Middle East Technical University Department of Computer Engineering

CENG 382

Analysis of Dynamic Systems

Fall 2018-2019

Homework Assignment 4

Due date: January 17 2018, Thursday, 23:55

Newton-Raphson is a numerical approximation method to the roots of a function. Let $f(x)$ be a well-defined function. To approximate one root of the function we use the following process:

$$x_{n+1} \coloneqq x_n - \frac{f(x_n)}{f'(x_n)}$$

To use the method, we first start with an initial value. In each iteration, we approximate more and more to the true root. This method can be generalized to the complex functions as:

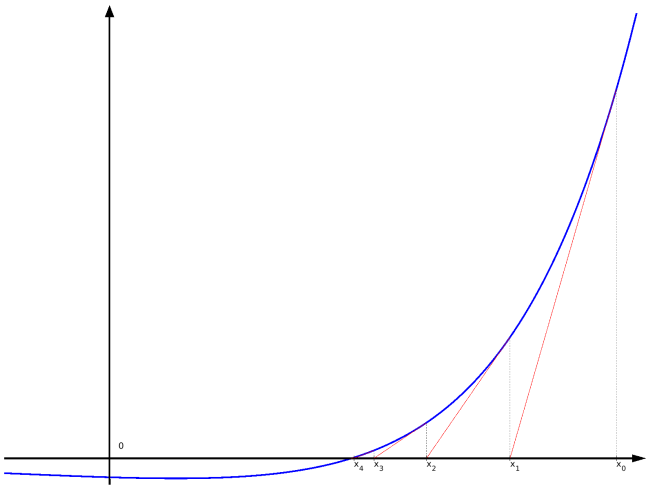


Figure 1: Starting with x_0 we get closer to the root at each iteration

$$z_{n+1} \coloneqq z_n - \frac{f(z_n)}{f'(z_n)}$$

where z_n is a complex number.
Since this method finds the roots based on the initial guess, different guesses may lead to different roots. In the complex plan, the set of all initial guesses that causes the method to converge to a particular root is called a *basin of attraction*. The boundaries of basins of attraction usually lead to fractal patterns in the complex plane which are called as the Newton fractals.

Fig. 2 and 3 illustrate fractals of

$$f(z) = (e^{z^3} - e^8)(z^3 - 8)$$

acquired from different regions of the complex plane.

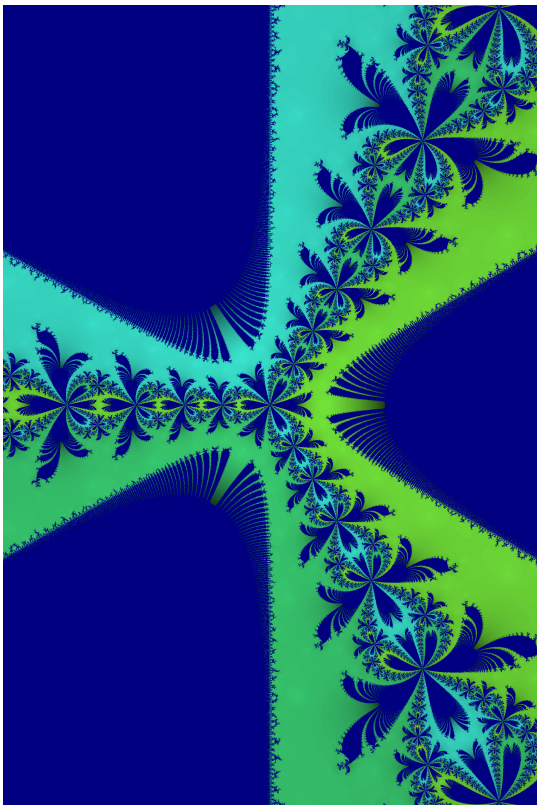


Figure 2: $f(z) = (e^{z^3} - e^8)(z^3 - 8)$'s fractal around $z = 0$.

