Welfare Accounting*

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Abstract

This paper develops a welfare accounting decomposition that identifies and quantifies the origins of welfare gains and losses in general economies with heterogeneous individuals and disaggregated production. The decomposition — exclusively based on preferences and technologies — first separates efficiency from redistribution considerations. Efficiency comprises i) exchange efficiency, which captures allocative efficiency gains due to reallocating consumption and factor supply across individuals, and ii) production efficiency, which captures allocative efficiency gains due to adjusting intermediate inputs and factors, as well as technical efficiency gains from primitive changes in technologies, factor endowments, and good endowments. Leveraging the decomposition, we provide a new characterization of efficiency conditions in disaggregated production economies with heterogeneous individuals that carefully accounts for non-interior solutions, extending classic efficiency results in Lange (1942) or Mas-Colell et al. (1995). In competitive economies, prices (and wedges) are directly informative about the welfare-relevant statistics that shape the welfare accounting decomposition, which allows us to characterize a generalized Hulten's theorem and a new converse Hulten's theorem. We present several minimal examples and four applications to workhorse models in macroeconomics: i) the Armington model, ii) the Diamond-Mortensen-Pissarides model, iii) the Hsieh-Klenow model, and iv) the New Keynesian model.

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1 Introduction

Identifying the sources of welfare gains and losses is critical to assess the impact of shocks and the desirability of policy interventions. This is a challenging task, however, especially in realistic economies where different individuals have different preferences, consumption baskets, and factor supply patterns, and where production technologies may rely on multiple factors and a complex network of intermediate inputs.

In light of these complexities, this paper introduces a decomposition of welfare assessments that applies to general economies with heterogeneous individuals and disaggregated production. This approach — which we refer to as *welfare accounting* — is useful to i) identify and quantify the ultimate origins of welfare gains and losses induced by changes in allocations, technologies, and goods or factor endowments and ii) characterize efficiency conditions.

We consider a static environment in which heterogeneous individuals consume different goods and supply different factors, and goods can be produced using other goods and factors. Our results allow for elastic and fixed factors and make no assumptions about the homotheticity of utility and production technologies. Critically and in contrast to existing work, the welfare accounting decomposition is exclusively based on preferences and technologies, and does not rely on assumptions about the (optimizing) behavior of agents, firm objectives, individual budget constraints, prices, or the notion of equilibrium.

Welfare Accounting Decomposition. The welfare accounting decomposition applies to welfare assessments under general social welfare functions. First, we separate welfare assessments into efficiency and redistribution components.[1] A central property of this decomposition is that the efficiency component does not depend on the choice of social welfare function — only redistribution does. For that reason, we study aggregate efficiency in Sections 3 through 5, and redistribution in Section 6.

Aggregate efficiency consists of *exchange efficiency* and *production efficiency*. Exchange efficiency captures efficiency welfare gains and losses due to the reallocation of consumption and factor supply among individuals. Theorem 1 decomposes exchange efficiency into two components. First, *cross-sectional consumption efficiency* measures welfare gains associated with reallocating consumption of a good from individuals who value it less to individuals who value it more, for a given level of aggregate consumption of the good. Second, *cross-sectional factor supply efficiency* measures welfare gains from reallocating the supply of a factor from individuals for whom supplying it is more costly to individuals for whom supplying it is less costly, for a given level of aggregate supply of the factor.

Production efficiency captures efficiency welfare gains and losses associated with the economy's

[1] The efficiency/redistribution decomposition leverages the results of Dávila and Schaab (2022). While that paper takes the mapping between allocations and policies or shocks as given and focuses on how different planners trade off different normative considerations in dynamic stochastic economies, this paper exploits resource constraints and production technologies to identify the ultimate origins of welfare gains and losses.

production side. It comprises allocative efficiency gains due to adjusting intermediate inputs and factors as well as technical efficiency gains from primitive changes in technologies and endowments. Theorem 2 decomposes production efficiency into seven components. First, *cross-sectional intermediate input efficiency* measures the welfare gains from reallocating intermediate inputs from less to more socially desirable uses, for a given level of aggregate intermediate use. Second, *aggregate intermediate input efficiency* measures the welfare gains from adjusting the share of good supply that is consumed instead of used in production, for a given level of aggregate good supply. Third, *cross-sectional factor efficiency* measures the welfare gains from reallocating factors from less to more socially desirable uses, for a given level of aggregate factor use. Fourth, *aggregate factor efficiency* measures the welfare gains from adjusting the supply of factors. Finally, the *technology change*, *good endowment change*, and *factor endowment change* components measure the direct welfare gains from primitive changes in technologies or endowments.

A central contribution of the welfare accounting decomposition is to identify the precise welfare-relevant statistics that translate physical changes in allocations, technologies, and endowments into welfare changes. These statistics are i) marginal rates of substitution (MRS), which measure the value of increases in individual consumption or factor supply; ii) aggregate marginal rates of substitution ($AMRS$), which measure the social value of changes in aggregate consumption or factor supply; iii) marginal welfare products (MWP), which measure the value of increasing the use of an input or factor in production; iv) aggregate marginal welfare products ($AMWP$), which measure the social value of changes in aggregate intermediate use or factor use; and v) marginal social values of goods (MSV), which measure the social value of having an additional unit of a particular good (either as an endowment or through a change in technology). When combined with changes in allocations or primitives, these statistics are sufficient to compute the welfare impact of any perturbation. The MSV of goods is a central object for welfare accounting because it is the sole determinant of the efficiency gains from pure technological change, and it governs marginal welfare products, which in turn determine each component of production efficiency.

Efficiency Conditions. In Section 4, we leverage the welfare accounting decomposition to characterize efficiency conditions, generalizing the classic efficiency conditions in Lange (1942), Samuelson (1947), and Mas-Colell et al. (1995). This is, to our knowledge, the first general characterization of efficiency conditions for disaggregated production economies with heterogeneous individuals.

Theorems 3 and 4 summarize the necessary conditions for exchange and production efficiency. Exchange efficiency requires the equalization of marginal rates of substitution for all individuals who consume a good or supply a factor, allowing individuals who do not consume a good (or supply a factor) to have lower (higher) marginal rates of substitution. Cross-sectional intermediate input and factor efficiency require the equalization of MWP across all uses of an input or a factor, allowing for potentially lower MWP when a good or factor is not used to produce another. Aggregate intermediate input efficiency requires the equalization of $AMWP$ with $AMRS$ for all mixed goods. For pure final goods, the $AMRS$ must be higher than the highest MWP of using the good in production. For pure intermediate goods, the $AMWP$ must be higher than the highest individual MRS when consuming it. Finally, aggregate factor efficiency requires the equalization of $AMWP$ with $AMRS$ for all factors in positive elastic supply.

A central message of this paper is that properly accounting for the non-negativity constraints that define feasible allocations is critical to characterize efficiency conditions and trace the origins of welfare gains and losses. This is particularly important when production is disaggregated — since disaggregated production networks are typically sparse — and when individuals are heterogeneous and consume different (disjoint) consumption bundles. In particular, we show that the classic efficiency conditions apply to interior links between mixed goods and/or elastic factors, but fail to hold otherwise, in particular when pure intermediate goods are involved. In general, we show that MWP and MRS are the appropriate objects to characterize efficiency conditions, rather than MRS and MRT (marginal rates of transformation), as in the classic approach.

After identifying the conditions for allocative efficiency, Theorem 5 characterizes the MSV of goods — which determines the technology change component of the welfare accounting decomposition — at an efficient allocation. Since efficiency ensures that the value of a good must be the same whether it is consumed or used as an input, we show that MSV exactly corresponds to $AMRS$ for final goods, to $AMWP$ for intermediate goods, and to both for mixed goods.

Competitive Economies. Our results until Section 5 require no assumptions about the (optimizing) behavior of agents, individual budget constraints, prices, or notions of equilibrium. It is nonetheless valuable to specialize the welfare accounting decomposition to competitive economies, which we do in Section 5, since prices are directly informative about the welfare-relevant statistics. Starting from our baseline environment, we assume that individuals maximize utility and technologies are operated by firms that minimize costs. To allow for distortions, we saturate all choice margins with wedges.

Theorem 6 characterizes the MSV of goods in competitive economies with wedges: It equals the competitive price augmented by an aggregate goods wedge term that captures average distortions in consumption and intermediate input use. Intuitively, the presence of aggregate consumption and intermediate input wedges implies that there are over- or under-produced goods. Hence, the MSV of goods that ultimately increase the supply of under-produced (over-produced) goods is higher (lower) than the price. Theorem 6 provides a converse result to Hulten's theorem that has been missing from the existing literature: The condition that ensures that prices equal the MSV of a good is that aggregate goods wedges are zero.

We provide a new general Hulten's theorem, which applies to frictionless competitive economies with heterogeneous individuals, elastic and fixed factors, arbitrary preferences and technologies, and arbitrary social welfare functions. Its generality allows us to systematically discuss the many qualifications associated with this result. In particular, we show that Hulten's theorem applies to frictionless competitive economies and efficient interior economies, but need not hold in efficient non-interior economies. Moreover, we show that Hulten's theorem is fundamentally a result about aggregate efficiency, not about final output or welfare.

Theorem 7 specializes the allocative efficiency components of the welfare accounting decomposition to competitive economies with wedges. A central takeaway from this analysis is that equalization of marginal revenue products is not sufficient for cross-sectional intermediate input or factor efficiency: Efficiency requires the equalization of marginal welfare products across uses of an intermediate input or a factor, while competition — when intermediate input or factor use wedges are zero — only enforces the equalization of marginal revenue products across uses.

Redistribution and Applications. Our analysis up to Section 5 focuses on efficiency. However, perturbations with identical efficiency implications may have different distributional implications. Theorem 9 in Section 6 decomposes redistribution gains or losses into four components: *Cross-sectional consumption* and *factor supply redistribution* capture redistribution gains due to the reallocation of consumption and factor supply shares, for given aggregate levels of consumption and factor supply. And *aggregate consumption* and *factor supply redistribution* capture redistribution gains due to changes in aggregate consumption and factor supply, for given shares. Critically, the choice of social welfare function will directly impact the welfare gains from redistribution and its components.

Finally, we illustrate how the welfare accounting results introduced in this paper can be used to identify the origins of welfare gains and losses in four workhorse models in macroeconomics and trade. Our first application shows how an increase in tariffs contributes negatively to exchange efficiency via cross-sectional consumption efficiency in the simplest endowment economy (Armington, 1969). This application also illustrates subtle patterns in cross-sectional consumption redistribution. Our second application shows how the aggregate efficiency gain induced by an improvement in a matching technology in a Diamond-Mortensen-Pissarides (DMP) model is due to cross-sectional factor efficiency gains large enough to compensate for aggregate intermediate input efficiency losses due to increased vacancy postings. This application illustrates how to use the welfare accounting decomposition in a random search economy, which differs substantially from competitive economies. Our third application illustrates how an increase in markup dispersion generates cross-sectional factor efficiency losses in Hsieh and Klenow (2009) economy. Our final application shows how to use the welfare accounting decomposition to identify the welfare gains from optimal monetary stabilization policy in a macroeconomic model with household and sectoral heterogeneity. We compute the optimal monetary policy response to a technology shock in a static, multi-sector heterogeneous-agent New Keynesian model with a rich input-output production structure. We contrast the aggregate efficiency welfare gains from stabilization policy with its impact on redistribution and decompose the former into its allocative efficiency components.

Related Literature. Our results are most closely related to classic studies of efficiency — see Lange (1942), Samuelson (1947) or, for a modern treatment, Section 16.F of Mas-Colell et al. (1995). This work proves the welfare theorems by first characterizing conditions for efficiency in a planned economy and then showing that allocations in frictionless competitive economies satisfy these conditions.[2] While the classic approach to efficiency assumes that all goods are final or mixed, our general results show that allowing for pure intermediate goods substantially changes the nature of efficiency conditions.[3] Moreover, while Lange (1942) characterizes efficiency conditions, neither that paper nor subsequent literature presents welfare decompositions of the form introduced in Theorems 1 and 2.

The welfare accounting decomposition introduced in this paper is also related to the vast literature on growth accounting and productivity measurement that follows Solow (1957) and includes Hall (1990), Basu (1995), Basu and Fernald (1997, 2002), Basu et al. (2006), Basu et al. (2022), and Baqaee and Farhi (2020), among many others. At times and to different degrees, this body of work draws connections between output growth and welfare gains — see for instance Basu and Fernald (2002), Basu et al. (2022), or Baqaee and Burstein (2022a).[4] A common challenge for this literature is to aggregate among heterogeneous individuals. By using the approach introduced in Dávila and Schaab (2022), we are able to make aggregate welfare assessments and to separate efficiency from redistribution considerations without relying on prices. This in turn allows us to characterize the welfare accounting decomposition exclusively in terms of preferences and technologies, making no assumptions about the (optimizing) behavior or budget constraints of agents, prices, or notions of equilibrium. This contrasts our results from Baqaee and Farhi (2020), whose decomposition is based on markups, prices, and cost minimization, as well as Baqaee and Burstein (2022b), whose welfare results also rely on prices. More broadly, our paper continues an agenda that seeks to understand the origins of welfare gains in general economies.

Our results build on the literature on multi-sector production networks.[5] A central result of this literature is Hulten's theorem (Hulten, 1978), which characterizes the aggregate impact of technological change in terms of prices (Domar weights). Instead of imposing a competitive structure, we provide a characterization of the impact of technological change exclusively based on preferences and technologies, identifying the MSV of goods as the relevant object. Liu (2019) presents a statistic

[2]While the classic proofs of the first welfare theorem provide useful insights into the relation between competition and efficiency, they are not the most general — see instead Arrow (1951) and Debreu (1951). Our approach is subject to the same advantages and disadvantages as the classic approach — see Geanakoplos (1989) for a discussion.

[3]By emphasizing the critical role played by pure intermediate goods, our results connect to the recent work on global value chains — see Antràs and Chor (2022) for a recent survey.

[4]In Section C.5 of the Online Appendix, we discuss the relation between welfare accounting, as developed in this paper, and growth accounting. Growth accounting measures the contribution of different inputs to final output, indirectly computing technological growth as a residual. Instead, welfare accounting attributes aggregate welfare gains to different sources. We also show how one could use the welfare accounting decomposition to conduct growth accounting.

[5]This literature includes, among many others, Gabaix (2011), Jones (2011), Acemoglu et al. (2012), Bigio and La'O (2020), Liu (2019), Baqaee and Farhi (2018, 2020), Acemoglu and Azar (2020), and Kopytov et al. (2022). See Carvalho and Tahbaz-Salehi (2019) and Baqaee and Rubbo (2022) for recent surveys.

that summarizes the social value of subsidizing inputs and factors. While related, our characterization of MSV differs because it i) makes no assumptions about optimizing behavior, budget constraints, or prices, and ii) considers a perturbation in the level of output rather than price subsidies. By specializing the MSV of goods to competitive environments, we provide the most general Hulten-style result to date. We show that Hulten's theorem is fundamentally a result about aggregate efficiency — not about final output or welfare — that applies to frictionless competitive economies — not efficient economies. Bigio and La'O (2020) show that Hulten's theorem is valid for production efficiency, rather than output, in an environment with a single individual and elastic factor supply.

Finally, our results also relate to the work that defines measures of changes in living standards, potentially refining popular notions like GDP. See Nordhaus and Tobin (1973) for an earlier account of these ideas and Fleurbaey (2009), Jones and Klenow (2016), and Basu et al. (2022) for modern treatments. The welfare accounting decomposition can be used to show that GDP changes only correspond to welfare changes in very specific scenarios since welfare assessments must also account for exchange efficiency, factor supply costs, and redistribution.

2 Environment and Social Welfare

We first introduce preferences, technologies, and resource constraints, and then define feasible allocations and perturbations. We conclude this section by describing how to separate efficiency from redistribution considerations when making welfare assessments.

2.1 Preferences, Technologies, and Resource Constraints

We consider a static economy populated by a finite number $I \geq 1$ of individuals, indexed by $i \in \mathcal{I} = \{1, \ldots, I\}$.[6] There are $J \geq 1$ goods, indexed by $j, \ell \in \mathcal{J} = \{1, \ldots, J\}$ and $F \geq 0$ factors, indexed by $f \in \mathcal{F} = \{1, \ldots, F\}$. Goods are produced using goods and factors as inputs, while factors are directly supplied by individuals. Goods and factors may also appear as (predetermined) endowments.

An individual i derives utility from consuming goods and (dis)utility from supplying factors, according to the utility function

$$(\textit{Preferences}) \qquad V^i = u^i\left(\left\{c^{ij}\right\}_{j\in\mathcal{J}}, \left\{n^{if,s}\right\}_{f\in\mathcal{F}}\right), \tag{1}$$

where c^{ij} denotes individual i's final consumption of good j and $n^{if,s}$ denotes individual i's supply of factor f (the superscript s stands for supply).

[6] In the body of the paper, we exclusively consider static economies with a finite number of individuals, goods and factors. Section C.1 of the Online Appendix extends our results to dynamic stochastic economies without accumulation technologies. In ongoing work (Dávila and Schaab, 2024b), we extend the approach of this paper to economies with accumulation technologies, which opens a new set of nontrivial considerations. Our results straightforwardly generalize to economies with a continuum of individuals, goods, and factors.

Goods are produced using technologies that take goods and factors as inputs. The production technology for good j, denoted by $G^j(\cdot) \geq 0$, is given by

$$(\textit{Technologies}) \qquad y^{j,s} = G^j\left(\left\{x^{j\ell}\right\}_{\ell\in\mathcal{J}}, \left\{n^{jf,d}\right\}_{f\in\mathcal{F}}; \theta\right), \tag{2}$$

where $y^{j,s}$ denotes the amount produced (output) of good j, $x^{j\ell}$ denotes the amount of good ℓ used in the production of good j, and $n^{jf,d}$ denotes the amount of factor f used in the production of good j (the superscript d stands for demand). We use the index $\ell \in \mathcal{J}$ to refer to goods used as intermediates. For clarity, we typically use L to denote the number of intermediate inputs, although $L = J$. We parametrize $G^j(\cdot;\theta)$ by θ to consider perturbations to technology, as described below.

The resource constraint for good j is

$$(\textit{Resource Constraints: Goods}) \qquad y^{j,s} + \bar{y}^{j,s}(\theta) = c^j + x^j, \tag{3}$$

where $c^j = \sum_i c^{ij}$ represents the total amount of good j consumed (aggregate consumption), $x^j = \sum_\ell x^{\ell j}$ represents the amount of good j used as an intermediate input in production (aggregate intermediate use), and $\bar{y}^{j,s}(\theta) = \sum_i \bar{y}^{ij,s}(\theta)$ represents the aggregate endowment of good j, where $\bar{y}^{ij,s}(\theta)$ denotes individual i's endowment of good j. We parametrize $\bar{y}^{j,s}(\theta)$ and $\bar{y}^{ij,s}(\theta)$ by θ to consider perturbations to good endowments. When needed, we denote the aggregate supply of good j by $y^j = y^{j,s} + \bar{y}^{j,s}(\theta)$.

The resource constraint for factor f is

$$(\textit{Resource Constraints: Factors}) \qquad n^{f,s} + \bar{n}^{f,s}(\theta) = n^{f,d}, \tag{4}$$

where $n^{f,s} = \sum_i n^{if,s}$ and $n^{f,d} = \sum_j n^{jf,d}$ respectively represent the aggregate elastic supply and the aggregate factor use of factor f, and $\bar{n}^{f,s}(\theta) = \sum_i \bar{n}^{if,s}(\theta)$ represents the aggregate endowment of factor f, where $\bar{n}^{if,s}(\theta)$ denotes individual i's endowment of factor f. We parametrize $\bar{n}^{f\cdot s}(\theta)$ and $\bar{n}^{if,s}(\theta)$ by θ to consider perturbations to factor endowments. When needed, we denote the aggregate supply of factor f by $n^f = n^{f,s} + \bar{n}^{f,s}(\theta)$.

2.2 Feasible Allocations and Perturbations

Definition 1 describes a feasible allocation. Binding non-negativity constraints play a central role in our analysis.

Definition 1. *(Feasible allocation). An allocation* $\left\{c^{ij}, n^{if,s}, x^{j\ell}, n^{jf,d}, y^{j,s}\right\}$ *is feasible if equations* (2) *through* (4) *hold and the non-negativity constraints* $c^{ij} \geq 0$, $n^{if,s} \geq 0$, $x^{j\ell} \geq 0$, $n^{jf,d} \geq 0$, *and* $y^{j,s} \geq 0$ *are satisfied.*

We assume that preferences and technologies are differentiable and that all variables are smooth functions of a perturbation parameter $\theta \in [0,1]$, so derivatives such as $\frac{dc^{ij}}{d\theta}$, $\frac{dx^{j\ell}}{d\theta}$, or $\frac{dn^{jf,d}}{d\theta}$ are well-defined.[7] Feasible perturbations $d\theta$ have a dual interpretation. First, a perturbation may capture exogenous changes in technologies or endowments, but also changes in policies (e.g., taxes, subsidies, transfers, etc.) or any other primitive of a fully specified model (e.g., trade costs, markups, bargaining power, etc.). Under this interpretation, the mapping between allocations and θ emerges endogenously and accounts for equilibrium effects. Second, a perturbation may alternatively capture changes in feasible allocations directly chosen by a planner. This second interpretation is useful to characterize the set of efficient allocations, as we explain in Section 4.

