# Model Checking Quantum Robots

## Suggestions for Formally Verifying Quantum Robots

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1. Acknowledgments
2. Model Checking
3. Interesting Research Projects???
   - Other Quantum Formal Verification Topics

# Table of Contents

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2. Model Checking
3. Interesting Research Projects???
   - Other Quantum Formal Verification Topics

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# Table of Contents

1 Acknowledgments

2 Model Checking

3 Interesting Research Projects???
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# Milestones in Model Checking (1)

- Traditional Model Checking via Linear Temporal Logic (LTL) or Computational Tree Logic (CTL):
  1. Clarke, Jr., E. M., Grumberg, O., and Peled, D. A. Model Checking. MIT Press, Cambridge, MA, 1999.
  2. Baier, C., and Katoen, J.-P. Principles of Model Checking. MIT Press, Cambridge, MA, 2008.
- Symbolic Model Checking. McMillan, K. L. Symbolic Model Checking: An approach to the state explosion problem. PhD thesis, Carnegie Mellon University, Pittsburgh, PA, May 1992. Or, DOI: 10.1007/978-1-4615-3190-6
- Bounded Model Checking. Biere, A., Cimatti, A., Clarke, E. M., Strichman, O., and Zhu, Y. Bounded model checking. In Advances in Computers, vol. 58 of Advances in Computers. Elsevier Science Publishing, New York, NY, 2003, pp. 117–148.

# Milestones in Model Checking (2)

- CounterExample-Guided Abstraction Refinement (CEGAR). Clarke, E., Grumberg, O., Jha, S., Lu, Y., and Veith, H. Counterexample-guided abstraction refinement. In Proceedings of the 12th International Conference on Computer Aided Verification (CAV 2000) (Chicago, IL, July 15?19 2000), vol. 1855 of Lecture Notes in Computer Science, Springer-Verlag Berlin Heidelberg, pp. 154?169.
- Property Directed Reachability (PDR), or Incremental Construction of Inductive Clauses for Indubitable Correctness (IC3). See `http://theory.stanford.edu/~arbrad/`.

# Table of Contents

1 Acknowledgments

2 Model Checking

3 Interesting Research Projects???
Other Quantum Formal Verification Topics

1. Acknowledgments
2. Model Checking
3. Interesting Research Projects???
   - Other Quantum Formal Verification Topics

# Additional Formal Verification Techniques for Quantum Robots (1)

- Reachability analysis:
    -
    - Possible solution: Quantum partially-observable Markov decision processes (QMDPs)
    - Reference: Shenggang Ying and Mingsheng Ying, "Reachability Analysis of Quantum Markov Decision Processes," *arXiv*, Cornell University, Ithaca, NY, July 9, 2014. Available online from *arXiv* as Version 2 at: `http://arxiv.org/abs/1406.6146v2`; May 30, 2015 was the last accessed date.
- Quantum Equivalence Checking:
    - Given 2 models of a quantum robot, determine if they are functionally equivalent.
    - Use quantum information decision diagrams (QuIDD), global-phase equivalence, and relative-phase equivalence (Viamontes et al., 2009).
    - Ditto for tensor calculus, dynamic tensor products, partial

# Additional Formal Verification Techniques for Quantum Robots (2)

- SMT-based quantum formal verification:
  - Use SMT solver (decision procedure for satisfiability modulo theories) as reasoning engine for formal verification
  - Use reasoning/computational engine for hybrid model checking and theorem proving or model checking and equivalence checking
  - Use fragments of 1st-order logic, such as differential dynamic logics, for hybrid systems verification; see work by Prof. André Platzer at `http://symbolaris.com/`.
  - Get the quantum logic equivalent of these.
  - References:

1. Platzer, A. Logical Analysis of Hybrid Systems: Proving Theorems for Complex Dynamics. Springer- Verlag Berlin Heidelberg, Heidelberg, Germany, 2010.

# Additional Formal Verification Techniques for Quantum Robots (3)

- Exploit equivalence of maximum satisfiability (Max-SAT), pseudo-boolean optimization (PBO) , and wighted PBO:
  1. Use meta-algorithms via algorithmic portfolio optimization to select the "best" or set of good solutions (Max-SAT, Max-SMT, PBO, or Weighted PBO).
  2. Engineer quantum logic variant/equivalent.
  3. Turn numerical models in the time domain to algebraic models in the frequency domain, via transform methods (e.g., Fourier transform, Laplace transform, and z-transform) and approximations via parameterization and (quasi-) linearization
  4. The meta-algorithms enable us to plug-and-play (or plug-and-pray) components for quantum formal verification.

# Additional Formal Verification Techniques for Quantum Robots (4)

- Use Quantum Model Order Reduction for Approximation:
  1. Quantum Model Order Reduction approximates components of quantum robots as continuous-time dynamical systems.
  2. Reference: [1] Viamontes, G. F., Markov, I. L., and Hayes, J. P. Quantum Circuit Simulation. Springer Science+Business Media, B.V., Dordrecht, The Netherlands, 2009.