Volatility and Informativeness*

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Abstract

We explore the relation between price volatility and price informativeness in financial markets. We identify two channels (noise reduction and equilibrium learning) through which changes in informativeness are associated with changes in volatility. When informativeness is sufficiently high (low), volatility and informativeness positively (negatively) comove in equilibrium for any change in primitives. We provide conditions on primitives that guarantee that volatility and informativeness comove positively or negatively for any change in parameters. We recover stock-specific primitives for US stocks, and find that most stocks lie in the region of the parameter space in which informativeness and volatility comove negatively.

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1 Introduction

There is a long tradition in economics, commonly traced back to Hayek (1945), which emphasizes the role of financial markets aggregating dispersed information. Under this view, prices not only convey scarcity, but they also reveal the dispersed information held by investors about the underlying fundamentals of the economy. Within this paradigm, price informativeness, understood as the precision of the signal about future payoffs revealed by asset prices, defines a natural measure of the ability of financial markets to aggregate information. While price informativeness is a complex equilibrium object that can only be inferred, price volatility is an alternative equilibrium object that is easily computable and regularly scrutinized.[1] In this paper, we explore the relation between both variables with the goal of characterizing the conditions under which changes in asset price fluctuations can be unambiguously interpreted as a reflection of more or less informative asset markets.

Because price volatility and price informativeness are jointly determined in equilibrium, this paper adopts an unconventional methodological approach. It initially explores the equilibrium relation between both endogenous variables to subsequently shed light on conventional comparative static exercises conducted within fully specified models.

Our first main result shows that the equilibrium relation between price informativeness and price volatility in financial markets for the class of models with linear asset demands and additive noise—which we refer to as the fundamental relation—is uniquely characterized by i) the variance of the innovation to asset payoffs and ii) the signal-to-price demand sensitivity, which corresponds to the ratio of investors' demand sensitivities to private information and to asset prices. Exploiting this relation, we identify two different channels through which changes in price informativeness that leave the fundamental relation otherwise unchanged are associated with changes in price volatility.[2] We refer to the first channel as the *noise reduction* channel. Through this channel, an increase in price informativeness is directly associated with a reduction in price volatility, since less noise is incorporated into the price. We refer to the second channel as the *equilibrium learning* channel. Through this channel, which is inactive when investors do not learn from asset prices, an increase in price informativeness changes investors' behavior by varying their equilibrium signal-to-price (demand) sensitivities. In principle, the sign of the equilibrium learning channel can be positive or negative.

To further characterize the behavior of signal-to-price sensitivities, we specialize our analysis to a general CARA-Gaussian environment. The additional structure allows us to show that the signal-to-price sensitivity is strictly increasing in the level of price informativeness, implying that the noise reduction channel and the equilibrium learning channel operate in opposite directions. Intuitively, an increase in price informativeness tilts investors' demands toward putting more weight on the price as a

[1] The relevant definition of price volatility for our analysis corresponds to the conditional idiosyncratic volatility of asset prices given past public information, as it will become clear in Section 2.

[2] Since price informativeness is an endogenous object, considering changes in price informativeness that do not otherwise affect the fundamental relation is only possible for a subset of all the model parameters. This type of changes is nonetheless useful for definitional purposes. We eventually consider changes in parameters that, at the same time, change price informativeness and shift the fundamental relation.

signal about asset payoffs, making investors' demands more correlated, which increases the sensitivity of prices to the aggregate payoff realization and, consequently, price volatility. The additional structure also allows us to express the fundamental relation between price informativeness and price volatility exclusively as a function of a subset of primitives. The only primitives that explicitly enter into the fundamental relation are i) the prior volatility of the innovation to asset payoffs, ii) the precision of investors' private signals about asset payoffs, and iii) the ratio of precisions of the signal contained in the price for an investor relative to an external observer.

Our second main result shows that, under a simple and plausible parameter restriction that limits the precision of the signal conveyed by the equilibrium price to be sufficiently low for an investor relative to an external observer, the fundamental relation between price informativeness and price volatility has a positive (negative) slope whenever price informativeness is sufficiently high (low). This result implies that any change in the subset of parameters that do not enter the fundamental relation directly must induce a positive comovement between price informativeness and volatility when prices are sufficiently informative and a negative comovement when price informativeness is sufficiently low. This result also implies that any change in the subset of parameters that at the same time shifts the fundamental relation upwards and increases price informativeness must also induce a positive comovement between equilibrium price informativeness and volatility when prices are sufficiently informative. Alternatively, we show that a change in the subset of parameters that shifts the fundamental relation upwards and increases price informativeness induces a negative comovement between informativeness volatility when prices are barely informative. When interpreted through the lens of our two-channel decomposition, when prices are sufficiently informative, the equilibrium learning channel becomes overwhelmingly important and dominates the noise reduction channel. However, when prices are sufficiently uninformative, the noise reduction channel dominates.

While the results derived in the general case provide interesting insights into the nature of the relation between price volatility and informativeness, understanding the exact comovement between both variables for any set of parameters and their independent behavior requires the study of fully specified models. Therefore, we specialize our results to three applications that allow us to interpret conventional comparative statics through the lens of the fundamental relation. First, we study a model in which heterogeneity in investors' priors provides the source of aggregate noise in the economy. Second, we study a model with a finite number of agents in which the law of large numbers breaks down and, hence, an average of investors' idiosyncratic shocks acts as an additional aggregate source of noise. Finally, we study a model in which uncertainty about the aggregate level of hedging needs is the source of aggregate noise. These applications illustrate the different values that the ratio of precisions of the signal contained in the price for an investor relative to an external observer can take. For instance, in the first application, investors' private trading motives are not useful to forecast the level of aggregate noise, which implies that price informativeness is identical for investors in the model relative to an external observer. In the case of strategic traders, price informativeness is higher for an external observer relative to investors in the model, while in the model with stochastic hedging needs, price informativeness is higher for investors in the model relative to an external observer.

Our third main result shows that whenever prices are sufficiently informative (uninformative), it is indeed the case that changes in *any* underlying parameter, including those that explicitly appear in the fundamental relation, induce a positive (negative) comovement between price informativeness and volatility across all applications considered. For instance, an increase in the precision of investors' private signals about the asset payoff increases price informativeness and, at the same time, shifts the fundamental relation upwards, since investors are more responsive to their private signals for any level of informativeness. Alternatively, an increase in the precision of investors' priors about the asset payoff decreases price informativeness and, at the same time, shifts the fundamental relation downwards, since investors are less responsive to their acquired information. In both cases, whenever the slope of the fundamental relation is positive (informativeness is high), informativeness and volatility positively comove, whereas if the fundamental relation has a negative slope (informativeness is low), informativeness and volatility react in different directions. Our results show that increases in price volatility are associated with increases (decreases) in the informational content of asset prices when price informativeness is sufficiently high (low).[3] Finding an unambiguous positive or negative comovement between price volatility and informativeness after changes in all primitives, even for specific regions of the parameter space, may come as a surprise since, by reading the existing literature (e.g., Vives (2008)), one may conclude that there is no systematic relation between these variables.

Propositions 2 and 3 formalize the relation between volatility and informativeness as a function of the level of price informativeness, which, despite being a meaningful variable, is an equilibrium object. The next natural step is to characterize the region in which volatility and informativeness positively or negatively comove as a function of primitives. With that goal, we specialize our general results to study comparative statics in all three applications. Several insights emerge from studying the comparative statistics in each individual application. In particular, we show that when prices are sufficiently informative, consistent with our general results, a reduction in the magnitude of aggregate noise increases price informativeness and, perhaps surprisingly, price volatility. While a reduction in the amount of trading due to noise directly makes prices more informative, once the equilibrium learning channel becomes sufficiently strong price informativeness and price volatility positively comove. Changes in the variance of the aggregate common belief, the level of hedging needs, or the number of traders induce similar effects.

Subsequently, in Section 5, our final main result explicitly characterizes the positive and negative comovement regions as a function of primitives. We show that positive and negative comovement regions can be characterized as a function of *ratios* of primitives, in particular, ratios of the precisions of i) private signals, ii) the asset payoff, and iii) the source of noise, among other primitives. Finally, in the context of our leading application with heterogeneous priors, which is the most tightly parameterized, we use data from US stocks between 1963 to 2017 to recover stock-specific parameter ratios that allow us to determine whether the recovered primitives for a given stock are in the positive comovement region, the negative comovement region, or in neither. We leverage the identification results developed

[3] We show that a region in which price volatility and informativeness positively comove always exists. However, we show that the region in which price volatility and informativeness negatively comove may be empty in some applications.

in Davila and Parlatore (2018) to provide a structural interpretation to regressions of asset prices on earnings.

Our empirical findings imply that most of the stocks in our final sample (roughly 55) are in the negative comovement region. For these stocks, our results imply that if we were to observe an increase in price volatility, this would be associated with a decrease in price informativeness, and vice versa. Intuitively, the large amount of idiosyncratic volatility unrelated to changes in earnings at the stock level suggests that price informativeness is low for most stocks, which, given our theoretical results, implies that most stocks feature negative comovement between price volatility and informativeness. Interestingly, we find that none of the remaining stocks are in the positive comovement region.

Related Literature This paper is most directly related to the literature that studies the role played by financial markets in aggregating dispersed information, going back to Hayek (1945), and following Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verrecchia (1981), among others. Biais, Glosten and Spatt (2005), Vives (2008), and Veldkamp (2011) provide recent reviews of this well-developed body of work. Although price informativeness is a central object of study in many of these papers, we provide, to our knowledge, the first systematic study of the relation between price volatility and price informativeness. While the textbook treatment of Vives (2008) separately discusses the comparative statics of price volatility and informativeness in a competitive model similar to the one that we consider, it does not explore further the relation between both variables.[4] Lee and Liu (2011) study comparative statics of price volatility and informativeness when varying the number of noise traders in a specific competitive model.

There exists a vast literature focused on the measurement of asset price volatility, including the seminal contribution of Engle (1982), which spurred a large amount of work in Financial Econometrics. Campbell et al. (2001) and Brandt et al. (2009) are well-known references within this vast literature. While these studies emphasize the implications of price volatility for diversification, as well as its relation with expected returns, these papers have not related their findings to whether prices are more or less informative. Our results seek to broaden the impact of these studies, by showing how to draw inferences for price informativeness from measures of price volatility. By modeling dispersed information and learning, our results also contrast to the vast literature studying excess volatility and predictability that follows Shiller (1981), mostly focused on a representative investor.

A small recent literature seeks to recover the behavior of price informativeness empirically. In particular, Bai, Philippon and Savov (2016) empirically test for the forecasting ability of financial markets by running cross-sectional regressions of future earnings on current market prices.[5] Their

[4] Since we consider a CARA-Gaussian setup, our results should be interpreted as a first-order approximation to more general environments (Ingersoll (1987), Huang and Litzenberger (1988)). There is scope to further understand the relation between volatility and informativeness in the context of non-linear models, as those studied by Barlevy and Veronesi (2000), Albagli, Hellwig and Tsyvinski (2014, 2015, 2017), Breon-Drish (2015), Chabakauri, Yuan and Zachariadis (2015), or Pálvölgyi and Venter (2015). There is scope to use a similar approach to the one developed in this paper to link outcomes of alternative allocative mechanisms (e.g., auctions) to measures of information aggregation.

[5] See Davila and Parlatore (2018) for a detailed analysis of how measures of price informativeness differ from forecastability/predictability measures.

measure has increased over the last decades. Farboodi et al. (2019) show that this increase is driven by large, growth stocks, for which data have become relatively more valuable. Davila and Parlatore (2018) develop a new methodology to structurally recover exact stock-specific measures of price informativeness.

Finally, we would like to highlight the high-level relation between our results and the work of Bergemann, Heumann and Morris (2015). They show in an abstract linear-quadratic environment that the information structure that yields maximal aggregate volatility is such that agents confound idiosyncratic and aggregate shocks, excessively responding to aggregate shocks. Their goal is to study how alternative information structures affect the moments (e.g., volatility) of endogenous variables in the economy. Instead, our goal is to understand the endogenous equilibrium relation between the signal-to-noise ratio associated with asset prices, which is an unobservable variable that captures the ability of financial markets to aggregate information, with the volatility of asset prices, which is an easily computable endogenous outcome of financial market trading.

Outline Section 2 describes the general setup and presents the fundamental relation between price informativeness and price volatility. Section 3 specializes our results to the CARA-Gaussian environment, while Section 4 introduces three canonical applications and provides full comparative statics on primitives exploiting our main results. Section 5 explicitly characterizes the set of primitives that guarantee positive and negative comovement and uses stock market data to recover model parameters and determine in which region different stocks lie and Section 6 concludes. The Appendix contains derivations, proofs, and additional results.

2 Fundamental Relation: General Environment

In this section, we characterize the equilibrium relation between price informativeness and price volatility when investors have linear asset demands and face additive noise.

2.1 General environment

Time is discrete, with periods denoted by $t = 0, 1, 2, ..., \infty$. There are two assets: a riskless asset in elastic supply with gross return $R > 1$ and a risky asset in fixed supply Q, which is traded at a price p_t in period t. The asset payoff, which accrues at the beginning of period $t + 1$, is given by

$$\theta_{t+1} = \mu_\theta + \rho\theta_t + \eta_t,$$

where μ_θ is a scalar, $|\rho| \leq 1$, and $\theta_0 = 0$, and where the innovations to the payoff, η_t, have mean zero, finite variance τ_η^{-1}, and are independently distributed.[6]

A set of investors, indexed by $i \in I$, trade both assets in period t. Before trading in period t, each investor i observes the already realized value of the asset payoff θ_t and a private signal s_t^i of the

[6] When $\rho = 0$, the model effectively behaves as if it were static. When $\rho = 1$, asset payoffs and prices are non-stationary and follow a random walk.

innovation to the future asset payoff η_t. Moreover, investors have additional motives for trading the risky asset that are orthogonal to the asset payoff. We denote by n_t^i investor i's additional trading motive in period t. These trading motives are private information of each investor.

We derive our first set of results under two assumptions. The first assumption imposes an additive informational structure, while the second assumption imposes a linear structure for investors' equilibrium asset demands. In Sections 3 and 4, we provide fully specified sets of primitives that are consistent with Assumptions 1 and 2.

Assumption 1. (Additive noise) *Each period t, every investor i receives an unbiased private signal s_t^i about the innovation to the payoff, η_t, of the form*

$$s_t^i = \eta_t + \varepsilon_{st}^i, \tag{1}$$

where ε_{st}^i, $\forall i \in I$, $\forall t$, are random variables with mean zero and finite variances, whose realizations are independent across investors and over time. Each period t, every investor i has a private trading need n_t^i, of the form

$$n_t^i = n_t + \varepsilon_{nt}^i, \tag{2}$$

where n_t is a random variable with finite mean, denoted by μ_n, and finite variance, and where ε_{nt}^i, $\forall i \in I$, $\forall t$, are random variables with mean zero and finite variances, whose realizations are independent across investors and over time.

Assumption 1 imposes a noise structure that is additive and independent across investors for the signals about the innovation to the future payoff η_t as well as for other sources of investors' private trading needs n_t^i. This assumption does not restrict the distribution of any random variable beyond the existence of finite first and second moments. Our second assumption describes the structure of the investors' net demands for the risky asset Δq_t^i.

Assumption 2. (Linear asset demands) *Investors' net asset demands satisfy*

$$\Delta q_t^i = \alpha_s^i s_t^i + \alpha_\theta^i \theta_t + \alpha_n^i n_t^i - \alpha_p^i p_t + \psi^i,$$

where α_s^i, α_θ^i, α_n^i, α_p^i, and ψ^i are individual demand coefficients, potentially determined in equilibrium.

Assumption 2 imposes that the net asset demand for the risky asset for a given investor is linear in his signal about the asset payoff and his private trading needs, as well as in the asset price p_t and the realized asset payoff θ_t. It also allows for an individual specific invariant component ψ^i. This linear structure arises endogenously under CARA utility and Gaussian uncertainty, as we show in Section 3. More broadly, linear asset demands can be interpreted as a linear approximation to general asset demand functions, so the results in Proposition 1 are valid generally up to a first-order approximation.

2.2 Equilibrium price characterization

Market clearing in the risky asset market implies that $\int \Delta q_t^i di = 0$ must hold each period t.[7] Assumptions 1 and 2, when combined with market clearing, imply that the equilibrium asset price

[7]To accommodate a continuum or a finite number of agents, all integrals in the paper represent Lebesgue integrals.

must satisfy

$$p_t = \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\theta_t + \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\eta_t + \frac{\overline{\alpha_n}}{\overline{\alpha_p}}n_t + \frac{\int_I \alpha_s^i \varepsilon_{st}^i di}{\overline{\alpha_p}} + \frac{\int_I \alpha_n^i \varepsilon_{nt}^i di}{\overline{\alpha_p}} + \frac{\overline{\psi}}{\overline{\alpha_p}},$$

where we denote the cross sectional averages of individual demand coefficients by $\overline{\alpha_\theta} = \int_I \alpha_\theta^i di$, $\overline{\alpha_s} = \int_I \alpha_s^i di$, $\overline{\alpha_n} = \int_I \alpha_n^i di$, $\overline{\alpha_p} = \int_I \alpha_p^i di$, and $\overline{\psi} = \int_I \psi^i di$. The linearity of net demands implies that the equilibrium asset price is also linear in the innovation to the asset payoff η_t, in the already realized payoff θ_t, and in the common component of investors' private trading needs n_t. When there is a continuum of investors, a law of large numbers guarantees that the terms $\frac{\int_I \alpha_s^i \varepsilon_{st}^i di}{\overline{\alpha_p}}$ and $\frac{\int_I \alpha_n^i \varepsilon_{nt}^i di}{\overline{\alpha_p}}$ vanish. Otherwise, these terms operate as additional sources of aggregate noise.

The equilibrium price p_t imperfectly reveals the innovation to the asset payoff η_t. The sensitivity of the equilibrium price to the realization of the innovation is modulated by the average weight that investors put on their private signals s_t^i. However, investors' demands also depend on their private trading motives n_t^i, which are orthogonal to the asset payoff. Since investors do not observe the common component of these additional trading motives, they cannot distinguish whether a high price is due to a high realization of the innovation to the asset payoff η_t or due to a high aggregate trading need unrelated to the asset payoff n_t. In this sense, investors' private trading motives act as noise, since they prevent their signals about the asset payoff from being revealed by their quantity demanded and, consequently, they prevent the price from being fully revealing. In our applications, we map the variable n_t to random heterogeneous priors and hedging needs, which become sources of noise in the filtering problem solved by investors.

Finally, we denote the unbiased signal about the innovation to the asset payoff η_t contained in the price by $\hat{p}_t$. We make use of the unbiased signal $\hat{p}_t$ in our definition of price informativeness. Formally, we define $\hat{p}_t$ as

$$\hat{p}_t = \frac{\overline{\alpha_p}}{\overline{\alpha_s}}\left(p_t - \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\theta_t - \frac{\overline{\alpha_n}}{\overline{\alpha_p}}\mathbb{E}\left[n_t\right] - \frac{\overline{\psi}}{\overline{\alpha_p}}\right) = \eta_t + \frac{\overline{\alpha_n}}{\overline{\alpha_s}}\left(n_t - \mathbb{E}\left[n_t\right]\right) + \frac{\int_I \alpha_s^i \varepsilon_{st}^i di}{\overline{\alpha_s}} + \frac{\int_I \alpha_n^i \varepsilon_{nt}^i di}{\overline{\alpha_s}}, \tag{3}$$

which guarantees that $\mathbb{E}\left[\hat{p}_t | \eta_t\right] = \eta_t$. The last three terms in Eq. (3) represent the noise contained in the price. The first of these three terms is the realization of the common component of the investors' private trading needs, adjusted by the ratio $\frac{\overline{\alpha_n}}{\overline{\alpha_s}}$, so it is expressed in payoff units. The final two terms in Eq. (3) capture the sources of aggregate noise that arise from the imperfect aggregation of idiosyncratic shocks when there is a finite number of investors. Note that our definition of $\hat{p}_t$ allows us to write $p_t = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\hat{p}_t + \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\theta_t + \frac{\overline{\alpha_n}}{\overline{\alpha_p}}\mathbb{E}\left[n_t\right] + \frac{\overline{\psi}}{\overline{\alpha_p}}$, which allows us to interpret $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ as $\frac{\partial p}{\partial \hat{p}}$.

2.3 Relating price informativeness and price volatility

Using the equilibrium price p_t and the unbiased signal about the asset payoff contained in the price $\hat{p}_t$, we can formally define our two objects of interest as follows.

Definition 1. (Price informativeness) *We define price informativeness as the precision of the unbiased signal of the innovation to the asset payoff η_t contained in the asset price, $\hat{p}_t$, defined in*

Eq. (3)*, from the perspective of an external observer. We denote price informativeness by*

$$\tau_{\hat{p}}^{e}=\left(\mathbb{V}ar\left[\hat{p}_t|\eta_t,\theta_t\right]\right)^{-1}. \tag{4}$$

Price informativeness is a variable that summarizes the ability of financial markets to disseminate information through prices. It is the relevant variable that captures how precise the price is as a signal of η_t from the perspective of an external observer who only observes the realization of the asset payoff θ_t. When price informativeness is high, an external observer receives a very precise signal about the asset payoff by observing the asset price p_t. On the contrary, when price informativeness is low, an external observer learns little about the asset payoff by observing the asset price p_t.

Definition 2. (Price volatility) *We define price volatility as the conditional variance of the asset price, given the past realizations of the asset payoff. We denote price volatility by*

$$\mathcal{V}\equiv\mathbb{V}ar\left[p_t|\,\theta_t\right].$$

For our purposes, price volatility is simply the idiosyncratic variance of asset prices conditional on the current publicly observed realization of the asset payoff. Our goal in this paper is to understand how price volatility and price informativeness are related in equilibrium to be able to make inferences about price informativeness, which is not directly observable and is hard to compute at high frequencies, from idiosyncratic conditional price volatility, which is easily computable. Characterizing the equilibrium relation between these two endogenous variables is the first step to understand how price informativeness and price volatility react to changes in primitives.

Our first set of results builds on the the law of total variance, which is an elementary identity that states that conditional price volatility can be decomposed into two components:

$$\mathbb{V}\mathrm{ar}\left[p_t|\,\theta_t\right]=\mathbb{E}\left[\mathbb{V}\mathrm{ar}\left[p_t|\,\eta_t,\theta_t\right]|\,\theta_t\right]+\mathbb{V}\mathrm{ar}\left[\mathbb{E}\left[p_t|\,\eta_t,\theta_t\right]|\,\theta_t\right].$$

The law of total variance asserts that the total variation in the equilibrium price p_t can be decomposed into two components, after conditioning on the innovation to the asset payoff η_t. The first component corresponds to the expectation over the different realizations of the innovation to the asset payoff η_t of the conditional variance of the equilibrium price p_t, given η_t. The second component corresponds to the variance of the conditional expectation of p_t, after learning η_t. Intuitively, the first component captures learnable uncertainty, captured by the best estimate of the residual error in p_t after learning η_t, while the second term captures residual uncertainty, which corresponds to the error from the best guess of p_t after learning η_t.

Under Assumptions 1 and 2, we can express both components as follows

$$\mathbb{E}\left[\mathbb{V}\mathrm{ar}\left[p_t|\,\eta_t,\theta_t\right]\right]=\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\left(\tau_{\hat{p}}^{e}\right)^{-1}\quad\text{and}\quad\mathbb{V}\mathrm{ar}\left[\mathbb{E}\left[p_t|\,\eta_t,\theta_t\right]\right]=\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\tau_\eta^{-1},$$

which allows us to establish the most general characterization of the relation between price informativeness and volatility in Proposition 1. Intuitively, the variation in $\mathbb{E}\left[p_t|\,\eta_t,\theta_t\right]$ is driven by the variance of the innovation to the asset payoff τ_η^{-1}, while the average residual variance is modulated by changes in price informativeness $\tau_{\hat{p}}^{e}$.