2.3 Social Welfare: Efficiency vs. Redistribution

We consider welfare assessments for welfarist planners, that is, planners with a social welfare function $\mathcal{W}(\cdot)$ given by

$$(\textit{Social Welfare Function}) \qquad W = \mathcal{W}\left(V^1, \ldots, V^i, \ldots, V^I\right), \tag{5}$$

where $\frac{\partial \mathcal{W}}{\partial V^i} > 0$, $\forall i$, and where individual utilities V^i are defined in (1).[8]

We leverage the welfare decomposition introduced in Dávila and Schaab (2022) to separate efficiency from redistribution considerations. Hence, a welfare assessment can be expressed as

$$\frac{dW}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \frac{dV^i}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \lambda^i \frac{\frac{dV^i}{d\theta}}{\lambda^i}, \tag{6}$$

where λ^i is an individual normalizing factor that allows us to express individual welfare gains or losses in units of a common welfare numeraire. In particular, since the units of λ^i are $\dim\left(\lambda^i\right) = \frac{\text{utils of individual } i}{\text{units of numeraire}}$, individual welfare gains or losses $\frac{dV^i}{d\theta}/\lambda^i$ are measured in units of the common welfare numeraire, with $\dim\left(\frac{dV^i}{d\theta}/\lambda^i\right) = \frac{\text{units of numeraire}}{\text{units of } \theta}$, $\forall i$. The only restriction when choosing the welfare numeraire is that λ^i must be strictly positive for all individuals.[9]

Lemma 1 derives Dávila and Schaab (2022)'s efficiency/redistribution decomposition in our environment. This is the unique decomposition in which a normalized welfare assessment can be expressed as Kaldor-Hicks efficiency and its complement.

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[7] To simplify the exposition, we assume throughout that i) consumption is (weakly) desirable but supplying factors is not, i.e., $\frac{\partial u^i}{\partial c^{ij}} \geq 0$ and $\frac{\partial u^i}{\partial n^{if,s}} \leq 0$; ii) the marginal products of using intermediates and factors are (weakly) positive, i.e., $\frac{\partial G^j}{\partial x^{j\ell}} \geq 0$ and $\frac{\partial G^j}{\partial n^{jf,d}} \geq 0$; and iii) the no-free-lunch property holds, i.e., $G^j(\cdot) = 0$ if $x^{j\ell} = 0$, $\forall \ell$, and $n^{jf,d} = 0$, $\forall f$. Many of our results, including the welfare accounting decomposition, do not require such restrictions.

[8] As in Boadway and Bruce (1984), Kaplow (2011), or Saez and Stantcheva (2016), we refer to the use of social welfare functions — typically traced back to Bergson (1938) and Samuelson (1947) — as the welfarist approach. This approach is widely used because it is Paretian, that is, it concludes that every Pareto-improving perturbation is desirable, with the converse being true under minimal assumptions (Kaplow and Shavell, 2001).

[9] While we derive our results for a general normalizing factor λ^i, the nominal unit (e.g., dollars) is the most natural welfare numeraire. In that case, λ^i is measured in $\frac{\text{utils of individual } i}{\text{dollars}}$ and $\frac{dV^i}{d\theta}/\lambda^i$ is measured in $\frac{\text{dollars}}{\text{units of } \theta}$. Alternative goods or bundles may be useful numeraires in particular applications. For instance, if good 1 is chosen as welfare numeraire, then $\lambda^i = \frac{\partial u^i}{\partial c^{i1}}$, $\forall i$.

Lemma 1. *(Efficiency/Redistribution Decomposition) A normalized welfare assessment for a welfarist planner can be decomposed into efficiency and redistribution components, Ξ^E and Ξ^{RD}, as follows:*

$$\underbrace{\frac{dW^\lambda}{d\theta}}_{\substack{\text{Welfare}\\ \text{Assessment}}} = \frac{\frac{dW}{d\theta}}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i} = \sum_i \omega^i \frac{\frac{dV^i}{d\theta}}{\lambda^i} = \underbrace{\sum_i \frac{\frac{dV^i}{d\theta}}{\lambda^i}}_{\Xi^E \text{ (Efficiency)}} + \underbrace{\mathbb{C}ov_i^\Sigma\left[\omega^i, \frac{\frac{dV^i}{d\theta}}{\lambda^i}\right]}_{\Xi^{RD} \text{ (Redistribution)}}, \tag{7}$$

where $\omega^i = \frac{\frac{\partial \mathcal{W}}{\partial V^i}\lambda^i}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i}$ and where $\mathbb{C}ov_i^\Sigma[\cdot,\cdot] = I \cdot \mathbb{C}ov_i^\Sigma[\cdot,\cdot]$ denotes a cross-sectional covariance-sum among all individuals.

The efficiency component Ξ^E corresponds to Kaldor-Hicks efficiency, that is, it is the unweighted sum of individual willingness-to-pay for the perturbation in units of the welfare numeraire. Hence, perturbations in which $\Xi^E > 0$ can be turned into Pareto improvements if transfers are feasible and costless. The redistribution component Ξ^{RD} captures the equity concerns embedded in a particular social welfare function: Ξ^{RD} is positive when the individuals relatively favored in a perturbation are those relatively preferred by the planner, that is, have a higher ω^i.

The decomposition of Lemma 1 satisfies several properties. Three are worth highlighting for our purposes. First, the efficiency component is invariant to i) the choice of social welfare function and ii) preference-preserving utility transformations. This property motivates the structure of our analysis, first studying efficiency and then redistribution. Second, efficient allocations feature a weakly negative efficiency component ($\Xi^E \leq 0$) for any feasible perturbation given endowments and technologies. This property allows us to use the efficiency component Ξ^E to characterize the set of efficient allocations, as we do in Section 4. Finally, since our economy is static, the efficiency component Ξ^E is exclusive driven by aggregate efficiency Ξ^{AE}, so going forward we use the fact that

$$\Xi^E = \Xi^{AE}.$$

In Section C.1 of the Online Appendix, we show that the decomposition of aggregate efficiency into exchange and production efficiency that we present next also applies to dynamic stochastic economies.

3 Welfare Accounting: Efficiency

This section develops the main welfare accounting result: a decomposition that identifies and quantifies the origins of efficiency welfare gains and losses. The aggregate efficiency component of a welfare assessment, Ξ^{AE}, can be decomposed into exchange and production efficiency components, $\Xi^{AE,X}$ and $\Xi^{AE,P}$, as follows:

$$\Xi^{AE} = \Xi^{AE,X} + \Xi^{AE,P}, \tag{8}$$

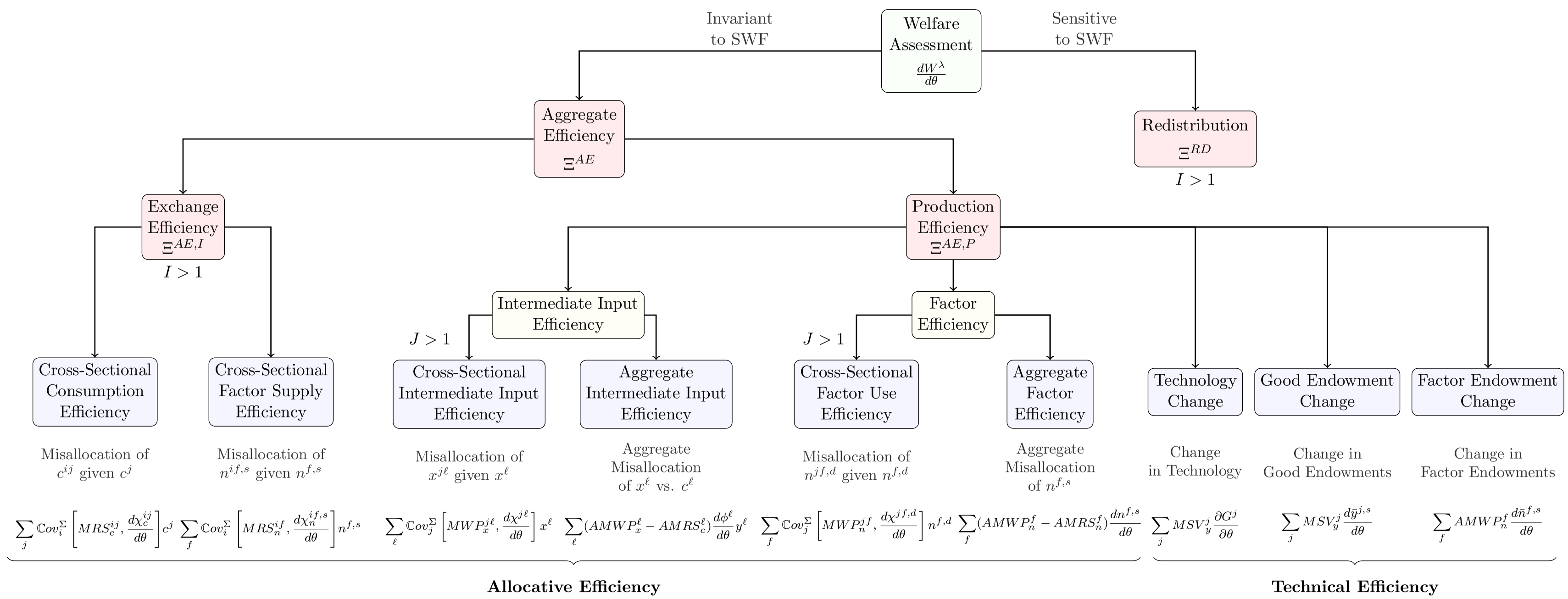


Figure 1: Welfare Accounting Decomposition

Note: This diagram illustrates the welfare accounting decomposition introduced in this paper. Lemma 1 decomposes welfare assessments into aggregate efficiency and redistribution components. Aggregate efficiency welfare gains comprise exchange efficiency and production efficiency. Theorem 1 decomposes exchange efficiency into cross-sectional consumption and factor supply efficiency. Theorem 2 decomposes production efficiency into intermediate input efficiency, factor efficiency, technology change, good endowment change, and factor endowment change components. Theorems 3 and 4 leverage this decomposition to characterize efficiency conditions. Theorem 5 characterizes the marginal social value of goods at efficient allocations. Theorems 6, 7, and 8 leverage this decomposition to characterize the technology change component and the allocative efficiency components in competitive economies with wedges. Theorem 9, which decomposes the redistribution component, is illustrated in Figure OA-2 in the Online Appendix, which is the complement of this figure. Figure OA-1 in the Online Appendix extends this diagram to dynamic stochastic economies.

where both components can be further decomposed, as illustrated in Figure 1 and explained in detail in the remainder of this paper. We study exchange efficiency in Section 3.1 and production efficiency in Section 3.2. We explore broader insights from the welfare accounting decomposition in Section 3.3 and illustrate each of its components with minimal examples in Section 3.4.

3.1 Exchange Efficiency

3.1.1 Allocation Shares: Consumption and Factor Supply

To study exchange efficiency, we first introduce consumption and factor supply allocations shares. Working with shares, instead of directly with levels, allows us to distinguish welfare gains and losses due to reallocation from those due to changes in aggregates.[10]

Formally, we define individual i's consumption share of good j, χ_c^{ij}, and individual i's factor supply share of factor f, $\chi_n^{if,s}$, as

$$\chi_c^{ij} := \begin{cases} \frac{c^{ij}}{c^j} & \text{if } c^j > 0 \\ \frac{\frac{dc^{ij}}{d\theta}}{\frac{dc^j}{d\theta}} & \text{if } c^j = 0 \quad \text{and} \quad \frac{dc^j}{d\theta} > 0 \\ 0 & \text{if } c^j = 0 \quad \text{and} \quad \frac{dc^j}{d\theta} = 0 \end{cases} \quad \text{and} \quad \chi_n^{if,s} := \begin{cases} \frac{n^{if,s}}{n^{f,s}} & \text{if } n^{f,s} > 0 \\ \frac{\frac{dn^{if,s}}{d\theta}}{\frac{dn^{f,s}}{d\theta}} & \text{if } n^{f,s} = 0 \quad \text{and} \quad \frac{dn^{f,s}}{d\theta} > 0 \\ 0 & \text{if } n^{f,s} = 0 \quad \text{and} \quad \frac{dn^{f,s}}{d\theta} = 0. \end{cases} \tag{9}$$

Individual consumption shares χ_c^{ij} represent either the share of aggregate consumption c^j consumed by individual i, when $c^j > 0$, or the share of the change in aggregate consumption $dc^j/d\theta$ consumed by individual i, when $c^j = 0$ and $dc^j/d\theta > 0$. Individual factor supply shares $\chi_n^{if,s}$ are defined analogously. The definitions of shares in equation (9) ensure that changes in individual consumption and factor supply can be expressed as

$$\frac{dc^{ij}}{d\theta} = \frac{d\chi_c^{ij}}{d\theta} c^j + \chi_c^{ij} \frac{dc^j}{d\theta} \quad \text{and} \quad \frac{dn^{if,s}}{d\theta} = \frac{d\chi_n^{if,s}}{d\theta} n^{f,s} + \chi_n^{if,s} \frac{dn^{f,s}}{d\theta}, \tag{10}$$

even when $c^j = 0$ or $n^{f,s} = 0$.

3.1.2 Exchange Efficiency Decomposition

Exchange efficiency captures efficiency welfare gains and losses associated with the reallocation of consumption and factor supply among individuals.

Theorem 1. *(Exchange Efficiency) The exchange efficiency component of aggregate efficiency, $\Xi^{AE,X}$, can be decomposed into i) cross-sectional consumption efficiency and ii) cross-sectional factor*

[10] The fact that reformulating the model in terms of shares is useful is a consequence of the linearity of the resource constraints, as explained in Dávila and Schaab (2024a).

supply efficiency, as

$$\Xi^{AE,X} = \underbrace{\sum_j \mathbb{C}ov_i^{\Sigma}\left[MRS_c^{ij}, \frac{d\chi_c^{ij}}{d\theta}\right]c^j}_{\substack{\text{Cross-Sectional}\\ \text{Consumption Efficiency}}} \underbrace{- \sum_f \mathbb{C}ov_i^{\Sigma}\left[MRS_n^{if}, \frac{d\chi_n^{if,s}}{d\theta}\right]n^{f,s}}_{\substack{\text{Cross-Sectional}\\ \text{Factor Supply Efficiency}}}$$

where individual i's marginal rates of substitution between good j and the numeraire, MRS_c^{ij}, and between factor f and the numeraire, MRS_n^{if}, are given by

$$MRS_c^{ij} = \frac{\frac{\partial u^i}{\partial c^{ij}}}{\lambda^i} \qquad \text{and} \qquad MRS_n^{if} = -\frac{\frac{\partial u^i}{\partial n^{if,s}}}{\lambda^i}, \tag{11}$$

and where $\mathbb{C}ov_i^{\Sigma}[\cdot,\cdot] = I \cdot \mathbb{C}ov_i^{\Sigma}[\cdot,\cdot]$ denotes a cross-sectional covariance-sum among all individuals.

Cross-sectional consumption efficiency measures the contribution to (Kaldor-Hicks) efficiency due to reallocating consumption of good j from individuals who value it less (with a lower MRS_c^{ij}) to individuals who value it more (with a higher MRS_c^{ij}), for a given level of aggregate consumption c^j.[11] Analogously, cross-sectional factor supply efficiency measures the contribution to (Kaldor-Hicks) efficiency due to reallocating the supply of factor f from individuals for whom increasing factor supply is more costly (with a higher MRS_n^{if}) to individuals for whom increasing factor supply is less costly (with a lower MRS_n^{if}), for a given aggregate (elastic) supply of factor f, $n^{f,s}$.

Corollary 1 presents several properties of practical relevance that exchange efficiency satisfies.

Corollary 1. *(Properties of Exchange Efficiency)*

(a) (Single Individual) In economies with a single individual ($I = 1$), exchange efficiency is zero.

(b) (No Elastic Factor Supply) In economies in which factors are not elastically supplied, so $n^{f,s} = 0$ for all factors, cross-sectional factor supply efficiency is zero.

(c) (Equalized MRS_c^{ij} or MRS_n^{if}) If marginal rates of substitution for good j (factor f) are identical across individuals for all goods (factors) with $c^j > 0$ ($n^{f,s} > 0$), then cross-sectional consumption (factor supply) efficiency is zero.

Since exchange efficiency welfare gains arise by reallocating consumption and factor supply across individuals, exchange efficiency must be zero in single individual economies.[12] Relatedly, in economies in which individuals do not derive (dis)utility from factor supply, cross-sectional factor supply efficiency is zero. Lastly, only when individuals value consuming the same good or supplying the same factor differently is there scope to find welfare gains from reallocating either.

[11] The marginal rate of substitution MRS_c^{ij} measures individual i's valuation in units of the welfare numeraire of a marginal increase in good j's consumption. Analogously, MRS_n^{if} measures individual i's cost in units of the welfare numeraire of a marginal increase in factor f's supply.

[12] Exchange efficiency and redistribution are completely different notions, even though both require individual heterogeneity. In particular, the choice of social welfare function does not affect exchange efficiency but it directly impacts redistribution.

3.2 Production Efficiency

3.2.1 Allocation Shares: Intermediate Input and Factor Use

To study production efficiency, we first introduce allocation shares for intermediate input and factor use. Once again, working with shares allows us to distinguish welfare gains and losses due to reallocation from those due to changes in aggregates.

Formally, we define good ℓ's *intermediate share*, ϕ_x^ℓ, and the *intermediate-use share* of good ℓ used to produce good j, $\chi_x^{j\ell}$, as

$$\phi_x^\ell := \begin{cases} \frac{x^\ell}{y^\ell} & \text{if } y^\ell > 0 \\ \frac{\frac{dx^\ell}{d\theta}}{\frac{dy^\ell}{d\theta}} & \text{if } y^\ell = 0 \quad \text{and} \quad \frac{dy^\ell}{d\theta} > 0 \\ 0 & \text{if } y^\ell = 0 \quad \text{and} \quad \frac{dy^\ell}{d\theta} = 0 \end{cases} \quad \text{and} \quad \chi_x^{j\ell} := \begin{cases} \frac{x^{j\ell}}{x^\ell} & \text{if } x^\ell > 0 \\ \frac{dx^{j\ell}}{dx^\ell} & \text{if } x^\ell = 0 \quad \text{and} \quad \frac{dx^\ell}{d\theta} > 0 \\ 0 & \text{if } x^\ell = 0 \quad \text{and} \quad \frac{dx^\ell}{d\theta} = 0. \end{cases} \tag{12}$$

Good ℓ's intermediate share, ϕ_x^ℓ, represents either the share of good ℓ's aggregate supply y^ℓ devoted to production, when $y^\ell > 0$, or the share of the change in good ℓ's aggregate supply $\frac{dy^\ell}{d\theta}$ devoted to production, when $y^\ell = 0$ and $\frac{dy^\ell}{d\theta} > 0$. Its complement defines the *aggregate consumption share* $\phi_c^\ell = 1 - \phi_x^\ell$. The intermediate-use share of good ℓ, $\chi_x^{j\ell}$, represents either the share of good ℓ's aggregate intermediate use devoted to the production of good j, when $x^\ell > 0$, or its counterpart in changes when $x^\ell = 0$ and $\frac{dx^\ell}{d\theta} > 0$.[13]

Finally, we also define the *intermediate-supply share* of good ℓ by $\xi^{j\ell} = \chi_x^{j\ell}\phi_x^\ell$, which corresponds to $\frac{x^{j\ell}}{y^\ell}$ when $y^\ell > 0$ or to its counterpart in changes when $y^\ell = 0$ and $\frac{dy^\ell}{d\theta} > 0$. These definitions of shares ensure that changes in intermediate use can be expressed as

$$\frac{dx^{j\ell}}{d\theta} = \frac{d\xi^{j\ell}}{d\theta} y^\ell + \xi^{j\ell}\frac{dy^\ell}{d\theta}, \qquad \text{where} \qquad \frac{d\xi^{j\ell}}{d\theta} = \frac{d\chi_x^{j\ell}}{d\theta}\phi_x^\ell + \chi_x^{j\ell}\frac{d\phi_x^\ell}{d\theta}, \tag{13}$$

even when $y^\ell = 0$ and $x^\ell = 0$. Expression (13) initially decomposes level changes in the use $x^{j\ell}$ of good ℓ in the production of good j into two terms. First, changes in the intermediate-supply share $\frac{d\xi^{j\ell}}{d\theta}$ change $x^{j\ell}$ in proportion to good ℓ's aggregate supply y^ℓ. Second, changes in good ℓ's aggregate supply $\frac{dy^\ell}{d\theta}$ change $x^{j\ell}$ in proportion to the intermediate-supply share $\xi^{j\ell}$. In turn, changes in the intermediate-supply share $\frac{d\xi^{j\ell}}{d\theta}$ can occur either due to reallocation of good ℓ across different intermediate uses — a change in the intermediate-use share $\chi_x^{j\ell}$ — or due to reallocation from consumption to production — a change in the intermediate share ϕ_x^ℓ.

[13] Depending on ϕ_x^ℓ, good ℓ can be i) *pure final*, when $\phi_x^\ell = 0$; ii) *pure intermediate*, when $\phi_x^\ell = 1$; or iii) *mixed*, when $\phi_x^\ell \in (0,1)$. Equivalently, good ℓ can be i) *final* when $\phi_x^\ell \in [0,1)$ or ii) *intermediate*, when $\phi_x^\ell \in (0,1]$, with mixed goods being simultaneously final and intermediate. These categorizations are only meaningful when $y^\ell > 0$ or $\frac{dy^\ell}{d\theta} > 0$. Depending on $\chi_x^{j\ell}$, an intermediate input ℓ is i) *specialized*, when $\chi_x^{j\ell} = 1$ for some j; or *diversified*, when $\chi_x^{j\ell} \in (0,1)$ for some j.

At last, we define the *factor use share* of factor f used to produce good j, $\chi_n^{jf,d}$, as

$$\chi_n^{jf,d} := \begin{cases} \frac{n^{jf,d}}{n^{f,d}} & \text{if } n^{f,d} > 0 \\ \frac{\frac{dn^{jf,d}}{d\theta}}{\frac{dn^{f,d}}{d\theta}} & \text{if } n^{f,d} = 0 \quad \text{and} \quad \frac{dn^{f,d}}{d\theta} > 0 \\ 0 & \text{if } n^{f,d} = 0 \quad \text{and} \quad \frac{dn^{f,d}}{d\theta} = 0. \end{cases} \tag{14}$$

The factor use share $\chi_n^{jf,d}$ represents the share of factor f's aggregate use $n^{f,d}$ devoted to the production of good j, or its counterpart in changes when $n^{f,d} = 0$ and $\frac{dn^{f,d}}{d\theta} > 0$.[14] In this case, equation (13) ensures that changes in factor use can be expressed as

$$\frac{dn^{jf,d}}{d\theta} = \frac{d\chi_n^{jf}}{d\theta} n^{f,d} + \chi_n^{jf,d} \frac{dn^{f,d}}{d\theta}, \tag{15}$$

even when $n^{jf,d} = 0$. Equation (15) decomposes level changes in the use $n^{jf,d}$ of factor f in the production of good j into a change in the factor use share, $\frac{dx_n^{jf}}{d\theta}$, and a change in the aggregate factor use, $\frac{d_n^{f,d}}{d\theta}$.

