Intergenerational Welfare Assessments*

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Abstract

This paper studies welfare assessments in economies with rich demographics. We introduce the notion of demographically disconnected economies, those with no date at which all individuals are concurrently alive. We identify the unique class of units that always enables meaningful welfare comparisons in such economies: those based on perpetual consumption, that is, consumption at all dates. Using these units, we uncover a novel possibility: feasible perturbations of Pareto efficient allocations can yield Kaldor-Hicks efficiency gains. We also introduce a decomposition that attributes intertemporal-sharing efficiency gains to financial frictions or demographic differences. These results allow us to derive new insights in three workhorse intergenerational models: (i) Samuelson (1958) two-date-life model, offering a novel rationale for social security; (ii) Diamond (1965) growth model, providing a new theory for capital taxation and capital over-/under-accumulation; and (iii) Samuelson (1958) three-date-life model, decomposing the efficiency gains from intergenerational transfers into frictional and demographic sources.

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1 Introduction

Making interpersonal welfare comparisons across individuals who are not concurrently alive is conceptually challenging but essential for evaluating policies in areas such as social security, public debt, education, and climate change. Yet, existing work on these questions typically avoids welfare comparisons across individuals from different generations. The alternative is to rely on the Pareto criterion, which — though widely accepted — is often inconclusive.

In this paper, we systematically study how to conduct welfare assessments in economies where individuals are born and die at different times. We introduce our results in an environment with a flexible demographic structure that accommodates arbitrary birth and death patterns. A key notion in our analysis is that of *demographically disconnected* economies — those with no date at which all individuals are concurrently alive.

Valid Unit for Interpersonal Welfare Comparisons. Meaningful interpersonal welfare comparisons require selecting a unit — a welfare numeraire — since individual utilities are ordinal and not directly comparable across individuals. Our first main result identifies the unique class of units that always enables such comparisons in demographically disconnected economies: those based on *perpetual consumption*, that is, consumption at all dates. Intuitively, only bundles that include consumption at all dates are always intrinsically valued by all individuals, alive and yet-to-be-born. For instance, individuals born at date 3 derive no value from date-0 consumption, while those certain to die by date 7 derive no value from date-9 consumption. Yet every individual assigns positive value to a bundle that includes consumption at all dates. This result contrasts with demographically connected economies, such as those with always-alive individuals. In those economies, other units — including date-0 consumption, date-t consumption, or bundles of consumption spanning multiple dates — can also serve as valid welfare numeraires.

Kaldor-Hicks Improvements from Pareto Efficient Allocations. Our second main result shows that it is possible to construct *feasible* perturbations of Pareto efficient allocations in demographically disconnected economies that feature Kaldor-Hicks efficiency gains.[1] That is, it is possible to move away from a Pareto efficient allocation in a demographically disconnected economy in a way that the sum of individual gains and losses is strictly positive. This result critically hinges on expressing Kaldor-Hicks efficiency gains in terms of perpetual consumption — a bundle whose elements, namely consumption at specific dates, are not valued by every individual. But there is no way around this: as already established, demographically disconnected economies necessitate welfare numeraires based on perpetual consumption.

This seemingly paradoxical result contrasts with economies where all individuals consume at every date. In such settings, no feasible perturbation of a Pareto efficient allocation can yield Kaldor-Hicks efficiency gains, regardless of the chosen welfare numeraire. In general, when a perturbation generates Kaldor-Hicks efficiency gains, the compensation principle calls for reallocating resources from winners to losers to engineer a Pareto improvement. Yet compensating transfers of perpetual consumption are *not feasible* in demographically disconnected economies because they would require transferring consumption away from the dead, violating consumption non-negativity constraints. That is, while welfare changes can be expressed and aggregated in terms of perpetual consumption, consumption non-negativity constraints prevent perpetual consumption from actually being transferred.

This result opens the door to reconsidering the role of government intervention in settings where demographics are relevant. Even when the first welfare theorem holds, finding feasible Kaldor-Hicks improvements from Pareto efficient allocations suggests that there may exist untapped sources of value in demographically disconnected economies that only government policies can potentially capture. Our applications present scenarios in which this insight can be used to rationalize specific policies, such as social security or capital taxation.

Intertemporal-Sharing Decomposition: Frictions vs. Demographics. Intertemporal-sharing efficiency gains arise when consumption is reallocated toward those individuals who value it more at specific dates. In economies with always-alive individuals, such valuation differences must be due to financial frictions. In demographically disconnected

[1] A perturbation features Kaldor-Hicks efficiency gains if the sum of individual welfare changes, expressed in a common unit, is strictly positive. This implies that the winners could hypothetically compensate the losers if transfers in the chosen unit were feasible.

economies, however, demographic differences themselves generate valuation differences — even in the absence of financial frictions — introducing a distinct source of intertemporal-sharing gains.

Our third main result shows how to systematically separate the sources of intertemporal-sharing gains into two components. The first component, *frictional intertemporal-sharing* (FIS), captures gains from reallocating consumption toward individuals whose intertemporal valuations differ due to financial frictions. The second component, *demographic intertemporal-sharing* (DIS), captures gains from reallocating consumption toward individuals whose intertemporal valuations differ due to demographic differences. To our knowledge, this is the first procedure that distinguishes whether efficiency gains arise because consumption is reallocated towards individuals who i) face financial frictions, addressing inefficiencies that individuals could in principle correct via contracts or markets, or ii) differ demographically, where gains require policy intervention.

Applications. Three applications demonstrate the practical relevance of our results. To highlight that our findings are not driven by well-understood double-infinity arguments (Shell, 1971; Geanakoplos, 1989), all three applications feature a finite terminal date.

Our first application conducts a welfare assessment of intergenerational transfers in the simplest overlapping generations (OLG) economy: the *two-date-life* version of Samuelson (1958)'s endowment economy. A central takeaway from this application is that a policy of young-to-old transfers — modeled to mimic a pay-as-you-go social security system — generates Kaldor-Hicks efficiency gains over the unique competitive equilibrium, which is Pareto efficient. Since this is a pure endowment economy with complete markets, all efficiency gains stem from demographic intertemporal-sharing (DIS), by reallocating consumption towards individuals whose valuations differ due to demographic differences.

Our analysis provides a novel justification for social security on the grounds that the sum of willingness-to-pay for it among all individuals, expressed in units of perpetual aggregate consumption, is strictly positive, even when it does not constitute a Pareto improvement. This application illustrates Proposition 1 by using perpetual aggregate consumption as welfare numeraire; Proposition 2 by showing that social security generates Kaldor-Hicks efficiency gains from a Pareto efficient allocation; and Proposition 3 by showing that these (intertemporal-sharing) gains reflect purely demographic differences

(DIS), as markets are complete and there are no financial frictions.

Our second application establishes that in the finite-horizon Diamond (1965)'s OLG growth model, the efficiency-maximizing capital tax is positive in low-capital-share economies, which over-accumulate capital, and negative (a subsidy) in high-capital-share economies, which under-accumulate capital, despite the Pareto efficiency of the competitive equilibrium. This is a new illustration of Proposition 2 demonstrating how to perturb a Pareto-efficient allocation in a production economy to yield Kaldor–Hicks efficiency gains. Interestingly, intertemporal-sharing considerations — again purely demographic (DIS) — attenuate the optimal tax relative to what investment efficiency alone would prescribe.

This second application revisits the efficiency of capital accumulation. Prior work has focused on identifying conditions for Pareto improvements, with Abel, Mankiw, Summers, and Zeckhauser (1989) calling for the use of social welfare functions.[2] Our analysis takes up this challenge by moving beyond the Pareto criterion and using welfare and efficiency gains to assess alternative capital-accumulation paths in a production economy. In doing so, it connects to the Golden Rule literature initiated by Phelps (1961), which asks whether an economy accumulates "too much" or "too little" capital. Our approach sheds new light on this classic question in two ways. First, it moves beyond steady-state comparisons by accounting for the effects of capital accumulation on aggregate consumption along the entire transition path. Second, it captures intertemporal-sharing gains arising from valuation heterogeneity across generations, rather than focusing solely on aggregate consumption.

Our third and last application revisits the welfare assessment of social security in the simplest OLG economy in which financial markets can be used to smooth consumption: the *three-date-life* version of Samuelson (1958)'s endowment economy. Starting from a hump-shaped consumption profile and absent financial markets, we consider a policy that transfers resources from middle-aged to old, mimicking again pay-as-you-go social security. A central takeaway is that the justification for this policy differs fundamentally from that

[2] Abel, Mankiw, Summers, and Zeckhauser (1989) conclude their study of efficiency in OLG economies with: *"The most important direction for future research is the evaluation of alternative dynamic paths using stronger criteria than the dynamic efficiency criterion. Our criterion is the dynamic analogue of the standard Pareto criterion. (...) The use of social welfare functions would make possible the evaluation of alternative social decision rules for determining the level of investment."*

of the young-to-old transfer in Application 1. Both interventions generate intertemporal-sharing gains, but for different reasons: in Application 3, frictional intertemporal-sharing (FIS) gains outweigh demographic intertemporal-sharing (DIS) losses, whereas in Application 1 all gains are demographic (DIS), with no contribution from FIS. This application underscores the importance of decomposing intertemporal-sharing gains into frictional and demographic components to understand the justification for policies in economies with incomplete markets and rich demographics, as we do in Proposition 3.

Related Literature. This paper contributes to the study of efficiency in OLG economies initiated in Samuelson (1958) and Diamond (1965), the two benchmark models underlying our applications. These topics routinely appear in macroeconomics textbooks, including Blanchard and Fischer (1989), De La Croix and Michel (2002), and Acemoglu (2009), among others. Spear and Young (2023) comprehensively review the development of these ideas.

A central normative insight of the OLG literature is that competitive equilibria may be inefficient — a possibility that requires both a double infinity of agents and goods and overlapping lifespans (Geanakoplos, 1989). Our results are orthogonal to this issue, and apply to finite-horizon economies, in which the first welfare theorem holds. Therefore, our results should be read as characterizing specific normative properties of Arrow-Debreu economies in which there is no commodity liked by all individuals.

Prior work has recognized the importance of the demographic structure in OLG economies. Weil (1989), in particular, points out that the arrival of new individuals (dynasties) unlinked to existing cohorts is necessary to generate bubbles, dynamic inefficiency, and Ricardian equivalence violations. While the continual birth of new individuals renders an economy demographically disconnected, the notion of demographically disconnected economies that we introduce in this paper is, to our knowledge, not present in prior work.[3]

A separate body of work studies intergenerational welfare criteria, including, for instance, the work by Calvo and Obstfeld (1988) and Eden (2023), among others. Our approach takes the social objective as given, but our emphasis on making interpersonal

[3] McKenzie (1959)'s notion of a connected economy refers to the interdependence of goods in preferences or production, ensuring that no group of goods is isolated from the rest of the economy. Demographically disconnected economies are typically connected in the sense of McKenzie (1959).

comparisons in a common unit leads us to establish that valid welfare numeraires must be based on perpetual consumption. A different approach is pursued by Aguiar, Amador, and Arellano (2023), who characterize Robust Pareto Improvements (RPI) in OLG economies, ensuring that the budget set of any agent is guaranteed to be weakly expanded at any state and time. An advantage of the RPI criterion is that it ensures a Pareto improvement regardless of how agents trade off consumption intertemporally or across states. Our results are complementary, as they are most useful for evaluating perturbations that do not constitute Pareto improvements. The importance of population growth for welfare gains has been emphasized by Jones and Klenow (2016) and Adhami, Bils, Jones, and Klenow (2024), among others. We hope that our results spur future research on the welfare implications of population dynamics.

2 Environment

Our notation follows Chapter 8 of Ljungqvist and Sargent (2018). Time is discrete, with dates indexed by $t \in \{0, \ldots, T\}$, where $T \leq \infty$. While prior work on demographically rich economies has emphasized the $T = \infty$ case, our results apply when T is finite or infinite.

Demographics. The economy is populated by a countable number of individuals, indexed by $i \in \mathcal{I} = \{1, \ldots, I\}$, where $1 \leq I \leq \infty$. Each individual i is associated with a date of birth $\tau_b^i \in \{-\infty, \ldots, T\}$, where a negative date of birth indicates that the individual is already alive at date 0, and a date of death $\tau_d^i \in \{1, \ldots, T\}$, with $\tau_b^i \leq \tau_d^i \leq T$.

Preferences. Individual i's preferences from date 0 onwards can be represented by

$$\text{(Preferences)} \qquad V^i = \sum_{t=0}^{T} \left(\beta^i\right)^{t-\tau_b^i} u_t^i\left(c_t^i\right), \tag{1}$$

where $\beta^i \in [0, 1)$ denotes individual i's discount factor, and $u_t^i(\cdot)$ and c_t^i respectively correspond to individual i's instantaneous utility and consumption at date t. When individual i is alive, that is, when $\tau_b^i \leq t \leq \tau_d^i$, $u_t^i(c_t^i)$ is well behaved, satisfying $\frac{\partial u_t^i}{\partial c_t^i} > 0$ and an Inada condition. When individual i is dead, $u_t^i(c_t^i) = 0$.

Table 1: Illustrating Demographic Disconnect

	Economy A		Economy B	
	$t=0$	$t=1$	$t=0$	$t=1$
$i=1$	✓	×	✓	×
$i=2$	✓	✓	✓	✓
$i=3$			×	✓

Note: Economy A has two individuals: $i \in \{1,2\}$. Economy B has three individuals: $i = \{1,2,3\}$. Checks (✓) indicate dates when an individual is alive, while crosses (×) indicate dates when an individual is not alive. Economy A is demographically connected since all individuals are concurrently alive at date 0. Economy B is demographically disconnected since there is no date at which all individuals are concurrently alive: individual $i=1$ is not alive at date 1 and individual $i=3$ is not alive at date 0.

Social Welfare Function. We study welfare assessments for welfarist planners, who evaluate outcomes using a social welfare function given by

$$\text{(Social Welfare Function)} \qquad W = \mathcal{W}\left(V^1, \ldots, V^i, \ldots, V^I\right), \tag{2}$$

where individual lifetime utilities V^i are defined in (1). We assume that $\frac{\partial \mathcal{W}}{\partial V^i} > 0$, $\forall i$, so that the planner values all individuals alive and yet-to-be-born at date 0.

Defining social welfare functions with multiple generations is analogous to doing so in static environments or with always-alive individuals (see e.g., Blanchard and Fischer, 1989), although ensuring that W is finite may require restricting $\mathcal{W}(\cdot)$ when $T = \infty$.[4] The welfarist approach is prevalent because it satisfies the Pareto principle — every Pareto improvement is desirable — whereas nonwelfarist approaches violate it (Kaplow and Shavell, 2001).

Extensions. For clarity, the main text considers a deterministic environment with a single consumption good. Section C of the Online Appendix shows how to accommodate demographic uncertainty, useful to model random deaths (Yaari, 1965; Blanchard, 1985), as well as non-demographic uncertainty, multiple consumption goods, and factor supplies.

[4]The most commonly used intergenerational social welfare functions are i) discounted utilitarian, which exponentially discounts future generations' utility, and ii) undiscounted utilitarian, which assigns equal weight to all individuals' utilities. We use both in our applications.

Demographic Disconnect. Throughout the paper, we rely on the notion of demographically disconnected economies, which we introduce here.

Definition. *(Demographic Disconnect) An economy is demographically disconnected if there is no date at which all individuals are concurrently alive.*

The defining feature of demographically disconnected economies is the absence of a date at which all individuals — both currently alive and those yet-to-be-born — assign positive value to consumption. Demographic disconnect is realistic and a common assumption when modeling demographics. We illustrate this notion through an example — summarized in Table 1 — that contrasts two economies with different demographic structures.

Example 1. *(Illustrating Demographic Disconnect)* Economy A has two individuals: $i = 1$, alive only at date 0, and $i = 2$, alive at dates 0 and 1. Since both individuals are alive at date 0, Economy A is demographically connected. Economy B is identical to economy A, except for the presence of a third individual $i = 3$, alive only at date 1. The presence of this third individual makes Economy B demographically disconnected, as there is no date at which all individuals are concurrently alive: individual $i = 1$ is not alive at date 1, and individual $i = 3$ is not alive at date 0. The demographics of Economy B are the same as in the two-date-life OLG model, which we study in Application 1.

Remarks. It is worth making three remarks on the notion of demographic disconnect.

Remark 1. *(Preference Disconnect)* Multi-good static economies mirror demographically disconnected economies when individuals assign zero value to consuming certain goods — a situation we term "*preference disconnect*". Section C.3 of the Online Appendix shows how our results translate to this setting. In reality, while demographic disconnect arises naturally in the absence of altruism — see Remark 2 — preference disconnect is harder to justify because all individuals value essential goods (e.g., water) at a point in time.

Remark 2. *(Intergenerational Altruism)* The definition of demographic disconnect presumes that individuals are self-interested, placing no value on the consumption of future generations. With intergenerational altruism, a demographically disconnected economy may become connected, as individuals who value their descendants assign positive value to consumption beyond their own lifetime. The notion of demographic disconnect extends to economies with intergenerational altruism by requiring that there be no date at which all individuals attach positive value to consumption, either privately or altruistically.

Remark 3. *(Demographic Disconnect Requires Births)* Demographic disconnect requires births after date 0: if all individuals were alive at date 0 — even if they die later — the economy would be connected at that date. Deaths are also required for disconnect in finite-horizon economies since otherwise all individuals would be concurrently alive at the terminal date T. But if $T = \infty$, this logic does not apply: demographic disconnect can arise even if everyone lives forever, so long as new individuals continue to be born over time.

3 Intergenerational Welfare Assessments

3.1 Welfare Numeraire

Our goal is to understand why a particular welfarist planner finds a given perturbation desirable. Formally, a perturbation corresponds to a smooth change in c_t^i as a function of a perturbation parameter $\theta \in [0, 1]$, so derivatives such as $\frac{dc_t^i}{d\theta}$ are well-defined. A perturbation may capture changes in policies, technologies, endowments, or other model primitives. It can also describe direct changes in feasible allocations selected by a planner.

A welfarist planner finds a perturbation $d\theta$ desirable (undesirable) if

$$\frac{dW}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \frac{dV^i}{d\theta} > (<)\, 0. \tag{3}$$

However, (3) does not immediately reveal how a welfarist planner makes tradeoffs across individuals, because individual utilities V^i are ordinal and therefore not directly comparable. Meaningful comparisons require choosing a common unit to express individual welfare gains, that is, a *welfare numeraire*. Formally, normalizing individual gains by a factor λ^i with $\dim(\lambda^i) = \frac{\text{utils of individual } i}{\text{units of welfare numeraire}}$ converts i's gains to units of the welfare numeraire, so

$$\frac{dW}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \lambda^i \frac{\frac{dV^i}{d\theta}}{\lambda^i}, \tag{4}$$

where $\dim\left(\frac{dV^i}{d\theta} / \lambda^i\right) = \frac{\text{units of welfare numeraire}}{\text{units of } \theta}$. For a welfare numeraire to be *valid*, all individuals — alive and yet-to-be-born — must assign it positive value, that is, $\lambda^i > 0$.