Proposition 1. (Fundamental relation)

a) Given Assumptions 1 and 2, price volatility $\mathcal{V}$ and price informativeness $\tau_{\hat{p}}^{e}$ satisfy the following relation

$$\mathcal{V}=\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^{2}\left(\tau_{\eta}^{-1}+\left(\tau_{\hat{p}}^{e}\right)^{-1}\right). \tag{5}$$

b) The equilibrium elasticity of price volatility to price informativeness is given by

$$\frac{d\log\mathcal{V}}{d\log\tau_{\hat{p}}^{e}}=\underbrace{2\frac{d\log\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)}{d\log\left(\tau_{\hat{p}}^{e}\right)}}_{\textit{Equilibrium Learning}}-\underbrace{\frac{\left(\tau_{\hat{p}}^{e}\right)^{-1}}{\tau_{\eta}^{-1}+\left(\tau_{\hat{p}}^{e}\right)^{-1}}}_{\textit{Noise Reduction}}. \tag{6}$$

We refer to Eq. (5) as the *fundamental relation* between price informativeness and price volatility. Part a) of Proposition 1 shows that this equilibrium relation features the exogenous primitive τ_{η}^{-1}, which corresponds to the variance of the innovation to the asset payoff, and the equilibrium object $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, which we refer to as the signal-to-price sensitivity and that in general depends on $\tau_{\hat{p}}^{e}$. By expressing $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ as a function of $\tau_{\hat{p}}^{e}$ and potentially other primitives, we identify two distinct channels that determine the relation between price informativeness and volatility at this level of generality in part b) of Proposition 1.[8]

We refer to the first channel as the *equilibrium learning* channel. If a high level of price informativeness is associated with a high (low) level of the signal-to-price sensitivity $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, this induces a positive (negative) relation between price informativeness and volatility. A high value of the signal-to-price sensitivity $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ amplifies the sensitivity of asset prices to aggregate shocks.[9] Intuitively, a high $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ implies that, on average, either investors react significantly to their private signals (high $\overline{\alpha_s}$), or that they have very steep – under the traditional economics convention that uses quantities in the horizontal axis – asset demand curves (low $\overline{\alpha_p}$), so investors barely adjust the quantity demanded even for large price changes, implying that equilibrium prices substantially react to the realization of the asset payoff. Alternatively, a low $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ implies that, on average, investors barely react to their private signals (low $\overline{\alpha_s}$), or that they have very flat —under the traditional economics convention—asset demand curves (high $\overline{\alpha_p}$), so investors significantly adjust the quantity demanded even for small price changes, implying that equilibrium prices are barely responsive to the realization of aggregate payoff shocks.

We refer to the second channel as the *noise reduction* channel. It is evident from Proposition 1 that, holding $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ constant, a high level of $\tau_{\hat{p}}^{e}$ is mechanically associated with a low level of $\mathcal{V}$. In fact, Eq. (5) implies that there exists an inverse relation between both variables. Intuitively, when prices are very informative, the noise in the price is low and the conditional variance of the price for a given realization of the asset payoff is necessarily low.

[8] Note that we express Equation (6) using a total derivative and not a partial derivative. This notation accounts for the fact that $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ may be related in equilibrium to $\tau_{\hat{p}}^{e}$.

[9] Note that $\mathbb{V}\mathrm{ar}\left[p\mid\theta_t\right]=\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^{2}\mathbb{V}\mathrm{ar}\left[\hat{p}\mid\theta_t\right]$, since the variance of the unbiased signal about the asset payoff can be expressed as $\mathbb{V}\mathrm{ar}\left[\hat{p}\mid\theta_t\right]=\tau_{\eta}^{-1}+\left(\tau_{\hat{p}}^{e}\right)^{-1}$. We can thus interpret asset price volatility as the volatility of the unbiased signal about the asset payoff, corrected by investors' endogenous responses through the signal-to-price sensitivity.

It is worth highlighting that part b) of Proposition 1 is not a comparative statics exercise, but a characterization of a relation between two endogenous variables that must be satisfied in any equilibrium, given the economy's parameters. There are scenarios in which changes in some primitives do not shift the locus defined in Eq. (5). In those cases, Eq. (5) can be interpreted as the possible combinations of $\mathcal{V}$ and $\tau^e_{\hat{p}}$ that can arise in equilibrium for different values of those primitives. In those scenarios, Proposition 1 implies that equilibria with high volatility are also equilibria with high (low) price informativeness whenever $\frac{d\log\mathcal{V}}{d\log\tau^e_{\hat{p}}} > 0\,(< 0)$. However, changes in parameters that shift the locus defined in Eq. (5) entail a shift of the fundamental relation and, in general, also a movement along the curve. Therefore, it is necessary to determine how $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ and $\tau^e_{\hat{p}}$ are related in equilibrium as a function of the model's parameters to further understand the relation between price informativeness and price volatility.

Before we study the link between $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ and the model's primitives in more detail, it is worth emphasizing that the fundamental relation can only have a positive slope when investors learn from asset prices. When investors do not learn from prices, changes in the level of price informativeness do not affect investors' behavior, so ${d\log\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)}/{d\log(\tau^e_{\hat{p}})} = 0$. In this case, only the noise reduction channel is active, and the relation between price informativeness and price volatility in Eq. (5) is monotonic and decreasing. However, as we show next, in the CARA-Gaussian case $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ is increasing in $\tau^e_{\hat{p}}$, so the equilibrium learning channel and the noise reduction channel operate in opposite directions.

3 Fundamental Relation: CARA-Gaussian Setup

In this section, we specialize our results to a canonical CARA-Gaussian environment, which endogenously satisfies Assumptions 1 and 2. This allows us to further characterize the relation between the signal-to-price sensitivity $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ and price informativeness $\tau^e_{\hat{p}}$. In the next section, we provide several fully specified models to completely characterize the fundamental relation between price informativeness and volatility as a function of primitives. We specialize the environment described in Section 2 along the following dimensions.

Timing and assets. Time is discrete, with periods denoted by $t = 0, 1, 2, \ldots, \infty$. There are two traded assets: a riskless asset in perfectly elastic supply with gross return $R > 1$ and a risky asset in fixed supply Q, which is traded at a price p_t in period t.

Preferences. A new set of investors, indexed by $i \in I$, is born in each period t. Investors born in period t trade in period t and consume their terminal wealth in period $t+1$. Each generation of investors lives two periods and has constant absolute risk aversion (CARA) preferences over their last period wealth. The expected utility of an investor i born in period t is given by

$$U\left(w^i_{t+1}\right) = -e^{-\gamma w^i_{t+1}}, \tag{7}$$

where Eq. (7) imposes that investors consume all their terminal wealth w^i_{t+1}. The parameter $\gamma > 0$ represents the coefficient of absolute risk aversion, $\gamma \equiv -\frac{U''}{U'}$.

Payoff process and signals. The asset payoff is given by

$$\theta_{t+1} = \mu_\theta + \rho\theta_t + \eta_t,$$

where μ_θ is a scalar, $|\rho| \leq 1$, and $\theta_0 = 0$, and where the innovations to the payoff, η_t, have mean zero, finite variance, τ_η^{-1}, and are independently and normally distributed. Before trading in period t, each investor i observes the current realized asset payoff θ_t. Each investor i receives a private signal s_t^i about the innovation to the asset payoff η_t, given by

$$s_t^i = \eta_t + \varepsilon_{st}^i \quad \text{with} \quad \varepsilon_{st}^i \sim N\left(0, \tau_s^{-1}\right).$$

Private trading needs. As before, the investors' privately observed trading motives are sources of aggregate noise in the economy that prevent the price from being fully revealing. In particular, every investor i privately observes n_t^i, which takes the form

$$n_t^i = n_t + \varepsilon_{nt}^i, \quad \text{with} \quad \varepsilon_{nt}^i \sim N\left(0, \tau_\varepsilon^{-1}\right),$$

where $n_t \sim N\left(\mu_n, \tau_n^{-1}\right)$, which can be interpreted as the aggregate sentiment in the economy, is orthogonal to ε_{nt}^i. We assume that the private trading needs of the investor are orthogonal to the asset payoff and that all error terms are independent of each other, of the common component of the private trading needs, and of the innovation to the asset payoff.

In the CARA-Gaussian setup presented in this section, all equilibria in linear strategies satisfy Assumption 2. As it is standard in this body of work, we focus on symmetric equilibria in linear strategies.[10] Investors in the model have more information than external observers because they receive a private signal about η_t and they observe their private trading need. For example, investors could learn about the aggregate noise in the price from their private trading need. If an investor's private trading need was perfectly informative about the aggregate trading need in the economy, the investor could perfectly observe the asset payoff by looking at the equilibrium price. Therefore, the amount of information that is contained in the price from an internal investor's perspective, which determines the investors' equilibrium learning, may differ from the informational content of prices from an external observer's point of view. To account for this discrepancy, we introduce the notion of internal price informativeness.

Definition 3. (Internal price informativeness) *We define internal price informativeness as the precision of the additional information contained in the unbiased signal of the innovation to the asset payoff η_t contained in the asset price, $\hat{p}_t$, defined in Eq. (3), from the perspective of an investor in the model. We denote internal price informativeness by*

$$\tau_{\hat{p}} = \left(\mathbb{V}ar\left[\hat{p}|\eta_t, \theta_t, n_t^i\right]\right)^{-1}. \tag{8}$$

[10]To ease the exposition, we describe our results in the text as if the model had a unique equilibrium, although we consider the possibility of multiplicity in the Appendix. If there were multiple equilibria, our analysis would be valid for locally stable equilibria as long as the economy does not jump from one equilibrium to another.

The notion of internal price informativeness becomes relevant in models in which investors' private trading needs are informative about the aggregate noise in the price, and in strategic environments. In the first case, internal price informativeness is higher than price informativeness for an external observer, since investors have additional information about the noise. In the second case, internal price informativeness is lower than price informativeness for an external observer. The new information contained in the price aggregates the signals of all investors from an external observer's perspective. Since one of these signals is the private signal observed by the investor, the price contains one new signal less for an strategic investor than for an external observer.

The following Lemma characterizes the equilibrium relation between $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ and $\tau_{\hat{p}}$ in the CARA-Gaussian setup that we consider.

Lemma 1. (Signal-to-price sensitivity) *In the CARA-Gaussian setup, the signal-to-price sensitivity can be expressed as a function of internal price informativeness $\tau_{\hat{p}}$ and primitives τ_s, τ_η, and $R^{-1}\rho$ as follows*

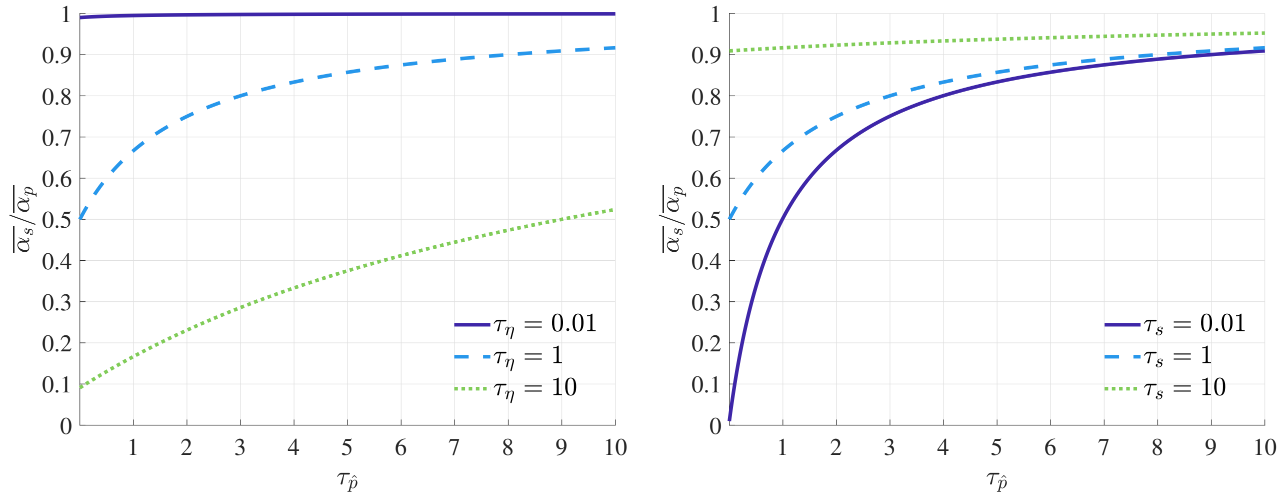
$$\frac{\overline{\alpha_s}}{\overline{\alpha_p}} = \frac{1}{1 - R^{-1}\rho} \frac{\tau_s + \tau_{\hat{p}}}{\tau_\eta + \tau_s + \tau_{\hat{p}}}, \tag{9}$$

where $\tau_{\hat{p}}$ is defined in Eq. (8).

Given that investors have three sources of information about the asset payoff (their prior, their private signal, and the price signal), the signal-to-price sensitivity corresponds to the share of information acquired from the new signals at the disposal of investors, discounted by $R^{-1}\rho$. Therefore, high values of τ_s and $\tau_{\hat{p}}$ are associated with high values of $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, while high values of the prior precision τ_η are associated with low values of $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$. Similarly, if the process for the asset payoff is highly persistent (ρ is high) or the investors' discount rate is small (R is low), the signal-to-price sensitivity $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ is high, since new information about the innovation η_t becomes more valuable.

It is useful to interpret $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ as the sensitivity of the equilibrium price p_t to a change in the realization of the innovation to the asset payoff η_t, since $\frac{\partial p_t}{\partial \eta_t} = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}$. Intuitively, a unit increase in the realization of η_t increases the value of the signals received by investors, increasing aggregate demand by $\overline{\alpha_s}$. This increase in aggregate demand increases the equilibrium price, which endogenously changes investors' demands, according to $\frac{1}{\overline{\alpha_p}}$, for two reasons: i) a reduction in demand for purely pecuniary considerations, and ii) an increase in demand for informational reasons, since a higher price leads investors to infer that other investors received high signals about the asset payoff. Since substitution effects dominate in our model, the first effect always dominates in equilibrium, so that asset demands are downward sloping ($\overline{\alpha_p} > 0$).

Figure 1a illustrates how the behavior of the signal-to-noise ratio varies with the strength of the prior precision τ_η, for a given internal price informativeness $\tau_{\hat{p}}$. If the asset payoff is extremely volatile ($\tau_\eta \to 0$), investors exclusively rely on the signals about the asset payoff at their disposal, and $\frac{\overline{\alpha_s}}{\overline{\alpha_p}} \to 1$. Alternatively, if investors' prior information is extremely accurate ($\tau_\eta \to \infty$), investors exclusively rely on their prior information, so changes in the realization of η_t barely move at all the equilibrium price, and $\frac{\overline{\alpha_s}}{\overline{\alpha_p}} \to 0$. Intuitively, the more precise the prior information τ_η held by investors, the less sensitive the asset price to the realization of η_t.



(a) Varying the prior precision about the innovation to the asset payoff, τ_η

(b) Varying the precision of investors' signals, τ_s

Figure 1: Signal-to-price sensitivity, $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$

Note: Figure 1 shows how the signal-to-price sensitivity $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, characterized in Eq. (9), varies as a function of internal price informativeness $\tau_{\hat{p}}$ for different values of τ_η and τ_s, respectively, when $\rho = 0$ and $R = 1.04$. This parameterization implies that $\frac{1}{1-R^{-1}\rho} = 1$. Figure 1a is drawn for $\tau_s = 1$ and Figure 1b is drawn for $\tau_\eta = 1$.

Figure 1b illustrates how the behavior of the signal-to-noise ratio varies with the strength of the precision of investors' private signals τ_s, for a given $\tau_{\hat{p}}$. If investors' signals are extremely precise ($\tau_s \to \infty$), investors trade one for one with their private signals, so $\frac{\overline{\alpha_s}}{\overline{\alpha_p}} \to 1$. Alternatively, if investors' signals are very inaccurate ($\tau_s \to 0$), investors exclusively rely on their prior information, so $\frac{\overline{\alpha_s}}{\overline{\alpha_p}} \to \frac{\tau_{\hat{p}}}{\tau_\eta + \tau_{\hat{p}}}$. For a given τ_s and τ_η, changes in $\tau_{\hat{p}}$ have the same effect as changes in τ_s given $\tau_{\hat{p}}$.

Note that Lemma 1 expresses the signal-to-price ratio $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ as a function of internal price informativeness, $\tau_{\hat{p}}$, so we need to further understand the relation between internal and external price informativeness to fully characterize the fundamental relation in Eq. (5). We do so in the following Lemma.

Lemma 2. (Relating internal price informativeness and price informativeness for an external observer) *In the CARA-Gaussian setup, there exists a scalar $\lambda > 0$ that can be expressed exclusively in terms of model primitives, such that*

$$\tau_{\hat{p}} = \lambda \tau_{\hat{p}}^e,$$

where $\tau_{\hat{p}}^e$ and $\tau_{\hat{p}}$ are respectively defined in Eqs. (4) and (8).

Lemma 2 shows that both notions of informativeness are related in this setup. Intuitively, when there is a continuum of investors and investors private trading needs reveal information about the aggregate noise, $\lambda > 1$ and $\tau_{\hat{p}}^e \leq \tau_{\hat{p}}$. If investors do not learn about the aggregate sources of noise from their own private trading needs, then $\tau_{\hat{p}} = \tau_{\hat{p}}^e$, as in the applications with heterogeneous priors and noise traders in Section 4. Alternatively, when there is a finite number of strategic investors N, investors perceive the price to be less informative than an external observer, because the price aggregates N new signals for an external observer, while for an investor in the model it only aggregates $N-1$ new signals, so $\lambda < 1$ and $\tau_{\hat{p}}^e \geq \tau_{\hat{p}}$.

Combining Lemmas 1 and 2, we specialize the fundamental relation between price volatility and price informativeness to the CARA-Gaussian environment in the following Lemma.

Lemma 3. (Fundamental relation CARA-Gaussian setup) *In the CARA-Gaussian setup, the fundamental relation between price volatility $\mathcal{V}$ and price informativeness $\tau_{\hat{p}}^e$ is given by*

$$\mathcal{V} = \left(\frac{1}{1 - R^{-1}\rho} \frac{\tau_s + \lambda \tau_{\hat{p}}^e}{\tau_\eta + \tau_s + \lambda \tau_{\hat{p}}^e} \right)^2 \left(\tau_\eta^{-1} + \left(\tau_{\hat{p}}^e \right)^{-1} \right), \tag{10}$$

where $\lambda = \frac{\tau_{\hat{p}}}{\tau_{\hat{p}}^e}$.

Lemma 3 represents the endogenous relation between $\mathcal{V}$ and $\tau_{\hat{p}}^e$ as a function of only three (combinations of) primitives: τ_η, τ_s, and λ, which allows us to explicitly characterize the properties of the fundamental relation. Note that the variance of the equilibrium price converges to the variance of the asset payoff when prices are infinitely informative. Alternatively, the equilibrium price is infinitely volatile when prices are totally uninformative. Formally,

$$\lim_{\tau_{\hat{p}}^e \to \infty} \mathcal{V} = \tau_\eta^{-1} \quad \text{and} \quad \lim_{\tau_{\hat{p}}^e \to 0} \mathcal{V} = \infty.$$

Note also that

$$\lim_{\tau_{\hat{p}}^e \to 0} \frac{d\mathcal{V}}{d\tau_{\hat{p}}^e} = -\infty \quad \text{and} \quad \lim_{\tau_{\hat{p}}^e \to \infty} \frac{d\mathcal{V}}{d\tau_{\hat{p}}^e} = 0.$$

Intuitively, for low levels of price informativeness, the noise reduction channel dominates the equilibrium learning channel, since learning is ineffective. When prices are infinitely informative, the noise reduction channel and the equilibrium learning channel perfectly cancel each other. These observations already provide some structure to the fundamental relation. Combining both sets of limits with the continuity of the relation, we conclude that the fundamental relation has an asymptote at $\tau_{\hat{p}}^e = 0$ and that it converges smoothly towards τ_η^{-1} when prices are sufficiently informative.

Whether the relation between price volatility and price informativeness in Eq. (5) is monotonic depends on the value of λ. In particular, when $\lambda < 2$, which encompasses the scenario in which internal and external price informativeness are equal, the fundamental relation is non-monotonic. The variable λ represents how much more new information is contained in the price for an investor relative to an external observer. If $\lambda > 2$, the investor learns more than twice as much as an external observer by using the price as a signal. Although one could argue that active investors may have better information about the noise embedded in asset prices, hence learning more from the price than external observers, it is not easy to argue why there should be a two-fold difference between both groups. In fact, most models considered in the literature on learning in financial markets (e.g., Veldkamp (2011) and Vives (2016)) implicitly adopt parameterizations that imply $\lambda = 1$. In two of our three applications, λ is also

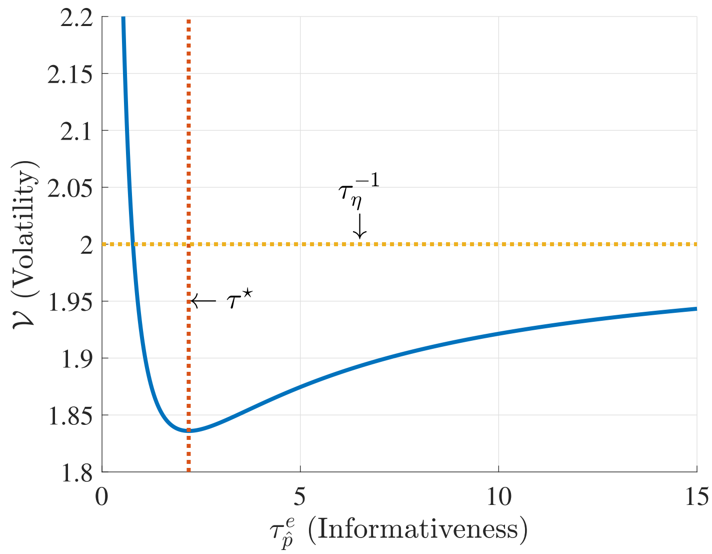


Figure 2: Fundamental relation

Note: Figure 2 plots price volatility as a function of price informativeness, as given by the fundamental relation in Eq. (10), for parameters $\tau_\eta = 0.5$, $\tau_s = 1$, $\lambda = 1$, $\rho = 0$, and $R = 1.04$. The vertical red dotted line represents the threshold $\underline{\tau}$ that delimits (to its right) the region in which the fundamental relation is downward sloping. The horizontal yellow dotted line depicts the limit τ_η^{-1} to which the fundamental relation converges when prices are perfectly informative.

weakly less than one. Therefore, in what follows, we focus on and state our formal results for the case $\lambda < 2$.[11]

We formally show that the fundamental relation is decreasing for sufficiently low values of $\tau_{\hat{p}}^{e}$ and increasing for sufficiently high values of $\tau_{\hat{p}}^{e}$. The following proposition formalizes this non-monotonicity.

Proposition 2. (Slope of fundamental relation) *The fundamental relation between price volatility and price informativeness is increasing (decreasing) if and only if price informativeness is high (low) enough. Formally, there exists a threshold $\tau^\star > 0$ such that*

$$\frac{d\mathcal{V}}{d\tau_{\hat{p}}^{e}} < 0 \quad \Longleftrightarrow \quad \tau_{\hat{p}}^{e} < \tau^\star \quad \textit{and} \quad \frac{d\mathcal{V}}{d\tau_{\hat{p}}^{e}} > 0 \quad \Longleftrightarrow \quad \tau_{\hat{p}}^{e} > \tau^\star,$$

where

$$\tau^\star = \frac{-\lambda\left(\tau_\eta - 2\tau_s\right) + \sqrt{\lambda\left(\lambda\tau_\eta\left(\tau_\eta - 8\tau_s\right) + 8\tau_s\left(\tau_\eta + \tau_s\right)\right)}}{2\left(2-\lambda\right)\lambda}. \tag{11}$$

Proposition 2 shows that, regardless of the source of noise in the model, the slope of Eq. (10) is positive when $\tau_{\hat{p}}^{e}$ is sufficiently large and negative otherwise. The threshold $\tau^\star$, which determines the lower boundary of the positive slope region, only depends on the precision of the innovation to the asset payoff, the precision of the private signal, and the value of λ. Interestingly, the threshold $\tau^\star$ only depends on the remaining model parameters indirectly through λ. In particular, the specific

[11]In previous versions of the paper, we also studied the case of classic noise traders, which also features $\lambda = 1$. The results of the $\lambda \geq 2$ case are available upon request.

source of noise may only affect $\tau^\star$ through λ. Exploiting our two-channel decomposition, we say that when prices are sufficiently informative, when $\tau^e_{\hat{p}} > \tau^\star$, the equilibrium learning channel dominates the noise reduction channel. On the contrary, when $\tau^e_{\hat{p}} < \tau^\star$, the noise reduction channel dominates the equilibrium learning channel. Figure 2 illustrates the shape of the fundamental relation between price volatility and price informativeness in Eq. (5) and the threshold $\tau^\star$ when $\lambda < 2$.

