Trading Costs and Informational Efficiency*

Eduardo Dávila† Cecilia Parlatore‡

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Abstract

We study the effect of trading costs on information aggregation and acquisition in financial markets. For a given precision of investors' private information, an irrelevance result emerges when investors are ex-ante identical: price informativeness is independent of the level of trading costs. When investors are ex-ante heterogeneous, anything goes, and a change in trading costs can increase or decrease price informativeness, depending on the source of heterogeneity. Our results are valid under quadratic, linear, and fixed costs. Through a reduction in information acquisition, trading costs reduce price informativeness. We discuss how our results inform the policy debate on financial transaction taxes/Tobin taxes.

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†Yale University/New York University, Stern School of Business, and NBER. Email: edavila@stern.nyu.edu

‡New York University, Stern School of Business. Email: cparlato@stern.nyu.edu

1 Introduction

There are costs associated with trading in financial markets, and the magnitude of these costs varies over time and across space. For instance, technological advances have dramatically reduced the cost of trading in many financial markets over the last decades. On the flip side, taxes on financial transactions that would increase the cost of trading at the margin along the lines of Tobin's proposal (Tobin, 1978) are periodically the subject of heated debates and sometimes implemented in practice.[1] Given that financial markets play an essential role in generating and aggregating dispersed information, it is natural to study whether changes in the level of trading costs make financial markets better or worse at aggregating information. It is also important to explore whether the ability to trade more or less cheaply encourages or discourages information acquisition in financial markets. In this paper, we seek to provide a systematic answer to these questions by formally studying the implications of trading costs for information aggregation and information acquisition in financial markets.

In our model, investors trade for two reasons. First, investors trade on private information, after receiving a private signal about asset payoffs. These trades contribute to making prices informative about asset payoffs. Second, investors also trade on the realization of a privately known prior, which is random across the population of investors and uncertain in the aggregate. The combination of trading based on private information with a non-payoff related source of aggregate uncertainty makes prices only partially informative about asset payoffs. The fact that prices are partially informative about asset payoffs forces anyone interested in recovering the information aggregated by asset prices to solve a filtering problem.[2] A key determinant of this filtering problem is price informativeness, formally defined as the precision of the unbiased signal about asset payoffs revealed by asset prices. In this paper, we formally characterize how price informativeness, which captures how good asset prices are at aggregating information, varies with the level of trading costs.

Two main results emerge from our initial analysis when investors' precision choices are predetermined and trading costs are quadratic. Our first main result is an irrelevance theorem that applies when investors are ex-ante homogeneous. We show that, for a given precision of investors' private signals, price informativeness is independent of the level of trading costs. The logic behind our main result is elementary and intuitive. The effect of trading costs on how prices aggregate information is a function of how the relevant signal-to-noise ratio contained in asset prices is affected. For example, an increase in trading costs necessarily reduces the amount of information based trading, reducing the informational content of prices. However, this increase in trading costs also reduces the amount of non-payoff relevant trading (due to investors' priors in our case), reducing the noise component of asset prices. When investors are ex-ante identical, the ratio of these two trading motives remains constant as trading costs change, leaving the aggregate signal-to-noise ratio unchanged, yielding the irrelevance result.[3]

[1]See Colliard and Hoffmann (2017) and Jinghan Cai, Jibao He, Wenxi Jiang and Wei Xiong (2017), who provide evidence on the consequences of introducing transaction taxes during the recent experiences of France and China, respectively.

[2]The exact nature of the non-payoff related source of aggregate uncertainty ("noise") is not essential for our results. In the Online Appendix, we derive our main results in a model in which we substitute investors' priors for hedging needs that are uncertain in the aggregate as the second source of trading. For multiple reasons that we highlight throughout the text, our leading formulation is the most workable.

[3]Like other irrelevance results, e.g., Modigliani and Miller (1958), our irrelevance result is pedagogical in nature. Part

Our second main result is a characterization of how trading costs affect price informativeness when investors are ex-ante heterogeneous. We show that anything-goes outside of the homogeneity benchmark, since the sign of the relation between changing trading costs and price informativeness is ambiguous. We identify the cross-sectional covariance of relative demand sensitivities to information and noise with the overall demand sensitivity to trading costs as the key endogenous object that determines how trading costs affect price informativeness. An increase in trading costs decreases (increases) price informativeness when the investors who trade relatively more (less) aggressively on their private information also have an overall demand that is more sensitive to trading costs.

While our characterization in term of endogenous objects is valid in general, we explicitly study how price informativeness reacts to changes in trading costs when investors differ along a single dimension. First, we show that when investors differ in the precision of the private signals about the fundamental, price informativeness decreases with trading costs. Intuitively, better informed investors are disproportionately more responsive to private information but also more responsive overall to trading costs, due to their overall lower perceived risk. Therefore, an increase in trading costs disproportionately reduces the informed trades, reducing price informativeness. Second, we show that when investors differ in the precision of their prior, price informativeness increases with trading costs. Intuitively, investors with tighter priors are more responsive to the realization of the non-payoff relevant component of prices and also more responsive overall to trading costs, due to their overall lower perceived risk. Therefore, an increase in trading costs disproportionately reduces the non-payoff relevant trades, increasing price informativeness. Finally, we show that trading costs do not modify price informativeness when investors are heterogeneous in their risk aversion. Intuitively, in our framework, the cross-sectional distribution of relative demand sensitivities to information and noise is not affected by investors' risk aversion, leaving price informativeness unchanged.

Folk wisdom often associates high trading costs with low price informativeness. Our two main results overturn this logic in two different ways. First, we show that under ex-ante homogeneity, changes in trading costs will not affect the ability of financial markets to aggregate information at all. Our result emphasizes that not only informed trades will be reduced when trading costs are higher, but that also trades that contribute to making the price noisy will be reduced too. Second, we show that heterogeneity among investors is necessary to make trading costs affect price informativeness. In particular, we show that it is possible to generate an ambiguous relation between the level of trading costs and price informativeness merely by considering one-dimensional heterogeneity. Our results highlight that the form in which investors are heterogeneous is also essential to understand the relation between trading costs and price informativeness.

Next, we illustrate how trading costs affect price informativeness in the context of four different applications. These results play a dual role. First, the specific applications allow us to model several scenarios of practical relevance. We consider environments meant to capture institutional and retail investors, informed and uninformed investors, elastic noise traders, and classic noise traders. Second, we use the applications to illustrate how the relation between trading costs and informativeness is

of our contribution lies on identifying the set of assumptions (investor homogeneity and exogenous information precision) that must be violated for trading costs to affect price informativeness.

determined when there are multiple dimensions of heterogeneity. For instance, although Theorem 2 shows that heterogeneity in risk aversion in isolation implies that trading costs do not affect price informativeness, when combined with other sources of heterogeneity, variation in risk aversion makes price informativeness respond to changes in trading costs.

Our applications with noise traders allow us to highlight the importance of how economic "noise" is modeled when studying information aggregation. Classic noise trading, as in Grossman and Stiglitz (1980), is often modeled as an exogenous stochastic demand or supply shock and it is often justified as standing in for hedging needs of unmodeled traders. We show that this formulation is a special case of ours when a group of investors is fully inelastic to prices and trading costs. Although a classic noise trading formulation may be a useful shortcut at times, it is not satisfactory when we seek to understand the effects of trading costs on price informativeness: it is silent on how noise traders react to changes in the level of trading costs, a form of Lucas (1976) Critique. Our results suggest that one must be cautious drawing conclusion from models with classic noise traders.

Subsequently, we allow investors to choose the precision of their private signal about the fundamental in the environment with ex-ante homogeneous investors, in which we know that trading costs do not modify price informativeness for a given set of precisions of private information. In that scenario, we show that an increase in trading costs endogenously reduces the precision of the signal about the fundamental chosen by investors, and consequently price informativeness. Intuitively, high trading costs make it harder for a given investor to profit from acquiring private information. Since investors anticipate that they will be able to profit less from having better information, they choose less precise signals, which reduces equilibrium price informativeness.[4] We can draw two conclusions from this exercise. First, trading costs have sharply different implications for information aggregation and information acquisition. Second, trading costs tend to reduce the endogenous precision of signals about the fundamental, decreasing equilibrium price informativeness.

At last, we show that our main results extend to environments with linear and fixed trading costs. For both types of trading costs, we derive an irrelevance result under ex-ante homogeneity and directional results under one-dimensional heterogeneity, which again take the anything-goes form. The challenge with linear and fixed costs is that some investors will find it optimal not to trade at all, generating inaction regions. Formally, to solve for a linear equilibrium, we need to expand the information set of investors so that they can observe the measures of buyers and sellers and augment the sources of aggregate noise accordingly to avoid the perfect revelation of information. This is a significant technical contribution by itself. To our knowledge, this is the first paper to solve for a Rational Expectations Equilibrium (REE) model with rich heterogeneity with linear and fixed costs, which endogenously generate inaction regions.[5] Conceptually, the irrelevance and directional results with linear and fixed costs are a logical

[4]In an earlier version of this paper, we allowed investors to choose the precision of a private signal about the aggregate non-payoff relevant component (noise). In that case, investors also choose less precise signals about the noise when they face higher trading costs, which also reduce the equilibrium level of price informativeness.

[5]Formally, the closest results are those of Yuan (2005, 2006), and Bond and Garcıa (2018), who solve for a REE with kinked asset demands for two groups of agents. See also the discussion in Vayanos and Wang (2012). Even though the equilibrium of the models with linear and fixed costs that we study can be fully solved and characterized generally, we resort to a small tax/small heterogeneity limit to provide analytical characterizations of comparative statics.

extension of those with quadratic costs, after accounting for the fact that changes in linear and fixed trading costs exclusively change the information and noise aggregated into the price at the extensive margin by varying the set of active investors.

Before concluding, we briefly discuss several practical implications of our results, with the goal to focus policy discussions and to facilitate future empirical work in the area. In particular, we discuss several testable predictions of our theory and we elaborate on how our results contribute to the policy debate on transaction taxes that follows Tobin (1978).

This paper lies at the intersection of two major strands of literature. On the one hand, we share the emphasis of the work that studies the role played by financial markets in aggregating and originating information, following Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981). From a modeling perspective, our benchmark formulation with a continuum of investors is closest to the large economy model in Admati (1985).[6] Our results on endogenous information acquisition are related to the large literature that follows Verrecchia (1982) and Kyle (1989), with recent contributions by Hellwig and Veldkamp (2009) and Van Nieuwerburgh and Veldkamp (2010). Biais, Glosten and Spatt (2005), Vives (2008), and Veldkamp (2009) provide thorough reviews of this line of work. These papers abstract from explicitly modeling trading costs, which is the focus of our paper.

Our results also relate to the body of literature that studies the effects of transaction costs/taxes on financial markets, following Constantinides (1986) and Amihud and Mendelson (1986). More recent contributions include Vayanos (1998); Vayanos and Vila (1999), Gârleanu and Pedersen (2013), and Abel, Eberly and Panageas (2013), among others. These papers focus on the implications of trading costs for volume or prices, while we focus on the effects on information aggregation and information acquisition. We refer the reader to Vayanos and Wang (2012) for a recent survey of this vast literature.[7]

Only a handful of papers feature both technological trading costs and learning, like ours. Vives (2016) shows in a linear-quadratic market game that introducing a quadratic trading cost can be welfare improving by reducing the degree of private information acquisition. Subrahmanyam (1998) and Dow and Rahi (2000) discuss the effect of quadratic trading costs in models of trading with strategic agents. The inherent asymmetry among investors embedded in these papers explains their findings regarding the effects of trading costs. Budish, Cramton and Shim (2015) show that a tax on trading is a coarse instrument to reduce high-frequency trading in a model with learning. In the context of a model of bilateral trading with information acquisition but without information aggregation, Dang and Morath (2015) compare profit and transaction taxes.

Investors in our model trade on private information and on a non-payoff relevant component which is random in the aggregate. Our formulation is similar to the one studied by Ganguli and Yang (2009) and by Manzano and Vives (2011), who use hedging needs as the additional source of trading need and emphasize the emergence of multiple equilibria. We explain how our results related to theirs in detail

[6]We exclusively consider a CARA-Gaussian setup, so our results should be interpreted as a first-order approximation to more general environments (Ingersoll (1987), Huang and Litzenberger (1988)). There is scope to understand how nonlinearities, like those studied by Barlevy and Veronesi (2000), Albagli, Tsyvinski and Hellwig (2012), Breon-Drish (2015), Chabakauri, Yuan and Zachariadis (2015), or Pálvölgyi and Venter (2017), interact with our findings.

[7]A small number of papers address normative issues regarding financial transaction taxes. See Scheinkman and Xiong (2003) and Davila (2014) for two examples of environments in which investors do not learn from prices.

when describing a version of our model with hedging needs in the Online Appendix. Goldstein, Li and Yang (2014) find that multiple equilibria may arise when market segmentation leads to heterogeneous hedging needs.

The remainder of the paper is organized as follows. Section 2 describes the baseline environment and Section 3 characterizes the equilibrium of the model, introducing our main results. Section 4 studies four distinct applications and Section 5 allows for endogenous information acquisition. Sections 6 and 7 respectively extend the results to environments with linear and fixed trading costs. Section 8 discusses practical implications of our results and Section 9 concludes. The Appendix contains derivations and proofs. The Online Appendix contains additional derivations and results.

2 Baseline environment

We initially study a competitive model of financial market trading with investors whose trades are motivated by private information about asset payoffs and by heterogeneous priors. Within this framework, we systematically study how trading costs affect price informativeness.

Preferences There are two dates $t = 1, 2$ and a unit measure of investors, indexed by i. Investors choose their portfolio allocation at date 1 and consume at date 2. Investors maximize constant absolute risk aversion (CARA) expected utility. Therefore, the expected utility of investor i is given by

$$\mathbb{E}\left[U_i\left(w_{2i}\right)\right] \quad \text{with} \quad U_i\left(w_{2i}\right) = -e^{-\gamma_i w_{2i}}, \tag{1}$$

where Eq. (1) imposes that investors consume all their terminal wealth w_{2i}. The parameter $\gamma_i \equiv -\frac{U_i''}{U_i'} > 0$ represents the coefficient of absolute risk aversion.

Investment opportunities There are two assets in the economy, a riskless asset and a risky asset. The riskless asset is in elastic supply and pays a gross interest rate R, normalized to 1. The risky asset is in exogenously fixed supply Q, has a random payoff θ, and is traded in a competitive market at date 1 at price p. This price is quoted in terms of an underlying consumption good (dollar), which acts as numeraire. Each investor i is endowed with q_{0i} units of the risky asset at date 1, where $\int q_{0i} di = Q$, since investors must hold as a whole the total supply of the asset Q. Similarly, market clearing at date 1 implies that $\int \Delta q_{1i} di = 0$, where Δq_{1i} denotes investor i's change in holdings of the risky asset. Investors face no constraints when choosing portfolios: they can borrow and short sell freely.

Random heterogeneous priors From the perspective of investor i, the per unit asset payoff at date 2, denoted by θ, is normally distributed as follows

$$\theta \sim N\left(\hat{\theta}_i, \tau_{\theta i}^{-1}\right),$$

where $\hat{\theta}_i$ denotes the prior expected value for investor i, which is cross-sectionally distributed as follows

$$\hat{\theta}_i = \overline{\theta} + \varepsilon_{\hat{\theta} i},$$

where

$$\varepsilon_{\hat{\theta} i} \sim N\left(0, \tau_{\hat{\theta} i}^{-1}\right) \qquad \text{and} \qquad \overline{\theta} \sim N\left(\mu_{\overline{\theta}}, \tau_{\overline{\theta}}^{-1}\right), \tag{2}$$

and the realizations of $\varepsilon_{\hat{\theta}i}$ are independent across investors. This formulation implies that the realized average prior mean is unknown, introducing a second source of aggregate uncertainty in addition to the uncertainty about the payoff of the risky asset. At times, we refer to θ as the fundamental and to $\overline{\theta}$ as the aggregate sentiment in the economy.

There are different ways to justify heterogeneity in priors: they may capture intrinsic differences in beliefs (optimistic versus pessimistic investors), they may be the result of having observed different private signals in the past, or they could also be interpreted as reflecting heterogeneous private valuations for the risky asset. For our purposes, modeling random heterogeneous priors that vary in the aggregate simply introduces an additional trading motive that prevents prices from being fully revealing.[8]

Information structure Investors do not observe the actual realization of the risky asset payoff, θ. However, every investor receives a private signal s_i about the asset payoff θ, with the following structure

$$s_i = \theta + \varepsilon_{si},$$

where

$$\varepsilon_{si} \sim N\left(0, \tau_{si}^{-1}\right),$$

and the realizations of ε_{si} are independent across investors. We allow for the precision of the private signal to be different for each investor. For now, we take the precisions of investors' private signals, τ_{si}, as a primitive.

Investors do not observe the aggregate sentiment $\overline{\theta}$ in the economy either. Investors only know their own prior, $\hat{\theta}_i$, which is private information of investor i. Importantly, we assume that investors take their priors as given and do not use them to learn about the priors of other investors.

Trading costs We initially consider the case in which investors face quadratic trading costs. In this case, a change in the asset holdings of the risky asset $\Delta q_{1i} \equiv q_{1i} - q_{0i}$ incurs a trading cost, in terms of the numeraire, due at the same time the transaction occurs, for both the buyer and the seller of

$$\frac{c}{2}\left(\Delta q_{1i}\right)^2.$$

Modeling trading costs as quadratic in the size of the trade preserves tractability. Whether c corresponds to the use of economic resources (a trading cost) or whether it corresponds to a transfer (a transaction tax) does not affect the relation between trading costs and price informativeness.

The consumption/wealth of a given investor i at $t = 2$ is given by the random payoff of the risky asset holdings $q_{1i}\theta$ in addition to the investment return on the riskless asset. This includes the net purchase or sale of the risky asset $(q_{0i} - q_{1i})\,p$ and the total trading cost $-\frac{c}{2}\left(\Delta q_{1i}\right)^2$. Formally, the final wealth of investor i is

$$w_{2i} = q_{1i}\theta + q_{0i}p - q_{1i}p - \frac{c}{2}\left(\Delta q_{1i}\right)^2. \tag{3}$$

[8]In previous versions of this paper, we modeled investors' additional trading motives as arising from a random hedging need, instead of using heterogeneous priors. Both formulations yield similar insights. The current formulation is substantially more tractable, since it guarantees equilibrium existence, features a unique equilibrium when investors are ex-ante homogeneous, and allows us to clearly show that trading costs can increase or decrease price informativeness, even with one-dimensional heterogeneity. We re-derive our main results in the context of a fully microfounded model in which hedging needs provide the additional trading motive in the Online Appendix.

We restrict our attention to rational expectations equilibria in which net asset demands are linear in an investor's private signal, his prior, and the price.

Definition. (Equilibrium) A rational expectations equilibrium in linear strategies with quadratic trading costs consists of a linear net portfolio demand Δq_{1i} for every investor i and a price function p such that: a) each investor i chooses Δq_{1i} to maximize his expected utility subject to his wealth accumulation constraint in Eq. (3) and given his information set and b) the price function p is such that the market for the risky asset clears, that is $\int \Delta q_{1i} di = 0$.[9]

We would like to conclude the description of the environment with three remarks.

Remark 1. *There are four relevant dimensions of ex-ante heterogeneity among investors.*

Ex-ante, investors can have different precisions of their private signals τ_{si}, different precisions of their priors $\tau_{\hat{\theta}i}$, different risk aversion γ_i, and different initial holdings of the risky asset q_{0i}. Ex-post, once the aggregate sentiment and the fundamental are realized, investors also differ in their priors about the fundamental $\hat{\theta}_i$ and in the realizations of their signal s_i.

Remark 2. *Uncertainty about the level of aggregate sentiment in the economy makes the filtering problem non-trivial, given that there are no exogenous noise traders in the model.*

The filtering problem faced by investors is non-trivial because the aggregate sentiment is unobservable and random in the aggregate. In particular, if $\tau_{\overline{\theta}} \to \infty$, the aggregate sentiment becomes deterministic, making the equilibrium price fully revealing. In order to have a meaningful filtering problem, many papers studying learning introduce an unmodeled stochastic demand shock or, equivalently, a shock to the number of shares available: this modeling approach is often referred to as having "noise traders". Allowing for noise traders in its standard form – as in Grossman and Stiglitz (1980) – is not appropriate to study the effects of trading costs. In particular, in those models it is hard to understand how the behavior of noise traders varies with the level of trading costs, which is a form of Lucas (1976) Critique. We explicitly compare the predictions of our formulation with those of classic noise trading in the applications studied in Section 4.

Remark 3. *Interpretation of trading costs.* Trading costs in our model can be directly mapped to trading fees charged by exchanges. These fees capture the technological costs of trading and participating in an exchange. For instance, linear costs can be mapped to the marginal cost charged by a competitive constant returns to scale sector that enables trading among investors. Relatedly, fixed costs can be interpreted as participation costs.[10] Trading costs, in particular of the linear form, also map directly to financial transaction taxes. Importantly, trading costs in our model do not correspond to measures of price impact, which are endogenous costs of trading and depend on the trading costs studied in this paper.[11]

[9]Because we adopt a formulation with a continuum of investors, as Admati (1985), our investors do not suffer from the schizophrenia critique of Hellwig (1980).

[10]See Vayanos and Wang (2012) for additional discussions on how to interpret and map trading costs of the form studied here.

[11]In an earlier version of the paper, we studied a version of our model with a finite number of strategic investors, in which changes in trading costs endogenously induce changes in price impact.

3 Price informativeness and trading costs

To characterize the equilibrium, we first study the portfolio problem of an individual investor i. Subsequently, we exploit market clearing to characterize the equilibrium of the model and define price informativeness, which is our main object of interest. We then introduce our main results: an irrelevance theorem for the case of ex-ante homogeneous investors and a characterization of how trading costs affect price informativeness when investors are ex-ante heterogeneous.

Investors' portfolio choice Because of the CARA-Gaussian structure of preferences and returns, the demand for the risky asset of every investor i is given by the solution to a mean-variance problem in q_{1i}. Note that an investor i knows the actual realization of his prior when trading, although that realization is not known to other investors. In particular, investor i chooses q_{1i} to solve

$$\max_{q_{1i}} \left(\mathbb{E}_i\left[\theta|\hat{\theta}_i, s_i, p\right] - p\right) q_{1i} - \frac{\gamma_i}{2}\mathbb{V}ar_i\left[\theta|\hat{\theta}_i, s_i, p\right] q_{1i}^2 - \frac{c}{2}\left(\Delta q_{1i}\right)^2. \tag{4}$$

The first term in the objective function of investor i represents the expected payoff of holding q_{1i} units of the risky asset. This expected payoff increases with the investor's expected value of the fundamental, $\mathbb{E}\left[\theta|\,\hat{\theta}_i, s_i, p\right]$, which is itself increasing in the realizations of the private signal s_i and prior $\hat{\theta}_i$, and decreases with the price he has to pay for the risky asset, p. The second term captures the utility loss suffered by a risk-averse investor who faces uncertainty about the asset payoff. The last term represents the trading cost that the investor must pay to adjust his asset holdings from q_{0i} to q_{1i}.

The optimal risky asset demand for investor i is therefore given by

$$q_{1i} = \frac{\mathbb{E}_i\left[\theta|\,\hat{\theta}_i, s_i, p\right] - p + cq_{0i}}{\gamma_i\mathbb{V}ar_i\left[\theta|\,\hat{\theta}_i, s_i, p\right] + c}. \tag{5}$$

Intuitively, investor i demands more shares of the risky asset when the expected asset payoff $\mathbb{E}\left[\theta|\,\hat{\theta}_i, s_i, p\right]$ is high, when the price of the risky asset is low, and when the perceived risk of the asset $\mathbb{V}ar_i\left[\theta|\,\hat{\theta}_i, s_i, p\right]$ is low. More risk averse investors demand fewer shares of the risky asset.

In an equilibrium in linear strategies, we guess (and subsequently verify) that investor i's optimal net portfolio demand takes the form

$$\Delta q_{1i} = \alpha_{si}s_i + \alpha_{\theta i}\hat{\theta}_i - \alpha_{pi}p + \psi_i,$$

where α_{si}, $\alpha_{\theta i}$, and α_{pi} are non-negative scalars, while ψ_i can take positive or negative values. The coefficients α_{si}, $\alpha_{\theta i}$, and α_{pi} respectively represent the demand sensitivities of investor i to his private signal, his prior about the fundamental, and the price. All of these sensitivities account for the informational content of the relevant variable. In particular, the price sensitivity α_{pi} accounts for the pecuniary cost of acquiring the asset and for the informational content of prices.

Market clearing in the asset market implies that the equilibrium price takes the form

$$p = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta + \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\overline{\theta} + \frac{\overline{\psi}}{\overline{\alpha_p}}, \tag{6}$$

where $\overline{\alpha_s} = \int \alpha_{si}di$, $\overline{\alpha_\theta} = \int \alpha_{\theta i}di$, and $\overline{\alpha_p} = \int \alpha_{pi}di$ are aggregate demand sensitivities to the fundamental, to the aggregate sentiment, and to the price, respectively, and $\overline{\psi} = \int \psi_i di$. A higher fundamental value of the asset θ and a higher aggregate sentiment $\overline{\theta}$ are associated with a higher asset price in equilibrium. The last term in Eq. (6) embeds both the unconditional expected payoff of the risky asset and a risk premium.

The price p contains information about the fundamental value of the asset and about the aggregate sentiment in the economy. Let $\hat{p} = \frac{\overline{\alpha_p}}{\overline{\alpha_s}} p - \frac{\overline{\alpha_\theta}}{\overline{\alpha_s}} \mu_{\overline{\theta}} - \frac{\overline{\psi}}{\overline{\alpha_s}} = \theta + \frac{\overline{\alpha_\theta}}{\overline{\alpha_s}} \left(\overline{\theta} - \mu_{\overline{\theta}} \right)$ be the unbiased signal about the fundamental θ contained in the price p for an external observer. Therefore, the distribution of $\hat{p}$ as a function of θ corresponds to

$$\hat{p} |\, \theta \sim N \left(\theta, \tau_{\hat{p}}^{-1} \right),$$

where

$$\tau_{\hat{p}} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}} \right)^2 \tau_{\overline{\theta}}. \tag{7}$$

We formally define price informativeness as follows.

Definition. (Price Informativeness) We define price informativeness as the precision of the unbiased signal of the payoff θ contained in the asset price from the perspective of an external observer. Formally, price informativeness corresponds to $\tau_{\hat{p}}$, as defined in Eq. (7).

Price informativeness is the key object of study of the paper. Price informativeness provides a direct measure of the ability of financial markets to aggregate dispersed information, along the lines of Hayek (1945). Aggregating dispersed information is one of the major roles played by financial markets. Consequently, any decision-maker who uses the information about the fundamental contained in the price would be able to make more accurate decisions when price informativeness is high, and vice versa.[12]

After solving the filtering problem, investor i's conditional expectation and conditional variance of the fundamental value of the asset take the form

$$\mathbb{E}_i \left[\theta |\, \hat{\theta}_i, s_i, p \right] = \frac{\tau_{\theta i} \hat{\theta}_i + \tau_{si} s_i + \tau_{\hat{p}} \hat{p}}{\tau_{\theta i} + \tau_{si} + \tau_{\hat{p}}}, \tag{8}$$

$$\mathbb{V}ar_i \left[\theta |\, \hat{\theta}_i, s_i, p \right] = \frac{1}{\tau_{\theta i} + \tau_{si} + \tau_{\hat{p}}}. \tag{9}$$

The expectation in Eq. (8) is a weighted average of investor i's prior about the fundamental, $\hat{\theta}_i$, the private signal, s_i, and the signal contained in prices, $\hat{p}$. When prices are completely uninformative ($\tau_{\hat{p}} \to 0$), observing the asset price does not reveal any information about the asset payoff θ. Alternatively, when $\tau_{\hat{p}} \to \infty$, asset prices are arbitrarily precise and observing the asset price perfectly reveals the realization of θ. Without aggregate risk regarding the average sentiment, that is, $\tau_{\overline{\theta}} \to \infty$, it follows directly from Eq. (7) that the equilibrium price is fully revealing and that the Grossman (1976) paradox applies. The variance in Eq. (9) is higher when the prior precision, the precision of the private signal, and the level of price informativeness are low.

[12]There exists a large literature following Blackwell's informativeness criterion (Blackwell, 1953) that seeks to define when a given signal is more informative than other in the sense of being more valuable to a given decision-maker. In the environment that we consider, our definition of price informativeness induces a complete order of price signals for a decision-maker with a quadratic objective function around the fundamental.

Equilibrium The equilibrium of the model is fully characterized by combining investors' portfolio decisions, given by Eq. (5), with the market clearing condition for the risky asset, accounting for the filtering problem solved by the investors. When forming their expectations about the fundamental, investors use all the information available to them. Therefore, each investor i effectively observes two signals about the fundamental θ: the private signal s_i and the public signal revealed by the price p.

The conjectured coefficients of investors' net demands satisfy the following conditions in equilibrium

$$\alpha_{si} = \frac{1}{\kappa_i}\frac{\tau_{si}}{\tau_{\theta i}+\tau_{si}+\tau_{\hat{p}}}, \qquad \alpha_{pi} = \frac{1}{\kappa_i}\left(1 - \frac{\tau_{\hat{p}}}{\tau_{\theta i}+\tau_{si}+\tau_{\hat{p}}}\frac{\overline{\alpha_p}}{\overline{\alpha_s}}\right), \tag{10}$$
$$\alpha_{\theta i} = \frac{1}{\kappa_i}\frac{\tau_{\theta i}}{\tau_{\theta i}+\tau_{si}+\tau_{\hat{p}}}, \qquad \psi_i = -\frac{1}{\kappa_i}\left(\frac{\tau_{\hat{p}}}{\tau_{\theta i}+\tau_{si}+\tau_{\hat{p}}}\left(\frac{\overline{\alpha_\theta}}{\overline{\alpha_s}}\mu_{\overline{\theta}} + \frac{\overline{\psi}}{\overline{\alpha_s}}\right) + \gamma_i \mathbb{V}ar_i\left[\theta |\, \hat{\theta}_i, s_i, p\right] q_{0i}\right),$$

where we define $\kappa_i \equiv \gamma_i \mathbb{V}ar_i\left[\theta |\, \hat{\theta}_i, s_i, p\right] + c$.

The coefficient α_{si}, which determines the sensitivity of the demand for the risky asset with respect to an investor i's private signal, is increasing in the precision of this investor's signal τ_{si}.[13] When the signal is more informative, investors put more weight on their signals since a higher realization of the signal increases the expected payoff of the asset. The coefficient $\alpha_{\theta i}$ determines the sensitivity of the demand for the risky asset with respect to his prior. For a given $\tau_{\hat{p}}$, when the prior is more precise, investors put more weight on their prior belief. The coefficient α_{pi}, which determines the sensitivity of the demand for the risky asset with respect to the asset price, features a substitution effect and an information effect. When $\tau_{\hat{p}} \to 0$, there is no information effect and $\alpha_{pi} \to \frac{1}{\kappa_i}$. In this case, the elasticity of investor i's portfolio demand to the price is given by $\frac{1}{\kappa_i}$, as in the model without learning: this is the standard substitution effect caused by price changes. When prices convey some information ($\tau_{\hat{p}} > 0$), an information effect arises. Investors are less sensitive to price changes since high prices induce investors to infer that the expected asset payoff is high and vice versa. The precision of the information contained in asset prices $\tau_{\hat{p}}$ relative to the information in private signals τ_{si} determines the relative sensitivity of the investor's demand to the asset price α_{pi}. Finally, the coefficient ψ_i determines the autonomous demand for the risky asset, which does not depend on private signals, prices or hedging needs. This term is driven by the unconditional expected value of the asset and the asset risk premium.

We conclude our formal characterization of the equilibrium establishing its existence and uniqueness properties.

Lemma 1. (Existence/Uniqueness) *An equilibrium always exists. When investors are ex-ante identical, the equilibrium is unique.*

The upshot of introducing noise in the form of random heterogeneous priors is that we can guarantee equilibrium existence for any set of primitives, and uniqueness under ex-ante homogeneity and some forms of heterogeneity, as illustrated by our applications. As it is standard in this type of environments, multiple equilibria may exist when investors are ex-ante heterogeneous. Therefore, in general, every result derived when investors are heterogeneous should be interpreted as local for a given equilibrium. To ease the exposition, from now on, we leave out any reference to multiplicity and proceed as if there is a single equilibrium.

[13]Note that this argument relies on having a continuum of investors, which allows us to keep $\tau_{\hat{p}}$ fixed as τ_{si} varies.

Irrelevance theorem and directional results Importantly, the equilibrium values of α_{si}, $\alpha_{\theta i}$, and α_{pi} are directly modulated by κ_i, which is a measure of investors' tolerance to risk and trading costs. The fact that κ_i enters multiplicatively in all three coefficients implies that the ratio $\frac{\alpha_{si}}{\alpha_{\theta i}}$ does not depend directly on the level of trading costs. In turn, this implies that the relative contribution of an individual's trade to price informativeness is not directly affected by trading costs. This observation is useful to establish our first result.

Theorem 1. (Irrelevance result with ex-ante identical investors) *When investors are ex-ante identical, price informativeness is independent of the level of trading costs. Formally, the precision of the unbiased signal about the fundamental revealed by the asset price $\tau_{\hat{p}}$ is independent of c, that is,*

$$\frac{d\tau_{\hat{p}}}{dc} = 0, \quad \forall c.$$

Theorem 1 establishes the first main irrelevance result of the paper. Theorem 1 shows that price informativeness is independent of the level of trading costs when investors are ex-ante homogeneous. That is, two identical economies with different levels of trading costs c will have equally informative prices. Although this result may come as a surprise, its logic is elementary: High trading costs make investors less willing to trade on both their private information and their prior beliefs, leaving unchanged the total relative demand sensitivities to fundamental information and noise and, consequently, the signal-to-noise ratio in asset prices.[14] Therefore, price informativeness is not affected by changes in the level of trading costs. Theorem 1 provides a natural benchmark to understand the role of trading costs on the informational efficiency of the economy: only departures from ex-ante homogeneity across investors can generate an effect of trading costs on information aggregation.

On the one hand, this irrelevance result overturns the conventional wisdom that high trading costs reduce price informativeness. For instance, Vives (2016) uses this conventional view to argue in favor of a transaction tax to reduce price informativeness in a model with classic noise traders. It is also often argued that hard-to-short securities are likely to have less informative prices (e.g., Lin, Liu and Sun (2015)). Our irrelevance result shows that the conditions under which the conventional wisdom apply are not obvious and must implicitly rely on some form of heterogeneity. On the other hand, this irrelevance result is no more than a knife-edge case, so we move on to study how changes in trading costs affect price informativeness when investors are heterogeneous. Our results when investors are heterogeneous exploit the following Lemma.

Lemma 2. (Directional characterization) *When the difference in relative-to-the-average sensitivities between information and private trading motives, $\frac{\alpha_{si}}{\overline{\alpha}_s} - \frac{\alpha_{\theta i}}{\overline{\alpha}_\theta}$, is positively (negatively) correlated in the cross-section of investors with the magnitude of investors' individual demand semi-elasticity to trading costs $\frac{1}{\kappa_i}$, an increase in trading costs c decreases (increases) price informativeness in a given equilibrium. Formally, the sign of $\frac{d\tau_{\hat{p}}}{dc}$ is determined by*

$$\operatorname{sgn}\left(\frac{d\tau_{\hat{p}}}{dc}\right) = -\operatorname{sgn}\left(\mathbb{C}ov_x\left[\frac{\alpha_{si}}{\overline{\alpha}_s} - \frac{\alpha_{\theta i}}{\overline{\alpha}_\theta}, \frac{1}{\kappa_i}\right]\right), \tag{11}$$

[14] The linear structure of asset demands guarantees that a change in the level of trading costs has an identical impact on the demand sensitivities to private information and prior beliefs. This property is not obviously true a priori, even though it does hold in CARA-Normal economies with quadratic transactions costs.

where $\text{sgn}(\cdot)$ *denotes the sign function and* $\mathbb{C}ov_x[\cdot,\cdot]$ *denotes a cross-sectional covariance.*

In Lemma 2, independently of the primitives of the economy, the equilibrium objects α_{si}, $\alpha_{\theta i}$, and κ_i are sufficient statistics to determine how changes in the level of trading costs affect price informativeness. Although every right hand side element in Eq. (11) is endogenous, this characterization illustrates the economic mechanisms at play. Note that, because there is a continuum of investors, the cross-sectional covariance $\mathbb{C}ov_x[\cdot,\cdot]$ in Eq. (11) corresponds to a scalar, due to a Law of Large Numbers.

