A User Guide to Dynamic-Stochastic Weights*

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In this user guide, we explain how to use Dynamic Stochastic Weights (DS-Weights) — introduced in Dávila and Schaab (2022) — in the context of several applications. This guide is a living document, which will be expanded and updated regularly.

This user guide has a dual purpose. Its first purpose is to explain the mechanics of how to define and use DS-weights in specific environments. Its second purpose is to show that, by using DS-weights, it is possible to obtain new economic insights into welfare assessments in a way that was not possible before.

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1 Application #1: Linear Labor Income Taxation with Stochastic Earnings

In their Handbook chapter, Piketty and Saez (2013) argue that the linear labor income taxation model considerably simplifies the exposition while capturing the key equity-efficiency trade-off that underlies the literature on optimal labor income taxation. This model is typically traced back to Sheshinski (1972) following the nonlinear income tax analysis of Mirrlees (1971). While our results can also be extended to the more complex nonlinear case, here we focus on the linear case. See also Saez and Stantcheva (2016) and (Kaplow, 2007) for more on this model.

The baseline labor income taxation model assumes that individuals are ex-ante heterogeneous, either on productivity or preferences, but it does not allow for random earnings. This extension is taken up by Varian (1980) and Eaton and Rosen (1980), among others — see Kaplow (2007) for a detailed discussion of these papers. However, Piketty and Saez (2013) write that

> *"Therefore, the random earnings model generates both the same equity-efficiency trade-off and the same type of optimal tax formula."*

In this application, we show that the deterministic earnings model features a tradeoff between Aggregate Efficiency and Redistribution, but that the stochastic earnings model trades off Aggregate Efficiency, Risk-Sharing, and Redistribution, and this may have important consequences for the determination of optimal taxes.

1.1 Deterministic earnings

Environment. We initially consider a single-period setting in which individuals have to a make consumption-labor decision. We assume that individuals have preferences of the form

$$u_i\left(c^i, n^i\right),$$

where c^i denotes consumption and n^i denotes hours worked. We assume that $\frac{\partial u_i\left(c^i,n^i\right)}{\partial c^i} > 0$, $\frac{\partial u_i\left(c^i,n^i\right)}{\partial n^i} < 0$, and any other needed regularity conditions. When individuals face a linear labor income tax, the budget constraint of individual i is given by

$$c^i = (1-\tau)\, w^i n^i + g,$$

where τ is the constant linear tax rate and g is a uniform per-capita grant, i.e., a demogrant.[1]

Individual optimality. The problem that an individual faces can be expressed in terms of the following Lagrangian:

$$\mathcal{L}^i = u_i\left(c^i, n^i\right) - \lambda^i\left(c^i - (1-\tau)\, w^i n^i - g\right),$$

whose solution is characterized by

$$(1-\tau)\, w^i \frac{\partial u_i\left(c^i, n^i\right)}{\partial c^i} + \frac{\partial u_i\left(c^i, n^i\right)}{\partial n^i} = 0.$$

[1] We could alternatively have written an individual's budget constraint as $c^i = w^i n^i - T\left(w^i n^i\right)$, where $T\left(w^i n^i\right) = \tau w^i n^i - g$

Hence, we can define an indirect utility function for individual i in terms of τ and g as

$$V_i(\tau, g) = \max u_i\left(c^i, n^i\right) \quad \text{s.t.} \quad c^i = (1-\tau)\, w^i n^i + g.$$

Under the assumption that g can be written as a function of τ, as in $g(\tau)$, we can write the total change in indirect utility for individual i as[2]

$$\frac{dV_i}{d\tau} = \frac{\partial u_i\left(c^i, n^i\right)}{\partial c^i} \underbrace{\left(-w^i n^i + \frac{dg}{d\tau}\right)}_{=\frac{du_{i|c}}{d\tau}}.$$

Optimal linear income tax. The government chooses τ and g to maximize a particular social welfare function $\mathcal{W}(\cdot)$ subject to a revenue constraint and to the constraints that represent individual behavior. Formally,

$$W(\tau) = \mathcal{W}\left(\{V_i(\tau, g(\tau))\}_{i \in I}\right),$$

where the mapping $g(\tau)$ is defined by the government's budget constraint, which takes the form

$$E = \tau \int w^i n^i \left((1-\tau)\, w^i, g\right) di - g, \tag{1}$$

where the function $n^i\left((1-\tau)\, w^i, g\right)$ denotes individual i's Marshallian labor supply. That is, it is evident that from Equation (1) it is possible to express g as a function of τ.

A welfare assessment for a welfarist planner takes the form

$$\begin{aligned} \frac{dW(\tau)}{d\tau} &= \int \frac{\partial \mathcal{W}}{\partial V_i} \frac{dV_i(\tau, g(\tau))}{d\tau} di \\ &= \int \frac{\partial \mathcal{W}}{\partial V_i} \frac{\partial u_i\left(c^i, n^i\right)}{\partial c^i} \left(-w^i n^i + \frac{dg}{d\tau}\right) di. \end{aligned}$$

Since this is a static environment, a normalized DS-Planner will only need to rely on normalized individual weights, which take the form

$$\tilde{\omega}^i = \frac{\frac{\partial \mathcal{W}}{\partial V_i} \frac{\partial u_i\left(c^i, n^i\right)}{\partial c^i}}{\int \frac{\partial \mathcal{W}}{\partial V_i} \frac{\partial u_i\left(c^i, n^i\right)}{\partial c^i} di}.$$

[2]Note that $\frac{dT\left(w^i n^i\right)}{d\tau} = -w^i n^i + \frac{dg}{d\tau}$.

Given these normalized individual weights, we can express the welfare assessments as follows:

$$
\begin{aligned}
\frac{dW^{DS}}{d\tau} &= \int \tilde{\omega}^i \left(-\underbrace{w^i n^i}_{z^i} + \underbrace{\frac{dg}{d\tau}}_{=Z-\tau\frac{dZ(1-\tau)}{d(1-\tau)}} \right) \\
&= \int \tilde{\omega}^i \left(-z^i + Z - \tau \frac{dZ\left(1-\tau\right)}{d\left(1-\tau\right)} \right) \\
&= \int \left(-z^i + Z - \tau \frac{dZ\left(1-\tau\right)}{d\left(1-\tau\right)} \right) di + \mathbb{C}ov_i \left[\tilde{\omega}^i, Z - z^i - \tau \frac{dZ\left(1-\tau\right)}{d\left(1-\tau\right)} \right] \\
&= \underbrace{-\tau \frac{dZ\left(1-\tau\right)}{d\left(1-\tau\right)}}_{=\Xi_{AE} \text{ (Agg. Efficiency)}} + \underbrace{\mathbb{C}ov_i \left[\tilde{\omega}^i, -z^i \right]}_{=\Xi_{RD} \text{ (Redistribution)}} . \qquad (2)
\end{aligned}
$$

In the optimal tax literature, it is typical to try to provide an expression for τ in terms of interpretable objects. In fact, solving for τ in Equation (2) yields exactly Equation (3) in (Piketty and Saez, 2013). We will not insist on doing that there, since our goal is to highlight the use the aggregate additive decomposition introduced in our paper. Note that $\Xi_{AE} = 0$ and $\Xi_{RD} > 0$ when $\tau = 0$. Under regularity conditions, we expect Ξ_{AE} to become more negative as τ increases, while Ξ_{RD} will become less positive, which yields a well-behaved solution for τ.