Proposition 2 implies that any change in the subset of parameters that do not enter the fundamental relation directly must induce a positive comovement between price informativeness and volatility when prices are sufficiently informative and a negative comovement otherwise. When interpreted through the lens of our two-channel decomposition, when prices are sufficiently informative, the equilibrium learning channel, which is driven by the change in investors' equilibrium behavior induced by learning, becomes overwhelmingly important and dominates the noise reduction channel, and vice versa. Proposition 2 also implies that to fully characterize the relation between price informativeness and price volatility across equilibria whenever there is a change in the subset of parameters that at the same time shifts the fundamental relation upwards or downwards and increases or decreases price informativeness, it is necessary to look at fully specified models in which the source of noise is full specified. In the following section, we consider three different ways of modeling noise: heterogeneous priors, hedging needs, and a finite number of investors.

4 Positive and Negative Comovement Regions

While the results derived in the general CARA-Gaussian setup provide interesting insights into the nature of the relation between price volatility and informativeness, understanding the exact behavior of both variables across equilibria requires the study of fully specified models. In this section, we seek to draw conclusions about the comovement of both variables by studying three representative applications that illustrate the different values that the ratio of precisions of the signal contained in the price for an investor relative to an external observer, λ, can take.

First, we study a model with $\lambda = 1$, in which the aggregate source of noise in the economy is driven by heterogeneity in investors' priors. Second, we study a model with $\lambda < 1$, in which a finite number of investors interact strategically. In this case, the law of large numbers breaks down and there are additional sources of aggregate noise coming from the average of the realized idiosyncratic realizations. Finally, we study a model with $\lambda > 1$, in which uncertainty about the aggregate level of hedging needs is the source of private trading needs.

In the three models that we consider in this section, it is always the case that when price informativeness is high enough, price informativeness and price volatility positively comove after any parameter change. Moreover, it is also the case that when price informativeness is low enough, there may exist a region in which price volatility and informativeness negatively comove for any parameter change. Proposition 3 formalizes these results. It is worth highlighting that our results apply to comparative static exercises that are valid for changes in *any of the underlying model parameters*, including those that appear in the fundamental relation.

Proposition 3. (Positive and negative comovement regions) *In all of the applications studied below:*

a) [Positive comovement region] Price volatility and price informativeness positively comove across equilibria if price informativeness is high enough. Formally, there exists a threshold $\overline{\tau} \in [\tau^{\star}, \infty)$ *such that, if* $\tau_{\hat{p}}^{e} \geq \overline{\tau}$, $\mathcal{V}$, *and* $\tau_{\hat{p}}^{e}$ *move in the same direction after any parameter change.*

b) [Negative comovement region] Price volatility and price informativeness negatively comove across equilibria if price informativeness is low enough. Formally, there exists a threshold $\underline{\tau} \in [0, \tau^{\star}]$ *such that, if* $\tau_{\hat{p}}^{e} < \underline{\tau}$, $\mathcal{V}$, *and* $\tau_{\hat{p}}^{e}$ *move in opposite directions after any parameter change.*

Proposition 3 follows by combining Proposition 2 with the fact that price informativeness i) increases with the precision of private information, τ_s, ii) decreases with the precision of the innovation to the asset payoff, τ_η, iii) increases with an increase in λ that leaves τ_n unchanged, and iv) increases with the precision of aggregate noise, τ_n, if $\tau_{\hat{p}}^{e} > \overline{\tau}$, in all applications. Consequently, Proposition 3 implies that parameter changes that shift the fundamental relation upwards (downwards) are associated with increases (decreases) in price volatility when price informativeness is high enough. Alternatively, Proposition 3 implies that parameter changes that shift the fundamental relation upwards (downwards) are associated with decreases (increases) in price volatility when price informativeness is low enough.

Which economic forces underlie the positive comovement finding in the region in which prices are very informative? For instance, an increase in the precision of investors' private signals about the asset payoff shifts the fundamental relation upwards because investors are more responsive to their information for any level of price informativeness, as we describe above when explaining Lemma 1. As expected, price informativeness also increases when investors receive more precise signals. When the equilibrium learning channel dominates, the upward shift in the fundamental relation and the increase in price informativeness guarantee an increase in price volatility, which yields the desired comovement. Alternatively, an increase in the precision of investors' priors about the fundamental shifts the fundamental relation downwards, since investors are less responsive to their information. As expected, price informativeness decreases when investors rely more on their priors. When the equilibrium learning channel dominates, the downward shift in the fundamental relation and the decrease in price informativeness guarantee a decrease in volatility, which yields again the same comovement. Similar arguments apply to changes in λ that do not involve τ_n, and to changes in τ_n, which may or may not modify λ.[12]

When price informativeness is sufficiently low, an increase in the precision of investors' private signals about the asset payoff shifts the fundamental relation upwards while increasing price informativeness. In principle, it may be that volatility increases or decreases with that change, since the fundamental relation is downward sloping. However, exploiting the fact, illustrated in Figure 2, that the fundamental relation has an asymptote at $\tau_{\hat{p}} = 0$, it is possible to show that there may exist a region of the

[12] Note that the only scenario in which the tighter threshold $\overline{\tau}$ becomes relevant is when considering changes in τ_n in cases in which investors partially infer the value of the aggregate trading need from their idiosyncratic realization, as it occurs in our application with hedging needs. Otherwise, the threshold $\tau^{\star}$, which defines the region in which the fundamental relation is upward sloping in Proposition 2, remains the relevant threshold for part a) in Proposition 3.

parameter space such that volatility and informativeness comove negatively for any parameter changes. Intuitively, when informativeness is sufficiently low, the noise reduction channel dominates for any change in primitives. While the positive comovement region always exists, the negative comovement region may or may not exist (that is, $\underline{\tau} = 0$) in specific applications, as we illustrate next.

Finding an unambiguous positive or negative comovement between both variables for any change in primitives, even for specific regions of the parameter space, may come as a surprise, since the prior about the sign of the relationship should not be obvious ex-ante. To further explore the results already derived, we formally introduce and perform comparative statics in all three applications. Subsequently, in Section 5, we further characterize the positive and negative comovement regions as a function of a subset of ratios of model primitives in all three applications. For our leading application, we eventually recover from observables the required stock-specific parameters that determine whether a given stock is in the positive or negative comovement regions.

4.1 Application 1: Disagreement

We consider a CARA-Gaussian setup as the one described in the previous section in which investors' private trading motives are given by differences in their beliefs about the expected payoff. This application yields a particularly tractable equilibrium characterization.[13] More specifically, there is a continuum of investors who have heterogeneous prior beliefs over the distribution of the innovation to the asset payoff. In particular, from an investor i's perspective, the innovation to the asset payoff η_t is distributed according to

$$\eta_t \sim_i N\left(\overline{\eta}_t^i, \tau_\eta^{-1}\right),$$

where $\overline{\eta}_t^i$ denotes investor i's prior expected payoff. We assume that investors' prior expected payoff innovations are random and distributed according to

$$\overline{\eta}_t^i = n_t + \varepsilon_{ut}^i,$$

where

$$\varepsilon_{ut}^i \overset{iid}{\sim} N\left(0, \tau_u^{-1}\right) \qquad \text{and} \qquad n_t \sim N\left(\mu_n, \tau_n^{-1}\right),$$

with $\varepsilon_{ut}^i \perp n_t$, $\varepsilon_{st}^i \perp n_t$, $\varepsilon_{ut}^i \perp \eta_t$, $\varepsilon_{st}^i \perp \eta_t$, $n_t \perp \eta_t$, and $\varepsilon_{ut}^i \perp \varepsilon_{st}^j$ for all $i, j \in I$, $i \neq j$, and $n_t \perp \eta_t$ for all t.

This formulation implies that an investor's prior mean has two components: an aggregate component, n_t, which can be interpreted as a measure of sentiment in the economy, and an idiosyncratic component ε_{ut}^i, which reflects an individual investors' beliefs. We assume that investors take their priors as given and do not use them to learn about the priors of others investors. Consequently, they do not infer anything about n_t from their own priors. However, investors know the distribution of priors in the economy and use this knowledge to learn from the price. The fact that the realized average prior mean n_t is unknown introduces an additional source of aggregate uncertainty, besides the innovation to the asset payoff.

[13] Davila and Parlatore (2017) introduce this specific model, which is closely related to De Long et al. (1990).

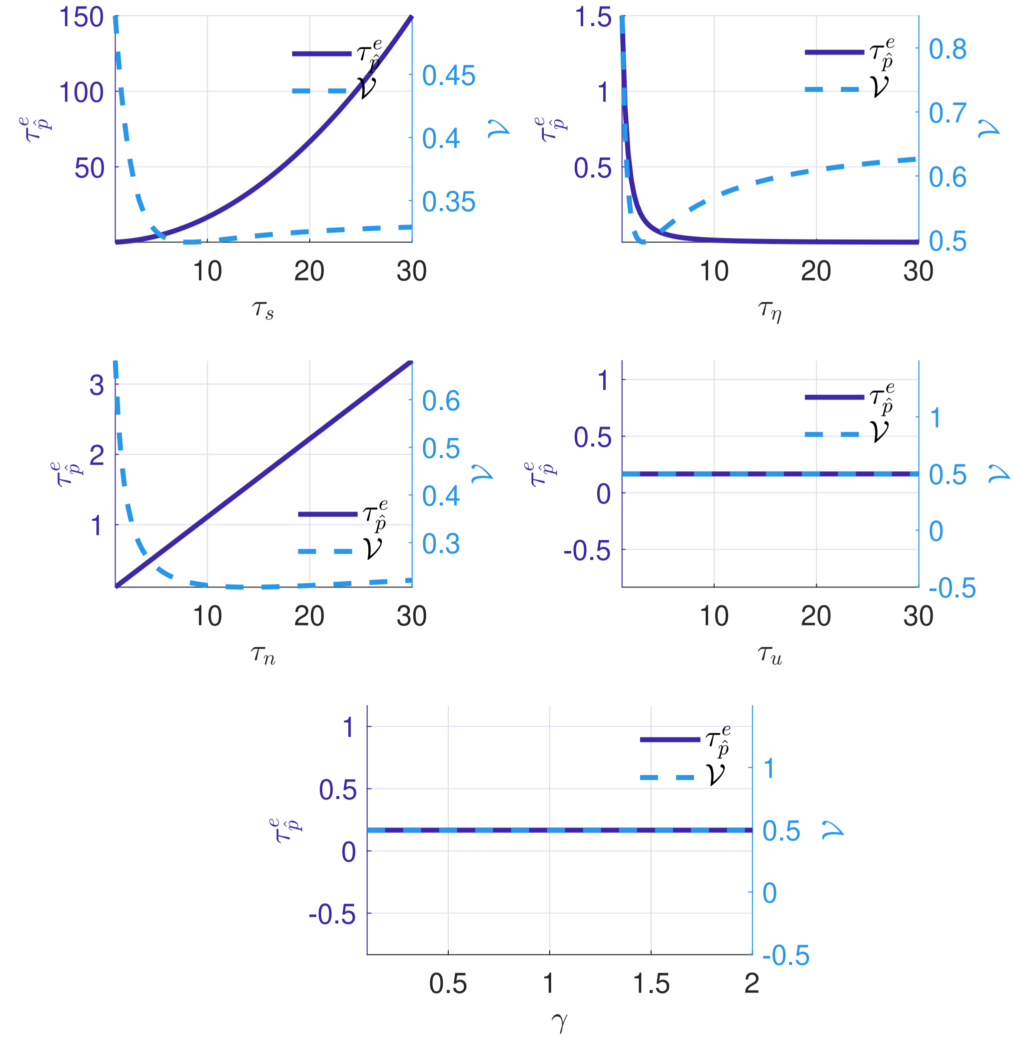


Figure 3: Comparative statics: Application 1

Note: Figure 3 shows comparative statics of price informativeness $\tau^e_{\hat{p}}$ and price volatility $\mathcal{V} = \mathbb{V}ar\left[p_t|\,\theta_t\right]$ as a function of all five primitives of the model considered in Application 1. All plots feature two y-axis: the left y-axis corresponds to the values of $\tau^e_{\hat{p}}$, while the right y-axis corresponds to the values of $\mathcal{V} = \mathbb{V}ar\left[p_t|\,\theta_t\right]$. The parameters of this model are the following: τ_s, precision of private signals about the innovation to the asset payoff, τ_η, precision of the innovation to the asset payoff, τ_n, precision of the average prior, τ_u, precision of investors' dispersion of heterogeneous beliefs, and γ, investors' coefficient of absolute risk aversion. The reference values are $\tau_s = 1$, $\tau_\eta = 3$, $\tau_n = 1.5$, $\tau_u = 1$, and $\gamma = 0.5$.

Consistently with our definition of λ in Lemma 2, the value of λ is unity in this application, so $\tau_{\hat{p}}^e = \tau_{\hat{p}}$. Therefore, we can express the fundamental relation for this application as

$$\mathcal{V} = \left(\frac{1}{1 - R^{-1}\rho} \frac{\tau_s + \tau_{\hat{p}}^e}{\tau_\eta + \tau_s + \tau_{\hat{p}}^e} \right)^2 \left(\tau_\eta^{-1} + \left(\tau_{\hat{p}}^e \right)^{-1} \right). \tag{12}$$

In this application, we can explicitly compute the equilibrium values of price volatility and price informativeness as a function of primitives. Formally, $\tau_{\hat{p}}^e$ and $\mathcal{V}$ are given by

$$\tau_{\hat{p}}^e = \left(\frac{\tau_s}{\tau_\eta} \right)^2 \tau_n \quad \text{and} \quad \mathcal{V} = \left(\frac{1}{1 - R^{-1}\rho} \frac{\tau_s + \left(\frac{\tau_s}{\tau_\eta} \right)^2 \tau_n}{\tau_\eta + \tau_s + \left(\frac{\tau_s}{\tau_\eta} \right)^2 \tau_n} \right)^2 \left(\tau_\eta^{-1} + \left(\frac{\tau_\eta}{\tau_s} \right)^2 \tau_n^{-1} \right).$$

Figure 3 shows the comparative statics of $\tau_{\hat{p}}^e$ and $\mathcal{V}$ as a function of the five primitives of the model: τ_s, τ_η, τ_n, τ_u, and γ. Interestingly, in this application, both price volatility and informativeness are independent of investors' risk aversion, γ, and of the dispersion in investors' priors, τ_u, although there are other equilibrium variables that do depend on γ or τ_u, for example, the risk premium and trading volume.

Figure 3 illustrates the existence of positive and negative comovement regions, as established in Proposition 2. For instance, when investors have precise private signals (τ_s is high), price informativeness $\tau_{\hat{p}}^e$ is high, and over that region volatility and informativeness comove positively. Intuitively, an increase in τ_s increases equilibrium price informativeness and shifts up the locus defined in Eq. (12). When price informativeness is high enough, which holds for high values of τ_s, the fundamental relation in Eq. (12) is increasing, so volatility necessarily increases in equilibrium. Similarly, when investors' prior precision τ_η is low or the amount of aggregate noise is low (τ_n is high), $\tau_{\hat{p}}^e$ is in the positive comovement region and any change in parameters moves the locus defined in Eq. (12) and equilibrium price informativeness in the same direction, which implies that $\tau_{\hat{p}}^e$ and $\mathcal{V}$ positively comove. Therefore, if price informativeness is in the positive comovement region, a change in any of the model's parameters leads to positive comovement between price volatility and price informativeness.

Alternatively, when investors have imprecise private signals (τ_s is low), price informativeness $\tau_{\hat{p}}^e$ is low, and over that region volatility and informativeness comove negatively. Intuitively, an increase in τ_s increases equilibrium price informativeness and shifts up the locus in Eq. (12), which pushes price volatility in an indeterminate direction. However, when price informativeness is low enough, the slope of the fundamental relation is sufficiently negative that it is the case that price volatility goes down, establishing the negative comovement. Similarly reasoning applies to the comparative statics for τ_η and τ_n. It is worth highlighting that while price informativeness varies monotonically with all the parameters, price volatility is non-monotonic for some changes on primitives. This is a necessary feature of the model in order to have both a positive and a negative comovement region.

4.2 Application 2: Strategic Traders

While the previous applications feature a continuum of price-taking investors, we now allow for strategic behavior. We specialize the environment presented in the previous section to a finite number of investors,

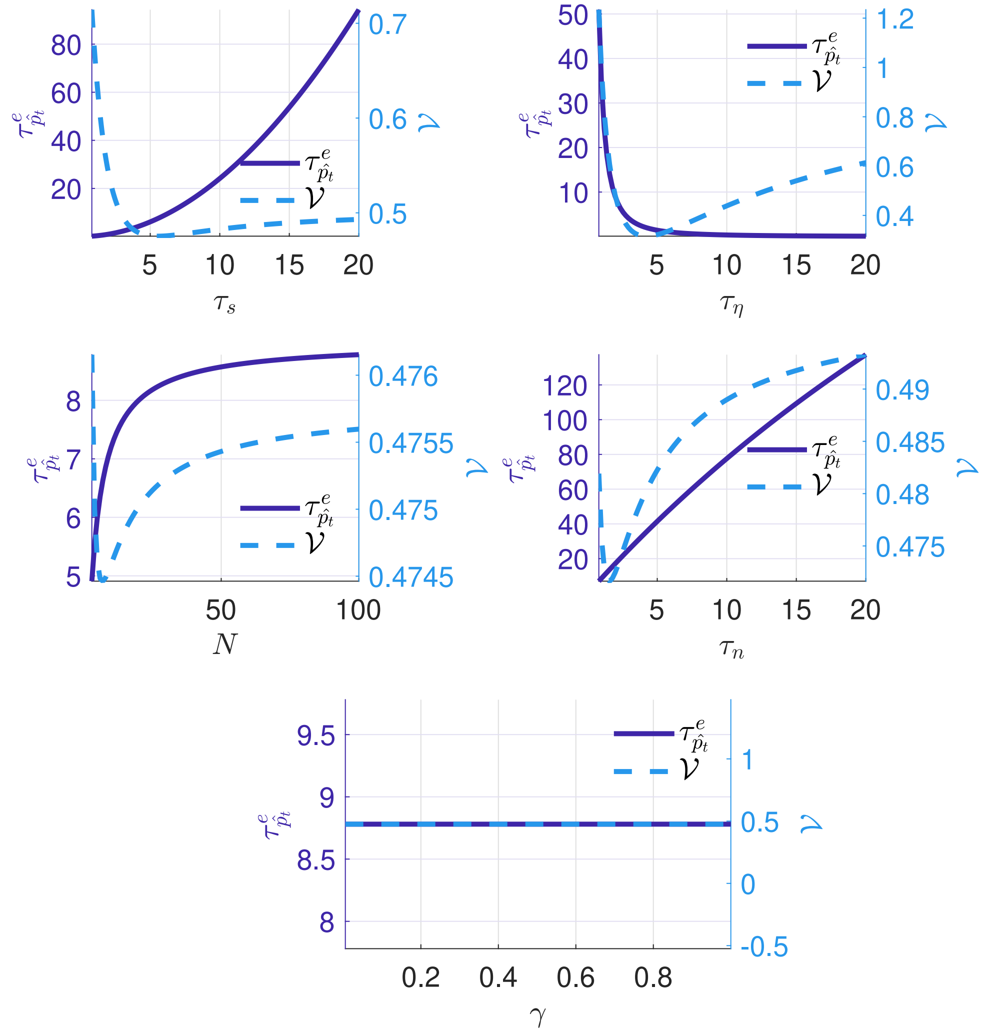


Figure 4: Comparative statics: Application 2

Note: Figure 4 shows comparative statics of price informativeness $\tau^e_{\hat{p}}$ and price volatility $\mathcal{V} = \mathbb{V}ar\left[p_t \middle| \theta_t\right]$ as a function of all five primitives of the model considered in Application 2. All plots feature two y-axis: the left y-axis corresponds to the values of $\tau^e_{\hat{p}}$, while the right y-axis corresponds to the values of $\mathcal{V} = \mathbb{V}ar\left[p_t \middle| \theta_t\right]$. The parameters of this model are the following: τ_s, precision of private signals about the innovation to the asset payoff, τ_η, precision of the innovation to the asset payoff, N, number of investors, τ_n, precision of aggregate noise, and γ, risk aversion. The reference values are $\tau_s = 6$, $\tau_\eta = 2$, $\tau_n = 1$, $\gamma = 0.5$, and $N = 100$.

N, who have heterogeneous priors over the value of the asset.[14] In particular, from the perspective of investor i, the asset payoff θ is distributed according to

$$\theta_{t+1} = \mu_\theta + \rho\theta_t + \eta_t,$$

where $\theta_0 = 0$,

$$\eta_t \sim_i N\left(\overline{\eta}_t^i, \tau_\eta^{-1}\right), \quad \text{and} \quad \overline{\eta}_t^i \overset{\text{i.i.d}}{\sim} N\left(0, (N+1)\,\tau_n^{-1}\right),$$

where the variance of noise increases with the number of investors to ensure the economy converges to the competitive economy in Application 1. In a symmetric equilibrium in linear strategies, we postulate net demand functions given by

$$\Delta q_t^i = \alpha_s s_t^i + \alpha_\theta \theta_t + \alpha_n \overline{\eta}_t^i - \alpha_p p_t + \psi,$$

where α_s, α_θ, α_n, and α_p are positive scalars, while ψ can take positive or negative values. Market clearing in the asset market implies that the equilibrium price takes the form

$$p_t = \frac{\alpha_s}{\alpha_p}\left(\eta_t + \frac{\sum_{i=1}^N \varepsilon_{st}^i}{N}\right) + \frac{\alpha_\theta}{\alpha_p}\theta_t + \frac{\alpha_\eta}{\alpha_p}\frac{\sum_{i=1}^N \overline{\eta}_t^i}{N} + \frac{\psi}{\alpha_p},$$

In this case, the price is not fully revealing because the noise contained in the signals on which the investors' trade and the noise contained in the investors' priors do not wash out in the aggregate. There is aggregate uncertainty coming from the realized signals and priors. Consistently with our definition of λ in Lemma 2, in this application,

$$\lambda = \frac{N-1}{N} < 1,$$

so $\tau_{\hat{p}} < \tau_{\hat{p}}^e$, and the fundamental relation can expressed as

$$\mathcal{V} = \left(\frac{1}{1 - R^{-1}\rho}\frac{\tau_s + \frac{N-1}{N}\tau_{\hat{p}}^e}{\tau_\eta + \tau_s + \frac{N-1}{N}\tau_{\hat{p}}^e}\right)^2 \left(\tau_\eta^{-1} + \left(\tau_{\hat{p}}^e\right)^{-1}\right).$$

Note that when investors behave strategically, the price is more informative for an external observer than for an investor inside the model. The price aggregates the N signals received by the active investors, which is all new information for an external observer. However, since an investor already knows the realization of his own signal, from the investors' perspective the price conveys new information only about $N-1$ new signals.

Figure 4 shows the comparative statics of $\tau_{\hat{p}}^e$ and $\mathcal{V}$ as a function of the five primitives of the model: τ_s, τ_η, N, τ_n, and γ. As in the disagreement model with a continuum of agents, price informativeness and volatility are invariant to the level of risk aversion γ. Figure 4 also illustrates the existence of positive and negative comovement regions, as established in Proposition 2. The intuition behind the results is identical to the one provided in Application 1. Interestingly, changes in the value of aggregate noise τ_n induce a positive comovement between volatility and informativeness when price informativeness is high

[14] The heterogeneity in beliefs introduces the additional trading motive needed to escape the no-trade theorem—see, for instance, Brunnermeier (2001).

enough. On the other hand, when price informativeness is low enough and in the negative comovement region, volatility and informativeness move in different directions.

