Elliott Daimler
MATH 4010
Dr. Chastkofsky
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Homework 5, Section 8.2 Solutions

1

Claim: Suppose that N is a subgroup of G and that N has the property that if a and b are any elements of G, then ab belongs to N if and only if ba belongs to N. Then N is a normal subgroup.

2

Claim: The inner automorphisms of a group G are a normal subgroup of $\mathrm{Aut}(G)$.

Proof. Since the inner automorphisms of G form a subgroup, then the inverse of each automorphism φ in $\mathrm{Inn}(G)$ is also in the subgroup.

$$\varphi_g \circ \varphi_x \circ \varphi_g^{-1} \circ \varphi_h^{-1} = gxg^{-1}hxh^{-1}gx^{-1}g^{-1}$$

□

3

Claim: If K is a characteristic subgroup of N and N is a normal subgroup of G, then K is a normal subgroup of G.

4

Claim: Suppose that G is a group all of whose subroups are normal. If a, b, are elements of G, then $ab = ba^k$ for some integer k.