Volatility and Informativeness*

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Abstract

This paper studies the relation between volatility and informativeness in financial markets. We identify two channels (noise-reduction and equilibrium-learning) that determine the volatility-informativeness relation. When informativeness is sufficiently high (low), volatility and informativeness positively (negatively) comove in equilibrium. We identify conditions on primitives that guarantee that volatility and informativeness comove positively or negatively. We introduce the comovement score, a statistic that measures the distance of a given asset to the positive/negative comovement regions. Empirically, comovement scores i) have trended downwards over the last decades, ii) are positively related to value and idiosyncratic volatility and negatively to size and institutional ownership.

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1 Introduction

A long tradition in economics and finance, commonly traced back to Hayek (1945), emphasizes the role of financial markets in aggregating dispersed information. Under this view, prices not only convey scarcity, but also reveal the dispersed information held by investors about the underlying fundamentals of the economy. Within this paradigm, price informativeness, understood as the precision of the signal about future payoffs revealed by asset prices, defines a natural measure of the ability of financial markets to aggregate information. While price informativeness is a complex equilibrium object that can only be inferred, price volatility is an alternative equilibrium object that is regularly scrutinized.[1] In particular, price volatility — because it is easy to compute — is typically used as a proxy for informativeness. For instance, in a recent review of the literature on idiosyncratic equity volatility, Campbell et al. (2022) state that *"idiosyncratic volatility serves as an empirical proxy for the flow of firm-specific information"*. An important practical conclusion of our analysis will be that the relation between volatility and informativeness is nuanced and far from obvious, and that using volatility to make inferences about informativeness is only justified in particular circumstances.

In this paper, we systematically explore the relation between volatility and informativeness with the goal of understanding the conditions under which changes in price volatility can be interpreted as a reflection of more or less informative asset markets. We do so theoretically and empirically. Because price volatility and price informativeness are jointly determined in equilibrium, we adopt an unconventional methodological approach, by initially studying the equilibrium relation between both endogenous variables before understanding comparative statics.

Our first set of results is theoretical. First, in the context of a tractable model of competitive trading in financial markets, we show that the equilibrium relation between price informativeness and price volatility — which refer to as the "volatility-informativeness" relation — is shaped by two different channels, the *equilibrium-learning* channel and the *noise-reduction* channel, which operate in opposite directions.[2] Through the equilibrium-learning channel, which is inactive when investors do not learn from asset prices, an increase in price informativeness tilts investors' demands toward putting more weight on the price as a signal about asset payoffs, making investors' demands more correlated, which increases the sensitivity of prices to the aggregate payoff realization and, consequently, price volatility. Alternatively, through the noise-reduction channel, an increase in

[1]The relevant definition of price volatility for our analysis corresponds to the conditional idiosyncratic volatility of asset prices given past public information, as it will become clear in Section E. In our context, there is a one-to-one mapping between conditional price volatility and conditional return volatility. To simplify the exposition, we typically use the term price volatility.

[2]Since price informativeness is an endogenous object, considering changes in price informativeness that do not otherwise affect the volatility-informativeness relation is only possible for a subset of all the model parameters. This type of changes is nonetheless useful for definitional purposes. We eventually consider changes in parameters that, at the same time, change price informativeness and shift the volatility-informativeness relation.

price informativeness is directly associated with a reduction in price volatility, since less noise is incorporated into the price.

Second, we show that the volatility-informativeness relation is upward-sloping (downward-sloping) whenever price informativeness is sufficiently high (low). Through the lens of our two-channel decomposition, when prices are sufficiently informative, the equilibrium-learning channel becomes overwhelmingly important and dominates the noise-reduction channel. However, when prices are sufficiently uninformative, the noise-reduction channel dominates. This result implies that any change among the subset of parameters that do not enter the volatility-informativeness relation directly must induce a positive (negative) comovement between price informativeness and volatility when price informativeness is sufficiently high (low).

Third, we characterize regions in which price informativeness and price volatility positively or negatively comove for changes in *any* of the underlying model parameters. Initially, we show that whenever prices are sufficiently informative (uninformative), changes in any underlying parameter, including those that explicitly appear in the volatility-informativeness relation, induce a positive (negative) comovement between price informativeness and volatility across all applications considered. Next, we characterize the conditions under which volatility and informativeness positively or negatively comove as a function of primitives. We do so by defining positive, negative, and ambiguous comovement regions in terms of model primitives. Interestingly, these regions can be defined exclusively in terms of two scale-invariant ratios of precisions: a signal-to-payoff ratio and a noise-to-payoff ratio. Finding an unambiguously positive or negative comovement between price volatility and informativeness for any change in model primitives, even for specific regions of the parameter space, may come as a surprise since, by reading the existing literature (e.g., Vives (2008)), one may conclude that there is no systematic relation between these variables.

Subsequently, building on our theoretical results, we develop an empirical framework to analyze the relation between volatility and informativeness. First, we introduce the notion of comovement score, which is a statistic that measures the relative distance of a given asset to the positive/negative comovement regions. The comovement score is useful because it defines a continuous measure of (the likelihood of) comovement between volatility and informativeness for all stocks, including those in the ambiguous comovement region. The definition of the comovement score is meant to capture the idea that stocks closer to the positive (negative) comovement region are more likely to experience positive (negative) comovement between volatility and informativeness.

Second, we show how to empirically measure stock-specific comovement scores. In particular, we describe how to estimate a stock's signal-to-payoff/noise-to-payoff pair, and how to use such estimates to compute comovement scores. We formally show that a specific combination of parameters and R-squareds from linear regressions of changes in asset prices on changes in asset payoffs allows us to consistently estimate signal-to-payoff and noise-to-payoff ratios, which in turn can be used to estimate comovement scores. The empirical measurement of the comovement score opens the door to using price volatility to measure price informativeness in applied work. More specifically, our analysis suggests that stocks with estimated comovement scores closer to 1 are better candidates to use volatility as a proxy for informativeness, while for stocks with estimated comovement scores closer to 0 increases in volatility are more likely to reflect decreases in price informativeness.

Using quarterly data between 1961 and 2017, we apply our estimation procedure to recover a panel of stock-specific comovement scores by running rolling time-series regressions. We find that roughly 14% of the estimated comovement scores are in or near the negative comovement region (29% of the scores are lower than 0.5), while roughly 40% are in or near the positive comovement region (71% of the scores are higher than 0.5). Our estimation exercise allows us to uncover both cross-sectional and time-series patterns about the behavior of comovement scores. In the time series, we find that both the mean and median of the distribution of comovement scores have steadily decreased since the mid-1980s. Through the lens of our framework, these results imply that changes in volatility are less likely to reflect changes in informativeness in the same direction in recent times. We also find an increase in the cross-sectional dispersion of comovement scores in recent times, which supports the idea that making inferences about informativeness from volatility is not straightforward, and needs to be studied on a case-by-case basis. In the cross section, we find that stocks that i) are small, ii) have a high book-to-market ratio, iii) have high idiosyncratic volatility and iv) have a lower institutional ownership share have higher comovement scores. We show that these cross-sectional patterns are remarkably stable over time, and that the conditional distributions of stocks according to the characteristics that we study also remain unchanged over time. Finally, consistent with our theoretical framework, we show that stocks with higher comovement scores are more likely to experience positive comovement between price volatility and price informativeness.

Related Literature. This paper is most directly related to the literature that studies the role played by financial markets in aggregating dispersed information, going back to Hayek (1945), and following Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verrecchia (1981), among others. Biais, Glosten and Spatt (2005), Vives (2008), and Veldkamp (2011) provide recent reviews of this body of work. Although price informativeness and price volatility are important equilibrium objects that have been studied in prior literature, we provide, to our knowledge, the first systematic study of the relation between both endogenous variables. In particular, while the existing literature typically presents expressions for price informativeness and volatility separately, in terms of primitives, and presents comparative statics for each of these two quantities separately, it does not explore further the relation between both variables. For instance, the textbook treatment of Vives (2008) separately discusses the comparative statics of price volatility and informativeness. Similarly, Wang (1993) and Lee and Liu (2011) study comparative statics of price volatility and informativeness when varying the number of noise traders. In contrast, our results characterize how volatility and informativeness comove for any change in the model primitives, identifying regions in which both variables comove unambiguously in a particular direction.

There exists a vast literature focused on the measurement of asset price volatility, including the seminal contribution of Engle (1982), which spurred a large amount of work in Financial Econometrics. Campbell et al. (2001, 2022) and Brandt et al. (2009) are well-known references within this vast literature.[3] While these studies emphasize the implications of price volatility for diversification and its relation with expected returns, they have not related their findings to whether prices are more or less informative, besides arguing at times (see our reference above) that volatility may be a valid proxy for informativeness. Our results highlight that using volatility to make inferences about informativeness is only justified in specific circumstances.

A growing literature seeks to understand the behavior of price informativeness empirically. In particular, Bai, Philippon and Savov (2016) explore whether financial markets have become more informative over time. Dávila and Parlatore (2020) show how to identify and estimate exact stock-specific measures of price informativeness in a large class of environments. Farboodi et al. (2019) show that financial markets are more informative because of large, growth stocks, for which data have become relatively more valuable. Kacperczyk, Sundaresan and Wang (2018) find that price informativeness increases with ownership by foreign investors. In contrast to this literature, we introduce a new statistic that determines how changes in price volatility can be used to infer changes in price informativeness.

Finally, we would like to highlight the high-level relation between our results and the work of Bergemann, Heumann and Morris (2015). They show in an abstract linear-quadratic environment that the information structure that yields maximal aggregate volatility is such that agents confound idiosyncratic and aggregate shocks, excessively responding to aggregate shocks. Their goal is to study how alternative information structures affect the moments (e.g., volatility) of endogenous variables in the economy. Instead, our goal is to understand the endogenous equilibrium relation between the signal-to-noise ratio associated with asset prices, which is an unobservable variable that captures the ability of financial markets to aggregate information, with the volatility of asset prices, which is an easily computable endogenous outcome of financial market trading.

[3] By modeling dispersed information and learning, our results also contrast with the vast literature studying excess volatility and predictability that follows Shiller (1981), mostly focused on a representative investor.

Outline. Section 2 describes the model environment and characterizes the equilibrium. Section 3 theoretically explores the relation between volatility and informativeness, formally characterizing comovement regions. Section 4 develops an empirical framework to analyze the relation between volatility and informativeness, measuring comovement scores and describing empirical patterns. Section 5 concludes. The Appendix contains derivations, proofs, and additional results.

2 Model

In this section, we first describe the baseline environment and then characterize the equilibrium.

2.1 Environment

Timing and assets. Time is discrete, with dates denoted by $t = 0, 1, 2, \ldots, \infty$. There are two traded assets: a riskless asset in perfectly elastic supply with gross return R, normalized to 1, and a risky asset in fixed supply Q, which is traded at a price p_t at date t.

Preferences. At each date t, a continuum of investors in unit measure — indexed by $i \in I$ — is born with wealth w_0^i. Each generation of investors lives two dates and maximizes expected utility with constant absolute risk aversion (CARA) preferences over terminal wealth. Hence, investors born at date t trade at date t and consume their date $t+1$ wealth. Formally, the flow utility of an investor over terminal wealth is given by

$$U(w) = -e^{-\gamma w},$$

where the parameter $\gamma \equiv \frac{-U''(\cdot)}{U'(\cdot)} > 0$ represents the coefficient of absolute risk aversion.

Payoff process and signals. The risky asset payoff is given by

$$\theta_{t+1} = \mu_\theta + \rho\theta_t + \eta_t,$$

where μ_θ is a scalar, $|\rho| \leq 1$ and $\theta_0 = 0$, and where the innovations to the payoff, η_t, are independently distributed across dates. Note that the innovation to the $t+1$ payoff, η_t, is indexed by t — instead of $t+1$ — to indicate that investors can potentially learn about the realization of η_t at date t. The actual realization of the payoff at date t, θ_{t+1}, is only observed by the investors after they trade at that date. However, before trading at date t, each investor i receives a private signal s_t^i about the innovation to the asset payoff η_t, given by

$$s_t^i = \eta_t + \varepsilon_{st}^i \quad \text{with} \quad \varepsilon_{st}^i \sim N\left(0, \tau_s^{-1}\right),$$

where $\varepsilon_{st}^i \perp \varepsilon_{st}^j$ for all $i \neq j$, and $\eta_t \perp \varepsilon_{st}^i$ for all t and all i.

Noisy heterogeneous priors. Investors have heterogeneous prior beliefs over the distribution of the innovation to the asset payoff. In particular, from an investor i's perspective, the innovation to the asset payoff η_t is distributed according to

$$\eta_t \sim_i N\left(\overline{\eta}_t^i, \tau_\eta^{-1}\right),$$

where $\overline{\eta}_t^i$ denotes investor i's prior expected payoff. We assume that investors' prior expected payoff innovations are random and distributed according to

$$\overline{\eta}_t^i = n_t + \varepsilon_{ut}^i,$$

where

$$n_t \sim N\left(\mu_n, \tau_n^{-1}\right) \quad \text{with} \quad \varepsilon_{ut}^i \overset{iid}{\sim} N\left(0, \tau_u^{-1}\right),$$

where $\varepsilon_{ut}^i \perp n_t$, $\varepsilon_{st}^i \perp n_t$, $\varepsilon_{ut}^i \perp \eta_t$, $\varepsilon_{st}^i \perp \eta_t$, $n_t \perp \eta_t$, and $\varepsilon_{ut}^i \perp \varepsilon_{st}^j$ for all $i, j \in I$, $i \neq j$, and $n_t \perp \eta_t$ for all t. Our formulation implies that an investor's prior mean has two components: an aggregate component, n_t, which can be interpreted as a measure of market-wide sentiment, and an idiosyncratic component ε_{ut}^i, which reflects the particular perceptions of individual investors.

We assume that investors take their priors as given and do not use them to learn about the priors of other investors.[4] Consequently, they do not infer anything about n_t from their own priors. However, investors know the distribution of priors in the economy and use this knowledge to learn from the price. The fact that the realized average prior mean n_t is unknown introduces an additional source of aggregate uncertainty besides the innovation to the asset payoff, which prevents the equilibrium from being fully revealing. We refer to n_t as aggregate sentiment or aggregate noise, indistinctly.We conclude the description of the environment with two remarks.

Remark 1. *(Repeated static economy)* It is important to highlight that the baseline model introduced in this section is effectively a repeated static economy. In particular, since investors are short-lived, the demand for the risky asset is the result of one-period-ahead optimization and only depends on the future through the equilibrium price at the following date. While it is helpful to study a dynamic model to guide our empirical analysis in Section 4, the particular dynamic formulation that we adopt here allows us to keep the model tractable.

Remark 2. *(Alternative sources of noise/general environments)* In order to streamline the exposition, we initially study the relation between volatility and informativeness in the baseline

[4]It is possible to assume instead that investors use their priors to learn about the priors of other investors — see e.g., Dávila and Parlatore (2021).

model introduced in this section, in which the aggregate component of investors' priors is the source of aggregate uncertainty that prevents the equilibrium price from being fully-revealing. However, our results relating between volatility and informativeness hold more generally. In Section E of the Online Appendix, we show that the relation between volatility and informativeness characterized in Proposition 1 below holds exactly in any model with linear demands and additive noise, and approximately in a more general class of models. Moreover, in Section F of the Online Appendix, we extend our results to environments with alternative sources of aggregate noise. In particular, we consider environments in which noise emerges from strategic trading or hedging needs.

2.2 Equilibrium

An investor i born at date t chooses a risky asset demand $q^i_{1,t}$ to maximize

$$\max_{q^i_{1,t}} \mathbb{E}\left[-e^{-\gamma w^i_{1,t+1}} \middle| \mathcal{I}^i_t\right],$$

subject to the wealth accumulation equation

$$w^i_{1,t+1} = \left(\theta_{t+1} + p_{t+1}\right) q^i_{1,t} + w^i_0 - p_t q^i_{1,t}, \tag{1}$$

where $\mathcal{I}^i_t = \left\{\theta_t, s^i_t, \overline{\eta}^i_t, p_t\right\}$ denotes the information set of investor i at date t. As it is customary, we focus on stationary equilibria in linear strategies.

Definition. (Equilibrium) *A stationary rational expectations equilibrium in linear strategies consists of linear risky asset demands $q^i_{1,t}$ for each investor i born at date t of the form*

$$q^i_{1,t} = \alpha_s s^i_t + \alpha_n \overline{\eta}^i_t - \alpha_p p_t + \psi^i,$$

where α_s, α_n, α_p, and ψ^i are determined in equilibrium, and a price function p_t such that at each date t: i) each investor i chooses $q^i_{1,t}$ to maximize expected utility subject to the wealth accumulation equations, defined in Equation (1) and ii) the price function p_t is such that the market for the risky asset clears, i.e., $\int q^i_{1,t} di = Q$.

In equilibrium, the price of the risky asset at date t is given by

$$p_t = \frac{\alpha_\theta}{\alpha_p}\theta_t + \frac{\alpha_s}{\alpha_p}\eta_t + \frac{\alpha_n}{\alpha_p}n_t + \frac{\overline{\psi}}{\alpha_p}, \tag{2}$$

where $\overline{\psi} \equiv \int \psi^i di - Q$. The linearity of asset demands implies that the equilibrium asset price is also linear in the innovation to the asset payoff η_t, in the already realized payoff θ_t, and in the common component of investors' private trading needs n_t.[5] Moreover, the stationarity of the environment guarantees that the coefficients of the equilibrium pricing equation are time-invariant.

The equilibrium price p_t imperfectly reveals the innovation to the asset payoff η_t. The sensitivity of the equilibrium price to the realization of the innovation is modulated by the weight α_s that investors put on their private signals s_t^i. However, investors' demands also depend on their priors, which are orthogonal to the asset payoff. Since investors do not observe the common component of the beliefs, they cannot distinguish whether a high price is due to a high realization of the innovation to the asset payoff η_t or due to a high realization of the aggregate sentiment n_t. It is in this sense that investors' heterogeneous beliefs act as noise, since they prevent the price from being fully revealing.

Our definition of price informativeness is based on the unbiased signal about the innovation to the asset payoff η_t contained in the price, which we denote by π_t. Formally, π_t is given by

$$\pi_t = \frac{\alpha_p}{\alpha_s}\left(p_t - \frac{\alpha_\theta}{\alpha_p}\theta_t - \frac{\alpha_n}{\alpha_p}\mathbb{E}\left[n_t\right] - \frac{\overline{\psi}}{\alpha_p}\right) = \eta_t + \frac{\alpha_n}{\alpha_s}\left(n_t - \mathbb{E}\left[n_t\right]\right), \tag{3}$$

which guarantees that $\mathbb{E}\left[\pi_t|\eta_t\right] = \eta_t$. The last term in Equation (3) represents the noise contained in the price given by the realization of the aggregate sentiment, adjusted by the ratio $\frac{\alpha_n}{\alpha_s}$, so it is expressed in payoff units.

In the Appendix, we fully characterize the equilibrium of the model, showing that it is uniquely determined. We formally state this result in Lemma 1 below.

Lemma 1. (Uniqueness of Equilibrium) *There exists a unique stationary rational expectations equilibrium in linear strategies.*

3 Relating Volatility and Informativeness

In this section, we provide a systematic theoretical analysis of the relation between volatility and informativeness. First, using the equilibrium price p_t, introduced in Equation (2), and the unbiased signal about the asset payoff contained in the price π_t, introduced in Equation (3), we provide formal definitions of our two objects of interest, price volatility and price informativeness. Next, we characterize and study in detail the volatility-informativeness relation, which is a condition that both variables must satisfy in equilibrium. Finally, we characterize the parameter regions such that price volatility and price informativeness unambiguously comove, either positively and negatively,

[5] Since there is a continuum of investors, a law of large numbers guarantees that the terms $\frac{\int \alpha_s \varepsilon_{st}^i di}{\alpha_p}$ and $\frac{\int \alpha_n \varepsilon_{nt}^i di}{\alpha_p}$ vanish, making the price independent of the idiosyncratic noise in investors' signals and priors. Otherwise, these terms operate as additional sources of aggregate noise. In the version of the model with a finite number of investors that we present in the Online Appendix, the realization of the idiosyncratic components of the investors' signals and priors act as the source of aggregate noise.

for any change in the model's parameters. We refer to these as the (volatility-informativeness) comovement regions.

3.1 Definitions

Definition. (Price volatility) *We define price volatility as the conditional variance of the asset price. Formally, we denote price volatility by*

$$\mathcal{V} \equiv \mathbb{V}ar\left[p_t \middle| \theta_t \right].$$

For our purposes, price volatility is simply the idiosyncratic variance of asset prices conditional on the current publicly observed realization of the asset payoff. In our setup, there is a one-to-one mapping between price volatility and return volatility, since investors observe past prices. To simplify the exposition, we only condition on θ_t and use the term price volatility but, formally, our analysis is valid replacing $\mathcal{V}$ by $\mathbb{V}\mathrm{ar}\left[\frac{p_t}{p_{t-1}} \middle| \theta_t, p_{t-1} \right]$.

Definition. (Price informativeness) *We define price informativeness as the precision of the unbiased signal of the innovation to the asset payoff η_t contained in the asset price, π_t, defined in Equation (3), from the perspective of an external observer. Formally, we denote price informativeness by*

$$\tau_\pi \equiv \left(\mathbb{V}ar\left[\pi_t | \eta_t, \theta_t \right] \right)^{-1}. \tag{4}$$

Price informativeness, as defined here, is a variable that summarizes the ability of financial markets to disseminate information through prices. It is the relevant variable that captures how precise the price is as a signal of the innovation to future payoffs η_t from the perspective of an external observer who only observes the realization of the asset payoff θ_t. When price informativeness is high, an external observer receives a very precise signal about the asset payoff by observing the asset price p_t. On the contrary, when price informativeness is low, an external observer learns little about the asset payoff by observing the asset price p_t. It is worth highlighting why it is helpful to understand how volatility and informativeness are related before formally exploring their relation.

Remark 3. *(Importance of understanding the relation between volatility and informativeness)* Changes in price informativeness, especially at high frequencies, are hard to measure. A main objective of this paper is to characterize how price volatility and price informativeness are related in equilibrium to be able to make inferences about movements in price informativeness, which is not directly observable, from movements in price volatility, which can be easily computed. An added benefit of our exercise is to correct the common misconception that idiosyncratic volatility can be used as a proxy for price informativeness without any further qualifications. As we show in the following section, whether price volatility and price informativeness positively or negatively comove depends on the parameters of the economy.

3.2 Volatility-Informativeness Relation

Our first set of results builds on the law of total variance, which is an elementary identity that, applied here, implies that conditional price volatility can be decomposed as follows.

$$\mathbb{V}\mathrm{ar}\left[p_t \middle| \theta_t \right] = \mathbb{E}\left[\mathbb{V}\mathrm{ar}\left[p_t \middle| \eta_t, \theta_t \right] \middle| \theta_t \right] + \mathbb{V}\mathrm{ar}\left[\mathbb{E}\left[p_t \middle| \eta_t, \theta_t \right] \middle| \theta_t \right].$$

The law of total variance asserts that the total variation in the equilibrium price p_t can be decomposed into two components, after conditioning on the innovation to the asset payoff η_t. The first component corresponds to the expectation over the different realizations of the innovation to the asset payoff η_t of the conditional variance of the equilibrium price p_t, given η_t. The second component corresponds to the variance of the conditional expectation of p_t, after learning η_t. Intuitively, the first component captures learnable uncertainty, captured by the best estimate of the residual error in p_t after learning η_t, while the second term captures residual uncertainty, which corresponds to the error from the best guess of p_t after learning η_t.

Using the equilibrium price in Equation (2), we can express both components as follows:

$$\mathbb{E}\left[\mathbb{V}\mathrm{ar}\left[p_t \middle| \eta_t, \theta_t \right] \right] = \left(\frac{\alpha_s}{\alpha_p} \right)^2 \tau_\pi^{-1} \quad \text{and} \quad \mathbb{V}\mathrm{ar}\left[\mathbb{E}\left[p_t \middle| \eta_t, \theta_t \right] \right] = \left(\frac{\alpha_s}{\alpha_p} \right)^2 \tau_\eta^{-1},$$

which allows us to characterize the relation between price volatility and price informativeness in Proposition 1 below. Intuitively, the variation in $\mathbb{E}\left[p_t \middle| \eta_t, \theta_t \right]$ is driven by the variance of the innovation to the asset payoff τ_η^{-1}, while the average residual variance is modulated by changes in price informativeness τ_π.

Proposition 1. (Volatility-informativeness relation: demand sensitivities formulation)
a) Price volatility $\mathcal{V}$ and price informativeness τ_π satisfy the following relation:

$$\mathcal{V} = \left(\frac{\alpha_s}{\alpha_p} \right)^2 \left(\tau_\eta^{-1} + \tau_\pi^{-1} \right). \tag{5}$$

b) The equilibrium elasticity of price volatility to price informativeness is given by

$$\frac{d \log \mathcal{V}}{d \log \tau_\pi} = \underbrace{2 \frac{d \log \left(\frac{\alpha_s}{\alpha_p} \right)}{d \log \left(\tau_\pi \right)}}_{\textit{equilibrium-learning}} - \underbrace{\frac{\tau_\pi^{-1}}{\tau_\eta^{-1} + \tau_\pi^{-1}}}_{\textit{noise-reduction}}. \tag{6}$$

We refer to Equation (5) as the *volatility-informativeness relation*. Since both volatility and informativeness are endogenous objects, the volatility-informativeness relation simply characterizes the locus over which price volatility and price informativeness will lie in equilibrium for a given set of parameters.

Part a) of Proposition 1 shows that this equilibrium relation features the exogenous primitive τ_η^{-1}, which corresponds to the variance of the innovation to the asset payoff, and the equilibrium object $\frac{\alpha_s}{\alpha_p}$, which we refer to as the signal-to-price sensitivity and depends on τ_π.[6] After accounting for the fact that $\frac{\alpha_s}{\alpha_p}$ is a function of τ_π and other primitives, we identify two distinct channels that determine the relation between price informativeness and volatility in part b) of Proposition 1.

We refer to the first channel as the *equilibrium-learning* channel. If a high level of price informativeness is associated with a high (low) level of the signal-to-price sensitivity $\frac{\alpha_s}{\alpha_p}$, this induces a positive (negative) relation between price informativeness and volatility. A high value of the signal-to-price sensitivity $\frac{\alpha_s}{\alpha_p}$ amplifies the sensitivity of asset prices to aggregate shocks.[7] Intuitively, a high $\frac{\alpha_s}{\alpha_p}$ implies that, on average, either investors react significantly to their private signals (high α_s), or that they have very steep — under the traditional economics convention that uses quantities in the horizontal axis — asset demand curves (low α_p), so investors barely adjust the quantity demanded even for large price changes, implying that equilibrium prices substantially react to the realization of the asset payoff. Alternatively, a low $\frac{\alpha_s}{\alpha_p}$ implies that, on average, investors barely react to their private signals (low α_s), or that they have very flat — under the traditional economics convention — asset demand curves (high α_p), so investors significantly adjust the quantity demanded even for small price changes, implying that equilibrium prices are barely responsive to the realization of aggregate payoff shocks.

We refer to the second channel as the *noise-reduction* channel. It is evident from Proposition 1 that, holding $\frac{\alpha_s}{\alpha_p}$ constant, a high level of τ_π is mechanically associated with a low level of $\mathcal{V}$. In fact, Equation (5) implies that there exists an inverse relation between both variables. Intuitively, when prices are very informative, the noise in the price is low and the conditional variance of the price for a given realization of the asset payoff is necessarily low.

It is worth highlighting that part b) of Proposition 1 is not a comparative statics exercise, but a characterization of a relation between two endogenous variables that must be satisfied in any equilibrium, given the economy's parameters. There are scenarios in which changes in some primitives do not shift the locus defined in Equation (5). In those cases, Equation (5) can be

[6] Note that we express Equation (6) using a total derivative and not a partial derivative. This notation accounts for the fact that $\frac{\alpha_s}{\alpha_p}$ is related in equilibrium to τ_π.

[7] Note that $\mathbb{V}\mathrm{ar}\left[p\middle|\,\theta_t\right] = \left(\frac{\alpha_s}{\alpha_p}\right)^2 \mathbb{V}\mathrm{ar}\left[\pi\middle|\,\theta_t\right]$, since the variance of the unbiased signal about the asset payoff can be expressed as $\mathbb{V}\mathrm{ar}\left[\pi\middle|\,\theta_t\right] = \tau_\eta^{-1} + \tau_\pi^{-1}$. We can thus interpret asset price volatility as the volatility of the unbiased signal about the asset payoff, corrected by investors' endogenous responses through the signal-to-price sensitivity.

interpreted as the possible combinations of $\mathcal{V}$ and τ_π that can arise in equilibrium for different values of those primitives. In those scenarios, Proposition 1 implies that equilibria with high volatility are also equilibria with high (low) price informativeness whenever $\frac{d \log \mathcal{V}}{d \log \tau_\pi} > 0\,(< 0)$. However, changes in parameters that shift the locus defined in Equation (5) entail a shift of the volatility-informativeness relation and, in general, also a movement along the curve. Therefore, it is necessary to determine how $\frac{\alpha_s}{\alpha_p}$ and τ_π are related in equilibrium as a function of the model's primitives to further understand the comovement between price informativeness and price volatility.

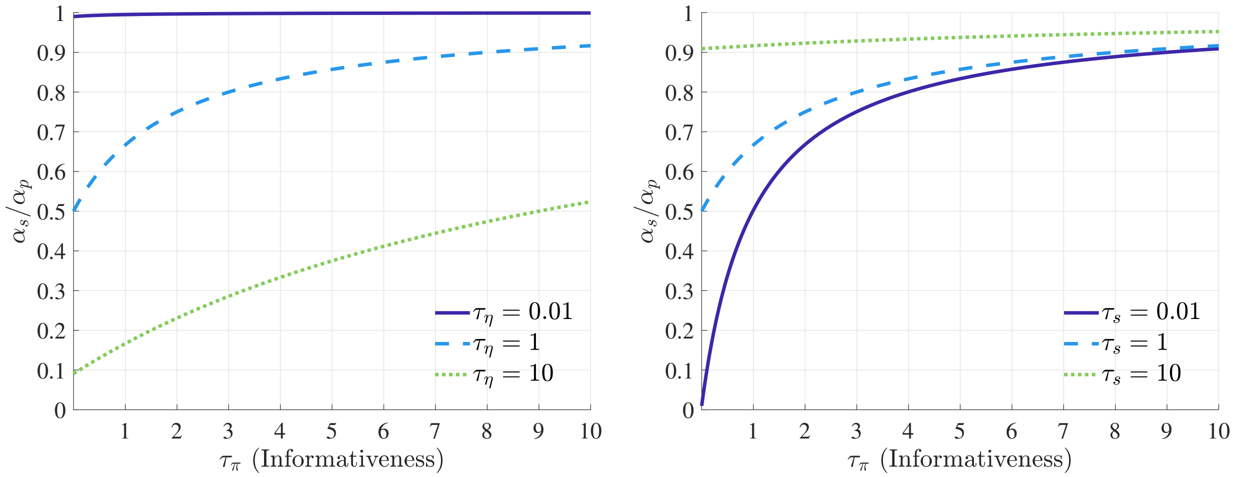
Lemma 2. (Signal-to-price sensitivity) *The signal-to-price sensitivity can be expressed as a function of price informativeness τ_π and primitives τ_s, τ_η, and ρ as follows:*

$$\frac{\alpha_s}{\alpha_p} = \frac{1}{1-\rho} \frac{\tau_s + \tau_\pi}{\tau_\eta + \tau_s + \tau_\pi}. \tag{7}$$

Given that investors have three sources of information about the asset payoff (their prior, their private signal, and the price signal), the signal-to-price sensitivity corresponds to the share of information acquired from the new signals at the disposal of investors, discounted by ρ. Therefore, high values of τ_s and τ_π are associated with high values of $\frac{\alpha_s}{\alpha_p}$, while high values of the prior precision τ_η are associated with low values of $\frac{\alpha_s}{\alpha_p}$. Similarly, if the process for the asset payoff is highly persistent (ρ is high), the signal-to-price sensitivity $\frac{\alpha_s}{\alpha_p}$ is high, since new information about the innovation η_t becomes more valuable.

