Elliott Daimler
MATH 4010
Dr. Chastkofsky
MWF 10:20am
March 4, 2022

Bonus Exercise on Linear Groups

1

Claim: The automorphism group of $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$ is isomorphic to $GL(3, 2)$.

Proof. For readability, call $G = \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$. The group G is generated by $(1, 0, 0), (0, 1, 0), (0, 0, 1)$, which, when treated as vectors in $\mathbb{Z}_2^3$, form a basis. The set of non-singular matrices in $\mathrm{GL}(3, 2)$ are closed under multiplication, and due to their non-singularity are closed under inverses. Their kernel is the zero vector $(0, 0, 0)$, which is the identity element in G. The composition of non-singular matrices in $\mathrm{GL}(3, 2)$ with a vector with entries in $\mathbb{Z}_2$ span the vector space $\mathbb{Z}_2^3$, so $\mathrm{GL}(3, 2)$ is isomorphic to $\mathrm{Aut}(G)$. $\square$