1.2 Random earnings

Environment. We now consider an environment in which individual earnings are partly due to a random process involving luck, in addition to ability and effort. We still assume that individuals exclusively consume during a single period in which they have to a make consumption-labor decision. We assume that individuals have expected utility preferences of the form

$$\int u^i \left(c^i \left(\varepsilon \right), n^i \left(\varepsilon \right) \right) dF \left(\varepsilon | i \right),$$

where ε denotes a shock to individual abilities, so the wage is now a function of ε, as in $w^i \left(\varepsilon \right)$. We assume that the distribution of the shock ε can be different for different individuals, according to $dF \left(\varepsilon \right) | i$. In this case, an individual's budget constraint takes the form

$$c^i \left(\varepsilon \right) = \left(1 - \tau \right) w^i \left(\varepsilon \right) n^i \left(\varepsilon \right) + g.$$

We assume that there is a distribution of ex-ante types given by $\nu \left(i \right)$, where $\int d\nu \left(i \right) = 1$.

Individual optimality. Ex-post, the consumption-hours decision of a given individual is identical to the problem without risk. Hence, we can define an indirect utility function for individual i in terms of τ and g as

$$V_i \left(\tau, g \right) = \max \int u_i \left(c^i \left(\varepsilon \right), n^i \left(\varepsilon \right) \right) dF \left(\varepsilon | i \right) \quad \text{s.t.} \quad c^i \left(\varepsilon \right) = \left(1 - \tau \right) w^i \left(\varepsilon \right) n^i \left(\varepsilon \right) + g.$$

However, we can write the total change in indirect utility for individual i as

$$\frac{dV_i}{d\tau} = \int \frac{\partial u_i\left(c^i(\varepsilon), n^i(\varepsilon)\right)}{\partial c^i} \underbrace{\left(-w^i(\varepsilon) n^i(\varepsilon) + \frac{dg}{d\tau}\right)}_{=\frac{du_{i|c}(\varepsilon)}{d\tau}} \underbrace{dF(\varepsilon|i)}_{f(\varepsilon|i)d\varepsilon}.$$

Optimal linear income tax. The government chooses τ and g to maximize a particular social welfare function $\mathcal{W}(\cdot)$ subject to a revenue constraint and to the constraints that represent individual behavior. Formally,

$$W(\tau) = \mathcal{W}\left(\{V_i(\tau, g(\tau))\}_{i\in I}\right),$$

where the mapping $g(\tau)$ is defined by the government's budget constraint, which takes the form

$$E = \tau \iint w^i(\varepsilon) n^i\left((1-\tau) w^i(\varepsilon), g\right) dF(\varepsilon|i) d\nu(i) - g,$$

where the function $n^i\left((1-\tau) w^i, g\right)$ denotes individual i's Marshallian labor supply. We can write $dF(\varepsilon|i) = f(\varepsilon|i) d\varepsilon$.

A welfare assessment for a welfarist planner takes the form

$$\begin{aligned}\frac{dW(\tau)}{d\tau} &= \int \frac{\partial \mathcal{W}}{\partial V_i} \frac{dV_i(\tau, g(\tau))}{d\tau} di \\ &= \int \frac{\partial \mathcal{W}}{\partial V_i} \int \frac{\partial u_i\left(c^i(\varepsilon), n^i(\varepsilon)\right)}{\partial c^i} \left(-w^i(\varepsilon) n^i(\varepsilon) + \frac{dg}{d\tau}\right) f(\varepsilon|i) d\varepsilon di.\end{aligned}$$

In this environment, a DS-planner can compute individual and stochastic weights, since there is a single date. The normalized individual and stochastic weights take the form

$$\begin{aligned}\tilde{\omega}^i &= \frac{\frac{\partial \mathcal{W}}{\partial V_i} \int \frac{\partial u_i\left(c^i(\varepsilon), n^i(\varepsilon)\right)}{\partial c^i} f(\varepsilon|i) d\varepsilon}{\int \frac{\partial \mathcal{W}}{\partial V_i} \int \frac{\partial u_i(c^i(\varepsilon), n^i(\varepsilon))}{\partial c^i} f(\varepsilon|i) d\varepsilon di} \\ \tilde{\omega}^i(\varepsilon) &= \frac{\frac{\partial u_i\left(c^i(\varepsilon), n^i(\varepsilon)\right)}{\partial c^i} f(\varepsilon|i)}{\int \frac{\partial u_i(c^i(\varepsilon), n^i(\varepsilon))}{\partial c^i} f(\varepsilon|i) d\varepsilon}.\end{aligned}$$

Given these normalized weights, we can express the aggregate welfare assessment as

$$\frac{\frac{dW(\tau)}{d\tau}}{\iint \frac{\partial \mathcal{W}}{\partial V_i} \frac{\partial u_i(c^i(\varepsilon), n^i(\varepsilon))}{\partial c^i} dF(\varepsilon|i) di} = \int \tilde{\omega}^i \int \tilde{\omega}^i(\varepsilon) \left(-w^i(\varepsilon) n^i(\varepsilon) + \frac{dg}{d\tau}\right) d\varepsilon di.$$

As in the deterministic earning model, we can write $\frac{dg}{d\tau} = Z - \tau \frac{dZ(1-\tau)}{d(1-\tau)}$. In this case,

$$\begin{aligned}\frac{du_{i|c}}{d\tau} &= -w^i(\varepsilon) n^i(\varepsilon) + \frac{dg}{d\tau} \\ &= -z^i(\varepsilon) + Z - \tau \frac{dZ(1-\tau)}{d(1-\tau)}.\end{aligned}$$