It is useful to interpret $\frac{\alpha_s}{\alpha_p}$ as the sensitivity of the equilibrium price p_t to a change in the realization of the innovation to the asset payoff η_t, since $\frac{\partial p_t}{\partial \eta_t} = \frac{\alpha_s}{\alpha_p}$. Intuitively, a unit increase in the realization of η_t increases the value of the signals received by investors, increasing aggregate demand by α_s. This increase in aggregate demand increases the equilibrium price, which endogenously changes investors' demands, according to $\frac{1}{\alpha_p}$, for two reasons: i) a reduction in demand for purely pecuniary considerations, and ii) an increase in demand for informational reasons, since a higher price leads investors to infer that other investors received high signals about the asset payoff. Since substitution effects dominate in our model, the first effect always dominates in equilibrium, so that asset demands are downward sloping ($\alpha_p > 0$).

Figure 1a illustrates how the behavior of the signal-to-price sensitivity varies with the strength of the prior precision τ_η, for a given price informativeness τ_π. If the asset payoff is extremely volatile ($\tau_\eta \to 0$), investors exclusively rely on the signals about the asset payoff at their disposal, and $\frac{\alpha_s}{\alpha_p} \to 1$. Alternatively, if investors' prior information is extremely accurate ($\tau_\eta \to \infty$), investors exclusively rely on their prior information, so changes in the realization of η_t barely move at all the equilibrium price, and $\frac{\alpha_s}{\alpha_p} \to 0$. Intuitively, the more precise the prior information held by investors (higher τ_η), the less sensitive the asset price to the realization of η_t.



(a) Varying the prior precision about the innovation to the asset payoff, τ_η

(b) Varying the precision of investors' signals, τ_s

Figure 1: Understanding the signal-to-price sensitivity, $\frac{\alpha_s}{\alpha_p}$

Note: Figure 1 shows how the signal-to-price sensitivity $\frac{\alpha_s}{\alpha_p}$, characterized in Equation (7), varies as a function of price informativeness τ_π for different values of τ_η and τ_s, respectively, when $\rho = 0$. Figure 1a is computed using $\tau_s = 1$ and Figure 1b is computed using $\tau_\eta = 1$.

Figure 1b illustrates how the behavior of the signal-to-price sensitivity varies with the strength of the precision of investors' private signals τ_s, for a given price informativeness τ_π. If investors' signals are extremely precise ($\tau_s \to \infty$), investors trade one-for-one with their private signals, so $\frac{\alpha_s}{\alpha_p} \to 1$. Alternatively, if investors' signals are very inaccurate ($\tau_s \to 0$), investors exclusively rely on their prior information, so $\frac{\alpha_s}{\alpha_p} \to \frac{\tau_\pi}{\tau_\eta + \tau_\pi}$. For a given τ_s and τ_η, changes in τ_π have the same effect as changes in τ_s given τ_π.

Combining Proposition 1 with Lemma 2, we can express the volatility-informativeness relation in terms of model primitives.

Proposition 2. (Volatility-informativeness relation: model primitives formulation) *The volatility-informativeness relation can be expressed in terms of primitives as follows:*

$$\mathcal{V} = \left(\frac{1}{1-\rho} \frac{\tau_s + \tau_\pi}{\tau_\eta + \tau_s + \tau_\pi} \right)^2 \left(\tau_\eta^{-1} + \tau_\pi^{-1} \right). \tag{8}$$

Corollary 2 represents the endogenous relation between $\mathcal{V}$ and τ_π as a function of only two (combinations of) primitives: τ_η and τ_s, which allows us to explicitly characterize the properties of the volatility-informativeness relation. Note that the variance of the equilibrium price converges to the variance of the asset payoff when prices are infinitely informative. Alternatively, the equilibrium

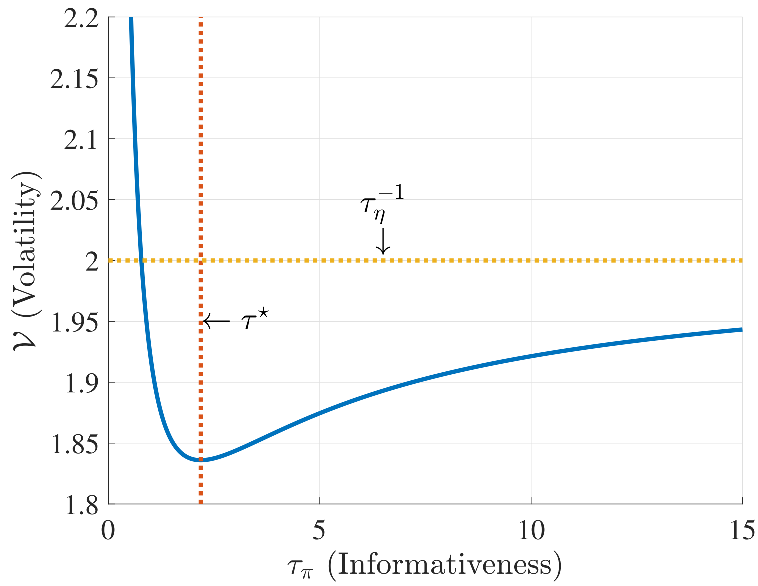


Figure 2: Volatility-informativeness relation

Note: Figure 2 plots price volatility as a function of price informativeness, as given by the volatility-informativeness relation in Equation (8), for parameters $\tau_\eta = 0.5$, $\tau_s = 1$, $\lambda = 1$, $\rho = 0$, and $R = 1.04$. The vertical red dotted line represents the threshold $\tau^\star$ that delimits the upward and downward-sloping regions of the volatility-informativeness relation. The horizontal yellow dotted line depicts the limit τ_η^{-1} to which the volatility-informativeness relation converges when prices are perfectly informative.

price is infinitely volatile when prices are totally uninformative. Formally,

$$\lim_{\tau_\pi \to \infty} \mathcal{V} = \left(\frac{1}{1-\rho}\right)^2 \tau_\eta^{-1} \quad \text{and} \quad \lim_{\tau_\pi \to 0} \mathcal{V} = \infty.$$

Note also that

$$\lim_{\tau_\pi \to 0} \frac{d\mathcal{V}}{d\tau_\pi} = -\infty \quad \text{and} \quad \lim_{\tau_\pi \to \infty} \frac{d\mathcal{V}}{d\tau_\pi} = 0.$$

Intuitively, for low levels of price informativeness, the noise-reduction channel dominates the equilibrium-learning channel, since learning is ineffective. When prices are infinitely informative, the noise-reduction channel and the equilibrium-learning channel perfectly cancel each other. Combining both sets of limits with the continuity of the relation, we conclude that the volatility-informativeness relation has an asymptote at $\tau_\pi = 0$ and that it converges smoothly towards $\left(\frac{1}{1-\rho}\right)^2 \tau_\eta^{-1}$ when prices are sufficiently informative.

We formally show that the volatility-informativeness relation is decreasing for sufficiently low values of τ_π and increasing for sufficiently high values of τ_π. The following proposition formalizes this nonmonotonicity.

Proposition 3. (Slope of volatility-informativeness relation) *The volatility-informativeness relation is increasing (decreasing) in τ_π if and only if price informativeness is high (low) enough.*

Formally, there exists a threshold $\tau^\star > 0$ *such that*

$$\frac{d\mathcal{V}}{d\tau_\pi} = \begin{cases} < 0, & \text{if } \tau_\pi < \tau^\star \\ > 0, & \text{if } \tau_\pi > \tau^\star \end{cases},$$

where the threshold $\tau^\star$ *is given by*

$$\tau^\star \equiv \frac{-(\tau_\eta - 2\tau_s) + \sqrt{\tau_\eta^2 + 8\tau_s^2}}{2}. \tag{9}$$

Proposition 3 shows that the slope of Equation (8) is positive when τ_π is sufficiently large and negative otherwise. The threshold $\tau^\star$, which determines the boundary between the positive and negative slope regions, only depends on the precision of the innovation to the asset payoff and the precision of the private signal. Exploiting our two-channel decomposition, we say that when prices are sufficiently informative, when $\tau_\pi > \tau^\star$, the equilibrium-learning channel dominates the noise-reduction channel. On the contrary, when $\tau_\pi < \tau^\star$, the noise-reduction channel dominates the equilibrium-learning channel. Figure 2 illustrates the shape of the volatility-informativeness relation in Equation (5) and the threshold $\tau^\star$.

Proposition 3 implies that any change among the subset of parameters that do not enter the volatility-informativeness relation directly must induce a positive comovement between price informativeness and volatility when prices are sufficiently informative and a negative comovement otherwise. When interpreted through the lens of our two-channel decomposition, when prices are sufficiently informative, the equilibrium-learning channel, which is driven by the change in investors' equilibrium behavior induced by learning, becomes overwhelmingly important and dominates the noise-reduction channel, and vice versa.

The slope of the volatility-informativeness relation is not enough to characterize how price informativeness and price volatility are related in equilibrium since there are parameters that move such relation as well as price informativeness. In the next section, we characterize comovement regions between price informativeness and price volatility in terms of primitives. Before doing so, it is worth including a remark on the notion of price informativeness considered in this section.

Remark 4. *(Notion of price informativeness)* The notion of price informativeness defined above in Equation (4) is exactly the precision of the price as a signal of the innovation to the asset payoff. This measure allows us to isolate the ability of prices to aggregate the information that is dispersed in the economy. A different variable that is at times used to measure the informational content of prices is the posterior variance of the future payoff conditional on the price (and the current payoff), given by $\mathbb{V}_P \equiv \mathbb{V}\text{ar}\left[\eta_t | p_t, \theta_t\right]$. The posterior variance $\mathbb{V}_P$ corresponds to the residual uncertainty about future payoffs after observing the price. When uncertainty is Gaussian, $\mathbb{V}_P$ and τ_π are related as follows.

$$\mathbb{V}_P = \frac{1}{\tau_\eta + \tau_\pi}. \tag{10}$$

From a theoretical perspective, the one-to-one mapping in Equation (10) implies that using τ_π or $\mathbb{V}_P$ to measure price informativeness will yield the same results. However, Equation (10) also illustrates the challenge faced by the posterior variance to empirically identify the precision of the price as a signal: $\mathbb{V}_P$ confounds the effect of uncertainty about future payoffs (τ_η^{-1}) with the precision of the price as a signal about future payoffs, τ_π. For example, $\mathbb{V}_P$ can be low because the payoff is not very volatile (high τ_η) or because asset prices give very precise information about future payoffs (high τ_π). Therefore, measuring informativeness using $\mathbb{V}_P$ is inadequate to capture the amount of information contained in asset prices, which is ultimately what we care about in this paper.

3.3 Comovement Regions

In our model, it is possible to explicitly compute price informativeness in terms of primitives, as follows:

$$\tau_\pi = \left(\frac{\tau_s}{\tau_\eta}\right)^2 \tau_n. \tag{11}$$

Hence, when combined with Equation (8), it becomes evident that different primitives may impact volatility and informativeness differently. In particular, without knowing which particular change in primitives drives changes in volatility and informativeness, it is in general not possible to know whether volatility and informativeness move in the same or opposite directions. In this section, we bypass this concern by characterizing regions in which price informativeness and price volatility positively or negatively comove for changes in *any* of the underlying model parameters.

First, in Proposition 4, we show that price informativeness is high (low) enough when price informativeness and price volatility positively (negatively) comove after any parameter change, including those that appear in the volatility-informativeness relation.

Proposition 4. (Comovement regions: price informativeness formulation) *There exist thresholds $\tau^\star > 0$ and $\underline{\tau} \in [0, \tau^\star]$ such that*

a) [Positive comovement region] If $\tau_\pi > \tau^\star$, price volatility $\mathcal{V}$ and price informativeness τ_π positively comove across equilibria after any parameter change.

b) [Negative comovement region] If $\tau_\pi < \underline{\tau}$, price volatility $\mathcal{V}$ and price informativeness τ_π negatively comove across equilibria after any parameter change.

c) [Ambiguous comovement region] If $\tau_\pi \in [\underline{\tau}, \tau^\star]$, price volatility $\mathcal{V}$ and price informativeness τ_π may comove positively or negatively across equilibria, depending on which parameter changes.

Proposition 4 identifies three regions depending on the comovement between price informativeness and price volatility: a positive comovement region, a negative comovement region, and an ambiguous comovement region. In the positive comovement region price informativeness is above the threshold $\tau^\star$ above which the volatility-informative relation is upward sloping. Intuitively, why is there positive comovement when prices are very informative? For instance, an increase in the precision of investors' private signals about the asset payoff shifts the volatility-informativeness relation upwards because investors are more responsive to their information for any level of price informativeness, as we describe above when explaining Lemma 2. As expected, price informativeness also increases when investors receive more precise signals. When the equilibrium-learning channel dominates, the upward shift in the volatility-informativeness relation and the increase in price informativeness guarantee an increase in price volatility, which yields the positive comovement described above. Alternatively, an increase in the precision of investors' priors about the fundamental shifts the volatility-informativeness relation downwards, since investors are less responsive to their information. As expected, price informativeness decreases when investors rely more on their priors. When the equilibrium-learning channel dominates, the downward shift in the volatility-informativeness relation and the decrease in price informativeness guarantee a decrease in volatility, which yields again positive comovement.

In the negative comovement region, price informativeness is low and below the threshold $\underline{\tau} < \tau^\star$. In this case, the volatility-informativeness relation is downward sloping. In principle, this implies that the comovement between price volatility and price informativeness is ambiguous since variables that increase price informativeness also shift the volatility-informativeness relation upwards, and vice-versa. However, exploiting the fact, illustrated in Figure 2, that the volatility-informativeness relation has an asymptote at $\tau_\pi = 0$, it is possible to show that there exists a region of the parameter space such that volatility and informativeness comove negatively for any parameter changes. Intuitively, when informativeness is sufficiently low, the noise-reduction channel dominates for any change in primitives. Finally, in the ambiguous comovement region, whether price volatility and price informativeness comove positively or negatively depends on the underlying parameter that is leading to the change.

The characterization in Proposition 4 in terms of price informativeness is useful to understand the comovement regions through the equilibrium-learning and noise-reduction channels. Next, we show in Proposition 5 that it is possible to characterize whether a given asset is in the positive, negative, or ambiguous comovement region as a function of a subset of model primitives. These results lay the foundation for our empirical analysis in Section 4.

Proposition 5. (Comovement regions: model primitives formulation)

a) [Positive comovement region] Price volatility $\mathcal{V}$ and price informativeness τ_π positively comove *in response to any parameter change if*

$$\frac{\tau_n}{\tau_\eta} > \frac{\sqrt{1+8\frac{\tau_s}{\tau_\eta}} - 1 + 2\frac{\tau_s}{\tau_\eta}}{2\left(\frac{\tau_s}{\tau_\eta}\right)^2}. \tag{12}$$

b) [Negative comovement region] Price volatility $\mathcal{V}$ *and price informativeness* τ_π *negatively comove in response to any parameter change if*

$$\frac{\tau_n}{\tau_\eta} < \frac{\frac{\tau_s}{\tau_\eta} - 2 + \sqrt{\left(2-\frac{\tau_s}{\tau_\eta}\right)^2 + 8\left(\frac{\tau_s}{\tau_\eta}\right)^2}}{4\left(\frac{\tau_s}{\tau_\eta}\right)^2}. \tag{13}$$

c) [Ambiguous comovement region] Whenever neither of the inequalities in Equations (12) and (13) are satisfied, price volatility $\mathcal{V}$ *and price informativeness* τ_π *may comove positively or negatively, depending on which parameter changes.*

Equations (12) and (13) explicitly characterize conditions on model primitives that guarantee a positive or negative comovement between price volatility and price informativeness in response to any change in model primitives. Interestingly, these conditions can be expressed exclusively in terms of two ratios of precisions, which are scale-invariant. In particular, both Equations (12) and (13) remain valid regardless of the values of investors' risk aversion γ, the dispersion of investors' heterogeneous beliefs τ_u, or the supply of the risky asset Q. When either $\frac{\tau_s}{\tau_\eta}$, which measures the ratio of the precision of private information to the precision of investors' prior information, or $\frac{\tau_n}{\tau_\eta}$, which measures the relative volatility of the innovation to the asset payoff relative to the volatility of sentiment, is sufficiently large, the economy features positive comovement. In these cases, either due to investors receiving precise signals or to aggregate noise being low, price informativeness is high enough to guarantee that the economy is in the positive comovement region. When Equation (13) is satisfied, either $\frac{\tau_s}{\tau_\eta}$ or $\frac{\tau_n}{\tau_\eta}$ is small enough such that price informativeness is small enough to guarantee that the economy is in the negative comovement region.

Figure 3 graphically illustrates the combinations of signal-to-payoff $\frac{\tau_s}{\tau_\eta}$ nad noise-to-payoff $\frac{\tau_n}{\tau_\eta}$ precisions that delimit the positive and negative comovement regions. In terms of magnitudes, our model implies that when investors' private signals and priors about the innovation are of equal precision, i.e., $\frac{\tau_s}{\tau_\eta} = 1$, volatility and informativeness positively comove whenever the variance of the aggregate component of beliefs is less than one half of the variance of the innovation to the asset payoff ($\frac{1}{\tau_n} < \frac{1}{2}\frac{1}{\tau_\eta}$) and negatively comove whenever the variance of the aggregate component of beliefs is greater than twice the variance of the innovation to the asset payoff ($\frac{1}{\tau_n} > 2\frac{1}{\tau_\eta}$), regardless of the value of the remaining parameters of the model. Moreover, our model implies that when the variance of the innovation to the asset payoff and the variance of the aggregate component

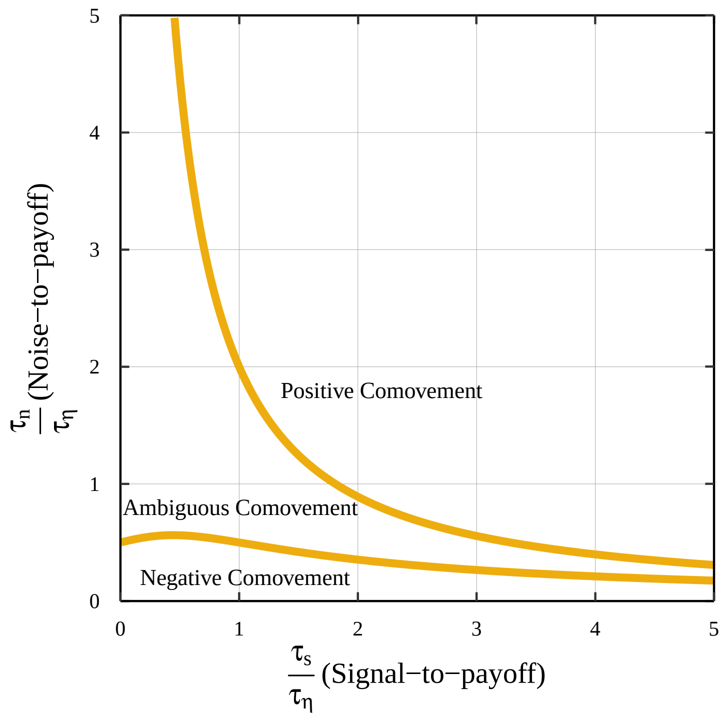


Figure 3: Comovement regions

Note: Figure 3 shows the combination of ratios of primitives $\frac{\tau_n}{\tau_\eta}$ and $\frac{\tau_s}{\tau_\eta}$ that are consistent with an economy that is in the positive, negative, or ambiguous comovement region, as defined in Proposition 5. The highest solid yellow line determines the lower bound of the positive comovement region. The lowest solid yellow line represents the upper bound of the negative comovement region. The region in between both solid yellow lines determines the ambiguous comovement region.

of beliefs are of equal magnitude, i.e., $\frac{\tau_n}{\tau_\eta} = 1$, volatility and informativeness positively comove provided that the precision of investors' private signals is greater than 2.2 times the precision of their prior about the innovation to the asset payoff ($\tau_s > 2.2\tau_\eta$). Figure 3 illustrates all remaining possible combinations.

Finding an unambiguously positive or negative comovement between both variables for any change in primitives, even for specific regions of the parameter space, may come as a surprise, since the prior about the sign of the relationship should not be obvious ex-ante. It is worth highlighting that our results apply to comparative static exercises that are valid for changes in any of the underlying model parameters, including those that appear in the volatility-informativeness relation. Figure 4 shows the comparative statics for τ_π and $\mathcal{V}$ as a function of the five primitives of the model: τ_s, τ_η, τ_n, τ_u, and γ. Interestingly, both price volatility and informativeness are independent of investors' risk aversion, γ, and of the dispersion in investors' priors, τ_u, although there are other equilibrium variables that do depend on γ or τ_u, for example, the risk premium and trading volume.

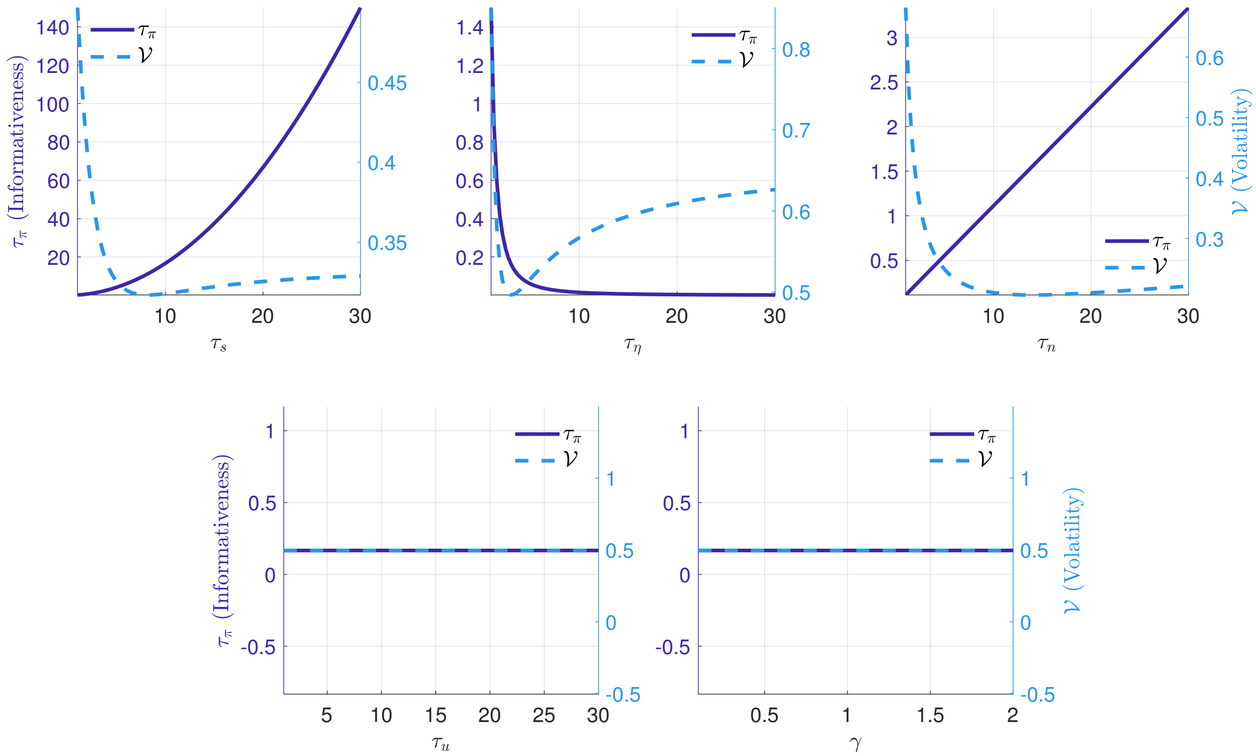


Figure 4: Comparative statics

Note: Figure 4 shows comparative statics of price informativeness τ_π and price volatility $\mathcal{V} = \mathbb{V}ar\left[p_t | \theta_t\right]$ as a function of all five primitives of the model considered. All plots feature two y-axes: the left y-axis corresponds to the values of τ_π, while the right y-axis corresponds to the values of $\mathcal{V} = \mathbb{V}ar\left[p_t | \theta_t\right]$. The parameters of this model are the following: τ_s, precision of private signals about the innovation to the asset payoff; τ_η, precision of the innovation to the asset payoff; τ_n, precision of the average prior; τ_u, precision of investors' dispersion of heterogeneous beliefs; and γ, investors' coefficient of absolute risk aversion. The reference values are $\tau_s = 1$, $\tau_\eta = 3$, $\tau_n = 1.5$, $\tau_u = 1$, and $\gamma = 0.5$.

4 Comovement Score: Definition, Measurement, and Validation

In this section, building on our theoretical results, we develop an empirical framework to analyze the relation between volatility and informativeness. First, we introduce the notion of comovement score. The comovement score is a statistic that measures the relative distance of a given asset to the positive/negative comovement regions. This measure is useful because, in practice, many stocks will fall in the ambiguous comovement region characterized in Proposition 5. Next, we go on to empirically recover stock-specific estimates of the comovement score for the cross-section of US stocks over time. We find that roughly half of our observations fall in the ambiguous region, with comovement scores between 0 and 1. Empirically, we find that comovement scores have trended downwards in recent decades, correlate positively to value and idiosyncratic volatility, and negatively to size and institutional ownership. Finally, we show that stocks with higher comovement scores are more likely to experience positive comovement, which is consistent with the theoretical framework developed in the paper.

Going forward, we use the index j to denote a given individual stock and $\boldsymbol{\tau}^j$ to denote the signal-to-payoff/noise-to-payoff pair of a given stock j, formally given by $\boldsymbol{\tau}^j = \left(\frac{\tau_s^j}{\tau_\eta^j}, \frac{\tau_n^j}{\tau_\eta^j}\right)$.

4.1 Comovement Score: Definition

In Proposition 5, we show that, depending on primitives, a given stock j can be in the positive comovement region, the negative comovement region, or the ambiguous comovement region. In practice, as we will show shortly, many stocks will be assigned to the ambiguous comovement region. Hence, in order to define a continuous measure of (the likelihood of) comovement between volatility and informativeness, we introduce the "comovement score". The definition of the comovement score is meant to capture the idea that stocks closer to the positive (negative) comovement region are more likely to experience positive (negative) comovement between volatility and informativeness.

Definition. (Comovement score) *The comovement score for a given asset j, $\mathcal{S}(j) \in [0,1]$, is defined as the relative distance in the space of signal-to-payoff and noise-to-payoff primitives from the positive comovement region. Formally,*

$$\mathcal{S}(j) = \begin{cases} 0, & \text{if } \boldsymbol{\tau}^j \in \mathcal{C}_- \\ \frac{d_-(j)}{d_+(j)+d_-(j)}, & \text{if } \boldsymbol{\tau}^j \notin \mathcal{C}_- \cup \mathcal{C}_+ \\ 1, & \text{if } \boldsymbol{\tau}^j \in \mathcal{C}_+ \end{cases},$$

where $d_+(j)$ and $d_-(j)$ respectively denote the minimum distances from the positive and negative comovement regions, given by

$$d_+(j) = \min_{\boldsymbol{\tau}} \left\{ d\left(\boldsymbol{\tau}^j; \boldsymbol{\tau}\right) : \boldsymbol{\tau} \in \partial_+ \right\} \quad \text{and} \quad d_-(j) = \min_{\boldsymbol{\tau}} \left\{ d\left(\boldsymbol{\tau}^j; \boldsymbol{\tau}\right) : \boldsymbol{\tau} \in \partial_- \right\},$$

where $d(\boldsymbol{\tau}^j; \boldsymbol{\tau})$ denotes the Euclidean distance in the noise-to-payoff/signal-to-payoff space, that is,

$$d\left(\boldsymbol{\tau}^j; \boldsymbol{\tau}'\right) = \sqrt{\left(\frac{\tau_s^j}{\tau_\eta^j} - \frac{\tau_s'}{\tau_\eta'}\right)^2 + \left(\frac{\tau_n^j}{\tau_\eta^j} - \frac{\tau_n'}{\tau_\eta'}\right)^2},$$

and where ∂_+ and ∂_- denote the boundaries of the positive and negative comovement regions, characterized in Proposition 5, and respectively, given by

$$\partial_+ = \left\{ \boldsymbol{\tau} : \frac{\tau_n}{\tau_\eta} = \frac{\sqrt{1+8\frac{\tau_s}{\tau_\eta}} - 1 + 2\frac{\tau_s}{\tau_\eta}}{2\left(\frac{\tau_s}{\tau_\eta}\right)^2} \right\} \quad \text{and} \quad \partial_- = \left\{ \boldsymbol{\tau} : \frac{\tau_n}{\tau_\eta} = \frac{\frac{\tau_s}{\tau_\eta} - 2 + \sqrt{\left(2 - \frac{\tau_s}{\tau_\eta}\right)^2 + 8\left(\frac{\tau_s}{\tau_\eta}\right)^2}}{4\left(\frac{\tau_s}{\tau_\eta}\right)^2} \right\}.$$

Conceptually, it is straightforward to compute the comovement score. First, if given primitives, a stock is assigned to the positive comovement region, then $\mathcal{S}(j) = 1$; or $\mathcal{S}(j) = 0$ if assigned to the negative comovement region. For those stocks in the ambiguous comovement region, one computes the minimum euclidean distance to the boundaries of the positive and negative comovement regions. Finally, the comovement score simply corresponds to the relative distance from the positive comovement region. That is, when the distance to the negative comovement region $d_-(j)$ is large relative to $d_+(j) + d_-(j)$, then a given asset j is closer in relative terms to the positive comovement region, and $\mathcal{S}(j) \to 1$. Alternatively, when the distance to the positive comovement region $d_+(j)$ is large relative to $d_+(j) + d_-(j)$, then a given asset j is closer in relative terms to the negative comovement region, so $\mathcal{S}(j) \to 0$. At times, it is helpful to report the comovement score in the form of log-odds, which we denote by $\mathcal{O}(j) \equiv \ln\left(\frac{\mathcal{S}(j)}{1-\mathcal{S}(j)}\right)$ and refer to as "comovement log-odds".[8]

4.2 Comovement Score: Measurement

Next, we show how to empirically measure stock-specific comovement scores. Since modeling log-payoffs as difference-stationary is often perceived as a better assumption when dealing with actual data (see e.g., Campbell (2017)), in Proposition 6 we show how to recover the stock-specific primitives necessary to compute comovement scores in a reformulation of our baseline model that is difference-stationary in logs once linearized, formally described in Section G of the Online Appendix. More specifically, in this reformulation of the model, the (log) payoff θ_t follows a unit-root process given by

$$\Delta\theta_{t+1} = \mu_{\Delta\theta} + \eta_t, \tag{14}$$

where $\Delta\theta_{t+1} = \theta_{t+1} - \theta_t$, and $\mu_{\Delta\theta}$ is a scalar. As in the baseline model, investors receive linear signals about the innovation η_t, while the realized payoff θ_t is common knowledge to all investors before the (log) price p_t is determined and that the realized (log) payoff at date $t+1$, θ_{t+1}, is only revealed to investors at date $t+1$.