Unique Class of Valid Welfare Numeraires. In demographically disconnected economies, there is a unique class of units in which it is always possible to make interpersonal welfare comparisons: those based on perpetual consumption, that is, consumption at all dates. Identifying this class in Proposition 1 is the first main result of the paper.

Proposition 1. *(Unique Class of Valid Welfare Numeraires) Only welfare numeraires based on perpetual consumption are always valid in demographically disconnected economies.*

Intuitively, only bundles that include consumption at all dates are intrinsically valued by all individuals, alive and yet-to-be-born. For instance, individuals born at date 3 do not value date-0 consumption, while individuals certain to die by date 7 do not value date-9 consumption. Yet everyone values a bundle that includes consumption at all dates, even if they do not value every element of the bundle. By contrast, in demographically connected economies — such as those with always-alive individuals — many welfare numeraires are possible. Any date on which all individuals are concurrently alive, as well as bundles spanning multiple such dates, qualifies as a valid welfare numeraire in those settings.

Perpetual Aggregate Consumption. Actually computing welfare gains requires selecting a specific bundle based on perpetual consumption. Following Lucas (1987) and Alvarez and Jermann (2004), in our applications we adopt *perpetual aggregate consumption* — the bundle that consists of aggregate consumption at each date — as welfare numeraire. In this case, the normalizing factor λ^i, which corresponds to i's marginal utility for perpetual aggregate consumption, $(c_0, c_1, c_2, \ldots)$, takes the form

$$\lambda^i = \sum_t \left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i}{\partial c_t^i} c_t, \tag{5}$$

where $c_t = \sum_i c_t^i$ denotes date t aggregate consumption.[5] The upshot of using this numeraire is that welfare and efficiency gains are easily interpretable, as further explained in Remark 6 below. Hence, if $\frac{\frac{dV^i}{d\theta}}{\lambda^i} = 0.05$, individual i's marginal willingness-to-pay for the perturbation equals 5% of perpetual aggregate consumption.

[5]Section C.4 of the Online Appendix derives (5) from a global representation of welfare gains.

Efficiency vs. Redistribution. After expressing individual welfare gains in a common unit, it is useful to normalize the overall assessment by $\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i$, as follows:

$$\frac{dW^\lambda}{d\theta} = \frac{\frac{dW}{d\theta}}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i} = \sum_i \omega^i \frac{dV^{i|\lambda}}{d\theta} \quad \text{where} \quad \frac{dV^{i|\lambda}}{d\theta} = \frac{\frac{dV^i}{d\theta}}{\lambda^i} \quad \text{and} \quad \omega^i = \frac{\frac{\partial \mathcal{W}}{\partial V^i}\lambda^i}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i}. \tag{6}$$

Under this normalization, $\frac{dW^\lambda}{d\theta}$ expresses the overall welfare change in units of the welfare numeraire distributed equally across individuals, with each receiving $1/I$. Equation (6) reveals that welfare assessments can be interpreted as a weighted sum across individuals of lifetime welfare gains. The individual weights ω^i — which satisfy $\frac{1}{I}\sum_i \omega^i = 1$ — define how a planner trades off welfare gains across individuals.[6]

Given a welfare numeraire, a normalized welfare assessment admits a unique decomposition into Kaldor-Hicks efficiency and redistribution (Dávila and Schaab, 2025), given by

$$\frac{dW^\lambda}{d\theta} = \sum_i \omega^i \frac{dV^{i|\lambda}}{d\theta} = \underbrace{\sum_i \frac{dV^{i|\lambda}}{d\theta}}_{\Xi^E \text{ (Efficiency)}} + \underbrace{\mathbb{C}ov_i^\Sigma\left[\omega^i, \frac{dV^{i|\lambda}}{d\theta}\right]}_{\Xi^{RD} \text{ (Redistribution)}}, \tag{7}$$

where $\mathbb{C}ov_i^\Sigma[\cdot,\cdot] = I \cdot \mathbb{C}ov_i[\cdot,\cdot]$ denotes a cross-sectional covariance-sum among individuals.

The efficiency component Ξ^E corresponds to Kaldor-Hicks efficiency gains — the sum of individual gains in units of the welfare numeraire. This notion is useful for at least three reasons. First, it is invariant to preference-preserving transformations of utilities, unlike the overall welfare assessment. Second, it is independent of the social welfare function, which only affects the redistribution component through the ω^i's. Third, it satisfies the compensation principle, that is, perturbations with $\Xi^E > 0$ can be turned into Pareto improvements provided that transfers of the welfare numeraire are feasible and costless. For this reason, Kaldor-Hicks efficiency is typically useful to define Pareto frontiers. The redistribution component Ξ^{RD} captures the equity concerns embedded in a particular social welfare function: Ξ^{RD} is positive when the individuals relatively favored in a perturbation are those relatively preferred by the planner, that is, have a higher individual weight ω^i.

[6]For example, if $\omega^i = 1.3$, the planner perceives providing individual i with a 1% increase in perpetual aggregate consumption as equivalent to providing everyone a 1.3% increase.

Before proceeding, note that i's lifetime welfare gains can be expressed as

$$\frac{dV^{i|\lambda}}{d\theta}=\frac{\frac{dV^i}{d\theta}}{\lambda^i}=\sum_t \omega_t^i \frac{dV_t^{i|\lambda}}{d\theta} \quad \text{where} \quad \omega_t^i=\frac{\left(\beta^i\right)^{t-\tau_b^i}\lambda_t^i}{\sum_t \left(\beta^i\right)^{t-\tau_b^i}\lambda_t^i} \quad \text{and} \quad \frac{dV_t^{i|\lambda}}{d\theta}=\frac{\frac{dV_t^i}{d\theta}}{\lambda_t^i}=\frac{1}{c_t}\frac{dc_t^i}{d\theta}, \tag{8}$$

where $\lambda_t^i=\frac{\partial u_t^i}{\partial c_t^i}c_t$ is a date-specific welfare numeraire that allows us to express date-t gains in a common unit, in this case aggregate date-t consumption. This expression reveals that individual i's welfare gains correspond to a discounted sum of date-t gains, where the dynamic weight ω_t^i acts as a discount factor, measuring how individual i trades off date-t consumption against perpetual consumption.[7]

Remarks. It is worth making three remarks on the choice of welfare numeraire.

Remark 4. *(Perpetual Unit vs. Aggregate Consumption)* In Section C.4 of the Online Appendix we present λ^i when using as welfare numeraire unit perpetual consumption — the bundle of one unit of consumption at each date, $(1, 1, \ldots)$. In the absence of growth, unit and perpetual aggregate consumption yield identical results. In our applications, we find minimal quantitative differences between the two.

Remark 5. *(Consumption- vs. Money-Based Welfare Numeraires)* In economies with prices, welfare gains could be expressed in monetary terms using money-based (money-metric) welfare numeraires. While useful in specific settings, such numeraires need not be well-defined when markets are missing or frictional. Therefore, because we aim to evaluate perturbations independently of the market structure, we must resort to consumption-based welfare numeraires, which can always be defined.

Remark 6. *(Interpretation of Efficiency Component Depending on Welfare Numeraire)* Although Propositions 2 and 3 hold for any welfare numeraire in the class identified in Proposition 1, the precise value of efficiency gains Ξ^E depends on the choice of welfare numeraire. Using perpetual aggregate consumption is particularly natural because it offers a clear interpretation: for example, $\Xi^E = 0.12$ means that the perturbation is equivalent to a 12% permanent increase in aggregate consumption allocated uniformly across all individuals.

[7] For example, if $\omega_t^i = 0.1$, the planner perceives a 1% increase in aggregate consumption assigned to individual i at date t as equivalent to a 0.1% increase in perpetual aggregate consumption assigned to that individual. Dynamic weights sum to one over time for each individual: $\sum_t \omega_t^i = 1$, $\forall i$.

3.2 Kaldor-Hicks Improvements from Pareto Efficient Allocations

With always-alive individuals, no feasible perturbation of a Pareto efficient allocation can feature Kaldor-Hicks efficiency gains, regardless of the welfare numeraire.[8] If such a perturbation existed, the planner could implement it and reallocate the welfare numeraire to engineer a Pareto improvement, contradicting the Pareto efficiency of the original allocation.

Proposition 2, the second main result of the paper, shows that the logic just outlined does not extend to demographically disconnected economies.

Proposition 2. *(Kaldor-Hicks Improvements from Pareto Efficient Allocations) There exist feasible perturbations from Pareto efficient allocations in demographically disconnected economies that feature Kaldor-Hicks efficiency gains:* $\Xi^E > 0$.

To understand Proposition 2, it is useful to characterize the set of Pareto efficient allocations by solving the Pareto problem. Formally, this involves maximizing

$$\max_{\chi_t^i} \sum_i \alpha^i V^i,$$

where $\alpha^i > 0$ are Pareto weights, V^i is defined in (1), and $\chi_t^i = \frac{c_t^i}{c_t}$ denotes individual i's share of aggregate consumption c_t at date t. This problem is subject to resource constraints

$$\sum_i \chi_t^i = 1, \quad \forall t, \tag{9}$$

and consumption non-negativity constraints

$$\chi_t^i \geq 0, \quad \forall i, t. \tag{10}$$

Denoting by $\eta_t > 0$ the (normalized) Lagrange multiplier on the aggregate date-t resource constraint introduced in (9), the solution to the planning problem can be expressed as

$$\omega^i \omega_t^i = \eta_t \quad \text{if} \quad c_t^i > 0 \tag{11}$$

$$\omega^i \omega_t^i < \eta_t \quad \text{if} \quad c_t^i = 0, \tag{12}$$

[8] Pareto efficient allocations are feasible allocations — satisfying resource constraints and production or accumulation technologies — that solve the Pareto problem (Ljungqvist and Sargent, 2018).

where ω^i and ω^i_t are defined in (6) and (8).[9] Intuitively, the planner reallocates date-t consumption to equalize its social marginal value — given by $\omega^i\omega^i_t$ — across individuals alive at that date. Aggregating across dates the optimality conditions for individual i yields

$$\omega^i \underbrace{\sum_t \omega^i_t}_{=1} = \omega^i = \sum_t \eta_t \mathbb{I}\left[\omega^i_t > 0\right], \tag{13}$$

where $\mathbb{I}[\cdot]$ denotes the indicator function. Equation (13) implies that individual weights ω^i differ across individuals with different lifespans but are equalized among those with identical lifespans. Consequently, (11) implies that dynamic weights ω^i_t at a particular date also differ across individuals with different lifespans. This immediately implies that feasible perturbations — reallocating consumption at a given date — with $\Xi^E > 0$ exist, consistent with Proposition 2. By the same logic, dynamic weights ω^i_t are equalized across individuals with identical lifespans, so any efficiency gains generated by reallocating resources at Pareto efficient allocations must be due to demographic differences.

We summarize the implications of Proposition 2 in four remarks.

Remark 7. *(ω^i not Equalized: Demographic Pecking Order at Planning Solutions)* Because individual weights ω^i are not equalized across individuals after solving the Pareto problem, the planner has a pecking order over individuals tied to differences in lifespans. That is, the planner would rather assign a unit of the welfare numeraire to some individuals than to others. Equation (13) shows that lifespan is the main determinant of this ranking, giving more weight, all else equal, to longer-lived individuals, as illustrated in our applications. No such pecking order arises in economies with always-alive individuals or in economies with a connected welfare numeraire, where $\omega^i = 1$.[10]

Remark 8. *(ω^i_t not Equalized: Efficiency Gains from Pareto Efficient Allocations)* Because dynamic weights ω^i_t differ across individuals at specific dates at Pareto efficient allocations, reallocating consumption from individuals with low to high dynamic weights at a given date is a feasible perturbation that generates Kaldor-Hicks efficiency gains. At first

[9]While the full planning problem may include additional resource constraints or production and accumulation technologies, it is sufficient to consider (9) and (10) to establish our result.

[10]A welfare numeraire is *connected* when every individual assigns positive value to all of its components. Although we emphasize demographically disconnected economies, Proposition 2 holds whenever some individuals fail to value at least one component of the bundle that defines the welfare numeraire.

glance, it may appear paradoxical that the sum of individual gains and losses can be positive starting from a Pareto efficient allocation. This result is due to using as welfare numeraire a bundle whose elements, namely consumption at specific dates, are not valued by every individual. This is unavoidable in demographically disconnected economies — see Proposition 1 — but can be avoided in connected economies by choosing a welfare numeraire based on consumption at dates when all individuals are concurrently alive, a connected welfare numeraire. In fact, no feasible perturbation can generate efficiency gains starting from Pareto efficient allocations in economies with always-alive individuals or with a connected welfare numeraire.

Remark 9. *(Infeasible Compensating Transfers)* A perturbation with positive Kaldor-Hicks efficiency gains calls for reallocating resources from winners to losers to engineer a Pareto improvement. Yet the compensating transfers that would turn $\Xi^E > 0$ perturbations in Proposition 2 into a Pareto improvement are *not feasible*, even though the planner can freely reallocate consumption across all individuals at all times. Why? In demographically disconnected economies, implementing such transfers would require transferring perpetual consumption away from dead individuals, violating consumption non-negativity at dates when they already consume zero. Hence, while welfare gains can be expressed and aggregated in terms of perpetual consumption, consumption non-negativity constraints prevent perpetual consumption from actually being transferred away from some individuals.

Remark 10. *(Is Proposition 2 a Negative Result for Kaldor-Hicks Efficiency?)* One may interpret Proposition 2 as a negative result for Kaldor-Hicks efficiency, in the sense that it fails to serve as a statistic to identify potential Pareto improvements with transfers in demographically disconnected economies, unlike in standard settings. This is a valid interpretation that underscores the distinctive normative properties of demographically disconnected economies that we identify in this paper. Our preferred perspective, however, is more constructive: feasible Kaldor-Hicks improvements starting from Pareto efficient allocations reveal untapped sources of value in demographically disconnected economies that only government policies can capture, as illustrated in our applications.

3.3 Intertemporal-Sharing: Frictions vs. Demographics

Aggregate-Efficiency vs. Intertemporal-Sharing. In dynamic economies with heterogeneous individuals, the (Kaldor-Hicks) efficiency gains Ξ^E defined in (7) are due to either changes in discounted aggregate consumption — aggregate-efficiency, Ξ^{AE} — or reallocating consumption toward individuals who value it more at specific dates — intertemporal-sharing, Ξ^{IS}. Dávila and Schaab (2025) show how to separate both, as follows:

$$\Xi^E = \sum_i \sum_t \omega_t^i \frac{dV_t^{i|\lambda}}{d\theta} = \underbrace{\sum_t \omega_t \sum_i \frac{dV_t^{i|\lambda}}{d\theta}}_{\Xi^{AE} \text{ (Aggregate-Efficiency)}} + \underbrace{\sum_t \mathbb{C}ov^{\Sigma}_{i|\omega_t^i>0}\left[\omega_t^i, \frac{dV_t^{i|\lambda}}{d\theta}\right]}_{\Xi^{IS} \text{ (Intertemporal-Sharing)}}, \tag{14}$$

where $\omega_t = \frac{1}{I_t}\sum_i \omega_t^i$ defines an aggregate time discount factor with $I_t = \sum_i \mathbb{I}\{i \mid \omega_t^i > 0\}$, and where $\mathbb{C}ov^{\Sigma}_{i|\omega_t^i>0}[\cdot,\cdot] = I_t \cdot \mathbb{C}ov^{\Sigma}_{i|\omega_t^i>0}[\cdot,\cdot]$ denotes a cross-sectional covariance-sum among individuals alive at date t.[11]

In particular, intertemporal-sharing Ξ^{IS} reflects consumption reallocations toward individuals with higher dynamic weights ω_t^i at specific dates. In economies with always-alive individuals, cross-sectional differences in dynamic weights ω_t^i at a specific date can only stem from financial frictions. Absent such frictions, individuals' valuations are equalized, yielding $\omega_t^i = \omega_t$, $\forall i, t$, and $\Xi^{IS} = 0$. In demographically disconnected economies, however, differences in ω_t^i can arise even without financial frictions, as noted in Remark 8. Even when all financial arrangements are feasible, so intertemporal valuations are identical whenever any two individuals coexist, their valuations of date-t consumption relative to perpetual consumption typically differ because of lifespan differences, resulting in intertemporal-sharing gains. Example 2 illustrates this possibility.

Example 2. *(Illustrating Demographic-Driven Differences in ω_t^i)* Consider an economy with three-dates and two individuals, in which individual $i = 1$ is alive at dates 0, 1, and 2, and individual $i = 2$ is alive at dates 1 and 2. Both individuals have identical discount factor β and utility $u(\cdot)$ when alive. In this case, dynamic weights at date 1, ω_1^i, are given by

[11] This is the unique decomposition in which efficiency gains Ξ^E can be expressed as the discounted sum — using an aggregate discount factor — of aggregate gains at each date, Ξ^{AE}, and its complement, Ξ^{IS}.

$$\omega_1^1 = \frac{\beta u'\left(c_1^1\right)c_1}{u'\left(c_0^1\right)c_0 + \beta u'\left(c_1^1\right)c_1 + (\beta)^2 u'\left(c_2^1\right)c_2} = \overbrace{\frac{\beta u'\left(c_1^1\right)c_1}{u'\left(c_1^1\right)c_1 + (\beta)^2 u'\left(c_2^1\right)c_2}}^{\text{Frictional}} \times \overbrace{\frac{u'\left(c_1^1\right)c_1 + (\beta)^2 u'\left(c_2^1\right)c_2}{u'\left(c_0^1\right)c_0 + \beta u'\left(c_1^1\right)c_1 + (\beta)^2 u'\left(c_2^1\right)c_2}}^{\text{Demographic}} \tag{15}$$

$$\omega_1^2 = \frac{\beta u'\left(c_1^2\right)c_1}{\beta u'\left(c_1^2\right)c_1 + (\beta)^2 u'\left(c_2^2\right)c_2} = \underbrace{\frac{\beta u'\left(c_1^2\right)c_1}{\beta u'\left(c_1^2\right)c_1 + (\beta)^2 u'\left(c_2^2\right)c_2}}_{\text{Frictional}} \times \underbrace{1}_{\text{Demographic}}, \tag{16}$$

where the frictional/demographic labels are consistent with the definition in (18) below. While frictionless trade between dates 1 and 2 would ensure that the frictional terms in (15) and (16) are equalized, in general $\omega_1^1 \neq \omega_1^2$, so consumption reallocations at date 1 generate efficiency gains. This example illustrates that both financial frictions and demographics determine dynamic weights and, in turn, intertemporal-sharing gains.