In general, when investors are heterogeneous, an increase in trading costs can increase or decrease price informativeness, depending on the sign of $-\mathbb{C}ov_x\left[\frac{\alpha_{si}}{\alpha_s}-\frac{\alpha_{\theta i}}{\alpha_\theta},\frac{1}{\kappa_i}\right]$. This is the negative of the cross-sectional covariance of two terms. The first term corresponds to the difference between relative sensitivities to private signals (information) about the fundamental and relative sensitivities to priors (noise). The second term corresponds to the semi-elasticity of investor i's net demand with respect to the trading cost, since

$$-\frac{\partial\Delta q_{1i}/\partial c}{\Delta q_{1i}}=\frac{1}{\kappa_i}.$$

When $\frac{1}{\kappa_i}$ is high, investors trade aggressively and their overall demand is highly sensitive to trading costs. Therefore, when the investors whose demands are relatively more responsive to information than to private trading motives, that is, those with a high $\frac{\alpha_{si}}{\alpha_s}-\frac{\alpha_{\theta i}}{\alpha_\theta}$, are also more sensitive to changes in trading costs, that is, those for which $\frac{1}{\kappa_i}$ is high, Lemma 2 implies that high trading costs reduce price informativeness. In this case, an increase in trading costs disproportionately reduces the fraction of trades due to information, which reduces price informativeness. Alternatively, when the investors whose demands are relatively more responsive to private trading motives than to information, that is, those with a low $\frac{\alpha_{si}}{\alpha_s}-\frac{\alpha_{\theta i}}{\alpha_\theta}$, are also more sensitive to changes in trading costs, that is, those for which $\frac{1}{\kappa_i}$ is high, Lemma 2 implies that high trading costs increase price informativeness. In this case, an increase in trading costs disproportionately reduces the fraction of non-payoff relevant trades, which increases price informativeness.

While Lemma 2 applies generally, it does not provide a formal result relating assumption on primitives to the impact of trading costs on price informativeness. Theorem 2 formally characterizes how changes in trading costs affect price informativeness when investors are heterogeneous across a single dimension.

Theorem 2. (Anything-goes: directional results under one-dimensional heterogeneity) *Let investors differ only in one of the three following dimensions: precision of their private signal about the fundamental, precision of their prior, or risk aversion. If investors differ in:*

a) the precision of their private signal, τ_{si}*, price informativeness decreases with trading costs,* $\frac{d\tau_{\hat{p}}}{dc}<0,\forall c$*;*

b) the precision of their prior, $\tau_{\theta i}$*, price informativeness increases with trading costs,* $\frac{d\tau_{\hat{p}}}{dc}>0,\forall c$*;*

c) risk aversion, price informativeness is unaffected by trading costs, $\frac{d\tau_{\hat{p}}}{dc}=0,\forall c$*.*

Theorem 2 presents the second main result of the paper. It shows that heterogeneity among investors, even along a single dimension, is sufficient to break down the irrelevance result in either direction.[15] The

[15]One-dimensional heterogeneity across initial asset holdings also implies that price informativeness is unaffected by trading costs. This is a consequence of the fact that asset demands are independent of initial asset holdings in CARA-Gaussian setups.

intuition behind the results can be traced back to Lemma 2.[16]

First, when some investors have access to relatively more precise information, those better informed are relatively more responsive to their private information (have a relatively high α_{si}) that others. At the same time, these better informed investors are overall more responsive to trading costs, since their conditional variance $\mathbb{V}ar_i\left[\theta|\,\hat{\theta}_i, s_i, p\right]$, which measures their perceived riskiness of the asset, is relatively lower, due to their more precise information. This makes $\kappa_i = \gamma_i \mathbb{V}ar_i\left[\theta|\,\hat{\theta}_i, s_i, p\right] + c$ lower, and their demand sensitivity to trading costs $\frac{1}{\kappa_i}$, higher. Therefore, exploiting the logic developed in Lemma 2, heterogeneity in the precision of signals endogenously generates a positive cross-sectional covariance between the sensitivity to private signals, $\frac{\alpha_{si}}{\alpha_s}$, and the sensitivity to trading costs, $\frac{1}{\kappa_i}$, which implies that high trading costs are associated with low price informativeness.

Second, when investors differ on their perceived prior precision, those with tighter/more precise priors are relatively more responsive to the realization of their prior (have a relatively high $\alpha_{\theta i}$) than others. At the same time, these investors with more precise priors are overall more responsive to trading costs, since their perceived riskiness of the asset, given by the conditional variance $\mathbb{V}ar_i\left[\theta|\,\hat{\theta}_i, s_i, p\right]$, is relatively lower, due to their perception of having more precise information about the fundamental. This makes $\kappa_i = \gamma_i \mathbb{V}ar_i\left[\theta|\,\hat{\theta}_i, s_i, p\right] + c$ lower, and their demand sensitivity to trading costs $\frac{1}{\kappa_i}$, higher. Therefore, exploiting the logic developed in Lemma 2, heterogeneity in the precision of priors endogenously generates a positive cross-sectional covariance between the sensitivity to investors' priors, $\frac{\alpha_{\theta i}}{\alpha_\theta}$, and the sensitivity to trading costs, $\frac{1}{\kappa_i}$, which implies that high trading costs are associated with high price informativeness.

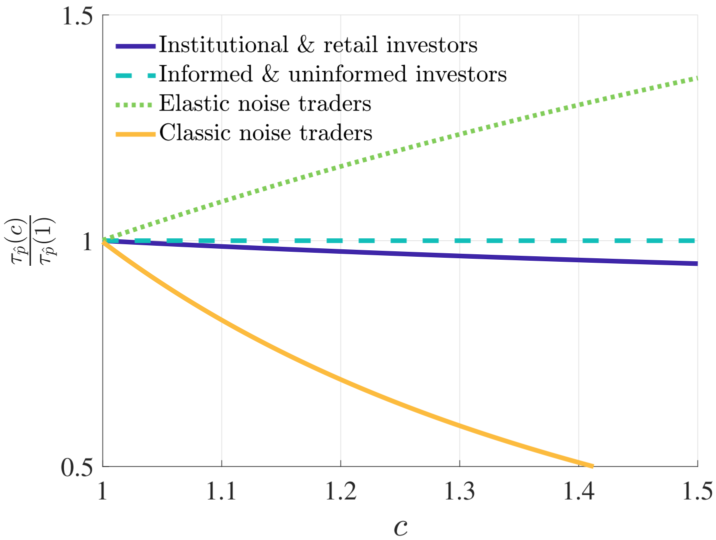
Finally, note that not every form of heterogeneity breaks down the irrelevance result. In particular, heterogeneity in investors' risk aversion leaves price informativeness unaffected by changes in the level of trading costs. In that case, it immediately follows from Eq. (10) that $\frac{\alpha_{si}}{\alpha_s} = \frac{\alpha_{\theta i}}{\alpha_\theta}$, which guarantees that $\mathbb{C}ov_x\left[\frac{\alpha_{si}}{\alpha_s} - \frac{\alpha_{\theta i}}{\alpha_\theta}, \frac{1}{\kappa_i}\right] = 0$. Intuitively, even though more (less) risk tolerant investors become more (less) sensitive to trading costs, there is no systematic pattern among investors relating the sensitivity to trading costs and the informational content of their trades. That is, the relative amount of information and noise that each investor contributes to the price is uncorrelated in the cross-section with the heterogeneity in the sensitivity of investors demands to changes in trading costs. Next, we provide further intuition for our results in the context of four applications.

4 Applications

We now present multiple applications of our baseline environment. This section has a dual goal. First, the specific applications allow us to model several scenarios of practical relevance. In particular, we consider environments meant to capture institutional and retail investors, informed and uninformed investors, elastic noise traders, and classic noise traders. Second, we can illustrate the relation between trading costs and informativeness when there are multiple dimensions of heterogeneity. For instance, although

[16] In previous versions of this paper, under a specific form of hedging needs, we showed that one-dimensional heterogeneity induced a negative association between trading costs and price informativeness. Theorem 2 clearly shows that the relation between trading costs and price informativeness is in general ambiguous and that it depends on the specifics of the model considered.

Theorem 2 shows that heterogeneity in risk aversion in isolation does not affect price informativeness, when combined with other sources of heterogeneity, variation in risk aversion becomes relevant to determine how price informativeness reacts to changes in trading costs. These applications further highlight the ambiguous relationship between trading costs and price informativeness, consistent with our results for one-dimensional heterogeneity in Theorem 2.



Note: Figure 1 shows how price informativeness varies with the level of trading costs (c) for the four different applications considered in Section 4 of the paper. To better illustrate the results, the value of price informativeness for each of the applications is normalized by the level of price informativeness in each application when $c = 1$. This figure graphically extends the anything-goes result from Theorem 2 to cases with multidimensional heterogeneity, since it shows that price informativeness can increase, decrease, or remain constant when trading costs increase. We use the following parameters for each of the applications. Application 1: $\mu = 0.7$, $\tau_\theta = 1$, $\tau_{\overline{\theta}} = 1$, $\tau_s^I = 1$, $\gamma^I = 1.2$, and $\gamma^R = 1.5$. Application 2: $\mu = 0.7$ and $\tau_{\overline{\theta}} = 1$. Application 3: $\mu = 0.6$, $\tau_{\overline{\theta}} = 1$, $\tau_s^I = 1$, and $\gamma = 1.5$. Application 4: $\mu = 0.4$ and $\tau_\theta^N = 0.25$.

Figure 1: Price Informativeness Across Applications

4.1 Institutional and retail investors

In our first application, we model a group of investors with access to better information and with higher risk tolerance (institutional investors), relative to another group of investors who are less informed and less willing to bear risk (retail investors). Formally, suppose that there are two groups of investors that differ along two different dimensions: the precision of their private information and their risk aversion. We refer to one group as institutional investors. These investors receive private signals about the fundamental, $\tau_s^I > 0$, and have low risk aversion, $\gamma^I > 0$. We refer to the other group as retail investors. These investors do not have any private information about the fundamental, $\tau_s^R = 0$, and they have high risk aversion, $\gamma^R > \gamma^I$. There is a fraction μ of institutional investors and a fraction $1 - \mu$ of retail investors.

The equilibrium demand coefficients for institutional investors satisfy

$$\alpha_s^I = \frac{\tau_s^I}{\kappa^I} \quad \text{and} \quad \alpha_\theta^I = \frac{\tau_\theta}{\kappa^I},$$

while those of retail investors satisfy

$$\alpha_s^R = 0 \quad \text{and} \quad \alpha_\theta^R = \frac{\tau_\theta}{\kappa^R},$$

where $\kappa^I = \gamma^I \left(\tau_\theta + \tau_s^I + \tau_{\hat{p}}\right)^{-1} + c$ and $\kappa^R = \gamma^R \left(\tau_\theta + \tau_{\hat{p}}\right)^{-1} + c$. Since $\frac{1}{\kappa^R} < \frac{1}{\kappa^I}$, institutional investors' asset demand is more sensitive to trading costs than retail investors. Moreover, institutional investors contribute more to the informational content of the price than retail investors, since

$$\frac{\alpha_s^I}{\alpha_s} - \frac{\alpha_\theta^I}{\alpha_\theta} > \frac{\alpha_s^R}{\alpha_s} - \frac{\alpha_\theta^R}{\alpha_\theta}.$$

Both observations allow us to exploit Lemma 2 to conclude that an increase in trading costs decreases price informativeness in this scenario, that is, $\frac{d\tau_{\hat{p}}}{dc} < 0$. Intuitively, institutional investors are better informed and more sensitive to trading costs, so an increase in trading costs disproportionately reduces the share of informed trades.

4.2 Perfectly informed and uninformed investors

Our second application models a group of investors with extremely precise information who trades with another group of investors who confidently rely on a signal that has no relation to the fundamental payoff of the asset. Formally, suppose that there are two groups of investors that differ along two different dimensions: the precision of their private signal and the precision of their prior. We refer to one group as informed investors. These investors perfectly observe the fundamental, $\tau_s^I = \infty$, and have a non-degenerate prior, $\tau_\theta^I \in (0, \infty)$. We refer to the other group as uninformed investors. These investors fully trust their prior, $\tau_\theta^U = \infty$, and find their private signal completely uninformative, $\tau_s^U = 0$. There is a fraction μ of informed investors and a fraction $1 - \mu$ of uninformed investors.

The equilibrium demand coefficients for informed investors satisfy

$$\alpha_s^I = \frac{1}{\kappa^I} \quad \text{and} \quad \alpha_\theta^I = 0,$$

while those of uninformed investors satisfy

$$\alpha_s^U = 0 \quad \text{and} \quad \alpha_\theta^U = \frac{1}{\kappa^U},$$

where $\kappa^I = \kappa^U = c$. Note that both groups of investors perceive the asset as safe and bear no residual uncertainty after observing their priors and signals, hence, they behave as if they were risk neutral and react to trading costs and the price in the same way.

Informed investors exclusively trade on their signal, while uninformed investors only trade on their prior. Since all investors fully trust the information at their disposal, neither informed nor uninformed traders face any uncertainty, so differences in risk aversion do not affect the equilibrium. Moreover, investors do not learn from the price, since they believe they have nothing to learn, given that their signals are perfectly informative. However, all investors respond to changes in the price and in trading costs, in the same way, $\kappa^I = \kappa^U = c$. From Lemma 2, this implies that an increase in trading costs does not affect price informativeness in this scenario, that is, $\frac{d\tau_{\hat{p}}}{dc} = 0$.

4.3 Elastic noise traders

In our third application, we consider one group of investors who act as noise traders, in the sense that they purely trade on their prior beliefs that are unrelated to fundamental, but they are responsive to the level of trading costs. These noise traders trade against partially informed investors, who only rely on their private signals, with precision $\tau_s^I \in (0, \infty)$, and have a flat prior, $\tau_\theta^I = 0$. These investors add no noise to the price. Formally, the elastic noise traders do not get any signals, but fully trust their prior, i.e., $\tau_s^U = 0$ and $\tau_\theta^U = \infty$. We assume that all investors have the same risk aversion γ and we respectively denote by μ and $1 - \mu$ the measures of informed traders and noise traders.

The equilibrium demand coefficients for informed investors satisfy

$$\alpha_s^I = \frac{\tau_s^I}{\kappa^I} \quad \text{and} \quad \alpha_\theta^I = 0,$$

while those of uninformed investors satisfy

$$\alpha_s^U = 0 \quad \text{and} \quad \alpha_\theta^U = \frac{1}{\kappa^U},$$

where $\kappa^I = \gamma \left(\tau_s^I + \tau_{\hat{p}}\right)^{-1} + c$ and $\kappa^U = c$. Since $\frac{1}{\kappa^I} < \frac{1}{\kappa^U}$, uninformed investors' asset demand is more sensitive to trading costs, at the same time that they only contribute noise to price. At the same time, informed investors contribute more to the informational content of the price, but their demands are less sensitive to the level of trading costs, so

$$\mathbb{C}ov_x \left[\frac{\alpha_s^i}{\overline{\alpha_s}} - \frac{\alpha_\theta^i}{\overline{\alpha_\theta}}, \frac{1}{\kappa^i} \right] < 0.$$

Both observations allow us to exploit Lemma 2 to conclude that an increase in trading costs increases price informativeness in this scenario, that is, $\frac{d\tau_{\hat{p}}}{dc} > 0$. We will contrast this result below with the case of classic noise traders, who do not trade on information while submitting asset demands that are fully insensitive to the cost of trading, in our fourth and final application.

4.4 Classic noise traders

Our final application captures the classic noise trader formulation, which has been exploited by a large literature following Grossman and Stiglitz (1980). Formally, we consider two groups of investors. A fraction μ of investors are fundamental investors and trade after having observed the true value of the asset θ, i.e., $\tau_s^F = \infty$ and $\tau_\theta^F = 0$. The remaining $1 - \mu$ investors are noise traders whose demand is exogenous and given by $\overline{\theta_N} \sim N\left(0, \left(\tau_\theta^N\right)^{-1}\right)$. Crucially, these investors do not react to the price or to trading costs. Hence, the relevant demand coefficients for fundamental investors are given by

$$\alpha_s^F = \frac{1}{c} \quad \text{and} \quad \alpha_\theta^F = 0,$$

while we can interpret noise traders as having demand coefficients of the form

$$\alpha_s^N = 0 \quad \text{and} \quad \alpha_\theta^N = 1.$$

In this case, price informativeness is given by

$$\tau_{\hat{p}} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}\right)^2 \tau_\theta^N = \left(\frac{\mu}{1-\mu}\frac{1}{c}\right)^2 \tau_\theta^N.$$

It follows from the expression above that price informativeness decreases with trading costs, so $\frac{d\tau_{\hat{p}}}{dc} < 0$. Intuitively, since noise traders do not react to trading costs, the amount of noise contained in the price remains constant. However, the amount of information decreases because the proportion of informed trades by fundamental investors decreases as trading costs increase. The widespread presumption that an increase in trading costs unequivocally decreases price informativeness follows from this framework. However, once one takes a deeper look at where noise in prices is coming from and considers noise being elastic, the effect of trading costs on informativeness depends on the specific setup, as we illustrate in the applications in this section.

Comparing Applications 4.3 and 4.4 indicates that existing common wisdom is driven by the fact that noise traders do not react to trading costs at all. Given its importance, we highlight this result in the following remark.

Remark. (Comparison with standard noise trading formulations) Our irrelevance argument crucially depends on the fact that all investors are symmetrically affected by the change in trading costs. At times, for tractability, models of learning in financial markets assume an ad-hoc supply/demand shock, often referred to as "noise trading". Taken at face value, this assumption leads us to believe that high trading costs are associated with low price informativeness. In these models, an increase in trading costs reduces the amount of information in asset prices because only informed investors react to this change, while noise traders' demand is fully inelastic. The classic noise trading formulation can be viewed as a particular case of our model in which a group of investors inelastically trades on private trading motives. Theorem 2 shows that increasing trading costs in an economy with a set of perfectly inelastic investors who do not trade on information makes prices less informative.

5 Endogenous information acquisition

So far, our analysis has treated the precision of investors' private information as a primitive of the model. In this section, we allow investors to optimally choose the precision of their private signals about the fundamental θ and show that higher trading costs are associated with lower price informativeness.[17] To isolate the effects coming from information acquisition, we focus our attention on the case with ex-ante identical investors, which generates a symmetric equilibrium that satisfies the irrelevance result shown in Theorem 1. Hence, any effects on price informativeness must be driven by investors' information acquisition decision.

The model with exogenous precisions can be interpreted as capturing the short-run response to changes in trading costs, when investors have not adjusted their information gathering technology. The model with endogenous information acquisition can be interpreted as capturing long-run responses, after investors are able to adjust how they gather information.

[17] In previous versions of this paper, we allowed investors to receive a private signal about the source of aggregate noise. In that scenario, investors also acquire less information about that additional signal when trading costs increase.

Environment The exact timing of the investors' choices is represented in Figure 2. As in the baseline model, investors choose their portfolio allocation q_{1i} at date 1, after observing the realizations of their private signal s_i and their prior $\hat{\theta}_i$, while filtering the information contained in the asset price. Now, at date 0, every investor chooses the precision of his private signal τ_{si} at a cost $\lambda(\tau_{si})$, where $\lambda(\cdot)$ is continuous and twice differentiable function that satisfies $\lambda'(\cdot) > 0$, $\lambda''(\cdot) \geq 0$, and the Inada condition $\lim_{\tau_{si}\to\infty} \lambda'(\tau_{si}) = \infty$. To simplify the analysis, we set $q_{0i} = 0$ and $\mu_\theta = \mu_{\bar{\theta}} = 0$.

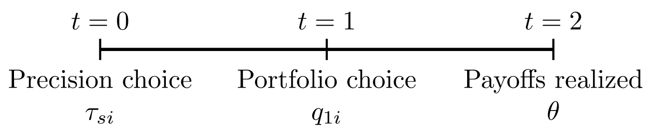


Figure 2: Timeline for endogenous information acquisition

The equilibrium of this augmented game is subgame perfect, that is, it takes into account the equilibrium played in the trading stage. We continue to restrict our attention to equilibria in linear strategies in the trading stage. Since investors could potentially choose different precisions at date 0, there may be multiple equilibria in the trading subgame, so we must allow for a non-degenerate probability distribution over the subgame equilibria when looking at the information acquisition decision of investors.

Let $\mathcal{S}(\boldsymbol{\tau_s})$ be the set of all stable equilibria when the precisions of the private signals chosen by the investors is $\boldsymbol{\tau_s} \equiv \{\tau_{si}\}_i$. Let $\{\pi_h(\boldsymbol{\tau_s})\}_{h\in\mathcal{S}(\boldsymbol{\tau_s})}$ be the probability distribution over all stable equilibria in the trading stage, with $\sum_h \pi_h(\boldsymbol{\tau_s}) = 1$. As we show in Lemma 1, when investors are ex-ante identical, the equilibrium in the trading game is unique with $\frac{\alpha_s}{\alpha_\theta} = \frac{\tau_s}{\tau_\theta}$. We define by $\boldsymbol{\tau_{s,-i}} \equiv \{\tau_{sj}\}_{j\neq i}$ the set of private information precisions for all investors different from i.

Definition. (Equilibrium) An equilibrium in the information acquisition game is a set of precision choices for each investor i, $\boldsymbol{\tau_s^\star} = \{\tau_{si}^\star\}_i$ and a probability distribution over the equilibria in the trading stage, $\{\pi_h(\boldsymbol{\tau_s})\}_{h\in\mathcal{S}(\boldsymbol{\tau_s})}$ such that each investor chooses the precision of his private signal $\tau_{si}^\star$ to maximize $V\left(\tau_{si}; \boldsymbol{\tau_{s,-i}^\star}\right)$, as defined in Eq. (12), taking as given the precision choices of other investors $\boldsymbol{\tau_{s,-i}^\star}$.

Investors' information choice Each investor i takes the equilibrium of the model starting at date 1 and the other investors' precision choices, $\boldsymbol{\tau_{s,-i}}$ as given when choosing the precision of his private signal. Specifically, an investor i optimally chooses τ_{si} by solving

$$\max_{\tau_{si}} V(\tau_{si}; \boldsymbol{\tau_{s,-i}}), \quad \text{where} \quad V(\tau_{si}; \boldsymbol{\tau_{s,-i}}) = \sum_{h\in\mathcal{S}(\boldsymbol{\tau_s})} \pi_h(\tau_{si}; \boldsymbol{\tau_{s,-i}}) \mathbb{E}\left[v_i^h(\tau_{si}; \boldsymbol{\tau_{s,-i}})\right] - \lambda(\tau_{si}), \tag{12}$$

and where $\mathbb{E}\left[v_i^h(\tau_{si}; \boldsymbol{\tau_{s,-i}})\right]$ corresponds to the mean-variance utility of investor i if equilibrium $h \in \mathcal{S}(\boldsymbol{\tau_s})$ arises in the trading game in which the profile of private information precisions is given by $\boldsymbol{\tau_s}$. More specifically, $\mathbb{E}\left[v_i^h(\tau_{si}; \boldsymbol{\tau_{s,-i}})\right]$ is given by

$$\mathbb{E}\left[v_i^h(\tau_{si}; \boldsymbol{\tau_{s,-i}})\right] = \mathbb{C}ov\left[\left(\mathbb{E}_i^h\left[\theta|\,\hat{\theta}_i, s_i, p^h\right] - p^h\right), q_{1i}^{h\star}\right] - \frac{1}{2}\left(\gamma \mathbb{V}ar_i^h\left[\theta|\,\hat{\theta}_i, s_i, p^h\right] + c\right)\mathbb{E}\left[\left(q_{1i}^{h\star}\right)^2\right],$$

where $q_{1i}^{h\star}$ and p^h correspond to the date 1 outcomes in equilibrium h, which are a function of the precision choices of all investors.[18] The conditional moments of θ take into account the trading equilibrium played, h, and the precision of investor i's information, and the expectation $\mathbb{E}[\cdot]$ is taken over the realization of the prior, $\hat{\theta}_i$, the realization of the signal, s_i, and the equilibrium played if there are multiple equilibria in the trading stage of the game.

Best responses and equilibrium determination In an interior solution, the first order condition of the investor's problem in Eq. (12) characterizes the best response of investor i when the second order condition for the investor's problem holds. Formally, the best response $\tau_{si}(\boldsymbol{\tau_{s,-i}})$ is given by the solution to

$$\sum_{h\in\mathcal{S}(\boldsymbol{\tau_s})} \pi_h(\tau_{si};\boldsymbol{\tau_{s,-i}}) \left(\underbrace{\frac{\partial \mathbb{C}ov\left[\left(\mathbb{E}_i^h\left[\theta|\,\hat{\theta}_i, s_i, p\right] - p\right), q_{1i}^{h\star}\right]}{\partial \tau_{si}}}_{\Delta \text{ in accuracy}} - \underbrace{\frac{\gamma}{2}\frac{\partial \mathbb{V}ar_i^h\left[\theta|\,\hat{\theta}_i, s_i, p\right]}{\partial \tau_{si}}\mathbb{E}\left[\left(q_{1i}^{h\star}\right)^2\right]}_{\Delta \text{ in perceived risk}} \right) \tag{13}$$

$$= \sum_{h\in\mathcal{S}(\boldsymbol{\tau_s})} \pi_h(\tau_{si};\boldsymbol{\tau_{s,-i}}) \left(\underbrace{\frac{\gamma}{2}\mathbb{V}ar_i^h\left[\theta|\,\hat{\theta}_i, s_i, p\right]\frac{\partial \mathbb{V}ar\left[q_{1i}^{h\star}\right]}{\partial \tau_{si}}}_{\Delta \text{ in risk taking}} + \underbrace{\frac{c}{2}\frac{\partial \mathbb{V}ar\left[q_{1i}^{h\star}\right]}{\partial \tau_{si}}}_{\Delta \text{ in trading costs}} \right) + \underbrace{\lambda'(\tau_{si})}_{\Delta \text{ in information cost}}.$$

The left hand side of Eq. (13) represents the expected marginal benefit of increasing the precision of the private signal. It has two terms. First, increasing the precision of the signal about the fundamental changes the accuracy of the demand function submitted by an investor i (in each equilibrium h). An investor wants to have a high demand for the risky asset when it offers a good return, and vice versa. Second, increasing the precision of the signal about the fundamental reduces the level of risk perceived by the investor. The right hand side of Eq. (13) represents the expected marginal cost of increasing the precision of the private signal. It has three terms. The first term captures the change in risk born by the investor when the expected final asset holdings change. The second term corresponds to the marginal change in trading costs. The last term is the marginal cost of increasing the precision of the signal. Note that since investors are infinitesimal, an individual investor's choice does not affect the set of equilibria in the trading stage nor the probability of an equilibrium being played.

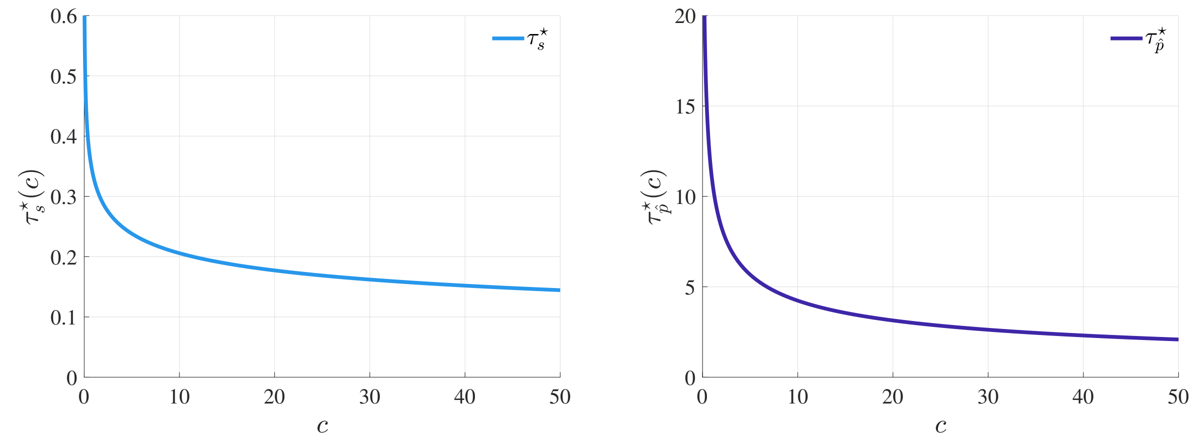
A higher precision of the private signal received by investors increases the accuracy of their demand and reduces their perceived variance of the fundamental. Then, by inspecting Eq. (13), we can see that, since investors can benefit less from acquiring information when trading costs are higher, information acquisition decreases with trading costs. This is the main result of this section, formally stated in Theorem 3.

Theorem 3. (Negative effect of trading costs on endogenous information acquisition) *When investors are ex-ante identical, an increase in trading costs decreases the precision of the information acquired about the asset payoff in an interior symmetric equilibrium. This reduction in information acquisition also generates a reduction in price informativeness, Formally,*

$$\frac{d\tau_{si}^{\star}}{dc} < 0, \quad \textit{and} \quad \frac{d\tau_{\hat{p}}}{dc} < 0.$$

[18] Our choice of objective function is standard in these environments. It is studied and justified in Veldkamp (2009) and Van Nieuwerburgh and Veldkamp (2010). They show that the expected utility case delivers analogous qualitative insights.

As Theorem 3 shows, higher trading costs induce investors to choose less precise signals in equilibrium, which makes prices less informative. Figure 3 further illustrates the effect of trading costs on the equilibrium information acquisition choices and price informativeness.



(a) Equilibrium information precision

(b) Equilibrium price informativeness

Note: The left figure shows investors' optimal information acquisition choice in a symmetric equilibrium as a function of the level of trading costs c. The right figure shows the equilibrium price informativeness implied by the equilibrium choice of $\tau_s^\star$. In both figures, we set $\lambda(x) = 0.01\frac{x^2}{2}$, $\gamma = 1$, $\tau_\theta = 0.1$, $\tau_{\bar{\theta}} = 1$, and $\tau_{\hat{\theta}} = 0.1$.

Figure 3: Equilibrium comparative statics

Our irrelevance result derived in the case of exogenously given information precisions for identical investors does not extend to situations in which investors acquire information. Intuitively, the presence of trading costs makes acquiring information less profitable for every individual investor. In equilibrium, even though the reduction on the precision of information acquired by every other investor in the economy due to the trading costs increases the incentives for an individual investor to acquire information (he learns less from the price), this effect is not large enough to overcome the original reduction of information precision choice caused by the higher trading cost. While intuitive, to our knowledge, the derivation of these results is novel.

6 Linear costs

In this section, we adapt the baseline environment from Section 2 to incorporate linear trading costs. Modeling linear (and fixed) costs introduces significant technical challenges, due to the to the impact of the presence of inaction regions on the filtering problem solved by investors.[19] To overcome these challenges, we modify the baseline model in three dimensions. First, we introduce a finite number of investor types, each facing a type-specific shock to their beliefs about the asset payoff. Second, we augment the information set of investors to include the measures of sellers and buyers of each type.

[19]Inaction regions arise because investors whose initial asset holdings are close to their optimal level of asset holdings experience a second-order welfare gain from adjusting their portfolios, but face a first-order welfare loss associated with trading. When trading costs are quadratic, the welfare loss is second-order, so it is optimal for (almost) every investor to have a non-zero net trading position.

Finally, we introduce random hedging needs to prevent the price from being fully revealing. The modified environment guarantees the existence of an equilibrium in linear strategies, as described below.

Environment We consider an economy with preferences and investment opportunities identical to those described in the baseline model in Section 2. We denote the asset payoff by θ. We assume that there are $n = \{1, ..., N\}$ types of investors, each of them in unit measure. We denote the set of type n investors by I^n. We index individual investors of any given type by i.

An investor i of type n has risk aversion γ_n and an initial endowment of the risky asset given by q_0^n, normalized to 0. Moreover, every investor i of type n receives a private signal s_i^n about the asset payoff θ of the form

$$s_i^n = \theta + \varepsilon_{si}^n, \quad \text{where} \quad \varepsilon_{si}^n \sim N\left(0, \tau_{sn}^{-1}\right),$$

where the realizations of ε_{si}^n are independent across investors. Every investor i of type n has a prior about the asset payoff given by

$$\theta \sim N\left(\hat{\theta}_i^n, (\tau_{\theta n})^{-1}\right)$$

where $\hat{\theta}_i^n$ denotes the prior expected asset payoff for a given investor. This prior mean is also stochastic and it is distributed according to

$$\hat{\theta}_i^n = \overline{\theta_n} + \varepsilon_{\hat{\theta}i}^n,$$

where

$$\varepsilon_{\hat{\theta}i}^n \sim N\left(0, (\tau_{\hat{\theta}n})^{-1}\right) \quad \text{and} \quad \overline{\theta_n} \sim N\left(\omega_\theta, \tau_{\overline{\theta}}^{-1}\right),$$

and where all random variables are independent of each other.

Finally, investors have an endowment at date 2 which is correlated with the asset payoff. This endowment is given by $n_{2i}^n = h_i^n \theta$, where $h_i^n = \delta + \varepsilon_{hi}^n$ with

$$\varepsilon_{hi}^n \sim N\left(0, (\tau_{hn})^{-1}\right) \quad \text{and} \quad \delta \sim N\left(\omega_\delta, \tau_\delta^{-1}\right).$$

The random variable h_i^n denotes the hedging need of an investor i of type n and δ is the aggregate hedging need in the economy, which is not observable.

This formulation implies that there are $N + 2$ sources of aggregate uncertainty in the economy: N coming from the average prior means of the asset payoff for each investor type $n = \{1, \ldots, N\}$, the aggregate hedging need in the economy δ, and the asset payoff θ. As in the previous sections, we assume that investors take their private trading motives as given and do not use them to make inferences about the noise in the price, which allows us to provide equilibrium existence in Lemma 3.[20]

Investors face a linear trading cost $\phi \geq 0$ per share traded of the risky asset. In this case, a change in the asset holdings of the risky asset $|\Delta q_{1i}^n|$ incurs a trading cost, in units of the numeraire, of

$$\phi \left|\Delta q_{1i}^n\right|,$$

so the final wealth of an investor i of type n is

$$w_{2i}^n = n_{2i}^n + q_{1n}^n \theta + q_0^n p - q_{1i}^n p - \phi \left|\Delta q_{1i}^n\right|. \tag{14}$$

[20] It is well-known from Ganguli and Yang (2009) and Manzano and Vives (2011) that equilibrium existence is not guaranteed if investors use private trading motives to make inferences about the noise.

There are two benefits of modeling trading costs as linear. First, linear costs overcome the problem of order slicing associated with any nonlinear trading cost. This is the reason why policy discussions regarding transaction taxes revolve around linear taxes. Second, linear costs can be derived as the compensation to a group of perfectly competitive outside agents that operate a constant returns to scale technology that enables trading. This interpretation allows us to directly interpret changes in trading costs as reductions in the physical cost of trading that are passed through as lower trading fees to investors.

Each investor of type n is characterized ex-ante by the set of parameters $\lambda_n = \left\{\tau_{sn}, \tau_{\theta n}, \tau_{\hat{\theta} n}, \tau_{hn}, \gamma_n, q_0^n\right\}$. When there are linear trading costs, some investors will find the cost of trading too high and choose not to trade altogether. The set of active investors within each group depends on the realization of the aggregate variables. This dependence implies that the equilibrium price is non-linear in the fundamental, which prevents us from solving investors' filtering problem as in standard models. To overcome this challenge and guarantee the existence of an equilibrium in linear strategies, we expand the information set of investors and assume that all investors observe the measures of buyers and sellers of each type, μ_B^n and μ_S^n.[21] As it will become clear when characterizing the equilibrium below, these measures contain information about the fundamental value of the asset and act as public signals.

Definition. (Equilibrium) A rational expectations equilibrium in linear strategies with linear trading costs consists of a linear net portfolio demand Δq_{1i}^n for each investor i of type n and a price function p such that: a) each investor i of each type n chooses Δq_{1i}^n to maximize his expected utility subject to his wealth accumulation constraint in Eq. (14) and given his information set, which includes the measures of buyers and sellers of each type, μ_B^n and μ_S^n, and b) the price function p is such that the market for the risky asset clears, that is $\sum_n \int_{I^n} \Delta q_{1i}^n di = 0$.

