policy* algorithm.

Differently from the Temporal Differences algorithms that aim at evaluating a fixed policy, Q-learning also deals with the control problem of choosing the policy. Q-learning is normally programmed so that its current policy is based on its current action-value estimates. An example is the ϵ -greedy policy, in which at each time step, the greedy action is picked with probability $1 - \epsilon$ and a random action is picked with probability ϵ .

This algorithm is only defined for discrete environments and it is proven to converge to the optimal action-values with a probability of 1, as long as all the states and actions are visited multiple times [31]. This requirement might not be feasible when the number of states is very large and it is impossible when the number of states is infinite such as when the MDP's state-space is continuous. Using a learned Q-function approximator instead of simply storing the action-values in tabular form can solve this issue, as it is possible to learn a mapping function that generalises over the whole state-space without having to visit all the possible states.

Deep Q-Learning (DQN) [17] implements the Q-function approximator using a deep neural network. This was demonstrated to achieve state-of-the-art performance in learning to play Atari games directly from the pixel data [16]. DQN takes advantage of the ability of deep learning to learn from high-dimensional data representations to be able to directly feed large state-spaces into the agent. Deep neural networks rely on the assumption that they are fed independent samples, however in reinforcement learning the samples are correlated as they are generated by a temporal sequence. An

experience replay is used to solve this. The transition samples are saved in memory and a random batch is selected to train the neural network at each time-step, thus removing correlations due to temporal proximity of the samples. This also allows the update to be based on a batch, rather than a single sample, which improves the sample efficiency⁴.

One issue of the Q-learning algorithm is that using the maximum action value of the next visited state in the update step results in a positive bias in the estimation of the action values, which can lead to poor performance in some stochastic environments. Double Q-learning [33] deals with this problem by splitting the samples randomly between two separate sets of action value estimates and by updating the values of each set of estimates using the values of the other, which can be proven to remove the positive bias⁵. The same problem persists in deep Q-learning and a similar solution has been devised in Double Deep Q-Learning [29]. Another version of DQN that achieved improved performance in typical benchmarks such as playing Atari games is Dueling DQN [30], which operates on the concept of expressing action values as a sum of state values and advantages: $Q_\pi(s, a) = V_\pi(s) + A_\pi(s, a)$.

While the original Q-learning algorithm can only be applied to systems with discrete state and action spaces, other algorithms based on Q-learning have been proposed to deal with continuous state and action spaces, such as Wire Fitted Neural Network Q-learning [12].

2.4 Problem of large state-spaces

Very large systems correspond to MDPs with a very large number of states, which can be difficult to solve using reinforcement learning algorithms. The number of possible state-transitions is the square of the number of states, thus the complexity of controlling a system grows very quickly as the size of a system increases, and learning an optimal policy becomes very difficult and often infeasible.

Various approaches have been used to solve this issue. One general approach is to learn representations of the states to improve generalisation to unseen states or to decrease the dimensionality of the states by compression. DQN [16] exploits the ability of deep learning to learn from high-dimensional data and to generalise to solve this issue, though this is often not sufficient. [22] jointly learns state and action embeddings to improve generalisation. In World Models [13], a compressed latent representation of the states is explicitly learned by an autoencoder, which is then fed into the reinforcement learning model instead of the original states. Other methods rely on state aggregation [10], in which states with similar state-transition probabilities and rewards are grouped to form a higher-level state. This technique can also be applied to hierarchical reinforcement learning to define the states of the global learner [6].

Hierarchical reinforcement learning [8] is another approach that reduces the impact of large state spaces. Hierarchical RL provides temporal abstraction by subdividing the main task into sub-tasks. For example, the task of commuting to work can be subdivided into sub-tasks such as opening a door, turning right, walking to the tube station, etc. The agent is composed of hierarchies of sub-agents in which each sub-agent is fed a compressed or reduced state space in input and outputs high-level actions to its "children" sub-agents that perform the lower-level actions required to complete the received higher-level action. Examples of hierarchical reinforcement learning algorithms are based on MAXQ value decomposition [11], Options [27] and Hierarchies of Machines (HAM) [20].

Model-based reinforcement learning [18] has been receiving increasing attention lately. This class of algorithms is at the intersubsection of planning algorithms and model-free reinforcement learning as it approximates the model of the environment during the learning process, which can be used to

⁴How efficiently the samples are used by the algorithm to learn. A more sample efficient algorithm requires fewer interactions with the environment to learn an optimal policy.

⁵It actually introduces a small negative bias, thus tending to underestimate the action values.

more efficiently learn from new samples. Dyna [25] learns a model of the environment and uses it to simulate samples for Q-learning other than those obtained by actual interaction with the environment. AlphaZero [24], which achieved state-of-the-art performance in playing Chess, leverages full knowledge of the model⁶ to perform a Monte-Carlo tree search for every step in the real-world environment. [13] trains an RNN model that predicts the following state at each-time step. Though this class of algorithms does not explicitly tackle the problem of large state-spaces, the improvements in sample efficiency can still solve the issue by allowing to train a model with fewer interactions with the environment.

⁶The model is given at the start rather than learnt.

Chapter 3

State-Decomposition method

3.1 State-Decomposition method

The main topic of this project is the development of the state-decomposition method. This is suited to control problems of environments that are composed of many sub-problems that only seldom interact with each other. Separate RL agents can be trained for these separate sub-problems independently, thus treating them as separate problems. The separate agents are then combined at a later stage to take into account the interactions between the sub-problems.

The first step of the process is to identify the subsystems as groups of states. This is done by examining the state-transition probability matrix. Given a finite MDP, this is defined as the matrix where each entry P_{ij} is the probability of transitioning from state $S_t = i$ to the next state $S_{t+1} = j$:

$$\begin{bmatrix} P(S_{t+1} = s_0 | S_t = s_0) & P(S_{t+1} = s_1 | S_t = s_0) & \dots & P(S_{t+1} = s_n | S_t = s_0) \\ P(S_{t+1} = s_0 | S_t = s_1) & P(S_{t+1} = s_1 | S_t = s_1) & \dots & P(S_{t+1} = s_n | S_t = s_1) \\ \dots & \dots & \dots & \dots \\ P(S_{t+1} = s_0 | S_t = s_n) & P(S_{t+1} = s_1 | S_t = s_n) & \dots & P(S_{t+1} = s_n | S_t = s_n) \end{bmatrix} \quad (3.1)$$

where n is the number of states in \mathcal{S} .

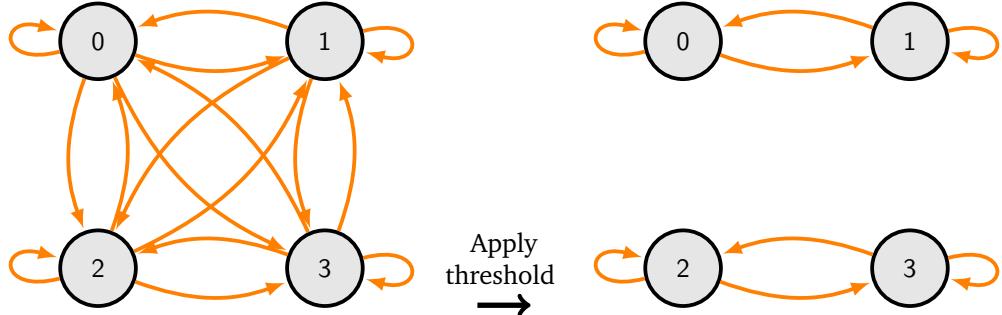


Figure 3.1: Applying a threshold to the state transition matrix and normalising the probabilities can cause the MDP to separate into separate MDPs as it reduces the number of possible transitions.

The state-decomposition method is based on the fact that some state-transition probabilities are negligible compared to others and could be regarded as 0 to obtain a sparse version of the state-transition matrix. Specifically, every element smaller than a threshold ϵ can be set to 0. The probabilities of the matrix can then be normalised so that each row of the matrix has a sum of 1. This step is unnecessary and can be skipped in the implementation; however, it is necessary to obtain a valid MDP from the state-transition matrix after applying the threshold. Following this procedure, the MDP could be formed by multiple independent MDPs with smaller state-spaces, as shown in Figure-3.1. In

this case, by reordering the states it is possible to express the state-transition matrix as a block matrix in the form:

$$\begin{bmatrix} B_1 & 0 & \dots & 0 \\ 0 & B_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & B_n \end{bmatrix} \quad (3.2)$$

where the matrices B_1, B_2, \dots, B_n are the state-transition matrices of the newly formed separate MDPs.

Once the states of the original MDP have been separated into multiple sub-spaces, it is possible to proceed with training the RL agent. In the first stage of training, for each of these sub-spaces, we train a separate DQN agent. To do so, we consider for each sub-space's agent only those state transitions that start and end within the same sub-space. Once the sub-spaces' agents converge¹, these should be combined to form a global agent that also takes into account transitions between the different sub-spaces. This is done by feeding the action values of the sub-spaces' agents, which are the outputs of their neural networks, as inputs of another neural network, as in Figure-3.2. This forms a bigger neural network which is the Q-function approximator of a new global DQN agent, which is then trained using all possible transitions. This will be referred to as the second stage of training.

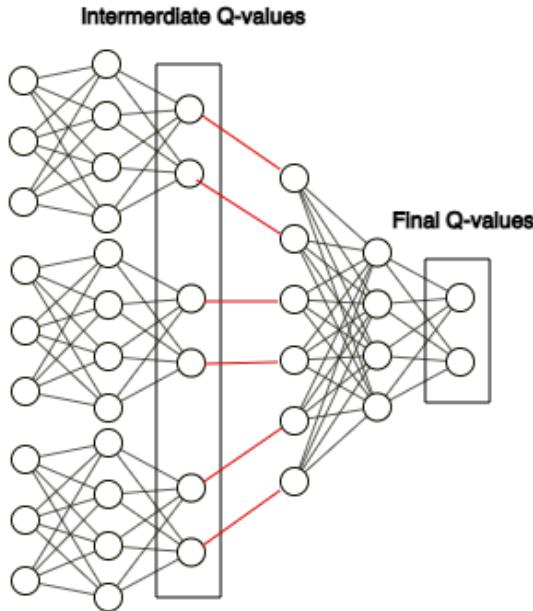


Figure 3.2: The NNs of the decomposed states' agents are merged as input of another NN to form a new bigger NN.

This method operates on the assumption that it would require fewer samples to achieve convergence for RL agents operating on many smaller MDPs than on one single MDP having as the number of states the sum of the number of states of the smaller MDPs. This is because the number of possible transitions in an MDP is the square of the number of states, so by dividing the system into separate independent sub-problems we reduce the complexity of the control problem². By then combining the neural networks as previously explained, we take into account interactions between the sub-problems, allowing us to achieve the optimal policy for the original MDP. For this method to be

¹Or earlier. The time when we should do this is to be researched in this project.

²By decreasing the number of possible transitions, as $\sum_{i=0}^n x_i^2 \leq (\sum_{i=0}^n x_i)^2$

successful, it is assumed that the weights of the neural networks of the sub-spaces' agents are not far from the values they take once the global agent converges in the final stage of training. This is equivalent to saying that the first stage of training provides a very good initialisation of the weights of the network used in the second stage. Although we use DQN agents, this method could be generalised to any agents that use a function approximator to evaluate the action-values given in input the states. This method however wouldn't be as useful if no approximator is being used, and the action-values are simply stored in a table, such as in the original Q-learning algorithm. In this case, applying the state-decomposition would simply ignore the transitions between different sub-spaces without changing the structure of the agent at all in the first stage of training.

3.1.1 Decomposing the state transition matrix

The decomposition into sub-spaces of states given the state-transition table is done using Algorithm-1. The Python code for this is given in the Appendix.

Algorithm 1: State-decomposition algorithm.

```

stateTransMatrix: input 2D array representing the state-transition matrix
subspaces : output data structure representing the decomposed sub-spaces

subspaces = initialise as one subspace for each group;
for  $i \leftarrow 0$  to  $\text{height}(\text{stateTransMatrix}) - 1$  do
    for  $j \leftarrow 0$  to  $\text{width}(\text{stateTransMatrix}) - 1$  do
        if  $\text{stateTransMatrix}[i][j] > \epsilon$  then  $\text{joinSubspacesContaining}(i, j)$ ;
        end
    end
end
```

Sometimes we know the number of sub-spaces that the original MDP should be decomposed into. For example, we might know the characteristics of the system and the number of subsystems it can be considered to have, such as how many sub-networks compose a certain communication network. In such case, the function that returns the decomposed sub-spaces can be run repeatedly while updating the threshold value ϵ at each iteration using binary search, until the MDP is decomposed in the correct

number of sub-spaces. This is given in Algorithm-2 and the Python code is in the Appendix.