Hence, we can express a welfare assessment as

$$
\begin{aligned}
\frac{dW^{DS}}{d\tau} &= \int \tilde{\omega}^i \left(\int \tilde{\omega}^i \left(\varepsilon\right) \left(-z^i \left(\varepsilon\right) + \frac{dg}{d\tau} \right) d\varepsilon \right) di \\
&= \int \tilde{\omega}^i \left(\int \tilde{\omega}^i \left(\varepsilon\right) \left(-z^i \left(\varepsilon\right) + Z - \tau \frac{dZ\left(1-\tau\right)}{d\left(1-\tau\right)} \right) d\varepsilon \right) di \\
&= \iint \tilde{\omega}^i \left(\varepsilon\right) \left(-z^i \left(\varepsilon\right) + Z - \tau \frac{dZ\left(1-\tau\right)}{d\left(1-\tau\right)} \right) d\varepsilon di \\
&\quad + \underbrace{\mathbb{C}ov_i \left[\tilde{\omega}^i, \int \tilde{\omega}^i \left(\varepsilon\right) \left(-z^i \left(\varepsilon\right) + Z - \tau \frac{dZ\left(1-\tau\right)}{d\left(1-\tau\right)} \right) d\varepsilon \right]}_{=\Xi_{RD} \text{ (Redistribution)}} \\
&= \iint \frac{\tilde{\omega}^i \left(\varepsilon\right)}{f\left(\varepsilon|i\right)} \left(-z^i \left(\varepsilon\right) + Z - \tau \frac{dZ\left(1-\tau\right)}{d\left(1-\tau\right)} \right) \underbrace{f\left(\varepsilon|i\right) d\varepsilon di}_{=dF(\varepsilon,i)} + \Xi_{RD} \\
&= \mathbb{E}_{\varepsilon,i} \left[\frac{\tilde{\omega}^i \left(\varepsilon\right)}{f\left(\varepsilon|i\right)} \left(-z^i \left(\varepsilon\right) + Z - \tau \frac{dZ\left(1-\tau\right)}{d\left(1-\tau\right)} \right) \right] + \Xi_{RD} \\
&= \iint \left(-z^i \left(\varepsilon\right) + Z - \tau \frac{dZ\left(1-\tau\right)}{d\left(1-\tau\right)} \right) f\left(\varepsilon|i\right) d\varepsilon di \\
&\quad + \mathbb{C}ov_{\varepsilon,i} \left[\tilde{\omega}^i \left(\varepsilon\right), -z^i \left(\varepsilon\right) + Z - \tau \frac{dZ\left(1-\tau\right)}{d\left(1-\tau\right)} \right] + \Xi_{RD} \\
&= \underbrace{-\tau \frac{dZ\left(1-\tau\right)}{d\left(1-\tau\right)}}_{=\Xi_{AE} \text{ (Agg. Efficiency)}} + \underbrace{\mathbb{C}ov_{\varepsilon,i} \left[\frac{\tilde{\omega}^i \left(\varepsilon\right)}{f\left(\varepsilon|i\right)}, -z^i \left(\varepsilon\right) \right]}_{=\Xi_{RS} \text{ (Risk-Sharing)}} + \underbrace{\mathbb{C}ov_i \left[\tilde{\omega}^i, \int \tilde{\omega}^i \left(\varepsilon\right) \left(-z^i \left(\varepsilon\right) \right) dF\left(\varepsilon|i\right) \right]}_{=\Xi_{RD} \text{ (Redistribution)}}
\end{aligned} \tag{3}
$$

Note that $\iint \tilde{\omega}^i \left(\varepsilon\right) d\varepsilon di = 1$, since $\int \tilde{\omega}^i \left(\varepsilon\right) d\varepsilon = 1$. Note that $\mathbb{E}_{\varepsilon,i} \left[\frac{\tilde{\omega}^i(\varepsilon)}{f(\varepsilon|i) d\varepsilon di} \right] = \iint \frac{\tilde{\omega}^i(\varepsilon)}{f(\varepsilon|i) d\varepsilon di} f\left(\varepsilon|i\right) d\varepsilon di = 1$. Note also that $\iint f\left(\varepsilon|i\right) d\varepsilon di = 1$ and that it is critical that there is no aggregate-risk in this economy. Typically, we have that $\Xi_{AE} < 0$, while $\Xi_{RD} > 0$. The sign of Ξ_{RS} is ambiguous, and depends on the joint distribution of shocks and ex-ante heterogeneity.

Note that the redistribution component can be decomposed, following the subdecomposition in Section 6.1 of Dávila and Schaab (2022), as

$$
\begin{aligned}
\Xi_{RD} &= \mathbb{C}ov_i \left[\tilde{\omega}^i, \int \tilde{\omega}^i \left(\varepsilon\right) \left(-z^i \left(\varepsilon\right) + Z - \tau \frac{dZ\left(1-\tau\right)}{d\left(1-\tau\right)} \right) d\varepsilon \right] \\
&= \mathbb{C}ov_i \left[\tilde{\omega}^i, - \int \tilde{\omega}^i \left(\varepsilon\right) \left(z^i \left(\varepsilon\right) \right) dF\left(\varepsilon|i\right) \right] \\
&= \mathbb{C}ov_i \left[\tilde{\omega}^i, - \left(\int \left(z^i \left(\varepsilon\right) \right) dF\left(\varepsilon|i\right) + \mathbb{C}ov_i \left[\tilde{\omega}^i \left(\varepsilon\right), z^i \left(\varepsilon\right) \right] \right) \right] \\
&= \underbrace{\mathbb{C}ov_i \left[\tilde{\omega}^i, - \int \left(z^i \left(\varepsilon\right) \right) dF\left(\varepsilon|i\right) \right]}_{\text{Expected Redistribution}} + \underbrace{\mathbb{C}ov_i \left[\tilde{\omega}^i, \mathbb{C}ov_i \left[\tilde{\omega}^i \left(\varepsilon\right), -z^i \left(\varepsilon\right) \right] \right]}_{\text{Redistributive Insurance}}.
\end{aligned}
$$

Summary of new insights.

- The equity-efficiency tradeoff is not the same in the random earnings case when compared to the deterministic earnings case.
- The optimal tax in the stochastic earnings case includes a new motive for taxation: risk-sharing. It is

important to note that risk-sharing is part of efficiency.

- Hence, the optimal tax for NR (No-Redistribution) DS-Planner will always be $\tau^\star = 0$ in the deterministic earnings case, but it will typically be different from zero in the random earnings case.