There are two steps to measure the comovement score for a stock j. First, one must recover the stock's noise-to-payoff/signal-to-payoff pair. Second, one needs to compute the minimum distance between the estimated primitives and the comovement boundaries as defined in Proposition 5.

Proposition 6. (Comovement score estimation) *Let $\overline{\beta}$, β_0, and β_1 denote the coefficients of the following regression of log asset price changes, denoted by Δp_t, on changes on log asset payoffs,*

[8] Log-odds/logit transformations are common in models that deal with probabilities/likelihoods — see e.g., Greene (2003). In our case, working with comovement log-odds enhances the variation of comovement scores in the ambiguous comovement region.

denoted by $\Delta\theta_t$:

$$\Delta p_t = \overline{\beta} + \beta_0 \Delta\theta_t + \beta_1 \Delta\theta_{t+1} + e_t, \tag{R1}$$

where $\Delta p_t \equiv p_t - p_{t-1}$ denotes the change in the ex-dividend log-price at date t and θ_t denotes the log asset-payoff realized at date t. Let $\overline{\zeta}$ and ζ_0 denote the coefficients of the following regression of log price changes on changes on lagged log asset payoffs:

$$\Delta p_t = \overline{\zeta} + \zeta_0 \Delta\theta_t + e_t^{\zeta}. \tag{R2}$$

Let $R^2_{\Delta\theta_{t+1},\Delta\theta_t}$ and $R^2_{\Delta\theta_t}$ be the R-squareds of Regressions R1 and R2, respectively.

a) **(Recovering stock-specific noise-to-payoff/signal-to-payoff pair)** *Then, it is possible to find consistent estimates of $\frac{\tau_s}{\tau_\eta}$ and $\frac{\tau_n}{\tau_\eta}$ as follows:*

$$\frac{\tau_s}{\tau_\eta} = \frac{\beta_1}{1-\beta_1} - \frac{R^2_{\Delta\theta_{t+1},\Delta\theta_t} - R^2_{\Delta\theta_t}}{1 - R^2_{\Delta\theta_{t+1},\Delta\theta_t}} \tag{15}$$

$$\frac{\tau_n}{\tau_\eta} = \left(\frac{\beta_1}{1-\beta_1} - \frac{R^2_{\Delta\theta_{t+1},\Delta\theta_t} - R^2_{\Delta\theta_t}}{1 - R^2_{\Delta\theta_{t+1},\Delta\theta_t}}\right)^{-2} \frac{R^2_{\Delta\theta_{t+1},\Delta\theta_t} - R^2_{\Delta\theta_t}}{1 - R^2_{\Delta\theta_{t+1},\Delta\theta_t}}. \tag{16}$$

b) **(Recovering stock-specific comovement scores)** *Combining Equations (15) and (16) with the definition of the comovement score in Proposition 5, it is possible to recover stock-specific comovement scores.*

Proposition 6 shows that it is possible to recover the noise-to-payoff/signal-to-payoff ratios $\frac{\tau_s}{\tau_\eta}$ and $\frac{\tau_n}{\tau_\eta}$ that allow us to assign a given stock to a comovement region by using regressions of price changes on changes on asset payoffs. In practice, recovering both ratios simply requires using the coefficient β_1 from Regression R1, and the R-squareds from both regressions. We describe the data used to compute estimates of these variables next. Combining Equations (15) and (16) with the definition of the comovement score in Proposition 5, it is possible to recover stock-specific comovement scores.

Data Description/Estimation. We provide here a brief description of the data and the sample selection procedure. The companion R notebooks include a more detailed description. We obtain information on stock prices and earnings (our payoff measure) from the CRSP/Compustat dataset, as distributed by WRDS. Our sample selection procedure follows the conventional approach described in Bali, Engle and Murray (2016). We conduct our analysis using quarterly data, available from 1961 to 2017. To match the timing of our model and to ensure that the accounting data were public on the trading date, we merge the Compustat data with CRSP data three months ahead, although our findings are robust to using alternative windows. We use the personal consumption

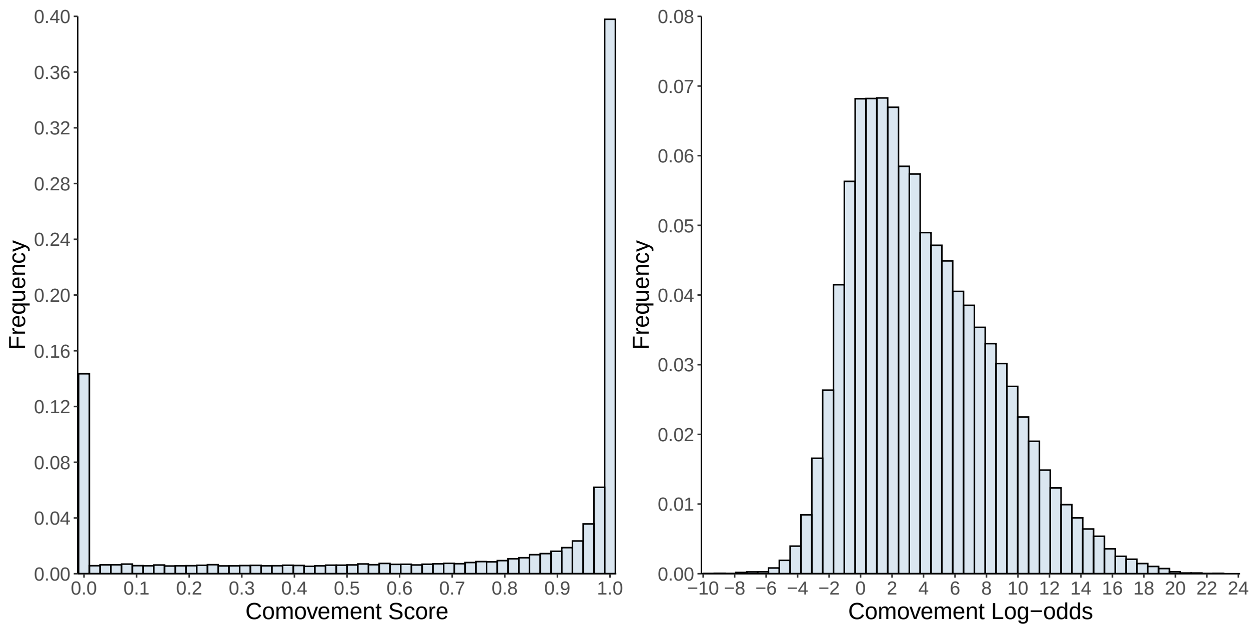


Figure 5: Relative-frequency histograms

Note: Figure 5 presents relative-frequency histograms of the estimated comovement scores. The left panel of Figure 5 shows the distribution of comovement scores for all periods and stocks. The right panel of Figure 5 shows the associated distribution of comovement log-odds. Note that comovement scores are estimated for rolling windows of 40-quarters using data between 1961 and 2017. Comovement log-odds, $\mathcal{O}(j)$, are computed from comovement scores, $\mathcal{S}(j)$, as follows: $\mathcal{O}(j) \equiv \ln\left(\frac{\mathcal{S}(j)}{1-\mathcal{S}(j)}\right)$. By construction, log-odds are not defined when $\mathcal{S}(j)$ is 0 or 1.

expenditure index (PCEPI), obtained from FRED, to deflate all nominal variables.

We implement Proposition 6 by running time-series regressions for each individual stock — indexed by j here — over rolling windows of 40 quarters. We denote by p_t^j the log price of stock j, adjusted for splits. We use earnings — as measured by EBIT — as the relevant measure of payoffs, since stock-level measures of dividends are problematic for various reasons.[9] We winsorize payoff and price values at the 2.5th and 97.5th percentile to reduce the impact of outliers. Since earnings can be negative, we compute $\Delta\theta_t^j$ directly as a growth rate as follows: when the lagged payoff is positive, the growth rate is defined as ${}^{\text{payoff}}/{}_{\text{payoff}_{t-1}} - 1$. When the lagged payoff is negative, the growth rate is defined as ${}^{\text{payoff}_t}/{}_{|\text{payoff}_{t-1}|} + 1$. We disregard the observations (less than 0.1%) for which the lagged payoff is exactly zero. Formally, in a given rolling window, we run time-series regressions of the form

$$\Delta p_t^j = \overline{\beta}^j + \beta_0^j \Delta\theta_t^j + \beta_1^j \Delta\theta_{t+1}^j + d_t^{j,q} + \varepsilon_t^j \quad \Rightarrow \quad R^{2,j}_{\Delta\theta_{t+1},\Delta\theta_t} \tag{17}$$

$$\Delta p_t^j = \overline{\zeta}^j + \zeta_0^j \Delta\theta_t^j \qquad\qquad\qquad + d_t^{j,q} + \hat{\varepsilon}_t^j \quad \Rightarrow \quad R^{2,j}_{\Delta\theta_t}\ , \tag{18}$$

where Δp_t^j is a measure of capital gains, $\Delta\theta_t^j$ and its one-period ahead counterpart $\Delta\theta_{t+1}^j$ are measures of earnings growth, and $d_t^{j,q}$ denote stock-specific quarterly dummies. The introduction of $d_t^{j,q}$ accounts for seasonality patterns. We estimate the regression coefficients and errors using OLS and recover β_1^j, $R^{2,j}_{\Delta\theta_{t+1},\Delta\theta_t}$, and $R^{2,j}_{\Delta\theta_t}$, which can be combined according to Proposition 6 to find estimates of $\frac{\tau_s^j}{\tau_\eta^j}$ and $\frac{\tau_n^j}{\tau_\eta^j}$ per stock in each rolling window. Consistently with the theory, we restrict our attention to stocks for which the estimates of β_1, $\frac{\tau_s}{\tau_\eta}$, and $\frac{\tau_n}{\tau_\eta}$ are strictly positive, with contiguous observations, and whose maximum leverage score across observations is lower than 0.95.[10]

Empirical Findings. Figure 5 presents relative-frequency histograms of the estimated comovement scores. The left panel of Figure 5 shows the actual estimated comovement scores for the whole sample. The right panel of Figure 5 shows the implied comovement log-odds, which are computed directly from the comovement scores. By considering comovement log-odds we are able to better illustrate that there is a well-behaved distribution of comovement scores in the ambiguous comovement region, in which $\mathcal{S}(j) \in (0,1)$.

We find that roughly 14% of the estimated comovement scores are in or near the negative comovement region ($\mathcal{S}(j) \approx 0$). For these stocks, increases in volatility can be interpreted as decreases in informativeness. We also find that roughly 40% of the comovement scores are in or near the positive comovement region ($\mathcal{S}(j) \approx 1$). In these cases, increases in volatility reflect increases

[9] In the log-difference specification, it is equivalent to use EBIT or EBIT-per-share as a measure of payoffs.

[10] For the study of the relation between comovement scores and measured positive comovement in the next subsection we discard observations with implausibly large estimates of informativeness.

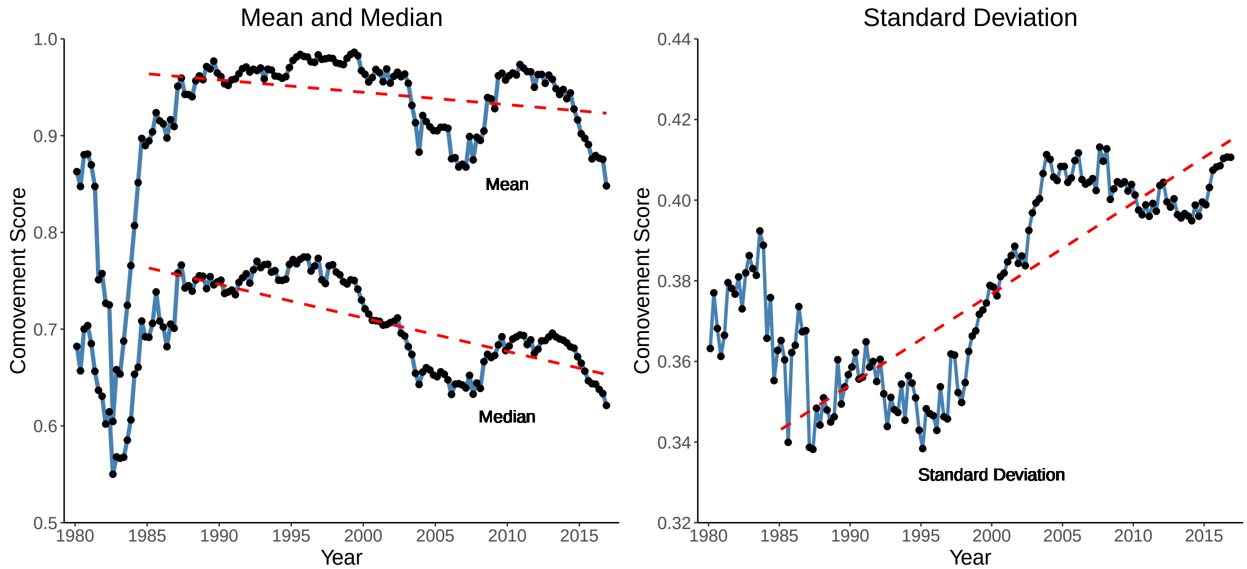


Figure 6: Comovement scores over time: aggregate findings

Note: The left panel of Figure 6 shows the time-series evolution of the cross-sectional mean and median comovement scores. The right panel of Figure 6 shows the time-series evolution of the the cross-sectional standard deviation of comovement scores. The red dashed lines show linear trends starting in 1985. In both panels, the dots correspond to the average within a quarter of the comovement scores computed using quarterly data.

in informativeness through the lens of our analysis. For the remaining stocks in our sample, which lie in the ambiguous comovement region, whether one should feel more or less comfortable inferring informativeness from volatility depends on the stock's comovement score. More specifically, more extreme comovement scores, either closer to 0 or closer to 1, are more likely to allow us to make inferences about price informativeness from price volatility. Overall, we find that 71% of estimated scores are above 0.5 while 29% are lower or equal than 0.5. In what follows, we describe in detail the time series evolution of comovement scores in the aggregate and depending on cross-sectional characteristics.

Comovement Scores over Time: Aggregate Results. An advantage of computing time-varying stock-specific comovement scores is that we are able to show how the distribution of comovement scores evolves over time. Tables OA-1 and OA-2 in the Appendix include detail information on the time evolution of the distribution of comovement scores. To better illustrate the results, we show the behavior of the median, mean, and standard deviation of the cross-sectional distribution of informativeness between 1980 and 2017 graphically in Figure 6.

We find that both the mean and median of the distribution of comovement scores have steadily decreased since the mid-1980s. The median moved from roughly 0.75 to roughly 0.65 between 1985 and 2017, while the mean experienced a similar decrease from being close to 1 to roughly 0.85. The quickly varying statistics from the earlier dates of the sample are due to the smaller sample sizes

Table 1: Cross-sectional results

	Estimate	Std. Error	t-stat
Size	-0.05762	0.00135	-42.62
Value	0.15424	0.00659	23.42
Turnover	0.00092	0.00025	3.63
Idiosyncratic Volatility	1.91267	0.06569	29.12
Institutional Ownership	-0.31201	0.01241	-25.13

Note: Table 1 reports the estimates ($\widehat{a_1^c}$) of panel regressions of comovement scores on cross-sectional characteristics (in twentiles) with year fixed effects (ξ_t): $\mathcal{S}_t^b = a_0^c + a_1^c c_t^b + \xi_t + \epsilon_t^b$, where $\mathcal{S}_t^b$ denotes the average comovement score bin (twentile) in a given period, c_t^b denotes the value of the given characteristic per bin (twentile) in a given period, ξ_t denotes a year fixed effect, a_0^c and a_1^c are parameters, and ϵ_b^t is an error term. Figures OA-3 through OA-7 in the Online Appendix provide the graphical counterpart of the results in this table. Size is measured as the natural log of stock market capitalization, value is measured as the ratio between a stock's book value and its market capitalization, turnover is measured as the ratio between trading volume and shares outstanding, idiosyncratic volatility is measured as the standard deviation — over a 30 month period — of the difference between the returns of a stock and the market return, and institutional ownership is measured as the proportion of a stock held by institutional investors.

in those periods. For that reason, we emphasize the steady decrease that starts in the mid-1980s.

Through the lens of our framework, these results imply that volatility and informativeness are less likely to positively comove in recent times. Moreover, the results on cross-sectional volatility of the comovement score estimates imply that there is in principle higher variation in the cross-section in terms of the volatility-informativeness relation. This fact supports the idea that making inferences about informativeness from volatility is not straightforward, and needs to be studied on a case-by-case basis.

Comovement Scores: Cross-sectional Results. By computing stock-specific comovement scores, we are able to establish a new set of cross-sectional patterns relating comovement scores and stock characteristics. We focus on five stock characteristics that have been widely used to explain patterns in the cross section of stock returns — see, e.g., Bali, Engle and Murray (2016). These are i) size, measured as the natural log of stocks market capitalization; ii) value, measured as the ratio between a stock's book value and its market capitalization; iii) turnover, measured as the ratio between trading volume and shares outstanding; iv) idiosyncratic volatility, measured as the standard deviation — over a 30 month period — of the difference between the returns of a stock and the market return; and v) institutional ownership, measured as the proportion of shares held by institutional investors.

In Table 1, we report the estimates of panel regressions of comovement scores (in twentiles) on each of the five explanatory variables, using year fixed effects. The coefficients that we report can be interpreted as a weighted average of the slopes of running year-by-year regressions of comovement scores on a given explanatory variable (size, value, turnover, return volatility,

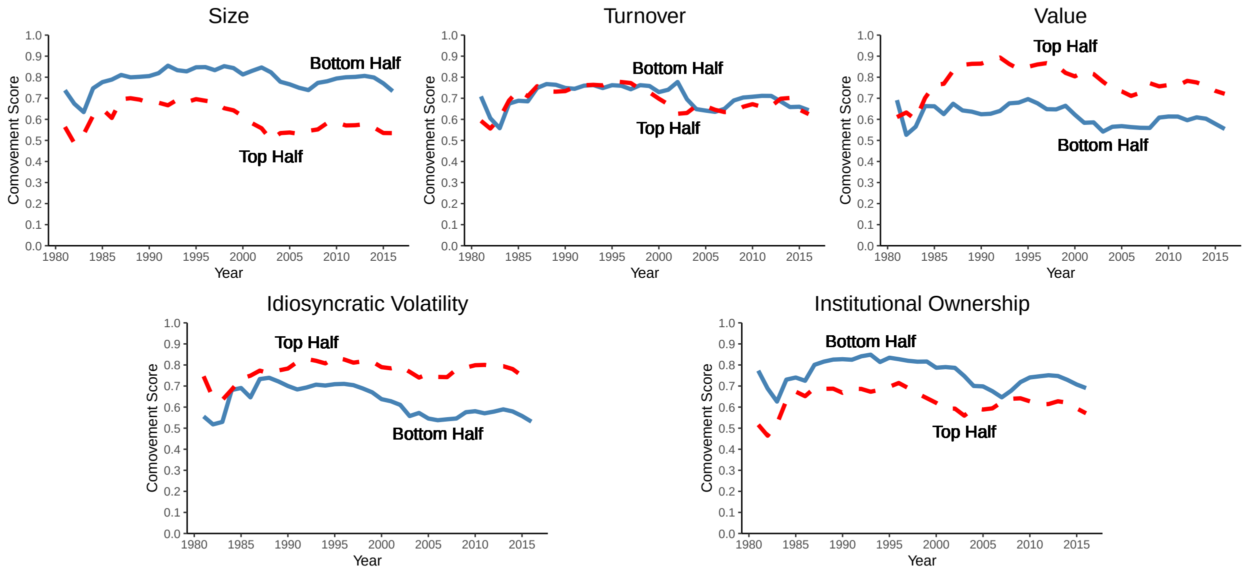


Figure 7: Comovement scores: cross-sectional results

Note: Each of the panels in Figure 7 shows the time-series evolution of the mean comovement score for the top and bottom of the distribution of comovement scores for each characteristic (size, turnover, value, idiosyncratic volatility, and institutional ownership). The red dashed lines correspond to the top half of the distribution, while the solid blue lines correspond to the bottom half of the distribution. In this figure, stocks are assigned to each half every quarter and observations are aggregated at the annual level.

institutional ownership). Figures OA-3 through OA-7 in the Online Appendix provide an alternative graphical illustration of our results. These figures show that the cross-sectional relations identified in Table 2 are stable over time.

Our cross-sectional analysis yields several robust patterns. First, we find a strong negative cross-sectional relation between a stock's size (market capitalization) and comovement scores; that is, large stocks have lower comovement scores informativeness. Second, we find a strong positive cross-sectional relation between a stock's book-to-market ratio and comovement scores; that is, value stocks have higher comovement scores. Third, we find a strong positive cross-sectional relation between a stock's turnover and comovement scores; that is, stocks that trade frequently have higher comovement scores. Fourth, we find a positive and strong cross-sectional relation between a stock's idiosyncratic return volatility and comovement scores; that is, stocks whose returns are more volatile have higher comovement scores. Finally, we find a strong negative cross-sectional relation between a stock's institutional ownership share and comovement scores, that is, stocks owned mostly by institutional investors have lower comovement scores.

Comovement Scores over Time: Cross-sectional Results. Finally, we look at the evolution of the distribution of comovement scores over time across the five stock characteristics described above: size, turnover, value, idiosyncratic volatility, and institutional ownership. Figure 7 shows the average and the standard deviation of the top and bottom halves of the distribution of comovement scores along each of the five characteristics. We find that the gap in comovement scores among top and bottom halves of the distributions of different characteristics is rather stable across time, only slightly widening for size and idiosyncratic volatility.

It is important to highlight that comovement score and price informativeness are completely different notions, as we explain in the following remark.

Remark 5. **(Comovement score vs. price informativeness)** The comovement score is useful to learn about the likelihood of an increase in a stock's price volatility reflecting an increase in price informativeness. For stocks with extreme values of the comovement score, it is justified to use price volatility to make inferences about price informativeness. However, the comovement score is not a measure of the level of price informativeness. High comovement scores can be associated with high or low informativeness depending on the primitives of the economy, as can be seen clearly from Figure 2.

4.3 Comovement Score: Validation

Finally, we proceed to determine whether a central relation implied by our model, that is, that stocks with higher comovement scores are more likely to experience positive comovement between volatility and informativeness, holds empirically. To do this, we construct an empirical measure of comovement between estimates of changes in volatility and informativeness one quarter ahead for each of the estimated comovement scores and then test whether higher comovement scores are associated with a stronger measured positive comovement between changes in volatility and informativeness.

For each stock/rolling window for which we have computed a comovement score, we first estimate idiosyncratic return volatility for that rolling window and one quarter ahead using daily data and then recover the estimates of price informativeness for the exact same time periods. To do so, we rely on Equation (2) to provide a structural interpretation to Regression R1, from which it follows that price informativeness can be consistently recovered as $\frac{\beta_1^2}{\mathbb{V}ar[e_t]}$. With the estimates of volatility and informativeness at hand, we define, for each stock/window, a positive comovement variable that takes the value of 1 when the product of the one period ahead changes in volatility and informativeness has positive sign and the value of 0 when such product has negative sign. We then group the stocks in our sample into twentiles according to their comovement score and compute the fraction of stocks within each twentile that experiences positive comovement between volatility and informativeness, that is, we compute the average of the positive comovement variable.[11]

[11] Note that our test is designed to be out-of-sample. That is, we compute comovement scores using past data, but we compute changes in volatility and changes in informativeness using future data.

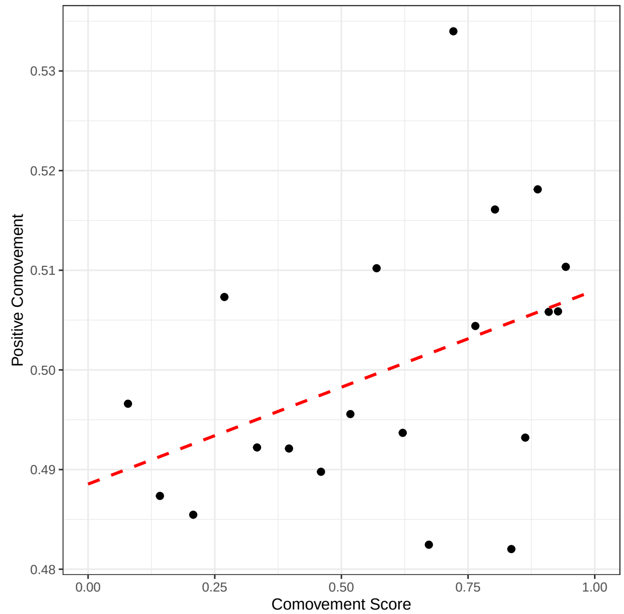


Figure 8: Relation between comovement scores and measured volatility-informativeness comovement

Note: Figure 8 shows the relation between comovement scores, recovered as explained in Subsection E.1, and estimated measures of actual comovement between volatility and informativeness, recovered as explained in this subsection. The R-squared of this relation is $R^2 = 16.5\%$, with a slope coefficient of 0.019 and a standard error of 0.103. In this figure, comovement scores are aggregated into twentiles, after winsorizing the top and bottom 5% of the distribution of estimated comovement scores, which we separately compare to show that stocks with comovement scores close to 1 experience higher positive comovement that stocks with comovement scores close to 0. The total number of unique comovement score observations used to build this figure is 21202.

Figure 8 shows the relation between the average comovement score by twentiles of the distribution of comovement scores and the fraction of stocks that experience positive comovement between volatility and informativeness among each group. This figure shows that there exists a positive correlation between the comovement score and the likelihood of positive comovement, which is consistent with the theoretical predictions of our framework.

5 Conclusion

This paper systematically studies the equilibrium relation between price informativeness and price volatility, identifying two different channels (noise-reduction and equilibrium-learning) through which changes in price informativeness are associated with changes in price volatility. We show that whenever prices are sufficiently informative (uninformative), changes in parameters induce a positive (negative) comovement between price informativeness and price volatility in response to changes in any of the model primitives. Moreover, we characterize simple conditions in terms of primitives that allow us to assign stocks to positive, negative, or ambiguous comovement regions.

Building on our theoretical analysis, we introduce the notion of the comovement score, which is a statistic that measures the relative distance of a given asset to the positive/negative comovement regions. Empirically, we compute stock-specific comovement scores and describe their time series and cross-sectional behavior. We find that comovement scores trend downwards over the last decades. In the cross-section, we find that comovement scores are positively related to value and idiosyncratic volatility and negatively related to size and institutional ownership. Finally, consistent with our theoretical framework, we show that stocks with higher comovement scores are more likely to experience positive comovement between volatility and informativeness.

An important practical takeaway from this paper is that the relation between volatility and informativeness is nuanced and far from obvious, and that making inferences about price informativeness by measuring price/return volatility is only justified in particular circumstances. By introducing the comovement score, we show how it is possible to determine in practical scenarios whether price volatility is a valid proxy for price informativeness. We hope that future work further explores in detail the relation between volatility and informativeness in more general environments, both theoretically and empirically.

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Appendix

A Variable Definitions

Table 2 below summarizes the notation and variable definitions used in the paper.

Table 2: Notation summary and variable description

Variable	Description
γ	CARA coefficient
ω_t^i	initial endowment of investor i born at date t
$w_{1,t+1}^i$	date $t+1$ wealth of investor i born at date t
θ_t	risky asset payoff at date t
μ_θ	drift in risky asset payoff process
η_t	date t innovation to the asset payoff
ρ	persistence coefficient in risky asset payoff process
s_t^i	private signal about η_t received by investor i born at date t
ε_{st}^i	idiosyncratic noise in s_t^i
τ_s^{-1}	variance of noise in s_t^i
$\overline{\eta}_t^i$	prior mean of innovation η_t of investor i born at date t
τ_η^{-1}	prior variance or η_t for an investor
n_t	average prior mean of innovation η_t (aggregate sentiment)
μ_n	expected average prior mean of innovation η_t, $\mathbb{E}\left[n_t\right] = \mu_n$
τ_n^{-1}	variance of prior mean of innovation η_t, $\mathbb{V}ar\left[n_t\right] = \tau_n^{-1}$
ε_{ut}^i	idiosyncratic component of beliefs $\overline{\eta}_t^i$
τ_u^{-1}	variance of idiosyncratic component of beliefs, $\mathbb{V}ar\left[\varepsilon_{ut}^i\right] = \tau_u^{-1}$
$q_{1,t}^i$	quantity demanded by an investor i born at date 1
p_t	risky asset price at date t
α_s	individual demand sensitivity to private signal
α_θ	individual demand sensitivity to previous realization of asset payoff
α_p	individual demand sensitivity to price
π_t	unbiased signal about η_t contained in price p_t
τ_π	price informativeness, $\tau_\pi \equiv \left(\mathbb{V}\mathrm{ar}\left[\pi_t \vert \eta_t, \theta_t\right]\right)^{-1}$
$\mathcal{V}$	price volatility, $\mathcal{V} \equiv \mathbb{V}\mathrm{ar}\left[p_t \vert\, \theta_t\right]$

B Proofs and Derivations: Section 2

Characterization of equilibrium Given the CARA-Gaussian setup that we consider in Section 2 and the wealth accumulation constraint in Equation 1, the demand for the risky asset of an investor i is given by the solution to

$$\max_{q_{1,t}^i} \left(\mathbb{E}\left[\theta_{t+1} + p_{t+1} | \mathcal{I}_t^i\right] - p_t\right) q_{1,t}^i - \frac{\gamma}{2} \mathbb{V}ar\left[\left(\theta_{t+1} + p_{t+1}\right) q_{1,t}^i | \mathcal{I}_t^i\right],$$

where $\mathcal{I}_t^i = \left\{\theta_t, s_t^i, \overline{\eta}_t^i, p_t\right\}$ is the information set of investor i at date t. Then, the net asset demand of investor i is

$$q_{1,t}^i = \frac{\mathbb{E}\left[\theta_{t+1} + p_{t+1} | \mathcal{I}_t^i\right] - p_t - \gamma \mathbb{V}ar\left[\theta_{t+1} + p_{t+1} | \mathcal{I}_t^i\right]}{\gamma \mathbb{V}ar\left[\theta_{t+1} + p_{t+1} | \mathcal{I}_t^i\right]}. \tag{A.1}$$

In a stationary symmetric equilibrium in linear strategies, we guess and subsequently verify that the net demand of an investor i born at date t is of the form

$$q_{1,t}^i = \alpha_\theta \theta_t + \alpha_s s_t^i + \alpha_n \eta_t^i - \alpha_p p_t + \psi^i.$$

Market clearing in the asset market is given by $\int_I q_{1t}^i di = Q$, which yields the following equilibrium price function:

$$p_t = \frac{\alpha_\theta}{\alpha_p}\theta_t + \frac{\alpha_s}{\alpha_p}\left(\eta_t + \int \varepsilon_{st}^i di\right) + \frac{\alpha_n}{\alpha_p}\left(n_t + \int \varepsilon_{nt}^i di\right) + \frac{\overline{\psi}}{\alpha_p},$$

where $\overline{\psi} \equiv \int \psi^i di - Q$. Note that the equilibrium price contains information about the innovation to the payoff, η_t. We can define the unbiased signal about η_t contained in the price as

$$\pi_t \equiv \frac{\alpha_p}{\alpha_s}\left(p_t - \frac{\alpha_\theta}{\alpha_p}\theta_t - \frac{\alpha_n}{\alpha_p}\mathbb{E}\left[n_t\right] - \frac{\overline{\psi}}{\alpha_p}\right) = \eta_t + \frac{\alpha_n}{\alpha_s}\left(n_t - \mathbb{E}\left[n_t\right]\right)$$

and we define the precision of this signal as $\tau_\pi = \mathbb{V}ar\left[\pi_t | \eta_t\right]^{-1}$.