Algorithm 2: State-decomposition into given number of sub-spaces

```

stateTransMatrix : input 2D array representing the state-transition matrix
nSubspacesDesired: input integer representing the number of sub-spaces to decompose into
subspaces : output data structure representing the decomposed sub-spaces
maxIterations  $\leftarrow$  50;
changeFactor  $\leftarrow$  10;
 $\epsilon \leftarrow 1.0$ ;
upperBound  $\leftarrow$  NONE;
lowerBound  $\leftarrow$  NONE;
for  $i \leftarrow 1$  to maxIterations do
    if upperBound and lowerBound are defined then
        |  $\epsilon \leftarrow (\text{upperBound} + \text{lowerBound})/2$ ;
    end
    if only upperBound is defined then  $\epsilon \leftarrow \text{upperBound}/\text{changeFactor}$ ;
    if only lowerBound is defined then  $\epsilon \leftarrow \text{lowerBound} * \text{changeFactor}$ ;
    subspaces  $\leftarrow$  stateDecomposition (stateTransMatrix,  $\epsilon$ );
    if nSubspaces(subspaces) == nSubspacesDesired - 1 then break;
    if nSubspaces(subspaces) < nSubspacesDesired - 1 then lowerBound =  $\epsilon$ ;
    if nSubspaces(subspaces) > nSubspacesDesired - 1 then upperBound =  $\epsilon$ ;
end
```

ONE OR TWO EXAMPLES TO DEMONSTRATE HOW THE STATE DECOMPOSITION WORKS
ALSO MENTION HOW YOU CAN DECOMPOSE USING TRANSITION TABLES AT DIFFERENT STAGES.
FOR EXAMPLE, DECOMPOSING BY DESTINATION IS LIKE DECOMPOSING USING A RANDOM POLICY,
WHILE DECOMPOSIIGN BY POSITION IS LIKE DECOMPOSITION USING THE OPTIMAL POLICY
EXPLANATION OF HAVING BASICALLY DIFFERNT PARTS OF THE BRAIN DOING DIFFERENT THINGS

Chapter 4

Design and testing of the proposed decomposition technique

The state-decomposition method is going to be implemented and tested in multiple varieties to assess the effect it has when applied to reinforcement learning algorithms.

4.1 Tools

Python The programming language chosen for this project is Python. This is motivated by Python being a high-level language that allows writing compact, readable code and by the widespread availability of resources and packages for it (such as the OpenAI Gym toolkit previously described). Additionally, Python can be run on Colab notebooks.

Google Colaboratory Google Colab is a platform from Google which is widely used for machine learning research and prototyping of new models. Colab combines a notebook platform that many Python developers are familiar with and the high-performance of cloud computing. The user can choose to run a program using CPUs, GPUs, or TPUs¹. Examples of the available GPUs are Nvidia K80 and Nvidia P100D. The disadvantage of Colab compared to cloud computing platforms such as Google Cloud and Amazon AWS is that the user is not guaranteed full access to the hardware accelerators (such as GPUs) and thus the performance can vary. The session length is also limited to 12 hours. These problems are mitigated by purchasing a "Pro" license for the price of \$10 per month, which prioritises access to GPUs and increases the maximum session length to 24 hours. The level of performance offered by Colab Pro is enough for this project. The main advantage is an easy to use platform thanks to its notebook format, rather than the less user-friendly access via SSH in the Terminal required by other cloud-computing platforms.

Keras It was decided to implement the deep-learning models using the deep-learning API Keras [28]. This is a high-level interface that uses Tensorflow [5] for its backend. Keras allows to code deep-learning models in a very readable and compact way. Having Tensorflow as its backend, it can run on CPUs, GPUs, and TPUs, taking advantage of the high-performance hardware offered by Google Colab.

OpenAI Gym As this project deals with the development and evaluation of a new method for reinforcement learning algorithms, it was decided to apply it in a "toy environment" that would simplify experimentation. OpenAI offers an open-source Python toolkit called "OpenAI Gym" [19], which is currently the most commonly used toolkit to benchmark reinforcement learning algorithms. This toolkit offers a variety of different environments, ranging from classical control problems such as

¹Tensor Processing Unit: a custom hardware accelerator developed by Google to accelerate the linear algebra operations that occur in machine learning. [1]

"CartPole-v0" [4], to more complicated environments such as playing Atari games using the pixel data as sensory input or solving robotic manipulation problems. The "OpenAI Gym" offers environments with a wide range of complexities and all combinations of discrete and continuous state and action spaces.

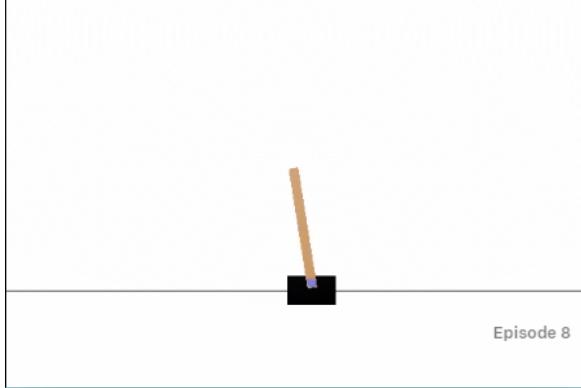


Figure 4.1: The CartPole-v1 environment is one of the most well-known environments of the OpenAI Gym library in which the goal is to balance the cartpole while keeping it within the boundaries of the screen by applying a leftwards or a rightwards force at each time-step. Screenshot of [4]

4.1.1 Obtaining the state transition matrix

The first step in the state-decomposition method is to obtain the state-transition matrix. Since we are testing the algorithm in software simulations, the state-transition matrix can be easily obtained from the source code of the environments and it can be considered as given. This is usually not the case, so at a later stage we shall test methods to approximate the state-transition matrix. This can be done by interacting with the environment and sampling the $\langle S, A, R, S' \rangle$ tuples that are generated. Since the state-transition probabilities are dependent on the action, the state-transition matrix is dependent on the chosen policy. As we are conducting this before the training and we have no information about the optimal policy, we choose to use the most general possible policy to obtain the state-transition table, that is, the actions are all chosen with equal probability independently of the current state. The number of iterations required to construct the state-transition matrix is dependant on the number of states, actions, the probability of visiting each different state and the required level of accuracy of the state-transition matrix estimate.

4.2 The need for a suitable environment

The state-transition matrices and their decomposed versions were obtained for 4 different OpenAI Gym environments: "Cartpole-v1"², "FrozenLake8x8-v0", "CliffWalking-v0" and "Taxi-v3". The state-decomposition algorithm is aimed at environments whose state-spaces decompose into similarly sized sub-spaces, as this is the case in which the method would be potentially very effective. If one of the state-spaces is much greater than the others and similar in size to the original state-space, the reduction in complexity of the training in the first stage would not compensate for the more complex multi-stage nature of the state-decomposition method. Having sub-spaces that are too small (in the extreme case having just one state) limits the amount of learning that can be achieved in the first stage of training, simply delaying this to the second stage. The MDPs of "Cartpole-v1", "FrozenLake8x8-v0", "CliffWalking-v0", always decompose into one big sub-space and sub-spaces formed by only one state. This occurs because these environments are not formed of almost independent sub-problems that rarely interact with each other.

²This environment has continuous states which are discretised using a linear quantiser to apply the decomposition algorithm.

4.2. THE NEED FOR A SUITABLE ENVIRONMENT testing of the proposed decomposition technique

The "Taxi-v3" environment, shown on the left in Figure-4.2 is a more suitable candidate. The environment presents:

- 500 discrete spaces, corresponding to 25 taxi positions in the 5x5 grid, 4 possible destinations, and 5 possible passenger positions (1 of which is inside the taxi).
- 6 discrete actions: 4 moves (up, down, left, right), pick-up passenger, and drop-off passenger.
- Rewards: +20 for a successful drop-off, -1 at each time step (also when hitting a wall), and -10 for illegal pick-up and drop-off actions.

An episode terminates when the passenger is successfully dropped off or after 200 time-steps. Applying the state-decomposition to this environment can form 4 equally sized sub-spaces even with a threshold of 0, meaning that these are 4 completely independent sub-spaces that can be treated as independent Markov Decision Processes. These 4 sub-spaces correspond to the 4 possible destinations of the passenger, which never change during a single episode. Thus, this is a more specific and easier problem than the one we are trying to solve, where the subsystems, though rarely, do interact with each other. Thus, this environment is not suitable for testing the state-decomposition algorithm as good performance of the algorithm in this simpler environment does not imply good performance in the more general scenario.

These problems in finding a suitable environment suggest that the best solution would be to create a custom environment with the characteristics that would allow better testing of the method. The "Taxi-v3" environment is composed of 4 independent MDPs. Modifying its transition-probabilities so that interaction between them is possible³, would make this environment suitable for testing the decomposition algorithm. For this purpose, I propose a modified version called "TaxiTraps", with added "traps" located on the edges of certain squares. These activate with a low probability p_{trap} when the Taxi moves over them. If a trap does not activate, the taxi simply moves over it as in the original "Taxi-v3" environment. When the trap does activate, the behaviour of the environment is modified in two ways:

1. The destination of the Taxi changes. This makes it possible for the destination of a passenger to change during an episode, thus providing the transitions between different sub-spaces that exist in the general problem that the state-decomposition method is aimed at.
2. A highly negative reward is released. This was added to ensure that the optimal policy in the "TaxiTraps" environment is different than in the original Taxi environment. Since in the first step of the state-decomposition method the transitions between different state sub-spaces are ignored, the agents are trained in an environment equivalent to the original "Taxi-v3". If the optimal policy is the same in "Taxi-v3" and "TaxiTraps", the optimal policy would not change between the first and second training stage of training, which is unlikely to happen in a general scenario. Thus, ensuring that the optimal policy changes between training stages is required for the results of testing to generalise to other environments.

³Making transition-probabilities between spaces of different subsystems non-zero.

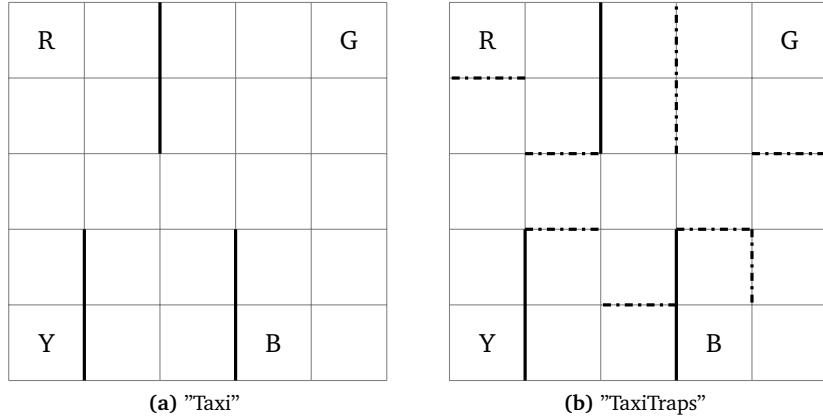


Figure 4.2: On the left, the original "Taxi" environment. On the right, the "TaxiTraps" environment.

During testing it was noted that such environments would take a very high number of iterations to train and would often get stuck for long periods of time at reward -200, which is when the agent has learned to not commit an illegal moves but it hasn't learned to drop-off the passenger successfully. Due to the limited computing power of my tools, this was a problem as it did not allow for significant testing with a high number of trials. Thus it was decided to use a simpler environment for easier experimentation and one that would also allow for easier interpretation of the results. I thus introduced the 'Grid' environment, which is a simple 25 by 25 grid with two destinations, as shown in 4.3. At the start of the episode, the Taxi is initialised in a random position, while the passenger is always assumed to be inside the Taxi. The pick-up and drop-off are eliminated, so that the learning process only deals with learning how to navigate in the grid. The two destinations are located at opposite corners. throughout an episode the destination is always the same as in the initial state. To summarise, the environment has:

- 1250 discrete states, corresponding to 625 taxi positions in the 25x25 grid and 2 possible destinations.
- 4 discrete actions: move up, down, left and right.
- Rewards: +20 for being in the same position as the destination, -1 at each time step (also when hitting a boundary and thus not moving).

This is a perfectly decomposable environment, as there are no possible transitions between states having different destinations. For a complete evaluation of the state-decomposition method, it needs to be tested in a more generic environment, with non-zero albeit small transition probabilities between the decomposed sub-spaces. This was achieved by introducing a probability ϵ of changing the destination whenever the current destination is the one closest to the current position in the grid. This modified 'Grid' environment will be referred to as 'GridEps'. The following table summarises the transition probabilities in 'GridEps'.

		next state	
		destination 1	destination 2
state	triangle 1 - destination 1	1- ϵ	ϵ
	triangle 1 - destination 2	0	1
	triangle 2 - destination 1	1	0
	triangle 2 - destination 2	ϵ	1- ϵ

Table 4.1: State transition probabilities in the 'GridEps' environment.