Sources of Intertemporal-Sharing: Frictions vs. Demographics. Our third main result shows how to systematically separate the sources of intertemporal-sharing gains into two components. The first component, *frictional intertemporal-sharing* (FIS), captures gains from reallocating consumption toward individuals whose intertemporal valuations differ due to financial frictions, addressing inefficiencies that individuals could in principle correct via contracts or markets. The second component, *demographic intertemporal-sharing* (DIS), captures gains from reallocating consumption toward individuals whose intertemporal valuations differ due to demographic differences, where gains require policy intervention. This is, to our knowledge, the first decomposition to date that separates these sources.

Our approach is based on three steps.

Step #1: Cross-Sectional to Pairwise Covariances. First, we rewrite the cross-sectional covariance-sum over all individuals at date t as a sum of pairwise covariance-sums. To do so, we define the set of pairs of individuals alive at date t by $\mathcal{A}_t = \{(i,j) \mid \{i,j\} \text{ are alive at } t\}$. Then, the date-$t$ intertemporal-sharing component can be written as

$$\underbrace{\mathbb{C}ov^{\Sigma}_{i|\omega_t^i>0}\left[\omega_t^i, \frac{dV_t^{i|\lambda}}{d\theta}\right]}_{\text{Cross-Sectional Covariance at } t} = \underbrace{\frac{1}{I_t}\sum_{(i,j)\in\mathcal{A}_t} \mathbb{C}ov^{\Sigma}_{(i,j)\in\mathcal{A}_t}\left[\omega_t^{(i,j)}, \frac{dV_t^{(i,j)|\lambda}}{d\theta}\right]}_{\text{Sum of Pairwise Covariances at } t}, \tag{17}$$

where $\mathbb{C}ov^{\Sigma}_{(i,j)\in\mathcal{A}_t}[\cdot,\cdot]$ denotes a pairwise-covariance-sum (with just two elements).

Step #2: Multiplicative Decomposition of Pairwise Weights. Second, we multiplicatively decompose the dynamic weight of individual i in the pair (i,j) at date t into a frictional dynamic weight and a demographic dynamic weight, as follows:

$$\underbrace{\omega_t^{i,(i,j)}}_{\substack{\text{Pairwise Dynamic}\\ \text{Weight}}} = \underbrace{\omega_t^{i,(i,j)|f}}_{\text{Frictional}} \underbrace{\omega^{i,(i,j)|d}}_{\text{Demographic}}, \tag{18}$$

where

$$\omega_t^{i,(i,j)|f} = \frac{\left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i}{\partial c_t^i} c_t}{\sum_{t|\omega_t^i,\omega_t^j>0} \left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i}{\partial c_t^i} c_t} \qquad \text{(Frictional-Dynamic Weight)}$$

$$\omega^{i,(i,j)|d} = \frac{\sum_{t|\omega_t^i,\omega_t^j>0} \left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i}{\partial c_t^i} c_t}{\sum_t \left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i}{\partial c_t^i} c_t}. \qquad \text{(Demographic-Dynamic Weight)}$$

Individual i's frictional dynamic weight captures i's valuation for date-t consumption relative to the dates when *both* individuals i and j are concurrently alive. The demographic dynamic weight reflects i's valuation for consumption during these overlapping dates relative to perpetual consumption. It is, by construction, time-independent.[12] Without financing frictions, individuals i and j equalize marginal valuations over their common lifespans, implying $\omega_t^{i,(i,j)|f} = \omega_t^{j,(i,j)|f}$, motivating the "frictional" label. By contrast, demographic differences typically imply $\omega^{i,(i,j)|d} \neq \omega^{j,(i,j)|d}$ even under frictionlessly intertemporal trade, justifying the "demographic" label. When any two individuals overlap at a single date — as in Applications 1 and 2 — the dynamic weight is purely demographic, so $\omega_t^{i,(i,j)|f} = 1$.

[12] Heuristically, Equation (18) can be described as

$$\omega_t^{i,(i,j)} = \underbrace{\frac{i's \text{ date-}t \text{ value}}{i'\text{s value over overlapping dates } (i,j)}}_{\omega_t^{i,(i,j)|f} \text{ (Frictional)}} \times \underbrace{\frac{i'\text{s value over overlapping dates } (i,j)}{i'\text{s lifetime value}}}_{\omega^{i,(i,j)|d} \text{ (Demographic)}},$$

where numerators and denominator capture marginal valuations over the appropriate dates.

Step #3: Formula for Covariance of a Product. Finally, we decompose the pairwise covariances in (17) into contributions due to cross-sectional variation in the frictional and demographic components, using Bohrnstedt and Goldberger (1969)' formula for covariances of products. This is the third main result of the paper, which we present in Proposition 3.

Proposition 3. *(Sources of Intertemporal-Sharing: Frictional Intertemporal-Sharing vs. Demographic Intertemporal-Sharing) Intertemporal-sharing gains can be decomposed into frictional and demographic components, as follows:*

$$\Xi^{IS} = \underbrace{\sum_t \frac{1}{I_t} \sum_{(i,j)\in\mathcal{A}_t} \mathbb{C}ov^{\Sigma}_{(i,j)\in\mathcal{A}_t}\left[\omega_t^{i,(i,j)|f}, \frac{dV_t^{(i,j)|\lambda}}{d\theta}\right]}_{\Xi^{FIS} \text{ (Frictional Intertemporal-Sharing)}} + \underbrace{\sum_t \frac{1}{I_t} \sum_{(i,j)\in\mathcal{A}_t} \mathbb{C}ov^{\Sigma}_{(i,j)\in\mathcal{A}_t}\left[\omega^{i,(i,j)|d}, \frac{dV_t^{(i,j)|\lambda}}{d\theta}\right]}_{\Xi^{DIS} \text{ (Demographic Intertemporal-Sharing)}}, \tag{19}$$

where $\mathbb{C}ov^{\Sigma}_{(i,j)\in\mathcal{A}_t}[\cdot,\cdot]$ *denotes a pairwise-covariance-sum.*

This decomposition identifies the distinct sources of intertemporal-sharing gains. The frictional component, Ξ^{FIS}, captures gains attributable to cross-sectional valuation over dates in which individuals are concurrently alive. These gains could materialize through improved financial arrangements between individuals. The demographic component, Ξ^{DIS}, reflects valuation differences arising purely from demographics. These gains *cannot* be realized by rearranging consumption between any two individuals. By separating these two forces, Proposition 3 identifies whether gains from specific reallocations stem from correctable financial frictions or demographic features that inherently call for policy.

Properties. Proposition 4 presents properties of the intertemporal-sharing decomposition. Application 3 illustrates the use of the decomposition in a fully-specified scenario.

Proposition 4. *(Properties of Intertemporal-Sharing Decomposition)*

a) (Financial Frictions) When any pair of individuals alive at any date t have different valuations over consumption at the dates when they are concurrently alive, which occurs when there are financial frictions, then $\Xi^{FIS} \neq 0$.

b) (Demographic Differences) When all individuals have different birth and death dates, the demographic component is generically non-zero, even when there are no financial frictions, so $\Xi^{DIS} \neq 0$.

c) (Frictionless Borrowing/Saving) When any pair of individuals alive at any date t have identical valuations over consumption at the dates when they are concurrently alive — a condition satisfied when there are no financial frictions — then $\Xi^{FIS} = 0$ and $\Xi^{IS} = \Xi^{DIS}$.

d) (Common Demographics) When all individuals have identical birth and death dates, then $\Xi^{DIS} = 0$ and $\Xi^{IS} = \Xi^{FIS}$.

e) (Frictionless Borrowing/Saving and Common Demographics) When any pair of individuals alive at any date t have identical valuations over consumption at the dates when they are concurrently alive — a condition satisfied when there are no financial frictions — and all individuals have identical birth and death dates, then $\Xi^{IS} = \Xi^{FIS} = \Xi^{DIS} = 0$.

Proposition 4 establishes key properties of the intertemporal-sharing decomposition. Part a) establishes that financial frictions lead to a non-zero frictional component Ξ^{FIS}, while Part c) concludes that frictional gains vanish under frictionless borrowing and saving. Part b) establishes that demographic differences lead to a non-zero demographic component Ξ^{DIS}, while Part d) concludes that demographic gains vanish under common demographics. Lastly, when financial arrangements are frictionless and all individuals share identical lifespans, there are no intertemporal-sharing gains of any kind, so the total, frictional, and demographic components all vanish, and $\Xi^{IS} = \Xi^{FIS} = \Xi^{DIS} = 0$.

4 Application 1: Social Security

We now present three applications that demonstrate the practical relevance of our results. The first application assesses the welfare implications of intergenerational transfers in the simplest OLG economy: the two-date-life version of Samuelson (1958)'s endowment economy. The main takeaway from this application is that a policy of young-to-old transfers — designed to mirror pay-as-you-go social security — is a Kaldor-Hicks improvement over the unique competitive equilibrium, which is Pareto efficient. Moreover, we show that all efficiency gains are solely due to demographic intertemporal-sharing, that is, $\Xi^E = \Xi^{IS} = \Xi^{DIS} > 0$.

Our analysis provides a novel justification for social security on the grounds that the sum of the willingness-to-pay for it in units of perpetual aggregate consumption among all individuals from all generations is strictly positive, even without constituting a Pareto improvement. This application illustrates Propositions 1 through 3, as it uses perpetual aggregate consumption as welfare numeraire, shows that social security generates Kaldor-Hicks efficiency gains from a Pareto efficient allocation, and establishes that such gains are solely due to demographics, as markets are complete and there are no financial frictions.

4.1 Environment, Equilibrium, and Policy

We consider a single-good endowment economy with dates $t \in \{0, \ldots, T\}$, where $T < \infty$. A single individual i is born at each date, where $i \in \{-1, \ldots, T\}$. Since individuals are uniquely associated with a date of birth, we also use t — as a superscript — to index individuals. Individuals born at dates $t \in \{0, \ldots, T-1\}$ are alive at two dates, first as "young" and then as "old". Individuals born at dates $t \in \{-1, T\}$ are alive at one date: these are the "initial old" and the "terminal young", respectively.[13]

Preferences and Endowments. The lifetime utility of an individual born at date $t \in \{0, \ldots, T-1\}$ is

$$V^t = u\left(c_t^t\right) + \beta u\left(c_{t+1}^t\right), \tag{20}$$

where c_t^t and c_{t+1}^t denote individual t's consumption at dates t and $t+1$, respectively. The lifetime utilities of individuals born at dates $t \in \{-1, T\}$ are given by

$$V^{-1} = \beta u\left(c_0^{-1}\right) \qquad \text{and} \qquad V^T = u\left(c_T^T\right), \tag{21}$$

respectively. Individuals have endowments of the consumption good, denoted $\left\{e_t^t, e_{t+1}^t\right\}$ for individuals $t \in \{0, \ldots, T-1\}$, as well as e_0^{-1} and e_T^T for individuals $t \in \{-1, T\}$, respectively.

Competitive Equilibrium. To highlight the absence of financial frictions, we consider an equilibrium with once-and-for-all trading. Equivalently, individuals could trade a risk-

[13] All applications omit population growth for simplicity; adding it is straightforward.

free asset at all dates. The budget constraint for individuals $t \in \{0, \dots, T-1\}$ is

$$p_t c_t^t + p_{t+1} c_{t+1}^t = p_t e_t^t + p_{t+1} e_{t+1}^t, \tag{22}$$

where p_t denotes the price of date-t consumption. The budget constraints for individuals $t \in \{-1, T\}$ are

$$p_0 c_0^{-1} = p_0 e_0^{-1} \qquad \text{and} \qquad p_T c_T^T = p_T e_T^T, \tag{23}$$

respectively. The resource constraint for date-t consumption is

$$c_t^{t-1} + c_t^t = e_t^{t-1} + e_t^t. \tag{24}$$

The equilibrium definition is standard, and presented in the Appendix. The unique competitive equilibrium of this economy is autarky: $c_t^i = e_t^i$. The supporting prices (interest rates) satisfy

$$\frac{p_{t+1}}{p_t} = \beta \frac{u'\left(e_{t+1}^t\right)}{u'\left(e_t^t\right)}, \tag{25}$$

normalizing $p_0 = 1$ when needed. Intuitively, young and old do not trade because the old are no longer alive in the future and there is a terminal date. Since $T < \infty$, this is an Arrow–Debreu economy, so the first welfare theorem applies and the competitive equilibrium is Pareto efficient.

Shares, Growth, and Young-to-Old Transfer. It is useful to formulate the model in terms of shares. At date t, individual i's consumption and endowment shares are $\chi_{t,c}^i = \frac{c_t^i}{c_t}$ and $\chi_{t,e}^i = \frac{e_t^i}{e_t}$, where aggregate consumption and endowment are $c_t = c_t^{t-1} + c_t^t$ and $e_t = e_t^{t-1} + e_t^t$. By construction, shares add up to 1.

The aggregate endowment — and, in equilibrium, aggregate consumption — grows at a constant rate g, so $e_t = (1+g)^t e_0$. We model pay-as-you-go social security as a young-to-old (YTO) transfer. Starting from baseline endowment shares $\bar{\chi}_{t,e}^t$ and $\bar{\chi}_{t,e}^{t-1}$, a policy indexed by a perturbation parameter $\theta \geq 0$ reallocates endowments from young to old individuals at each date according to

(Young-to-Old Transfer) $$\chi_{t,e}^t = \bar{\chi}_{t,e}^t - \theta \qquad \text{and} \qquad \chi_{t,e}^{t-1} = \bar{\chi}_{t,e}^{t-1} + \theta.$$

Calibration. Preferences are $u(c) = \log(c)$. One date in the model corresponds to 25 years, so $\beta = (0.98)^{25} = 0.60$. We normalize the date-0 aggregate endowment to $e_0 = 1$, and set baseline endowment shares to

$$\text{Young:} \quad \bar{\chi}_{t,e}^{t} = 0.75 \qquad \text{and} \qquad \text{Old:} \quad \bar{\chi}_{t,e}^{t-1} = 0.25,$$

so young individuals consume more than old in the absence of policy. The economy grows at a rate $g = 0.64 = (1.02)^{25} - 1$, implying 2% annual growth. For ease of visualization, we assume that $T = 5$, so the economy runs for 125 years.

4.2 Welfare Assessments and Weights

Welfare Assessments. We study welfare assessments for utilitarian planners, for whom welfare assessments satisfy

$$\frac{dW}{d\theta} = \sum_{t=-1}^{T} \alpha^t \frac{dV^t(\theta)}{d\theta}, \tag{26}$$

where α^t is the Pareto weight associated with the individual born at date t. We consider two cases: i) an undiscounted utilitarian planner (our benchmark), who assigns equal weight to all individuals' lifetime utilities, so $\alpha^t = 1$, and ii) a discounted utilitarian planner, who exponentially discounts the utilities of future generations, with $\alpha^t = (\bar{\alpha})^t$ for a constant $\bar{\alpha} \in (0, 1)$, which we take to be $\bar{\alpha} = \beta$, the individuals' discount factor.

We use perpetual aggregate consumption — the bundle of aggregate consumption at each date — as the welfare numeraire. The normalizing factor λ^t, defined in (5), for individuals $t \in \{0, \ldots, T-1\}$ is therefore

$$\lambda^t = u'\left(c_t^t\right) c_t + \beta u'\left(c_{t+1}^t\right) c_{t+1}, \tag{27}$$

as well as $\lambda^{-1} = \beta u'\left(c_0^{-1}\right) c_0$ and $\lambda^T = u'\left(c_T^T\right) c_T$ for individuals $t \in \{-1, T\}$, where c_t denotes aggregate consumption at date t.

Individual Weights. Individual weights uncover the value a planner assigns to the welfare gains of different individuals in units of perpetual aggregate consumption. That is, a planner distributing one unit of perpetual aggregate consumption would assign it to the individual whose weight is greatest. Policies that reallocate perpetual aggregate

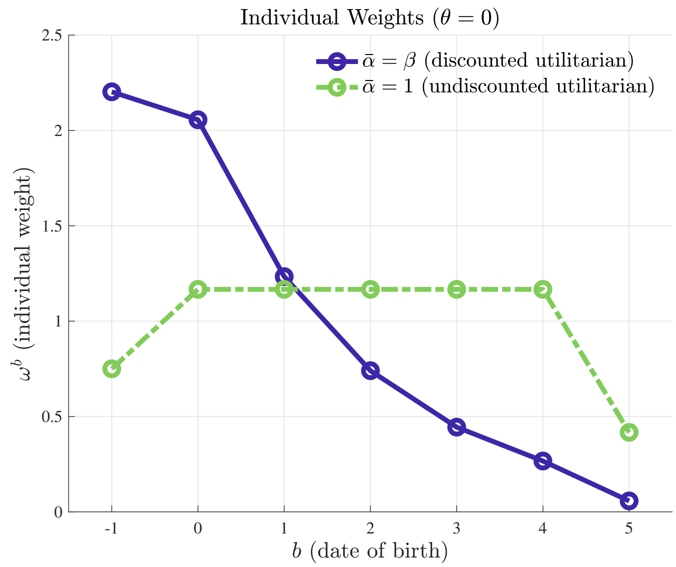


Figure 1: Individual Weights (Application 1)

Note. This figure shows individual weights as a function of an individual's date of birth for the undiscounted (dashed green line) and discounted (solid blue line) utilitarian planners in the absence of policy ($\theta = 0$).

consumption from those with low weights to those with high weights yield redistribution gains.

Individual weights, defined in (6), are given by

$$\omega^t = \frac{\alpha^t \lambda^t}{\frac{1}{I}\sum_{t=-1}^{T} \alpha^t \lambda^t}, \tag{28}$$

where λ^t is defined in (27) and where $I = T + 2$. Individual weights are thus shaped by i) marginal utilities of perpetual aggregate consumption and ii) Pareto weights. In general, the former depend on individual lifespans, discount factors, and aggregate and individual consumption levels. With log utility, we have that

$$u'\left(c_t^t\right) c_t = \left(\chi_{t,c}^t\right)^{-1},$$

so λ^t depends inversely on individual consumption shares.[14]

[14] With isoelastic utility and EIS denoted by ψ, individual t's marginal utility of aggregate consumption at date t is

$$u'\left(c_t^t\right) c_t = \left(\chi_{t,c}^t\right)^{-\frac{1}{\psi}} \left(c_t\right)^{1-\frac{1}{\psi}},$$

which depends on aggregate consumption in addition to consumption shares. An increase in aggregate consumption c_t holding the consumption share $\chi_{t,c}^t$ fixed, has two effects: a direct increase in consumption (substitution effect), and reduction in the value of consumption (income/valuation effect), with the net effect captured by $1 - \frac{1}{\psi}$. When $\psi > (<)\, 1$, individuals have a high (low) willingness to substitute

Figure 1 displays individual weights by birth date in the absence of social security ($\theta = 0$). The undiscounted utilitarian planner assigns lower weights to shorter-lived individuals $t \in \{-1, T\}$ and higher weights to longer-lived individuals, because they enjoy consumption at more dates. The initial-old has a higher individual weight than the terminal young, reflecting the higher consumption share of the young in our calibration. For the discounted utilitarian planner ($\bar{\alpha} = \beta < 1$), the same forces remain, but individual weights now decline with birth date because later generations are discounted more heavily. As a result, the undiscounted utilitarian planner values gains for longer-lived individuals the most, with the initial old next and the terminal young last. The discounted utilitarian planner, in contrast, simply favors earlier-born generations.

