

The Fiddler Solution

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1 Problem

There are several rectangular prisms with integer edge lengths that have an internal diagonal of length 2024. What is the greatest volume among these prisms?

2 Solution

We are looking to find the integer solution for a , b , and c that maximizes the product abc , subject to the constraint that $a^2 + b^2 + c^2 = 2024^2$. First, note that 2024^2 is a multiple of 4, and all integer squares have residues of either 0 or 1 modulo 4. It follows that a^2 , b^2 , and c^2 all have a residue of 0 modulo 4 or, equivalently, that all three are even. Therefore, we can simplify the problem by dividing each of a , b , c , and 2024 by two. In fact, we can do this three times, and instead find d , e , and f that maximize the product def , subject to the constraint that $d^2 + e^2 + f^2 = 253^2$. The solution to the original problem will have $a = 8d$, $b = 8e$, and $c = 8f$, and the maximum volume will be $abc = 8^3 \cdot def = 512def$.

At this point, the problem is solved computationally, giving $(d, e, f) = (138, 138, 161)$, $(a, b, c) = (1104, 1104, 1288)$, and a maximal volume of 1569835008.