Characterization of equilibrium The demand for the risky asset of an investor i in group n is given by the solution to

$$\max_{q_{1i}^n} \left(\mathbb{E}_n\left[\theta|\mathcal{I}_i^n\right] - p\right) q_{1i}^n + p q_0^n - h_i^n \gamma_n \mathbb{V}\mathrm{ar}_n\left[\theta|\mathcal{I}_i^n\right] q_{1i}^n - \frac{\gamma_n}{2} \mathbb{V}\mathrm{ar}_n\left[\theta|\mathcal{I}_i^n\right] \left(q_{1i}^n\right)^2 - \phi \left|\Delta q_{1i}^n\right|,$$

where $\mathcal{I}_i^n = \left\{\hat{\theta}_i^n, h_i^n, s_i^n, p, \left\{\mu_B^j, \mu_S^j\right\}_{j=\{1,\ldots,N\}}\right\}$ denotes the information set of an investor i of type n and $\Delta q_{1i}^n \equiv q_{1i}^n - q_0^n$. The optimal portfolio choice of an investor i of type n, which features an inaction region, is given by

$$\Delta q_{1i}^n = \begin{cases} \Delta q_{1i}^{n+} = \frac{\mathbb{E}_n\left[\theta|\mathcal{I}_i^n\right] - h_i^n \gamma_n \mathbb{V}\mathrm{ar}_n\left[\theta|\mathcal{I}_i^n\right] - p - \phi}{\gamma_n \mathbb{V}\mathrm{ar}_n\left[\theta|\mathcal{I}_i^n\right]} - q_0^n, & \text{if} \quad \Delta q_{1i}^{n+} > 0 \\ 0, & \text{if} \quad \Delta q_{1i}^{n+} \leq 0, \text{ and } \Delta q_{1i}^{n-} \geq 0 \\ \Delta q_{1i}^{n-} = \frac{\mathbb{E}_n\left[\theta|\mathcal{I}_i^n\right] - h_i^n \gamma_n \mathbb{V}\mathrm{ar}_n\left[\theta|\mathcal{I}_i^n\right] - p + \phi}{\gamma_n \mathbb{V}\mathrm{ar}_n\left[\theta|\mathcal{I}_i^n\right]} - q_0^n, & \text{if} \quad \Delta q_{1i}^{n-} < 0. \end{cases} \tag{15}$$

The measure of active buyers and sellers of each type of investor depends on the realization of the asset payoff θ, on the realization of the type specific sentiment, $\overline{\theta}_n$, and on the aggregate hedging need

[21] Since the measure of active investors corresponds to the sum of measures of buyers and sellers, that is, $\mu_A^n = \mu_B^n + \mu_S^n$, knowledge about two out of the three measures is sufficient to back out the other. The realism of this assumption varies with the environment considered. At times, it may be easy to argue that investors have a good understanding of demand and supply pressures among different market participants.

in the economy, δ. Therefore, these measures contain information about θ. We respectively denote by $\hat{\mu}_B^n$ and $\hat{\mu}_S^n$ the unbiased signals about the asset payoff θ contained in the measures of buyers and sellers of type n. We formally describe how to calculate such signals starting from the information set of an investor i in the Appendix. In an equilibrium in linear strategies, we postulate (and subsequently verify) that investors' net demand functions for buyers and sellers, Δq_{1i}^{n+} and Δq_{1i}^{n-}, are respectively given by

$$\Delta q_{1i}^{n+} = \alpha_s^n s_i^n + \alpha_\theta^n \hat{\theta}_i^n - \alpha_h^n h_i^n - \alpha_p^n p + \sum_{j=1}^{N} \left(\alpha_{\mu B}^{jn} \hat{\mu}_B^j + \alpha_{\mu S}^{jn} \hat{\mu}_S^j \right) + \psi^{n+}$$
$$\Delta q_{1i}^{n-} = \alpha_s^n s_i^n + \alpha_\theta^n \hat{\theta}_i^n - \alpha_h^n h_i^n - \alpha_p^n p + \sum_{j=1}^{N} \left(\alpha_{\mu B}^{jn} \hat{\mu}_B^j + \alpha_{\mu S}^{jn} \hat{\mu}_S^j \right) + \psi^{n-},$$

where α_s^n, α_θ^n, α_h^n, α_p^n, $\alpha_{\mu B}^{jn}$, and $\alpha_{\mu S}^{jn}$ are non-negative scalars, while ψ^{n+} and ψ^{n-} can take positive or negative values. Investors' net demands are linear in the private signals s_i, their prior $\hat{\theta}_i^n$, their private hedging needs h_i^n, the price p, and in the signals contained in the measures of buyers and sellers of each type j, respectively $\hat{\mu}_B^j$ and $\hat{\mu}_S^j$, but not necessarily in the measures μ_B^j and μ_S^j themselves.

Lemma 3. (Existence) *An equilibrium in linear strategies with linear tradings costs generically exist.*

By expanding the information set of the investors at the time of trading, the equilibrium price remains linear in the fundamental, which allows investors to solve a linear-gaussian filtering problem. We explicitly characterize the equilibrium step-by-step in the Appendix. Without expanding the information set to allow investors to observe the measures of buyers and sellers of each type, it is not possible to solve for a linear equilibrium, since the measures of active investors of each type depend on the realization of θ. If these measures are not observed by investors, this dependence makes the price not linear in θ, making it impossible to solve the filtering problem.

Price informativeness and linear trading costs In the Appendix, we show that the set of buyers and the set of sellers contain the same unbiased signal of the price, i.e., $\hat{\mu}_B^n = \hat{\mu}_S^n$.[22] Therefore, we can assume without loss of generality that $\alpha_{\mu S}^{jn} = 0$ for all j, n. Given this, it's worth highlighting that we need $N+2$ sources of aggregate noise to avoid perfect revelation of information, since investors have $N+1$ public signals: the N measures of buyers/sellers and the price.

Market clearing in the asset market implies

$$p = \frac{\overline{\alpha_s}}{\overline{\alpha_p}} \theta + \sum_{n=1}^{N} \frac{\alpha_\theta^n \mu_A^n}{\overline{\alpha_p}} \overline{\theta_n} - \frac{\overline{\alpha_h}}{\overline{\alpha_p}} \delta + \sum_{n=1}^{N} \frac{\overline{\alpha_{\mu B}^n}}{\overline{\alpha_p}} \hat{\mu}_B^n + \frac{\overline{\psi}}{\overline{\alpha_p}}, \tag{16}$$

where $\overline{\alpha_s} = \sum_{n=1}^N \alpha_s^n \mu_A^n$, $\overline{\alpha_h} = \sum_{n=1}^N \alpha_h^n \mu_A^n$, $\overline{\alpha_{\mu B}^n} = \sum_{j=1}^N \alpha_{\mu B}^{nj} \mu_A^n$, $\overline{\alpha_p} = \sum_{n=1}^N \alpha_p^n \mu_A^n$, $\overline{\psi}$ is a constant term defined in the Appendix, and $\mu_A^n = \mu_B^n + \mu_S^n$ is the measure of active investors of type n. Since $\hat{\mu}_B^n$ forms part of investors' information set, the unbiased signal of θ contained in the price for an external observer who observes all public signals but only learns from the price can be expressed as a linear transformation

[22] This result is by no means obvious, as carefully explained in the Appendix. Even though investors need to know the measure of buyers and sellers to be able to conjecture the equilibrium price, the informational content of both measures is identical, because of the symmetry of the normal distribution.

of the price as follows

$$\hat{p} = \frac{\overline{\alpha_p}}{\overline{\alpha_s}} \left(p - \sum_{n=1}^{N} \frac{\alpha_\theta^n \mu_A^n}{\overline{\alpha_p}} \omega_\theta + \frac{\overline{\alpha_h}}{\overline{\alpha_p}} \omega_\delta - \sum_{n=1}^{N} \frac{\overline{\alpha_{\mu B}^n}}{\overline{\alpha_p}} \hat{\mu}_B^n - \frac{\overline{\psi}}{\overline{\alpha_p}} \right)$$
$$= \theta + \sum_{n=1}^{N} \frac{\alpha_\theta^n \mu_A^n}{\overline{\alpha_s}} \left(\overline{\theta_n} - \omega_\theta \right) - \frac{\overline{\alpha_h}}{\overline{\alpha_s}} \left(\delta - \omega_\delta \right).$$

For a given realization of the set of aggregate states $\left\{ \theta, \left\{ \overline{\theta_n} \right\}_{n=1}^{N}, \delta \right\}$, price informativeness corresponds to

$$\tau_{\hat{p}} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_N}} \right)^2, \quad \text{where} \quad \left(\overline{\alpha_N} \right)^2 \equiv \sum_{n=1}^{N} \left(\alpha_\theta^n \mu_A^n \right)^2 \tau_{\overline{\theta}}^{-1} + \left(\overline{\alpha_h} \right)^2 \tau_\delta^{-1}.$$

Unlike in the case of quadratic costs, the level of price informativeness depends in general on the realization of the set of aggregate states through the measure of active investors of each type. However, since the measures of buyers and sellers of each type are publicly observed, investors and external observers alike can back out the degree of price informativeness in the economy without uncertainty. After formally defining price informativeness, we are ready to establish a new irrelevance result.

Theorem 4. (Irrelevance result with ex-ante identical investors and linear trading costs) *In an economy with linear trading costs, when investors are ex-ante identical, price informativeness is independent of the level of trading costs. Formally, the precision of the unbiased signal about the fundamental revealed by the asset price $\tau_{\hat{p}}$ is independent of ϕ, that is*

$$\frac{d\tau_{\hat{p}}}{d\phi} = 0, \quad \forall \phi.$$

Theorem 4 shows that our irrelevance argument is not specific to assuming quadratic trading costs, applying also when trading costs are linear. When trading costs are linear, an increase in trading costs is associated with a reduction in trading on both intensive and extensive margins – some investors cease to trade altogether. However, because the decrease in trading at the extensive margin reduces both fundamental and sentiment trades in equal proportions, price informativeness remains unchanged when ϕ varies. This result is valid for any realization of the set of aggregate states. In fact, when investors are ex-ante identical, there exists a linear equilibrium even without augmenting the investors' information set. In this equilibrium, the asset price is independent of the measures of buyers and sellers, eliminating the potential dependence of price informativeness on the realizations of aggregate states. It is trivial to prove the more general irrelevance result with both linear and quadratic trading costs, given by $\phi \left| \Delta q_{1i} \right| + \frac{c}{2} \left| \Delta q_{1i} \right|^2$.

We introduce the directional results as in Section 3. We first characterize an intermediate result that expresses the response of informativeness to trading costs as a function of a few high-level endogenous variables, and then use this result to show that the relation between linear trading costs and price informativeness is ambiguous and it depends on the source of investor heterogeneity.

Lemma 4. ***(*Directional characterization with linear trading costs)** *When the difference between the marginal relative contribution to the average sensitivities to information and noise of a change in the share of active investors of each group, $\frac{\frac{\partial \overline{\alpha_s}}{\partial \mu_A^n}}{\overline{\alpha_s}} - \frac{\frac{\partial \overline{\alpha_N}}{\partial \mu_A^n}}{\overline{\alpha_N}}$, is positively (negatively) correlated in the cross-section* *of investors with the marginal impact of an increase in linear costs on the measure of active investors, $\frac{\frac{\partial \mu_A^n}{\partial \phi}}{\mu_A^n}$, an increase in trading costs ϕ increases (decreases) price informativeness in a given equilibrium. Formally, the sign of $\frac{d\tau_{\hat{p}}}{d\phi}$, for a given realization of the set of aggregate states $\left\{\theta, \left\{\overline{\theta_n}\right\}_{n=1}^N, \delta\right\}$, is determined by*

$$\operatorname{sgn}\left(\frac{d\tau_{\hat{p}}}{d\phi}\right) = \operatorname{sgn}\left(\mathbb{C}ov_n\left[\frac{\frac{\partial \overline{\alpha_s}}{\partial \mu_A^n}}{\overline{\alpha_s}} - \frac{\frac{\partial \overline{\alpha_N}}{\partial \mu_A^n}}{\overline{\alpha_N}}, \frac{\frac{\partial \mu_A^n}{\partial \phi}}{\mu_A^n}\right]\right), \tag{17}$$

where $\operatorname{sgn}(\cdot)$ *denotes the sign function and* $\mathbb{C}ov_n[\cdot,\cdot]$ *denotes a cross-sectional covariance calculated among the active investors of types $n = \{1, \ldots, N\}$.*

The first term inside the covariance corresponds to the difference between the marginal relative contribution of a change in the share of active investors of each group n to the information and noise contained in the price, respectively. The second term captures the marginal change of an increase in linear trading costs on the share of active investors of each group n, capturing the extensive margin impact of a change in ϕ – note that $\frac{\partial \mu_A^n}{\partial \phi} < 0$. Lemma 4 shows that when investors are ex-ante heterogeneous, the response of price informativeness to a change in linear trading costs depends on the cross-sectional covariance across groups between the relative contribution of information to noise of each group and their extensive margin response to changes in linear costs. Intuitively, if the fraction of active investors of types that (marginally) contribute relatively more information to the price drop out of the market relatively more (less) when the linear trading cost increases, price informativeness will decrease (increase) when linear trading costs increase. Somewhat unexpectedly, given that linear costs reduce trading at the intensive and extensive margins, linear trading costs affect price informative only through the extensive margin participation of investors. This is due to the fact that, in our model, the demand sensitivities to information and noise for a given type of investor, α_s^n and α_θ^n are invariant to the level of linear costs ϕ.

Consistent with our anything-goes result under quadratic costs, the relation between linear trading costs and price informativeness is in general ambiguous. We formalize how one-dimensional heterogeneity among investors determines the relation between trading costs and price informativeness in the following theorem. In order to provide an explicit analytical characterization, we focus on the tractable case in which heterogeneity among investors is small and linear costs go to zero, although the ambiguous comparative statics naturally extend outside that limiting case.

Theorem 5. (Directional results under one-dimensional heterogeneity with linear trading costs) *Let investors types differ only in one of the three following dimensions: precision of their private signal about the fundamental, precision of their prior, or risk aversion. Let the heterogeneity in parameter $z \in \{\tau_s, \tau_\theta, \gamma\}$ be given by $z_n = z + \eta h_n$ where $\eta > 0$ and h_n is a type-specific scalar. In the limit, when the heterogeneity across investors is small and the linear trading cost is small, i.e., for sufficiently small values of η when $\phi \to 0$, if investors differ in:*

a) The precision of their private signal, $\tau_{sn} = \tau_s + \eta h_n$, price informativeness can increase or decrease with linear trading costs. More specifically, there exist thresholds $\tau_{s1}^\star$ and $\tau_{s2}^\star$ such that

$$\frac{d\tau_{\hat{p}}}{d\phi}\begin{cases} < 0, & \textit{if} \quad \tau_s \in (\tau_{s1}^\star, \tau_{s2}^\star) \\ \geq 0, & \quad\textit{otherwise} \end{cases}.$$

b) The precision of their prior, $\tau_{\theta n} = \tau_\theta + \eta h_n$, price informativeness can increase or decrease with linear trading costs. More specifically, there exist thresholds $\tau^\star_{\theta,1}$, $\tau^\star_{\theta,2}$, and $\tau^\star_{\theta,3}$ such that

$$\frac{d\tau_{\hat{p}}}{d\phi}\begin{cases} < 0, & \text{if} \quad \tau_\theta \in (\tau^\star_{\theta 1}, \tau^\star_{\theta 2}) \bigcup (\tau^\star_{\theta 3}, \infty) \\ \geq 0, & \text{otherwise} \end{cases}.$$

c) Risk aversion, $\gamma_n = \gamma + \eta h_n$, price informativeness decreases with linear trading costs. Formally,

$$\frac{d\tau_{\hat{p}}}{d\phi} < 0,$$

where all the thresholds in this theorem are a function of the primitives of the economy.

The effect of one-dimensional heterogeneity on the relation between trading costs and price informativeness is ambiguous and it depends on the specific parameter in which investors differ.[23] It should not be too surprising to find an ambiguous relation when there are inaction regions, since the shape of the distribution becomes an important primitive. However, this ambiguity persists even in the limit in which the distribution behaves approximately as uniform, which suggests that the anything-goes result is not only driven by assumptions on the shape of the distribution of investors.

We formally show in the Appendix (Lemma 8) that the way in which the marginal impact of an increase in linear costs on the measure of active investors, $\frac{\frac{\partial \mu^n_A}{\partial \phi}}{\mu^n_A}$, varies across types in the limit considered is driven by two effects. The first effect (concentration effect) accounts for how concentrated the distribution of investors with zero net trades is across types, which depends on the equilibrium distribution of $\sqrt{\tau_{\Delta n}}$ across types. Intuitively, an increase in trading costs will reduce the set of active traders by more when there are many marginal traders. The second effect (threshold sensitivity effect) depends on the sensitivity of investors' inaction thresholds across types, which depends on the equilibrium distribution of $\frac{1}{\kappa_n}$ across types. Intuitively, an increase in trading costs will reduce the set of active traders by more when their inaction thresholds vary significantly with the level of trading costs. For all three parameters considered here, both effects work in opposite directions, which opens the door to an ambiguous relation between trading costs and price informativeness.

We now explain our results in all three cases. First, when investors differ in the precision of their private signals, τ_{sn}, investors with more precise signals contribute relatively more information than noise to the price. However, the marginal impact of an increase in linear costs on the measure of active investors for types with more precise private signals may be higher or lower than for less informed investors, depending non-trivially on the level of τ_s. On the one hand, the concentration effect implies that the mass of investors with more precise private signals that stops trading with a small linear trading cost is lower.[24] On the other hand, the threshold sensitivity effect implies that investors with more precise signals have more sensitive thresholds. If one could shut down the concentration effect, we would recover the negative relation between informativeness and trading costs from the quadratic trading cost model.

[23]Note that price informativeness does not depend on the realizations of the set of aggregate states when $\eta \to 0$ and $\phi \to 0$, since the set of inactive investors vanishes. The same logic applies to the fixed cost case.

[24]This effect is at the same time determined by the net impact of two mechanisms: investors with precise signals have more dispersed net trades, because they are more aggressive, but they have less dispersed net trades because their signals are concentrated. The first mechanism always dominates in the environment considered here.

However, whenever the concentration effect is at play, it may be the case that higher linear trading costs are associated with higher price informativeness even when investors only differ on the precision of their private signals.

Second, when investors differ in the precision of their prior, $\tau_{\theta n}$, investors with more precise priors contribute relatively more noise than information to the price. As in the previous case, the marginal impact of an increase in linear costs on the measure of active investors for types with more precise priors may be higher or lower than for investors with less precise priors, depending non-trivially on the level of τ_θ. On the one hand, the concentration effect implies that the mass of investors with more precise priors that stops trading with a small linear trading cost is lower. On the other hand, the threshold sensitivity effect implies that investors with more precise priors have more sensitive thresholds. If one could shut down the concentration effect, we would recover the positive relation between informativeness and trading costs from the quadratic trading cost model. However, whenever the concentration effect is at play, it may be the case that higher linear trading costs are associated with lower price informativeness even when investors only differ on the precision of their private signals.

Third, when investors are heterogeneous in their risk aversion, γ_n, investors with higher risk aversion contribute relatively more noise than information to the price. At the same time, the marginal impact of an increase in linear costs on the measure of active investors for types with higher risk aversion turns out to be lower than for investors with lower risk aversion. On the one hand, the concentration effect implies that the mass of investors with higher risk aversion that stops trading with a small linear trading cost is higher. On the other hand, the threshold sensitivity effect implies that investors with higher risk aversion have less sensitive thresholds. It turns out that the threshold sensitivity effect dominates in this case, implying that the relation between informativeness and trading costs is negative when investors differ in their risk aversion.

7 Fixed costs

In this section, we adapt the baseline environment from Section 2 to incorporate fixed trading costs. The findings with fixed costs are very similar to those with linear trading costs, so we follow closely the exposition and the discussion of the results in Section 6.

Environment We consider an environment identical to the one in Section 6, with the only difference that investors now face a fixed cost of trading $\chi \geq 0$, instead of a linear cost. Formally, an investor who changes the asset holdings of the risky asset incurs a trading cost, in units of the numeraire, of χ, so the final wealth of an investor i of type n is

$$w_{2i}^n = n_{2i} + q_{1i}\theta - \Delta q_{1i}^n p - \chi \cdot \mathbf{1}\left[\Delta q_{1i}^n \neq 0\right], \tag{18}$$

where $\mathbf{1}\left\{|\Delta q_{1i}^n| \neq 0\right\}$ denotes the indicator function that takes the value one if $|\Delta q_{1i}^n|$ is different from zero and zero otherwise. Fixed costs of trading can capture participation costs or other trading costs that are invariant to the size of the trade.

As in the case of linear costs, we expand the information set of investors and assume that all investors observe the measures of buyers and sellers of each type, μ_B^n and μ_S^n. We also assume that investors have two private trading motives: random heterogeneous priors for each type and random hedging needs.

Definition. (Equilibrium) A rational expectations equilibrium in linear strategies with fixed trading costs consists of a linear net portfolio demand Δq_{1i}^n for each investor i of type n and a price function p such that: a) each investor i of each type n chooses Δq_{1i}^n to maximize his expected utility subject to his wealth accumulation constraint in Eq. (18) and given his information set, which includes the measures of buyers and sellers of each type j, μ_B^j and μ_S^j, and b) the price function p is such that the market for the risky asset clears, that is $\sum_n \int_{I^n} \Delta q_{1i}^n di = 0$.

Characterization of equilibrium The demand for the risky asset of an investor i in group n is given by the solution to

$$\max_{q_{1i}^n} \left(\mathbb{E}_n\left[\theta|\mathcal{I}_i^n\right] - p\right) q_{1i}^n - \gamma_n \mathbb{V}\mathrm{ar}_n\left[\theta|\mathcal{I}_i^n\right] h_i^n q_{1i}^n - \frac{\gamma_n}{2}\mathbb{V}\mathrm{ar}_n\left[\theta|\mathcal{I}_i^n\right]\left(q_{1i}^n\right)^2 - \chi \cdot \mathbf{1}\left\{\Delta q_{1i}^n \neq 0\right\},$$

where $\mathcal{I}_i^n = \left\{\hat{\theta}_i^n, s_i^n, h_i^n, p, \left\{\mu_B^j, \mu_S^j\right\}_{j=\{1,\ldots,N\}}\right\}$ denotes the information set of an investor i. The optimal portfolio choice of an investor i of type n, which features an inaction region, is given by

$$\Delta q_{1i}^n = \begin{cases} \frac{\mathbb{E}_n\left[\theta|\mathcal{I}_i^n\right]-p}{\gamma_n \mathbb{V}\mathrm{ar}_n\left[\theta|\mathcal{I}_i^n\right]} - h_i^n - q_{0i}^n, & \text{if} \quad W_1^n\left(\mathcal{I}_i\right) - W_0^n\left(\mathcal{I}_i\right) > \chi \\ 0, & \text{otherwise,} \end{cases}$$

where $W_1^n\left(\mathcal{I}_i^n\right)$ and $W_0^n\left(\mathcal{I}_i^n\right)$, defined in the Appendix, respectively denote the indirect utility of an investor if he participates in the market and if he does not.

As in the case in which investors face linear costs, the measure of active buyers and sellers of each type of investor depends on the realization of the set of aggregate states $\left\{\theta, \left\{\overline{\theta_n}\right\}_{n=1}^N, \delta\right\}$. This implies that the measures $\left\{\mu_n^B, \mu_n^S\right\}_n$ contain information about θ. As in the previous section, we respectively denote by $\hat{\mu}_B^n$ and $\hat{\mu}_S^n$ the unbiased signals about the fundamental contained in the measure of buyers and sellers of type n. In an equilibrium in linear strategies, we postulate (and subsequently verify) net demand functions for active investors of type n to be

$$\Delta q_{1i}^n = \alpha_s^n s_i^n + \alpha_\theta^n \hat{\theta}_i^n - \alpha_h^n h_i^n - \alpha_p^n p + \sum_{j=1}^N \left(\alpha_{\mu B}^{jn} \hat{\mu}_B^j + \alpha_{\mu S}^{jn} \hat{\mu}_S^j\right) + \psi^n,$$

where α_s^n, α_θ^n, α_h^n, α_p^n, $\alpha_{\mu B}^{jn}$, and $\alpha_{\mu S}^{jn}$ are non-negative scalars, while ψ^n can take positive or negative values. We explicitly characterize the equilibrium coefficients in the Appendix.

Lemma 5. (Existence) *An equilibrium in linear strategies with fixed tradings costs generically exists.*

As in the linear cost case, augmenting investors' information sets to account for the measure of buyers and sellers is enough to guarantee the existence of an equilibrium with fixed costs of trading. This is to our knowledge the first characterization of a linear equilibrium with fixed trading costs in a model with rich investor heterogeneity and learning.

Price informativeness and fixed trading costs Building on the same logic used to characterize the equilibrium with linear costs, we can exploit market clearing to express the equilibrium asset price as follows,

$$p=\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta+\sum_{n=1}^{N}\frac{\alpha_\theta^n\mu_A^n}{\overline{\alpha_p}}\overline{\theta_n}-\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\delta+\sum_{n=1}^{N}\frac{\overline{\alpha_{\mu B}^n}}{\overline{\alpha_p}}\hat{\mu}_B^n+\frac{\overline{\psi}}{\overline{\alpha_p}},$$

where $\overline{\alpha_s}=\sum_n\alpha_s^n\mu_A^n$, $\overline{\alpha_{\mu B}^n}=\sum_j\alpha_{\mu B}^{nj}\mu_A^n$, $\overline{\alpha_h}=\sum_n\alpha_h^n\mu_A^n$, $\overline{\alpha_p}=\sum_n\alpha_p^n\mu_A^n$, $\overline{\psi}$ is a constant term defined in the Appendix, and $\mu_A^n=\mu_B^n+\mu_B^n$ is the measure of active investors of type n. Since $\hat{\mu}_B^n$ is observable, the unbiased signal of θ contained in the price for an external observer who observes all public signals but only learns from the price can be expressed as a linear transformation of the price as follows

$$\begin{aligned}\hat{p}&=\frac{\overline{\alpha_p}}{\overline{\alpha_s}}\left(p-\sum_{n=1}^{N}\frac{\alpha_\theta^n\mu_A^n}{\overline{\alpha_p}}\omega_\theta+\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\omega_\delta-\sum_{n=1}^{N}\frac{\overline{\alpha_{\mu B}^n}}{\overline{\alpha_p}}\hat{\mu}_B^n-\frac{\overline{\psi}}{\overline{\alpha_p}}\right).\\&=\theta+\sum_{n=1}^{N}\frac{\alpha_\theta^n\mu_A^n}{\overline{\alpha_s}}\left(\overline{\theta_n}-\omega_\theta\right)-\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\left(\delta-\omega_\delta\right).\end{aligned}$$

For a given realization of the set of aggregate states $\left\{\theta,\left\{\overline{\theta_n}\right\}_{n=1}^{N},\delta\right\}$, price informativeness corresponds to

$$\tau_{\hat{p}}=\left(\frac{\overline{\alpha_s}}{\overline{\alpha_N}}\right)^2,\quad\text{where}\quad\left(\overline{\alpha_N}\right)^2\equiv\sum_{n=1}^{N}\left(\alpha_\theta^n\mu_A^n\right)^2\tau_{\overline{\theta}}^{-1}+\left(\overline{\alpha_h}\right)^2\tau_\delta^{-1},$$

and where μ_A^n is the measure of active investors of type n. As in the case of linear costs, the level of price informativeness depends in general on the realization of the set of aggregate states through the measure of active investors of each type. After formally defining price informativeness, we are ready to establish the final irrelevance result.

Theorem 6. (Irrelevance result with ex-ante identical investors and fixed trading costs) *In an economy with fixed trading costs, when investors are ex-ante identical, price informativeness is independent of the level of trading costs. Formally, the precision of the unbiased signal about the fundamental revealed by the asset price $\tau_{\hat{p}}$ does not depend on χ, that is,*

$$\frac{d\tau_{\hat{p}}}{d\chi}=0,\quad\forall\chi.$$

Theorem 6 shows that our irrelevance argument extends to the case of fixed costs. As in the case of linear costs, some investors find it optimal to stop trading altogether when they face fixed costs. Therefore, an increase in trading costs is associated with a reduction in trading along the extensive margin. However, because the decrease in trading at the extensive margin reduces both fundamental and sentiment trades in equal proportions when investors are ex-ante identical, price informativeness remains unchanged when χ varies. The same arguments given when studying linear costs regarding the validity of the irrelevance result for any realization of the aggregate states apply here too. Finally, we characterize an intermediate result that expresses the change in informativeness to trading costs as a function of a few high-level endogenous variables, and then use this result to show that the relation between fixed trading costs and price informativeness is ambiguous and it depends on the source of investor heterogeneity.

Lemma 6. (Directional characterization with fixed trading costs) *When the difference between the marginal relative contribution to the average sensitivities to information and noise of a change in the share of active investors of each group,* $\frac{\frac{\partial \overline{\alpha_s}}{\partial \mu_A^n}}{\overline{\alpha_s}} - \frac{\frac{\partial \overline{\alpha_N}}{\partial \mu_A^n}}{\overline{\alpha_N}}$*, is positively (negatively) correlated in the cross-section of investors with the marginal impact of an increase in fixed costs on the measure of active investors,* $\frac{\frac{\partial \mu_A^n}{\partial \chi}}{\mu_A^n}$*, an increase in trading costs* χ *increases (decreases) price informativeness in a given equilibrium. Formally, the sign of* $\frac{d\tau_{\hat{p}}}{d\chi}$*, for a given realization of the set of aggregates states* $\left\{\theta, \left\{\overline{\theta_n}\right\}_{n=1}^{N}, \delta\right\}$*, is determined by*

$$\operatorname{sgn}\left(\frac{d\tau_{\hat{p}}}{d\chi}\right) = \operatorname{sgn}\left(\mathbb{C}ov_n\left[\frac{\frac{\partial \overline{\alpha_s}}{\partial \mu_A^n}}{\overline{\alpha_s}} - \frac{\frac{\partial \overline{\alpha_N}}{\partial \mu_A^n}}{\overline{\alpha_N}}, \frac{\frac{\partial \mu_A^n}{\partial \chi}}{\mu_A^n}\right]\right),$$

where $\operatorname{sgn}(\cdot)$ *denotes the sign function and* $\mathbb{C}ov_n[\cdot,\cdot]$ *denotes a cross-sectional covariance calculated among the active investors of types* $n = \{1, \ldots, N\}$*.*

The expression that determines the sign of the relation between price informativeness and fixed trading costs is identical to the one with linear trading costs in Eq. (17). The economics behind these results are analogous to those in the linear cost case. Both expressions are identical because, as described above, linear trading costs affect price informative only through the extensive margin participation of investors, the only margin affected when investors face fixed costs.

Theorem 7. (Directional results under one-dimensional heterogeneity with fixed trading costs) *Let investors types differ only in one of the three following dimensions: precision of their private signal about the fundamental, precision of their prior, or risk aversion. Let the heterogeneity in parameter* $z \in \{\tau_s, \tau_\theta, \gamma\}$ *be given by* $z_n = z + \eta h_n$ *where* $\eta > 0$*. In the limit, when the heterogeneity across investors is small and the fixed trading cost is small, i.e., for sufficiently small values of* η *when* $\chi \to 0$*, if investors differ in:*

a) The precision of their private signal $\tau_{sn} = \tau_s + \eta h_n$*, price informativeness can increase or decrease with fixed trading costs. More specifically, there exist thresholds* $\tau_{s1}^\star$ *and* $\tau_{s2}^\star$ *such that*

$$\frac{d\tau_{\hat{p}}}{d\chi}\begin{cases} < 0, & \text{if} \quad \tau_s \in (\tau_{s1}^\star, \tau_{s2}^\star) \\ \geq 0, & \quad\text{otherwise} \end{cases}.$$

b) The precision of their prior $\tau_{\theta n} = \tau_\theta + \eta h_n$*, price informativeness can increase or decrease with fixed trading costs. More specifically, there exist thresholds* $\tau_{\theta,1}^\star$*,* $\tau_{\theta,2}^\star$*, and* $\tau_{\theta,3}^\star$ *such that*

$$\frac{d\tau_{\hat{p}}}{d\chi}\begin{cases} < 0, & \text{if} \quad \tau_\theta \in (\tau_{\theta 1}^\star, \tau_{\theta 2}^\star) \bigcup (\tau_{\theta 3}^\star, \infty) \\ \geq 0, & \quad\text{otherwise} \end{cases}.$$

c) Risk aversion $\gamma_n = \gamma + \eta h_n$*, price informativeness decreases with fixed trading costs. Formally,*

$$\frac{d\tau_{\hat{p}}}{d\chi} < 0.$$

where all the thresholds in this theorem are a function of the remaining parameters of the economy.

As in the linear case, the effect of one-dimensional heterogeneity on the relation between trading costs and price informativeness is ambiguous and it depends on the specific parameter in which investors differ. By focusing on the tractable case in which heterogeneity among investors is small and fixed costs go to zero, we are able to derive Theorem 7, with fixed trading costs, to exactly mimic Theorem 5, with linear trading costs. The economic mechanisms underlying both results are exactly identical, so we refer the reader to our discussion of Theorem 7 in page 31 for a detailed explanation of this result.

8 Practical implications

Before concluding, with the goal to focus existing policy discussions and to facilitate future empirical work in the area, we summarize several practical implications of our findings. First, our results contribute to the ongoing policy debate on the desirability of implementing financial transaction (Tobin) taxes. Tobin taxes are often advocated on the grounds that they would reduce noise/speculative trading (Summers and Summers, 1989; Stiglitz, 1989). As in our model, a financial transaction tax is conceived as a linear trading cost paid by investors. If a planner could differentiate between noise/speculative trades and fundamental-driven trades, she could set investor specific taxes. However, in practice, as in our model, all investors end up facing the same transaction tax. Our results show that the impact of a transaction tax on price informativeness is ambiguous and subtle, and crucially depends on the sources of noise and on the sources of heterogeneity in the economy. Therefore, one cannot argue that a transaction tax would make prices more or less informative without further discussing specific forms of heterogeneity, as discussed next. It is somewhat surprising that our irrelevance and directional results have been absent from policy discussions to this date.

Second, our results imply that one would expect price informativeness to be less affected by trading costs/taxes in markets with homogeneous participants. It is therefore necessary to consider specific forms of heterogeneity. For instance, our first application in Section 4 shows that environments in which sophisticated investors with better information and higher risk tolerance coexist with investors who are less informed and less risk tolerant unsophisticated are likely to feature a negative relation between the level of trading costs/taxes and price informativeness: this may be a good description of the stock market. Alternatively, specialized secondary markets in which most of the trading is done by professional investors with similar risk attitudes and information processing abilities seem to map better to a model with ex-ante identical investors. In those environments, our results predict that price informativeness should in principle not be affected by the level of trading costs/transaction taxes.

Third, even when considering markets with ex-ante identical investors, our results with endogenous information acquisition imply that it is important to distinguish between the short-term impact of a change in trading costs, in which investors have not had enough time to adjust the form in which they acquire information, relative to the long-term impact. Our results imply that over longer horizons price informativeness should decline after increases in trading costs, with the strength of this effect modulated by the ability of investors to adjust information choices. This interpretation associating a reduction in trading costs with an increase in information acquisition can be used to rationalize the rise in the share of trading-type financial activities in aggregate GDP since the mid-1970s, as documented by Philippon

(2015) and Greenwood and Scharfstein (2013).

Finally, our results point out to a few endogenous but potentially measurable variables that are sufficient to determine how price informativeness varies with the level of trading costs. Lemmas 2, 4, and 6 show that recovering the distribution of demand sensitivities to information and to non-payoff relevant trading sources, along with demand sensitivities to trading costs and extensive margin responses for the whole population of investors is sufficient to compute our directional characterizations directly. Undertaking that non-trivial measurement effort will enable to test the the results introduced in this paper.

9 Conclusion

In this paper, we provide a systematic formal analysis of the effects of trading costs on information aggregation and information acquisition in financial markets. When investors are ex-ante identical, an irrelevance result emerges when investors' information precisions are predetermined: price informativeness is independent of the level of trading costs. Intuitively, a change in trading costs equally reduces the amount of payoff relevant and non-payoff relevant trading, leaving the market signal-to-noise ratio unchanged. When investors are ex-ante heterogeneous, anything-goes, and a change in trading costs can increase or decrease price informativeness, depending on the source of heterogeneity. Through a reduction in information acquisition, trading costs reduce price informativeness. We study the cases of quadratic, linear, and fixed costs, which allows us to consider how trading costs modify investors' trading decisions at the intensive and extensive margins. Our results provide a clear framework to develop tests on the impact of trading cost on price informativeness and have the potential to discipline future policy discussions on the impact of financial transaction taxes.

