

The Riddler Solution

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May 14th 2021

This problem can be solved through dynamic programming, by working backwards from the end state. Define a player's rank as 1 if they win and the number of players in the round they are eliminated in otherwise. Let $V_0^N(q)$ be the rank that a player will finish in, if that player is given a value of q at the start of her turn in a round when there are N players. More generally, let $V_i^N(q)$ be the rank that the player i turns after the current player will ultimately finish in when the current player is given a value of q at the start of his turn in a round of N players. Although the starting player in a round is forced to say 1, we can denote the (absent) starting value of the player by $q = 0$. The boundary conditions are as follows:

$$V_i^N(20) = \begin{cases} N, & \text{if } i = 0 \\ 1, & \text{if } i = 1 \text{ and } N = 2 \\ V_{i-1}^{N-1}(0), & \text{if } i \neq 0 \text{ and } N > 2 \end{cases} \quad (1)$$

The first case indicates that a player which is given a value of 20 is eliminated immediately, in which case that player's rank is N , the number of players in the round. The second case indicates that, in the round of $N = 2$ players, the player not eliminated wins the game. The last case indicates that, for the players not eliminated in a round of $N > 2$ players, the remaining players will place in accordance with the results of the round of $N - 1$.

Optimal play requires that a player try to minimize their rank. Mathematically, a player given a value of q in a round with N players will pick the value q' that minimizes $V_{N-1}^N(q')$, since this is the rank of the player $N - 1$ turns after the next player (i.e., the player herself). Let this optimal action q' be denoted by $A^N(q)$ when a player is given value q in a round with N players. Then:

$$A^N(q) = \begin{cases} 1, & \text{if } q = 0 \\ \arg \min_{q' \in \{q+j | j \in \{1,2,3,4\}, q+j \leq 20\}} V_{N-1}^N(q'), & \text{if } 0 < q < 20 \end{cases} \quad (2)$$

$$V_i^N(q) = V_{i-1}^N(A^N(q)) \quad (3)$$

In the latter equation, the subscript is taken modulo N . These equations were applied in a python program, from which we obtain that:

- The starting player loses the two player round.
- The three player game is ultimately won by player B, with player A coming in second place.
- The four player game is ultimately won by player C, which is followed by players B, D, and finally A.