2 Application #2: Linear Capital Tax in the Neoclassical Growth Model with Uninsurable Idiosyncratic Shocks.

In their highly influential contribution, Davila et al. (2012) study the welfare properties of the one-sector neoclassical growth model with uninsurable idiosyncratic shocks and precautionary savings. After illustrating the role played by pecuniary externalities in a version of the model with ex-ante identical individuals, the paper then revisits the main results in an economy with initial wealth heterogeneity. The paper motivates the study of the model with initial wealth heterogeneity by arguing that is better connected to the infinite-horizon studied later:

> *" (...) with sufficient dispersion in initial wealth, it would not be possible to find a Pareto improvement by altering aggregate saving. However, the main point of considering initial wealth inequality here is that it provides a useful link to the analysis of the infinite-horizon model studied in the sections to follow."*

The paper then justifies the use of a utilitarian objective as follows:

"There, initial (as of time 0) wealth is identical across agents, but as a result of uninsurable earnings shocks, wealth levels will diverge over time; in a laissez-faire steady state, there is a nontrivial joint distribution over asset levels and employment status. Thus, in that setting, as in the model studied in the previous section, there is a natural planner objective, namely, ex ante expected utility—which will be equal for all agents, though realized utility, of course, differs across consumers. Since ex ante expected utility amounts to a probability-weighted average, it can be thought of as a utilitarian objective: the planner is "behind the veil of ignorance." This means that an ex post desire for redistribution from the consumption-rich to the consumption-poor reflects the ex ante insurance aim. Now turning back to our two-period model, based on the previous discus- sion, we can think of it as the last two periods of a long-horizon model, which then means that the appropriate planner objective is the utilitarian one. Thus, whether more or less aggregate saving is called for in the second-to-last period is more readily answered: we only need to sum the effects on welfare across all consumers."

In what follows, we will formally illustrate that the welfare assessments of a utilitarian in this economy involve aggregate efficiency, risk-sharing, and redistribution considerations, but not intertemporal-sharing. For simplicity, we illustrate our results in the context of varying a linear capital income tax.

2.1 Environment

We study the same environment as in Section 2 of Davila et al. (2012), only augmented to allow for additional individual heterogeneity. In particular, we allow for arbitrary ex-ante heterogeneity in preferences, via β_i and $u_i(\cdot)$, as well as in the initial endowment, via m^i.[3]

This economy is populated by a unit measure of individuals. An individual i solves

$$\max_{c_0^i, c_1^i(\varepsilon), a^i} u_i\left(c_0^i\right) + \beta_i \int u_i\left(c_1^i(\varepsilon)\right) dG_i(\varepsilon),$$

[3]Note that the index i exclusively captures individual heterogeneity, as in Section G.6 of the Online Appendix in Dávila and Schaab (2022).

subject to budget constraints

$$c_0^i = m^i - (1+\tau)\, a^i + T^i$$
$$c_1^i(\varepsilon) = r \cdot a^i + w \cdot e^i(\varepsilon) + \underbrace{\Pi^i}_{=0},\ \forall \varepsilon.$$

Individuals face idiosyncratic stocks, indexed by ε, and distributive according to $dG(\cdot)$.[4] We denote initial wealth by m^i, savings by a^i, and the idiosyncratic realization of labor productivity by $e^i(\varepsilon)$. The interest rate and the wage are denoted by r and w, respectively. Consumption is denoted by c_0^i and $c_1^i(\varepsilon)$.

We allow for a linear tax on capital, τ, whose proceeds are rebated according to T^i. We typically consider i) targeted rebates or ii) uniform rebates, that is,

$$T^i = \begin{cases} \tau a^i, & \text{targeted rebate} \\ \tau K, & \text{uniform rebate} \end{cases}$$

In either case, note that $\int T^i di = \tau K$, and this implies that

$$\frac{d\int T^i di}{d\tau} = \int \frac{dT^i}{d\tau} di = K + \tau \frac{dK}{d\tau}.$$

In this economy, aggregate capital and labor are given by

$$K = \int a^i di$$
$$L = \iint e^i(\varepsilon)\, dG_i(\varepsilon)\, di,$$

which are scalars under a law of large numbers. Competitive firms produce at date 1 using a Cobb-Douglas technology.[5] Hence, given the absence of aggregate uncertainty, r and w are pinned down by

$$r = F_K(K, L)$$
$$w = F_L(K, L).$$

The optimality condition of individual i is given by an Euler equation of the form[6]

$$(1+\tau)\, u_i'\left(c_0^i\right) = \beta r \int u_i'\left(c_1^i(\varepsilon)\right) dG_i(\varepsilon) \Rightarrow 1 + \tau = \beta r \int \frac{u_i'\left(c_1^i(\varepsilon)\right)}{u_i'\left(c_0^i\right)} dG_i(\varepsilon).$$

[4] In Davila et al. (2012), they assume that the idiosyncratic shock ε, can simply take two values,

$$e^i(\varepsilon) = \begin{cases} e^1, & \text{with } \pi\left(e^1\right) \\ e^2, & \text{with } \pi\left(e^2\right) \end{cases}.$$

[5] In a competitive equilibrium, profits are given by

$$\Pi = f(K, L) - rK - wL = (F_L - w)\, L + (F_K - r)\, K = 0.$$

Given the CRS assumption, the distribution of profits is irrelevant, since they are zero.

[6] Below, we will use the fact that

$$(1+\tau)\, \tilde{\omega}_0^i = r \tilde{\omega}_1^i \iff (1+\tau)\, \tilde{\omega}_0^i = r \tilde{\omega}_1^i \int \tilde{\omega}_1^i(\varepsilon)\, d\varepsilon \iff (1+\tau) \int \tilde{\omega}_0^i di = r \int \tilde{\omega}_1^i \int \tilde{\omega}_1^i(\varepsilon)\, d\varepsilon di,$$

since $\int \tilde{\omega}_1^i(\varepsilon)\, d\varepsilon = 1$.

2.2 Welfare Assessments

Useful preliminary computations. Note that

$$\frac{dV^i}{d\tau} = u_i'\left(c_0^i\right)\frac{dc_0^i}{d\tau} + \beta \int u_i'\left(c_1^i\left(\varepsilon\right)\right)\frac{dc_1^i\left(\varepsilon\right)}{d\tau}dG\left(\varepsilon\right),$$

where

$$\begin{aligned}\frac{dc_0^i}{d\tau} &= -\left(1+\tau\right)\frac{da^i}{d\tau} - a^i + \frac{dT^i}{d\tau}\\ \frac{dc_1^i\left(\varepsilon\right)}{d\tau} &= r\frac{da^i}{d\tau} + \frac{dr}{d\tau}a^i + \frac{dw}{d\tau}e^i\left(\varepsilon\right),\end{aligned}$$

and

$$\frac{dT^i}{d\tau} = \begin{cases}\tau\frac{da^i}{d\tau} + a^i, & \text{targeted rebate}\\ \tau\frac{dK}{d\tau} + K, & \text{uniform rebate.}\end{cases}$$