The reason that the transitions were implemented in this way, rather than I THAT IM DUMBO, THERE IS NO REASON FOR THIS, JUST ME RUSHING THINGS CUZ I HAD TO DO EXAMS

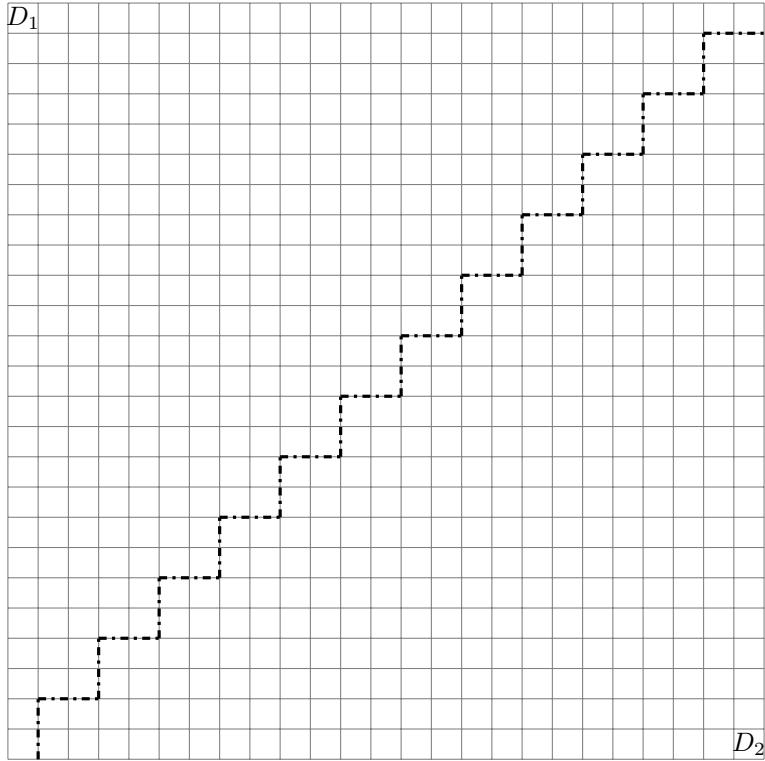


Figure 4.3: 'Grid' environment. The dotted lines have no effect on the environment, they are just used to separate what will be referred to as the upper-left triangle and the lower-right triangle.

4.3 Implementation of the DQN agents

Some initial testing of the state-decomposition method has been conducted in the "Taxi-v3" environment. Here, the state-decomposition method modified to skip the second stage of joining the networks together showed equivalent performance as a single DQN agent. The equivalence is to be expected since in this environment the sub-problems are completely independent and it does not prove anything about the algorithm in general. One issue of the testing conducted until now is the format in which the states were fed into the Neural Networks. The OpenAI Gym library represents discrete states and actions as single integers. Such a representation is optimal when using tabular Q-learning, as the representations of states and actions simply act as indexes and don't need to be meaningful. However, when using a Q-function approximator, such a meaningless representation impedes the generalisation properties in unseen states that make DQN so useful. Representing the states as one-hot encoded versions of the original integer has also been tested. Though this could allow using shallower neural networks as the decoding of the states is simpler, this is still not a meaningful representation of the states. The current state of the code of the state-decomposition agent is given in the Appendix. To take full advantage of the generalisation to unseen states property of DQN, the format in which states are input to the agents will be modified to be a meaningful representation. This could be in a vector format, where each element of the vector represents a different aspect, such as position, destination, etc.

4.4 Benchmark DQN agent

The aim of this project is to determine whether state-decomposition techniques can be used to increase the efficiency of reinforcement learning algorithms. Doing so requires comparing the performance of an algorithm where state-decomposition was applied and that of the same algorithm without state-decomposition, while choosing all other parameters to be the same or to be the values

that make for the fairest comparison. The number of possible choices to be made in these algorithms is such that providing a fair comparison can be a non-trivial task, as discussed in the following subsection.

The chosen benchmark agent is a basic DQN agent. The value updates occur with discount factor $\gamma = 0.95$ WHY??? (I think even though the episode are finite so could use gamma=1, using a lower gamma enhances the stability of the algorithm). The agents acts according to an ϵ -greedy policy. This means that at each agent-environment interaction, the agent picks the value that currently has the highest estimated action value with probability $1 - \epsilon$ or a random action with probability ϵ (RANDOM OR ONE OF THE OTHERS? NOT big difference anyways). ϵ is initialised at 1, meaning that in the first iteration the action is completely random and decays by a factor 0.999 after each update of the action values. ϵ is set to stop decreasing at $\epsilon = 0.01$, this is to ensure that the algorithm doesn't stop exploring by having too small an ϵ .

The neural network that approximates the Q-values is fully connected and it has 4 layers. The input state is fed as an integer to an encoding layer⁴, which then feeds to the first fully-connected layer, consisting of 512 neurons with "ReLU"⁵ activation function. This is then followed by 2 similar layers, having 256 neurons each. The last layer consists of a number of neurons corresponding to the number of actions in the environment where the agent is being used. Each output of the network corresponds to the action value of an action, thus the activation function was chosen to be linear, that is, the outputs of the layer are equal to the outputs of the neurons. This is to allow the outputs to take any possible values, without upper or lower bounds so that the network is theoretically capable of mapping any state to any possible action-value. The network is updated using "Minimum Square Error" as its loss function, that is, at each update step, the network weights are updated to reduce the function $L = \sum_{i=1}^n (y_i - f_\theta(x_i))^2$ where x_i are the inputs, y_i the corresponding outputs and f_θ is the function implemented by the neural network with weights θ . The network updates are done using the optimiser Adam [14], which was shown to have better performance than simple gradient descent, through the use of weight momentums.

Each iteration with the environment is saved in a replay memory. The maximum size of this is set to be big enough so that all samples are saved, without having to discard them in the later stages of training. After each interaction with the environment, one update step of the neural network using a batch of 128 samples, randomly picked from the replay memory.

COULD ADD IMAGE OF NETWORK

4.5 Ensuring fair comparison

When comparing the performance obtained using the state-decomposition technique with a benchmark method that doesn't use it, we would like ideally to isolate the effect of the state-decomposition effect, independent to other changes to other parameters. Unfortunately, it is not possible to do so due to the complexity of the relationship between various hyperparameters. However, it is possible to make reasonable choices such the comparison is as fair as possible. In this project, it was aimed to keep all hyperparameters constant in all trials. Different experiments that involve different network architectures, such as using 2 neural networks instead of 1 always ensure that the total number of weights in each agent is approximately equal. Other factors such the encoding method of the state are always kept equal, unless the objective of the experiment is to investigate their effect on the performance.

⁴This is fixed and pre-programmed and doesn't include any learned parameters.

⁵"ReLU" stands for Rectified Linear Unit. It represents the function $f(x) = \max(0, x)$. Neural networks rely on the non-linear activation functions such as "ReLU" in order to be able to learn non-linear functions.

Chapter 5

Results

Introduce the chapter

5.1 Evaluation method

The main aspect of these algorithms that will be evaluated is the sample efficiency, which is how efficiently the algorithm uses the samples that it experiences to learn a policy. This affects how quickly the algorithm can learn an optimal policy and it can be measured by plotting the total reward per episode vs the number of training steps. It is also important to consider the fact that the learning process is stochastic and the performance varies between trials, hence we normally express the reward per episode using a confidence bound.

EXPLAIN HOW EXPERIEMNTS WERE RUN, NUMBER OF OF TRIALS, USE OF CONFIDENCE BOUNDS, USE OF SAMPLE GRAPHS and also probelm with sample graphs so why have to use shorter versions

5.2 Effect of different state representations on the performance

Although it doesn't necessarily impact the state-decomposition method, it was initially necessary to decide how to encode the state to be fed into the neural network. As a first approach, the state was fed directly as an integer between 0 and $no.states - 1$. This is the most compact encoding of the state which is what one would use in standard Q-learning, in which the state is used merely as an index in a look-up table rather than as the input of a neural network. Though extremely compact, this representation of the state is very hard to decode for the neural network and its structure does not directly reveal aspects of the speed directly¹ and led to difficulties in achieving convergence in a reasonable number of iterations. Thus it was decided the encode the state in in three alternative representations:

1. **Compact** The idea is to give a physical meaning to the state. The state is represented by a vector of values, each representing a different aspect of the physical meaning of the state. In the 'Grid' environemnt, these are the row in which the Taxi is currently located, the column and its current destination. (NEED TO SAY FOR SPEICIFC ENV)
2. **One-hot encoding** The idea is to make it easier for the neural network to learn to decode the state. The state is in this method simply converted to a one-hot encoded version.
3. **Hybrid encoding** This combines the previous two methods. The destination is a represented by a single bit, while the position of the state is represented by a one-hot-encoded vector. This results in a state representation that is a vector of size one bit greater than the number of possible positions in the environment.

¹As the state is represented by an integer, this has not physical meaning before decoding. However, the state could be represented for example by a vector of values, each having a physical meaning such as horizontal position, etc.

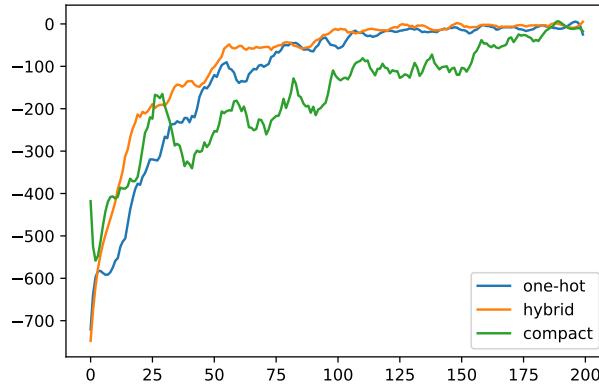


Figure 5.1: Effect of different state encoding methods on the performance of DeltaSwitch. IN WCHIH ENVIRONMENT??? SHOULD SHOW FOR DQN INSTEAD

Figure-5.1 shows that in this environment (WHICH ENVIRONMENT?) the hybrid encoding achieves the best performance, followed by the standard one-hot encoding and lastly by the compacy encoding. Thus, it was decided to use the hybrid encoding for all experiments in order to expedite experimentation time. This was actually a majot issue during the project, as the training time of the algorithms is long and any increase in performance is wellcome as it allows to tun more trials to obtain higher quality results and to also run different experiments.

5.3 First approach

Need to describe initial results using the original method, which didn't allow communication between different sub-networks and was a 2-stage method. Also the environment was too complicated and it was not easy to identify problems.

Don't even know if I should talk about these old methods because I don't even know what I achieved with them

The first meaningful results that I obtained used the original state-decomposition algorithm applied in a modified 'TaxiTraps' environment. The changes made the environment simpler. The number of possible destinations was reduced to 2, while the passenger is initialised to be always inside the Taxi, and the actions of picking-up and dropping-off were removed, so that the learning task only involves learning to navigate. The probability of activating the Traps was set to 1%. In this case, though the learning was

- Original TaxiTraps was problematic because the rewards used to get stuck at -200 (this is because of multi-sub-task nature of the task)
- Even with OneHot encoding it was so complicated that rewards would not improve, also extremely slow training, not good for my purposes

5.4 Initial results with Taxi2 and original state decomposition algorithm

5.5 Effect of varying the neural network's size

It is important to note that the one of the factors influencing the speed of learning of deep reinforcement learning algorithm is the size of the neural network. Figure-5.2 shows that the performance seems to improve as the network size increases, at least for the range of dimensions that I experimented with. When the network is too small, performance degrades as the network is not complex enough to represent the function that maps from states to action-values of the environment, thus

rewards are lower after any number of episodes compared to other networks. In the figure, the three bigger networks are all complex enough to reach convergence, whoever the biggest one does so quicker. This might be due to increased flexibility and the existance of mutliple sets of weights that achieve the same optimal function representation. While this leads to faster convergence and is not a problem in the environments used in this report, it might lead to problems in complex environments that rely on generalisation of unseen states. This is because using an overly big network can easily cause overfitting, thus hurting the generalisation propertties to unseesn states. Thus, for the sake of this project, it was decided to use networks that are complex enough to capture the dynamics of the environment, while limiting the size so that the project could be relevant to other scenarios. The figure also shows that the difference in performance of 'medium2', which has twice the number of trainable weights of 'medium' is not very significant. I chose to show this comparison because in the environments used in this project, the decomposition divides the state-space into two sub-spaces, which means that there are two NNs corresponding to the sub-spaces instead of the one original NN. In order to make the comparison fair, I make the total number of trainable parameters approximately equal in experiments that are being compared. It is arguable whether this is a fair comparison, but the fact that the performance hardly varies when doubling the number of trainable parameters demonstrates that this is hardly an issue.

