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MATH 4010
Dr. Chastkofsky
MWF 10:20am
April 20, 2022

Bonus Assignment on Separability Solutions

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a

Using the Freshman's Dream property of characteristic p in $\mathbb{Z}_p$, we have $f(x) = (a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0)^p$, so $g(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ and $f(x) = (g(x))^p$.

b

Since F is finite, it has characteristic p. Define a map $\phi(x) = x^p$; since $\mathbb{Z}_p$ is characteristic p, this map is injective. Further, since F is finite and ϕ maps from the set to itself injectively, it is also surjective. So ϕ is bijective and thus for $x^i p$ in F, we have $\phi^{-1}(x^i p) = x^i$. We can then map the polynomial $f(x)$ by $\phi(a_i x^{ip}) = a_i x^i$. Defining $g(x) = \phi(a_i x^{ip})$ for $0 \geq i \geq n$ and again applying the property in part a), we have $f(x) = (g(x))^p$.