Dynamic Weights. Dynamic weights define individual marginal rates of substitution between date-t and perpetual aggregate consumption, revealing whose consumption the planner values more at each date. Reallocating consumption from individuals with lower to higher dynamic weights at a given date generates intertemporal-sharing gains.

For individuals $t \in \{0, \ldots, T-1\}$, dynamic weights, defined in (8), are given by

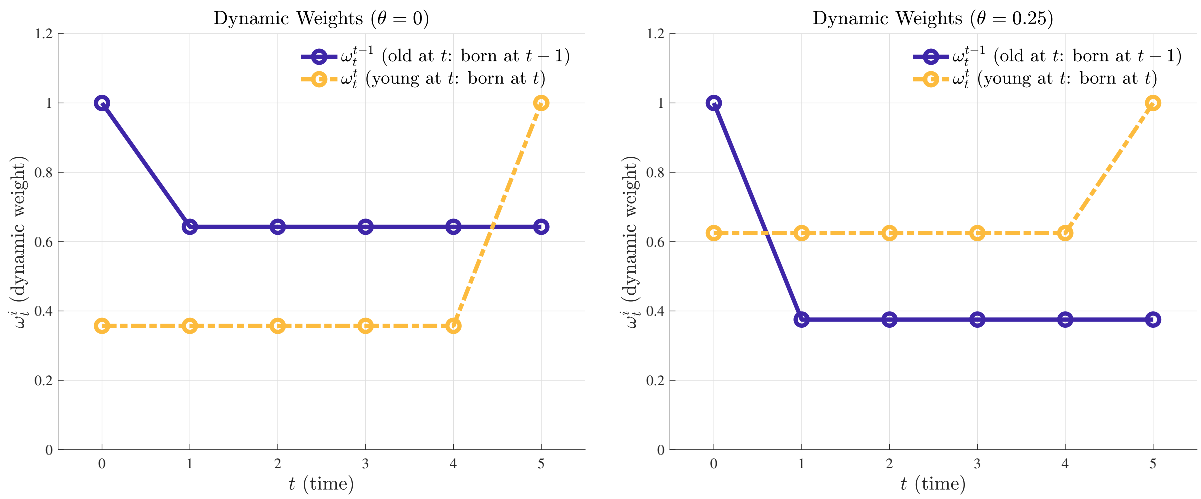
$$\omega_t^t = \frac{u'\left(c_t^t\right) c_t}{u'\left(c_t^t\right) c_t + \beta u'\left(c_{t+1}^t\right) c_{t+1}} \quad \text{and} \quad \omega_{t+1}^t = \frac{\beta u'\left(c_{t+1}^t\right) c_{t+1}}{u'\left(c_t^t\right) c_t + \beta u'\left(c_{t+1}^t\right) c_{t+1}}. \tag{29}$$

For individuals alive at only one date, $t \in \{-1, T\}$, dynamic weights are $\omega_0^{-1} = \omega_T^T = 1$, as they are willing to exchange one unit of perpetual aggregate consumption for one unit of consumption when alive.

The left panel of Figure 2 displays dynamic weights in the absence of policy ($\theta = 0$). In this case, old individuals have a higher dynamic weight than young individuals whenever both live for two dates, reflecting the higher consumption share of the young in our calibration. Therefore, reallocating consumption from young to old at, say, date $t = 2$ generates intertemporal-sharing gains because $\omega_2^1 = 0.64 > 0.36 = \omega_2^2$. This pattern holds at all dates except the terminal one.

The right panel of Figure 2 displays dynamic weights after implementing the welfare-maximizing transfer for the undiscounted utilitarian planner ($\theta = 0.25$). In this case, young individuals have a higher dynamic weight than old individuals whenever both live

intertemporally, so the first (second) effect dominates. With growth, these effects shape both individual and dynamic weights, even when consumption shares are constant.



(a) No transfer ($\theta = 0$) (b) Positive transfer ($\theta = 0.25$)

Figure 2: Dynamic Weights (Application 1)

Note. The left panel shows dynamic weights in the absence of policy ($\theta = 0$), so $\chi_{t,c}^{t} = 0.75$ and $\chi_{t,c}^{t-1} = 0.25$. The right panel shows dynamic weights after implementing the welfare-maximizing transfer for the undiscounted utilitarian planner ($\theta = 0.25$, so $\chi_{t,c}^{t} = \chi_{t,c}^{t-1} = \frac{1}{2}$. The dashed yellow lines display the dynamic weights of young individuals and the solid blue lines display the dynamic weights of old individuals at each date t (horizontal axis).

for two dates. While consumption shares are now equal, as every individual consumes one-half of the aggregate endowment, consumption when old is valued less due to time discounting ($\beta < 1$). As a result, a young-to-old transfer initially generates efficiency gains, but only up to a point. The shape of the dynamic weights described here explains the pattern of intertemporal-sharing gains shown in Figure 4.

4.3 Policy Experiment: Young-to-Old Transfer

Individual Welfare Gains. To understand the aggregate welfare implications of the young-to-old (YTO) transfer, it is helpful to first examine its impact on each individual. Figure 3 displays marginal welfare gains, expressed in units of perpetual aggregate consumption, for every individual as a function of the YTO transfer. The initial old always benefits from the policy while the terminal young is worse off, simply because their consumption increases and decreases, respectively. All other individuals initially benefit from the policy, as it improves consumption smoothing, but only up to $\theta = 0.125$.

Four takeaways emerge from Figure 3. First, a YTO transfer is not a Pareto improve-

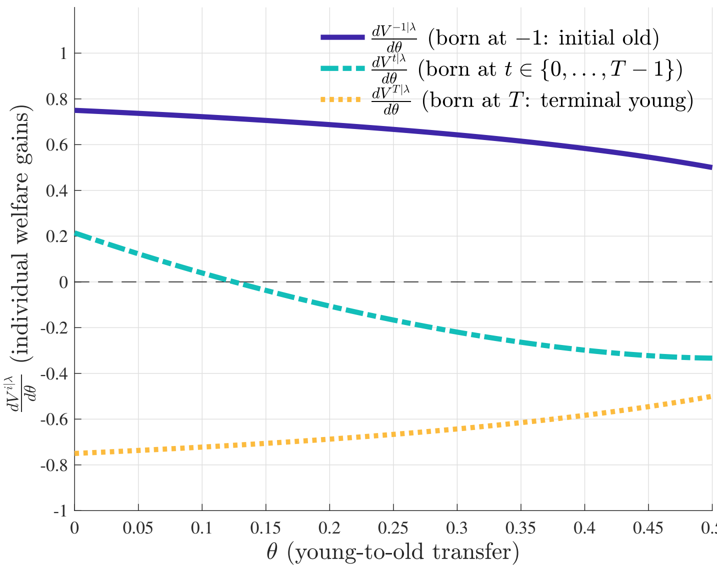


Figure 3: Individual Welfare Gains (Application 1)

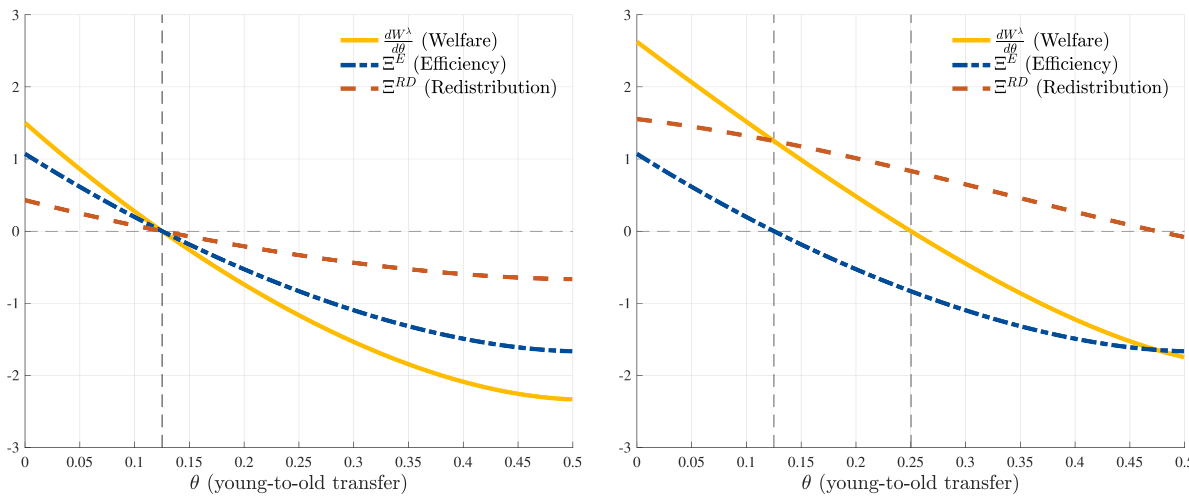
Note. This figure shows normalized welfare gains, defined in (6), for different individuals as a function of the young-to-old transfer. The dashed green line corresponds to the normalized individual welfare gains of all individuals with two-date-lives. The solid blue line represents the gains of the initial old born at date -1. The dotted yellow line corresponds to the gains of the terminal young born at date T.

ment: the terminal young is always worse off. Achieving a Pareto improvement would require compensating terminal young, but individuals born at dates $\{-1, \ldots, T-2\}$ already consume zero at the terminal date, and transfers from the terminal old to the terminal young would eliminate smoothing gains for all. Ideally, one would transfer resources from the initial old to the terminal young, but this economy lacks a technology to do so.[15]

Second, while the YTO transfer can never constitute a Pareto improvement for any finite T, its aggregate gains grow with T, as more generations benefit from smoother consumption while the losses to the terminal young remain fixed. It is well-understood that if $T = \infty$, the terminal young "disappear", enabling a Pareto improvement.

Third, the consumption smoothing gains identified here, which underpin the efficiency gains in Figure 4, would disappear if individual utilities were linear in consumption, as in Samuelson (1958). This result further differentiates our results from double-infinite arguments, which also apply with linear utilities.

[15] Introducing a linear storage technology fundamentally changes both the characterization of the competitive equilibrium and the assessment of policies. Detailed results for this case are available upon request. Application 2 considers instead the more realistic case of neoclassical accumulation.



(a) Undiscounted Utilitarian

(b) Discounted utilitarian

Figure 4: Welfare Assessment and Sources of Welfare Gains (Application 1)

Note. This figure shows aggregate welfare gains as a function of the YTO transfer, decomposed into efficiency and redistribution components, defined in (7). The left panel shows the welfare assessment of an undiscounted utilitarian planner, with $\bar{\alpha} = 1$, and the right panel shows the welfare assessment of a discounted utilitarian planner, with $\bar{\alpha} = \beta$. Since this is an endowment economy with fixed aggregate consumption, all efficiency gains arise from intertemporal-sharing, so $\Xi^E = \Xi^{IS}$. Because individuals overlap for a single date, these intertemporal-sharing gains are entirely demographic, so $\Xi^{IS} = \Xi^{DIS}$.

Finally, we would like to highlight that our contribution relative to existing work lies in the ability to express individual welfare gains in a common unit — perpetual aggregate consumption, leveraging Proposition 1. This allows us to add up individual gains to make aggregate assessments, as we do next.

Welfare Assessment and Sources of Welfare Gains. Figure 4 displays aggregate welfare gains as a function of the YTO transfer, decomposed into efficiency and redistribution components, as in (7). Because this is an endowment economy with fixed aggregate consumption, all efficiency gains arise from intertemporal-sharing. And since individuals overlap for a single date, Proposition 3 implies that these intertemporal-sharing gains are entirely demographic. Therefore, $\Xi^E = \Xi^{IS} = \Xi^{DIS}$.

A small YTO transfer generates both efficiency and redistribution gains. Adding up the individual welfare gains shown in Figure 3 yields the efficiency gains Ξ^E displayed in Figure 4. The gain to the initial old exactly offsets the loss to the terminal young, so the overall efficiency gains reflect the consumption-smoothing improvements for individuals

$t \in \{0, \ldots, T-1\}$.[16] Quantitatively, $\Xi^E\Big|_{\theta=0} \approx -0.8 + 0.2 \times 5 + 0.8 = 1$, so a back-of-the-envelope calculation implies that the efficiency gains from a YTO transfer $\theta = 0.125$ are equivalent to a $1 \times 0.125 \approx 12.5\%$ increase in perpetual aggregate consumption equally distributed across all individuals. Because efficiency gains are independent of the social welfare function, $\theta = 0.125$ is the efficiency-maximizing transfer for *any* welfarist planner.

A small YTO transfer also generates redistribution gains for both discounted and undiscounted utilitarian planners. The undiscounted utilitarian planner assigns the lowest individual weight to the terminal young, so the YTO transfer, which relatively favors all other individuals, yields positive redistribution gains. For this planner, redistribution and efficiency components are coincidentally maximized at a YTO transfer $\theta = 0.125$. A discounted utilitarian planner assigns an even lower individual weight to the terminal young, thus finding larger redistribution gains and larger YTO transfer $\theta = 0.25$ optimal.

This application shows that a pay-as-you-go social security policy generates Kaldor-Hicks efficiency gains over the unique competitive equilibrium, which is Pareto efficient. It thus offers a concrete illustration of Proposition 2 applied to one of the foundational OLG models. This phenomenon arises because the economy is demographically disconnected, making it infeasible for the winners to compensate the losers in units of perpetual aggregate consumption despite the aggregate gain in value when expressed in that unit. While the social security policy that we study does not constitute a Pareto improvement, our analysis provides a normative justification for a limited level of social security, as the aggregate willingness-to-pay — expressed in units of perpetual aggregate consumption — by all individuals from all generations is strictly positive.

5 Application 2: Capital Taxation

This application analyzes the welfare consequences of taxing capital in the finite-horizon Diamond (1965)'s overlapping-generations growth model — the canonical OLG economy with capital. We show that capital taxes or subsidies can generate Kaldor-Hicks efficiency

[16] The sources of welfare gains shift discontinuously when $T = \infty$. When $T < \infty$, aggregate consumption is fixed, so efficiency gains arise only from intertemporal-sharing. When $T = \infty$, the YTO transfer also increases aggregate consumption at infinity, yielding aggregate efficiency gains. Thus gains in double-infinite economies can be due to i) intertemporal-sharing, if individuals value consumption-smoothing, and ii) aggregate-efficiency, even with linear utilities.

gains in this economy, even though the competitive equilibrium is Pareto efficient.

The main takeaway from this application is that the efficiency-maximizing capital tax is positive in low-capital-share economies but negative (i.e. a subsidy) in high-capital-share economies. This illustrates that it is possible to perturb Pareto efficient allocations in a production economy to generate efficiency gains, a new instance of Proposition 2. More broadly, this application illustrates how the results of the paper can be used to define notions of over- and under-accumulation of capital in economies that are Pareto optimal.

5.1 Environment, Equilibrium, and Policy

We consider an economy with dates $t \in \{0, \ldots, T\}$, where $T < \infty$. A single individual i is born at each date, where $i \in \{-1, \ldots, T\}$. As in Application 1, since individuals are uniquely associated with a date of birth, we also use t — as a superscript — to index individuals. Individuals born at dates $t \in \{0, \ldots, T-1\}$ are alive at two dates, first as "young" and then as "old". Individuals born at dates $t \in \{-1, T\}$ are alive at one date: these are the "initial old" and the "terminal young", respectively.

Individuals. The lifetime utilities of all individuals are the same as in Application 1: see (20) and (21). When young, individuals supply one unit of labor inelastically and decide how much to consume (c_t^t) and save. The only savings vehicle available is capital. Young individuals can either purchase k_t units of capital on the secondary market at a price q_t or invest ι_t in new capital. The budget constraint of a young individual at date t is

$$c_t^t + \iota_t + q_t k_t = w_t, \tag{30}$$

where w_t denotes the wage. Capital accumulates neoclassically according to

$$k_{t+1} = (1 - \delta)\, k_t + \iota_t, \tag{31}$$

where δ is the depreciation rate.

When old, individuals no longer work. They rent out the capital they have accumulated to firms and subsequently sell it on the secondary market. The budget constraint of an old individual at date $t+1$ is therefore given by

$$c_{t+1}^{t} = (1-\tau)\left(q_{t+1}+d_{t+1}\right)k_{t+1} + T_{t+1}, \tag{32}$$

where d_{t+1} is the rental rate of capital, τ is the capital tax, and T_{t+1} is a lump-sum rebate.

Firms. At each date t, a representative firm produces y_t units of the consumption good with a constant-returns production function $y_t = f\left(k_t, n_t\right)$ that uses capital and labor. The firm choose inputs to maximize profits: $y_t - w_t n_t - d_t k_t$.

Government. The government budget must balance at each date. Revenues from the tax on capital are rebated lump-sum to the old individuals, which requires $T_t = \tau\left(q_t + d_t\right)k_t$.

Competitive Equilibrium. The equilibrium definition is standard, and presented in the Appendix. In equilibrium, young individuals must be indifferent between acquiring capital via new investment on purchasing it on the secondary market. The secondary market price must therefore satisfy $q_t = 1-\delta$ for dates $t \leq T-1$. At the final date, capital has no productive use, so $q_T = 0$. Firm optimization requires $\frac{\partial f}{\partial k_t} = d_t$ and $\frac{\partial f}{\partial n_t} = w_t$.

Calibration. Preferences are $u\left(c\right) = \log\left(c\right)$. One date in the model corresponds to 25 years, so $\beta = \left(0.98\right)^{25} = 0.60$. Consistent with this choice, we set $\delta = 1$, so capital depreciates fully between dates. The production technology is Cobb-Douglas, $f\left(k,n\right) = k^a n^{1-a}$, where a maps to the capital share. We contrast two calibrations with different values of a:

$$a = \begin{cases} 0.2 & \Rightarrow \text{low-capital-share/high-labor-share} \\ 0.4 & \Rightarrow \text{high-capital-share/low-labor-share.} \end{cases}$$

We assume that $T = 10$, so the economy runs for 250 years. The initial capital stock is set to $k_{-1} = 0.01$, so this economy initially undergoes a transition phase with growth before converging to a steady state under both calibrations.