Given our guesses for the demand functions and the resulting linear structure of prices we have

$$\begin{aligned}
\mathbb{E}\left[\theta_{t+1} + p_{t+1} | \mathcal{I}_t^i\right] &= \left(1 + \frac{\alpha_\theta}{\alpha_p}\right) \mathbb{E}\left[\theta_{t+1} | \mathcal{I}_t^i\right] + \frac{\alpha_s}{\alpha_p}\mathbb{E}\left[\eta_{t+1}\right] + \frac{\alpha_n}{\alpha_p}\mathbb{E}\left[n_{t+1}\right] + \frac{\overline{\psi}}{\alpha_p} \\
&= \left(1 + \frac{\alpha_\theta}{\alpha_p}\right)\left(\mu_\theta + \rho\theta_t + \mathbb{E}\left[\eta_t | \mathcal{I}_t^i\right]\right) + \frac{\alpha_s}{\alpha_p}\mathbb{E}\left[\eta_{t+1}\right] + \frac{\alpha_n}{\alpha_p}\mu_n + \frac{\overline{\psi}}{\alpha_p},
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{V}ar\left[\theta_{t+1} + p_{t+1} | \mathcal{I}_t^i\right] &= \left(1 + \frac{\alpha_\theta}{\alpha_p}\right)^2 \mathbb{V}ar\left[\theta_{t+1} | \mathcal{I}_t^i\right] + \left(\frac{\alpha_s}{\alpha_p}\right)^2 \mathbb{V}ar\left[\eta_{t+1}\right] + \frac{\alpha_n}{\alpha_p}^2 \mathbb{V}ar\left[n_{t+1}\right] \\
&= \left(1 + \frac{\alpha_\theta}{\alpha_p}\right)^2 \mathbb{V}ar\left[\eta_t | \mathcal{I}_t^i\right] + \left(\frac{\alpha_s}{\alpha_p}\right)^2 \mathbb{V}ar\left[\eta_{t+1}\right] + \left(\frac{\alpha_n}{\alpha_p}\right)^2 \tau_n^{-1}.
\end{aligned}$$

The normal linear structure of the signals and the price implies we can express investors ex-post variances about the innovation to the asset payoff after solving their filtering problem as

$$\mathbb{V}\mathrm{ar}\left[\eta_t \left| s_t^i, \overline{\eta}_t^i, p_t, \theta_t\right.\right] = \left(\tau_\eta + \tau_s + \tau_\pi\right)^{-1}.$$

Similarly, investors' means can be expressed as

$$\mathbb{E}\left[\eta_t \left| s_t^i, \overline{\eta}_t^i, p_t\right.\right] = \frac{\tau_\eta \overline{\eta}_t^i + \tau_s s_t^i + \tau_\pi \pi_t}{\tau_\eta + \tau_s + \tau_\pi}.$$

Using these expressions in the equilibrium demand in Equation (A.1) and matching coefficients we have

$$\alpha_s = \frac{1+\frac{\alpha_\theta}{\alpha_p}}{\kappa}\frac{\tau_s}{\tau_\eta+\tau_s+\tau_\pi}, \quad \alpha_p = \frac{1}{\kappa}\left(1-\left(1+\frac{\alpha_\theta}{\alpha_p}\right)\frac{\tau_\pi}{\tau_\eta+\tau_s+\tau_\pi}\frac{\alpha_p}{\alpha_s}\right), \tag{A.2}$$

$$\alpha_n = \frac{1+\frac{\alpha_\theta}{\alpha_p}}{\kappa}\tau_\eta, \quad \text{and} \quad \alpha_\theta = \frac{1+\frac{\alpha_\theta}{\alpha_p}}{\kappa}\left(\rho-\frac{\tau_\pi}{\tau_\eta+\tau_s+\tau_\pi}\frac{\alpha_\theta}{\alpha_s}\right) \tag{A.3}$$

where

$$\kappa \equiv \gamma\mathbb{V}ar\left[\theta_{t+1}+p_{t+1}|\mathcal{I}_t^i\right] = \gamma\left(\left(1+\frac{\alpha_\theta}{\alpha_p}\right)^2\mathbb{V}ar\left[\eta_t|\mathcal{I}_t^i\right]+\left(\frac{\alpha_s}{\alpha_p}\right)^2\mathbb{V}ar\left[\eta_{t+1}\right]+\left(\frac{\alpha_\eta}{\alpha_p}\right)^2\tau_n^{-1}\right).$$

Lemma 1 (Uniqueness of Equilibrium)

Proof. Solving the system in Equations (A.2) and (A.3), we have

$$\frac{\alpha_\theta}{\alpha_s} = \frac{\rho-\frac{\tau_\pi}{\tau_\eta+\tau_s+\tau_\pi}\frac{\alpha_\theta}{\alpha_s}}{\frac{\tau_s}{\tau_\eta+\tau_s+\tau_\pi}} \Rightarrow \frac{\alpha_\theta}{\alpha_s} = \frac{\tau_\eta+\tau_s+\tau_\pi}{\tau_s+\tau_\pi}\rho \tag{A.4}$$

$$\frac{\alpha_n}{\alpha_s} = \frac{\frac{\tau_\eta}{\tau_\eta+\tau_s+\tau_\pi}}{\frac{\tau_s}{\tau_\eta+\tau_s+\tau_\pi}} \Rightarrow \frac{\alpha_n}{\alpha_s} = \frac{\tau_\eta}{\tau_s} \tag{A.5}$$

$$\frac{\alpha_\theta}{\alpha_p} = \frac{\left(1+\frac{\alpha_\theta}{\alpha_p}\right)\left(\rho-\frac{\tau_\pi}{\tau_\eta+\tau_s+\tau_\pi}\frac{\alpha_\theta}{\alpha_s}\right)}{1-\left(1+\frac{\alpha_\theta}{\alpha_p}\right)\frac{\tau_\pi}{\tau_\eta+\tau_s+\tau_\pi}\frac{\alpha_p}{\alpha_s}} \Rightarrow \frac{\alpha_\theta}{\alpha_p} = \frac{\rho}{1-\rho} \tag{A.6}$$

$$\frac{\alpha_s}{\alpha_p} = \frac{\left(1+\frac{\alpha_\theta}{\alpha_p}\right)\frac{\tau_s}{\tau_\eta+\tau_s+\tau_\pi}}{1-\left(1+\frac{\alpha_\theta}{\alpha_p}\right)\frac{\tau_\pi}{\tau_\eta+\tau_s+\tau_\pi}\frac{\alpha_p}{\alpha_s}} \Rightarrow \frac{\alpha_s}{\alpha_p} = \frac{1}{1-\rho}\frac{\tau_s+\tau_\pi}{\tau_\eta+\tau_s+\tau_\pi}. \tag{A.7}$$

Then, the equilibrium demand sensitivities are uniquely determined and given by

$$\alpha_s = \frac{1}{\kappa(1-\rho)}\frac{\tau_s}{\tau_\eta+\tau_s+\tau_\pi} \tag{A.8}$$

$$\alpha_p = \frac{1}{\kappa}\frac{\tau_s}{\tau_s+\tau_\pi} \tag{A.9}$$

$$\alpha_n = \frac{1}{\kappa(1-\rho)}\frac{\tau_\eta}{\tau_\eta+\tau_s+\tau_\pi} \tag{A.10}$$

$$\alpha_\theta = \frac{\rho}{\kappa(1-\rho)}\frac{\tau_s}{\tau_s+\tau_\pi}. \tag{A.11}$$

□

Proposition 1. (Volatility-informativeness relation: demand sensitivities formulation)

Proof. a) From the Law of Total Variance we have

$$\mathbb{V}\text{ar}\left[p_t|\,\theta_t\right] = \mathbb{V}\text{ar}\left[\mathbb{E}\left[p_t|\,\eta_t,\theta_t\right]|\,\theta_t\right]+\mathbb{E}\left[\mathbb{V}\text{ar}\left[p_t|\,\eta_t,\theta_t\right]|\,\theta_t\right].$$

Moreover, using the equilibrium price in Equation (2) we have

$$\mathbb{E}\left[\mathbb{V}\text{ar}\left[p_t|\,\eta_t,\theta_t\right]\right] = \left(\frac{\alpha_n}{\alpha_p}\right)^2\tau_n^{-1} \quad \text{and} \quad \mathbb{V}\text{ar}\left[\mathbb{E}\left[p_t|\,\eta_t,\theta_t\right]\right] = \left(\frac{\alpha_s}{\alpha_p}\right)^2\tau_\eta^{-1}.$$

Using that price informativeness is given by $\tau_\pi = \left(\frac{\alpha_s}{\alpha_n}\right)^2 \tau_n$, price volatility can be expressed as follows

$$\mathcal{V} = \left(\frac{\alpha_s}{\alpha_p}\right)^2 \left(\tau_\eta^{-1} + \tau_\pi^{-1}\right),$$

where $\tau_\eta = \mathbb{V}\text{ar}\left[\eta_t\right]^{-1}$ and τ_π denote precisions (inverse of variances).

b) Differentiating $\mathcal{V}$ with respect to τ_π in Equation (5), we have that

$$\begin{aligned}\frac{d\mathcal{V}}{d\tau_\pi} &= 2\frac{\alpha_s}{\alpha_p}\frac{d\left(\frac{\alpha_s}{\alpha_p}\right)}{d\tau_\pi}\left(\tau_\eta^{-1} + \tau_\pi^{-1}\right) - \left(\frac{\alpha_s}{\alpha_p}\right)^2 \left(\tau_\pi\right)^{-2} \\ &= \frac{\mathcal{V}}{\tau_\pi}\left(2\frac{d\log\left(\frac{\alpha_s}{\alpha_p}\right)}{d\log\left(\tau_\pi\right)} - \frac{\tau_\pi^{-1}}{\tau_\eta^{-1} + \tau_\pi^{-1}}\right),\end{aligned}$$

which corresponds to Equation (6) in the text. □

Lemma 2. (Signal-to-price sensitivity)

Proof. Equation (7) in the text follows directly from Equation (A.8) and Equation (A.9). □

Proposition 2. (Volatility-informativeness relation: model primitives formulation)

Proof. The proof follows directly from part a) of Proposition 1 and Lemmas 2. □

Proposition 3. (Slope of volatility-informativeness relation)

Proof. From Equation (7) it follows that

$$\frac{d\log\left(\frac{\alpha_s}{\alpha_p}\right)}{d\log\left(\tau_\pi\right)} = \frac{\tau_\eta}{\tau_\eta + \tau_s + \tau_\pi}\frac{\tau_\pi}{\tau_s + \tau_\pi}.$$

Therefore, from Lemma 2 it follows that

$$\begin{aligned}\frac{d\log\mathcal{V}}{d\log\tau_\pi} &= 2\frac{d\log\left(\frac{\alpha_s}{\alpha_p}\right)}{d\log\left(\tau_\pi\right)} - \frac{\tau_\pi^{-1}}{\tau_\eta^{-1} + \tau_\pi^{-1}} = 2\frac{\tau_\eta}{\tau_\eta + \tau_s + \tau_\pi}\frac{\tau_\pi}{\tau_s + \tau_\pi} - \frac{\tau_\eta}{\tau_\eta + \tau_\pi} \\ &= \tau_\eta\frac{\left(\tau_\pi\right)^2 + \tau_\pi\left(\tau_\eta - 2\tau_s\right) - \tau_s\left(\tau_\eta + \tau_s\right)}{\left(\tau_\eta + \tau_s + \tau_\pi\right)\left(\tau_s + \tau_\pi\right)\left(\tau_\eta + \tau_\pi\right)}.\end{aligned}$$

We can then conclude that

$$\text{sgn}\left(\frac{d\log\mathcal{V}}{d\log\tau_\pi}\right) = \text{sgn}\left[\left(\tau_\pi\right)^2 + \tau_\pi\left(\tau_\eta - 2\tau_s\right) - \tau_s\left(\tau_\eta + \tau_s\right)\right].$$

Note that the expression on the right hand side is a convex quadratic function of τ_π with only one positive root given by

$$\tau^\star \equiv \frac{-\left(\tau_\eta - 2\tau_s\right) + \sqrt{\tau_\eta^2 + 8\tau_s^2}}{2}. \qquad \text{(A.12)}$$

Then,

$$\frac{d\mathcal{V}}{d\tau_\pi} < 0 \iff \tau_\pi < \tau^\star \quad \text{and} \quad \frac{d\mathcal{V}}{d\tau_\pi} > 0 \iff \tau_\pi > \tau^\star.$$

$\square$

C Proofs and Derivations: Section 3.3

Characterization of comovement regions

We use the following result:

$$\frac{d\mathcal{V}}{d\tau_\pi} = \frac{1}{1-\rho}\frac{\alpha_s}{\alpha_p}\frac{(\tau_\pi)^{-2}}{(\tau_\eta+\tau_s+\tau_\pi)^2}\Big((\tau_\pi)^2 + (\tau_\eta - 2\tau_s)\,\tau_\pi - \tau_s\,(\tau_\eta+\tau_s)\Big), \tag{A.13}$$

which follows directly from Equation (A.22).

Proposition 4. (Comovement regions: price informativeness formulation)

Proof. From the characterization of equilibrium in the previous section we have that the variance of π, which we denote by τ_π^{-1} and whose inverse we adopt as the relevant measure of price informativeness, is given by

$$\tau_\pi = \left(\frac{\alpha_s}{\alpha_\eta}\right)^2 \tau_n = \left(\frac{\tau_s}{\tau_\eta}\right)^2 \tau_n.$$

The change in volatility when a parameter x changes is given by

$$\frac{d\mathcal{V}}{dx} = \frac{\partial\mathcal{V}}{\partial x} + \frac{d\mathcal{V}}{d\tau_\pi}\frac{d\tau_\pi}{dx},$$

where $\frac{\partial\mathcal{V}}{\partial x}$ and $\frac{d\tau_\pi}{dx}$ can be obtained from Equation (5) and Equation (7) using the definition of τ_π. Note that

$$\frac{d\tau_\pi}{d\tau_u} = 0 \quad \text{and} \quad \frac{\partial\mathcal{V}}{\partial\tau_u} = 0,$$

$$\frac{d\tau_\pi}{d\gamma} = 0 \quad \text{and} \quad \frac{\partial\mathcal{V}}{\partial\gamma} = 0,$$

and

$$\frac{d\tau_\pi}{d\rho} = 0 \quad \text{and} \quad \frac{\partial\mathcal{V}}{\partial\rho} > 0,$$

so the comovement in this proposition is weak.

a) **Positive comovement**

For changes in τ_n, we have that $\frac{d\tau_\pi}{d\tau_n} = \left(\frac{\tau_s}{\tau_\eta}\right)^2 > 0$, and $\frac{\partial\mathcal{V}}{\partial\tau_n} = 0$. Hence, $\frac{d\mathcal{V}}{d\tau_\pi} > 0$ is a sufficient and necessary condition for $\frac{d\mathcal{V}}{d\tau_n} = \frac{d\mathcal{V}}{d\tau_\pi}\frac{d\tau_\pi}{d\tau_n}$ to be positive.

For changes in τ_s, it follows that $\frac{d\tau_\pi}{d\tau_s} = \frac{2\tau_\pi}{\tau_s} > 0$, and

$$\frac{\partial\mathcal{V}}{\partial\tau_s} = 2\frac{\alpha_s}{\alpha_p}\frac{1}{1-\rho}\frac{\tau_\eta}{(\tau_\eta+\tau_s+\tau_\pi)^2}\left(\tau_\eta^{-1}+\tau_\pi^{-1}\right) > 0.$$

Then, $\frac{\partial\mathcal{V}}{\partial\tau_\pi} > 0$ is a sufficient condition for $\frac{d\mathcal{V}}{d\tau_s} = \frac{\partial\mathcal{V}}{\partial\tau_s} + \frac{d\mathcal{V}}{d\tau_\pi}\frac{d\tau_\pi}{d\tau_s}$ to be positive.

Similarly, for changes in τ_η, we have that $\frac{d\tau_\pi}{d\tau_\eta} < 0$, and

$$\frac{\partial\mathcal{V}}{\partial\tau_\eta} = -2\frac{\alpha_s}{\alpha_p}\frac{1}{1-\rho}\frac{\tau_s+\tau_\pi}{(\tau_\eta+\tau_s+\tau_\pi)^2}\left(\tau_\eta^{-1}+\tau_\pi^{-1}\right) - \left(\frac{\alpha_s}{\alpha_p}\right)^2\tau_\eta^{-2}.$$

Then, $\frac{d\mathcal{V}}{d\tau_\pi} > 0$ is a sufficient condition for $\frac{d\mathcal{V}}{d\tau_\eta} = \frac{\partial\mathcal{V}}{\partial\tau_\eta} + \frac{d\mathcal{V}}{d\tau_\pi}\frac{d\tau_\pi}{d\tau_\eta}$ to be negative.

Therefore, if $\tau_\pi > \tau^\star$ an increase in price volatility reflects a weak increase in price informativeness for any parameter change.

b) **Negative comovement**

For changes in τ_n, price volatility and price informativeness negatively comove if $\tau_\pi < \tau^\star$.

For changes in τ_s, it follows that

$$\frac{d\mathcal{V}}{d\tau_s} = \frac{2}{1-\rho}\frac{\alpha_s}{\alpha_p}\frac{1}{(\tau_\eta+\tau_s+\tau_\pi)^2}\frac{\tau_s}{\tau_\pi}\frac{1}{\tau_\eta^2}\left(\left(\frac{\tau_n}{\tau_\eta}\right)^2\tau_s^2 - \tau_n\tau_s - \tau_\eta(\tau_\eta-\tau_n)\right).$$

Note that there are two case. If $\tau_\eta < \tau_n$ the expression above is positive and there cannot be negative comovement between price volatility and price informativeness. In this case, we define $\tau_{\tau_s} = 0$. If $\tau_\eta > \tau_n$, there exists a threshold $\underline{s}$ such that for all $\tau_s < \underline{s}$, $\frac{d\mathcal{V}}{d\tau_s}$ and price informativeness and price volatility negatively comove when τ_s changes. This threshold is given by

$$\underline{s} \equiv \frac{\tau_n + \sqrt{\tau_n^2 + 4\left(\frac{\tau_n}{\tau_\eta}\right)^2\tau_\eta(\tau_\eta-\tau_n)}}{2\left(\frac{\tau_n}{\tau_\eta}\right)^2} = \frac{\tau_\eta^2}{2\tau_n}\left(1+\sqrt{1+4\tau_\eta^{-1}(\tau_\eta-\tau_n)}\right).$$

This implies that if $\tau_\eta > \tau_n$, price informativeness and price volatility negatively comove when τ_s changes for all $\tau_\pi < \tau_{\tau_s}$, where

$$\tau_{\tau_s} \equiv \tau_\eta\frac{-\left(1-\frac{\tau_s}{\tau_\eta}\right)+\sqrt{\left(1-\frac{\tau_s}{\tau_\eta}\right)^2+4\left(\frac{\tau_s}{\tau_\eta}\right)^2}}{2}. \tag{A.14}$$

For changes in τ_η we have that

$$\frac{d\mathcal{V}}{d\tau_\eta} = -\frac{2}{1-\rho}\frac{\alpha_s}{\alpha_p}\frac{\tau_\pi^{-1}}{(\tau_\eta+\tau_s+\tau_\pi)^2}\frac{1}{\tau_\eta}\left(2\left(\frac{\tau_s}{\tau_\eta}\right)^4\tau_n^2 + (2\tau_\eta-\tau_s)\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n - \tau_s^2\right).$$

Then,

$$\operatorname{sgn}\left(\frac{d\mathcal{V}}{d\tau_\eta}\right) = -\operatorname{sgn}\left(2\left(\frac{\tau_s}{\tau_\eta}\right)^4\tau_n^2 + (2\tau_\eta-\tau_s)\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n - \tau_s^2\right).$$

Since

$$\lim_{\tau_\eta\to\infty} 2\left(\frac{\tau_s}{\tau_\eta}\right)^4\tau_n^2 + (2\tau_\eta-\tau_s)\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n - \tau_s^2 = -\tau_s^2,$$

there exists a threshold $\underline{\eta} > 0$ such that $\frac{d\mathcal{V}}{d\tau_\eta}$ is positive for all $\tau_\eta > \underline{\eta}$. This implies that there exists a threshold τ_{τ_η} such that $\frac{d\mathcal{V}}{d\tau_\eta} > 0$ for all $\tau_\pi < \tau_{\tau_\eta}$ where

$$\tau_{\tau_\eta} \equiv \tau_\eta\frac{-\left(2-\frac{\tau_s}{\tau_\eta}\right)+\sqrt{\left(2-\frac{\tau_s}{\tau_\eta}\right)^2+8\left(\frac{\tau_s}{\tau_\eta}\right)^2}}{4}. \tag{A.15}$$

Therefore, price volatility and price informativeness negative comove is $\tau_\pi < \underline{\tau} \equiv \min\left\{\tau_{\tau_s}, \tau_{\tau_\eta}\right\}$, where $\underline{\tau} < \tau^\star$ since it is necessary for $\frac{d\mathcal{V}}{d\tau_\pi} < 0$ for price informativeness and price volatility to comove negatively for changes in τ_s and τ_η.

c) **Ambiguous comovement**

Finally, if neither Equation (A.16) nor Equation (A.17) are satisfied, price volatility and price informativeness positively comove after some parameter changes and negatively comove after others. This is the ambiguous comovement region. □

Proposition 5. (Comovement regions: model primitives formulation)

Proof. a) **Positive comovement**

Using that price informativeness is given by $\tau_\pi = \left(\frac{\tau_s}{\tau_\eta}\right)^2 \tau_n$ and $\tau^\star$ is given by equation (A.12), $\tau_\pi > \tau^\star$ can be written in terms of primitives as

$$\frac{\tau_n}{\tau_\eta} > \frac{\sqrt{1 + 8\left(\frac{\tau_s}{\tau_\eta}\right)^2} - 1 + 2\frac{\tau_s}{\tau_\eta}}{2\left(\frac{\tau_s}{\tau_\eta}\right)^2},$$

which is exactly Equation (12) in the text.

b) **Negative comovement**

If $\tau_\eta > \tau_n$, using the definition for τ_{τ_s} in Equation (A.14) in the proof of the previous proposition and that $\tau_\pi = \left(\frac{\tau_s}{\tau_\eta}\right)^2 \tau_n$, condition $\tau_\pi < \tau_{\tau_s}$ becomes

$$\frac{\tau_n}{\tau_\eta} < \frac{-\left(1 - \frac{\tau_s}{\tau_\eta}\right) + \sqrt{\left(1 - \frac{\tau_s}{\tau_\eta}\right)^2 + 4\left(\frac{\tau_s}{\tau_\eta}\right)^2}}{2\left(\frac{\tau_s}{\tau_\eta}\right)^2}. \tag{A.16}$$

Similarly, using the definition for τ_{τ_η} in Equation (A.15) above, condition $\tau_\pi < \tau_{\tau_\eta}$ becomes

$$\frac{\tau_n}{\tau_\eta} < \frac{\frac{\tau_s}{\tau_\eta} - 2 + \sqrt{\left(2 - \frac{\tau_s}{\tau_\eta}\right)^2 + 8\left(\frac{\tau_s}{\tau_\eta}\right)^2}}{4\left(\frac{\tau_s}{\tau_\eta}\right)^2}. \tag{A.17}$$

Therefore, putting Equations (A.16) and (A.17) together with the condition $\frac{\tau_n}{\tau_\eta} < 1$, we have that the negative comovement region is characterized by (A.17) since for all $x > 0$

$$\min\left\{\frac{x - 1 + \sqrt{5x^2 - 2x + 1}}{2x^2}, \frac{\frac{\tau_s}{\tau_\eta} - 2 + \sqrt{(2-x)^2 + 8x^2}}{4x^2}, 1\right\} = \frac{x - 2 + \sqrt{(2-x)^2 + 8x^2}}{4x^2},$$

which is exactly Equation (13) in the text. ☐

D Proofs: Section 4

Proposition 6. (Comovement score estimation)

Proof. a) Using the equilibrium expression for the price in Equation and Regression R1, and the equilibrium coefficients in Equations (A.39) - (A.43) we have

$$\beta_1 = \frac{\alpha_s}{\alpha_p} = \frac{\tau_s + \tau_\pi}{\tau_s + \tau_\eta + \tau_\pi},$$
$$\beta_0 = \left(\frac{\alpha_\theta}{\alpha_p} - \frac{\alpha_s}{\alpha_p}\right) \frac{\tau_\eta}{\tau_s + \tau_\eta + \tau_\pi},$$
$$\mathbb{V}ar\left[\varepsilon_t\right] = \mathbb{V}ar\left[\frac{\alpha_n}{\alpha_p} n_t\right] = \left(\frac{\frac{\tau_\eta}{\tau_s+\tau_\eta+\tau_\pi}}{\frac{\tau_s}{\tau_s+\tau_\pi}}\right)^2 \tau_n^{-1}, \quad \text{and}$$
$$\mathbb{V}ar\left[\varepsilon_t^\zeta\right] = \mathbb{V}ar\left[\frac{\alpha_s}{\alpha_p}\eta_t + \frac{\alpha_n}{\alpha_p} n_t\right] = \left(\frac{\tau_s + \tau_\pi}{\tau_s + \tau_\eta + \tau_\pi}\right)^2 \tau_\eta^{-1} + \left(\frac{\frac{\tau_\eta}{\tau_s+\tau_\eta+\tau_\pi}}{\frac{\tau_s}{\tau_s+\tau_\pi}}\right)^2 \tau_n^{-1}.$$

Note that

$$\frac{\beta_1}{1-\beta_1} = \frac{\tau_s + \tau_\pi}{\tau_\eta}$$

and

$$\frac{\mathbb{V}ar\left[\varepsilon_t^\zeta\right] - \mathbb{V}ar\left[\varepsilon_t\right]}{\mathbb{V}ar\left[\varepsilon_t\right]} = \frac{\left(\frac{\alpha_s}{\alpha_p}\right)^2 \tau_\eta^{-1}}{\left(\frac{\alpha_n}{\alpha_p}\right)^2 \tau_n^{-1}} = \left(\frac{\tau_s}{\tau_\eta}\right)^2 \frac{\tau_n}{\tau_\eta} = \frac{\tau_\pi}{\tau_\eta}.$$

Then, one can write

$$\frac{\tau_s}{\tau_\eta} = \frac{\beta_1}{1-\beta_1} - \frac{\mathbb{V}ar\left[\varepsilon_t^\zeta\right] - \mathbb{V}ar\left[\varepsilon_t\right]}{\mathbb{V}ar\left[\varepsilon_t\right]}$$

and

$$\frac{\tau_n}{\tau_\eta} = \left(\frac{\beta_1}{1-\beta_1} - \frac{\mathbb{V}ar\left[\varepsilon_t^\zeta\right] - \mathbb{V}ar\left[\varepsilon_t\right]}{\mathbb{V}ar\left[\varepsilon_t\right]}\right)^{-2} \frac{\mathbb{V}ar\left[\varepsilon_t^\zeta\right] - \mathbb{V}ar\left[\varepsilon_t\right]}{\mathbb{V}ar\left[\varepsilon_t\right]}.$$

Noting that

$$\frac{\mathbb{V}ar\left[\varepsilon_t^\zeta\right] - \mathbb{V}ar\left[\varepsilon_t\right]}{\mathbb{V}ar\left[\varepsilon_t\right]} = \frac{R^2_{\Delta\theta_{t+1},\Delta\theta_t} - R^2_{\Delta\theta_t}}{1 - R^2_{\Delta\theta_{t+1},\Delta\theta_t}},$$

where $R^2_{\Delta\theta_{t+1},\Delta\theta_t} \equiv 1 - \frac{\mathbb{V}\mathrm{ar}[\varepsilon_t]}{\mathbb{V}\mathrm{ar}[\Delta p_t]}$ and $R^2_{\Delta\theta_t} \equiv 1 - \frac{\mathbb{V}\mathrm{ar}[\varepsilon_t^\zeta]}{\mathbb{V}\mathrm{ar}[\Delta p_t]}$ denote the R-squared of the regressions R1 and R2, respectively.

Under the conditions specified, it is straight forward to show that the OLS estimates are consistent. □

Online Appendix

In this Online Appendix, we show that the results derived on the paper apply more generally. We start by deriving the volatility-informativeness in a general model with additive noise and linear demands. Then, we specialize the analysis to CARA-Normal models and consider two alternative sources of noise to the heterogeneous beliefs considered in the main body of the paper: strategic traders and hedging needs.

E Volatility-Informativeness Relation: General Environment

In this section, we characterize the equilibrium relation between price informativeness and price volatility in models in which investors have linear asset demands and face additive noise. For completeness, we reproduce part of the analysis and results presented in the main body of the paper in this Online Appendix.

E.1 General Environment

Time is discrete, with dates denoted by $t = 0, 1, 2, \ldots, \infty$. There are two assets: a riskless asset in perfectly elastic supply with gross return $R > 1$ and a risky asset in fixed supply Q, which is traded at a price p_t at date t. The asset payoff, which accrues at the beginning of date $t+1$, is given by

$$\theta_{t+1} = \mu_\theta + \rho\theta_t + \eta_t,$$

where μ_θ is a scalar, $|\rho| \leq 1$, and $\theta_0 = 0$, and where the innovations to the payoff, η_t, have mean zero, finite variance τ_η^{-1}, and are independently distributed.[12] Note that the innovation to the $t+1$ payoff difference, η_t, is indexed by t — instead of $t+1$ — to indicate that investors can potentially learn about the realization of η_t at date t.

A set of investors, indexed by $i \in I$, trade both assets at each date t. Before trading at date t, each investor i observes the already realized value of the asset payoff θ_t and a private signal s_t^i of the innovation to the future asset payoff η_t. Moreover, investors have additional motives for trading the risky asset that are orthogonal to the asset payoff. We denote by n_t^i investor i's additional trading motive at date t. These trading motives are private information of each investor.

We derive our first set of results under two assumptions. The first assumption imposes an additive informational structure, while the second assumption imposes a linear structure for investors' equilibrium asset demands. In Section F, we provide fully specified sets of primitives that are consistent with Assumptions 1 and 2.

Assumption 1. (Additive noise) *Each date t, every investor i receives an unbiased private signal s_t^i about the innovation to the payoff, η_t, of the form*

$$s_t^i = \eta_t + \varepsilon_{st}^i, \tag{A.18}$$

where ε_{st}^i, $\forall i \in I$, $\forall t$, are random variables with mean zero and finite variances, whose realizations are independent across investors and over time. Each date t, every investor i has a private trading need n_t^i, of the form

$$n_t^i = n_t + \varepsilon_{nt}^i, \tag{A.19}$$

where n_t is a random variable with finite mean, denoted by μ_n, and finite variance, and where ε_{nt}^i, $\forall i \in I$, $\forall t$, are random variables with mean zero and finite variances, whose realizations are independent across investors and over time.