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Appendix

A Proofs

A.1 Section 3: Price informativeness and trading costs

Investors' portfolio problem

Under the assumptions of CARA utility and normal uncertainty, an investor i solves the following mean-variance problem

$$\max_{q_{1i}} \mathbb{E}_i\left[w_{2i}\right] - \frac{\gamma_i}{2}\mathbb{V}ar_i\left[w_{2i}\right],$$

where w_{2i} is the investor's terminal wealth given by Eq. (3) in the text. After discarding constants, investor i solves Eq. (4) in the text, with an optimality condition given by

$$\begin{aligned} q_{1i} &= \underbrace{\frac{\gamma_i \mathbb{V}ar_i\left[\theta|\,\hat{\theta}_i, s_i, p\right]}{\gamma_i \mathbb{V}ar_i\left[\theta|\,\hat{\theta}_i, s_i, p\right]+c}}_{\equiv \omega_i(c)} \underbrace{\frac{\mathbb{E}_i\left[\theta|\,\hat{\theta}_i, s_i, p\right]-p}{\gamma_i \mathbb{V}ar_i\left[\theta|\,\hat{\theta}_i, s_i, p\right]}}_{\equiv \hat{q}_{1i}} + \underbrace{\frac{c}{\gamma_i \mathbb{V}ar_i\left[\theta|\,\hat{\theta}_i, s_i, p\right]+c}}_{\equiv 1-\omega_i(c)} q_{0i}, \\ &= \omega_i\left(c\right)\hat{q}_{1i} + \left(1-\omega_i\left(c\right)\right)q_{0i}. \end{aligned}$$

where $\hat{q}_{1i}$ corresponds to the expression for investors' demands in the absence of trading costs. We can write the net demand for the risky asset of investor i as

$$\Delta q_{1i} = \omega_i\left(c\right)\left(\hat{q}_{1i} - q_{0i}\right).$$

In an equilibrium in linear strategies we guess (and verify) that the optimal net asset demand of investor i takes the form

$$\Delta q_{1i} = \alpha_{si}s_i + \alpha_{\theta i}\hat{\theta}_i - \alpha_{pi}p + \psi_i, \tag{A1}$$

where α_{si}, $\alpha_{\theta i}$, and α_{pi} are non-negative scalars, and ψ_i can take positive and negative values. The market clearing condition $\int \Delta q_{1i} di = 0$ implies that the equilibrium price takes the form

$$p = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta + \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\overline{\theta} + \frac{\overline{\psi}}{\overline{\alpha_p}}, \tag{A2}$$

where we define

$$\overline{\alpha_s} \equiv \int \alpha_{si}di, \qquad \overline{\alpha_\theta} \equiv \int \alpha_{\theta i}di, \qquad \overline{\alpha_p} \equiv \int \alpha_{pi}di, \quad \text{and} \quad \overline{\psi} \equiv \int \psi_i di.$$

We assume a Strong Law of Large Numbers, as described in the Appendix of Vives (2008), which guarantees that $\int \alpha_{si}\varepsilon_{si}di \to 0$ and $\int \alpha_{\theta i}\varepsilon_{\theta i}di \to 0$ almost surely, so that we can write $\int \alpha_{si}s_i di = \overline{\alpha_s}\theta$ and $\int \alpha_{\theta i}\theta_i di = \overline{\alpha_\theta}\overline{\theta}$ in Eq. (A2). Using the distribution of $\overline{\theta}$, defined in Eq. (2) in the text, we can write the conditional distribution of the equilibrium price p given the fundamental θ as follows

$$p|\theta \sim N\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta + \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\mu_{\overline{\theta}} + \frac{\overline{\psi}}{\overline{\alpha_p}}, \left(\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)^2 \tau_{\overline{\theta}}^{-1}\right).$$

We denote by $\hat{p} = \frac{\overline{\alpha_p}}{\overline{\alpha_s}}p - \frac{\overline{\alpha_\theta}}{\overline{\alpha_s}}\mu_{\overline{\theta}} - \frac{\overline{\psi}}{\overline{\alpha_s}}$ the unbiased signal of θ contained in the price, which is distributed as follows

$$\hat{p}|\theta \sim N\left(\theta, \left(\tau_{\hat{p}}\right)^{-1}\right), \quad \text{where} \quad \tau_{\hat{p}} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}\right)^2 \tau_{\overline{\theta}},$$

which denotes the precision of the unbiased signal of θ contained in the price. Solving the optimal filtering problem – as described in the Online Appendix – from the perspective of investor i allows us to write

$$\mathbb{E}_i\left[\theta|\,\hat{\theta}_i, s_i, p\right] = \frac{\tau_{\theta i}\hat{\theta}_i + \tau_{si}s_i + \tau_{\hat{p}}\hat{p}}{\tau_{\theta i} + \tau_{si} + \tau_{\hat{p}}},$$
$$\mathbb{V}ar_i\left[\theta|\,\hat{\theta}_i, s_i, p\right] = \frac{1}{\tau_{\theta i} + \tau_{si} + \tau_{\hat{p}}}.$$

The expected value and the variance of θ, conditional on the investor's prior, his private signal and the equilibrium price, are the inputs in the portfolio decision of an investor, as described in Eq. (5) in the text.

We define κ_i, to simplify notation, as

$$\kappa_i \equiv \gamma_i \mathbb{V}ar_i\left[\theta|\,\hat{\theta}_i, s_i, p\right] + c.$$

Matching coefficients with our initial guess in Eq. (A1), we show that α_{si}, $\alpha_{\theta i}$, α_{pi}, and ψ_i must satisfy

$$\alpha_{si} = \frac{1}{\kappa_i}\frac{\tau_{si}}{\tau_{\theta i} + \tau_{si} + \tau_{\hat{p}}}, \tag{A3}$$
$$\alpha_{\theta i} = \frac{1}{\kappa_i}\frac{\tau_{\theta i}}{\tau_{\theta i} + \tau_{si} + \tau_{\hat{p}}}, \tag{A4}$$
$$\alpha_{pi} = \frac{1}{\kappa_i}\left(1 - \frac{\tau_{\hat{p}}}{\tau_{\theta i} + \tau_{si} + \tau_{\hat{p}}}\frac{\overline{\alpha_p}}{\overline{\alpha_s}}\right), \quad \text{and} \tag{A5}$$
$$\psi_i = -\frac{1}{\kappa_i}\left(\frac{\tau_{\hat{p}}}{\tau_{\theta i} + \tau_{si} + \tau_{\hat{p}}}\left(\frac{\overline{\alpha_\theta}}{\overline{\alpha_s}}\mu_{\overline{\theta}} + \frac{\overline{\psi}}{\overline{\alpha_s}}\right) + \gamma_i \mathbb{V}ar_i\left[\theta|\hat{\theta}_i, s_i, p\right]q_{0i}\right). \tag{A6}$$

Combining Eq. (A3) and Eq. (A4) allows us to characterize $\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}$, and consequently $\tau_{\hat{p}}$ and $\mathbb{V}ar_i\left[\theta|\,\hat{\theta}_i, s_i, p\right]$, as a function of primitives. $\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}} = \frac{\int \alpha_{si}di}{\int \alpha_{\theta i}di}$ is implicitly characterized by

$$\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}} = \frac{\int \frac{1}{\kappa_i}\frac{\tau_{si}}{\tau_{\theta i}+\tau_{si}+\tau_{\hat{p}}}di}{\int \frac{1}{\kappa_i}\frac{\tau_{\theta i}}{\tau_{\theta i}+\tau_{si}+\tau_{\hat{p}}}di}.$$

Proof of Lemma 1. (Existence/Uniqueness)

Proof. Since $\tau_{\hat{p}}$ depends only on $\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}$, the solution to the system in Eqs. (A3)-(A6) and, hence, an equilibrium, is fully characterized by $\left(\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}\right)^\star$, which is given by the fixed point of

$$J(x) = \frac{\int \frac{\tau_{si}}{\gamma_i + c\left(\tau_{\theta i}+\tau_{si}+x^2\tau_{\overline{\theta}}\right)}di}{\int \frac{\tau_{\theta i}}{\gamma_i + c\left(\tau_{\theta i}+\tau_{si}+x^2\tau_{\overline{\theta}}\right)}di}, \tag{A7}$$

where we used that the equilibrium individual demand sensitivities depend on $\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}$ through $\tau_{\hat{p}}$. $J(x)$ determines the aggregate ratio $\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}$ when investors expect the signal-to-noise ratio in the price to be x. The fixed point of Eq. (A7) can also be found as the value that satisfies $H\left(\left(\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}\right)^\star\right) = 0$, where

$$H(x) \equiv -x + \frac{\int \frac{\tau_{si}}{\gamma_i + c\left(\tau_{\theta i}+\tau_{si}+x^2\tau_{\overline{\theta}}\right)}di}{\int \frac{\tau_{\theta i}}{\gamma_i + c\left(\tau_{\theta i}+\tau_{si}+x^2\tau_{\overline{\theta}}\right)}di}. \tag{A8}$$

Note that

$$\lim_{x\to 0} H(x) > 0 \quad \text{and} \quad \lim_{x\to\infty} H(x) = -\infty.$$

Therefore, an equilibrium always exists, since $H(\cdot)$ is a continuous function.

Even though a non-negative root of $H(\cdot)$ always exists, there may be multiple equilibria. We adopt a conventional notion of stability. The function $H(x)$ is defined such that if $H(x_0) > 0$, then $J(x_0) > x_0$, which implies that if investors in the model expect the signal-to-noise ratio to be x_0, the realized value of this ratio will be $x_1 = J(x_0) > x_0$. Let $x^\star$ be a solution to $H(x^\star) = 0$. Then, we will say that the equilibrium $x^\star$ is stable if for all $x_0 \in (x^\star - \varepsilon_\delta, x^\star + \varepsilon_\delta)$ and for some $\varepsilon_\delta > 0$ the sequence $\{x_m\}_{m=0}^{\infty}$ where $x_m = J(x_{m-1})$ for $m > 1$ converges to $x^\star$. This sequence will converge only if $|J'(x^\star)| < 1$, which implies $H'(x^\star) < 0$. Hence, in all stable equilibria, $H'(x^\star) < 0$, that is,

$$H'(x) = -1 - 2c\tau_{\hat{p}} \int \left(\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{\theta i}}{\overline{\alpha_\theta}} \right) \frac{1}{\kappa_i} \mathbb{V}ar_i \left(\theta | \, s_i, \hat{\theta}_i, p \right) di < 0.$$

When investors are ex-ante identical,

$$H(x) \equiv -x + \frac{\frac{\tau_s}{\gamma + c\left(\tau_\theta + \tau_s + x^2 \tau_{\overline{\theta}}\right)}}{\frac{\tau_\theta}{\gamma + c\left(\tau_\theta + \tau_s + x^2 \tau_{\overline{\theta}}\right)}} = -x + \frac{\tau_s}{\tau_\theta},$$

which implies that $H(x) = 0$ has a unique solution given by $\left(\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}\right)^\star = \frac{\tau_s}{\tau_\theta}$. Note that this equilibrium is stable since $H'(x) = -1 < 0$. □

Proof of Theorem 1. (Irrelevance result with ex-ante identical investors)

Proof. Recall that $\tau_{\hat{p}} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}\right)^\star \tau_{\overline{\theta}}$. When investors are ex-ante identical, $\left(\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}\right)^\star = \frac{\tau_s}{\tau_\theta}$, which is independent of c. □

Proof of Lemma 2. (Directional characterization)

Proof. From the definition of $\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}$ we know that, in equilibrium, $\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}$ is a root of H, where $H(\cdot)$ is defined as in Eq. (A8). Moreover, in any stable equilibrium, $H' < 0$. Using the Implicit Function Theorem, we have that

$$\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}\right)}{dc} = \frac{\left.\frac{\partial H}{\partial c}\right|_{\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}}}{-\left.\frac{\partial H}{\partial x}\right|_{\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}}}, \tag{A9}$$

where

$$\frac{\partial H}{\partial c} = \frac{\int \left(-\frac{1}{\kappa_i} \alpha_{si} \overline{\alpha_\theta} + \frac{1}{\kappa_i} \alpha_{\theta i} \overline{\alpha_s} \right) di}{\left(\overline{\alpha_\theta}\right)^2} = -\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}} \int \left(\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{\theta i}}{\overline{\alpha_\theta}} \right) \frac{1}{\kappa_i} di = -\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}} \mathbb{C}ov_x \left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{\theta i}}{\overline{\alpha_\theta}}, \frac{1}{\kappa_i} \right],$$

where $\mathbb{C}ov_x[z_i, y_i]$ denotes the cross-sectional covariance between z and y in the population of investors. Since in any stable equilibria $H' < 0$, Eq. (A9) implies that

$$\operatorname{sgn}\left(\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}\right)}{dc} \right) = -\operatorname{sgn}\left(\mathbb{C}ov_x \left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{\theta i}}{\overline{\alpha_\theta}}, \frac{1}{\kappa_i} \right] \right).$$

Using that $\tau_{\hat{p}} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}\right)^2 \tau_{\overline{\theta}}$, Eq. (11) follows immediately. □

Proof of Theorem 2. (Anything-goes: directional results under one-dimensional heterogeneity)

Proof. The proof of this theorem uses Lemma 2 and the result that the covariance of two monotone increasing functions is positive (see Schmidt (2003)). The structure of the proof is the same for all dimensions of heterogeneity considered.

From Lemma 2 we know that

$$\operatorname{sgn}\left(\frac{d\tau_{\hat{p}}}{dc} \right) = -\operatorname{sgn}\left(\mathbb{C}ov_x \left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{\theta i}}{\overline{\alpha_\theta}}, \frac{1}{\kappa_i} \right] \right). \tag{A10}$$

There are four sources of heterogeneity across investors τ_{si}, $\tau_{\theta i}$, γ_i, and q_{0i}. Given the expression in Eq. (A10) and the definition of α_{si} and $\alpha_{\theta i}$ in Eq. (A3) and Eq. (A4), respectively, the only relevant dimensions of heterogeneity that affect price informativeness are τ_{si}, $\tau_{\theta i}$, and γ_i.

a) Suppose that investors only differ in the precision of their private information τ_{si}. In that case, we can write

$$\mathbb{C}ov_x\left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{\theta i}}{\overline{\alpha_\theta}}, \frac{1}{\kappa_i}\right] = \mathbb{C}ov_x\left[F_s\left(\tau_{si}\right), G_s\left(\tau_{si}\right)\right],$$

where $F_s\left(\tau_{si}\right) = \frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{\theta i}}{\overline{\alpha_\theta}} = \frac{1}{\overline{\alpha_s}}\frac{\tau_{si}}{\gamma + c(\tau_\theta + \tau_{si} + \tau_{\hat{p}})} - \frac{1}{\overline{\alpha_\theta}}\frac{\tau_\theta}{\gamma + c(\tau_\theta + \tau_{si} + \tau_{\hat{p}})}$ and $G_s\left(\tau_{si}\right) = \frac{1}{\kappa_i} = \frac{1}{\gamma\frac{1}{\tau_\theta + \tau_{si} + \tau_{\hat{p}}} + c}$. Note that the functions $F_s\left(\cdot\right)$ and $G_s\left(\cdot\right)$ are increasing in τ_{si}, since

$$\begin{aligned}
\frac{\partial F\left(\tau_{si}\right)}{\partial \tau_{si}} &= \frac{1}{\overline{\alpha_s}}\frac{\partial \alpha_{si}}{\partial \tau_{si}} - \frac{1}{\overline{\alpha_\theta}}\frac{\partial \alpha_{\theta i}}{\partial \tau_{si}} \\
&= \frac{1}{\overline{\alpha_s}}\frac{\gamma_i + c\left(\tau_{\theta i} + \tau_{\hat{p}}\right)}{\left(\gamma_i + c\left(\tau_{si} + \tau_{\theta i} + \tau_{\hat{p}}\right)\right)^2} - \frac{1}{\overline{\alpha_\theta}}\left(-\frac{c\tau_{\theta i}}{\left(\gamma_i + c\left(\tau_{si} + \tau_{\theta i} + \tau_{\hat{p}}\right)\right)^2}\right) > 0,
\end{aligned}$$

and

$$\frac{\partial G\left(\tau_{si}\right)}{\partial \tau_{si}} = \frac{1}{\left(\kappa_i\right)^2}\gamma_i \mathbb{V}ar_i\left(\theta \middle| s_i, \hat{\theta}_i, p\right)^2 > 0.$$

Therefore, since the covariance of two monotone increasing functions is positive (Schmidt, 2003),

$$\mathbb{C}ov_x\left[F_s\left(\tau_{si}\right), G_s\left(\tau_{si}\right)\right] > 0.$$

b) Similarly, when investors only differ in the precision of their priors $\tau_{\theta i}$, we can write

$$\mathbb{C}ov_x\left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{\theta i}}{\overline{\alpha_\theta}}, \frac{1}{\kappa_i}\right] = \mathbb{C}ov_x\left[F_\theta\left(\tau_{\theta i}\right), G_\theta\left(\tau_{\theta i}\right)\right],$$

where $F_\theta\left(\tau_{\theta i}\right) = \frac{1}{\overline{\alpha_s}}\frac{\tau_s}{\gamma + c(\tau_{\theta i} + \tau_s + \tau_{\hat{p}})} - \frac{1}{\overline{\alpha_\theta}}\frac{\tau_{\theta i}}{\gamma + c(\tau_{\theta i} + \tau_s + \tau_{\hat{p}})}$ and $G_\theta\left(\tau_{\theta i}\right) = \frac{1}{\kappa_i} = \frac{1}{\gamma\frac{1}{\tau_{\theta i} + \tau_s + \tau_{\hat{p}}} + c}$. Note that

$$\begin{aligned}
\frac{\partial F\left(\tau_{\theta i}\right)}{\partial \tau_{\theta i}} &= \frac{1}{\overline{\alpha_s}}\frac{\partial \alpha_{si}}{\partial \tau_{\theta i}} - \frac{1}{\overline{\alpha_\theta}}\frac{\partial \alpha_{\theta i}}{\partial \tau_{\theta i}} \\
&= -\frac{1}{\overline{\alpha_s}}\frac{c\tau_{si}}{\left(\gamma_i + c\left(\tau_{si} + \tau_{\theta i} + \tau_{\hat{p}}\right)\right)^2} - \frac{1}{\overline{\alpha_\theta}}\frac{\gamma_i + c\left(\tau_{si} + \tau_{\hat{p}}\right)}{\left(\gamma_i + c\left(\tau_{si} + \tau_{\theta i} + \tau_{\hat{p}}\right)\right)^2} < 0,
\end{aligned}$$

and

$$\frac{\partial G\left(\tau_{\theta i}\right)}{\partial \tau_{\theta i}} = \frac{1}{\left(\kappa_i\right)^2}\gamma_i \mathbb{V}ar_i\left(\theta \middle| s_i, \hat{\theta}_i, p\right)^2 > 0.$$

Then,

$$\mathbb{C}ov_x\left[F_\theta\left(\tau_{\theta i}\right), G_\theta\left(\tau_{\theta i}\right)\right] < 0.$$

c) Finally, if investors only differ in their risk aversion γ_i, $\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}} = \frac{\tau_s}{\tau_\theta}$, which is independent of c. □

A.2 Section 5: Endogenous information acquisition

Given the other investors' choices $\boldsymbol{\tau_{s,-i}}$, an investor i chooses τ_{si} to solve

$$\max_{\tau_{si} \geq 0} \sum_{h \in \mathcal{S}(\boldsymbol{\tau_s})} \pi_h\left(\left\{\tau_{si}, \boldsymbol{\tau_{s,-i}}\right\}\right) \mathbb{E}\left[v_i^h\left(\left\{\tau_{si}, \boldsymbol{\tau_{s,-i}}\right\}\right)\right] - \lambda\left(\tau_{si}\right),$$

where $\mathbb{E}\left[v_i^h\left(\boldsymbol{\tau_s}\right)\right]$ is the mean-variance utility of investor i if equilibrium $h \in \mathcal{S}\left(\boldsymbol{\tau_s}\right)$ is played in the trading game when the profile of private information precisions is given by $\boldsymbol{\tau_s}$. Formally, $\mathbb{E}\left[v_i^h\left(\boldsymbol{\tau_s}\right)\right]$ is given by

$$\begin{aligned}
\mathbb{E}\left[v_i^h\left(\boldsymbol{\tau_s}\right)\right] &= \mathbb{E}\left[\left(\mathbb{E}_i^h\left[\theta \middle| \hat{\theta}_i, s_i, p\right] - p\right)\right]\mathbb{E}\left[q_{1i}^{h\star}\right] + \mathbb{C}ov\left[\left(\mathbb{E}_i^h\left[\theta \middle| \hat{\theta}_i, s_i, p\right] - p\right), q_{1i}^{h\star}\right] - \frac{1}{2}\left(\gamma \mathbb{V}ar_i^h\left[\theta \middle| \hat{\theta}_i, s_i, p\right] + c\right)\mathbb{E}\left[\left(q_{1i}^{h\star}\right)^2\right] \\
&= \mathbb{C}ov\left[\left(\mathbb{E}_i^h\left[\theta \middle| \hat{\theta}_i, s_i, p\right] - p\right), q_{1i}^{h\star}\right] - \frac{1}{2}\left(\gamma \mathbb{V}ar_i^h\left[\theta \middle| \hat{\theta}_i, s_i, p\right] + c\right)\mathbb{E}\left[\left(q_{1i}^{h\star}\right)^2\right] \\
&= \frac{1}{2}\mathbb{C}ov\left[\left(\mathbb{E}_i^h\left[\theta \middle| \hat{\theta}_i, s_i, p\right] - p\right), q_{1i}^{h\star}\right],
\end{aligned}$$

where the equilibrium h considered affects the conditional moments and the optimal trading choice of investor i through the expectation operator $\mathbb{E}_i^h[\cdot]$, which depends on price informativeness.[25] Moreover, we use the fact that $\mathbb{E}\left[q_{1i}^{h\star}\right]=\frac{1}{\kappa_i}\mathbb{E}\left[\mathbb{E}_i^h\left[\theta|\,\hat{\theta}_i,s_i,p\right]-p\right]=0$, since $q_{0i}=0$ and $\mu_\theta=\mu_{\overline{\theta}}=0$.

Given the other investors' precision decisions, $\boldsymbol{\tau_{s,-i}}$, the optimal precision choice of investor i, $\tau_{si}^\star$, is given by the solution to

$$\sum_{h\in\mathcal{S}\left(\left\{\tau_{si}^\star,\boldsymbol{\tau_{s,-i}}\right\}\right)}\pi_h\left(\boldsymbol{\tau_s}\right)H_h\left(\tau_{si}^\star;\boldsymbol{\tau_{s,-i}}\right)=0,$$

where

$$H_h\left(\tau_{si}^\star;\boldsymbol{\tau_{s,-i}}\right)=\frac{1}{2}\frac{\partial\mathbb{C}ov\left[\left(\mathbb{E}_i^h\left[\theta|\,\hat{\theta}_i,s_i,p\right]-p\right),q_{1i}^{h\star}\right]}{\partial\tau_{si}}-\lambda'\left(\tau_{si}\right).$$

The second order condition of the information choice problem is given by

$$\sum_{h\in\mathcal{S}\left(\left\{\tau_{si}^\star,\boldsymbol{\tau_{s,-i}}\right\}\right)}\pi_h\left(\boldsymbol{\tau_s}\right)\frac{\partial H_h\left(\tau_{si}^\star;\boldsymbol{\tau_{s,-i}}\right)}{\partial\tau_{si}}=\sum_{h\in\mathcal{S}\left(\left\{\tau_{si}^\star,\boldsymbol{\tau_{s,-i}}\right\}\right)}\pi_h\left(\boldsymbol{\tau_s}\right)\frac{\partial^2\mathbb{C}ov\left[\left(\mathbb{E}_i^h\left[\theta|\,\hat{\theta}_i,s_i,p\right]-p\right),q_{1i}^{h\star}\right]}{\partial\tau_{si}^2}-\lambda''\left(\tau_{si}\right)<0.$$

Proof of Theorem 3. (Negative effect of trading costs on endogenous information acquisition)

Proof. Recall that the equilibrium in the trading stage is unique when investors are ex-ante identical. Therefore, since investors are infinitesimal, when $\tau_{sj}=\tau_s^\star$ for all $j\neq i$ the FOC for the information acquisition stage is given by

$$H\left(\tau_{si}^\star;\boldsymbol{\tau_{s,-i}}\right)=\frac{1}{2}\frac{\partial\mathbb{C}ov\left[\left(\mathbb{E}_i^h\left[\theta|\,\hat{\theta}_i,s_i,p\right]-p\right),q_{1i}^{h\star}\right]}{\partial\tau_{si}}-\lambda'\left(\tau_{si}\right)=0.$$

The implicit function theorem implies that for any equilibrium in the trading stage in an interior symmetric equilibrium of the information acquisition game

$$\frac{d\tau_{si}^\star}{dc}=\left.\frac{\frac{\partial H(\tau_{si};\tau_s^\star)}{\partial c}}{-\frac{\partial H(\tau_{si};\tau_s^\star)}{\partial\tau_{si}}}\right|_{\tau_{si}=\tau_s^\star}<0,$$

because the second order condition is $\left.\frac{\partial H(\tau_{si};\tau_s^\star)}{\partial\tau_{si}}\right|_{\tau_{si}=\tau_s^\star}<0$ and

$$\left.\frac{\partial H\left(\tau_{si}\right)}{\partial c}\right|_{\tau_{si}=\tau_s^\star}=\frac{\partial^2\mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|\,\hat{\theta}_i,s_i,p\right]-p\right),q_{1i}^\star\right]}{\partial\tau_{si}\partial c}<0.$$

To see the last result note that

$$\mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|\,\hat{\theta}_i,s_i,p\right]-p\right),q_{1i}^\star\right]=\frac{1}{\kappa_i}\mathbb{V}ar\left[\mathbb{E}\left[\theta|\,\hat{\theta}_i,s_i,p\right]-p\right]$$

where

$$\mathbb{V}ar\left[\mathbb{E}\left[\theta|\,\hat{\theta}_i,s_i,p\right]-p\right]=\left(1-\tau_\theta\mathbb{V}ar\left[\theta|\,\hat{\theta}_i,s_i,p\right]-\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\tau_\theta^{-1}+\left(\left(\tau_\theta+\tau_{\hat{p}}\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}\right)\mathbb{V}ar\left[\theta|\,\hat{\theta}_i,s_i,p\right]-\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)^2\tau_{\overline{\theta}}^{-1}$$
$$+\tau_{si}\mathbb{V}ar\left[\theta|\,\hat{\theta}_i,s_i,p\right]^2+\tau_\theta^2\mathbb{V}ar\left[\theta|\,\hat{\theta}_i,s_i,p\right]^2\tau_{\hat{\theta}}^{-1}$$

[25] As described in Veldkamp (2009), we are formally assuming that investors' preferences correspond to $\mathbb{E}\left[u_i\left(\mathbb{E}\left[U_i\left(w_{2i}\right)|\hat{\theta}_i,s_i,p\right]\right)\right]$, where $U_i\left(w_{2i}\right)=-e^{-\gamma_i w_{2i}}$ and $u_i\left(x\right)=-\ln\left(-x\right)$.

is independent of c since $\tau_{sj} = \tau_s^\star$ for all $j \neq i$. Hence,

$$
\begin{aligned}
\frac{\partial H(\tau_{si})}{\partial c} &= \frac{\partial^2 \left(\frac{1}{\kappa_i}\mathbb{V}ar\left[\mathbb{E}\left[\theta|\,\hat{\theta}_i, s_i, p\right] - p\right]\right)}{\partial \tau_{si}\partial c} \\
&= \frac{\partial^2 \left(\frac{1}{\kappa_i}\right)}{\partial \tau_{si}\partial c}\mathbb{V}ar\left[\mathbb{E}\left[\theta|\,\hat{\theta}_i, s_i, p\right] - p\right] + \frac{\partial\left(\frac{1}{\kappa_i}\right)}{\partial c}\frac{\partial \mathbb{V}ar\left[\mathbb{E}\left[\theta|\,\hat{\theta}_i, s_i, p\right] - p\right]}{\partial \tau_{si}} \\
&= -\frac{1}{\kappa_i}\frac{\partial \left(\frac{1}{\kappa_i}\mathbb{V}ar\left[\mathbb{E}\left[\theta|\,\hat{\theta}_i, s_i, p\right] - p\right]\right)}{\partial \tau_{si}}.
\end{aligned}
$$

When the FOC holds,

$$
\frac{\partial \left(\frac{1}{\kappa_i}\mathbb{V}ar\left[\mathbb{E}\left[\theta|\,\hat{\theta}_i, s_i, p\right] - p\right]\right)}{\partial \tau_{si}} = \lambda'(\tau_{si}) > 0,
$$

which implies that

$$
\left.\frac{\partial H(\tau_{si})}{\partial c}\right|_{\tau_{si}=\tau_s^\star} = \frac{\partial^2 \left(\frac{1}{\kappa_i}\mathbb{V}ar\left[\mathbb{E}\left[\theta|\,\hat{\theta}_i, s_i, p\right] - p\right]\right)}{\partial \tau_{si}\partial c} < 0.
$$

□

A.3 Section 6: Linear costs

Characterization of the sets of buyers and sellers

Let B_n, S_n, and A_n be the sets of buyers, sellers, and active investors of type n, respectively. An investor $i \in I^n$ will be a buyer in the asset market if

$$
\Delta q_{1i}^{n+} > 0,
$$

where Δq_{1i}^{n+} is defined in Eq. (15) in the text. Combining the equilibrium price in Eq. (16) in the text with investors assets demands, we can express equilibrium asset demands for buyers as follows

$$
\begin{aligned}
\Delta q_{1i}^{n+} =& \alpha_s^n \varepsilon_{si}^n + \alpha_\theta^n \varepsilon_{\hat{\theta}i} - \alpha_h^n \varepsilon_{hi}^n + \left(\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\alpha_s}\theta - \left(\frac{\alpha_h^n}{\overline{\alpha_h}} - \frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\alpha_h}\delta + \left(1 - \frac{\alpha_p^n}{\overline{\alpha_p}}\mu_A^n\right)\alpha_\theta^n\overline{\theta_n} \\
&- \frac{\alpha_p^n}{\overline{\alpha_p}}\sum_{j\neq n}\mu_A^j\alpha_\theta^j\overline{\theta_j} + \sum_{j=1}^N\left(\left(\frac{\alpha_{\mu B}^{jn}}{\overline{\overline{\alpha_{\mu B}^j}}} - \frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\alpha_{\mu B}^j}\hat{\mu}_B^j + \left(\frac{\alpha_{\mu S}^{jn}}{\overline{\overline{\alpha_{\mu S}^j}}} - \frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\alpha_{\mu S}^j}\hat{\mu}_S^j\right) + \left(\frac{\psi^{n+}}{\overline{\psi}} - \frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\psi}.
\end{aligned}
$$

Consequently, we can express the distribution of Δq_{1i}^{n+} as follows

$$
\Delta q_{1i}^{n+}\Big|\,\theta, \delta, \left\{\overline{\theta_j}, \mu_B^j, \mu_S^j\right\}_{j=1}^N \sim N\left(\Delta_n^+, (\tau_{\Delta n})^{-1}\right),
$$

where

$$
\begin{aligned}
\Delta_n^+ \equiv& \left(\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\alpha_s}\theta - \left(\frac{\alpha_h^n}{\overline{\alpha_h}} - \frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\alpha_h}\delta + \left(1 - \frac{\alpha_p^n}{\overline{\alpha_p}}\mu_A^n\right)\alpha_\theta^n\overline{\theta_n} - \frac{\alpha_p^n}{\overline{\alpha_p}}\sum_{j\neq n}\alpha_\theta^j\mu_A^j\overline{\theta_j} \\
&+ \sum_{j=1}^N\left(\left(\frac{\alpha_{\mu B}^{jn}}{\overline{\overline{\alpha_{\mu B}^j}}} - \frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\alpha_{\mu B}^j}\hat{\mu}_B^j + \left(\frac{\alpha_{\mu S}^{jn}}{\overline{\overline{\alpha_{\mu S}^j}}} - \frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\alpha_{\mu S}^j}\hat{\mu}_S^j\right) + \left(\frac{\psi^{n+}}{\overline{\psi}} - \frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\psi},
\end{aligned} \tag{A11}
$$

and

$$
(\tau_{\Delta n})^{-1} = (\alpha_s^n)^2\tau_{sn}^{-1} + (\alpha_h^n)^2(\tau_{hn})^{-1} + (\alpha_\theta^n)^2(\tau_{\hat{\theta}n})^{-1}. \tag{A12}
$$

Since for a given aggregate state $\left\{\theta, \delta, \left\{\overline{\theta_j}\right\}_{j=1}^N\right\}$ and measures $\left\{\mu_B^j, \mu_S^j\right\}_{j=1}^N$ all investors i from type n with $\Delta q_{1i}^{n+} > 0$ are buyers, the measure of buyers of type n is given by

$$\mu_B^n = 1 - \Phi\left(-\sqrt{\tau_{\Delta n}}\Delta_n^+\right) = \Phi\left(\sqrt{\tau_{\Delta n}}\Delta_n^+\right).$$

Similarly, an investor $i \in I^n$ will be a seller in the asset market if

$$\Delta q_{1i}^{n-} < 0,$$

where

$$\Delta q_{1i}^{n-} \Big| \theta, \delta, \left\{\overline{\theta_j}, \mu_B^j, \mu_S^j\right\}_{j=1}^N \sim N\left(\Delta_n^-, (\tau_{\Delta n})^{-1}\right),$$

and

$$\Delta_n^- \equiv \left(\frac{\alpha_s^n}{\alpha_s} - \frac{\alpha_p^n}{\alpha_p}\right)\overline{\alpha_s}\theta - \left(\frac{\alpha_h^n}{\alpha_h} - \frac{\alpha_p^n}{\alpha_p}\right)\overline{\alpha_h}\delta + \left(1 - \frac{\alpha_p^n}{\alpha_p}\mu_A^n\right)\alpha_\theta^n\overline{\theta_n} - \frac{\alpha_p^n}{\alpha_p}\sum_{j\neq n}\alpha_\theta^j\mu_A^j\overline{\theta_j} \tag{A13}$$
$$+\sum_{j=1}^N\left(\left(\frac{\alpha_{\mu B}^{jn}}{\overline{\alpha_{\mu B}^j}} - \frac{\alpha_p^n}{\alpha_p}\right)\overline{\alpha_{\mu B}^j}\hat{\mu}_B^j + \left(\frac{\alpha_{\mu S}^{jn}}{\overline{\alpha_{\mu S}^j}} - \frac{\alpha_p^n}{\alpha_p}\right)\overline{\alpha_{\mu S}^j}\hat{\mu}_S^j\right) + \left(\frac{\psi^{n-}}{\overline{\psi}} - \frac{\alpha_p^n}{\alpha_p}\right)\overline{\psi}.$$

Therefore, since for a given aggregate state $\left\{\theta, \delta, \left\{\overline{\theta_j}\right\}_{j=1}^N\right\}$ and measures $\left\{\mu_B^j, \mu_S^j\right\}_{j=1}^N$ all investors i from type n with $\Delta q_{1i}^{n-} < 0$ are sellers, the set of sellers of type n is given by

$$\mu_S^n = \Phi\left(-\sqrt{\tau_{\Delta n}}\Delta_n^-\right) = 1 - \Phi\left(\sqrt{\tau_{\Delta n}}\Delta_n^-\right).$$

