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MATH 4010
Dr. Chastkofsky
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Bonus Assignment on 8.4 Solutions

1

Claim: Let K and N be subgroups of a group G, with N normal in G, and let $NK = \{nk|n \in N, k \in K\}$. Then $K/(N \cap K)$ is isomorphic to NK/N.

Proof. By Exercise 20 of section 8.2, NK is a subgroup of G that contains both K and N. We will complete the remainder of the proof in three parts.

a

First, to show that N is a normal subgroup of NK, let $g \in G$ be given. Since N is normal in G, we have gng^{-1}. Now let $a \in NK$ be given. Since NK is a subgroup of G, we have ana^{-1} where $g = a \in NK$, so N is a subgroup of NK.

b

Next, consider the map $f : K \to NK/N$ given by $f(k) = Nk$. This can be rewritten as $Nk = nk$ for $n \in N$, $k \in K$. The kernel of f is all $k \in K$ such that $f(k) = N$, or $\{k \in K|nk \in N\}$. Since every n is in N, this is $\{k \in K|k \in N\}$ or $K \cap N$. Every $k \in K$ is also in NK, so f is surjective.

c

Last, by the First Isomorphism Theorem, the map $f : K \to NK/N$ with kernel $N \cap K$ implies that the quotient group $K/(N \cap K)$ is isomorphic to NK/N. □