Welfare Accounting*

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Abstract

This paper identifies and quantifies the origins of welfare gains in economies with heterogeneous individuals and disaggregated production. It does so by introducing a welfare accounting decomposition — exclusively based on preferences, technologies, and resource constraints — that systematically computes the contribution to efficiency of i) changes in allocations and ii) primitive changes in technologies and endowments, both through exchange and production. We leverage the decomposition to provide a new characterization of efficiency conditions that carefully accounts for binding non-negativity constraints, in particular with pure intermediate goods, extending classical efficiency results. In competitive economies, the welfare accounting decomposition can be expressed in terms of prices (and wedges), which allows us to characterize a new converse Hulten's theorem and a generalized Hulten's theorem. We present four applications to workhorse models in macroeconomics: i) the Armington model, ii) the Diamond-Mortensen-Pissarides model, iii) the Hsieh-Klenow model, and iv) a multi-sector heterogeneous-agent New-Keynesian model.

JEL Codes: E61, D60

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1 Introduction

Understanding the origins of welfare gains is essential for assessing the impact of shocks and the desirability of policy interventions. This paper tackles this question by introducing a decomposition of welfare assessments in economies with heterogeneous individuals and disaggregated production. This approach — which we refer to as *welfare accounting* — serves two primary purposes: i) to identify and quantify the precise sources of welfare gains induced by changes in allocations, technologies, or endowments and ii) to characterize the associated efficiency conditions.

Unlike other approaches, such as Baqaee and Farhi (2020), the welfare accounting decomposition is constructed solely from preferences, technologies, and resource constraints, without invoking assumptions on individual or firm optimizing behavior, budget constraints, prices, or equilibrium notions. This defining feature of our approach yields a unified and general framework that can be applied across radically different economic environments — including competitive, strategic, search, bargaining, contracting, and centralized planning settings — to systematically quantify and compare the sources of welfare gains, as illustrated in our applications.

Welfare Accounting Decomposition. We consider a static economy with heterogeneous individuals who consume goods and supply factors. Goods are produced using other goods and factors. The first main contribution of this paper is to develop a decomposition of welfare assessments for general social welfare functions, summarized in Figure 1.

We begin by separating welfare assessments into two components: i) (Kaldor-Hicks) efficiency, analyzed in the body of the paper because it is invariant to the choice of social welfare function, and ii) redistribution, examined in the Appendix.[1] We further decompose efficiency gains into *exchange* and *production efficiency*. The key guiding principle that makes our decomposition of efficiency gains unique is that, at each step, changes in levels are broken down into i) changes in shares, capturing cross-sectional reallocations of goods or factors, and ii) changes in aggregates.

Theorem 1 decomposes exchange efficiency into i) *cross-sectional consumption efficiency*, capturing gains associated with reallocating consumption across individuals, for given

[1] This efficiency-redistribution decomposition leverages the results of Dávila and Schaab (2025). That paper takes as given the mapping between allocations and policies or shocks, whereas in this paper we exploit production technologies and resource constraints to identify the primitive origins of welfare gains.

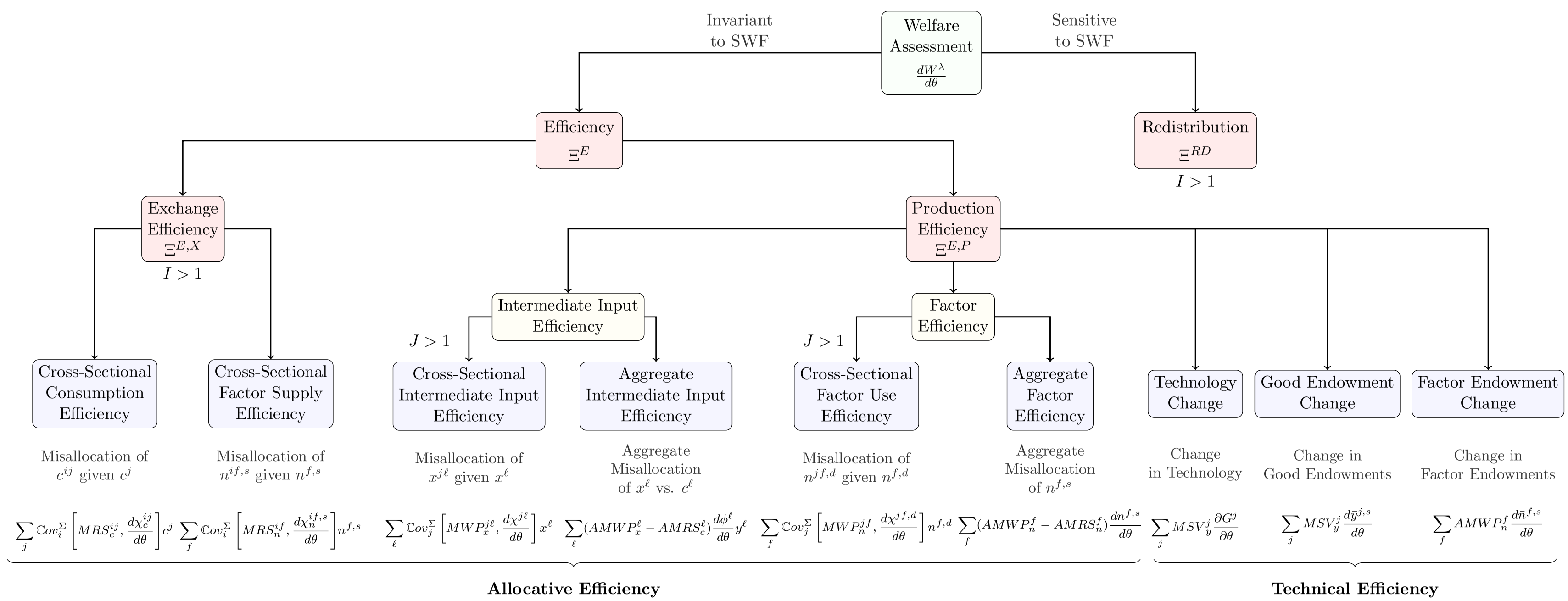


Figure 1: Welfare Accounting Decomposition

Note: This diagram illustrates the welfare accounting decomposition introduced in this paper. Lemma 1 decomposes welfare assessments into efficiency and redistribution components. Efficiency gains comprise exchange and production efficiency. Theorem 1 decomposes exchange efficiency into cross-sectional consumption and factor supply efficiency. Theorem 2 decomposes production efficiency into intermediate input efficiency, factor efficiency, technology change, good endowment change, and factor endowment change components. Theorems 3 and 4 leverage this decomposition to characterize efficiency conditions. Theorem 5 characterizes the technology change component in competitive economies with wedges.

aggregate consumption, and ii) *cross-sectional factor supply efficiency*, capturing gains from reallocating factor supply across individuals, for given aggregate factor supply.

Theorem 2 decomposes production efficiency, which comprises allocative efficiency gains due to adjusting intermediate inputs and factors as well as technical efficiency gains from primitive changes in technologies and endowments. For intermediate inputs, *cross-sectional intermediate input efficiency* captures the gains from reallocating intermediate inputs across uses, for given aggregate intermediate use, while *aggregate intermediate input efficiency* captures the gains from adjusting the share of good supply that is consumed instead of used in production, for given aggregate good supply. For factors, *cross-sectional factor use efficiency* captures the gains from reallocating factors across uses, for given aggregate factor use, while *aggregate factor efficiency* captures the gains from adjusting overall factor supply. Finally, the *technology*, *good endowment*, and *factor endowment change* components measure the direct gains from primitive changes in technologies or endowments.

The welfare accounting decomposition identifies the variables that translate changes in allocations, technologies, and endowments into welfare changes. First, the marginal rate of substitution (MRS) and its aggregate counterpart ($AMRS$) capture the social value of increasing individual or aggregate consumption or factor supply, respectively. Second, the marginal welfare product (MWP) and its aggregate counterpart ($AMWP$) capture the social value of increasing the particular or aggregate use of an input or factor in production, respectively. Finally, the marginal social value of goods (MSV) captures the social value of an additional unit of a good. MSV's are central to welfare accounting because they alone determine gains from pure technological change and govern marginal welfare products, which in turn determine each component of production efficiency.

Why is it useful to decompose welfare and efficiency assessments? What is the purpose of the decomposition? Within a given economy, each allocative efficiency term traces efficiency gains to a specific violation of efficiency conditions in the planner's problem. For instance, a positive (negative) cross-sectional factor use efficiency term indicates that a perturbation reallocates factors towards more (less) efficient uses, revealing an underlying inefficiency in the use of factors. Similarly, a positive (negative) aggregate factor efficiency term indicates that a perturbation increases the supply of initially under-supplied (over-supplied) factors, revealing an underlying inefficiency in the aggregate supply of factors. Perhaps more importantly, because the decomposition is derived solely from preferences, technologies, and resource constraints, it enables formal comparisons of the origins of efficiency gains *across* economies that may organize production and exchange very differently. For example, our results allow us to formalize the idea that unemployment losses in the Diamond-Mortensen-Pissarides search economy (Application 2), misallocation losses in the Hsieh and Klenow (2009) economy (Application 3), and the losses from price dispersion in a New Keynesian economy (Application 4), are of *identical* nature — cross-sectional factor use efficiency — even when allocations are determined through very different procedures: search, flexible prices, and sticky prices, respectively.

Efficiency Conditions. The second main contribution of this paper is to provide a complete characterization of efficiency conditions for disaggregated production economies with heterogeneous individuals, thereby generalizing the classical efficiency conditions in Lange (1942) and Mas-Colell et al. (1995). Theorems 3 and 4 characterize the necessary conditions for exchange and production efficiency by requiring that each component of the welfare accounting decomposition be non-positive for all feasible perturbations.

Although efficiency conditions naturally take the form of inequalities when non-negativity constraints bind, this is not the main insight from our analysis. The central takeaway from our characterization is that, even when non-negativity constraints *do not bind* for a given decision, the associated efficiency conditions (which hold with equality) are altered once one properly accounts for constraints *binding elsewhere*. This insight is central to economies with pure intermediate goods, which are those goods whose consumption non-negativity constraints bind for all individuals — see Section 4.3, in particular Examples 1 and 2.

Formally, we show that the classical efficiency conditions expressed in terms of MRS and MRT cease to be valid when pure intermediate goods are involved, whereas the conditions we identify in terms of MWP and MRS remain valid. Our characterization also highlights how computing efficiency conditions in production economies with pure intermediate goods is significantly more challenging than allocating goods efficiently across individuals. In particular, economies with pure intermediate goods require the computation of an inverse matrix. Intuitively, when all goods are mixed/final, the value of a good can be read off its consumption value, but computing the value of a pure intermediate good necessitates knowledge of the production network. This result allows us to speak to the socialist calculation debate (Lange, 1936; Lerner, 1944; Hayek, 1945) in Section 4.4.

Competitive Economies. Until Section 5, our results make no assumptions about individual behavior, budget constraints, prices, or equilibrium notions. In Section 5, we specialize the welfare accounting decomposition to competitive economies with wedges, in which prices reveal relevant information for welfare accounting.

The third main contribution of this paper is a characterization of the marginal social value (MSV) of goods in competitive economies with wedges. The MSV of a good equals its competitive price augmented by an aggregate wedge term that captures average distortions in consumption and intermediate input use. Intuitively, the MSV of a good that ultimately increases the supply of goods that are under-produced (over-produced) due to the presence of wedges is higher (lower) than its price. This characterization allows us to i) present a new converse result to Hulten's theorem that has been missing from the existing literature, ii) qualify the conditions under which Hulten's theorem holds, and iii) characterize the relation between marginal revenue product and marginal welfare product equalization.

The converse Hulten's theorem identifies a condition on wedges that ensures that prices equal the MSV of a good, regardless of whether an economy is frictionless or not. We show that Hulten's theorem fails to hold only in *non-interior* efficient economies, applying to interior efficient economies and to all frictionless competitive economies.[2] This result further underscores the importance of carefully analyzing non-negativity constraints in disaggregated economies. We also show that Hulten's theorem is, at its core, a result about efficiency (via production efficiency), only becoming a result about output or welfare under specific circumstances. This result expands on Bigio and La'O (2020), who have already shown that Hulten's theorem is valid for efficiency, rather than output, in an environment with a single individual and elastic factor supply. At last, we also show that efficiency requires the equalization of marginal welfare products across uses of an intermediate input or a factor, while competition only enforces the equalization of marginal revenue products.

Applications. Finally, we illustrate how the welfare accounting decomposition can be put to use to identify the origins of welfare gains in four workhorse models in macroeconomics and trade.

Our first application shows that an increase in tariffs contributes negatively to exchange efficiency via cross-sectional consumption efficiency in the simplest endowment economy

[2]We refer to allocations/economies with binding non-negativity constraints as non-interior, and to those with not binding non-negativity constraints as interior.

(Armington, 1969). Our second application shows that the efficiency gains from an improvement in the matching technology in a Diamond-Mortensen-Pissarides (DMP) model stem from cross-sectional factor use efficiency gains large enough to offset the aggregate intermediate input efficiency losses caused by higher vacancy postings. This application illustrates how the welfare accounting decomposition can be applied to a random search economy, which differs substantially from competitive economies. Our third application illustrates how an increase in markup dispersion generates cross-sectional factor use efficiency losses in Hsieh and Klenow (2009) economy.

Our final application shows how to use the welfare accounting decomposition to identify the welfare gains from optimal monetary stabilization policy in a macroeconomic model with household and sectoral heterogeneity. We compute the optimal monetary policy response to a technology shock in a static, multi-sector heterogeneous-agent New Keynesian model with a rich input-output production structure. We contrast the efficiency gains from stabilization policy with its impact on redistribution and decompose the former into its production efficiency components. Quantitatively, we show that "cross-sectional" production efficiency terms are more important than "aggregate" terms in a standard calibration of the New Keynesian model.

Related Literature. Our characterization of efficiency conditions is closely related to classical studies of efficiency — see Lange (1942) or, for a modern treatment, Section 16.F of Mas-Colell et al. (1995). While existing work has assumed that all goods are final or mixed, we show that allowing for pure intermediate goods substantially changes the nature of efficiency conditions. Even though it is understood that particular efficiency conditions ensure exchange and production efficiency, this literature has not explored welfare decompositions that allow to quantitatively separate the sources of efficiency gains for a given perturbation.

The welfare accounting decomposition relates to the vast literature on growth accounting and productivity measurement that follows Solow (1957) and includes Hall (1990), Basu (1995), Basu and Fernald (1997, 2002), Basu et al. (2006), Basu et al. (2022), and Baqaee and Farhi (2020), among many others. The unique feature of our decomposition is the fact that it is exclusively based on preferences, technologies, and resource constraints, making no assumptions about individual behavior, budget constraints, prices, or equilibrium notions. This contrasts our results with Baqaee and Farhi (2020), who present a decomposition based on markups, prices, and cost minimization.

Our results build on the production networks literature.[3] A central result of this literature is Hulten's theorem (Hulten, 1978), which characterizes the aggregate impact of technological change in terms of prices (Domar weights). Instead of imposing a competitive structure, we provide a characterization of the impact of technological change exclusively based on preferences and technologies, identifying the MSV of goods as the relevant object. By specializing the MSV of goods to competitive environments, we are able to i) present a new converse Hulten's theorem, characterizing the conditions under which Hulten's theorem applies even for economies with frictions, and ii) qualify the conditions under which Hulten's theorem applies. Liu (2019) presents a statistic that summarizes the social value of subsidizing inputs and factors. While related, our characterization of MSV differs because it i) makes no assumptions about optimizing behavior, budget constraints, or prices, and ii) considers a perturbation in the level of output rather than price subsidies.

2 Environment and Social Welfare

We first introduce preferences, technologies, and resource constraints, and then define feasible allocations and perturbations. We conclude this section by describing how to separate efficiency from redistribution considerations when making welfare assessments.

2.1 Preferences, Technologies, and Resource Constraints

We consider a static economy populated by a finite number $I \geq 1$ of individuals, indexed by $i \in \mathcal{I} = \{1, \ldots, I\}$. There are $J \geq 1$ goods, indexed by $j, \ell \in \mathcal{J} = \{1, \ldots, J\}$ and $F \geq 0$ factors, indexed by $f \in \mathcal{F} = \{1, \ldots, F\}$. Goods are produced using goods and factors as inputs, while factors are directly supplied by individuals. Goods and factors may also appear as (predetermined) endowments.

An individual i derives utility from consuming goods and (dis)utility from supplying factors, according to the utility function

$$\text{(Preferences)} \qquad V^i = u^i \left(\left\{ c^{ij} \right\}_{j \in \mathcal{J}}, \left\{ n^{if,s} \right\}_{f \in \mathcal{F}} \right), \tag{1}$$

[3] This literature includes, among many others, Gabaix (2011), Jones (2011), Acemoglu et al. (2012), Liu (2019), Bigio and La'O (2020), Acemoglu and Azar (2020), La'O and Tahbaz-Salehi (2022), and Kopytov et al. (2022). By emphasizing the critical role played by pure intermediate goods, our results connect to the recent work on global value chains — see Antràs and Chor (2022) for a recent survey.

where c^{ij} denotes individual i's final consumption of good j and $n^{if,s}$ denotes individual i's supply of factor f (the superscript s stands for supply).

Goods are produced using technologies that take goods and factors as inputs. The production technology for good j, denoted by $G^j(\cdot) \geq 0$, is given by

$$\text{(Technologies)} \qquad y^{j,s} = G^j\left(\left\{x^{j\ell}\right\}_{\ell \in \mathcal{J}}, \left\{n^{jf,d}\right\}_{f \in \mathcal{F}}; \theta\right), \tag{2}$$

where $y^{j,s}$ denotes the amount produced (output) of good j, $x^{j\ell}$ denotes the amount of good ℓ used in the production of good j, and $n^{jf,d}$ denotes the amount of factor f used in the production of good j (the superscript d stands for demand). We use the index $\ell \in \mathcal{J}$ to refer to goods used as intermediates. We parametrize $G^j(\cdot;\theta)$ by θ to consider perturbations to technology, as described below.

The resource constraint for good j is

$$\text{(Resource Constraints: Goods)} \qquad y^{j,s} + \bar{y}^{j,s}(\theta) = c^j + x^j, \tag{3}$$

where $c^j = \sum_i c^{ij}$ represents the total amount of good j consumed (aggregate consumption), $x^j = \sum_\ell x^{\ell j}$ represents the amount of good j used as an intermediate input in production (aggregate intermediate use), and $\bar{y}^{j,s}(\theta) = \sum_i \bar{y}^{ij,s}(\theta)$ represents the aggregate endowment of good j, where $\bar{y}^{ij,s}(\theta)$ denotes individual i's endowment of good j. We parametrize $\bar{y}^{j,s}(\theta)$ and $\bar{y}^{ij,s}(\theta)$ by θ to consider perturbations to goods' endowments. When needed, we denote the aggregate supply of good j by $y^j = y^{j,s} + \bar{y}^{j,s}(\theta)$.

The resource constraint for factor f is

$$\text{(Resource Constraints: Factors)} \qquad n^{f,s} + \bar{n}^{f,s}(\theta) = n^{f,d}, \tag{4}$$

where $n^{f,s} = \sum_i n^{if,s}$ and $n^{f,d} = \sum_j n^{jf,d}$ respectively represent the aggregate elastic supply and the aggregate factor use of factor f, and $\bar{n}^{f,s}(\theta) = \sum_i \bar{n}^{if,s}(\theta)$ represents the aggregate endowment of factor f, where $\bar{n}^{if,s}(\theta)$ denotes individual i's endowment of factor f. We parametrize $\bar{n}^{f \cdot s}(\theta)$ and $\bar{n}^{if,s}(\theta)$ by θ to consider perturbations to factor endowments. When needed, we denote the aggregate supply of factor f by $n^f = n^{f,s} + \bar{n}^{f,s}(\theta)$.

2.2 Feasible Allocations and Perturbations

Here we define a feasible allocation. Non-negativity constraints are critical for our results.

Definition. *(Feasible allocation). An allocation* $\left\{c^{ij}, n^{if,s}, x^{j\ell}, n^{jf,d}, y^{j,s}\right\}$ *is feasible if equations* (2) *through* (4) *hold and the non-negativity constraints* $c^{ij} \geq 0$, $n^{if,s} \geq 0$, $x^{j\ell} \geq 0$, $n^{jf,d} \geq 0$, *and* $y^{j,s} \geq 0$ *are satisfied.*

We assume that preferences and technologies are differentiable and that all variables are smooth functions of a perturbation parameter $\theta \in [0,1]$, so derivatives such as $\frac{dc^{ij}}{d\theta}$ or $\frac{dn^{jf,d}}{d\theta}$ are well-defined. We describe (standard) regularity conditions on preferences and technologies in the Appendix.

Feasible perturbations $d\theta$ have a dual interpretation. First, a perturbation may capture exogenous changes in technologies or endowments, but also changes in policies (e.g., taxes, subsidies, transfers, etc.) or any other primitive of a fully specified model (e.g., trade costs, markups, bargaining power, etc.). Under this interpretation, the mapping between allocations and θ emerges endogenously and accounts for equilibrium effects. Second, a perturbation may alternatively capture changes in feasible allocations directly chosen by a planner. This second interpretation is useful to characterize the set of efficient allocations, as in Section 4.

2.3 Social Welfare: Efficiency vs. Redistribution

We consider welfare assessments for welfarist planners, that is, planners with a social welfare function $\mathcal{W}(\cdot)$ given by

$$\text{(Social Welfare Function)} \qquad W = \mathcal{W}\left(V^1, \ldots, V^i, \ldots, V^I\right), \tag{5}$$

where $\frac{\partial \mathcal{W}}{\partial V^i} > 0$, $\forall i$, and where individual utilities V^i are defined in (1).[4] A welfare assessment can be expressed as

$$\frac{dW}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \frac{dV^i}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \lambda^i \frac{\frac{dV^i}{d\theta}}{\lambda^i}, \tag{6}$$

where λ^i is an individual normalizing factor that allows us to express individual welfare gains or losses in units of a common welfare numeraire. In particular, since the units of λ^i are

[4]The welfarist approach is widely used because it is Paretian, that is, it concludes that Pareto-improving perturbations are desirable, and because nonwelfarist approaches violate the Pareto principle (Kaplow and Shavell, 2001).

$\dim(\lambda^i) = \frac{\text{utils of individual } i}{\text{units of numeraire}}$, individual welfare gains or losses $\frac{dV^i}{d\theta}/\lambda^i$ are measured in units of the common welfare numeraire. The only restriction when choosing the welfare numeraire is that λ^i must be strictly positive for all individuals.[5]

Lemma 1 derives Dávila and Schaab (2025)'s efficiency/redistribution decomposition in our environment. This is the unique decomposition in which a normalized welfare assessment can be expressed as Kaldor-Hicks efficiency, Ξ^E, and its complement, Ξ^{RD}.

Lemma 1. *(Efficiency/Redistribution Decomposition) A normalized welfare assessment for a welfarist planner can be decomposed into efficiency and redistribution components, Ξ^E and Ξ^{RD}, as*

$$\underbrace{\frac{dW^\lambda}{d\theta}}_{\substack{\text{Welfare}\\ \text{Assessment}}} = \frac{\frac{dW}{d\theta}}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i} = \sum_i \omega^i \frac{\frac{dV^i}{d\theta}}{\lambda^i} = \underbrace{\sum_i \frac{\frac{dV^i}{d\theta}}{\lambda^i}}_{\Xi^E \text{ (Efficiency)}} + \underbrace{\mathbb{C}ov_i^\Sigma\left[\omega^i, \frac{\frac{dV^i}{d\theta}}{\lambda^i}\right]}_{\Xi^{RD} \text{ (Redistribution)}}, \tag{7}$$

where $\omega^i = \frac{\frac{\partial \mathcal{W}}{\partial V^i}\lambda^i}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i}$ and where $\mathbb{C}ov_i^\Sigma[\cdot,\cdot] = I \cdot \mathbb{C}ov_i[\cdot,\cdot]$ denotes a cross-sectional covariance-sum among all individuals.

The efficiency component Ξ^E corresponds to Kaldor-Hicks efficiency, that is, it is the sum of individual willingness-to-pay for the perturbation in units of the welfare numeraire. Hence, perturbations in which $\Xi^E > 0$ can be turned into Pareto improvements if transfers are feasible and costless. The redistribution component Ξ^{RD} captures the equity concerns embedded in a particular social welfare function: Ξ^{RD} is positive when individuals relatively favored in a perturbation are relatively preferred by the planner, that is, have a higher ω^i.

Two properties of this decomposition are worth highlighting. First, the efficiency component is invariant to i) the choice of social welfare function and ii) preference-preserving utility transformations, hence our focus on efficiency, relegating the study of the redistribution component to the Appendix. Second, efficient allocations feature a weakly negative efficiency component ($\Xi^E \leq 0$) for any feasible perturbation given endowments and technologies when the welfare numeraire could (hypothetically) be freely transferred. This property allows us to use Ξ^E to characterize the set of efficient allocations in Section 4.

[5]For instance, if good 1 is chosen as welfare numeraire, then $\lambda^i = \frac{\partial u^i}{\partial c^{i1}}$, $\forall i$. Alternatively, if a nominal unit (dollars) exists, it can alternatively be used as welfare numeraire.

3 Welfare Accounting: Efficiency

This section introduces the welfare accounting decomposition that identifies and quantifies the origins of efficiency gains. The efficiency component of a welfare assessment, Ξ^{E}, can be decomposed into exchange and production efficiency, $\Xi^{E,X}$ and $\Xi^{E,P}$, as follows:

$$\underbrace{\Xi^{E}}_{\text{Efficiency}} = \underbrace{\Xi^{E,X}}_{\substack{\text{Exchange}\\ \text{Efficiency}}} + \underbrace{\Xi^{E,P}}_{\substack{\text{Production}\\ \text{Efficiency}}}, \tag{8}$$

where both $\Xi^{E,X}$ and $\Xi^{E,P}$ can be further decomposed, as illustrated in Figure 1 and explained in detail in the remainder of this section.

3.1 Exchange Efficiency

Working with shares, rather than levels, is critical to distinguish gains due to reallocation from those due to changes in aggregates. We repeatedly apply this logic at each step. Formally, changes in individual consumption, c^{ij}, and factor supply, $n^{if,s}$, can be expressed as

$$\frac{dc^{ij}}{d\theta} = \frac{d\chi_{c}^{ij}}{d\theta} c^{j} + \chi_{c}^{ij} \frac{dc^{j}}{d\theta} \quad \text{and} \quad \frac{dn^{if,s}}{d\theta} = \frac{d\chi_{n}^{if,s}}{d\theta} n^{f,s} + \chi_{n}^{if,s} \frac{dn^{f,s}}{d\theta}, \tag{9}$$

where individual i's consumption share of good j is given by $\chi_{c}^{ij} = c^{ij}/c^{j}$, and individual i's factor supply share of factor f is given by $\chi_{n}^{if,s} = n^{if,s}/n^{f,s}$.[6]

Exchange efficiency captures efficiency gains associated with the reallocation of consumption and factor supply among individuals.

Theorem 1. *(Exchange Efficiency) Exchange efficiency, $\Xi^{E,X}$, can be decomposed into i) cross-sectional consumption efficiency and ii) cross-sectional factor supply efficiency, as*

$$\Xi^{E,X} = \underbrace{\sum_{j} \mathbb{C}ov_{i}^{\Sigma}\left[MRS_{c}^{ij}, \frac{d\chi_{c}^{ij}}{d\theta}\right] c^{j}}_{\substack{\text{Cross-Sectional}\\ \text{Consumption Efficiency}}} \underbrace{- \sum_{f} \mathbb{C}ov_{i}^{\Sigma}\left[MRS_{n}^{if}, \frac{d\chi_{n}^{if,s}}{d\theta}\right] n^{f,s}}_{\substack{\text{Cross-Sectional}\\ \text{Factor Supply Efficiency}}},$$

where individual i's marginal rates of substitution between good j and the numeraire, MRS_{c}^{ij},

[6] All definitions of shares in the body of the paper assume that denominators are positive. See Section B of the Appendix for formal definitions when denominators are zero.

and between factor f and the numeraire, MRS_n^{if}, are given by

$$MRS_c^{ij} = \frac{\frac{\partial u^i}{\partial c^{ij}}}{\lambda^i} \qquad \textit{and} \qquad MRS_n^{if} = -\frac{\frac{\partial u^i}{\partial n^{if,s}}}{\lambda^i}. \tag{10}$$

Cross-sectional consumption efficiency measures the contribution to efficiency due to reallocating consumption of good j from individuals with low to high MRS_c^{ij}, for given aggregate consumption c^j. Analogously, cross-sectional factor supply efficiency measures the contribution to efficiency due to reallocating the supply of factor f from individuals with high to low MRS_n^{if}, for given aggregate (elastic) supply of factor, $n^{f,s}$.

Corollary 1 presents several properties of practical relevance that exchange efficiency satisfies.

Corollary 1. *(Properties of Exchange Efficiency)*

(a) (Single Individual) In economies with a single individual ($I = 1$), exchange efficiency is zero.

(b) (No Elastic Factor Supply) In economies in which factors are not elastically supplied, so $n^{f,s} = 0$ for all factors, cross-sectional factor supply efficiency is zero.

(c) (Equalized MRS_c^{ij} or MRS_n^{if}) If marginal rates of substitution for good j (factor f) are identical across individuals for all goods (factors) with $c^j > 0$ ($n^{f,s} > 0$), then cross-sectional consumption (factor supply) efficiency is zero.