Since all parameters are known in the economy, knowing the measure of sellers and buyers of each type reveals information about the fundamental θ. By inverting the normal c.d.f. $\Phi(\cdot)$, one can recover Δ_n^+ and Δ_n^-, and using Eqs. (A11) and (A13), one can recover the unbiased linear signals contained in these measures, denoted by $\hat{\mu}_S^n$ and $\hat{\mu}_B^n$. By inspecting (A11) and (A13), it is clear that $\hat{\mu}_S^n = \hat{\mu}_B^n$, where $\hat{\mu}_B^n$ is given by

$$z_n\hat{\mu}_B^n = \theta + \frac{\left(\frac{\alpha_h^n}{\alpha_h} - \frac{\alpha_p^n}{\alpha_p}\right)\overline{\alpha_h}}{\left(\frac{\alpha_s^n}{\alpha_s} - \frac{\alpha_p^n}{\alpha_p}\right)\overline{\alpha_s}}(\delta - \omega_\delta) + \frac{\left(1 - \frac{\alpha_p^n}{\alpha_p}\mu_A^n\right)\alpha_\theta^n}{\left(\frac{\alpha_s^n}{\alpha_s} - \frac{\alpha_p^n}{\alpha_p}\right)\overline{\alpha_s}}\left(\overline{\theta_n} - \omega_\theta\right)$$
$$- \frac{\frac{\alpha_p^n}{\alpha_p}\sum_{j\neq n}\alpha_\theta^j\mu_A^j\left(\overline{\theta_j} - \omega_\theta\right)}{\left(\frac{\alpha_s^n}{\alpha_s} - \frac{\alpha_p^n}{\alpha_p}\right)\overline{\alpha_s}} + \sum_{j=1}^N\left(\frac{\left(\frac{\alpha_{\mu B}^{jn}}{\overline{\alpha_{\mu B}^j}} - \frac{\alpha_p^n}{\alpha_p}\right)}{\left(\frac{\alpha_s^n}{\alpha_s} - \frac{\alpha_p^n}{\alpha_p}\right)}\frac{\overline{\alpha_{\mu B}^j}}{\overline{\alpha_s}}\hat{\mu}_B^j + \frac{\left(\frac{\alpha_{\mu S}^{jn}}{\overline{\alpha_{\mu S}^j}} - \frac{\alpha_p^n}{\alpha_p}\right)}{\left(\frac{\alpha_s^n}{\alpha_s} - \frac{\alpha_p^n}{\alpha_p}\right)}\frac{\overline{\alpha_{\mu S}^j}}{\overline{\alpha_s}}\hat{\mu}_S^j\right),$$

where z_n is a constant that guarantees that $\hat{\mu}_B^n$ (equivalently $\hat{\mu}_S^n$) are unbiased for all n. Given the measure of active investors, the measures of buyers and sellers contain the same information, so we can set $\alpha_{\mu S}^{jn} = 0$ for all j, n, which implies that

$$z_n\hat{\mu}_B^n = \theta + \frac{\left(\frac{\alpha_h^n}{\alpha_h} - \frac{\alpha_p^n}{\alpha_p}\right)\overline{\alpha_h}}{\left(\frac{\alpha_s^n}{\alpha_s} - \frac{\alpha_p^n}{\alpha_p}\right)\overline{\alpha_s}}(\delta - \omega_\delta) + \frac{\left(1 - \frac{\alpha_p^n}{\alpha_p}\mu_A^n\right)\alpha_\theta^n}{\left(\frac{\alpha_s^n}{\alpha_s} - \frac{\alpha_p^n}{\alpha_p}\right)\overline{\alpha_s}}\left(\overline{\theta_n} - \omega_\theta\right) \tag{A14}$$
$$- \frac{\frac{\alpha_p^n}{\alpha_p}\sum_{j\neq n}\alpha_\theta^j\mu_A^j\left(\overline{\theta_j} - \omega_L\right)}{\left(\frac{\alpha_s^n}{\alpha_s} - \frac{\alpha_p^n}{\alpha_p}\right)\overline{\alpha_s}} + \sum_{j=1}^N\frac{\left(\frac{\alpha_{\mu B}^{jn}}{\overline{\alpha_{\mu B}^j}} - \frac{\alpha_p^n}{\alpha_p}\right)}{\left(\frac{\alpha_s^n}{\alpha_s} - \frac{\alpha_p^n}{\alpha_p}\right)}\frac{\overline{\alpha_{\mu B}^j}}{\overline{\alpha_s}}\hat{\mu}_B^j.$$

Note that this expression can be written in matrix form as follows

$$
\begin{aligned}
z \odot \hat{\boldsymbol{\mu}}^B &= \theta \mathbf{1}_{N\times 1} + B\left(\overline{\boldsymbol{\theta}} - \omega_\theta \mathbf{1}_{N\times 1}\right) \\
&+ \begin{bmatrix}
\frac{\left(\frac{\alpha^{11}_{\mu B}}{\overline{\alpha^1_{\mu B}}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)}{\left(\frac{\alpha^1_s}{\overline{\alpha_s}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)}\frac{\overline{\alpha^1_{\mu B}}}{\overline{\alpha_s}} & \frac{\left(\frac{\alpha^{21}_{\mu B}}{\overline{\alpha^2_{\mu B}}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)}{\left(\frac{\alpha^1_s}{\overline{\alpha_s}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)}\frac{\overline{\alpha^2_{\mu B}}}{\overline{\alpha_s}} & \cdots & \frac{\left(\frac{\alpha^{N1}_{\mu B}}{\overline{\alpha^N_{\mu B}}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)}{\left(\frac{\alpha^1_s}{\overline{\alpha_s}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)}\frac{\overline{\alpha^N_{\mu B}}}{\overline{\alpha_s}} \\
\vdots & \ddots & \cdots & \frac{\left(\frac{\alpha^{N2}_{\mu B}}{\overline{\alpha^N_{\mu B}}}-\frac{\alpha^2_p}{\overline{\alpha_p}}\right)}{\left(\frac{\alpha^2_s}{\overline{\alpha_s}}-\frac{\alpha^2_p}{\overline{\alpha_p}}\right)}\frac{\overline{\alpha^N_{\mu B}}}{\overline{\alpha_s}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\left(\frac{\alpha^{1N}_{\mu B}}{\overline{\alpha^1_{\mu B}}}-\frac{\alpha^N_p}{\overline{\alpha_p}}\right)}{\left(\frac{\alpha^N_s}{\overline{\alpha_s}}-\frac{\alpha^N_p}{\overline{\alpha_p}}\right)}\frac{\overline{\alpha^1_{\mu B}}}{\overline{\alpha_s}} & \cdots & \cdots & \frac{\left(\frac{\alpha^{NN}_{\mu B}}{\overline{\alpha^N_{\mu B}}}-\frac{\alpha^N_p}{\overline{\alpha_p}}\right)}{\left(\frac{\alpha^N_s}{\overline{\alpha_s}}-\frac{\alpha^N_p}{\overline{\alpha_p}}\right)}\frac{\overline{\alpha^N_{\mu B}}}{\overline{\alpha_s}}
\end{bmatrix} \hat{\boldsymbol{\mu}}^B + \begin{bmatrix}
\frac{\left(\frac{\alpha^1_h}{\overline{\alpha_h}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)}{\left(\frac{\alpha^1_s}{\overline{\alpha_s}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)} \\
\vdots \\
\vdots \\
\frac{\left(\frac{\alpha^N_h}{\overline{\alpha_h}}-\frac{\alpha^N_p}{\overline{\alpha_p}}\right)}{\left(\frac{\alpha^N_s}{\overline{\alpha_s}}-\frac{\alpha^N_p}{\overline{\alpha_p}}\right)}
\end{bmatrix} \frac{\overline{\alpha_h}}{\overline{\alpha_s}}\left(\delta - \omega_\delta\right),
\end{aligned}
$$

where $\odot$ denotes the Hadamard product (element-wise multiplication), $z = [z_1, \ldots, z_N]^T$, $\hat{\boldsymbol{\mu}}^B = \left[\hat{\mu}^1_B, \ldots, \hat{\mu}^N_B\right]^T$, $\overline{\boldsymbol{\theta}} = \left[\overline{\theta_1}, \ldots, \overline{\theta_N}\right]^T$, $\mathbf{1}_{N\times 1}$ is an N by 1 vector of ones, and

$$
B \equiv \begin{bmatrix}
\frac{\left(1-\frac{\alpha^1_p}{\overline{\alpha_p}}\mu^1_A\right)\alpha^1_\theta}{\left(\frac{\alpha^1_s}{\overline{\alpha_s}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)\overline{\alpha_s}} & \frac{\frac{\alpha^1_p}{\overline{\alpha_p}}\alpha^2_\theta\mu^2_A}{\left(\frac{\alpha^1_s}{\overline{\alpha_s}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)\overline{\alpha_s}} & \cdots & \frac{\frac{\alpha^1_p}{\overline{\alpha_p}}\alpha^N_\theta\mu^N_A}{\left(\frac{\alpha^1_s}{\overline{\alpha_s}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)\overline{\alpha_s}} \\
\frac{\frac{\alpha^2_p}{\overline{\alpha_p}}\alpha^1_\theta\mu^1_A}{\left(\frac{\alpha^2_s}{\overline{\alpha_s}}-\frac{\alpha^2_p}{\overline{\alpha_p}}\right)\overline{\alpha_s}} & \frac{\left(1-\frac{\alpha^2_p}{\overline{\alpha_p}}\mu^2_A\right)\alpha^2_\theta}{\left(\frac{\alpha^2_s}{\overline{\alpha_s}}-\frac{\alpha^2_p}{\overline{\alpha_p}}\right)\overline{\alpha_s}} & \cdots & \frac{\frac{\alpha^2_p}{\overline{\alpha_p}}\alpha^N_\theta\mu^N_A}{\left(\frac{\alpha^2_s}{\overline{\alpha_s}}-\frac{\alpha^2_p}{\overline{\alpha_p}}\right)\overline{\alpha_s}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\frac{\alpha^n_p}{\overline{\alpha_p}}\alpha^1_\theta\mu^1_A}{\left(\frac{\alpha^n_s}{\overline{\alpha_s}}-\frac{\alpha^n_p}{\overline{\alpha_p}}\right)\overline{\alpha_s}} & \frac{\frac{\alpha^n_p}{\overline{\alpha_p}}\alpha^2_\theta\mu^2_A}{\left(\frac{\alpha^n_s}{\overline{\alpha_s}}-\frac{\alpha^n_p}{\overline{\alpha_p}}\right)\overline{\alpha_s}} & \cdots & \frac{\left(1-\frac{\alpha^N_p}{\overline{\alpha_p}}\mu^N_A\right)\alpha^N_\theta}{\left(\frac{\alpha^N_s}{\overline{\alpha_s}}-\frac{\alpha^N_p}{\overline{\alpha_p}}\right)\overline{\alpha_s}}
\end{bmatrix}.
$$

Then,

$$
\hat{\boldsymbol{\mu}}^B = A\left(\theta \mathbf{1}_{N\times 1} + B\left(\overline{\boldsymbol{\theta}} - \omega_\theta \mathbf{1}_{N\times 1}\right) + C\left(\delta - \omega_\delta\right)\right),
$$

where

$$
A \equiv \begin{bmatrix}
z_1 - \frac{\left(\frac{\alpha^{11}_{\mu B}}{\overline{\alpha^1_{\mu B}}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)}{\left(\frac{\alpha^1_s}{\overline{\alpha_s}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)}\frac{\overline{\alpha^1_{\mu B}}}{\overline{\alpha_s}} & -\frac{\left(\frac{\alpha^{21}_{\mu B}}{\overline{\alpha^2_{\mu B}}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)}{\left(\frac{\alpha^1_s}{\overline{\alpha_s}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)}\frac{\overline{\alpha^2_{\mu B}}}{\overline{\alpha_s}} & \cdots & -\frac{\left(\frac{\alpha^{N1}_{\mu B}}{\overline{\alpha^N_{\mu B}}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)}{\left(\frac{\alpha^1_s}{\overline{\alpha_s}}-\frac{\alpha^1_p}{\overline{\alpha_p}}\right)}\frac{\overline{\alpha^N_{\mu B}}}{\overline{\alpha_s}} \\
\vdots & \ddots & \cdots & -\frac{\left(\frac{\alpha^{N2}_{\mu B}}{\overline{\alpha^N_{\mu B}}}-\frac{\alpha^2_p}{\overline{\alpha_p}}\right)}{\left(\frac{\alpha^2_s}{\overline{\alpha_s}}-\frac{\alpha^2_p}{\overline{\alpha_p}}\right)}\frac{\overline{\alpha^N_{\mu B}}}{\overline{\alpha_s}} \\
\vdots & \vdots & \ddots & \vdots \\
-\frac{\left(\frac{\alpha^{1N}_{\mu B}}{\overline{\alpha^1_{\mu B}}}-\frac{\alpha^N_p}{\overline{\alpha_p}}\right)}{\left(\frac{\alpha^N_s}{\overline{\alpha_s}}-\frac{\alpha^N_p}{\overline{\alpha_p}}\right)}\frac{\overline{\alpha^1_{\mu B}}}{\overline{\alpha_s}} & \cdots & \cdots & z_N - \frac{\left(\frac{\alpha^{NN}_{\mu B}}{\overline{\alpha^N_{\mu B}}}-\frac{\alpha^N_p}{\overline{\alpha_p}}\right)}{\left(\frac{\alpha^N_s}{\overline{\alpha_s}}-\frac{\alpha^N_p}{\overline{\alpha_p}}\right)}\frac{\overline{\alpha^N_{\mu B}}}{\overline{\alpha_s}}
\end{bmatrix}^{-1},
$$

and

$$
C \equiv \begin{bmatrix} \dfrac{\left(\frac{\alpha_h^1}{\overline{\alpha_h}} - \frac{\alpha_p^1}{\overline{\alpha_p}}\right)}{\left(\frac{\alpha_s^1}{\overline{\alpha_s}} - \frac{\alpha_p^1}{\overline{\alpha_p}}\right)} \\ \vdots \\ \vdots \\ \dfrac{\left(\frac{\alpha_h^N}{\overline{\alpha_h}} - \frac{\alpha_p^N}{\overline{\alpha_p}}\right)}{\left(\frac{\alpha_s^N}{\overline{\alpha_s}} - \frac{\alpha_p^N}{\overline{\alpha_p}}\right)} \end{bmatrix} \frac{\overline{\alpha_h}}{\overline{\alpha_s}}.
$$

The constants $z_1, ..., z_N$ are such that $\sum_m A_{im} = 1$, $\forall i$, so the signals in $\hat{\boldsymbol{\mu}}_{\boldsymbol{B}}$ are unbiased. Then,

$$
\hat{\mu}_B^n = \theta + \sum_j (AB)_{nj} \left(\overline{\theta}_j - \omega_\theta\right) + (AC)_n (\delta - \omega_\delta),
$$

where $(AB)_{nj}$ denotes the element (n, j) of matrix AB and $(AC)_n$ is element n of vector AC.

The public signals in vector $\hat{\boldsymbol{\mu}}^B$ are correlated among themselves. The linear filtering solved by investor i in group n, which can be found in the Online Appendix, implies that

$$
\begin{aligned}
\mathbb{E}_n \left[\theta | \mathcal{I}_i^n\right] &= \frac{\tau_{\theta n} \hat{\theta}_i^n + \tau_{sn} s_i^n + \mathbf{1}_{1\times(N+1)} \tilde{\Lambda}^{-1} \Gamma^T \boldsymbol{x_1}}{\tau_{\theta n} + \tau_{sn} + \sum_{h=1}^{N+1} \tilde{\Lambda}_{hh}^{-1}} \\
\mathbb{V}\mathrm{ar}_n \left[\theta | \mathcal{I}_i^n\right] &= \frac{1}{\tau_{\theta n} + \tau_{sn} + \sum_{h=1}^{N+1} \tilde{\Lambda}_{hh}^{-1}},
\end{aligned}
\tag{A15}
$$

for $i \in I^n$, $n = 1, ..., N$, where Λ and Γ are, respectively, the diagonal eigenvalue matrix and eigenvector matrix in the eigen-decomposition of Ω, $\tilde{\Lambda}$ is a normalization of Λ described in the Online Appendix, and where the vector of $N+1$ unbiased public signals, $\boldsymbol{x_1} \equiv \left[\hat{p}, \hat{\boldsymbol{\mu}}^B\right]^T$ is such that

$$
\boldsymbol{x_1} | \, \theta \sim N\left(\theta \mathbf{1}_{(N+1)\times 1}, \Omega\right),
$$

with

$$
\Omega = \begin{bmatrix} \frac{\alpha_\theta^1 \mu_A^1}{\overline{\alpha_s}} & \frac{\alpha_\theta^2 \mu_A^2}{\overline{\alpha_s}} & \cdots & \frac{\alpha_\theta^N \mu_A^N}{\overline{\alpha_s}} & \frac{\overline{\alpha_h}}{\overline{\alpha_s}} \\ (AB)_{11} & (AB)_{12} & \cdots & (AB)_{1N} & (AC)_1 \\ \vdots & \cdots & \ddots & \vdots & \vdots \\ (AB)_{N1} & (AB)_{N2} & \cdots & (AB)_{NN} & (AC)_N \end{bmatrix} \begin{bmatrix} \tau_{\overline{\theta}}^{-1} & 0 & \cdots & \cdots & 0 \\ 0 & \tau_{\overline{\theta}}^{-1} & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \cdots & \vdots \\ 0 & \cdots & 0 & \tau_{\overline{\theta}}^{-1} & 0 \\ 0 & 0 & \cdots & \cdots & \tau_\delta^{-1} \end{bmatrix} \begin{bmatrix} \frac{\alpha_\theta^1 \mu_A^1}{\overline{\alpha_s}} & \frac{\alpha_\theta^2 \mu_A^2}{\overline{\alpha_s}} & \cdots & \frac{\alpha_\theta^N \mu_A^N}{\overline{\alpha_s}} & \frac{\overline{\alpha_h}}{\overline{\alpha_s}} \\ (AB)_{11} & (AB)_{12} & \cdots & (AB)_{1N} & (AC)_1 \\ \vdots & \cdots & \ddots & \vdots & \vdots \\ (AB)_{N1} & (AB)_{N2} & \cdots & (AB)_{NN} & (AC)_N \end{bmatrix}^T.
$$

The posterior mean is linear in all the signals and the posterior prior is independent of the realization of the signals, as it is usual in Bayesian updating problems with linear Gaussian structures.

Using the expression in Eq. (A15) and the first order conditions of the investors' problems we have that

$$
\begin{aligned}
\alpha_s^n &= \frac{\tau_{sn}}{\gamma_n}, \quad \alpha_\theta^n = \frac{\tau_{\theta n}}{\gamma_n}, \quad \alpha_h^n = 1 \\
\alpha_{\mu B}^{jn} &= \frac{K_{(j+1)}^n}{\kappa_n}, \quad \alpha_{\mu S}^{jn} = 0, \quad \alpha_p^n = \frac{1 - \frac{\overline{\alpha_p}}{\overline{\alpha_s}} K_1^n}{\kappa_n} \\
\psi^{n+} &= \frac{1}{\kappa_n} \left(-\kappa_n q_0^n - \phi - K_1^n \left(\frac{\overline{\alpha_\theta}}{\overline{\alpha_s}} \omega_\theta - \frac{\overline{\alpha_h}}{\overline{\alpha_s}} \omega_\delta + \sum_{j=1}^N \frac{\overline{\alpha_{\mu B}^j}}{\overline{\alpha_s}} \hat{\mu}_B^j + \frac{\overline{\psi}}{\overline{\alpha_s}}\right)\right) \\
\psi^{n-} &= \frac{1}{\kappa_n} \left(-\kappa_n q_0^n + \phi - K_1^n \left(\frac{\overline{\alpha_L}}{\overline{\alpha_s}} \omega_\theta - \frac{\overline{\alpha_h}}{\overline{\alpha_s}} \omega_\delta + \sum_{j=1}^N \frac{\overline{\alpha_{\mu B}^j}}{\overline{\alpha_s}} \hat{\mu}_B^j + \frac{\overline{\psi}}{\overline{\alpha_s}}\right)\right),
\end{aligned}
$$

where K_i^n is the ith element of the Kalman gain vector for an investor of type n, given by $K^n = \frac{\mathbf{1}_{1\times(N+1)}\tilde{\Lambda}^{-1}\Gamma^T}{\tau_{\theta n}+\tau_{sn}+\sum_{h=1}^{N+1}\tilde{\Lambda}_{hh}^{-1}}$, $\kappa_n \equiv \gamma_n \mathbb{V}\mathrm{ar}_n\left[\theta|\mathcal{I}_i^n\right]$, and where without loss of generality we are imposing that $\alpha_{\mu S}^{jn} = 0$.

The precision of the unbiased signal of θ contained in the price from the perspective of an external observer who observes all public signals but only learns from the price, which we denote by $\tau_{\hat{p}}$, is the relevant measure of price informativeness. Price informativeness is given by

$$\tau_{\hat{p}} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_N}}\right)^2,$$

where

$$(\overline{\alpha_N})^2 \equiv \sum_{j=1}^{N}\left(\alpha_\theta^j \mu_A^j\right)^2 \tau_{\overline{\theta}}^{-1} + \left(\sum_{j=1}^{N}\mu_A^j\right)^2 \tau_\delta^{-1}.$$

Linear trading costs affect price informativeness through the set of active investors A_n. The direct effect of this change is through the extensive margin, since

$$\alpha_s^n = \frac{\tau_{sn}}{\gamma_n} \quad \text{and} \quad \alpha_\theta^n = \frac{\tau_{\theta n}}{\gamma_n}$$

do not depend on the set of active investors of each type.

Equilibrium price characterization

In an equilibrium in linear strategies, we conjecture (and subsequently verify) net demand functions for buyers (Δq_{1i}^{n+}) and sellers (Δq_{1i}^{n-}) of type n respectively given by

$$\Delta q_i^{n+} = \alpha_s^n s_i^n + \alpha_\theta^n \hat{\theta}_i^n - \alpha_h^n h_i^n - \alpha_p^n p + \sum_{j=1}^{N}\left(\alpha_{\mu B}^{jn}\hat{\mu}_B^j + \alpha_{\mu S}^{jn}\hat{\mu}_S^j\right) + \psi^{n+}$$
$$\Delta q_i^{n-} = \alpha_s^n s_i^n + \alpha_\theta^n \hat{\theta}_i^n - \alpha_h^n h_i^n - \alpha_p^n p + \sum_{j=1}^{N}\left(\alpha_{\mu B}^{jn}\hat{\mu}_B^j + \alpha_{\mu S}^{jn}\hat{\mu}_S^j\right) + \psi^{n-},$$

where $\hat{\mu}_n^B$ and $\hat{\mu}_n^S$ are the unbiased signals about the fundamental contained in the measures of buyers and sellers of type n, respectively, and α_θ^n, α_s^n, and α_p^n are positive scalars, while ψ^{n+} and ψ^{n-} can take positive or negative values. We define as B_n, S_n and A_n the sets of buyers, sellers, and active investors of type n, respectively. Market clearing in the asset market is given by

$$\sum_{n=1}^{N}\int_{A_n}\Delta q_i^n di = \sum_{n=1}^{N}\int_{B_n}\Delta q_i^{n+} di + \sum_{n=1}^{N}\int_{S_n}\Delta q_i^{n-} di = 0,$$

which allows us to express the equilibrium price p as

$$\begin{aligned}
p = &\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta + \frac{\sum_{n=1}^{N}\alpha_\theta^n\overline{\theta_n}\mu_A^n}{\overline{\alpha_p}} - \frac{\overline{\alpha_h}}{\overline{\alpha_p}}\delta + \frac{1}{\overline{\alpha_p}}\sum_{n=1}^{N}\left(\int_{B_n}\left(\alpha_s^n\varepsilon_{si}^n + \alpha_\theta^n\varepsilon_{\hat{\theta}i}^n - \alpha_h^n\varepsilon_{hi}^n\right)di + \int_{S_n}\left(\alpha_s^n\varepsilon_{si}^n + \alpha_\theta^n\varepsilon_{\hat{\theta}i}^n - \alpha_h^n\varepsilon_{hi}^n\right)di\right)\\
&+ \sum_{n=1}^{N}\left(\frac{\overline{\alpha_{\mu B}^n}}{\overline{\alpha_p}}\hat{\mu}_B^n + \frac{\overline{\alpha_{\mu S}^n}}{\overline{\alpha_p}}\hat{\mu}_S^n\right) + \sum_{n=1}^{N}\left(\psi^{n+}\mu_B^n + \psi^{n-}\mu_S^n\right)\\
= &\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta + \frac{\sum_{n=1}^{N}\alpha_\theta^n\overline{\theta_n}\mu_A^n}{\overline{\alpha_p}} - \frac{\overline{\alpha_h}}{\overline{\alpha_p}}\delta + \sum_{n=1}^{N}\left(\frac{\overline{\alpha_{\mu B}^n}}{\overline{\alpha_p}}\hat{\mu}_B^n + \frac{\overline{\alpha_{\mu S}^n}}{\overline{\alpha_p}}\hat{\mu}_S^n\right) + \frac{\overline{\psi}}{\overline{\alpha_p}},
\end{aligned}$$

where $\overline{\alpha_s} = \sum_{n=1}^{N} \alpha_s^n \mu_A^n$, $\overline{\alpha_h} = \sum_{n=1}^{N} \alpha_h^n \mu_A^n$, $\overline{\alpha_p} = \sum_{n=1}^{N} \alpha_p^n \mu_A^n$, and

$$\overline{\psi} = \sum_{n=1}^{N} \left(\psi^{n+}\mu_B^n + \psi^{n-}\mu_S^n + m^{n+} + m^{n-}\right),$$

where m^{n+} and m^{n-} are given by:

$$m^{n+} = \int_{B_n} \left(\alpha_s^n \varepsilon_{si}^n + \alpha_\theta^n \varepsilon_{\hat{\theta}i}^n - \alpha_h^n \varepsilon_{hi}^n\right) di$$
$$m^{n-} = \int_{S_n} \left(\alpha_s^n \varepsilon_{si}^n + \alpha_\theta^n \varepsilon_{\hat{\theta}i}^n - \alpha_h^n \varepsilon_{hi}^n\right) di.$$

We can further express m^{n+} and m^{n-} as follows. Note that

$$w_n \equiv \alpha_s^n \varepsilon_{si}^n + \alpha_\theta^n \varepsilon_{\hat{\theta}i}^n - \alpha_h^n \varepsilon_{hi}^n \sim N\left(0, (\tau_{\Delta n})^{-1}\right),$$

where $\tau_{\Delta n}$ is defined in Eq. (A12). Moreover, given the characterization of the sets of buyers and sellers in the previous subsection, we have that

$$\int_{B_n} w_n di = \mathbb{E}\left[w_n | w_n > \Delta_n^+\right] \quad \text{and} \quad \int_{S_n} w_n di = \mathbb{E}\left[w_n | w_n < \Delta_n^-\right],$$

which are the expected values of a truncated normal at Δ_n^+ and Δ_n^-, respectively. Then,

$$m^{n+} = \frac{\Phi'\left(\frac{\Delta_n^+}{\sqrt{\tau_{\Delta n}}}\right)}{1 - \Phi\left(\frac{\Delta_n^+}{\sqrt{\tau_{\Delta n}}}\right)} \frac{1}{\sqrt{\tau_{\Delta n}}}$$
$$m^{n-} = \frac{-\Phi'\left(\frac{\Delta_n^-}{\sqrt{\tau_{\Delta n}}}\right)}{\Phi\left(\frac{\Delta_n^-}{\sqrt{\tau_{\Delta n}}}\right)} \frac{1}{\sqrt{\tau_{\Delta n}}}.$$

Since Δ_n^+ and Δ_n^- can be recovered from the measures of buyers and sellers of type n as

$$\Delta_n^+ = \frac{\Phi^{-1}\left(\mu_B^n\right)}{\sqrt{\tau_{\Delta n}}} \quad \text{and} \quad \Delta_n^- = -\frac{\Phi^{-1}\left(\mu_S^n\right)}{\sqrt{\tau_{\Delta n}}},$$

m^{n+} and m^{n-} are effectively observed given investors' information set, and the price is linear in θ,$\left\{\overline{\theta}_n\right\}_n$, δ, and $\{\hat{\mu}_B^n, \hat{\mu}_S^n\}_n$, as guessed above.

Proof of Lemma 3 (Existence)

Proof. Our existence proof uses Brouwer's fixed point theorem. Note that equilibrium price informativeness $\tau_{\hat{p}}$ can be expressed explicitly as a function of the measures of active buyers and sellers, $\boldsymbol{y} = \left[\mu_B^1, ..., \mu_B^N, \mu_S^1, ..., \mu_S^N\right]$, as follows:

$$\tau_{\hat{p}} = \frac{\left(\sum_{j=1}^{N} \frac{\tau_{sj}}{\gamma_j} \mu_A^j\right)^2}{\sum_{j=1}^{N} \left(\frac{\tau_{\theta j}}{\gamma_j} \mu_A^j\right)^2 \tau_{\overline{\theta}}^{-1} + \left(\sum_{j=1}^{N} \mu_A^j\right)^2 \tau_\delta^{-1}}.$$

Then, an equilibrium is fully characterized by the solution to the following system of equations

$$\mu_B^n = \Phi\left(\sqrt{\tau_{\Delta n}}\Delta_n^+\right), \forall n = 1, ..., N \tag{A16}$$
$$\mu_S^n = \Phi\left(-\sqrt{\tau_{\Delta n}}\Delta_n^-\right), \forall n = 1, ..., N,$$

where Δ_n^+ and Δ_n^- are functions of $\boldsymbol{y}$ and $\tau_{\Delta n}$ known in equilibrium given investors' information set. We can rewrite the system in Eq. (A16) as the fixed point of a mapping $\boldsymbol{T} : [0,1]^{2N} \to [0,1]^{2N}$, where $\boldsymbol{T}$ is continuous in $\boldsymbol{y}$.[26] An application of Brouwer's fixed point theorem implies that a solution to Eq. (A16) exists. $\square$

Claim. (Stability) In equilibrium, price informativeness is given by the fixed point of

$$H_L(x) \equiv -x + \frac{\left(\sum_{j=1}^{N} \frac{\tau_{sj}}{\gamma_j} \mu_A^j(x)\right)^2}{\sum_{j=1}^{N} \left(\frac{\tau_{\theta j}}{\gamma_j} \mu_A^j(x)\right)^2 \tau_\theta^{-1} + \left(\sum_{j=1}^{N} \mu_A^j(x)\right)^2 \tau_\delta^{-1}} = 0.$$

In a stable equilibrium, $H_L'(x^\star) < 0$.

Proof. Local stability follows from Eq. (A17) using local continuity and the implicit function theorem. In an equilibrium, $\tau_{\hat{p}}$ is implicitly characterized by the fixed point $J_L(x) = x$, where $J_L(x)$ is defined by

$$J_L(x) \equiv \frac{\left(\sum_{j=1}^{N} \frac{\tau_{sj}}{\gamma_j} \mu_A^j(x)\right)^2}{\sum_{j=1}^{N} \left(\frac{\tau_{\theta j}}{\gamma_j} \mu_A^j(x)\right)^2 \tau_\theta^{-1} + \left(\sum_{j=1}^{N} \mu_A^j(x)\right)^2 \tau_\delta^{-1}}, \tag{A17}$$

and where we used the contribution to the equilibrium aggregate demand sensitivities of each group, which depend on x through the set of active investors $\mu_A^n(x)$ and through the equilibrium demand sensitivities. Using the implicit function theorem, we know that $\mu_A^j(x)$ is a continuous and differentiable function of x in a neighborhood of $\tau_{\hat{p}}$. The function $J_L(x)$ determines price informativeness when investors expect the price informativeness to be x.

We adopt a conventional notion of stability. The function $H_L(x)$ is such that if $H_L(x_0) > 0$, then $J_L(x_0) > x_0$, which implies that if investors in the model expect the signal-to-noise ratio to be x_0, the realized value of this ratio will be $x_1 > x_0$. Let $x^\star$ be a solution to $H_L(x^\star) = 0$. Then, we will say that the equilibrium $x^\star$ is stable if for all $x_0 \in (x^\star - \varepsilon_\delta, x^\star - \varepsilon_\delta)$ for some $\varepsilon_\delta > 0$ the sequence $\{x_m\}_{m=0}^{\infty}$ where $x_m = J_L(x_{m-1})$ for $m > 1$ converges to $x^\star$. This sequence will converge only if $|J_L'(x^\star)| < 1$, which implies $H_L'(x^\star) < 0$. Hence, in all stable equilibria, $H_L'(x^\star) < 0$. $\square$

Proof of Theorem 4 (Irrelevance theorem with ex-ante identical investors and linear trading costs)

When investors are ex-ante identical, price informativeness is given by $\tau_{\hat{p}} = \frac{\tau_s^2}{\frac{\tau_\theta^2}{N}\tau_\theta^{-1} + \gamma^2 \tau_\delta^{-1}}$, which is independent of the level of the linear trading cost ϕ.

Proof of Lemma 4 (Directional characterization with linear trading costs)

Using the implicit function theorem we have that

$$\frac{\partial \tau_{\hat{p}}}{\partial \phi} = \left.\frac{\frac{\partial H_L}{\partial \phi}}{-H_L'}\right|_{\tau_{\hat{p}}},$$

[26] When $\boldsymbol{y} = 0$, we define the mapping $\boldsymbol{T}$ as the limit implied by Eq. (A16).

where

$$\left.\frac{\partial H_L}{\partial \phi}\right|_{\tau_{\hat{p}}=x} = \frac{2\overline{\alpha_s}(x)\sum_{j=1}^{N}\alpha_s^j\frac{\partial \mu_A^j(x)}{\partial \phi}\left(\overline{\alpha_N}(x)\right)^2 - 2\left(\overline{\alpha_s}(x)\right)^2\sum_{j=1}^{N}\mu_A^j(x)\left(\left(\alpha_\theta^j\right)^2\frac{\partial \mu_A^j(x)}{\partial \phi}\tau_{\overline{\theta}}^{-1} + \sum_{n=1}^{N}\frac{\partial \mu_A^n(x)}{\partial \phi}\tau_\delta^{-1}\right)}{\left(\overline{\alpha_N}(x)\right)^4}$$

$$= 2\left(\frac{\overline{\alpha_s}(x)}{\overline{\alpha_N}(x)}\right)^2\left(\frac{\sum_{j=1}^{N}\alpha_s^j\frac{\partial \mu_A^j(x)}{\partial \phi}}{\overline{\alpha_s}(x)} - \frac{\sum_{j=1}^{N}\frac{\partial \mu_A^j(x)}{\partial \phi}\left(\left(\alpha_\theta^j\right)^2\mu_A^j(x)\tau_{\overline{\theta}}^{-1} + \sum_{n=1}^{N}\mu_A^n(x)\tau_\delta^{-1}\right)}{\left(\overline{\alpha_N}(x)\right)^2}\right)$$

$$= 2\left(\frac{\overline{\alpha_s}(x)}{\overline{\alpha_N}(x)}\right)^2\sum_{j=1}^{N}\left(\frac{\alpha_s^j}{\overline{\alpha_s}(x)} - \frac{\left(\alpha_\theta^j\right)^2\mu_A^j(x)\tau_{\overline{\theta}}^{-1} + \sum_{n=1}^{N}\mu_A^n(x)\tau_\delta^{-1}}{\left(\overline{\alpha_N}(x)\right)^2}\right)\frac{\partial \mu_A^j(x)}{\partial \phi},$$

where

$$\left(\overline{\alpha_N}(x)\right)^2 \equiv \sum_{j=1}^{N}\left(\frac{\tau_{\theta j}}{\gamma_j}\mu_A^j(x)\right)^2\tau_{\overline{\theta}}^{-1} + \left(\sum_{j=1}^{N}\mu_A^j(x)\right)^2\tau_\delta^{-1}.$$

Hence,

$$\left.\frac{\partial H_L}{\partial \phi}\right|_{\tau_{\hat{p}}=x} = 2\left(\frac{\overline{\alpha_s}(x)}{\overline{\alpha_N}(x)}\right)^2\sum_{j=1}^{N}\mu_A^j(x)\,\mathbb{C}\mathrm{ov}_n\left(\frac{\alpha_s^n}{\overline{\alpha_s}(x)} - \frac{\left(\alpha_\theta^n\right)^2\mu_A^n(x)\tau_{\overline{\theta}}^{-1} + \sum_{j=1}^{N}\mu_A^j(x)\tau_\delta^{-1}}{\left(\overline{\alpha_N}(x)\right)^2}, \frac{\frac{\partial \mu_A^n(x)}{\partial \phi}}{\mu_A^n(x)}\right)$$

Note that $\frac{\partial H_L}{\partial \phi}$ keeps price informativeness constant as ϕ changes. Therefore,

$$\left.\frac{\partial \tau_{\hat{p}}}{\partial \phi}\right|_{\tau_{\hat{p}}=x} = \frac{2\left(\frac{\overline{\alpha_s}(x)}{\overline{\alpha_N}(x)}\right)^2\sum_{j=1}^{N}\mu_A^j(x)\,\mathbb{C}\mathrm{ov}_n\left(\frac{\alpha_s^n}{\overline{\alpha_s}(x)} - \frac{\left(\alpha_\theta^n\right)^2\mu_A^n(x)\tau_{\overline{\theta}}^{-1} + \sum_{j=1}^{N}\mu_A^j(x)\tau_\delta^{-1}}{\left(\overline{\alpha_N}(x)\right)^2}, \frac{\frac{\partial \mu_A^n(x)}{\partial \phi}}{\mu_A^n(x)}\right)}{-H_L'}.$$

Note that

$$\frac{\partial\left(\overline{\alpha_N}\right)^2}{\partial \mu_A^n} = 2\overline{\alpha_N}\frac{\partial \overline{\alpha_N}}{\partial \mu_A^j} = 2\left(\alpha_\theta^n\right)^2\mu_A^n\tau_{\overline{\theta}}^{-1} + 2\sum_{j=1}^{N}\mu_A^j\tau_\delta^{-1}.$$

Since the denominator is positive in any stable equilibrium, we have that

$$\mathrm{sgn}\left(\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_\theta}}\right)}{d\phi}\right) = \mathrm{sgn}\left(\mathbb{C}ov_n\left(\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{\left(\alpha_\theta^n\right)^2\mu_A^n\tau_{\overline{\theta}}^{-1} + \sum_{j=1}^{N}\mu_A^j\tau_\delta^{-1}}{\left(\overline{\alpha_N}\right)^2}, \frac{\frac{\partial \mu_A^n}{\partial \phi}}{\mu_A^n}\right)\right)$$

$$= \mathrm{sgn}\left(\mathbb{C}ov_n\left(\frac{\frac{\partial \overline{\alpha_s}}{\partial \mu_A^n}}{\overline{\alpha_s}} - \frac{\frac{\partial \overline{\alpha_N}}{\partial \mu_A^n}}{\overline{\alpha_N}}, \frac{\frac{\partial \mu_A^n}{\partial \phi}}{\mu_A^n}\right)\right).$$

The first term inside the covariance is the difference between the marginal contribution of group n to the information and noise in the price, respectively. The second term is the effect of linear trading costs on the extensive margin of group n. When this covariance is positive, an increase in linear trading costs induces less investors to stop trading in groups that contribute relatively more information than noise to the price. In this case, price informativeness increases with linear trading costs.