Equilibrium Characterization. In the Appendix, we display the equilibrium paths under the two calibrations. A *higher* capital share a leads to *lower* capital accumulation, both in the steady state and along the transition. This occurs because, as a increases, a larger share of output accrues to the old as capital income and a smaller share to the young as labor income. Because only the young save, this reduction in labor income depresses aggregate savings, lowering investment and capital accumulation:

$$a \uparrow \quad \Rightarrow \quad \text{labor share} \downarrow \quad \Rightarrow \quad \text{savings} \downarrow \quad \Rightarrow \quad \text{investment} \downarrow \quad \Rightarrow \quad k \downarrow \ .$$

5.2 Welfare Assessments and Weights

Welfare Assessments. We study welfare assessments for utilitarian planners, for whom the welfare assessment of a change in the capital tax rate $d\tau$ is given by

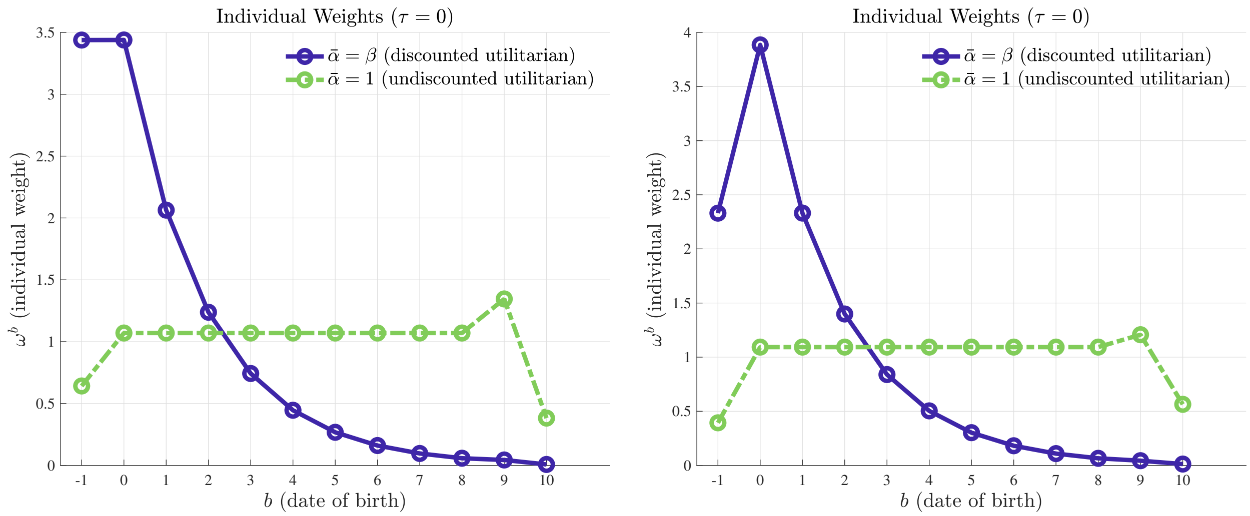
$$\frac{dW}{d\tau} = \sum_{t=-1}^{T} \alpha^t \frac{dV^t(\tau)}{d\tau}, \tag{33}$$

where α^t denotes the Pareto weight associated with the individual born at date t. As in Application 1, we focus on undiscounted and discounted utilitarian planners. We also use perpetual aggregate consumption as welfare numeraire, so (27) and its counterpart for individuals $t \in \{-1, T\}$ apply here unchanged.

Individual Weights. Individual weights uncover the value assigned by a planner to the welfare gains of different individuals in units of the perpetual aggregate consumption. They are defined exactly as in (28).

Figure 5 displays individual weights by birth date in the absence of the capital tax ($\tau = 0$). Qualitatively, individual weights are similar in high- and low-capital-share economies. Shorter-lived individuals have lower individual weights, all else equal, as in Application 1. A discounted utilitarian planner assigns lower weights to individuals born later because of discounting. In contrast to Application 1, individuals born at $T-1$ have higher individual weights than other long-lived individuals since they have a lower consumption share when old at date T, because the date-T young do not invest.

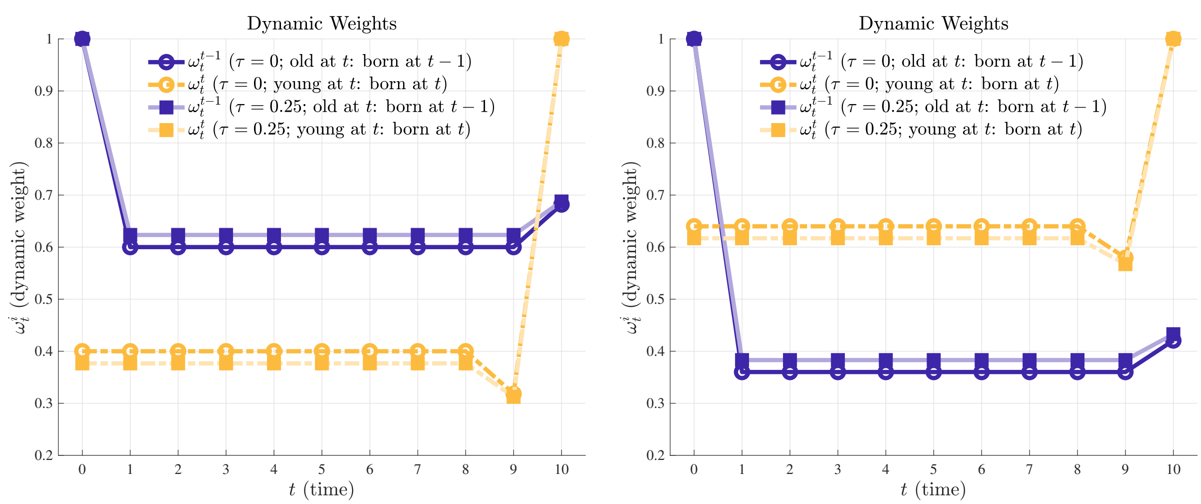
Dynamic Weights. Dynamic weights — defined as in (27) — define marginal rates of substitution between date-t and perpetual aggregate consumption for each individual.



(a) Low-capital-share (b) High-capital-share

Figure 5: Individual Weights (Application 2)

Note. This figure shows individual weights as a function of an individual's date of birth for the undiscounted (dashed green line) and discounted (solid blue line) utilitarian planners when $\tau = 0$ for the low-capital-share economy ($a = 0.2$, left panel) and the high-capital-share economy ($a = 0.4$, right panel).



(a) Low-capital-share (b) High-capital-share

Figure 6: Dynamic Weights (Application 2)

Note. The figure shows dynamic weights for the low-capital-share economy ($a = 0.2$) in the left panel and the high-capital-share economy ($a = 0.4$) in the right panel, in the absence of a capital tax ($\tau = 0$) and after the efficiency-maximizing capital tax for the low-capital-share economy is implemented ($\tau = 0.25$).

The left panel of Figure 6 shows the dynamic weights for the low-capital-share economy ($a = 0.2$). In the absence of a capital tax ($\tau = 0$), old individuals have a higher dynamic weight than young individuals whenever both have two-date-lives. This reflects that the labor share is high in this calibration, so young individuals, who earn labor income, consume more than old individuals. As the capital tax increases, young individuals save and invest less, consuming more when young. This widens the consumption gap between the young and the old, further increasing the gap in dynamic weights.

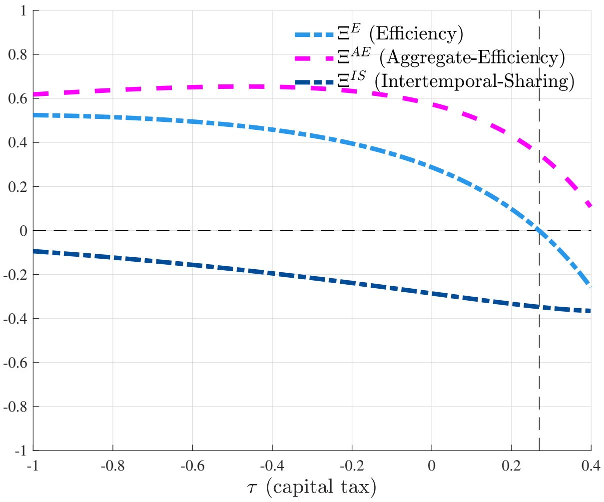
The right panel of Figure 6 shows the dynamic weights for the high-capital-share economy ($a = 0.4$). In the absence of a capital tax ($\tau = 0$), old individuals have a lower dynamic weight than young individuals whenever both have two-date-lives. This reflects that the labor share is low in this calibration, so young individuals, who earn labor income, consume less than old individuals. As the capital tax increases, young individuals save less and increase their current consumption, narrowing the consumption gap between the young and the old and thereby reducing the gap in dynamic weights.

Therefore, increasing (reducing) the capital tax starting from $\tau = 0$ generates intertemporal-sharing losses in the low-capital-share economy and gains in the high-capital-share economy, as further explained below and consistent with Figure 7. As in Application 1, intertemporal-sharing gains and losses are entirely demographic, because individuals overlap for a single date. Hence, $\Xi^{IS} = \Xi^{DIS}$, given Proposition 3.

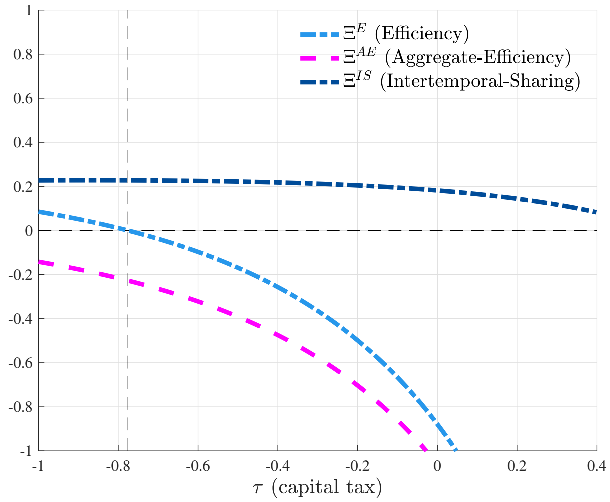
5.3 Policy Experiment: Capital Tax

Figure 7 shows aggregate welfare gains from capital taxation, decomposed into efficiency and redistribution components (bottom panels) and, within efficiency, into aggregate-efficiency and intertemporal-sharing gains (top panels). Unlike in Application 1, both sources of efficiency are present here because capital taxes affect the path of aggregate consumption.

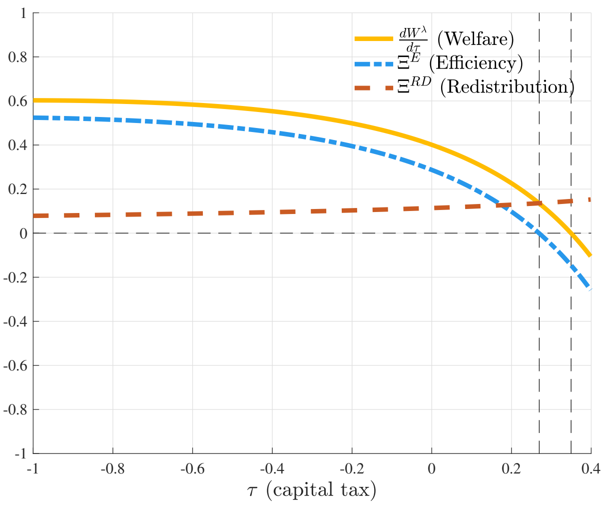
Three main conclusions emerge. First, the efficiency-maximizing capital tax is positive in the low-capital-share economy ($\tau^\star = 0.25$) and negative in the high-capital-share economy ($\tau^\star = -0.7$), with the sign in each case driven by aggregate (investment) efficiency considerations. In the low-capital-share economy, higher capital taxes raise aggregate consumption at all but the final date, indicating over-investment and over-accumulation of capital. In the high-capital-share economy, the opposite holds: higher capital taxes lower aggregate consumption at all but the initial date, reflecting under-



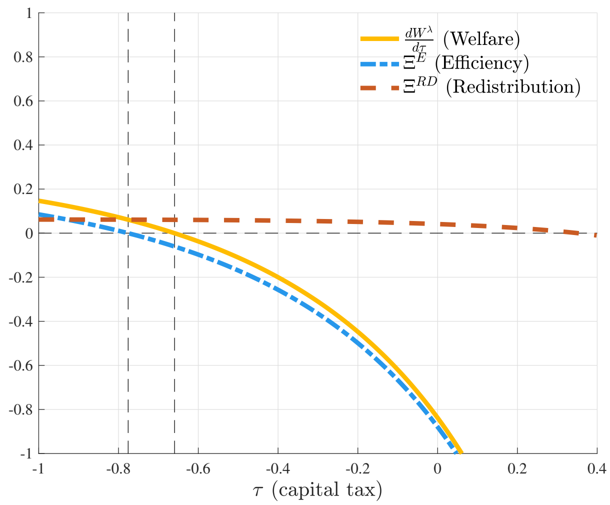
(a) Efficiency Decomposition: Low-capital-share



(b) Efficiency Decomposition: High-capital-share



(c) Welfare Decomposition: Low-capital-share



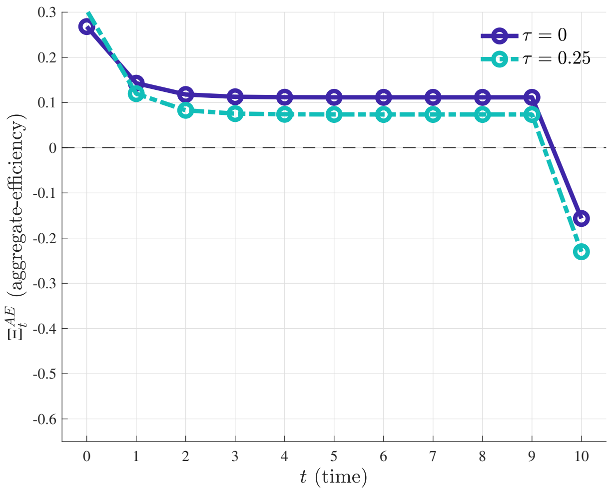
(d) Welfare Decomposition: High-capital-share

Figure 7: Welfare Assessment and Sources of Welfare Gains (Application 2)

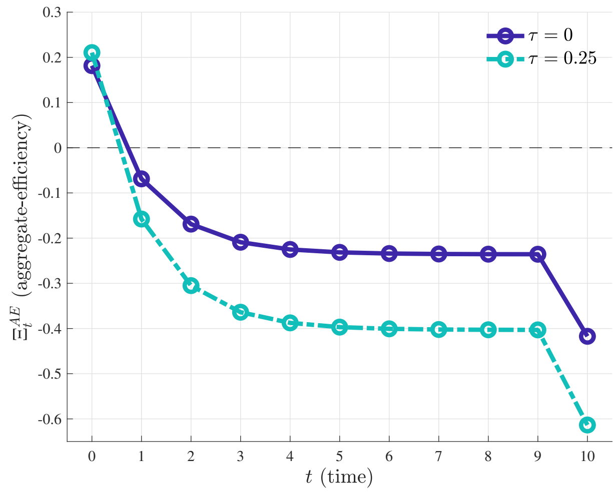
Note. The top panels of this figure show efficiency gains as a function of the tax, decomposed into aggregate-efficiency and intertemporal-sharing components, defined in (14). The bottom panels of this figure show aggregate welfare gains as a function of the tax, decomposed into efficiency and redistribution components, defined in (7). The bottom panels show the welfare assessments of an undiscounted utilitarian planner, with $\bar{\alpha} = 1$. The left panels correspond to the low-capital-share economy ($a = 0.2$) and the right panels to the high-capital-share economy ($a = 0.4$). Because individuals overlap for a single date, intertemporal-sharing gains are entirely demographic, so $\Xi^{IS} = \Xi^{DIS}$.

investment and under-accumulation of capital.

Second, intertemporal-sharing offsets aggregate efficiency in both economies, so the optimal capital tax is smaller in magnitude than what aggregate-efficiency alone would imply. In the low-capital-share economy, higher capital taxes increase the consumption



(a) Aggregate Efficiency: Low-capital-share



(b) Aggregate Efficiency: High-capital-share

Figure 8: Term Structure of Aggregate Efficiency Gains (Application 2)

Note. This figure shows the term structure of aggregate-efficiency gains under two scenarios: no capital tax, $\tau = 0$, and a positive tax, $\tau = 0.25$. Formally, we can write $\Xi^{AE} = \sum_t \omega_t \Xi_t^{AE}$, where $\omega_t = \frac{1}{I} \sum_i \omega_t^i$.

of the young — who already consume more than the old — thereby widening the consumption gap and generating intertemporal-sharing losses. In the high-capital-share economy, the opposite occurs: higher capital taxes increase the consumption of the old relative to the young, narrowing the gap and generating intertemporal-sharing gains. These intertemporal-sharing gains are purely demographic, since individuals overlap for a single date, so $\Xi^{IS} = \Xi^{DIS}$, as in Application 1.

Third, because the last young generation — most harmed by a tax increase — has the smallest normalized welfare weight, higher capital taxes generate positive redistribution gains in both economies under an undiscounted utilitarian criterion. Intuitively, higher capital taxes reduce capital accumulation, lowering the consumption of the final individual who consumes the remaining capital stock.

We would like to conclude this application with two remarks.

Remark 11. *(Term Structure of Aggregate-Efficiency Gains)* Figure 8 sheds light on aggregate efficiency by displaying the term structure of aggregate-efficiency gains. This clarifies when changes in aggregate consumption actually occur. In both low- and high-capital-share economies, a capital tax discourages investment and raises aggregate consumption at date 0. In the low-capital-share economy, increasing τ raises consumption at all but the final date, reflecting over-accumulation of capital: discouraging investment boosts aggregate consumption at almost all times. The high-capital-share economy shows the reverse pattern — an increase in τ lowers consumption at almost all dates—indicating under-accumulation of capital. In both cases, a higher τ reduces capital accumulation, lowering aggregate consumption at the terminal date T, when the capital stock is consumed.

Remark 12. *(Social Welfare Functions, Pareto Criterion, and the Golden Rule)* Prior work has focused on characterizing conditions under which Pareto improvements are possible in economies like the one considered here. The economy we study is Pareto efficient: a capital tax hurts the terminal young and a subsidy the initial old. Yet we identify Kaldor–Hicks efficiency gains from reducing (increasing) the aggregate capital stock in low- (high-) capital-share economies: this is a concrete illustration of Proposition 2 in a production economy. Our results thus address the challenge posed by Abel, Mankiw, Summers, and Zeckhauser (1989) — see Footnote 2 — by moving beyond the Pareto criterion and evaluating capital accumulation using a social welfare function. In doing so, our results connect to the Golden Rule literature (Phelps, 1961), which asks whether an economy accumulates "too much" or "too little" capital. Our approach sheds new light on the Golden Rule question in two ways. First, aggregate-efficiency extends beyond steady-state considerations to account for the entire transition path. Second, intertemporal-sharing captures efficiency gains arising from heterogeneity in valuations across different generations.

6 Application 3: Frictions vs. Demographics

This application revisits the welfare assessment of intergenerational transfers in the simplest OLG economy in which financial markets can be used to smooth consumption: the *three-date-life* version of Samuelson (1958)'s endowment economy. Starting from a hump-shaped consumption profile and absent financial markets, we consider a policy that transfers resources from middle-aged to old, mimicking again a pay-as-you-go social security system.