Note that exchange efficiency and redistribution are completely different notions, even though both require individual heterogeneity. In particular, the choice of social welfare function does not affect exchange efficiency but it directly impacts redistribution.

3.2 Production Efficiency

To distinguish gains due to reallocation from those due to changes in aggregates, we define good ℓ's *i) intermediate share*, $\phi_x^\ell = x^\ell / y^\ell$, which represents the share of good ℓ's aggregate supply y^ℓ devoted to production; ii) *intermediate-use share* used to produce good j, $\chi_x^{j\ell} = x^{j\ell}/x^\ell$, which represents the share of good ℓ's aggregate intermediate use devoted to the production of good j; and *iii) intermediate-supply share* used to produce j by $\xi^{j\ell} = \chi_x^{j\ell}\phi_x^\ell = x^{j\ell}/y^\ell$, as the product of the two.

Hence, changes in intermediate use are given by

$$\frac{dx^{j\ell}}{d\theta} = \frac{d\xi^{j\ell}}{d\theta} y^{\ell} + \xi^{j\ell} \frac{dy^{\ell}}{d\theta}, \qquad \text{where} \qquad \frac{d\xi^{j\ell}}{d\theta} = \frac{d\chi_x^{j\ell}}{d\theta} \phi_x^{\ell} + \chi_x^{j\ell} \frac{d\phi_x^{\ell}}{d\theta}. \tag{11}$$

We also define the *factor use share* of factor f used to produce good j, $\chi_n^{jf,d} = n^{jf,d}/n^{f,d}$, so changes in factor use shares are given by

$$\frac{dn^{jf,d}}{d\theta} = \frac{d\chi_n^{jf,d}}{d\theta} n^{f,d} + \chi_n^{jf,d} \frac{dn^{f,d}}{d\theta}. \tag{12}$$

3.2.1 Network Propagation: Goods Inverse Matrix

To study production efficiency it is necessary to understand how perturbations propagate through the production network of goods. Lemma 2 introduces the goods inverse matrix $\boldsymbol{\Psi}_y$, which characterizes the ultimate change in the aggregate supply of goods induced by unit impulses in the supply of goods.[7]

Lemma 2. *(Goods Inverse Matrix). Changes in good j's aggregate supply $\frac{dy^j}{d\theta}$ can be expressed in terms of changes in intermediate-supply shares $\frac{d\xi^{j\ell}}{d\theta}$, changes in factor use $\frac{dn^{jf,d}}{d\theta}$, changes in the good endowment $\frac{d\bar{y}^{j,s}}{d\theta}$, and changes in technology $\frac{\partial G^j}{\partial\theta}$, as*

$$\frac{dy^j}{d\theta} = \underbrace{\sum_{\ell} \frac{\partial G^j}{\partial x^{j\ell}} \xi^{j\ell} \frac{dy^{\ell}}{d\theta}}_{\text{Propagation}} + \underbrace{\sum_{\ell} \frac{\partial G^j}{\partial x^{j\ell}} \frac{d\xi^{j\ell}}{d\theta} y^{\ell} + \sum_{f} \frac{\partial G^j}{\partial n^{jf,d}} \frac{dn^{jf,d}}{d\theta} + \frac{d\bar{y}^{j,s}}{d\theta} + \frac{\partial G^j}{\partial\theta}}_{\text{Impulse}}. \tag{13}$$

Equivalently, in matrix form,

$$\frac{d\boldsymbol{y}}{d\theta} = \underbrace{\boldsymbol{\Psi}_y}_{\text{Propagation}} \underbrace{\left(\boldsymbol{G}_x \frac{d\boldsymbol{\xi}}{d\theta} \boldsymbol{y} + \boldsymbol{G}_n \frac{d\mathring{\boldsymbol{n}}^d}{d\theta} + \frac{d\bar{\boldsymbol{y}}^s}{d\theta} + \boldsymbol{G}_\theta \right)}_{\text{Impulse}} \qquad \textit{where} \qquad \underbrace{\boldsymbol{\Psi}_y = (\boldsymbol{I}_J - \boldsymbol{G}_x \boldsymbol{\xi})^{-1}}_{\text{Goods Inverse}}, \tag{14}$$

where $\frac{d\boldsymbol{y}}{d\theta}$ and $\frac{d\bar{\boldsymbol{y}}^s}{d\theta}$ respectively denote the $J \times 1$ vectors of $\frac{dy^j}{d\theta}$ and $\frac{d\bar{y}^{j,s}}{d\theta}$, and $\boldsymbol{\Psi}_y = (\boldsymbol{I}_J - \boldsymbol{G}_x \boldsymbol{\xi})^{-1}$ defines the $J \times J$ goods inverse matrix. The remaining matrices are defined in Appendix A.

Lemma 2 characterizes how the aggregate supply of goods ultimately changes in response to the four "impulse" terms of equation (13), which represent the first-round impact of the

[7] We introduce two related propagation matrices in the Appendix: the intermediate inverse matrix $\boldsymbol{\Psi}_x$, which characterizes network propagation for changes in the level of intermediates; and the proportional goods inverse matrix $\tilde{\boldsymbol{\Psi}}_y = \hat{\boldsymbol{y}}^{-1} \boldsymbol{\Psi}_y \hat{\boldsymbol{y}}$, where $\hat{\boldsymbol{y}} = \text{diag}(\boldsymbol{y})$, which characterizes network propagation for proportional impulses in the supply of goods.

perturbation on the supply of goods. A perturbation that changes intermediate-supply shares by $\frac{d\xi^{j\ell}}{d\theta}$ raises at impact the amount of good ℓ used as input for good j in proportion to y^{ℓ}, which in turn increases output at impact by $\frac{\partial G^j}{\partial x^{j\ell}}$. Similarly, a perturbation that changes the use of factor f in the production of good j by $\frac{dn^{jf,d}}{d\theta}$ increases output at impact by $\frac{\partial G^j}{\partial n^{jf,d}}$. Changes in the endowment or the technology used to produce good j simply increase aggregate supply at impact by $\frac{d\bar{y}^{j,s}}{d\theta}$ or $\frac{\partial G^j}{\partial \theta}$, respectively.

Such first-round changes in the level of aggregate supply in turn induce further changes in the level of intermediate inputs, which in turn induce further changes in aggregate supply. These knock-on effects through the production network are captured by the goods inverse matrix $\boldsymbol{\Psi}_y$. Under minimal regularity conditions — described in the Appendix — $\boldsymbol{\Psi}_y$ admits the series representation

$$\boldsymbol{\Psi}_y = (\boldsymbol{I}_J - \boldsymbol{G}_x \boldsymbol{\xi})^{-1} = \boldsymbol{I}_J + \boldsymbol{G}_x \boldsymbol{\xi} + (\boldsymbol{G}_x \boldsymbol{\xi})^2 + (\boldsymbol{G}_x \boldsymbol{\xi})^3 + \dots \, . \tag{15}$$

The first term in the expansion, $\boldsymbol{I}_J$, represents the first round of aggregate supply changes just described. As aggregate supply adjusts, the level of intermediate inputs $x^{j\ell}$ changes in proportion to the intermediate-supply share $\xi^{j\ell}$, or $\boldsymbol{\xi}$ in matrix form. In turn, changes in the level of intermediate inputs translate into a second round of changes in aggregate supply in proportion to the marginal products of each input $\frac{\partial G^j}{\partial x^{j\ell}}$, or $\boldsymbol{G}_x$ in matrix form. This explains the second term $\boldsymbol{G}_x \boldsymbol{\xi}$ in (15), which generates knock-on effects in proportion to $(\boldsymbol{G}_x \boldsymbol{\xi})^2$ and so on.

The following remark highlights how the goods inverse matrix differs from propagation matrices identified in the literature.

Remark 1. *(Goods Inverse Matrix is Purely Technological)* While propagation matrices abound in the study of models with rich production structures — see e.g. Carvalho and Tahbaz-Salehi (2019) — the goods inverse matrix introduced in Lemma 2 is distinct in the sense that it is purely a technological object at a given allocation.[8] That is, $\boldsymbol{\Psi}_y$ is exclusively based on production technologies. This is important because $\boldsymbol{\Psi}_y$ will be a key input when characterizing efficiency conditions in Section 4. In competitive economies, the goods inverse matrix $\boldsymbol{\Psi}_y$ will be related to well-known Leontief-style inverses that depend on prices (and wedges), as explained in Section 5.

[8]The allocations at which the goods inverse matrix is computed could themselves depend on the underlying economic or market structure.

3.2.2 Defining $AMRS$, MSV, MWP, and $AMWP$

Decomposing production efficiency requires defining the sets of variables that translate changes in allocations, technologies, and endowments into welfare changes: $AMRS$, MSV, MWP, and $AMWP$.

Definition. *(Aggregate Marginal Rate of Substitution). The aggregate marginal rate of substitution (AMRS) between good j and the numeraire and between factor f and the numeraire is given by*

$$AMRS_c^j = \sum_i \chi_c^{ij} MRS_c^{ij} \quad \text{and} \quad AMRS_n^f = \sum_i \chi_n^{if,s} MRS_n^{if,s}. \tag{16}$$

The aggregate marginal rate of substitution for good j corresponds to the efficiency gain associated with increasing aggregate consumption of good j by a unit, making individuals consume in proportion to their consumption shares. The aggregate marginal rate of substitution for factor f corresponds to the welfare cost associated with increasing the aggregate supply of factor f by a unit, making individuals supply the factor in proportion to their factor supply shares.

Definition. *(Marginal Social Value of Goods). The marginal social value of good j, MSV_y^j, is defined as the j'th element of the $1 \times J$ vector $\boldsymbol{MSV}_y$, given by*

$$\boldsymbol{MSV}_y = \boldsymbol{AMRS}_c \boldsymbol{\phi}_c \boldsymbol{\Psi}_y, \tag{17}$$

where $\boldsymbol{AMRS}_c$ is a $1 \times J$ vector of $AMRS_c^j$; $\boldsymbol{\phi}_c$ is the $J \times J$ diagonal matrix of aggregate consumption shares, with $\phi_c^j = \frac{c^j}{y^j}$; and $\boldsymbol{\Psi}_y$ is the $J \times J$ goods inverse matrix defined in (14).

The marginal social value of good j captures the efficiency gain associated with having an additional unit of that good. While a unit impulse in the supply of goods generates an ultimate increase in the aggregate supply of goods given by the goods inverse matrix $\boldsymbol{\Psi}_y$, only the aggregate consumption share $\boldsymbol{\phi}_c$ is consumed by individuals. And $\boldsymbol{AMRS}_c$ captures the gain associated with increasing aggregate consumption, so the marginal social value of an impulse in the supply of goods is the product of these three objects. The definition of MSV highlights that the social value of a good emanates from the final consumption — potentially of other goods — it ultimately generates.

Definition. *(Marginal Welfare Product). The marginal welfare products (MWP) of input ℓ and factor f for technology j are given by*

$$MWP_x^{j\ell} = MSV_y^j \frac{\partial G^j}{\partial x^{j\ell}} \quad \text{and} \quad MWP_n^{jf} = MSV_y^j \frac{\partial G^j}{\partial n^{jf,d}}. \tag{18}$$

Marginal welfare products capture the efficiency gain associated with using an input or factor in the production of a good. Marginal increases in $x^{j\ell}$ or $n^{jf,d}$ increase output at impact by their technological marginal products, $\frac{\partial G^j}{\partial x^{j\ell}}$ and $\frac{\partial G^j}{\partial n^{jf,d}}$. As just described, the social value of a unit impulse in the supply of goods is summarized by the marginal social value of goods, MSV_y^j. Hence, marginal welfare products of inputs and factors are given by the product of physical marginal products and the marginal social value of the goods produced.

Definition. *(Aggregate Marginal Welfare Product). The aggregate marginal welfare product ($AMWP$) of good j and factor f, respectively, are given by*

$$AMWP_x^{\ell} = \sum_j \chi_x^{j\ell} MWP_x^{j\ell} \quad \text{and} \quad AMWP_n^{f} = \sum_j \chi_n^{jf,d} MWP_n^{jf}. \tag{19}$$

The aggregate marginal welfare product for good ℓ corresponds to the efficiency gain associated with increasing the aggregate intermediate use of good ℓ in proportion to the intermediate use shares. The aggregate marginal welfare product for factor f corresponds to the welfare gain associated with increasing the use of factor f in proportion to the factor use shares.

3.2.3 Production Efficiency Decomposition

Production efficiency gains ultimately correspond to higher aggregate consumption and lower aggregate factor supply, since $\Xi^{E,P}$ is given by

$$\Xi^{E,P} = \sum_j AMRS_c^j \frac{dc^j}{d\theta} - \sum_f AMRS_n^f \frac{dn^{f,s}}{d\theta}.$$

Part of the contribution of Theorem 2 is to express changes in aggregate consumption net of factor supply costs in terms of changes in the allocation of intermediates and factors (allocative efficiency), and primitive changes in technologies and endowments (technical efficiency).

Theorem 2. *(Production Efficiency). Production efficiency $\Xi^{E,P}$ can be decomposed into i) cross-sectional intermediate input efficiency, ii) aggregate intermediate input efficiency, iii)* *cross-sectional factor use efficiency, iv) aggregate factor efficiency, v) technology change, vi) good endowment change, and vii) factor endowment change, as*

$$\Xi^{E,P} = \overbrace{\underbrace{\sum_{\ell} \mathbb{C}ov_j^{\Sigma}\left[MWP_x^{j\ell}, \frac{d\chi_x^{j\ell}}{d\theta}\right] x^{\ell}}_{\substack{\text{Cross-Sectional} \\ \text{Intermediate Input Efficiency}}} + \underbrace{\sum_{\ell}\left(AMWP_x^{\ell} - AMRS_c^{\ell}\right)\frac{d\phi_x^{\ell}}{d\theta} y^{\ell}}_{\substack{\text{Aggregate} \\ \text{Intermediate Input Efficiency}}}}^{\text{Intermediate Input Efficiency}}$$

$$+ \overbrace{\underbrace{\sum_{f} \mathbb{C}ov_j^{\Sigma}\left[MWP_n^{jf}, \frac{d\chi_n^{jf,d}}{d\theta}\right] n^{f,d}}_{\substack{\text{Cross-Sectional} \\ \text{Factor Use Efficiency}}} + \underbrace{\sum_{f}\left(AMWP_n^{f} - AMRS_n^{f}\right)\frac{dn^{f,s}}{d\theta}}_{\substack{\text{Aggregate} \\ \text{Factor Efficiency}}}}^{\text{Factor Efficiency}}$$

$$+ \underbrace{\sum_j MSV_y^j \frac{\partial G^j}{\partial \theta}}_{\substack{\text{Technology} \\ \text{Change}}} + \underbrace{\sum_j MSV_y^j \frac{d\bar{y}^{j,s}}{d\theta}}_{\substack{\text{Good Endowment} \\ \text{Change}}} + \underbrace{\sum_f AMWP_n^f \frac{d\bar{n}^{f,s}}{d\theta}}_{\substack{\text{Factor Endowment} \\ \text{Change}}}.$$

Each component of the production efficiency decomposition quantifies the contribution to Kaldor-Hicks efficiency of changes in allocations or primitives. Cross-sectional components correspond to covariances across uses, measuring gains from reallocating intermediate inputs or factors from low to high marginal welfare product uses, for given levels of aggregate intermediate use or factor use.

The aggregate intermediate input efficiency component measures the gains from adjusting the share of aggregate goods supply devoted to final consumption relative to production, for a given level of aggregate goods supply. Hence, for good ℓ it is shaped by the product of the difference $AMWP_x^{\ell} - AMRS_c^{\ell}$ and the change in the intermediate use share, $\frac{d\phi_x^{\ell}}{d\theta} y^{\ell}$. The aggregate factor efficiency component measures the gains from adjusting the elastic supply of factors. Hence, for factor f it is shaped by the product of the difference between $AMWP_n^f - AMRS_n^f$ and the change in the factor supply, $\frac{dn^{f,s}}{d\theta}$.

The final three components measure welfare gains due to primitive changes in technology and endowments, for given allocations. The gain from changes in the technology or endowment of good j is given by its marginal social value, MSV_y^j. The gain from changes in the endowment of factor f is simply given by the marginal gain associated with increasing factor use, $AMWP_n^f$.

Corollary 2 presents several properties that production efficiency satisfies.

Corollary 2. *(Properties of Production Efficiency)*

(a) *(Single Good Economies) In economies with a single good ($J = 1$), cross-sectional intermediate input efficiency and cross-sectional factor use efficiency are zero.*

(b) *(No Intermediate Input Economies) In economies with no intermediate goods ($x^{j\ell} = \xi^{j\ell} = 0$), cross-sectional and aggregate intermediate input efficiency are zero.*

(c) *(Fixed Factor Supply Economies) In economies in which all factors are in fixed supply ($\frac{dn^{f,s}}{d\theta} = 0$), aggregate factor efficiency is zero.*

(d) *(Specialized Intermediate/Factor Economies) In economies in which all intermediate inputs (factors) are specialized with $\chi_x^{j\ell} = 1$ ($\chi_n^{jf} = 1$) for some j, cross-sectional intermediate input (factor use) efficiency is zero.*

(e) *(Equalized $MWP_x^{j\ell}$ or MWP_n^{jf}) If marginal welfare products for good ℓ (factor f) are identical across uses for all goods (factors) with $x^\ell > 0$ ($n^{f,d} > 0$), then cross-sectional intermediate (factor use) efficiency is zero.*

3.3 Remarks on the Welfare Accounting Decomposition

We highlight several insights that emerge from the welfare accounting decomposition.

Remark 2. *(Technological and Preference Origins of Efficiency Gains).* Theorems 1 and 2 trace the origins of efficiency gains under any perturbation to changes in the allocation of resources and to primitive changes in technology and endowments. Since this decomposition is purely based on preferences, technologies, and resource constraints, it is useful to quantify, compare, and contrast different economic environments, e.g. competitive, strategic, search, bargaining, contracting, etc.

Remark 3. *(Social Value of Technology).* Theorem 2 identifies the efficiency gains from pure technological change with MSV_y^j, without making assumptions about the individual behavior, budget constraints, prices, or equilibrium notions. The technology change component of the welfare accounting decomposition is always positive if technology improves since $MSV_y^j > 0$. However, a technological improvement can feature a negative efficiency component if its impact on allocative efficiency is sufficiently negative, which can only happen at inefficient allocations.

Remark 4. *(Rationale and Uniqueness of the Decomposition).* By design, the allocative efficiency components of the welfare accounting decomposition are systematically expressed in terms of changes in allocation shares, with the exception of aggregate factor efficiency. Working with shares allows us to separate changes due to reallocation (holding consumption, factor supply, goods supply, intermediate input use, or factor use fixed) from changes in aggregates (aggregate factor supply, technology, or endowments). This separation would not possible if the decomposition were expressed in levels. Consequently, this is the unique decomposition that, at each step, separates changes in variables into components reflecting changes in shares and changes in aggregates.

4 Pareto Efficient Allocations

In this section, we leverage the welfare accounting decomposition to characterize the set of Pareto efficient allocations.

4.1 Efficiency Conditions

An allocation is Pareto efficient if there is no feasible perturbation in which any of the allocative efficiency components in Theorems 1 and 2 is positive. While a version of the exchange efficiency conditions in Theorem 3 already appears in Mas-Colell et al. (1995), the production efficiency conditions in Theorem 4 are novel, yielding a new set of insights.

Theorem 3. *(Efficiency Conditions: Exchange Efficiency). An efficient allocation satisfies the following exchange efficiency conditions:*

(a) (Cross-sectional consumption efficiency) For goods with $c^j > 0$, it must be that $MRS_c^{ij} = AMRS_c^j$, $\forall i$ s.t. $\chi_c^{ij} > 0$; and $MRS_c^{ij} \leq AMRS_c^j$, $\forall i$ s.t. $\chi_c^{ij} = 0$.

(b) (Cross-sectional factor supply efficiency) For factors with $n^{f,s} > 0$, it must be that $MRS_n^{if} = AMRS_n^f$, $\forall i$ s.t. $\chi_n^{if,s} > 0$; and $MRS_n^{if} \geq AMRS_n^f$, $\forall i$ s.t. $\chi_n^{if,s} = 0$.

Pareto efficiency requires the equalization of MRS_c^{ij} across all consumers of good j, with MRS_c^{ij} potentially lower for individuals for whom $c^{ij} = 0$. Otherwise, it is feasible and welfare-improving to reallocate consumption from low to high MRS_c^{ij} individuals, for given aggregate consumption c^j. At the corner where individual i does not consume good j, it is not feasible to reallocate consumption away from individual i, even though marginal rates of substitution are not equalized. The same logic applies to factor supply.

Theorem 4. *(Efficiency Conditions: Production Efficiency). An efficient allocation satisfies the following production efficiency conditions:*

(a) (Cross-sectional intermediate input efficiency) For goods with $x^{\ell} > 0$, it must be that $MWP_x^{j\ell} = AMWP_x^{\ell}$, $\forall j$ s.t. $\chi_x^{j\ell} > 0$; and $MWP_x^{j\ell} \leq AMWP_x^{\ell}$, $\forall j$ s.t. $\chi_x^{j\ell} = 0$.

(b) (Aggregate intermediate input efficiency) For goods with $y^{\ell} > 0$, it must be that $\max_j \left\{MWP_x^{j\ell}\right\} \leq AMRS_c^{\ell}$, $\forall \ell$ s.t. $\phi_x^{\ell} = 0$; $AMWP_x^{\ell} = AMRS_c^{\ell}$, $\forall \ell$ s.t. $\phi_x^{\ell} \in (0,1)$; and $AMWP_x^{\ell} \geq \max_i \left\{MRS_c^{i\ell}\right\}$, $\forall \ell$ s.t. $\phi_x^{\ell} = 1$.

(c) (Cross-sectional factor use efficiency) For factors with $n^{f,d} > 0$, it must be that $MWP_n^{jf} = AMWP_n^{f}$, $\forall j$ s.t. $\chi_n^{jf} > 0$; and $MWP_n^{jf} \leq MWP_n^{f}$, $\forall j$ s.t. $\chi_n^{jf} = 0$.

(d) (Aggregate factor efficiency) For factors with $n^{f,d} > 0$, it must be that $AMWP_n^{f} = AMRS_n^{f}$, $\forall f$ s.t. $n^{f,s} > 0$; and $AMWP_n^{f} \leq \min_i \left\{MRS_n^{if}\right\}$, $\forall f$ s.t. $n^{f,s} = 0$.

Pareto efficiency requires the equalization of $MWP_x^{j\ell}$ across all uses of good ℓ in production. Otherwise, it is feasible and welfare-improving to reallocate intermediate inputs from low to high $MWP_x^{j\ell}$ uses, for given aggregate intermediate input use x^{ℓ}. When good ℓ is not used to produce good j, $MWP_x^{j\ell}$ must be weakly lower. The same logic applies to the allocation of a factor across uses in 4c).

Pareto efficiency also requires the equalization of the marginal rate of substitution from consuming good ℓ with its marginal welfare product as an input for mixed goods with $\phi_x^{\ell} \in (0,1)$, with inequalities for pure final ($\phi_x^{\ell} = 0$) and pure intermediate goods ($\phi_x^{\ell} = 1$). Similarly, efficiency requires the equalization of the marginal welfare product of elastic factor f with its marginal rate of substitution, which captures the utility cost of supplying the factor, whenever a factor is elastically supplied.

Theorems 3 and 4 highlight that carefully incorporating non-negativity constraints is critical to characterize efficiency conditions in disaggregated economies. These issues become more relevant at finer levels of disaggregation, since heterogeneous individuals typically do not consume most goods and production networks with heterogeneous producers become increasingly sparse. We elaborate on these issues in the remainder of this section.

4.2 Classical Efficiency Conditions: Interior Economies

Section 16.F of Mas-Colell et al. (1995) summarizes the classical efficiency conditions — typically traced back to Lange (1942). Theorems 3 and 4 generalize these classical conditions to general environments with disaggregated production.

Definition. *(Classical Efficiency Conditions). The classical (production) efficiency conditions for an intermediate link $j\ell$ and a factor link jf hold if*

$$MRS_c^{ij}\frac{\partial G^j}{\partial x^{j\ell}} = MRS_c^{i\ell} \quad \text{and} \quad MRS_c^{ij}\frac{\partial G^j}{\partial n^{jf,d}} = MRS_n^{if}. \tag{20}$$

Critically, the classical approach exclusively studies interior production economies, in which every good is mixed and used in the production of every other good, i.e., $\chi_x^{j\ell} \in (0,1)$ and $\phi_x^{\ell} \in (0,1)$. In that case, the classical efficiency conditions in (20) imply i) equalized marginal rates of substitution across individuals, ii) equalized marginal rates of transformation (MRT) across goods, and iii) the equalization of MRS with MRT. Corollary 3 shows that classical efficiency conditions are a special case of Theorems 3 and 4 in interior economies.

Corollary 3. *(Interior economies). In interior economies, the efficiency conditions of Theorems 3 and 4 collapse to those in Section 16.F of Mas-Colell et al. (1995).*

Next, we show that the classical efficiency conditions are typically invalid in disaggregated production economies that are not interior.

4.3 Failure of Classical Efficiency Conditions: Non-Interior Economies

What then distinguishes the conditions for production efficiency in non-interior economies, and why do the classical conditions not apply to these environments?

Consider increasing $x^{j\ell}$, the use of good ℓ in the production of good j. Assuming this is a feasible perturbation, efficiency requires that its social cost — the marginal social value of good ℓ — is equalized with its social benefit — the marginal social value of good j multiplied by the marginal product $\frac{\partial G^j}{\partial x^{j\ell}}$. The classical efficiency conditions (20) use marginal rates of substitution to measure the social benefit (20 LHS) and cost (20 RHS). This is appropriate for interior efficient economies where all goods are mixed, since $MSV = MRS$ for final goods as we showed above. When j or ℓ is a pure intermediate, however, marginal rates of substitution no longer represent the good's marginal social value, even at an efficient allocation. Since pure intermediates are not consumed, efficiency requires their MRS to be lower than their MSV. The marginal social value of a pure intermediate instead derives from the consumption value it eventually generates downstream as it is used in the production of other goods throughout the network.

There is a second, more mechanical reason why the classical efficiency conditions do not extend to non-interior economies. If good ℓ is not used in the production of good j, efficiency at the $j\ell$ link then requires that $MWP_x^{j\ell}$ be lower than the marginal social value of good ℓ.

We summarize the implications of Theorems 3 and 4 for non-interior economies in two corollaries. Corollary 4 concludes that the classical efficiency conditions hold at the level of an intermediate input link whenever that link itself is interior.

Corollary 4. *(Classical Efficiency Conditions Hold for Interior Links). The classical efficiency conditions hold for the $j\ell$ and jf links when*

(a) a mixed good ℓ is used to produce a mixed (or a pure final) good j,

(b) an elastically supplied factor f is used to produce a mixed (or a pure final) good j.

Intuitively, the classical efficiency conditions (20) extend to all interior links $j\ell$ and jf because the MSV of mixed goods coincides with their MRS, even when there are non-interior links elsewhere in the network. Corollary 5 characterizes the scenarios in which the classical conditions fail to hold.

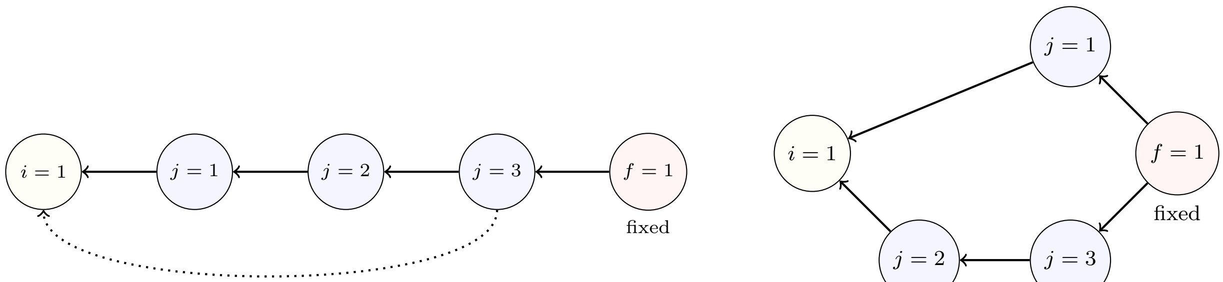
Corollary 5. *(Scenarios in which Classical Efficiency Conditions Do Not Hold). The classical efficiency conditions generically fail to hold for links $j\ell$ and jf that feature pure intermediate goods, i.e.,*

(a) a mixed good ℓ is used to produce a pure intermediate good j,

(b) a pure intermediate good ℓ is used to produce any good j,

(c) a factor f is used to produce a pure intermediate good j.

Trivially, the classical conditions also fail to hold for links $j\ell$ and jf when good ℓ and factor f are not used in the production of good j.



(a) Example 1

(b) Example 2

Figure 2: Scenarios in which Classical Efficiency Conditions Do Not Hold

Note: This figure illustrates Corollary 5 in two simple scenarios. The left panel shows a mixed good (good 3) used to produce a pure intermediate (good 2), as well as a a pure intermediate (good 2) used to produce a final good (good 1). The right panel shows a factor used to produce both a pure intermediate (good 3) and a final good (good 1).

The first and third items of Corollary 5 highlight that the classical efficiency conditions may fail at links in which the efficiency conditions take the form of an *equality*, as long as an intermediate good is produced. This observation implies that properly characterizing production efficiency is more subtle than simply considering a set of inequalities, as in the case of exchange efficiency.