[12] When $\rho = 0$, the model effectively behaves as if it were static. When $\rho = 1$, asset payoffs and prices are non-stationary and follow a random walk.

Assumption 1 imposes a noise structure that is additive and independent across investors for the signals about the innovation to the future payoff η_t as well as for other sources of investors' private trading needs n_t^i. This assumption does not restrict the distribution of any random variable beyond the existence of finite first and second moments. Our second assumption describes the structure of the investors' net demands for the risky asset Δq_t^i.

Assumption 2. (Linear asset demands) *Investors' net asset demands satisfy*

$$\Delta q_t^i = \alpha_s^i s_t^i + \alpha_\theta^i \theta_t + \alpha_n^i n_t^i - \alpha_p^i p_t + \psi^i,$$

where α_s^i, α_θ^i, α_n^i, α_p^i, and ψ^i are individual demand coefficients, potentially determined in equilibrium.

Assumption 2 imposes that the net asset demand for the risky asset for a given investor is linear in his signal about the asset payoff and his private trading needs, as well as in the asset price p_t and the realized asset payoff θ_t. It also allows for an individual specific invariant component ψ^i. This linear structure arises endogenously under CARA utility and Gaussian uncertainty, as we show in Section F in the paper. More broadly, linear asset demands can be interpreted as a linear approximation to general asset demand functions, so the results in Proposition 7 are valid generally up to a first-order approximation.

E.2 Equilibrium Price Characterization

Market clearing in the risky asset market implies that $\int \Delta q_t^i di = 0$ must hold at each date t.[13] Assumptions 1 and 2, when combined with market clearing, imply that the equilibrium asset price must satisfy

$$p_t = \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\theta_t + \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\eta_t + \frac{\overline{\alpha_n}}{\overline{\alpha_p}}n_t + \frac{\int_I \alpha_s^i \varepsilon_{st}^i di}{\overline{\alpha_p}} + \frac{\int_I \alpha_n^i \varepsilon_{nt}^i di}{\overline{\alpha_p}} + \frac{\overline{\psi}}{\overline{\alpha_p}},$$

where we denote the cross sectional averages of individual demand coefficients by $\overline{\alpha_\theta} = \int_I \alpha_\theta^i di$, $\overline{\alpha_s} = \int_I \alpha_s^i di$, $\overline{\alpha_n} = \int_I \alpha_n^i di$, $\overline{\alpha_p} = \int_I \alpha_p^i di$, and $\overline{\psi} = \int_I \psi^i di$. The linearity of net demands implies that the equilibrium asset price is also linear in the innovation to the asset payoff η_t, in the already realized payoff θ_t, and in the common component of investors' private trading needs n_t. When there is a continuum of investors, a law of large numbers guarantees that the terms $\frac{\int_I \alpha_s^i \varepsilon_{st}^i di}{\overline{\alpha_p}}$ and $\frac{\int_I \alpha_n^i \varepsilon_{nt}^i di}{\overline{\alpha_p}}$ vanish. Otherwise, these terms operate as additional sources of aggregate noise.

The equilibrium price p_t imperfectly reveals the innovation to the asset payoff η_t. The sensitivity of the equilibrium price to the realization of the innovation is modulated by the average weight that investors put on their private signals s_t^i. However, investors' demands also depend on their private trading motives n_t^i, which are orthogonal to the asset payoff. Since investors do not observe the common component of these additional trading motives, they cannot distinguish whether a high price is due to a high realization of the innovation to the asset payoff η_t or due to a high aggregate trading need unrelated to the asset payoff n_t. In this sense, investors' private trading motives act as noise, since they prevent their signals about the asset payoff from being revealed by their quantity demanded and, consequently, they prevent the price from being fully revealing. In our applications, we map the variable n_t to random heterogeneous priors and hedging needs, which become sources of noise in the filtering problem solved by investors.

Finally, we denote the unbiased signal about the innovation to the asset payoff η_t contained in the price by π_t. We make use of the unbiased signal π_t in our definition of price informativeness. Formally, we define π_t as

$$\pi_t = \frac{\overline{\alpha_p}}{\overline{\alpha_s}}\left(p_t - \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\theta_t - \frac{\overline{\alpha_n}}{\overline{\alpha_p}}\mathbb{E}\left[n_t\right] - \frac{\overline{\psi}}{\overline{\alpha_p}}\right) = \eta_t + \frac{\overline{\alpha_n}}{\overline{\alpha_s}}\left(n_t - \mathbb{E}\left[n_t\right]\right) + \frac{\int_I \alpha_s^i \varepsilon_{st}^i di}{\overline{\alpha_s}} + \frac{\int_I \alpha_n^i \varepsilon_{nt}^i di}{\overline{\alpha_s}}, \tag{A.20}$$

[13] To accommodate a continuum or a finite number of agents, all integrals in the paper represent Lebesgue integrals.

which guarantees that $\mathbb{E}\left[\pi_t|\eta_t\right] = \eta_t$. The last three terms in Equation (A.20) represent the noise contained in the price. The first of these three terms is the realization of the common component of the investors' private trading needs, adjusted by the ratio $\frac{\overline{\alpha_n}}{\overline{\alpha_s}}$, so it is expressed in payoff units. The final two terms in Equation (A.20) capture the sources of aggregate noise that arise from the imperfect aggregation of idiosyncratic shocks when there is a finite number of investors. Note that our definition of π_t allows us to write $p_t = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\pi_t + \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\theta_t + \frac{\overline{\alpha_n}}{\overline{\alpha_p}}\mathbb{E}\left[n_t\right] + \frac{\overline{\psi}}{\overline{\alpha_p}}$, which allows us to interpret $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ as $\frac{\partial p}{\partial \pi}$.

E.3 Relating Price Informativeness and Price Volatility

Using the equilibrium price p_t and the unbiased signal about the asset payoff contained in the price π_t, we can formally define our two objects of interest as follows.

Definition 1. (Price informativeness) *We define price informativeness as the precision of the unbiased signal of the innovation to the asset payoff η_t contained in the asset price, π_t, defined in Equation (A.20), from the perspective of an external observer. We denote price informativeness by*

$$\tau_\pi \equiv \left(\mathbb{V}ar\left[\pi_t|\eta_t, \theta_t\right]\right)^{-1}. \tag{A.21}$$

Price informativeness is a variable that summarizes the ability of financial markets to disseminate information through prices. It is the relevant variable that captures how precise the price is as a signal of η_t from the perspective of an external observer who only observes the realization of the asset payoff θ_t. When price informativeness is high, an external observer receives a very precise signal about the asset payoff by observing the asset price p_t. On the contrary, when price informativeness is low, an external observer learns little about the asset payoff by observing the asset price p_t.

Definition 2. (Price volatility) *We define price volatility as the conditional variance of the asset price. We denote price volatility by*

$$\mathcal{V} \equiv \mathbb{V}ar\left[p_t|\,\theta_t\right].$$

For our purposes, price volatility is simply the idiosyncratic variance of asset prices conditional on the current publicly observed realization of the asset payoff. In our setup, there is a one-to-one mapping between price volatility and return volatility, since investors observe past prices. To simplify the exposition, we only condition on θ_t and use the term price volatility but, formally, our analysis is valid replacing $\mathcal{V}$ by $\mathbb{V}\text{ar}\left[\left.\frac{p_t}{p_{t-1}}\right|\theta_t, p_{t-1}\right]$.

One goal of this paper is to understand how price volatility and price informativeness are related in equilibrium to be able to make inferences about price informativeness, which is not directly observable, from conditional price volatility, which is easily computable. Characterizing the equilibrium relation between these two endogenous variables is the first step to understand how price informativeness and price volatility react to changes in primitives.

Our first set of results builds on the law of total variance, which is an elementary identity that, applied here, implies that conditional price volatility can be decomposed into two components:

$$\mathbb{V}\text{ar}\left[p_t|\,\theta_t\right] = \mathbb{E}\left[\mathbb{V}\text{ar}\left[p_t|\,\eta_t, \theta_t\right]|\,\theta_t\right] + \mathbb{V}\text{ar}\left[\mathbb{E}\left[p_t|\,\eta_t, \theta_t\right]|\,\theta_t\right].$$

The law of total variance asserts that the total variation in the equilibrium price p_t can be decomposed into two components, after conditioning on the innovation to the asset payoff η_t. The first component corresponds to the expectation over the different realizations of the innovation to the asset payoff η_t of the conditional variance of the equilibrium price p_t, given η_t. The second component corresponds to the variance of the conditional expectation of p_t, after learning η_t. Intuitively, the first component captures learnable uncertainty, captured by the best estimate of the residual error in p_t after learning η_t, while the second term captures residual uncertainty, which corresponds to the error from the best guess of p_t after learning η_t.

Under Assumptions 1 and 2, we can express both components as follows

$$\mathbb{E}\left[\mathbb{V}\mathrm{ar}\left[p_t \mid \eta_t, \theta_t\right]\right] = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 \tau_\pi^{-1} \quad \text{and} \quad \mathbb{V}\mathrm{ar}\left[\mathbb{E}\left[p_t \mid \eta_t, \theta_t\right]\right] = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 \tau_\eta^{-1},$$

which allows us to establish the most general characterization of the relation between price informativeness and volatility in Proposition 7. Intuitively, the variation in $\mathbb{E}\left[p_t \mid \eta_t, \theta_t\right]$ is driven by the variance of the innovation to the asset payoff τ_η^{-1}, while the average residual variance is modulated by changes in price informativeness τ_π.

Proposition 7. (General volatility-informativeness relation)

a) Given Assumptions 1 and 2, price volatility $\mathcal{V}$ and price informativeness τ_π satisfy the following relation:

$$\mathcal{V} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 \left(\tau_\eta^{-1} + \tau_\pi^{-1}\right). \tag{A.22}$$

b) The equilibrium elasticity of price volatility to price informativeness is given by

$$\frac{d \log \mathcal{V}}{d \log \tau_\pi} = \underbrace{2 \frac{d \log \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)}{d \log \left(\tau_\pi\right)}}_{\textit{equilibrium-learning}} - \underbrace{\frac{\tau_\pi^{-1}}{\tau_\eta^{-1} + \tau_\pi^{-1}}}_{\textit{noise-reduction}}. \tag{A.23}$$

We refer to Equation (A.22) as the *volatility-informativeness relation* between price informativeness and price volatility. Note that this result is exactly the same one we derived in the main body of the paper under CARA-Normal assumptions.

Part a) of Proposition 7 shows that this equilibrium relation features the exogenous primitive τ_η^{-1}, which corresponds to the variance of the innovation to the asset payoff, and the equilibrium object $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, which we refer to as the signal-to-price sensitivity and in general depends on τ_π.[14] By expressing $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ as a function of τ_π and potentially other primitives, we identify two distinct channels that determine the relation between price informativeness and volatility at this level of generality in part b) of Proposition 7.

We refer to the first channel as the *equilibrium-learning* channel. If a high level of price informativeness is associated with a high (low) level of the signal-to-price sensitivity $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, this induces a positive (negative) relation between price informativeness and volatility. A high value of the signal-to-price sensitivity $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ amplifies the sensitivity of asset prices to aggregate shocks.[15] Intuitively, a high $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ implies that, on average, either investors react significantly to their private signals (high $\overline{\alpha_s}$), or that they have very steep — under the traditional economics convention that uses quantities in the horizontal axis — asset demand curves (low $\overline{\alpha_p}$), so investors barely adjust the quantity demanded even for large price changes, implying that equilibrium prices substantially react to the realization of the asset payoff. Alternatively, a low $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ implies that, on average, investors barely react to their private signals (low $\overline{\alpha_s}$), or that they have very flat — under the traditional economics convention — asset demand curves (high $\overline{\alpha_p}$), so investors significantly adjust the quantity demanded even for small price changes, implying that equilibrium prices are barely responsive to the realization of aggregate payoff shocks.

We refer to the second channel as the noise-reduction channel. It is evident from Proposition 7 that, holding $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ constant, a high level of τ_π is mechanically associated with a low level of $\mathcal{V}$. In fact, Equation

[14] Note that we express Equation (A.23) using a total derivative and not a partial derivative. This notation accounts for the fact that $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ may be related in equilibrium to τ_π.

[15] Note that $\mathbb{V}\mathrm{ar}\left[p \mid \theta_t\right] = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 \mathbb{V}\mathrm{ar}\left[\hat{p} \mid \theta_t\right]$, since the variance of the unbiased signal about the asset payoff can be expressed as $\mathbb{V}\mathrm{ar}\left[\hat{p} \mid \theta_t\right] = \tau_\eta^{-1} + \tau_\pi^{-1}$. We can thus interpret asset price volatility as the volatility of the unbiased signal about the asset payoff, corrected by investors' endogenous responses through the signal-to-price sensitivity.

(A.22) implies that there exists an inverse relation between both variables. Intuitively, when prices are very informative, the noise in the price is low and the conditional variance of the price for a given realization of the asset payoff is necessarily low.

It is worth highlighting that part b) of Proposition 7 is not a comparative statics exercise, but a characterization of a relation between two endogenous variables that must be satisfied in any equilibrium, given the economy's parameters. There are scenarios in which changes in some primitives do not shift the locus defined in Equation (A.22). In those cases, Equation (A.22) can be interpreted as the possible combinations of $\mathcal{V}$ and τ_π that can arise in equilibrium for different values of those primitives. In those scenarios, Proposition 7 implies that equilibria with high volatility are also equilibria with high (low) price informativeness whenever $\frac{d \log \mathcal{V}}{d \log \tau_\pi} > 0\,(< 0)$. However, changes in parameters that shift the locus defined in Equation (A.22) entail a shift of the volatility-informativeness relation and, in general, also a movement along the curve. Therefore, it is necessary to determine how $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ and τ_π are related in equilibrium as a function of the model's parameters to further understand the relation between price informativeness and price volatility.

Before we study the link between $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ and the model's primitives in more detail, it is worth emphasizing that the volatility-informativeness relation can only have a positive slope when investors learn from asset prices. When investors do not learn from prices, changes in the level of price informativeness do not affect investors' behavior, so ${}^{d \log\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)}/_{d \log(\tau_\pi)} = 0$. In this case, only the noise-reduction channel is active, and the relation between price informativeness and price volatility in Equation (A.22) is monotonic and decreasing. However, as we show next, in the CARA-Gaussian case $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ is increasing in τ_π, so the equilibrium-learning channel and the noise-reduction channel operate in opposite directions.

F Volatility-Informativeness Relation: CARA-Normal Applications

In this section, we specialize our results to a canonical CARA-Gaussian environment with general aggregate noise, which endogenously satisfies Assumptions 1 and 2. This allows us to further characterize the relation between the signal-to-price sensitivity $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ and price informativeness τ_π. We then provide two alternative sources of aggregate noise to show the robustness of our results. We first present an application in which there is a finite number of strategic investors with heterogeneous beliefs. Second, we consider the case in which the investors private trading needs are giving by heterogeneous hedging needs that are random in the aggregate.

F.1 General Aggregate Noise

Timing and assets. Time is discrete, with dates denoted by $t = 0, 1, 2, \ldots, \infty$. There are two traded assets: a riskless asset in perfectly elastic supply with gross return $R = 1$ and a risky asset in fixed supply Q, which is traded at a price p_t at date t.

Preferences. A new set of investors, indexed by $i \in I$, is born at each date t. Investors born at date t trade at date t and consume their terminal wealth at date $t+1$. Each generation of investors lives two dates and has constant absolute risk aversion (CARA) preferences over terminal wealth. The flow utility of an investor i born at date t is given by

$$U\left(w_{1,t}^i\right) = -e^{-\gamma w_{1,t}^i}, \tag{A.24}$$

where Equation (A.24) imposes that investors consume all their terminal wealth $w_{1,t}^i$. The parameter $\gamma > 0$ represents the coefficient of absolute risk aversion, $\gamma \equiv -\frac{U''}{U'}$.

Payoff process and signals. The asset payoff is given by

$$\theta_{t+1} = \mu_\theta + \rho\theta_t + \eta_t,$$

where μ_θ is a scalar, $|\rho| \leq 1$, and $\theta_0 = 0$, and where the innovations to the payoff, η_t, have mean zero, finite variance, τ_η^{-1}, and are independently and normally distributed. Before trading in at date t, each investor i observes the current realized asset payoff θ_t. Each investor i receives a private signal s_t^i about the innovation to the asset payoff η_t, given by

$$s_t^i = \eta_t + \varepsilon_{st}^i \quad \text{with} \quad \varepsilon_{st}^i \sim N\left(0, \tau_s^{-1}\right).$$

Private trading needs. As before, the investors' privately observed trading motives are sources of aggregate noise in the economy that prevent the price from being fully revealing. In particular, every investor i privately observes n_t^i, which takes the form

$$n_t^i = n_t + \varepsilon_{nt}^i, \quad \text{with} \quad \varepsilon_{nt}^i \sim N\left(0, \tau_\varepsilon^{-1}\right),$$

where $n_t \sim N\left(\mu_n, \tau_n^{-1}\right)$, which can be interpreted as the aggregate sentiment in the economy, is orthogonal to ε_{nt}^i. We assume that the private trading needs of the investor are orthogonal to the asset payoff and that all error terms are independent of each other, of the common component of the private trading needs, and of the innovation to the asset payoff.

In the CARA-Gaussian setup presented in this section, all equilibria in linear strategies satisfy Assumption 2. As it is standard in this body of work, we focus on symmetric equilibria in linear strategies.[16] Investors in the model have more information than external observers because they receive a private signal about η_t and they observe their private trading need. For example, investors could learn about the aggregate noise in the price from their private trading need. If an investor's private trading need were perfectly informative about the aggregate trading need in the economy, the investor could perfectly observe the asset payoff by looking at the equilibrium price. Therefore, the amount of information that is contained in the price from an internal investor's perspective, which determines the investor's equilibrium-learning, may differ from the informational content of prices from an external observer's point of view. To account for this discrepancy, we introduce the notion of internal price informativeness.

Definition 3. (Internal price informativeness) *We define internal price informativeness as the precision of the additional information contained in the unbiased signal of the innovation to the asset payoff η_t contained in the asset price, π_t as defined in Equation (A.20), from the perspective of an investor in the model. Formally, we define internal price informativeness as*

$$\tau_\pi^I \equiv \left(\mathbb{V}ar\left[\pi_t | \eta_t, \theta_t, n_t^i\right]\right)^{-1}. \tag{A.25}$$

The notion of internal price informativeness becomes relevant in models in which investors' private trading needs are informative about the aggregate noise in the price, and in strategic environments. In the first case, internal price informativeness is higher than price informativeness for an external observer, since investors have additional information about the noise. In the second case, internal price informativeness is lower than price informativeness for an external observer. The new information contained in the price aggregates the signals of all investors from an external observer's perspective. Since one of these signals is the private signal observed by the investor, the price contains one new signal fewer for an strategic investor than for an external observer.

[16] To ease the exposition, we describe our results in the text as if the model had a unique equilibrium, although we consider the possibility of multiplicity in the Appendix. If there were multiple equilibria, our analysis would be valid for locally stable equilibria as long as the economy does not jump from one equilibrium to another.

Using the first order condition of the investor's problem in Equation (A.1) and using the linear guess for the net demand and matching coefficients we have that the equilibrium demand sensitivities are given by

$$\begin{aligned}\overline{\alpha_s} &= \frac{1}{\kappa\left(1-\rho\right)}\frac{\tau_s}{\tau_{\eta|s,\pi}}, \quad \overline{\alpha_p} = \frac{1}{\kappa}\frac{\tau_s}{\tau_s+\tau_\pi^I}, \\ \overline{\alpha_n\alpha_\theta} &= \frac{\chi}{\kappa\left(1-\rho\right)}\frac{\tau_s}{\tau_s+\tau_\pi^I\frac{\hat{\tau}_\varepsilon}{\hat{\tau}_\varepsilon+\tau_n}}, \quad \text{and} \quad = \frac{\rho}{\kappa\left(1-\rho\right)}\frac{\tau_s}{\tau_s+\tau_\pi^I},\end{aligned} \tag{A.26}$$

where

$$\kappa \equiv \gamma\left(\left(1+\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)^2 \mathbb{V}ar\left[\eta_t|\mathcal{I}_t^i\right] + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 \mathbb{V}ar\left[\eta_{t+1}\right] + \left(\frac{\overline{\alpha_\eta}}{\overline{\alpha_p}}\right)^2 \tau_n^{-1}\right)$$

and where we denote by χ the loading of the private trading need on the investors' utility, which will vary across applications, and $\hat{\tau}_\varepsilon$ is the precision of the investors' private trading need as a signal of the aggregate noise contained in the price.

Note that the equilibrium coefficients are expressed as a function of internal price informativeness, τ_π^I, so we need to further understand the relation between internal and external price informativeness to fully characterize the volatility-informativeness relation in Equation (A.22). We do so in the following Lemma.

Lemma 3. (Relating internal price informativeness and price informativeness for an external observer) *In the CARA-Gaussian setup, there exists a scalar $\lambda > 0$ that can be expressed exclusively in terms of model primitives, such that*

$$\tau_\pi^I = \lambda\tau_\pi,$$

where τ_π and τ_π^I are respectively defined in Equations (A.21) and (A.25).

Lemma 3 shows that both notions of informativeness are related in this setup. Intuitively, when there is a continuum of investors and investors' private trading needs reveal information about the aggregate noise, $\lambda > 1$ and $\tau_\pi < \tau_\pi^I$. If investors do not learn about the aggregate sources of noise from their own private trading needs, then $\tau_\pi^I = \tau_\pi$, as in the applications with heterogeneous priors in Section 2 in the main body of the paper. Alternatively, when there is a finite number of strategic investors N, investors perceive the price to be less informative than an external observer, because the price aggregates N new signals for an external observer, while for an investor in the model it only aggregates $N-1$ new signals, so $\lambda < 1$ and $\tau_\pi > \tau_\pi^I$.

Combining the equilibrium characterization and 3, we specialize the volatility-informativeness relation between price volatility and price informativeness to the CARA-Gaussian environment in the following Lemma.

Lemma 4. (Volatility-informativeness relation CARA-Gaussian setup) *In the CARA-Gaussian setup, the volatility-informativeness relation between price volatility $\mathcal{V}$ and price informativeness τ_π is given by*

$$\mathcal{V} = \left(\frac{1}{1-R^{-1}\rho}\frac{\tau_s+\lambda\tau_\pi}{\tau_\eta+\tau_s+\lambda\tau_\pi}\right)^2\left(\tau_\eta^{-1}+\tau_\pi^{-1}\right), \tag{A.27}$$

where $\lambda = \frac{\tau_\pi^I}{\tau_\pi}$.

Lemma 4 represents the endogenous relation between $\mathcal{V}$ and τ_π as a function of only three (combinations of) primitives: τ_η, τ_s, and λ, which allows us to explicitly characterize the properties of the volatility-informativeness relation.

Whether the relation between price volatility and price informativeness in Equation (A.22) is monotonic depends on the value of λ. In particular, when $\lambda < 2$, which encompasses the scenario in which internal and external price informativeness are equal, the volatility-informativeness relation is non-monotonic. The variable λ represents how much more new information is contained in the price for an investor relative to an external observer. If $\lambda > 2$, the investor learns more than twice as much as an external observer by using the price as a signal. Although one could argue that active investors may have better information about the noise embedded in asset prices, hence learning more from the price than external observers, it is not easy to argue why there should be a two-fold difference between both groups. In fact, most models considered in the literature on learning in financial markets (e.g., Veldkamp (2011) and Vives (2016)) implicitly adopt parameterizations that imply $\lambda = 1$. In two of our three applications, λ is also weakly less than one. Therefore, in what follows, we focus on and state our formal results for the case $\lambda < 2$.[17]

We formally show that the volatility-informativeness relation is decreasing for sufficiently low values of τ_π and increasing for sufficiently high values of τ_π. The following proposition formalizes this non-monotonicity.

Proposition 8. (Slope of volatility-informativeness relation) *The volatility-informativeness relation between price volatility and price informativeness is increasing (decreasing) if and only if price informativeness is high (low) enough. Formally, there exists a threshold $\tau^\star > 0$ such that*

$$\frac{d\mathcal{V}}{d\tau_\pi} < 0 \quad \iff \quad \tau_\pi < \tau^\star \quad \text{and} \quad \frac{d\mathcal{V}}{d\tau_\pi} > 0 \quad \iff \quad \tau_\pi > \tau^\star,$$

where

$$\tau^\star \equiv \frac{-\lambda\left(\tau_\eta - 2\tau_s\right) + \sqrt{\lambda\left(\lambda\tau_\eta\left(\tau_\eta - 8\tau_s\right) + 8\tau_s\left(\tau_\eta + \tau_s\right)\right)}}{2\left(2-\lambda\right)\lambda}. \tag{A.28}$$

Proposition 8 shows that, regardless of the source of noise in the model, the slope of Equation (A.27) is positive when τ_π is sufficiently large and negative otherwise. The threshold $\tau^\star$, which determines the lower boundary of the positive slope region, only depends on the precision of the innovation to the asset payoff, the precision of the private signal, and the value of λ. Interestingly, the threshold $\tau^\star$ only depends on the remaining model parameters indirectly through λ. In particular, the specific source of noise may only affect $\tau^\star$ through λ. Exploiting our two-channel decomposition, we say that when prices are sufficiently informative, when $\tau_\pi > \tau^\star$, the equilibrium-learning channel dominates the noise-reduction channel. On the contrary, when $\tau_\pi < \tau^\star$, the noise-reduction channel dominates the equilibrium-learning channel.

Proposition 8 implies that any change among the subset of parameters that do not enter the volatility-informativeness relation directly must induce a positive comovement between price informativeness and volatility when prices are sufficiently informative and a negative comovement otherwise. When interpreted through the lens of our two-channel decomposition, when prices are sufficiently informative, the equilibrium-learning channel, which is driven by the change in investors' equilibrium behavior induced by learning, becomes overwhelmingly important and dominates the noise-reduction channel, and vice versa. Proposition 8 also implies that to fully characterize the relation between price informativeness and price volatility across equilibria whenever there is a change among the subset of parameters that at the same time shifts the volatility-informativeness relation upwards or downwards and increases or decreases price informativeness, it is necessary to look at fully specified models in which the source of noise is explicitly modeled. In the following subsections, we consider two alternative ways of modeling aggregate noise to the one in the main body of the paper: a finite number of investors and heterogeneous hedging needs.

F.2 Strategic Traders

While the application with heterogeneous beliefs in the main body of the paper features a continuum of price-taking investors, we now allow for strategic behavior.[18] We specialize the environment presented in

[17] In previous versions of the paper, we also studied the case of classic noise traders, which also features $\lambda = 1$.

[18] There is a large literature that explores the strategic behavior in models of trading with dispersed information, following Kyle (1985). See Du and Zhu (2017), who explore the optimal frequency of trading, and Kacperczyk, Nosal and Sundaresan (2020), who theoretically analyze the relation between market power and price informativeness, for recent contributions

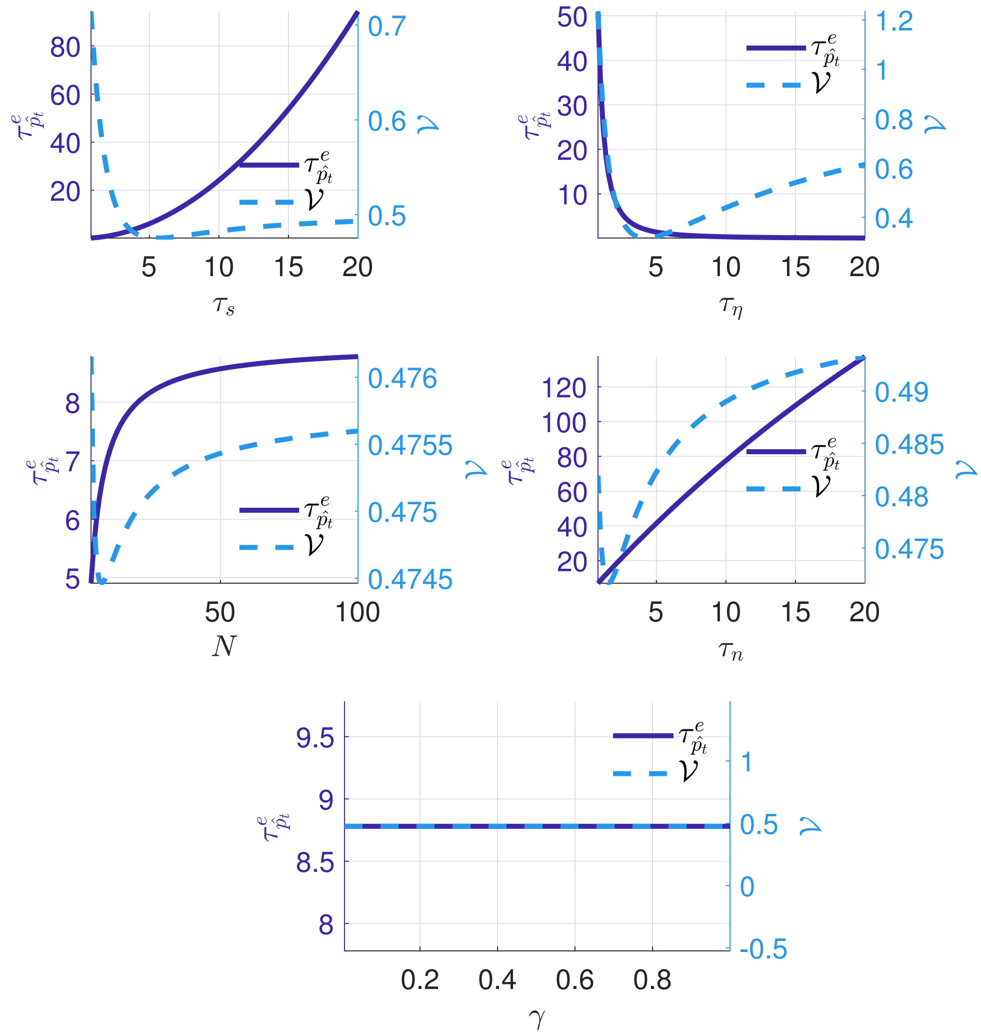

Figure OA-1: Comparative statics: Strategic Traders

Note: Figure OA-1 shows comparative statics of price informativeness τ_π and price volatility $\mathcal{V} = \mathbb{V}ar\left[p_t | \theta_t\right]$ as a function of all five primitives of the model considered in Application 2. All plots feature two y-axes: the left y-axis corresponds to the values of τ_π, while the right y-axis corresponds to the values of $\mathcal{V} = \mathbb{V}ar\left[p_t | \theta_t\right]$. The parameters of this model are the following: τ_s, precision of private signals about the innovation to the asset payoff, τ_η, precision of the innovation to the asset payoff, N, number of investors, τ_n, precision of aggregate noise, and γ, risk aversion. The reference values are $\tau_s = 6$, $\tau_\eta = 2$, $\tau_n = 1$, $\gamma = 0.5$, and $N = 100$.

the previous section to a finite number of investors, N, who have heterogeneous priors over the value of the asset.[19] In particular, from the perspective of investor i, the asset payoff θ is distributed according to

$$\theta_{t+1} = \mu_\theta + \rho\theta_t + \eta_t,$$

where $\theta_0 = 0$,

$$\eta_t \sim_i N\left(\overline{\eta}_t^i, \tau_\eta^{-1}\right), \quad \text{and} \quad \overline{\eta}_t^i \overset{\text{i.i.d}}{\sim} N\left(0, (N+1)\,\tau_n^{-1}\right),$$

where the variance of noise increases with the number of investors to ensure the economy converges to the competitive economy in the main body of the paper. In a symmetric equilibrium in linear strategies, we postulate net demand functions given by

$$\Delta q_t^i = \alpha_s s_t^i + \alpha_\theta \theta_t + \alpha_n \overline{\eta}_t^i - \alpha_p p_t + \psi,$$

where α_s, α_θ, α_n, and α_p are positive scalars, while ψ can take positive or negative values. Market clearing in the asset market implies that the equilibrium price takes the form

$$p_t = \frac{\alpha_s}{\alpha_p}\left(\eta_t + \frac{\sum_{i=1}^N \varepsilon_{st}^i}{N}\right) + \frac{\alpha_\theta}{\alpha_p}\theta_t + \frac{\alpha_\eta}{\alpha_p}\frac{\sum_{i=1}^N \overline{\eta}_t^i}{N} + \frac{\psi}{\alpha_p},$$

In this case, the price is not fully revealing because the noise contained in the signals on which the investors' trade and the noise contained in the investors' priors do not wash out in the aggregate. There is aggregate uncertainty coming from the realized signals and priors.