This application includes a new comparative static exercise on the number of investors. In this model, price informativeness is increasing in the number of investors N. However, price volatility is non-monotonic in the number of investors, initially decreasing in N in the negative comovement region, and finally increasing with N once price informativeness is sufficiently high. Finally, as in Application 1, note that while price informativeness varies monotonically with all the parameters, price volatility is non-monotonic on changes on primitives, which is consistent with the existence of positive and negative comovement regions.

4.3 Application 3: Hedging Needs

In this application, we use aggregate hedging needs as an alternative formulation for investors' private trading needs. In particular, we assume that the asset payoff has both a learnable and an unlearnable component. Formally, we assume that

$$\theta_{t+1} = \mu_\theta + \rho\theta_t + \eta_t,$$

where $\theta_0 = 0$,

$$\eta_t = \eta_t^l + \eta_t^u,$$

and

$$\eta_t^l \sim N\left(0, \tau_\eta^{-1}\right) \quad \text{and} \quad \eta_t^u \sim N\left(\overline{\eta}, \tau_u^{-1}\right).$$

The random variables η_t^u and η_t^l, which represent the unlearnable and learnable components of the innovation to the asset payoff, are orthogonal to each other. The realized asset payoff θ_t is observable in period t.

We further assume that investors born in generation t have an endowment ω_{t+1}^i realized in period $t+1$ which is potentially correlated with the unlearnable component of the asset payoff η_{t+1}^u and is independent of the learnable component. Investors' hedging needs, given by the correlation between the investors' endowment and the asset payoff, are given by a random variable h_t^i, which is distributed as follows

$$h_t^i \equiv \mathbb{C}ov\left(\theta_{t+1}, \omega_{t+1}^i \middle| \theta_t\right) = \mathbb{C}ov\left(\eta_t^u, \omega_{t+1}^i\right) = n_t + \varepsilon_{ht}^i,$$

where

$$n_t \sim N\left(0, \tau_n^{-1}\right) \quad \text{and} \quad \varepsilon_{ht}^i \overset{iid}{\sim} N\left(\overline{\theta}, \tau_h^{-1}\right).$$

The hedging needs h_t^i are private information of investor i and have two components: an aggregate component n_t^i and an idiosyncratic component ε_{ht}^i. Investors only observe their total hedging need h_t^i and cannot distinguish between the aggregate and idiosyncratic components.

Investors receive a private signal of the learnable component of the asset's payoff

$$s_t^i = \eta_t^l + \varepsilon_{st}^i, \quad \text{with} \quad \varepsilon_{st}^i \overset{iid}{\sim} N\left(\overline{\theta}, \tau_s^{-1}\right).$$

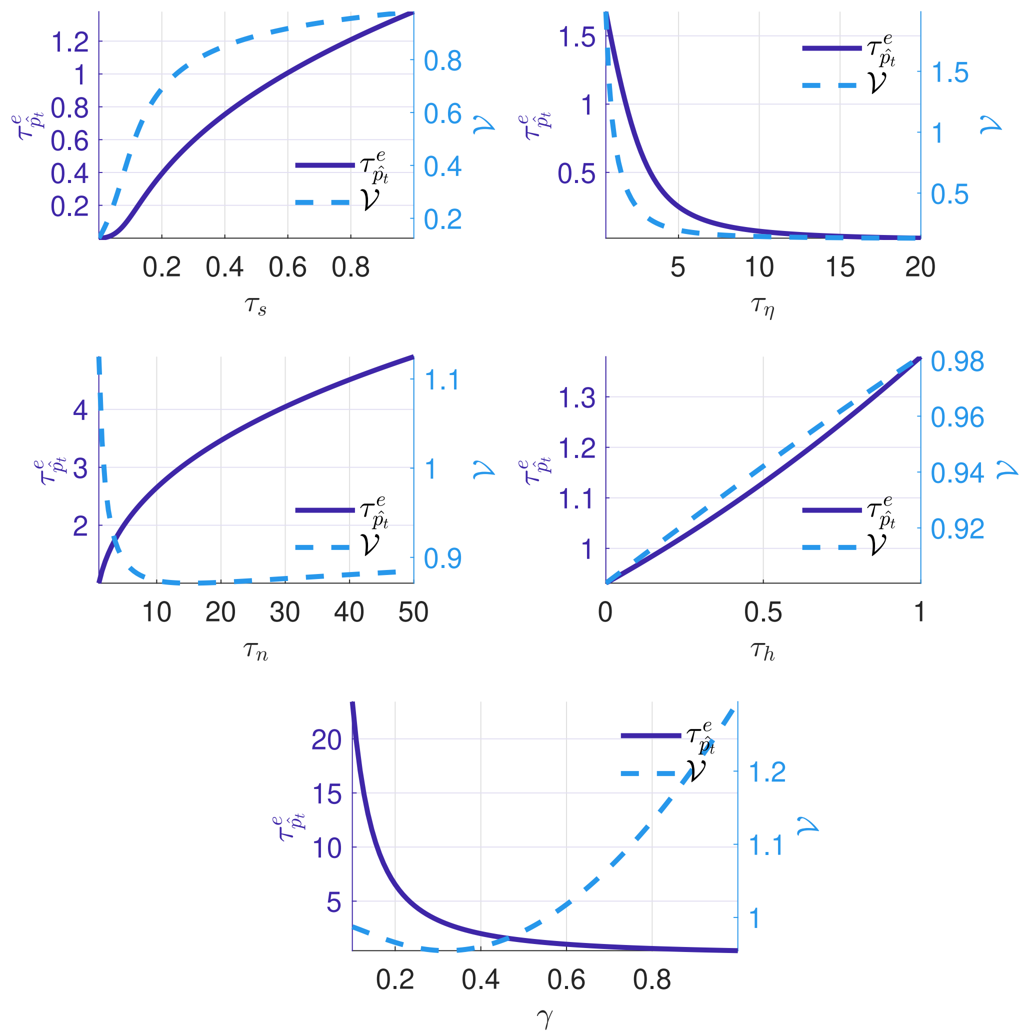


Figure 5: Comparative statics: Application 3

Note: Figure 5 shows comparative statics of price informativeness $\tau^e_{\hat{p}}$ and price volatility $\mathcal{V} = \mathbb{V}ar\left[p_t \mid \theta_t\right]$ as a function of all five primitives of the model considered in Application 3. All plots feature two y-axis: the left y-axis corresponds to the values of $\tau^e_{\hat{p}}$, while the right y-axis corresponds to the values of $\mathcal{V} = \mathbb{V}ar\left[p_t \mid \theta_t\right]$.The parameters of this model are the following: τ_s, precision of private signals about the innovation to the asset payoff, τ_η, precision of the innovation to the asset payoff, τ_n, precision of aggregate hedging term, τ_h, precision of individual hedging need, and γ, investors' coefficient of absolute risk aversion. The reference values are $\tau_s = 1$, $\tau_\eta = 1$, $\tau_n = 2$, $\tau_h = 1$, and $\gamma = 0.5$.

Depending on parameters, this model can potentially feature multiple equilibria, as described in detail in Davila and Parlatore (2017). Consistent with our definition of λ in Lemma 2, in this application,

$$\lambda = \frac{\tau_h + \tau_n}{\tau_n} > 1,$$

so the fundamental relation can be expressed as

$$\mathcal{V} = \left(\frac{1}{1 - R^{-1}\rho} \frac{\tau_s + \frac{\tau_n + \tau_h}{\tau_n} \tau_{\hat{p}}^e}{\tau_\eta + \tau_s + \frac{\tau_n + \tau_h}{\tau_n} \tau_{\hat{p}}^e} \right)^2 \left(\tau_\eta^{-1} + \left(\tau_{\hat{p}}^e \right)^{-1} \right).$$

Note that when investors are aware that the source of aggregate noise has a common component, the price is less informative for an external observer than for an individual investor. In this case, an individual investor can make use of the realization of his hedging need to partially infer the level of aggregate hedging needs, which allows him to better filter the information conveyed by the price.

Figure 5 shows the comparative statics of $\tau_{\hat{p}}^e$ and $\mathcal{V}$ as a function of the five primitives of the model: τ_s, τ_η, τ_n, τ_h, and γ. In this model, all five primitives determine the equilibrium values of $\tau_{\hat{p}}^e$ and $\mathcal{V}$. As in the previous applications, and consistently with Proposition 3, Figure 5 shows that, when price informativeness is high enough, changes in τ_s, τ_η, τ_h, τ_n, and γ move price volatility and price informativeness in the same direction. Interestingly, a negative comovement region between price volatility and informativeness does not exist in this application, that is, the threshold $\underline{\tau}$, defined in Proposition 3, is equal to zero. Figure 5 shows that even when price informativeness is arbitrarily small, changes in parameters other than τ_n and γ imply a positive comovement between volatility and informativeness.

5 Comovement Regions: Characterization and Measurement

Propositions 2 and 3 are formalized in terms of price informativeness, which, despite being a meaningful variable, is an equilibrium object. Our theoretical results beget the question of which precise combinations of primitives are consistent with a negative or positive comovement between volatility and informativeness. To that end, we now show that it is possible to characterize whether a given economy is in the positive or negative comovement region as a function of a subset of model primitives in all three applications. Subsequently, for our leading application, we recover from observables the required stock-specific parameters that determine whether a given economy features positive or negative comovement. We empirically find that most stocks are in the negative comovement region.

5.1 Explicit Characterization of Comovement Regions

For simplicity, we state Proposition 4 only for our leading application, which is the most tightly parameterized, although we study the remaining applications in the Appendix.

Proposition 4. (Explicit characterization of comovement regions) *Consider our leading application, which features a continuum of investors with private trading motives based on differences in beliefs, introduced in Section 4.1. In that case,* *a) Sufficiently large values of the "signal-to-payoff" ratio of precisions, $\frac{\tau_s}{\tau_\eta}$, and the "noise-to-payoff" ratio of precisions, $\frac{\tau_n}{\tau_\eta}$, guarantee that the economy is in the positive comovement region and that volatility and informativeness positively comove for any change in primitives. Explicitly, the economy is in the positive comovement region when*

$$\frac{\tau_n}{\tau_\eta} \geq \frac{\sqrt{1+8\frac{\tau_s}{\tau_\eta}}-1+2\frac{\tau_s}{\tau_\eta}}{2\left(\frac{\tau_s}{\tau_\eta}\right)^2}. \tag{13}$$

b) Sufficiently low values of the "signal-to-payoff" ratio of precisions, $\frac{\tau_s}{\tau_\eta}$, and the "noise-to-payoff" ratio of precisions, $\frac{\tau_n}{\tau_\eta}$, guarantee that the economy is in the negative comovement region and that volatility and informativeness negatively comove for any change in primitives. Explicitly, the economy is in the negative comovement region when

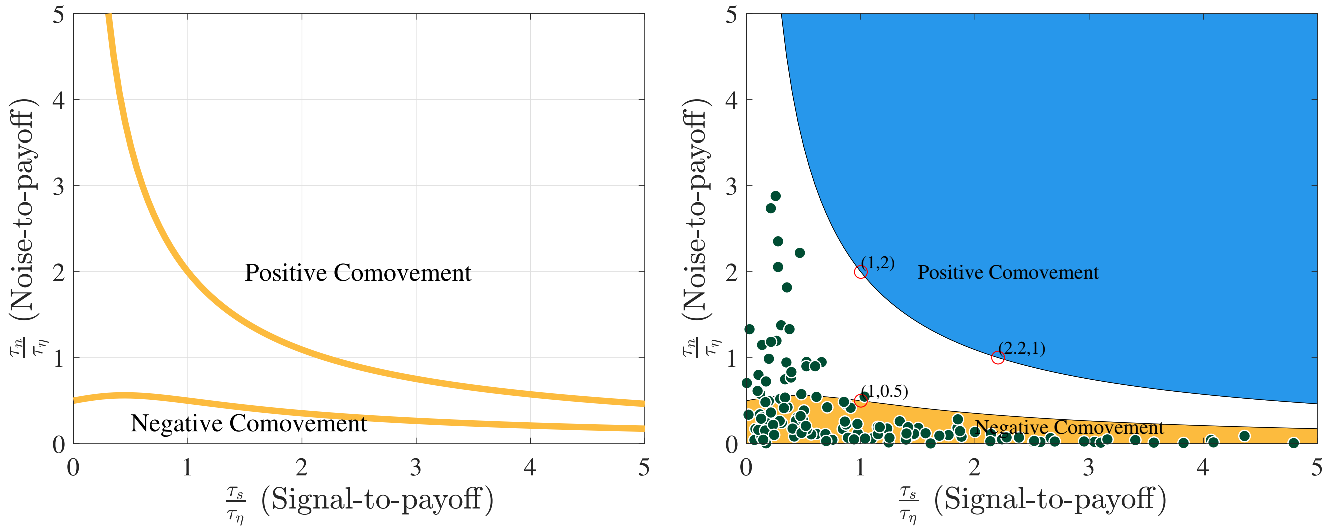
$$\frac{\tau_n}{\tau_\eta} < \min\left\{1, \frac{\left(\frac{\tau_s}{\tau_\eta}-1\right)+\sqrt{5\left(\frac{\tau_s}{\tau_\eta}\right)^2-2\frac{\tau_s}{\tau_\eta}+1}}{2\left(\frac{\tau_s}{\tau_\eta}\right)^2}, \frac{-\left(2-\frac{\tau_s}{\tau_\eta}\right)+\sqrt{\left(2-\frac{\tau_s}{\tau_\eta}\right)^2+8\left(\frac{\tau_s}{\tau_\eta}\right)^2}}{4\left(\frac{\tau_s}{\tau_\eta}\right)^2}\right\}. \tag{14}$$

Eqs. (13) and (14) provide an explicit characterization of conditions on model primitives that guarantee a positive or negative comovement between price informativeness and price volatility. Interestingly, our characterization can be expressed exclusively in terms of two ratios of precisions, which allows us to provide a sense of the magnitudes implied by the model in a scale-invariant form. In particular, both Eqs. (13) and (14) remain valid regardless of the values of investors' risk aversion γ, the dispersion of investors' heterogeneous beliefs τ_u, or the supply of the risky asset. As we show in the Appendix, the conditions that determine whether an economy features positive or negative comovement in equilibrium can be expressed exclusively as a function of ratios of primitives in all applications.[15]

Figure 6a graphically illustrates the combinations of noise-to-payoff $\frac{\tau_n}{\tau_\eta}$ and signal-to-payoff $\frac{\tau_s}{\tau_\eta}$ precisions that delimit the positive and negative comovement regions. When either $\frac{\tau_s}{\tau_\eta}$, which measures the ratio of the precision of private information to the investors' prior information, or $\frac{\tau_n}{\tau_\eta}$, which measures the relative volatility of the innovation to the asset payoff relative to the volatility of sentiment, are sufficiently large, the economy features positive comovement. Intuitively, when τ_s is high, investors receive precise signals and price informativeness is high, which puts the economy in the positive comovement region. Similarly, when τ_n is high, aggregate noise is low, which also implies high price informativeness, guaranteeing that the economy is in the positive comovement region. When either $\frac{\tau_s}{\tau_\eta}$ or $\frac{\tau_n}{\tau_\eta}$ are small, informativeness is small and the economy is in the negative comovement region.

In terms of magnitudes, our model implies that when investors' private signals and priors about the innovation are of equal precision, i.e., $\frac{\tau_s}{\tau_\eta}=1$, volatility and informativeness positively comove whenever the variance of the aggregate component of beliefs is less than one half of the variance of the innovation to the asset payoff ($\frac{1}{\tau_n}<\frac{1}{2}\frac{1}{\tau_\eta}$) and negatively comove whenever the variance of the aggregate component of beliefs is greater than twice the variance of the innovation to the asset payoff ($\frac{1}{\tau_n}>2\frac{1}{\tau_\eta}$), regardless

[15] The characterization of the positive and negative comovement regions for the other applications follows immediately from Propositions 7 and 8 in the Appendix.



(a) Explicit characterization

(b) Measurement

Figure 6: Comovement regions

Note: Figure 6a shows the combination of ratios of primitives $\frac{\tau_n}{\tau_\eta}$ and $\frac{\tau_s}{\tau_\eta}$ that are consistent with an economy that is in the positive or negative comovement region, as defined in Proposition 4, and that features positive or negative comovement, as defined in Proposition 3. The highest dashed line determines the lower bound of the positive comovement region. The lowest dashed line represents the upper bound of the negative comovement region.

Figure 6b shows the combination of ratios of primitives $\frac{\tau_n}{\tau_\eta}$ and $\frac{\tau_s}{\tau_\eta}$ that are consistent with a stock in the positive and negative comovement regions, as defined in Proposition 4, and that features positive or negative comovement, as defined in Proposition 3. The orange shaded region in the bottom left quadrant represents the negative comovement region and the blue shaded region in the upper right quadrant represents the positive comovement region.

Each dot corresponds to the measures of $\frac{\tau_n}{\tau_\eta}$ and $\frac{\tau_s}{\tau_\eta}$ recovered from each individual stock, using annual stock price and earnings data from CRSP/Compustat from 1961 to 2017.

of the value of the remaining parameters of the model. Moreover, our model implies that when the variance of the innovation to the asset payoff and the variance of the aggregate component of beliefs are of equal magnitude, i.e., $\frac{\tau_n}{\tau_\eta} = 1$, volatility and informativeness positively comove provided that the precision of investors' private signals is greater than 2.2 times the precision of their prior about the innovation to the asset payoff ($\tau_s > 2.2\tau_\eta$). Figure 6a illustrates all remaining possible combinations. In the next section, we recover these ratios of primitives at the stock level.

5.2 Comovement Regions in the Data

In our final proposition, we recover stock-specific measures of $\frac{\tau_n}{\tau_\eta}$ and $\frac{\tau_s}{\tau_\eta}$, which allows us to determine in practice if a given stock is in the positive or negative comovement region. To do so, we follow closely the methodology developed in Davila and Parlatore (2018), which provides formal identification results for a class of models that includes those considered in this paper.

Since modeling payoff measures (dividends or earnings) as non-stationary is often perceived as a better assumption (see e.g., Campbell (2017)), we show in Proposition 5 how to recover primitives under the assumption that payoffs follow a random walk. In the Appendix, we show how to extend this proposition to the case in which the payoff process is stationary.

Proposition 5. (Recovering stock-specific primitives) *Let β_0, β_1, and β_2 denote the coefficients of the following regression of asset price changes on changes on asset payoffs,*

$$\Delta p_t = \beta_0 + \beta_1 \Delta\theta_t + \beta_2 \Delta\theta_{t+1} + \varepsilon_t, \tag{R1}$$

where p_t denotes the ex-dividend price at the beginning of period t and θ_t denotes the measure of asset payoffs realized over period t. Let ζ_0 and ζ_1 denote the coefficients of the following regression R2 of price changes on changes on lagged asset payoffs

$$\Delta p_t = \zeta_0 + \zeta_1 \Delta\theta_t + \varepsilon_t^\zeta. \tag{R2}$$

Let $\tau_{\hat{p}}^R$ denote a measure of relative price informativeness, defined and computed as described in the Appendix. It is then possible to find measures of $\frac{\tau_n}{\tau_\eta}$ and $\frac{\tau_s}{\tau_\eta}$ as follows

$$\frac{\tau_s}{\tau_\eta} = \frac{\beta_2}{1 + R^{-1}\zeta_1 - \beta_2} - \tau_{\hat{p}}^R, \quad \textit{and} \tag{15}$$

$$\frac{\tau_n}{\tau_\eta} = \left(\frac{\beta_2}{1 + R^{-1}\zeta_1 - \beta_2} - \tau_{\hat{p}}^R\right)^{-2} \tau_{\hat{p}}^R, \tag{16}$$

where R corresponds to the risk-free rate.

Proposition 5 shows that it is possible to recover the desired ratios by using outcomes of regressions of price changes and changes on asset payoffs. In particular, recovering the ratios $\frac{\tau_n}{\tau_\eta}$ and $\frac{\tau_s}{\tau_\eta}$ involves using the coefficients β_1 and β_2 from Regression R1, the coefficient ζ_1 from Regression R2, and the R-squared of both regressions to recover $\tau_{\hat{p}}^R$, as shown in the Appendix. We describe the data used to compute estimates of these variables next.

Table 1: Summary Statistics (All Observations)

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
Market Cap.	21,599	7,957.04	27,052.34	1.17	230.89	1,104.34	4,435.83	658,736.50
Earnings	21,599	848.67	2,996.27	−85,918.67	27.48	129.29	525.75	71,147.22

Note: Table 1 presents summary statistics for the full sample of $21,366$ stock-year observations. It provides information on the sample mean, median, and standard deviation, as well as the minimum, the maximum and the 25th and 75th percentiles of the distribution of market capitalization and total earnings. All variables are expressed in millions of dollars in 2008.

Data Description We conduct our analysis using annual data from 1961 to 2017. We obtain stock price data from the Center for Research in Security Prices (CRSP) to calculate stocks market values, data on reported earnings, to use as a measure of asset payoffs, from CRSP/Compustat Merged (CCM), and a personal consumption deflator index from FRED. We initially restrict our sample to stocks with at least 40 observations.[16]

Table 1 shows summary statistics for the full sample of $21,599$ stock-year observations. The data exhibits considerable variation in terms of market capitalization and total earnings. The distribution of market capitalization across firms and periods has a mean of roughly 8 billion, a median of 1.1 billion, and a standard deviation of 27 billion. The minimum market capitalization in a given period is of roughly 1 million and the maximum is 658 billion. The distribution of total earnings across firms and periods has a mean of 842 million, a median of 130 million, and a standard deviation of 3 billion.

Table 2 shows summary statistics at the stock level for the 461 stocks that form the full sample. In particular, this table summarizes the differences in the distribution of earnings across stocks. The mean earnings across stocks have a mean of 793 million, a median of 169 million and a large standard deviation of 2.1 billion. The median standard deviation in earnings is 676 million and it exhibits a standard deviation of 1.9 billion. These summary statistics show that there is significant heterogeneity in the earnings process in the cross-section of firms as the mean and, more importantly, the volatility of payoffs varies considerably across stocks.

The summary statistics presented in Tables 1 and 2 show that the process for earnings is substantially different across stocks. Moreover, there is no reason to believe that the amount of information available is the same across stocks. Therefore, our results about the comovement of volatility and informativeness are only meaningful when interpreted at the stock level. This is an important argument that invalidates the use of cross-sectional regressions, as discussed in Davila and Parlatore (2018). We describe next the exact empirical implementation of the results.

[16]The frequency of the data, the exact payoff variable used, the restrictions on the sample, the stability of estimated parameters, as well as many other details of the empirical implementation may be subject to discussion. Given that these are not central to this paper, we refer the reader to Davila and Parlatore (2018) for a systematic justification of the approach used here for the empirical implementation.

Table 2: Summary Statistics (Mean and Standard Deviation of Earnings)

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
Mean Earnings	461	793.82	2,131.77	−6.93	43.27	169.74	563.20	22,828.67
St. Dev. Earnings	461	676.36	1,856.12	1.44	32.91	142.16	519.37	16,242.50

Note: Table 2 presents summary statistics for the full sample of 466 stocks. It provides information on the sample mean, median, and standard deviation, as well as the minimum, the maximum and the 25th and 75th percentiles of the distribution of the variance of earnings. All variables are expressed in millions of dollars in 2008.