3.2.2 Network Propagation: Goods Inverse Matrix

To study production efficiency it is necessary to understand how perturbations propagate through the production network of goods. Lemma 2 introduces the goods inverse matrix $\boldsymbol{\Psi}_y$, which characterizes the ultimate change in the aggregate supply of goods induced by unit impulses in the supply of goods.[15]

Lemma 2. *(Goods Inverse Matrix). Changes in good j's aggregate supply can be expressed in terms of changes in intermediate-supply shares $\frac{d\xi^{j\ell}}{d\theta}$, changes in factor use $\frac{dn^{jf,d}}{d\theta}$, changes in the good endowment $\frac{d\bar{y}^{j,s}}{d\theta}$, and changes in technology $\frac{\partial G^j}{\partial \theta}$, as*

$$\frac{dy^j}{d\theta} = \underbrace{\sum_\ell \frac{\partial G^j}{\partial x^{j\ell}} \xi^{j\ell} \frac{dy^\ell}{d\theta}}_{\text{Propagation}} + \underbrace{\sum_\ell \frac{\partial G^j}{\partial x^{j\ell}} \frac{d\xi^{j\ell}}{d\theta} y^\ell + \sum_f \frac{\partial G^j}{\partial n^{jf,d}} \frac{dn^{jf,d}}{d\theta} + \frac{d\bar{y}^{j,s}}{d\theta} + \frac{\partial G^j}{\partial \theta}}_{\text{Impulse}}. \tag{16}$$

Equivalently, in matrix form,

$$\frac{d\boldsymbol{y}}{d\theta} = \underbrace{\boldsymbol{\Psi}_y}_{\text{Propagation}} \underbrace{\left(\boldsymbol{G}_x \frac{d\boldsymbol{\xi}}{d\theta} \boldsymbol{y} + \boldsymbol{G}_n \frac{d\mathring{\boldsymbol{n}}^d}{d\theta} + \frac{d\bar{\boldsymbol{y}}^s}{d\theta} + \boldsymbol{G}_\theta \right)}_{\text{Impulse}} \quad \textit{where} \quad \underbrace{\boldsymbol{\Psi}_y = (\boldsymbol{I}_J - \boldsymbol{G}_x \boldsymbol{\xi})^{-1}}_{\text{Goods Inverse}}, \tag{17}$$

[14] A factor f is i) *specialized*, when $\chi_n^{jf,d} = 1$ for some j; or *diversified*, when $\chi_n^{jf,d} \in (0,1)$ for some j.

[15] In Appendix C.3, we introduce two related propagation matrices: the intermediate inverse matrix $\boldsymbol{\Psi}_x$, which characterizes network propagation for changes in the level of intermediates; and the proportional goods inverse matrix $\tilde{\boldsymbol{\Psi}}_y = \hat{\boldsymbol{y}}^{-1} \boldsymbol{\Psi}_y \hat{\boldsymbol{y}}$, where $\hat{\boldsymbol{y}} = \text{diag}(\boldsymbol{y})$, which characterizes network propagation for proportional impulses in the supply of goods. To simplify the exposition, we exclusively use the goods inverse matrix in the body of the paper, but all three matrices are useful to understand network propagation, as explained in the Appendix.

where $\frac{d\boldsymbol{y}}{d\theta}$ and $\frac{d\bar{\boldsymbol{y}}^s}{d\theta}$ respectively denote the $J \times 1$ vectors of $\frac{dy^j}{d\theta}$ and $\frac{d\bar{y}^{j,s}}{d\theta}$, and $\boldsymbol{\Psi}_y = (\boldsymbol{I}_J - \boldsymbol{G}_x\boldsymbol{\xi})^{-1}$ defines the $J \times J$ goods inverse matrix. The remaining matrices are defined in Appendix A.

Lemma 2 characterizes how the aggregate supply of goods ultimately changes in response to changes in intermediate-supply shares, factor use, goods endowments, and technology, accounting for network propagation. Consider the four "impulse" terms of equation (16), which represent the first-round impact of the perturbation on the supply of goods. First, a perturbation that changes intermediate-supply shares by $\frac{d\xi^{j\ell}}{d\theta}$ raises at impact the amount of good ℓ used as input for good j in proportion to y^ℓ, which in turn increases output at impact by $\frac{\partial G^j}{\partial x^{j\ell}}$. Similarly, a perturbation that changes the use of factor f in the production of good ℓ by $\frac{dn^{\ell f,d}}{d\theta}$ increases output at impact by $\frac{\partial G^j}{\partial n^{jf,d}}$. Changes in the endowment or the technology used to produce good j simply increase aggregate supply at impact by $\frac{d\bar{y}^{j,s}}{d\theta}$ or $\frac{\partial G^j}{\partial \theta}$, respectively.

Such first-round changes in the level of aggregate supply in turn induce further changes in the level of intermediate inputs, which in turn induce further changes in aggregate supply. These knock-on effects through the production network are captured by the goods inverse matrix $\boldsymbol{\Psi}_y$. Under minimal regularity conditions — described in Section C.3 of the Online Appendix — $\boldsymbol{\Psi}_y$ admits the series representation

$$\boldsymbol{\Psi}_y = (\boldsymbol{I}_J - \boldsymbol{G}_x\boldsymbol{\xi})^{-1} = \boldsymbol{I}_J + \boldsymbol{G}_x\boldsymbol{\xi} + (\boldsymbol{G}_x\boldsymbol{\xi})^2 + (\boldsymbol{G}_x\boldsymbol{\xi})^3 + \dots \,. \tag{18}$$

The first term in the expansion, $\boldsymbol{I}_J$, represents the first round of aggregate supply changes we just described. As aggregate supply adjusts, the level of intermediate inputs $x^{j\ell}$ changes in proportion to the intermediate-supply share $\xi^{j\ell}$, or $\boldsymbol{\xi}$ in matrix form. In turn, changes in the level of intermediate inputs translate into a second round of changes in aggregate supply in proportion to the marginal products of each input $\frac{\partial G^j}{\partial x^{j\ell}}$, or $\boldsymbol{G}_x$ in matrix form. This explains the second term $\boldsymbol{G}_x\boldsymbol{\xi}$ in (18), which generates knock-on effects in proportion to $(\boldsymbol{G}_x\boldsymbol{\xi})^2$ and so on. We refer to the conclusion of this fixed point of network propagation as the ultimate change in the aggregate supply of goods induced by the perturbation. We conclude with the following remark.

Remark 1. *(Goods Inverse Matrix is Purely Technological)* While propagation matrices abound in the study of models with rich production structures — see e.g. Carvalho and Tahbaz-Salehi (2019) — the goods inverse matrix introduced in Lemma 2 is distinct in the sense that it is purely a technological object. That is, $\boldsymbol{\Psi}_y$ is exclusively based on production technologies, regardless of preferences, equilibrium assumptions, etc. This is important because $\boldsymbol{\Psi}_y$ will be a key input when characterizing efficiency conditions in Section 4. In competitive economies, the goods inverse matrix $\boldsymbol{\Psi}_y$ will be related to Leontief-style inverses, which depend on prices, as explained in Section 5.

3.2.3 Welfare-Relevant Statistics

In order to characterize production efficiency, we must first introduce three sets of welfare-relevant statistics. These objects represent the welfare impact — through efficiency — of specific perturbations in the consumption, factor supply, goods supply, intermediate use, and factor use. First, we introduce *aggregate marginal rates of substitution* ($AMRS$).

Definition 2. *(Aggregate Marginal Rate of Substitution). We define the aggregate marginal rate of substitution (AMRS) between good j and the numeraire and between factor f and the numeraire as*

$$AMRS_c^j = \sum_i \chi_c^{ij} MRS_c^{ij} \quad \text{and} \quad AMRS_n^f = \sum_i \chi_n^{if,s} MRS_n^{if,s}, \tag{19}$$

where consumption and factor supply shares χ_c^{ij} and $\chi_n^{if,s}$ are defined in (9) and individual marginal rates of substitution MRS_c^{ij} and $MRS_n^{if,s}$ are defined in (11). We denote the $1 \times J$ and $1 \times F$ vectors of $AMRS_c^j$ and $AMRS_n^f$ by $\boldsymbol{AMRS}_c$ and $\boldsymbol{AMRS}_n$.

Aggregate marginal rates of substitution for goods and factors are cross-sectional weighted averages of individual marginal rates of substitution. For goods with $c^j > 0$ or $\frac{dc^j}{d\theta} > 0$, $AMRS_c^j$ corresponds to the welfare gain associated with increasing aggregate consumption of good j by a unit, making individuals consume in proportion to their consumption shares. For factors with $n^{f,s} > 0$ or $\frac{dn^{f,s}}{d\theta} > 0$, $AMRS_n^f$ corresponds to the welfare cost associated with increasing the aggregate supply of factor f by a unit, making individuals supply the factor in proportion to their factor supply shares.

Second, we introduce the *marginal social value* of goods (MSV).

Definition 3. *(Marginal Social Value of Goods). We define the marginal social value of good j, MSV_y^j, as the j'th element of the $1 \times J$ vector $\boldsymbol{MSV}_y$, given by*

$$\boldsymbol{MSV}_y = \boldsymbol{AMRS}_c \boldsymbol{\phi}_c \boldsymbol{\Psi}_y, \tag{20}$$

where $\boldsymbol{AMRS}_c$ is defined in (19), $\boldsymbol{\phi}_c$ is the $J \times J$ diagonal matrix of aggregate consumption shares defined in Appendix A, and $\boldsymbol{\Psi}_y$ is the $J \times J$ goods inverse matrix defined in (17).[16]

The marginal social value of good j captures the efficiency gain associated with having an additional unit of that good in the economy. As just described in Section 3.2.2, a unit impulse in the supply of goods generates an ultimate increase in the aggregate supply of goods given by the goods inverse matrix $\boldsymbol{\Psi}_y$. However, a fraction of the aggregate supply of goods is used in the production of other goods, so only the aggregate consumption share $\boldsymbol{\phi}_c$ is consumed by individuals. And the $\boldsymbol{AMRS}_c$ captures the welfare gain associated with increasing aggregate consumption, so the marginal social

[16] When $c^j = \frac{dc^j}{d\theta} = 0$ or $n^{f,s} = \frac{dn^{f,s}}{d\theta} = 0$, the definition of shares in (9) implies that $AMRS_c^j = 0$ and $AMRS_n^f = 0$, so these $AMRS$ cannot correspond to the welfare gain or loss associated with changing aggregate consumption or factor supply. This is purely a notational convention to simplify the exposition: we will show that the production efficiency decomposition does not depend on the values of $AMRS_c^j$ and $AMRS_n^f$ in those cases.

value of an impulse in the supply of goods is the product of these three objects. The definition of MSV highlights that the social value of a good emanates from the consumption — potentially of other goods — it ultimately generates.

Third, we introduce marginal welfare products (MWP).

Definition 4. *(Marginal Welfare Product). We define the marginal welfare products (MWP) of input ℓ and factor f for technology j as*

$$MWP_x^{j\ell} = MSV_y^j \frac{\partial G^j}{\partial x^{j\ell}} \quad \text{and} \quad MWP_n^{jf} = MSV_y^j \frac{\partial G^j}{\partial n^{jf,d}}, \tag{21}$$

where the marginal social value of good j, MSV_y^j, is defined in (20).

Marginal welfare products correspond to the welfare gain associated with increasing the use of an input or factor in the production of a good. Marginal increases in $x^{j\ell}$ or $n^{jf,d}$ increase output at impact by their physical marginal products, $\frac{\partial G^j}{\partial x^{j\ell}}$ and $\frac{\partial G^j}{\partial n^{jf,d}}$. As just described, the social value of a unit impulse in the supply of goods is summarized by the marginal social value of goods, MSV_y^j. Hence, marginal welfare products of inputs and factors are given by the product of physical marginal products and the marginal social value of the goods produced.

Finally, we introduce aggregate marginal welfare products ($AMWP$).

Definition 5. *(Aggregate Marginal Welfare Product). We define the aggregate marginal welfare product ($AMWP$) of good j and factor f, respectively, as*

$$AMWP_x^{\ell} = \sum_j \chi_x^{j\ell} MWP_x^{j\ell} \quad \text{and} \quad AMWP_n^{f} = \sum_j \chi_n^{jf,d} MWP_n^{jf}, \tag{22}$$

where intermediate input use and factor use shares $\chi_x^{j\ell}$ and $\chi_n^{jf,d}$ are defined in (12) and (14) and marginal welfare products in (21).

The aggregate marginal welfare product of an input or factor is a cross-sectional weighted average of marginal welfare products. For inputs with $x^{\ell} > 0$ or $\frac{dx^{\ell}}{d\theta} > 0$, it corresponds to the welfare gain associated with increasing the aggregate intermediate use of good ℓ in proportion to the intermediate use shares. For factors with $n^{f,d} > 0$ or $\frac{dn^{f,d}}{d\theta} > 0$, it corresponds to the welfare gain associated with increasing the factor use of factor f in proportion to the factor use shares.[17]

At last, note that the marginal social value of a good can be expressed in terms of aggregate marginal rates of substitution and aggregate marginal welfare products as

$$MSV_y^j = \phi_c^j AMRS_c^j + \phi_x^j AMWP_x^j. \tag{23}$$

[17] When $x^{\ell} = \frac{dx^{j\ell}}{d\theta} = 0$ or $n^{f,d} = \frac{dn^{f,d}}{d\theta} = 0$, $AMWP$'s as defined here do not correspond to the welfare gain or loss associated with changing aggregate intermediate or factor use. This is inconsequential, since the welfare accounting decomposition does not depend on the values of $AMWP_x^{\ell}$ and $AMWP_n^{f}$ in those cases.

This equation, which provides an alternative definition for MSV_y^j, shows that the value of a good corresponds to the value of consuming its aggregate consumption share ϕ_c^j and using its aggregate intermediate use share ϕ_x^j in production. This definition is recursive since $AMWP_x^j$ is a function of the marginal social value of all goods.

3.2.4 Production Efficiency Decomposition

Production efficiency captures efficiency welfare gains associated with the economy's production side. It comprises i) allocative efficiency gains due to adjusting inputs and factors and ii) technical efficiency gains from primitive changes in technologies and factor endowments.[18]

Theorem 2. *(Production Efficiency). Production efficiency $\Xi^{AE,P}$ can be decomposed into i) cross-sectional intermediate input efficiency, ii) aggregate intermediate input efficiency, iii) cross-sectional factor efficiency, iv) aggregate factor efficiency, v) technology change, vi) good endowment change, and vii) factor endowment change, as*

$$\Xi^{AE,P} = \overbrace{\underbrace{\sum_{\ell} \mathbb{C}ov_j^{\Sigma}\left[MWP_x^{j\ell}, \frac{d\chi_x^{j\ell}}{d\theta}\right] x^{\ell}}_{\substack{\text{Cross-Sectional}\\ \text{Intermediate Input Efficiency}}} + \underbrace{\sum_{\ell}\left(AMWP_x^{\ell} - AMRS_c^{\ell}\right)\frac{d\phi_x^{\ell}}{d\theta} y^{\ell}}_{\substack{\text{Aggregate}\\ \text{Intermediate Input Efficiency}}}}^{\text{Intermediate Input Efficiency}}$$

$$+ \overbrace{\underbrace{\sum_{f} \mathbb{C}ov_j^{\Sigma}\left[MWP_n^{jf}, \frac{d\chi_n^{jf,d}}{d\theta}\right] n^{f,d}}_{\substack{\text{Cross-Sectional}\\ \text{Factor Efficiency}}} + \underbrace{\sum_{f}\left(AMWP_n^{f} - AMRS_n^{f}\right)\frac{dn^{f,s}}{d\theta}}_{\substack{\text{Aggregate}\\ \text{Factor Efficiency}}}}^{\text{Factor Efficiency}}$$

$$+ \underbrace{\sum_j MSV_y^j \frac{\partial G^j}{\partial \theta}}_{\substack{\text{Technology}\\ \text{Change}}} + \underbrace{\sum_j MSV_y^j \frac{d\bar{y}^{j,s}}{d\theta}}_{\substack{\text{Good Endowment}\\ \text{Change}}} + \underbrace{\sum_f AMWP_n^f \frac{d\bar{n}^{f,s}}{d\theta}}_{\substack{\text{Factor Endowment}\\ \text{Change}}},$$

where marginal welfare products, $MWP_x^{j\ell}$ and MWP_n^{jf}, aggregate marginal rates of substitution, $AMRS_c^{\ell}$ and $AMRS_n^{f}$, aggregate marginal welfare products, $AMWP_x^{\ell}$ and $AMWP_n^{f}$, and the marginal social value of goods, MSV_y^j, are defined in Section 3.2.3.

[18] Production efficiency gains ultimately correspond to higher aggregate consumption and lower aggregate factor supply. In fact, $\Xi^{AE,P}$ is given by

$$\Xi^{AE,P} = \sum_j AMRS_c^j \frac{dc^j}{d\theta} - \sum_f AMRS_n^f \frac{dn^{f,s}}{d\theta}.$$

This formulation shows that production efficiency can be interpreted as higher aggregate consumption/value added after appropriately netting the cost of supplying factors — see Nordhaus and Tobin (1973) for the importance of subtracting the cost of supplying factors to connect aggregate consumption/value added/GDP and welfare. In that sense, part of the contribution of Theorem 2 is to express changes in aggregate consumption net of factor supply costs in terms of changes in the allocation of intermediates, factors, technologies, and factor endowments.

First, cross-sectional intermediate input efficiency measures welfare gains from reallocating intermediate inputs from low to high marginal welfare product uses, for a given level of aggregate intermediate use. Hence, for good ℓ it corresponds to the covariance across uses between $MWP_x^{j\ell}$ and the change in the intermediate use shares, $\frac{dx_x^{j\ell}}{d\theta}$, in proportion to the good's aggregate intermediate use, x^ℓ.

Second, aggregate intermediate input efficiency measures the welfare gains from adjusting the share of aggregate goods supply devoted to final consumption relative to production, for a given level of aggregate goods supply. Hence, for good ℓ it corresponds to the difference between $AMWP_x^\ell$ and $AMRS_c^\ell$, which captures the net welfare impact of reducing consumption of good ℓ and using it in production, multiplied by the change in the intermediate use share, $\frac{d\phi_x^\ell}{d\theta}$, in proportion to the aggregate supply of the good, y^ℓ.

Third, cross-sectional factor efficiency measures the welfare gains from reallocating factors from low to high marginal welfare product uses, for a given level of aggregate factor use. Hence, for factor f it corresponds to the covariance across uses between MWP_n^{jf} and the change in the factor use shares, $\frac{d\chi_n^{jf,d}}{d\theta}$, in proportion to the aggregate use of the factor, $n^{f,d}$.

Fourth, aggregate factor efficiency measures the welfare gains from adjusting the (elastic) supply of factors. Hence, for factor f it corresponds to the difference between $AMWP_n^f$ and $AMRS_n^f$, which captures the net welfare impact of supplying an additional unit of factor f and putting it to use, multiplied by the change in factor supply, $\frac{dn^{f,s}}{d\theta}$.[19]

The final three components of the production efficiency decomposition measure welfare gains due to primitive changes in technology and endowments. The technology change component measures the welfare gains from having more supply of goods (at no cost) for given allocation shares and elastic factor supplies. Hence, for good j it corresponds to the output change induced by the change in technology, $\frac{\partial G^j}{\partial\theta}$, valued at its marginal social value MSV_y^j. A change in the technology used to produce good j is identical to a change in the endowment of the good (at no cost), which is also valued at its marginal social value MSV_y^j, defining the good endowment change component. Finally, the factor endowment change component measures the welfare gains from having more factors (at no cost) for given allocation shares and elastic factor supplies. Hence, for factor f it corresponds to the change in the supply of factor f, $\frac{d\bar{n}^{f,s}}{d\theta}$ valued at the welfare gain associated with increasing factor use, $AMWP_n^f$.

Corollary 2 shows that production efficiency satisfies several desirable properties. These properties are helpful to quickly analyze particular economies, as we do in Section 3.4.

Corollary 2. *(Properties of Production Efficiency).*

(a) (Single Good Economies) In economies with a single good ($J = 1$), cross-sectional intermediate input efficiency and cross-sectional factor efficiency are zero.

[19] In general, aggregate factor efficiency must be expressed in terms aggregate factor supply and not factor use. If the endowment of an elastically supplied factor is zero or does not change in a given perturbation, then $\frac{dn^{f,s}}{d\theta} = \frac{dn^{f,d}}{d\theta}$.

(b) *(No Intermediate Input Economies) In economies with no intermediate goods ($x^{j\ell} = \xi^{j\ell} = 0$), cross-sectional and aggregate intermediate input efficiency are zero.*

(c) *(Fixed Factor Supply Economies) In economies in which all factors are in fixed supply ($\frac{dn^{f,s}}{d\theta} = 0$), aggregate factor efficiency is zero.*

(d) *(Specialized Intermediate/Factor Economies) In economies in which all intermediate inputs (factors) are specialized with $\chi_x^{j\ell} = 1$ ($\chi_n^{jf} = 1$) for some j, cross-sectional intermediate input (factor use) efficiency is zero.*

(e) *(Equalized $MWP_x^{j\ell}$ or MWP_n^{jf}) If marginal welfare products for good ℓ (factor f) are identical across uses for all goods (factors) with $x^\ell > 0$ ($n^{f,d} > 0$), then cross-sectional intermediate (factor) efficiency is zero.*

Since both cross-sectional intermediate input and factor efficiency rely on reallocating intermediate inputs and factors towards different uses in production, it is necessary to have at least two goods that can be produced. Relatedly, economies with no intermediate inputs cannot feature cross-sectional or aggregate intermediate input efficiency gains, since $\frac{dx_x^{j\ell}}{d\theta} = 0$ and $\frac{d\phi_x^\ell}{d\theta} = 0$, while economies with factors in fixed supply, cannot feature aggregate factor efficiency gains, since $\frac{dn^{f,s}}{d\theta} = 0$. Finally, cross-sectional intermediate input (factor) use efficiency must be zero when i) intermediate inputs (factors) are specialized, since there is no scope for reallocating intermediate input (factor) shares towards alternative uses, or ii) the social value of using a good (factor) is identical across uses, since there is no scope to find welfare gains from reallocating goods (factors).

