

Reachability Analysis of Quantum Markov Decision Processes

It Is Mandatory For Quantum Robots

Zhiyang Ong

Department of Electrical and Computer Engineering
Dwight Look College of Engineering,
Texas A&M University
College Station, TX

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- 1 Acknowledgments
- 2 Classical Reachability Analysis
- 3 Interesting Research Projects???
Other Quantum Formal Verification Topics

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Reference: Shenggang Ying and Mingsheng Ying, “Reachability Analysis of Quantum Markov Decision Processes,” in *arXiv*, Cornell University, Ithaca, NY, July 9, 2014. Available online from *arXiv* as Version 2 at: <http://arxiv.org/abs/1406.6146> and <http://arxiv.org/abs/1406.6146v2>; May 30, 2015 was the last accessed date.

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Classical Reachability Analysis

- Reachability Analysis is :
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 - E.g., A small bounded Petri Net can have many states, which are represented in the state/reachability graph. Use symmetry or stubborn set reductions, followed by creating and checking CTL formulae and predicates. Determine if a target state/marking is reachable from the initial state/marking, not necessarily via the minimal path. Check CTL properties/predicates for testing liveness properties.
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Problem Statement

- Verify functional correctness of quantum systems.
- Input: Description of system behavior
 - i.e., specifications based on quantum automata
- Input: Functional linear-time properties
 - Linear-time invariants of quantum automata-based model
- Output: Boolean flag indicating correct/incorrect system behavior/functionality
- Examine only a finite number of representative elements of the state space of a quantum system
 - Or at most countably infinitely many representative elements

Shortcomings of Traditional Model Checking

- Cannot formalize behavior/functionality of quantum systems
 - Discrete state spaces of classical systems are finite or countably finite
 - Continuous state spaces of quantum systems cannot be addressed by discrete state spaces
 - State spaces of quantum systems are continuous, even for finite-dimensional quantum systems
 - Need to examine a finite number of representative elements (in an orthonormal basis) of the state space of a quantum system
 - Or, at most, examine countably infinitely many representative elements of this state space
 - Always preserve the linear algebraic structure of the representative elements [& linear-time properties]
- Cannot formally describe temporal properties of quantum systems



Prior Work

- Almost all previous work use model checking to verify quantum communication protocols
- Use quantum process algebra to verify quantum communication systems, including quantum error correction codes
- Use simulation tools for quantum systems to verify their behavior/functionality, especially their correctness and safety properties

Design Decisions

- Quantum model checking framework for the formal verification of generic quantum engineering systems
 - Not just quantum communication systems
- Use a formal method based on modeling quantum systems with quantum automaton
 - Exploit similar work in quantum Markov chains, quantum dot automata, & quantum cellular automata
- Only consider linear-time properties of generic quantum systems
 - Describe these linear-time properties as infinite sequences of sets of atomic propositions, just like LTL model checking
- Extend this to verify safety properties for reversible automata
- Extend this to verify ω -properties for reversible Büchi automata
- Meet requirements for correctness, safety, & reliability

Key Contributions of (Ying 2014)

- Almost all previous work use model checking to verify quantum communication protocols
- Use quantum process algebra to verify quantum communication systems, including quantum error correction codes
- Use simulation tools for quantum systems to verify their behavior/functionality, especially their correctness and safety properties
- Extend this to verify safety properties for reversible automata
- Extend this to verify ω -properties for reversible Büchi automata
- Meet requirements for correctness, safety, & reliability

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Other Quantum Formal Verification Topics

Additional Formal Verification Techniques for Quantum Robots (1)

- Reachability analysis:
 - Possible solution: Quantum partially-observable Markov decision processes (QMDPs)
 - Reference: Shenggang Ying and Mingsheng Ying, "Reachability Analysis of Quantum Markov Decision Processes," *arXiv*, Cornell University, Ithaca, NY, July 9, 2014. Available online from *arXiv* as Version 2 at: <http://arxiv.org/abs/1406.6146v2>; May 30, 2015 was the last accessed date.
- Quantum Equivalence Checking:
 - Given 2 models of a quantum robot, determine if they are functionally equivalent.
 - Use quantum information decision diagrams (QuIDD), global-phase equivalence, and relative-phase equivalence (Viamontes et al., 2009).
 - Ditto for tensor calculus, dynamic tensor products, partial



Additional Formal Verification Techniques for Quantum Robots (2)

- SMT-based quantum formal verification:
 - Use SMT solver (decision procedure for satisfiability modulo theories) as reasoning engine for formal verification
 - Use reasoning/computational engine for hybrid model checking and theorem proving or model checking and equivalence checking
 - Use fragments of 1st-order logic, such as differential dynamic logics, for hybrid systems verification; see work by Prof. André Platzer at <http://symbolaris.com/>.
 - Get the quantum logic equivalent of these.
 - References:
 - ① Platzer, A. Logical Analysis of Hybrid Systems: Proving Theorems for Complex Dynamics. Springer- Verlag Berlin Heidelberg, Heidelberg, Germany, 2010.



Additional Formal Verification Techniques for Quantum Robots (3)

- Exploit equivalence of maximum satisfiability (Max-SAT), pseudo-boolean optimization (PBO), and weighted PBO:
 - ① Use meta-algorithms via algorithmic portfolio optimization to select the “best” or set of good solutions (Max-SAT, Max-SMT, PBO, or Weighted PBO).
 - ② Engineer quantum logic variant/equivalent.
 - ③ Turn numerical models in the time domain to algebraic models in the frequency domain, via transform methods (e.g., Fourier transform, Laplace transform, and z-transform) and approximations via parameterization and (quasi-) linearization
 - ④ The meta-algorithms enable us to plug-and-play (or plug-and-pray) components for quantum formal verification.

Additional Formal Verification Techniques for Quantum Robots (4)

- Use Quantum Model Order Reduction for Approximation:
 - ① Quantum Model Order Reduction approximates components of quantum robots as continuous-time dynamical systems.
 - ② Reference: [1] Viamontes, G. F., Markov, I. L., and Hayes, J. P. Quantum Circuit Simulation. Springer Science+Business Media, B.V., Dordrecht, The Netherlands, 2009.