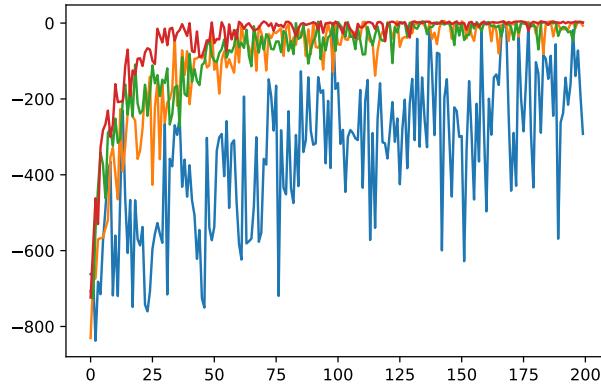


Figure 5.2: Performance of DQN with NNs of different sizes in the Taxi2 environment. 'medium2' has twice the number of trainable weights of 'medium'.

5.6 A simple approach to decomposition: 'DeltaSwitch'

Though the original idea involves the use of a combinining neural network on top of the neural networks correspodning to each one of the sub-spaces, I decided to experiment with a simpler approach in order to first validate the idea of state-decomposition in the simplest possible application. For this purpose I introduced a network architecture that I called 'DeltaSwitch'. This consists of one neural network corresponding to each sub-space, as in the orginal proposed architecture. The difference is that the action-values are combined using a simple weighed sum, that is the output action-values of the combined architecture are given by

$$q(s, a) = \sum_{i=1}^N \delta q_i(s, a) + (1 - N\delta) \cdot q_{\{j:s \in S_j\}}(s, a) \quad (5.1)$$

where $q_i(s, a)$ is the action-value of action a given by the network of the sub-space S_i with index i , δ is a small constant that satisfies $\delta \in [0, \frac{1}{N}]$ and N is the number of sub-spaces. This approach is also simpler in the fact that it only consists of stage of training. In fact, all the networks are combined from the start and this is done in a fixed way, as δ is a non-learned constant. The combining by weighed average effectively substiutes the combining neural network and acts as a switch when $\delta = 0$, as it

effectively selects the output of the network corresponding to sub-space of the current space, thus the name 'DeltaSwitch'. If the special architecture of 'DeltaSwitch' is ignored, the algorithm is identical to the basic DQN algorithm used as a benchmark, as the state decomposition acts directly on the 'switch'. δ is chosen to be small, meaning that each sample used in an update of the network causes significant change in the weights of the corresponding network, while only slightly changing those of other networks. This can be confirmed by deriving the formulas of the updated of weights given the used loss function and optimiser, however it is sufficient to understand that those weights that have a smaller effect on the output are updated by a smaller amount in the backpropagation update. As the network is always treated as a whole, the target values required in the update of the network are allowed to use all the possible networks. This means that each network is trained using target values generated by other networks, thus the networks are not independent during the training stage. This is important as it was previously observed that not allowing this exchange of information during training leads to instability and lack of convergence (WHERE WAS THIS OBSERVED).

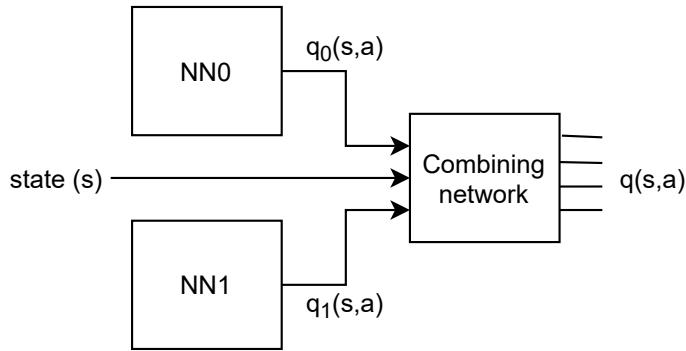


Figure 5.3: Combining 'NN0' and 'NN1' using another NN.

The idea of this approach is that it might not be necessary to combine the action-values of the networks in a complicated manner and that doing so using another neural network might be complicated and lead to stability issues and lack of convergence.

The first environment where this was tested was the 'Taxi2' environment. In this environment there seems to be no difference in performance between DQN and DeltaSwitch with differing δ values, as shown in Figure-5.4.

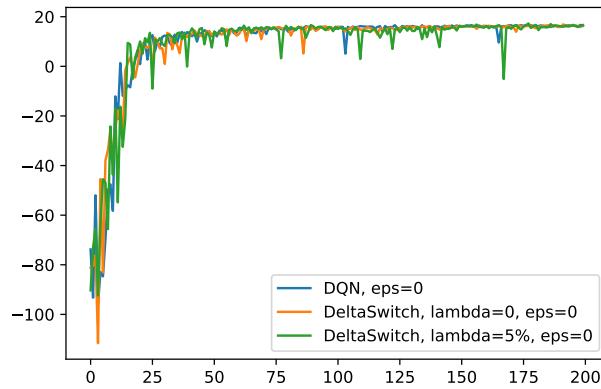


Figure 5.4: Comparison of DQN and DeltaSwitch with $\delta = 0$ in the Taxi2 environment.

Similar results were obtained for other environments, however, these environments are complex and it is difficult to interpret the results. For this reason it was deemed important to repeat the experiment in an environment easier to interpret. This experiment was repeated in the 'Grid' environment, by

setting the environment's ϵ equal to 0, meaning that the destination is chosen at the start and it doesn't change for the whole duration of the episode. Figure-5.5 shows that DeltaSwitch using $\delta = 0$ presents some performance gain compared to DQN. This gain is especially noticeable when the state is decomposed by position, meaning that one sub-space is formed by all those states in which the current position in the upper-left triangle of the Grid, while the other sub-space is formed by those positions in the lower-right triangle. Decomposing by destination means that the possible states are divided in two sub-spaces, corresponding to the two destinations.

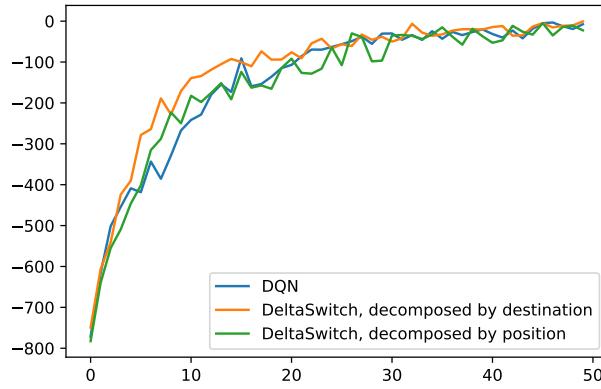


Figure 5.5: Comparison of DQN and DeltaSwitch with $\delta = 0$ in the Grid environment.

This result shows that 'DeltaSwitch' can present some performance gains compared to a normal DQN algorithm in a perfectly decomposable environment. This environment is perfectly decomposable by destination, as the destination never changes, meaning that there are no possible transitions between states having different destinations. It is also perfectly decomposable by position, as when adopting the optimal policy, the agent never moves from one of the triangles to the other, since the destination is always the one closest to the current position. This experiment validates the idea of the state decomposition as having a performance gain in such specific environment mean that it might be possible to also have a performance gain in other environments that are not perfectly decomposable. For this kind of perfectly decomposable environment one can't find a reason to set $\delta \neq 0$ as the two sub-spaces are completely independent. Figure-5.6 shows how the performance of 'DeltaSwitch' degrades in the 'Grid' environment when increasing δ . In fact, as shown later, there is never a reason to set $\delta \neq 0$, as even when the sub-spaces are not completely independent, as communication between them is allowed during training, the action values of the networks are accurate, so the weighted sum should act as a mere switch rather than combining multiple values. The idea of a weighted average was actually initially devised to consider the case when communication between networks does not occur during training. However, in this case the training fails so using $\delta > 0$ is never useful.

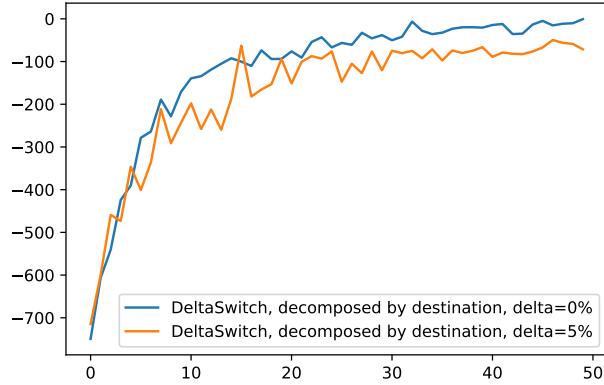


Figure 5.6: Comparison of DeltaSwitch with different δ .

Thus we determined that a 'DeltaSwitch' agent with $\delta = 0$ can lead to performance benefits. We now shall explore the performance of such an agent in environments that are not perfectly decomposable. This is done by running experiments in the 'GridEps' environment with $\epsilon > 0$. Figure-(WHATFIGURE!!!!) compares the performance of training a DQN agent and 'DeltaSwitch' agents, decomposing by destination and by position. It can be seen that for for $\epsilon \leq 0.3$ 'DeltaSwitch' with decomposition by destination learns noticeably faster than the other two algorithms. The difference is especially large for $\epsilon = 0.3$. This might be due to this environemnt being the harder to solve, giving a greater margin of improvement over DQN. For higher ϵ values, there isn't a statistically relevant difference between the three algorithms. This is probably due to the fact that the environment is not as decomposable anymore, as the transition probabilities between the sub-spaces are larger.

This is an important result, as it shows that while the state-decomposition method is applicable to perfectly decomposable environments, or environments that are almost fully-decomposable, the performance gain disappears once the transition probabilities between the sub-spaces are too large.

INSERT FIGURES

Though these experiments shows that the state-decomposition can be a viable way to reduce training times in certain environments, it does not explain the meachanism in which it does so. The following subsections highlight some of the experiments that were used to shed light on this mechanism.

5.7 Effect of using reduced encoding

As I used the 'hybrid' encoding described in a previous subsection, the input to the first layer of the network is a vectors of '0's and '1's of size the number of possible positions in the grid plus one. However, since I am using $\delta = 0$ and each network only contributes to the output when the input state is in the network's sub-space, it is possible to reduce the size of the input vector to only account for the states that are relevant to each network. For example, when decomposing 'Grid' by position, the two networks only deal with 312 and 313 positions instead of the original 625. This means that instead of the original vector of $625 + 1 = 626$ elements, it is sufficient to feed the networks with vectors of $312 + 1 = 313$ and $313 + 1 = 314$ elements respectively. I then ran experiments to determine whether such a reduction in input size would improve the performance. Figure-5.7 shows that the performance gain when reducing the input size seems to be marginal and might be caused by the randomness of the trials.

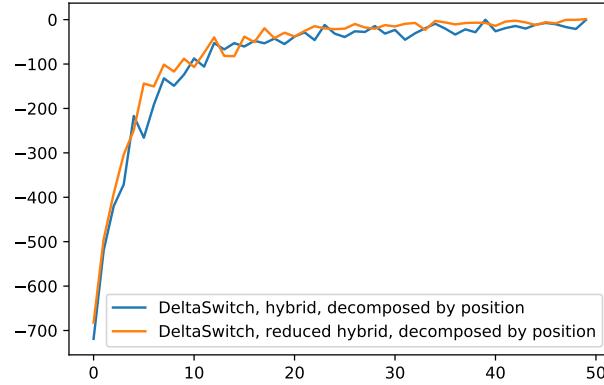


Figure 5.7: Effect of reducing the input size.

WAIT THIS IS WRONG, MAYBE NOT ACTUALLY, BUT IS FOR WHEN THERE IS A WALL, SO BE CAREFUL

This result suggests that the change in size of the input of the network does not significantly affect its ability to represent a mapping, as long as the number of inputs for which it has to learn this stays constant. Effectively, when using the full input vector size, although the number of inputs is greater, almost half of these are always 0 when the network is active, thus these inputs mathematically have no effect on the network and it is intuitive that the results should be similar when using the reduced input vector.

5.8 Samples plots

5.9 Effect of wall / no wall

The experiments with DeltaSwitch were repeated in a modified version of the 'Grid' environment. This version has a wall that separates the upper-left triangle of the grid from the other triangle. This was done to test the effect of having decomposition upon convergence, versus having decomposition with any possible policy. When there is no wall, the decomposition by position only occurs upon convergence, that is when the optimal policy is being used, as there is no reason why the agent would act to cross the boundary between the two sub-spaces. However, it is physically possible to cross this boundary and this is very likely to occur during the training stage. Conversely, when a wall is present, it is not physically possible to cross this boundary, regardless of the policy.

It is intuitive that when the wall is present, the learning process is faster, as it impedes the agent from moving further from its destination. It is interesting to notice that as shown in Figure-5.8, the performance of 'DeltaSwitch' and the benchmark DQN algorithms is equivalent in this environment. Any differences in the figure are small enough to be attributed to the randomness of the training, as they are not consistent throughout the graph, with the lines crossing each other multiple times.