Recovering Primitives We empirically implement Proposition 5 by running the following specifications in differences for each of the stocks (indexed by j) in our sample:

$$\Delta M_t^j = \beta_0^j + \beta_1^j \Delta E_t^j + \beta_2^j \Delta E_{t+1}^j + \varepsilon_{\Delta t}^j \tag{17}$$

$$\Delta M_t^j = \zeta_0^j + \zeta_1^j \Delta E_t^j + \hat{\varepsilon}_{\Delta t}^j, \tag{18}$$

where M_t^j denotes a stock total market capitalization and E_t^j denotes total earnings. As shown in the Appendix, we can recover relative price informativeness using the R-squareds of the regressions in Eqs. (17) and (18), respectively $R^{2j}_{|\Delta\theta_t,\Delta\theta_{t-1}}$ and $R^{2j}_{|\Delta\theta_{t-1}}$. Therefore, by using the OLS estimates of β_1^j, β_2^j, and ζ_1^j, we can apply the results in Proposition 5 and make use of Eqs. (15) and (16) to recover ratios $\frac{\tau_n^j}{\tau_\eta^j}$ and $\frac{\tau_s^j}{\tau_\eta^j}$ that determine whether for stock j price informativeness and price volatility positively or negatively comove. For the stocks for which statistical tests imply that the earnings process is stationary, we use the stationary version of Proposition 5, as described in the Appendix.

Figure 6b illustrates the main empirical findings of the paper. Figure 6b shows the non-negative estimated values of the pairs $\left(\frac{\tau_n^j}{\tau_\eta^j}, \frac{\tau_s^j}{\tau_\eta^j}\right)$ for the stocks in our sample in conjunction with the positive and negative comovement regions defined in Proposition 4.[17] It is evident from the figure that roughly 55% of the final set of stocks are in the negative comovement region. For these stocks, our model implies that a decrease (increase) in price volatility is associated with an increase (decrease) in price informativeness. Interestingly, there are no stocks whose estimated primitives lie in the positive comovement region. We recover ratios of primitives for the remaining 45% of stocks in the intermediate region between the positive and negative comovement regions. For these stocks, our framework shows that it is not possible to find an unambiguous prediction for how volatility and informativeness comove after a change in primitives. Intuitively, the large amount of idiosyncratic volatility unrelated to changes in earnings at the stock level suggests that price informativeness is low for most stocks, which given our theoretical results implies that most stocks feature negative comovement between price volatility and informativeness.

[17] Our methodology does not impose that the estimates of the ratios must be non-negative. The fact that we find some ratios with negative values is a symptom of model misspecification. The representation of the results in Figure 6b implicitly disregards these observations.

6 Conclusion

This paper has systematically characterized the equilibrium relation between price informativeness and price volatility in models of financial market trading, identifying two different channels (noise reduction and equilibrium learning) through which changes in price informativeness are associated with changes in price volatility. Our main results establish that whenever prices are sufficiently informative (uninformative), changes in parameters induce a positive (negative) comovement between price informativeness and price volatility in response to a change in primitives. We characterize simple conditions in terms of primitives that identify whether volatility and informativeness positively or negatively comove for a given stock for different applications of our general framework. In the context of our leading application, we use data on US stocks to recover stock-specific estimates of such primitives. This allows us to determine whether individual stocks are in the region of the parameter space in which informativeness and volatility comove positively or negatively. Our empirical findings conclude that most stocks feature negative comovement between price informativeness and price volatility. In practical terms, our results imply that stocks with more volatile prices are likely to be less informative, and vice versa.

There is scope to extend the framework that we develop in this paper to explore in detail the relation between price informativeness and volatility in more general environments that incorporate multiple assets or richer wealth effects. The overall approach of relating easily computable statistics to interesting unobservables that capture the ability of markets to aggregate information seems to be of broader applicability to other contexts in which information dispersion is relevant, for instance, macro environments or auctions.

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Appendix

A Proofs: Section 2

Assumption 2 and market clearing imply that

$$\int \Delta q_t^i di = \int \alpha_\theta^i di \theta_t + \int \alpha_s^i s_t^i di + \int \alpha_n^i n_t^i di - \int \alpha_p^i di p_t + \int \psi^i di = 0,$$

so the equilibrium price must satisfy

$$p_t = \frac{\int \alpha_s^i s_t^i di}{\overline{\alpha_p}} + \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\theta_t + \frac{\int \alpha_n^i n_t^i di}{\overline{\alpha_p}} + \frac{\overline{\psi}}{\overline{\alpha_p}},$$

where we define cross sectional averages $\overline{\alpha_\theta} = \int \alpha_\theta^i di$, $\overline{\alpha_p} = \int \alpha_p^i di$, and $\overline{\psi} = \int \psi_i di$.[18] Using the additive structure of the signals in Eqs. (1) and (2), we can further write

$$p_t = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\eta_t + \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\theta_t + \frac{\overline{\alpha_n}}{\overline{\alpha_p}}n_t + \frac{\int \alpha_s^i \varepsilon_{st}^i di}{\overline{\alpha_p}} + \frac{\int \alpha_n^i \varepsilon_{nt}^i di}{\overline{\alpha_p}} + \frac{\overline{\psi}}{\overline{\alpha_p}}, \tag{A.1}$$

where we define cross sectional averages $\overline{\alpha_s} = \int \alpha_{si} di$ and $\overline{\alpha_n} = \int \alpha_{ni} di$. Under a strong law of large numbers, see, e.g., Vives (2008), the terms $\frac{\int \alpha_s^i \varepsilon_{st}^i di}{\overline{\alpha_p}}$ and $\frac{\int \alpha_n^i \varepsilon_{nt}^i di}{\overline{\alpha_p}}$ vanish when there is a continuum of investors.

Proposition 1. (Fundamental relation between price informativeness and price volatility)

Proof. a) Eq. (A.1) can be written as

$$p = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\left(\eta_t + \frac{\overline{\alpha_\theta}}{\overline{\alpha_s}}\theta_t + \frac{\overline{\alpha_n}}{\overline{\alpha_s}}n_t + \frac{\int \alpha_s^i \varepsilon_{st}^i di}{\overline{\alpha_s}} + \frac{\int \alpha_n^i \varepsilon_{nt}^i di}{\overline{\alpha_s}}\right) + \frac{\overline{\psi}}{\overline{\alpha_p}}.$$

Consistently with the definition of price informativeness in Eq. (4), the inverse of price informativeness can be written as

$$\left(\tau_{\hat{p}}^e\right)^{-1} = \mathbb{V}\text{ar}\left[\hat{p} | \eta_t, \theta_t\right] = \mathbb{V}\text{ar}\left[\frac{\overline{\alpha_n}}{\overline{\alpha_s}}n_t + \frac{\int \alpha_s^i \varepsilon_{st}^i di}{\overline{\alpha_s}} + \frac{\int \alpha_n^i \varepsilon_{nt}^i di}{\overline{\alpha_s}}\right].$$

Using this definition of $\left(\tau_{\hat{p}}^e\right)^{-1}$, price volatility can be expressed as follows

$$\mathcal{V} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 \left(\tau_\eta^{-1} + \left(\tau_{\hat{p}}^e\right)^{-1}\right),$$

where $\tau_\eta = \mathbb{V}\text{ar}\left[\eta_t\right]^{-1}$ and $\tau_{\hat{p}}^e$ denote precisions (inverse of variances).

b) Differentiating $\mathcal{V}$ with respect to $\tau_{\hat{p}}^e$ in Eq. (5), we find that

$$\begin{aligned}
\frac{d\mathcal{V}}{d\tau_{\hat{p}}^e} &= 2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)}{d\tau_{\hat{p}}^e}\left(\tau_\eta^{-1} + \left(\tau_{\hat{p}}^e\right)^{-1}\right) - \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\left(\tau_{\hat{p}}^e\right)^{-2} \\
&= \frac{\mathcal{V}}{\tau_{\hat{p}}^e}\left(2\frac{d\log\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)}{d\log\left(\tau_{\hat{p}}^e\right)} - \frac{\left(\tau_{\hat{p}}^e\right)^{-1}}{\tau_\eta^{-1} + \left(\tau_{\hat{p}}^e\right)^{-1}}\right),
\end{aligned}$$

which corresponds to Eq. (6) in the text. □

[18] To simplify the notation, we omit the set of agents over which integrals are defined whenever there is no ambiguity.

B Proofs: Section 3

Characterization of equilibrium In the CARA-Gaussian setup that we consider in Section 3, the demand for the risky asset of an investor i is given by the solution to

$$\max_{q_t^i} \left(\mathbb{E}\left[\theta_{t+1} + R^{-1} p_{t+1} | \mathcal{I}_t^i \right] - p_t \right) q_t^i - \frac{\gamma}{2} \mathbb{V}ar\left[\omega_{t+1}^i + \theta_{t+1} + R^{-1} p_{t+1} | \mathcal{I}_t^i \right] \left(q_t^i \right)^2,$$

where $\mathcal{I}_t^i = \{\theta_t, s_t^i, n_t^i, p_t\}$ is the information set of investor i at time t and ω_{t+1}^i denotes investors' period 1 endowment. Then, the net asset demand of investor i is

$$\Delta q_t^i \equiv q_{t+1}^i - q_t^i = \frac{\mathbb{E}\left[\theta_{t+1} + R^{-1} p_{t+1} | \mathcal{I}_t^i\right] - p_t - \hat{n}_t^i - \gamma \mathbb{V}ar\left[\theta_{t+1} + R^{-1} p_{t+1} | \mathcal{I}_t^i\right] q_t^i}{\gamma \mathbb{V}ar\left[\theta_{t+1} + R^{-1} p_{t+1} | \mathcal{I}_t^i\right]},$$

where $\hat{n}_t^i$ stands for any additional term of investors' asset demands. In a symmetric equilibrium in linear strategies,

$$\Delta q_{1t}^i = \alpha_s s_t^i + \alpha_n n_t^i - \alpha_p p_t + \psi^i,$$

where n_t^i is a transformation of $\hat{n}_t^i$. Market clearing in the asset market is given by $\int_I \Delta q_{1t}^i di = 0$, which yields the following equilibrium price function:

$$p_t = \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}} \theta_t + \frac{\overline{\alpha_s}}{\overline{\alpha_p}} \left(\eta_t + \int \varepsilon_{st}^i di \right) + \frac{\overline{\alpha_n}}{\overline{\alpha_p}} \left(n_t + \int \varepsilon_{nt}^i di \right) + \frac{\overline{\psi}}{\overline{\alpha_p}},$$

where overlined variables are defined as in page A. When investors learn from their private trading motives, the information contained in the price for investor i taking into account the informational content of his private trading needs is

$$\hat{p}_t + \frac{\overline{\alpha_n}}{\overline{\alpha_s}} \left(\mathbb{E}\left[n_t\right] - \mathbb{E}\left[n_t | \, n_t^i \right] \right) \Big| \, \eta_t, \theta_t, n_t^i \sim N\left(\eta_t, \tau_{\hat{p}}^{-1} \right),$$

where

$$\mathbb{E}\left[n_t | \, n_t^i \right] = \frac{\tau_n \mu_n + \hat{\tau}_\varepsilon n_t^i}{\tau_n + \hat{\tau}_\varepsilon} \quad \text{and} \quad \tau_{\hat{p}}^{-1} = \left(\frac{\overline{\alpha_n}}{\overline{\alpha_s}} \right)^2 \left((\tau_n + \hat{\tau}_\varepsilon)^{-1} + \frac{\hat{\tau}_\varepsilon^{-1}}{N-1} \right) + \frac{\tau_s^{-1}}{N-1},$$

where N is the number of investors in the economy and $\hat{\tau}_\varepsilon$ is the precision of the investor's private trading motive as a signal of the aggregate noise. If investors learn from their private trading motive, $\hat{\tau}_\varepsilon = \tau_\varepsilon$. If investors do not learn from their private trading motive, $\hat{\tau}_\varepsilon = 0$. If there is a continuum of investors, then $N \to \infty$.

Given our guesses for the demand functions and the linear structure of prices we have

$$\theta_{t+1} + R^{-1} p_{t+1} = \theta_{t+1} + R^{-1} \left(\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}} \theta_{t+1} + \frac{\overline{\alpha_s}}{\overline{\alpha_p}} \eta_{t+1} + \frac{\overline{\alpha_n}}{\overline{\alpha_p}} n_{t+1} + \frac{\overline{\psi}}{\overline{\alpha_p}} \right),$$

$$\begin{aligned}
\mathbb{E}\left[\theta_{t+1} + R^{-1} p_{t+1} | \mathcal{I}_t^i \right] &= \left(1 + R^{-1} \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}} \right) \mathbb{E}\left[\theta_{t+1} | \mathcal{I}_t^i \right] + R^{-1} \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}} \mathbb{E}\left[\eta_{t+1}\right] + \frac{\overline{\alpha_n}}{\overline{\alpha_p}} \mathbb{E}\left[n_{t+1}\right] + \frac{\overline{\psi}}{\overline{\alpha_p}} \right) \\
&= \left(1 + R^{-1} \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}} \right) \left(\mu_\theta + \rho \theta_t + \mathbb{E}\left[\eta_t | \mathcal{I}_t^i \right] \right) + R^{-1} \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}} \mathbb{E}\left[\eta_{t+1}\right] + \frac{\overline{\alpha_n}}{\overline{\alpha_p}} \mu_n + \frac{\overline{\psi}}{\overline{\alpha_p}} \right),
\end{aligned}$$

and

$$\mathbb{V}ar\left[\theta_{t+1}+R^{-1}p_{t+1}|\mathcal{I}_t^i\right]=\left(1+R^{-1}\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)^2\mathbb{V}ar\left[\theta_{t+1}|\mathcal{I}_t^i\right]+R^{-1}\left(\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\mathbb{V}ar\left[\eta_{t+1}\right]+\left(\frac{\overline{\alpha_n}}{\overline{\alpha_p}}\right)^2\mathbb{V}ar\left[n_{t+1}\right]\right)$$
$$=\left(1+R^{-1}\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)^2\mathbb{V}ar\left[\eta_t|\mathcal{I}_t^i\right]+\left(R^{-1}\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\mathbb{V}ar\left[\eta_{t+1}\right]+\left(R^{-1}\frac{\overline{\alpha_n}}{\overline{\alpha_p}}\right)^2\tau_n^{-1}.$$

Given the normal linear structure of the signals, we can express investors ex-post variances about the innovation to the asset payoff after solving their filtering problem as

$$\mathbb{V}\mathrm{ar}\left[\eta_t\left|s_t^i,n_t^i,p_t,\theta_t\right.\right]=\tau_{\eta|s,\hat{p}}^{-1},\quad \text{where}\quad \tau_{\eta|s,\hat{p}}\equiv\left(\tau_\eta+\tau_s+\tau_{\hat{p}}\right)^{-1}.$$

Similarly, investors' means can be expressed as

$$\mathbb{E}\left[\eta_t\left|s_t^i,n_t^i,p_t\right.\right]=\frac{\tau_\eta\mathbb{E}_i\left[\eta_t\right]+\tau_s s_t^i+\tau_{\hat{p}}\left(\hat{p}_t+\frac{\overline{\alpha_n}}{\overline{\alpha_s}}\left(\mathbb{E}\left[n_t\right]-\mathbb{E}\left[n_t|\,n_t^i\right]\right)\right)}{\tau_\eta+\tau_s+\tau_{\hat{p}}},$$

where $\mathbb{E}_i\left[\eta_t\right]$ is investor i's prior mean about the innovation to the asset payoff η_t. Matching coefficients, we have[19]

$$\overline{\alpha_s}=\frac{1+R^{-1}\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}}{\kappa}\frac{\tau_s}{\tau_{\eta|s,\hat{p}}},\quad \overline{\alpha_p}=\frac{1}{\kappa}\left(1-\left(1+R^{-1}\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)\frac{\tau_{\hat{p}}}{\tau_{\eta|s,\hat{p}}}\frac{\overline{\alpha_p}}{\overline{\alpha_s}}\right),\tag{A.2}$$
$$\overline{\alpha_n}=\frac{1+R^{-1}\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}}{\kappa}\left(\pi-\frac{\tau_{\hat{p}}}{\tau_{\eta|s,\hat{p}}}\frac{\overline{\alpha_n}}{\overline{\alpha_s}}\frac{\hat{\tau}_\varepsilon}{\tau_n+\hat{\tau}_\varepsilon}\right),\quad\text{and}\quad \overline{\alpha_\theta}=\frac{1+R^{-1}\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}}{\kappa}\left(\rho-\frac{\tau_{\hat{p}}}{\tau_{\eta|s,\hat{p}}}\frac{\overline{\alpha_\theta}}{\overline{\alpha_s}}\right)\tag{A.3}$$

where

$$\kappa\equiv\gamma\left(\left(1+R^{-1}\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)^2\mathbb{V}ar\left[\eta_t|\mathcal{I}_t^i\right]+\left(R^{-1}\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\mathbb{V}ar\left[\eta_{t+1}\right]+\left(R^{-1}\frac{\overline{\alpha_\eta}}{\overline{\alpha_p}}\right)^2\tau_n^{-1}\right)$$

and where we denote by π the loading of the private trading need on the investors' utility, which will vary across applications, and $\hat{\tau}_\varepsilon$ is the precision of the investors' private trading need as a signal of the aggregate noise contained in the price.

From Equations (A.2) and (A.3), we have

$$\frac{\overline{\alpha_\theta}}{\overline{\alpha_s}}=\frac{\rho-\frac{\tau_{\hat{p}}}{\tau_{\eta|s,\hat{p}}}\frac{\overline{\alpha_\theta}}{\overline{\alpha_s}}}{\frac{\tau_s}{\tau_{\eta|s,\hat{p}}}}\Rightarrow\frac{\overline{\alpha_\theta}}{\overline{\alpha_s}}=\rho\frac{\tau_{\eta|s,\hat{p}}}{\tau_s+\tau_{\hat{p}}}\tag{A.4}$$
$$\frac{\overline{\alpha_n}}{\overline{\alpha_s}}=\frac{\pi-\frac{\tau_{\hat{p}}}{\tau_{\eta|s,\hat{p}}}\frac{\overline{\alpha_n}}{\overline{\alpha_s}}\frac{\hat{\tau}_\varepsilon}{\tau_n+\hat{\tau}_\varepsilon}}{\frac{\tau_s}{\tau_{\eta|s,\hat{p}}}}\Rightarrow\frac{\overline{\alpha_n}}{\overline{\alpha_s}}=\frac{\pi\tau_{\eta|s,\hat{p}}}{\tau_s+\tau_{\hat{p}}\frac{\hat{\tau}_\varepsilon}{\hat{\tau}_\varepsilon+\tau_n}}\tag{A.5}$$
$$\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}=\frac{\left(1+R^{-1}\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)\left(\rho-\frac{\tau_{\hat{p}}}{\tau_{\eta|s,\hat{p}}}\frac{\overline{\alpha_\theta}}{\overline{\alpha_s}}\right)}{1-\left(1+R^{-1}\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)\frac{\tau_{\hat{p}}}{\tau_{\eta|s,\hat{p}}}\frac{\overline{\alpha_p}}{\overline{\alpha_s}}}\Rightarrow\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}=\frac{\rho}{1-R^{-1}\rho}\tag{A.6}$$

and

$$\frac{\overline{\alpha_s}}{\overline{\alpha_p}}=\frac{\left(1+R^{-1}\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)\frac{\tau_s}{\tau_{\eta|s,\hat{p}}}}{1-\left(1+R^{-1}\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)\frac{\tau_{\hat{p}}}{\tau_{\eta|s,\hat{p}}}\frac{\overline{\alpha_p}}{\overline{\alpha_s}}}\Rightarrow\frac{\overline{\alpha_s}}{\overline{\alpha_p}}=\frac{1}{1-R^{-1}\rho}\frac{\tau_s+\tau_{\hat{p}}}{\tau_{\eta|s,\hat{p}}}.\tag{A.7}$$

[19]Note that the term $\frac{\tau_{\hat{p}}}{\tau_{\theta|s,\hat{p}}}\frac{\overline{\alpha_p}}{\overline{\alpha_s}}$ captures the impact of the information contained in the equilibrium price on investors' demand.

Then, the equilibrium demand sensitivities are given by

$$\overline{\alpha_s} = \frac{1}{\kappa\left(1-R^{-1}\rho\right)}\frac{\tau_s}{\tau_{\eta|s,\hat{p}}} \tag{A.8}$$

$$\overline{\alpha_p} = \frac{1}{\kappa}\frac{\tau_s}{\tau_s+\tau_{\hat{p}}} \tag{A.9}$$

$$\overline{\alpha_n} = \frac{\pi}{\kappa\left(1-R^{-1}\rho\right)}\frac{\tau_s}{\tau_s+\tau_{\hat{p}}\frac{\hat{\tau}_\varepsilon}{\hat{\tau}_\varepsilon+\tau_n}} \tag{A.10}$$

$$\overline{\alpha_\theta} = \frac{\rho}{\kappa\left(1-R^{-1}\rho\right)}\frac{\tau_s}{\tau_s+\tau_{\hat{p}}}. \tag{A.11}$$

Lemma 1. (Signal-to-price sensitivity)

Proof. Eq. (9) in the text follows directly from Eq. (A.8) and Eq. (A.9). □

Lemma 2. (Relating internal price informativeness and price informativeness for an external observer)

Proof. Suppose that there is a continuum of investors. In this case, the unbiased signal contained in the price from the perspective of an investor i is

$$\hat{p}_t^i = \eta_t + \frac{\overline{\alpha_n}}{\overline{\alpha_s}}\left(n_t - \mathbb{E}\left[n_t|\, n_t^i\right]\right),$$

and internal price informativeness is given by

$$\tau_{\hat{p}} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2 \mathbb{V}ar\left[n_t|\, n_t^i\right]^{-1} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2\left(\tau_n+\hat{\tau}_\varepsilon\right),$$

where $\hat{\tau}_\varepsilon$ is the precision of the investors' private trading motive as a signal of the aggregate private trading need.

An external observer does not have any private trading need from which he can learn about the aggregate noise contained in the price. Hence, the unbiased signal about the innovation to the asset payoff contained in the price from the perspective of an external observer is

$$\hat{p}_t = \eta_t + \frac{\overline{\alpha_n}}{\overline{\alpha_s}}\left(n_t - \mathbb{E}\left[n_t\right]\right),$$

and external price informativeness is given by

$$\tau_{\hat{p}}^e = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2 \mathbb{V}ar\left[n_t\right]^{-1} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2\tau_n.$$

Hence,

$$\tau_{\hat{p}} = \lambda\tau_{\hat{p}}^e, \quad \text{where} \quad \lambda \equiv \frac{\mathbb{V}ar\left[n_t\right]}{\mathbb{V}ar\left[n_t|\, n_t^i\right]} = \frac{\tau_n+\hat{\tau}_\varepsilon}{\tau_n}.$$

□

Lemma 3. (Fundamental relation CARA-Gaussian setup)

Proof. It follows by direct substitution of the result in Proposition 1a) and the results of Lemmas 1 and 2. □

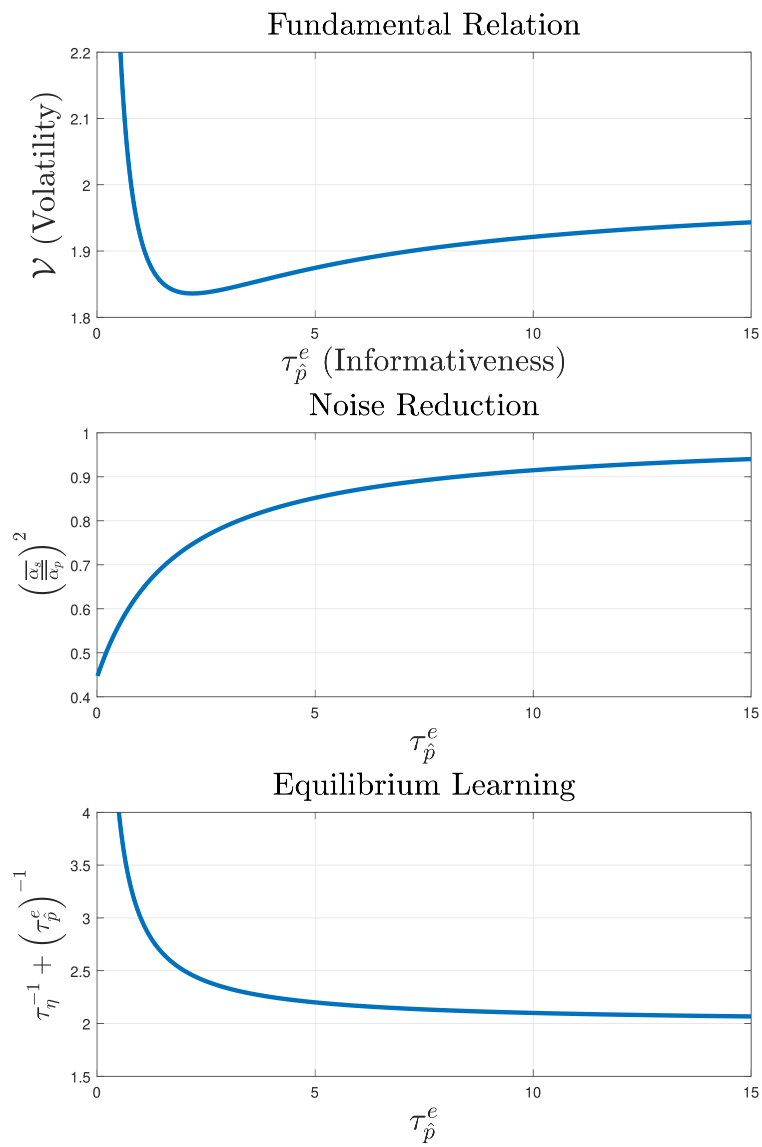

Figure A.1: Fundamental relation decomposition

Note: Figure A.1 plots the fundamental relation and its determinants for parameters $\tau_\eta = 0.5$, $\tau_s = 1$, $\lambda = 1$, $\rho = 0$, and $R = 1.04$.