Hence

$$\frac{dc_0^i}{d\tau} = \begin{cases}-\frac{da^i}{d\tau}, & \text{targeted rebate}\\ -\frac{da^i}{d\tau} + K - a^i + \tau\left(\frac{dK}{d\tau} - \frac{da^i}{d\tau}\right), & \text{uniform rebate.}\end{cases}$$

and

$$\int \frac{dc_0^i}{d\tau}di = \underbrace{-\int\left(1+\tau\right)\frac{da^i}{d\tau}di}_{=-(1+\tau)\frac{dK}{d\tau}} - \underbrace{\int a^i di}_{=K} + \underbrace{\int \frac{dT^i}{d\tau}di}_{K+\tau\frac{dK}{d\tau}} = -\frac{dK}{d\tau}$$

$$\begin{aligned}\iint \frac{dc_1^i\left(\varepsilon\right)}{d\tau}dF\left(\varepsilon,i\right) &= r\iint \frac{da^i}{d\tau}dF\left(\varepsilon,i\right) + \iint \frac{dr}{d\tau}a^i dF\left(\varepsilon,i\right) + \iint \frac{dw}{d\tau}e^i\left(\varepsilon\right)dF\left(\varepsilon,i\right)\\ &= r\underbrace{\iint \frac{da^i}{d\tau}dF\left(\varepsilon,i\right)}_{=\frac{dK}{d\tau}} + \frac{dr}{d\tau}\underbrace{\iint a^i dF\left(\varepsilon,i\right)}_{=K} + \frac{dw}{d\tau}\underbrace{\iint e^i\left(\varepsilon\right)dF\left(\varepsilon,i\right)}_{=L}\\ &= r\frac{dK}{d\tau}.\end{aligned}$$

The last line follows from the zero-profit condition, since

$$\frac{d\Pi}{d\tau} = \left(F_L - w\right)\frac{dL}{d\tau} + \left(F_K - r\right)\frac{dK}{d\tau} - K\frac{dr}{d\tau} - L\frac{dw}{d\tau} = -\left(K\frac{dr}{d\tau} + L\frac{dw}{d\tau}\right) = 0.$$

The fact that the *distributive pecuniary effects* of a policy change, $K\frac{dr}{d\tau} + L\frac{dw}{d\tau}$, add up to zero is a manifestation of a more general results: see Equation (20) and the associated discussion in Dávila and Korinek (2018).

Finally, Davila et al. (2012) show that

$$\frac{dr}{d\tau} > 0 \quad \text{and} \quad \frac{dw}{d\tau} < 0.$$

DS-weights definition. Using the definition of DS-planner introduced in DS, and the individual multiplicative decomposition introduced in Lemma 1 of that paper, we can write the aggregate welfare assessments of a change in the capital tax τ as follows:

$$\frac{dW}{d\tau} = \int \tilde{\omega}^i \left(\tilde{\omega}_0^i \frac{dc_0^i}{d\tau} + \tilde{\omega}_1^i \int \tilde{\omega}_1^i \left(\varepsilon\right) \frac{dc_1^i\left(\varepsilon\right)}{d\tau} d\varepsilon \right) di.$$

The individual component of DS-weights (for utilitarian planner) is given by[7]

$$\tilde{\omega}^i = \frac{u_i'\left(c_0^i\right) + \beta \int u_i'\left(c_1^i\left(\varepsilon\right)\right) dF\left(\varepsilon\right)}{\int \left(u_i'\left(c_0^i\right) + \beta \int u_i'\left(c_1^i\left(\varepsilon\right)\right) dF\left(\varepsilon\right)\right) di}.$$

The dynamic component of DS-weights is given by

$$\tilde{\omega}_0^i = \frac{u_i'\left(c_0^i\right)}{u_i'\left(c_0^i\right) + \beta \int u_i'\left(c_1^i\left(\varepsilon\right)\right) dF\left(\varepsilon\right)}$$
$$\tilde{\omega}_1^i = \frac{\beta \int u_i'\left(c_1^i\left(\varepsilon\right)\right) dF\left(\varepsilon\right)}{u_i'\left(c_0^i\right) + \beta \int u_i'\left(c_1^i\left(\varepsilon\right)\right) dF\left(\varepsilon\right)}.$$

Note that, for any value of τ, we have that $\tilde{\omega}_0^i$ is identical for all individuals, as well as $\tilde{\omega}_1^i$.

Finally, the stochastic component of DS-weights is given by

$$\tilde{\omega}_1^i\left(\varepsilon\right) = \frac{u_i'\left(c_1^i\left(\varepsilon\right)\right) \frac{dF(\varepsilon)}{d\varepsilon}}{\int u_i'\left(c_1^i\left(\varepsilon\right)\right) dF\left(\varepsilon\right)}.$$

Aggregate additive decomposition. The aggregate additive decomposition of the welfare assessments associated with changing the capital tax τ takes the form