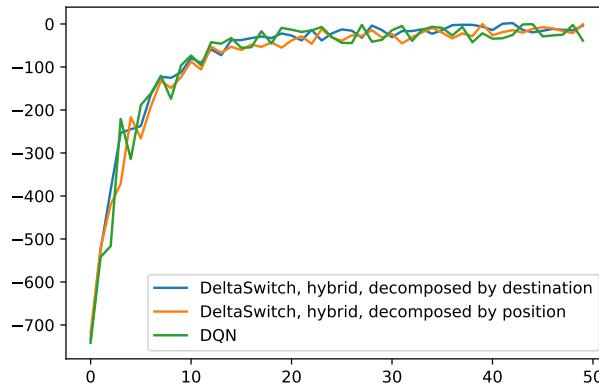


Figure 5.8: Performance of DQN and DeltaSwitch with different decomposition in the 'Grid' environment with wall.

These results suggest that 'DeltaSwitch' loses its advantage towards DQN when the decomposition is 'complete', that is the task can be divided into 2 completely independent tasks, such as when the wall is present. It could be that the advantage of 'DeltaSwitch' when this is not the case is in identifying semi-independent tasks to expedite training, however, when these are completely independent this already occurs in DQN.

5.10 Effect of decomposing by destination or by position

Another important thing to notice is the difference in performance when decomposing by position and when decomposing by destination. It must be noted that in the standard 'Grid' environment, decomposing by position is to apply a decomposition that only exists upon convergence, as previously discussed, while decomposing by destination is to apply a decomposition that always exists. The second difference is that the experiments were run using the hybrid encoding, in which most of the input vector length represents the position and only one bit represents the destination. The graphs in previous figures show that the decomposition by position tends to have superior performance in the 'Grid' and in the 'GridEps' environments. As I already discussed the effect of a decomposition that relies on the optimal policy in the previous subsection, I will now discuss the effect of decomposing in a way that reduces the number of possible active input elements. I define a possible active input element as an element that can be both a 0 or a 1, given the state-decomposition. For example, given the hybrid encoding with 625 elements corresponding to the positions in the grid and 1 element corresponding to the destination, decomposing by position gives 312 (or 313) active position elements and 1 active destination element, while decomposing by position leaves 625 position elements while making the 1 destination element inactive. Thus, decomposing by position greatly reduces the number of active elements, while decomposing by destination only changes the status of 1 element. This might justify the reduced performance of the decomposition by position. A further experiment was run to validate this hypothesis, using standard one-hot encoding instead of the hybrid encoding. This is because with standard one-hot-encoding, both types of decomposition roughly half the number of active neurons.

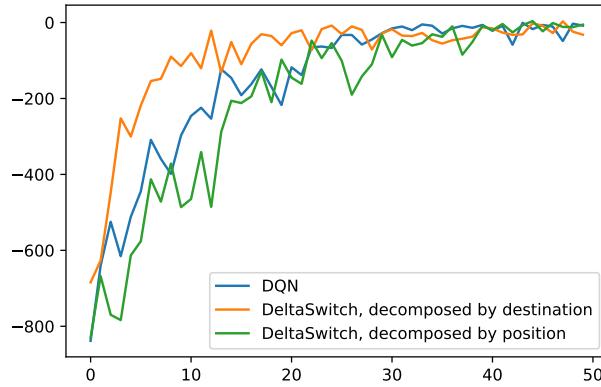


Figure 5.9: Performance of DQN and 'DeltaSwitch' when the state is one-hot encoded.

Figure-5.9 shows that when using standard one-hot encoding, the decomposition by destination is much better than that by position which is infact slightly worse now than DQN. Comparing with the experiments run with the hybrid encoding, one can see that the performance increases further when using 'DeltaSwitch' with decomposition by destination, while it degrades when decomposing by position. This can be attributed to the change only involving the "destination" subsection of the input state. Thus, when the decomposition is by position, the change in performance going from hybrid to one-hot encoding is equivalent to the one that occurs for DQN, which also gets slightly worse. Conversely, going from hybrid to one-hot allows the decomposition to have the previously described effect of making some of the inputs inactive.

Need to explain the different between one-hot inputs and normal inputs and how normal inputs despite being fewer are much more complicated so that a single input that can take 1000 values is more complicated than 1000 that can only take one value. Also explain that one-hot allows inactive inputs to be ignored, while normal doesn't.

5.11 A different approach based on transfer learning

The idea of decomposition relies on the fact that the optimal policy to be followed in a perfectly decomposable environment should be similar to that of a similar environment, having a small probability of transitions between states of the different sub-spaces. Another class of technique that leverages similarity between environments is transfer learning. Using transfer learning in reinforcement learning means to use the knowledge obtained by having trained an RL agent in a certain environment in order to more efficiently learn in a new similar environment. The 'DeltaSwitch' algorithm demonstrates how it is possible to achieve some performance gain by state-decomposition and it does so in one stage of training. The original idea was to first train multiple agents that deal with each of the sub-spaces and then combine them using another neural network. This idea is very similar to training an agent in the perfectly decomposable environment and then using its knowledge, more efficiently learn in the more complex environment. Effectively this two stage approach is to do transfer learning from a simpler fully decomposable environment to the original environment.

The following methods were all tested by using the two networks from a 'DeltaSwitch' agent that was previously trained in a Grid environment, decomposed by position (SHOULD I SAY WHY BY POSITION), which gives two neural networks, will I call 'NN0' and 'NN1', which correspond to the two sub-spaces. The transfer learning was then applied to a 'GridEps' environment.

The first approach was to combine 'NN0' and 'NN1' using another neural network. The weights of 'NN0' and 'NN1' are set to be fixed. This network is also fed the state input. The idea is that this network can act as a clever switch instead of the 'rudimentary' switch used in 'DeltaSwitch', in order to take into account the transition between sub-spaces. This algorithm gives the results in Figure-5.10, which shows that this algorithm, despite using 'NN0' and 'NN1' which were previously trained,

achieves lower rewards than a standard DQN algorithm for both values of ϵ . This gap increases for higher values of ϵ . This means that this method is unsuccessful in leverage the knowledge obtained from 'NN0' and 'NN1'

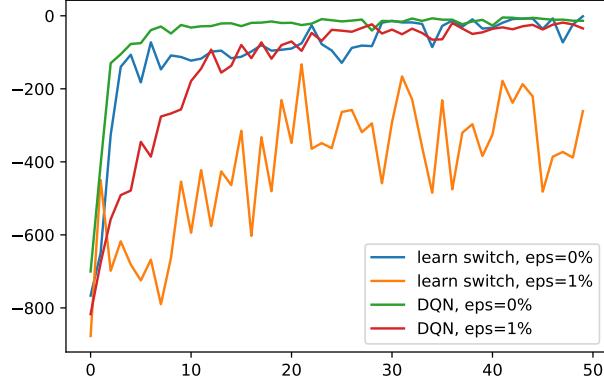


Figure 5.10: Performance of combining 'NN0' and 'NN1' using another NN that is also fed the current state as input in a 'GridEps' environment with $\epsilon = 0\%, 1\%$.

As this method is applied to environments where the probability of transitions between different subspaces is small, it is fair to assume that directly using the original 'DeltaSwitch' agent that 'NN0' and 'NN1' are taken from should give near-optimal performance. Thus, at least in the start, the transfer learning method should achieve much higher rewards than a standard DQN. This was achieved by 'forcing' the initialisation of the combining network, whereas before this was randomly initialised. This is done by training the combining NN in a supervised manner so that the combined network ('NN0', 'NN1' and combining network) is initialised to behave exactly as the 'DeltaSwitch' of 'NN0' and 'NN1'. The training samples have as inputs all the possible states and as outputs the corresponding outputs of 'DeltaSwitch'. After training for multiple epochs², with MSE as the loss function, the combined network can be utilised in the environment as before and RL process can be started. Figure-5.11 shows the results of doing so with different NN sizes. The performance is much better than without initialisation in the start. Using the smallest network does not allow to capture the environment's dynamics and fails to converge. Only the biggest NN allow to achieve performance comparable to DQN in the last part of training and in this case the network used is actually bigger than that in DQN.

²In the experiment, the network was trained for 1000 epochs, in order to ensure an accurate initialisation. However, such high accuracy might not be needed.

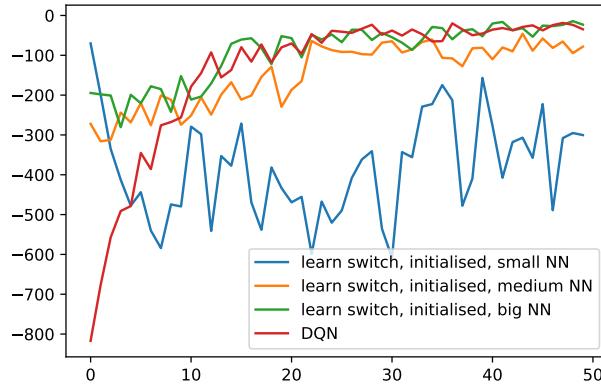


Figure 5.11: Performance of combining 'NNO' and 'NN1' with another NN when initialising the combined network to behave as the original 'DeltaSwitch'. Tested with different network sizes

Requiring a combining network of a similar size to that of the DQN agent means that the function that this NN has to represent is similarly difficult to that of the DQN's network. Thus it seems that the initialisation of the network only serves the purpose to limit negative rewards at the start of training, but it doesn't have a lasting effect in making the learning easier. I thought that the issue might be that though Q-learning is guaranteed to converge after sufficient amounts of exploration, it always finds the optimal policy, so it is likely to completely discard the initial policy provided when initialising the combined network. I thus experimented with SARSA, as its updates are given by the formula

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)] \quad (5.2)$$

that is, SARSA is an on-policy algorithm, meaning that the action values are updated using the values of the states they were actually visited (NOT CLEAR). My intuition is that doing so is more likely to preserve the initial policy provided by the initialisation of the combined network, which would increase the speed of learning at the expense of the performance upon convergence (MIGHT WANNA PROVE THIS SOMEHOW, GET A PAPER THAT AGREES). However, using SARSA failed to converge, thus I tried with expected SARSA, which has the updates

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\{R_{t+1} + \gamma E_\pi[Q(S_{t+1}, A_{t+1})] - Q(S_t, A_t)\} \quad (5.3)$$

where π is the policy at the time of the update. Figure-5.12 shows that using expected SARSA seems to improve the initial performance, as rewards never fall below -200.

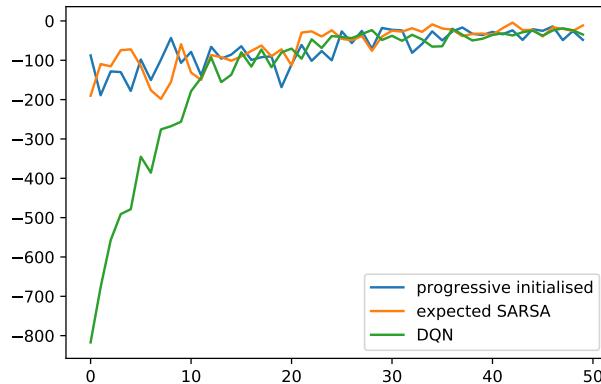


Figure 5.12: Comparison of using Expected SARSA after combining, progressive combined network and standard DQN.

I THINK THE SARSA RESULTS MIGHT BE BETTER JUST BECAUSE OF BETTER INITIALISATION AS I ALWAYS USED THE SAME NN0 AND NN1 which is unfair

Another transfer learning idea which proved to be useful is called 'progressive neural network' [23]. This is a kind of neural network that was devised for transfer learning, as it shows better performance in transferring knowledge between tasks and it avoids "forgetting", which is a common problem in transfer learning, as seen when using Q-learning. The progressive neural network was devised for learning successive related tasks. In the first task, a neural network is used as normal. Then, in the second task, the neural network is formed by the neural network trained in the first task with its weights frozen and a new 'column' of layers. Each layer in the new column receives in input the outputs of the previous activation layer of the same column and the outputs of the activation layer of the network of the previous task. This combined network is then trained in the new task and the process is repeated for all the tasks, each new column receiving in input the outputs of all the previously trained networks. Figure-5.13 shows the structure of a progressive network used on a third task when transferring knowledge from two previously trained tasks.