We illustrate Corollary 5 in two simple examples — see also Figure 2.

Example 1. *(Pure Intermediates).* Example 1 features a single individual ($I = 1$), three goods ($J = 3$), and a single factor in fixed supply ($F = 1$). The individual's preferences are $V^1 = u^1\left(c^{11}, c^{13}\right)$, which implies that $MRS^{12} = 0$. Technologies for each of the goods are $y^1 = G^1\left(x^{12}\right)$, $y^2 = G^2\left(x^{23}\right)$, and $y^3 = G^3\left(n^{31,d}\right)$, which already imposes that many marginal products are zero, e.g., $\frac{\partial G^1}{\partial x^{13}} = 0$.

The welfare accounting decomposition for this economy only features aggregate intermediate input efficiency: exchange efficiency is zero since $I = 1$, cross-sectional intermediate input and factor use efficiency are zero since all inputs and factors are specialized, and aggregate factor efficiency is zero since the single factor is in fixed supply.[9] Plugging into

[9]Formally, we assume here that the efficient production structure is as in Figure 2a. The full set of efficiency conditions also features inequalities to ensure that, for example, it is not efficient to consume good 2 or use it in the production of good 3.

Theorem 2,

$$\Xi^{E}=\Xi^{E,P}=\sum_{\ell}\left(AMWP_{x}^{\ell}-AMRS_{c}^{\ell}\right)\frac{d\phi_{x}^{\ell}}{d\theta}y^{\ell}=\left(\underbrace{MRS_{c}^{11}\frac{\partial G^{1}}{\partial x^{12}}\frac{\partial G^{2}}{\partial x^{23}}}_{AMWP_{x}^{3}}-\underbrace{MRS_{c}^{13}}_{AMRS_{c}^{3}}\right)\frac{d\phi_{x}^{3}}{d\theta}y^{3}.$$

For the mixed good 3 with $\phi_x^3 \in (0,1)$, aggregate intermediate input efficiency requires that $AMWP_x^3 = AMRS_c^3$, or equivalently $MRS_c^{11}\frac{\partial G^1}{\partial x^{12}}\frac{\partial G^2}{\partial x^{23}} = MRS_c^{13}$. The classical efficiency condition would instead require $MRS_c^{12}\frac{\partial G^2}{\partial x^{23}} = MRS_c^{13}$, which is invalid since good 2 is a pure intermediate and $MRS_c^{11}\frac{\partial G^1}{\partial x^{12}} > MRS_c^{12} = 0$. At the efficient allocation, the classical condition would lead one to conclude good 3's intermediate use is inefficiently high. This illustrates Corollary 5a.

This example also illustrates Corollary 5b since it features a pure intermediate (good 2) that is used in the production of another good. Since $\phi_x^2 = 1$, aggregate intermediate input efficiency requires that $MRS_c^{11}\frac{\partial G^1}{\partial x^{12}} > MRS^{12} = 0$, i.e., the consumption value of good 2 must be lower than its production value. The classical efficiency condition $MRS_c^{11}\frac{\partial G^1}{\partial x^{12}} = MRS_c^{12}$ would lead one to conclude that, at the efficient allocation, $MSV_y^2 = AMWP_x^2 = AMRS_c^2$, which would be incorrect.

Example 2. *(Factor Used to Produce Pure Intermediate).* Example 2 features one individual ($I = 1$), three goods ($J = 3$), and one factor in fixed supply ($F = 1$). Preferences are $V^1 = u^1\left(c^{11}, c^{12}\right)$ and technologies for each of the goods are $y^1 = G^1\left(n^{11,d}\right)$, $y^2 = G^2\left(x^{23}\right)$, and $y^3 = G^3\left(n^{31,d}\right)$.

The welfare accounting decomposition for this economy only features cross-sectional factor use efficiency: exchange efficiency is zero since $I = 1$, cross-sectional intermediate input efficiency is zero since all inputs are specialized, aggregate factor efficiency is zero since the single factor is in fixed supply, and aggregate intermediate input efficiency is zero since $\phi_c^1 = \phi_x^2 = \phi_x^3 = 1$ by construction. Therefore,

$$\Xi^{E}=\Xi^{E,P}=\mathbb{C}ov_{j}^{\Sigma}\left[MWP_{n}^{j1},\frac{d\chi_{n}^{j1,d}}{d\theta}\right]n^{1,d}=\left(MSV_{y}^{1}\frac{\partial G^{1}}{\partial n^{11,d}}\frac{d\chi_{n}^{11,d}}{d\theta}+MSV_{y}^{3}\frac{\partial G^{3}}{\partial n^{31,d}}\frac{d\chi_{n}^{31,d}}{d\theta}\right)n^{1,d},$$

where $MSV_y^1 = MRS_c^{11}$ and $MSV_y^3 = MRS_c^{12}\frac{\partial G^2}{\partial x^{23}}$. Since labor is in fixed supply but used in the production of two goods, a feasible perturbation is $\frac{d\chi_n^{11,d}}{d\theta} = -\frac{d\chi_n^{31,d}}{d\theta}$. Cross-sectional factor use efficiency therefore requires that $MRS_c^{11}\frac{\partial G^1}{\partial n^{11,d}} = MRS_c^{12}\frac{\partial G^2}{\partial x^{23}}\frac{\partial G^3}{\partial n^{11,d}}$. The classical efficiency condition would instead associate the marginal social value of pure intermediate good 3 with its MRS and require $MRS_c^{11}\frac{\partial G^1}{\partial n^{11,d}} = MRS_c^{13}\frac{\partial G^3}{\partial n^{31,d}}$. Since $MRS_c^{12}\frac{\partial G^2}{\partial x^{23}} > MRS_c^{13} = 0$ at the efficient allocation, the classical condition would lead one to conclude the use of labor in the production of good 3 is inefficiently high, illustrating Corollary 5c.

We conclude the study of non-interior economies with a remark that highlights the importance of characterizing efficiency conditions in terms of MWP and MRS instead of MRS and MRT.

Remark 1. *($MWP \gtreqless MRS$ generalizes $MRS \gtreqless MRT$).* One central takeaway from this section is that MWP and MRS are the appropriate objects to characterize efficiency conditions, rather than MRS and MRT, as in the classical approach. For instance, when good ℓ is mixed or factor f is in elastic supply, efficiency requires that

$$MWP_x^{j\ell} = MRS_c^{i\ell} \quad \text{and} \quad MWP_n^{jf} = MRS_n^{if}, \tag{21}$$

for all i such that $\chi_c^{ij} > 0$ and for all j such that $\chi_x^{j\ell} > 0$, but the classical efficiency conditions in (20) would not be valid if j is a pure intermediate. More generally, the correct inequalities that characterize production efficiency — see Theorem 2 — can be written in terms of MWP and MRS, but not MRS and MRT.

4.4 Planning Problem, Lagrange Multipliers, and Socialist Calculation Debate

We have emphasized that the welfare accounting decomposition can be leveraged to derive efficiency conditions directly. Equivalently, each allocative efficiency component maps directly into an optimality condition of the planning problem.

Definition. *(Planning Problem). The planning problem — formally stated in Appendix E.1 — maximizes the social welfare function in (5), with preferences V^i defined in (1), subject to technologies and resource constraints, as well as non-negativity constraints.*

There are two reasons why studying the planning problem is useful. First, it provides an equivalent characterization of the efficiency conditions in Theorems 3 and 4. As we show in the Appendix, the restriction to feasible perturbations that underlies our characterization of efficiency conditions is implied by the Kuhn-Tucker multipliers on the constraints of the planning problem. Second, and more importantly, each component of the decomposition can be interpreted as a particular perturbation of the planning problem, as we show in the Appendix. Hence, the planning problem provides a justification for the different components of the welfare accounting decomposition.

Two implications of our new characterization of efficiency conditions are worth highlighting.

Remark 2. *(MSV_y^j and $AMWP_n^f$ as Lagrange Multipliers on Resource Constraints).* The planning problem provides an interpretation of the technology change (and good endowment change) and factor endowment change components of the welfare accounting decomposition in terms of the Lagrange multipliers on goods and factors resource constraints: ζ_y^j and ζ_n^f, since $\zeta_y^j = MSV_y^j$ when $y^j \neq 0$ and $\zeta_n^f = AMWP_n^f$ when $n^{f,d} \neq 0$. To our knowledge, our results provide the first characterization of the Lagrange multipliers of the planning problem in general disaggregated economies.

Remark 3. *(Socialist Calculation Debate with Intermediate Goods).* Our characterization of efficiency conditions directly speaks to the socialist calculation debate, which discusses the feasibility of central planning — see e.g. Lange (1936), Lerner (1944), or Hayek (1945). Our results illustrate how computing efficiency conditions in production economies is significantly more challenging than efficiently allocating goods across individuals, especially in economies that feature pure intermediates. In particular, our results imply that computing MSV_y^j for pure intermediates requires knowledge of the entire production network — to compute the goods inverse matrix $\boldsymbol{\Psi}_y$ — while computing MSV_y^j for mixed or pure final goods only requires knowledge of aggregate individual valuations via marginal rates of substitution. Intuitively, the value of goods that are consumed by individuals can be ascertained from individual valuations, even when these goods also used to produce, while pure intermediates only derive value once eventually consumed.

This observation can be used to support the hypothesis that the losses associated with planning increase with the complexity of production networks, in particular when these feature pure intermediate goods. It is thus not a a surprise that Friedman and Friedman (1980) chose a pencil — a good with a complex production structure that relies on pure intermediates — as the example to praise the virtues of competitive markets.

5 Competitive Economies

Our results so far have made no assumptions about the individual behavior, budget constraints, prices, or notions of equilibrium. We now specialize the welfare accounting decomposition to competitive economies with and without wedges. This provides new insights by shedding light on the relation between efficiency and competition and by relating prices and wages to the welfare determinants that we have identified in this paper.

5.1 Competitive Equilibrium with Wedges

We now assume that individuals maximize utility and technologies are operated with the objective of minimizing costs and maximizing profits. To allow for distortions, we saturate all choices with wedges, which we take as primitives. For simplicity, we set $\bar{y}^{j,s} = 0$.

Individual i faces a budget constraint of the form

$$\sum_j p^j \left(1 + \tau_c^{ij}\right) c^{ij} = \sum_f w^f \left(1 + \tau_n^{if,s}\right) \left(n^{if,s} + \bar{n}^{if,s}\right) + \sum_j \nu^{ij} \pi^j + T^{ij}, \tag{22}$$

where p^j denotes the price of good j, w^f denotes factor f's per unit compensation, $\nu^{ij}\pi^j$ denotes the profit associated with the operation of technology j received by individual i, and T^{ij} is a lump-sum transfer that rebates wedges back to individuals. Individual i faces individual-specific consumption and factor supply wedges τ_c^{ij} and $\tau_n^{if,s}$.

Firms operate technologies to minimize costs, which defines the cost functions

$$\mathcal{C}^j \left(y^j; \left\{w^f\right\}_f, \left\{p^\ell\right\}_\ell\right) = \min_{n^{jf,d}, x^{j\ell}} \sum_f w^f \left(1 + \tau_n^{jf,d}\right) n^{jf,d} + \sum_\ell p^\ell \left(1 + \tau_x^{j\ell}\right) x^{j\ell}, \tag{23}$$

subject to equation (2), facing technology-specific factor wedges $\tau_n^{jf,d}$ and technology-specific intermediate input wedges $\tau_x^{j\ell}$. We assume that the supply of good j can be expressed as the solution to a profit maximization problem given by

$$\pi^j = \max_{y^j} p^j \left(1 + \tau_y^j\right) y^j - \mathcal{C}^j \left(y^j; \left\{w^f\right\}_f, \left\{p^\ell\right\}_\ell\right), \tag{24}$$

where τ_y^j denotes a markup wedge for technology j.

The definition of competitive equilibrium with wedges is standard, so we include it in the Appendix. In a competitive equilibrium, individuals equalize marginal rates of substitution with prices or wages cum wedges, while firms equalize marginal revenue products with marginal costs cum wedges, whenever non-negative constraints are slack. We can compactly represent the optimality conditions in matrix form as

$$\begin{aligned} \boldsymbol{MRS}_c &\leq \boldsymbol{p}\left(\mathbf{1}_c + \boldsymbol{\tau}_c\right) \\ \boldsymbol{MRS}_n &\geq \boldsymbol{w}\left(\mathbf{1}_{n^s} + \boldsymbol{\tau}_{n^s}\right) \end{aligned} \quad \text{and} \quad \begin{aligned} \boldsymbol{pG}_x &\leq \boldsymbol{p}\left(\mathbf{1}_x + \boldsymbol{\tau}_x\right) \\ \boldsymbol{pG}_n &\leq \boldsymbol{w}\left(\mathbf{1}_{n^d} + \boldsymbol{\tau}_{n^d}\right), \end{aligned} \tag{25}$$

where all matrices are defined in Appendix A. The matrices $\boldsymbol{\tau}_x$ and $\boldsymbol{\tau}_{n^d}$ include markup wedges τ_y^j in addition to intermediate input use wedges $\tau_x^{j\ell}$ and factor use wedges $\tau_n^{jf,d}$.

We refer to economies with no wedges ($\tau_c^{ij} = \tau_n^{if,s} = \tau_x^{j\ell} = \tau_n^{jf,d} = \tau_y^j = 0$) as frictionless competitive economies. In these economies, the First Welfare Theorem holds, so any competitive equilibrium allocation is efficient. Conditions (25) link prices to marginal rates of substitution and (physical) marginal products, an insight that we exploit repeatedly in this section.

5.2 MSV and Converse Hulten's Theorem

Characterizing the marginal social value of goods in competitive economies with wedges is critical because it directly determines the efficiency gains from technology change as well as marginal welfare products, which in turn govern all production efficiency components.

Theorem 5. *(MSV in Competitive Economies with Wedges). In competitive economies with wedges, the marginal social value of goods, defined via a $1 \times J$ matrix $\boldsymbol{MSV}_y$, is given by*

$$\boldsymbol{MSV}_y = \boldsymbol{p} + \boldsymbol{p}\bar{\boldsymbol{\tau}}_y \boldsymbol{\Psi}_y \quad \textit{where} \quad \bar{\boldsymbol{\tau}}_y = \boldsymbol{\phi}_x \bar{\boldsymbol{\tau}}_x + \boldsymbol{\phi}_c \bar{\boldsymbol{\tau}}_c, \tag{26}$$

where $\boldsymbol{p}$ denotes the $1 \times J$ vector of prices, $\bar{\boldsymbol{\tau}}_x$ and $\bar{\boldsymbol{\tau}}_c$ denote $J \times J$ diagonal matrices of aggregate intermediate input and consumption wedges, with elements given by $\bar{\tau}_x^j = \sum_\ell \chi_x^{\ell j} \tau_x^{\ell j}$ and $\bar{\tau}_c^j = \sum_i \chi_c^{ij} \tau_c^{ij}$, $\boldsymbol{\phi}_x$ and $\boldsymbol{\phi}_c$ are $J \times J$ diagonal matrices of aggregate intermediate use and consumption shares, $\bar{\boldsymbol{\tau}}_y$ defines the aggregate goods wedge, and $\boldsymbol{\Psi}_y$ is the goods inverse matrix defined in (14).

Equation (26) shows that the marginal social value of goods equals the vector of prices augmented by a term that captures the average of the aggregate wedges in consumption and intermediate input use.[10] Aggregate consumption and intermediate input use wedges are weighted averages of individual consumption wedges, $\bar{\tau}_c^j = \sum_i \chi_c^{ij} \tau_c^{ij}$, and intermediate input use wedges, $\bar{\tau}_x^j = \sum_\ell \chi_x^{\ell j} \tau_x^{\ell j}$. The aggregate goods wedge is in turn a weighted average of the two.

In order to understand why $\boldsymbol{MSV}_y$ takes this form in competitive economies, it is useful to start from its definition, $\boldsymbol{MSV}_y = \boldsymbol{AMRS}_c \boldsymbol{\phi}_c \boldsymbol{\Psi}_y$, and proceed gradually. First, using the optimality conditions for individual consumption, $\boldsymbol{MSV}_y$ can be written as

$$\boldsymbol{MSV}_y = \boldsymbol{p}\boldsymbol{\phi}_c \boldsymbol{\Psi}_y + \underbrace{(\boldsymbol{AMRS}_c - \boldsymbol{p})}_{\boldsymbol{p}\bar{\boldsymbol{\tau}}_c} \boldsymbol{\phi}_c \boldsymbol{\Psi}_y. \tag{27}$$

Intuitively, a unit impulse in aggregate supply ultimately increases aggregate consumption by $\boldsymbol{\phi}_c \boldsymbol{\Psi}_y$, for given allocation shares and factor supplies. The social value of this change in aggregate consumption can be split into its market value and the deviation between the true social value, given by $\boldsymbol{AMRS}_c$, and the market value. This difference is precisely determined the aggregate consumption wedge, $\bar{\boldsymbol{\tau}}_c$.

Next, the market value of the change in aggregate consumption, can be expressed as

$$\boldsymbol{p}\boldsymbol{\phi}_c \boldsymbol{\Psi}_y = \boldsymbol{p} + \underbrace{(\boldsymbol{p}\boldsymbol{G}_x \boldsymbol{\chi}_x - \boldsymbol{p})}_{\boldsymbol{p}\bar{\boldsymbol{\tau}}_x} \boldsymbol{\phi}_x \boldsymbol{\Psi}_y. \tag{28}$$

Intuitively, the ultimate change in aggregate consumption induced by a unit impulse in aggregate supply, $\boldsymbol{\phi}_c \boldsymbol{\Psi}_y$, can be expressed as the ultimate change in aggregate supply net of aggregate intermediate use.

Formally, (28) uses the following physical identity, which follows from (15):

$$\boldsymbol{\phi}_c \boldsymbol{\Psi}_y = \boldsymbol{\Psi}_y - \boldsymbol{\phi}_x \boldsymbol{\Psi}_y = \boldsymbol{I}_J + \boldsymbol{G}_x \boldsymbol{\xi} \boldsymbol{\Psi}_y - \boldsymbol{\phi}_x \boldsymbol{\Psi}_y = \boldsymbol{I}_J + (\boldsymbol{G}_x \boldsymbol{\chi}_x - \boldsymbol{I}_J)\, \boldsymbol{\phi}_x \boldsymbol{\Psi}_y,$$

where the ultimate change in aggregate supply, $\boldsymbol{\Psi}_y$, is decomposed into the unit impulse, $\boldsymbol{I}_J$, and knock-on effects, $\boldsymbol{G}_x \boldsymbol{\xi} \boldsymbol{\Psi}_y$. Hence, the ultimate market value of a unit impulse in aggregate supply corresponds to the sum of the market value of the impulse, given by $\boldsymbol{p}$, and the market value of the knock-on effects net of aggregate intermediate use, given by $\boldsymbol{p}\boldsymbol{G}_x \boldsymbol{\chi}_x - \boldsymbol{p}$. This difference is precisely determined by the aggregate intermediate input wedge, $\bar{\boldsymbol{\tau}}_x$.

[10] In sum form, we can express an element of $\boldsymbol{MSV}_y$ as $MSV_y^\ell = p^\ell + \sum_j p^j \bar{\tau}_y^j \psi_y^{j\ell}$, where $\bar{\tau}_y^j = \phi_c^j \bar{\tau}_c^j + \phi_x^j \bar{\tau}_x^j$.

Combining (27) and (28), we can reformulate (26) as

$$\boldsymbol{MSV}_y = \boldsymbol{p} + \underbrace{(\boldsymbol{p}\boldsymbol{G}_x\boldsymbol{\chi}_x - \boldsymbol{p})}_{=\boldsymbol{p}\bar{\boldsymbol{\tau}}_x}\boldsymbol{\phi}_x\boldsymbol{\Psi}_y + \underbrace{(\boldsymbol{AMRS}_c - \boldsymbol{p})}_{=\boldsymbol{p}\bar{\boldsymbol{\tau}}_c}\boldsymbol{\phi}_c\boldsymbol{\Psi}_y.$$

This expression illustrates that aggregate consumption (intermediate input use) of good j is too low when $\bar{\tau}_c^j > 0$ ($\bar{\tau}_x^j > 0$), and aggregate supply of good j is too low when $\bar{\tau}_y^j = \phi_c^j\bar{\tau}_c^j + \phi_x^j\bar{\tau}_x^j > 0$. Hence, the marginal social value of goods that ultimately increase the aggregate supply of goods with positive aggregate goods wedges is higher than the price.

While prices capture the marginal social value of goods in frictionless competitive economies — see Corollary 7 below, Theorem 5 allows us to establish a converse result that has been missing from the existing literature, and that we state as Corollary 6.

Corollary 6. *(Converse Hulten's Theorem: Condition for $\boldsymbol{MSV}_y = \boldsymbol{p}$) The condition that ensures $\boldsymbol{MSV}_y = \boldsymbol{p}$ is that aggregate goods wedges are zero, that is,*

$$\bar{\boldsymbol{\tau}}_y = \boldsymbol{\phi}_c\bar{\boldsymbol{\tau}}_c + \boldsymbol{\phi}_x\bar{\boldsymbol{\tau}}_x = 0. \tag{29}$$

While frictionless competition guarantees that (29) is satisfied, this condition may also hold otherwise, possibly at inefficient allocations. In particular, prices will capture the marginal social value of goods as long as aggregate goods wedges are zero, even when intermediate input and consumption wedges are non-zero ($\tau_x \neq 0$ and $\tau_c \neq 0$) and the competitive equilibrium is inefficient. Aggregate goods wedges can be zero when aggregate consumption and intermediate use wedges cancel out, or when both are zero. In turn, aggregate consumption and intermediate use wedges can be zero when its elements cancel out, or when all its constituents are zero. For cancellations to occur, it must be that some wedges are positive and other negative.

5.3 Hulten's Theorem Revisited

Theorem 5 allows us to revisit the impact of technology changes in the frictionless competitive case. This is the widely studied Hulten's theorem (Hulten, 1978), a result that has played a prominent role in the study of the macroeconomic impact of microeconomic shocks and growth accounting (Gabaix, 2011; Acemoglu et al., 2012; Bigio and La'O, 2020; Baqaee and Farhi, 2020).

Corollary 7. *(Hulten's Theorem Revisited). In frictionless competitive economies, the efficiency impact of a proportional Hicks-neutral technology change j is*

$$\frac{1}{\sum_j p^j c^j} \Xi^E = \underbrace{\frac{p^j y^j}{\sum_j p^j c^j}}_{\text{Sales Share}}, \tag{30}$$

where $\frac{p^j y^j}{\sum_j p^j c^j}$ is the Domar weight or sales share of good j in $\sum_j p^j c^j$.

Corollary 7 provides a general Hulten-like result that applies to frictionless competitive economies with heterogeneous individuals, elastic factor supplies, arbitrary preferences and technologies, and arbitrary social welfare functions. Its generality allows us to systematically present the many qualifications associated with this result in three remarks.

Remark 1. *(Normalizations behind Domar Weights).* Comparing Theorem 5 and Corollary 7 highlights why Hulten's theorem is typically stated in terms of Domar weights. First, considering proportional Hicks-neutral technology shocks implies that $\frac{\partial G^j}{\partial \theta} = y^j$, which ensures that the numerator of the Domar weight in (30) is $p^j y^j$. Second, Hulten's theorem is typically stated using nominal GDP as numeraire, which ensures that the denominator of the Domar weight in (30) is $\sum_j p^j c^j$. These are valid normalizations that transform the condition $MSV_y^j = p^j$ into a Domar weight.

Remark 2. *(Welfare vs. Efficiency vs. Production Efficiency vs. Output).* In economies with a single individual ($I = 1$) and in which supplying factors causes no disutility ($\partial u^i / \partial n^{if,s} = 0$), changes in final output, production efficiency, efficiency, and welfare coincide, which justifies the typical formulation of Hulten's theorem in terms of final output. Corollary 7 highlights that Hulten's theorem is, at its core, a result about efficiency (via production efficiency). Why is this the case? In economies with a single individual, redistribution and exchange efficiency are zero, so efficiency and welfare coincide and are exclusively determined by production efficiency.[11] And when supplying factors causes no disutility, there is no need to subtract the social cost of supplying factors to transform final output changes into welfare changes, so production efficiency exclusively captures changes in final output (i.e. aggregate consumption).

Remark 3. *(Efficient vs. Frictionless Competitive vs. Efficient Interior Economies).* Corollary 7 states that Hulten's Theorem applies to frictionless competitive economies, rather than

[11]In fact, Bigio and La'O (2020) have already shown that Hulten's theorem is valid for efficiency in an $I = 1$ environment with elastic labor supply.

efficient economies, as often formulated. One reason that may explain why Hulten's theorem is often formulated in terms of efficiency is that prior to the results in Section 4 there had been no general characterization of efficiency conditions that dealt with non-interior allocations. Why is this relevant? When an allocation is efficient, all allocative efficiency components are necessarily zero, which guarantees that efficiency gains are exclusively due to technology and endowment changes. But efficiency is not enough to guarantee that $\boldsymbol{MSV}_y = \boldsymbol{p}$. The converse Hulten's Theorem shows that this occurs when $\bar{\boldsymbol{\tau}}_y = 0$, a condition that holds in frictionless competitive economies, but that need not hold in efficient economies. That is, there may exist efficient *non-interior* allocations in which $\bar{\boldsymbol{\tau}}_y \neq 0$ and Hulten's theorem does not hold. This occurs because in efficient *non-interior* allocations input prices need not reflect marginal welfare products. Therefore, while Hulten's theorem applies to i) frictionless competitive economies and ii) efficient interior allocations, it can fail in efficient non-interior allocations. We illustrate this possibility in Example 3.

Example 3. *(Failure of Hulten's Theorem in Non-Interior Efficient Equilibrium).* We consider the same environment as in Example 1, and focus on a technology change for good 2, so $\frac{\partial G^2}{\partial \theta} \neq 0$. For simplicity, we set all wedges to zero, with the exception of $\tau_x^{12} \neq 0$. The competitive equilibrium of this economy is efficient, with the relevant efficiency condition here being $MRS_c^{11} \frac{\partial G^1}{\partial x^{12}} > 0$. In this case, competition ensures that $p^1 \frac{\partial G^1}{\partial x^{12}} = p^2 \left(1 + \tau_x^{12}\right)$. But note that

$$MSV_y^2 = MRS_c^{11} \frac{\partial G^1}{\partial x^{12}} = p^1 \frac{\partial G^1}{\partial x^{12}} = p^2 \left(1 + \tau_x^{12}\right) \neq p^2,$$

so prices do not capture the marginal social value of goods and Hulten's theorem fails in this efficient economy. This example illustrates that $\bar{\tau}_y^2 = \bar{\tau}_x^2 = \tau_x^{12} = 0$ is the condition that ensures $MSV_y^2 = p^2$, not efficiency. Baqaee and Farhi (2020) already provide an example analogous to this one in which Hulten's theorem fails, justifying this failure in that revenue- and cost-based Domar weights are not equal. Our result complement theirs in the sense that we establish that this failure of Hulten's in efficient economies can *only* occur at non-interior allocations. Our result further underscores the importance of carefully dealing with non-interior allocations when studying disaggregated economies.

5.4 Marginal Revenue Product vs. Marginal Welfare Product

In frictionless competitive economies, marginal revenue products are equalized across all uses and the cross-sectional factor use efficiency component is zero. However, equalization of marginal revenue products is not sufficient to ensure that the cross-sectional factor use efficiency component is zero in competitive economies with wedges, even when factor use wedges are zero. A similar logic applies to cross-sectional input efficiency. Why is this the case?

As explained in Section 4, efficiency requires the equalization of marginal welfare products across uses of a factor, while competition when factor use wedges are zero ensures the equalization of marginal revenue products across uses. If $MSV_y^j \neq p^j$ for some goods that use a particular factor, the marginal welfare products of that factor won't be equalized across uses, allowing for cross-sectional factor use efficiency to be non-zero. We illustrate this possibility in Example 4.

Example 4. *(Marginal Welfare Product vs. Marginal Revenue Product).* We consider the same environment as in Example 2. All wedges are zero except $\tau_x^{23} \neq 0$. In this case, competition ensures that $MRS_c^{11} = p^1$ and $MRS_c^{12} = p^2$, as well as $p^1 \frac{\partial G^1}{\partial n^{11,d}} = w^1$ and $p^3 \frac{\partial G^3}{\partial n^{31,d}} = w^1$. The only equilibrium condition with a wedge is $p^2 \frac{\partial G^2}{\partial x^{23}} = (1 + \tau_x^{23})\, p^3$. Consequently, competition implies that marginal revenue products are equalized across uses, so $MRP_n^{11} = MRP_n^{31}$. Therefore,

$$p^1 \frac{\partial G^1}{\partial n^{11,d}} = p^3 \frac{\partial G^3}{\partial n^{31,d}} \implies p^1 \frac{\partial G^1}{\partial n^{11,d}} = \frac{1}{1+\tau_x^{23}} p^2 \frac{\partial G^2}{\partial x^{23}} \frac{\partial G^3}{\partial n^{31,d}}.$$

However, this condition is inconsistent with cross-sectional factor use efficiency,

$$p^1 \frac{\partial G^1}{\partial n^{11,d}} = p^2 \frac{\partial G^2}{\partial x^{23}} \frac{\partial G^3}{\partial n^{31,d}},$$

which requires the equalization of marginal welfare products. This discrepancy is due to the fact that marginal social value of good 3 does not equal its price, since $\bar{\tau}_y^3 = \tau_x^{23} > 0$.