Proposition 2. (Slope of fundamental relation)

Proof. From Eq. (9) using Lemma 2 it follows that

$$\frac{d\log\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)}{d\log\left(\tau_{\hat{p}}^e\right)} = \frac{\tau_\eta}{\tau_\eta + \tau_s + \lambda\tau_{\hat{p}}^e}\frac{\lambda\tau_{\hat{p}}^e}{\tau_s + \lambda\tau_{\hat{p}}^e}$$

Therefore, from Lemma 3 it follows that

$$\begin{aligned}\frac{d\log\mathcal{V}}{d\log\tau_{\hat{p}}^e} &= 2\frac{d\log\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)}{d\log\left(\tau_{\hat{p}}^e\right)} - \frac{\left(\tau_{\hat{p}}^e\right)^{-1}}{\tau_\eta^{-1} + \left(\tau_{\hat{p}}^e\right)^{-1}} = 2\frac{\tau_\eta}{\tau_\eta + \tau_s + \lambda\tau_{\hat{p}}^e}\frac{\lambda\tau_{\hat{p}}^e}{\tau_s + \lambda\tau_{\hat{p}}^e} - \frac{\tau_\eta}{\tau_\eta + \tau_{\hat{p}}^e}\\ &= \tau_\eta\left(\frac{(2-\lambda)\,\lambda\left(\tau_{\hat{p}}^e\right)^2 + \lambda\tau_{\hat{p}}^e\left(\tau_\eta - 2\tau_s\right) - \tau_s\left(\tau_\eta + \tau_s\right)}{\left(\tau_\eta + \tau_s + \lambda\tau_{\hat{p}}^e\right)\left(\tau_s + \lambda\tau_{\hat{p}}^e\right)\left(\tau_\eta + \tau_{\hat{p}}^e\right)}\right).\end{aligned}$$

We can then conclude that

$$\operatorname{sgn}\left(\frac{d\log\mathcal{V}}{d\log\tau_{\hat{p}}^e}\right) = \operatorname{sgn}\left[(2-\lambda)\,\lambda\left(\tau_{\hat{p}}^e\right)^2 + \lambda\tau_{\hat{p}}^e\left(\tau_\eta - 2\tau_s\right) - \tau_s\left(\tau_\eta + \tau_s\right)\right].$$

Note that, since $\lambda < 2$, the expression on the right hand side is a convex quadratic function of $\tau_{\hat{p}}^e$ with only one positive root given by

$$\tau^\star \equiv \frac{-\lambda\left(\tau_\eta - 2\tau_s\right) + \sqrt{\lambda\left(\lambda\tau_\eta\left(\tau_\eta - 8\tau_s\right) + 8\tau_s\left(\tau_\eta + \tau_s\right)\right)}}{2\,(2-\lambda)\,\lambda}.$$

Then,

$$\frac{d\mathcal{V}}{d\tau_{\hat{p}}^e} < 0 \quad\Longleftrightarrow\quad \tau_{\hat{p}}^e < \tau^\star \quad\text{and}\quad \frac{d\mathcal{V}}{d\tau_{\hat{p}}^e} > 0 \quad\Longleftrightarrow\quad \tau_{\hat{p}}^e > \tau^\star.$$

□

C Proofs: Section 4

In the next three subsections we will use the following result

$$\frac{d\mathcal{V}}{d\tau_{\hat{p}}^e} = \frac{1}{1-R^{-1}\rho}\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{\left(\tau_{\hat{p}}^e\right)^{-2}}{\left(\tau_\eta + \tau_s + \lambda\tau_{\hat{p}}^e\right)^2}\left(\lambda\,(2-\lambda)\left(\tau_{\hat{p}}^e\right)^2 + \lambda\left(\tau_\eta - 2\tau_s\right)\tau_{\hat{p}}^e - \tau_s\left(\tau_\eta + \tau_s\right)\right),$$

which follows directly from Eq. (5). In order to prove Proposition 3, we establish three propositions, one for each application.

Proposition 3. (Positive and negative comovement regions)

Proof. It follows from Propositions 6, 7, and 8 below. □

C.1 Disagreement

Characterization of equilibrium In a symmetric equilibrium in linear strategies, we postulate net demand functions given by

$$\Delta q_t^i = \alpha_s s_t^i + \alpha_\eta \overline{\eta}_t^i + \alpha_\theta \theta_t - \alpha_p p_t + \psi,$$

where α_θ, α_s, and α_p are positive scalars, while ψ can take positive or negative values. This implies that the equilibrium price takes the form

$$p_t = \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\theta_t + \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\eta_t + \frac{\overline{\alpha_\eta}}{\overline{\alpha_p}}n_t + \frac{\overline{\psi}}{\overline{\alpha_p}},$$

where overlined variables are defined as in page A. In this case, the asset price depends on both the aggregate sentiment in the economy n_t, and the actual innovation to the payoff realization η_t, which are both unobservable. Therefore, if an investor faces a high price in the asset market, it can be because the sentiment in the economy is high or because the asset payoff is high. The price is not fully revealing because the sentiment in the economy is random and not observed by the investors. However, the price contains information about the innovation to asset payoff η_t. The unbiased signal of η_t contained in the price is given by

$$\hat{p} = \frac{\overline{\alpha_p}}{\overline{\alpha_s}}\left(p - \frac{\overline{\alpha_\eta}}{\overline{\alpha_p}}\mu_n - \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\theta_t - \frac{\overline{\psi}}{\overline{\alpha_p}}\right) = \eta_t + \frac{\overline{\alpha_\eta}}{\overline{\alpha_s}}\left(n_t - \mu_n\right).$$

Then, the variance of $\hat{p}$, which we denote by $(\tau_{\hat{p}})^{-1}$ and whose inverse we adopt as the relevant measure of price informativeness, is given by

$$\tau_{\hat{p}} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_\eta}}\right)^2 \tau_n = \left(\frac{\tau_s}{\tau_\eta}\right)^2 \tau_n.$$

Note that since investors do not learn about the aggregate sentiment in the economy from their own prior, they have the same information that an external observer has. Then, $\lambda = 1$ and

$$\tau_{\hat{p}}^e = \tau_{\hat{p}}.$$

The equilibrium is characterized by Eq. (A.8), Eq. (A.9), Eq. (A.10), and Eq. (A.11). This characterization maps to the model in Section 3 by setting $\lambda = 1$, $\hat{\tau}_\epsilon = 0$, and $\pi = \frac{\tau_\eta}{\tau_{\eta|s,\hat{p}}}$.

Proposition 6. (Comovement disagreement) *a) Price volatility and price informativeness positively comove (weakly) across equilibria if price informativeness is high enough. Formally, if $\tau_{\hat{p}}^e \geq \tau^\star$, $\mathcal{V}$ and $\tau_{\hat{p}}^e$ move in the same direction after any parameter change.*

b) Price volatility and price informativeness negatively comove (weakly) across equilibria if price informativeness is low enough. Formally, there exists a threshold $\underline{\tau} \in [0, \tau^\star]$ such that, if $\tau_{\hat{p}}^e < \underline{\tau}$, $\mathcal{V}$ and $\tau_{\hat{p}}^e$ move in opposite directions after any parameter change. Moreover, if $\tau_\eta < \tau_n$ then $\underline{\tau} > 0$.

Proof. The change in volatility when a parameter x changes is given by

$$\frac{d\mathcal{V}}{dx} = \frac{\partial \mathcal{V}}{\partial x} + \frac{dV}{d\tau_{\hat{p}}^e}\frac{d\tau_{\hat{p}}^e}{dx},$$

where $\frac{\partial \mathcal{V}}{\partial x}$ and $\frac{d\tau_{\hat{p}}^e}{dx}$ can be obtained from Eq. (5) and Eq. (9) using the definition of $\tau_{\hat{p}}^e$. Note that

$$\frac{d\tau_{\hat{p}}}{d\tau_u} = 0 \quad \text{and} \quad \frac{\partial \mathcal{V}}{\partial \tau_u} = 0,$$

$$\frac{d\tau_{\hat{p}}}{d\gamma} = 0 \quad \text{and} \quad \frac{\partial \mathcal{V}}{\partial \gamma} = 0,$$

and

$$\frac{d\tau_{\hat{p}}}{d\left(R^{-1}\rho\right)} = 0 \quad \text{and} \quad \frac{\partial \mathcal{V}}{\partial \left(R^{-1}\rho\right)} > 0,$$

so the comovements in this proposition are weak.

a) **Positive comovement**

For changes in τ_s, it follows that $\frac{d\tau_{\hat{p}}^e}{d\tau_s} = \frac{2\tau_{\hat{p}}^e}{\tau_s} > 0$, and

$$\begin{aligned}\frac{\partial \mathcal{V}}{\partial \tau_s} &= 2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{\partial\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)}{\partial \tau_s}\left(\tau_\eta^{-1} + \left(\tau_{\hat{p}}^e\right)^{-1}\right) \\ &= 2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-R^{-1}\rho}\frac{\tau_\eta}{\left(\tau_\eta + \tau_s + \tau_{\hat{p}}^e\right)^2}\left(\tau_\eta^{-1} + \left(\tau_{\hat{p}}^e\right)^{-1}\right) > 0.\end{aligned}$$

Then, $\frac{\partial \mathcal{V}}{\partial \tau_{\hat{p}}^e} > 0$ is a sufficient condition for $\frac{d\mathcal{V}}{d\tau_s} = \frac{\partial \mathcal{V}}{\partial \tau_s} + \frac{d\mathcal{V}}{d\tau_{\hat{p}}^e}\frac{d\tau_{\hat{p}}^e}{d\tau_s}$ to be positive.

Similarly, for changes in τ_η, we have that $\frac{d\tau_{\hat{p}}^e}{d\tau_\eta} < 0$, and

$$\begin{aligned}\frac{\partial \mathcal{V}}{\partial \tau_\eta} &= 2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{\partial\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)}{\partial \tau_\eta}\left(\tau_\eta^{-1} + \left(\tau_{\hat{p}}^e\right)^{-1}\right) - \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\tau_\eta^{-2} \\ &= -2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-R^{-1}\rho}\frac{\tau_s + \tau_{\hat{p}}^e}{\left(\tau_\eta + \tau_s + \tau_{\hat{p}}^e\right)^2}\left(\tau_\eta^{-1} + \left(\tau_{\hat{p}}^e\right)^{-1}\right) - \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\tau_\eta^{-2}.\end{aligned}$$

Then, $\frac{d\mathcal{V}}{d\tau_{\hat{p}}^e} > 0$ is a sufficient condition for $\frac{d\mathcal{V}}{d\tau_n} = \frac{\partial \mathcal{V}}{\partial \tau_\eta} + \frac{d\mathcal{V}}{d\tau_{\hat{p}}^e}\frac{d\tau_{\hat{p}}^e}{d\tau_\eta}$ to be negative.

For changes in τ_n, we have that $\frac{d\tau_{\hat{p}}^e}{d\tau_n} = \left(\frac{\tau_s}{\tau_\eta}\right)^2 > 0$, and $\frac{\partial \mathcal{V}}{\partial \tau_n} = 0$. Hence, $\frac{d\mathcal{V}}{d\tau_{\hat{p}}^e} > 0$ is a sufficient and necessary condition for $\frac{d\mathcal{V}}{d\tau_n} = \frac{d\mathcal{V}}{d\tau_{\hat{p}}^e}\frac{d\tau_{\hat{p}}^e}{d\tau_n}$ to be positive.

Therefore, if $\tau_{\hat{p}}^e > \tau^\star$ an increase in price volatility reflects a weak increase in price informativeness for any parameter change.

b) **Negative comovement**

For changes in τ_s, it follows that

$$\frac{d\mathcal{V}}{d\tau_s} = \frac{2}{1-R^{-1}\rho}\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{\left(\tau_\eta + \tau_s + \tau_{\hat{p}}^e\right)^2}\frac{\tau_s}{\tau_{\hat{p}}^e}\frac{1}{\tau_\eta^2}\left(\left(\frac{\tau_n}{\tau_\eta}\right)^2\tau_s^2 - \tau_n\tau_s - \tau_\eta(\tau_\eta - \tau_n)\right).$$

Then, if $\tau_\eta > \tau_n$ there exists a threshold $\underline{s}$ such that for all $\tau_s \leq \underline{s}$, $\frac{d\mathcal{V}}{d\tau_s}$ and price informativeness and price volatility negatively comove when τ_s changes. This threshold is given by

$$\underline{s} \equiv \frac{\tau_n + \sqrt{\tau_n^2 + 4\left(\frac{\tau_n}{\tau_\eta}\right)^2\tau_\eta(\tau_\eta - \tau_n)}}{2\left(\frac{\tau_n}{\tau_\eta}\right)^2} = \frac{\tau_\eta^2}{2\tau_n}\left(1 + \sqrt{1 + 4\tau_\eta^{-1}(\tau_\eta - \tau_n)}\right).$$

This implies that if $\tau_\eta > \tau_n$, price informativeness and price volatility negatively comove when τ_s changes for all $\tau^e_{\hat{p}} < \tau_{\tau_s}$, where

$$\tau_{\tau_s} \equiv \tau_\eta \frac{-\left(1-\frac{\tau_s}{\tau_\eta}\right)+\sqrt{\left(1-\frac{\tau_s}{\tau_\eta}\right)^2+4\left(\frac{\tau_s}{\tau_\eta}\right)^2}}{2}$$

In terms of parameters, this region is given by

$$\frac{\tau_n}{\tau_\eta} < \frac{-\left(1-\frac{\tau_s}{\tau_\eta}\right)+\sqrt{\left(1-\frac{\tau_s}{\tau_\eta}\right)^2+4\left(\frac{\tau_s}{\tau_\eta}\right)^2}}{2\left(\frac{\tau_s}{\tau_\eta}\right)^2}$$

$$\frac{\tau_s}{\tau_\eta} < \frac{1}{2\frac{\tau_n}{\tau_\eta}}\left(1+\sqrt{1+4\left(1-\frac{\tau_n}{\tau_\eta}\right)}\right). \tag{A.12}$$

For changes in τ_η we have that

$$\frac{d\mathcal{V}}{d\tau_\eta} = -\frac{2}{1-R^{-1}\rho}\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{\left(\tau^e_{\hat{p}}\right)^{-1}}{\left(\tau_\eta+\tau_s+\tau^e_{\hat{p}}\right)^2}\frac{1}{\tau_\eta}\left(2\left(\frac{\tau_s}{\tau_\eta}\right)^4\tau_n^2+(2\tau_\eta-\tau_s)\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n-\tau_s^2\right).$$

Then, we have that

$$\operatorname{sgn}\left(\frac{d\mathcal{V}}{d\tau_\eta}\right) = -\operatorname{sgn}\left(2\left(\frac{\tau_s}{\tau_\eta}\right)^4\tau_n^2+(2\tau_\eta-\tau_s)\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n-\tau_s^2\right).$$

Since

$$\lim_{\tau_\eta\to\infty} 2\left(\frac{\tau_s}{\tau_\eta}\right)^4\tau_n^2+(2\tau_\eta-\tau_s)\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n-\tau_s^2 = -\tau_s^2,$$

there exists a threshold $\underline{\eta} > 0$ such that $\frac{d\mathcal{V}}{d\tau_\eta}$ is positive for all $\tau_\eta > \underline{\eta}$. This implies that there exists a threshold τ_{τ_η} such that $\frac{d\mathcal{V}}{d\tau_\eta} > 0$ for all $\tau^e_{\hat{p}} < \tau_{\tau_\eta}$ where

$$\tau_{\tau_\eta} \equiv \tau_\eta \frac{-\left(2-\frac{\tau_s}{\tau_\eta}\right)+\sqrt{\left(2-\frac{\tau_s}{\tau_\eta}\right)^2+8\left(\frac{\tau_s}{\tau_\eta}\right)^2}}{4}.$$

In this case, price informativeness and price volatility negatively comove when τ_η changes. In terms of parameters, this expression is

$$\frac{\tau_n}{\tau_\eta} < \frac{-\left(2-\frac{\tau_s}{\tau_\eta}\right)+\sqrt{\left(2-\frac{\tau_s}{\tau_\eta}\right)^2+8\left(\frac{\tau_s}{\tau_\eta}\right)^2}}{4\left(\frac{\tau_s}{\tau_\eta}\right)^2}. \tag{A.13}$$

For changes in τ_n, price volatility and price informativeness negatively comove if $\tau^e_{\hat{p}} < \tau^\star$. Note that τ_{τ_s} and τ_{τ_η} are lower than $\tau^\star$ since it is necessary for $\frac{d\mathcal{V}}{d\tau^e_{\hat{p}}} < 0$ for price informativeness and price volatility to comove negatively for changes in τ_s and τ_η.

Therefore, $\underline{\tau} = \min\left\{\tau_{\tau_s}, \tau_{\tau_\eta}\right\}$ if $\tau_\eta > \tau_n$, and $\underline{\tau} = 0$ otherwise. □

C.2 Strategic traders

The equilibrium demand sensitivities are given by by Eq. (A.8), Eq. (A.9), Eq. (A.10), and Eq. (A.11) setting $\pi = \frac{\tau_\eta}{\tau_{\eta|s,p}}$, $\hat{\tau}_\varepsilon = 0$, $\lambda = \frac{N-1}{N}$, and $\kappa = \gamma \mathbb{V}\text{ar}\left[\theta | s_i, \hat{p}\right] + \chi = \frac{\gamma}{\tau_\eta + \tau_s + \tau_{\hat{p}}} + \chi$, where $\chi = \frac{1}{(N-1)\alpha_p}$ is the price impact of an investor. In equilibrium, $\frac{\overline{\alpha_s}}{\overline{\alpha_n}} = \frac{\tau_s}{\tau_\eta}$, and external price informativeness is given by

$$\begin{aligned}
\tau_{\hat{p}}^e &= N \left(\tau_s^{-1} + \left(\frac{\overline{\alpha_n}}{\overline{\alpha_s}} \right)^2 (N+1)\, \tau_n^{-1} \right)^{-1} \\
&= N \left(\tau_s^{-1} + \tau_s^{-2} \tau_\eta^2 (N+1)\, \tau_n^{-1} \right)^{-1} \\
&= \frac{N}{\frac{\tau_s}{\tau_\eta} \frac{\tau_n}{\tau_\eta} + N + 1} \left(\frac{\tau_s}{\tau_\eta} \right)^2 \tau_n,
\end{aligned} \tag{A.14}$$

and internal price informativeness is

$$\tau_{\hat{p}} = (N-1) \left(\tau_s^{-1} + \left(\frac{\overline{\alpha_n}}{\overline{\alpha_s}} \right)^2 (N+1)\, \tau_n^{-1} \right)^{-1} = \frac{N-1}{\frac{\tau_s}{\tau_\eta} \frac{\tau_n}{\tau_\eta} + N + 1} \left(\frac{\tau_s}{\tau_\eta} \right)^2 \tau_n.$$

Lemma 4. (Comparative statics strategic traders) Price informativeness is increasing in τ_s, τ_n, and N and decreasing in τ_η.

Proof. From the definition of price informativeness in Eq. (A.14) we have that

$$\frac{d\tau_{\hat{p}}^e}{d\tau_s} = N \frac{1}{\tau_\eta^2} \frac{2\tau_s \left(\frac{\tau_s}{\tau_\eta} \frac{\tau_n}{\tau_\eta} + N + 1 \right) - \tau_s \frac{\tau_s}{\tau_\eta} \frac{\tau_n}{\tau_\eta}}{\left(\frac{\tau_s}{\tau_\eta} \frac{\tau_n}{\tau_\eta} + N + 1 \right)^2} \tau_n = N \frac{\frac{\tau_s}{\tau_\eta} \frac{\tau_n}{\tau_\eta} + 2(N+1)}{\left(\frac{\tau_s}{\tau_\eta} \frac{\tau_n}{\tau_\eta} + N + 1 \right)^2} \frac{\tau_s}{\tau_\eta} \frac{\tau_n}{\tau_\eta} > 0.$$

Moreover,

$$\begin{aligned}
\frac{d\tau_{\hat{p}}^e}{d\tau_n} &= \frac{N(N+1)}{\left(\frac{\tau_s}{\tau_\eta} \frac{\tau_n}{\tau_\eta} + N + 1 \right)^2} \left(\frac{\tau_s}{\tau_\eta} \right)^2 > 0, \\
\frac{d\tau_{\hat{p}}^e}{dN} &= \frac{\frac{\tau_s}{\tau_\eta} \frac{\tau_n}{\tau_\eta} + 1}{\left(\frac{\tau_s}{\tau_\eta} \frac{\tau_n}{\tau_\eta} + N + 1 \right)^2} \left(\frac{\tau_s}{\tau_\eta} \right)^2 \tau_n > 0,
\end{aligned}$$

and

$$\frac{d\tau_{\hat{p}}^e}{d\tau_\eta} = -\frac{2N(N+1)}{\left(\frac{\tau_s}{\tau_\eta} \frac{\tau_n}{\tau_\eta} + N + 1 \right)^2} \frac{\tau_s^2}{\tau_\eta^2} \frac{\tau_n}{\tau_\eta} < 0.$$

□

Proposition 7. (Comovement strategic traders) *a) Price volatility and price informativeness positively comove (weakly) across equilibria if price informativeness is high enough. Formally, there exists $\overline{\tau} \in (\tau^\star, \infty)$ such that if $\tau_{\hat{p}}^e \geq \overline{\tau}$, $\mathcal{V}$ and $\tau_{\hat{p}}^e$ move in the same direction after any parameter change.*

b) Price volatility and price informativeness negatively comove (weakly) across equilibria if price informativeness is low enough. Formally, there exists a threshold $\underline{\tau} \in [0, \tau^\star]$ such that, if $\tau_{\hat{p}}^e < \underline{\tau}$, $\mathcal{V}$ and $\tau_{\hat{p}}^e$ move in opposite directions after any parameter change.