Proof of Theorem 5 (Directional results under one-dimensional heterogeneity with linear trading costs)

The proof of this theorem uses Lemma 4 and the result that the covariance of two monotone increasing functions is positive (see Schmidt (2003)). The structure of the proof is the same for all dimensions of heterogeneity considered. We will denote by $z_n = z + \eta h_n$ the group specific parameter over which investor groups differ. Then, we can write

$$\mathbb{C}ov_n \left[\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{\left(\alpha_\theta^n\right)^2 \mu_A^n \tau_{\overline{\theta}}^{-1} + \sum_{j=1}^N \mu_A^j \tau_\delta^{-1}}{\left(\overline{\alpha_N}\right)^2}, \frac{\frac{\partial \mu_A^n}{\partial \phi}}{\mu_A^n} \right] = \mathbb{C}ov_n \left[F_L\left(z_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega\right), G_L\left(z_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega\right) \right],$$

where $\overline{\boldsymbol{\alpha}}$ is the vector of all aggregate demand sensitivities, Ω denotes the variance-covariance matrix of the vector of public signals, and

$$F_L\left(z_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega\right) \equiv \frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{\left(\alpha_\theta^n\right)^2 \mu_A^n \tau_{\overline{\theta}}^{-1} + \overline{\alpha_h} \tau_\delta^{-1}}{\left(\overline{\alpha_N}\right)^2} \quad \text{and} \quad G_L\left(z_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega\right) \equiv \frac{\frac{\partial \mu_A^n}{\partial \phi}}{\mu_A^n},$$

where we used that $\overline{\alpha_h} = \sum_{j=1}^N \mu_A^j(x)$.

The main part of the proof consists of characterizing $\frac{\partial F_L}{\partial z_n}$ and $\frac{\partial G_L}{\partial z_n}$. To characterize these partial derivatives, we use the following intermediate results.[27]

Lemma 7. *When the linear cost is small, the set of active investors is the same across groups, i.e.,*

$$\lim_{\phi \to 0} \frac{\partial \mu_A^n\left(z_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega\right)}{\partial z_n} = 0.$$

Proof. The measure of active investors of type n is given by

$$\mu_A^n = \left(1 - \Phi\left(-\sqrt{\tau_{\Delta n}} \Delta_n^+\right) + \Phi\left(-\sqrt{\tau_{\Delta n}} \Delta_n^-\right)\right). \tag{A18}$$

Suppose investors differ only in dimension z with $z_n = z + \eta f_n$ for $f_n \in \mathbb{R}$ and $z \in \{\gamma, \tau_s, \tau_\theta\}$. Then,

$$\begin{aligned} \frac{\partial \mu_A^n\left(z_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega\right)}{\partial z_n} &= \left(\Phi'\left(-\sqrt{\tau_{\Delta n}} \Delta_n^+\right) \Delta_n^+ - \Phi'\left(-\sqrt{\tau_{\Delta n}} \Delta_n^-\right) \Delta_n^-\right) \frac{1}{2} \frac{1}{\sqrt{\tau_{\Delta n}}} \frac{\partial \tau_{\Delta n}}{\partial z_n} \\ &\quad + \sqrt{\tau_{\Delta n}} \left(\Phi'\left(-\sqrt{\tau_{\Delta n}} \Delta_n^+\right) \frac{\partial \Delta_n^+}{\partial z_n} - \Phi'\left(-\sqrt{\tau_{\Delta n}} \Delta_n^-\right) \frac{\partial \Delta_n^-}{\partial z_n}\right), \end{aligned}$$

which can be rewritten as

$$\begin{aligned} \frac{\partial \mu_A^n\left(z_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega\right)}{\partial z_n} &= \left(\Phi'\left(-\sqrt{\tau_{\Delta n}} \Delta_n^+\right) \Delta_n^+ - \Phi'\left(-\sqrt{\tau_{\Delta n}} \Delta_n^-\right) \Delta_n^-\right) \frac{1}{2} \frac{1}{\sqrt{\tau_{\Delta n}}} \frac{\partial \tau_{\Delta n}}{\partial z_n} \\ &\quad + \sqrt{\tau_{\Delta n}} \left(\left(-\Phi'\left(-\sqrt{\tau_{\Delta n}} \Delta_n^-\right) + \Phi'\left(-\sqrt{\tau_{\Delta n}} \Delta_n^+\right)\right) \frac{\partial \Delta_n^+}{\partial z_n} - 2\Phi'\left(\sqrt{\tau_{\Delta n}} \Delta_n^-\right) \frac{\partial \left(\frac{\phi}{\kappa_n}\right)}{\partial z_n}\right) \end{aligned}$$

using that

$$\Delta_n^- = \Delta_n^+ + \psi^{n-} - \psi^{n+} \quad \text{and} \quad \psi^{n-} - \psi^{n+} = 2\frac{\phi}{\kappa_n}.$$

Note that $\frac{\partial \mu_A^n(z_n; \overline{\boldsymbol{\alpha}})}{\partial z_n}$ keeps the aggregate demand sensitivities constant. Taking limits when the linear cost goes to zero gives

$$\lim_{\phi \to 0} \frac{\partial \mu_A^n}{\partial z_n} = 0,$$

since

$$\lim_{\phi \to 0} \Delta_n^- = \lim_{\phi \to 0} \Delta_n^+ = \Delta_n, \quad \text{and} \quad \lim_{\phi \to 0} \phi \frac{\partial \left(\frac{1}{\kappa_n}\right)}{\partial z_n} = 0.$$

□

[27]Note that all the partial derivatives that we compute are meant to keep aggregates constant. These partial derivatives seek to characterize how these functions change in the cross section of investors. The same logic applies to the fixed cost case.

Lemma 8. *The change in the extensive margin for different groups of investors when heterogeneity is small and linear costs go to zero is*

$$\lim_{\phi\to 0}\lim_{\eta\to 0}\frac{\partial G_L\left(z_n;\overline{\boldsymbol{\alpha}},\tau_{\hat{p}},\Omega\right)}{\partial z_n}=\lim_{\phi\to 0}\lim_{\eta\to 0}-\frac{\left(\frac{\partial\sqrt{\tau_{\Delta n}}}{\partial z_n}\frac{1}{\kappa_n}+\frac{\partial\left(\frac{1}{\kappa_n}\right)}{\partial z_n}\sqrt{\tau_{\Delta n}}\right)\frac{2}{\sqrt{2\pi}}}{\mu_A^n}.$$

Proof. Using the definition of μ_A^n in Eq. (A18) we have

$$G_L\left(z_n;\overline{\boldsymbol{\alpha}},\tau_{\hat{p}},\Omega\right)=\frac{\frac{\partial\mu_A^n}{\partial\phi}}{\mu_A^n}=\frac{\Phi'\left(-\sqrt{\tau_{\Delta n}}\Delta_n^+\right)\sqrt{\tau_{\Delta n}}\frac{\partial\Delta_n^+}{\partial\phi}-\Phi'\left(-\sqrt{\tau_{\Delta n}}\Delta_n^-\right)\sqrt{\tau_{\Delta n}}\frac{\partial\Delta_n^-}{\partial\phi}}{\mu_A^n}$$

where

$$\frac{\partial\Delta_n^+}{\partial\phi}=\sum_{n=1}^N\frac{\partial\Delta_n^+}{\partial\mu_A^n}\frac{\partial\mu_A^n}{\partial\phi}+\frac{\partial\psi^{n+}}{\partial\phi}.$$

Using that

$$\Delta_n^-=\Delta_n^++\psi^{n-}-\psi^{n+}=\Delta_n^++2\frac{\phi}{\kappa_n}$$

we have

$$\frac{\partial\Delta_n^+}{\partial\mu_A^n}=\frac{\partial\Delta_n^-}{\partial\mu_A^n}.$$

Then,

$$\begin{aligned}\frac{\partial\mu_A^n}{\partial\phi}=&\left(\Phi'\left(-\sqrt{\tau_{\Delta n}}\Delta_n^+\right)-\Phi'\left(-\sqrt{\tau_{\Delta n}}\Delta_n^-\right)\right)\sqrt{\tau_{\Delta n}}\left(\sum_{n=1}^N\frac{\partial\Delta_n^+}{\partial\mu_A^n}\frac{\partial\mu_A^n}{\partial\phi}\right)\\&+\Phi'\left(-\sqrt{\tau_{\Delta n}}\Delta_n^+\right)\sqrt{\tau_{\Delta n}}\frac{\partial\psi^{n+}}{\partial\phi}-\Phi'\left(-\sqrt{\tau_{\Delta n}}\Delta_n^-\right)\sqrt{\tau_{\Delta n}}\frac{\partial\psi^{n-}}{\partial\phi},\end{aligned}$$

which is the same as

$$\begin{aligned}\frac{\partial\mu_A^n}{\partial\phi}=&\left(\Phi'\left(-\sqrt{\tau_{\Delta n}}\Delta_n^+\right)-\Phi'\left(-\sqrt{\tau_{\Delta n}}\Delta_n^-\right)\right)\sqrt{\tau_{\Delta n}}\left(\sum_{n=1}^N\frac{\partial\Delta_n^+}{\partial\mu_A^n}\frac{\partial\mu_A^n}{\partial\phi}\right)\\&-\left(\Phi'\left(-\sqrt{\tau_{\Delta n}}\Delta_n^+\right)+\Phi'\left(-\sqrt{\tau_{\Delta n}}\Delta_n^-\right)\right)\sqrt{\tau_{\Delta n}}\frac{1}{\kappa_n},\end{aligned}$$

where we used that

$$\frac{\partial\psi^{n+}}{\partial\phi}=-\frac{1}{\kappa_n}\quad\text{and}\quad\frac{\partial\psi^{n-}}{\partial\phi}=\frac{1}{\kappa_n}.$$

Note that

$$\lim_{\phi\to 0}\lim_{\eta\to 0}\left(\Phi'\left(-\sqrt{\tau_{\Delta n}}\Delta_n^+\right)-\Phi'\left(-\sqrt{\tau_{\Delta n}}\Delta_n^-\right)\right)=0$$

and

$$\lim_{\phi\to 0}\lim_{\eta\to 0}\Phi''\left(-\sqrt{\tau_{\Delta n}}\Delta_n^+\right)=\lim_{\phi\to 0}\lim_{\eta\to 0}\Phi''\left(-\sqrt{\tau_{\Delta n}}\Delta_n^-\right)=0.$$

Then, using that $\lim_{\phi\to 0}\frac{\partial\mu_A^n}{\partial z_n}=0$ and that $\lim_{\phi\to 0}\mu_A^n=1$ we have

$$\begin{aligned}\lim_{\phi\to 0}\lim_{\eta\to 0}\frac{\partial G_L\left(z_n;\overline{\boldsymbol{\alpha}},\tau_{\hat{p}},\Omega\right)}{\partial z_n}&=\lim_{\phi\to 0}\lim_{\eta\to 0}-\frac{\partial\left(\sqrt{\tau_{\Delta n}}\frac{1}{\kappa_n}\right)}{\partial z_n}\left(\Phi'(0)+\Phi'(0)\right)\\&=\lim_{\phi\to 0}\lim_{\eta\to 0}-\left(\frac{\partial\sqrt{\tau_{\Delta n}}}{\partial z_n}\frac{1}{\kappa_n}+\frac{\partial\left(\frac{1}{\kappa_n}\right)}{\partial z_n}\sqrt{\tau_{\Delta n}}\right)\frac{2}{\sqrt{2\pi}}.\end{aligned}$$

There are two effects that determine how the measure of active investors changes with the linear trading costs across groups. First, different groups will have different dispersion in the distribution of their net realized demands. When this dispersion is low (higher $\tau_{\Delta n}$), the fraction of active investors decreases more with linear trading costs. Second, the scale of the net demands is different across groups. Groups with higher scales (lower κ_n) react more in the extensive margin to changes in linear trading costs. These two effects are captured by the two terms in brackets in the expression above. $\square$

a) Suppose that investor groups are heterogeneous only in the precision of their private information, with $\tau_{sn} = \tau_s + \eta f_n$. Recall that

$$(\overline{\alpha_N})^2 \equiv \sum_{j=1}^{N} \left(\frac{\tau_\theta}{\gamma} \mu_A^j \right)^2 \tau_{\overline{\theta}}^{-1} + (\overline{\alpha_h})^2 \tau_\delta^{-1}$$

and $\overline{\alpha_h} = \sum_{j=1}^{N} \mu_A^j$. Then,

$$\frac{\partial (\overline{\alpha_N})^2}{\partial \mu_A^n} = 2 (\alpha_\theta^n)^2 \mu_A^n \tau_{\overline{\theta}}^{-1} + 2\overline{\alpha_h} \tau_\delta^{-1}$$

and

$$\frac{\partial F_L (\tau_{sn}; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega)}{\partial \tau_{sn}} = \frac{1}{\gamma} \frac{1}{\overline{\alpha_s}} - \frac{(\alpha_\theta^n)^2 \tau_{\overline{\theta}}^{-1}}{(\overline{\alpha_N})^2} \frac{\partial \mu_A^n}{\partial \tau_{sn}}.$$

Using Lemma 7 we have

$$\lim_{\phi \to 0} \lim_{\eta \to 0} \frac{\partial F_L (\tau_{sn}; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega)}{\partial \tau_{sn}} = \frac{1}{\gamma} \frac{1}{\overline{\alpha_s}} > 0.$$

Moreover, from Lemma 8 we have

$$\lim_{\phi \to 0} \lim_{\eta \to 0} \frac{\partial G_L (\tau_{sn}; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega)}{\partial \tau_{sn}} = \lim_{\phi \to 0} \lim_{\eta \to 0} - \left(\frac{\partial \sqrt{\tau_{\Delta n}}}{\partial \tau_{sn}} \frac{1}{\kappa_n} + \frac{\partial \left(\frac{1}{\kappa_n} \right)}{\partial \tau_{sn}} \sqrt{\tau_{\Delta n}} \right) \frac{2}{\sqrt{2\pi}},$$

which is the same as

$$\begin{aligned}
\lim_{\phi \to 0} \lim_{\eta \to 0} \frac{\partial G_L \left(\tau_{sn}; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega \right)}{\partial \tau_{sn}} &= \lim_{\phi \to 0} \lim_{\eta \to 0} -\frac{1}{\gamma} \sqrt{\tau_{\Delta n}} \left(-\frac{1}{2} \frac{1}{\gamma} \tau_{\Delta n} \frac{1}{\kappa_n} + 1 \right) \frac{2}{\sqrt{2\pi}} \\
&= \lim_{\phi \to 0} \lim_{\eta \to 0} -\frac{1}{\gamma} \sqrt{\tau_{\Delta n}} \left(-\frac{1}{2} \left(\tau_{sn} + \tau_\theta^2 \tau_{\hat{\theta}}^{-1} + \gamma^2 \tau_h^{-1} \right)^{-1} \left(\tau_{sn} + \tau_\theta + \sum_{h=1}^{N+1} \tilde{\Lambda}_{hh}^{-1} \right) + 1 \right) \frac{2}{\sqrt{2\pi}} \\
&= \lim_{\phi \to 0} \lim_{\eta \to 0} -\frac{1}{\gamma} \sqrt{\tau_{\Delta n}} \left(\tau_{sn} + \tau_\theta^2 \tau_{\hat{\theta}}^{-1} + \gamma^2 \tau_h^{-1} \right)^{-1} \left(-\frac{1}{2} \left(\tau_{sn} + \tau_\theta + \sum_{h=1}^{N+1} \tilde{\Lambda}_{hh}^{-1} \right) + \tau_{sn} + \tau_\theta^2 \tau_{\hat{\theta}}^{-1} + \gamma^2 \tau_h^{-1} \right) \frac{2}{\sqrt{2\pi}} \\
&= -\frac{1}{\gamma} \sqrt{\tau_\Delta} \left(\tau_s + \tau_\theta^2 \tau_{\hat{\theta}}^{-1} + \gamma^2 \tau_h^{-1} \right)^{-1} \left(\frac{1}{2} \left(\tau_s - \tau_\theta - \frac{\tau_s^2}{\frac{\tau_\theta^2 \tau_{\overline{\theta}}^{-1}}{N} + \gamma^2 \tau_\delta^{-1}} \right) + \tau_\theta^2 \tau_{\hat{\theta}}^{-1} + \gamma^2 \tau_h^{-1} \right) \frac{2}{\sqrt{2\pi}}
\end{aligned}$$

since

$$\frac{\partial \sqrt{\tau_{\Delta n}}}{\partial \tau_{sn}} = -\frac{1}{2} \frac{1}{\sqrt{\tau_{\Delta n}}} (\tau_{\Delta n})^2 \frac{1}{\gamma^2}, \quad \frac{\partial \left(\frac{1}{\kappa_n} \right)}{\partial \tau_{sn}} = \frac{1}{\gamma},$$

and

$$\lim_{\eta \to 0} \sum_{h=1}^{N+1} \tilde{\Lambda}_{hh}^{-1} = \tau_{\hat{p}} = \frac{N^2 \tau_s^2}{N \tau_\theta^2 \tau_{\overline{\theta}}^{-1} + \gamma^2 N^2 \tau_\delta^{-1}}.$$

Since

$$S_s(\tau_s) \equiv \frac{1}{2} \left(\tau_s - \tau_\theta - \frac{\tau_s^2}{\frac{\tau_\theta^2 \tau_{\overline{\theta}}^{-1}}{N} + \gamma^2 \tau_\delta^{-1}} \right) + \tau_\theta^2 \tau_{\hat{\theta}}^{-1} + \gamma^2 \tau_h^{-1}$$

is quadratic and concave in τ_s. Then, there exist thresholds $\tau_{s1}^{\star}$ and $\tau_{s2}^{\star}$ such that:

i) for all $\tau_s \in (0, \tau_{s1}^{\star}) \bigcup (\tau_{s2}^{\star}, \infty)$

$$\operatorname{sgn}\left(\mathbb{C}ov_n\left(\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{(\alpha_\theta^n)^2 \mu_A^n \tau_{\overline{\theta}}^{-1} + \overline{\alpha_h}\tau_\delta^{-1}}{\overline{\alpha_\theta^2}}, \frac{\frac{\partial \mu_A^n}{\partial \phi}}{\mu_A^n}\right)\right) > 0$$

and price informativeness increases with linear trading costs as investors who contribute less to the information contained in the price are the ones who disproportionately exit the market (recall that $\frac{\partial \mu_A^n}{\partial \phi} < 0$) .

ii) for all $\tau_s \in (\tau_{s1}^{\star}, \tau_{s2}^{\star})$

$$\operatorname{sgn}\left(\mathbb{C}ov_n\left(\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{(\alpha_\theta^n)^2 \mu_A^n \tau_{\overline{\theta}}^{-1} + \overline{\alpha_h}\tau_\delta^{-1}}{\overline{\alpha_\theta^2}}, \frac{\frac{\partial \mu_A^n}{\partial \phi}}{\mu_A^n}\right)\right) < 0$$

and price informativeness decreases with linear trading costs as investors who contribute more to the information contained in the price are the ones who disproportionately exit the market.

b) Suppose that investor groups are heterogeneous only in the precision of their prior, with $\tau_{\theta n} = \tau_\theta + \eta f_n$. In this case,

$$\frac{\partial F_L(\tau_{\theta n}; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega)}{\partial \tau_{\theta n}} = -\frac{2\alpha_\theta^n}{(\overline{\alpha_N})^2}\frac{1}{\gamma_n}\mu_A^n \tau_{\overline{\theta}}^{-1} - \frac{(\alpha_\theta^n)^2 \tau_{\overline{\theta}}^{-1}}{(\overline{\alpha_N})^2}\frac{\partial \mu_A^n}{\partial \tau_{\theta n}},$$

and using Lemma 7

$$\lim_{\phi\to 0}\lim_{\eta\to 0}\frac{\partial F_L(\tau_{\theta n}; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega)}{\partial \tau_{\theta n}} = -\frac{2\alpha_\theta^n}{\overline{\alpha_\theta^2}}\frac{1}{\gamma_n}\mu_A^n \tau_{\overline{\theta}}^{-1} < 0.$$

Moreover, from Lemma 8 we have

$$\lim_{\phi\to 0}\lim_{\eta\to 0}\frac{\partial G_L(\tau_{\theta n}; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega)}{\partial \tau_{\theta n}} = \lim_{\phi\to 0}\lim_{\eta\to 0} -\left(\frac{\partial\sqrt{\tau_{\Delta n}}}{\partial \tau_{\theta n}}\frac{1}{\kappa_n} + \frac{\partial\left(\frac{1}{\kappa_n}\right)}{\partial \tau_{\theta n}}\sqrt{\tau_{\Delta n}}\right)\frac{2}{\sqrt{2\pi}},$$

which is the same as

$$\begin{aligned}
\lim_{\phi\to 0}\lim_{\eta\to 0}\frac{\partial G_L(\tau_{\theta n}; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega)}{\partial \tau_{\theta n}} &= \lim_{\phi\to 0}\lim_{\eta\to 0} -\frac{1}{\gamma}\sqrt{\tau_{\Delta n}}\left(-\frac{1}{\gamma}\tau_{\Delta n}\frac{\tau_{\theta n}}{\tau_{\hat{\theta}}}\frac{1}{\kappa_n} + 1\right)\frac{2}{\sqrt{2\pi}} \\
&= \lim_{\phi\to 0}\lim_{\eta\to 0} -\frac{1}{\gamma^3}\frac{1}{\sqrt{\tau_{\Delta n}}}\left(-\left(\tau_s + \tau_{\theta n} + \sum_{h=1}^{N+1}\tilde{\Lambda}_{hh}^{-1}\right)\frac{\tau_{\theta n}}{\tau_{\hat{\theta}}} + \tau_s + \tau_{\theta n}^2\tau_{\hat{\theta}}^{-1} + \gamma^2(\tau_h)^{-1}\right)\frac{2}{\sqrt{2\pi}} \\
&= -\frac{1}{\gamma^3}\frac{1}{\sqrt{\tau_{\Delta n}}}\left(\left(1 - \frac{\tau_\theta}{\tau_{\hat{\theta}}}\right)\tau_s - \frac{N^2\tau_s^2}{N\tau_\theta^2\tau_{\overline{\theta}}^{-1} + \gamma^2 N^2\tau_\delta^{-1}}\frac{\tau_\theta}{\tau_{\hat{\theta}}} + \gamma^2(\tau_h)^{-1}\right)\frac{2}{\sqrt{2\pi}},
\end{aligned}$$

since

$$\frac{\partial\sqrt{\tau_{\Delta n}}}{\partial \tau_{\theta n}} = -\frac{1}{\sqrt{\tau_{\Delta n}}}(\tau_{\Delta n})^2\frac{1}{\gamma^2}\frac{\tau_{\theta n}}{\tau_{\hat{\theta}}}, \quad \frac{\partial\left(\frac{1}{\kappa_n}\right)}{\partial \tau_{\theta n}} = \frac{1}{\gamma}, \quad \text{and} \quad \lim_{\eta\to 0}\sum_{h=1}^{N+1}\Lambda_{hh}^{-1} = \tau_{\hat{p}} = \frac{N^2\tau_s^2}{N\tau_\theta^2\tau_{\overline{\theta}}^{-1} + \gamma^2 N^2\tau_\delta^{-1}}.$$

Let

$$S_\theta(\tau_\theta) \equiv -N^2\tau_s^2\frac{\tau_\theta}{\tau_{\hat{\theta}}} + \left(\tau_s - \frac{\tau_\theta}{\tau_{\hat{\theta}}}\tau_s + \gamma^2(\tau_h)^{-1}\right)\left(N\tau_\theta^2\tau_{\overline{\theta}}^{-1} + \gamma^2 N^2\tau_\delta^{-1}\right).$$

Note that $S_\theta(\cdot)$ is a cubic function of τ_θ and that $S_\theta(0) = \left(\tau_s + \gamma^2 (\tau_h)^{-1}\right)\gamma^2 N^2 \tau_\delta^{-1} > 0$, $S'_\theta(0) = -N^2 \frac{\tau_s}{\tau_{\hat{\theta}}}\left(\tau_s + \gamma^2 \tau_\delta^{-1}\right) < 0$, $S''_\theta(0) = \left(\tau_s + \gamma^2 (\tau_h)^{-1}\right) 2N\tau_{\overline{\theta}}^{-1} > 0$ and $\lim_{\tau_\theta \to \infty} S_\theta(\tau_\theta) = -\infty$. Then, generically, there exist three thresholds $\tau^\star_{\theta,1}$, $\tau^\star_{\theta,2}$, and $\tau^\star_{\theta,3}$ such that

i) If $\tau_\theta \in [0, \tau^\star_{\theta 1}) \bigcup (\tau^\star_{\theta 2}, \tau^\star_{\theta 3})$, $\frac{\partial \mu_A^n}{\partial \phi}$ is decreasing in $\tau_{\theta n}$ and

$$\operatorname{sgn}\left(\mathbb{C}ov_n\left(\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{(\alpha_\theta^n)^2 \mu_A^n \tau_{\overline{\theta}}^{-1} + \overline{\alpha_h}\tau_\delta^{-1}}{(\overline{\alpha_N})^2}, \frac{\frac{\partial \mu_A^n}{\partial \phi}}{\mu_A^n}\right)\right) > 0$$

which implies price informativeness increases with linear trading costs.

ii) If $\tau_\theta \in (\tau^\star_{\theta 1}, \tau^\star_{\theta 2}) \bigcup (\tau^\star_{\theta 3}, \infty)$, $\frac{\partial \mu_A^n}{\partial \phi}$ is increasing in $\tau_{\theta n}$ and

$$\operatorname{sgn}\left(\mathbb{C}ov_n\left(\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{(\alpha_\theta^n)^2 \mu_A^n \tau_{\overline{\theta}}^{-1} + \overline{\alpha_h}\tau_\delta^{-1}}{\alpha_N^2}, \frac{\frac{\partial \mu_A^n}{\partial \phi}}{\mu_A^n}\right)\right) < 0$$

which implies price informativeness decreases with linear trading costs.

c) Suppose that investor groups are heterogeneous only in their risk aversion, with $\gamma_n = \gamma + \eta f_n$. Then,

$$\frac{\partial F_L(\gamma_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega)}{\partial \gamma_n} = -\frac{1}{\gamma_n}\frac{\alpha_s^n}{\overline{\alpha_s}} + 2\frac{1}{\gamma_n}\frac{(\alpha_\theta^n)^2 \mu_A^n}{(\overline{\alpha_N})^2}\tau_{\overline{\theta}}^{-1} - \frac{(\alpha_\theta^n)^2}{(\overline{\alpha_N})^2}\frac{\partial \mu_A^n}{\partial \gamma_n}\tau_{\overline{\theta}}^{-1}.$$

Taking limits when heterogeneity and the linear trading cost go to zero, we have

$$\lim_{\phi \to 0}\lim_{\eta \to 0}\frac{\partial F_L(\gamma_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega)}{\partial \gamma_n} = -\frac{1}{\gamma}\frac{\alpha_s}{\overline{\alpha_s}} + 2\frac{1}{\gamma}\frac{(\alpha_\theta)^2}{(\overline{\alpha_N})^2}\tau_{\overline{\theta}}^{-1} = \frac{1}{\gamma}\frac{-(N-2)\tau_\theta^2\tau_{\overline{\theta}}^{-1} - \gamma^2 N^2 \tau_\delta^{-1}}{N\tau_\theta^2 \tau_{\overline{\theta}}^{-1} + \gamma^2 N^2 \tau_\delta^{-1}} < 0,$$

where we used Lemma 7 and that $\lim_{\phi \to 0}\lim_{\eta \to 0}\mu_A^n = 1$, $\lim_{\eta \to 0}\alpha_s^n = \alpha_s = \overline{\alpha_s}$ and $\lim_{\eta \to 0}(\overline{\alpha_N})^2 = \frac{1}{\gamma^2}\left(N\tau_\theta^2\tau_{\overline{\theta}}^{-1} + \gamma^2 N^2 \tau_\delta^{-1}\right)$.

Moreover, from Lemma 8 we have

$$\lim_{\phi \to 0}\lim_{\eta \to 0}\frac{\partial G_L(\gamma_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega)}{\partial \gamma_n} = \lim_{\phi \to 0}\lim_{\eta \to 0} - \left(\frac{\partial\sqrt{\tau_{\Delta n}}}{\partial \gamma_n}\frac{1}{\kappa_n} + \frac{\partial\left(\frac{1}{\kappa_n}\right)}{\partial \gamma_n}\sqrt{\tau_{\Delta n}}\right)\frac{2}{\sqrt{2\pi}},$$

which is the same as

$$\begin{aligned}
\lim_{\phi \to 0}\lim_{\eta \to 0}\frac{\partial G_L(\gamma_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega)}{\partial \gamma_n} &= \lim_{\phi \to 0}\lim_{\eta \to 0} - \left(\frac{\tau_{\Delta n}}{\gamma_n^2}\left(\tau_s + \tau_\theta^2 \tau_{\hat{\theta}}^{-1}\right) - 1\right)\frac{1}{\gamma_n}\frac{\sqrt{\tau_{\Delta n}}}{\kappa_n}\frac{2}{\sqrt{2\pi}} \\
&= \lim_{\phi \to 0}\lim_{\eta \to 0} - \left(\tau_s + \tau_\theta^2\tau_{\hat{\theta}}^{-1} - \left(\tau_s + \tau_\theta^2\tau_{\hat{\theta}}^{-1} + \gamma_n^2 \tau_h^{-1}\right)\right)\frac{(\tau_{\Delta n})^{\frac{3}{2}}}{\gamma_n^3}\frac{1}{\kappa_n}\frac{2}{\sqrt{2\pi}} \\
&= \lim_{\phi \to 0}\lim_{\eta \to 0}\gamma_n^2 \tau_h^{-1}\frac{(\tau_{\Delta n})^{\frac{3}{2}}}{\gamma_n^3}\frac{1}{\kappa_n}\frac{2}{\sqrt{2\pi}} > 0
\end{aligned}$$

since

$$\frac{\partial\sqrt{\tau_{\Delta n}}}{\partial \gamma_n} = \frac{1}{2\sqrt{\tau_{\Delta n}}}2\frac{\tau_{\Delta n}^2}{\gamma_n^3}\left(\tau_s + \tau_\theta \tau_{\hat{\theta}}^{-1}\right) = \frac{(\tau_{\Delta n})^{\frac{3}{2}}}{\gamma_n^3}\left(\tau_s + \tau_\theta^2 \tau_{\hat{\theta}}^{-1}\right),$$

and

$$\frac{\partial\left(\frac{1}{\kappa_n}\right)}{\partial \gamma_n} = -\frac{\left(\tau_s + \tau_\theta + \sum_{h=1}^{N+1}\tilde{\Lambda}_{hh}^{-1}\right)}{\gamma_n^2} = -\frac{1}{\gamma_n}\frac{1}{\kappa_n}.$$

Table A1: Summary of effects, Theorem 5

	Information to noise	Extensive Margin	Concentration and thresholds	Price Informativeness
	$\frac{\partial F}{\partial z}$	$\frac{\partial G}{\partial z}$	$\frac{\partial\sqrt{\tau_{\Delta n}}}{\partial z_n}$ and $\frac{\partial\left(\frac{1}{\kappa_n}\right)}{\partial z_n}$	$\frac{d\tau_{\hat{p}}}{d\phi}$
τ_s	>0	$\gtreqless 0$	>0 and <0	$\gtreqless 0$
τ_θ	<0	$\gtreqless 0$	>0 and <0	$\gtreqless 0$
γ	<0	>0	<0 and >0	<0

Therefore,

$$\lim_{\phi\to0}\lim_{\eta\to0}\mathbb{C}ov_n\left(\frac{\alpha_s^n}{\overline{\alpha_s}}-\frac{\left(\alpha_\theta^n\right)^2\mu_A^n\tau_{\overline{\theta}}^{-1}+\overline{\alpha_h}\tau_\delta^{-1}}{\overline{\alpha_\theta^2}},\frac{\frac{\partial\mu_A^n}{\partial\phi}}{\mu_A^n}\right)<0.$$

Table A1 summarizes the results.