$$\begin{aligned}
dW &= \int \tilde{\omega}^i \left(\tilde{\omega}_0^i \frac{dc_0^i}{d\tau} + \tilde{\omega}_1^i \int \tilde{\omega}_1^i\left(\varepsilon\right) \frac{dc_1^i\left(\varepsilon\right)}{d\tau} d\varepsilon \right) di \\
&= \int \left(\tilde{\omega}_0^i \frac{dc_0^i}{d\tau} + \tilde{\omega}_1^i \int \tilde{\omega}_1^i\left(\varepsilon\right) \frac{dc_1^i\left(\varepsilon\right)}{d\tau} d\varepsilon \right) di + \underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i, \tilde{\omega}_0^i \frac{dc_0^i}{d\tau} + \tilde{\omega}_1^i \int \tilde{\omega}_1^i\left(\varepsilon\right) \frac{dc_1^i\left(\varepsilon\right)}{d\tau} d\varepsilon\right]}_{=\Xi_{RD}} \\
&= \int \tilde{\omega}_0^i di \int \frac{dc_0^i}{d\tau} di + \int \tilde{\omega}_1^i di \iint \tilde{\omega}_1^i\left(\varepsilon\right) \frac{dc_1^i\left(\varepsilon\right)}{d\tau} d\varepsilon di + \underbrace{\mathbb{C}ov_i\left[\tilde{\omega}_0^i, \frac{dc_0^i}{d\tau}\right] + \mathbb{C}ov_i\left[\tilde{\omega}_1^i, \int \tilde{\omega}_1^i\left(\varepsilon\right) \frac{dc_1^i\left(\varepsilon\right)}{d\tau} d\varepsilon di\right]}_{=\Xi_{IS}} + \Xi_{RD} \\
&= \int \tilde{\omega}_0^i di \int \frac{dc_0^i}{d\tau} di + \int \tilde{\omega}_1^i di \left(\iint \frac{\tilde{\omega}_1^i\left(\varepsilon\right)}{\frac{dF(\varepsilon)}{d\varepsilon}} \frac{dc_1^i\left(\varepsilon\right)}{d\tau} \underbrace{dF\left(\varepsilon\right) di}_{dF(\varepsilon,i)} \right) + \Xi_{IS} + \Xi_{RD} \\
&= \int \tilde{\omega}_0^i di \int \frac{dc_0^i}{d\tau} di + \int \tilde{\omega}_1^i di \left(\iint \frac{\tilde{\omega}_1^i\left(\varepsilon\right)}{\frac{dF(\varepsilon)}{d\varepsilon}} dF\left(\varepsilon, i\right) \iint \frac{dc_1^i\left(\varepsilon\right)}{d\tau} dF\left(\varepsilon, i\right) + \mathbb{C}ov_{\varepsilon,i}\left[\frac{\tilde{\omega}_1^i\left(\varepsilon\right)}{\frac{dF(\varepsilon)}{d\varepsilon}}, \frac{dc_1^i\left(\varepsilon\right)}{d\tau}\right] \right) + \Xi_{IS} + \Xi_{RD} \\
&= \underbrace{\int \tilde{\omega}_0^i di \int \frac{dc_0^i}{d\tau} di + \int \tilde{\omega}_1^i di \iint \frac{\tilde{\omega}_1^i\left(\varepsilon\right)}{\frac{dF(\varepsilon)}{d\varepsilon}} dF\left(\varepsilon, i\right) \iint \frac{dc_1^i\left(\varepsilon\right)}{d\tau} dF\left(\varepsilon, i\right)}_{=\Xi_{AE}} + \underbrace{\int \tilde{\omega}_1^i di \mathbb{C}ov_{\varepsilon,i}\left[\frac{\tilde{\omega}_1^i\left(\varepsilon\right)}{\frac{dF(\varepsilon)}{d\varepsilon}}, \frac{dc_1^i\left(\varepsilon\right)}{d\tau}\right]}_{=\Xi_{RS}} + \Xi_{IS} + \Xi_{RD}.
\end{aligned}$$

[7] It is straightforward to consider a general welfarist planner: see

Summing up, we have

$$\Xi_{AE} = \int \tilde{\omega}_0^i di \int \frac{dc_0^i}{d\tau} di + \int \tilde{\omega}_1^i di \iint \frac{\tilde{\omega}_1^i(\varepsilon)}{\frac{dF(\varepsilon)}{d\varepsilon}} dF(\varepsilon, i) \iint \frac{dc_1^i(\varepsilon)}{d\tau} dF(\varepsilon, i)$$

$$\Xi_{RS} = \int \tilde{\omega}_1^i di \mathbb{C}ov_{\varepsilon,i} \left[\frac{\tilde{\omega}_1^i(\varepsilon)}{\frac{dF(\varepsilon)}{d\varepsilon}}, \frac{dc_1^i(\varepsilon)}{d\tau} \right]$$

$$\Xi_{IS} = \mathbb{C}ov_i \left[\tilde{\omega}_0^i, \frac{dc_0^i}{d\tau} \right] + \mathbb{C}ov_i \left[\tilde{\omega}_1^i, \int \tilde{\omega}_1^i(\varepsilon) \frac{dc_1^i(\varepsilon)}{d\tau} d\varepsilon di \right]$$

$$\Xi_{RD} = \mathbb{C}ov_i \left[\tilde{\omega}^i, \tilde{\omega}_0^i \frac{dc_0^i}{d\tau} + \tilde{\omega}_1^i \int \tilde{\omega}_1^i(\varepsilon) \frac{dc_1^i(\varepsilon)}{d\tau} d\varepsilon \right]$$

Exploring the different components. *Redistribution*. First, we study the redistribution term, Ξ_{RD}. Note that

$$\begin{aligned}
\tilde{\omega}_0^i \frac{dc_0^i}{d\tau} + \tilde{\omega}_1^i \int \tilde{\omega}_1^i(\varepsilon) \frac{dc_1^i(\varepsilon)}{d\tau} d\varepsilon &= \tilde{\omega}_0^i \left(-(1+\tau) \frac{da^i}{d\tau} - a^i + \frac{dT^i}{d\tau} \right) + \tilde{\omega}_1^i \int \tilde{\omega}_1^i(\varepsilon) \left(r\frac{da^i}{d\tau} + \frac{dr}{d\tau} a^i + \frac{dw}{d\tau} e^i(\varepsilon) \right) d\varepsilon \\
&= \tilde{\omega}_0^i \left(-a^i + \frac{dT^i}{d\tau} \right) + \tilde{\omega}_1^i \int \tilde{\omega}_1^i(\varepsilon) \left(\frac{dr}{d\tau} a^i + \frac{dw}{d\tau} e^i(\varepsilon) \right) d\varepsilon \\
&\quad + \underbrace{\left[-(1+\tau)\tilde{\omega}_0^i + r\tilde{\omega}_1^i \int \tilde{\omega}_1^i(\varepsilon) d\varepsilon \right]}_{=0} \frac{da^i}{d\tau} = 0 \\
&= \tilde{\omega}_0^i \left(-a^i + \frac{dT^i}{d\tau} \right) + \tilde{\omega}_1^i \int \tilde{\omega}_1^i(\varepsilon) \left(\frac{dr}{d\tau} a^i + \frac{dw}{d\tau} e^i(\varepsilon) \right) d\varepsilon,
\end{aligned}$$

where

$$\tilde{\omega}_0^i \left(-a^i + \frac{dT^i}{d\tau} \right) = \begin{cases} \tilde{\omega}_0^i \tau \frac{da^i}{d\tau}, & \text{targeted rebate} \\ \tilde{\omega}_0^i \left(\tau \frac{dK}{d\tau} + K - a^i \right), & \text{uniform rebate} \end{cases}$$

Hence, with the targeted rebate, the non-pecuniary component of Ξ^{RD} at $\tau = 0$ is zero. With the uniform rebate, individuals with assets below average, $K - a^i > 0$, benefit from the increase in the tax, and vice versa. In general, the redistribution term has three components:

1. $\tilde{\omega}_0^i \left(K - a^i \right)$: related to the presence of a non-targeted rebate, favoring individuals with low assets
2. $\tilde{\omega}_0^i \tau \frac{da^i}{dk}$: related to the losses associated with taxation and how the may differentially affect different individuals (this term is 0 when $\tau = 0$)
3. $\tilde{\omega}_1^i \int \tilde{\omega}_1^i(\varepsilon) \left(\frac{dr}{d\tau} a^i + \frac{dw}{d\tau} e^i(\varepsilon) \right) d\varepsilon$: these are the distributive pecuniary effects of the policy

Intertemporal-sharing. Second, note that the intertemporal-sharing term is always zero, since $\tilde{\omega}_0^i$ and $\tilde{\omega}_1^i$ are identical for all individuals.