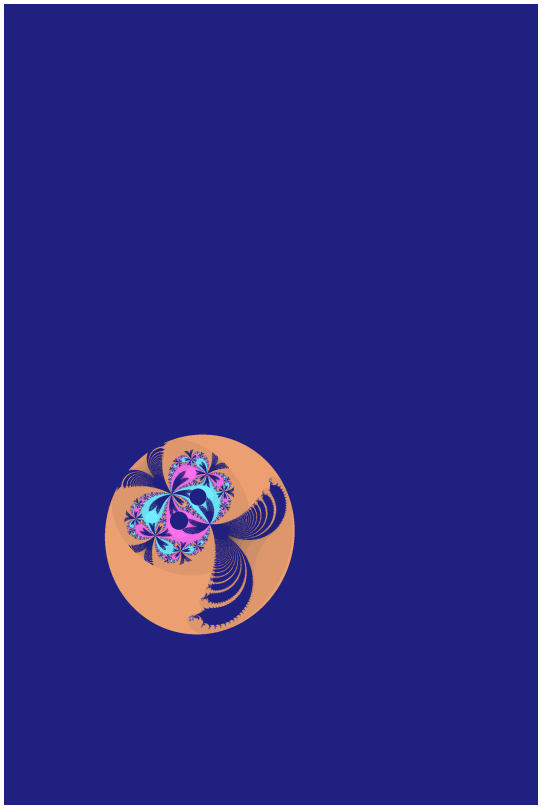


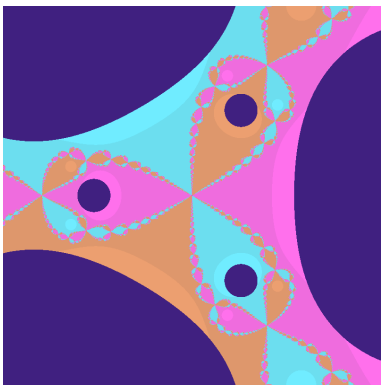
Figure 3: $f(z)$'s fractal from the rectangular region between $z_1 = -0.7 + 1.70i$ and $z_2 = -0.5 + 2i$.

Generalized Newton fractals

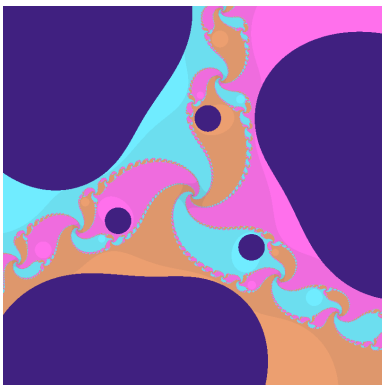
You can use the generalized Newton fractals to generate more complicated patterns. These fractals are acquired by the iterations of the kind

$$z_{n+1} \coloneqq z_n - \alpha \frac{f(z_n)}{f'(z_n)}$$

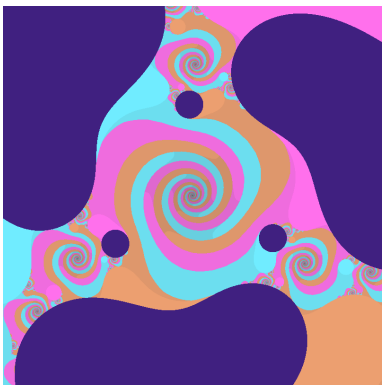
where α is a complex number. See Fig. 4a, 4b and 4c.



(a) $\alpha = 1$



(b) $\alpha = 1.2 + i$



(c) $\alpha = 0.4 + i$

Figure 4: Generalized Newton fractals of $f(z) = z^3 - 8$ for $x, y \in [-2, 2]$ where $z = x + yi$.

Question

Using MATLAB, compute Newton fractals for **three** different complex functions that you choose and write a report showing the fractal pattern for each function. Use different color for different basins of attractions of each function.

Submission

You will submit a report and `assgn4.m` that generates the fractals given in your report.

Regulations

1. It is the student's responsibility to check the validity of their submitted files.
2. **Late Submission:** Not allowed.
3. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.