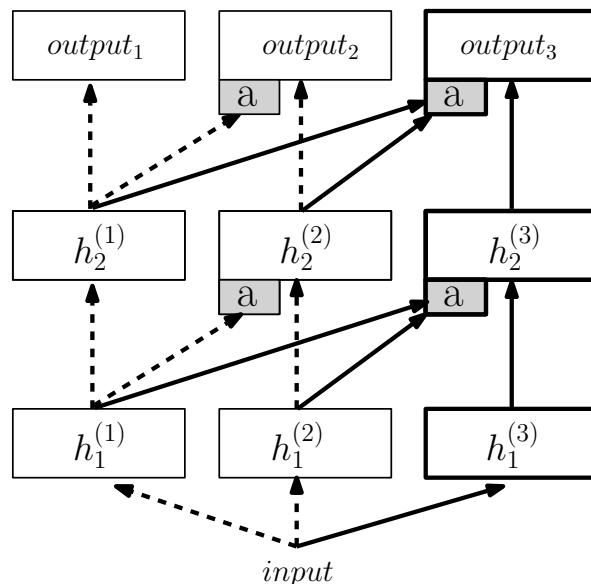


Figure 5.13: Structure of progressive network when training the third task. Figure taken from [23].

The architecture that I propose is very similar. Instead of connecting the networks one after the other, I simply connect one new network to all the previously trained networks that correspond to the subspaces. That is, I take 'NN0' and 'NN1', I freeze their weights and I connect the outputs of all their layers to every input of a new network. This new network has thus the same number of layers as 'NN0' and 'NN1'. (INSERT DIAGRAM) This was initially tested with random initialisation of the combined network, leading to slow convergence and worse performance than DQN. However, when initialising the combined network similarly to what was previously done in the other combining methodology, the performance is similar to what was achieved in the other methodology using expected SARSA, as shown in Figure-5.12.

5.12 Techniques to learn the 'switch'

While the transfer learning techniques in the previous subsection show superior performance to DQN, this is an unfair comparison, as they rely on the previously trained networks 'NN0' and 'NN1'. However, the two combining techniques can be modified to work in one stage, thus simply providing a decomposition rather than doing transfer learning.

SHOULD I TALK ABOUT RUNNING DQN FIRST, and LEARN SWITCH AFTER

The first proposed technique is to use a combination neural network right from the start, instead of training 'NN0' and 'NN1' first using a 'DeltaSwitch' agent. 'NN0' and 'NN1' are forced to learn the action values corresponding to the decomposed sub-spaces by including their outputs in the combined network's output and thus in the loss function. Understanding this requires some detail on how target values are normally generated. In the standard DQN algorithm, at each update step, a batch of samples is selected from the replay memory. Each sample provides information about what the target value³ of a state-action pair. The target value of those state-action pairs that are not in any of the batch's samples are simply set to be equal to current output of the network for the corresponding states. This way, the network tends to improve for the state-action pairs in the batch's samples, while penalising changes in other pairs. (MAYBE SHOULD PUT THIS IN DQN subsection) A similar method is used to deal with the fact that only certain samples should update 'NN0' and the others should update 'NN1'. For those transitions samples that start in a state that is in sub-space 0, the target value for the output of 'NN0' is set as normal using the formula of Q-learning updates, while the target value of 'NN1' is simply set to be what 'NN1' currently outputs for the considered state. The same is true the other way around. The output of the combining network is treated the same way as the output of the network in DQN. This methodology allows the combining network to learn the true action-values for all states, while 'NN0' and 'NN1' learn them for the states in the assigned sub-space. However, as seen in the previous subsection, the combining neural network did not seem to be able to make use of the information provided by 'NN0' and 'NN1'. As here 'NN0' and 'NN1' are not provided pre-trained, one can only expect the results to be worse. This is confirmed by Figure-5.14⁴, which shows that in 'GridEps' with $\epsilon = 1\%$, this architecture performs worse than standard DQN.

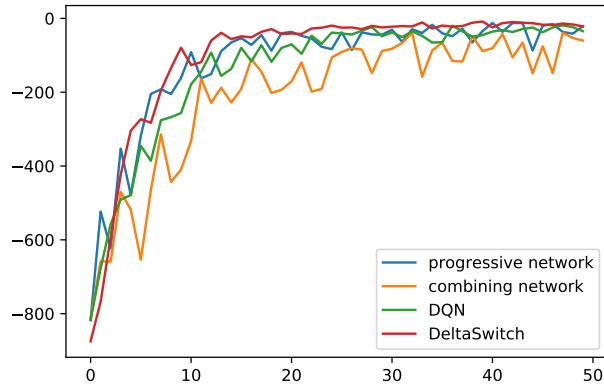


Figure 5.14: Performance of progressive network and combining network when 'NN0' and 'NN1' are not pre-trained. Standard DQN and DeltaSwitch are shown for comparison.

The second proposed technique is based on the modified progressive network from the previous subsection. 'NN0' and 'NN1' instead of being pre-trained and having frozen weights are trained in a similar fashion as what was done in the first proposed technique in this subsection. That is, the progressive network is composed of three columns, 'NN0', 'NN1' and the combining network which at every layer also receives the outputs of the previous layer in 'NN0' and 'NN1', as shown by Figure-5.15. The only difference between this method and the first proposed method is given by the connections between 'NN0', 'NN1' and the combining network, but the way the target values are generated is completely equivalent. This second technique achieves better results than the previous one. Figure-5.14 shows that the performance is slightly better than DQN and slightly worse the 'DeltaSwitch'. This means that this technique is capable of leveraging the state decomposition in order to speed up

³Target value refers to the values used at the output to calculate the loss function, as the neural network is trained in a supervised manner.

⁴All the experiments in this figure were run by applying decomposition by position.

the training. (ARE WE SURE THAT IT'S NOT JUST BECAUSE OF HAVING MORE PARAMETERS?) This better performance might be due to the greater number of connections between the networks, acting at all the layers. In the previous technique, the only connection between 'NN0', 'NN1' and the combining network was given by the outputs of 'NN0' and 'NN1'. This makes it easy for the combining network to completely ignore 'NN0' and 'NN1', thus making the training of 'NN0' and 'NN1' something that complicates the training without helping it in any way, thus slowing it down.

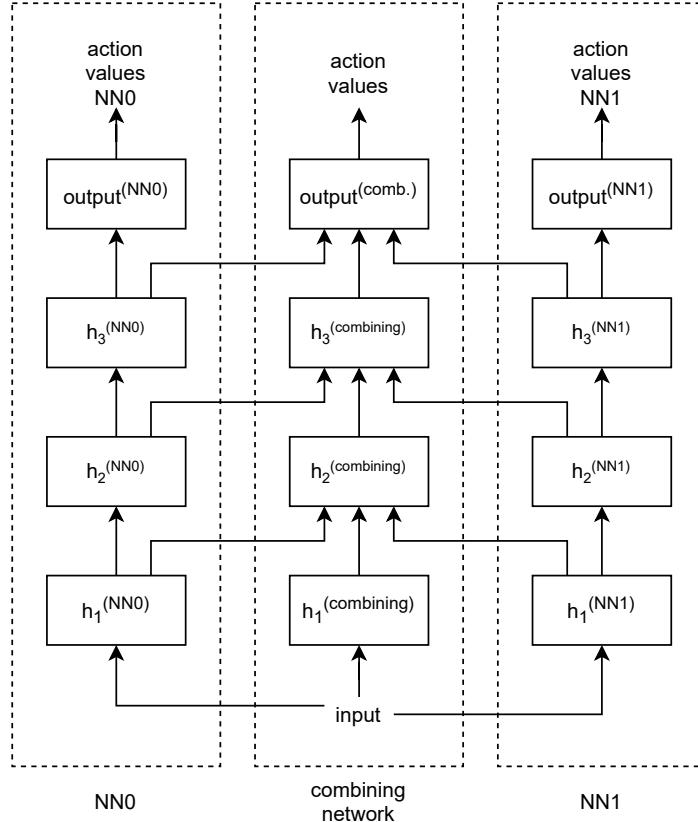


Figure 5.15: Modified progressive neural network. The layers of the combining neural network receive as inputs the respective outputs of the previous layers of frozen 'NN0', frozen 'NN1' and the combining network itself. h indicates hidden layers of the network.

At this point one could ask why even use these more complicated architectures when 'DeltaSwitch' seems to provide better performance. The reason is that while 'DeltaSwitch' works well when using the kind of state-decompositions that we consider in this project, there are also different ways to decompose a state-space that would require the use of a combination network. For example, consider having a state-space represented by a $2n$ -dimensional vector, composed of two n -dimensional vectors. The dynamics of the model are such that transitions in which both sub-vectors change are very rare. Using the original decomposition method, this would not decompose. However, using alternative decomposition methods we could for example decompose by simply splitting the vector into the two sub-vectors, such as each of two neural networks deals with changes in one of the sub-vectors.

COULD ADD SAMPLE GRAPHS

5.13 Can have separate subsection for things that didn't work

The mechanism by which the algorithm improves performance is potentially different for small problems and for big problems. I think that in small problems the decomposition makes it so that there are some inputs that can basically be ignored. The performance seems to only improve when these

ifnoresd states exist. This increases the number of aparameters / number of states raio and has a similar effect to increasing the network size. Howver, in a big enironemnt that relies on generalisation I think it would be different and the mains mechanism would be that generalisation has to occur over a smaler subspace thus converging faster.

Chapter 6

Applicability of the proposed technique

The previous section shows that given certain conditions, the state-decomposition method can give a performance gain in terms of the speed of learning of a reinforcement learning agent. Additionally, such methods are able to leverage information from simpler environments to learn faster in a more complicated version of said environments. This section aims to provide a more real-life like examples to illustrate the potential of the state-decomposition method in scenarios where using a single global agent could be deemed wasteful, given the better efficiency of the proposed methods.

In general, techniques based on the state-decomposition method could be developed for any kind of problem that is decomposable. Usually, such a characteristic occurs in systems composed of multiple sub-systems, where the various components seldom interact with each other. It is useful to discuss a possible application of the methodology to better show how such a technique could be applied to a real-life scenario.

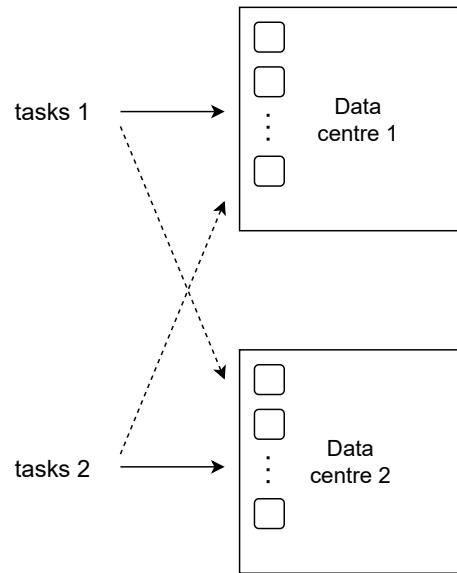


Figure 6.1: Allocation of resources problem for two interconnected data centres.

Consider a scenario where we have two data centres, each of which has multiple computers, as shown in Figure-6.1. Each of these data centres receives tasks (jobs) that can be assigned to the computers and completed. Each of these jobs requires a random time to process on its allocated

machine. At each time-step there can be no task or 1 task arriving to one of the data centres. The resource allocation problem deals with assigning these tasks to machines in the two data centres in order to minimise the average waiting time to completion of the tasks. Most of the time it is most efficient for the data-centre that receives the task to fulfil it, as sending the task to the other data-centres incurs in additional delays. However, sometimes a data-centre is busy enough to justify the additional delay incurred in transferring tasks between data-centres. This problem can be carefully formulated as a Markov Decision Process and solved using reinforcement learning. The MDP can be defined as follows:

- The state of the MDP indicates the number of tasks waiting for each machine, that is, a computer in a data centre. It also indicates the presence of eventual incoming tasks in the data centres. Thus the state can be represented by a vector having an element for each machine, which take integer values indicating the number of jobs waiting for processing in each of the machines. The vector would also include additional two elements, taking values 0 or 1, indicating the presence of new incoming tasks in each of the data centres.
- The actions to be taken by the controller are to assign the tasks to the machines. At each time-step, the controller can allocate the received task to any machine, either in the local or in the remote data centre.
- The rewards can be positive values that occur when tasks are completed. In order to encourage resolution of a task in a local machine one the rewards can be set to be higher when the task is completed in the data centre that received the task than when it is completed by the other data centre.

If the difference of the rewards upon completion of tasks between local and remote data centres is set to be very high, the controller will allocate most tasks locally. The fact that the controller is unlikely to assign tasks to the remote data centre makes it so that the equivalent transitions are unlikely to happen and that the state-space can be decomposed accordingly.

The simplest way of applying a state-decomposition in this scenario would be with a 'DeltaSwitch' agent having two neural networks, one for when a task is received by one data centre and another NN for when a task is received by the other data centre. Though this might be effective, it does not use the decomposition previously discussed based on decomposing by 'ignoring' state transitions below a certain threshold. In this case, an intuitive decomposition of treating two relatively separate problems as such is used.