6 Applications

In this section, we illustrate how the welfare accounting decomposition can be used to trace the origins of welfare gains to changes in allocations and primitives in four workhorse models in macroeconomics and trade. Our first application shows how an increase in tariffs contributes negatively to exchange efficiency via cross-sectional consumption efficiency in the simplest endowment economy (Armington, 1969). This application also illustrates subtle patterns in redistribution. Our second application shows how the efficiency gain induced by an improvement in a matching technology in a Diamond-Mortensen-Pissarides (DMP) model is due to cross-sectional factor use efficiency gains that are large enough to compensate for aggregate intermediate input efficiency losses due to an increase in vacancy postings. This application illustrates how to use the welfare accounting decomposition in economies that are not competitive. Our third application illustrates how an increase in markup dispersion generates cross-sectional factor use efficiency losses in a Hsieh and Klenow (2009) economy. Our final application shows how to use the welfare accounting decomposition to identify the welfare gains from optimal monetary stabilization policy in a macroeconomic model with household and sectoral heterogeneity.

6.1 Armington (1969) Model

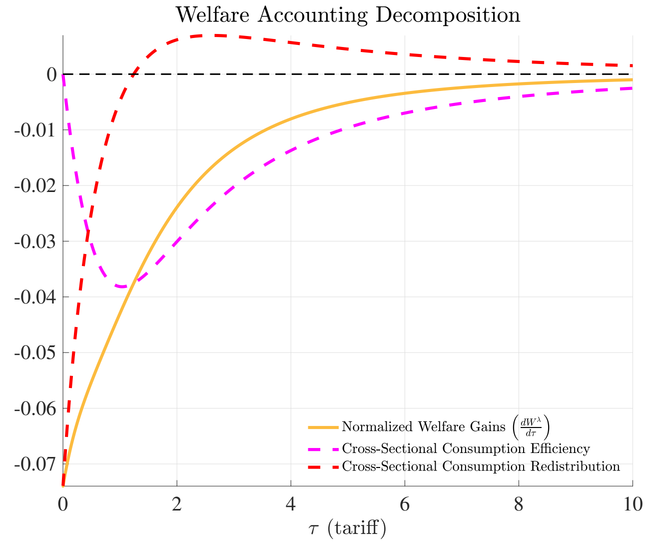
Environment. We consider the simplest Armington (1969) economy, which has $I = 2$ individuals (here representing countries), $J = 2$ goods, and $F = 2$ inelastically supplied factors.[12] Each country produces a single good with their domestic factor — normalized so that $\bar{n}^{if,s} = 1$ — but consumes both goods. Country i has preferences given by

$$V^i = \left(\sum_j \left(c^{ij} \right)^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}},$$

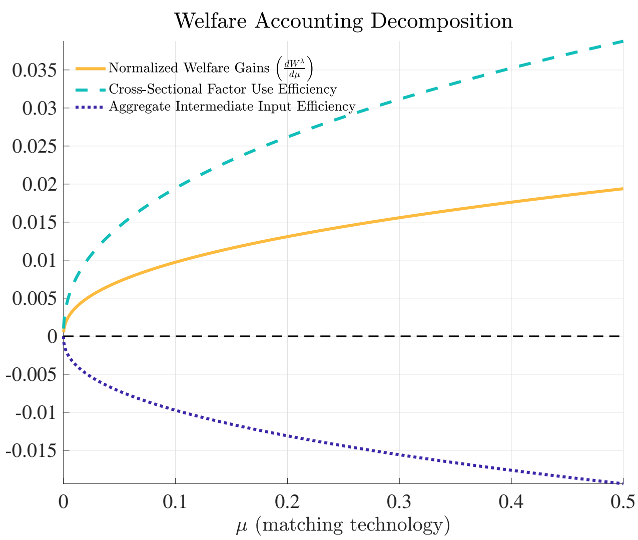
and faces the budget constraint

$$\sum_j p^j \left(1 + \tau^{ij} \right) c^{ij} = w^{if} + \sum_j T^{ij}, \quad \text{where} \quad T^{ij} = p^j \tau^{ij} c^{ij}. \tag{31}$$

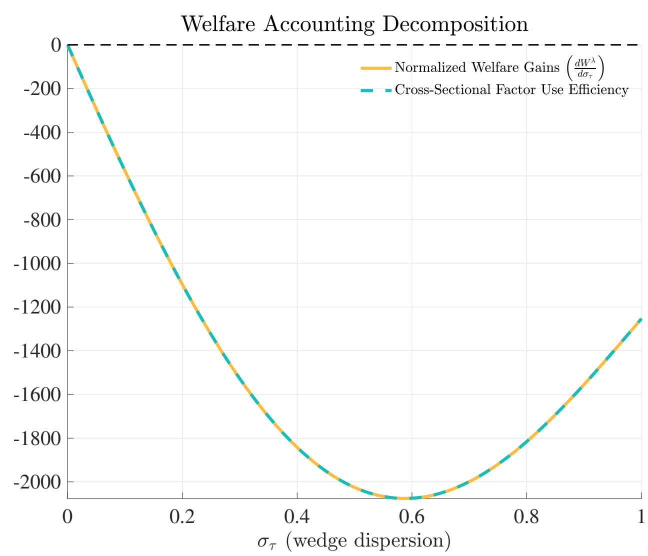
[12] As we show in the Appendix, this economy is isomorphic to an economy without factors in which each country has a predetermined endowment of their home good.



(a) Armington Model



(b) Diamond-Mortensen-Pissarides Model



(c) Hsieh-Klenow Model

Figure 3: Welfare Accounting Decomposition: Applications

Note: This figure illustrates the welfare accounting decomposition for the first three applications. The top panel shows that an increase in tariffs decreases exchange efficiency through cross-sectional consumption efficiency in an Armington model. It also show that cross-sectional consumption redistribution is positive, since the tariff increase hurts more the country with lower consumption. The bottom left panel shows that the efficiency gain induced by an improvement in a matching technology in a DMP model is due to cross-sectional factor use efficiency gains that are large enough to compensate for aggregate intermediate input efficiency losses due to increase vacancy postings. The bottom right panel shows that all welfare losses due to the increase in the dispersion of wedges/markups — typically referred to as misallocation — are attributed to production efficiency via cross-sectional factor use efficiency in Hsieh and Klenow (2009) economy.

Since the iceberg costs τ^{ij} are rebated they should be interpreted as tariffs rather than physical costs, although it is straightforward to consider the alternative case. Goods are competitively produced according to constant returns to scale technologies — which justifies the absence of profits in (31) — given by

$$y^1 = A^1 n^{11,d} \quad \text{and} \quad y^2 = A^2 n^{22,d},$$

so each country uses the domestic factor to exclusively produce the domestic good. An equilibrium is characterized by allocations c^{ij}, prices p^j, and wages w^{if} such that both countries choose consumption optimally, countries produce competitively, and all markets clear. Resource constraints in this economy are given by $\sum_i c^{ij} = y^j$, $\forall j$, and $n^{f,d} = \bar{n}^{f,s} = 1$, $\forall f$.

Our parameterization assumes that $\sigma = 2$, $A^1 = 1$, $A^2 = 50$, $\tau^{ii} = 0$, and $\tau^{ij} = \tau^{ji} = \tau$. We use aggregate world consumption as welfare numeraire, and consider a social welfare function $\sum_i \left(V^i\right)^{\frac{\sigma-1}{\sigma}}$, which only impacts redistribution, but not efficiency assessments.

Results. The top panel in Figure 3 illustrates the welfare impact of a multilateral increase in tariffs τ — see also Figure OA-1. The welfare accounting decomposition yields insights for both efficiency and redistribution.

First, a multilateral increase in tariffs always features a negative exchange efficiency component, due to cross-sectional consumption efficiency. This occurs because the increase in tariffs reallocates consumption toward each country's domestic good, which is the one with a relatively lower MRS_c^{ij} as long as $\tau > 0$. Note that $\Xi^{E,X} = 0$ at $\tau = 0$, since this economy is efficient in the absence of tariffs.

Second, an increase in tariffs eventually makes both countries worse off, but initially benefits country 2, because p^2/p^1 increases in equilibrium. Since country 2 is more productive and consumes more of both goods than country 1 in equilibrium, the planner attaches a lower individual weight to country 2, so $\omega^1 > \omega^2$. Hence, initially, the increase in tariffs benefits the country relatively less preferred by the planner and harms redistribution, with $\Xi^{RD} < 0$. However, once tariffs are large enough, further increases in tariffs make both countries worse off. Around $\tau \approx 1.2$, the marginal increase in τ hurts country 2 disproportionately more. From this level of tariffs onwards, $\Xi^{RD} > 0$, since country 1 — the relatively preferred by the planner — is hurt by less.

6.2 DMP Model

Environment. We consider a stylized version of the textbook labor search model, as in e.g. Pissarides (2000). We consider a two date economy, $t \in \{0, 1\}$, populated by a single/representative individual ($I = 1$) endowed with a unit supply of labor ($F = 1$), which can be used in technology $j = 1$ (unemployment) or $j = 2$ (employment). Each of these technologies produces perfectly substitutable goods (or equivalently, a single final good), so the preferences of the representative individual can be written as

$$V = c_0 + \beta c_1, \quad \text{where} \quad c_t = c_t^1 + c_t^2, \tag{32}$$

where c_t^j denotes consumption of the good produced by technology j at date t. Both technologies have constant returns to scale and are given by

$$y_t^1 = G_t^1\left(n_t^1\right) = z^1 n_t^1 = z^1 \chi_{t,n}^1 \qquad \text{and} \qquad y_t^2 = G_t^2\left(n_t^2\right) = z^2 n_t^2 = z^2 \chi_{t,n}^2, \tag{33}$$

where $\chi_{t,n}^1$ and $\chi_{t,n}^2$ respectively denote the employment and unemployment rates, and where $z^2 > z^1$.

Moreover, there exists a third "vacancy-generating" technology ($J = 3$) at date 0 that takes the final good and generates vacancies, as follows

$$y_t^3 = v_t = G_t^3\left(x_t\right) = \frac{1}{\kappa_t} x_t, \tag{34}$$

where κ_t captures the marginal cost of vacancy posting. Vacancies can be interpreted as a good that no individual desires to consume, which means that in a first-best environment vacancies should be zero. Hence, the resource constraints in this model can be expressed as

$$y_t^1 + y_t^2 = c_t + x_t \qquad \text{and} \qquad \chi_{t,n}^1 + \chi_{t,n}^2 = 1. \tag{35}$$

Equation (32) through (35) are sufficient to characterize the efficiency conditions for this economy. Since $z^2 > z^1$, efficiency requires full employment, with $\chi_{t,n}^1 = 0$ and $\chi_{t,n}^2 = 1$, as well as, $v_t = 0$.

However, we consider a standard random search equilibrium in which employment only adjusts according to

$$\chi^1_{t+1,n} - \chi^1_{t,n} = \varphi\left(1 - \chi^1_{t,n}\right) - m\left(\chi^1_{t,n}, v_t\right),$$

where φ denotes the job destruction rate and the matching function $m(\cdot)$ is given by

$$m\left(\chi^1_{t,n}, v_t\right) = \mu\left(\chi^1_{t,n}\right)^{\alpha}(v_t)^{1-\alpha}.$$

As usual in this class of models, labor market tightness is defined as $\theta_t = \frac{v_t}{\chi^1_{t,n}}$. We formally describe the (standard) characterization of the equilibrium in the Appendix and describe the welfare impact of a change in the matching technology μ. Our parameterization assumes that $\beta = 0.99$, $z^1 = 0$, $z^2 = 1$, $\eta = 0.5$, $\alpha = 0.7$, $\varphi = 0.036$, $b_0 = b_1 = 0$, $\kappa_0 = 0.1$, with $\chi^1_{0,n} = 0.037$.

Results We consider a standard search equilibrium in this economy — see Online Appendix — and explore the welfare implications of improvements in the matching technology μ. The effects are illustrated in the bottom left panel in Figure 3 — see also Figure OA-2. Several insights emerge.

First, the technology change component of the welfare accounting decomposition is zero even though the matching technology improves. This occurs because the matching technology does not change the production frontier of the economy, and it is simply a mechanism to determine how factors of production are allocated. Second, as the matching technology improves, firms post more vacancies at date 0, which translates into higher employment at date 1. The increase in employment drives the positive cross-sectional factor use efficiency component — as discussed above, at the first-best, unemployment should be zero. However, the additional vacancies posted make the aggregate intermediate input efficiency component negative. This occurs because posting vacancies entails using a technology that produces no final output, and it only contributes to reallocating factors, something that could be done freely in the absence of search frictions.

Hence, even though the improvement in the matching technology generates welfare efficiency gains, the welfare accounting decomposition shows that these gains combine positive and negative effects. More generally, this application illustrates how adjustment cost functions will typically generate a negative aggregate intermediate input efficiency component.

6.3 Hsieh and Klenow (2009) Model

Environment. We consider a simplified version of the Hsieh and Klenow (2009) economy, with a representative individual ($I = 1$) — whose index we drop — and a single final good, which we index by $j = 1$. Individual preferences are given by $V = u(c^1)$, where the final good is produced according to the technology

$$y^1 = \left(\sum_{j=2}^{J} \left(x^{1j}\right)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$

where ϵ denotes the elasticity of substitution between the $J-1$ intermediate inputs. Each intermediate input $j \geq 2$ is produced according to the technology

$$y^j = A^j n^{j1,d},$$

where a single factor not elastically supplied ($F = 1$) — whose index we also drop — can be used to produce the different intermediates. Formally, resource constraints in this economy can be written as

$$c^1 = y^1, \quad y^j = x^{1j}, \ \forall j \geq 2, \quad \text{and} \quad \sum_{j=2}^{J} \chi_n^{j,d} = 1.$$

If the final good is produced competitively, and the intermediate inputs are chosen under monopolistic competition subject to wedges τ^j (which can be interpreted as markups) the equilibrium factor use shares $\chi_n^{j,d}$, can be expressed as

$$\chi_n^{j,d} = \frac{\left(A^j\right)^{\epsilon-1}\left(\tau^j\right)^{-\epsilon}}{\sum_{j=2}^{J}\left(A^j\right)^{\epsilon-1}\left(\tau^j\right)^{-\epsilon}},$$

Our parameterization — designed to mimic Hsieh and Klenow (2009) — assumes that $(\log A^j, \log \tau^j) \sim \mathcal{N}(\mu_A, \mu_\tau, \sigma_A^2, \sigma_\tau^2, \sigma_{\tau A})$, where $\mu_A = 0.5$, $\mu_\tau = 1.1$, $\sigma_A = 0.95$, $\sigma_\tau = 0.63$, $\sigma_{\tau A} = 0.36$, $J = 211,304$, and $\epsilon = 3$. We explore the welfare implications of an increase in markup dispersion through σ_τ.

Results. The bottom left panel in Figure 3 illustrates the welfare impact of a change in markup dispersion — typically referred to as misallocation. Since all intermediate inputs in this economy are fully specialized and there is a single final good, no welfare changes are attributed to intermediate input efficiency. And since the single factor is fixed, aggregate factor efficiency is also zero. Hence, all welfare losses due to the increase in the dispersion of markups are attributed to production efficiency via cross-sectional factor use efficiency. Given our calibration of the model, chosen to mimic Hsieh and Klenow (2009), these effects are quantitatively large. Since there is a single representative individual, both exchange efficiency and redistribution are zero.

6.4 New Keynesian Model

This application shows how the welfare accounting decomposition can be used to identify the welfare gains from optimal monetary stabilization policy. To that end, we develop a static, multi-sector heterogeneous agent New Keynesian model with an input-output production network — a static "HANK-IO" model (Schaab and Tan, 2023). This model builds on La'O and Tahbaz-Salehi (2022) and Rubbo (2023) but allows for household heterogeneity in addition to sectoral heterogeneity.

Environment. There are I (types of) households indexed by i. Each has mass μ^i, with $\sum_i \mu^i = 1$. There are N production sectors indexed by j. Each comprises a continuum of firms indexed by $\ell \in [0,1]$. Each firm produces a distinct good, indexed by $j\ell$.

The preferences of household i are given by

$$V^i = \frac{1}{1-\gamma}(c^i)^{1-\gamma} - \frac{1}{1+\varphi}(n^i)^{1+\varphi}, \quad \text{where} \tag{36}$$

$$c^i = \left(\sum_j (\Gamma_c^{ij})^{\frac{1}{\eta_c}} (c^{ij})^{\frac{\eta_c-1}{\eta_c}}\right)^{\frac{\eta_c}{\eta_c-1}} \quad \text{and} \quad c^{ij} = \left(\int_0^1 (c^{ij\ell})^{\frac{\epsilon^j-1}{\epsilon^j}} d\ell\right)^{\frac{\epsilon^j}{\epsilon^j-1}},$$

where c^i denotes a final consumption aggregator, c^{ij} denotes a sectoral consumption aggregator, and $c^{ij\ell}$ is household i 's consumption of good $j\ell$. Each household is endowed with a unique labor factor and n^i denotes hours of work. The household budget constraint is given by $\sum_j \int_0^1 p^{j\ell} c^{ij\ell} d\ell = W^i n^i + T^i$, where $p^{j\ell}$ is the price of good $j\ell$, W^i is the wage paid to factor i, and T^i is a lump-sum transfer that accounts for profits. Household optimization implies $(n^i)^{\varphi}(c^i)^{\gamma} = W^i/P^i$.

Firm ℓ in sector j produces according to the nested CES production technology

$$y^{j\ell} = A^j \left((1-\vartheta^j)^{\frac{1}{\eta}} (n^{j\ell})^{\frac{\eta-1}{\eta}} + (\vartheta^j)^{\frac{1}{\eta}} (x^{j\ell})^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \quad \text{where} \quad n^{j\ell} = \left(\sum_i (\Gamma_w^{ji})^{\frac{1}{\eta_w}} (n^{j\ell i})^{\frac{\eta_w-1}{\eta_w}} \right)^{\frac{\eta_w}{\eta_w-1}}, \tag{37}$$

$$x^{j\ell} = \left(\sum_\ell (\Gamma_x^{j\ell})^{\frac{1}{\eta_x}} (x^{j\ell\ell})^{\frac{\eta_x-1}{\eta_x}} \right)^{\frac{\eta_x}{\eta_x-1}} \quad \text{and} \quad x^{j\ell\ell} = \left(\int_0^1 (x^{j\ell\ell\ell'})^{\frac{\epsilon^\ell-1}{\epsilon^\ell}} d\ell' \right)^{\frac{\epsilon^\ell}{\epsilon^\ell-1}}.$$

We denote by A^j a sector-specific, Hicks-neutral technology shifter, ϑ^j governs sector j's intermediate input share, and η is the elasticity of substitution between labor and inputs. Firm ℓ in sector j uses a bundle of labor $n^{j\ell}$ that is itself a CES aggregate of its use of labor factors i, $n^{j\ell i}$. It also uses a bundle of intermediate inputs $x^{j\ell}$, which is a CES aggregate of sectoral bundles $x^{j\ell\ell}$, where $x^{j\ell\ell\ell'}$ denotes firm $j\ell$'s use of good $\ell\ell'$ in production.

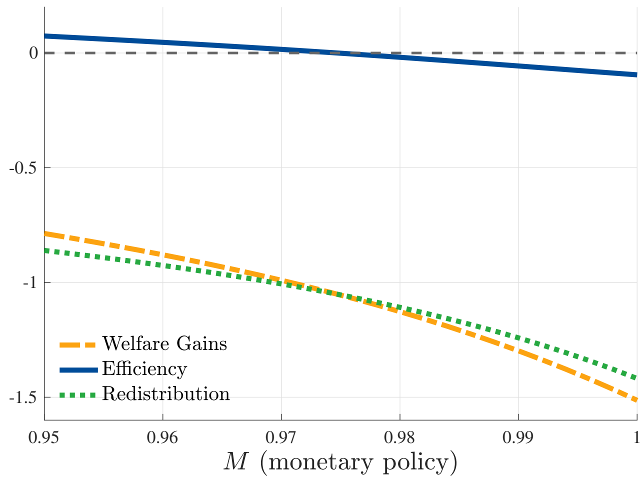
Firms are monopolistically competitive. They choose labor and inputs to minimize costs, and prices to maximize profits. Each firm ℓ is small and takes as given aggregate and sectoral variables. Profits are $\Pi^{j\ell} = (1-\tau^j)\, p^{j\ell} y^{j\ell} - \sum_\ell \int_0^1 p^{\ell\ell'} x^{j\ell\ell\ell'} d\ell' - \sum_i W^i n^{j\ell i} = (1-\tau^j)\, p^{j\ell} y^{j\ell} - mc^j y^{j\ell}$, where τ^j is a revenue tax. Marginal cost mc^j is uniform across firms in each sector as we show in Appendix F.4.1. If prices are flexible, firms set prices as a markup over marginal cost, $p^{j\ell} = p^j = \frac{\epsilon^j}{\epsilon^j-1} \frac{1}{1-\tau^j} mc^j$. To introduce nominal rigidities, we assume that only a fraction $\delta^j \in [0,1]$ of firms in sector j can reset their prices in response to a shock. Otherwise, prices remain fixed at some initial level $\bar{p}^j$, which we specify in the Appendix. The sectoral price distribution is thus given by

$$p^{j\ell} = \begin{cases} \frac{\epsilon^j}{\epsilon^j-1} \frac{1}{1-\tau^j} mc^j & \text{for } \ell \in [0, \delta^j] \\ \bar{p}^j & \text{for } \ell \in (\delta^j, 1]\,. \end{cases} \tag{38}$$

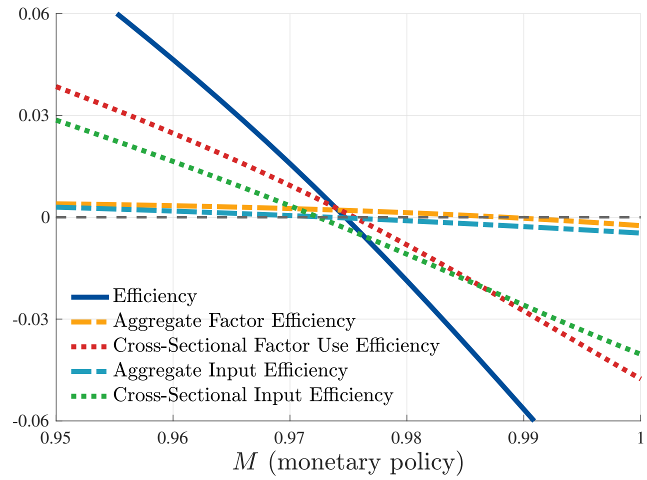
We model monetary policy by assuming that aggregate nominal expenditures are constrained by a cash-in-advance constraint of the form $\sum_j \int_0^1 p^{j\ell} y^{j\ell} d\ell \leq M$, where M is the monetary policy instrument. Finally, the markets for goods and labor factors have to clear, requiring

$$y^{j\ell} = \sum_i \mu_i c^{ij\ell} + \sum_\ell \int_0^1 x^{\ell\ell' j\ell} d\ell' \quad \text{and} \quad \mu^i n^i = \sum_j \int_0^1 n^{j\ell i} d\ell. \tag{39}$$

We formally define competitive equilibrium in Appendix F.4.2.



(a) Welfare Gains



(b) Efficiency Gains

Figure 4: Welfare Accounting Decomposition: New Keynesian Model

Note: This figure illustrates the welfare accounting decomposition for the New Keynesian application when varying monetary policy in response to an unanticipated positive technology shock.

Calibration. We calibrate a model with $N = 66$ sectors and $I = 10$ household types, corresponding to deciles of the income distribution, as in Schaab and Tan (2023). We use data from the Consumer Expenditure Survey to calibrate Γ_c^{ij} so the model matches consumption expenditure shares. Similarly, we use data from the American Community Survey and the BEA's I-O and GDP tables to calibrate ϑ^j, $\Gamma_x^{j\ell}$, and Γ_w^{ji} so the model matches sectoral input-output data and payroll shares. We calibrate ϵ^j to match sectoral markup data from Baqaee and Farhi (2020) and δ^j to match Pasten et al. (2017)'s data on sectoral price rigidities. We allow revenue taxes τ^j to offset initial markups and study the case with $\tau^j = 0$ in Appendix F.4.4. Finally, we assume an equal-weighted utilitarian social welfare function. Appendix F.4.3 presents a detailed discussion of our calibration.

Results. We study monetary policy in response to a 2% technology shock that is uniform across sectors. When households and sectors are symmetric, Divine Coincidence holds and there exists an optimal monetary policy M^* that closes output and inflation gaps. Through the lens of the welfare accounting decomposition, Divine Coincidence implies that each allocative efficiency term of Theorems 1 and 2 is zero. We discuss this case in Appendix F.4.4.

When households and sectors are heterogeneous, Divine Coincidence fails. Figure 4 plots the welfare accounting decomposition, treating M as the perturbation parameter (θ).[13] The

[13]Even though households are heterogeneous, exchange efficiency is zero because goods and factor supply

left panel decomposes welfare gains (yellow) into gains from efficiency (blue) and redistribution (green). The blue line intersects 0 at around $M^{AE} = 0.974$, which is the policy that maximizes efficiency. Redistribution is negative at this point, indicating that the redistribution motive of the utilitarian social welfare function calls for a more contractionary policy (lower M).

The right panel decomposes efficiency into its four allocative efficiency components: cross-sectional and aggregate factor and intermediate input efficiency. Several additional insights emerge. First, factor and input efficiency are both quantitatively important determinants of the production efficiency gains from monetary policy. Second, at $M^{AE} = 0.974$, aggregate (light blue) and cross-sectional (green) input efficiency are negative. These two motives call for more contractionary policy. Third, aggregate (yellow) and cross-sectional (red) factor use efficiency are positive at $M^{AE} = 0.974$, calling for more expansionary policy. The policy that maximizes efficiency trades off and balances these considerations.

It is well understood that stabilizing inflation (which maps to cross sectional factor use efficiency) is more important than stabilizing the output gap (which maps to aggregate factor efficiency) for welfare in standard calibrations of the New Keynesian model (Woodford, 2003). Our results preserve this conclusion and also show that the cross-sectional component also dominates the aggregate component for intermediate input efficiency. Lastly, Appendix F.4.4 illustrates the role of revenue taxes. When they are not available to offset initial markup distortions, aggregate input and factor efficiency become quantitatively more important and call for expansionary policy.

7 Conclusion

This paper introduces a welfare accounting decomposition that identifies and quantifies the origins of welfare gains induced by changes in allocations or primitive changes in technologies or endowments. The decomposition is distinctive in two respects. First, it is derived solely from preferences, technologies, and resource constraints, without invoking prices, budget constraints, or equilibrium notions. Second, at each step, it systematically separates changes in levels into i) changes in shares, capturing cross-sectional reallocations of goods or factors, and ii) changes in aggregates. We illustrate its use through four applications to workhorse models in macroeconomics and trade.

The approach extends naturally to dynamic stochastic economies without accumulation

markets are frictionless.

technologies. In ongoing work, we extend the approach of this paper to economies with accumulation technologies, which opens a new set of nontrivial considerations.

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Online Appendix

A Matrix Definitions

This section defines all matrices used in the body of the paper and in this Appendix. To simplify the exposition, we represent all matrices for the $I = 2$, $J = 3$, $F = 2$ case, although we define matrix dimensions for the general case. For clarity, we typically use L to denote the number of intermediate inputs, although $L = J$.