Proof. Note that

$$\frac{d\tau_{\hat{p}}}{d\left(R^{-1}\rho\right)}=0 \quad \text{and} \quad \frac{d\mathcal{V}}{d\left(R^{-1}\rho\right)}>0,$$

so the comovements in this proposition are weak. Moreover,

$$\frac{d\tau_{\hat{p}}^{e}}{d\tau_{n}}<0 \quad \text{and} \quad \frac{d\mathcal{V}}{d\tau_{n}}=0.$$

Hence, movements in τ_n will induce positive (negative) comovements between price informativeness and price volatility if and only if $\tau_{\hat{p}}^{e} > (<)\, \tau^{\star}$.

a) **Positive comovement**

From Lemma 4 and using that

$$\frac{\overline{\alpha_s}}{\overline{\alpha_p}}=\frac{1}{1-R^{-1}\rho}\frac{\tau_s+\frac{N-1}{N}\tau_{\hat{p}}^{e}}{\tau_\eta+\tau_s+\frac{N-1}{N}\tau_{\hat{p}}^{e}},$$

we have

$$\begin{aligned}&\frac{d\tau_{\hat{p}}^{e}}{d\tau_s}>0, \quad \frac{d\tau_{\hat{p}}^{e}}{d\tau_\eta}<0, \quad \frac{d\tau_{\hat{p}}^{e}}{dN}>0 \quad \text{and,}\\ &\frac{d\mathcal{V}}{d\tau_s}<0, \quad \frac{d\mathcal{V}}{d\tau_\eta}<0, \quad \frac{d\mathcal{V}}{dN}>0.\end{aligned}$$

Therefore, whenever $\tau_{\hat{p}}^{e} > \tau^{\star}$, price informativeness and price volatility positively comove.

b) **Negative comovement**

For changes in τ_s we have

$$\frac{d\mathcal{V}}{d\tau_s}=\frac{\partial\mathcal{V}}{\partial\tau_s}+\frac{d\mathcal{V}}{d\tau_{\hat{p}}^{e}}\frac{d\tau_{\hat{p}}^{e}}{d\tau_s},$$

where we can write

$$\begin{aligned}\frac{d\mathcal{V}}{d\tau_s}&=2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-R^{-1}\rho}\frac{\tau_\eta}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_{\hat{p}}^{e}\right)^2}\left(\frac{\tau_\eta+\tau_{\hat{p}}^{e}}{\tau_\eta\tau_{\hat{p}}^{e}}\right)\\ &+\left(2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-R^{-1}\rho}\frac{\frac{N-1}{N}\tau_\eta}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_{\hat{p}}^{e}\right)^2}\left(\frac{\tau_\eta+\tau_{\hat{p}}^{e}}{\tau_\eta\tau_{\hat{p}}^{e}}\right)-\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\left(\tau_{\hat{p}}^{e}\right)^{-2}\right)N\frac{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+2\left(N+1\right)}{\left(\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1\right)^2}\tau_n\\ \frac{d\mathcal{V}}{d\tau_s}&=\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-R^{-1}\rho}\frac{\left(\tau_{\hat{p}}^{e}\right)^{-2}}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_{\hat{p}}^{e}\right)^2}\left(\begin{array}{c}2\left(\tau_\eta+\tau_{\hat{p}}^{e}\right)\tau_{\hat{p}}^{e}+\\ \left(2\frac{N-1}{N}\left(\tau_\eta+\tau_{\hat{p}}^{e}\right)\tau_{\hat{p}}^{e}-\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_{\hat{p}}^{e}\right)\right)N\frac{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+2(N+1)}{\left(\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1\right)^2}\tau_n\end{array}\right)\end{aligned}$$

Using the definition of $\tau_{\hat{p}}^{e}$ we have $\lim_{\tau_s\to 0}\tau_{\hat{p}}^{e}=0$. Then, taking limits when $\tau_s\to 0$, we have

$$\lim_{\tau_s\to 0}\frac{d\mathcal{V}}{d\tau_s}=\lim_{\tau_s\to 0}\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-R^{-1}\rho}\frac{\left(\tau_{\hat{p}}^{e}\right)^{-2}}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau_{\hat{p}}^{e}\right)^2}\left(-\tau_\eta N\frac{2\left(N+1\right)}{\left(N+1\right)^2}\right)=-\infty.$$

Then there exists a threshold $\hat{s}$ such that $\frac{d\mathcal{V}}{d\tau_s}<0$ for $\tau_s<\hat{s}$, which implies there exists a threshold τ_{τ_s} such that for $\tau_{\hat{p}}^{e}<\tau_{\tau_s}$ price volatility and price informativeness negatively comove.

For changes in τ_η we have

$$\frac{d\mathcal{V}}{d\tau_\eta} = -\frac{1}{\tau_\eta}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\left(\frac{2}{\tau_\eta+\tau_s+\frac{N-1}{N}\tau^e_{\hat{p}}}\frac{\tau_\eta+\tau^e_{\hat{p}}}{\tau^e_{\hat{p}}}+\frac{1}{\tau_\eta}\right)-\left(\begin{array}{c} 2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-R^{-1}\rho}\frac{\frac{N}{N-1}\tau_\eta}{\left(\tau_\eta+\tau_s+\frac{N}{N-1}\tau^e_{\hat{p}}\right)^2}\frac{\tau_\eta+\tau^e_{\hat{p}}}{\tau_\eta\tau^e_{\hat{p}}}- \\ \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\left(\tau^e_{\hat{p}}\right)^{-2} \end{array}\right)\frac{2N\left(N+1\right)}{\left(\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1\right)^2}\frac{\tau_s^2}{\tau_\eta^2}\frac{\tau_n}{\tau_\eta}$$

$$= -2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-R^{-1}\rho}\frac{\tau_\eta\left(\tau^e_{\hat{p}}\right)^{-1}}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau^e_{\hat{p}}\right)^2}\left(\begin{array}{c} \left(\frac{\tau_s}{\tau_\eta}+\frac{N-1}{N}\frac{\tau^e_{\hat{p}}}{\tau_\eta}\right)\left(1+\frac{\tau^e_{\hat{p}}}{\tau_\eta}\right)+ \\ \left(2\frac{N}{N-1}\left(1+\frac{\tau^e_{\hat{p}}}{\tau_\eta}\right)-\left(1+\frac{\tau_s}{\tau_\eta}+\frac{N}{N-1}\frac{\tau^e_{\hat{p}}}{\tau_\eta}\right)\left(\tau^e_{\hat{p}}\right)^{-1}\right)\frac{N(N+1)}{(\tau_s\tau_n+N+1)^2}\tau_s^2\frac{\tau_n}{\tau_\eta} \end{array}\right)-\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{\tau_\eta}\right)^2$$

Using the definition of $\tau^e_{\hat{p}}$ we have $\lim_{\tau_\eta\to\infty}\tau^e_{\hat{p}}=0$. Hence, taking limits when $\tau_\eta\to\infty$ we have

$$\lim_{\tau_\eta\to\infty}\frac{d\mathcal{V}}{d\tau_\eta} = \lim_{\tau_\eta\to\infty} 2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-R^{-1}\rho}\frac{\left(\tau^e_{\hat{p}}\right)^{-2}}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau^e_{\hat{p}}\right)^2}\frac{N\left(N+1\right)}{\left(\tau_s\tau_n+N+1\right)^2}\tau_s^2\tau_n=\infty$$

since

$$\lim_{\tau_\eta\to\infty}\frac{\left(\tau^e_{\hat{p}}\right)^{-2}}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau^e_{\hat{p}}\right)^2} = \lim_{\tau_\eta\to\infty}\left(\frac{N}{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1}\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau^e_{\hat{p}}\right)\right)^{-2}=\infty.$$

Hence, there exists a threshold $\overline{\eta}$ such that for all $\tau_\eta>\overline{\eta}$ we have $\frac{d\mathcal{V}}{d\tau_\eta}>0$. Then, there exists a threshold τ_{τ_η} such that for all $\tau^e_{\hat{p}}<\tau_{\tau_\eta}$ informativeness and volatility negatively comove after changes in τ_η where

$$\tau_{\tau_\eta}\equiv\frac{N}{\tau_s\tau_n+\left(N+1\right)\overline{\eta}^2}\left(\tau_s\right)^2\tau_n.$$

For changes in N we have

$$\frac{d\mathcal{V}}{dN} = 2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-R^{-1}\rho}\frac{\tau_\eta\tau^e_{\hat{p}}\left(\frac{1}{N^2}\right)}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau^e_{\hat{p}}\right)^2}\left(\frac{\tau_\eta+\tau^e_{\hat{p}}}{\tau_\eta\tau^e_{\hat{p}}}\right)$$
$$+\left(2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-R^{-1}\rho}\frac{\frac{N-1}{N}\tau_\eta}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau^e_{\hat{p}}\right)^2}\left(\frac{\tau_\eta+\tau^e_{\hat{p}}}{\tau_\eta\tau^e_{\hat{p}}}\right)-\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\left(\tau^e_{\hat{p}}\right)^{-2}\right)\frac{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+1}{\left(\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1\right)^2}\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n$$

$$\frac{d\mathcal{V}}{dN} = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-R^{-1}\rho}\frac{\left(\tau^e_{\hat{p}}\right)^{-2}}{\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau^e_{\hat{p}}\right)^2}\left(\begin{array}{c} 2\left(\tau_\eta+\tau^e_{\hat{p}}\right)\left(\frac{\tau^e_{\hat{p}}}{N}\right)^2+ \\ \left(2\frac{N-1}{N}\left(\tau_\eta+\tau^e_{\hat{p}}\right)\tau^e_{\hat{p}}-\left(\tau_\eta+\tau_s+\frac{N-1}{N}\tau^e_{\hat{p}}\right)\left(\tau_s+\frac{N-1}{N}\tau^e_{\hat{p}}\right)\right)\frac{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+1}{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+N+1}\frac{\tau^e_{\hat{p}}}{N} \end{array}\right).$$

Moreover,

$$\lim_{N\to0}\frac{d\mathcal{V}}{dN} = \lim_{N\to0}\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1-R^{-1}\rho}\frac{\left(\tau^e_{\hat{p}}\right)^{-2}\left(\frac{1}{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+1}\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n\right)^2}{\left(\tau_\eta+\tau_s+\frac{1}{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+1}\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n\right)^2}\left(-\frac{\left(\tau_\eta+\tau_s-\frac{1}{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+1}\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n\right)}{\frac{1}{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+1}\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n}\right)=-\infty$$

because $\lim_{N\to0}\tau^e_{\hat{p}}=0$ and $\lim_{N\to0}\frac{\tau^e_{\hat{p}}}{N}=\frac{1}{\frac{\tau_s}{\tau_\eta}\frac{\tau_n}{\tau_\eta}+1}\left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n$. Hence, there exists a threshold $\underline{N}$ such that for all $N<\underline{N}$ we have $\frac{d\mathcal{V}}{dN}<0$. This implies that there exists a threshold τ_N for all $\tau^e_{\hat{p}}<\tau_N$ informativeness and volatility negatively comove after changes in N. Note that for an equilibrium to exists we need $N\geq3$. The threshold $\underline{N}$ depends on all other parameters in the economy. When $\tau^e_{\hat{p}}$ is low, $\underline{N}$ is larger. Moreover, for all N there exist parameters such that $\frac{d\mathcal{V}}{dN}<0$ when $\tau^e_{\hat{p}}$ is low enough.

Therefore, for all $\tau^e_{\hat{p}}<\underline{\tau}$ informativeness and volatility (weakly) negatively comove for any parameter change, where $\underline{\tau}=\min\left\{\tau_{\tau_s},\tau_{\tau_\eta},\tau_N\right\}$ since $\max\left\{\tau_{\tau_s},\tau_{\tau_\eta},\tau_N\right\}<\tau^\star$. $\square$

C.3 Hedging needs

In this case, $\lambda = \frac{\tau_n+\tau_h}{\tau_n}$. Therefore, $\lambda < 2$ implies $\tau_h < \tau_n$. If $\tau_h < \tau_n$, there exists $\tau^\star$ such that

$$\frac{d\mathcal{V}}{d\tau_{\hat{p}}^e} > 0 \quad \forall \tau_{\hat{p}}^e > \tau^\star.$$

Moreover, the equilibrium demand sensitivities are given by Eq. (A.8), Eq. (A.9), Eq. (A.10), and Eq. (A.11) setting $\pi = 1$ and $\hat{\tau}_\varepsilon = \tau_h$ and

$$\kappa = \gamma\left(\left(1 + R^{-1}\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)^2 \mathbb{V}ar\left[\eta_t^l | \mathcal{I}_t^i\right] + \left(R^{-1}\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 \mathbb{V}ar\left[\eta_{t+1}^l\right] + \left(R^{-1}\frac{\overline{\alpha_\eta}}{\overline{\alpha_p}}\right)^2 \tau_n^{-1} + \tau_u^{-1}\right).$$

Then, in equilibrium,

$$\tau_{\hat{p}}^e = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2 \tau_n, \tag{A.15}$$

where $\frac{\overline{\alpha_s}}{\overline{\alpha_n}}$ solves the following fixed point

$$J\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right) = \frac{\overline{\frac{\tau_s}{\tau_\eta + \tau_s + \frac{\tau_n+\tau_h}{\tau_n}\tau_{\hat{p}}^e}}}{\gamma - \frac{\frac{\tau_n+\tau_h}{\tau_n}\tau_{\hat{p}}^e}{\tau_\eta+\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_{\hat{p}}^e}\frac{\overline{\alpha_n}}{\overline{\alpha_s}}\frac{\tau_h}{\tau_n+\tau_h}}, \tag{A.16}$$

where we used the equilibrium demand sensitivities which depend on $\frac{\overline{\alpha_s}}{\overline{\alpha_n}}$ directly and through $\tau_{\hat{p}}^e$.[20] $J(x)$ determines the ratio $\frac{\overline{\alpha_s}}{\overline{\alpha_n}}$ when investors expect the signal-to-noise ratio in the price to be x. The fixed point of Eq. (A.16) can also be found as the solution to

$$\hat{H}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right) \equiv -\gamma\left(\tau_n + \tau_h\right)\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^3 + \tau_h\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2 - \gamma\left(\tau_s + \tau_\eta\right)\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right) + \tau_s = 0. \tag{A.17}$$

The polynomial $\hat{H}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)$ always has a positive root but there may be multiple equilibria (generically, one or three). We adopt a conventional notion of stability. The function $\hat{H}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)$ is defined such that if $\hat{H}(x_0) > 0$, then $J(x_0) > x_0$, which implies that if investors in the model expect the signal-to-noise ratio to be x_0, the realized value of this ratio will be $x_1 > x_0$. Let x^* be a solution to $\hat{H}(x^*) = 0$. Then, we will say that the equilibrium x^* is stable if for all $x_0 \in (x^* - \delta, x^* - \delta)$ for some $\delta > 0$ the sequence $\{x_m\}_{m=0}^\infty$ where $x_m = J(x_{m-1})$ for $m > 1$ converges to x^*. This sequence will converge only if $J'(x^*) < 1$, which is equivalent to $\hat{H}'(x^*) < 0$. Hence, in all stable equilibria, $\hat{H}'(x^*) < 0$. Finally, note that when $\tau_s = 0$, the only root of $\hat{H}(x^*)$ is at $x^* = 0$.

Lemma 5. (Comparative statics hedging needs) *In any stable equilibrium, the signal to noise ration $\frac{\overline{\alpha_s}}{\overline{\alpha_n}}$ increases with τ_s and τ_h and it decreases with τ_η, τ_n, and γ.*

[20] We could have alternatively adopted similar notions of trading with stochastic hedging needs, as in Ganguli and Yang (2009) and Manzano and Vives (2011).

Proof. From Eq. (A.17) we have

$$\begin{aligned}
\frac{\partial \hat{H}}{\partial \tau_h} &= -\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^3 + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2 = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2\left(-\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)+1\right) > 0,\\
\frac{\partial \hat{H}}{\partial \tau_s} &= -\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right) + 1 > 0,\\
\frac{\partial \hat{H}}{\partial \gamma} &= -\left(\tau_n + \tau_h\right)\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^3 - \left(\tau_s + \tau_\eta\right)\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right) < 0, \quad \text{and}\\
\frac{\partial \hat{H}}{\partial \tau_\eta} &= -\gamma\frac{\overline{\alpha_s}}{\overline{\alpha_n}} < 0, \quad \text{and}\\
\frac{\partial \hat{H}}{\partial \tau_n} &= -\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^3 < 0,
\end{aligned}$$

since $\frac{\overline{\alpha_s}}{\overline{\alpha_n}} < \frac{1}{\gamma}$. Using the implicit function theorem and that in any stable equilibrium $\hat{H}' < 0$ we have

$$\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_h} > 0, \quad \frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_s} > 0, \quad \frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\gamma} < 0, \quad \frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_\eta} < 0, \quad \text{and} \quad \frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_n} < 0.$$

□

Proposition 8. (Comovement hedging needs) *a) Price volatility and price informativeness positively comove (weakly) across equilibria if price informativeness is high enough. Formally, there exists $\overline{\tau} \in (\tau^\star, \infty)$ such that if $\tau_{\hat{p}}^e \geq \overline{\tau}$, $\mathcal{V}$ and $\tau_{\hat{p}}^e$ move in the same direction after any parameter change.*

b) Price volatility and price informativeness negatively comove (weakly) across equilibria if price informativeness is low enough for changes in τ_n and γ. For changes in τ_s, τ_h, and τ_η price informativeness and price volatility always comove positively. Hence, there does not exist a negative comovement region.

Proof. Note that

$$\frac{d\tau_{\hat{p}}}{d\left(R^{-1}\rho\right)} = 0 \quad \text{and} \quad \frac{\partial \mathcal{V}}{\partial\left(R^{-1}\rho\right)} > 0,$$

so the comovements in this proposition are weak. Moreover,

$$\frac{d\tau_{\hat{p}}^e}{d\gamma} = 2\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\gamma}\tau_n < 0 \quad \text{and} \quad \frac{\partial \mathcal{V}}{\partial \gamma} = 0.$$

Hence, price informativeness and price volatility positively (negatively) comove after a change in γ if $\tau_{\hat{p}}^e > (<)\tau^\star$.

a) **Positive comovement.** Using Lemma 5 and the definition of equilibrium price informativeness in Eq. (A.15), we get

$$\begin{aligned}
\frac{d\tau_{\hat{p}}^e}{d\tau_s} &= 2\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_s}\tau_n > 0\\
\frac{d\tau_{\hat{p}}^e}{d\tau_h} &= 2\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_h}\tau_n > 0\\
\frac{d\tau_{\hat{p}}^e}{d\tau_\eta} &= 2\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_\eta}\tau_n < 0.
\end{aligned}$$

Then, since

$$\mathcal{V}=\left(\frac{1}{1-R^{-1}\rho}\right)^{2}\left(\frac{\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_{\hat{p}}^{e}}{\tau_\eta+\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_{\hat{p}}^{e}}\right)^{2}\left(\tau_\eta^{-1}+\left(\tau_{\hat{p}}^{e}\right)^{-1}\right),$$

we have

$$\frac{\partial\mathcal{V}}{\partial\tau_s}=\left(\frac{1}{1-R^{-1}\rho}\right)^{2}2\frac{\left(\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_{\hat{p}}^{e}\right)\tau_\eta}{\left(\tau_\eta+\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_{\hat{p}}^{e}\right)^{3}}\left(\tau_\eta^{-1}+\left(\tau_{\hat{p}}^{e}\right)^{-1}\right)>0,$$

$$\frac{\partial\mathcal{V}}{\partial\tau_h}=\left(\frac{1}{1-R^{-1}\rho}\right)^{2}2\frac{\left(\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_{\hat{p}}^{e}\right)\tau_\eta\frac{\tau_{\hat{p}}^{e}}{\tau_n}}{\left(\tau_\eta+\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_{\hat{p}}^{e}\right)^{3}}\left(\tau_\eta^{-1}+\left(\tau_{\hat{p}}^{e}\right)^{-1}\right)>0,\quad\text{and}$$

$$\frac{\partial\mathcal{V}}{\partial\tau_\eta}=-\left(\frac{1}{1-R^{-1}\rho}\right)^{2}\left(\frac{\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_{\hat{p}}^{e}}{\tau_\eta+\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_{\hat{p}}^{e}}\right)^{2}\left(2\frac{\tau_\eta^{-1}+\left(\tau_{\hat{p}}^{e}\right)^{-1}}{\tau_\eta+\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_{\hat{p}}^{e}}+\frac{1}{\tau_\eta^{2}}\right)<0.$$

Hence, if $\tau_{\hat{p}}^{e}>\tau^{\star}$ price volatility and price informativeness (weakly) comove when τ_s, τ_η, or τ_h change.

For changes in τ_n we have that

$$\begin{aligned}\frac{\partial\mathcal{V}}{\partial\tau_n}&=-\left(\frac{1}{1-R^{-1}\rho}\right)^{2}2\frac{\left(\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_{\hat{p}}^{e}\right)\tau_\eta\frac{\tau_h}{\tau_n^{2}}\tau_{\hat{p}}^{e}}{\left(\tau_\eta+\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_{\hat{p}}^{e}\right)^{3}}\left(\tau_\eta^{-1}+\left(\tau_{\hat{p}}^{e}\right)^{-1}\right)\\&=-\left(\frac{1}{1-R^{-1}\rho}\right)^{2}2\frac{\left(\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_{\hat{p}}^{e}\right)\tau_\eta\frac{\tau_h}{\tau_n^{2}}\tau_{\hat{p}}^{e}}{\left(\tau_\eta+\tau_s+\frac{\tau_n+\tau_h}{\tau_n}\tau_{\hat{p}}^{e}\right)^{3}}\left(\tau_\eta^{-1}+\left(\tau_{\hat{p}}^{e}\right)^{-1}\right),\end{aligned}$$

which is increasing in $\tau_{\hat{p}}^{e}$ with $\lim_{\tau_{\hat{p}}^{e}\to\infty}\frac{\partial\mathcal{V}}{\partial\tau_n}=0$. Also,

$$\frac{\partial\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{\partial\tau_n}=\frac{\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^{3}}{-3\gamma\left(\tau_n+\tau_h\right)\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^{2}+2\tau_h\frac{\overline{\alpha_s}}{\overline{\alpha_n}}-\gamma\left(\tau_s+\tau_\eta\right)}<0$$

since the denominator is negative in any stable equilibrium

$$\begin{aligned}\frac{d\tau_{\hat{p}}^{e}}{d\tau_n}&=2\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\frac{\partial\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{\partial\tau_n}\tau_n+\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^{2}=\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^{2}\left(2\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)}{d\tau_n}\frac{\tau_n}{\frac{\overline{\alpha_s}}{\overline{\alpha_n}}}+1\right)\\&=\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^{2}\left(\frac{2\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^{2}\tau_n}{-3\gamma\left(1+\frac{\tau_h}{\tau_n}\right)\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^{2}\tau_n+2\tau_h\frac{\overline{\alpha_s}}{\overline{\alpha_n}}-\gamma\left(\tau_s+\tau_\eta\right)}+1\right)\\&=\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^{2}\left(\frac{-\gamma\left(1+3\frac{\tau_h}{\tau_n}\right)\tau_{\hat{p}}^{e}+2\tau_h\frac{\overline{\alpha_s}}{\overline{\alpha_n}}-\gamma\left(\tau_s+\tau_\eta\right)}{-3\gamma\left(1+\frac{\tau_h}{\tau_n}\right)\tau_{\hat{p}}^{e}+2\tau_h\frac{\overline{\alpha_s}}{\overline{\alpha_n}}-\gamma\left(\tau_s+\tau_\eta\right)}\right).\end{aligned}$$

Note that the numerator can be written as

$$-\gamma\left(1+3\frac{\tau_h}{\tau_n}\right)\tau_{\hat{p}}^{e}+2\tau_h\frac{\overline{\alpha_s}}{\overline{\alpha_n}}-\gamma\left(\tau_s+\tau_\eta\right)=-\gamma\left(1+3\frac{\tau_h}{\tau_n}\right)\tau_{\hat{p}}^{e}+2\tau_h\sqrt{\frac{\tau_{\hat{p}}^{e}}{\tau_n}}-\gamma\left(\tau_s+\tau_\eta\right).$$

This is a concave quadratic function of $\sqrt{\tau_{\hat{p}}^{e}}$ which is negative at $\sqrt{\tau_{\hat{p}}^{e}}=0$. Then, if $4\frac{\tau_h^{2}}{\tau_n}-4\gamma^{2}\left(1+3\frac{\tau_h}{\tau_n}\right)\left(\tau_s+\tau_\eta\right)<0$ we have $\frac{d\tau_{\hat{p}}^{e}}{d\tau_n}>0$ for all $\tau_{\hat{p}}^{e}$ and if $4\frac{\tau_h^{2}}{\tau_n}-4\gamma^{2}\left(1+3\frac{\tau_h}{\tau_n}\right)\left(\tau_s+\tau_\eta\right)\geq 0$ it is less than $\frac{d\tau_{\hat{p}}^e}{d\tau_n} > 0$ if

$$\tau_{\hat{p}}^e > \left(\frac{-2\frac{\tau_h}{\sqrt{\tau_n}} - \sqrt{4\frac{\tau_h^2}{\tau_n} - 4\gamma^2 \left(1 + 3\frac{\tau_h}{\tau_n}\right)(\tau_s + \tau_\eta)}}{-2\gamma \left(1 + 3\frac{\tau_h}{\tau_n}\right)} \right)^2 \equiv \overline{n},$$

or if

$$\tau_{\hat{p}}^e < \left(\frac{-2\frac{\tau_h}{\sqrt{\tau_n}} + \sqrt{4\frac{\tau_h^2}{\tau_n} - 4\gamma^2 \left(1 + 3\frac{\tau_h}{\tau_n}\right)(\tau_s + \tau_\eta)}}{-2\gamma \left(1 + 3\frac{\tau_h}{\tau_n}\right)} \right)^2 \equiv \underline{n}.$$

Then, when $\lambda < 2$, there exists a threshold $\widetilde{\tau}$ such that

$$\frac{d\mathcal{V}}{d\tau_n} > 0$$

for all $\tau_{\hat{p}}^e > \widetilde{\tau}$, where $\widetilde{\tau} = \tau^\star$ if $4\frac{\tau_h^2}{\tau_n} - 4\gamma^2 \left(1 + 3\frac{\tau_h}{\tau_n}\right)(\tau_s + \tau_\eta) < 0$ and $\widetilde{\tau} = \max\{\tau^\star, \overline{n}\}$ otherwise.