In the case where the difference in rewards between completing a task locally or remotely is infinite, the tasks would always be solved locally. In this case the problem of deciding which data centre the task should be processed in is inexistant. In this case we could simply have two separate agents for the two data centres that learn how to assign tasks within the data centres, without dealing with their interconnection. However, as the difference in rewards decreases, it becomes optimal to use the interconnection. The 'learn the switch' and transfer learning techniques discussed in the previous chapter in this case would be useful. The networks trained to assign tasks internally within the two data-centres can be combined in order to learn a global agent that can assign tasks in both in the most optimal manner. This architecture is illustrated in Figure-6.2.

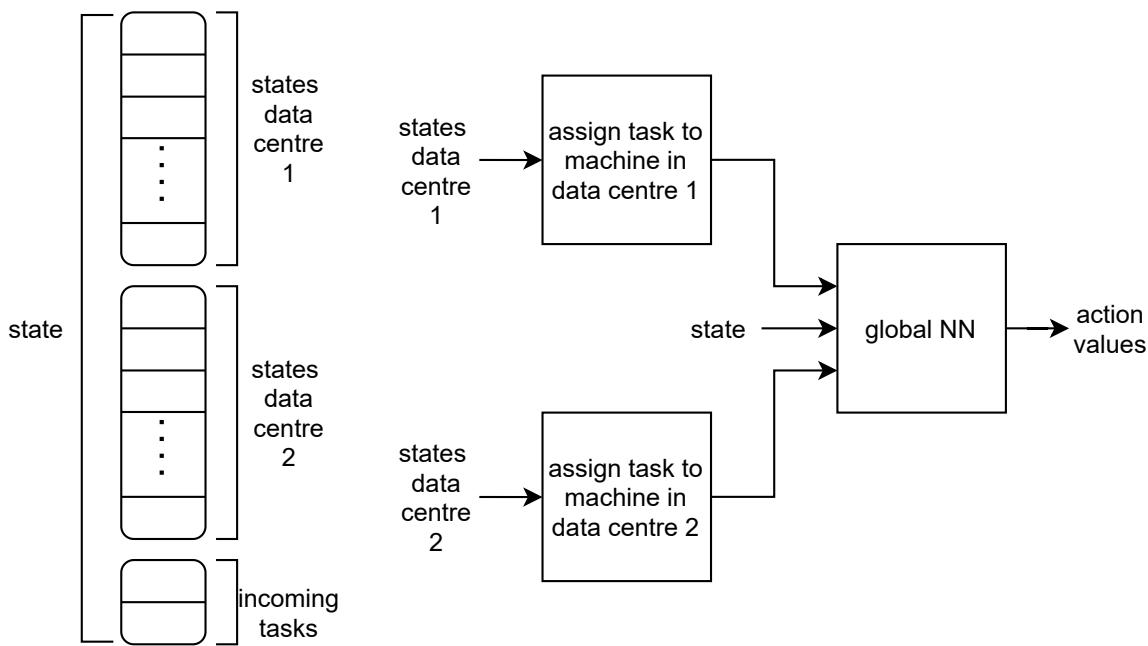


Figure 6.2: Possible state-decomposition agent.

This last methodology is very similar to what would be done in a hierarchical architecture. This would have an agent that explicitly decides whether a task should be assigned to data centre 1 or 2. Given this knowledge a sub-agent trained to decide how to assign the task internally within the data centre would be used and the outputs of these two would be combined to give the full action description. The difference with the state-decomposition method is that while in the latter the full-action description is given by the combination network and all other components merely provide implicit information to it, in the hierarchical approach the action description is formed explicitly by combining the outputs of multiple 'layers' of sub-agents. Additionally, the hierarchical approach deals with the problem on 'high level' to 'low level' approach, in which 'high level' actions are decided first and 'low level' actions are determined using the 'high level' actions, while the state-decomposition approach feeds the knowledge of the possible 'low level' agents to a global agent that deals with all levels of action.

Chapter 7

Conclusions and Further Work

- ONE POSSIBLE LIMITATION IS THAT I TRIED ALL THIS STUFF IN RELATIVELY SIMPLE SCENARIOS, WHERE THE NETWROSK DOES NOT NECESSARILY NEED TO GENERALISE. I THINK THAT IN THE CASE OF LIKE A VIDEO-GAME OR SOMETHING LIKE THAT, THE STATE-DECOMPOSITION WOULD BE EVEN MORE HELPFUL, AS IT WOULD MEAN THAT THE NETWORKS HAVE TO GENERALISE TO A SMALLER SUB-SPACE THAN THE ORIGINAL STATE-SPACE.

- could try to use state-decomposition with policy algorithms

- actually I think that what makes a difference is more like a better decoding

- if you think about it isn't deltaswitch just a type of hierarchical learning? What's the difference?

-DeltaSwitch is basically a hierarchical agent that works for environments that are not perfectly decomposable too

- Need to explain that this project focussed on decomposition using transition probabilities, however this is not the only kind and actually the example given in the data centre part might give an even better decomposition.

- other limitations of my work, small number of samples and incomplete range of scenarios to be tested due to time constraints and computational constraints

The time at which the agents of the sub-spaces should be joined together to form the global agent is not defined. Different strategies can be compared, such as waiting for the sub-agents' action-values to converge or defining different minimum rates of change of the average episode rewards going below which would start the second stage of training.

Disadvantages of state-decomposition: need to determine decomposition, more complicated system, not guaranteed to be faster

7.1 Extensions

Compression and embeddings of states The state-decomposition algorithm divides the original state-space of the system into smaller groups of states. It can be supposed that the variance of the states within these sub-spaces is smaller than in the original state space. Thus, the states that are input in the sub-spaces' agents could be successfully compressed into a more compact representation which could positively affect the performance of the algorithm. More generally, it could be beneficial to learn custom embeddings for the states of each sub-space, having the aim of better generalisation rather than state-space reduction.

Applying state-decomposition to continuous state-spaces The state-decomposition method is only applicable to discrete state spaces. A possible extension would investigate methods to decompose continuous state-spaces. A state-transition matrix could be obtained by discretising the states and the state-space would be decomposed accordingly, while the agents could still be fed the original continuous states as input.

Applying threshold to transitions between whole sub-spaces The original state-decomposition method in this project consists of temporarily ignoring improbable state transitions by applying a threshold to the state-transition matrix. There are scenarios in which groups of states are connected by state-transitions that have a high probability but it would still be useful to consider them as separate sub-spaces. Consider for example Figure-7.1, where an MDP is composed of nearly independent sub-spaces that are only connected by the blue transitions. In such a scenario it could be useful to determine the sub-spaces by limiting the number of possible non-zero transition probabilities between any two sub-spaces rather than applying a probability threshold to every single transition. Alternatively, we could apply a threshold to transition probabilities between sub-spaces instead of single states.

Offline training The samples obtained from the pre-training interaction required to obtain the state-transition matrix could be saved and later added to the experience replay memories of the DQN agents. Offline pre-training could also be conducted using these samples as this could improve sample efficiency. Similarly, transition samples between different sub-spaces are not used in the first stage of training. These could be used to conduct offline pre-training of the merged network.

Sharing information between sub-agents YE basically the link between this and hierarchical innit

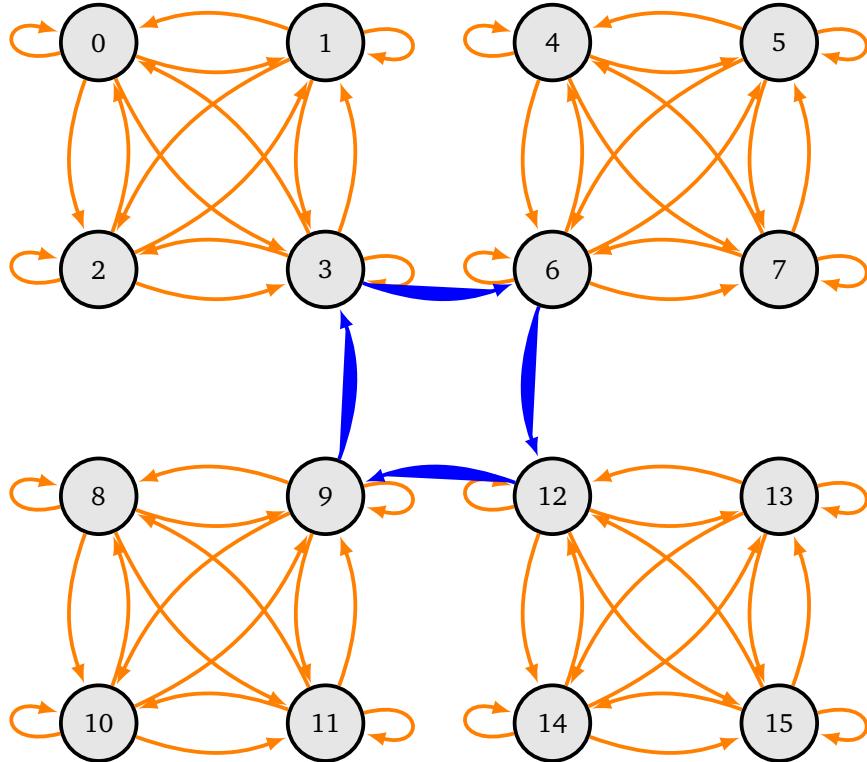


Figure 7.1: MDP composed of smaller sub-spaces of states connected by few but probable transitions.

Using state-decomposition for automatic sub-goal discovery The idea of state-decomposition, especially when implemented by considering transitions between whole sub-spaces could be applied to the automatic discovery of sub-goals in hierarchical reinforcement learning. A similar idea has been adopted by [37] and [15], in which the graph of the MDP is divided into sub-spaces by cutting through the least possible number of state-transitions. The intuition is that sub-goals, such as driving towards the passenger in "Taxi-v3" or driving to the destination after picking it up, are often connected by bottleneck states, such as states 3, 6, 9 and 12 in Figure-7.1.

7.1.1 Application to real-life scenarios

Resource allocation problem, 2 data centres, allocate job to 2nd data-centre Driving taxi Alibaba thing

Bibliography

- [1] Cloud tensor processing units (TPUs). pages 13
- [2] Google AI blog: Scalable deep reinforcement learning for robotic manipulation. pages 2
- [3] Markov decision process. Page Version ID: 1002665350. pages 4
- [4] OpenAI gym: the CartPole-v0 environment. pages 14
- [5] TensorFlow. pages 13
- [6] Mehran Asadi and Manfred Huber. State space reduction for hierarchical reinforcement learning. 2004. pages 7
- [7] Ravi B Kiran, Ibrahim Sobh, Talpaert Victor, Patrick Mannion, Ahmad A. Al Sallab, Senthil Yogamani, and Patrick Pérez. Deep reinforcement learning for autonomous driving: A survey. *arXiv.org*, Feb 2020. pages 2
- [8] Andrew G. Barto and Sridhar Mahadevan. Recent advances in hierarchical reinforcement learning. 13(1):41–77. pages 7
- [9] R. Bellman. Dynamic programming. 153(3731):34–37. pages 5
- [10] Thomas L. Dean, Robert Givan, and Sonia Leach. Model Reduction Techniques for Computing Approximately Optimal Solutions for Markov Decision Processes. *arXiv e-prints*, page arXiv:1302.1533, February 2013. pages 7
- [11] Thomas G. Dietterich. Hierarchical reinforcement learning with the MAXQ value function decomposition. pages 7
- [12] Chris Gaskett, David Wettergreen, and Alexander Zelinsky. Q-learning in continuous state and action spaces. In Norman Foo, editor, *Advanced Topics in Artificial Intelligence*, volume 1747, pages 417–428. Springer Berlin Heidelberg. Series Title: Lecture Notes in Computer Science. pages 7
- [13] David Ha and Jürgen Schmidhuber. World Models. *arXiv e-prints*, page arXiv:1803.10122, March 2018. pages 7, 8
- [14] Diederik P. Kingma and Jimmy Ba. Adam: A method for stochastic optimization. pages 18
- [15] Ishai Menache, Shie Mannor, and Nahum Shimkin. Q-cut—dynamic discovery of sub-goals in reinforcement learning. In Tapio Elomaa, Heikki Mannila, and Hannu Toivonen, editors, *Machine Learning: ECML 2002*, Lecture Notes in Computer Science, pages 295–306. Springer. pages 38
- [16] Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Alex Graves, Ioannis Antonoglou, Daan Wierstra, and Martin Riedmiller. Playing Atari with Deep Reinforcement Learning. *arXiv e-prints*, page arXiv:1312.5602, December 2013. pages 6, 7