Risk-sharing. Third, we study the risk-sharing term, Ξ_{RS}. In this case, note that

$$\Xi_{RS} = \int \tilde{\omega}_1^i di \mathbb{C}ov_{\varepsilon,i} \left[\frac{\tilde{\omega}_1^i(\varepsilon)}{\frac{dF(\varepsilon)}{d\varepsilon}}, r\frac{da^i}{d\tau} + \frac{dr}{d\tau} a^i + \frac{dw}{d\tau} e^i(\varepsilon) \right].$$

Interestingly, the $r\frac{da^i}{d\tau}$ term impact risk-sharing, even though individuals have an envelope condition on a^i. Note that if agents are ex-ante identical, then $\tilde{\omega}_1^i(\varepsilon)$ is constant across i's, and a^i and $\frac{da^i}{d\tau}$ is the same across individuals, so

$$\Xi_{RS} = \int \tilde{\omega}_1^i di \mathbb{C}ov_{\varepsilon,i}\left[\frac{\tilde{\omega}_1(\varepsilon)}{\frac{dF(\varepsilon)}{d\varepsilon}}, \frac{dw}{d\tau}e^i(\varepsilon)\right].$$

Since $\frac{dw}{d\tau} < 0$, and agents with high $e^i(\varepsilon)$ have low $\tilde{\omega}_1^i(\varepsilon)$, it is straightforward to show that $\Xi_{RS} > 0$ in this case.

Aggregate Efficiency. Finally, the aggregate efficiency term can be written as

$$\Xi_{AE} = \int \tilde{\omega}_0^i di \underbrace{\int dc_0^i di}_{=-\frac{dK}{d\tau}} + \int \tilde{\omega}_1^i di \iint \frac{\tilde{\omega}_1^i(\varepsilon)}{\frac{dF(\varepsilon)}{d\varepsilon}} dF(\varepsilon, i) \underbrace{\iint \frac{dc_1^i(\varepsilon)}{d\tau} dF(\varepsilon, i)}_{=r\frac{dK}{d\tau}}.$$

So the aggregate efficiency term satisfies an aggregate Euler equation of the form

$$\begin{aligned}
\Xi_{AE} &= \left[-\int \tilde{\omega}_0^i di + r\int \tilde{\omega}_1^i di \iint \frac{\tilde{\omega}_1^i(\varepsilon)}{\frac{dF(\varepsilon)}{d\varepsilon}} dF(\varepsilon, i)\right]\frac{dK}{d\tau} \\
&= \left[-\int \tilde{\omega}_0^i di + r\int \tilde{\omega}_1^i di \iint \frac{\tilde{\omega}_1^i(\varepsilon)}{\frac{dF(\varepsilon)}{d\varepsilon}} dF(\varepsilon)\, di\right]\frac{dK}{d\tau} \\
&= \left[-\int \tilde{\omega}_0^i di + r\int \tilde{\omega}_1^i di \iint \tilde{\omega}_1^i(\varepsilon)\, d\varepsilon di\right]\frac{dK}{d\tau}.
\end{aligned}$$

If markets were complete, this equation would be zero. But what is the sign of this term if markets are incomplete? In general, we can substitute in the individual Euler equations to find[8]

$$\Xi_{AE} = \int \tilde{\omega}_0^i di \tau \frac{dK}{d\tau}.$$

From here, we can conclude that , since $\frac{dK}{d\tau} < 0$ and $\tau > 0$

Summary of new insights.

1. It is always the case that the intertemporal-sharing term $\Xi_{IS} = 0$, because all agents can freely trade in capital, which in this economy is a risk-free security.

2. The redistribution term Ξ_{RS} cannot be immediately signed, and depends on:

   (a) How the tax revenues are rebated,

   (b) The potential differential impact of tax distortions among individuals (this is 0 when $\tau = 0$),

   (c) The distributive pecuniary effects of the policy.

3. When individuals are ex-ante identical, it is easy to show that the risk-sharing term is positive, so $\Xi_{RS} > 0$.

4. When $\tau = 0$, it is the case that $\Xi_{AE} = 0$. But when $\tau > 0$, we have that Ξ_{AE}. Hence this model yields a simple theory of capital taxation. An increase in τ is beneficial in terms of risk-sharing, but this is

[8]Note that since the individual Euler equations imply that

$$\tau \int \tilde{\omega}_0^i di = -\int \tilde{\omega}_0^i di + r\int \tilde{\omega}_1^i \int \tilde{\omega}_1^i(\varepsilon)\, d\varepsilon di.$$

costly in terms of aggregate efficiency. When agents are ex-ante identical, this is the only tradeoff. If agents are ex-ante heterogeneous, there may be other considerations that impact the desirability of a policy change.

5. An AE (Aggregate Efficiency) DS-Planner (Dávila and Schaab, 2022) always finds that the optimal capital tax is 0.

6. A NR (No-Redistribution) DS-Planner — which exclusively maximizes efficiency and perceives that $\Xi_{RD} = 0$ — will find that a positive optimal capital tax is optimal.

3 Application #3: Optimal Deposit Insurance

In this application, we include a proof of Proposition 9 in Dávila and Goldstein (2021), which in turn provides an alternative characterization — based on DS-weights — of Equation (26) in Proposition 2 of that paper. We use the exact some notation as in that paper.

This application illustrates how to use instantaneous social welfare functions, as well as how to use DS-weights in environments with multiple equilibria.

3.1 Derivation

We start from an instantaneous social welfare function, so $ISWF = \int V(j, \delta, R_1)\, dH(j)$, where

$$
\begin{aligned}
V(\tau, \delta, R_1) = &\int_{\underline{s}}^{\hat{s}(R_1)} \zeta(j, s)\, U\left(C^F(\tau, s)\right) dF(s) \\
&+ \int_{\hat{s}(R_1)}^{s^*(\delta, R_1)} \left(\pi \zeta(j, s)\, U\left(C^F(\tau, s)\right) + (1-\pi)\, \zeta(j, s)\, U\left(C^N(\tau, s)\right)\right) dF(s) \\
&+ \int_{s^*(\delta, R_1)}^{\overline{s}} \zeta(j, s)\, U\left(C^N(\tau, s)\right) dF(s),
\end{aligned}
$$

where $\zeta(j, s)$ denotes instantaneous Pareto weights. Given this ISWF, we can express $\frac{dV(j,\delta,R_1)}{d\delta} = \frac{d\mathbb{E}_s[\zeta(j,s)U(C_t(j,s))]}{d\delta}$ as