3.3 Insights from Welfare Accounting Decomposition

We present several of the insights that emerge from the welfare accounting decomposition in a series of remarks.

Remark 2. *(Technological and preference origins of welfare gains and losses).* Theorems 1 and 2 trace the origins of efficiency gains and losses under any perturbation to the reallocation of resources and to primitive changes in technology and endowments. Their main conceptual contribution is to characterize the welfare-relevant social valuations for each of these changes. In fact, Theorems 1 and 2 identify a small set of variables — MRS, MWP, $AMRS$, $AMWP$ and MSV — that are sufficient to translate physical changes in allocations, technologies, and endowments into welfare gains and losses. Importantly, this decomposition is written purely in terms of preferences and technologies, and makes no reference to prices, individual budget constraints, or notions of equilibrium. It therefore directly applies not only to competitive environments, but also to, for instance, bargaining, search, or imperfectly competitive environments, as we highlight in our applications.

Remark 3. *(Social Value of Technology).* Theorem 2 identifies the efficiency gains from pure technological change with the marginal social value of goods, MSV_y^j, without making assumptions about the (optimizing) behavior or budget constraints of individuals, prices, or equilibrium notions. In fact, since MSV_y^j can be computed at the original allocation, Theorem 2 characterizes the efficiency gains from technology changes without the need to specify, compute, or measure a perturbation.[20] The technology change component of the welfare accounting decomposition is always positive since $MSV_y^j > 0$. However, a technological improvement may decrease aggregate efficiency overall if its impact on allocative efficiency is sufficiently negative, which can only happen at inefficient allocations — see Section 4.

Remark 4. *(Allocative vs. Technical Efficiency; Efficiency vs. Misallocation).* We refer to the welfare gains due to exchange efficiency and the first four components of production efficiency as allocative efficiency gains, because these involve changes in allocations (allocation shares and factor supplies). We could have alternatively used the term misallocation. That is, a perturbation that increases, say, cross-sectional or aggregate factor efficiency can be described as reducing cross-sectional or aggregate factor misallocation. In fact, the factor efficiency components are the marginal counterpart of the notions of misallocation in Hsieh and Klenow (2009), as shown in Section 7. Since the welfare gains associated with technology and endowment changes do not involve changes in allocations but instead capture the pure effect of changes in primitives, we refer to these as technical efficiency gains.

Remark 5. *(Shares, Efficiency Conditions, and Planning Problem).* By design, the allocative efficiency components of the welfare accounting decomposition — with the exception of aggregate factor efficiency — are written in terms of changes in allocation shares. Working with shares allows us to separate changes due to reallocation (holding consumption, factor supply, goods supply, intermediate input use, or factor use fixed) from changes in aggregates (aggregate factor supply, technology, or endowments).[21] Moreover, each allocative efficiency component maps directly into a particular optimality condition of the planning problem for this economy, whose solution we formally characterize in Section C.2 of the Online Appendix. This occurs because at an efficient allocation, reallocating resources cannot generate efficiency gains. We characterize these efficiency conditions next in Section 4.

Remark 6. *(Informational Requirements).* What are the informational requirements to implement the welfare accounting decomposition, either by computing it in a structural model or by empirically measuring its components? To compute exchange efficiency, it is sufficient to know i) aggregate consumption and factor supply, ii) changes in individual consumption and factor supply shares, and iii) individual marginal rates of substitution. Conditional on these objects, the economy's production structure does not independently determine exchange efficiency. To compute production efficiency, it is sufficient to know i) total aggregate supply, intermediate use, and factor use; ii) changes in

[20] By contrast, computing the welfare gains from exchange, intermediate input, and factor efficiency requires knowledge of changes in allocations, which must be computed within a model or measured empirically.

[21] This separation is not possible working with levels, since perturbations that change the level of aggregate supply necessarily change consumption and/or intermediate input use levels, via (3), while perturbations that change the level of factor supply necessarily change factor use levels, via (4). Our results can be seen as an application of the non-envelope theorem result in Dávila and Schaab (2024a) that exploits the linearity of resource constraints.

intermediate use, and factor use shares, and changes in aggregate factor supply, technology, and endowments; iii) marginal welfare products; iv) aggregate marginal welfare products and marginal rates of substitution; and v) the marginal social value of goods. Conditional on these objects, the distribution of consumption and factor supply does not independently determine production efficiency. In Section 5, we show how prices can be used to infer these objects.

3.4 Examples: Minimal Welfare Accounting Economy

We conclude this section by applying Theorems 1 and 2 to simple economies. This is helpful to illustrate the economic forces that underlie each of the components of the decomposition. Figure 2 summarizes the minimal welfare accounting economy, which is the simplest economy in which each component of the welfare accounting decomposition can take non-zero values. In Appendix D, we present seven special cases of this economy in which particular components of the welfare accounting decomposition are non-zero. Table 1 summarizes these special cases.

4 Efficiency Conditions

In this section, we leverage the welfare accounting decomposition to characterize and study efficient allocations. This is, to our knowledge, the first general characterization of efficiency conditions for disaggregated production economies with heterogeneous individuals.

4.1 Exchange Efficiency

We adopt the conventional definition of (Pareto) efficiency: an allocation is efficient when there is no perturbation that makes every individual (weakly) better off. Equivalently, given Theorems 1 and 2, an allocation is efficient if there is no feasible perturbation for which any of the allocative efficiency components are positive. Theorems 3 and 4 respectively provide the necessary conditions for exchange and production efficiency.[22]

Theorem 3. *(Efficiency Conditions: Exchange Efficiency). An efficient allocation must satisfy the following exchange efficiency conditions:*

(a) *(Cross-sectional consumption efficiency) For goods with $c^j > 0$, it must be that*

$$MRS_c^{ij} = \begin{cases} = AMRS_c^j & \forall i \;\; s.t. \;\; \chi_c^{ij} > 0 \\ \leq AMRS_c^j & \forall i \;\; s.t. \;\; \chi_c^{ij} = 0. \end{cases} \tag{24}$$

[22]To simplify the exposition, we assume in the body of the paper that $y^j > 0$ and $n^{f,d} > 0$. We allow efficient allocations to feature $y^j = 0$ and $n^{f,d} = 0$ in the Appendix.

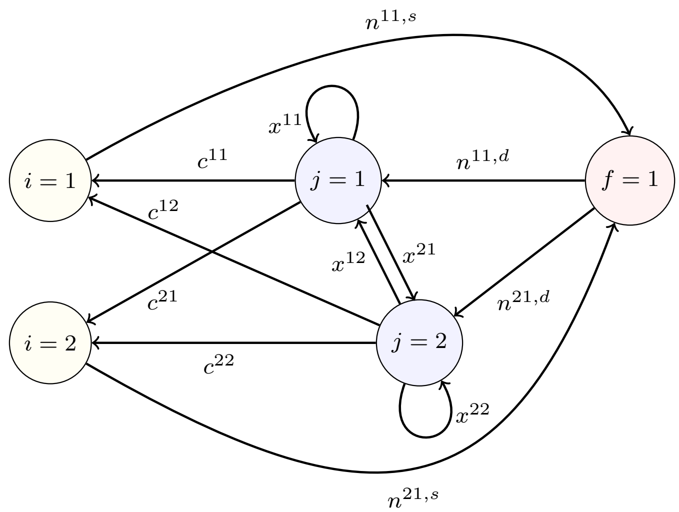


Figure 2: Minimal Welfare Accounting Economy

Note: This figure illustrates the minimal economy in which all components of the welfare accounting decomposition can take non-zero values. We summarize special cases of this economy in Table 1 and study them in Appendix D.

	Exchange Efficiency		Production Efficiency			
	Cross-Sectional Consumption Efficiency	Cross-Sectional Factor Supply Efficiency	Cross-Sectional Intermediate Input Efficiency	Aggregate Intermediate Input Efficiency	Cross-Sectional Factor Efficiency	Aggregate Factor Efficiency
Vertical	×	×	×	×	×	×
Robinson Crusoe	×	×	×	×	×	✓
Horizontal	×	×	×	×	✓	×
Roundabout	×	×	×	✓	×	×
Diversified Intermediate	×	×	✓	✓	×	×
Multiple Factor Suppliers	×	✓	×	×	×	✓
Edgeworth Box	✓	×	×	×	×	×

Table 1: Summary of Minimal Welfare Accounting Special Cases

Note: This table illustrates the components of the welfare accounting decomposition that can be non-zero in special cases of the minimal welfare accounting economy introduced in Figure 2. All economies are formally defined in Appendix D.

(b) *(Cross-sectional factor supply efficiency) For factors with $n^{f,s} > 0$, it must be that*

$$MRS_n^{if} = \begin{cases} = AMRS_n^f & \forall i \quad s.t. \quad \chi_n^{if,s} > 0 \\ \geq AMRS_n^f & \forall i \quad s.t. \quad \chi_n^{if,s} = 0. \end{cases} \tag{25}$$

Efficiency requires the equalization of MRS_c^{ij} across all consumers of good j, with MRS_c^{ij} potentially lower for individuals for whom $c^{ij} = 0$. Otherwise, it is feasible and welfare-improving to reallocate consumption from low to high MRS_c^{ij} individuals, for given aggregate consumption c^j. At the corner where individual i does not consume good j, it is not feasible to reallocate consumption away from individual i, even though marginal rates of substitution are not equalized. Similarly, efficiency requires the equalization of MRS_n^{if} across all suppliers of factor f, with MRS_n^{if} potentially lower for individuals for whom $n^{if,s} = 0$. Otherwise, it is feasible and welfare-improving to reallocate factor supply from high to low MRS_n^{if} individuals, for given aggregate factor supply $n^{f,s}$. At the corner where individual i does not supply factor f, it is not feasible to reallocate factor supply away from individual i, even though marginal rates of substitution are not equalized.

4.2 Production Efficiency

While the exchange efficiency conditions in Theorem 3 are arguably standard (see e.g. Mas-Colell et al. (1995)), the production efficiency conditions in Theorem 4 are novel.

Theorem 4. *(Efficiency Conditions: Production Efficiency). An efficient allocation must satisfy the following production efficiency conditions:*

(a) *(Cross-sectional intermediate input efficiency) For goods with $x^\ell > 0$, it must be that*

$$MWP_x^{j\ell} = \begin{cases} = AMWP_x^\ell & \forall j \quad s.t. \quad \chi_x^{j\ell} > 0 \\ \leq AMWP_x^\ell & \forall j \quad s.t. \quad \chi_x^{j\ell} = 0. \end{cases} \tag{26}$$

(b) *(Aggregate intermediate input efficiency) For goods with $y^\ell > 0$, it must be that*

$$\begin{aligned} \max_j \left\{MWP_x^{j\ell}\right\} &\leq AMRS_c^\ell && \forall \ell \;\; s.t. \;\; \phi_x^\ell = 0. \\ AMWP_x^\ell &= AMRS_c^\ell && \forall \ell \;\; s.t. \;\; \phi_x^\ell \in (0,1) \\ AMWP_x^\ell &\geq \max_i \left\{MRS_c^{i\ell}\right\} && \forall \ell \;\; s.t. \;\; \phi_x^\ell = 1. \end{aligned} \tag{27}$$

(c) *(Cross-sectional factor efficiency) For factors with $n^{f,d} > 0$, it must be that*

$$MWP_n^{jf} = \begin{cases} = AMWP_n^f & \forall j \quad s.t. \quad \chi_n^{jf} > 0 \\ \leq AMWP_n^f & \forall j \quad s.t. \quad \chi_n^{jf} = 0. \end{cases} \tag{28}$$

(d) *(Aggregate factor efficiency) For factors with $n^{f,d} > 0$, it must be that*

$$\begin{aligned} AMWP_n^f &= AMRS_n^f && \forall f \;\; s.t. \;\; n^{f,s} > 0 \\ AMWP_n^f &\leq \min_i \left\{ MRS_n^{if} \right\} && \forall f \;\; s.t. \;\; n^{f,s} = 0. \end{aligned} \tag{29}$$

While the formal statement of the conditions for production efficiency is somewhat involved, the underlying economics are simple. First, cross-sectional intermediate input efficiency requires the equalization of $MWP_x^{j\ell}$ across all uses of good ℓ in production. Otherwise, it is feasible and welfare-improving to reallocate intermediate inputs from low to high $MWP_x^{j\ell}$ uses, for given aggregate intermediate input use x^ℓ. When good ℓ is not used to produce good j, $MWP_x^{j\ell}$ must be weakly lower.

Second, aggregate intermediate input efficiency for mixed goods with $\phi_x^\ell \in (0,1)$ requires the equalization of the marginal rate of substitution from consuming good ℓ with its marginal welfare product as an input. For pure final goods with $\phi_x^\ell = 0$, the marginal rate of substitution from consuming good ℓ must be higher than its highest marginal welfare product if used as an input. For pure intermediate goods with $\phi_x^\ell = 1$, the marginal welfare product of good ℓ must be higher than its highest marginal rate of substitution if consumed. If these conditions are not satisfied, it is feasible and welfare-improving to reallocate good ℓ from final consumption to intermediate input use, or vice versa, for a given level of aggregate supply y^ℓ.

A similar logic applies to factors. Third, cross-sectional factor efficiency requires the equalization of MWP_n^{jf} across all uses of factor f, with MWP_n^{jf} potentially lower when factor f is not used to produce good j. Otherwise, it is feasible and welfare-improving to reallocate factors from low to high MWP_n^{jf} uses, for a given level of fixed aggregate factor use $n^{f,d}$.

Finally, aggregate factor efficiency requires the equalization of the marginal welfare product of elastic factor f with its marginal rate of substitution, which captures the utility cost of supplying the factor. When factor f is not elastically supplied, its marginal welfare product must be weakly lower than the lowest marginal rate of substitution, which captures the cheapest cost of supplying the factor.

Theorems 3 and 4 highlight that carefully incorporating non-negativity constraints is critical to characterize the conditions for allocative efficiency in disaggregated economies. These issues become more relevant at finer levels of disaggregation, since heterogeneous individuals typically do not consume most goods and production networks with heterogeneous producers become increasingly sparse. We elaborate on these issues in subsections 4.5 and 4.6.

4.3 MSV under Efficiency

The marginal social value of goods is a central object for welfare accounting. It is a key determinant of marginal welfare products and thus governs each component of production efficiency.

It is furthermore the single determinant of the technology change (and good endowment change) component of the welfare accounting decomposition. Theorem 5 characterizes the marginal social value of goods at efficient allocations.[23]

Theorem 5. *(MSV under Efficiency). At an allocation that satisfies aggregate intermediate input efficiency, the marginal social value of good j is given by*

$$MSV_y^j = \begin{cases} AMRS_c^j & \text{if} \quad \phi_c^j > 0 \\ AMWP_x^j & \text{if} \quad \phi_x^j > 0. \end{cases} \tag{30}$$

At an allocation that additionally satisfies cross-sectional consumption and cross-sectional intermediate input efficiency, the marginal social value of good j is given by

$$MSV_y^j = \begin{cases} MRS_c^{ij} & \forall i \ \ s.t. \ \chi_c^{ij} > 0 \quad \text{if} \quad \phi_c^j > 0 \\ MWP_x^{\ell j} & \forall \ell \ \ s.t. \ \chi_x^{\ell j} > 0 \quad \text{if} \quad \phi_x^j > 0. \end{cases} \tag{31}$$

The marginal social value of a good derives from its consumption value when the good is final and from its production value when the good is used as an input. Aggregate intermediate input efficiency guarantees that these are equalized for mixed goods, i.e., $AMRS_c^j = AMWP_x^j$ for j mixed. When j is a final good with $\phi_c^j > 0$, therefore, its marginal social value equals its consumption value $AMRS_c^j$. When j is an intermediate good with $\phi_x^j > 0$, its marginal social value equals its production value $AMWP_x^j$. And when good j is mixed with $\phi_c^j > 0$ and $\phi_x^j > 0$, consumption and production value must be equalized, so $MSV_y^j = AMRS_c^j = AMWP_x^j$.

Conversely, the marginal social value of a pure final (pure intermediate) good is not equal to its production (consumption) value. As long as aggregate intermediate input efficiency is satisfied, $MSV_y^j > AMRS_c^j$ when j is a pure intermediate with $\phi_x^j = 1$ and $MSV_y^j > AMWP_x^j$ when j is a pure final good with $\phi_c^j = 1$.

Cross-sectional consumption efficiency furthermore guarantees that $MRS_c^{ij} = AMRS_c^j$ are equalized across all individuals i that consume good j ($\chi_c^{ij} > 0$). The MSV of a final good must therefore coincide with the valuation of each individual. Similarly, cross-sectional intermediate input efficiency guarantees that $MWP_x^{\ell j} = AMWP_x^j$ are equalized for good j across all its intermediate uses ℓ ($\chi_x^{\ell j} > 0$). The MSV of goods used as intermediate inputs must then coincide with the marginal welfare product of each use. More broadly, efficiency requires that the value of using a good must be equalized across all uses and coincide with the MSV of the good.

[23] Characterizing the factor endowment change component under efficiency is straightforward. When $n^{f,d} > 0$, efficiency requires that $AMWP_n^f = MWP_n^{jf}$, $\forall j$ with $\chi_n^{jf,d} > 0$.

4.4 Planning Problem, Lagrange Multipliers, and Socialist Calculation Debate

We have emphasized that the welfare accounting decomposition can be leveraged to derive efficiency conditions directly. An equivalent alternative approach is to set up the planning problem.

Definition 6. *(Planning Problem). The planning problem — formally stated in Appendix C.2 — maximizes the social welfare function in (5), with preferences V^i defined in (1), subject to technologies and resource constraints, defined in (2), (3) and (4), as well as non-negativity constraints.*

There are two reasons why studying the planning problem is useful. First, it provides an equivalent characterization of the efficiency conditions in Theorems 3 and 4. As we show in the Appendix, the restriction to feasible perturbations that underlies our characterization of efficiency conditions is implied by the Kuhn-Tucker multipliers on the constraints of the planning problem. Second, and more importantly for this paper, the planning problem provides a justification for the welfare accounting decomposition. As we show in the Appendix, each of the components of the decomposition can be interpreted as a particular perturbation of the planning problem.

Two implications of our new characterization of efficiency conditions are worth highlighting.

Remark 7. *(MSV_y^j and $AMWP_n^f$ as Lagrange Multipliers on Resource Constraints).* The planning problem provides an interpretation of the technology change (and good endowment change) and factor endowment change components of the welfare accounting decomposition in terms of the Lagrange multipliers on goods and factors resource constraints: ζ_y^j and ζ_n^f, since $\zeta_y^j = MSV_y^j$ when $y^j \neq 0$ and $\zeta_n^f = AMWP_n^f$ when $n^{f,d} \neq 0$. To our knowledge, our results provide the first characterization of the Lagrange multipliers of the planning problem in general disaggregated economies.[24]

Remark 8. *(Socialist Calculation Debate with Intermediate Goods).* Our characterization of efficiency conditions directly speaks to the socialist calculation debate, which discusses the feasibility of central planning — see e.g. Lange (1936), Lerner (1944), or Hayek (1945). Our results illustrate how computing efficiency conditions in production economies is significantly harder than efficiently allocating goods across individuals, especially in economies that feature pure intermediates. In particular, our results imply that computing MSV_y^j for pure intermediates requires knowledge of the entire production network — to compute the goods inverse matrix $\boldsymbol{\Psi}_y$ — while computing MSV_y^j for mixed or pure final goods only requires knowledge of aggregate individual valuations via marginal rates of substitution. Intuitively, the value of goods that are consumed by individuals can be ascertained from individual valuations, even when these goods also used to produce, while pure intermediates only derive value once eventually consumed. This observation can be used to support the hypothesis that the losses associated with planning increase with the complexity of production networks, in particular when these feature pure intermediate goods.[25]

[24] As we explain in Section 5, these multipliers can be expressed in terms of prices and wedges in competitive economies.

[25] It is thus not a a surprise that Friedman and Friedman (1980) chose a pencil — a good with a complex production structure that relies on pure intermediates — as the example to praise the virtues of competitive markets. See also Read (1958).

4.5 Interior Economies: Revisiting Lange (1942) and Mas-Colell et al. (1995)

The classic approach to characterizing efficiency conditions is typically traced back to Lange (1942) — see also Samuelson (1947) — and is summarized in a modern treatment in Section 16.F of Mas-Colell et al. (1995). A central contribution of our paper is to show that Theorems 3 and 4 generalize these classic conditions to general environments with disaggregated production.

Definition 7. *(Classic Efficiency Conditions). The classic (production) efficiency conditions for an intermediate link $j\ell$ and a factor link jf hold if*

$$MRS_c^{ij}\frac{\partial G^j}{\partial x^{j\ell}} = MRS_c^{i\ell} \quad \text{and} \quad MRS_c^{ij}\frac{\partial G^j}{\partial n^{jf,d}} = MRS_n^{if}. \tag{32}$$

Critically, the classic approach exclusively studies interior production economies, in which every good is mixed and used in the production of every other good, i.e., $\chi_x^{j\ell} \in (0,1)$ and $\phi_x^{\ell} \in (0,1)$.[26] In that case, the classic efficiency conditions in (32) imply i) equalized marginal rates of substitution across individuals, ii) equalized marginal rates of transformation (MRT) across goods, and iii) the equalization of MRS with MRT.[27] In Corollary 3, we show that the classic efficiency conditions emerge as a special case of Theorems 3 and 4 in interior economies. We then show in subsection 4.6 that the classic efficiency conditions are typically invalid in disaggregated production economies that are not interior.