A.4 Section 7: Fixed costs

Characterization of the sets of buyers and sellers

Note that $W_1^n\left(\mathcal{I}_i\right)\equiv\frac{\gamma_n\mathbb{V}\mathrm{ar}_n[\theta|\mathcal{I}_i]}{2}\left(q_{1i}^{n\star}\right)^2+pq_0^n$ measures the welfare of an investor i of type n if he chooses to participate in the asset market and optimally chooses to trade $q_{1i}^{n\star}-q_{0i}^n$ units of the risky asset. Analogously, $W_0^n\left(\mathcal{I}_i\right)\equiv\mathbb{E}_n\left[\theta|\mathcal{I}_i\right]q_{0i}^n-\frac{\gamma_n}{2}\mathbb{V}\mathrm{ar}_n\left[\theta|\,\mathcal{I}_i\right]\left(q_{0i}^n\right)^2$ measures the welfare of an investor i of type n if he does not participate in the asset market. Since $q_{0i}^n=0$ for all investors, we have that an investor i of type n will choose to participate in the asset market if

$$\Delta W_n\equiv W_1^n\left(\mathcal{I}_i\right)-W_0^n\left(\mathcal{I}_i\right)=\frac{\gamma_n}{2}\mathbb{V}\mathrm{ar}_n\left[\theta|\mathcal{I}_i\right]\left(\Delta q_{1i}^n\right)^2>\chi,$$

where, in an equilibrium in linear strategies,

$$\begin{aligned}\Delta q_{1i}^n=&\alpha_s^n\varepsilon_{si}^n+\alpha_\theta^n\varepsilon_{\hat{\theta}i}^n+\left(\frac{\alpha_s^n}{\overline{\alpha_s}}-\frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\alpha_s}\theta-\left(\frac{\alpha_h^n}{\overline{\alpha_h}}-\frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\alpha_h}\delta+\left(1-\frac{\alpha_p^n}{\overline{\alpha_p}}\mu_A^n\right)\alpha_\theta^n\overline{\theta}_n-\frac{\alpha_p^n}{\overline{\alpha_p}}\sum_{j\neq n}\mu_A^j\alpha_\theta^j\overline{\theta_j}\\&+\sum_{j=1}^N\left(\left(\frac{\alpha_{\mu B}^{jn}}{\overline{\alpha_{\mu B}^j}}-\frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\alpha_{\mu B}^j\hat{\mu}_B^j}+\left(\frac{\alpha_{\mu S}^{jn}}{\overline{\alpha_{\mu S}^j}}-\frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\alpha_{\mu S}^j\hat{\mu}_S^j}\right)+\left(\frac{\psi^n}{\overline{\psi}}-\frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\psi}.\end{aligned}$$

Note that

$$\Delta q_{1i}^n|\theta,\delta,\left\{\overline{\theta}_j,\hat{\mu}_A^j,\mu_A^j\right\}_{j=1}^N\sim N\left(\overline{\Delta_n},\tau_{\Delta n}^{-1}\right)$$

with

$$\begin{aligned}\overline{\Delta_n}=&\left(\frac{\alpha_s^n}{\overline{\alpha_s}}-\frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\alpha_s}\theta-\left(\frac{\alpha_h^n}{\overline{\alpha_h}}-\frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\alpha_h}\delta+\left(1-\frac{\alpha_p^n}{\overline{\alpha_p}}\mu_A^n\right)\alpha_\theta^n\overline{\theta}_n-\frac{\alpha_p^n}{\overline{\alpha_p}}\sum_{j\neq n}\mu_A^j\alpha_\theta^j\overline{\theta_j}\\&+\sum_{j=1}^N\left(\left(\frac{\alpha_{\mu B}^{jn}}{\overline{\alpha_{\mu B}^j}}-\frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\alpha_{\mu B}^j\hat{\mu}_B^j}+\left(\frac{\alpha_{\mu S}^{jn}}{\overline{\alpha_{\mu S}^j}}-\frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\alpha_{\mu S}^j\hat{\mu}_S^j}\right)+\left(\frac{\psi^n}{\overline{\psi}}-\frac{\alpha_p^n}{\overline{\alpha_p}}\right)\overline{\psi}\end{aligned}$$

and

$$\left(\tau_{\Delta n}\right)^{-1}=\left(\alpha_s^n\right)^2\tau_{sn}^{-1}+\left(\alpha_h^n\right)^2\left(\tau_{hn}\right)^{-1}+\left(\alpha_\theta^n\right)^2\left(\tau_{\hat{\theta}n}\right)^{-1}.$$

Then, the set of active buyers of type n is

$$B_n = \left\{ i : \Delta q_{1i}^n > \sqrt{\frac{2}{\kappa_n}\chi} \right\},$$

where $\kappa_n \equiv \gamma_n \mathbb{V}\mathrm{ar}_n\left[\theta|\mathcal{I}_i\right]$. The measure of active buyers of type n is

$$\mu_B^n = 1 - \Phi\left(\sqrt{\tau_{\Delta n}} \left(\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n} \right) \right).$$

Similarly, the set of active sellers is

$$S_n = \left\{ i : \Delta q_{1i}^n < -\sqrt{\frac{2}{\kappa_n}\chi} \right\},$$

and the measure of active sellers is

$$\mu_S^n = \Phi\left(\sqrt{\tau_{\Delta n}} \left(-\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n} \right) \right).$$

Since all parameters are known in the economy, knowing the measure of buyers and sellers of each type reveals information about the fundamental θ, through $\overline{\Delta_n}$. Hence, the information contained in the set of buyers is the same as the one contained in the set of sellers, *i.e.*, $\hat{\mu}_B = \hat{\mu}_S$. Hence, without loss of generality we can set $\alpha_{\mu S}^{jn} = 0$ for all n, j. Then, the linear signals contained in the measure of buyers of type n is given by the system,

$$z_n \hat{\mu}_B^n = \theta + \frac{\left(\frac{\alpha_h^n}{\overline{\alpha_h}} - \frac{\alpha_p^n}{\overline{\alpha_p}} \right) \overline{\alpha_h}}{\left(\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{\alpha_p^n}{\overline{\alpha_p}} \right) \overline{\alpha_s}} \delta + \frac{\left(1 - \frac{\alpha_p^n}{\overline{\alpha_p}} \mu_A^n \right) \alpha_\theta^n}{\left(\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{\alpha_p^n}{\overline{\alpha_p}} \right) \overline{\alpha_s}} \left(\overline{\theta}_n - \mu_{\overline{\theta}} \right) - \frac{\frac{\alpha_p^n}{\overline{\alpha_p}} \sum_{j \neq n} \alpha_\theta^j \mu_A^j \overline{\theta_j}}{\left(\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{\alpha_p^n}{\overline{\alpha_p}} \right) \overline{\alpha_s}} \left(\overline{\theta_j} - \mu_{\overline{\theta}} \right) + \sum_{j=1}^{N} \left(\frac{\left(\frac{\alpha_{\mu B}^{jn}}{\overline{\alpha_{\mu B}^j}} - \frac{\alpha_p^n}{\overline{\alpha_p}} \right)}{\left(\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{\alpha_p^n}{\overline{\alpha_p}} \right)} \frac{\overline{\alpha_{\mu B}^j}}{\overline{\alpha_s}} \hat{\mu}_B^j \right) \quad \forall n,$$

where z_n are such that the signals $\hat{\mu}_B^n$ are unbiased. Note that this unbiased signal is the same as the one in Eq. (A14) for the linear cost case. Hence, the filtering problem solved by the investors in this case is the same as the one in the linear cost case. Using the expressions in Eq. (A15), we have that the equilibrium coefficients are

$$\alpha_s^n = \frac{\tau_{sn}}{\gamma_n}, \quad \alpha_\theta^n = \frac{\tau_{\theta n}}{\gamma_n}, \quad \alpha_h^n = 1, \quad \alpha_{\mu B}^{jn} = \frac{K_{j+1}^n}{\gamma_n \mathbb{V}\mathrm{ar}_n \left[\theta | \hat{\theta}_i^n, s_i^n, h_i^n, \hat{p}, \left\{ \mu_B^j \right\}_{j=1}^N \right]}$$

$$\alpha_p^n = \frac{1 - \frac{\overline{\alpha_p}}{\overline{\alpha_s}} K_1^n}{\kappa_n}, \quad \text{and}$$

$$\psi = \frac{1}{\kappa_n} \left(-\kappa_n q_0^n - K_1^n \left(-\frac{\overline{\alpha_h}}{\overline{\alpha_s}} \omega_\delta + \frac{\overline{\alpha_\theta}}{\overline{\alpha_s}} \omega_{\overline{\theta}} + \sum_j \frac{\overline{\alpha_{\mu B}^j}}{\overline{\alpha_s}} \hat{\mu}_B^j + \frac{\overline{\psi}}{\overline{\alpha_s}} \right) \right)$$

where K_i^n is the ith element of the Kalman gain vector in Eq. (A15) for an investor of type n, where as in the cases with quadratic and linear costs the overlined variables are averages of the individual demand parameters.

The precision of the unbiased signal of θ from the perspective of an external observer who observes the measures of buyers and sellers of each type, which we denote by $\tau_{\hat{p}}$, is the relevant measure of price informativeness. Price informativeness is given by

$$\tau_{\hat{p}} = \frac{\left(\overline{\alpha_s}\right)^2}{\left(\overline{\alpha_N}\right)^2},$$

where

$$\left(\overline{\alpha_N}\right)^2 \equiv \sum_{j=1}^{N} \left(\alpha_\theta^j \mu_A^j\right)^2 \tau_{\overline{\theta}}^{-1} + \left(\overline{\alpha_h}\right)^2 \tau_\delta^{-1}$$

with $\overline{\alpha_h} = \sum_{j=1}^{N} \mu_A^j$.

Equilibrium price characterization

The characterization in this subsection follows the one for linear costs. In an equilibrium in linear strategies, we conjecture net demand functions for active investors of type n given by

$$\Delta q_{1i}^n = \alpha_s^n s_i + \alpha_\theta^n \hat{\theta}_i^n - \alpha_h^n h_i^n - \alpha_p^n p + \sum_{j=1}^{N} \left(\alpha_{\mu B}^{jn} \hat{\mu}_B^j + \alpha_{\mu S}^{jn} \hat{\mu}_S^j\right) + \psi^n,$$

where $\hat{\mu}_n^B$ and $\hat{\mu}_n^S$ are the unbiased signals about the fundamental contained in the measures of buyers and sellers of type n, respectively, and α_θ^n, α_s^n, and α_p^n are positive scalars, while ψ^n can take positive or negative values. We define as B_n, S_n and A_n the sets of buyers, sellers, and active investors of type n, respectively. Market clearing in the asset market is given by

$$\sum_{n=1}^{N} \int_{A_n} \Delta q_{1i}^n di = \sum_{n=1}^{N} \int_{B_n} \Delta q_{1i}^n di + \sum_{n=1}^{N} \int_{S_n} \Delta q_{1i}^n di = 0,$$

which implies

$$\begin{aligned}
p &= \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta + \frac{\sum_{n=1}^{N} \alpha_\theta^n \overline{\theta}_n \mu_A^n}{\overline{\alpha_p}} - \frac{\overline{\alpha_h}}{\overline{\alpha_p}}\delta + \frac{1}{\overline{\alpha_p}} \sum_{n=1}^{N} \left(\int_{B_n} \left(\alpha_s^n \varepsilon_{si}^n + \alpha_\theta^n \varepsilon_{\hat{\theta}i}^n - \alpha_h^n \varepsilon_{hi}^n\right) di + \int_{S_n} \left(\alpha_s^n \varepsilon_{si}^n + \alpha_\theta^n \varepsilon_{\hat{\theta}i}^n - \alpha_h^n \varepsilon_{hi}^n\right) di \right) \\
&\quad + \sum_{n=1}^{N} \left(\frac{\overline{\alpha_{\mu B}^n}}{\overline{\alpha_p}} \hat{\mu}_B^n + \frac{\overline{\alpha_{\mu S}^n}}{\overline{\alpha_p}} \hat{\mu}_S^n \right) + \sum_{n=1}^{N} \psi^n \mu_A^n \\
&= \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta + \frac{\sum_{n=1}^{N} \alpha_\theta^n \overline{\theta}_n \mu_A^n}{\overline{\alpha_p}} + \frac{\overline{\alpha_h}}{\overline{\alpha_p}}\delta + \sum_{n=1}^{N} \left(\frac{\overline{\alpha_{\mu B}^n}}{\overline{\alpha_p}} \hat{\mu}_B^n + \frac{\overline{\alpha_{\mu S}^n}}{\overline{\alpha_p}} \hat{\mu}_S^n \right) + \frac{\overline{\psi}}{\overline{\alpha_p}},
\end{aligned}$$

where $\overline{\alpha_s} = \sum_{n=1}^{N} \alpha_s^n \mu_A^n$, $\overline{\alpha_h} = \sum_{n=1}^{N} \alpha_h^n \mu_A^n$, $\overline{\alpha_p} = \sum_{n=1}^{N} \alpha_p^n \mu_A^n$, and

$$\overline{\psi} = \sum_{n=1}^{N} \left(\psi^n \mu_A^n + m^{n+} + m^{n-}\right),$$

with

$$\begin{aligned}
m^{n+} &= \int_{B_n} \left(\alpha_s^n \varepsilon_{si}^n + \alpha_\theta^n \varepsilon_{\hat{\theta}i}^n - \alpha_h^n \varepsilon_{hi}^n\right) di \\
m^{n-} &= \int_{S_n} \left(\alpha_s^n \varepsilon_{si}^n + \alpha_\theta^n \varepsilon_{\hat{\theta}i}^n - \alpha_h^n \varepsilon_{hi}^n\right) di.
\end{aligned}$$

Note that

$$w_n \equiv \alpha_s^n \varepsilon_{si}^n + \alpha_\theta^n \varepsilon_{\hat{\theta}i}^n - \alpha_h^n \varepsilon_{hi}^n \sim N\left(0, \left(\tau_{\Delta n}\right)^{-1}\right).$$

Moreover, given the characterization of the sets of buyers and sellers in the previous subsection, we have that

$$\int_{B_n} w_n di = \mathbb{E}\left[w_n | w_n > \sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right] \quad \text{and} \quad \int_{S_n} w_n di = \mathbb{E}\left[w_n | w_n < -\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right],$$

which are the expected values of a truncated normal at $\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}$ and $-\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}$, respectively. Then,

$$m^{n-} = \frac{\Phi'\left(\frac{\sqrt{\frac{2}{\kappa_n}\chi}-\overline{\Delta_n}}{\sqrt{\tau_{\Delta n}}}\right)}{1-\Phi\left(\frac{\sqrt{\frac{2}{\kappa_n}\chi}-\overline{\Delta_n}}{\sqrt{\tau_{\Delta n}}}\right)}\frac{1}{\sqrt{\tau_{\Delta n}}} \quad \text{and} \quad m^{n+} = \frac{-\Phi'\left(\frac{-\sqrt{\frac{2}{\kappa_n}\chi}-\overline{\Delta_n}}{\sqrt{\tau_{\Delta n}}}\right)}{\Phi\left(\frac{-\sqrt{\frac{2}{\kappa_n}\chi}-\overline{\Delta_n}}{\sqrt{\tau_{\Delta n}}}\right)}\frac{1}{\sqrt{\tau_{\Delta n}}}.$$

Since Δ_n can be recovered from the measures of buyers as

$$\overline{\Delta_n} = \sqrt{\frac{2}{\kappa_n}\chi} - \frac{\Phi^{-1}\left(1-\mu_B^n\right)}{\sqrt{\tau_{\Delta n}}},$$

m^{n+} and m^{n-} are constants given the information set of the investors and the price is linear in θ,$\left\{\overline{\theta}_n\right\}_n$, δ, and $\{\hat{\mu}_B^n, \hat{\mu}_S^n\}_n$, as guessed above.

Proof of Lemma 5 (Existence)

Proof. The proof is analogous to the one in the previous section for linear trading costs. Note that equilibrium price informativeness $\tau_{\hat{p}}$ can be expressed explicitly as a function of the measures of active buyers and sellers, $\boldsymbol{y} = \left[\mu_B^1, ..., \mu_B^N, \mu_S^1, ..., \mu_S^N\right]$, as follows:

$$\tau_{\hat{p}} = \frac{\left(\sum_{j=1}^N \frac{\tau_{sj}}{\gamma_j}\mu_A^j\right)^2}{\sum_{j=1}^N \left(\frac{\tau_{\theta j}}{\gamma_j}\mu_A^j\right)^2 \tau_{\overline{\theta}}^{-1} + \left(\sum_{j=1}^N \mu_A^j\right)^2 \tau_\delta^{-1}}.$$

Then, an equilibrium is fully characterized by the solution to the following system of equations

$$\begin{aligned} \mu_B^n &= 1-\Phi\left(\sqrt{\tau_{\Delta n}}\left(\sqrt{\frac{2}{\kappa_n}\chi}-\overline{\Delta_n}\right)\right), \forall n = 1, ..., N \\ \mu_S^n &= \Phi\left(\sqrt{\tau_{\Delta n}}\left(-\sqrt{\frac{2}{\kappa_n}\chi}-\overline{\Delta_n}\right)\right), \forall n = 1, ..., N, \end{aligned} \tag{A19}$$

where $\overline{\Delta_n}$ and κ_n are functions of $\boldsymbol{y}$ and $\tau_{\Delta n}$ is a constant. We can rewrite the system in Eq. (A19) as the fixed point of a mapping $\boldsymbol{T} : [0,1]^{2N} \to [0,1]^{2N}$, where $\boldsymbol{T}$ is continuous in $\boldsymbol{y}$.[28] An application of Brouwer's fixed point theorem implies that a solution to Eq. (A19) exists. □

Claim. (Stability) In equilibrium, price informativeness is given by the fixed point of

$$H_F(x) \equiv -x + \frac{\left(\sum_{j=1}^N \frac{\tau_{sj}}{\gamma_j}\mu_A^j(x)\right)^2}{\sum_{j=1}^N \left(\frac{\tau_{\theta j}}{\gamma_j}\mu_A^j(x)\right)^2 \tau_{\overline{\theta}}^{-1} + \left(\sum_{j=1}^N \mu_A^j(x)\right)^2 \tau_\delta^{-1}} = 0. \tag{A20}$$

In a stable equilibrium, $H_F'(x^\star) < 0$.

Proof. The proof is identical to the one in the linear cost case. □

[28] When $\boldsymbol{y} = 0$, we define the mapping $\boldsymbol{T}$ as the limit implied by Eq. (A19).

Proof of Theorem 6 (Irrelevance result with ex-ante identical investors and fixed trading costs)

When all investors are ex-ante identical, $\tau_{\hat{p}} = \frac{\tau_s^2}{\frac{\tau_\theta^2}{N}\tau_{\overline{\theta}}^{-1}+\gamma^2\tau_\delta^{-1}}$ which is independent of χ.

Proof of Lemma 6

From Eq. (A20), using the implicit function theorem we have

$$\frac{d\tau_{\hat{p}}}{d\chi} = \frac{\frac{\partial H_F}{\partial \chi}}{-H_F'},$$

where $H_F' < 0$ in any stable equilibrium. Note that $\frac{\partial H_F}{\partial \chi}$ keeps price informativeness constant as χ changes. Moreover,

$$\begin{aligned}
\left.\frac{\partial H_F}{\partial \chi}\right|_{\tau_{\hat{p}}=x} &= \frac{2\overline{\alpha_s}(x)\sum_{j=1}^N \alpha_s^j \frac{\partial \mu_A^j(x)}{\partial \chi}\left(\overline{\alpha_N}(x)\right)^2 - 2\overline{\alpha_s}(x)^2 \sum_{j=1}^N \mu_A^j(x)\left(\left(\alpha_\theta^j\right)^2 \frac{\partial \mu_A^j(x)}{\partial \chi}\tau_{\overline{\theta}}^{-1} + \sum_{n=1}^N \frac{\partial \mu_A^n(x)}{\partial \chi}\tau_\delta^{-1}\right)}{\left(\overline{\alpha_N}(x)\right)^4} \\
&= \frac{2\overline{\alpha_s}(x)\sum_{j=1}^N \alpha_s^j \frac{\partial \mu_A^j(x)}{\partial \chi}\left(\overline{\alpha_N}(x)\right)^2 - 2\overline{\alpha_s}(x)^2 \sum_{j=1}^N \frac{\partial \mu_A^j(x)}{\partial \chi}\left(\left(\alpha_\theta^j\right)^2 \mu_A^j(x)\tau_{\overline{\theta}}^{-1} + \sum_{n=1}^N \mu_A^n(x)\tau_\delta^{-1}\right)}{\left(\overline{\alpha_N}(x)\right)^4} \\
&= 2\left(\frac{\overline{\alpha_s}(x)}{\overline{\alpha_N}(x)}\right)^2 \sum_{j=1}^N \left(\left(\frac{\alpha_s^j}{\overline{\alpha_s}(x)} - \frac{\left(\alpha_\theta^j\right)^2 \mu_A^j(x)\tau_{\overline{\theta}}^{-1} + \overline{\alpha_h}\tau_\delta^{-1}}{\left(\overline{\alpha_N}(x)\right)^2}\right)\frac{\partial \mu_A^j(x)}{\partial \chi}\right).
\end{aligned}$$

Hence,

$$\operatorname{sgn}\left(\frac{d\tau_{\hat{p}}}{d\chi}\right) = \operatorname{sgn}\left(\mathbb{C}ov_n\left[\frac{\frac{\partial \overline{\alpha_s}}{\partial \mu_A^n}}{\overline{\alpha_s}} - \frac{\frac{\partial \overline{\alpha_N}}{\partial \mu_A^n}}{\overline{\alpha_N}}, \frac{\frac{\partial \mu_A^n}{\partial \chi}}{\mu_A^n}\right]\right),$$

since

$$\frac{\partial\left(\overline{\alpha_N}\right)^2}{\partial \mu_A^n} = 2\left(\alpha_\theta^n\right)^2 \mu_A^n \tau_{\overline{\theta}}^{-1} + 2\overline{\alpha_h}\tau_\delta^{-1}.$$

Proof of Theorem 7

The proof of this theorem follows the same structure as the proof of Theorem 5 in the case of linear trading costs. We use Lemma 6 and the result that the covariance of two monotone increasing functions is positive (see Schmidt (2003)). We will denote by $z_n = z + \eta f_n$ the group specific parameter over which investor groups differ. Then, analogous to the case with linear costs, we can write

$$\mathbb{C}ov_n\left[\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{\left(\alpha_\theta^n\right)^2 \mu_A^n \tau_{\overline{\theta}}^{-1} + \overline{\alpha_h}\tau_\delta^{-1}}{\left(\overline{\alpha_N}\right)^2}, \frac{\frac{\partial \mu_A^n}{\partial \chi}}{\mu_A^n}\right] = \mathbb{C}ov_n\left[F_F\left(z_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}\right), G_F\left(z_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}\right)\right],$$

where $\overline{\boldsymbol{\alpha}}$is the vector of all aggregate demand sensitivities, Ω is the variance-covariance matrix of the vector of public signals, and

$$F_F\left(z_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega\right) \equiv \frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{\left(\alpha_\theta^n\right)^2 \mu_A^n \tau_{\overline{\theta}}^{-1} + \overline{\alpha_h}\tau_\delta^{-1}}{\left(\overline{\alpha_N}\right)^2} \quad \text{and} \quad G_F\left(z_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}\right) \equiv \frac{\frac{\partial \mu_A^n}{\partial \chi}}{\mu_A^n}.$$

The main part of the proof consists of characterizing $\frac{\partial F_F}{\partial z_n}$ and $\frac{\partial G_F}{\partial z_n}$. To characterize these partial derivatives, we use the following intermediate results.

Lemma 9. *When the fixed trading cost is small, the set of active investors is the same across groups, i.e.,*

$$\lim_{\chi \to 0} \frac{\partial \mu_A^n \left(z_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega\right)}{\partial z_n} = 0.$$

Proof. The measure of active investors of type n is given by

$$\mu_A^n = \left(1 - \Phi\left(\sqrt{\tau_{\Delta n}}\left(\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right) + \Phi\left(\sqrt{\tau_{\Delta n}}\left(-\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right)\right) \tag{A21}$$

Suppose investors differ only in dimension with $z_n = z + \eta f_n$ for $f_n \in \mathbb{R}$ and $z \in \{\gamma_n, \tau_{sn}, \tau_{\theta n}\}$. Then,

$$\begin{aligned}\frac{\partial \mu_A^n \left(z_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega\right)}{\partial z_n} =& \frac{1}{2\sqrt{\tau_{\Delta n}}}\left(-\Phi'\left(\sqrt{\tau_{\Delta n}}\left(\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right)\left(\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right) - \Phi'\left(\sqrt{\tau_{\Delta n}}\left(-\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right)\left(\sqrt{\frac{2}{\kappa_n}\chi} + \overline{\Delta_n}\right)\right)\frac{\partial \tau_{\Delta n}}{\partial z_n} + \\ &+ \left(\Phi'\left(\sqrt{\tau_{\Delta n}}\left(\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right) - \Phi'\left(\sqrt{\tau_{\Delta n}}\left(-\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right)\right)\left(\sqrt{\tau_{\Delta n}}\frac{\partial \overline{\Delta_n}}{\partial z_n}\right)\end{aligned}$$

and taking limits when $\chi \to 0$ we have

$$\lim_{\chi \to 0} \frac{\partial \mu_A^n \left(z_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega\right)}{\partial z_n} = 0.$$

$\square$

Lemma 10. *The change in the extensive margin across different groups of investors when heterogeneity is small and fixed costs go to zero is*

$$\lim_{\chi \to 0}\lim_{\eta \to 0} \frac{\partial G_F \left(z_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega\right)}{\partial z_n} = \lim_{\chi \to 0}\lim_{\eta \to 0} -2\Phi'(0)\sqrt{\frac{1}{2\chi}}\frac{\partial \sqrt{\frac{\tau_{\Delta n}}{\kappa_n}}}{\partial z_n}.$$

Proof. Using the definition of μ_A^nin Eq. (A21) we have $\mu_A^n = \left(1 - \Phi\left(\sqrt{\tau_{\Delta n}}\left(\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right) + \Phi\left(\sqrt{\tau_{\Delta n}}\left(-\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right)\right)$

$$\begin{aligned}\frac{\partial \mu_A^n}{\partial \chi} =& -\left(\Phi'\left(\sqrt{\tau_{\Delta n}}\left(\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right) + \Phi'\left(\sqrt{\tau_{\Delta n}}\left(-\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right)\right)\sqrt{\tau_{\Delta n}}\sqrt{\frac{1}{2\kappa_n\chi}} \\ &+ \left(\Phi'\left(\sqrt{\tau_{\Delta n}}\left(\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right) - \Phi'\left(\sqrt{\tau_{\Delta n}}\left(-\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right)\right)\sqrt{\tau_{\Delta n}}\sum_j \frac{\partial \overline{\Delta_j}}{\partial \mu_A^j}\frac{\partial \mu_A^j}{\partial \chi}.\end{aligned}$$

Note that

$$\begin{aligned}\sum_n \frac{\partial \overline{\Delta_n}}{\partial \mu_A^n}\frac{\partial \mu_A^n \left(x_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega\right)}{\partial \chi} =& -\sum_n \frac{\partial \overline{\Delta_n}}{\partial \mu_A^n}\left(\Phi'\left(\sqrt{\tau_{\Delta n}}\left(\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right) + \Phi'\left(\sqrt{\tau_{\Delta n}}\left(-\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right)\right)\sqrt{\tau_{\Delta n}}\sqrt{\frac{1}{2\kappa_n\chi}} \\ &+ \sum_n \frac{\partial \overline{\Delta_n}}{\partial \mu_A^n}\left(\Phi'\left(\sqrt{\tau_{\Delta n}}\left(\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right) - \Phi'\left(\sqrt{\tau_{\Delta n}}\left(-\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right)\right)\sqrt{\tau_{\Delta n}}\sum_j \frac{\partial \overline{\Delta_j}}{\partial \mu_A^j}\frac{\partial \mu_A^j}{\partial \chi}.\end{aligned}$$

$$\sum_n \frac{\partial \overline{\Delta_n}}{\partial \mu_A^n}\frac{\partial \mu_A^n}{\partial \chi} = \frac{-\sum_n \frac{\partial \overline{\Delta_n}}{\partial \mu_A^n}\left(\Phi'\left(\sqrt{\tau_{\Delta n}}\left(\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right) + \Phi'\left(\sqrt{\tau_{\Delta n}}\left(-\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right)\right)\sqrt{\tau_{\Delta n}}\sqrt{\frac{1}{2\kappa_n\chi}}}{1 - \sum_n \frac{\partial \overline{\Delta_n}}{\partial \mu_A^n}\left(\Phi'\left(\sqrt{\tau_{\Delta n}}\left(\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right) - \Phi'\left(\sqrt{\tau_{\Delta n}}\left(-\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right)\right)\sqrt{\tau_{\Delta n}}}.$$

Note that

$$\lim_{\chi \to 0} \Phi'\left(\sqrt{\tau_{\Delta n}}\left(\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right) - \Phi'\left(\sqrt{\tau_{\Delta n}}\left(-\sqrt{\frac{2}{\kappa_n}\chi} - \overline{\Delta_n}\right)\right) = 0$$

and

$$\lim_{\chi\to 0}\Phi''\left(\sqrt{\frac{2}{\kappa_n}\chi}-\sqrt{\tau_{\Delta n}}\overline{\Delta_n}\right)=\lim_{\chi\to 0}\Phi''\left(-\sqrt{\frac{2}{\kappa_n}\chi}-\sqrt{\tau_{\Delta n}}\overline{\Delta_n}\right)=0.$$

Then, since $\lim_{\chi\to 0}\mu_A^n\left(z_n;\overline{\boldsymbol{\alpha}},\tau_{\hat{p}},\Omega\right)=1$ and $\lim_{\chi\to 0}\frac{\partial\mu_A^n\left(z_n;\overline{\boldsymbol{\alpha}},\tau_{\hat{p}},\Omega\right)}{\partial z_n}=0$,we have

$$\begin{aligned}\lim_{\chi\to 0}\lim_{\eta\to 0}\frac{\partial G_F\left(z_n;\overline{\boldsymbol{\alpha}},\tau_{\hat{p}},\Omega\right)}{\partial z_n}&=\lim_{\chi\to 0}\lim_{\eta\to 0}-\left(\Phi'\left(\sqrt{\tau_{\Delta n}}\left(\sqrt{\frac{2}{\kappa_n}\chi}-\overline{\Delta_n}\right)\right)+\Phi'\left(\sqrt{\tau_{\Delta n}}\left(-\sqrt{\frac{2}{\kappa_n}\chi}-\overline{\Delta_n}\right)\right)\right)\sqrt{\frac{1}{2\chi}}\frac{\partial\sqrt{\frac{\tau_{\Delta n}}{\kappa_n}}}{\partial z_n}\\&=\lim_{\chi\to 0}\lim_{\eta\to 0}-2\Phi'\left(0\right)\sqrt{\frac{1}{2\chi}}\frac{\partial\sqrt{\frac{\tau_{\Delta n}}{\kappa_n}}}{\partial z_n},\end{aligned}$$

where the derivative with respect to z_n keeps the aggregate demand sensitivities, the informational content of the measures and price informativeness constant. $\square$

a) Suppose that investor groups are heterogeneous only in the precision of their private information, with $\tau_{sn}=\tau_s+\eta f_n$.

$$\frac{\partial F_F\left(\tau_{sn};\overline{\boldsymbol{\alpha}},\tau_{\hat{p}},\Omega\right)}{\partial\tau_{sn}}=\frac{1}{\gamma}\frac{1}{\overline{\alpha_s}}-\frac{\left(\alpha_\theta^n\right)^2\tau_{\overline{\theta}}^{-1}}{\overline{\alpha_\theta^2}}\frac{\partial\mu_A^n}{\partial\tau_{sn}}.$$

Using Lemma 9 we have

$$\lim_{\chi\to 0}\lim_{\eta\to 0}\frac{\partial F_F\left(\tau_{sn};\overline{\boldsymbol{\alpha}},\tau_{\hat{p}},\Omega\right)}{\partial\tau_{sn}}=\frac{1}{\gamma}\frac{1}{\overline{\alpha_s}}>0.$$

Moreover, from Lemma 10 we have

$$\lim_{\chi\to 0}\lim_{\eta\to 0}\frac{\partial G_F\left(\tau_{sn};\overline{\boldsymbol{\alpha}},\tau_{\hat{p}},\Omega\right)}{\partial\tau_{sn}}=\lim_{\chi\to 0}\lim_{\eta\to 0}-2\Phi'\left(0\right)\sqrt{\frac{1}{2\chi}}\frac{\partial\sqrt{\frac{\tau_{\Delta n}}{\kappa_n}}}{\partial\tau_{sn}},$$

which implies

$$\operatorname{sgn}\left(\lim_{\chi\to 0}\lim_{\eta\to 0}\frac{\partial G_F\left(\tau_{sn};\overline{\boldsymbol{\alpha}},\tau_{\hat{p}},\Omega\right)}{\partial\tau_{sn}}\right)=-\operatorname{sgn}\left(\lim_{\chi\to 0}\lim_{\eta\to 0}\frac{\partial\sqrt{\frac{\tau_{\Delta n}}{\kappa_n}}}{\partial\tau_{sn}}\right),$$

where

$$\operatorname{sgn}\left(\lim_{\chi\to 0}\lim_{\eta\to 0}\frac{\partial\sqrt{\frac{\tau_{\Delta n}}{\kappa_n}}}{\partial\tau_{sn}}\right)=\operatorname{sgn}\left(\frac{1}{2}\left(\tau_s-\tau_\theta-\frac{\tau_s^2}{\frac{\tau_\theta^2\tau_{\overline{\theta}}^{-1}}{N}+\gamma^2\tau_\delta^{-1}}\right)+\tau_\theta^2\tau_{\hat{\theta}}^{-1}+\gamma^2\tau_h^{-1}\right),$$

since

$$\frac{\partial\sqrt{\tau_{\Delta n}}}{\partial\tau_{sn}}=-\frac{1}{2}\frac{1}{\sqrt{\tau_{\Delta n}}}\left(\tau_{\Delta n}\right)^2\frac{1}{\gamma^2},\qquad\frac{\partial\left(\frac{1}{\kappa_n}\right)}{\partial\tau_{sn}}=\frac{1}{\gamma},$$

and

$$\lim_{\eta\to 0}\sum_{h=1}^{N+1}\tilde{\Lambda}_{hh}^{-1}=\tau_{\hat{p}}=\frac{N^2\tau_s^2}{N\tau_\theta^2\tau_{\overline{\theta}}^{-1}+\gamma^2N^2\tau_\delta^{-1}}.$$

Then,

$$\operatorname{sgn}\left(\lim_{\chi\to 0}\lim_{\eta\to 0}\mathbb{C}ov_n\left(\frac{\alpha_s^n}{\overline{\alpha_s}}-\frac{\left(\alpha_\theta^n\right)^2\mu_A^n}{\overline{\alpha_\theta^2}},\frac{\frac{\partial\mu_A^n}{\partial\chi}}{\mu_A^n}\right)\right)=-\operatorname{sgn}\left(\frac{1}{2}\left(\tau_s-\tau_\theta-\frac{\tau_s^2}{\frac{\tau_\theta^2\tau_{\overline{\theta}}^{-1}}{N}+\gamma^2\tau_\delta^{-1}}\right)+\tau_\theta^2\tau_{\hat{\theta}}^{-1}+\gamma^2\tau_h^{-1}\right),$$

Note that this condition is exactly the same condition on parameters as the one with linear costs. Hence, as in the case with linear costs, there exist thresholds $\tau_{s1}^\star$ and $\tau_{s2}^\star$ such that:

i) for all $\tau_s \in (0, \tau_{s1}^\star) \bigcup (\tau_{s2}^\star, \infty)$

$$\operatorname{sgn}\left(\mathbb{C}ov_n\left(\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{(\alpha_\theta^n)^2 \mu_A^n \tau_{\overline{\theta}}^{-1} + \overline{\alpha_h}\tau_\delta^{-1}}{\overline{\alpha_\theta^2}}, \frac{\frac{\partial \mu_A^n}{\partial \chi}}{\mu_A^n}\right)\right) > 0$$

and price informativeness increases with fixed trading costs as investors who contribute less to the information contained in the price are the ones who disproportionately exit the market.

ii) for all $\tau_s \in (\tau_{s1}^\star, \tau_{s2}^\star)$

$$\operatorname{sgn}\left(\mathbb{C}ov_n\left(\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{(\alpha_\theta^n)^2 \mu_A^n \tau_{\overline{\theta}}^{-1} + \overline{\alpha_h}\tau_\delta^{-1}}{\overline{\alpha_\theta^2}}, \frac{\frac{\partial \mu_A^n}{\partial \chi}}{\mu_A^n}\right)\right) < 0$$

and price informativeness decreases with fixed trading costs as investors who contribute more to the information contained in the price are the ones who disproportionately exit the market.

b) Suppose that investor groups are heterogeneous only in the precision of their prior, with $\tau_{\theta n} = \tau_\theta + \eta f_n$. In this case,

$$\frac{\partial F_F(\tau_{\theta n}; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega)}{\partial \tau_{\theta n}} = -\frac{2\alpha_\theta^n}{\overline{\alpha_\theta^2}} \frac{1}{\gamma_n} \mu_A^n - \frac{(\alpha_\theta^n)^2}{\overline{\alpha_\theta^2}} \tau_{\overline{\theta}}^{-1} \frac{\partial \mu_A^n}{\partial \tau_{\theta n}},$$

and using Lemma 7

$$\lim_{\chi \to 0} \lim_{\eta \to 0} \frac{\partial F_F(\tau_{\theta n}; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega)}{\partial \tau_{\theta n}} = -\frac{2\alpha_\theta^n}{\overline{\alpha_\theta^2}} \frac{1}{\gamma} \mu_A^n < 0.$$