The equilibrium demand sensitivities are given by Equations (A.26) setting $\chi = \frac{\tau_\eta}{\tau_{\eta|s,p}}$, $\hat{\tau}_\varepsilon = 0$, $\lambda = \frac{N-1}{N}$, and $\kappa = \gamma \mathbb{V}\text{ar}\left[\theta_t | s_t^i, \pi_t\right] + \xi = \frac{\gamma}{\tau_\eta + \tau_s + \tau_\pi^I} + \xi$, where $\xi = \frac{1}{(N-1)\overline{\alpha_p}}$ is the price impact of an investor. In equilibrium, $\frac{\overline{\alpha_s}}{\alpha_n} = \frac{\tau_s}{\tau_\eta}$, price informativeness is given by

$$\tau_\pi = \frac{N}{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta} + N + 1}\left(\frac{\tau_s}{\tau_\eta}\right)^2 \tau_n, \tag{A.29}$$

and internal price informativeness is

$$\tau_\pi^I = \frac{N-1}{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta} + N + 1}\left(\frac{\tau_s}{\tau_\eta}\right)^2 \tau_n.$$

Therefore, the volatility-informativeness relation can expressed as

$$\mathcal{V} = \left(\frac{1}{1-\rho}\frac{\tau_s + \frac{N-1}{N}\tau_\pi}{\tau_\eta + \tau_s + \frac{N-1}{N}\tau_\pi}\right)^2 \left(\tau_\eta^{-1} + \tau_\pi^{-1}\right).$$

Note that when investors behave strategically, the price is more informative for an external observer than for an investor inside the model. The price aggregates the N signals received by the active investors, which is all new information for an external observer. However, since an investor already knows the realization of his own signal, from the investor's perspective the price conveys new information only about $N-1$ new signals.

Figure OA-1 shows the comparative statics of τ_π and $\mathcal{V}$ as a function of the five primitives of the model: τ_s, τ_η, N, τ_n, and γ. As in the disagreement model with a continuum of agents, price informativeness and volatility are invariant to the level of risk aversion γ. Figure OA-1 also illustrates the existence of

[19] The heterogeneity in beliefs introduces the additional trading motive needed to escape the no-trade theorem — see, for instance, Brunnermeier (2001).

positive and negative comovement regions, as established in Proposition 3. The intuition behind the results is identical to the one provided in the application with heterogeneous beliefs in the main body of the paper. Interestingly, changes in the value of aggregate noise τ_n induce a positive comovement between volatility and informativeness when price informativeness is high enough. On the other hand, when price informativeness is low enough and in the negative comovement region, volatility and informativeness move in different directions.

This application includes a new comparative static exercise on the number of investors. In this model, price informativeness is increasing in the number of investors N. However, price volatility is non-monotonic in the number of investors, initially decreasing in N in the negative comovement region, and finally increasing with N once price informativeness is sufficiently high. Finally, as in the main application in the paper, note that while price informativeness varies monotonically with all the parameters, price volatility is non-monotonic in changes on primitives, which is consistent with the existence of positive and negative comovement regions.

Lemma 5. (Comparative statics strategic traders) Price informativeness is increasing in τ_s, τ_n, and N and decreasing in τ_η.

Proof. From the definition of price informativeness in Equation (A.29) we have that

$$\frac{d\tau_\pi}{d\tau_s} = N\frac{1}{\tau_\eta^2}\frac{2\tau_s\left(\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1\right)-\tau_s\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}}{\left(\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1\right)^2}\tau_n = N\frac{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+2\left(N+1\right)}{\left(\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1\right)^2}\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta} > 0.$$

Moreover,

$$\frac{d\tau_\pi}{d\tau_n} = \frac{N\left(N+1\right)}{\left(\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1\right)^2}\left(\frac{\tau_s}{\tau_\eta}\right)^2 > 0,$$

$$\frac{d\tau_\pi}{dN} = \frac{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+1}{\left(\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1\right)^2}\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n > 0,$$

and

$$\frac{d\tau_\pi}{d\tau_\eta} = -\frac{2N\left(N+1\right)}{\left(\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1\right)^2}\frac{\tau_s^2}{\tau_\eta^2}\frac{\tau_n}{\tau_\eta} < 0.$$

□

Proposition 9. (Comovement strategic traders) *a) Price volatility and price informativeness positively comove (weakly) across equilibria if price informativeness is high enough. Formally, there exists $\overline{\tau} \in (\tau^\star, \infty)$ such that if $\tau_\pi > \overline{\tau}$, $\mathcal{V}$ and τ_π move in the same direction after any parameter change.*

b) Price volatility and price informativeness negatively comove (weakly) across equilibria if price informativeness is low enough. Formally, there exists a threshold $\underline{\tau} \in [0, \tau^\star]$ such that, if $\tau_\pi < \underline{\tau}$, $\mathcal{V}$ and τ_π move in opposite directions after any parameter change.

Proof. Note that

$$\frac{d\tau_\pi}{d\rho} = 0 \quad \text{and} \quad \frac{d\mathcal{V}}{d\rho} > 0,$$

so the comovements in this proposition are weak. Moreover,

$$\frac{d\tau_\pi}{d\tau_n} < 0 \quad \text{and} \quad \frac{d\mathcal{V}}{d\tau_n} = 0.$$

Hence, movements in τ_n will induce positive (negative) comovement between price informativeness and price volatility if and only if $\tau_\pi > (<)\, \tau^\star$.

a) **Positive comovement**

From Lemma 5 and using that

$$\frac{\overline{\alpha_s}}{\overline{\alpha_p}} = \frac{1}{1-\rho}\frac{\tau_s + \frac{N-1}{N}\tau_\pi}{\tau_\eta + \tau_s + \frac{N-1}{N}\tau_\pi},$$

we have

$$\begin{aligned}\frac{d\tau_\pi}{d\tau_s} &> 0, \quad \frac{d\tau_\pi}{d\tau_\eta} < 0, \quad \frac{d\tau_\pi}{dN} > 0 \quad \text{and,}\\ \frac{d\mathcal{V}}{d\tau_s} &> 0, \quad \frac{d\mathcal{V}}{d\tau_\eta} < 0, \quad \frac{d\mathcal{V}}{dN} > 0.\end{aligned}$$

Therefore, whenever $\tau_\pi > \tau^\star$, price informativeness and price volatility positively comove.

b) **Negative comovement**

For changes in τ_s we have

$$\frac{d\mathcal{V}}{d\tau_s} = \frac{\partial\mathcal{V}}{\partial\tau_s} + \frac{d\mathcal{V}}{d\tau_\pi}\frac{d\tau_\pi}{d\tau_s},$$

where we can write

$$\begin{aligned}\frac{d\mathcal{V}}{d\tau_s} &= 2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-\rho}\frac{\tau_\eta}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_\pi\right)^2}\left(\frac{\tau_\eta+\tau_\pi}{\tau_\eta\tau_\pi}\right)\\ &+\left(2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-\rho}\frac{\frac{N-1}{N}\tau_\eta}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_\pi\right)^2}\left(\frac{\tau_\eta+\tau_\pi}{\tau_\eta\tau_\pi}\right)-\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2(\tau_\pi)^{-2}\right)N\frac{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+2(N+1)}{\left(\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1\right)^2}\tau_n\end{aligned}$$

$$\frac{d\mathcal{V}}{d\tau_s} = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-\rho}\frac{(\tau_\pi)^{-2}}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_\pi\right)^2}\left(\begin{array}{c}2(\tau_\eta+\tau_\pi)\tau_\pi+\\ \left(2\frac{N-1}{N}(\tau_\eta+\tau_\pi)\tau_\pi-\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_\pi\right)\left(\tau_s+\frac{N-1}{N}\tau_\pi\right)\right)N\frac{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+2(N+1)}{\left(\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1\right)^2}\tau_n\end{array}\right)$$

Using the definition of τ_π we have $\lim_{\tau_s\to 0}\tau_\pi = 0$. Then, taking limits when $\tau_s \to 0$, we have

$$\lim_{\tau_s\to 0}\frac{d\mathcal{V}}{d\tau_s} = \lim_{\tau_s\to 0}\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-\rho}\frac{(\tau_\pi)^{-2}}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_\pi\right)^2}\left(-\tau_\eta N\frac{2(N+1)}{(N+1)^2}\right) = -\infty.$$

Then there exists a threshold $\hat{s}$ such that $\frac{d\mathcal{V}}{d\tau_s} < 0$ for $\tau_s < \hat{s}$, which implies there exists a threshold τ_{τ_s} such that for $\tau_\pi < \tau_{\tau_s}$ price volatility and price informativeness negatively comove.

For changes in τ_η we have

$$\begin{aligned}\frac{d\mathcal{V}}{d\tau_\eta} &= -\frac{1}{\tau_\eta}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\left(\frac{2}{\tau_\eta+\tau_s+\frac{N-1}{N}\tau_\pi}\frac{\tau_\eta+\tau_\pi}{\tau_\pi}+\frac{1}{\tau_\eta}\right)-\left(\begin{array}{c}2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-R^{-1}\rho}\frac{\frac{N}{N-1}\tau_\eta}{\left(\tau_\eta+\tau_s+\frac{N}{N-1}\tau_\pi\right)^2}\frac{\tau_\eta+\tau_\pi}{\tau_\eta\tau_\pi}-\\ \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2(\tau_\pi)^{-2}\end{array}\right)\frac{2N(N+1)}{\left(\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1\right)^2}\frac{\tau_s^2}{\tau_\eta^2}\frac{\tau_n}{\tau_\eta}\\ &= -2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-\rho}\frac{\tau_\eta\tau_\pi^{-1}}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_\pi\right)^2}\left(\begin{array}{c}\left(\frac{\tau_s}{\tau_\eta}+\frac{N-1}{N}\frac{\tau_\pi}{\tau_\eta}\right)\left(1+\frac{\tau_\pi}{\tau_\eta}\right)+\\ \left(2\frac{N}{N-1}\left(1+\frac{\tau_\pi}{\tau_\eta}\right)-\left(1+\frac{\tau_s}{\tau_\eta}+\frac{N}{N-1}\frac{\tau_\pi}{\tau_\eta}\right)\tau_\pi^{-1}\right)\frac{N(N+1)}{(\tau_s\tau_n+N+1)^2}\tau_s^2\frac{\tau_n}{\tau_\eta}\end{array}\right)-\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{\tau_\eta}\right)^2\end{aligned}$$

Using the definition of τ_π we have $\lim_{\tau_\eta\to\infty}\tau_\pi = 0$. Hence, taking limits when $\tau_\eta \to \infty$ we have

$$\lim_{\tau_\eta\to\infty}\frac{d\mathcal{V}}{d\tau_\eta} = \lim_{\tau_\eta\to\infty}2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-\rho}\frac{(\tau_\pi)^{-2}}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_\pi\right)^2}\frac{N(N+1)}{(\tau_s\tau_n+N+1)^2}\tau_s^2\tau_n = \infty$$

since

$$\lim_{\tau_\eta\to\infty}\frac{(\tau_\pi)^{-2}}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_\pi\right)^2}=\lim_{\tau_\eta\to\infty}\left(\frac{N}{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1}\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_\pi\right)\right)^{-2}=\infty.$$

Hence, there exists a threshold $\overline{\eta}$ such that for all $\tau_\eta>\overline{\eta}$ we have $\frac{d\mathcal{V}}{d\tau_\eta}>0$. Then, there exists a threshold τ_{τ_η} such that for all $\tau_\pi<\tau_{\tau_\eta}$ informativeness and volatility negatively comove after changes in τ_η where

$$\tau_{\tau_\eta}\equiv\frac{N}{\tau_s\tau_n+(N+1)\,\overline{\eta}^2}(\tau_s)^2\tau_n.$$

For changes in N we have

$$\begin{aligned}\frac{d\mathcal{V}}{dN}&=2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-\rho}\frac{\tau_\eta\tau_\pi\left(\frac{1}{N^2}\right)}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_\pi\right)^2}\left(\frac{\tau_\eta+\tau_\pi}{\tau_\eta\tau_\pi}\right)\\&\quad+\left(2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-\rho}\frac{\frac{N-1}{N}\tau_\eta}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_\pi\right)^2}\left(\frac{\tau_\eta+\tau_\pi}{\tau_\eta\tau_\pi}\right)-\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2(\tau_\pi)^{-2}\right)\frac{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+1}{\left(\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1\right)^2}\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n\\\frac{d\mathcal{V}}{dN}&=\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-\rho}\frac{(\tau_\pi)^{-2}}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_\pi\right)^2}\left(\begin{array}{c}2(\tau_\eta+\tau_\pi)\left(\frac{\tau_\pi}{N}\right)^2+\\\left(2\frac{N-1}{N}(\tau_\eta+\tau_\pi)\tau_\pi-\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_\pi\right)\left(\tau_s+\frac{N-1}{N}\tau_\pi\right)\right)\frac{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+1}{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1}\frac{\tau_\pi}{N}\end{array}\right).\end{aligned}$$

Moreover,

$$\lim_{N\to0}\frac{d\mathcal{V}}{dN}=\lim_{N\to0}\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-\rho}\frac{(\tau_\pi)^{-2}\left(\frac{1}{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+1}\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n\right)^2}{\left(\tau_\eta+\tau_s+\frac{1}{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+1}\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n\right)^2}\left(-\frac{\left(\tau_\eta+\tau_s-\frac{1}{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+1}\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n\right)}{\frac{1}{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+1}\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n}\right)=-\infty$$

because $\lim_{N\to0}\tau_\pi=0$ and $\lim_{N\to0}\frac{\tau_\pi}{N}=\frac{1}{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+1}\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n$. Hence, there exists a threshold $\underline{N}$ such that for all $N<\underline{N}$ we have $\frac{d\mathcal{V}}{dN}<0$. This implies that there exists a threshold τ_N such that for all $\tau_\pi<\tau_N$ informativeness and volatility negatively comove after changes in N. Note that for an equilibrium to exists we need $N\geq3$. The threshold $\underline{N}$ depends on all other parameters in the economy. When τ_π is low, $\underline{N}$ is larger. Moreover, for all N there exist parameters such that $\frac{d\mathcal{V}}{dN}<0$ when τ_π is low enough.

Therefore, for all $\tau_\pi<\underline{\tau}$ informativeness and volatility (weakly) negatively comove for any parameter change, where $\underline{\tau}=\min\left\{\tau_{\tau_s},\tau_{\tau_\eta},\tau_N\right\}$ since $\max\left\{\tau_{\tau_s},\tau_{\tau_\eta},\tau_N\right\}<\tau^\star$. □

F.3 Hedging Needs

In this second application, we use aggregate hedging needs as an alternative formulation for investors' private trading needs. In particular, we assume that the asset payoff has both a learnable and an unlearnable component. Formally, we assume that

$$\theta_{t+1}=\mu_\theta+\rho\theta_t+\eta_t,$$

where $\theta_0=0$,

$$\eta_t=\eta_t^l+\eta_t^u,$$

and

$$\eta_t^l\sim N\left(0,\tau_\eta^{-1}\right)\quad\text{and}\quad\eta_t^u\sim N\left(\overline{\eta},\tau_u^{-1}\right).$$

The random variables η_t^u and η_t^l, which represent the unlearnable and learnable components of the innovation to the asset payoff, are orthogonal to each other. The realized asset payoff θ_t is observable at t.

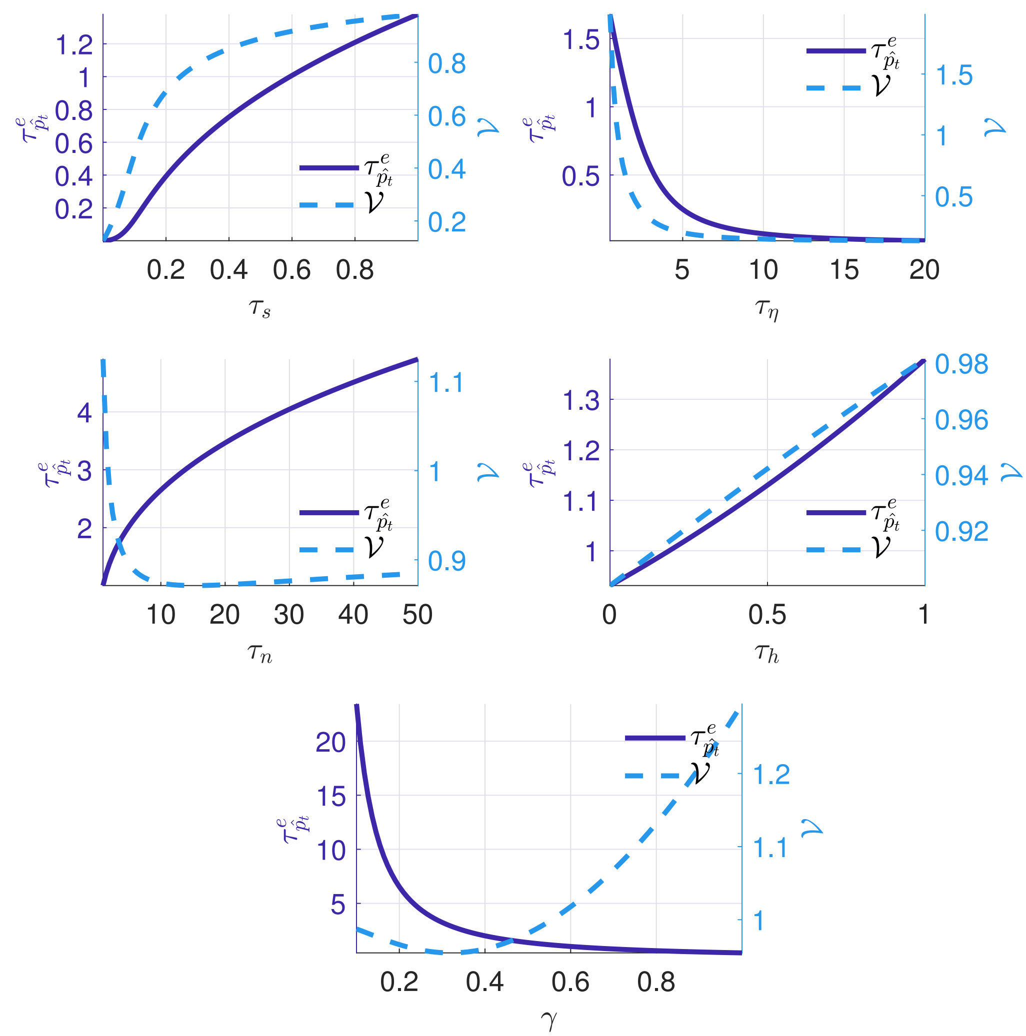

Figure OA-2: Comparative statics: Hedging Needs

Note: Figure OA-2 shows comparative statics of price informativeness τ_π and price volatility $\mathcal{V} = \mathbb{V}ar\left[p_t | \theta_t\right]$ as a function of all five primitives of the model considered in Application 3. All plots feature two y-axes: the left y-axis corresponds to the values of τ_π, while the right y-axis corresponds to the values of $\mathcal{V} = \mathbb{V}ar\left[p_t | \theta_t\right]$. The parameters of this model are the following: τ_s; precision of private signals about the innovation to the asset payoff; τ_η, precision of the innovation to the asset payoff, τ_n; precision of aggregate hedging term, τ_h; precision of individual hedging need; and γ, investors' coefficient of absolute risk aversion. The reference values are $\tau_s = 1$, $\tau_\eta = 1$, $\tau_n = 2$, $\tau_h = 1$, and $\gamma = 0.5$.

We further assume that investors born in generation t have an endowment ω_{t+1}^i realized at date $t+1$ which is potentially correlated with the unlearnable component of the asset payoff η_{t+1}^u and is independent of the learnable component. Investors' hedging needs, given by the correlation between the investors' endowment and the asset payoff, are given by a random variable h_t^i, which is distributed as follows

$$h_t^i \equiv \mathbb{C}ov\left(\theta_{t+1}, \omega_{t+1}^i \middle| \theta_t\right) = \mathbb{C}ov\left(\eta_t^u, \omega_{t+1}^i\right) = n_t + \varepsilon_{ht}^i,$$

where

$$n_t \sim N\left(0, \tau_n^{-1}\right) \quad \text{and} \quad \varepsilon_{ht}^i \overset{iid}{\sim} N\left(\overline{\theta}, \tau_h^{-1}\right).$$

The hedging needs h_t^i are private information of investor i and have two components: an aggregate component n_t^i and an idiosyncratic component ε_{ht}^i. Investors only observe their total hedging need h_t^i and cannot distinguish between the aggregate and idiosyncratic components.

Investors receive a private signal of the learnable component of the asset's payoff

$$s_t^i = \eta_t^l + \varepsilon_{st}^i, \quad \text{with} \quad \varepsilon_{st}^i \overset{iid}{\sim} N\left(\overline{\theta}, \tau_s^{-1}\right).$$

Depending on parameters, this model can potentially feature multiple equilibria, as described in detail in Dávila and Parlatore (2021). Consistent with our definition of λ in Lemma 3, in this application,

$$\lambda = \frac{\tau_h + \tau_n}{\tau_n} > 1,$$

so the volatility-informativeness relation can be expressed as

$$\mathcal{V} = \left(\frac{1}{1-\rho} \frac{\tau_s + \frac{\tau_n + \tau_h}{\tau_n}\tau_\pi}{\tau_\eta + \tau_s + \frac{\tau_n + \tau_h}{\tau_n}\tau_\pi}\right)^2 \left(\tau_\eta^{-1} + \tau_\pi^{-1}\right).$$

Note that when investors are aware that the source of aggregate noise has a common component, the price is less informative for an external observer than for an individual investor. In this case, an individual investor can use the realization of his hedging need to partially infer the level of aggregate hedging needs, which allows him to better filter the information conveyed by the price.

Figure OA-2 shows the comparative statics of τ_π and $\mathcal{V}$ as a function of the five primitives of the model: τ_s, τ_η, τ_n, τ_h, and γ. In this model, all five primitives determine the equilibrium values of τ_π and $\mathcal{V}$. As in the previous applications, and consistently with Proposition 4, Figure OA-2 shows that, when price informativeness is high enough, changes in τ_s, τ_η, τ_h, τ_n, and γ move price volatility and price informativeness in the same direction. Interestingly, a negative comovement region between price volatility and informativeness does not exist in this application, that is, the threshold $\underline{\tau}$, defined in Proposition 4, is equal to zero. Figure OA-2 shows that even when price informativeness is arbitrarily small, changes in parameters other than τ_n and γ imply a positive comovement between volatility and informativeness.

In this case, $\lambda = \frac{\tau_n + \tau_h}{\tau_n}$. Therefore, $\lambda < 2$ implies $\tau_h < \tau_n$. If $\tau_h < \tau_n$, there exists $\tau^\star$ such that

$$\frac{d\mathcal{V}}{d\tau_\pi} > 0 \quad \forall \tau_\pi > \tau^\star.$$

Moreover, the equilibrium demand sensitivities are given by Equation (A.8), Equation (A.9), Equation (A.10), and Equation (A.11) setting $\pi = 1$ and $\hat{\tau}_\varepsilon = \tau_h$ and

$$\kappa = \gamma\left(\left(1 + \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)^2 \mathbb{V}ar\left[\eta_t^l | \mathcal{I}_t^i\right] + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 \mathbb{V}ar\left[\eta_{t+1}^l\right] + \left(\frac{\overline{\alpha_\eta}}{\overline{\alpha_p}}\right)^2 \tau_n^{-1} + \tau_u^{-1}\right).$$

Then, in equilibrium,

$$\tau_\pi = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2 \tau_n, \tag{A.30}$$

where $\frac{\overline{\alpha_s}}{\overline{\alpha_n}}$ solves the following fixed point:

$$J\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right) = \frac{\frac{\tau_s}{\tau_\eta+\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_\pi}}{\gamma - \frac{\frac{\tau_n+\tau_h}{\tau_n}\tau_\pi}{\tau_\eta+\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_\pi}\frac{\overline{\alpha_n}}{\overline{\alpha_s}}\frac{\tau_h}{\tau_n+\tau_h}}, \tag{A.31}$$

where we used the equilibrium demand sensitivities that depend on $\frac{\overline{\alpha_s}}{\overline{\alpha_n}}$ directly and through τ_π.[20] $J(x)$ determines the ratio $\frac{\overline{\alpha_s}}{\overline{\alpha_n}}$ when investors expect the signal-to-noise ratio in the price to be x. The fixed point of Equation (A.31) can also be found as the solution to

$$\hat{H}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right) \equiv -\gamma\left(\tau_n+\tau_h\right)\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^3 + \tau_h\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2 - \gamma\left(\tau_s+\tau_\eta\right)\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right) + \tau_s = 0. \tag{A.32}$$

The polynomial $\hat{H}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)$ always has a positive root but there may be multiple equilibria (generically, one or three).[21] We adopt a conventional notion of stability. The function $\hat{H}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)$ is defined such that if $\hat{H}(x_0) > 0$, then $J(x_0) > x_0$, which implies that if investors in the model expect the signal-to-noise ratio to be x_0, the realized value of this ratio will be $x_1 > x_0$. Let x^* be a solution to $\hat{H}(x^*) = 0$. Then, we will say that the equilibrium x^* is stable if for all $x_0 \in (x^* - \delta, x^* - \delta)$ for some $\delta > 0$, the sequence $\{x_m\}_{m=0}^{\infty}$ where $x_m = J(x_{m-1})$ for $m > 1$ converges to x^*. This sequence will converge only if $J'(x^*) < 1$, which is equivalent to $\hat{H}'(x^*) < 0$. Hence, in all stable equilibria, $\hat{H}'(x^*) < 0$. Finally, note that when $\tau_s = 0$, the only root of $\hat{H}(x^*)$ is at $x^* = 0$.

Lemma 6. (Comparative statics hedging needs) *In any stable equilibrium, the signal to noise ration $\frac{\overline{\alpha_s}}{\overline{\alpha_n}}$ increases with τ_s and τ_h and it decreases with τ_η, τ_n, and γ.*

Proof. From Equation (A.32) we have

$$\begin{aligned}
\frac{\partial \hat{H}}{\partial \tau_h} &= -\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^3 + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2 = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2\left(-\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)+1\right) > 0,\\
\frac{\partial \hat{H}}{\partial \tau_s} &= -\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right) + 1 > 0,\\
\frac{\partial \hat{H}}{\partial \gamma} &= -\left(\tau_n+\tau_h\right)\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^3 - \left(\tau_s+\tau_\eta\right)\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right) < 0, \quad \text{and}\\
\frac{\partial \hat{H}}{\partial \tau_\eta} &= -\gamma\frac{\overline{\alpha_s}}{\overline{\alpha_n}} < 0, \quad \text{and}\\
\frac{\partial \hat{H}}{\partial \tau_n} &= -\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^3 < 0,
\end{aligned}$$

[20] We could have alternatively adopted similar notions of trading with stochastic hedging needs, as in Ganguli and Yang (2009) and Manzano and Vives (2011).

[21] Goldstein, Li and Yang (2014) find that multiple equilibria may also arise when market segmentation leads to heterogeneous hedging needs.

since $\frac{\overline{\alpha_s}}{\overline{\alpha_n}} < \frac{1}{\gamma}$. Using the implicit function theorem and that in any stable equilibrium $\hat{H}' < 0$, we have

$$\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_h} > 0, \quad \frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_s} > 0, \quad \frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\gamma} < 0, \quad \frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_\eta} < 0, \quad \text{and} \quad \frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_n} < 0.$$

□

Proposition 10. (Comovement hedging needs) *a) Price volatility and price informativeness positively comove (weakly) across equilibria if price informativeness is high enough. Formally, there exists $\overline{\tau} \in (\tau^\star, \infty)$ such that if $\tau_\pi > \overline{\tau}$, $\mathcal{V}$ and τ_π move in the same direction after any parameter change.*

b) Price volatility and price informativeness negatively comove (weakly) across equilibria if price informativeness is low enough for changes in τ_n and γ. For changes in τ_s, τ_h, and τ_η price informativeness and price volatility always comove positively. Hence, there does not exist a negative comovement region.

Proof. Note that

$$\frac{d\tau_\pi}{d\rho} = 0 \quad \text{and} \quad \frac{\partial \mathcal{V}}{\partial \rho} > 0,$$

so the comovements in this proposition are weak. Moreover,

$$\frac{d\tau_\pi}{d\gamma} = 2\frac{\overline{\alpha_s}}{\overline{\alpha_n}} \frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\gamma} \tau_n < 0 \quad \text{and} \quad \frac{\partial \mathcal{V}}{\partial \gamma} = 0.$$

Hence, price informativeness and price volatility positively (negatively) comove after a change in γ if $\tau_\pi > (<) \tau^\star$.

a) **Positive comovement.** Using Lemma 6 and the definition of equilibrium price informativeness in Equation (A.30), we get

$$\frac{d\tau_\pi}{d\tau_s} = 2\frac{\overline{\alpha_s}}{\overline{\alpha_n}} \frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_s} \tau_n > 0$$

$$\frac{d\tau_\pi}{d\tau_h} = 2\frac{\overline{\alpha_s}}{\overline{\alpha_n}} \frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_h} \tau_n > 0$$

$$\frac{d\tau_\pi}{d\tau_\eta} = 2\frac{\overline{\alpha_s}}{\overline{\alpha_n}} \frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_\eta} \tau_n < 0.$$

Then, since

$$\mathcal{V} = \left(\frac{1}{1-\rho}\right)^2 \left(\frac{\tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_\pi}{\tau_\eta + \tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_\pi}\right)^2 \left(\tau_\eta^{-1} + \tau_\pi^{-1}\right),$$

we have

$$\frac{\partial \mathcal{V}}{\partial \tau_s} = \left(\frac{1}{1-\rho}\right)^2 2\frac{\left(\tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_\pi\right)\tau_\eta}{\left(\tau_\eta + \tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_\pi\right)^3} \left(\tau_\eta^{-1} + \tau_\pi^{-1}\right) > 0,$$

$$\frac{\partial \mathcal{V}}{\partial \tau_h} = \left(\frac{1}{1-\rho}\right)^2 2\frac{\left(\tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_\pi\right)\tau_\eta \frac{\tau_\pi}{\tau_n}}{\left(\tau_\eta + \tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_\pi\right)^3} \left(\tau_\eta^{-1} + \tau_\pi^{-1}\right) > 0, \quad \text{and}$$

$$\frac{\partial \mathcal{V}}{\partial \tau_\eta} = -\left(\frac{1}{1-\rho}\right)^2 \left(\frac{\tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_\pi}{\tau_\eta + \tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_\pi}\right)^2 \left(2\frac{\tau_\eta^{-1} + \tau_\pi^{-1}}{\tau_\eta + \tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_\pi} + \frac{1}{\tau_\eta^2}\right) < 0.$$

Hence, if $\tau_\pi > \tau^\star$ price volatility and price informativeness (weakly) comove when τ_s, τ_η, or τ_h change.