Corollary 3. *(Revisiting Lange (1942)/Mas-Colell et al. (1995)). In interior economies, the efficiency conditions of Theorems 3 and 4 collapse to those in Section 16.F of Mas-Colell et al. (1995).*

By construction, all (production) non-negativity constraints are slack in interior economies. Since $\chi_x^{j\ell} \in (0,1)$ and $\phi_x^{\ell} \in (0,1)$, it follows directly from Theorems 3 and 4 (conditions (26) and (27)) that

[26]The classic approach typically allows for consumption or factor supply to be zero for some (but not all) individuals. For that reason, our contribution is to study non-interior economies in the production sense. Mas-Colell et al. (1995) justify their restriction to interior production economies as follows:

> *"(..) every commodity is both an input and an output of the production process. Because this is unrealistic, we emphasize that no more than expositional ease is involved here. Recall that for expositional ease we are not imposing any boundary constraints on the vectors of inputs/outputs."*

Our results show that exclusively considering interior economies is insufficient to properly understand efficiency conditions in disaggregated economies.

[27]Recall that we define marginal rates of substitution in units of the numeraire in this paper, i.e., $MRS_c^{ij} = \frac{\partial u^i}{\partial c^{ij}}/\lambda^i$. If condition (32) holds, then $MRS_c^{i\ell}/MRS_c^{ij} = \frac{\partial G^j}{\partial x^{j\ell}}$ must be equal across individuals since marginal products do not depend on i. This implies that two individuals' valuation of good ℓ, expressed in units of good j, is equalized. Since (32) applies for all j and ℓ, it also implies the equalization of MRS in units of the welfare numeraire. To derive the equalization of MRT, notice that (32) can be rewritten as

$$MRS_c^{ij}\frac{\partial G^j}{\partial x^{j\ell}} = MRS_c^{ij'}\frac{\partial G^{j'}}{\partial x^{j'\ell}} \quad \Longrightarrow \quad MRS_c^{ij}/MRS_c^{ij'} = \frac{\partial G^{j'}}{\partial x^{j'\ell}}\Big/\frac{\partial G^j}{\partial x^{j\ell}} \equiv MRT^{jj',\ell}$$

where the RHS defines the marginal rate of transformation (MRT). Condition (32) therefore implies both $MRS = MRT$ (after a change of units) and the equalization of MRT across uses since the LHS does not depend on ℓ. A similar argument applies to factor use.

$MWP_x^{j\ell} = MRS_c^{i\ell}$, $\forall i, j$ for every good ℓ. Similarly for factors, conditions (28) and (29) imply that $MRS_n^{if} = MWP_x^{jf}$, $\forall i, j$ for every factor f. Both sets of conditions imply that the classic efficiency conditions in equation (32) are satisfied for all links.

4.6 Non-Interior Economies

What then distinguishes the conditions for production efficiency in economies that are not interior, and why do the classic conditions not apply to these environments?

Consider increasing $x^{j\ell}$, the use of good ℓ in the production of good j. Assuming this is a feasible perturbation, efficiency requires that its social cost — the marginal social value of good ℓ — is equalized with its social benefit — the marginal social value of good j multiplied by the marginal product $\frac{\partial G^j}{\partial x^{j\ell}}$. The classic efficiency conditions (32) use marginal rates of substitution to measure the social benefit (32 LHS) and cost (32 RHS). This is appropriate for interior efficient economies where all goods are mixed, since $MSV = MRS$ for final goods as we showed above. When j or ℓ is a pure intermediate, however, marginal rates of substitution no longer represent the good's marginal social value, even at an efficient allocation (Theorem 5). Since pure intermediates are not consumed, efficiency requires their MRS to be lower than their MSV. The marginal social value of a pure intermediate instead derives from the consumption value it eventually generates downstream as it is used in the production of other goods throughout the network.

There is a second, more mechanical reason why the classic efficiency conditions do not extend to non-interior economies. If good ℓ is not used in the production of good j, the associated efficiency condition is determined by the inequality in (26): efficiency at the $j\ell$ link then requires that $MWP_x^{j\ell}$ be lower than the marginal social value of good ℓ.

We summarize the implications of Theorems 3 and 4 for non-interior economies in two corollaries. Corollary 4 concludes that the classic efficiency conditions still hold at the level of an intermediate input link, as long as that link itself is interior.

Corollary 4. *(Classic Efficiency Conditions Hold for Interior Links). The classic efficiency conditions hold for the $j\ell$ and jf links when*

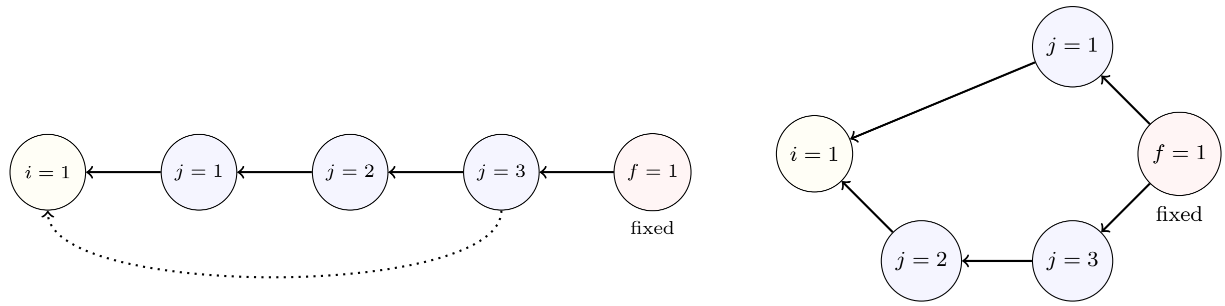
(a) a mixed good ℓ is used to produce a mixed (or a pure final) good j

(b) an elastically supplied factor f is used to produce a mixed (or a pure final) good j.

Intuitively, the classic efficiency conditions (32) extend to all interior links $j\ell$ and jf because the MSV of mixed goods coincides with their MRS, even when there are non-interior links elsewhere in the network. Corollary 5 characterizes the scenarios in which the classic conditions fail to hold.

Corollary 5. *(Scenarios in which Classic Efficiency Conditions Do Not Hold). The classic efficiency conditions generically*[28] *fail to hold for links $j\ell$ and jf that feature pure intermediate goods, i.e.,*

[28]The qualifier generically captures that it is always possible to find production structures for which these results hold.



(a) Example 1 (b) Example 2

Figure 3: Scenarios in which Classic Efficiency Conditions Do Not Hold

Note: This figure illustrates Corollary 5 in two simple scenarios. The left panel shows a mixed good (good 3) used to produce a pure intermediate (good 2), as well as a a pure intermediate (good 2) used to produce a final good (good 1). The right panel shows a factor used to produce both a pure intermediate (good 3) and a final good (good 1).

(a) a mixed good ℓ is used to produce a pure intermediate good j

(b) a pure intermediate good ℓ is used to produce any good j

(c) a factor f is used to produce a pure intermediate good j.

Trivially, the classic conditions also fail to hold for links $j\ell$ and jf when good ℓ and factor f are not used in the production of good j.

The first and third items of Corollary 5 highlight that the classic efficiency conditions may fail at links in which the efficiency conditions take the form of an equality, as long as an intermediate good is produced. This observation implies that properly characterizing production efficiency is more subtle than simply considering a set of inequalities, as in the case of exchange efficiency.

We illustrate Corollary 5 in two simple examples — see also Figure 3.

Example 1. *(Pure Intermediates).* Example 1 features a single individual ($I = 1$), three goods ($J = 3$), and a single factor in fixed supply ($F = 1$). The individual's preferences are $V^1 = u^1\left(c^{11}, c^{13}\right)$, which implies that $MRS^{12} = 0$. Technologies for each of the goods are $y^1 = G^1\left(x^{12}\right)$, $y^2 = G^2\left(x^{23}\right)$, and $y^3 = G^3\left(n^{31,d}\right)$, which already imposes that many marginal products are zero, e.g., $\frac{\partial G^1}{\partial x^{13}} = 0$.

The welfare accounting decomposition for this economy only features aggregate intermediate input efficiency: exchange efficiency is zero since $I = 1$, cross-sectional intermediate input and factor efficiency are zero since all inputs and factors are specialized, and aggregate factor efficiency is zero since the single factor is in fixed supply.[29] Plugging into Theorem 2,

$$\Xi^{AE} = \Xi^{AE,P} = \sum_{\ell}\left(AMWP_x^{\ell} - AMRS_c^{\ell}\right)\frac{d\phi_x^{\ell}}{d\theta}y^{\ell} = \left(\underbrace{MRS_c^{11}\frac{\partial G^1}{\partial x^{12}}\frac{\partial G^2}{\partial x^{23}}}_{AMWP_x^3} - \underbrace{MRS_c^{13}}_{AMRS_c^3}\right)\frac{d\phi_x^3}{d\theta}y^3.$$

[29]Formally, we assume here that the efficient production structure is as in Figure 3a. The full set of efficiency conditions also features inequalities to ensure that, for example, it is not efficient to consume good 2 or use it in the production of good 3.

For the mixed good 3 with $\phi_x^3 \in (0,1)$, aggregate intermediate input efficiency requires that $AMWP_x^3 = AMRS_c^3$, or equivalently $MRS_c^{11}\frac{\partial G^1}{\partial x^{12}}\frac{\partial G^2}{\partial x^{23}} = MRS_c^{13}$. The classic efficiency condition would instead require $MRS_c^{12}\frac{\partial G^2}{\partial x^{23}} = MRS_c^{13}$, which is invalid since good 2 is a pure intermediate and $MRS_c^{11}\frac{\partial G^1}{\partial x^{12}} > MRS_c^{12} = 0$. At the efficient allocation, the classic condition would lead one to conclude good 3's intermediate use is inefficiently high. This illustrates Corollary 5a.

This example also illustrates Corollary 5b since it features a pure intermediate (good 2) that is used in the production of another good. Since $\phi_x^2 = 1$, aggregate intermediate input efficiency requires that $MRS_c^{11}\frac{\partial G^1}{\partial x^{12}} > MRS^{12} = 0$, i.e., the consumption value of good 2 must be lower than its production value. The classic efficiency condition $MRS_c^{11}\frac{\partial G^1}{\partial x^{12}} = MRS_c^{12}$ would lead one to conclude that, at the efficient allocation, $MSV_y^2 = AMWP_x^2 = AMRS_c^2$, which would be incorrect.

Example 2. *(Factor Used to Produce Pure Intermediate).* Example 2 features one individual ($I = 1$), three goods ($J = 3$), and one factor in fixed supply ($F = 1$). Preferences are $V^1 = u^1\left(c^{11}, c^{12}\right)$ and technologies for each of the goods are $y^1 = G^1\left(n^{11,d}\right)$, $y^2 = G^2\left(x^{23}\right)$, and $y^3 = G^3\left(n^{31,d}\right)$.

The welfare accounting decomposition for this economy only features cross-sectional factor efficiency: exchange efficiency is zero since $I = 1$, cross-sectional intermediate input efficiency is zero since all inputs are specialized, aggregate factor efficiency is zero since the single factor is in fixed supply, and aggregate intermediate input efficiency is zero since $\phi_c^1 = \phi_x^2 = \phi_x^3 = 1$ by construction. Therefore,

$$\Xi^{AE} = \Xi^{AE,P} = \mathbb{C}ov_j^{\Sigma}\left[MWP_n^{j1}, \frac{d\chi_n^{j1,d}}{d\theta}\right]n^{1,d} = \left(MSV_y^1\frac{\partial G^1}{\partial n^{11,d}}\frac{d\chi_n^{11,d}}{d\theta} + MSV_y^3\frac{\partial G^3}{\partial n^{31,d}}\frac{d\chi_n^{31,d}}{d\theta}\right)n^{1,d}$$

where $MSV_y^1 = MRS_c^{11}$ and $MSV_y^3 = MRS_c^{12}\frac{\partial G^2}{\partial x^{23}}$. Since labor is in fixed supply but used in the production of two goods, a feasible perturbation is $\frac{d\chi_n^{11,d}}{d\theta} = -\frac{d\chi_n^{31,d}}{d\theta}$. Cross-sectional factor efficiency therefore requires that $MRS_c^{11}\frac{\partial G^1}{\partial n^{11,d}} = MRS_c^{12}\frac{\partial G^2}{\partial x^{23}}\frac{\partial G^3}{\partial n^{11,d}}$. The classic efficiency condition would instead associate the marginal social value of pure intermediate good 3 with its MRS and require $MRS_c^{11}\frac{\partial G^1}{\partial n^{11,d}} = MRS_c^{13}\frac{\partial G^3}{\partial n^{31,d}}$. Since $MRS_c^{12}\frac{\partial G^2}{\partial x^{23}} > MRS_c^{13} = 0$ at the efficient allocation, the classic condition would lead one to conclude the use of labor in the production of good 3 is inefficiently high, illustrating Corollary 5c.

We conclude the study of non-interior economies with a remark that highlights the importance of characterizing efficiency conditions in terms of MWP and MRS instead of MRS and MRT.

Remark 9. ($MWP \gtreqless MRS$ generalizes $MRS \gtreqless MRT$). One central takeaway from this section is that MWP and MRS are the appropriate objects to characterize efficiency conditions, rather than MRS and MRT, as in the classic approach. For instance, when good ℓ is mixed or factor f is in elastic supply, efficiency requires that

$$MWP_x^{j\ell} = MRS_c^{i\ell} \quad \text{and} \quad MWP_n^{jf} = MRS_n^{if}, \tag{33}$$

for all i such that $\chi_c^{ij} > 0$ and for all j such that $\chi_x^{j\ell} > 0$, but the classic efficiency conditions in (32) would not be valid if j is a pure intermediate. More generally, the correct inequalities that characterize production efficiency — see Theorem 2 — can be written in terms of MWP and MRS, but not MRS and MRT. This insight is useful to understand the distinction between marginal revenue products and marginal welfare products in Section 5.3.

5 Competitive Economies

Our results so far have made no assumptions about the (optimizing) behavior of agents, individual budget constraints, prices, or notions of equilibrium. In this section, we specialize the welfare accounting decomposition to competitive economies with and without wedges. This provides new insights by shedding light on the relation between efficiency and competition and by relating prices to the welfare-relevant statistics we have identified in this paper.

5.1 Competitive Equilibrium with Wedges

Starting from the physical environment described in Section 2, we now assume that individuals maximize utility and technologies are operated with the objective of minimizing costs and maximizing profits. To allow for distortions, we saturate all choices with wedges, which we take as primitives. For simplicity, we set $\bar{y}^{j,s} = 0$.

Individual i faces a budget constraint of the form

$$\sum_j p^j \left(1 + \tau_c^{ij}\right) c^{ij} = \sum_f w^f \left(1 + \tau_n^{if,s}\right) \left(n^{if,s} + \bar{n}^{if,s}\right) + \sum_j \nu^{ij} \pi^j + T^{ij}, \tag{34}$$

where p^j denotes the price of good j, w^f denotes factor f's compensation per unit supplied, $\nu^{ij}\pi^j$ denotes the profit associated with the operation of technology j received by individual i, and T^{ij} is a lump-sum transfer that rebates wedges back to individuals. Individual i faces individual-specific consumption and factor supply wedges τ_c^{ij} and $\tau_n^{if,s}$.

Firms operate technologies to minimize costs, which defines the cost functions

$$\mathcal{C}^j \left(y^j; \left\{w^f\right\}_f, \left\{p^\ell\right\}_\ell\right) = \min_{n^{jf,d}, x^{j\ell}} \sum_f w^f \left(1 + \tau_n^{jf,d}\right) n^{jf,d} + \sum_\ell p^\ell \left(1 + \tau_x^{j\ell}\right) x^{j\ell}, \tag{35}$$

subject to equation (2), facing technology-specific factor wedges $\tau_n^{jf,d}$ and technology-specific intermediate input wedges $\tau_x^{j\ell}$. We assume that the supply of good j can be expressed as the solution to a profit maximization problem given by

$$\pi^j = \max_{y^j} p^j \left(1 + \tau_y^j\right) y^j - \mathcal{C}^j \left(y^j; \left\{w^f\right\}_f, \left\{p^\ell\right\}_\ell\right) \tag{36}$$

where τ_y^j denotes a markup wedge for technology j.

Definition 8. *(Competitive Equilibrium with Wedges). A competitive equilibrium with wedges comprises a feasible allocation* $\left\{c^{ij}, n^{if,s}, x^{j\ell}, n^{jf,d}, y^{j,s}\right\}$ *and prices* $\left\{p^{j}, w^{f}\right\}$ *that satisfy resource constraints (3) and (4), such that individuals optimize,*

$$MRS_{c}^{ij} \leq p^{j}\left(1+\tau_{c}^{ij}\right), \quad \forall i, \forall j \quad \text{and} \quad MRS_{n}^{if} \geq w^{f}\left(1+\tau_{n}^{if,s}\right), \quad \forall i, \forall f,$$

where the equations hold with equality when $c^{ij} > 0$ *and* $n^{if,s} > 0$*, respectively, and firms minimize costs and maximize profits,*

$$p^{j}\frac{\partial G^{j}}{\partial x^{j\ell}} \leq p^{\ell}\frac{1+\tau_{x}^{j\ell}}{1+\tau_{y}^{j}}, \quad \forall j, \forall \ell \quad \text{and} \quad p^{j}\frac{\partial G^{j}}{\partial n^{jf,d}} \leq w^{f}\frac{1+\tau_{n}^{jf,d}}{1+\tau_{y}^{j}}, \quad \forall j, \forall f,$$

where the equations hold with equality when $x^{j\ell} > 0$ *and* $n^{jf,d} > 0$*, respectively.*[30]

In a competitive equilibrium, individuals equalize marginal rates of substitution with prices or wages cum wedges, while firms equalize marginal revenue products with marginal costs cum wedges.[31] We can compactly represent the optimality conditions in matrix form as

$$\begin{array}{lcl} \boldsymbol{MRS}_{c} \leq \boldsymbol{p}\left(\mathbf{1}_{c}+\boldsymbol{\tau}_{c}\right) & & \boldsymbol{pG}_{x} \leq \boldsymbol{p}\left(\mathbf{1}_{x}+\boldsymbol{\tau}_{x}\right) \\ & \text{and} & \\ \boldsymbol{MRS}_{n} \geq \boldsymbol{w}\left(\mathbf{1}_{n^{s}}+\boldsymbol{\tau}_{n^{s}}\right) & & \boldsymbol{pG}_{n} \leq \boldsymbol{w}\left(\mathbf{1}_{n^{d}}+\boldsymbol{\tau}_{n^{d}}\right), \end{array} \tag{37}$$

where all matrices are defined in Appendix A. The matrices $\boldsymbol{\tau}_{x}$ and $\boldsymbol{\tau}_{n^{d}}$ include markup wedges τ_{y}^{j} in addition to intermediate input use wedges $\tau_{x}^{j\ell}$ and factor use wedges $\tau_{n}^{jf,d}$. We refer to economies with no wedges ($\tau_{c}^{ij} = \tau_{n}^{if,s} = \tau_{x}^{j\ell} = \tau_{n}^{jf,d} = \tau_{y}^{j} = 0$) as frictionless competitive economies. In these economies, the First Welfare Theorem holds, so any competitive equilibrium allocation is efficient.[32]

Prices and wages (cum wedges) are helpful to recover the welfare-relevant statistics in competitive economies. Conditions (37) link prices to marginal rates of substitution and (physical) marginal products, an insight that we exploit repeatedly in this section.

5.2 Marginal Social Value of Output in Competitive Economies

5.2.1 Competitive Economies with Wedges

Characterizing the marginal social value of goods in competitive economies with wedges is critical because it directly determines the efficiency gains from technology change as well as marginal welfare products, which in turn govern all production efficiency components.

[30] In this section, we implicitly choose the nominal numeraire (i.e. the unit in which prices, wages, and profits are defined) to be the welfare numeraire. This is without loss of generality since we can always renormalize MRS.

[31] In parallel to the definition of marginal welfare products, we define marginal revenue products as $MRP_{x}^{j\ell} = p^{j}\frac{\partial G^{j}}{\partial x^{j\ell}}$ and $MRP_{n}^{jf} = p^{j}\frac{\partial G^{j}}{\partial n^{jf,d}}$. In matrix form, $\boldsymbol{MRP}_{x} = \boldsymbol{pG}_{x}$ and $\boldsymbol{MRP}_{n} = \boldsymbol{pG}_{n}$.

[32] While the general proofs of the First Welfare Theorem by Arrow (1951) and Debreu (1951) apply to the economy considered here, our results provide an alternative constructive proof. Under standard convexity assumptions, a Second Welfare Theorem also holds.

Theorem 6. *(MSV in Competitive Economies with Wedges). In competitive economies with wedges, the marginal social value of goods, defined via a* $1 \times J$ *matrix* $\boldsymbol{MSV}_y$*, is given by*

$$\boldsymbol{MSV}_y = \boldsymbol{p} + \boldsymbol{p}\bar{\boldsymbol{\tau}}_y \boldsymbol{\Psi}_y \quad \text{where} \quad \bar{\boldsymbol{\tau}}_y = \boldsymbol{\phi}_x \bar{\boldsymbol{\tau}}_x + \boldsymbol{\phi}_c \bar{\boldsymbol{\tau}}_c, \tag{38}$$

where $\boldsymbol{p}$ *denotes the* $1 \times J$ *vector of prices,* $\bar{\boldsymbol{\tau}}_x$ *and* $\bar{\boldsymbol{\tau}}_c$ *denote* $J \times J$ *diagonal matrices of aggregate intermediate input and consumption wedges, with elements given by* $\bar{\tau}_x^j = \sum_\ell \chi_x^{\ell j} \tau_x^{\ell j}$ *and* $\bar{\tau}_c^j = \sum_i \chi_c^{ij} \tau_c^{ij}$*,* $\boldsymbol{\phi}_x$ *and* $\boldsymbol{\phi}_c$ *are* $J \times J$ *diagonal matrices of aggregate intermediate use and consumption shares,* $\bar{\boldsymbol{\tau}}_y$ *defines the aggregate goods wedge, and* $\boldsymbol{\Psi}_y$ *is the goods inverse matrix defined in* (17)*.*[33]

Equation (38) shows that the marginal social value of goods equals the vector of prices augmented by a term that captures the average of the aggregate wedges in consumption and intermediate input use. Aggregate consumption and intermediate input use wedges are weighted averages of individual consumption wedges, $\bar{\tau}_c^j = \sum_i \chi_c^{ij} \tau_c^{ij}$, and intermediate input use wedges, $\bar{\tau}_x^j = \sum_\ell \chi_x^{\ell j} \tau_x^{\ell j}$. The aggregate goods wedge is in turn a weighted average of the two.