Moreover, from Lemma 10 we have

$$\operatorname{sgn}\left(\lim_{\chi \to 0} \lim_{\eta \to 0} \frac{\partial G_F(\tau_{\theta n}; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega)}{\partial \tau_{\theta n}}\right) = -\operatorname{sgn}\left(\lim_{\chi \to 0} \lim_{\eta \to 0}\left(\frac{\partial \sqrt{\tau_{\Delta n}}}{\partial \tau_{\theta n}} \frac{1}{\kappa_n} + \frac{\partial\left(\frac{1}{\kappa_n}\right)}{\partial \tau_{\theta n}} \sqrt{\tau_{\Delta n}}\right)\right),$$

which is the same a

$$\begin{aligned}
\lim_{\chi \to 0} \lim_{\eta \to 0} \frac{\partial \sqrt{\frac{\tau_{\Delta n}}{\kappa_n}}}{\partial \tau_{\theta n}} &= \lim_{\chi \to 0} \lim_{\eta \to 0} -\frac{1}{\gamma} \sqrt{\tau_{\Delta n}} \left(-\frac{1}{\gamma} \tau_{\Delta n} \frac{\tau_{\theta n}}{\tau_\varepsilon} \frac{1}{\kappa_n} + 1\right) \frac{2}{\sqrt{2\pi}} \\
&= \lim_{\chi \to 0} \lim_{\eta \to 0} -\frac{1}{\gamma^3} \frac{1}{\sqrt{\tau_{\Delta n}}} \left(-\left(\tau_s + \tau_{\theta n} + \sum_{h=1}^{N+1} \Lambda_{hh}^{-1}\right) \frac{\tau_{\theta n}}{\tau_\varepsilon} + \tau_s + \tau_{\theta n}^2 \tau_{\hat{\theta}}^{-1} + \gamma^2 (\tau_h)^{-1}\right) \frac{2}{\sqrt{2\pi}} \\
&= -\frac{1}{\gamma^3} \frac{1}{\sqrt{\tau_{\Delta n}}} \left(\left(1 - \frac{\tau_\theta}{\tau_\varepsilon}\right) \tau_s - \frac{N^2 \tau_s^2}{N \tau_\theta^2 \tau_{\overline{\theta}}^{-1} + \gamma^2 N^2 \tau_\delta^{-1}} \frac{\tau_\theta}{\tau_\varepsilon} + \gamma^2 (\tau_h)^{-1}\right) \frac{2}{\sqrt{2\pi}},
\end{aligned}$$

since

$$\frac{\partial \sqrt{\tau_{\Delta n}}}{\partial \tau_{\theta n}} = -\frac{1}{\sqrt{\tau_{\Delta n}}} (\tau_{\Delta n})^2 \frac{1}{\gamma^2} \frac{\tau_{\theta n}}{\tau_\varepsilon}, \quad \frac{\partial\left(\frac{1}{\kappa_n}\right)}{\partial \tau_{\theta n}} = \frac{1}{\gamma}, \quad \text{and} \quad \lim_{\eta \to 0} \sum_{h=1}^{N+1} \tilde{\Lambda}_{hh}^{-1} = \tau_{\hat{p}} = \frac{N^2 \tau_s^2}{N \tau_\theta^2 \tau_{\overline{\theta}}^{-1} + \gamma^2 N^2 \tau_\delta^{-1}}.$$

Note that the parametric condition that determines the sign of $\lim_{\chi \to 0} \lim_{\eta \to 0} \frac{\partial G_F(\tau_{\theta n}; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega)}{\partial \tau_{\theta n}}$ is the same as in the case with linear trading costs. Then, generically, there exist three thresholds $\tau_{\theta,1}^\star$, $\tau_{\theta,2}^\star$, and $\tau_{\theta,3}^\star$ such that

i) If $\tau_\theta \in [0, \tau_{\theta 1}^\star) \bigcup (\tau_{\theta 2}^\star, \tau_{\theta 3}^\star)$, $\frac{\partial \mu_A^n}{\partial \chi}$ is decreasing in $\tau_{\theta n}$ and

$$\operatorname{sgn}\left(\mathbb{C}ov_n\left(\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{(\alpha_\theta^n)^2 \mu_A^n \tau_{\overline{\theta}}^{-1} + \overline{\alpha_h}\tau_\delta^{-1}}{(\overline{\alpha_N})^2}, \frac{\frac{\partial \mu_A^n}{\partial \chi}}{\mu_A^n}\right)\right) > 0,$$

which implies price informativeness increases with linear trading costs.

ii) If $\tau_\theta \in (\tau^\star_{\theta 1}, \tau^\star_{\theta 2}) \bigcup (\tau^\star_{\theta 3}, \infty)$, $\frac{\partial \mu_A^n}{\partial \chi}$ is increasing in $\tau_{\theta n}$ and

$$\operatorname{sgn}\left(\mathbb{C}ov_n\left(\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{\left(\alpha_\theta^n\right)^2 \mu_A^n \tau_{\overline{\theta}}^{-1} + \overline{\alpha_h}\tau_\delta^{-1}}{\left(\overline{\alpha_N}\right)^2}, \frac{\frac{\partial \mu_A^n}{\partial \chi}}{\mu_A^n}\right)\right) < 0,$$

which implies price informativeness decreases with linear trading costs.

c) Suppose that investor groups are heterogeneous only in their risk aversion, with $\gamma_n = \gamma + \eta f_n$. Then,

$$\frac{\partial F_F\left(\gamma_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega\right)}{\partial \gamma_n} = -\frac{1}{\gamma_n}\frac{\alpha_s^n}{\overline{\alpha_s}} + 2\frac{1}{\gamma_n}\frac{\left(\alpha_\theta^n\right)^2 \mu_A^n}{\left(\overline{\alpha_N}\right)^2}\tau_{\overline{\theta}}^{-1} - \frac{\left(\alpha_\theta^n\right)^2}{\left(\overline{\alpha_N}\right)^2}\frac{\partial \mu_A^n}{\partial \gamma_n}\tau_{\overline{\theta}}^{-1}.$$

Taking limits when heterogeneity and the linear trading cost go to zero, we have

$$\lim_{\chi \to 0}\lim_{\eta \to 0}\frac{\partial F_F\left(\gamma_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega\right)}{\partial \gamma_n} = -\frac{1}{\gamma}\frac{\alpha_s}{\overline{\alpha_s}} + 2\frac{1}{\gamma}\frac{\left(\alpha_\theta\right)^2}{\left(\overline{\alpha_N}\right)^2}\tau_{\overline{\theta}}^{-1} = \frac{1}{\gamma}\frac{-\left(N-2\right)\tau_\theta^2\tau_{\overline{\theta}}^{-1} - \gamma^2 N^2 \tau_\delta^{-1}}{N\tau_\theta^2\tau_{\overline{\theta}}^{-1} + \gamma^2 N^2 \tau_\delta^{-1}} < 0,$$

where we used Lemma 9 and that $\lim_{\chi \to 0}\lim_{\eta \to 0}\mu_A^n = 1$, $\lim_{\eta \to 0}\alpha_s^n = \alpha_s = \overline{\alpha_s}$, and $\lim_{\eta \to 0}\left(\overline{\alpha_N}\right)^2 = \frac{1}{\gamma^2}\left(N\tau_\theta^2\tau_{\overline{\theta}}^{-1} + \gamma^2 N^2 \tau_\delta^{-1}\right)$.

Moreover, from Lemma 10 we have

$$\operatorname{sgn}\left(\lim_{\chi \to 0}\lim_{\eta \to 0}\frac{\partial G_F\left(\gamma_n; \overline{\boldsymbol{\alpha}}, \tau_{\hat{p}}, \Omega\right)}{\partial \gamma_n}\right) = -\operatorname{sgn}\left(\lim_{\chi \to 0}\lim_{\eta \to 0}\frac{\partial \sqrt{\frac{\tau_{\Delta n}}{\kappa_n}}}{\partial \gamma_n}\right),$$

where

$$\lim_{\chi \to 0}\lim_{\eta \to 0}\frac{\partial \sqrt{\frac{\tau_{\Delta n}}{\kappa_n}}}{\partial \gamma_n} = \lim_{\chi \to 0}\lim_{\eta \to 0}\left(\frac{\partial \sqrt{\tau_{\Delta n}}}{\partial \gamma_n}\frac{1}{\kappa_n} + \frac{\partial\left(\frac{1}{\kappa_n}\right)}{\partial \gamma_n}\sqrt{\tau_{\Delta n}}\right),$$

which is the same as

$$\lim_{\chi \to 0}\lim_{\eta \to 0}\frac{\partial \sqrt{\frac{\tau_{\Delta n}}{\kappa_n}}}{\partial \gamma_n} = \lim_{\chi \to 0}\lim_{\eta \to 0} -\gamma_n^2 \tau_h^{-1}\frac{\left(\tau_s + \tau_\theta^2 \tau_{\hat{\theta}}^{-1} + \gamma_n^2 \tau_h^{-1}\right)^{-1}}{\gamma_n}\frac{\sqrt{\tau_{\Delta n}}}{\kappa_n} < 0.$$

Therefore,

$$\operatorname{sgn}\left(\lim_{\chi \to 0}\lim_{\eta \to 0}\mathbb{C}ov_n\left(\frac{\alpha_s^n}{\overline{\alpha_s}} - \frac{\left(\alpha_\theta^n\right)^2 \mu_A^n \tau_{\overline{\theta}}^{-1} + \overline{\alpha_h}\tau_\delta^{-1}}{\overline{\alpha_\theta^2}}, \frac{\frac{\partial \mu_A^n}{\partial \chi}}{\mu_A^n}\right)\right) < 0,$$

and price informativeness decreases with fixed trading costs.

Online Appendix

B Filtering

Before observing any public information, an investor of type n who received a signal s_i^n has a prior distribution over θ given by

$$\theta \sim N\left(x_0^n, \Sigma_0^n\right),$$

where

$$x_0^n \equiv \frac{\tau_{\theta n}\hat{\theta}_i^n + \tau_{sn} s_i^n}{\tau_{\theta n} + \tau_{sn}} \quad \text{and} \quad \Sigma_0^n \equiv \left(\tau_{\theta n} + \tau_{sn}\right)^{-1}.$$

We can define the vector of public signals as

$$\boldsymbol{x_1} \equiv \left[\hat{p}, \hat{\mu}_B^1, \ldots, \hat{\mu}_B^N\right]^T,$$

where

$$\boldsymbol{x_1} = \theta \mathbf{1}_{(N+1)\times 1} + \begin{bmatrix} \frac{\alpha_\theta^1 \mu_A^1}{\overline{\alpha_s}} & \frac{\alpha_\theta^2 \mu_A^2}{\overline{\alpha_s}} & \cdots & \frac{\alpha_\theta^N \mu_A^N}{\overline{\alpha_s}} & \frac{\overline{\alpha_h}}{\overline{\alpha_s}} \\ (AB)_{11} & (AB)_{12} & \cdots & (AB)_{1N} & (AC)_1 \\ \vdots & \cdots & \ddots & \vdots & \vdots \\ (AB)_{N1} & (AB)_{N2} & \cdots & (AB)_{NN} & (AC)_N \end{bmatrix} \begin{bmatrix} \overline{\theta_1} - \omega_\theta \\ \overline{\theta_2} - \omega_\theta \\ \vdots \\ \overline{\theta_N} - \omega_\theta \\ \delta - \omega_\delta \end{bmatrix},$$

and

$$\boldsymbol{x_1} | \, \theta \sim N\left(\theta \mathbf{1}_{(N+1)\times 1}, \Omega\right),$$

where

$$\Omega = \begin{bmatrix} \frac{\alpha_\theta^1 \mu_A^1}{\overline{\alpha_s}} & \frac{\alpha_\theta^2 \mu_A^2}{\overline{\alpha_s}} & \cdots & \frac{\alpha_\theta^N \mu_A^N}{\overline{\alpha_s}} & \frac{\overline{\alpha_h}}{\overline{\alpha_s}} \\ (AB)_{11} & (AB)_{12} & \cdots & (AB)_{1N} & (AC)_1 \\ \vdots & \cdots & \ddots & \vdots & \vdots \\ (AB)_{N1} & (AB)_{N2} & \cdots & (AB)_{NN} & (AC)_N \end{bmatrix} \begin{bmatrix} \tau_{\overline{\theta}}^{-1} & 0 & \cdots & \cdots & 0 \\ 0 & \tau_{\overline{\theta}}^{-1} & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \cdots & \vdots \\ 0 & \cdots & 0 & \tau_{\overline{\theta}}^{-1} & 0 \\ 0 & 0 & \cdots & \cdots & \tau_\delta^{-1} \end{bmatrix} \begin{bmatrix} \frac{\alpha_\theta^1 \mu_A^1}{\overline{\alpha_s}} & \frac{\alpha_\theta^2 \mu_A^2}{\overline{\alpha_s}} & \cdots & \frac{\alpha_\theta^N \mu_A^N}{\overline{\alpha_s}} & \frac{\overline{\alpha_h}}{\overline{\alpha_s}} \\ (AB)_{11} & (AB)_{12} & \cdots & (AB)_{1N} & (AC)_1 \\ \vdots & \cdots & \ddots & \vdots & \vdots \\ (AB)_{N1} & (AB)_{N2} & \cdots & (AB)_{NN} & (AC)_N \end{bmatrix}^T.$$

The $N+1$ signals in $\boldsymbol{x_1}$ are correlated. However, we can transform this vector in a vector of $N+1$ uncorrelated signals about θ as follows. Let

$$\boldsymbol{y} \equiv \Gamma^T \boldsymbol{x_1},$$

where Γ is the eigenvector matrix in the eigen-decomposition of Ω, i.e., $\Omega = \Gamma \Lambda \Gamma^T$, where Λ is the diagonal eigenvalue matrix. Let $\hat{\boldsymbol{y}}$ be a vector whose nth element is $\widetilde{y}_n = \frac{y_n}{\bar{e}_n}$, where $\bar{e}_n$ is the sum of the elements in the nth eigenvector. Note that

$$\hat{\boldsymbol{y}} | \, \theta \sim N\left(\theta \mathbf{1}_{(N+1)\times 1}, \tilde{\Lambda}\right),$$

where $\tilde{\Lambda}^{-1} = E\Lambda^{-1}E$, E is the diagonal matrix with element $E_{nn} = \bar{e}_n$, and the precision of the public signals is the same for all investors types.

Hence, using this transformation, we have that

$$\mathbb{E}_n\left[\theta | \, \mathcal{I}_i^n\right] = \frac{\tau_{\theta n}\hat{\theta}_i^n + \tau_{sn} s_i^n + \sum_{h=1}^{N+1} \tilde{\Lambda}_{hh}^{-1} \hat{\boldsymbol{y}}_h}{\tau_{\theta n} + \tau_{sn} + \sum_{h=1}^{N+1} \tilde{\Lambda}_{hh}^{-1}},$$

and

$$\mathbb{V}\mathrm{ar}_n\left[\theta|\,\mathcal{I}_i^n\right]=\left(\tau_{\theta n}+\tau_{sn}+\sum_{h=1}^{N+1}\tilde{\Lambda}_{hh}^{-1}\right)^{-1}.$$

Using the definition of $\hat{\boldsymbol{y}}$, gives

$$\begin{aligned}\mathbb{E}_n\left[\theta|\,\mathcal{I}_i^n\right]&=\frac{\tau_{\theta n}\hat{\theta}_i^n+\tau_{sn}s_i^n+\mathbf{1}_{1\times(N+1)}\tilde{\Lambda}^{-1}\hat{\boldsymbol{y}}}{\tau_{\theta n}+\tau_{sn}+\sum_{h=1}^{N+1}\tilde{\Lambda}_{hh}^{-1}}\\&=\frac{\tau_{\theta n}\hat{\theta}_i^n+\tau_{sn}s_i^n+\mathbf{1}_{1\times(N+1)}\tilde{\Lambda}^{-1}\Gamma^T\tilde{\boldsymbol{x}}_{\mathbf{1}}}{\tau_{\theta n}+\tau_{sn}+\sum_{h=1}^{N+1}\tilde{\Lambda}_{hh}^{-1}}.\end{aligned}$$

Hence, matching coefficients from the first order condition of an investor of type n we have

$$\begin{aligned}\alpha_s^n&=\frac{\tau_{sn}}{\gamma_n},\quad\alpha_\theta^n=\frac{\tau_{\theta n}}{\gamma_n},\quad\alpha_h^n=1\\\alpha_{\mu B}^{jn}&=\frac{K_{(j+1)}^n}{\kappa_n},\quad\alpha_p^n=\frac{1-\frac{\overline{\alpha_p}}{\overline{\alpha_s}}K_1^n}{\kappa_n}\\\psi^{n+}&=\frac{1}{\kappa_n}\left(-\kappa_n q_0^n-\phi-K_1^n\left(\frac{\overline{\alpha_\theta}}{\overline{\alpha_s}}\omega_\theta-\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\omega_\delta+\sum_{j=1}^N\frac{\overline{\alpha_{\mu B}^j}}{\overline{\alpha_s}}\hat{\mu}_B^j+\frac{\overline{\psi}}{\overline{\alpha_s}}\right)\right)\\\psi^{n-}&=\frac{1}{\kappa_n}\left(-\kappa_n q_0^n+\phi-K_1^n\left(\frac{\overline{\alpha_L}}{\overline{\alpha_s}}\omega_\theta-\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\omega_\delta+\sum_{j=1}^N\frac{\overline{\alpha_{\mu B}^j}}{\overline{\alpha_s}}\hat{\mu}_B^j+\frac{\overline{\psi}}{\overline{\alpha_s}}\right)\right),\end{aligned}$$

where $\kappa_n=\gamma_n\mathbb{V}\mathrm{ar}_n\left[\theta|\mathcal{I}_i^n\right]$ and K_j^n is the jth element of the Kalman K gain for an investor of type n, which is given by

$$K^n=\frac{1}{\tau_{\theta n}+\tau_{sn}+\sum_{h=1}^{N+1}\tilde{\Lambda}_{hh}^{-1}}\left[\mathbf{1}_{1\times(N+1)}\tilde{\Lambda}^{-1}\Gamma^T\right].$$

C Hedging needs

The setup is the same as in the benchmark model with the difference that each investor i is endowed with n_{2i} units of the consumption good at date 2. To simplify the analysis we set $q_{0i}=0$ for all investors. Then, each investor i solves

$$\max_{q_{1i}}\left(\mathbb{E}\left[\theta|\mathcal{I}_i\right]-p-\gamma\mathbb{C}ov\left[\theta,n_{2i}|\mathcal{I}_i\right]\right)q_{1i}-\frac{\gamma}{2}\mathbb{V}ar\left[\theta|\mathcal{I}_i\right]q_{1i}^2-T\left(q_{1i}\right).$$

We consider two cases for the correlation between the endowment and the asset payoff.

1. Learnable endowment: $n_{2i}=h_i\theta+z_i$, where $z_i\overset{iid}{\sim}N\left(0,\tau_\eta^{-1}\right)$ and $z_i\perp\theta$. This implies that

$$\mathbb{C}ov\left[\theta,n_{2i}|\mathcal{I}_i\right]=h_i\mathbb{V}ar\left[\theta|\mathcal{I}_i\right],$$

   which depends on the equilibrium price.

2. Unlearnable endowment: Suppose that $\theta=\theta^l+\theta^u$, where θ^l and θ^u are learnable and unlearnable components of the asset payoff with

$$\theta^l\sim N\left(\overline{\theta},\tau_\theta\right)\quad\text{and}\quad\theta^u\sim N\left(0,1\right).$$

   In this case, the signal structure is $s_i=\theta^l+\varepsilon_{si}$. Moreover, suppose $n_{2i}=h_i\theta^u+z_i$, where $z\perp\theta^u,\theta^l$. Then,

$$\mathbb{C}ov\left[\theta,n_{2i}|\mathcal{I}_i\right]=h_i\mathbb{V}ar\left[\theta^u\right]=h_i,$$

   which is independent of the equilibrium price.

In both cases, we assume $h_i = \delta + u_i$, where

$$\delta \sim N\left(0, \tau_\delta^{-1}\right) \quad \text{and} \quad u_i \sim N\left(0, \tau_{hi}^{-1}\right),$$

with $u_i \perp u_j$ for all $i \neq j$. The aggregate hedging need is random and not observed by the investors. Hence, investors cannot distinguish whether a high price is due to a high realization of the fundamental or due to a low realization of δ, and the price is not fully revealing.

C.1 Learnable endowment

Suppose $n_{2i} = h_i\theta + z_i$ where $h_i = \delta + u_i$ with

$$u_i \overset{iid}{\sim} N\left(0, \tau_h^{-1}\right), \quad z_i \overset{iid}{\sim} N\left(0, \tau_\eta^{-1}\right), \quad \text{and} \quad \delta \sim N\left(\overline{\delta}, \tau_\delta^{-1}\right).$$

When costs as quadratic and given by $T\left(\Delta q_{1i}\right) = \frac{c}{2}\left(\Delta q_{1i}\right)^2$, the FOC is

$$\Delta q_{1i} = \frac{\mathbb{E}_i\left[\theta|\mathcal{I}_i\right] - p - \gamma_i \mathbb{C}ov_i\left[\theta, n_{2i}|\mathcal{I}_i\right]}{\gamma_i \mathbb{V}ar_i\left[\theta|\mathcal{I}_i\right] + c}.$$

The information set of an investor is given by $\mathcal{I}_i = \{s_i, h_i, p\}$. Then,

$$\Delta q_{1i} = \frac{\mathbb{E}_i\left[\theta|s_i, h_i, p\right] - p - \gamma_i h_i \mathbb{V}ar_i\left[\theta|s_i, h_i, p\right]}{\gamma_i \mathbb{V}ar_i\left[\theta|s_i, h_i, p\right] + c}.$$

In an equilibrium in linear strategies,

$$\Delta q_{1i} = \alpha_{si}s_i - \alpha_{hi}h_i - \alpha_{pi}p + \psi_i.$$

Market clearing implies

$$p = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta - \frac{\overline{\alpha_h}}{\overline{\alpha_p}}\delta + \frac{\overline{\psi}}{\overline{\alpha_p}}.$$

The filtering problem solved by the investor (see the next section in this Online Appendix) implies

$$\mathbb{E}\left[\theta|\, s_i, h_i, p\right] = \frac{\tau_\theta\overline{\theta} + \tau_{si}s_i + \tau_{\hat{p}i}\left(\hat{p} + \frac{\overline{\alpha_h}}{\overline{\alpha_s}}\frac{\tau_{hi}}{\tau_\delta + \tau_{hi}}h_i\right)}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}} \quad \text{and} \quad \mathbb{V}ar_i\left[\theta|s_i, h_i, p\right] = \frac{1}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}},$$

where $\hat{p} = \frac{\overline{\alpha_p}}{\overline{\alpha_s}}\left(p - \frac{\overline{\psi}}{\overline{\alpha_p}}\right)$ and $\tau_{\hat{p}i}^{-1} = \mathbb{V}ar_i\left[\hat{p}|\theta, h_i\right] = \left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)^2\left(\tau_\delta + \tau_{hi}\right)^{-1}$.

Matching coefficients we have

$$\alpha_{si} = \frac{\tau_{si}\mathbb{V}ar_i\left[\theta|s_i, h_i, p\right]}{\kappa_i}, \quad \alpha_{hi} = \frac{\left(\gamma_i - \tau_{\hat{p}i}\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\frac{\tau_{hi}}{\tau_\delta + \tau_{hi}}\right)\mathbb{V}ar_i\left[\theta|s_i, h_i, p\right]}{\kappa_i}.$$

$$\alpha_{pi} = \frac{1 - \tau_{\hat{p}i}\frac{\overline{\alpha_p}}{\overline{\alpha_s}}\mathbb{V}ar_i\left[\theta|s_i, h_i, p\right]}{\kappa_i}, \quad \text{and} \quad \psi_i = -\frac{\left(\tau_{\hat{p}i}\frac{\overline{\psi}}{\overline{\alpha_s}} + \gamma_i q_{0i}\right)\mathbb{V}ar_i\left[\theta|s_i, h_i, p\right]}{\kappa_i},$$

where $\kappa_i = \gamma\mathbb{V}ar\left[\theta|s_i, h_i, p\right] + c$.

Lemma 11. *In any stable equilibrium,*

$$\operatorname{sgn}\left(\frac{\partial\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{\partial c}\right) = -\operatorname{sgn}\left(\mathbb{C}ov_x\left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}, \frac{1}{\kappa_i}\right]\right).$$

Proof. External price informativeness is given by

$$\tau_{\hat{p}} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 \tau_\delta,$$

where the equilibrium ratio $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$ is characterized by $H(x) = 0$, where

$$H(x) \equiv \frac{1}{x} - \frac{\int \frac{\gamma - x\tau_{hi}}{\gamma_i + c(\tau_\theta + \tau_{si} + x^2(\tau_\delta + \tau_{hi}))} di}{\int \frac{\tau_{si}}{\gamma_i + c(\tau_\theta + \tau_{si} + x^2(\tau_\delta + \tau_{hi}))} di}.$$

Taking limits we have $\lim_{x\to 0} H(x) = \infty$ and $\lim_{x\to\infty} H(x) = \infty$. Hence, as we know from the literature, and equilibrium may not always exist (Ganguli and Yang, 2009; Manzano and Vives, 2011); see also the results in Cespa and Vives (2015), in which multiplicity arises because investors have information on the two factors driving the equilibrium price. In any stable equilibrium $H' < 0$. Moreover, using the Implicit Function Theorem we know that

$$\begin{aligned}
\frac{\partial\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{\partial c} &= -\frac{\frac{\partial H}{\partial c}}{H'(x)} \\
&= -\frac{\frac{\int \frac{-\tau_{si}\mathbb{V}ar_i[\theta|s_i,h_i,p]}{(\kappa_i)^2} di\overline{\alpha_h} - \alpha_s \int \frac{(\gamma_i - x\tau_{hi})\mathbb{V}ar_i[\theta|s_i,h_i,p]}{(\kappa_i)^2} di}{(\overline{\alpha_s})^2}}{H'(x)} \\
&= -\frac{\frac{\overline{\alpha_h}}{\overline{\alpha_s}} \int \left(\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}\right) \frac{1}{\kappa_i} di}{-H'(x)} \\
&= -\frac{\frac{\overline{\alpha_s}}{\overline{\alpha_h}} \mathbb{C}ov_x\left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}, \frac{1}{\kappa_i}\right]}{-H'(x)},
\end{aligned}$$

which yields the result since in any stable equilibrium $H' < 0$. □

Theorem 8. *(Directional results under one-dimensional heterogeneity) When endowments are learnable and there are quadratic trading costs, if investor differ in only one of the following dimensions a) the precision of information τ_{si}, b) in the precision of the private hedging, τ_{hi}, or c) in risk aversion, γ_i, an increase in trading costs decrease price informativeness.*

Proof. Note that a) $\frac{\partial\left(\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}\right)}{\partial \tau_{si}} > 0$ and $\frac{\partial\left(\frac{1}{\kappa_i}\right)}{\partial \tau_{si}} > 0$, b) $\frac{\partial\left(\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}\right)}{\partial \tau_{hi}} > 0$ and $\frac{\partial\left(\frac{1}{\kappa_i}\right)}{\partial \tau_{hi}} > 0$, and $\frac{\partial\left(\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}\right)}{\partial \gamma_i} < 0$ and $\frac{\partial\left(\frac{1}{\kappa_i}\right)}{\partial \gamma_i} < 0$. Since the covariance of two monotone increasing functions is positive (see Schmidt (2003)), if investors only differ in one dimension,

$$\operatorname{sgn}\left(\mathbb{C}ov_x\left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}, \frac{1}{\kappa_i}\right]\right) > 0$$

and Lemma 11 implies the result. □

C.2 Unlearnable correlation

When costs as quadratic and given by $T(\Delta q_{1i}) = \frac{c}{2}(\Delta q_{1i})^2$, the FOC is then,

$$\Delta q_{1i} = \frac{\mathbb{E}_i[\theta|s_i, h_i, p] - p - \gamma_i h_i - \gamma_i \mathbb{V}ar_i[\theta|s_i, h_i, p]\, q_{0i}}{\gamma_i \mathbb{V}ar_i[\theta^l|s_i, h_i, p] + \gamma_i + c}.$$

In an equilibrium in linear strategies,

$$\Delta q_{1i} = \alpha_{si} s_i - \alpha_{hi} h_i - \alpha_{pi} p + \psi_i.$$

Market clearing implies

$$p = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta - \frac{\overline{\alpha_h}}{\overline{\alpha_p}}\delta + \frac{\overline{\psi}}{\overline{\alpha_p}}.$$

This implies

$$\mathbb{E}\left[\theta^l \middle| s_i, h_i, p\right] = \frac{\tau_\theta \overline{\theta} + \tau_{si} s_i + \tau_{\hat{p}i}\left(\hat{p} + \frac{\overline{\alpha_h}}{\overline{\alpha_s}}\frac{\tau_{hi}}{\tau_\delta + \tau_{hi}} h_i\right)}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}},$$

where $\hat{p} = \frac{\overline{\alpha_p}}{\overline{\alpha_s}}\left(p - \frac{\overline{\psi}}{\overline{\alpha_p}}\right)$ and $\tau_{\hat{p}i}^{-1} = \mathbb{V}ar\left[\hat{p}|\theta, h_i\right] = \left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)^2 \left(\tau_\delta + \tau_{hi}\right)^{-1}$.

Matching coefficients we have

$$\alpha_{si} = \frac{\tau_{si}\mathbb{V}ar_i\left[\theta^l|s_i, h_i, p\right]}{\kappa_i}, \quad \alpha_{hi} = \frac{\left(\gamma_i - \tau_{\hat{p}i}\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\frac{\tau_{hi}}{\tau_\delta + \tau_{hi}}\right)\mathbb{V}ar\left[\theta^l|s_i, h_i, p\right]}{\kappa_i}.$$

$$\alpha_{pi} = \frac{1 - \tau_{\hat{p}i}\frac{\overline{\alpha_p}}{\overline{\alpha_s}}\mathbb{V}ar\left[\theta^l|s_i, h_i, p\right]}{\kappa_i}, \quad \text{and} \quad \psi_i = -\frac{\left(\tau_{\hat{p}i}\frac{\overline{\psi}}{\overline{\alpha_s}} + \gamma q_{0i}\right)\left(1 + \mathbb{V}ar\left[\theta^l|s_i, h_i, p\right]\right)}{\kappa_i},$$

where $\kappa_i = \gamma \mathbb{V}ar\left[\theta|s_i, h_i, p\right] + \gamma + c$.

The equilibrium hinges on finding the equilibrium value of $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$. In this case, $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$ it is given by the solution to the following nonlinear equation

$$\hat{H}(x) \equiv \frac{1}{x} - \frac{\int \frac{1}{\kappa_i}\left(\gamma_i - x\frac{\tau_{hi}}{\tau_\theta + \tau_{si} + x^2(\tau_\delta + \tau_{hi})}\right)di}{\int \frac{1}{\kappa_i}\frac{\tau_{si}}{\tau_\theta + \tau_{si} + x^2(\tau_\delta + \tau_{hi})}di} = 0.$$

Taking limits we have $\lim_{x\to 0}\hat{H}(x) = \infty$ and $\lim_{x\to\infty}\hat{H}(x) = -\infty$. Hence, since $\hat{H}(x)$ is continuous in $(0, \infty)$, an equilibrium always exists. Moreover, in any stable equilibrium, $\hat{H}'(x) < 0$.

Lemma 12. *In any stable equilibrium,*

$$\operatorname{sgn}\left(\frac{\partial\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{\partial c}\right) = -\operatorname{sgn}\left(\mathbb{C}ov_x\left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}, \frac{1}{\kappa_i}\right]\right).$$

Proof. Using the Implicit Function Theorem we have

$$\begin{aligned}
\frac{\partial\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{\partial c} &= -\frac{\frac{\partial \hat{H}}{\partial c}}{\hat{H}'(x)} \\
&= -\frac{\frac{\int \frac{-\tau_{si}\mathbb{V}ar_i[\theta|s_i,h_i,p]}{(\kappa_i)^2}di\overline{\alpha_h} - \overline{\alpha_s}\int \frac{\gamma_i - x\tau_{hi}\mathbb{V}ar_i[\theta|s_i,h_i,p]}{(\kappa_i)^2}di}{(\overline{\alpha_s})^2}}{\hat{H}'(x)} \\
&= -\frac{\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\int\left(\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}\right)\frac{1}{\kappa_i}di}{-\hat{H}'(x)} \\
&= -\frac{\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\mathbb{C}ov_x\left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}, \frac{1}{\kappa_i}\right]}{-\hat{H}'(x)}.
\end{aligned}$$

which yields the result since in any stable equilibrium $H' < 0$. ☐

Theorem 9. *(Directional results under one-dimensional heterogeneity) When endowments are unlearnable and there are quadratic trading costs, if investor differ in only one of the following dimensions a) the precision of information τ_{si}, b) in the precision of the private hedging, τ_{hi}, or c) in risk aversion, γ_i, an increase in trading costs decrease price informativeness.*

Proof. Note that a) $\frac{\partial\left(\frac{\alpha_{si}}{\overline{\alpha_s}}-\frac{\alpha_{hi}}{\overline{\alpha_h}}\right)}{\partial\tau_{si}} > 0$ and $\frac{\partial\left(\frac{1}{\kappa_i}\right)}{\partial\tau_{si}} > 0$, b) $\frac{\partial\left(\frac{\alpha_{si}}{\overline{\alpha_s}}-\frac{\alpha_{hi}}{\overline{\alpha_h}}\right)}{\partial\tau_{hi}} > 0$ and $\frac{\partial\left(\frac{1}{\kappa_i}\right)}{\partial\tau_{hi}} > 0$, and $\frac{\partial\left(\frac{\alpha_{si}}{\overline{\alpha_s}}-\frac{\alpha_{hi}}{\overline{\alpha_h}}\right)}{\partial\gamma_i} < 0$ and $\frac{\partial\left(\frac{1}{\kappa_i}\right)}{\partial\gamma_i} < 0$. Since the covariance of two monotone increasing functions is positive (see Schmidt (2003)), if investors only differ in one dimension,

$$\text{sgn}\left(\mathbb{C}ov_x\left[\frac{\alpha_{si}}{\overline{\alpha_s}}-\frac{\alpha_{hi}}{\overline{\alpha_h}},\frac{1}{\kappa_i}\right]\right) > 0$$

and Lemma 12 implies the result. □

C.3 Filtering with hedging needs

Investors observe two pieces of information about the fundamental θ, the private signal s_i and the public signal p. Moreover, the realization of their individual hedging need reveals information about the aggregate hedging need in the economy δ and, thus, about the noise contained in the price. In the equilibrium in linear strategies, the unbiased signal of the fundamental contained in the price can be summarized in $\hat{p} = \theta - \frac{\overline{\alpha_h}}{\overline{\alpha_s}}\delta$. The linear system that characterizes the unknown fundamentals and the information observed by an individual investor is the following

$$\begin{bmatrix} s_i \\ h_i \\ \hat{p} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -\frac{\overline{\alpha_h}}{\overline{\alpha_s}} \end{bmatrix}\begin{bmatrix} \theta \\ \delta \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \varepsilon_{si} \\ \varepsilon_{hi} \end{bmatrix}$$

where

$$\begin{bmatrix} \theta \\ \delta \end{bmatrix} \sim N\left(\begin{bmatrix} \overline{\theta} \\ 0 \end{bmatrix},\begin{bmatrix} \tau_\theta^{-1} & 0 \\ 0 & \tau_\delta^{-1} \end{bmatrix}\right)$$

and

$$\begin{bmatrix} \varepsilon_{si} \\ \varepsilon_{hi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix},\begin{bmatrix} \tau_{si}^{-1} & 0 \\ 0 & \tau_{hi}^{-1} \end{bmatrix}\right).$$

A standard application of the Kalman filter yields

$$\mathbb{E}\left[\begin{bmatrix} \theta \\ \delta \end{bmatrix}\middle| s_i,h_i,p\right] = \frac{1}{\tau_\theta+\tau_{si}+\tau_{\hat{p}i}}\begin{bmatrix} \tau_\theta\overline{\theta}+\tau_{si}s_i+\tau_{\hat{p}i}\hat{p}+\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\tau_{hi}h_i \\ \tau_{hi}h_i-\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\left(\tau_{si}+\tau_\theta\right)\hat{p}+\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\tau_{si}s_i+\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\tau_\theta\overline{\theta} \end{bmatrix}$$

and

$$\mathbb{V}ar\left[\begin{bmatrix} \theta \\ \delta \end{bmatrix}\middle| s_i,h_i,p\right] = \frac{1}{\tau_\theta+\tau_{si}+\tau_{\hat{p}i}}\begin{bmatrix} 1 & \frac{\overline{\alpha_s}}{\overline{\alpha_h}} \\ \frac{\overline{\alpha_s}}{\overline{\alpha_h}} & \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 \end{bmatrix},$$

where

$$\tau_{\hat{p}i} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2\left(\tau_\delta+\tau_{hi}\right) \quad \text{and} \quad \tau_{\hat{p}} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2\tau_\delta.$$

Note that we can write $\mathbb{E}\left[\theta| s_i,h_i,p\right]$ as follows

$$\mathbb{E}\left[\theta| s_i,h_i,p\right] = \frac{\tau_\theta\overline{\theta}+\tau_{si}s_i+\tau_{\hat{p}i}\hat{p}+\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\tau_{hi}h_i}{\tau_\theta+\tau_{si}+\tau_{\hat{p}i}} = \frac{\tau_\theta\overline{\theta}+\tau_{si}s_i+\tau_{\hat{p}i}\left(\hat{p}+\frac{1}{\frac{\overline{\alpha_s}}{\overline{\alpha_h}}}\frac{\tau_{hi}}{\tau_\delta+\tau_{hi}}h_i\right)}{\tau_\theta+\tau_{si}+\tau_{\hat{p}i}},$$

where $\mathbb{E}\left[\delta| h_i\right] = \frac{\tau_{hi}}{\tau_\delta+\tau_{hi}}h_i$.