Hence, if $\tau_{\hat{p}}^e > \widetilde{\tau}$ price informativeness and price volatility weakly positively comove for any parameter change.

b) **Negative comovement**

For changes in τ_n we have from part a) of this proof that for $\tau_n < \underline{n}$, $\frac{d\tau_{\hat{p}}^e}{d\tau_n} > 0$. Moreover, we know that $\frac{\partial \mathcal{V}}{\partial \tau_n} < 0$ and $\frac{d\mathcal{V}}{d\tau_{\hat{p}}^e} < 0$ for all $\tau_{\hat{p}}^e < \tau^\star$. Hence, since $\frac{d\mathcal{V}}{d\tau_n} = \frac{\partial \mathcal{V}}{\partial \tau_n} + \frac{d\mathcal{V}}{d\tau_{\hat{p}}^e}\frac{d\tau_{\hat{p}}^e}{d\tau_n}$, there exists a threshold $\tau_{\tau_n} < \tau^\star$ such that for all $\tau_{\hat{p}}^e < \tau^\star$ we have $\frac{d\mathcal{V}}{d\tau_n} < 0$ and price informativeness and price volatility negatively comove when τ_n changes.

For changes in τ_s, we have

$$\frac{d\mathcal{V}}{d\tau_s} = 2\left(\frac{1}{1 - R^{-1}\rho}\right)^2 \frac{\left(\tau_s + \frac{\tau_n + \tau_h}{\tau_n}\tau_{\hat{p}}^e\right)\left(\tau_{\hat{p}}^e\right)^{-1}}{\left(\tau_\eta + \tau_s + \frac{\tau_n + \tau_h}{\tau_n}\tau_{\hat{p}}^e\right)^3} \left(\begin{array}{c} \tau_\eta + \tau_{\hat{p}}^e + \\ \left(\frac{\tau_n + \tau_h}{\tau_n}\left(2 - \frac{\tau_n + \tau_h}{\tau_n}\right)\frac{\left(\tau_{\hat{p}}^e\right)^2}{\tau_s} - \frac{\tau_n + \tau_h}{\tau_n}\frac{\tau_{\hat{p}}^e}{\tau_s}(2\tau_s - \tau_\eta) - (\tau_\eta + \tau_s)\right) \frac{\tau_s}{\frac{\overline{\alpha_s}}{\alpha_n}} \frac{d\left(\frac{\overline{\alpha_s}}{\alpha_n}\right)}{d\tau_s} \end{array} \right).$$

We know that $\lim_{\tau_s \to 0} \frac{\overline{\alpha_s}}{\alpha_n} = 0$. Moreover, using the definition of $\frac{\overline{\alpha_s}}{\alpha_n}$ in Eq. (A.17), we have that

$$\frac{\tau_s}{\frac{\overline{\alpha_s}}{\alpha_n}} = \gamma \left(\tau_\eta + \tau_s + (\tau_n + \tau_h)\left(\frac{\overline{\alpha_s}}{\alpha_n}\right)^2 \right) - \tau_h \frac{\overline{\alpha_s}}{\alpha_n},$$

which allows us to compute $\lim_{\tau_s \to 0} \frac{\tau_s}{\frac{\overline{\alpha_s}}{\alpha_n}} = \gamma\tau_\eta$ and $\lim_{\tau_s \to 0} \frac{\tau_{\hat{p}}^e}{\tau_s} = 0$. Therefore,

$$\lim_{\tau_s \to 0} \frac{d\mathcal{V}}{d\tau_s} = 2\left(\frac{1}{1 - R^{-1}\rho}\right)^2 \frac{\tau_n^{-1}}{(\tau_\eta)^2} \left(\frac{\tau_n + \tau_h}{\tau_n} - \gamma^2\tau_\eta\right),$$

where we use the fact that $\lim_{\tau_s \to 0} \frac{\tau_s}{\frac{\overline{\alpha_s}}{\alpha_n}} \frac{\partial\left(\frac{\overline{\alpha_s}}{\alpha_n}\right)}{\partial \tau_s} = 1$. Hence, $\frac{d\mathcal{V}}{d\tau_s} > 0$ and price informativeness and price volatility positively comove for changes in τ_s.

For τ_η, we have

$$\frac{d\mathcal{V}}{d\tau_\eta} = -\left(\frac{1}{1 - R^{-1}\rho}\right)^2 \left(\frac{\tau_s + \frac{\tau_n + \tau_h}{\tau_n}\tau_{\hat{p}}^e}{\tau_\eta + \tau_s + \frac{\tau_n + \tau_h}{\tau_n}\tau_{\hat{p}}^e}\right)^2 \left(2\frac{\left(\tau_\eta^{-1} + \left(\tau_{\hat{p}}^e\right)^{-1}\right)}{\left(\tau_\eta + \tau_s + \frac{\tau_n + \tau_h}{\tau_n}\tau_{\hat{p}}^e\right)} + \frac{1}{\tau_\eta^2}\right)$$
$$+ \left(2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{1}{1 - R^{-1}\rho}\frac{\frac{\tau_n + \tau_h}{\tau_n}\tau_\eta}{\left(\tau_\eta + \tau_s + \frac{\tau_n + \tau_h}{\tau_n}\tau_{\hat{p}}^e\right)^2}\left(\frac{\tau_\eta + \tau_{\hat{p}}^e}{\tau_\eta\tau_{\hat{p}}^e}\right) - \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\left(\tau_{\hat{p}}^e\right)^{-2}\right) 2\frac{\overline{\alpha_s}}{\alpha_n}\frac{\partial\left(\frac{\overline{\alpha_s}}{\alpha_n}\right)}{\partial \tau_\eta}\tau_n.$$

Since $\lim_{\tau_\eta \to \infty} \frac{\overline{\alpha_s}}{\overline{\alpha_n}} = 0$ and $\lim_{\tau_\eta \to \infty} \frac{\overline{\alpha_s}}{\overline{\alpha_n}} \tau_\eta = \frac{\tau_s}{\gamma}$, it is the case that

$$\begin{aligned}\lim_{\tau_\eta \to \infty} \frac{d\mathcal{V}}{d\tau_\eta} &= \lim_{\tau_\eta \to \infty} \left(\frac{1}{1 - R^{-1}\rho} \right)^2 \frac{\tau_s}{(\tau_\eta + \tau_s)^3} \frac{1}{\left(\tau_{\hat{p}}^e\right)^2 \tau_\eta^2} \left(\left(-\tau_\eta^2 \tau_s \left(\tau_\eta + \tau_s\right) \right) 2\gamma \frac{\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2}{\left(-H'\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)\right)} \tau_n \right) \\ &= \lim_{\tau_\eta \to \infty} \left(\frac{1}{1 - R^{-1}\rho} \right)^2 \frac{\tau_s}{\left(1 + \frac{\tau_s}{\tau_\eta}\right)^3} \frac{1}{\left(\tau_{\hat{p}}^e\right)^2 \tau_\eta^2} \left(-\left(1 + \frac{\tau_s}{\tau_\eta}\right) \frac{2\tau_s \tau_n}{\tau_n + \tau_h} \right) = -\infty\end{aligned}$$

Then, $\tau_{\tau_\eta} = 0$ and when τ_η changes, volatility and informativeness always positively comove.

Finally, for changes in τ_h, we have

$$\frac{d\mathcal{V}}{d\tau_h} = \frac{2\frac{\overline{\alpha_s}}{\overline{\alpha_p}}}{1 - R^{-1}\rho} \frac{\left(\tau_{\hat{p}}^e\right)^{-1}}{\left(\tau_\eta + \tau_s + \frac{\tau_n + \tau_h}{\tau_n} \tau_{\hat{p}}^e\right)^2} \left[\begin{array}{c} \frac{1}{\tau_n}\left(\tau_\eta + \tau_{\hat{p}}^e\right)\tau_{\hat{p}}^e + \\ \left(\frac{\tau_n+\tau_h}{\tau_n}\left(2 - \frac{\tau_n+\tau_h}{\tau_n}\right)\left(\tau_{\hat{p}}^e\right)^2 + \frac{\tau_n+\tau_h}{\tau_n}\left(\tau_\eta - 2\tau_s\right)\tau_{\hat{p}}^e - \tau_s\left(\tau_\eta + \tau_s\right) \right) \frac{\overline{\alpha_s}}{\overline{\alpha_n}} \frac{\left(-\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)+1\right)}{\left(-H'\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)\right)} \end{array} \right]$$

and

$$\lim_{\tau_h \to 0} \frac{d\mathcal{V}}{d\tau_h} = \left(\frac{1}{1 - R^{-1}\rho} \right)^2 \frac{2\left(\tau_s + \tau_{\hat{p}}^e\right)}{\left(\tau_\eta + \tau_s + \tau_{\hat{p}}^e\right)^3} \left(\tau_{\hat{p}}^e\right)^{-1} \left(\frac{1}{\tau_n}\left(\tau_\eta + \tau_{\hat{p}}^e\right)\tau_{\hat{p}}^e + \left(\left(\tau_{\hat{p}}^e\right)^2 + \left(\tau_\eta - 2\tau_s\right)\tau_{\hat{p}}^e - \tau_s\left(\tau_\eta + \tau_s\right) \right) \frac{\overline{\alpha_s}}{\overline{\alpha_n}} \frac{\left(-\gamma\frac{\overline{\alpha_s}}{\overline{\alpha_n}} + 1\right)}{\left(-H'\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)\right)} \right),$$

where $\lim_{\tau_h \to 0} \tau_{\hat{p}}^e \in (0, \infty)$. Note that

$$\operatorname{sgn}\left(\lim_{\tau_h \to 0} \frac{d\mathcal{V}}{d\tau_h} \right) = \operatorname{sgn}\left(\frac{1}{\tau_n}\left(\tau_\eta + \tau_{\hat{p}}^e\right)\tau_{\hat{p}}^e + \left(\left(\tau_{\hat{p}}^e\right)^2 + \left(\tau_\eta - 2\tau_s\right)\tau_{\hat{p}}^e - \tau_s\left(\tau_\eta + \tau_s\right) \right) \frac{\overline{\alpha_s}}{\overline{\alpha_n}} \frac{\left(-\gamma\frac{\overline{\alpha_s}}{\overline{\alpha_n}} + 1\right)}{\left(-H'\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)\right)} \right),$$

which is positive for low enough values of price informativeness since $-\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right) + 1 > 0$ and in any stable equilibrium $H'\left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right) < 0$. Hence, $\tau_{\tau_h} = 0$ and price volatility and price informative always comove positively for changes in τ_h.

Hence, for $\tau_{\hat{p}}^e < \underline{\tau} = \min\left\{\tau_{\tau_s}, \tau_{\tau_\eta}, \tau_{\tau_n}, \tau_{\tau_h}, \tau_\gamma\right\} = 0$ and there is no negative comovement region. $\square$

D Proofs: Section 5

Proposition 4. (Explicit characterization of comovement regions)

Proof. Using Proposition 2 and the characterization of equilibrium price informativeness in the previous section of this Appendix, we can write the positive and negative comovement regions in terms of parameters for Application 1. In this case, price volatility is given by

$$\mathcal{V} = \left(\frac{1}{1 - R^{-1}\rho} \right)^2 \left(\frac{\tau_s + \tau_{\hat{p}}^e}{\tau_\eta + \tau_s + \tau_{\hat{p}}^e} \right)^2 \left(\tau_\eta^{-1} + \left(\tau_{\hat{p}}^e\right)^{-1} \right)$$

and price informativeness is given by

$$\tau_{\hat{p}}^e = \left(\frac{\tau_s}{\tau_\eta} \right)^2 \tau_n.$$

There are three parameters that determine price informativeness and price volatility in equilibrium, the precision of private information, τ_s, the precision of the investors' prior, τ_η, and the precision of the aggregate sentiment, τ_n. The term $R^{-1}\rho$ affects price volatility but leaves informativeness unchanged. All other parameters leave the price distribution unchanged.

The threshold $\tau^\star$ in Proposition 2 is given by

$$\tau^\star \equiv \frac{-(\tau_\eta - 2\tau_s) + \sqrt{\tau_\eta^2 + 8\tau_s^2}}{2}.$$

Since $\lambda = 1$, $\overline{\tau} = \tau^\star$ and, using the definition of $\tau_{\hat{p}}^e$, the positive comovement region is characterized by $\tau_{\hat{p}}^e > \tau^\star$, which, in terms of primitives, is given by

$$\frac{\tau_n}{\tau_\eta} \geq \frac{\sqrt{1 + 8\left(\frac{\tau_s}{\tau_\eta}\right)^2} - 1 + 2\frac{\tau_s}{\tau_\eta}}{2\left(\frac{\tau_s}{\tau_\eta}\right)^2}.$$

Moreover, using the results in Eq. (A.12) and Eq. (A.13) in the proof Prop. 6 we have that the negative comovement region is characterized by the following conditions: $\frac{\tau_n}{\tau_\eta} < 1$,

$$\frac{\tau_n}{\tau_\eta} < \frac{\left(\frac{\tau_s}{\tau_\eta} - 1\right) + \sqrt{5\left(\frac{\tau_s}{\tau_\eta}\right)^2 - 2\frac{\tau_s}{\tau_\eta} + 1}}{2\left(\frac{\tau_s}{\tau_\eta}\right)^2},$$

and

$$\frac{\tau_n}{\tau_\eta} < \frac{-\left(2 - \frac{\tau_s}{\tau_\eta}\right) + \sqrt{\left(2 - \frac{\tau_s}{\tau_\eta}\right)^2 + 8\left(\frac{\tau_s}{\tau_\eta}\right)^2}}{4\left(\frac{\tau_s}{\tau_\eta}\right)^2}.$$

Note that $\frac{\tau_n}{\tau_\eta} < 1$ will always be satisfied if the other two conditions hold. Then, the negative comovement region is characterized by

$$\frac{\tau_n}{\tau_\eta} < \min\left\{\frac{\left(\frac{\tau_s}{\tau_\eta} - 1\right) + \sqrt{5\left(\frac{\tau_s}{\tau_\eta}\right)^2 - 2\frac{\tau_s}{\tau_\eta} + 1}}{2\left(\frac{\tau_s}{\tau_\eta}\right)^2}, \frac{-\left(2 - \frac{\tau_s}{\tau_\eta}\right) + \sqrt{\left(2 - \frac{\tau_s}{\tau_\eta}\right)^2 + 8\left(\frac{\tau_s}{\tau_\eta}\right)^2}}{4\left(\frac{\tau_s}{\tau_\eta}\right)^2}\right\}.$$

□

Proposition 5. (Recovering stock-specific primitives)

Proof. From the characterization of the equilibrium described above, we can express $\frac{\alpha_s}{\alpha_p}$ and $\frac{\alpha_s}{\alpha_\eta}$ as follows:

$$\frac{\overline{\alpha_s}}{\overline{\alpha_p}} = \left(1 + R^{-1}\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)\frac{\tau_s + \tau_{\hat{p}}}{\tau_\eta + \tau_s + \tau_{\hat{p}}} = \left(1 + R^{-1}\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)\frac{\frac{\tau_s}{\tau_\eta} + \tau_{\hat{p}}^R}{1 + \frac{\tau_s}{\tau_\eta} + \tau_{\hat{p}}^R} \tag{A.18}$$

$$\frac{\overline{\alpha_s}}{\overline{\alpha_\eta}} = \frac{\tau_s}{\tau_\eta}.$$

Under the stated assumptions, we can therefore interpret the coefficients β_1 and ζ_1 as follows

$$\beta_2 = \frac{\overline{\alpha_s}}{\overline{\alpha_p}} \quad \text{and} \quad \zeta_1 = \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}. \tag{A.19}$$

Therefore, Equations (A.18) and Equation (A.19) imply that $\frac{\tau_s}{\tau_\eta} + \tau_{\hat{p}}^R$ can be recovered as follows

$$\beta_2 = \left(1 + R^{-1}\zeta_1\right)\frac{\frac{\tau_s}{\tau_\eta} + \tau_{\hat{p}}^R}{1 + \frac{\tau_s}{\tau_\eta} + \tau_{\hat{p}}^R} \Rightarrow \frac{\tau_s}{\tau_\eta} + \tau_{\hat{p}}^R = \frac{\beta_2}{1 + R^{-1}\zeta_1 - \beta_2},$$

which allows to express $\frac{\tau_s}{\tau_\eta}$ as

$$\frac{\tau_s}{\tau_\eta} = \frac{\beta_2}{1+R^{-1}\zeta_1-\beta_2}\tau_\eta - \tau_{\hat{p}}^R = \tau_{\hat{p}}^R\left(\frac{\beta_2}{1+R^{-1}\zeta_1-\beta_2}\frac{1}{\tau_{\hat{p}}^R}-1\right).$$

Finally, exploiting the relation $\tau_{\hat{p}} = \left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n$, $\frac{\tau_n}{\tau_\eta}$ can be recovered as follows

$$\tau_{\hat{p}} = \left(\frac{\tau_s}{\tau_\eta}\right)^2\tau_n = \left(\frac{\beta_2}{1+R^{-1}\zeta_1-\beta_2}-\tau_{\hat{p}}^R\right)^2\tau_n \Rightarrow \frac{\tau_n}{\tau_\eta} = \left(\frac{\beta_2}{1+R^{-1}\zeta_1-\beta_2}-\tau_{\hat{p}}^R\right)^{-2}\tau_{\hat{p}}^R,$$

where R can be mapped to the risk-free rate.

Finally, it is possible to show (see Davila and Parlatore (2018)) that $\tau_{\hat{p}}^R$ can be recovered using the following expression

$$\tau_{\hat{p}}^R = \frac{\tau_{\hat{p}}}{\tau_\eta} = 2\frac{R^2_{|\Delta\theta_{t+1},\Delta\theta_t} - R^2_{|\Delta\theta_t}}{1-R^2_{|\Delta\theta_{t+1},\Delta\theta_t}},$$

where $R^2_{|\Delta\theta_{t+1},\Delta\theta_t} \equiv 1-\frac{\mathbb{V}\mathrm{ar}[\varepsilon_t]}{\mathbb{V}\mathrm{ar}[\Delta p_t]}$ denotes the R-squared of the following regression of changes on lagged fundamentals

$$\Delta p_t = \beta_0 + \beta_1\Delta\theta_t + \beta_2\Delta\theta_{t+1} + \varepsilon_t, \tag{R1}$$

and where $R^2_{|\Delta\theta_t}$ denotes the R-squared of the following regression R2 of price changes on changes on fundamentals

$$\Delta p_t = \zeta_0 + \zeta_1\Delta\theta_t + \varepsilon_t^\zeta. \tag{R2}$$

□

Stationary case When the payoff process satisfies $\rho < 1$ we can recover $\frac{\tau_s}{\tau_\eta}$ and $\frac{\tau_n}{\tau_\eta}$ following an almost identical approach. Let's define Regressions R3 and R4 as follows:

$$p_t = \beta_0 + \beta_1\theta_t + \beta_2\theta_{t+1} + \varepsilon_t \tag{R3}$$

$$p_t = \zeta_0 + \zeta_1\theta_t + \varepsilon_t^\zeta. \tag{R4}$$

Note that Regression R3 can be written as

$$p_t = \beta_0 + (\beta_1 + \rho\beta_2)\,\theta_t + \beta_2\eta_t + \varepsilon_t,$$

so we can express $\frac{\alpha_s}{\alpha_p}$ and $\frac{\alpha_s}{\alpha_\eta}$ as follows:

$$\frac{\overline{\alpha_s}}{\overline{\alpha_p}} = \left(1+R^{-1}\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)\frac{\tau_s+\tau_{\hat{p}}}{\tau_\eta+\tau_s+\tau_{\hat{p}}} = \left(1+R^{-1}\frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\right)\frac{\frac{\tau_s}{\tau_\eta}+\tau_{\hat{p}}^R}{1+\frac{\tau_s}{\tau_\eta}+\tau_{\hat{p}}^R} \tag{A.20}$$

$$\frac{\overline{\alpha_s}}{\overline{\alpha_\eta}} = \frac{\tau_s}{\tau_\eta}.$$

Under the stated assumptions, we can therefore interpret the coefficients β_1 and $\zeta_1$as follows

$$\beta_2 = \frac{\overline{\alpha_s}}{\overline{\alpha_p}} \quad \text{and} \quad \zeta_1 = \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}. \tag{A.21}$$

Therefore, Equations (A.20) and Equation (A.21) imply that $\frac{\tau_s}{\tau_\eta} + \tau_{\hat{p}}^R$ can be recovered as follows

$$\beta_2 = \left(1 + R^{-1}\zeta_1\right) \frac{\frac{\tau_s}{\tau_\eta} + \tau_{\hat{p}}^R}{1 + \frac{\tau_s}{\tau_\eta} + \tau_{\hat{p}}^R} \Rightarrow \frac{\tau_s}{\tau_\eta} + \tau_{\hat{p}}^R = \frac{\beta_2}{1 + R^{-1}\zeta_1 - \beta_2},$$

which allows to express τ_s:

$$\tau_s = \frac{\beta_2}{1 + R^{-1}\zeta_1 - \beta_2}\tau_\eta - \tau_{\hat{p}} = \tau_{\hat{p}} \left(\frac{\beta_2}{1 + R^{-1}\zeta_1 - \beta_2} \frac{1}{\tau_{\hat{p}}^R} - 1 \right),$$

Finally, exploiting the relation $\tau_{\hat{p}} = \left(\frac{\tau_s}{\tau_\eta}\right)^2 \tau_n$, $\frac{\tau_n}{\tau_\eta}$ can be recovered as follows

$$\tau_{\hat{p}} = \left(\frac{\tau_s}{\tau_\eta}\right)^2 \tau_n = \left(\frac{\beta_2}{1 + R^{-1}\zeta_1 - \beta_2} - \tau_{\hat{p}}^R\right)^2 \tau_n \Rightarrow \tau_n = \left(\frac{\beta_2}{1 + R^{-1}\zeta_1 - \beta_2} - \tau_{\hat{p}}^R\right)^{-2} \tau_{\hat{p}},$$

where R can be mapped to the risk-free rate. The only meaningful difference with the random walk case, in addition to running regressions in levels rather than differences, is that, as shown in Davila and Parlatore (2018), $\tau_{\hat{p}}^R$ is now given by

$$\tau_{\hat{p}}^R = \frac{\tau_{\hat{p}}}{\tau_\eta} = \frac{R^2_{|\theta_{t+1},\theta_t} - R^2_{|\theta_t}}{1 - R^2_{|\theta_{t+1},\theta_t}},$$

where $R^2_{|\theta_{t+1},\theta_t}$ and $R^2_{|\theta_t}$ respectively denote the R-squared of Regressions R3 and R4.