In order to understand why $\boldsymbol{MSV}_y$ takes this form in competitive economies, it is useful to start from its definition, $\boldsymbol{MSV}_y = \boldsymbol{AMRS}_c \boldsymbol{\phi}_c \boldsymbol{\Psi}_y$, and proceed gradually. First, using the optimality conditions for individual consumption, $\boldsymbol{MSV}_y$ can be written as

$$\boldsymbol{MSV}_y = \boldsymbol{p}\boldsymbol{\phi}_c \boldsymbol{\Psi}_y + \underbrace{(\boldsymbol{AMRS}_c - \boldsymbol{p})}_{\boldsymbol{p}\bar{\boldsymbol{\tau}}_c} \boldsymbol{\phi}_c \boldsymbol{\Psi}_y. \tag{39}$$

Intuitively, a unit impulse in aggregate supply ultimately increases aggregate consumption by $\boldsymbol{\phi}_c \boldsymbol{\Psi}_y$, for given allocation shares and factor supplies. The social value of this change in aggregate consumption can be split into its market value and the deviation between the true social value, given by $\boldsymbol{AMRS}_c$, and the market value. This difference is precisely determined the aggregate consumption wedge, $\bar{\boldsymbol{\tau}}_c$.

Next, the market value of the change in aggregate consumption, can be expressed as

$$\boldsymbol{p}\boldsymbol{\phi}_c \boldsymbol{\Psi}_y = \boldsymbol{p} + \underbrace{(\boldsymbol{p}\boldsymbol{G}_x \boldsymbol{\chi}_x - \boldsymbol{p})}_{\boldsymbol{p}\bar{\boldsymbol{\tau}}_x} \boldsymbol{\phi}_x \boldsymbol{\Psi}_y. \tag{40}$$

Intuitively, the ultimate change in aggregate consumption induced by a unit impulse in aggregate supply, $\boldsymbol{\phi}_c \boldsymbol{\Psi}_y$, can be expressed as the ultimate change in aggregate supply net of aggregate intermediate use.[34] Hence, the ultimate market value of a unit impulse in aggregate supply corresponds to the sum of the market value of the impulse, given by $\boldsymbol{p}$, and the market value of

[33] In sum form, we can express an element of $\boldsymbol{MSV}_y$ as $MSV_y^\ell = p^\ell + \sum_j p^j \bar{\tau}_y^j \psi_y^{j\ell}$, where $\bar{\tau}_y^j = \phi_c^j \bar{\tau}_c^j + \phi_x^j \bar{\tau}_x^j$.

[34] Formally, (40) uses the following physical identity, which follows from (18):

$$\boldsymbol{\phi}_c \boldsymbol{\Psi}_y = \boldsymbol{\Psi}_y - \boldsymbol{\phi}_x \boldsymbol{\Psi}_y = \boldsymbol{I}_J + \boldsymbol{G}_x \boldsymbol{\xi} \boldsymbol{\Psi}_y - \boldsymbol{\phi}_x \boldsymbol{\Psi}_y = \boldsymbol{I}_J + (\boldsymbol{G}_x \boldsymbol{\chi}_x - \boldsymbol{I}_J)\, \boldsymbol{\phi}_x \boldsymbol{\Psi}_y,$$

where the ultimate change in aggregate supply, $\boldsymbol{\Psi}_y$, is decomposed into the unit impulse, $\boldsymbol{I}_J$, and knock-on effects, $\boldsymbol{G}_x \boldsymbol{\xi} \boldsymbol{\Psi}_y$.

the knock-on effects net of aggregate intermediate use, given by $\boldsymbol{p}\boldsymbol{G}_x\boldsymbol{\chi}_x - \boldsymbol{p}$. This difference is precisely determined by the aggregate intermediate input wedge, $\bar{\boldsymbol{\tau}}_x$.

Combining (39) and (40), we can reformulate (38) as

$$\boldsymbol{MSV}_y = \boldsymbol{p} + \underbrace{(\boldsymbol{p}\boldsymbol{G}_x\boldsymbol{\chi}_x - \boldsymbol{p})}_{=\boldsymbol{p}\bar{\boldsymbol{\tau}}_x}\boldsymbol{\phi}_x\boldsymbol{\Psi}_y + \underbrace{(\boldsymbol{AMRS}_c - \boldsymbol{p})}_{=\boldsymbol{p}\bar{\boldsymbol{\tau}}_c}\boldsymbol{\phi}_c\boldsymbol{\Psi}_y.$$

This expression illustrates that aggregate consumption (intermediate input use) of good j is too low when $\bar{\tau}_c^j > 0$ ($\bar{\tau}_x^j > 0$), and aggregate supply of good j is too low when $\bar{\tau}_y^j = \phi_c^j\bar{\tau}_c^j + \phi_x^j\bar{\tau}_x^j > 0$. Hence, the marginal social value of goods that ultimately increase the aggregate supply of goods with positive aggregate goods wedges is higher than the price.

Given Theorem 6, the technology change component of the welfare accounting decomposition is simply given by

$$\boldsymbol{MSV}_y\boldsymbol{G}_\theta = \sum_j MSV_y^j\frac{\partial G^j}{\partial\theta} = \sum_j\left(p^j + \sum_\ell p^\ell\bar{\tau}_y^\ell\psi_y^{\ell j}\right)\frac{\partial G^j}{\partial\theta}.$$

The following remarks discuss insights that emerge from Theorem 6 for competitive economies with wedges. We then revisit its implications for frictionless competitive economies in Section 5.2.2.

Remark 10. *(Condition for $\boldsymbol{MSV}_y = \boldsymbol{p}$).* It is well understood that prices capture the social value of technology change in frictionless competitive economies — see Corollary 6 below. Theorem 6 implies a converse result that has been missing from the existing literature: The condition that ensures $\boldsymbol{MSV}_y = \boldsymbol{p}$ is that aggregate goods wedges are zero, that is,

$$\bar{\boldsymbol{\tau}}_y = \boldsymbol{\phi}_c\bar{\boldsymbol{\tau}}_c + \boldsymbol{\phi}_x\bar{\boldsymbol{\tau}}_x = 0. \tag{41}$$

While frictionless competition guarantees that (41) is satisfied, this condition may also hold otherwise, possibly at inefficient allocations. For instance, prices will capture the marginal social value of goods as long as aggregate goods wedges are zero, even when intermediate input and consumption wedges are non-zero ($\tau_x \neq 0$ and $\tau_c \neq 0$) and the competitive equilibrium is inefficient.[35]

Remark 11. *(Invariance of $\boldsymbol{MSV}_y$ to Factor Wedges).* Theorem 6 also implies that the marginal social value of goods does not depend directly on factor supply or factor use wedges. This result underscores the asymmetry between consumption and intermediate input distortions on the one hand and factor supply and use distortions on the other. Because $\boldsymbol{MSV}_y$ enters in the definition of marginal welfare products, all production efficiency components are non-zero when $\bar{\boldsymbol{\tau}}_y \neq 0$, but only factor efficiency components directly depend on factor wedges, as we show in Theorem 7 below.

Remark 12. *($\boldsymbol{MSV}_y$ and Network Propagation).* Theorem 6 has two important implications for

[35] Aggregate goods wedges can be zero when aggregate consumption and intermediate use wedges cancel out, or when both are zero. In turn, aggregate consumption and intermediate use wedges can be zero when its elements cancel out, or when all its constituents are zero. For cancellations to occur, it must be that some wedges are positive and other negative.

network propagation. First, when $\bar{\boldsymbol{\tau}}_y = 0$, the marginal social value of goods can be read exclusively off prices and does not require knowledge of the entire production network. This observation is made at times in frictionless competitive economies — see Corollary 6 — which Theorem 6 shows applies more generally. Second, when $\bar{\boldsymbol{\tau}}_y = 0$, the goods inverse matrix $\boldsymbol{\Psi}_y$ contains the necessary information on network propagation to determine $\boldsymbol{MSV}_y$. While it is possible to characterize $\boldsymbol{\Psi}_y$ in terms of prices, allocations, and intermediate input wedges — as we do in Appendix C.3 — this is only relevant insofar as it captures ultimate changes in aggregate supply.[36]

Remark 13. *(Relation to Cost-Based Domar Weights).* A central result of Baqaee and Farhi (2020) is that cost-based Domar weights summarize the impact of pure technological change on final output in an environment with a single individual, factors in fixed supply, and markup wedges. Their result is a special case of Theorem 6. Formally, under the assumptions in that paper,

$$\underbrace{\frac{1}{\sum_j p^j c^j}}_{\text{Normalization}} \underbrace{\boldsymbol{MSV}_y \boldsymbol{G}_\theta}_{\substack{\text{Technology Change}\\ \text{Component}}} = \underbrace{\frac{1}{\sum_j p^j c^j} \boldsymbol{p}\hat{\boldsymbol{c}}}_{\substack{\text{Final Expenditure}\\ \text{Share}}} \underbrace{\tilde{\boldsymbol{\Psi}}_y}_{\substack{\text{Cost-Based}\\ \text{Leontief Inverse}}}, \tag{42}$$

where $\hat{\boldsymbol{c}} = \operatorname{diag}(\boldsymbol{c})$ and $\tilde{\boldsymbol{\Psi}}_y$ is the proportional goods inverse, which in turn maps to the intermediate input block of the cost-based Leontief inverse defined in Baqaee and Farhi (2020) — see Appendix C.3. Relative to equation (42), Theorem 6 illustrates how competitive forces guarantee that $MSV_y^j = p^j$ when $\bar{\boldsymbol{\tau}}_y = 0$. Crucially, away from the assumptions in Baqaee and Farhi (2020), Theorem 6 highlights that cost-based Domar weights cease to capture the efficiency gains from pure technological change, for instance in the presence of aggregate consumption wedges.

5.2.2 Frictionless Competitive Economies: Hulten's Theorem Revisited

Theorem 6 allows us to revisit the impact of technology changes in the frictionless competitive case. This is the widely studied Hulten's theorem (Hulten, 1978), a result that has played a prominent role in the study of the macroeconomic impact of microeconomic shocks and growth accounting (Gabaix, 2011; Acemoglu et al., 2012; Baqaee and Farhi, 2020; Bigio and La'O, 2020).[37]

Corollary 6. *(Hulten's Theorem Revisited). In frictionless competitive economies, the aggregate efficiency impact of a proportional Hicks-neutral technology change j is*

$$\frac{1}{\sum_j p^j c^j} \Xi^{AE} = \underbrace{\frac{p^j y^j}{\sum_j p^j c^j}}_{\text{Sales Share}}, \tag{43}$$

[36] Only intermediate input wedges directly enter $\boldsymbol{\Psi}_y$, which echoes existing insights highlighting the outsized role that intermediate input distortions play in production — see e.g. Ciccone (2002) or Jones (2011).

[37] Hulten's theorem is typically stated as "for efficient economies and under minimal assumptions, the impact on aggregate TFP of a microeconomic TFP shock is equal to the shocked producer's sales as a share of GDP (Domar weight)" (Baqaee and Farhi, 2019).

where $\frac{p^j y^j}{\sum_j p^j c^j}$ *is the Domar weight or sales share of good* j *in* $\sum_j p^j c^j$.

Corollary 6 provides, to our knowledge, the most general Hulten-style result to date, which applies to frictionless competitive economies with heterogeneous individuals, elastic factor supplies, arbitrary preferences and technologies, and arbitrary social welfare functions. Its generality allows us to systematically discuss the many qualifications associated with this result in the following remarks.

Remark 14. *(Welfare vs. Aggregate Efficiency vs. Production Efficiency vs. Output).* Hulten's theorem is typically formulated in terms of final output (often via TFP). This is in contrast to Corollary 6, which highlights that Hulten's theorem is at its core a result about aggregate efficiency (via production efficiency) and neither about final output nor welfare. Why is this the case? In economies with a single individual ($I = 1$) and in which supplying factors causes no disutility ($\partial u^i / \partial n^{if,s} = 0$), changes in final output, production efficiency, aggregate efficiency, and welfare coincide, which has justified the use of Hulten's theorem as a result about final output. In economies with a single individual, redistribution and exchange efficiency are zero, so aggregate efficiency and welfare coincide and are exclusively determined by production efficiency. And when supplying factors causes no disutility, there is no need to subtract the social cost of supplying factors to transform final output changes into welfare changes, so production efficiency exclusively captures changes in final output (i.e. aggregate consumption).[38] Corollary 6 highlights that, in frictionless competitive economies, sales shares capture the impact of technology on aggregate efficiency, not final output or overall welfare.[39]

Remark 15. *(Efficient vs. Frictionless Competitive vs. Efficient Interior Economies).* Note that stating that Hulten's theorem applies to efficient economies would be incorrect. Corollary 6 shows instead that Hulten's theorem applies to frictionless competitive economies, which is a subset of efficient economies.[40] Why is this the case? When an allocation is efficient, all allocative efficiency components are necessarily zero, which guarantees that aggregate efficiency is exclusively due to technology and endowment changes. But efficiency is not enough to guarantee that $\boldsymbol{MSV}_y = \boldsymbol{p}$:

[38] It is common to state that Hulten's theorem does not apply to economies with elastic factor supplies. For instance, Baqaee and Farhi (2018) state that "Hulten's theorem fails when factors supplies are elastic". While this is true when Hulten's theorem is formulated in terms of final output, Corollary 6 highlights that Hulten's theorem does apply to economies with elastic factors when formulated in terms of aggregate efficiency. Bigio and La'O (2020) already show that Hulten's theorem is valid for aggregate efficiency in an environment with a single individual and elastic labor supply; see also Basu and Fernald (2002).

[39] Away from frictionless competition, Hulten's Theorem applies to production efficiency (i.e. sales shares capture the production efficiency impact of a proportional Hicks-neutral technology change) if i) all production wedges and aggregate consumption wedges are zero and ii) aggregate goods wedges are zero at an allocation that satisfies production efficiency. Away from frictionless competition, Hulten's theorem applies to production efficiency (i.e. sales shares capture the production efficiency impact of a proportional Hicks-neutral technology change) if i) all production wedges and aggregate consumption wedges are zero and ii) aggregate goods wedges are zero at an allocation that satisfies production efficiency.

[40] This logic applies regardless of whether Hulten's theorem is expressed in terms of aggregate efficiency or final output. The fact that frictionless competition is a more stringent condition than efficiency is well understood (Edgeworth, 1881; Debreu and Scarf, 1963). One reason that explains why the existing literature has been imprecise about the scope of Hulten's theorem is that prior to the results in Section 4 there had been no characterization of efficiency conditions for general disaggregated production economies with heterogeneous individuals.

Corollary 6 shows this occurs when $\bar{\boldsymbol{\tau}}_y = 0$, a condition that holds in frictionless competitive economies. That is, there may exist efficient *non-interior* allocations in which $\bar{\boldsymbol{\tau}}_y \neq 0$ and Hulten's theorem does not hold. This occurs because in efficient *non-interior* allocations input prices need not reflect marginal welfare products. Therefore, while Hulten's theorem applies to i) frictionless competitive economies and ii) efficient interior allocations, it can fail in efficient non-interior allocations.[41] This result further underscores the importance of carefully dealing with non-interior allocations when studying disaggregated economies.

Example 3. *(Failure of Hulten's Theorem in an Efficient Equilibrium).* We consider the same environment as in Example 1, and focus on a technology change for good 2, so $\frac{\partial G^2}{\partial \theta} \neq 0$. For simplicity, we set all wedges to zero, with the exception of $\tau_x^{12} \neq 0$. The competitive equilibrium of this economy is efficient, with the relevant efficiency condition here being $MRS_c^{11} \frac{\partial G^1}{\partial x^{12}} > 0$. In this case, competition ensures that $p^1 \frac{\partial G^1}{\partial x^{12}} = p^2 \left(1 + \tau_x^{12}\right)$. But note that

$$MSV_y^2 = MRS_c^{11} \frac{\partial G^1}{\partial x^{12}} = p^1 \frac{\partial G^1}{\partial x^{12}} = p^2 \left(1 + \tau_x^{12}\right) \neq p^2,$$

so prices do not capture the marginal social value of goods and Hulten's theorem fails in this efficient economy. This example illustrates that $\bar{\tau}_y^2 = \bar{\tau}_x^2 = \tau_x^{12} = 0$ is the condition that ensures $MSV_y^2 = p^2$, not efficiency.

Remark 16. *(Normalizations behind Domar Weights).* Comparing Theorem 6 and Corollary 6 highlights why Hulten's theorem is typically stated in terms of Domar weights. First, considering proportional Hicks-neutral technology shocks implies that $\frac{\partial G^j}{\partial \theta} = y^j$, which ensures that the numerator of the Domar weight in (43) is $p^j y^j$. Second, Hulten's theorem is typically stated using nominal GDP as numeraire, which ensures that the denominator of the Domar weight in (43) is $\sum_j p^j c^j$. These are arbitrary normalization; in fact, normalizing by the aggregate value of aggregate supply $\sum_j p^j y^j$ would define alternative weights that add up to one.

5.3 Allocative Efficiency in Competitive Economies

In this subsection, we specialize the allocative efficiency components of the welfare accounting decomposition to competitive economies with wedges.

Theorem 7. *(Production Efficiency in Competitive Economies). In competitive economies with*

[41] Baqaee and Farhi (2020) provide an example — analogous to our Example 3 — of an efficient (non-interior) allocation in which Hulten's theorem fails, because "revenue-based and cost-based Domar weights are not equal". Our results show that the failure of Hulten's theorem at an efficient allocation is possible only because the economy studied is not competitive and non-interior.

wedges, in the absence of technology and endowment changes, production efficiency is given by

$$
\begin{aligned}
\Xi^{AE,P} = & \underbrace{\sum_{\ell} \mathbb{C}ov_j^{\Sigma}\left[\tau_x^{j\ell}, \frac{d\chi_x^{j\ell}}{d\theta}\right] p^{\ell} x^{\ell} + \sum_{\ell} \mathbb{C}ov_j^{\Sigma}\left[\left(MSV_y^j - p^j\right)\frac{\partial G^j}{\partial x^{j\ell}}, \frac{d\chi_x^{j\ell}}{d\theta}\right] x^{\ell}}_{\text{Cross-Sectional Intermediate Input Efficiency}} \\
& + \underbrace{\sum_{\ell}\left(p^{\ell}\left(\bar{\tau}_x^{\ell} - \bar{\tau}_c^{\ell}\right) + \sum_j \left(MSV_y^j - p^j\right)\frac{\partial G^j}{\partial x^{j\ell}}\chi_x^{j\ell}\right)\frac{d\phi_x^{\ell}}{d\theta} y^{\ell}}_{\text{Aggregate Intermediate Input Efficiency}} \\
& + \underbrace{\sum_{f} \mathbb{C}ov_j^{\Sigma}\left[\tau_{n^d}^{jf}, \frac{d\chi_n^{jf,d}}{d\theta}\right] w^f n^{f,d} + \sum_{f} \mathbb{C}ov_j^{\Sigma}\left[\left(MSV_y^j - p^j\right)\frac{\partial G^j}{\partial n^{jf,d}}, \frac{d\chi_n^{jf,d}}{d\theta}\right] n^{f,d}}_{\text{Cross-Sectional Factor Efficiency}} \\
& + \underbrace{\sum_{f}\left(w^f\left(\bar{\tau}_{n^s}^{f} - \bar{\tau}_{n^d}^{f}\right) + \sum_j \left(MSV_y^j - p^j\right)\frac{\partial G^j}{\partial n^{jf,d}}\chi_n^{jf,d}\right)\frac{dn^{f,s}}{d\theta}}_{\text{Aggregate Factor Efficiency}}.
\end{aligned}
$$

Theorem 7 follows from imposing the equilibrium conditions in (37) into the production efficiency decomposition in Theorem 2. In line with Remark 15, Theorem 7 further underscores the asymmetry between aggregate goods wedges, which directly impact all production efficiency components (via the terms that contain $MSV_y^j - p^j$, since $\boldsymbol{MSV}_y - \boldsymbol{p} = \boldsymbol{p}\bar{\boldsymbol{\tau}}_y\boldsymbol{\Psi}_y$) and other wedges. Hence, any changes in inputs or factors that increase the supply of goods with high aggregate goods wedges have a separate impact on the aggregate efficiency components. Since these effects are identical across all components, we focus on describing the remaining terms.

First, cross-sectional intermediate input efficiency directly depends on the dispersion in intermediate input use wedges. Intuitively, reallocating intermediate inputs towards uses with higher wedges is valuable since the competitive equilibrium features too little of those input uses. Second, aggregate intermediate input efficiency directly depends on the difference between aggregate intermediate input and consumption wedges. Intuitively, if $\bar{\tau}_x^{\ell} > (<)\ \bar{\tau}_c^{\ell}$, the aggregate intermediate use of good ℓ is inefficiently high relative to its consumption use. Third, cross-sectional factor efficiency directly depends on the dispersion in factor use wedges. Intuitively, reallocating factors towards uses with higher wedges is valuable since the competitive equilibrium features too little of those factor uses. Finally, aggregate factor efficiency directly depends on the difference between aggregate factor supply and factor use wedges. Intuitively, if $\bar{\tau}_{n^s}^{f} > (<)\ \bar{\tau}_{n^d}^{f}$, the aggregate supply of factor f is inefficiently low (high) relative to its use. In the Appendix, we characterize the factor endowment change component.

Remark 17. (Equalization of Marginal Revenue Products Does Not Ensure Cross-Sectional Factor Efficiency). In frictionless competitive economies, marginal revenue products are equalized across all uses and the cross-sectional factor efficiency component is zero. However, equalization of marginal revenue products is not sufficient to ensure that the cross-sectional factor efficiency component is zero in competitive economies with wedges, even when factor use wedges are zero. A similar logic applies to cross-sectional input efficiency. Why is this the case? As explained in Section 4, efficiency requires the equalization of marginal welfare products across uses of a factor, while competition when factor use wedges are zero ensures the equalization of marginal revenue products across uses. If $MSV_y^j \neq p^j$ for some goods that use a particular factor, the marginal welfare products of that factor won't be equalized across uses, allowing for cross-sectional factor efficiency to be non-zero. We illustrate this possibility in Example 4.