For changes in τ_n we have that

$$\begin{aligned}\frac{\partial\mathcal{V}}{\partial\tau_n} &= -\left(\frac{1}{1-\rho}\right)^2 2\frac{\left(\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_\pi\right)\tau_\eta\frac{\tau_h}{\tau_n^2}\tau_\pi}{\left(\tau_\eta+\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_\pi\right)^3}\left(\tau_\eta^{-1}+\tau_\pi^{-1}\right)\\ &= -\left(\frac{1}{1-\rho}\right)^2 2\frac{\left(\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_\pi\right)\tau_\eta\frac{\tau_h}{\tau_n^2}\tau_\pi}{\left(\tau_\eta+\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_\pi\right)^3}\left(\tau_\eta^{-1}+\tau_\pi^{-1}\right),\end{aligned}$$

which is increasing in τ_π with $\lim_{\tau_\pi\to\infty}\frac{\partial\mathcal{V}}{\partial\tau_n}=0$. Also,

$$\frac{\partial\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{\partial\tau_n}=\frac{\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^3}{-3\gamma\left(\tau_n+\tau_h\right)\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2+2\tau_h\frac{\overline{\alpha_s}}{\overline{\alpha_n}}-\gamma\left(\tau_s+\tau_\eta\right)}<0$$

since the denominator is negative in any stable equilibrium. Moreover,

$$\begin{aligned}\frac{d\tau_\pi}{d\tau_n} &= 2\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\frac{\partial\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{\partial\tau_n}\tau_n+\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2=\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2\left(2\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_n}\frac{\tau_n}{\frac{\overline{\alpha_s}}{\overline{\alpha_n}}}+1\right)\\ &= \left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2\left(\frac{2\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2\tau_n}{-3\gamma\left(1+\frac{\tau_h}{\tau_n}\right)\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2\tau_n+2\tau_h\frac{\overline{\alpha_s}}{\overline{\alpha_n}}-\gamma\left(\tau_s+\tau_\eta\right)}+1\right)\\ &= \left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2\left(\frac{-\gamma\left(1+3\frac{\tau_h}{\tau_n}\right)\tau_\pi+2\tau_h\frac{\overline{\alpha_s}}{\overline{\alpha_n}}-\gamma\left(\tau_s+\tau_\eta\right)}{-3\gamma\left(1+\frac{\tau_h}{\tau_n}\right)\tau_\pi+2\tau_h\frac{\overline{\alpha_s}}{\overline{\alpha_n}}-\gamma\left(\tau_s+\tau_\eta\right)}\right).\end{aligned}$$

Note that the numerator can be written as

$$-\gamma\left(1+3\frac{\tau_h}{\tau_n}\right)\tau_\pi+2\tau_h\frac{\overline{\alpha_s}}{\overline{\alpha_n}}-\gamma\left(\tau_s+\tau_\eta\right)=-\gamma\left(1+3\frac{\tau_h}{\tau_n}\right)\tau_\pi+2\tau_h\sqrt{\frac{\tau_\pi}{\tau_n}}-\gamma\left(\tau_s+\tau_\eta\right).$$

This is a concave quadratic function of $\sqrt{\tau_\pi}$ which is negative at $\sqrt{\tau_\pi}=0$. Then, if $4\frac{\tau_h^2}{\tau_n}-4\gamma^2\left(1+3\frac{\tau_h}{\tau_n}\right)\left(\tau_s+\tau_\eta\right)<0$ we have $\frac{d\tau_\pi}{d\tau_n}>0$ for all τ_π and if $4\frac{\tau_h^2}{\tau_n}-4\gamma^2\left(1+3\frac{\tau_h}{\tau_n}\right)\left(\tau_s+\tau_\eta\right)>0$ it is less than $\frac{d\tau_\pi}{d\tau_n}>0$ if

$$\tau_\pi>\left(\frac{-2\frac{\tau_h}{\sqrt{\tau_n}}-\sqrt{4\frac{\tau_h^2}{\tau_n}-4\gamma^2\left(1+3\frac{\tau_h}{\tau_n}\right)\left(\tau_s+\tau_\eta\right)}}{-2\gamma\left(1+3\frac{\tau_h}{\tau_n}\right)}\right)^2\equiv\overline{n},$$

or if

$$\tau_\pi<\left(\frac{-2\frac{\tau_h}{\sqrt{\tau_n}}+\sqrt{4\frac{\tau_h^2}{\tau_n}-4\gamma^2\left(1+3\frac{\tau_h}{\tau_n}\right)\left(\tau_s+\tau_\eta\right)}}{-2\gamma\left(1+3\frac{\tau_h}{\tau_n}\right)}\right)^2\equiv\underline{n}.$$

Then, when $\lambda<2$, there exists a threshold $\widetilde{\tau}$ such that

$$\frac{d\mathcal{V}}{d\tau_n}>0$$

for all $\tau_\pi > \widetilde{\tau}$, where $\widetilde{\tau} = \tau^\star$ if $4\frac{\tau_h^2}{\tau_n} - 4\gamma^2 \left(1 + 3\frac{\tau_h}{\tau_n}\right)(\tau_s + \tau_\eta) < 0$ and $\widetilde{\tau} = \max\{\tau^\star, \overline{n}\}$ otherwise.

Hence, if $\tau_\pi > \widetilde{\tau}$ price informativeness and price volatility weakly positively comove for any parameter change.

b) **Negative comovement**

For changes in τ_n we have from part a) of this proof that for $\tau_n < \underline{n}$, $\frac{d\tau_\pi}{d\tau_n} > 0$. Moreover, we know that $\frac{\partial \mathcal{V}}{\partial \tau_n} < 0$ and $\frac{d\mathcal{V}}{d\tau_\pi} < 0$ for all $\tau_\pi < \tau^\star$. Hence, since $\frac{d\mathcal{V}}{d\tau_n} = \frac{\partial \mathcal{V}}{\partial \tau_n} + \frac{d\mathcal{V}}{d\tau_\pi}\frac{d\tau_\pi}{d\tau_n}$, there exists a threshold $\tau_{\tau_n} < \tau^\star$ such that for all $\tau_\pi < \tau^\star$ we have $\frac{d\mathcal{V}}{d\tau_n} < 0$ and price informativeness and price volatility negatively comove when τ_n changes.

For changes in τ_s, we have

$$\frac{d\mathcal{V}}{d\tau_s} = 2\left(\frac{1}{1-\rho}\right)^2 \frac{\left(\tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_\pi\right)\tau_\pi^{-1}}{\left(\tau_\eta + \tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_\pi\right)^3} \left(\begin{array}{c} \tau_\eta + \tau_\pi + \\ \left(\frac{\tau_n+\tau_h}{\tau_n}\left(2 - \frac{\tau_n+\tau_h}{\tau_n}\right)\frac{(\tau_\pi)^2}{\tau_s} - \frac{\tau_n+\tau_h}{\tau_n}\frac{\tau_\pi}{\tau_s}(2\tau_s - \tau_\eta) - (\tau_\eta + \tau_s)\right)\frac{\tau_s}{\frac{\overline{\alpha_s}}{\overline{\alpha_n}}}\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_s} \end{array}\right).$$

We know that $\lim_{\tau_s\to 0}\frac{\overline{\alpha_s}}{\overline{\alpha_n}} = 0$. Moreover, using the definition of $\frac{\overline{\alpha_s}}{\overline{\alpha_n}}$ in Equation (A.32), we have that

$$\frac{\tau_s}{\frac{\overline{\alpha_s}}{\overline{\alpha_n}}} = \gamma\left(\tau_\eta + \tau_s + (\tau_n + \tau_h)\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2\right) - \tau_h\frac{\overline{\alpha_s}}{\overline{\alpha_n}},$$

which allows us to compute $\lim_{\tau_s\to 0}\frac{\tau_s}{\frac{\overline{\alpha_s}}{\overline{\alpha_n}}} = \gamma\tau_\eta$ and $\lim_{\tau_s\to 0}\frac{\tau_\pi}{\tau_s} = 0$. Therefore,

$$\lim_{\tau_s\to 0}\frac{d\mathcal{V}}{d\tau_s} = 2\left(\frac{1}{1-\rho}\right)^2 \frac{\tau_n^{-1}}{(\tau_\eta)^2}\left(\frac{\tau_n + \tau_h}{\tau_n} - \gamma^2\tau_\eta\right),$$

where we use the fact that $\lim_{\tau_s\to 0}\frac{\tau_s}{\frac{\overline{\alpha_s}}{\overline{\alpha_n}}}\frac{\partial\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{\partial\tau_s} = 1$. Hence, $\frac{d\mathcal{V}}{d\tau_s} > 0$ and price informativeness and price volatility positively comove for changes in τ_s.

For τ_η, we have

$$\begin{aligned}\frac{d\mathcal{V}}{d\tau_\eta} = &-\left(\frac{1}{1-\rho}\right)^2\left(\frac{\tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_\pi}{\tau_\eta + \tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_\pi}\right)^2\left(2\frac{\left(\tau_\eta^{-1} + \tau_\pi^{-1}\right)}{\left(\tau_\eta + \tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_\pi\right)} + \frac{1}{\tau_\eta^2}\right) \\ &+ \left(2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-\rho}\frac{\frac{\tau_n+\tau_h}{\tau_n}\tau_\eta}{\left(\tau_\eta + \tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_\pi\right)^2}\left(\frac{\tau_\eta + \tau_\pi}{\tau_\eta\tau_\pi}\right) - \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2(\tau_\pi)^{-2}\right)2\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\frac{\partial\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{\partial\tau_\eta}\tau_n.\end{aligned}$$

Since $\lim_{\tau_\eta\to\infty}\frac{\overline{\alpha_s}}{\overline{\alpha_n}} = 0$ and $\lim_{\tau_\eta\to\infty}\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\tau_\eta = \frac{\tau_s}{\gamma}$, it is the case that

$$\begin{aligned}\lim_{\tau_\eta\to\infty}\frac{d\mathcal{V}}{d\tau_\eta} &= \lim_{\tau_\eta\to\infty}\left(\frac{1}{1-\rho}\right)^2\frac{\tau_s}{(\tau_\eta + \tau_s)^3}\frac{1}{(\tau_\pi)^2\tau_\eta^2}\left(\left(-\tau_\eta^2\tau_s(\tau_\eta + \tau_s)\right)2\gamma\frac{\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2}{\left(-H'\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)\right)}\tau_n\right) \\ &= \lim_{\tau_\eta\to\infty}\left(\frac{1}{1-\rho}\right)^2\frac{\tau_s}{\left(1 + \frac{\tau_s}{\tau_\eta}\right)^3}\frac{1}{(\tau_\pi)^2\tau_\eta^2}\left(-\left(1 + \frac{\tau_s}{\tau_\eta}\right)\frac{2\tau_s\tau_n}{\tau_n + \tau_h}\right) = -\infty\end{aligned}$$

Then, $\tau_{\tau_\eta} = 0$ and when τ_η changes, volatility and informativeness always positively comove.

Finally, for changes in τ_h, we have

$$\frac{d\mathcal{V}}{d\tau_h} = \frac{2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}}{1-\rho}\frac{\tau_\pi^{-1}}{\left(\tau_\eta + \tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_\pi\right)^2}\left[\begin{array}{c} \frac{1}{\tau_n}(\tau_\eta + \tau_\pi)\tau_\pi + \\ \left(\frac{\tau_n+\tau_h}{\tau_n}\left(2 - \frac{\tau_n+\tau_h}{\tau_n}\right)(\tau_\pi)^2 + \frac{\tau_n+\tau_h}{\tau_n}(\tau_\eta - 2\tau_s)\tau_\pi - \tau_s(\tau_\eta + \tau_s)\right)\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\frac{\left(-\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)+1\right)}{\left(-H'\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)\right)} \end{array}\right]$$

and

$$\lim_{\tau_h \to 0} \frac{d\mathcal{V}}{d\tau_h} = \left(\frac{1}{1-\rho}\right)^2 \frac{2\left(\tau_s+\tau_\pi\right)}{\left(\tau_\eta+\tau_s+\tau_\pi\right)^3}\tau_\pi^{-1}\left(\frac{1}{\tau_n}\left(\tau_\eta+\tau_\pi\right)\tau_\pi+\left(\left(\tau_\pi\right)^2+\left(\tau_\eta-2\tau_s\right)\tau_\pi-\tau_s\left(\tau_\eta+\tau_s\right)\right)\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\frac{\left(-\gamma\frac{\overline{\alpha_s}}{\overline{\alpha_n}}+1\right)}{\left(-H'\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)\right)}\right),$$

where $\lim_{\tau_h \to 0} \tau_\pi \in (0,\infty)$. Note that

$$\operatorname{sgn}\left(\lim_{\tau_h \to 0} \frac{d\mathcal{V}}{d\tau_h}\right) = \operatorname{sgn}\left(\frac{1}{\tau_n}\left(\tau_\eta+\tau_\pi\right)\tau_\pi+\left(\left(\tau_\pi\right)^2+\left(\tau_\eta-2\tau_s\right)\tau_\pi-\tau_s\left(\tau_\eta+\tau_s\right)\right)\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\frac{\left(-\gamma\frac{\overline{\alpha_s}}{\overline{\alpha_n}}+1\right)}{\left(-H'\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)\right)}\right),$$

which is positive for low enough values of price informativeness since $-\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)+1>0$ and in any stable equilibrium $H'\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)<0$. Hence, $\tau_{\tau_h}=0$ and price volatility and price informative always comove positively for changes in τ_h.

Hence, for $\tau_\pi < \underline{\tau} = \min\left\{\tau_{\tau_s}, \tau_{\tau_\eta}, \tau_{\tau_n}, \tau_{\tau_h}, \tau_\gamma\right\} = 0$ and there is no negative comovement region. □

G Log-linear Approximation

Environment

The economy is populated by a continuum of investors, indexed by $i \in I$, who live for two dates. Each investor i is born with wealth w_0 and has well-behaved expected utility preferences over his terminal wealth w_1^i, with flow utility given by $U\left(w_{1,t}^i\right)$, where $U'(\cdot)>0$ and $U''(\cdot)<0$.

There are two long-term assets in the economy: a risk-free asset in perfectly elastic supply, with gross return $R>1$, and a risky asset in fixed supply Q, whose date t (log) payoff is $\theta_t=\ln\left(X_t\right)$ and which trades at a (log) price $p_t=\ln\left(P_t\right)$. The process followed by θ_t is given by

$$\Delta\theta_{t+1}=\mu_{\Delta\theta}+\eta_t, \tag{A.33}$$

where $\Delta\theta_{t+1}=\theta_{t+1}-\theta_t$, $\mu_{\Delta\theta}$ is a scalar, and $\theta_0=0$. The realized payoff θ_t is common knowledge to all investors before the price p_t is determined. The realized payoff at date $t+1$, θ_{t+1}, is only revealed to investors at date $t+1$.

We assume that investors receive private signals about the innovation to the risky asset payoff. Formally, each investor receives a signal about the payoff innovation η_t given by

$$s_t^i=\eta_t+\varepsilon_{st}^i \quad \text{with} \quad \varepsilon_{st}^i \sim N\left(0,\tau_s^{-1}\right),$$

where $\varepsilon_{st}^i \perp \varepsilon_{st}^j$ for all $i \neq j$, and $\eta_t \perp \varepsilon_{st}^i$ for all t and all i.

We also assume that investors have additional private trading motives coming from heterogeneous priors that are random in the aggregate. Formally, each investor i born at date t has a prior over the innovations to the payoff difference η_t given by

$$\eta_t \sim_{i,t} N\left(\overline{n}_t^i,\tau_\eta^{-1}\right),$$

where

$$\overline{n}_t^i=n_t+\varepsilon_{\overline{n}t}^i \quad \text{with} \quad \varepsilon_{\overline{n}t}^i \overset{\text{iid}}{\sim} N\left(0,\tau_{\overline{n}}^{-1}\right),$$

and

$$n_t=\mu_n+\varepsilon_t^n \quad \text{with} \quad \varepsilon_t^n \overset{\text{iid}}{\sim} N\left(0,\tau_{\Delta n}^{-1}\right),$$

where μ_n is a scalar, and where $\varepsilon_t^n \perp \varepsilon_{\overline{n}t}^i$ for all t and all i. The variable n_t, which can be interpreted as the aggregate sentiment in the economy, is not observed and acts as a source of aggregate noise, preventing the asset price from being fully revealing. Without loss of generality, we assume that $u_{t+s} \sim_{i,t} N\left(0, \tau_\eta^{-1}\right)$ for all $s > 0$.

Each investor i born at date t optimally chooses a portfolio share in the risky asset, denoted by q_t^i, to solve

$$\max_{q_t^i} \mathbb{E}\left[U\left(w_{1,t}^i\right) \middle| \mathcal{I}_t^i\right] \tag{A.34}$$

subject to a wealth accumulation constraint

$$w_{1,t}^i = \left(R + q_t^i \left(\frac{X_{t+1} + P_{t+1}}{P_t} - R\right)\right) w_0, \tag{A.35}$$

where the information set of an investor i at date t is given by $\mathcal{I}_t^i = \left\{s_t^i, \overline{n}_t^i, \{X_s\}_{s \leq t}, \{P_s\}_{s \leq t}\right\}$.

Portfolio Demand Approximation

The optimality condition of an investor that maximizes the investor's expected utility subject to the wealth accumulation constraint is given by

$$\mathbb{E}\left[U'\left(w_{1,t}^i\right)\left(\frac{X_{t+1} + P_{t+1}}{P_t} - R\right) \middle| \mathcal{I}_t^i\right] = 0. \tag{A.36}$$

We approximate an investor's first-order condition in three steps.

First, we take a first-order Taylor expansion of an investor's future marginal utility $U'\left(w_{1,t}^i\right)$ around the current date t wealth level w_0^i. Formally, we approximate $U'\left(w_{1,t}^i\right)$ as follows

$$U'\left(w_{1,t}^i\right) \approx U'\left(w_0\right) + U''\left(w_0\right) \Delta w_{1,t}^i,$$

which allows us to express Equation (A.36) as

$$U'\left(w_0\right) \mathbb{E}_t^i \left[\frac{X_{t+1} + P_{t+1}}{P_t} - R\right] + U''\left(w_0\right) w_0 \mathbb{E}_t^i \left[\left((R-1) + q_t^i \left(\frac{X_{t+1} + P_{t+1}}{P_t} - R\right)\right)\left(\frac{X_{t+1} + P_{t+1}}{P_t} - R\right)\right] \approx 0.$$

Second, we impose that terms that involve the product of two or more net interest rates are negligible. In continuous time, these terms would be of order $(dt)^2$. Formally, it follows that

$$(R-1)\, \mathbb{E}_t^i \left[\frac{X_{t+1} + P_{t+1}}{P_t} - R\right] \approx 0 \quad \text{and} \quad \left(\mathbb{E}_t^i \left[\frac{X_{t+1} + P_{t+1}}{P_t} - R\right]\right)^2 \approx 0,$$

which allows us to express Equation (A.36) as

$$U'\left(w_0\right) \mathbb{E}_t^i \left[\frac{X_{t+1} + P_{t+1}}{P_t} - R\right] + U''\left(w_0\right) w_0 q_t^i \mathbb{V}\text{ar}_t^i \left[\frac{X_{t+1} + P_{t+1}}{P_t}\right] \approx 0.$$

Therefore, we can express an investor's risky portfolio share q_t^i as

$$q_t^i \approx \frac{1}{\gamma} \frac{\mathbb{E}_t^i \left[\frac{X_{t+1}+P_{t+1}}{P_t} - R\right]}{\mathbb{V}\text{ar}_t^i \left[\frac{X_{t+1}+P_{t+1}}{P_t}\right]}, \tag{A.37}$$

where $\gamma \equiv -\frac{w_0 U''(w_0)}{U'(w_0)}$ denotes the coefficient of relative risk aversion.

Third, as in Campbell and Shiller (1988), we take a log-linear approximation of returns around a predetermined dividend-price ratio. Formally, note that

$$\frac{X_{t+1}+P_{t+1}}{P_t} = e^{\ln\left(\frac{\left(1+\frac{P_{t+1}}{X_{t+1}}\right)\frac{X_{t+1}}{X_t}}{\frac{P_t}{X_t}}\right)},$$

which implies that

$$\begin{aligned}\ln\left(\frac{X_{t+1}+P_{t+1}}{P_t}\right) &= \ln\left(1+\frac{P_{t+1}}{X_{t+1}}\right)+\theta_{t+1}-\theta_t-(p_t-\theta_t)\\ &= \ln\left(1+e^{p_{t+1}-\theta_{t+1}}\right)+\Delta\theta_{t+1}-(p_t-\theta_t),\end{aligned}$$

where $y_t = \ln Y_t$ for any given variable Y_t. Following Campbell and Shiller (1988), we approximate the first term around a point $PX = e^{p-\theta}$, to find that

$$\begin{aligned}\ln\left(1+e^{p_{t+1}-\theta_{t+1}}\right) &\approx \ln(1+PX)+\frac{PX}{PX+1}\left(p_{t+1}-\theta_{t+1}-(p-\theta)\right).\\ &= k_0+k_1\left(p_{t+1}-\theta_{t+1}\right),\end{aligned}$$

where $k_1 \equiv \frac{PX}{PX+1}$ and $k_0 \equiv \ln(1+PX)-k_1(p-\theta)$.

Therefore, starting from Equation (A.37), we can express an investor's risky portfolio share q_t^i as

$$q_t^i \approx \frac{1}{\gamma}\frac{k_0+k_1\mathbb{E}_t^i\left[p_{t+1}-\theta_{t+1}\right]+\mathbb{E}_t^i\left[\Delta\theta_{t+1}\right]-(p_t-\theta_t)-r}{\mathbb{V}\mathrm{ar}\left[k_1\left(p_{t+1}-\theta_{t+1}\right)+\Delta\theta_{t+1}\right]},$$

where we define $r \equiv \ln R$ and we used that $e^y \approx 1+y$.

Forming expectations

In order to characterize the equilibrium it is necessary to characterize investors' expectations. We conjecture and subsequently verify that $k_1\mathbb{E}_t^i\left[p_{t+1}-\theta_{t+1}\right]+\mathbb{E}_t^i\left[\Delta\theta_{t+1}\right]$ is linear in s_t^i, $\overline{n}_t^i$, and θ_t and that $\mathbb{V}\mathrm{ar}\left[k_1\left(p_{t+1}-\theta_{t+1}\right)+\Delta\theta_{t+1}\right]$ is a constant. Under this conjecture, q_t^i is a linear function of s_t^i, θ_t, and $\overline{n}_t^i$, given by

$$q_t^i \approx \alpha_\theta\theta_t+\alpha_s s_t^i+\alpha_n\overline{n}_t^i-\alpha_p p_t+\psi.$$

This expression and the market clearing condition $\int q_t^i w_0^i di = Q$ imply that

$$p_t \approx \frac{\alpha_\theta}{\alpha_p}\theta_t+\frac{\alpha_s}{\alpha_p}\eta_t+\frac{\alpha_n}{\alpha_p}n_t+\frac{\psi}{\alpha_p}.$$

As in Vives (2008), we make use the Strong Law of Large Numbers, since the sequence of independent random variables $\left\{\varepsilon_{st}^i,\varepsilon_{\overline{n}t}^i\right\}$ has uniformly bounded variance and mean zero. This expression can also be written as

$$p_t \approx \left(\frac{\alpha_\theta}{\alpha_p}-\frac{\alpha_s}{\alpha_p}\right)\theta_t+\frac{\alpha_s}{\alpha_p}\theta_{t+1}+\frac{\alpha_n}{\alpha_p}n_t+\frac{\psi}{\alpha_p}-\frac{\alpha_s}{\alpha_p}\mu_{\Delta\theta}. \tag{A.38}$$

Investors in the model learn from the price. The information contained in the price for an investor in the model is

$$\pi_t = \frac{\alpha_p}{\alpha_s}\left(p_t-\left(\frac{\alpha_\theta}{\alpha_p}\theta_t+\frac{\alpha_n}{\alpha_p}\mu_n+\frac{\psi}{\alpha_p}\right)\right),$$

which has a precision

$$\tau_\pi \equiv \mathbb{V}\mathrm{ar}\left[\pi_t|\,\eta_t,\{\theta_s\}_{s\le t},p_{t-1}\right]^{-1} = \left(\frac{\alpha_s}{\alpha_n}\right)^2\tau_n.$$

Then,

$$\mathbb{E}_t^i\left[\eta_t\right] = \mathbb{E}\left[\eta_t|\mathcal{I}_t^i\right] = \frac{\tau_s s_t^i + \tau_\eta \overline{n}_t^i + \tau_\pi \pi_t}{\tau_s + \tau_\eta + \tau_\pi} = \frac{\tau_s s_t^i + \tau_\eta \overline{n}_t^i + \tau_\pi \frac{\alpha_p}{\alpha_s}\left(p_t - \frac{\alpha_\theta}{\alpha_p}\theta_t - \frac{\alpha_n}{\alpha_s}\mu_{\Delta n} - \frac{\psi}{\alpha_p}\right)}{\tau_s + \tau_\eta + \tau_\pi}$$

and

$$\mathbb{V}\mathrm{ar}\left[\eta_t|\,\mathcal{I}_t^i\right] = \left(\tau_s + \tau_\eta + \tau_\pi\right)^{-1},$$

where $\mathcal{I}_t^i = \left\{s_t^i, \overline{n}_t^i, \{p_s\}_{s\leq t}, \{\theta_s\}_{s\leq t}\right\}$. Note that these two expressions imply that our conjecture about q_t^i is satisfied. To see this, note that

$$\begin{aligned}
k_1\mathbb{E}_t^i\left[p_{t+1}-\theta_{t+1}\right] + \mathbb{E}_t^i\left[\Delta\theta_{t+1}\right] &= k_1\mathbb{E}_t^i\left[\frac{\alpha_\theta}{\alpha_p}\theta_{t+1} + \frac{\alpha_s}{\alpha_p}\eta_{t+1} + \frac{\alpha_n}{\alpha_p}n_{t+1} + \frac{\psi}{\alpha_p} - \theta_{t+1}\right] + \mu_{\Delta\theta} + \mathbb{E}_t^i\left[\eta_t\right] \\
&= k_1\left(\mathbb{E}_t^i\left[\left(\frac{\alpha_\theta}{\alpha_p} - 1\right)\theta_{t+1} + \frac{\alpha_s}{\alpha_p}\eta_{t+1}\right] + \frac{\alpha_n}{\alpha_p}\mu_n + \frac{\psi}{\alpha_p}\right) + \mu_{\Delta\theta} + \mathbb{E}_t^i\left[\eta_t\right] \\
&= k_1\left(\left(\frac{\alpha_\theta}{\alpha_p} - 1\right)\left(\mu_{\Delta\theta} + \mathbb{E}_t^i\left[\eta_t\right]\right) + \left(\frac{\alpha_\theta}{\alpha_p} - 1\right)\theta_t + \frac{\alpha_n}{\alpha_p}\mu_n + \frac{\psi}{\alpha_p}\right) + \mu_{\Delta\theta} + \mathbb{E}_t^i\left[\eta_t\right] \\
&= k_1\left(\left(\frac{\alpha_\theta}{\alpha_p} - 1 + \frac{1}{k_1}\right)\left(\mu_{\Delta\theta} + \mathbb{E}_t^i\left[\eta_t\right]\right) + \left(\frac{\alpha_\theta}{\alpha_p} - 1\right)\theta_t + \frac{\alpha_n}{\alpha_p}\mu_n + \frac{\psi}{\alpha_p}\right),
\end{aligned}$$

where we used that $\mathbb{E}_t^i\left[\eta_{t+1}\right] = 0$ and that $\mathbb{E}_t^i\left[\varepsilon_{t+1}^n\right] = 0$. Moreover,

$$\begin{aligned}
\mathbb{V}\mathrm{ar}_t^i\left[k_1\left(p_{t+1}-\theta_{t+1}\right) + \Delta\theta_{t+1}\right] &= \mathbb{V}\mathrm{ar}_t^i\left[k_1\left(\left(\frac{\alpha_\theta}{\alpha_p} - 1 + \frac{1}{k_1}\right)\Delta\theta_{t+1} + \frac{\alpha_s}{\alpha_p}\eta_{t+1} + \frac{\alpha_n}{\alpha_p}n_{t+1}\right)\right] \\
&= k_1^2\left(\frac{\alpha_\theta}{\alpha_p} - 1 + \frac{1}{k_1}\right)^2\mathbb{V}\mathrm{ar}_t^i\left[\eta_t\right] + k_1^2\left(\frac{\alpha_s}{\alpha_p}\right)^2\mathbb{V}\mathrm{ar}_t^i\left[\eta_{t+1}\right] + k_1^2\left(\frac{\alpha_n}{\alpha_p}\right)^2\mathbb{V}\mathrm{ar}_t^i\left[\varepsilon_t^n\right] \\
&= k_1^2\left(\frac{\alpha_\theta}{\alpha_p} - 1 + \frac{1}{k_1}\right)^2\left(\tau_s + \tau_\eta + \tau_\pi\right)^{-1} + k_1^2\left(\frac{\alpha_s}{\alpha_p}\right)^2\tau_\eta^{-1} + k_1^2\left(\frac{\alpha_n}{\alpha_p}\right)^2\tau_n^{-1}.
\end{aligned}$$

Using these expressions in the first-order condition and matching coefficients gives

$$\begin{aligned}
\alpha_\theta &= \frac{1}{\kappa}k_1\left(\frac{\alpha_\theta}{\alpha_p} - 1 + \frac{1}{k_1}\right)\left(1 - \frac{\tau_\pi\frac{\alpha_\theta}{\alpha_s}}{\tau_s + \tau_\eta + \tau_\pi}\right) \\
\alpha_s &= \frac{1}{\kappa}k_1\left(\frac{\alpha_\theta}{\alpha_p} - 1 + \frac{1}{k_1}\right)\frac{\tau_s}{\tau_s + \tau_\eta + \tau_\pi} \\
\alpha_n &= \frac{1}{\kappa}k_1\left(\frac{\alpha_\theta}{\alpha_p} - 1 + \frac{1}{k_1}\right)\frac{\tau_\eta}{\tau_s + \tau_\eta + \tau_\pi} \\
\alpha_p &= \frac{1}{\kappa}\left(1 - k_1\left(\left(\frac{\alpha_\theta}{\alpha_p} - 1 + \frac{1}{k_1}\right)\frac{\tau_\pi\frac{\alpha_p}{\alpha_s}}{\tau_s + \tau_\eta + \tau_\pi}\right)\right) \\
\psi &= \frac{1}{\kappa}\left(\ k_0 + k_1\left(-\left(\frac{\alpha_\theta}{\alpha_p} - 1 + \frac{1}{k_1}\right)\left(\frac{\tau_\pi\frac{\alpha_p}{\alpha_s}}{\tau_s+\tau_\eta+\tau_\pi}\right) + 1\right)\left(\frac{\alpha_n}{\alpha_p}\mu_n + \frac{\psi}{\alpha_p}\right) + \mu_{\Delta\theta} - r\ \right),
\end{aligned}$$

where $\kappa \equiv \gamma\mathbb{V}\mathrm{ar}\left[k_1\left(p_{t+1}-\theta_{t+1}\right) + \Delta\theta_{t+1}\right]$.