Allocations. We collect consumption allocations, c^{ij}, and individual endowments of goods, $\bar{y}^{ij,s}$, in the $IJ \times 1$ vectors $\mathring{\boldsymbol{c}}$ and $\mathring{\bar{\boldsymbol{y}}}^s$, as well as intermediate uses, $x^{j\ell}$, in the $JL \times 1$ vector $\mathring{\boldsymbol{x}}$, given by

$$\mathring{\boldsymbol{c}} = \begin{pmatrix} c^{11} \\ c^{21} \\ c^{12} \\ c^{22} \\ c^{13} \\ c^{23} \end{pmatrix}_{IJ\times 1}, \quad \mathring{\bar{\boldsymbol{y}}}^s = \begin{pmatrix} \bar{y}^{11,s} \\ \bar{y}^{21,s} \\ \bar{y}^{12,s} \\ \bar{y}^{22,s} \\ \bar{y}^{13,s} \\ \bar{y}^{23,s} \end{pmatrix}_{IJ\times 1}, \quad \mathring{\boldsymbol{x}} = \begin{pmatrix} x^{11} \\ x^{21} \\ x^{31} \\ x^{12} \\ x^{22} \\ x^{32} \\ x^{13} \\ x^{23} \\ x^{33} \end{pmatrix}_{JL\times 1}.$$

Similarly, we collect factor uses, $n^{jf,d}$, in the $JF \times 1$ vector $\mathring{\boldsymbol{n}}^d$, and elastic factor supplies, $n^{if,s}$, and individual endowments of factors, $\bar{n}^{if,s}$, in the $IF \times 1$ vectors $\mathring{\boldsymbol{n}}^s$ and $\mathring{\bar{\boldsymbol{n}}}^s$, given by

$$\mathring{\boldsymbol{n}}^d = \begin{pmatrix} n^{11,d} \\ n^{21,d} \\ n^{31,d} \\ n^{12,d} \\ n^{22,d} \\ n^{32,d} \end{pmatrix}_{JF\times 1}, \quad \mathring{\boldsymbol{n}}^s = \begin{pmatrix} n^{11,s} \\ n^{21,s} \\ n^{12,s} \\ n^{22,s} \end{pmatrix}_{IF\times 1}, \quad \mathring{\bar{\boldsymbol{n}}}^s = \begin{pmatrix} \bar{n}^{11,s} \\ \bar{n}^{21,s} \\ \bar{n}^{12,s} \\ \bar{n}^{22,s} \end{pmatrix}_{IF\times 1}.$$

Aggregate allocations. We collect aggregate consumption, c^j, aggregate intermediate use, x^j, aggregate produced supply, $y^{j,s}$, aggregate endowment, $\bar{y}^{j,s}$, and aggregate supply, y^j, of goods in $J \times 1$ vectors $\boldsymbol{c}$, $\boldsymbol{x}$, $\boldsymbol{y}^s$, $\bar{\boldsymbol{y}}^s$, and $\boldsymbol{y}$ given by

$$\boldsymbol{c} = \begin{pmatrix} c^1 \\ c^2 \\ c^3 \end{pmatrix}_{J\times 1}, \quad \boldsymbol{x} = \begin{pmatrix} x^1 \\ x^2 \\ x^3 \end{pmatrix}_{J\times 1}, \quad \boldsymbol{y}^s = \begin{pmatrix} y^{1,s} \\ y^{2,s} \\ y^{3,s} \end{pmatrix}_{J\times 1}, \quad \bar{\boldsymbol{y}}^s = \begin{pmatrix} \bar{y}^{1,s} \\ \bar{y}^{2,s} \\ \bar{y}^{3,s} \end{pmatrix}_{J\times 1}, \quad \boldsymbol{y} = \begin{pmatrix} y^1 \\ y^2 \\ y^3 \end{pmatrix}_{J\times 1}.$$

Similarly, we collect aggregate use, $n^{f,d}$, aggregate elastic supply $n^{f,s}$, aggregate endowment, $\bar{n}^{f,s}$, and aggregate supply, n^f, of factors in $F \times 1$ vectors $\boldsymbol{n}^d$, $\boldsymbol{n}^s$, $\bar{\boldsymbol{n}}^s$, and $\boldsymbol{n}$ given by

$$\boldsymbol{n}^d = \begin{pmatrix} n^{1,d} \\ n^{2,d} \end{pmatrix}_{F\times 1}, \quad \boldsymbol{n}^s = \begin{pmatrix} n^{1,s} \\ n^{2,s} \end{pmatrix}_{F\times 1}, \quad \bar{\boldsymbol{n}}^s = \begin{pmatrix} \bar{n}^{1,s} \\ \bar{n}^{2,s} \end{pmatrix}_{F\times 1}, \quad \boldsymbol{n} = \begin{pmatrix} n^{1} \\ n^{2} \end{pmatrix}_{F\times 1}.$$

Aggregates satisfy

$$\boldsymbol{c} = \mathbf{1}_c \mathring{\boldsymbol{c}}, \quad \boldsymbol{x} = \mathbf{1}_x \mathring{\boldsymbol{x}}, \quad \boldsymbol{y}^s = \mathbf{1}_{y^s} \mathring{\boldsymbol{y}}^s, \quad \bar{\boldsymbol{y}}^s = \mathbf{1}_{\bar{y}^s} \mathring{\bar{\boldsymbol{y}}}^s, \quad \boldsymbol{n}^d = \mathbf{1}_{n^d} \mathring{\boldsymbol{n}}^d \quad \boldsymbol{n}^s = \mathbf{1}_{n^s} \mathring{\boldsymbol{n}}^s, \quad \bar{\boldsymbol{n}}^s = \mathbf{1}_{\bar{n}^s} \mathring{\bar{\boldsymbol{n}}}^s,$$

where we define the following matrices of zeros and ones:

$$\mathbf{1}_c = \mathbf{1}_{y^s} = \mathbf{1}_{\bar{y}^s} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}_{J\times IJ}, \qquad \mathbf{1}_x = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}_{J\times JL}$$

$$\mathbf{1}_{n^d} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}_{F\times JF}, \qquad \mathbf{1}_{n^s} = \mathbf{1}_{\bar{n}^s} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}_{F\times IF}.$$

We can thus write resource constraints (3) and (4) as

$$\boldsymbol{y} = \boldsymbol{c} + \boldsymbol{x} \quad \text{and} \quad \boldsymbol{n} = \boldsymbol{n}^d, \qquad \text{where} \qquad \boldsymbol{y} = \boldsymbol{y}^s + \bar{\boldsymbol{y}}^s \quad \text{and} \quad \boldsymbol{n} = \boldsymbol{n}^s + \bar{\boldsymbol{n}}^s.$$

Allocation shares. We collect consumption shares, χ_c^{ij}, in a $IJ \times J$ matrix $\boldsymbol{\chi}_c$, factor use shares, $\chi_n^{jf,d}$, in a $JF \times F$ matrix $\boldsymbol{\chi}_{n^d}$, and factor supply shares, $\chi_n^{if,s}$, in a $IF \times F$ matrix, $\boldsymbol{\chi}_{n^s}$, given by

$$\boldsymbol{\chi}_c = \begin{pmatrix} \chi_c^{11} & 0 & 0 \\ \chi_c^{21} & 0 & 0 \\ 0 & \chi_c^{12} & 0 \\ 0 & \chi_c^{22} & 0 \\ 0 & 0 & \chi_c^{13} \\ 0 & 0 & \chi_c^{23} \end{pmatrix}_{IJ\times J}, \quad \boldsymbol{\chi}_{n^d} = \begin{pmatrix} \chi_n^{11,d} & 0 \\ \chi_n^{21,d} & 0 \\ \chi_n^{31,d} & 0 \\ 0 & \chi_n^{12,d} \\ 0 & \chi_n^{22,d} \\ 0 & \chi_n^{32,d} \end{pmatrix}_{JF\times F}, \quad \boldsymbol{\chi}_{n^s} = \begin{pmatrix} \chi_n^{11,s} & 0 \\ \chi_n^{21,s} & 0 \\ 0 & \chi_n^{12,s} \\ 0 & \chi_n^{22,s} \end{pmatrix}_{IF\times F}.$$

We collect intermediate-use shares, $\chi_x^{j\ell}$, and intermediate-supply shares, $\xi^{j\ell}$, in $JL \times J$ matrices $\boldsymbol{\chi}_x$ and $\boldsymbol{\xi}$, given by

$$\boldsymbol{\chi}_x = \begin{pmatrix} \chi_x^{11} & 0 & 0 \\ \chi_x^{21} & 0 & 0 \\ \chi_x^{31} & 0 & 0 \\ 0 & \chi_x^{12} & 0 \\ 0 & \chi_x^{22} & 0 \\ 0 & \chi_x^{32} & 0 \\ 0 & 0 & \chi_x^{13} \\ 0 & 0 & \chi_x^{23} \\ 0 & 0 & \chi_x^{33} \end{pmatrix}_{JL\times J}, \quad \boldsymbol{\xi} = \begin{pmatrix} \xi^{11} & 0 & 0 \\ \xi^{21} & 0 & 0 \\ \xi^{31} & 0 & 0 \\ 0 & \xi^{12} & 0 \\ 0 & \xi^{22} & 0 \\ 0 & \xi^{32} & 0 \\ 0 & 0 & \xi^{13} \\ 0 & 0 & \xi^{23} \\ 0 & 0 & \xi^{33} \end{pmatrix}_{JL\times J}.$$

We collect aggregate consumption and aggregate intermediate shares, ϕ_c^j and ϕ_x^j, in $J \times J$ diagonal matrices, $\boldsymbol{\phi}_c$ and $\boldsymbol{\phi}_x$, given by

$$\boldsymbol{\phi}_c = \begin{pmatrix} \phi_c^1 & 0 & 0 \\ 0 & \phi_c^2 & 0 \\ 0 & 0 & \phi_c^3 \end{pmatrix}_{J\times J}, \quad \boldsymbol{\phi}_x = \begin{pmatrix} \phi_x^1 & 0 & 0 \\ 0 & \phi_x^2 & 0 \\ 0 & 0 & \phi_x^3 \end{pmatrix}_{J\times J},$$

where $\boldsymbol{\phi}_c + \boldsymbol{\phi}_x = \boldsymbol{I}_J$, and

$$\boldsymbol{c} = \boldsymbol{\phi}_c \boldsymbol{y} \quad \text{and} \quad \boldsymbol{x} = \boldsymbol{\phi}_x \boldsymbol{y}.$$

We can thus write

$$\mathring{\boldsymbol{c}} = \boldsymbol{\chi}_c \boldsymbol{c}, \quad \mathring{\boldsymbol{n}}^d = \boldsymbol{\chi}_{n^d} \boldsymbol{n}^d, \quad \mathring{\boldsymbol{n}}^s = \boldsymbol{\chi}_{n^s} \boldsymbol{n}^s, \quad \mathring{\boldsymbol{x}} = \boldsymbol{\chi}_x \boldsymbol{x} = \boldsymbol{\xi} \boldsymbol{y}, \quad \boldsymbol{\xi} = \boldsymbol{\chi}_x \boldsymbol{\phi}_x.$$

Note that

$$\boldsymbol{1}_c \boldsymbol{\chi}_c = \boldsymbol{I}_J,\ \boldsymbol{1}_{n^s} \boldsymbol{\chi}_{n^s} = \boldsymbol{I}_F,\ \boldsymbol{1}_{n^d} \boldsymbol{\chi}_{n^d} = \boldsymbol{I}_F,\ \boldsymbol{1}_x \boldsymbol{\chi}_x = \boldsymbol{I}_J,\ \boldsymbol{1}_x \boldsymbol{\xi} = \boldsymbol{\phi}_x,$$

where $\boldsymbol{I}_J$ and $\boldsymbol{I}_F$ denote identity matrices of dimensions J and F respectively.

Marginal products/technology change. We collect marginal products of intermediates in a $J \times JL$ matrix $\boldsymbol{G}_x$, marginal products of factors in a $J \times JF$ matrix $\boldsymbol{G}_n$, and technology changes in a $J \times 1$ vector $\boldsymbol{G}_\theta$, given by

$$\boldsymbol{G}_x = \begin{pmatrix} \frac{\partial G^1}{\partial x^{11}} & 0 & 0 & \frac{\partial G^1}{\partial x^{12}} & 0 & 0 & \frac{\partial G^1}{\partial x^{13}} & 0 & 0 \\ 0 & \frac{\partial G^2}{\partial x^{21}} & 0 & 0 & \frac{\partial G^2}{\partial x^{22}} & 0 & 0 & \frac{\partial G^2}{\partial x^{23}} & 0 \\ 0 & 0 & \frac{\partial G^3}{\partial x^{31}} & 0 & 0 & \frac{\partial G^3}{\partial x^{32}} & 0 & 0 & \frac{\partial G^3}{\partial x^{33}} \end{pmatrix}_{J\times JL}$$

$$\boldsymbol{G}_n = \begin{pmatrix} \frac{\partial G^1}{\partial n^{11,d}} & 0 & 0 & \frac{\partial G^1}{\partial n^{12,d}} & 0 & 0 \\ 0 & \frac{\partial G^2}{\partial n^{21,d}} & 0 & 0 & \frac{\partial G^2}{\partial n^{22,d}} & 0 \\ 0 & 0 & \frac{\partial G^3}{\partial n^{31,d}} & 0 & 0 & \frac{\partial G^3}{\partial n^{32,d}} \end{pmatrix}_{J\times JF}, \quad \boldsymbol{G}_\theta = \begin{pmatrix} \frac{\partial G^1}{\partial \theta} \\ \frac{\partial G^2}{\partial \theta} \\ \frac{\partial G^3}{\partial \theta} \end{pmatrix}_{J\times 1}.$$

Marginal rates of substitution. We collect marginal rates of substitution in $1 \times IJ$ and $1 \times IF$ vectors $\boldsymbol{MRS}_c$ and $\boldsymbol{MRS}_n$, given by

$$\boldsymbol{MRS}_c = \begin{pmatrix} MRS_c^{11} & MRS_c^{21} & MRS_c^{12} & MRS_c^{22} & MRS_c^{13} & MRS_c^{23} \end{pmatrix}_{1\times IJ}$$
$$\boldsymbol{MRS}_n = \begin{pmatrix} MRS_n^{11} & MRS_n^{21} & MRS_n^{12} & MRS_n^{22} \end{pmatrix}_{1\times IF}.$$

The $1 \times J$ and $1 \times F$ vectors of aggregate marginal rates of substitution, $\boldsymbol{AMRS}_c$ and $\boldsymbol{AMRS}_n$, can be written as

$$\boldsymbol{AMRS}_c = \boldsymbol{MRS}_c \boldsymbol{\chi}_c \quad \text{and} \quad \boldsymbol{AMRS}_n = \boldsymbol{MRS}_n \boldsymbol{\chi}_{n^s}.$$

Marginal social value of goods. We collect the marginal social value of goods in $1 \times J$ vector $\boldsymbol{MSV}_y$, given by

$$\boldsymbol{MSV}_y = \begin{pmatrix} MSV_y^1 & MSV_y^2 & MSV_y^3 \end{pmatrix}_{1\times J}, \quad \text{where} \quad \boldsymbol{MSV}_y = \boldsymbol{AMRS}_c \boldsymbol{\phi}_c \boldsymbol{\Psi}_y.$$

Marginal welfare products. We collect marginal welfare products in $1 \times JL$ and $1 \times JF$ vectors $\boldsymbol{MWP}_x$ and $\boldsymbol{MWP}_n$, given by

$$\boldsymbol{MWP}_x = \boldsymbol{MSV}_y \boldsymbol{G}_x, \quad \text{and} \quad \boldsymbol{MWP}_n = \boldsymbol{MSV}_y \boldsymbol{G}_n.$$

The $1 \times J$ and $1 \times F$ vectors of aggregate marginal welfare products, $\boldsymbol{AMWP}_x$ and $\boldsymbol{AMWP}_n$, can be written as

$$\boldsymbol{AMWP}_x = \boldsymbol{MWP}_x \boldsymbol{\chi}_x \quad \text{and} \quad \boldsymbol{AMWP}_n = \boldsymbol{MWP}_n \boldsymbol{\chi}_{n^d}.$$

Goods inverse matrix. We define the elements of the $J \times J$ goods inverse $\boldsymbol{\Psi}_y$ as follows:

$$\boldsymbol{\Psi}_y = \begin{pmatrix} \psi_y^{11} & \psi_y^{12} & \psi_y^{13} \\ \psi_y^{21} & \psi_y^{22} & \psi_y^{23} \\ \psi_y^{31} & \psi_y^{32} & \psi_y^{33} \end{pmatrix}_{J\times J}, \quad \text{where} \quad \boldsymbol{\Psi}_y = \left(\boldsymbol{I}_J - \boldsymbol{G}_x \boldsymbol{\xi}\right)^{-1}.$$

Competitive economies. In competitive economies, we collect prices p^j in the $1 \times J$ vector $\boldsymbol{p}$ and wages w^f in $1 \times F$ vector $\boldsymbol{w}$, given by

$$\boldsymbol{p} = \begin{pmatrix} p^1 & p^2 & p^3 \end{pmatrix}_{1\times J}, \quad \boldsymbol{w} = \begin{pmatrix} w^1 & w^2 \end{pmatrix}_{1\times F}.$$

We also collect consumption wedges in a $J \times IJ$ vector, $\boldsymbol{\tau}_c$, factor supply wedges in a $F \times IF$ vector, $\boldsymbol{\tau}_{n^s}$, intermediate use wedges in a $J \times JL$ vector, $\boldsymbol{\tau}_x$, and factor demand wedges in a $F \times JF$, $\boldsymbol{\tau}_{n^d}$, given by

$$\boldsymbol{\tau}_c = \begin{pmatrix} \tau_c^{11} & \tau_c^{21} & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau_c^{12} & \tau_c^{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & \tau_c^{13} & \tau_c^{23} \end{pmatrix}_{J\times IJ}, \quad \boldsymbol{\tau}_{n^s} = \begin{pmatrix} \tau_n^{11,s} & \tau_n^{21,s} & 0 & 0 \\ 0 & 0 & \tau_n^{12,s} & \tau_n^{22,s} \end{pmatrix}_{F\times IF},$$

$$
\boldsymbol{\tau}_x = \begin{pmatrix} \frac{\tau_x^{11}-\tau_y^1}{1+\tau_y^1} & \frac{\tau_x^{21}-\tau_y^2}{1+\tau_y^2} & \frac{\tau_x^{31}-\tau_y^3}{1+\tau_y^3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\tau_x^{12}-\tau_y^1}{1+\tau_y^1} & \frac{\tau_x^{22}-\tau_y^2}{1+\tau_y^2} & \frac{\tau_x^{32}-\tau_y^3}{1+\tau_y^3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\tau_x^{13}-\tau_y^1}{1+\tau_y^1} & \frac{\tau_x^{23}-\tau_y^2}{1+\tau_y^2} & \frac{\tau_x^{33}-\tau_y^3}{1+\tau_y^3} \end{pmatrix}_{J\times JL},
$$

$$
\boldsymbol{\tau}_{n^d} = \begin{pmatrix} \frac{\tau_n^{11,d}-\tau_y^1}{1+\tau_y^1} & \frac{\tau_n^{12,d}-\tau_y^1}{1+\tau_y^1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\tau_n^{21,d}-\tau_y^2}{1+\tau_y^2} & \frac{\tau_n^{22,d}-\tau_y^2}{1+\tau_y^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\tau_n^{31,d}-\tau_y^3}{1+\tau_y^3} & \frac{\tau_n^{32,d}-\tau_y^3}{1+\tau_y^3} \end{pmatrix}_{F\times JF}.
$$

We use $\boldsymbol{I}_c$ and $\boldsymbol{I}_x$ to denote the following $J \times J$ indicator matrices:

$$
\boldsymbol{I}_c = \begin{pmatrix} 1\left[c^1>0\right] & 0 & 0 \\ 0 & 1\left[c^2>0\right] & 0 \\ 0 & 0 & 1\left[c^3>0\right] \end{pmatrix}_{J\times J}, \quad \boldsymbol{I}_x = \begin{pmatrix} 1\left[x^1>0\right] & 0 & 0 \\ 0 & 1\left[x^2>0\right] & 0 \\ 0 & 0 & 1\left[x^3>0\right] \end{pmatrix}_{J\times J}.
$$

We use and $\boldsymbol{I}_{n^s}$ to denote the following $F \times F$ indicator matrix:

$$
\boldsymbol{I}_{n^s} = \begin{pmatrix} 1\left[n^{1,s}>0\right] & 0 \\ 0 & 1\left[n^{2,s}>0\right] \end{pmatrix}_{F\times F}.
$$

We collect aggregate supply, aggregate consumption, and prices in $J \times J$ diagonal matrices $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{c}}$, and $\hat{\boldsymbol{p}}$, given by

$$
\hat{\boldsymbol{y}} = \operatorname{diag}(\boldsymbol{y}) = \begin{pmatrix} y^1 & 0 & 0 \\ 0 & y^2 & 0 \\ 0 & 0 & y^3 \end{pmatrix}_{J\times J}, \quad \hat{\boldsymbol{c}} = \operatorname{diag}(\boldsymbol{c}) = \begin{pmatrix} c^1 & 0 & 0 \\ 0 & c^2 & 0 \\ 0 & 0 & c^3 \end{pmatrix}_{J\times J}, \quad \hat{\boldsymbol{p}} = \operatorname{diag}(\boldsymbol{p}) = \begin{pmatrix} p^1 & 0 & 0 \\ 0 & p^2 & 0 \\ 0 & 0 & p^3 \end{pmatrix}_{J\times J}.
$$

In parallel to the definition of marginal welfare products, we define marginal revenue products as $MRP_x^{j\ell} = p^j \frac{\partial G^j}{\partial x^{j\ell}}$ and $MRP_n^{jf} = p^j \frac{\partial G^j}{\partial n^{jf,d}}$. In matrix form, $\boldsymbol{MRP}_x = \boldsymbol{p}\boldsymbol{G}_x$ and $\boldsymbol{MRP}_n = \boldsymbol{p}\boldsymbol{G}_n$.

B Shares Definitions

Here we provide formal definitions of shares that apply also when denominators can take zero value. We define individual i's consumption share of good j, χ_c^{ij}, and individual i's factor supply share of factor f, $\chi_n^{if,s}$, as

$$
\chi_c^{ij} := \begin{cases} \frac{c^{ij}}{c^j} & \text{if } c^j > 0 \\ \frac{\frac{dc^{ij}}{d\theta}}{\frac{dc^j}{d\theta}} & \text{if } c^j = 0 \quad \text{and} \quad \frac{dc^j}{d\theta} > 0 \\ 0 & \text{if } c^j = 0 \quad \text{and} \quad \frac{dc^j}{d\theta} = 0 \end{cases} \quad \text{and} \quad \chi_n^{if,s} := \begin{cases} \frac{n^{if,s}}{n^{f,s}} & \text{if } n^{f,s} > 0 \\ \frac{\frac{dn^{if,s}}{d\theta}}{\frac{dn^{f,s}}{d\theta}} & \text{if } n^{f,s} = 0 \quad \text{and} \quad \frac{dn^{f,s}}{d\theta} > 0 \\ 0 & \text{if } n^{f,s} = 0 \quad \text{and} \quad \frac{dn^{f,s}}{d\theta} = 0. \end{cases}
$$

Individual consumption shares χ_c^{ij} represent either the share of aggregate consumption c^j consumed by individual i, when $c^j > 0$, or the share of the change in aggregate consumption $dc^j/d\theta$ consumed by individual i, when $c^j = 0$ and $dc^j/d\theta > 0$. Individual factor supply shares $\chi_n^{if,s}$ are defined analogously.

We define good ℓ's *intermediate share*, ϕ_x^ℓ, and the *intermediate-use share* of good ℓ used to produce good j, $\chi_x^{j\ell}$, as

$$\phi_x^\ell := \begin{cases} \frac{x^\ell}{y^\ell} & \text{if } y^\ell > 0 \\ \frac{\frac{dx^\ell}{d\theta}}{\frac{dy^\ell}{d\theta}} & \text{if } y^\ell = 0 \quad \text{and} \quad \frac{dy^\ell}{d\theta} > 0 \\ 0 & \text{if } y^\ell = 0 \quad \text{and} \quad \frac{dy^\ell}{d\theta} = 0 \end{cases} \quad \text{and} \quad \chi_x^{j\ell} := \begin{cases} \frac{x^{j\ell}}{x^\ell} & \text{if } x^\ell > 0 \\ \frac{dx^{j\ell}}{dx^\ell} & \text{if } x^\ell = 0 \quad \text{and} \quad \frac{dx^\ell}{d\theta} > 0 \\ 0 & \text{if } x^\ell = 0 \quad \text{and} \quad \frac{dx^\ell}{d\theta} = 0. \end{cases}$$

Good ℓ's intermediate share, ϕ_x^ℓ, represents either the share of good ℓ's aggregate supply y^ℓ devoted to production, when $y^\ell > 0$, or the share of the change in good ℓ's aggregate supply $\frac{dy^\ell}{d\theta}$ devoted to production, when $y^\ell = 0$ and $\frac{dy^\ell}{d\theta} > 0$. Its complement defines the *aggregate consumption share* $\phi_c^\ell = 1 - \phi_x^\ell$. The intermediate-use share of good ℓ, $\chi_x^{j\ell}$, represents either the share of good ℓ's aggregate intermediate use devoted to the production of good j, when $x^\ell > 0$, or its counterpart in changes when $x^\ell = 0$ and $\frac{dx^\ell}{d\theta} > 0$.

Depending on ϕ_x^ℓ, good ℓ can be i) *pure final*, when $\phi_x^\ell = 0$; ii) *pure intermediate*, when $\phi_x^\ell = 1$; or iii) *mixed*, when $\phi_x^\ell \in (0,1)$. Equivalently, good ℓ can be i) *final* when $\phi_x^\ell \in [0,1)$ or ii) *intermediate*, when $\phi_x^\ell \in (0,1]$, with mixed goods being simultaneously final and intermediate. These categorizations are only meaningful when $y^\ell > 0$ or $\frac{dy^\ell}{d\theta} > 0$. Depending on $\chi_x^{j\ell}$, an intermediate input ℓ is i) *specialized*, when $\chi_x^{j\ell} = 1$ for some j; or *diversified*, when $\chi_x^{j\ell} \in (0,1)$ for some j.

Finally, we also define the *intermediate-supply share* of good ℓ by $\xi^{j\ell} = \chi_x^{j\ell}\phi_x^\ell$, which corresponds to $\frac{x^{j\ell}}{y^\ell}$ when $y^\ell > 0$ or to its counterpart in changes when $y^\ell = 0$ and $\frac{dy^\ell}{d\theta} > 0$. These definitions of shares ensure that changes in intermediate use can be expressed as

$$\frac{dx^{j\ell}}{d\theta} = \frac{d\xi^{j\ell}}{d\theta} y^\ell + \xi^{j\ell}\frac{dy^\ell}{d\theta}, \qquad \text{where} \qquad \frac{d\xi^{j\ell}}{d\theta} = \frac{d\chi_x^{j\ell}}{d\theta}\phi_x^\ell + \chi_x^{j\ell}\frac{d\phi_x^\ell}{d\theta}, \tag{OA1}$$

even when $y^\ell = 0$ and $x^\ell = 0$. Expression (OA1) initially decomposes level changes in the use $x^{j\ell}$ of good ℓ in the production of good j into two terms. First, changes in the intermediate-supply share $\frac{d\xi^{j\ell}}{d\theta}$ change $x^{j\ell}$ in proportion to good ℓ's aggregate supply y^ℓ. Second, changes in good ℓ's aggregate supply $\frac{dy^\ell}{d\theta}$ change $x^{j\ell}$ in proportion to the intermediate-supply share $\xi^{j\ell}$. In turn, changes in the intermediate-supply share $\frac{d\xi^{j\ell}}{d\theta}$ can occur either due to reallocation of good ℓ across different intermediate uses — a change in the intermediate-use share $\chi_x^{j\ell}$ — or due to reallocation from consumption to production — a change in the intermediate share ϕ_x^ℓ.

At last, we define the *factor use share* of factor f used to produce good j, $\chi_n^{jf,d}$, as

$$\chi_n^{jf,d} := \begin{cases} \frac{n^{jf,d}}{n^{f,d}} & \text{if } n^{f,d} > 0 \\ \frac{\frac{dn^{jf,d}}{d\theta}}{\frac{dn^{f,d}}{d\theta}} & \text{if } n^{f,d} = 0 \quad \text{and} \quad \frac{dn^{f,d}}{d\theta} > 0 \\ 0 & \text{if } n^{f,d} = 0 \quad \text{and} \quad \frac{dn^{f,d}}{d\theta} = 0. \end{cases}$$

The factor use share $\chi_n^{jf,d}$ represents the share of factor f's aggregate use $n^{f,d}$ devoted to the production of good j, or its counterpart in changes when $n^{f,d}=0$ and $\frac{dn^{f,d}}{d\theta}>0$. In this case, equation (OA1) ensures that changes in factor use can be expressed as

$$\frac{dn^{jf,d}}{d\theta}=\frac{d\chi_n^{jf,d}}{d\theta}n^{f,d}+\chi_n^{jf,d}\frac{dn^{f,d}}{d\theta}, \tag{OA2}$$

even when $n^{jf,d}=0$. Equation (OA2) decomposes level changes in the use $n^{jf,d}$ of factor f in the production of good j into a change in the factor use share, $\frac{dx_n^{jf}}{d\theta}$, and a change in the aggregate factor use, $\frac{d_n^{f,d}}{d\theta}$. A factor f is i) *specialized*, when $\chi_n^{jf,d}=1$ for some j; or *diversified*, when $\chi_n^{jf,d}\in(0,1)$ for some j.

C Proofs and Derivations

To simplify the exposition, we assume throughout that i) consumption is (weakly) desirable but supplying factors is not, i.e., $\frac{\partial u^i}{\partial c^{ij}}\geq 0$ and $\frac{\partial u^i}{\partial n^{if,s}}\leq 0$; ii) the marginal products of using intermediates and factors are (weakly) positive, i.e., $\frac{\partial G^j}{\partial x^{j\ell}}\geq 0$ and $\frac{\partial G^j}{\partial n^{jf,d}}\geq 0$; and iii) the no-free-lunch property holds, i.e., $G^j(\cdot)=0$ if $x^{j\ell}=0$, $\forall\ell$, and $n^{jf,d}=0$, $\forall f$. Many of our results, including the welfare accounting decomposition, do not require such restrictions.

