Reachability Analysis of Quantum Markov Decision Processes

It Is Mandatory For Quantum Robots

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Table of Contents

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Reference: Shenggang Ying and Mingsheng Ying, "Reachability Analysis of Quantum Markov Decision Processes," in *arXiv*, Cornell University, Ithaca, NY, July 9, 2014. Available online from *arXiv* as Version 2 at: `http://arxiv.org/abs/1406.6146` and `http://arxiv.org/abs/1406.6146v2`; May 30, 2015 was the last accessed date.

Warning!!!

- Research publications on formal verification or formal methods have lots of definitions.
- Exact/Approximate algorithms and heuristics for formal verification or formal methods are based on these definitions.
- Hence, exact definitions of terms in formal verification or formal methods are required for proving these algorithms and heuristics and theorems.

Table of Contents

Classical Reachability Analysis

- Reachability Analysis is :
  - Definition of reachable state in a Petri Net or FSM.
  - Define reachability analysis, based on the definition of the reachability of a state in a Petri Net or FSM.
  - E.g., A small bounded Petri Net can have many states, which are represented in the state/reachability graph. Use symmetry or stubborn set reductions, followed by creating and checking CTL formulae and predicates. Determine if a target state/marking is reachable from the initial state/marking, not necessarily via the minimal path. Check CTL properties/predicates for testing liveness properties.
  - Give more examples of reachability analysis
- Explain the importance of reachability analysis
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- References

[1] Yuan, J., Pixley, C., and Aziz, A. Constrained-Based

Problem Statement

- Models of concurrent and nondeterministic quantum systems need to be verified.
- Quantum Markov decision processes (qMDPs) can model such quantum systems.
- Question: How can we carry out reachability analysis on concurrent and nondeterministic quantum systems, modeled as qMDPs?
- Input: qMDP $\mathcal{M}$
- Input: state space $\mathcal{H}$, which is a Hilbert space
- Input: state space $B \in \mathcal{H}$
- Output: Scheduler $\mathfrak{S}$
- Output: Non-negative integer, n.

Shortcomings of Classical Reachability Analysis

- Classic Markov chains cannot capture concurrency.
- A Markov chain only allows one "choice" of action per state, which implies that all "rewards" of the Markov chain are the same.
- Cannot formalize behavior/functionality of quantum systems
    - Discrete state spaces of classical systems are finite or countably finite
    - Continuous state spaces of quantum systems cannot be addressed by discrete state spaces
    - State spaces of quantum systems are continuous, even for finite-dimensional quantum systems
    - Need to examine a finite number of representative elements (in an orthonormal basis) of the state space of a quantum system
    - Or, at most, examine countably infinitely many representative elements of this state space
    - Always preserve the linear algebraic structure of the representative elements [& linear-time properties]

Prior and Related Work

- Almost all previous work use model checking to verify quantum communication protocols
- Use quantum process algebra to verify quantum communication systems, including quantum error correction codes
- Use simulation tools for quantum systems to verify their behavior/functionality, especially their correctness and safety properties

Design Decisions

- Quantum model checking framework for the formal verification of generic quantum engineering systems
    - Not just quantum communication systems
- Use a formal method based on modeling quantum systems with quantum automaton
    - Exploit similar work in quantum Markov chains, quantum dot automata, & quantum cellular automata
- Only consider linear-time properties of generic quantum systems
    - Describe these linear-time properties as infinite sequences of sets of atomic propositions, just like LTL model checking
- Extend this to verify safety properties for reversible automata
- Extend this to verify ω-properties for reversible Büchi automata
- Meet requirements for correctness, safety, & reliability

Key Contributions of (Ying 2014)

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Table of Contents

Analysis on Decidability of Quantum Reachability Analysis In the Finite-Horizon

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Analysis on Decidability of Quantum Reachability Analysis In the Infinite-Horizon

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Table of Contents

Analysis on Complexity of Quantum Reachability Analysis In the Finite-Horizon

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Analysis on Complexity of Quantum Reachability Analysis In the Infinite-Horizon

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Table of Contents

Discussion and Suggested Future Work

- Extensions of