- [17] Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Andrei A. Rusu, Joel Veness, Marc G. Bellemare, Alex Graves, Martin Riedmiller, Andreas K. Fidjeland, Georg Ostrovski, Stig Petersen, Charles Beattie, Amir Sadik, Ioannis Antonoglou, Helen King, Dharshan Kumaran, Daan Wierstra, Shane Legg, and Demis Hassabis. Human-level control through deep reinforcement learning. 518(7540):529–533. pages 6
- [18] Thomas M. Moerland, Joost Broekens, and Catholijn M. Jonker. Model-based reinforcement learning: A survey. pages 7
- [19] OpenAI. Gym: A toolkit for developing and comparing reinforcement learning algorithms. pages 13
- [20] Ronald Parr and Stuart Russell. Reinforcement learning with hierarchies of machines. page 7. pages 7
- [21] Romain Paulus, Caiming Xiong, and Richard Socher. A deep reinforced model for abstractive summarization, November 2017. pages 2
- [22] Paul J. Pritz, Liang Ma, and Kin K. Leung. Jointly-Trained State-Action Embedding for Efficient Reinforcement Learning. *arXiv e-prints*, page arXiv:2010.04444, October 2020. pages 7
- [23] Andrei A. Rusu, Neil C. Rabinowitz, Guillaume Desjardins, Hubert Soyer, James Kirkpatrick, Koray Kavukcuoglu, Razvan Pascanu, and Raia Hadsell. Progressive neural networks. pages 30
- [24] David Silver, Thomas Hubert, Julian Schrittwieser, Ioannis Antonoglou, Matthew Lai, Arthur Guez, Marc Lanctot, Laurent Sifre, Dharshan Kumaran, Thore Graepel, Timothy Lillicrap, Karen Simonyan, and Demis Hassabis. Mastering chess and shogi by self-play with a general reinforcement learning algorithm. pages 2, 8
- [25] Richard S. Sutton. Dyna, an integrated architecture for learning, planning, and reacting. 2(4):160–163. pages 8
- [26] Richard S. Sutton and Andrew G. Barto. *Reinforcement learning: an introduction*. The MIT Press, 2018. pages 2, 3
- [27] Richard S. Sutton, Doina Precup, and Satinder Singh. Between MDPs and semi-MDPs: A framework for temporal abstraction in reinforcement learning. 112(1):181–211. pages 7
- [28] Keras Team. Simple. flexible. powerful. Accessed: 2021-01-23. pages 13
- [29] Hado van Hasselt, Arthur Guez, and David Silver. Deep reinforcement learning with double q-learning. pages 7
- [30] Ziyu Wang, Tom Schaul, Matteo Hessel, Hado van Hasselt, Marc Lanctot, and Nando de Freitas. Dueling network architectures for deep reinforcement learning. page 9. pages 7
- [31] Christopher J. C. H. Watkins and Peter Dayan. Q-learning. 8(3):279–292. pages 6
- [32] Christopher John Cornish Hellaby. Watkins. *Learning from delayed rewards*. PhD thesis, University of Cambridge, 1989. pages 6
- [33] Centrum Wiskunde. *DoubleQ-learning Hado van Hasselt Multi-agent and Adaptive Computation Group*. pages 7
- [34] Chao Yu, Jiming Liu, and Shamim Nemati. Reinforcement learning in healthcare: A survey. pages 2
- [35] Ziyao Zhang, Liang Ma, Kin K. Leung, Leandros Tassiulas, and Jeremy Tucker. Q-placement: Reinforcement-learning-based service placement in software-defined networks. *2018 IEEE 38th International Conference on Distributed Computing Systems (ICDCS)*, 2018. pages 2

- [36] Ziyao Zhang, Liang Ma, Konstantinos Poullarakis, Kin K. Leung, and Lingfei Wu. Dq scheduler: Deep reinforcement learning based controller synchronization in distributed sdn. *ICC 2019 - 2019 IEEE International Conference on Communications (ICC)*, 2019. pages 2
- [37] Özgür Şimşek, Alicia P. Wolfe, and Andrew G. Barto. Identifying useful subgoals in reinforcement learning by local graph partitioning. In *Proceedings of the 22nd international conference on Machine learning*, ICML '05, pages 816–823. Association for Computing Machinery. pages 38

Appendix A

Code

```
def stateDecomposition(st_table , thr):
    def joinGroups(i , j , d , groups):
        M = max(d[i] , d[j])
        m = min(d[i] , d[j])
        if d[i] != d[j]:
            groups[m] += groups[M]
            del groups[M]
        for ind , el in enumerate(d):
            if el > M: d[ind] -= 1
            if el == M: d[ind] = m

    n_states = st_table.shape[0]
    d = [i for i in range(n_states)]
    groups = [[i] for i in range(n_states)]

    #iterate through all the entries
    for i , j in np.ndindex(st_table.shape):
        if st_table[i][j] > thr: joinGroups(i , j , d , groups)

    return d , groups
```

Listing A.1: Function which is given the state-transition matrix in input and combines the thresholding of its values with the decomposition into separate MDPs, returning d which is a list indicating which sub-space each state belongs to (as an index) and $groups$ which is a list of lists containing the states of each sub-space.

```
def stateDecompositionWithNGroups(st_table , target_n_groups):
    limitIterations = 50 # number of iterations after which the algorithm stops
    firstChangeFactor = 10.0 # factor by which the threshold is multiplied/divided
                            # after first iteration
    thr = 1.0 # initial threshold

    lowerBound = None
    upperBound = None

    for i in range(limitIterations):
        if lowerBound == None and upperBound != None: thr = upperBound /
            firstChangeFactor
        if lowerBound != None and upperBound == None: thr = lowerBound *
            firstChangeFactor
        if lowerBound != None and upperBound != None: thr = (upperBound + lowerBound) /
            2.0
```

```

stateGroups = stateDecomposition(st_table, thr)
n_groups = len(stateGroups[1])
if n_groups == target_n_groups:
    print("Target number achieved")
    break
if n_groups < target_n_groups: lowerBound = thr
if n_groups > target_n_groups: upperBound = thr
print(thr)

return stateGroups, thr

```

Listing A.2: Function that given the state-transition and a target number of subspaces performs the state-decomposition with different thresholds in order to obtain the correct number of sub-spaces.

```

class StateDecompositionDQNAgent:
    def __init__(self, state_size, action_size, subnetIndexes, subnets):
        self.state_size = state_size
        self.action_size = action_size
        self.gamma = 0.95      # discount rate
        self.epsilon = 1.0     # exploration rate
        self.epsilon_min = 0.01
        self.epsilon_decay = 0.999
        self.learning_rate = 0.001
        self.subnets = subnets
        self.n_subnets = len(stateGroups[1])
        self.subnetIndex = self._subnetIndex_init(subnetIndexes)
        self.memory = [deque(maxlen=50000) for i in range(len(self.subnets)+1)]
        self.models = self._build_models()

    def _subnetIndex_init(self, subnetIndexes):
        ind0s = subnetIndexes
        ind1s = [np.where(self.subnets[ind0s[i]] == i)[0][0] for i in range(self.state_size)]
        return list(zip(ind0s, ind1s))

    def _number_to_one_hot(self, n_classes, x):
        one_hot_vect = np.eye(n_classes)[x]
        reshaped = np.reshape(one_hot_vect, (1, n_classes))
        return reshaped

    def _one_hot_subnet(self, state):
        ind0, ind1 = self.subnetIndex[state]
        n_classes = self.subnets[ind0].shape[0]
        return self._number_to_one_hot(n_classes, ind1)

    def _all_zeros(self, subnet):
        l = len(subnet)
        return np.zeros((1, l))

    def _one_hot_joined(self, state):
        ind0, _ = self.subnetIndex[state]
        one_hot_inputs = []
        for i, subnet in enumerate(self.subnets):
            if i == ind0:
                one_hot_inputs.append(self._one_hot_subnet(state))
            else:
                one_hot_inputs.append(self._all_zeros(subnet))
        return np.concatenate(one_hot_inputs, axis=1)

```

```

def _build_subnet_model(self, subnet):
    subLen = len(subnet)
    inputs = Input(shape=(subLen,), name="input{}".format(subLen))
    x = Dense(50, activation='relu', name="subnet{}_dense_0".format(subLen))(inputs)
    x = Dense(50, activation='relu', name="subnet{}_dense_1".format(subLen))(x)
    x = Dense(50, activation='relu', name="subnet{}_dense_2".format(subLen))(x)
    out = Dense(self.action_size, activation='linear', name="subnet{}_dense_3".format(subLen))(x)
    model = Model(inputs=inputs, outputs=out, name="model{}".format(subLen))
    model.compile(loss='mse',
                  optimizer=Adam(lr=self.learning_rate))
    return model

def _build_models(self):
    return [self._build_subnet_model(s) for s in self.subnets]

def join_models(self):
    def _joined_subnets(input):
        ind = 0
        outputs = []
        for i, subnet in enumerate(self.subnets):
            input_subnet = Lambda(lambda x: x[:, i:i+len(subnet)], name="lambda_{}".format(i))(input)
            return Lambda(lambda x: x[:, i:i+len(subnet)], name="lambda_{}".format(i))(input)
            outputs.append(self.models[i](input_subnet))
            i += len(subnet)
        return Concatenate(name="concatenate_int_outputs")(outputs)

    def _top_network(input):
        n_inputs = self.action_size * self.n_subnets
        x = Dense(24, activation='relu', input_shape=(None, n_inputs,), name="top_dense_0")(input)
        x = Dense(48, activation='relu', name="top_dense_1")(x)
        x = Dense(48, activation='relu', name="top_dense_2")(x)
        x = Dense(self.action_size, activation='linear', name="top_dense_3")(x)
        return x

    input = Input(shape=(self.state_size,), name="intermediate_input")
    int_outputs = _joined_subnets(input)
    output = _top_network(int_outputs)
    model = Model(inputs=input, outputs=output, name="joined_model")
    model.compile(loss='mse',
                  optimizer=Adam(lr=self.learning_rate)) #maybe should have
                                                #separate lr for the top model
    self.models.append(model)

def remember_subnet(self, state, action, reward, next_state, done):
    ind0, _ = self.subnetIndex[state]
    next_ind0, _ = self.subnetIndex[next_state]
    if ind0 == next_ind0:
        self.memory[ind0].append((state, action, reward, next_state, done))
        return ind0
    self.memory[self.n_subnets].append((state, action, reward, next_state, done))
    return -1

def remember_joined(self, state, action, reward, next_state, done):
    self.memory[self.n_subnets].append((state, action, reward, next_state, done))

```

```

def act_subnet(self, state):# We implement the epsilon-greedy policy
    if np.random.rand() > self.epsilon:
        one_hot_state = self._one_hot_subnet(state)
        model = self.models[self.subnetIndex[state][0]]
        act_values = model.predict(one_hot_state)
        return np.argmax(act_values[0]) # returns action
    return random.randrange(self.action_size)

def exploit(self, state): # When we test the agent we dont want it to explore
    # anymore, but to exploit what it has learnt
    act_values = self.models[self.n_subnets].predict(self._one_hot_joined(state))
    return np.argmax(act_values[0])

def act_joined(self, state):# We implement the epsilon-greedy policy
    if np.random.rand() > self.epsilon:
        return self.exploit(state) # returns action
    return random.randrange(self.action_size)

def replay(self, subnet_ind, batch_size, joined):
    if joined:
        index = self.n_subnets
    else:
        index = subnet_ind

    minibatch = random.sample(self.memory[index], batch_size)

    b = False
    for el in minibatch:
        if self.subnetIndex[el[0]][0] != subnet_ind or self.subnetIndex[el[3]][0]
           != subnet_ind:
            return True
    if b: print("WRONG")

    action_b = np.squeeze(np.array(list(map(lambda x: x[1], minibatch))))
    reward_b = np.squeeze(np.array(list(map(lambda x: x[2], minibatch))))
    done_b = np.squeeze(np.array(list(map(lambda x: x[4], minibatch)))))

    ### Q-learning
    if joined:
        state_b_one_hot = np.squeeze(np.array(list(map(lambda x: self.
            _one_hot_joined(x[0]), minibatch))))
        next_state_b_one_hot = np.squeeze(np.array(list(map(lambda x: self.
            _one_hot_joined(x[3]), minibatch))))
    else:
        state_b_one_hot = np.squeeze(np.array(list(map(lambda x: self.
            _one_hot_subnet(x[0]), minibatch))))
        next_state_b_one_hot = np.squeeze(np.array(list(map(lambda x: self.
            _one_hot_subnet(x[3]), minibatch))))

    pred = self.models[index].predict(next_state_b_one_hot)
    target = (reward_b + self.gamma * np.amax(pred, 1))
    target[done_b==1] = reward_b[done_b==1]
    target_f = self.models[index].predict(state_b_one_hot)

    for k in range(target_f.shape[0]):
        target_f[k][action_b[k]] = target[k]
    self.models[index].train_on_batch(state_b_one_hot, target_f)
    if self.epsilon > self.epsilon_min:

```

```
    self.epsilon *= self.epsilon_decay

def load(self, name):
    self.model.load_weights(name)
def save(self, name):
    self.model.save_weights(name)
```

Listing A.3: Current version of my state-decomposition agent. In this version the states are fed in the neural networks as one-hot-encoded integers.