C.1 Section 2

Proof of Lemma 1. (Efficiency/Redistribution Decomposition)

Proof. For any welfarist planner with social welfare function $\mathcal{W}(\cdot)$, we can express $\frac{dW}{d\theta}$ as

$$\frac{dW}{d\theta}=\sum_i\frac{\partial\mathcal{W}}{\partial V^i}\frac{dV^i}{d\theta}=\sum_i\frac{\partial\mathcal{W}}{\partial V^i}\lambda^i\frac{\frac{dV^i}{d\theta}}{\lambda^i},$$

where λ^i is an individual normalizing factor with units $\dim(\lambda^i)=\frac{\text{utils of individual } i}{\text{units of numeraire}}$ that allows us to express individual welfare assessments into a common unit/numeraire. We can therefore write

$$\frac{dW^\lambda}{d\theta}=\frac{\frac{dW}{d\theta}}{\frac{1}{I}\sum_i\frac{\partial\mathcal{W}}{\partial V^i}\lambda^i}=\sum_i\omega^i\frac{\frac{dV^i}{d\theta}}{\lambda^i}=\underbrace{\frac{1}{I}\sum_i\omega^i}_{=1}\sum_i\frac{\frac{dV^i}{d\theta}}{\lambda^i}+I\mathbb{C}ov_i\left[\omega^i,\frac{\frac{dV^i}{d\theta}}{\lambda^i}\right]=\underbrace{\sum_i\frac{\frac{dV^i}{d\theta}}{\lambda^i}}_{=\Xi^E}+\underbrace{\mathbb{C}ov_i^\Sigma\left[\omega^i,\frac{\frac{dV^i}{d\theta}}{\lambda^i}\right]}_{=\Xi^{RD}},$$

where $\omega^i=\frac{\frac{\partial\mathcal{W}}{\partial V^i}\lambda^i}{\frac{1}{I}\sum_i\frac{\partial\mathcal{W}}{\partial V^i}\lambda^i}$, which implies that $\frac{1}{I}\sum_i\omega^i=1$. □

C.2 Section 3

Proof of Theorem 1. (Exchange Efficiency)

Proof. Given the definition of V^i in equation (1), we can express $\frac{\frac{dV^i}{d\theta}}{\lambda^i}$ as

$$\frac{\frac{dV^i}{d\theta}}{\lambda^i} = \sum_j \frac{\frac{\partial u^i}{\partial c^{ij}}}{\lambda^i} \frac{dc^{ij}}{d\theta} + \sum_f \frac{\frac{\partial u^i}{\partial n^{if,s}}}{\lambda^i} \frac{dn^{if,s}}{d\theta} = \sum_j MRS_c^{ij} \frac{dc^{ij}}{d\theta} - \sum_f MRS_n^{if} \frac{dn^{if,s}}{d\theta}.$$

The marginal rate of substitution MRS_c^{ij} measures individual i's valuation in units of the welfare numeraire of a marginal increase in good j's consumption. Analogously, MRS_n^{if} measures individual i's cost in units of the welfare numeraire of a marginal increase in factor f's supply. Hence, from Lemma 1, it follows that

$$\Xi^E = \sum_i \frac{\frac{dV^i}{d\theta}}{\lambda^i} = \sum_j \sum_i MRS_c^{ij} \frac{dc^{ij}}{d\theta} - \sum_f \sum_i MRS_n^{if} \frac{dn^{if,s}}{d\theta}.$$

Given (9), we can write

$$\sum_i MRS_c^{ij} \frac{dc^{ij}}{d\theta} = \mathbb{C}ov_i^{\Sigma} \left[MRS_c^{ij}, \frac{d\chi_c^{ij}}{d\theta} \right] c^j + AMRS_c^j \frac{dc^j}{d\theta},$$

where $AMRS_c^j$ is defined in (16). Similarly, we can write

$$\sum_i MRS_n^{if,s} \frac{dn^{if,s}}{d\theta} = \mathbb{C}ov_i^{\Sigma} \left[MRS_n^{if,s}, \frac{d\chi_n^{if,s}}{d\theta} \right] n^{f,s} + AMRS_n^f \frac{dn^{f,s}}{d\theta},$$

where $AMRS_n^f$ is also defined in (16). Hence, exchange efficiency, $\Xi^{E,X}$, can be expressed as

$$\Xi^{E,X} = \underbrace{\mathbb{C}ov_i^{\Sigma} \left[MRS_c^{ij}, \frac{d\chi_c^{ij}}{d\theta} \right] c^j}_{\substack{\text{Cross-Sectional} \\ \text{Consumption Efficiency}}} \underbrace{- \mathbb{C}ov_i^{\Sigma} \left[MRS_n^{if,s}, \frac{d\chi_n^{if,s}}{d\theta} \right] n^{f,s}}_{\substack{\text{Cross-Sectional} \\ \text{Factor Supply Efficiency}}},$$

while production efficiency corresponds to

$$\Xi^{E,P} = \sum_j AMRS_c^j \frac{dc^j}{d\theta} - \sum_f AMRS_n^f \frac{dn^{f,s}}{d\theta}.$$

Alternatively, in matrix form, we can write

$$\Xi^E = \sum_i \frac{\frac{dV^i}{d\theta}}{\lambda^i} = \boldsymbol{MRS}_c \frac{d\mathring{\boldsymbol{c}}}{d\theta} - \boldsymbol{MRS}_n \frac{d\mathring{\boldsymbol{n}}^s}{d\theta},$$

where (9) can be expressed as

$$\frac{d\mathring{\boldsymbol{c}}}{d\theta} = \frac{d\boldsymbol{\chi}_c}{d\theta} \boldsymbol{c} + \boldsymbol{\chi}_c \frac{d\boldsymbol{c}}{d\theta} \quad \text{and} \quad \frac{d\mathring{\boldsymbol{n}}^s}{d\theta} = \frac{d\boldsymbol{\chi}_{n^s}}{d\theta} \boldsymbol{n}^s + \boldsymbol{\chi}_{n^s} \frac{d\boldsymbol{n}^s}{d\theta}.$$

Hence,

$$\boldsymbol{MRS}_c \frac{d\mathring{\boldsymbol{c}}}{d\theta} = \boldsymbol{MRS}_c \frac{d\boldsymbol{\chi}_c}{d\theta}\boldsymbol{c} + \boldsymbol{AMRS}_c \frac{d\boldsymbol{c}}{d\theta} \quad \text{and} \quad \boldsymbol{MRS}_n \frac{d\mathring{\boldsymbol{n}}^s}{d\theta} = \boldsymbol{MRS}_n \frac{d\boldsymbol{\chi}_{n^s}}{d\theta}\boldsymbol{n}^s + \boldsymbol{AMRS}_n \frac{d\boldsymbol{n}^s}{d\theta},$$

where $\boldsymbol{AMRS}_c = \boldsymbol{MRS}_c \boldsymbol{\chi}_c$ and $\boldsymbol{AMRS}_n = \boldsymbol{MRS}_n \boldsymbol{\chi}_{n^s}$. We can thus write

$$\Xi^E = \underbrace{\underbrace{\boldsymbol{MRS}_c \frac{d\boldsymbol{\chi}_c}{d\theta}\boldsymbol{c}}_{\substack{\text{Cross-Sectional} \\ \text{Consumption Efficiency}}} \underbrace{- \boldsymbol{MRS}_n \frac{d\boldsymbol{\chi}_{n^s}}{d\theta}\boldsymbol{n}^s}_{\substack{\text{Cross-Sectional} \\ \text{Factor Supply Efficiency}}}}_{\Xi^{E,X} \text{ (Exchange Efficiency)}} + \underbrace{\boldsymbol{AMRS}_c \frac{d\boldsymbol{c}}{d\theta} - \boldsymbol{AMRS}_n \frac{d\boldsymbol{n}^s}{d\theta}}_{\Xi^{E,P} \text{ (Production Efficiency)}}$$

□

Proof of Corollary 1. (Properties of Exchange Efficiency)

Proof. Proceeding item-by-item:

(a) When $I = 1$, $\mathbb{C}ov_i^{\Sigma}\left[MRS_c^{ij}, \frac{dc^{ij}}{d\theta}\right] = \mathbb{C}ov_i^{\Sigma}\left[MRS_n^{if,s}, \frac{dn^{if,s}}{d\theta}\right] = 0$, $\forall j$ and $\forall f$.

(b) When $n^{f,s} = 0$, $\mathbb{C}ov_i^{\Sigma}\left[MRS_n^{if,s}, \frac{d\chi_n^{if,s}}{d\theta}\right] n^{f,s} = 0$, $\forall f$.

(c) When MRS_c^{ij} is identical for all i, $\mathbb{C}ov_i^{\Sigma}\left[MRS_c^{ij}, \frac{dc^{ij}}{d\theta}\right] = 0$. When MRS_n^{if} is identical for all f, $\mathbb{C}ov_i^{\Sigma}\left[MRS_n^{if}, \frac{dn^{if,s}}{d\theta}\right] = 0$.

□

Proof of Lemma 2. (Goods Inverse Matrix)

Proof. Given (11) and (13) we can write $\frac{dy^j}{d\theta}$ and $\frac{dx^{j\ell}}{d\theta}$ in matrix form, as

$$\frac{d\boldsymbol{y}^s}{d\theta} = \boldsymbol{G}_x \frac{d\mathring{\boldsymbol{x}}}{d\theta} + \boldsymbol{G}_n \frac{d\mathring{\boldsymbol{n}}^d}{d\theta} + \boldsymbol{G}_\theta \quad \text{and} \quad \frac{d\mathring{\boldsymbol{x}}}{d\theta} = \frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y} + \boldsymbol{\xi}\frac{d\boldsymbol{y}}{d\theta}, \tag{OA3}$$

where

$$\frac{d\boldsymbol{y}}{d\theta} = \frac{d\boldsymbol{y}^s}{d\theta} + \frac{d\bar{\boldsymbol{y}}^s}{d\theta}.$$

Combining these expressions, we can express $\frac{d\boldsymbol{y}^s}{d\theta}$ as

$$\begin{aligned}
\frac{d\boldsymbol{y}^s}{d\theta} &= \boldsymbol{G}_x \left(\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y} + \boldsymbol{\xi}\left(\frac{d\boldsymbol{y}^s}{d\theta} + \frac{d\bar{\boldsymbol{y}}^s}{d\theta}\right)\right) + \boldsymbol{G}_n \frac{d\mathring{\boldsymbol{n}}^d}{d\theta} + \boldsymbol{G}_\theta \\
&= \boldsymbol{\Psi}_y \left(\boldsymbol{G}_x \frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y} + \boldsymbol{G}_n \frac{d\mathring{\boldsymbol{n}}^d}{d\theta} + \boldsymbol{G}_x \boldsymbol{\xi}\frac{d\bar{\boldsymbol{y}}^s}{d\theta} + \boldsymbol{G}_\theta\right),
\end{aligned}$$

where $\boldsymbol{\Psi}_y = (\boldsymbol{I}_J - \boldsymbol{G}_x\boldsymbol{\xi})^{-1}$. Finally, we use the fact that $\boldsymbol{\Psi}_y = \boldsymbol{I}_J + \boldsymbol{\Psi}_y \boldsymbol{G}_x \boldsymbol{\xi}$, so that we can express $\frac{d\boldsymbol{y}}{d\theta}$ as

$$\frac{d\boldsymbol{y}}{d\theta} = \frac{d\boldsymbol{y}^s}{d\theta} + \frac{d\bar{\boldsymbol{y}}^s}{d\theta} = \boldsymbol{\Psi}_y \left(\boldsymbol{G}_x \frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y} + \boldsymbol{G}_n \frac{d\mathring{\boldsymbol{n}}^d}{d\theta} + \frac{d\bar{\boldsymbol{y}}^s}{d\theta} + \boldsymbol{G}_\theta\right),$$

which corresponds to equation (14) in the text. □

Proof of Theorem 2. (Production Efficiency)

Proof. As shown above, we can express $\Xi^{E,P}$ in matrix form as

$$\Xi^{E,P} = \boldsymbol{AMRS}_c \frac{d\boldsymbol{c}}{d\theta} - \boldsymbol{AMRS}_n \frac{d\boldsymbol{n}^s}{d\theta}.$$

First, note that we can express the change in aggregate consumption, $\frac{d\boldsymbol{c}}{d\theta}$, as

$$\frac{d\boldsymbol{c}}{d\theta} = \frac{d\boldsymbol{y}}{d\theta} - \frac{d\boldsymbol{x}}{d\theta} = \frac{d\boldsymbol{y}}{d\theta} - \left(\boldsymbol{\phi}_x \frac{d\boldsymbol{y}}{d\theta} + \frac{d\boldsymbol{\phi}_x}{d\theta}\boldsymbol{y}\right) = \boldsymbol{\phi}_c \frac{d\boldsymbol{y}}{d\theta} - \frac{d\boldsymbol{\phi}_x}{d\theta}\boldsymbol{y},$$

where we use the fact that $\frac{d\boldsymbol{x}}{d\theta} = \boldsymbol{\phi}_x \frac{d\boldsymbol{y}}{d\theta} + \frac{d\boldsymbol{\phi}_x}{d\theta}\boldsymbol{y}$ and $\boldsymbol{\phi}_c = \boldsymbol{I}_J - \boldsymbol{\phi}_x$.

Next, note that $\frac{d\boldsymbol{y}}{d\theta}$ can be written as

$$\frac{d\boldsymbol{y}}{d\theta} = \boldsymbol{\Psi}_y \left(\boldsymbol{G}_x \frac{d\boldsymbol{\chi}_x}{d\theta}\boldsymbol{x} + \boldsymbol{G}_x\boldsymbol{\chi}_x \frac{d\boldsymbol{\phi}_x}{d\theta}\boldsymbol{y} + \boldsymbol{G}_n \frac{d\boldsymbol{\chi}_{n^d}}{d\theta}\boldsymbol{n}^d + \boldsymbol{G}_n\boldsymbol{\chi}_{n^d}\frac{d\boldsymbol{n}^d}{d\theta} + \frac{d\bar{\boldsymbol{y}}}{d\theta} + \boldsymbol{G}_\theta\right),$$

where we use the fact that

$$\frac{d\boldsymbol{\xi}}{d\theta} = \frac{d\boldsymbol{\chi}_x}{d\theta}\boldsymbol{x} + \boldsymbol{\chi}_x \frac{d\boldsymbol{\phi}_x}{d\theta}\boldsymbol{y} \quad \text{and} \quad \frac{d\mathring{\boldsymbol{n}}^d}{d\theta} = \frac{d\boldsymbol{\chi}_{n^d}}{d\theta}\boldsymbol{n}^d + \boldsymbol{\chi}_{n^d}\frac{d\boldsymbol{n}^d}{d\theta}.$$

This result allows us to express $\frac{d\boldsymbol{c}}{d\theta}$ as

$$\frac{d\boldsymbol{c}}{d\theta} = \boldsymbol{\phi}_c\boldsymbol{\Psi}_y\boldsymbol{G}_x \frac{d\boldsymbol{\chi}_x}{d\theta}\boldsymbol{x} + (\boldsymbol{\phi}_c\boldsymbol{\Psi}_y\boldsymbol{G}_x\boldsymbol{\chi}_x - \boldsymbol{I}_J)\frac{d\boldsymbol{\phi}_x}{d\theta}\boldsymbol{y} + \boldsymbol{\phi}_c\boldsymbol{\Psi}_y\boldsymbol{G}_n \frac{d\boldsymbol{\chi}_{n^d}}{d\theta}\boldsymbol{n}^d + \boldsymbol{\phi}_c\boldsymbol{\Psi}_y\boldsymbol{G}_n\boldsymbol{\chi}_{n^d}\frac{d\boldsymbol{n}^d}{d\theta} + \boldsymbol{\phi}_c\boldsymbol{\Psi}_y\left(\frac{d\bar{\boldsymbol{y}}^s}{d\theta} + \boldsymbol{G}_\theta\right)$$

Hence, combining this expression for $\frac{d\boldsymbol{c}}{d\theta}$ with the resource constraint for factors, which implies that $\frac{d\boldsymbol{n}^d}{d\theta} = \frac{d\boldsymbol{n}^s}{d\theta} + \frac{d\bar{\boldsymbol{n}}^s}{d\theta}$, we can express production efficiency exactly as in text, as follows:

$$\begin{aligned}\Xi^{E,P} = & \underbrace{\boldsymbol{MWP}_x \frac{d\boldsymbol{\chi}_x}{d\theta}\boldsymbol{x}}_{\substack{\text{Cross-Sectional}\\ \text{Intermediate Input Efficiency}}} + \underbrace{(\boldsymbol{AMWP}_x - \boldsymbol{AMRS}_c)\frac{d\boldsymbol{\phi}_x}{d\theta}\boldsymbol{y}}_{\substack{\text{Aggregate}\\ \text{Intermediate Input Efficiency}}} \\ & + \underbrace{\boldsymbol{MWP}_n \frac{d\boldsymbol{\chi}_{n^d}}{d\theta}\boldsymbol{n}^d}_{\substack{\text{Cross-Sectional}\\ \text{Factor Efficiency}}} + \underbrace{(\boldsymbol{AMWP}_n - \boldsymbol{AMRS}_n)\frac{d\boldsymbol{n}^s}{d\theta}}_{\substack{\text{Aggregate}\\ \text{Factor Efficiency}}} \\ & + \underbrace{\boldsymbol{MSV}_y \frac{d\bar{\boldsymbol{y}}^s}{d\theta}}_{\substack{\text{Technology}\\ \text{Change}}} + \underbrace{\boldsymbol{MSV}_y\boldsymbol{G}_\theta}_{\substack{\text{Good Endowment}\\ \text{Change}}} + \underbrace{\boldsymbol{AMRS}_n \frac{d\bar{\boldsymbol{n}}^s}{d\theta}}_{\substack{\text{Factor Endowment}\\ \text{Change}}},\end{aligned}$$

where

$$\boldsymbol{MWP}_x = \boldsymbol{MSV}_y\boldsymbol{G}_x,\ \boldsymbol{MWP}_n = \boldsymbol{MSV}_y\boldsymbol{G}_n,\ \boldsymbol{MSV}_y = \boldsymbol{AMRS}_c\boldsymbol{\phi}_c\boldsymbol{\Psi}_y.$$

□

Proof of Corollary 2. (Properties of Production Efficiency Decomposition)

Proof. Proceeding item-by-item:

(a) When $J = 1$, $\mathbb{C}ov_j^{\Sigma}\left[MWP_x^{j\ell}, \frac{d\chi_x^{j\ell}}{d\theta}\right] = 0$, $\forall \ell$.

(b) With no intermediate goods, $x^{\ell} = \frac{d\phi_x^{\ell}}{d\theta} = 0$, $\forall \ell$.

(c) If all factors are in fixed supply, $\frac{dn^{f,s}}{d\theta} = 0$, $\forall s$.

(d) If all intermediate inputs are specialized: $\frac{dx_x^{j\ell}}{d\theta} = 0$, $\forall j$, $\forall \ell$. If all factors are specialized, $\frac{dx_n^{jf,d}}{d\theta} = 0$, $\forall j$, $\forall f$.

(e) When marginal welfare products are equalized for intermediates: $\mathbb{C}ov_j^{\Sigma}\left[MWP_x^{j\ell}, \frac{d\chi_x^{j\ell}}{d\theta}\right] = 0$, $\forall \ell$; for factors: $\mathbb{C}ov_j^{\Sigma}\left[MWP_n^{jf}, \frac{d\chi_n^{jf,d}}{d\theta}\right] = 0$, $\forall f$.

□

C.3 Section 4

In the body of the paper, we assume that that $y^j > 0$ and $n^{f,d} > 0$, but here we also allow for $y^j = 0$ and $n^{f,d} = 0$.

Proof of Theorem 3. (Efficiency Conditions: Exchange Efficiency)

Proof. If MRS_c^{ij} is different across any two individuals with $\chi_c^{ij} > 0$ for good j with $c^j > 0$, then there exists a perturbation of consumption shares in which cross-sectional consumption efficiency is positive. If MRS_c^{ij} is less than $AMRS_c^j$ when $\chi_c^{ij} = 0$, then there is no feasible perturbation that reduces the share of consumption for individual i. The same logic applies to cross-sectional factor supply efficiency. □

Proof of Theorem 4. (Efficiency Conditions: Production Efficiency)

Proof. If $MWP_x^{j\ell}$ is different across any two intermediate uses of good ℓ two individuals with $\chi_x^{j\ell} > 0$, then there exists a perturbation of intermediate use shares in which cross-sectional intermediate input efficiency is positive. The same logic applies to cross-sectional factor use efficiency.

When $\phi_x^{\ell} \in (0, 1)$, then there exists a perturbation of ϕ_x^{ℓ} such that aggregate intermediate input efficiency is positive unless $AMWP_x^{\ell} = AMRS_c^{\ell}$. If $\phi_x^{\ell} = 0$, it must be that $AMWP_x^{\ell} \leq AMRS_c^{\ell}$ for the best possible combination of intermediate use shares, which is the one that allocates good ℓ to its highest marginal welfare product intermediate use. If $\phi_x^{\ell} = 1$, it must be that $AMWP_x^{\ell} \geq AMRS_c^{\ell}$ for the possible combinations of consumption shares, which is the one that allocates the consumption of good j to the individual with the highest $MRS_c^{i\ell}$.

When $n^{f,s} > 0$ (and $n^{f,d} > 0$), then there exists a perturbation of $n^{f,s}$ such that aggregate factor supply efficiency is positive unless $AMWP_n^f = AMRS_n^f$. If $n^{f,s} = 0$, it must be that $AMWP_n^f \leq AMRS_n^f$ for the best possible combination of factor supply shares, which is the one that allocates the consumption of good j to the individual with the lowest MRS_n^{if}. If $n^{f,s} = n^{f,d} = 0$, then it must be that the most costly way of supplying a factor is higher than the highest marginal welfare product of doing so, formally: $\max_j \left\{MWP_n^{jf}\right\} \leq \min_i \left\{MRS_n^{if}\right\}$.[14] $\square$

Proof of Corollary 3. (Interior Economies)

Proof. Recall that we define marginal rates of substitution in units of the numeraire, i.e., $MRS_c^{ij} = \frac{\partial u^i}{\partial c^{ij}}/\lambda^i$. If condition (20) holds, then $MRS_c^{i\ell}/MRS_c^{ij} = \frac{\partial G^j}{\partial x^{j\ell}}$ must be equal across individuals since marginal products do not depend on i. This implies that two individuals' valuation of good ℓ, expressed in units of good j, is equalized. Since (20) applies for all j and ℓ, it also implies the equalization of MRS in units of the welfare numeraire. To derive the equalization of MRT, notice that (20) can be rewritten as

$$MRS_c^{ij}\frac{\partial G^j}{\partial x^{j\ell}} = MRS_c^{ij'}\frac{\partial G^{j'}}{\partial x^{j'\ell}} \quad \implies \quad MRS_c^{ij}/MRS_c^{ij'} = \frac{\partial G^{j'}}{\partial x^{j'\ell}}/\frac{\partial G^j}{\partial x^{j\ell}} \equiv MRT^{jj',\ell}$$

where the RHS defines the marginal rate of transformation (MRT). Condition (20) therefore implies both $MRS = MRT$ (after a change of units) and the equalization of MRT across uses since the LHS does not depend on ℓ. A similar argument applies to factor use. $\square$

Proof of Corollary 4. (Classical Efficiency Conditions Hold for Interior Links)

Proof. Proceeding item-by-item:

(a) At an interior link, Theorem 4 and the fact that $MSV_y^j = MWP_x^{\ell j} = MRS_c^{ij}$ imply that both equations in (20) hold.

(b) The result follows then from the same logic as in Corollary 3.

$\square$

Proof of Corollary 5. (Scenarios in which Classical Efficiency Conditions Do Not Hold)

Proof. Proceeding item-by-item:

(a) If good j is a pure intermediate, then $MSV_y^j \neq AMRS_c^j$, which implies that the classical efficiency conditions cannot hold, since efficiency requires that $MSV_y^j \frac{\partial G^j}{\partial x^{j\ell}} = MRS_c^{ij}$.

(b) If good ℓ is a pure intermediate, then last condition of Theorem 4 already implies that the classical efficiency conditions cannot hold.

[14] When $n^{f,d} = 0$, the value of a marginal unit of endowment of factor f is simply $\max_j \left\{MWP_n^{jf}\right\}$.

(c) As in (a), $MSV_y^j \neq AMRS_c^j$, which implies that the classical efficiency conditions cannot hold, since efficiency requires that $MSV_y^j \frac{\partial G^j}{\partial n^{jf,d}} = MRS_n^{if}$.

$\square$

C.4 Section 5

Definition. *(Competitive Equilibrium with Wedges). A competitive equilibrium with wedges comprises a feasible allocation* $\left\{c^{ij}, n^{if,s}, x^{j\ell}, n^{jf,d}, y^{j,s}\right\}$ *and prices* $\left\{p^j, w^f\right\}$ *that satisfy resource constraints* (3) *and* (4)*, such that individuals optimize,*

$$MRS_c^{ij} \leq p^j \left(1 + \tau_c^{ij}\right), \quad \forall i, \forall j \quad \textit{and} \quad MRS_n^{if} \geq w^f \left(1 + \tau_n^{if,s}\right), \quad \forall i,\ \forall f,$$

where the equations hold with equality when $c^{ij} > 0$ *and* $n^{if,s} > 0$*, respectively, and firms minimize costs and maximize profits,*

$$p^j \frac{\partial G^j}{\partial x^{j\ell}} \leq p^\ell \frac{1 + \tau_x^{j\ell}}{1 + \tau_y^j}, \quad \forall j, \forall \ell \quad \textit{and} \quad p^j \frac{\partial G^j}{\partial n^{jf,d}} \leq w^f \frac{1 + \tau_n^{jf,d}}{1 + \tau_y^j}, \quad \forall j,\ \forall f,$$

where the equations hold with equality when $x^{j\ell} > 0$ *and* $n^{jf,d} > 0$*, respectively.*

In this section, we implicitly choose the nominal numeraire (i.e. the unit in which prices, wages, and profits are defined) to be the welfare numeraire. This is without loss of generality since we can always renormalize MRS.

Proof of Theorem 5. (MSV in Competitive Economies with Wedges)

Proof. In a competitive equilibrium with wedges, we can express aggregate marginal rates of substitution as

$$\boldsymbol{AMRS}_c = \boldsymbol{MRS}_c \boldsymbol{\chi}_c = \boldsymbol{p}\left(\boldsymbol{I}_c + \bar{\boldsymbol{\tau}}_c\right),$$

where $\boldsymbol{I}_c$ is $J \times J$ diagonal matrix in which the j'th element is 1 when $c^j > 0$ and 0 if $c^j = 0$, and where we define a $J \times J$ matrix of aggregate consumption wedges as $\bar{\boldsymbol{\tau}}_c = \boldsymbol{\tau}_c \boldsymbol{\chi}_c$. It is also the case that

$$\boldsymbol{p}\boldsymbol{G}_x \boldsymbol{\chi}_x = \boldsymbol{p}\left(\boldsymbol{I}_x + \boldsymbol{\tau}_x \boldsymbol{\chi}_x\right) = \boldsymbol{p}\left(\boldsymbol{I}_x + \bar{\boldsymbol{\tau}}_x\right),$$

where $\boldsymbol{I}_x$ is $J \times J$ diagonal matrix in which the j'th element is 1 when $x^j > 0$ and 0 if $x^j = 0$, and where we define a $J \times J$ matrix of aggregate intermediate use wedges as $\bar{\boldsymbol{\tau}}_x = \boldsymbol{\tau}_x \boldsymbol{\chi}_x$. Hence, we can express the marginal social value of goods as

$$\begin{aligned}\boldsymbol{MSV}_y = \boldsymbol{AMRS}_c \boldsymbol{\phi}_c \boldsymbol{\Psi}_y &= \boldsymbol{p}\left(\boldsymbol{I}_c + \bar{\boldsymbol{\tau}}_c\right)\boldsymbol{\phi}_c \boldsymbol{\Psi}_y = \boldsymbol{p}\boldsymbol{\phi}_c \boldsymbol{\Psi}_y + \boldsymbol{p}\bar{\boldsymbol{\tau}}_c \boldsymbol{\phi}_c \boldsymbol{\Psi}_y \\ &= \boldsymbol{p} + \boldsymbol{p}\left(\bar{\boldsymbol{\tau}}_x \boldsymbol{\phi}_x + \bar{\boldsymbol{\tau}}_c \boldsymbol{\phi}_c\right)\boldsymbol{\Psi}_y,\end{aligned}$$

where we use the fact that $\boldsymbol{I}_c\boldsymbol{\phi}_c = \boldsymbol{\phi}_c$ and that

$$\begin{aligned}\boldsymbol{p}\boldsymbol{\phi}_c\boldsymbol{\Psi}_y &= \boldsymbol{p}\left(\left(\boldsymbol{G}_x - \boldsymbol{1}_x\right)\boldsymbol{\xi}\boldsymbol{\Psi}_y + \boldsymbol{I}_J\right) = \left(\boldsymbol{p}\boldsymbol{G}_x - \boldsymbol{p}\boldsymbol{1}_x\right)\boldsymbol{\chi}_x\boldsymbol{\phi}_x\boldsymbol{\Psi}_y + \boldsymbol{p} \\ &= \left(\boldsymbol{p}\boldsymbol{G}_x\boldsymbol{\chi}_x - \boldsymbol{p}\right)\boldsymbol{\phi}_x\boldsymbol{\Psi}_y + \boldsymbol{p} = \left(\boldsymbol{p}\left(\boldsymbol{I}_x + \bar{\boldsymbol{\tau}}_x\right) - \boldsymbol{p}\right)\boldsymbol{\phi}_x\boldsymbol{\Psi}_y + \boldsymbol{p} = \boldsymbol{p}\bar{\boldsymbol{\tau}}_x\boldsymbol{\phi}_x\boldsymbol{\Psi}_y + \boldsymbol{p}.\end{aligned}$$

Given Theorem 5, the technology change component of the welfare accounting decomposition is simply given by

$$\boldsymbol{MSV}_y\boldsymbol{G}_\theta = \sum_j MSV_y^j \frac{\partial G^j}{\partial \theta} = \sum_j \left(p^j + \sum_\ell p^\ell \bar{\tau}_y^\ell \psi_y^{\ell j}\right)\frac{\partial G^j}{\partial \theta}.$$

□

Proof of Corollary 6. (Converse Hulten's Theorem)

Proof. Follows directly from Theorem 5. □

Proof of Corollary 7. (Hulten's Theorem Revisited)

Proof. Since frictionless competitive economies are efficient, Ξ^E simply equals technology change. When $\bar{\boldsymbol{\tau}}_c = \bar{\boldsymbol{\tau}}_x = 0$, it follows from Theorem 5 that $MSV_y^j = p^j$. Hence, $\Xi^E = p^j \frac{\partial G^j}{\partial \theta} = p^j y^j$, where we use the fact that $\frac{\partial G^j}{\partial \theta} = y^j$ for proportional Hicks-neutral technology changes. Simply dividing by $\sum_j p^j c^j$ yields equation (26) in the text. □

Relation to Cost-Based Domar Weights. A central result of Baqaee and Farhi (2020) is that cost-based Domar weights summarize the impact of pure technological change on final output in an environment with a single individual, factors in fixed supply, and markup wedges. Their result is a special case of Theorem 5. Formally, under the assumptions in that paper,

$$\underbrace{\frac{1}{\sum_j p^j c^j}}_{\text{Normalization}} \underbrace{\boldsymbol{MSV}_y\boldsymbol{G}_\theta}_{\substack{\text{Technology Change} \\ \text{Component}}} = \underbrace{\frac{1}{\sum_j p^j c^j}\boldsymbol{p}\hat{\boldsymbol{c}}}_{\substack{\text{Final Expenditure} \\ \text{Share}}} \underbrace{\tilde{\boldsymbol{\Psi}}_y}_{\substack{\text{Cost-Based} \\ \text{Leontief Inverse}}}, \tag{OA4}$$

where $\hat{\boldsymbol{c}} = \text{diag}(\boldsymbol{c})$ and $\tilde{\boldsymbol{\Psi}}_y$ is the proportional goods inverse, which in turn maps to the intermediate input block of the cost-based Leontief inverse defined in Baqaee and Farhi (2020) — see Appendix E.2. Relative to equation (OA4), Theorem 5 illustrates how competitive forces guarantee that $MSV_y^j = p^j$ when $\bar{\boldsymbol{\tau}}_y = 0$. Away from the assumptions in Baqaee and Farhi (2020), Theorem 5 highlights that cost-based Domar weights cease to capture the efficiency gains from pure technological change, for instance in the presence of aggregate consumption wedges.

