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MATH 4200
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Section 3.4 Solutions

Exercise 3.30

Theorem: Let $(X, \mathcal{T})$ be a topological space, and let $(Y, \mathcal{T}_Y)$ be a subspace. If $\mathcal{B}$ is a basis for $\mathcal{T}$, then $\mathcal{B}_Y = \{B \cap Y | B \in \mathcal{B}\}$ is a basis for $\mathcal{T}_Y$.

Proof. The set $\mathcal{B}$ is a basis for $\mathcal{T}$, so for any $x \in X$ there is a $B \in \mathcal{B}$ where $x \in B$. Since Y is a subset of X, for any $y \in X$ there is also a C where $y \in C$, so $y \in Y \cap C \in \mathcal{B}_Y$.

Next, let U_Y, $V_Y \in \mathcal{B}_Y$ be given where $U_Y \cap V_Y \neq \emptyset$, and let $p \in U_Y \cap V_Y$ be given. There are some $U \in \mathcal{B}$ and $V \in \mathcal{B}$ such that $U \cap Y = U_Y$, $V \cap Y = V_Y$, so $p \in U \cap V$. It follows that there is some $W \subset (U \cap V)$ where $p \in W$, and $(W \cap Y) \subset (U_Y \cap V_Y)$, so $p \in (W \cap Y)$. $\square$

Exercise 3.31

1

$D = \left\{\left(x, \frac{1}{2}\right) | 0 < x < 1\right\}$.

2

$E = \left\{\left(\frac{1}{2}, y\right) | 0 < y < 1\right\}$.

3

$F = \{(x, 1) | 0 < x < 1\}$.