Example 4. *(Marginal Welfare Product vs. Marginal Revenue Product).* We consider the same environment as in Example 2. All wedges are zero except $\tau_x^{23} \neq 0$. In this case, competition ensures that $MRS_c^{11} = p^1$ and $MRS_c^{12} = p^2$, as well as $p^1 \frac{\partial G^1}{\partial n^{11,d}} = w^1$ and $p^3 \frac{\partial G^3}{\partial n^{31,d}} = w^1$. The only equilibrium condition with a wedge is $p^2 \frac{\partial G^2}{\partial x^{23}} = \left(1 + \tau_x^{23}\right) p^3$. Consequently, competition implies that marginal revenue products are equalized across uses, so $MRP_n^{11} = MRP_n^{31}$. Therefore,

$$p^1 \frac{\partial G^1}{\partial n^{11,d}} = p^3 \frac{\partial G^3}{\partial n^{31,d}} \implies p^1 \frac{\partial G^1}{\partial n^{11,d}} = \frac{1}{1 + \tau_x^{23}} p^2 \frac{\partial G^2}{\partial x^{23}} \frac{\partial G^3}{\partial n^{31,d}}.$$

However, this condition is inconsistent with cross-sectional factor efficiency,

$$p^1 \frac{\partial G^1}{\partial n^{11,d}} = p^2 \frac{\partial G^2}{\partial x^{23}} \frac{\partial G^3}{\partial n^{31,d}},$$

which requires the equalization of marginal welfare products. This discrepancy is due to the fact that marginal social value of good 3 does not equal its price, since $\bar{\tau}_y^3 = \tau_x^{23} > 0$.

Theorem 8. *(Exchange Efficiency in Competitive Economies). In competitive economies with wedges, exchange efficiency is given by*

$$\Xi^{AE,X} = \underbrace{\sum_j \mathbb{C}ov_i^{\Sigma} \left[\tau_c^{ij}, \frac{d\chi_c^{ij}}{d\theta} \right] p^j c^j}_{\substack{\text{Cross-Sectional} \\ \text{Consumption Efficiency}}} - \underbrace{\sum_f \mathbb{C}ov_i^{\Sigma} \left[\tau_n^{if,s}, \frac{d\chi_n^{if,s}}{d\theta} \right] w^f n^{f,s}}_{\substack{\text{Cross-Sectional} \\ \text{Factor Supply Efficiency}}}. \tag{44}$$

Equation (44) highlights that cross-sectional dispersion in consumption and factor supply wedges is necessary for exchange efficiency to be non-zero. Intuitively, reallocating consumption towards individuals with higher consumption wedges is valuable since these individuals consume too little in equilibrium. Similarly, reallocating factor supply towards individuals with lower factor supply wedges is valuable since these individuals' factor supply is too high in equilibrium. Finally, note that intermediate input wedges, factor use wedges, or the aggregate levels of consumption and factor supply wedges do not determine exchange efficiency directly.

6 Welfare Accounting: Redistribution

Our analysis has so far focused on aggregate efficiency, which is invariant to the choice of a social welfare function, as explained in Section 2.3. However, two different perturbations with identical efficiency implications may have completely different distributional implications, as we explain next. Theorem 9 presents a decomposition of the redistribution component of the welfare accounting decomposition using the definitions of allocation shares for consumption and factor supply. Figure OA-2 in the Online Appendix, illustrates this decomposition, which complements Figure 1.

Theorem 9. *(Redistribution Decomposition). The redistribution component of the welfare accounting decomposition, Ξ^{RD}, can be decomposed into*

$$\Xi^{RD} = \overbrace{\sum_j \mathbb{C}ov_i^{\Sigma}\left[\omega^i, MRS_c^{ij}\frac{d\chi_c^{ij}}{d\theta}\right]c^j}^{\substack{\text{Cross-Sectional}\\ \text{Consumption Redistribution}}} + \overbrace{\sum_j \mathbb{C}ov_i^{\Sigma}\left[\omega^i, MRS_c^{ij}\chi_c^{ij}\right]\frac{dc^j}{d\theta}}^{\substack{\text{Aggregate}\\ \text{Consumption Redistribution}}}$$
$$\underbrace{-\sum_f \mathbb{C}ov_i^{\Sigma}\left[\omega^i, MRS_n^{if}\frac{d\chi_n^{if,s}}{d\theta}\right]n^{f,s}}_{\substack{\text{Cross-Sectional}\\ \text{Factor Supply Redistribution}}} \underbrace{-\sum_f \mathbb{C}ov_i^{\Sigma}\left[\omega^i, MRS_n^{if}\chi_n^{if,s}\right]\frac{dn^{f,s}}{d\theta}}_{\substack{\text{Aggregate}\\ \text{Factor Supply Redistribution}}}.$$

The cross-sectional terms capture redistribution gains or losses due to the reallocation of consumption and factor supply, for given c^j and $n^{f,s}$. In particular, cross-sectional consumption redistribution is positive for good j when individuals with high normalized individual weight ω^i — those relatively favored by the planner — see their consumption shares increase; MRS_c^{ij} captures potentially different marginal consumption values. The aggregate terms capture redistribution gains due to changes in aggregates, for given allocation shares. In particular, aggregate consumption redistribution is positive for good j when aggregate consumption increases and individuals with high ω^i consume a relatively larger share of the good. The logic is parallel for factor supply redistribution.

The cross-sectional terms parallel exchange efficiency since they are driven by changes in consumption or factor supply shares given aggregates, while the aggregate terms parallel production efficiency since they are driven by changes in aggregates consumption and factor supply. While it is possible to further decompose the aggregate terms, this is not particularly useful. Instead, in Theorem 10 in Appendix C.4 we provide an alternative decomposition in competitive economies with wedges based on distributive pecuniary effects and individual distortions.

7 Applications

In this section, we illustrate how the welfare accounting decomposition can be used to identify the origins of welfare gains and losses in four workhorse models in macroeconomics and trade. Our first application shows how an increase in tariffs contributes negatively to exchange efficiency via cross-sectional consumption efficiency in the simplest endowment economy (Armington, 1969). This application also illustrates subtle patterns in cross-sectional consumption redistribution. Our second application shows how the aggregate efficiency gain induced by an improvement in a matching technology in a Diamond-Mortensen-Pissarides (DMP) model is due to cross-sectional factor efficiency gains that are large enough to compensate for aggregate intermediate input efficiency losses due to an increase in vacancy postings. This application illustrates how to use the welfare accounting decomposition in economies that are not competitive. Our third application illustrates how an increase in markup dispersion generates cross-sectional factor efficiency losses in a Hsieh and Klenow (2009) economy. Our final application shows how to use the welfare accounting decomposition to identify the welfare gains from optimal monetary stabilization policy in a macroeconomic model with household and sectoral heterogeneity.

7.1 Armington (1969) Model

Environment. We consider the simplest Armington (1969) economy, which has $I = 2$ individuals (here representing countries), $J = 2$ goods, and $F = 2$ inelastically supplied factors.[42] Each country produces a single good with their domestic factor — normalized so that $\bar{n}^{if,s} = 1$ — but consumes both goods. Country i has preferences given by

$$V^i = \left(\sum_j \left(c^{ij} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

and faces the budget constraint

$$\sum_j p^j \left(1 + \tau^{ij} \right) c^{ij} = w^{if} + \sum_j T^{ij}, \quad \text{where} \quad T^{ij} = p^j \tau^{ij} c^{ij}. \tag{45}$$

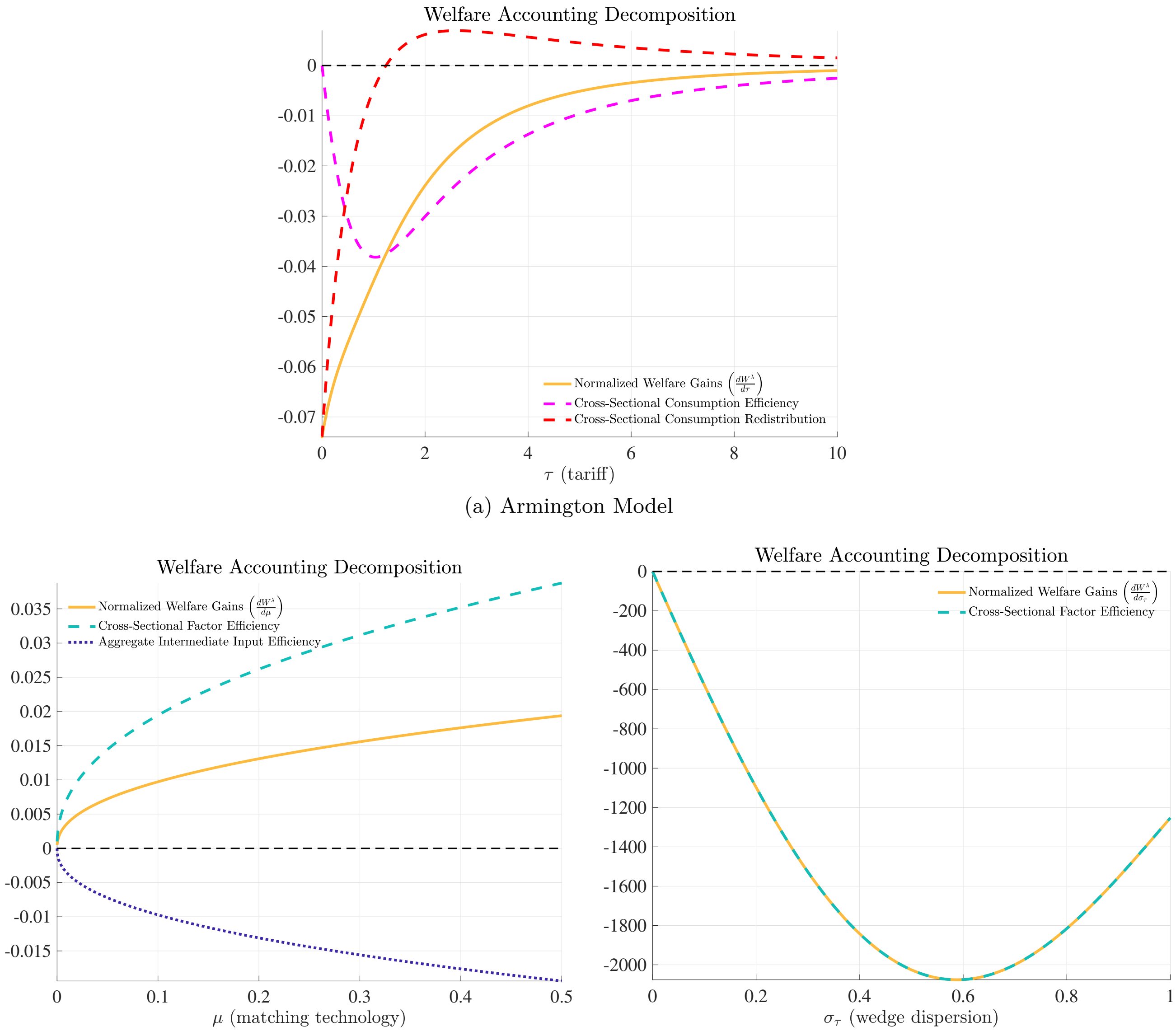
Since the iceberg costs τ^{ij} are rebated they should be interpreted as tariffs rather than physical costs, although it is straightforward to consider the alternative case. Goods are competitively produced according to constant returns to scale technologies — which justifies the absence of profits in (45) — given by

$$y^1 = A^1 n^{11,d} \quad \text{and} \quad y^2 = A^2 n^{22,d},$$

so each country uses the domestic factor to exclusively produce the domestic good. An equilibrium is characterized by allocations c^{ij}, prices p^j, and wages w^{if} such that both countries choose consumption optimally, countries produce competitively, and all markets clear. Resource constraints in this economy are given by $\sum_i c^{ij} = y^j$, $\forall j$, and $n^{f,d} = \bar{n}^{f,s} = 1$, $\forall f$.

Our parameterization assumes that $\sigma = 2$, $A^1 = 1$, $A^2 = 50$, $\tau^{ii} = 0$, and $\tau^{ij} = \tau^{ji} = \tau$. We use aggregate world consumption as welfare numeraire, and assume that the planner has a social welfare

[42] As we show in the Online Appendix, this economy is isomorphic to an economy without factors in which each country has a predetermined endowment of their home good.



(a) Armington Model

(b) Diamond-Mortensen-Pissarides Model

(c) Hsieh-Klenow Model

Figure 4: Welfare Accounting Decomposition: Applications

Note: This figure illustrates the welfare accounting decomposition for the first three applications. The top panel shows that an increase in tariffs decreases exchange efficiency through cross-sectional consumption efficiency in an Armington model. It also show that cross-sectional consumption redistribution is positive, since the tariff increase hurts more the country with lower consumption. The bottom left panel shows that the aggregate efficiency gain induced by an improvement in a matching technology in a DMP model is due to cross-sectional factor efficiency gains that are large enough to compensate for aggregate intermediate input efficiency losses due to increase vacancy postings. The bottom right panel shows that all welfare losses due to the increase in the dispersion of wedges/markups — typically referred to as misallocation — are attributed to production efficiency via cross-sectional factor efficiency in Hsieh and Klenow (2009) economy.

function given by $\sum_i \left(V^i\right)^{\frac{\sigma-1}{\sigma}}$.

Results. The top panel in Figure 4 illustrates the welfare impact of a multilateral increase in tariffs τ — see also Figure OA-4. The welfare accounting decomposition yields insights for both efficiency and redistribution.

First, a multilateral increase in tariffs always features a negative exchange efficiency component, due to cross-sectional consumption efficiency. This occurs because the increase in tariffs reallocates consumption toward each country's domestic good, which is the one with a relatively lower MRS_c^{ij} as long as $\tau > 0$. Note that $\Xi^{AE,X} = 0$ at $\tau = 0$, since this economy is efficient in the absence of tariffs.

Second, an increase in tariffs eventually makes both countries worse off, but initially benefits country 2, because p^2/p^1 increases in equilibrium. Since country 2 is more productive and consumes more of both goods than country 1 in equilibrium, the planner attaches a lower individual weight to country 2, so $\omega^1 > \omega^2$. Hence, initially, the increase in tariffs benefits the country relatively less preferred by the planner and harms redistribution, with $\Xi^{RD} < 0$. However, once tariffs are large enough, further increases in tariffs make both countries worse off. Around $\tau \approx 1.2$, the marginal increase in τ hurts country 2 disproportionately more. From this level of tariffs onwards, $\Xi^{RD} > 0$, since country 1 — the relatively preferred by the planner — is hurt by less.

7.2 DMP Model

Environment. We consider a stylized version of the textbook labor search model, as in e.g. Pissarides (2000). We consider a two date economy, $t \in \{0, 1\}$, populated by a single/representative individual ($I = 1$) endowed with a unit supply of labor ($F = 1$), which can be used in technology $j = 1$ (unemployment) or $j = 2$ (employment). Each of these technologies produces perfectly substitutable goods (or equivalently, a single final good), so the preferences of the representative individual can be written as

$$V = c_0 + \beta c_1, \quad \text{where} \quad c_t = c_t^1 + c_t^2, \tag{46}$$

where c_t^j denotes consumption of the good produced by technology j at date t. Both technologies have constant returns to scale and are given by

$$y_t^1 = G_t^1\left(n_t^1\right) = z^1 n_t^1 = z^1 \chi_{t,n}^1 \qquad \text{and} \qquad y_t^2 = G_t^2\left(n_t^2\right) = z^2 n_t^2 = z^2 \chi_{t,n}^2, \tag{47}$$

where $\chi_{t,n}^1$ and $\chi_{t,n}^2$ respectively denote the employment and unemployment rates, and where $z^2 > z^1$.

Moreover, there exists a third "vacancy-generating" technology ($J = 3$) at date 0 that takes the final good and generates vacancies, as follows

$$y_t^3 = v_t = G_t^3\left(x_t\right) = \frac{1}{\kappa_t} x_t, \tag{48}$$

where κ_t captures the marginal cost of vacancy posting. Vacancies can be interpreted as a good that no individual desires to consume, which means that in a first-best environment vacancies should be zero. Hence, the resource constraints in this model can be expressed as

$$y_t^1 + y_t^2 = c_t + x_t \qquad \text{and} \qquad \chi_{t,n}^1 + \chi_{t,n}^2 = 1. \tag{49}$$

Equation (46) through (49) are sufficient to characterize the efficiency conditions for this economy. Since $z^2 > z^1$, efficiency requires full employment, with $\chi_{t,n}^1 = 0$ and $\chi_{t,n}^2 = 1$, as well as, $v_t = 0$.

However, we consider a standard random search equilibrium in which employment only adjusts according to

$$\chi_{t+1,n}^1 - \chi_{t,n}^1 = \varphi\left(1 - \chi_{t,n}^1\right) - m\left(\chi_{t,n}^1, v_t\right),$$

where φ denotes the job destruction rate and the matching function $m(\cdot)$ is given by

$$m\left(\chi_{t,n}^1, v_t\right) = \mu\left(\chi_{t,n}^1\right)^{\alpha}(v_t)^{1-\alpha}.$$

As usual in this class of models, labor market tightness is defined as $\theta_t = \frac{v_t}{\chi_{t,n}^1}$. We formally describe the (standard) characterization of the equilibrium in the Online Appendix and describe the welfare impact of a change in the matching technology μ. Our parameterization assumes that $\beta = 0.99$, $z^1 = 0$, $z^2 = 1$, $\eta = 0.5$, $\alpha = 0.7$, $\varphi = 0.036$, $b_0 = b_1 = 0$, $\kappa_0 = 0.1$, with $\chi_{0,n}^1 = 0.037$.

Results We consider a standard search equilibrium in this economy — see Online Appendix — and explore the welfare implications of improvements in the matching technology μ. The effects are illustrated in the bottom left panel in Figure 4 — see also Figure OA-5. Several insights emerge.

First, the technology change component of the welfare accounting decomposition is zero even though the matching technology improves. This occurs because the matching technology does not change the production frontier of the economy, and it is simply a mechanism to determine how factors of production are allocated. Second, as the matching technology improves, firms post more vacancies at date 0, which translates into higher employment at date 1. The increase in employment drives the positive cross-sectional factor efficiency component — as discussed above, at the first-best, unemployment should be zero. However, the additional vacancies posted make the aggregate intermediate input efficiency component negative. This occurs because posting vacancies entails using a technology that produces no final output, and it only contributes to reallocating factors, something that could be done freely in the absence of search frictions. Hence, even though the improvement in the matching technology generates welfare efficiency gains, the welfare accounting decomposition shows that these gains combine positive and negative effects. More generally, this application illustrates how adjustment cost functions will typically generate a negative aggregate intermediate input efficiency component.

7.3 Hsieh and Klenow (2009) Model

Environment. We consider a simplified version of the Hsieh and Klenow (2009) economy, with a representative individual ($I = 1$) — whose index we drop — and a single final good, which we index by $j = 1$. Individual preferences are given by

$$V = u\left(c^1\right),$$

where the final good is produced according to the technology

$$y^1 = \left(\sum_{j=2}^{J}\left(x^{1j}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$

where ϵ denotes the elasticity of substitution between the $J-1$ intermediate inputs. Each intermediate input $j \geq 2$ is produced according to the technology

$$y^j = A^j n^{j1,d},$$

where a single factor not elastically supplied ($F = 1$) — whose index we also drop — can be used to produce the different intermediates. Formally, resource constraints in this economy can be written as

$$c^1 = y^1, \quad y^j = x^{1j},\ \forall j \geq 2, \quad \text{and} \quad \sum_{j=2}^{J}\chi_n^{j,d} = 1.$$

If the final good is produced competitively, and the intermediate inputs are chosen under monopolistic competition subject to wedges τ^j (which can be interpreted as markups) the equilibrium factor use shares $\chi_n^{j,d}$, can be expressed as

$$\chi_n^{j,d} = \frac{\left(A^j\right)^{\epsilon-1}\left(\tau^j\right)^{-\epsilon}}{\sum_{j=2}^{J}\left(A^j\right)^{\epsilon-1}\left(\tau^j\right)^{-\epsilon}},$$

Our parameterization — designed to mimic Hsieh and Klenow (2009) — assumes that $\left(\log A^j, \log \tau^j\right) \sim \mathcal{N}\left(\mu_A, \mu_\tau, \sigma_A^2, \sigma_\tau^2, \sigma_{\tau A}\right)$, where $\mu_A = 0.5$, $\mu_\tau = 1.1$, $\sigma_A = 0.95$, $\sigma_\tau = 0.63$, $\sigma_{\tau A} = 0.36$, $J = 211,304$, and $\epsilon = 3$. We explore the welfare implications of an increase in markup dispersion through σ_τ.

Results. The bottom left panel in Figure 4 illustrates the welfare impact of a change in markup dispersion — typically referred to as misallocation. Since all intermediate inputs in this economy are fully specialized and there is a single final good, no welfare changes are attributed to intermediate input efficiency. And since the single factor is fixed, aggregate factor efficiency is also zero. Hence, all welfare losses due to the increase in the dispersion of markups are attributed to production efficiency via cross-sectional factor efficiency. Given our calibration of the model, chosen to mimic Hsieh and Klenow (2009), these effects are quantitatively large. Since there is a single representative individual, both exchange efficiency and redistribution are zero.

7.4 New Keynesian Model

This application shows how the welfare accounting decomposition can be used to identify the welfare gains from optimal monetary stabilization policy. To that end, we develop a static, multi-sector heterogeneous agent New Keynesian model with an input-output production network — a static "HANK-IO" model (Schaab and Tan, 2023). This model builds on La'O and Tahbaz-Salehi (2022) and Rubbo (2023) but allows for household heterogeneity in addition to sectoral heterogeneity.