D Redistribution

Our analysis in the body of the paper exclusively focuses on efficiency. However, perturbations with identical efficiency implications may have different distributional implications. Theorem D decomposes redistribution gains or losses into four components: *Cross-sectional consumption* and *factor supply redistribution* capture redistribution gains due to the reallocation of consumption and factor supply shares, for given aggregate levels of consumption and factor supply. And *aggregate consumption* and *factor supply redistribution* capture redistribution gains due to changes in aggregate consumption and factor supply, for given shares. Critically, the choice of social welfare function will directly impact the welfare gains from redistribution and its components.

Theorem. *(Redistribution Decomposition). The redistribution component of the welfare accounting decomposition, Ξ^{RD}, can be decomposed into*

$$\begin{aligned}\Xi^{RD} = &\overbrace{\sum_j \mathbb{C}ov_i^{\Sigma}\left[\omega^i, MRS_c^{ij}\frac{d\chi_c^{ij}}{d\theta}\right]c^j}^{\substack{\text{Cross-Sectional}\\ \text{Consumption Redistribution}}} \quad + \overbrace{\sum_j \mathbb{C}ov_i^{\Sigma}\left[\omega^i, MRS_c^{ij}\chi_c^{ij}\right]\frac{dc^j}{d\theta}}^{\substack{\text{Aggregate}\\ \text{Consumption Redistribution}}} \\ &\underbrace{-\sum_f \mathbb{C}ov_i^{\Sigma}\left[\omega^i, MRS_n^{if}\frac{d\chi_n^{if,s}}{d\theta}\right]n^{f,s}}_{\substack{\text{Cross-Sectional}\\ \text{Factor Supply Redistribution}}} \underbrace{-\sum_f \mathbb{C}ov_i^{\Sigma}\left[\omega^i, MRS_n^{if}\chi_n^{if,s}\right]\frac{dn^{f,s}}{d\theta}}_{\substack{\text{Aggregate}\\ \text{Factor Supply Redistribution}}}.\end{aligned}$$

The cross-sectional terms capture redistribution gains or losses due to the reallocation of consumption and factor supply, for given c^j and $n^{f,s}$. In particular, cross-sectional consumption redistribution is positive for good j when individuals with high normalized individual weight ω^i — those relatively favored by the planner — see their consumption shares increase; MRS_c^{ij} captures potentially different marginal consumption values. The aggregate terms capture redistribution gains due to changes in aggregates, for given allocation shares. In particular, aggregate consumption redistribution is positive for good j when aggregate consumption increases and individuals with high ω^i consume a relatively larger share of the good. The logic is parallel for factor supply redistribution. The cross-sectional terms parallel exchange efficiency since they are driven by changes in consumption or factor supply shares given aggregates, while the aggregate terms parallel production efficiency since they are driven by changes in aggregates consumption and factor supply.

E Additional Results

E.1 Planning Problem

The Lagrangian of the planning problem can be expressed as

$$\begin{aligned}\mathcal{L} = &\mathcal{W}\left(V^1, \ldots, V^i, \ldots, V^I\right) \\ &-\sum_j \zeta_y^j\left(\sum_i c^{ij} + \sum_\ell x^{\ell j} - G^j\left(\left\{x^{j\ell}\right\}_\ell, \left\{n^{jf,d}\right\}_f\right)\right) - \sum_f \zeta_n^f\left(\sum_j n^{jf,d} - \sum_i n^{if,s} - \sum_i \bar{n}^{if,s}\right) \\ &+\sum_i\sum_j \kappa_c^{ij}c^{ij} + \sum_i\sum_f \kappa_n^{if,s}n^{if,s} + \sum_j\sum_\ell \kappa_x^{j\ell}x^{j\ell} + \sum_j\sum_f \kappa_n^{jf,d}n^{jf,d},\end{aligned}$$

where V^i is defined in (1). Hence, the first-order conditions can be derived from a perturbation of the form

$$d\mathcal{L} = \sum_j \sum_i \left(\alpha^i \frac{\partial u^i}{\partial c^{ij}} - \zeta_y^j + \kappa_c^{ij} \right) dc^{ij} + \sum_i \sum_f \left(\alpha^i \frac{\partial u^i}{\partial n^{if,s}} + \zeta_n^f + \kappa_n^{if,s} \right) dn^{if,s}$$
$$+ \sum_j \sum_\ell \left(\zeta_y^j \frac{\partial G^j}{\partial x^{j\ell}} - \zeta_y^\ell + \kappa_x^{j\ell} \right) dx^{j\ell} + \sum_j \sum_f \left(\zeta_y^j \frac{\partial G^j}{\partial n^{jf,d}} - \zeta_n^f + \kappa_n^{jf,d} \right) dn^{jf,d},$$

where we take good j' as numeraire, which allows us to substitute α^i for $\alpha^i \frac{\partial u^i}{\partial c^{ij'}} = \zeta_y^{j'} \Rightarrow \alpha^i = \left(\frac{\frac{\partial u^i}{\partial c^{j'}}}{\zeta_y^{j'}} \right)^{-1}$, and where we define $MWP_x^{j\ell} = \zeta_y^j \frac{\partial G^j}{\partial x^{j\ell}}$ and $MWP_n^{jf} = \zeta_y^j \frac{\partial G^j}{\partial n^{jf,d}}$. Formally, the Kuhn-Tucker conditions are

i) $\kappa_c^{ij} c^{ij} = 0 \Rightarrow \left(\zeta_y^j - MRS_c^{ij} \right) c^{ij} = 0$, with generically one of the two terms > 0;

ii) $\kappa_n^{if,s} n^{if,s} = 0 \Rightarrow \left(\zeta_n^f + MRS_n^{ij} \right) n^{if,s} = 0$, with generically one of the two terms > 0;

iii) $\kappa_x^{j\ell} x^{j\ell} = 0 \Rightarrow \left(\zeta_y^\ell - MWP_x^{j\ell} \right) x^{j\ell} = 0$, with generically one of the two terms > 0;

iv) $\kappa_n^{if,d} n^{jf,d} = 0 \Rightarrow \left(\zeta_n^f - MWP_n^{jf} \right) n^{jf,d} = 0$, with generically one of the two terms > 0.

By adding up the consumption optimality conditions for all individuals for good j :

$$\sum_i \left(\zeta_y^j - MRS_c^{ij} \right) c^{ij} = 0 \Rightarrow \sum_i MRS_c^{ij} c^{ij} - \zeta_y^j \sum_i c^{ij} \Rightarrow \sum_i MRS_c^{ij} c^{ij} = \zeta_y^j c^j.$$

If $c^j > 0$ (as long as one agent is consuming the good, so good j is final):

$$\zeta_y^j = \sum_i MRS_c^{ij} \frac{c^{ij}}{\sum_i c^{ij}} = \sum_i \chi_c^{ij} MRS_c^{ij} = AMRS_c^\ell.$$

If $c^j = 0$, we must have $\zeta_y^j > MRS_c^{ij}$, for all i, which means that $\zeta_y^j > \max_i \left\{ MRS_c^{ij} \right\}$. By adding up the intermediate good optimality conditions for all uses j of good ℓ:

$$\sum_j \left(MWP_x^{j\ell} - \zeta_y^\ell \right) x^{j\ell} = 0 \Rightarrow \sum_j MWP_x^{j\ell} x^{j\ell} - \zeta_y^\ell \sum_j x^{j\ell} \Rightarrow \sum_j MWP_x^{j\ell} x^{j\ell} = \zeta_y^\ell x^\ell.$$

If $x^\ell > 0$ (as long as one good j uses good ℓ as input, so good ℓ is intermediate):

$$\zeta_y^\ell = \sum_j MWP_x^{j\ell} \frac{x^{j\ell}}{\sum_j x^{j\ell}} = \sum_j \chi_x^{j\ell} MWP_x^{j\ell} = AMWP_x^\ell.$$

If $x^\ell = 0$, we must have $\zeta_y^\ell > MWP_x^{j\ell}$, for all j, which means that $\zeta_y^\ell > \max_j \left\{ MWP_x^{j\ell} \right\}$. Combining consumption and intermediate good optimality:

$$\sum_i MRS_c^{ij} c^{i\ell} + \sum_j MWP_x^{j\ell} x^{j\ell} = \zeta_y^\ell y^\ell,$$

so if $y^\ell > 0$, it must be that $\zeta_y^\ell = AMRS_c^\ell \phi_c^\ell + \sum_j \zeta_y^j \frac{\partial G^j}{\partial x^{j\ell}} \xi^{j\ell}$, which can be written in matrix form as

$$\boldsymbol{\zeta}_y = \boldsymbol{AMRS}_c \boldsymbol{\phi}_c \boldsymbol{\Psi}_y, \quad \text{where} \quad \boldsymbol{\Psi}_y = (\boldsymbol{I}_J - \boldsymbol{G}_x \boldsymbol{\xi})^{-1}.$$

Similarly, for factors, if $n^{f,s} > 0$ (as long as one agent is supplying factor f):

$$\zeta_n^f = \sum_i \frac{n^{if,s}}{n^{f,s}} MRS_n^{ij} = \sum_i \chi_n^{if,s} MRS_n^{ij} = AMRS_n^f$$

If $n^{f,s} = 0$, we must have $\zeta_n^f < MRS_n^{ij}$, for all i, which means that $\zeta_n^f < \max_i \{MRS_n^{ij}\}$. If $n^{jf,d} > 0$ (as long as factor f is used to produce a good j):

$$\zeta_n^f = \sum_j MWP_n^{jf} \frac{n^{jf,d}}{n^{f,d}} = \sum_j MWP_n^{jf} \chi_n^{jf,d} = AMWP_n^{jf}$$

If $n^{jf,d} = 0$, we must have $\zeta_n^f > \sum_j MWP_n^{jf} \chi_n^{jf,d}$, for all j, which means that $\zeta_n^f > \max_j \left\{MWP_n^{jf}\right\}$. If $n^{f,s} > 0$ and $n^{f,d} > 0$: $AMWP_n^f = AMRS_n^f$. If $n^{f,s} = 0$, it must be that $\zeta_n^f < MRS_n^{if}$, or $\zeta_n^f < \min_i \left\{MRS_n^{if}\right\}$. If $n^{f,d} = 0$, it must be that $MWP_n^{jf} < \zeta_n^f$, or $\max_j \left\{MWP_n^{jf}\right\} < \zeta_n^f$. Hence, for $n^{f,s} = 0 = n^{f,d}$, we must have that $\max_j \left\{MWP_n^{jf}\right\} < \min_i \left\{MRS_n^{if}\right\}$. Finally, for $y^j = 0$ to be optimal, it must be that $c^j = x^{\ell j} = 0$ on the use side and $x^{j\ell} = n^{jf,d} = 0$ on the input side. This condition can be written as

$$\max\left\{\max_i\left\{\frac{\partial u^i}{\partial c^{ij}}\right\}, \max_\ell\left\{\zeta_y^\ell \frac{\partial G^\ell}{\partial x^{\ell j}}\right\}\right\} < \zeta_y^j < \min\left\{\min_f\left\{\left(\frac{\partial G^j}{\partial n^{jf,d}}\right)^{-1} \zeta_n^f\right\}, \min_\ell\left\{\left(\frac{\partial G^j}{\partial x^{j\ell}}\right)^{-1} \zeta_y^\ell\right\}\right\}.$$

E.2 Propagation Matrices

Intermediate Inverse Matrix. Following similar steps as in the Proof of Lemma 2, we can express changes in intermediate input use as follows. Using both equations in (OA3), we can instead solve for $\frac{dx}{d\theta}$ as follows

$$\frac{d\boldsymbol{x}}{d\theta} = \frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y} + \boldsymbol{\xi}\left(\boldsymbol{G}_x \frac{d\mathring{\boldsymbol{x}}}{d\theta} + \boldsymbol{G}_n \frac{d\mathring{\boldsymbol{n}}^d}{d\theta} + \frac{d\bar{\boldsymbol{y}}^s}{d\theta} + \boldsymbol{G}_\theta\right),$$

so we can define a $JL \times JL$ propagation matrix in the space of intermediate links $\boldsymbol{\Psi}_x$:

$$\frac{d\boldsymbol{x}}{d\theta} = \underbrace{\boldsymbol{\Psi}_x}_{\text{Propagation}} \underbrace{\left(\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y} + \boldsymbol{\xi}\left(\boldsymbol{G}_n \frac{d\mathring{\boldsymbol{n}}^d}{d\theta} + \frac{d\bar{\boldsymbol{y}}^s}{d\theta} + \boldsymbol{G}_\theta\right)\right)}_{\text{Impulse}}, \quad \text{where} \quad \boldsymbol{\Psi}_x = (\boldsymbol{I}_{JL} - \boldsymbol{\xi}\boldsymbol{G}_x)^{-1}. \tag{OA5}$$

Propagation in the space of goods and the space of intermediate links is connected. In particular, Woodbury's identity implies that

$$\boldsymbol{\Psi}_x = \boldsymbol{I}_{JL} + \boldsymbol{\xi}\boldsymbol{\Psi}_y\boldsymbol{G}_x,$$

and it is also the case that

$$\boldsymbol{\Psi}_x \boldsymbol{\xi} = \boldsymbol{\xi} \boldsymbol{\Psi}_y,$$

connecting propagation in the space of goods and the space of intermediate links. Leveraging (OA5), it is possible to solve for changes in consumption as

$$\begin{aligned}
\frac{d\boldsymbol{c}}{d\theta} &= \frac{d\boldsymbol{y}}{d\theta} - \frac{d\boldsymbol{x}}{d\theta} = \boldsymbol{G}_x \frac{d\boldsymbol{x}}{d\theta} + \boldsymbol{G}_n \frac{d\mathring{\boldsymbol{n}}^d}{d\theta} + \boldsymbol{G}_\theta - \frac{d\boldsymbol{x}}{d\theta} \\
&= \left(\boldsymbol{G}_x - \mathbf{1}_x\right) \boldsymbol{\Psi}_x \frac{d\boldsymbol{\xi}}{d\theta} \boldsymbol{y} + \left(\left(\boldsymbol{G}_x - \mathbf{1}_x\right) \boldsymbol{\Psi}_x \boldsymbol{\xi} + \boldsymbol{I}_J\right) \boldsymbol{G}_n \frac{d\mathring{\boldsymbol{n}}^d}{d\theta} + \left(\left(\boldsymbol{G}_x - \mathbf{1}_x\right) \boldsymbol{\Psi}_x \boldsymbol{\xi} + \boldsymbol{I}_J\right) \boldsymbol{G}_\theta.
\end{aligned}$$

Proportional Goods Inverse Matrix. While the goods inverse is expressed in levels, at times, it may be useful to work with proportional propagation matrix. Starting from the definition of $\frac{dy}{d\theta}$, it follows that

$$\hat{\boldsymbol{y}}^{-1} \frac{d\boldsymbol{y}}{d\theta} = \hat{\boldsymbol{y}}^{-1} \boldsymbol{\Psi}_y \hat{\boldsymbol{y}} \left(\hat{\boldsymbol{y}}^{-1} \boldsymbol{G}_x \frac{d\boldsymbol{\xi}}{d\theta} \boldsymbol{y} + \hat{\boldsymbol{y}}^{-1} \boldsymbol{G}_n \frac{d\mathring{\boldsymbol{n}}^d}{d\theta} + \hat{\boldsymbol{y}}^{-1} \boldsymbol{G}_\theta \right) = \tilde{\boldsymbol{\Psi}}_y \left(\hat{\boldsymbol{y}}^{-1} \boldsymbol{G}_x \frac{d\boldsymbol{\xi}}{d\theta} \boldsymbol{y} + \hat{\boldsymbol{y}}^{-1} \boldsymbol{G}_n \frac{d\boldsymbol{n}^d}{d\theta} + \hat{\boldsymbol{y}}^{-1} \boldsymbol{G}_\theta \right),$$

where

$$\tilde{\boldsymbol{\Psi}}_y = \hat{\boldsymbol{y}}^{-1} \boldsymbol{\Psi}_y \hat{\boldsymbol{y}}.$$

In the competitive case, $\boldsymbol{\Psi}_y = \hat{\boldsymbol{y}} \left(\hat{\boldsymbol{p}} \hat{\boldsymbol{y}} - \left(\mathbb{I}_x + \tilde{\boldsymbol{\tau}}_x\right) \check{\boldsymbol{p}} \check{\boldsymbol{x}}\right)^{-1} \hat{\boldsymbol{p}}$ and $\tilde{\boldsymbol{\Psi}}_y = \left(\hat{\boldsymbol{p}} \hat{\boldsymbol{y}} - \left(\mathbb{I}_x + \tilde{\boldsymbol{\tau}}_x\right) \check{\boldsymbol{p}} \check{\boldsymbol{x}}\right)^{-1} \hat{\boldsymbol{p}} \hat{\boldsymbol{y}}$, where we define a $JL \times JL$ matrix of prices as $\check{\boldsymbol{p}} = \hat{\boldsymbol{p}} \otimes \boldsymbol{I}_J$, where $\tilde{\boldsymbol{\tau}}_x$ is a $J \times JL$ matrix analogous to $\bar{\boldsymbol{\tau}}_x$, but with the same ordering as the $J \times JL$ matrix $\mathbb{I}_x$, given by

$$\mathbb{I}_x = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}_{J \times JL},$$

and where we define an alternative $JL \times J$ matrix of intermediates uses $\check{\boldsymbol{x}}$, given by

$$\check{\boldsymbol{x}} = \begin{pmatrix} x^{11} & 0 & 0 \\ x^{21} & 0 & 0 \\ x^{31} & 0 & 0 \\ 0 & x^{12} & 0 \\ 0 & x^{22} & 0 \\ 0 & x^{32} & 0 \\ 0 & 0 & x^{13} \\ 0 & 0 & x^{23} \\ 0 & 0 & x^{33} \end{pmatrix}_{JL \times J}.$$

Regularity Conditions for Goods Inverse Matrix. In order to provide conditions under which the inversion step to define the goods inverse is valid, we can appeal to the Perron–Frobenius theory of non-negative matrices. If production functions have constant returns to scale, then by the homogeneous function theorem, we have that

$$y^{j,s}=\sum_k \frac{\partial G^j}{\partial x^{jk}}x^{jk}+\sum_f \frac{\partial G^j}{\partial n^{jf,d}}n^{jf,d} \quad \Rightarrow \quad 1=\sum_k \frac{\partial \log G^j}{\partial \log x^{jk}}+\sum_f \frac{\partial \log G^j}{\partial \log n^{jf,d}}.$$

This implies that the matrix (here represented for $J=2$ case)

$$\hat{\boldsymbol{y}}^{-1}\boldsymbol{G}_x\boldsymbol{\xi}\hat{\boldsymbol{y}}=\begin{pmatrix} \frac{\partial \log G^1}{\partial \log x^{11}} & \frac{\partial \log G^1}{\partial \log x^{12}} \\ \frac{\partial \log G^2}{\partial \log x^{21}} & \frac{\partial \log G^2}{\partial \log x^{22}} \end{pmatrix},$$

features rows whose sum can be written as

$$r^j=\sum_k \frac{\partial \log G^j}{\partial \log x^{jk}}<1.$$

Hence, this result implies that the spectral radius (maximum of the absolute value of eigenvalues) of $\hat{\boldsymbol{y}}^{-1}\boldsymbol{G}_x\boldsymbol{\xi}\hat{\boldsymbol{y}}$ is less than 1, so the Neumann series lemma concludes that the proportional goods inverse is well defined (Meyer, 2023). It is possible to derive bounds of convergence, so that the sectors with lowest and highest intermediate shares drive the speed of convergence. Convergence of the proportional goods inverse is sufficient for convergence of the the goods inverse. Hence, the goods inverse exists in economies with constant or decreasing returns to scale.

F Applications

F.1 Armington (1969) Model

Model Solution. First, note that country profits are given by $\pi^j=\left(p^jA^j-w^j\right)n^{jj,s}$, where we already impose that $j=f$. Hence, profit maximization requires that $p^j=\frac{w^j}{A^j}$. Without loss of generality, we normalize $p^1=1$, so $w^1=A^1$. We also assume that $\tau^{ii}=0$ and $\tau^{ij}=\tau^{ji}=\tau$.

Hence, exploiting Walras' law, an equilibrium of the model can be expressed as the solution to the system

$$\begin{aligned} \frac{c^{11}}{c^{12}}&=\left(\frac{1}{p^2\left(1+\tau\right)}\right)^{-\sigma} \quad \text{and} \quad \frac{c^{21}}{c^{22}}=\left(\frac{1+\tau}{p^2}\right)^{-\sigma} \\ A^1&=c^{11}+c^{21} \qquad\qquad \text{and} \quad A^2=c^{12}+c^{22} \\ p^2A^2&=c^{21}+p^2c^{22}, \end{aligned}$$

for $\{c^{11},c^{12},c^{21},c^{22},p^2\}$. If instead we had assumed that countries have endowments of goods, then their budget constraints take the form

$$\sum_j p^j\left(1+\tau^{ij}\right)c^{ij}=p^i\bar{y}^{i,s}+\sum_j T^{ij},$$

which is equivalent to the formulation in the text when $A^i=\bar{y}^{i,s}$. Hence, our parameterization implies that country 2's good is 50 times more abundant than country 1's.

Welfare Accounting Decomposition. Country i's welfare gains induced by a perturbation take the form

$$\frac{dV^{i|\lambda}}{d\tau} = \frac{\frac{dV^i}{d\tau}}{\lambda^i} = \sum_j \frac{\frac{\partial V^i}{\partial c^{ij}}}{\lambda^i} \frac{dc^{ij}}{d\tau} = \sum_j MRS_c^{ij} \frac{dc^{ij}}{d\tau} = \sum_j MRS_c^{ij} \frac{d\chi_c^{ij}}{d\tau} c^j,$$

where $MRS_c^{ij} = \frac{\partial V^i}{\partial c^{ij}} / \lambda^i$, $\frac{dc^{ij}}{d\tau} = \frac{d\chi_c^{ij}}{d\tau} c^j + \chi_c^{ij} \frac{dc^j}{d\tau}$, and $\frac{dc^j}{d\tau} = 0$.

We can therefore expresses the normalized welfare gain as

$$\frac{dW^\lambda}{d\tau} = \frac{\frac{dW}{d\tau}}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i} \lambda^i} = \underbrace{\sum_j \mathbb{C}ov_i^\Sigma \left[MRS_c^{ij}, \frac{d\chi_c^{ij}}{d\tau} \right] c^j}_{\text{Cross-Sectional Consumption Efficiency}} + \underbrace{\sum_j \mathbb{C}ov_i^\Sigma \left[\omega^i, MRS_c^{ij} \frac{d\chi_c^{ij}}{d\tau} \right] c^j}_{\text{Cross-Sectional Consumption Redistribution}},$$

where $\omega^i = \frac{\lambda^i \frac{\partial \mathcal{W}}{\partial V^i}}{\frac{1}{I}\sum_i \lambda^i \frac{\partial \mathcal{W}}{\partial V^i}}$. We choose aggregate world consumption as welfare numeraire, which implies that $\lambda^i = \sum_j \frac{\partial V^i}{\partial c^{ij}} c^j$. Similar results obtain if we choose unit world consumption, which implies that $\lambda^i = \sum_j \frac{\partial V^i}{\partial c^{ij}}$. Even though country 2's consumption is substantially higher than country 1 in the absence of tariffs, as shown in the middle plots in Figure OA-1, the linear homogeneity of the preferences imply that $\frac{\partial V^1}{\partial c^{11}} = \frac{\partial V^2}{\partial c^{21}}$ and $\frac{\partial V^1}{\partial c^{12}} = \frac{\partial V^2}{\partial c^{22}}$. Hence, to ensure that the planner attaches a higher weight to the country that consumes less (country 1), we use a social welfare function of the form $\mathcal{W}\left(V^1, V^2\right) = \sum_i \left(V^i\right)^{\frac{\sigma-1}{\sigma}}$, which implies that $\omega^i = \frac{\lambda^i \left(V^i\right)^{-\frac{1}{\sigma}}}{\frac{1}{I}\sum_i \lambda^i \left(V^i\right)^{-\frac{1}{\sigma}}}$. This is equivalent to expressing country preferences as $V^i = \sum_j \left(c^{ij}\right)^{\frac{\sigma-1}{\sigma}}$ and assuming a utilitarian social welfare function. The bottom two plots in Figure OA-1 illustrate the equilibrium values of ω^i and λ^i.

F.2 DMP Model

Model Solution. We consider the standard search equilibrium definition (Pissarides, 2000), whose notation we mostly follow. Job-filling and job-finding rates, respectively denoted by $q_0\left(\theta_0\right)$ and $p_0\left(\theta_0\right)$, are given by

$$q_0\left(\theta_0\right) = \frac{m\left(\chi_{0,n}^1, v_0\right)}{v_0} = \mu \theta_0^{-\alpha} \quad \text{and} \quad p_0\left(\theta_0\right) = \theta_0 q_0\left(\theta_0\right) = \frac{m\left(\chi_{0,n}^1, v_0\right)}{\chi_{0,n}^1} = \mu \theta_0^{1-\alpha}.$$

The value of an occupied job, denoted by J_0, is given by

$$J_0 = z^2 - w_0 + \beta\left[\left(1-\varphi\right) J_1 + \varphi V_1\right] \quad \text{and} \quad J_1 = z^2 - w_1,$$

where w_t denotes the wage. The value of a vacant job is given by

$$V_0 = -\kappa_0 + \beta\left[q_0\left(\theta_0\right) J_1 + \left(1 - q_t\left(\theta_t\right)\right) V_1\right] \quad \text{and} \quad V_1 = 0.$$

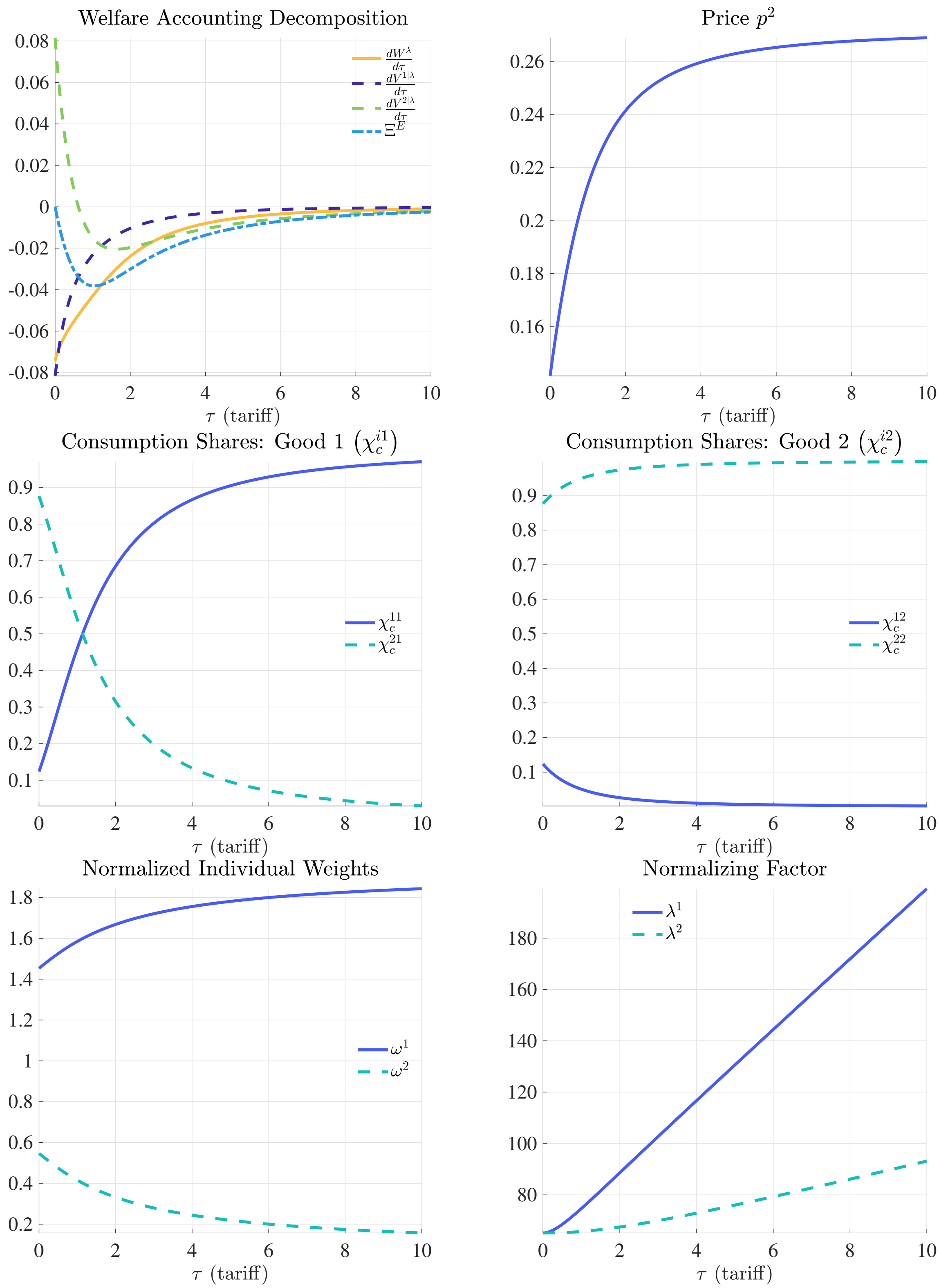


Figure OA-1: Armington Model

At an equilibrium with free-entry, $V_0 = 0$, so

$$J_1 = \frac{\kappa_0}{\beta q_0(\theta_0)} \quad \text{and} \quad J_0 = z^2 - w_0 + (1-\varphi)\frac{\kappa_0}{q_0(\theta_0)}.$$

The value of employed and unemployed workers, respectively denoted by E_t and U_t, are given by

$$\begin{aligned} E_0 &= w_0 + \beta\left[\varphi U_1 + (1-\varphi) E_1\right] && \text{and} \quad E_1 = w_1 \\ U_0 &= b_0 + \beta\left[p_t(\theta_t) E_1 + (1 - p_t(\theta_t)) U_1\right] && \text{and} \quad U_1 = b_1. \end{aligned}$$

The wage is determined by Nash bargaining, with

$$w_t = \arg\max_{w_t} (E_t - U_t)^{\eta} (J_t - V_t)^{1-\eta}.$$

The solution to this problem is

$$E_t - U_t = \eta(E_t - U_t + J_t - V_t) \quad \text{and} \quad J_t - V_t = (1-\eta)(E_t - U_t + J_t - V_t).$$

Given our parametrization, we have that $U_1 = V_1 = 0$, which means that $w_1 = E_1 = \eta(E_1 + J_1) = \eta z^2$ and that $J_1 = (1-\eta) z^2$.