The main takeaway is the underlying rationales that justify social security in Applications 1 and 3 are markedly distinct: the intertemporal-sharing gains in Application

3 arise solely from financial frictions (FIS), while the demographic intertemporal-sharing (DIS) component is negative, in sharp contrast to Application 1, in which all gains are demographic (DIS). This application underscores the importance of decomposing intertemporal-sharing gains into frictional and demographic components to understand the justification for policies in economies with incomplete markets and rich demographics, as we do in Proposition 3.

6.1 Environment, Equilibrium, and Policy

We consider a single-good endowment economy with dates $t \in \{0, \dots, T\}$, where $T < \infty$. A single individual i is born at each date, where $i \in \{-1, \dots, T\}$. Individuals typically live for three dates, so three individuals are alive at any date — one "young", one "middle-aged", and one "old". As in the previous applications, t as a superscript indexes individuals.

Preferences and Endowments. The lifetime utility of an individual born at date $t \in \{0, \dots, T-2\}$ is

$$V^t = u\left(c_t^t\right) + \beta u\left(c_{t+1}^t\right) + \beta^2 u\left(c_{t+2}^t\right), \tag{34}$$

where c_t^t, c_{t+1}^t, and c_{t+2}^t denote individual t's consumption at dates t, $t+1$, and $t+2$, respectively. These individuals have endowments of the consumption good, denoted $\left\{e_t^t, e_{t+1}^t, e_{t+2}^t\right\}$. The preferences and endowments of individuals born at dates $t \in \{-2, -1, T-1, T\}$ are defined analogously with zero endowments and preferences for consumption when not alive, as shown in the Appendix.

Competitive Equilibrium. The budget constraints of individuals $t \in \{0, \dots, T-2\}$ are

$$c_t^t = e_t^t + b_t^t, \quad c_{t+1}^t + (1 + r_{t+1})\, b_t^t = b_{t+1}^t + e_{t+1}^t, \quad \text{and} \quad c_{t+2}^t + (1 + r_{t+2})\, b_{t+1}^t = e_{t+2}^t, \tag{35}$$

where b_t^t and b_{t+1}^t denote individual t's borrowing at dates t and $t+1$ respectively, and $1 + r_{t+1}$ denotes the interest rate between dates t and $t+1$. Analogous budget constraints hold for individuals that live for only one or two dates. Individuals face borrowing constraints of the form $b_t^i \leq \bar{b}$. To more clearly illustrate our results, we set $\bar{b} = 0$, but similar results obtain as long as the borrowing constraint binds.

The resource constraint for date-t consumption can be written as

$$c_t^{t-2} + c_t^{t-1} + c_t^t = e_t^{t-2} + e_t^{t-1} + e_t^t. \tag{36}$$

The equilibrium definition is standard, and presented in the Appendix.

Shares and Middle-Aged-to-Old Transfer. We model a pay-as-you-go social security system as a middle-aged-to-old (MTO) transfer at all dates. Formally, we parametrize the model by initial endowment shares given by $\bar{\chi}_{t,e}^t$, $\bar{\chi}_{t,e}^{t-1}$, and $\bar{\chi}_{t,e}^{t-2}$. We then consider a policy — indexed by a perturbation parameter $\theta \geq 0$ — that transfers endowments from middle-aged to old individuals at each date according to

$$\text{(Middle-Aged-to-Old Transfer)} \qquad \chi_{t,e}^{t-1} = \bar{\chi}_{t,e}^{t-1} - \theta \qquad \text{and} \qquad \chi_{t,e}^{t-2} = \bar{\chi}_{t,e}^{t-2} + \theta.$$

Calibration. Preferences are $u(c) = \log(c)$. One date in the model corresponds to 20 years, so $\beta = (0.98)^{20} = 0.67$. We normalize $e_t = 1$, assume no aggregate endowment growth, and set endowment shares

$$\text{young:} \quad \bar{\chi}_{t,e}^t = 0.25, \qquad \text{middle-aged:} \quad \bar{\chi}_{t,e}^{t-1} = 0.5, \qquad \text{and} \qquad \text{old:} \quad \bar{\chi}_{t,e}^{t-1} = 0.25,$$

so middle-aged consume more that young and old at each date in the absence of policy. We assume that individuals born at dates $t \in \{-2, -1\}$ have no savings at date 0. For ease of visualization, we assume that $T = 10$, so the economy runs for 200 years.

6.2 Normalized Weights

Welfare Assessments. We study welfare assessments for utilitarian planners, for whom the welfare assessment of a perturbation $d\theta$ is given by

$$\frac{dW}{d\theta} = \sum_{t=-2}^{T} \alpha^t \frac{dV^t(\theta)}{d\theta}, \tag{37}$$

where α^t denotes the Pareto weight associated with the individual born at date t. Once again, we focus on undiscounted and discounted utilitarian planners and use perpetual aggregate consumption as the welfare numeraire, so the counterpart of (27) is now

$$\lambda^t = u'\left(c_t^t\right) c_t + \beta u'\left(c_{t+1}^t\right) c_{t+1} + \beta^2 u'\left(c_{t+2}^t\right) c_{t+2},$$

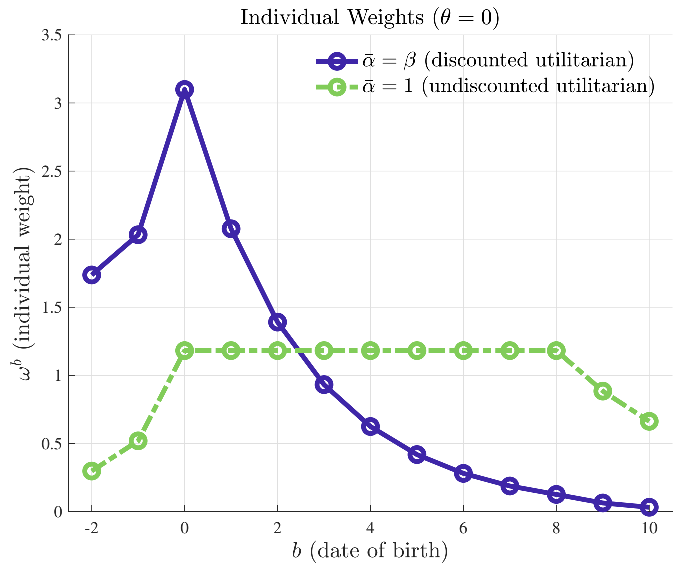


Figure 9: Individual Weights (Application 3)

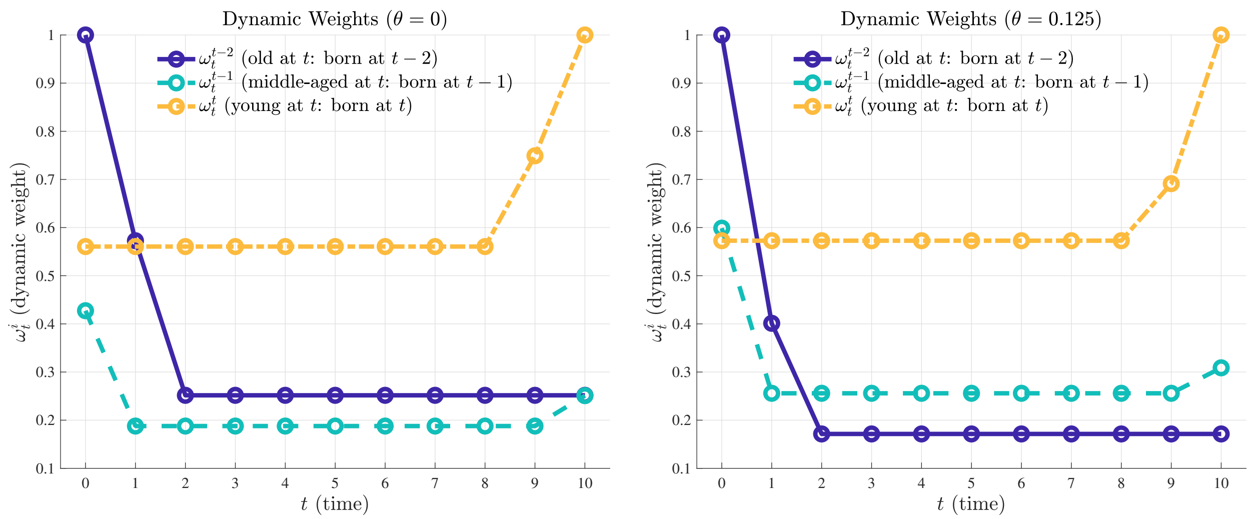
Note. This figure shows individual weights as a function of an individual's date of birth for the undiscounted (dashed green line) and discounted (solid blue line) utilitarian planners in the absence of policy ($\theta = 0$).

with equivalent expressions for shorter-lived individuals.

Individual Weights. Figure 9 displays individual weights — defined as in (28) — by birth date in the absence of policy. As in Application 1, longer-lived individuals have higher individual weights, all else equal. The initial old and middle-aged have a higher individual weight than the terminal middle-aged and young because the definition individuals utilities discounts the flow utilities of the latter. A discounted utilitarian planner assigns lower weights to individuals born later because of discounting.

Dynamic Weights. The left panel of Figure 10 displays dynamic weights in the absence of policy ($\theta = 0$). Among individuals with identical lifespans, the middle-aged have the lowest dynamic weights because they have the highest consumption, while the young have higher dynamic weights than the old, reflecting a preference for early-life consumption due to discounting. Hence, reallocating consumption from middle-aged to old (also to young) generates intertemporal-sharing gains, consistent with Figure 12.

The right panel of Figure 10 displays dynamic weights after implementing the welfare-maximizing transfer ($\theta = 0.125$). Once middle-aged and old consumption are equal,



(a) No transfer ($\theta = 0$) (b) Positive transfer ($\theta = 0.125$)

Figure 10: Dynamic Weights (Application 3)

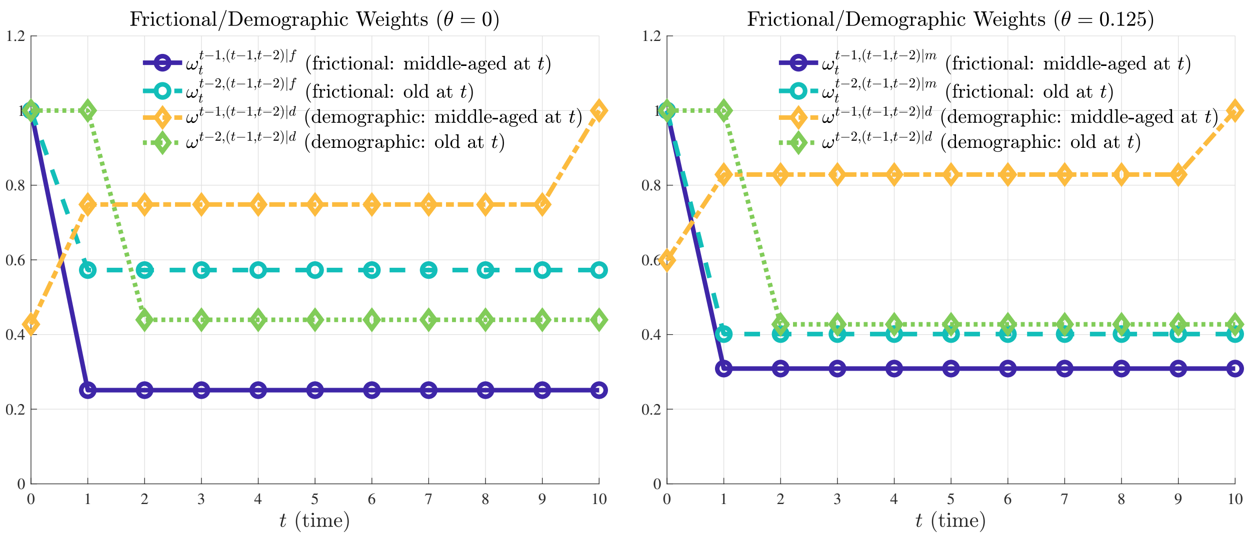
Note. The left panel shows dynamic weights in the absence of policy ($\theta = 0$, so $\chi_{t,c}^{t} = 0.25$, $\chi_{t,c}^{t-1} = 0.5$, and $\chi_{t,c}^{t-1} = 0.25$). The right panel shows dynamic weights after implementing the welfare-maximizing transfer for the undiscounted utilitarian planner ($\theta = 0.125$, so $\chi_{t,c}^{t} = 0.25$, $\chi_{t,c}^{t-1} = 0.375$, and $\chi_{t,c}^{t-1} = 0.375$). The dashed yellow lines denote the dynamic weights of young individuals and the solid blue lines denote the dynamic weights of old individuals at each date t (horizontal axis).

discounting makes the dynamic weight of the middle-aged exceed that of the old. This reverses the earlier pattern, so a MTO transfer generates efficiency losses at $\theta = 0.125$.

Decomposition of Dynamic Weights. The left panel of Figure 11 illustrates the multiplicative decomposition of dynamic weights into frictional and demographic components — introduced in (18) — for middle-aged and old individuals in the absence of policy ($\theta = 0$). Among individuals with identical lifespans, the frictional weight is higher for the old, who consume less than the middle-aged and hence value consumption more relative to consumption whenever both coexist.[17] Therefore, the MTO transfer

[17] It is useful to display the multiplicative decomposition of dynamic weights for middle-aged and old at date t:

$$
\begin{aligned}
\omega_t^{t-1} &= \overbrace{\frac{u'\left(c_{t-1}^{t-1}\right) + \beta u'\left(c_t^{t-1}\right)}{u'\left(c_{t-1}^{t-1}\right) + \beta u'\left(c_t^{t-1}\right) + \beta^2 u'\left(c_{t+1}^{t-1}\right)}}^{\text{Demographic}} \times \overbrace{\frac{\beta u'\left(c_t^{t-1}\right)}{u'\left(c_{t-1}^{t-1}\right) + \beta u'\left(c_t^{t-1}\right)}}^{\text{Frictional}} \qquad \text{(Middle-Aged at } t\text{)} \\
\omega_t^{t-2} &= \underbrace{\frac{\beta u'\left(c_{t-1}^{t-2}\right) + \beta^2 u'\left(c_t^{t-2}\right)}{u'\left(c_{t-2}^{t-2}\right) + \beta u'\left(c_{t-1}^{t-2}\right) + \beta^2 u'\left(c_t^{t-2}\right)}}_{\text{Demographic}} \times \underbrace{\frac{\beta u'\left(c_t^{t-2}\right)}{u'\left(c_{t-1}^{t-2}\right) + \beta u'\left(c_t^{t-2}\right)}}_{\text{Frictional}}, \qquad \text{(Old at } t\text{)}
\end{aligned}
$$



(a) No transfer ($\theta = 0$) (b) Positive transfer ($\theta = 0.125$)

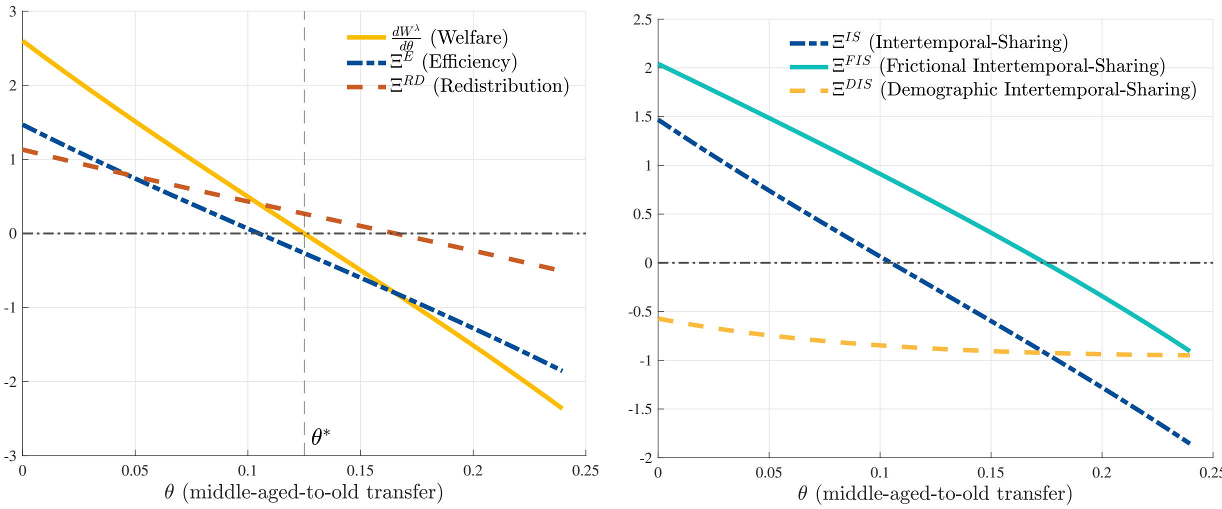
Figure 11: Frictional-Demographic Decomposition of Dynamic Weights (Application 3)

Note. This figure shows the multiplicative decomposition of pairwise dynamic weights between middle-aged and old, as defined in (18). The left panel is computed in the absence of policy ($\theta = 0$) while the right panel is computed after implementing the welfare-maximizing transfer for the undiscounted utilitarian planner ($\theta = 0.125$). The solid blue lines represent the frictional dynamic weights of the old, the dashed dark green lines represent the frictional dynamic weights of the middle-aged, the dash-dotted yellow lines represent the demographic dynamic weights of the young, and the dotted light green lines represent the demographic dynamic weights of the young at each date t (horizontal axis).

generates frictional (FIS) gains. By contrast, the demographic weight is higher for the middle-aged: because of discounting, the value of consumption when young and middle-aged is valued more than when middle-aged and old, justifying why the middle-aged have a higher demographic dynamic weight. This explains why the MTO transfer generates demographic (DIS) losses.

When the transfer is implemented, old individuals consume more, lowering their value for consumption when old and narrowing the gap in frictional weights. At the same time, the MTO transfer leaves the value of consumption when middle-aged and old roughly unchanged while increasing the value of consumption when young and middle-aged, which widens the gap in demographic weights. These results are illustrated in the right panel of Figure 11 and explain the sources of intertemporal-sharing gains in Figure 12.

where this expression uses $c_t = 1$, $\forall t$. Middle-aged and old coexist when the former are young and middle-aged and the latter are middle-aged and old, as reflected in the denominator of the frictional component and the numerator of the demographic component.



(a) Welfare Decomposition

(b) Intertemporal-Sharing Decomposition

Figure 12: Welfare Assessment and Sources of Welfare Gains (Application 3)

Note. The left panel shows aggregate welfare gains as a function of the MTO transfer, decomposed into efficiency and redistribution components, defined in (7), for the undiscounted utilitarian planner. Since this is an endowment economy with fixed aggregate consumption, all efficiency gains arise from intertemporal-sharing, so $\Xi^E = \Xi^{IS}$. The right panel shows the decomposition of intertemporal-sharing gains Ξ^{IS} into frictional and demographic components, Ξ^{IMS} and Ξ^{IDS}, defined in (19).