Then, the equilibrium coefficients are

$$\alpha_\theta = \frac{1}{\kappa}\frac{\tau_s}{\tau_s + \tau_\pi} \tag{A.39}$$

$$\alpha_s = \frac{1}{\kappa}\frac{\tau_s}{\tau_s + \tau_\eta + \tau_\pi} \tag{A.40}$$

$$\alpha_n = \frac{1}{\kappa}\frac{\tau_\eta}{\tau_s + \tau_\eta + \tau_\pi} \tag{A.41}$$

$$\alpha_p = \frac{1}{\kappa}\frac{\tau_s}{\tau_s + \tau_\pi} \tag{A.42}$$

$$\psi = \frac{1}{\kappa}\left(k_0 + k_1\left(-\frac{1}{k_1}\frac{\tau_\pi}{\tau_s + \tau_\pi} + 1\right)\frac{\alpha_n}{\alpha_p}\mu_n + \mu_{\Delta\theta} - r\right)\left(1 + \frac{\frac{1}{k_1}\frac{\tau_\pi}{\tau_s+\tau_\pi} - k_1}{\frac{\tau_s}{\tau_s+\tau_\pi}}\right)^{-1}. \tag{A.43}$$

H Comovement Scores: Additional Empirical Results

Figures OA-3 through OA-7 are the counterparts of the cross-sectional results presented in Table 1. Each figure shows scatter plots of cross-sectional regressions of comovement scores (in twentiles) on each of the five variables considered: size, value, turnover, idiosyncratic return volatility, and institutional ownership, for each of the years between 1981 and 2016.

Table OA-1 reports year-by-year summary statistics on the panel of comovement scores recovered. Finally, Table OA-2 reports the cross-sectional correlation between comovement scores measured in year t and comovement scores measured in prior years, following the methodology of Chapter 4 of Bali, Engle and Murray (2016). This table shows that comovement scores are highly persistent over time. As one might expect, the strength of the correlation decays over time, with one-year cross correlations consistently above 0.9, while five-year cross-correlations can be as low as 0.5.

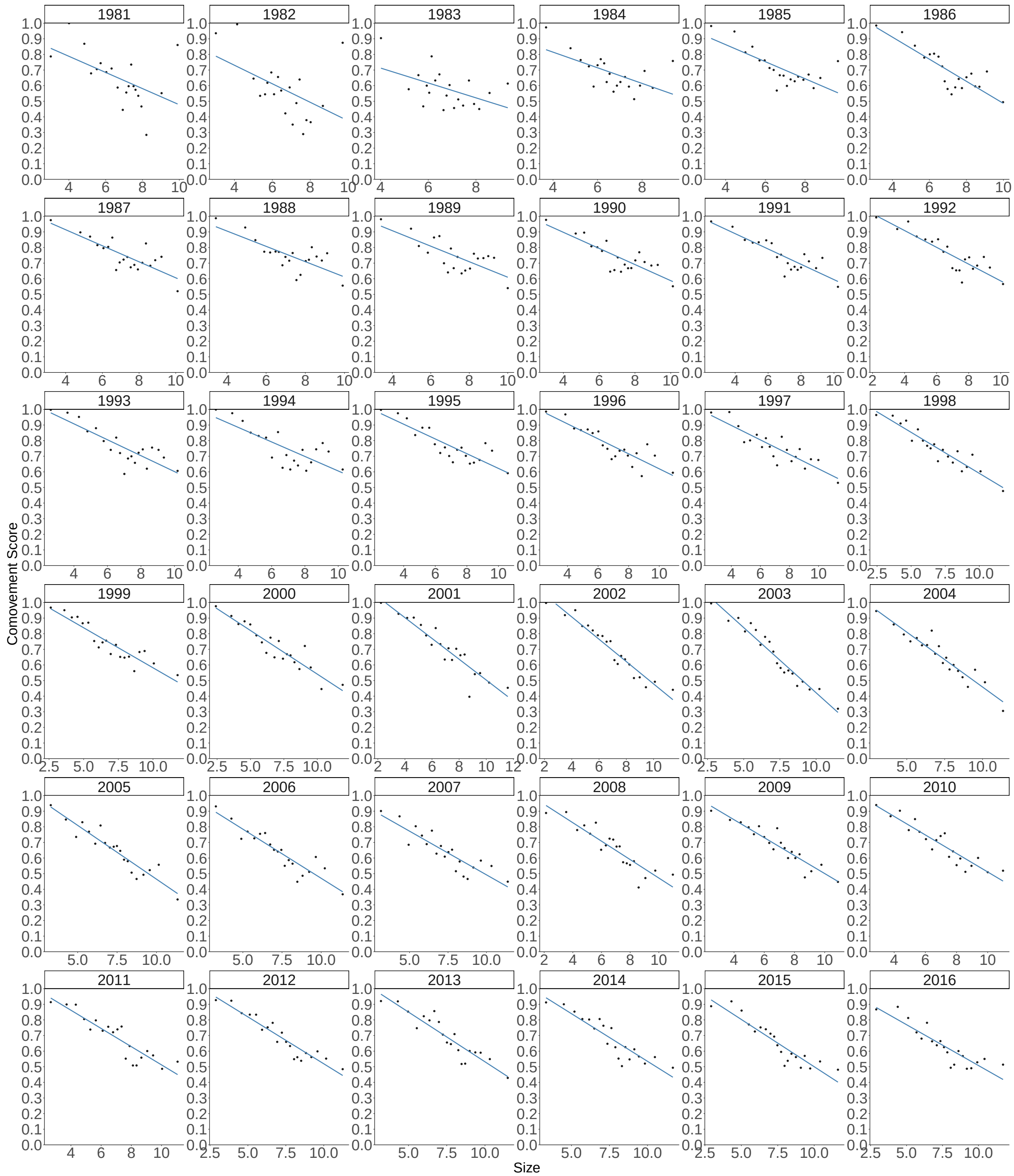

Figure OA-3: Comovement score and size

Note: Figure OA-3 shows year-by-year cross-sectional regressions of comovement scores(in twentiles) on size, defined as the log of market capitalization — see e.g. Bali, Engle and Murray (2016). The estimate reported in Table 1 can be interpreted as a weighted averaged of the year-by-year slope coefficient illustrated here.

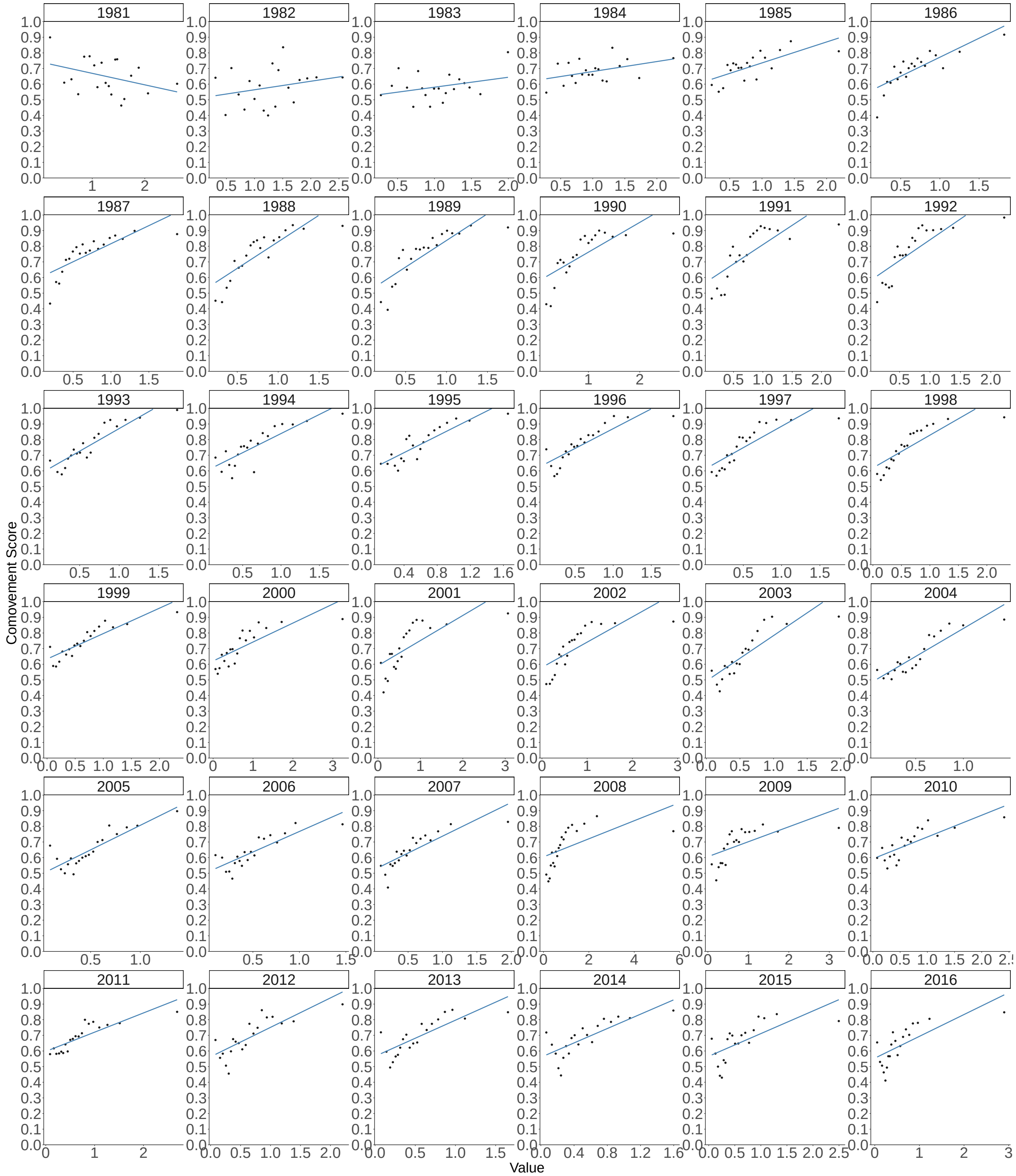

Figure OA-4: Comovement score and value

Note: Figure OA-4 shows year-by-year cross-sectional regressions of comovement scores (in twentiles) on value, defined as the ratio between a stock's book value and its market capitalization. The estimate reported in Table 1 can be interpreted as a weighted averaged of the year-by-year slope coefficient illustrated here.

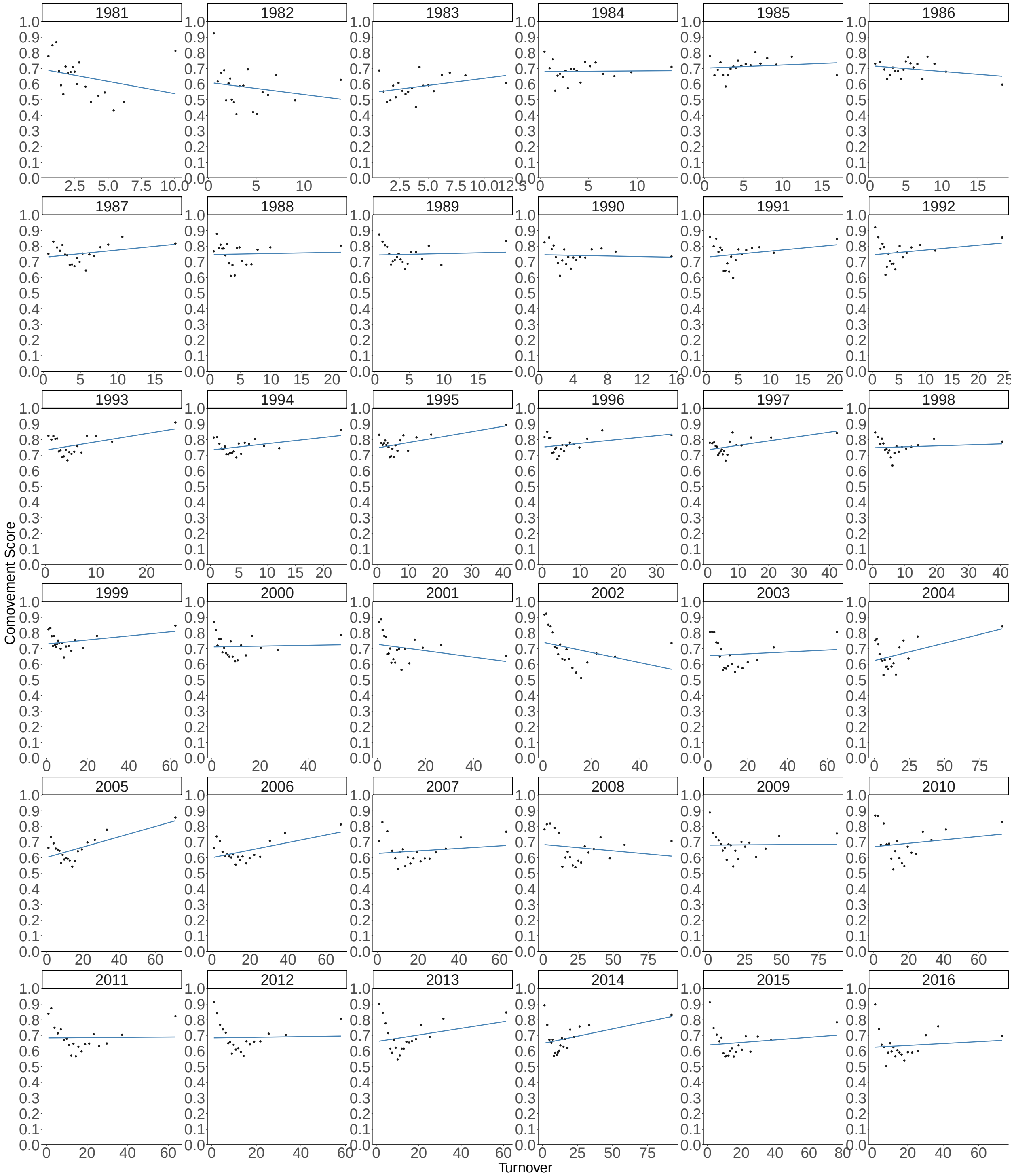

Figure OA-5: Comovement score and turnover

Note: Figure OA-5 shows year-by-year cross-sectional regressions of comovement scores (in twentiles) on turnover, defined as the ratio between trading volume and shares outstanding. The estimate reported in Table 1 can be interpreted as a weighted averaged of the year-by-year slope coefficient illustrated here.

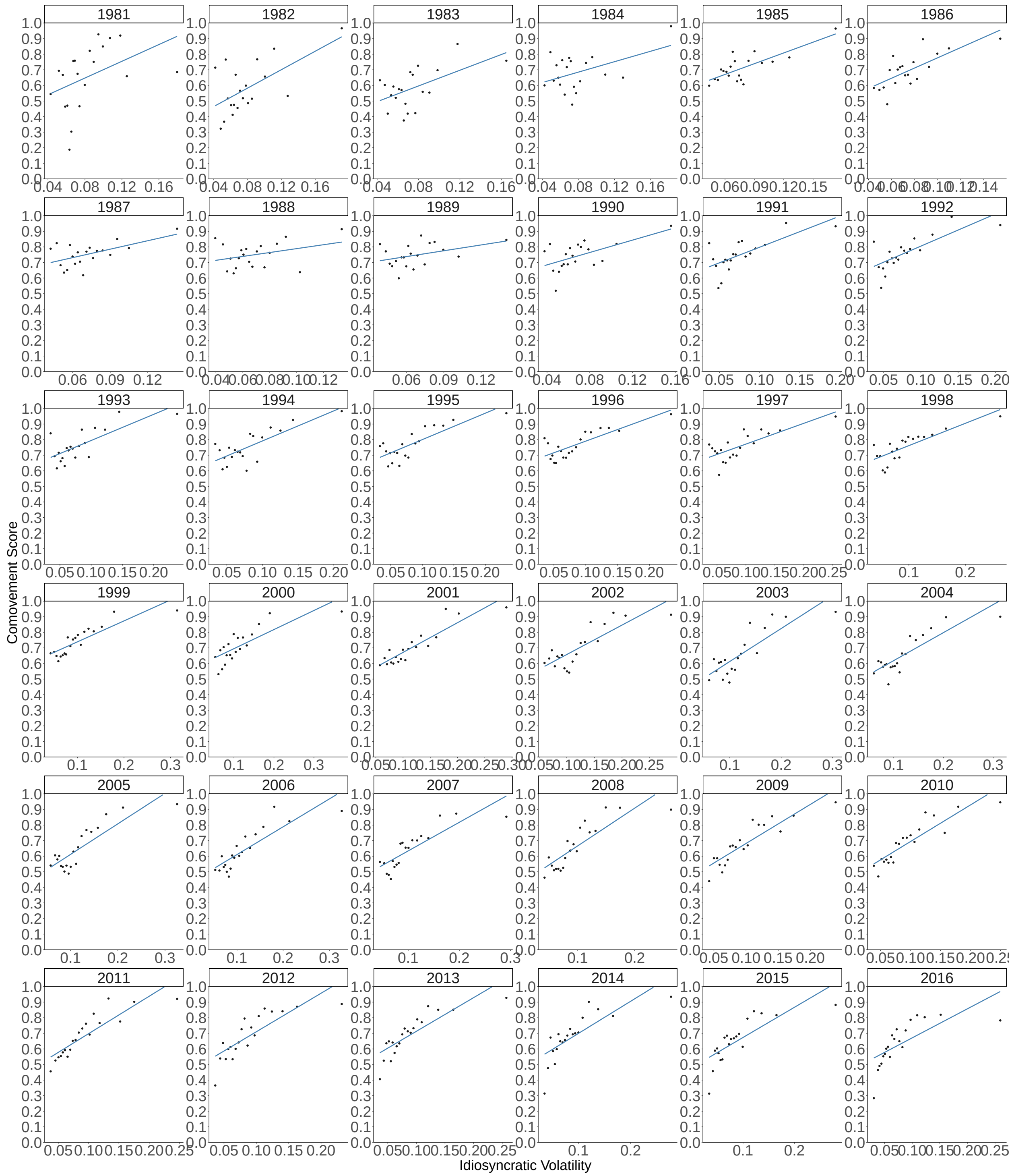

Figure OA-6: Comovement score and idiosyncratic return volatility

Note: Figure OA-6 shows year-by-year cross-sectional regressions of comovement scores (in twentiles) on idiosyncratic volatility, define as the standard deviation over a 30 month period of the difference between the returns of a stock and the market return. The estimate reported in Table 1 can be interpreted as a weighted averaged of the year-by-year slope coefficient illustrated here.

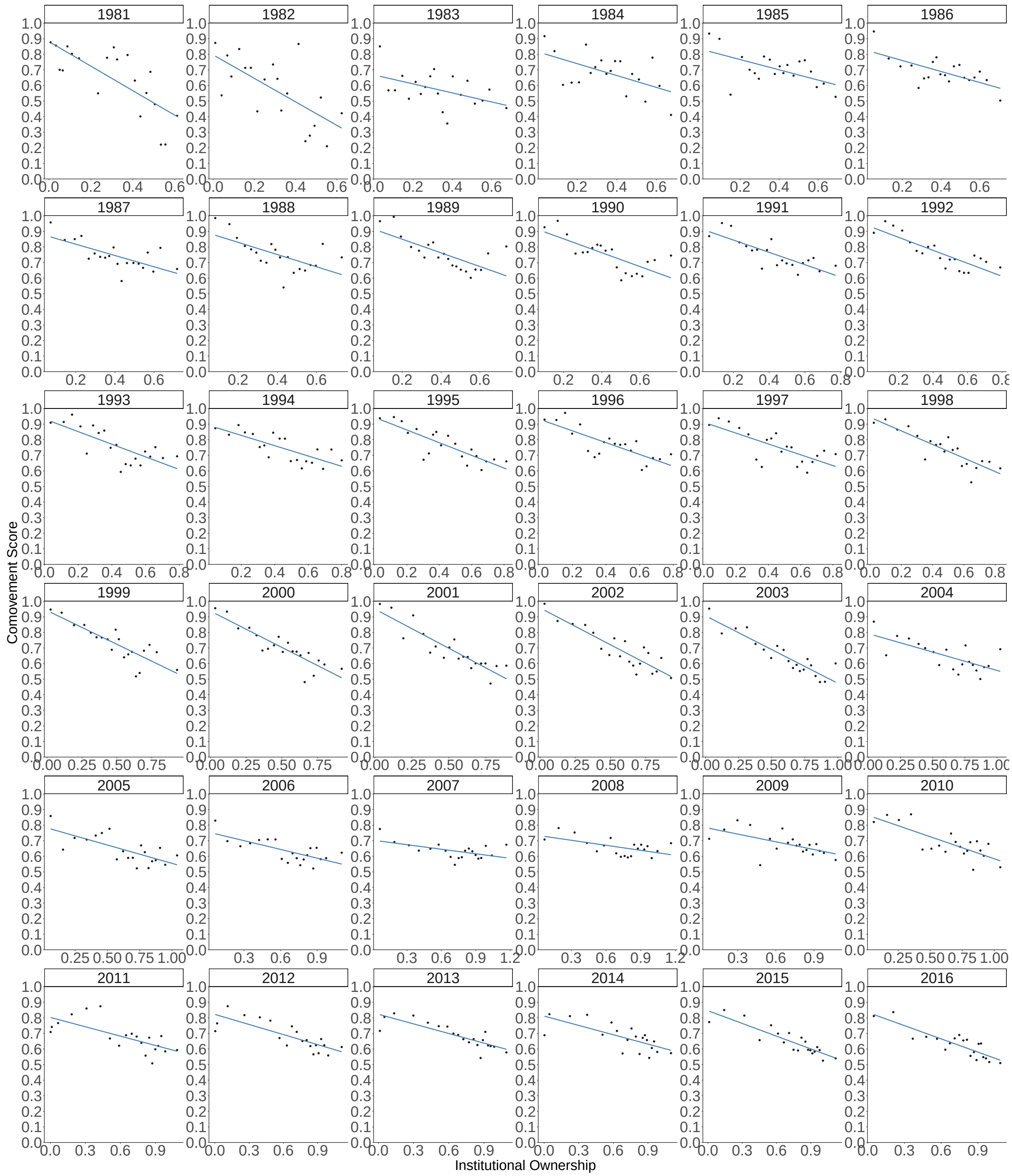

Figure OA-7: Comovement score and institutional ownership

Note: Figure OA-7 shows year-by-year cross-sectional regressions of comovement scores (in twentiles) on institutional ownership, defined as the proportion of a stock held by institutional investors. The estimate reported in Table 1 can be interpreted as a weighted averaged of the year-by-year slope coefficient illustrated here.

Table OA-1: Comovement scores: year-by-year summary statistics, annual data

t	$Median$	$Mean$	SD	$Skew$	$Kurt$	$P5$	$P25$	$P75$	$P95$	n
1980	0.8645	0.6860	0.3662	-0.8667	-0.8011	0	0.4020	0.9969	1	56
1981	0.8083	0.6658	0.3640	-0.7911	-0.8876	0	0.4371	0.9941	1	74
1982	0.6729	0.5704	0.3876	-0.3687	-1.4655	0	0.1290	0.9454	1	140
1983	0.7518	0.6042	0.3818	-0.4733	-1.3556	0	0.2554	0.9804	1	156
1984	0.8698	0.6960	0.3547	-0.8963	-0.7168	0	0.4124	0.9912	1	241
1985	0.9012	0.7148	0.3456	-0.9707	-0.5447	0	0.4684	0.9965	1	383
1986	0.9167	0.7126	0.3583	-0.9708	-0.6088	0	0.4603	0.9977	1	372
1987	0.9464	0.7601	0.3327	-1.2620	0.1272	0	0.5719	0.9987	1	435
1988	0.9610	0.7623	0.3366	-1.2793	0.1636	0	0.6184	0.9995	1	357
1989	0.9699	0.7478	0.3531	-1.1905	-0.1467	0	0.5997	0.9995	1	349
1990	0.9616	0.7572	0.3468	-1.2629	0.0639	0	0.5880	0.9993	1	354
1991	0.9715	0.7611	0.3498	-1.2713	0.0441	0	0.6208	0.9997	1	455
1992	0.9766	0.7726	0.3442	-1.3374	0.2166	0	0.6607	0.9998	1	496
1993	0.9696	0.7662	0.3467	-1.3292	0.2043	0	0.6347	0.9998	1	522
1994	0.9758	0.7687	0.3456	-1.3449	0.2542	0	0.6606	0.9998	1	506
1995	0.9847	0.7802	0.3394	-1.3966	0.4275	0	0.6350	0.9998	1	544
1996	0.9843	0.7814	0.3398	-1.4068	0.4433	0	0.6896	0.9998	1	584
1997	0.9853	0.7683	0.3474	-1.3036	0.1478	0	0.6206	0.9998	1	622
1998	0.9800	0.7684	0.3524	-1.3256	0.1334	0	0.6661	0.9999	1	631
1999	0.9842	0.7547	0.3664	-1.2212	-0.1980	0	0.5973	0.9999	1	603
2000	0.9754	0.7329	0.3697	-1.0704	-0.5049	0	0.4819	0.9998	1	562
2001	0.9814	0.7323	0.3724	-1.0482	-0.5776	0	0.4705	0.9999	1	548
2002	0.9733	0.7169	0.3862	-0.9837	-0.7408	0	0.3992	0.9999	1	579
2003	0.9524	0.6811	0.3983	-0.7773	-1.1171	0	0.3026	0.9998	1	632
2004	0.9504	0.6812	0.3992	-0.7895	-1.1039	0	0.3001	0.9998	1	675
2005	0.9331	0.6721	0.3997	-0.7323	-1.1878	0	0.2786	0.9997	1	683
2006	0.9115	0.6602	0.3984	-0.6691	-1.2627	0	0.2540	0.9993	1	699
2007	0.9065	0.6541	0.4024	-0.6386	-1.3095	0	0.2255	0.9994	1	711
2008	0.9457	0.6833	0.3963	-0.8029	-1.0727	0	0.3222	0.9996	1	762
2009	0.9630	0.6974	0.3947	-0.8665	-0.9812	0	0.3295	0.9998	1	764
2010	0.9760	0.7064	0.3903	-0.9087	-0.8882	0	0.3796	0.9998	1	753
2011	0.9812	0.7092	0.3876	-0.9134	-0.8655	0	0.3949	0.9998	1	753
2012	0.9704	0.7050	0.3918	-0.9100	-0.8973	0	0.3569	0.9998	1	765
2013	0.9642	0.7081	0.3871	-0.9306	-0.8466	0	0.3812	0.9998	1	733
2014	0.9461	0.6996	0.3883	-0.8948	-0.8979	0	0.3698	0.9999	1	722
2015	0.9060	0.6687	0.3969	-0.7394	-1.1463	0	0.2776	0.9996	1	695
2016	0.8878	0.6522	0.4016	-0.6440	-1.2898	0	0.2378	0.9993	1	645
2017	0.8747	0.6398	0.4061	-0.5907	-1.3566	0	0.2043	0.9994	1	572

Note: Table OA-1 reports year-by-year summary statistics on the panel of comovement scores recovered. It provides information on the median; mean; standard deviation; skewness; excess kurtosis; and 5th, 25th, 75th, and 95th percentiles of each yearly distribution, as well as the number of stocks in each year. Since our panel of comovement scores is quarterly, we average the measures of quarterly comovement scores at the yearly level before computing the summary statistics. Comovement scores in year t are computed over a rolling window of 40 quarters prior.

Table OA-2: Persistence of comovement scores

t	$\rho_{t,t-1}$	$\rho_{t,t-2}$	$\rho_{t,t-3}$	$\rho_{t,t-4}$	$\rho_{t,t-5}$
1980	0.9801	0.9904	0.9569	0.9411	0.8531
1981	0.9818	0.9525	0.9574	0.8967	0.8677
1982	0.9739	0.9307	0.9026	0.9168	0.8835
1983	0.9557	0.8415	0.7163	0.6307	0.4347
1984	0.9171	0.8707	0.7555	0.6337	0.4964
1985	0.8663	0.7424	0.7443	0.6135	0.5704
1986	0.9402	0.7487	0.6604	0.6623	0.5005
1987	0.9362	0.8528	0.6446	0.5614	0.5553
1988	0.9581	0.8861	0.7930	0.5763	0.5505
1989	0.9727	0.9086	0.8409	0.7497	0.5833
1990	0.9358	0.9032	0.8255	0.7331	0.6774
1991	0.9530	0.8577	0.8203	0.7796	0.7230
1992	0.9540	0.9100	0.8099	0.7777	0.7312
1993	0.9573	0.9191	0.8998	0.8140	0.8067
1994	0.9735	0.9204	0.8662	0.8797	0.8178
1995	0.9330	0.8921	0.8223	0.7591	0.7946
1996	0.9324	0.8526	0.8048	0.7426	0.6714
1997	0.9398	0.8385	0.7718	0.7288	0.6590
1998	0.9253	0.8262	0.6850	0.6541	0.5879
1999	0.9418	0.8541	0.8096	0.7006	0.6538
2000	0.9702	0.9155	0.8482	0.7857	0.6959
2001	0.9256	0.8498	0.7936	0.7654	0.7061
2002	0.9407	0.8056	0.7190	0.6526	0.6594
2003	0.9403	0.8973	0.7853	0.7095	0.6327
2004	0.9604	0.8662	0.8289	0.7385	0.6773
2005	0.9404	0.8679	0.7694	0.6934	0.6613
2006	0.9428	0.8437	0.7792	0.7001	0.6133
2007	0.9501	0.8797	0.7654	0.6913	0.6299
2008	0.9310	0.8389	0.7775	0.6629	0.5924
2009	0.8740	0.7828	0.7083	0.6775	0.6068
2010	0.9455	0.8027	0.7246	0.6550	0.6208
2011	0.9538	0.8860	0.7470	0.6905	0.6213
2012	0.9396	0.8656	0.7890	0.6490	0.5620
2013	0.9640	0.8872	0.8348	0.7497	0.5889
2014	0.9657	0.9217	0.8395	0.7866	0.7031
2015	0.9659	0.9256	0.8935	0.8219	0.7699
2016	0.9803	0.9312	0.8913	0.8622	0.7949
2017	0.9633	0.9285	0.8945	0.8437	0.8119

Note: Table OA-2 reports the cross-sectional correlation between comovement scores measured in year t and comovement scores measures in year $t-k$, where $k = \{1, 2, 3, 4, 5\}$. Since our panel of comovement scores is quarterly, we average the measures of quarterly comovement scores at the yearly level before computing the correlations. We start reporting the correlations in 1980, since that is the first year with more than 250 stocks. Comovement scores in year t are computed over a rolling window of 40 quarters prior.