The Riddler Classic Solution

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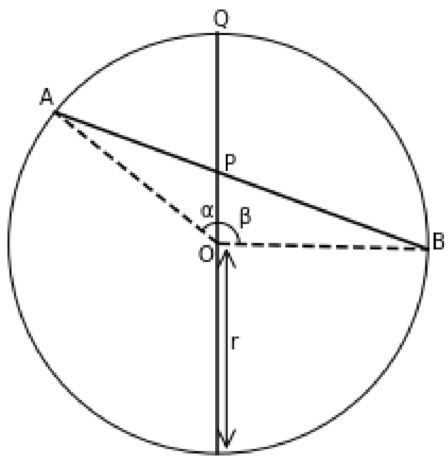
Problem Statement:

From Dean Ballard comes a matter of asymmetrical pizza:

Dean made a pizza to share with his three friends. Among the four of them, they each wanted a different amount of pizza. In particular, the ratio of their appetites was 1:2:3:4. Therefore, Dean wants to make two complete, straight cuts (i.e., chords) across the pizza, resulting in four pieces whose areas have a 1:2:3:4 ratio.

Where should Dean make the two slices?

Solution:
By putting the pieces of sizes 1 and 4 on one side and the pieces of size 2 and 3 on the other, the first cut can simply be made along a diameter. The second cut is more complicated. It is defined by two points on the circumference of a circle, and each of those points can be defined by an angle (see diagram below).



The areas of the top left and top right pieces can each be determined by subtracting the triangular area from the sector's area. Letting Δ_1 and Δ_2 be the top left and top right pieces' areas, respectively:

$$\Delta_1 = \frac{1}{2}\alpha r^2 - |\triangle OAP| = \frac{1}{2}\alpha r^2 - \frac{1}{2}|\overline{OP}|r\sin\alpha \tag{1}$$

$$\Delta_2 = \frac{1}{2}\beta r^2 - |\triangle OBP| = \frac{1}{2}\beta r^2 - \frac{1}{2}|\overline{OP}|r\sin\beta \tag{2}$$

Also, points A and B are at $(-r\sin\alpha, r\cos\alpha)$ and $(r\sin\beta, r\cos\beta)$, respectively, so that the line connecting points A and B has the equation:

$$y = \frac{\cos\beta - \cos\alpha}{\sin\beta + \sin\alpha}x + \frac{\cos\alpha\sin\beta + \cos\beta\sin\alpha}{\sin\beta + \sin\alpha}r. \tag{3}$$

It follows that:

$$\overline{OP} = \frac{\cos\alpha\sin\beta + \cos\beta\sin\alpha}{\sin\beta + \sin\alpha}r = \frac{\sin(\alpha+\beta)}{\sin\alpha + \sin\beta}r. \tag{4}$$

Plugging this into Eqs. (1) and (2) and solving for the values of α and β that yield $\Delta_1 = \frac{1}{10}\pi r^2$ and $\Delta_2 = \frac{2}{10}\pi r^2$ gives

$$\begin{aligned} \alpha &= 51.2^\circ \\ \beta &= 91.5^\circ, \end{aligned}$$

which are the angles shown in the diagram.