6.3 Policy Experiment: Middle-Aged-to-Old Transfer

The left panel of Figure 12 shows the efficiency-redistribution decomposition in (7) for an undiscounted utilitarian planner. As in Application 1, all efficiency gains arise entirely from intertemporal-sharing since this is an endowment economy with fixed aggregate consumption, so $\Xi^E = \Xi^{IS}$. Consistent with Figure 10, reallocating consumption from middle-aged individuals, who have lower dynamic weights, to old, who have higher, generates intertemporal-sharing gains. Redistribution gains arise because the MTO transfer harms only the terminal middle-aged, who carry lower individual weights, while benefiting others.

In contrast to Application 1, intertemporal-sharing gains now reflect both frictions and demographics, as illustrated in the right panel of Figure 12. The MTO transfer reallocates consumption from middle-aged individuals, who have lower frictional weights, to old, who have higher, thereby generating frictional gains, so $\Xi^{FIS} > 0$. This result shows that the MTO transfer substitutes for the trade that middle-aged and old would undertake if financial markets or contracts were available. At the same time, the MTO transfer reallocates consumption from middle-aged individuals, with higher demographic weights, to old, with lower, thereby generating demographic losses, so $\Xi^{DIS} < 0$. Interestingly, with three-date-lives, social security policy generates DIS losses, in contrast to Application 1, in which $\Xi^{DIS} > 0$. We elaborate on this point in our final remark.

Remark 13. *(Different Sources of Welfare Gains)* A central takeaway from this application is that the justification for a middle-aged-to-old transfer is fundamentally different from that of the young-to-old transfer in Application 1. Both policies generate intertemporal-sharing gains, but for different reasons. In Application 1, markets are complete and all gains arise purely from demographic differences. In this application, the gains from the policy arise because markets are incomplete, with demographic differences actually contributing efficiency losses. This result underscores the importance of decomposing intertemporal-sharing gains into frictional and demographic components to understand the rationale for policies in economies with incomplete markets and rich demographics.

7 Conclusion

This paper shows that welfare assessments in economies with rich demographics are far from trivial. After establishing that intergenerational comparisons must be based on perpetual consumption, we show i) that feasible perturbations of Pareto efficient allocations in demographically disconnected economies can generate positive Kaldor-Hicks efficiency gains, a phenomenon impossible when all individuals consume at every date, and ii) how to distinguish whether efficiency gains arise from financial frictions or demographic differences. Our applications quantify the sources of efficiency gains and identify new policy trade-offs, illustrating how our results yield new practical insights across multiple environments. We look forward to developing richer, quantitative applications to evaluate specific policy reforms in environments with frictions and complex demographic structures.

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Online Appendix

A Proofs and Derivations: Section 3

Normalized Welfare Gains. We can express an (unnormalized) welfare assessment $\frac{dW}{d\theta}$ as

$$\frac{dW}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \frac{dV^i}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \lambda^i \frac{\frac{dV^i}{d\theta}}{\lambda^i},$$

where $\lambda^i = \sum_t \left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i}{\partial c_t^i} c_t$. This expression and $\frac{\partial \mathcal{W}}{\partial V^i} > 0$ imply that $\frac{dW}{d\theta} > 0$ for Pareto-improving perturbations.

The normalized welfare assessment takes the form

$$\frac{dW^\lambda}{d\theta} = \frac{\frac{dW}{d\theta}}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i} = \sum_i \omega^i \frac{\frac{dV^i}{d\theta}}{\lambda^i}, \quad \text{where} \quad \omega^i = \frac{\frac{\partial \mathcal{W}}{\partial V^i}\lambda^i}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i}.$$

We can then express individual i's normalized lifetime welfare gains as

$$\frac{dV^{i|\lambda}}{d\theta} = \frac{\frac{dV^i}{d\theta}}{\lambda^i} = \sum_t \frac{\left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i}{\partial c_t^i} c_t}{\sum_t \left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i}{\partial c_t^i} c_t} \frac{1}{c_t} \frac{dc_t^i}{d\theta} = \sum_t \omega_t^i \frac{dV_t^{i|\lambda}}{d\theta},$$

where

$$\omega_t^i = \frac{\left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i}{\partial c_t^i} c_t}{\sum_t \left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i}{\partial c_t^i} c_t} \quad \text{and} \quad \frac{dV_t^{i|\lambda}}{d\theta} = \frac{1}{c_t}\frac{dc_t^i}{d\theta}.$$

Defining individual i's consumption share by $\chi_t^i = \frac{c_t^i}{c_t}$, we have that $c_t^i = \chi_t^i c_t$, which implies that $\frac{dc_t^i}{d\theta} = \frac{d\chi_t^i}{d\theta} c_t + \chi_t^i \frac{dc_t}{d\theta}$, so

$$\frac{dV_t^{i|\lambda}}{d\theta} = \frac{1}{c_t}\frac{dc_t^i}{d\theta} = \chi_t^i \frac{d\ln c_t^i}{d\theta} = d\chi_t^i + \chi_t^i \frac{dc_t}{c_t}.$$

Efficiency vs. Redistribution. For any two random variables x_i and y_i, it follows that $\sum_i x_i y_i = \frac{1}{I}\sum_i x_i \sum_i y_i + \mathbb{C}ov_i^\Sigma\left[x_i, y_i\right]$, where $\mathbb{C}ov_i^\Sigma\left[x_i, y_i\right] = I \cdot \mathbb{C}ov_i\left[x_i, y_i\right]$. Equation (7) follows from

$$\frac{dW^\lambda}{d\theta} = \sum_i \omega^i \frac{dV^{i|\lambda}}{d\theta} = \underbrace{\sum_i \frac{dV^{i|\lambda}}{d\theta}}_{\Xi^E} + \underbrace{\mathbb{C}ov_i^\Sigma\left[\omega^i, \frac{dV^{i|\lambda}}{d\theta}\right]}_{\Xi^{RD}},$$

where we use the fact that $\frac{1}{I}\sum_i \omega^i = 1$. This is the unique decomposition of the weighted sum $\sum_i \omega^i \frac{dV^{i|\lambda}}{d\theta}$ into an unweighted sum and its complement.

Proof of Proposition 1. (Unique Class of Welfare Numeraires)

Proof. The proof is constructive. As explained in Remark 5, only consumption based numeraires are in general valid regardless of the particular economic structure of the economy: competitive, strategic, search, etc, so to always be valid a numeraire must be based on consumption. Consumption at a given date or a bundle of consumption across different dates are valid welfare numeraires only when all individuals in the economy have a positive value for each of them. Formally, a welfare numeraire is valid if the individual normalizing factor λ^i, introduced in (4), is strictly positive, that is, $\lambda^i > 0$, $\forall i$. In demographically disconnected economies, consumption at a particular date cannot be a valid welfare numeraire, since there is always at least one individual who is not alive at that date. In principle, if $I < T$, one could choose a numeraire based on consumption over a subset of dates when all individuals are at least alive for one date. For instance, with three individuals and four dates, if $i = 1$ is alive at $\{0, 1\}$, $i = 2$ is alive $\{1, 2\}$, and $i = 3$ is alive at $\{2, 3\}$, numeraires based on bundles of consumption at $\{1, 2\}$, $\{0, 1, 2\}$, $\{1, 2, 3\}$, and $\{0, 1, 2, 3\}$ (perpetual consumption) are valid welfare numeraires. As I increases while the economy remains disconnected (in this example, say that $i = 4$ is alive only at $t = 0$ or $t = 4$), only numeraires based on perpetual consumption remain valid welfare numeraires. □

Proof of Proposition 2. (Kaldor-Hicks Improvements from Pareto Efficient Allocations)

Proof. The Pareto problem can be written as

$$\max_{\chi_t^i} \sum_i \alpha^i V^i,$$

subject to $\sum_i \chi_t^i = 1, \quad \forall t$ and $\chi_t^i \geq 0$, $\forall i, t$. The Lagrangian associated with this problem can be written as

$$\mathcal{L} = \sum_i \alpha^i V^i - \tilde{\eta}_t \left(\sum_i \chi_t^i - 1 \right) + \phi_t^i \chi_t^i,$$

where the optimality condition for χ_t^i is given by

$$\omega^i \omega_t^i = \eta_t > 0 \quad \text{if} \quad c_t^i > 0, \quad \text{and} \quad \omega^i \omega_t^i < \eta_t \quad \text{if} \quad c_t^i = 0,$$

where $\eta_t = \frac{\tilde{\eta}_t}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i}$. Aggregating the optimality conditions for individual i yields

$$\omega^i \underbrace{\sum_t \omega_t^i}_{=1} = \omega^i = \sum_t \eta_t \mathbb{I}\left[\omega_t^i > 0\right],$$

where $\mathbb{I}[\cdot]$ is an indicator function. This expression implies that ω^i is not equalized, and the optimality condition imply that ω_t^i are not equalized. But then note that

$$\Xi^E = \sum_i \frac{dV^{i|\lambda}}{d\theta} = \sum_i \sum_t \omega_t^i \frac{dV_t^{i|\lambda}}{d\theta} = \sum_t \sum_i \omega_t^i \frac{dV_t^{i|\lambda}}{d\theta},$$

so as long as ω_t^i are not equalized, it is possible to reallocate consumption so that $\Xi^E > 0$. □

A similar phenomenon occurs in demographically connected economies when using a "disconnected numeraire". In these economies, if the lifetime welfare is based on consumption at dates when all individuals are concurrently alive, then $\sum_{t\in\mathcal{T}} \omega_t^i = 1$, where $\mathcal{T}$ is the set of dates when all individuals are concurrently alive. In this case, $\omega^i = \sum_{t\in\mathcal{T}} \eta_t$, which implies that ω_t^i is equalized across all individuals whenever they are alive. If instead the lifetime welfare is based on consumption at dates when not all individuals are concurrently alive, the same proof as in the demographically disconnected economies applies.

Proof of Proposition 3. (Sources of Intertemporal-Sharing: Frictional Intertemporal-Sharing vs. Demographic Intertemporal-Sharing)

Proof. It follows that

$$\Xi^{IS} = \sum_t \mathbb{C}ov^{\Sigma}_{i|\omega_t^i>0}\left[\omega_t^i, \frac{dV_t^{i|\lambda}}{d\theta}\right] = \sum_t \frac{1}{I_t} \sum_{(i,j)\in\mathcal{A}_t} \mathbb{C}ov^{\Sigma}_{(i,j)\in\mathcal{A}_t}\left[\omega_t^{(i,j)}, \frac{dV_t^{(i,j)|\lambda}}{d\theta}\right],$$

where $\mathcal{A}_t$ denotes the set of all pairs of individuals alive at t and I_t is the number of alive individuals.[18] Using the fact that $\omega_t^{i,(i,j)} = \omega_t^{i,(i,j)|f} \omega^{i,(i,j)|d}$, as defined in (18), we can apply

[18]Expressing covariances in terms of pairwise covariances is common, for instance, in portfolio choice.

the result in Bohrnstedt and Goldberger (1969) to write Ξ^{IS} as

$$\Xi^{IS} = \sum_t \frac{1}{I_t} \sum_{(i,j)\in\mathcal{A}_t} \mathbb{C}ov^{\Sigma}_{(i,j)\in\mathcal{A}_t}\left[\omega_t^{(i,j)|f}, \frac{dV_t^{(i,j)|\lambda}}{d\theta}\right] + \sum_t \frac{1}{I_t} \sum_{(i,j)\in\mathcal{A}_t} \mathbb{C}ov^{\Sigma}_{(i,j)\in\mathcal{A}_t}\left[\omega^{(i,j)|d}, \frac{dV_t^{(i,j)|\lambda}}{d\theta}\right]$$
$$+ \sum_t \frac{1}{I_t} \sum_{(i,j)\in\mathcal{A}_t} \mathbb{C}osk^{\Sigma}_{(i,j)\in\mathcal{A}_t}\left[\omega_t^{(i,j)|f}, \omega^{(i,j)|d}, \frac{dV_t^{(i,j)|\lambda}}{d\theta}\right],$$

where, given three random variables X, Y, and Z, the coskewness operator is defined as $\mathbb{C}osk_i^{\Sigma}[X,Y,Z] = \mathbb{E}_i[(X-\mathbb{E}_i[X])(Y-\mathbb{E}_i[Y])(Z-\mathbb{E}_i[Z])]$ and $\mathbb{C}osk_i^{\Sigma}[\cdot,\cdot,\cdot] = I \cdot \mathbb{C}osk_i[\cdot,\cdot,\cdot]$. In this case, since all coskewness are computed pairwise (with two elements), they must be zero, which yields (19) in the text. □

Proof of Proposition 4. (Properties of Intertemporal-Sharing Decomposition)

Proof. a) In this case, the frictional dynamic weights $\omega_t^{i,(i,j)|f}$ are not equalized among individuals, so $\Xi^{FIS} \neq 0$ for generic perturbations.

b) In this case, the demographic dynamic weights $\omega_d^{i,(i,j)|d}$ are not equalized among individuals, so $\Xi^{DIS} \neq 0$ for generic perturbations.

c) In this case, the frictional dynamic weights $\omega_t^{i,(i,j)|f}$ is identical for all individuals, so $\Xi^{FIS} = 0$ for all perturbations.

d) In this case, the demographic dynamic weights $\omega_t^{i,(i,j)|d}$ is identical for all individuals, so $\Xi^{DIS} = 0$ for all perturbations.

e) Follows from c) and d). □

Aggregate-Efficiency/Intertemporal-Sharing Decomposition. It follows that

$$\Xi^{E} = \sum_i \sum_t \omega_t^i \frac{dV_t^{i|\lambda}}{d\theta} = \sum_t \sum_{i|\omega_t^i>0} \omega_t^i \frac{dV_t^{i|\lambda}}{d\theta} = \underbrace{\sum_t \omega_t \sum_{i|\omega_t^i>0} \frac{dV_t^{i|\lambda}}{d\theta}}_{\Xi^{AE}} + \underbrace{\sum_t \mathbb{C}ov^{\Sigma}_{i|\omega_t^i>0}\left[\omega_t^i, \frac{dV_t^{i|\lambda}}{d\theta}\right]}_{\Xi^{IS}},$$

where $\omega_t = \frac{1}{I_t}\sum_i \omega_t^i$ with $I_t = \sum_i \mathbb{I}\{i \mid \omega_t^i > 0\}$, and where $\mathbb{C}ov^{\Sigma}_{i|\omega_t^i>0}[\cdot,\cdot] = I \cdot \mathbb{C}ov^{\Sigma}_{i|\omega_t^i>0}[\cdot,\cdot]$.

B Redistribution

The redistribution component captures the equity concerns embedded in a particular Social Welfare Function. Ξ^{RD} is positive when the individuals relatively favored in a perturbation are those relatively preferred by the planner, i.e., have higher normalized individual weights ω^i. At times, it is convenient to distinguish whether redistribution gains take place i) within individuals from the same generation or ii) across individuals from different generations. To this end, we show how further decompose the redistribution component into *intra*-generational redistribution and *inter*-generational redistribution.

Proposition 5. *(Redistribution Decomposition) The redistribution component, Ξ^{RD}, can be decomposed into i) intra-generational redistribution and inter-generational redistribution, as follows:*

$$\Xi_{RD} = \underbrace{\sum_t |\mathcal{G}_t| \cdot \mathbb{C}ov_{i\in\mathcal{G}_t}\left[\omega^i, \frac{dV^{i|\lambda}}{d\theta}\right]}_{\Xi^{RD-intra}\ \textit{(Intra-Generational)}} + \underbrace{I \cdot \mathbb{C}ov_{\mathcal{G}}\left[\mathbb{E}_{i\in\mathcal{G}_t}\left[\omega^i\right], \mathbb{E}_{i\in\mathcal{G}_t}\left[\frac{dV^{i|\lambda}}{d\theta}\right]\right]}_{\Xi^{RD-inter}\ \textit{(Inter-Generational)}},$$

where $\mathcal{G}_t$ is the set of individuals born at date t (generation-t) and $\mathcal{G}_0$ is the set of individual alive at date 0. $\mathbb{C}ov_{\mathcal{G}}[\cdot,\cdot]$ denotes a covariance across generations that weights each generation by the probability $\frac{|\mathcal{G}_t|}{I}$ of a given individual coming from a particular generation t. Finally, the cross-sectional conditional expectation is given by $\mathbb{E}_{i\in\mathcal{G}_t}\left[X_i\right] = \frac{1}{|\mathcal{G}_t|}\sum_{i\in\mathcal{G}_t} X_i$.

This decomposition illustrates that welfare gains from redistribution have two sources: a weighted sum of within generations redistribution; and a covariance between the per-generation average normalized individual weight, and the per-generation average normalized lifetime welfare gains. This last term, $\Xi^{RD-inter}$, is positive when the generations relatively favored in a perturbation are those relatively preferred by the planner, i.e., have higher per-generation average normalized individual weights. This decomposition of Ξ^{RD} separates the component that is zero if the planner can costlessly transfer across individuals alive, $\Xi^{RD-intra}$, an its complement, $\Xi^{RD-inter}$. Even if transfers are generally not possible across generations, especially when individuals do not overlap, certain policies can still redistribute across generations. For example, policies aimed at mitigating climate change require current generations to make sacrifices that benefit future individuals that are yet-to-be-born. We illustrate this trade-off and how our welfare decomposition rationalizes the implied welfare effects in the following example.

Example 3. *(Aggregate Efficiency v.s. Intergenerational Redistribution)* Consider a two-date endowment economy with two individuals ($I = 2$). Individual A is alive at date 0, with preferences $V^A = u\left(c_0^A\right)$; and individual B is alive at date 1, with preferences $V^B = u\left(c_1^B\right)$. We study a perturbation that reallocates consumption as follows:

$$c_0^A = 1 - \theta \quad \text{and} \quad c_1^B = \frac{1}{2} + \varphi \cdot \theta; \qquad \theta \in (0,1)\,,\ \varphi \in (0,\infty)\,.$$

We interpret the negative consequences of climate change as follows. In the absence of policy intervention ($\theta = 0$), the current generation, individual A, consumes more than the unborn generation, individual B. Individual A can make an effort, indexed by θ, that reduces their consumption levels and allows the future generation to consume more. This effort represents a costly climate change mitigation policy. Lastly, φ summarizes the effectiveness of the policy.

