

Section 3.4 Solutions

Exercise 3.30

Theorem: Let (X, \mathcal{T}) be a topological space, and let (Y, \mathcal{T}_Y) be a subspace. If \mathcal{B} is a basis for \mathcal{T} , then $\mathcal{B}_Y = \{B \cap Y \mid B \in \mathcal{B}\}$ is a basis for \mathcal{T}_Y .

Proof. The set \mathcal{B} is a basis for \mathcal{T} , so for any $x \in X$ there is a $B \in \mathcal{B}$ where $x \in B$. Since Y is a subset of X , for any $y \in X$ there is also a C where $y \in C$, so $y \in Y \cap C \in \mathcal{B}_Y$.

Next, let $U_Y, V_Y \in \mathcal{B}_Y$ be given where $U_Y \cap V_Y \neq \emptyset$, and let $p \in U_Y \cap V_Y$ be given. There are some $U \in \mathcal{B}$ and $V \in \mathcal{B}$ such that $U \cap Y = U_Y$, $V \cap Y = V_Y$, so $p \in U \cap V$. It follows that there is some $W \subset (U \cap V)$ where $p \in W$, and $(W \cap Y) \subset (U_Y \cap V_Y)$, so $p \in (W \cap Y)$. \square

Exercise 3.31

1

$$D = \{(x, \tfrac{1}{2}) \mid 0 < x < 1\}.$$

2

$$E = \{(\tfrac{1}{2}, y) \mid 0 < y < 1\}.$$

3

$$F = \{(x, 1) \mid 0 < x < 1\}.$$