Elliott Daimler
MATH 4200
Dr. Gay
MWF 9:10am
October 20, 2022

Sections 6.1, 6.2 Solutions

Exercise 6.10

Let $(X, \mathcal{T})$ be a topological space made by an indiscrete topology on X, and let U be a proper nonempty subset of X. Since X is the only nonempty open set in $\mathcal{T}$, any covering $\mathcal{C}$ of U must be of the form $\mathcal{C} = X$, which is finite, so U is compact. The complement of U is not open in $(X, \mathcal{T})$, so U is not closed.

Exercise 6.11

Given some topological space $(X, \mathcal{T})$ and subsets A and B, in order for the intersection of A and B to be necessarily compact, we should impose the additional restriction that the intersection of A and B is closed. Then, by Theorem 6.7, $A \cup B$ is compact, and we can treat $A \cap B$ as a closed subspace of X; by Theorem 6.8, this subspace inherits compactness from $A \cup B$. We can extend the intersection to finitely many subsets of X with the condition that the resulting intersection is closed.

Exercise 6.16

Let $(\mathbb{Q}, \mathcal{T}_{std})$ be the topological subspace of rationals with the standard topology inherited from $\mathbb{R}_{std}$. For the constant π, the intervals $(-\infty, -\pi)$ and (π, ∞) are open in $\mathbb{R}_{std}$, so $(-\infty, -\pi) \cap \mathbb{Q}$ and $(\pi, \infty) \cap \mathbb{Q}$ are both open in $(\mathbb{Q}, \mathcal{T}_{std})$. Their complement $[-\pi, \pi] \cap \mathbb{Q}$, henceforth A, is bounded and closed in $(\mathbb{Q}, \mathcal{T}_{std})$, but we can construct an open cover of A that has no finite subcover.

Let $\mathcal{C} = \{(x, y) | x < y \quad and \quad x, y \in A\}$ be an open cover of A. For any finite subset $\mathcal{C}'$ of $\mathcal{C}$, there is some $y_0 < \pi$ such that for every interval $(x, y) \in \mathcal{C}'$, $y \leq y_0$ and there is an $x_0 > -\pi$ such that for every interal $(x, y) \in \mathcal{C}'$, $x \leq x_0$. But there are rationals $y_1, x_1 \in A$ where $y_0 < y_1 < \pi$ and $-\pi < x_1 < x_0$, so y_1 and x_1 are not contained in $\mathcal{C}'$ and $\mathcal{C}'$ does not cover A.