$$
\begin{aligned}
\frac{dV(j, \delta, R_1)}{d\delta} = &\overbrace{\int_{\underline{s}}^{\hat{s}} \zeta(j, s)\, U'\left(C_t^F(j, s)\right) \frac{\partial C_t^F(j, s)}{\partial \delta} dF(s) + \pi \int_{\hat{s}}^{s^*} \zeta(j, s)\, U'\left(C_t^F(j, s)\right) \frac{\partial C_t^F(j, s)}{\partial \delta} dF(s)}^{= q^F \mathbb{E}_s^F\left[\zeta(j,s) U'\left(C_t^F(j,s)\right) \frac{\partial C_t^F(j,s)}{\partial \delta}\right]} \\
&+ \left[\zeta^-(j, s^*)\, U\left(C_t^F(j, s^*)\right) - \zeta^+(j, s^*)\, U\left(C_t^N(j, s^*)\right)\right] \pi f(s^*) \frac{\partial s^*}{\partial \delta},
\end{aligned}
$$

Now, transforming instantaneous Pareto weights (defined over utilities) into dynamic-stochastic weights (defined over consumption) we can express $\frac{dV(j,\delta,R_1)}{d\delta}$ as

$$
\frac{d\tilde{V}(j, \delta, R_1)}{d\delta} = q^F \mathbb{E}_s^F\left[\omega_t(j, s) \frac{\partial C_t^F(j, s)}{\partial \delta}\right] + \left[\omega_t^F(j, s^*)\, C_t^F(j, s^*) - \omega_t^N(j, s^*)\, C_t^N(j, s^*)\right] \frac{\partial q^F}{\partial \delta},
$$

where we define $\frac{\partial q^F}{\partial \delta} = \pi f(s^*) \frac{\partial s^*}{\partial \delta}$, $\omega_t(j, s) = \zeta(j, s)\, U'\left(C_t^F(j, s)\right)$, $\omega_t^F(j, s^*) = \zeta^-(j, s^*) \frac{U\left(C_t^F(j,s^*)\right)}{C_t^F(j,s^*)}$, and $\omega_t^N(j, s^*) = \zeta^+(j, s^*) \frac{U\left(C_t^N(j,s^*)\right)}{C_t^N(j,s^*)}$. We can decompose the dynamic stochastic weights into an individual component and a stochastic component, as follows:

$$\omega_t^N(j,s^*) = \tilde{\omega}(j)\,\tilde{\omega}_t^N(j,s^*), \quad \omega_t^F(j,s^*) = \tilde{\omega}(j)\,\tilde{\omega}_t^F(j,s^*), \quad \omega_t(j,s) = \tilde{\omega}(j)\,\tilde{\omega}_t(j,s).$$

Hence, under the assumption that $\int dH(j) = 1$, which may require a normalization, we can now express a welfare assessment $\frac{dW}{d\delta}$ as follows:

$$\frac{dW}{d\delta} = \int \tilde{\omega}(j)\,\frac{d\tilde{V}(j,\delta,R_1)}{d\delta}dH(j) = \mathbb{E}_j\left[\tilde{\omega}(j)\,\frac{d\tilde{V}(j,\delta,R_1)}{d\delta}\right] = \int \frac{d\tilde{V}(j,\delta,R_1)}{d\delta}dH(j) + \Xi_{RD},$$

where we use the fact that $\int dH(j) = 1$ and where

$$\Xi_{RD} = \mathbb{C}ov_j\left[\tilde{\omega}(j),\frac{d\tilde{V}(j,\delta,R_1)}{d\delta}\right]. \tag{4}$$

Note that we can write $\int \frac{d\tilde{V}(j,\delta,R_1)}{d\delta}dH(j) = \mathbb{E}_j\left[\frac{d\tilde{V}(j,\delta,R_1)}{d\delta}\right]$ as

$$\begin{aligned}\int \frac{d\tilde{V}(j,\delta,R_1)}{d\delta}dH(j) &= -\frac{\partial q^F}{\partial\delta}\left(\mathbb{E}_j\left[\tilde{\omega}_t^N(j,s^*)\,C_t^N(j,s^*)\right] - \mathbb{E}_j\left[\tilde{\omega}_t^F(j,s^*)\,C_t^F(j,s^*)\right]\right)\\ &\quad + q^F\mathbb{E}_j\left[\mathbb{E}_s^F\left[\tilde{\omega}_t(j,s)\,\frac{\partial C_t^F(j,s)}{\partial\delta}\right]\right]\\ &= \Xi_{AE} + \Xi_{RS},\end{aligned}$$

where

$$\begin{aligned}\Xi_{AE} &= -\frac{\partial q^F}{\partial\delta}\left(\mathbb{E}_j\left[\tilde{\omega}_t^N(j,s^*)\right]\mathbb{E}_j\left[C_t^N(j,s^*)\right] - \mathbb{E}_j\left[\tilde{\omega}_t^F(j,s^*)\right]\mathbb{E}_j\left[C_t^F(j,s^*)\right]\right)\\ &\quad + q^F\mathbb{E}_s^F\left[\mathbb{E}_j\left[\tilde{\omega}_t(j,s)\right]\mathbb{E}_j\left[\frac{\partial C_t^F(j,s)}{\partial\delta}\right]\right] \qquad (5)\\ \Xi_{RS} &= -\frac{\partial q^F}{\partial\delta}\left(\mathbb{C}ov_j\left[\tilde{\omega}_t^N(j,s^*),C_t^N(j,s^*)\right] - \mathbb{C}ov\left[\tilde{\omega}_t^F(j,s^*),C_t^F(j,s^*)\right]\right)\\ &\quad + q^F\mathbb{E}_s^F\left[\mathbb{C}ov_j\left[\tilde{\omega}_t(j,s),\frac{\partial C_t^F(j,s)}{\partial\delta}\right]\right]. \qquad (6)\end{aligned}$$

When $\tilde{\omega}(j) = 1$, $\Xi_{RD} = 0$. And when $\tilde{\omega}_t^N(j,s^*) = \tilde{\omega}_t^F(j,s^*)$ and $\tilde{\omega}_t(j,s) = 1$, $\Xi_{RS} = 0$ and Ξ_{AE} is exactly given by

$$\frac{dW}{d\delta} = -\frac{\partial q^F}{\partial\delta}\int\left(C^N(j,s^*) - C^F(j,s^*)\right)dH(j) + q^F\mathbb{E}_s^F\left[\int\frac{\partial C^F(j,s)}{\partial\delta}dH(j)\right], \tag{7}$$

which is the exact counterpart of Equation (26) in Proposition 2.

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