Part2

1. Consider a relation V with attributes LMNOPQRST and functional dependencies W
   a. LPR^+ = LPRQST, not a superkey, violates BCNF
      LR^+ = LRST, not a superkey, violates BCNF
      M^+ = MLO, not a superkey, violates BCNF
      MR^+ = MRN, not a superkey, violates BCNF

   b. For the sake of conciseness, subsets that produce 'obvious' results that do not contribute to the answer have been omitted (i.e. subsets such as {L} have been omitted since it's closure is itself and does not give any information about the FDs).

      LPR^+ = LPRQST violates BCNF of V; chosen for decomposition
      Decomposing with LPR produces relations R_1 = LPRQST, R_2 = LMNOPR

      R_1 = LPRQST

L	**P**	**Q**	**R**	**S**	**T**	**Closure**	**FDs**
✓	✓		✓			LPR^+ = LPRQST	Superkey
✓			✓			LR^+ = LRST	Violates BCNF

      R_1 = LPRQST
      LR^+ = LRST violates BCNF of LPQRST
      Produced relations: R_3 = LRST, R_4 = LPRQ
      R_3 = LRST

L	**R**	**S**	**T**	**Closure**	**FDs**
✓	✓			LR^+ = LRST	Superkey LR^+ = ST

      R_4 = LPRQ

L	**P**	**R**	**Q**	**Closure**	**FDs**
✓	✓	✓		LPR^+ = LPRQST	Superkey LPR^+ = Q

      R_2 = LMNOPR

L	**M**	**N**	**O**	**P**	**R**	**Closure**	**FDs**
	✓					M^+ = MLO	Violates BCNF

$M^+ = MLO$ violates BCNF of LMNOPR

Produced relations: $R_5 = MLO$, $R_6 = MNPR$

$R_5 = MLO$

M	L	O	Closure	FDs
✓			$M^+ = MLO$	Superkey $M^+ = LO$

$R_6 = MNPR$

M	N	P	R	Closure	FDs
✓			✓	$MR^+ = MRN$	Violates BCNF

$R_6 = MNPR$, $MR^+ = MRN$, $R_7 = MRN$, $R_8 = MPR$

$R_7 = MRN$

M	R	N	Closure	FDs
✓	✓		$MR^+ = MRN$	Superkey $MR^+ = N$

$R_8 = MPR$

M	P	R	Closure	FDs

Final decomposition and projection of FDs:

$R_3 = LRST$,

$R_4 = LPQR$

$R_5 = LMO$

$R_7 = MNR$

$R_8 = MPR$

Projecting FDs onto relations:

R_3: $LR \rightarrow ST$

R_4: $LPR \rightarrow Q$

R_5: $M \rightarrow LO$

R_7: $MR \rightarrow N$

R_8: No FDs

2. Consider a relation P with attributes ABCDEFGH and functional dependencies T.
   T = {AB→CD, ACDE→BF, B→ACD, CD→AF, CDE→FG, EB→D}
   a. S1:
      AB → C
      AB → D
      ACDE → B
      ACDE → F
      B → A
      B → C
      B → D
      CD → A
      CD → F
      CDE → F
      CDE → G
      EB → D

      AB → C: A^+ = A, B^+ = BACDF, reduced to B → C
      AB → D: B^+ = BACDF, reduced to B → D
      ACDE → B: Nothing yields B, no reduction
      ACDE → F: CD^+ = CDAF, reduced to CD → F
      B → A: singleton no reduction
      B → C: singleton no reduction
      B → D: singleton no reduction
      CD → A: Nothing yields A, no reduction
      CD → F: Nothing yields F, no reduction
      CDE → F: CD^+ = CDAF, reduced to CD → F
      CDE → G: Nothing yields G, no reduction
      EB → D: B^+ = BACDF, reduced to B → D

      New set S2:
         a. ACDE → B
         b. B → A
         c. B → C
         d. B → D
         e. CD → F
         f. CD → A
         g. CDE → G

Try to eliminate FDs:

ACDE → B $ACDE^+_{S2-\{a\}}$ = ABCDEFG, therefore this FD is needed

B → A, $B^+_{S2-\{(b)\}}$ = BCDAF, removed since B → CD, CD → A

B → C, $B^+_{S2-\{(b),(c)\}}$ = BD, needed

B → D, $B^+_{S2-\{(b),(d)\}}$ = BC, needed

CD → F, $CD^+_{S2-\{(b),(e)\}}$ = CDA, needed

CD → A, $CD^+_{S2-\{(b),(f)\}}$ = CDF, needed

CDE → G, $CDE^+_{S2-\{(b),(g)\}}$ = CDEAFB, needed

Final set:

ACDE → B

B → C

B → D

CD → A

CD → F

CDE → G

b.

Attribute	LHS	RHS	Conclusion
A	✓	✓	Check
B	✓	✓	Check
C	✓	✓	Check
D	✓	✓	Check
E	✓	×	In all keys
F	×	✓	Not in any key
G	×	✓	Not in any key
H	×	×	In all keys

- Any attribute that does not appear in the RHS implies that it cannot be obtained by the FDs and therefore must be in the key
- Any attribute that only appears in the RHS must be inferred from some FD in the set and therefore cannot

$CDEH^+$ = ABCDEFGH, a superkey

BEH^+ = ABCDEFGH, a superkey

AEH^+ = AEH, not a superkey

CEH^+ = CEH, not a superkey

DEH^+ = DEH, not a superkey

$ACEH^+$ = ACEH, not a superkey

$ADEH^+$ = ADEH, not a superkey

All other possibilities must include CDEH or BEH and therefore are not minimal.
Keys: CDEH, BEH

c. Minimal basis:
   ACDE → B
   B → C
   B → D
   CD → A
   CD → F
   CDE → G

   Revised FDs after joining RHSs:
   ACDE → B
   B → CD
   CD → AF
   CDE → G

   Result set relations:
   R_1 {ACDEB}, R_2{BCD}, R_3{CDAF}, R_4{CDEG}, discard R_2 because it is in R_1

   Final set relations:
   R_1 {ACDEB}, R_3{CDAF}, R_4{CDEG}

d. Relation that violates BCNF: CD projects onto R_1 and produces ACD and it is not a superkey.
   Since there exists a relation that violates BCNF, this schema allows redundancy.