The efficiency gains, whose unique source is aggregate efficiency, are determined by the effectiveness parameter φ, and given by

$$\Xi^{AE} = \varphi - 1, \qquad \begin{cases} \Xi^{AE} \geq 0, & \text{if } \varphi \in [1,\infty) \\ \Xi^{AE} < 0, & \text{if } \varphi \in (0,1)\,. \end{cases}$$

Only when $\varphi > 1$ does the perturbation result in an increase in the total aggregate consumption across dates, which is interpreted as an aggregate efficiency gain. The other source of welfare gains is intergenerational redistribution. For instance, if $\varphi < 1$, the planner may still consider the transfer worthwhile, especially if she places a high enough weight on the unborn, even though this transfer reduces efficiency. To highlight the importance of individual weights, consider the case with a constant social generational discount factor where $\alpha^A = 1$ and $\alpha^B = \bar{\alpha} \in (0,1]$, set $\varphi = 1$, and use $u(c) = \log(c)$. Then,

$$\Xi^{RD} = \Xi^{RD-inter} > 0 \quad \text{if} \quad \begin{cases} \theta \in \left(0, \frac{1}{4}\right), & \text{if } \bar{\alpha} = 1 \\ \theta \in \left(0, \frac{\bar{\alpha}-0.5}{\bar{\alpha}+1}\right), & \text{if } \bar{\alpha} < 1. \end{cases}$$

In this special case, the only justification for implementing the policy is a redistribution motive. As the social generational discount rate rises — meaning the planner's discounting applied to unborn generations is smaller — the desirable level of effort to mitigate climate change also increases. In the extreme case where the planner attaches a very low social generational discount factor ($\bar{\alpha} \leq \frac{1}{2}$), mitigation of climate change is not desirable at all. Instead, the planner would prefer to redistribute away from unborn individuals by increasing the consumption of those currently alive, at the expense of future generations.

C Extensions

C.1 Stochastic Environment

Demographic and non-demographic uncertainty can be introduced using history-notation, as in Chapter 8 of Ljungqvist and Sargent (2018). Consider an economy populated by a countable number of individuals, indexed by $i \in \mathcal{I} = \{1, \ldots, I\}$, where $1 \leq I \leq \infty$. At each date $t \in \{0, \ldots, T\}$, where $0 \leq T \leq \infty$, there is a realization of a stochastic event $s_t \in S$. We denote the history of events up to date t by $s^t = (s_0, s_1, \ldots, s_t)$, and the probability of observing a particular sequence of events s^t by $\pi_t(s^t)$. The initial value of s_0 is predetermined, so $\pi_0(s_0) = 1$. At all dates and histories, individuals (potentially) consume a single good.

Births and deaths are now simply stochastic events captured by the realization of s^t. Formally, individual i *dies* at history s^t if $u_t^i(\cdot) = 0$ for all future histories. If births are random, individual i is *potentially born* at a date denoted $\tau_b^i \in \{-\infty, \ldots, T\}$, where this is the first date with a history in which $u_t^i(\cdot) > 0$. This notion allows us to index individuals by the first time in which they are potentially born.

Formally, preferences in this case can be expressed as

$$V^i = \sum_{t=0}^{T} \left(\beta^i\right)^{t-\tau_b^i} \sum_{s^t} \pi_t\left(s^t\right) u_t^i\left(c_t^i\left(s^t\right); s^t\right),$$

where $\beta^i \in [0, 1)$ denotes individual i's discount factor, and $u_t^i(\cdot; s^t)$ and $c_t^i(s^t)$ respectively correspond to individual i's instantaneous utility and consumption at history s^t at date t. Whenever individual i is not alive, $u_t^i(\cdot; s^t) = 0$. Whenever individual i is alive, $u_t^i(\cdot; s^t)$ is well behaved, so that $\frac{\partial u_t^i(\cdot)}{\partial c_t^i} > 0$ and an Inada condition applies.

Normalized date welfare gains are now

$$\frac{dV_t^{i|\lambda}}{d\theta} = \sum_{s^t} \omega_t^i\left(s^t\right) \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta},$$

where the normalized stochastic weight $\omega_t^i\left(s^t\right)$ is given by

$$\omega_t^i\left(s^t\right) = \frac{\pi_t\left(s^t\right) \frac{\partial u_t^i\left(s^t\right)}{\partial c_t^i} c_t\left(s^t\right)}{\sum_{s^t} \pi_t\left(s^t\right) \frac{\partial u_t^i\left(s^t\right)}{\partial c_t^i} c_t\left(s^t\right)}.$$

Normalized history welfare gains are defined as

$$\frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} = \frac{1}{c_t\left(s^t\right)} \frac{dc_t^i\left(s^t\right)}{d\theta}.$$

In this economy, efficiency can be decomposed into aggregate efficiency, risk-sharing, and intertemporal-sharing, as shown in Dávila and Schaab (2025).

C.2 Multiple Goods and Factors

It is straightforward to augment individual preferences to account for more goods and/or factors. Formally, we can consider preferences of the form

$$V^i = \sum_{t=0}^{T} \left(\beta^i\right)^{t-\tau_b^i} u_t^i\left(c_t^{ij}, n_t^{if}\right),$$

where individual i now has preferences over J consumption, indexed by $j \in \{1, \ldots, J\}$, and F factors, goods and $f \in \{1, \ldots, F\}$ factors. In this case, a version of Proposition 1 still applies where the welfare numeraire needs to be defined over perpetual bundles of particular goods or factors.

C.3 Preference Disconnect

As explained in Remark 1, an analogous notion to demographic disconnect can be defined in static multi-good economies. Formally, consider a static economy populated by a finite number of individuals, indexed by $i \in \mathcal{I} = \{1, \ldots, I\}$ with preferences over J consumption goods, indexed by $j \in \{1, \ldots, J\}$. In this case, individual i preferences are given by

$$V^i = u^i\left(\left\{c^{ij}\right\}_j\right).$$

In this case, an economy is *preference disconnected* if there is no good j such that $\frac{\partial u^i}{\partial c^{ij}} > 0$ for all individuals, assuming also an Inada condition for consumed goods. A version of Proposition 1 also applies to these economies: only welfare numeraires based on consumption of all goods are always valid in preference disconnected economies. A version of Proposition 2 also applies in this case.

C.4 Welfare Numeraire

C.4.1 Perpetual Aggregate Consumption: Global Formulation

Note that we can define an individual equivalent variation $\Lambda^i(\theta)$ as follows

$$\sum_t \left(\beta^i\right)^{t-\tau_b^i} u_t^i \left(c_t^i(0) + c_t(0)\Lambda^i(\theta)\right) = \sum_t \left(\beta^i\right)^{t-\tau_b^i} u_t^i \left(c_t^i(\theta)\right),$$

where $c_t^i(0)$ denotes consumption at a status-quo $\theta = 0$, and where $c_t^i(\theta)$ denotes the path of consumption for individual i as a function of perturbation parameter θ. We denote aggregate consumption at a status-quo $\theta = 0$ by $c_t(0)$. Note that differentiating the definition of $\Lambda^i(\theta)$, we can express $\frac{d\Lambda^i(\theta)}{d\theta}$ in differential equation form, as follows:

$$\sum_t \left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i\left(c_t^i(0) + c_t(0)\Lambda^i(\theta)\right)}{\partial c_t^i} c_t(0) \frac{d\Lambda^i(\theta)}{d\theta} = \underbrace{\sum_t \left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i\left(c_t^i(\theta)\right)}{\partial c_t^i} \frac{dc_t^i(\theta)}{d\theta}}_{=\frac{dV^i(\theta)}{d\theta}},$$

which is equivalent to

$$\frac{d\Lambda^i(\theta)}{d\theta} = \frac{\frac{dV^i(\theta)}{d\theta}}{\sum_t \left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i\left(c_t^i(0)+c_t(0)\Lambda^i(\theta)\right)}{\partial c_t^i} c_t(0)}.$$

But note that as $\theta \to 0$, then

$$\frac{d\Lambda^i(0)}{d\theta} = \frac{\frac{dV^i(0)}{d\theta}}{\sum_t \left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i\left(c_t^i(0)\right)}{\partial c_t^i} c_t(0)} = \frac{\frac{dV^i}{d\theta}}{\lambda^i},$$

which is the object that characterizes normalized individual welfare gains from a marginal perturbation.

C.4.2 Perpetual Unit Consumption

As discussed in Section 3, the two natural choices for welfare numeraire among those based on perpetual consumption are i) perpetual unit consumption, the bundle that pays one unit of the consumption good at each date, and ii) perpetual aggregate consumption, the bundle that pays aggregate consumption at each date. In the body of the text, we adopted perpetual aggregate consumption as welfare numeraire, but here we normalized aggregate welfare gains for the perpetual unit consumption welfare numeraire.

In this case, we can express an (unnormalized) welfare assessment $\frac{dW}{d\theta}$ as

$$\frac{dW}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \frac{dV^i}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \lambda^i \frac{\frac{dV^i}{d\theta}}{\lambda^i},$$

where our choice of welfare numeraire is such that $\lambda^i = \sum_t \left(\beta^i\right)^t \frac{\partial u_t^i}{\partial c_t^i}$. Hence, the normalized welfare assessment takes the form

$$\frac{dW^\lambda}{d\theta} = \frac{\frac{dW}{d\theta}}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i} = \sum_i \omega^i \frac{\frac{dV^i}{d\theta}}{\lambda^i}, \quad \text{where} \quad \omega^i = \frac{\frac{\partial \mathcal{W}}{\partial V^i}\lambda^i}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i}.$$

We can then express individual i's normalized lifetime welfare gains as

$$\frac{dV^{i|\lambda}}{d\theta} = \frac{\frac{dV^i}{d\theta}}{\lambda^i} = \sum_t \frac{\left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i}{\partial c_t^i}}{\sum_t \left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i}{\partial c_t^i}} \frac{dc_t^i}{d\theta} = \sum_t \omega_t^i \frac{dV_t^{i|\lambda}}{d\theta},$$

where

$$\omega_t^i = \frac{\left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i}{\partial c_t^i}}{\sum_t \left(\beta^i\right)^{t-\tau_b^i} \frac{\partial u_t^i}{\partial c_t^i}} \quad \text{and} \quad \frac{dV_t^{i|\lambda}}{d\theta} = \frac{dc_t^i}{d\theta}.$$

As explained in the text, in economies without aggregate consumption growth both numeraires yield identical results.

D Application 1: Additional Results

Definition. *(Competitive Equilibrium) Given endowments, a competitive equilibrium consists of consumption allocations and prices such that i) individuals maximize lifetime utility* (20) – (21) *subject to* (22) – (23)*, and ii) markets clear, so* (24) *holds at each date.*

E Application 2: Additional Results

Definition. *(Competitive Equilibrium) Given an initial capital stock* k_0 *and a capital tax* τ*, a competitive equilibrium comprises allocations* $\{y_t, c_t, \iota_t, n_t, k_t, T_t\}$ *and prices* $\{w_t, d_t, q_t\}$ *such that (i) individuals maximize lifetime utility* (20) – (21) *subject to* (30)*,* (31)*, and* (32)*, (ii) firms maximize profits, (iii) the government budget balances, and (iv) markets clear, that is,* $y_t = c_t + \iota_t$ *and* $1 = n_t$ *hold, where* $c_t = c_t^{t-1} + c_t^t$ *denotes date-t aggregate consumption.*

Figure OA-1 shows the equilibrium path for capital and aggregate consumption for both calibrations.

F Application 3: Additional Results

Definition. *(Competitive Equilibrium) Given endowments, a competitive equilibrium comprises consumption allocations and interest rates such that i) individuals maximize lifetime utility subject to budget constraints, and ii) markets clear at each date.*

Environment. Individuals born at dates $t = -1$ and $t = T - 1$ are alive for two dates. Their lifetime utility is given by

$$V^{-1} = \beta u\left(c_0^{-1}\right) + \beta^2 u\left(c_1^{-1}\right) \quad \text{and} \quad V^{T-1} = u\left(c_{T-1}^{T-1}\right) + \beta u\left(c_T^{T-1}\right).$$

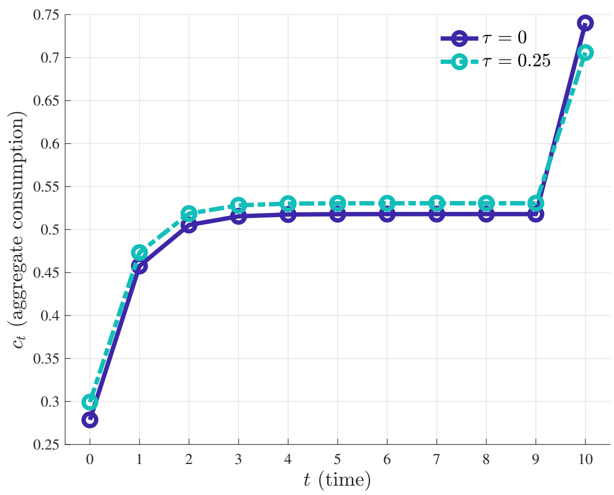
Individuals born at dates $t = -2$ and $t = T$ are alive for only one date. Their lifetime utility is given by

$$V^{-2} = \beta^2 u\left(c_0^{-2}\right) \qquad \text{and} \qquad V^T = u\left(c_T^T\right).$$

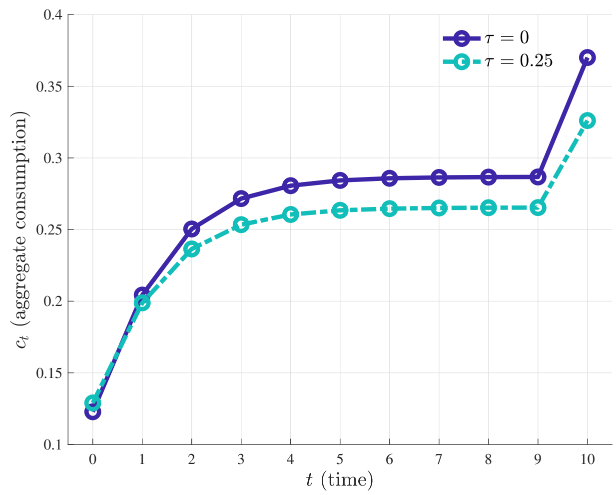
for individuals $t \in \{0, \ldots, T-2\}$, $\left\{e_0^{-1}, e_1^{-1}\right\}$ and $\left\{e_{T-1}^{T-1}, e_T^{T-1}\right\}$ for individuals $t \in \{-1, T-1\}$, as well as e_0^{-2} and e_T^T for individuals $t \in \{-2, T\}$.

As in Application 1, individual i's consumption and endowment shares at date t are $\chi_{t,c}^i = \frac{c_t^i}{c_t}$ and $\chi_{t,e}^i = \frac{e_t^i}{e_t}$, with aggregates given by $c_t = c_t^{t-2} + c_t^{t-1} + c_t^t$ and $e_t^{t-2} + e_t^{t-1} + e_t^t$, respectively.

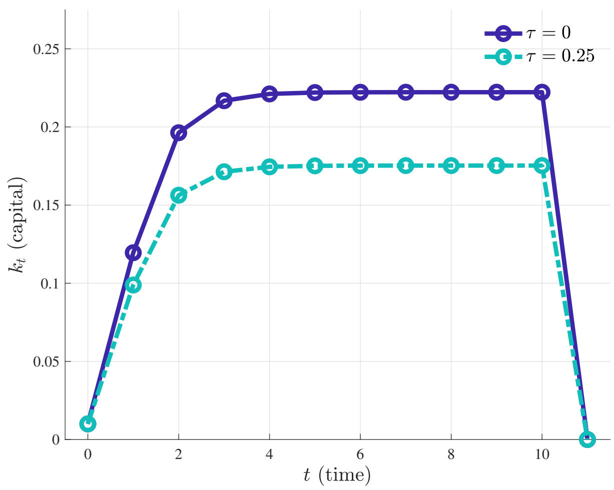
Individual Welfare Gains. Figure OA-2 shows normalized welfare gains of the MTO transfer for each individual. Similar to Application 1, the initial old (individual $t = -2$)



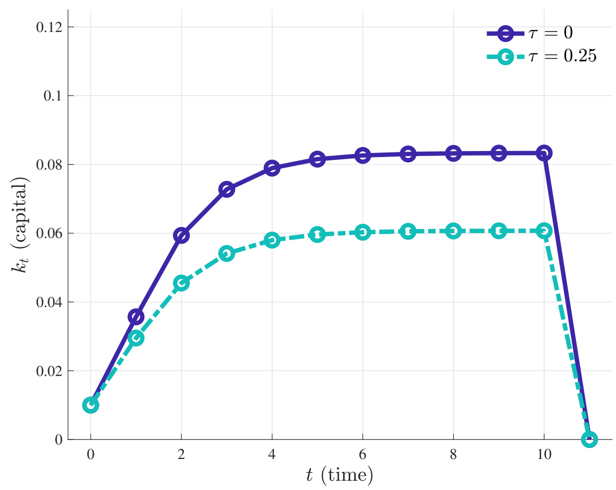
(a) Aggregate Consumption: Low-capital-share



(b) Aggregate Consumption: High-capital-share



(c) Capital: Low-capital-share



(d) Capital: High-capital-share

Figure OA-1: Equilibrium Dynamics (Application 2)

Note. This top panels of this figure show the equilibrium path of aggregate consumption, $c_t = \sum_i c_t^i$, for low-capital-share (left, $a = 0.2$) and high-capital-share (right, $a = 0.4$) economies. The bottom panels show the equilibrium path of capital for both calibrations.

always gains from the perturbation, whereas the terminal middle-aged (individual $t = T - 1$) always loses, with the terminal young remaining indifferent. All other individuals initially benefit from the MTO transfer since their consumption becomes smoother. Figure OA-2 also shows that the MTO transfer is not a Pareto improvement since the terminal middle-aged are worse off.

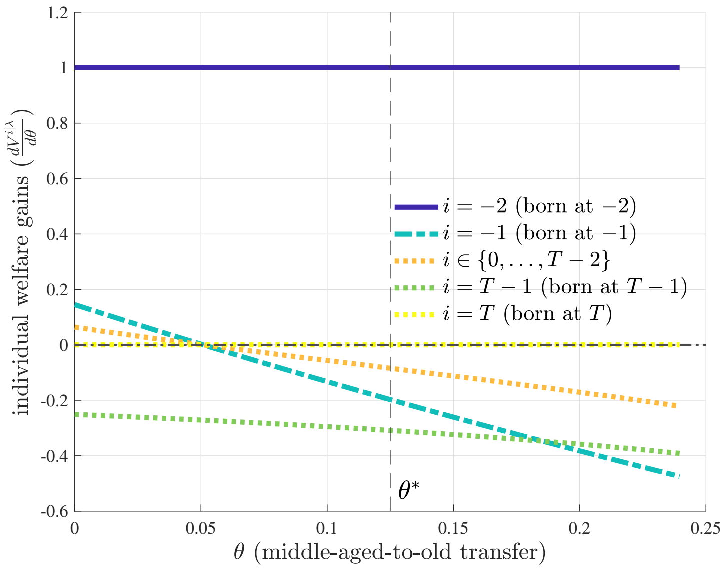


Figure OA-2: Individual Welfare Gains (Application 3)

Note. This figure shows the normalized individual welfare gains, given by $\frac{dV^{i|\lambda}}{d\theta}$.