Hence, the condition

$$(1-\eta) z^2 = \frac{\kappa_0}{\beta q_0(\theta_0)}$$

pins down equilibrium tightness θ_0. Given θ_0 and $\chi^1_{0,n}$, we can compute equilibrium vacancies, which is sufficient to compute the welfare accounting decomposition. Figure OA-2 illustrates how an improvement in the matching technology translates in higher vacancies posted at date 0, which in turn translates into lower unemployment at date 1.

Welfare Accounting. The welfare gain of a marginal change in μ can be written as

$$\frac{dW}{d\mu} = \frac{dc_0}{d\mu} + \beta\frac{dc_1}{d\mu}$$

Using unit perpetual consumption as lifetime welfare numeraire, we can express the normalized welfare gain as

$$\frac{dW^{\lambda}}{d\mu} = \frac{\frac{dW}{d\mu}}{\lambda} = \omega_0\frac{dc_0}{d\mu} + \omega_1\frac{dc_1}{d\mu},$$

where $\lambda = 1 + \beta$ and where $\omega_0 = \frac{1}{1+\beta}$ and $\omega_1 = \frac{\beta}{1+\beta}$. Note that

$$\frac{dc_0}{d\mu} = \phi_{0,c}\left(\frac{dy_0^1}{d\mu} + \frac{dy_0^2}{d\mu}\right) - \frac{d\phi_{x,0}}{d\mu}\left(y_0^1 + y_0^2\right) \quad \text{and} \quad \frac{dc_1}{d\mu} = \frac{dy_1^1}{d\mu} + \frac{dy_1^2}{d\mu},$$

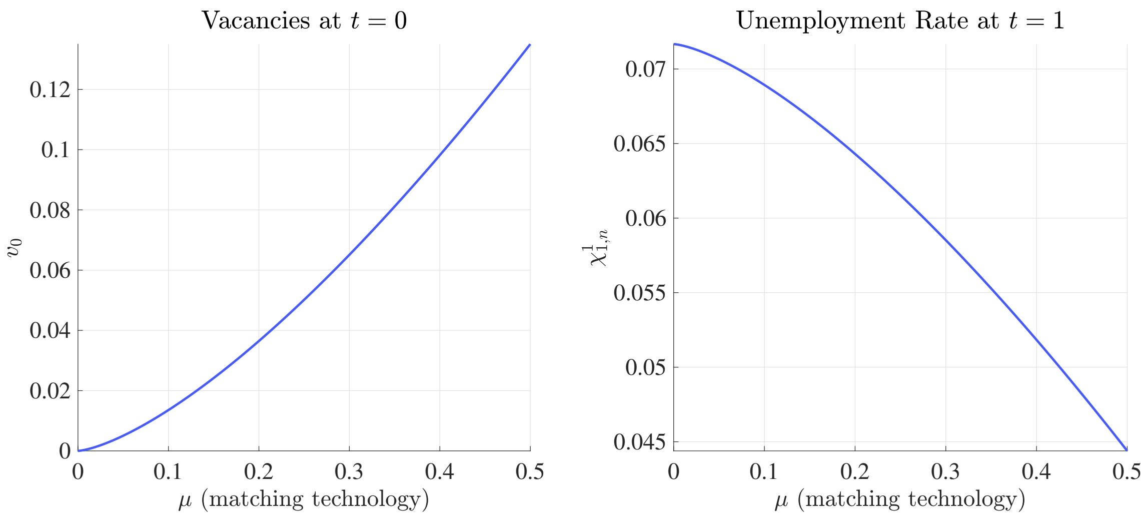


Figure OA-2: DMP Model

where $\frac{dy_t^j}{d\mu} = z^j \frac{d\chi_{t,n}}{d\mu}$, which allows us to write

$$\begin{aligned}\frac{dW^\lambda}{d\mu} &= \omega_1 \sum_j \phi_{1,c} z^j \frac{d\chi^j_{1,n}}{d\mu} - \omega_0 \frac{d\phi_{x,0}}{d\mu}\left(y_0^1 + y_0^2\right)\\ &= \underbrace{\omega_1 \mathbb{C}ov_j^\Sigma\left[MWP^j_{1,n}, \frac{d\chi^j_{1,n}}{d\mu}\right]}_{\text{Cross-Sectional Factor Use Efficiency}} - \underbrace{\omega_0 \frac{d\phi_{x,0}}{d\mu}\left(y_0^1 + y_0^2\right)}_{\text{Aggregate Intermediate Input Efficiency}},\end{aligned}$$

where marginal welfare products are given by $MWP^j_{1,n} = \phi_{1,c} z^j$.

F.3 Hsieh and Klenow (2009) Model

Welfare Accounting. Since the solution of the model is completely standard, we exclusively describe here how to characterize the welfare accounting decomposition. We consider a perturbation in σ_τ, which is associated with a welfare change given by

$$\frac{dW}{d\sigma_\tau} = \frac{\partial u}{\partial c^1} \frac{dc^1}{d\sigma_\tau}.$$

Using good 1 as numeraire, $\lambda = \frac{\partial u}{\partial c^1}$, so

$$\frac{dW^\lambda}{d\sigma_\tau} = \frac{dy^1}{d\sigma_\tau} = \sum_{j=2}^{J} \frac{\partial y^1}{\partial y^j} A^j \frac{d\chi^{j,d}_n}{d\sigma_\tau} = \sum_{j=2}^{J} MWP^j_n \frac{d\chi^{j,d}_n}{d\sigma_\tau} = \underbrace{\mathbb{C}ov_j^\Sigma\left[MWP^j_n, \frac{d\chi^{j,d}_n}{d\sigma_\tau}\right]}_{\text{Cross-Sectional Factor Use Efficiency}},$$

where the marginal welfare product of factor use $\chi^{j,d}_n$ is $MWP^j_n = \frac{\partial y^1}{\partial y^j} A^j$ and where $\frac{\partial y^1}{\partial y^j} = \left(\frac{y^j}{y^1}\right)^{-\frac{1}{\epsilon}}$.

F.4 New Keynesian Model

This Appendix presents additional model details in F.4.1, competitive equilibrium in F.4.2, a self-contained quantitative calibration in F.4.3, and additional numerical results in F.4.4.

F.4.1 Additional Model Details

Households. Household preferences (36) give rise to the usual CES demand functions

$$c^{ij} = \Gamma_c^{ij} \left(\frac{p^j}{P^i} \right)^{-\eta_c} c^i \qquad \text{and} \qquad c^{ij\ell} = \left(\frac{p^{j\ell}}{p^j} \right)^{-\epsilon^j} c^{ij}.$$

Under homothetic CES consumption preferences, each household i faces an ideal price index

$$P^i = \left[\sum_j \Gamma_c^{ij} (p^j)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}}.$$

Production. The production function (37) features three nests of CES aggregates. Taking as given prices and wages, firms choose inputs to minimize cost

$$\mathcal{C}^{j\ell} = \min_{\{x^{j\ell\ell\ell'}\}_{\ell\ell'},\, \{n^{j\ell i}\}_i} \sum_\ell \int_0^1 p^{\ell\ell'} x^{j\ell\ell\ell'} d\ell' + \sum_i W^i n^{j\ell i},$$

subject to the CES production structure in (37). This problem gives rise to labor demand

$$n^{j\ell} = (A^j)^{\eta-1} (1-\vartheta^j) \left(\frac{W^{j\ell}}{mc^j} \right)^{-\eta} y^{j\ell} \qquad \text{and} \qquad n^{j\ell i} = \Gamma_w^{ji} \left(\frac{W^i}{W^{j\ell}} \right)^{-\eta^w} n^{j\ell}$$

and intermediate input demand

$$x^{j\ell} = (A^j)^{\eta-1} \vartheta^j \left(\frac{p_x^{j\ell}}{mc^j} \right)^{-\eta} y^{j\ell}, \qquad x^{j\ell\ell} = \Gamma_x^{j\ell} \left(\frac{p^\ell}{p_x^{j\ell}} \right)^{-\eta_x} x^{j\ell} \qquad \text{and} \qquad x^{j\ell\ell\ell'} = \left(\frac{p^{\ell\ell'}}{p^\ell} \right)^{-\epsilon_\ell} x^{j\ell\ell}.$$

Nominal marginal cost is given by

$$mc^j = \frac{1}{A^j} \left[\left(1-\vartheta^j\right) \left(W^j\right)^{1-\eta} + \vartheta^j \left(p_x^j\right)^{1-\eta} \right]^{\frac{1}{1-\eta}},$$

which is symmetric across firms ℓ in sector j. Marginal cost is not affected by the revenue tax, which is the only wedge in this application. Finally, the cost indices are given by

$$W^j = \left[\sum_\ell \Gamma_w^{ji} (W^i)^{1-\eta_w} \right]^{\frac{1}{1-\eta_w}} \qquad \text{and} \qquad p_x^j = \left[\sum_\ell \Gamma_x^{j\ell} (p^\ell)^{1-\eta_x} \right]^{\frac{1}{1-\eta_x}}.$$

Since production functions are homogeneous of degree one, total cost is given by $\boldsymbol{C}^{j\ell} = mc^j y^{j\ell}$.

Sectoral Aggregation. Firms set prices according to (38). Aggregating to the sectoral level, the price of sector j 's good is

$$p^j = \left(\int_0^1 (p^{j\ell})^{1-\epsilon^j} d\ell\right)^{\frac{1}{1-\epsilon^j}} = \left[\int_0^{\delta^j} \left(\frac{\epsilon^j}{\epsilon^j - 1}\frac{1}{1-\tau^j} mc^j\right)^{1-\epsilon^j} d\ell + \int_{\delta^j}^1 (\bar{p}^j)^{1-\epsilon_j} d\ell\right]^{\frac{1}{1-\epsilon^j}}$$
$$= \frac{\epsilon^j}{\epsilon^j - 1}\frac{1}{1-\tau^j}\left[\delta^j (mc^j)^{1-\epsilon^j} + (1-\delta^j)(\bar{mc}^j)^{1-\epsilon^j}\right]^{\frac{1}{1-\epsilon^j}},$$

where the very first equality follows since

$$p^j c^{ij} = \int_0^1 p^{j\ell} c^{ij\ell} d\ell = \int_0^1 p^{j\ell}\left(\frac{p^{j\ell}}{p^j}\right)^{-\epsilon^j} c^{ij} d\ell \implies p^j = \left(\int_0^1 (p^{j\ell})^{1-\epsilon^j} d\ell\right)^{\frac{1}{1-\epsilon^j}}.$$

Aggregating the goods market clearing condition, we have

$$p^j y^j \equiv \int_0^1 p^{j\ell} y^{j\ell} d\ell = \sum_i \mu^i \int_0^1 p^{j\ell} c^{ij\ell} d\ell + \sum_\ell \int_0^1 \int_0^1 p^{j\ell} x^{\ell\ell' j\ell} d\ell' d\ell,$$

where $\int_0^1 p^{j\ell} y^{j\ell} d\ell$ denotes total nominal expenditures on sectoral good j. This also implies a resource constraint at the sectoral level, given by $y^j = \sum_i \mu^i c^{ij} + \sum_\ell \int_0^1 x^{\ell\ell j} d\ell$. All this relies on our assumption that all agents buying in sector j share the same homothetic demand aggregators over varieties ℓ. In particular, it implies that we also have

$$y^{j\ell} = \left(\frac{p^{j\ell}}{p^j}\right)^{-\epsilon^j} y^j \qquad \text{and} \qquad y^j = \left(\int_0^1 (y^{j\ell})^{\frac{\epsilon^j - 1}{\epsilon^j}} d\ell\right)^{\frac{\epsilon^j}{\epsilon^j - 1}}.$$

Fiscal Rebates. In the absence of fiscal policy, the rebate T^i that household i receives simply corresponds to total corporate profits plus the proceeds from the revenue tax. That is,

$$\sum_i \mu^i T^i = \sum_j \int_0^1 \Pi^{j\ell} d\ell + \sum_j \int_0^1 \tau^j p^{j\ell} y^{j\ell} d\ell = \sum_j \int_0^1 \left(p^{j\ell} - mc^j\right) y^{j\ell} d\ell$$

Assuming a uniform rebate, we simply have $T^i = \sum_j \int_0^1 (p^j - mc^j)\, y^{j\ell} d\ell$.

F.4.2 Equilibrium

Definition. (Competitive Equilibrium). Taking as given an initial price distribution $\left\{\bar{p}^{j\ell}\right\}_{j\ell}$, a realization of technology shocks $\{A^j\}_j$, revenue taxes $\{\tau^j\}_j$, and monetary policy M, a competitive equilibrium comprises an allocation $\left\{c^{ij\ell}, n^i, x^{i\ell\ell\ell'}, y^{j\ell}\right\}_{i,j\ell,\ell\ell'}$ and prices $\left\{p^{j\ell}, W^i\right\}_{i,j\ell}$ such that (i) households optimize consumption and labor supply, (ii) firms $\ell \in [0, \delta^j)$ in sector j reset their prices optimally, and (iii) markets for goods and factors clear

$$y^{j\ell} = \sum_i \mu^i c^{ij\ell} + \sum_\ell \int_0^1 x^{\ell\ell' j\ell} d\ell' \qquad \text{and} \qquad \mu^i n^i = \sum_j \int_0^1 n^{j\ell i} d\ell.$$

Notice that each sector features two representative firms ex post since all firms are symmetric ex ante and those firms that reset prices all choose the same reset price. At the sector level, there is consequently a representative price-adjusting firm and a representative fixed-price firm.

Computing competitive equilibrium requires an initial price distribution $\left\{\bar{p}^{j\ell}\right\}_{j\ell}$. We assume that initial prices are given by

$$\bar{p}^{j\ell} = \bar{p}^j = \frac{\epsilon^j}{\epsilon^j - 1}\frac{1}{1-\tau^j}\bar{mc}^j = \frac{\epsilon^j}{\epsilon^j - 1}\frac{1}{1-\tau^j}mc^j\left(1, \left\{\bar{p}^{\ell\ell'}\right\}_{\ell\ell'}, \left\{\bar{W}^i\right\}_i\right).$$

That is, $\bar{p}^j$ corresponds to the price firms in sector j would set if all technologies remain at their default level $A^j = \bar{A}^j$. This initialization is heuristically consistent with the zero-inflation steady state of a dynamic New Keynesian model. In the absence of technology shocks, therefore, no firm faces an incentive to adjust prices. If $A^j \neq \bar{A}^j$, a fraction δ^j of firms in each sector reset their price.

Numeraire. We take as our numeraire total nominal expenditures in the absence of shocks, i.e., $\bar{M} = \sum_j p^j y^j = 1$. Therefore, $\bar{M} = 1$ provides a benchmark stance for monetary policy. In the absence of technology shocks, setting $M = \bar{M} = 1$ implies production efficiency and therefore efficiency since all firms are symmetric.

Macro Block. To compute this model, it is particularly convenient to characterize a macro block by aggregating to the sectoral level. To that end, we aggregate several key equilibrium conditions. The aggregate labor market clearing condition (aggregated to the level of household type) is

$$\mu^i n^i = \sum_j \int_0^1 n^{j\ell i} d\ell = \sum_j \int_0^1 \Gamma_w^{ji}\left(\frac{W^i}{W^j}\right)^{-\eta_w} n^{j\ell} d\ell = \sum_j \Gamma_w^{ji}\left(\frac{W^i}{W^j}\right)^{-\eta_w}(A^j)^{\eta-1}(1-\vartheta^j)\left(\frac{W^j}{mc^j}\right)^{-\eta} D^j y^j,$$

where $D^j = \int_0^1 \left(\frac{p^{j\ell}}{p^j}\right)^{-\epsilon^j} d\ell$ is a measure of sectoral price dispersion. Aggregating the goods market clearing condition yields

$$y^{j\ell} = \sum_i \mu^i c^{ij\ell} + \left(\frac{p^{j\ell}}{p^j}\right)^{-\epsilon^j} \sum_\ell \Gamma_x^{\ell j}\left(\frac{p^j}{p_x^\ell}\right)^{-\eta_x} \int_0^1 (A^\ell)^{\eta-1}\vartheta^\ell \left(\frac{p_x^\ell}{mc^\ell}\right)^{-\eta} y^{\ell\ell'} d\ell'.$$

And plugging in for CES demand functions implies

$$y^j = \sum_i \mu^i c^{ij} + \sum_\ell \Gamma_x^{\ell j}\left(\frac{p^j}{p_x^\ell}\right)^{-\eta_x}(A^\ell)^{\eta-1}\vartheta^\ell\left(\frac{p_x^\ell}{mc^\ell}\right)^{-\eta} y^\ell D^\ell,$$

yielding sectoral goods market clearing conditions written as a fixed point in y^j.

Finally, the budget constraint can be written as

$$P^i c^i = W^i n^i + \sum_j (p^j - D^j mc^j) y^j.$$

Computationally, it is now easiest to solve the macro block as a separate system of equations. Firm-level allocations can then be obtained from CES demand functions.

F.4.3 Calibration

Our calibration broadly follows Schaab and Tan (2023) and is summarized in Table OA-1. It is based on 66 production sectors and 10 household types, which we associate with deciles of the household income distribution.

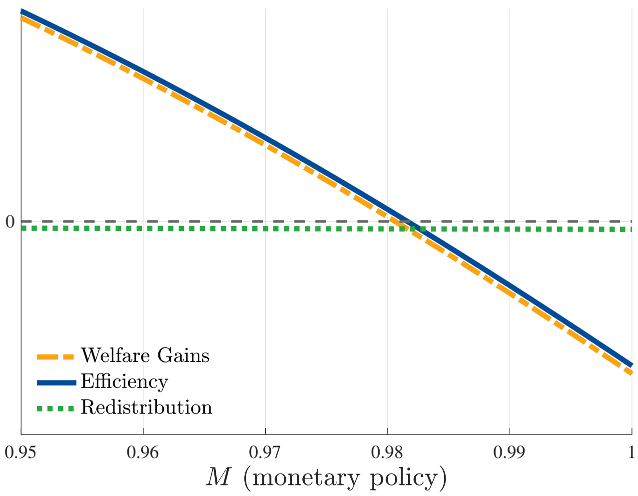
For household preferences, we set the coefficient of relative risk aversion to $\gamma = 2$ and the inverse Frisch elasticity to $\varphi = 2$. We use an elasticity of substitution of $\eta_c = 1$, so the consumption aggregator is Cobb-Douglas, and we calibrate the consumption weights Γ_c^{ij} to match consumption expenditure shares across household types in the CEX.

	Parameters	Value / Target	Source
	Household preferences		
γ	Relative risk aversion	2	Classical Efficiency
φ	Inverse Frisch elasticity	2	Classical Efficiency
η_c	Elasticity of substitution across goods	1	Cobb-Douglas
Γ_c^{ij}	CES consumption weights	Consumption expenditure shares	CEX
	Production and nominal rigidities		
η	Elasticity of substitution across inputs and labor	1	Cobb-Douglas
ϑ^j	CES input bundle weight	Sectoral input share	BEA
η_x	Elasticity of substitution across inputs	1	Cobb-Douglas
η_w	Elasticity of substitution across factors	1	Cobb-Douglas
Γ_x^{ij}	CES input use weights	Input-output network	BEA I-O
Γ_w^{ij}	CES factor use weights	Payroll shares	ACS
ϵ^j	Elasticities of substitution across varieties	Sectoral markups	Baqaee and Farhi (2020)
δ^j	Sectoral price adjustment probabilities	Price adjustment frequencies	Pasten et al. (2017)

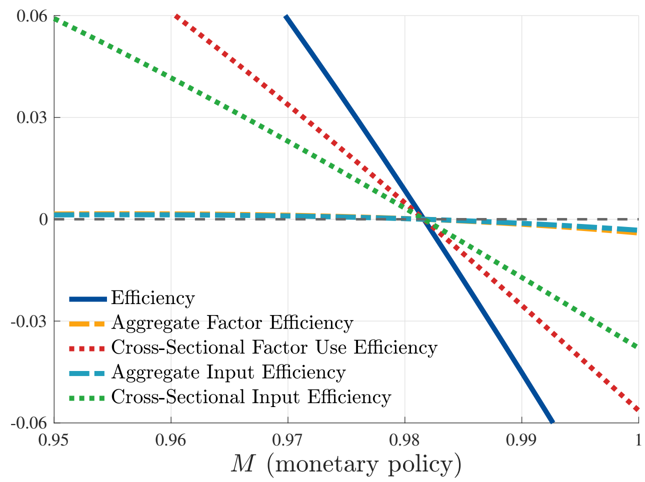
Table OA-1: List of Calibrated Parameters

On the production side, we set the elasticity of substitution between the labor and intermediate input bundles to $\eta = 1$. Therefore, ϑ^j and $1 - \vartheta^j$ correspond respectively to the input and labor shares in production, which we obtain from the BEA GDP-by-Industry data. We compute the input share ϑ^j as input expenditures relative to gross output, averaged between 1997 and 2015, and treat the labor share as its complement. We set the elasticities of substitution across intermediate inputs and factors to $\eta_x = \eta_w = 1$. We calibrate Γ_x^{ij} and Γ_w^{ij} to match data on input-output linkages and payroll shares. For the former, we use data from the BEA Input Output "Use" Table to compute input shares as a sector j's expenditures on goods from sector ℓ as a share of j's total expenditures on inputs, averaged between 1997 and 2015. We obtain payroll shares from a linked ACS-IO dataset as type i 's earnings from sector j as a share of total earnings, averaged between 1997 and 2015. We use data from Baqaee and Farhi (2020) on sectoral markups to calibrate the elasticity of substitution across sectoral varieties ϵ^j. Sectoral markups are computed as $\mu^j = \frac{\epsilon^j}{\epsilon^j - 1}$.

Finally, we use data from Pasten et al. (2017) on price adjustment frequencies to calibrate δ^j. They estimate monthly price adjustment frequencies using the data underlying the Bureau



(a) Welfare Gains



(b) Efficiency Gains

Figure OA-3: Optimal Monetary Policy under Divine Coincidence

of Labor Statistics' Producer Price Index for 754 industries from 2005 to 2011. First, we link these estimates to the 66 sectors in our data. Second, we obtain quarterly adjustment probabilities as $1-\left(1-\frac{\text{monthly adjustment frequency}}{100}\right)^3$. Finally, we bin these estimates into quintiles. This allows us to solve our model assuming that each of the 66 sectors consists of 5 firms.

F.4.4 Additional Results

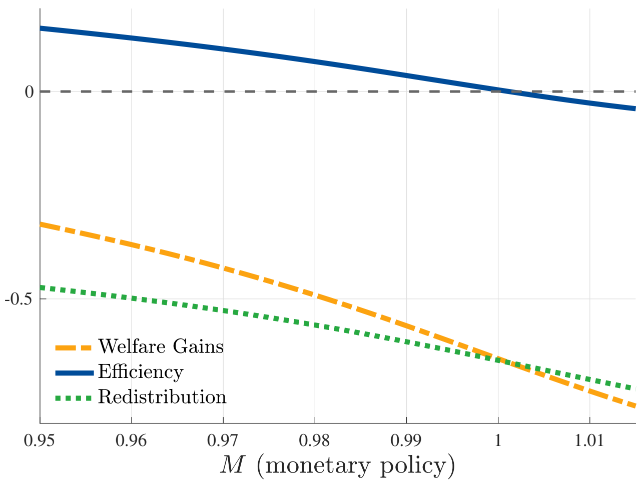
In this subsection, we present additional numerical results that are referenced in the main text.

Divine Coincidence. Consider an alternative calibration where households and sectors are symmetric, so there exist a representative household and a representative sector. Our model then collapses to the standard, one-sector New Keynesian model, albeit with roundabout production. Divine Coincidence holds in this model. That is, the optimal monetary policy response to an aggregate technology shock closes both output and inflation gaps. Figure OA-3 illustrates this benchmark from the perspective of our welfare accounting decomposition. In that context, Divine Coincidence implies that each allocative efficiency component is 0, indicating that optimal policy can attain an efficient allocation. Moreover, since households are symmetric, there is no scope for redistribution gains, so welfare and efficiency coincide.

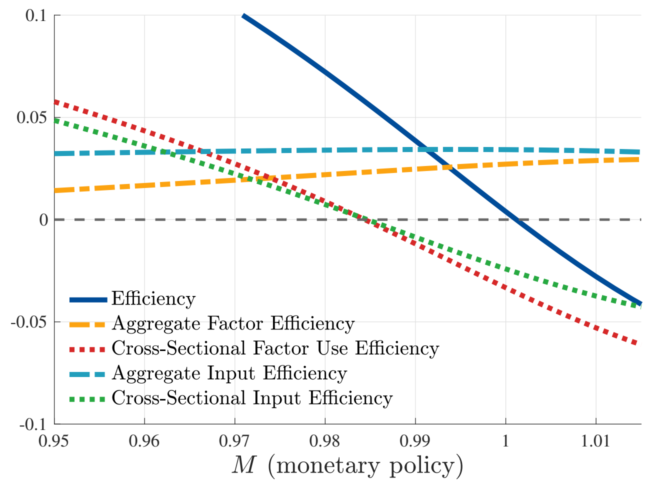
Importance of Markup Distortions. Figure 4 in the main text corresponds to a calibration of the model that assumes revenue taxes are available to eliminate initial markups. We reproduce our main experiment in Figure OA-4 below, assuming that revenue taxes are not available.

It is well known from the New Keynesian literature that monopolistic competition implies inefficiently low steady state employment. In that context, optimal monetary policy under discretion, which is heuristically comparable to the static optimization problem we consider, seeks to raise employment via expansionary monetary policy. We revisit this result from the perspective of our welfare accounting decomposition. Figure OA-4 demonstrates that, in the presence of initial markup distortions, aggregate factor and input use efficiency considerations push optimal monetary policy towards a more expansionary stance. In the one-sector New Keynesian model (without roundabout production), aggregate factor efficiency corresponds to the standard labor wedge. In this multi-sector variant, aggregate factor and input use efficiency formally capture that aggregate employment and aggregate activity are inefficiently low.

Cross-sectional factor use and intermediate input efficiency, on the other hand, push monetary policy towards a relatively more contractionary stance. Optimal policy therefore trades off the gains from stimulating aggregate activity in the presence of markup distortions against the cost of creating misallocation in the form of price dispersion, captured by cross-sectional factor use and intermediate input efficiency.



(a) Welfare Gains



(b) Efficiency Gains

Figure OA-4: Optimal Monetary Policy with Markup Distortions