Environment. There are I (types of) households indexed by i. Each has mass μ^i, with $\sum_i \mu^i = 1$. There are N production sectors indexed by j. Each comprises a continuum of firms indexed by $\ell \in [0,1]$. Each firm produces a distinct good, indexed by $j\ell$.

The preferences of household i are given by

$$V^i = \frac{1}{1-\gamma}(c^i)^{1-\gamma} - \frac{1}{1+\varphi}(n^i)^{1+\varphi}, \quad \text{where} \tag{50}$$

$$c^i = \left(\sum_j (\Gamma_c^{ij})^{\frac{1}{\eta_c}} (c^{ij})^{\frac{\eta_c - 1}{\eta_c}}\right)^{\frac{\eta_c}{\eta_c - 1}} \quad \text{and} \quad c^{ij} = \left(\int_0^1 (c^{ij\ell})^{\frac{\epsilon^j - 1}{\epsilon^j}} d\ell\right)^{\frac{\epsilon^j}{\epsilon^j - 1}},$$

where c^i denotes a final consumption aggregator, c^{ij} denotes a sectoral consumption aggregator, and $c^{ij\ell}$ is household i 's consumption of good $j\ell$. Each household is endowed with a unique labor factor and n^i denotes hours of work. The household budget constraint is given by $\sum_j \int_0^1 p^{j\ell} c^{ij\ell} d\ell = W^i n^i + T^i$, where $p^{j\ell}$ is the price of good $j\ell$, W^i is the wage paid to factor i, and T^i is a lump-sum transfer that accounts for profits. Household optimization implies $(n^i)^\varphi (c^i)^\gamma = W^i / P^i$.

Firm ℓ in sector j produces according to the nested CES production technology

$$y^{j\ell} = A^j \left((1-\vartheta^j)^{\frac{1}{\eta}} (n^{j\ell})^{\frac{\eta-1}{\eta}} + (\vartheta^j)^{\frac{1}{\eta}} (x^{j\ell})^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \quad \text{where} \quad n^{j\ell} = \left(\sum_i (\Gamma_w^{ji})^{\frac{1}{\eta_w}} (n^{j\ell i})^{\frac{\eta_w-1}{\eta_w}}\right)^{\frac{\eta_w}{\eta_w-1}}, \tag{51}$$

$$x^{j\ell} = \left(\sum_\ell (\Gamma_x^{j\ell})^{\frac{1}{\eta_x}} (x^{j\ell\ell})^{\frac{\eta_x-1}{\eta_x}}\right)^{\frac{\eta_x}{\eta_x-1}} \quad \text{and} \quad x^{j\ell\ell} = \left(\int_0^1 (x^{j\ell\ell\ell'})^{\frac{\epsilon^\ell-1}{\epsilon^\ell}} d\ell'\right)^{\frac{\epsilon^\ell}{\epsilon^\ell-1}}.$$

We denote by A^j a sector-specific, Hicks-neutral technology shifter, ϑ^j governs sector j''s intermediate input share, and η is the elasticity of substitution between labor and inputs. Firm ℓ in sector j uses a bundle of labor $\eta^{j\ell}$ that is itself a CES aggregate of its use of labor factors $i, \eta^{j\ell i}$. It also uses a bundle of intermediate inputs $x^{j\ell}$, which is a CES aggregate of sectoral bundles $x^{j\ell\ell}$, where $x^{j\ell\ell\ell'}$ denotes firm $j\ell$'s use of good $\ell\ell'$ in production.

Firms are monopolistically competitive. They choose labor and inputs to minimize costs, and prices to maximize profits. Each firm ℓ is small and takes as given aggregate and sectoral variables. Profits are $\Pi^{j\ell} = (1-\tau^j)\, p^{j\ell} y^{j\ell} - \sum_\ell \int_0^1 p^{\ell\ell'} x^{j\ell\ell\ell'} d\ell' - \sum_i W^i n^{j\ell i} = (1-\tau^j)\, p^{j\ell} y^{j\ell} - mc^j y^{j\ell}$, where τ^j is a revenue tax. Marginal cost mc^j is uniform across firms in each sector as we show in Appendix E.4.1. If prices are flexible, firms set prices as a markup over marginal cost, $p^{j\ell} = p^j = \frac{\epsilon^j}{\epsilon^j - 1} \frac{1}{1-\tau^j} mc^j$. To introduce nominal rigidities, we assume that only a fraction $\delta^j \in [0,1]$ of firms in sector j can reset their prices in response to a shock. Otherwise, prices remain fixed at some initial level $\bar{p}^j$, which we specify in the Appendix. The sectoral price distribution is thus given by

$$p^{j\ell} = \begin{cases} \frac{\epsilon^j}{\epsilon^j - 1} \frac{1}{1-\tau^j} mc^j & \text{for } \ell \in [0, \delta^j] \\ \bar{p}^j & \text{for } \ell \in (\delta^j, 1]\,. \end{cases} \tag{52}$$

We model monetary policy by assuming that aggregate nominal expenditures are constrained by a cash-in-advance constraint of the form $\sum_j \int_0^1 p^{j\ell} y^{j\ell} d\ell \leq M$, where M is the monetary policy instrument. Finally, the markets for goods and labor factors have to clear, requiring

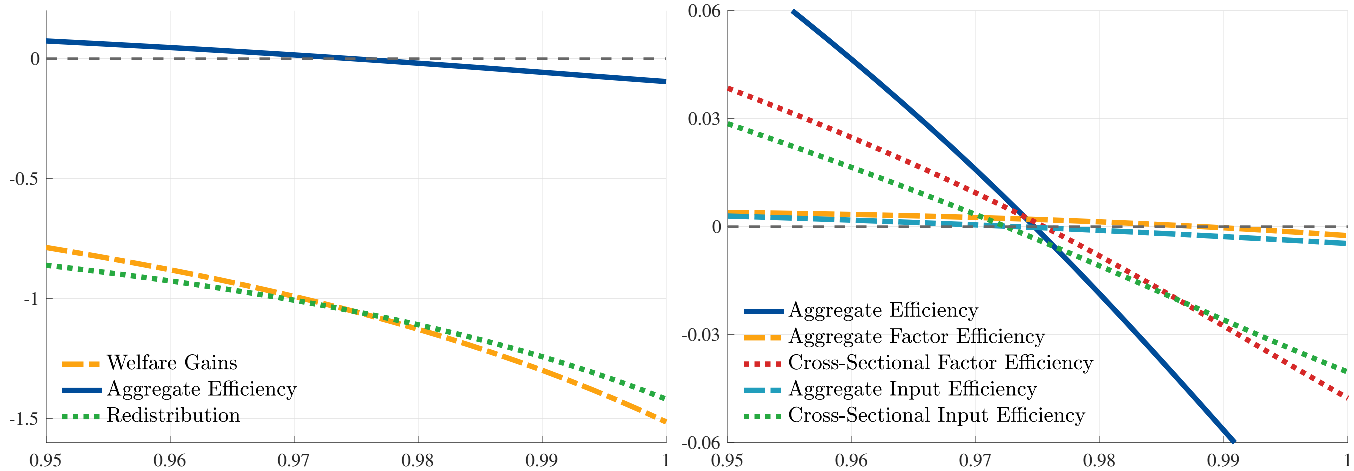
$$y^{j\ell} = \sum_i \mu_i c^{ij\ell} + \sum_\ell \int_0^1 x^{\ell\ell' j\ell} d\ell' \quad \text{and} \quad \mu^i n^i = \sum_j \int_0^1 n^{j\ell i} d\ell. \tag{53}$$

We formally define competitive equilibrium in Appendix E.4.2.

Calibration. We calibrate a model with $N = 66$ sectors and $I = 10$ household types, corresponding to deciles of the income distribution, as in Schaab and Tan (2023). We use data from the Consumer Expenditure Survey to calibrate Γ_c^{ij} so the model matches consumption expenditure shares. Similarly, we use data from the American Community Survey and the BEA's I-O and GDP tables to calibrate ϑ^j, $\Gamma_x^{j\ell}$, and Γ_w^{ji} so the model matches sectoral input-output data and payroll shares. We calibrate ϵ^j to match sectoral markup data from Baqaee and Farhi (2020) and δ^j to match Pasten et al. (2017)'s data on sectoral price rigidities. We allow revenue taxes τ^j to offset initial markups and study the case with $\tau^j = 0$ in Appendix E.4.4. Finally, we assume an equal-weighted utilitarian social welfare function. Appendix E.4.3 presents a detailed discussion of our calibration.

Results. We study monetary policy in response to a 2% technology shock that is uniform across sectors. When households and sectors are symmetric, Divine Coincidence holds and there exists an optimal monetary policy M^* that closes output and inflation gaps. Through the lens of the welfare accounting decomposition, Divine Coincidence implies that each allocative efficiency term of Theorems 1 and 2 is zero. We discuss this case in Appendix E.4.4.

When households and sectors are heterogeneous, Divine Coincidence fails. Figure 5 plots the welfare accounting decomposition, treating M as the perturbation parameter (θ). The left panel decomposes welfare gains (yellow) into gains from aggregate efficiency (blue) and redistribution



(a) Welfare Gains

(b) Efficiency Gains

Figure 5: Welfare Accounting Decomposition: New Keynesian Model

Note: This figure illustrates the welfare accounting decomposition for the New Keynesian application when varying monetary policy in response to an unanticipated positive technology shock.

(green). The blue line intersects 0 at around $M^{AE} = 0.974$, which is the policy that maximizes aggregate efficiency. Redistribution is negative at this point, indicating that the redistribution motive of the utilitarian social welfare function calls for a more contractionary policy (lower M).

The right panel decomposes aggregate efficiency into its four allocative efficiency components: cross-sectional and aggregate factor and intermediate input efficiency. Several additional insights emerge. First, factor and input efficiency are both quantitatively important determinants of the production efficiency gains from monetary policy. Second, at $M^{AE} = 0.974$, aggregate (light blue) and cross-sectional (green) input efficiency are negative. These two motives call for more contractionary policy. Third, aggregate (yellow) and cross-sectional (red) factor efficiency are positive at $M^{AE} = 0.974$, calling for more expansionary policy. The policy that maximizes efficiency trades off and balances these considerations.[43] Lastly, Appendix E.4.4 illustrates the role of revenue taxes. When they are not available to offset initial markup distortions, aggregate input and factor efficiency become quantitatively more important and call for expansionary policy.

8 Conclusion

This paper introduces a welfare accounting decomposition that is useful to identify and quantify the ultimate origins of welfare gains and losses induced by changes in allocations or primitive changes in

[43]It is well understood that stabilizing inflation (which maps to cross sectional factor efficiency) is more important than stabilizing the output gap (which maps to aggregate factor efficiency) for welfare in standard calibrations of the New Keynesian model (Rotemberg and Woodford, 1997; Woodford, 2003). Our results preserve this conclusion and also show that the cross-sectional component also dominates the aggregate component for intermediate input efficiency.

technologies or endowments. Importantly, this decomposition is written purely in terms of preferences and technologies, and makes no reference to prices, individual budget constraints, or notions of equilibrium. For that reason, it is also useful to characterize efficiency conditions, which allows us to provide a new characterization of efficiency conditions in disaggregated production economies with heterogeneous individuals that carefully accounts for non-interior solutions, extending classic efficiency results. In competitive economies, prices and wedges can be used to recover the welfare-relevant statistics required to implement the welfare accounting decomposition. We illustrate the use of the welfare accounting decomposition through several minimal examples and four applications to workhorse models in macroeconomics and trade.

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Online Appendix

A Matrix Definitions

This section defines all matrices used in the body of the paper and in this Appendix. To simplify the exposition, we represent all matrices for the $I = 2$, $J = 3$, $F = 2$ case, although we define matrix dimensions for the general case.

Allocations. We collect consumption allocations, c^{ij}, and individual endowments of goods, $\bar{y}^{ij,s}$, in the $IJ \times 1$ vectors $\mathring{\boldsymbol{c}}$ and $\mathring{\bar{\boldsymbol{y}}}^s$, as well as intermediate uses, $x^{j\ell}$, in the $JL \times 1$ vector $\mathring{\boldsymbol{x}}$, given by

$$\mathring{\boldsymbol{c}} = \begin{pmatrix} c^{11} \\ c^{21} \\ c^{12} \\ c^{22} \\ c^{13} \\ c^{23} \end{pmatrix}_{IJ\times 1}, \quad \mathring{\bar{\boldsymbol{y}}}^s = \begin{pmatrix} \bar{y}^{11,s} \\ \bar{y}^{21,s} \\ \bar{y}^{12,s} \\ \bar{y}^{22,s} \\ \bar{y}^{13,s} \\ \bar{y}^{23,s} \end{pmatrix}_{IJ\times 1}, \quad \mathring{\boldsymbol{x}} = \begin{pmatrix} x^{11} \\ x^{21} \\ x^{31} \\ x^{12} \\ x^{22} \\ x^{32} \\ x^{13} \\ x^{23} \\ x^{33} \end{pmatrix}_{JL\times 1}.$$

Similarly, we collect factor uses, $n^{jf,d}$, in the $JF \times 1$ vector $\mathring{\boldsymbol{n}}^d$, and elastic factor supplies, $n^{if,s}$, and individual endowments of factors, $\bar{n}^{if,s}$, in the $IF \times 1$ vectors $\mathring{\boldsymbol{n}}^s$ and $\mathring{\bar{\boldsymbol{n}}}^s$, given by

$$\mathring{\boldsymbol{n}}^d = \begin{pmatrix} n^{11,d} \\ n^{21,d} \\ n^{31,d} \\ n^{12,d} \\ n^{22,d} \\ n^{32,d} \end{pmatrix}_{JF\times 1}, \quad \mathring{\boldsymbol{n}}^s = \begin{pmatrix} n^{11,s} \\ n^{21,s} \\ n^{12,s} \\ n^{22,s} \end{pmatrix}_{IF\times 1}, \quad \mathring{\bar{\boldsymbol{n}}}^s = \begin{pmatrix} \bar{n}^{11,s} \\ \bar{n}^{21,s} \\ \bar{n}^{12,s} \\ \bar{n}^{22,s} \end{pmatrix}_{IF\times 1}.$$

Aggregate Allocations. We collect aggregate consumption, c^j, aggregate intermediate use, x^j, aggregate produced supply, $y^{j,s}$, aggregate endowment, $\bar{y}^{j,s}$, and aggregate supply, y^j, of goods in $J \times 1$ vectors $\boldsymbol{c}$, $\boldsymbol{x}$, $\boldsymbol{y}^s$, $\bar{\boldsymbol{y}}^s$, and $\boldsymbol{y}$ given by

$$\boldsymbol{c} = \begin{pmatrix} c^1 \\ c^2 \\ c^3 \end{pmatrix}_{J\times 1}, \quad \boldsymbol{x} = \begin{pmatrix} x^1 \\ x^2 \\ x^3 \end{pmatrix}_{J\times 1}, \quad \boldsymbol{y}^s = \begin{pmatrix} y^{1,s} \\ y^{2,s} \\ y^{3,s} \end{pmatrix}_{J\times 1}, \quad \bar{\boldsymbol{y}}^s = \begin{pmatrix} \bar{y}^{1,s} \\ \bar{y}^{2,s} \\ \bar{y}^{3,s} \end{pmatrix}_{J\times 1}, \quad \boldsymbol{y} = \begin{pmatrix} y^1 \\ y^2 \\ y^3 \end{pmatrix}_{J\times 1}.$$

Similarly, we collect aggregate use, $n^{f,d}$, aggregate elastic supply $n^{f,s}$, aggregate endowment, $\bar{n}^{f,s}$, and aggregate supply, n^f, of factors in $F \times 1$ vectors $\boldsymbol{n}^d$, $\boldsymbol{n}^s$, $\bar{\boldsymbol{n}}^s$, and $\boldsymbol{n}$ given by

$$\boldsymbol{n}^d = \begin{pmatrix} n^{1,d} \\ n^{2,d} \end{pmatrix}_{F\times 1}, \quad \boldsymbol{n}^s = \begin{pmatrix} n^{1,s} \\ n^{2,s} \end{pmatrix}_{F\times 1}, \quad \bar{\boldsymbol{n}}^s = \begin{pmatrix} \bar{n}^{1,s} \\ \bar{n}^{2,s} \end{pmatrix}_{F\times 1}, \quad \boldsymbol{n} = \begin{pmatrix} n^1 \\ n^2 \end{pmatrix}_{F\times 1}.$$

Aggregates satisfy

$$\boldsymbol{c} = \boldsymbol{1}_c \mathring{\boldsymbol{c}}, \quad \boldsymbol{x} = \boldsymbol{1}_x \mathring{\boldsymbol{x}}, \quad \boldsymbol{y}^s = \boldsymbol{1}_{y^s} \mathring{\boldsymbol{y}}^s, \quad \bar{\boldsymbol{y}}^s = \boldsymbol{1}_{\bar{y}^s} \mathring{\bar{\boldsymbol{y}}}^s, \quad \boldsymbol{n}^d = \boldsymbol{1}_{n^d} \mathring{\boldsymbol{n}}^d \quad \boldsymbol{n}^s = \boldsymbol{1}_{n^s} \mathring{\boldsymbol{n}}^s, \quad \bar{\boldsymbol{n}}^s = \boldsymbol{1}_{\bar{n}^s} \mathring{\bar{\boldsymbol{n}}}^s,$$

where we define the following matrices of zeros and ones:

$$\mathbf{1}_c = \mathbf{1}_{y^s} = \mathbf{1}_{\bar{y}^s} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}_{J\times IJ}, \qquad \mathbf{1}_x = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}_{J\times JL}$$

$$\mathbf{1}_{n^d} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}_{F\times JF}, \qquad \mathbf{1}_{n^s} = \mathbf{1}_{\bar{n}^s} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}_{F\times IF}.$$

We can thus write resource constraints (3) and (4) as

$$\boldsymbol{y} = \boldsymbol{c} + \boldsymbol{x} \quad \text{and} \quad \boldsymbol{n} = \boldsymbol{n}^d, \qquad \text{where} \qquad \boldsymbol{y} = \boldsymbol{y}^s + \bar{\boldsymbol{y}}^s \quad \text{and} \quad \boldsymbol{n} = \boldsymbol{n}^s + \bar{\boldsymbol{n}}^s.$$

Allocation shares. We collect consumption shares, χ_c^{ij}, in a $IJ \times J$ matrix $\boldsymbol{\chi}_c$, factor use shares, $\chi_n^{jf,d}$, in a $JF \times F$ matrix $\boldsymbol{\chi}_{n^d}$, and factor supply shares, $\chi_n^{if,s}$, in a $IF \times F$ matrix, $\boldsymbol{\chi}_{n^s}$, given by

$$\boldsymbol{\chi}_c = \begin{pmatrix} \chi_c^{11} & 0 & 0 \\ \chi_c^{21} & 0 & 0 \\ 0 & \chi_c^{12} & 0 \\ 0 & \chi_c^{22} & 0 \\ 0 & 0 & \chi_c^{13} \\ 0 & 0 & \chi_c^{23} \end{pmatrix}_{IJ\times J}, \quad \boldsymbol{\chi}_{n^d} = \begin{pmatrix} \chi_n^{11,d} & 0 \\ \chi_n^{21,d} & 0 \\ \chi_n^{31,d} & 0 \\ 0 & \chi_n^{12,d} \\ 0 & \chi_n^{22,d} \\ 0 & \chi_n^{32,d} \end{pmatrix}_{JF\times F}, \quad \boldsymbol{\chi}_{n^s} = \begin{pmatrix} \chi_n^{11,s} & 0 \\ \chi_n^{21,s} & 0 \\ 0 & \chi_n^{12,s} \\ 0 & \chi_n^{22,s} \end{pmatrix}_{IF\times F}.$$

We collect intermediate-use shares, $\chi_x^{j\ell}$, and intermediate-supply shares, $\xi^{j\ell}$, in $JL \times J$ matrices $\boldsymbol{\chi}_x$ and $\boldsymbol{\xi}$, given by

$$\boldsymbol{\chi}_x = \begin{pmatrix} \chi_x^{11} & 0 & 0 \\ \chi_x^{21} & 0 & 0 \\ \chi_x^{31} & 0 & 0 \\ 0 & \chi_x^{12} & 0 \\ 0 & \chi_x^{22} & 0 \\ 0 & \chi_x^{32} & 0 \\ 0 & 0 & \chi_x^{13} \\ 0 & 0 & \chi_x^{23} \\ 0 & 0 & \chi_x^{33} \end{pmatrix}_{JL\times J}, \quad \boldsymbol{\xi} = \begin{pmatrix} \xi^{11} & 0 & 0 \\ \xi^{21} & 0 & 0 \\ \xi^{31} & 0 & 0 \\ 0 & \xi^{12} & 0 \\ 0 & \xi^{22} & 0 \\ 0 & \xi^{32} & 0 \\ 0 & 0 & \xi^{13} \\ 0 & 0 & \xi^{23} \\ 0 & 0 & \xi^{33} \end{pmatrix}_{JL\times J}.$$

We collect aggregate consumption and aggregate intermediate shares, ϕ_c^j and ϕ_x^j, in $J \times J$ diagonal matrices, $\boldsymbol{\phi}_c$ and $\boldsymbol{\phi}_x$, given by

$$\boldsymbol{\phi}_c = \begin{pmatrix} \phi_c^1 & 0 & 0 \\ 0 & \phi_c^2 & 0 \\ 0 & 0 & \phi_c^3 \end{pmatrix}_{J\times J}, \quad \boldsymbol{\phi}_x = \begin{pmatrix} \phi_x^1 & 0 & 0 \\ 0 & \phi_x^2 & 0 \\ 0 & 0 & \phi_x^3 \end{pmatrix}_{J\times J},$$

where $\boldsymbol{\phi}_c + \boldsymbol{\phi}_x = \boldsymbol{I}_J$, and

$$\boldsymbol{c} = \boldsymbol{\phi}_c \boldsymbol{y} \quad \text{and} \quad \boldsymbol{x} = \boldsymbol{\phi}_x \boldsymbol{y}.$$

We can thus write

$$\mathring{\boldsymbol{c}} = \boldsymbol{\chi}_c \boldsymbol{c}, \quad \mathring{\boldsymbol{n}}^d = \boldsymbol{\chi}_{n^d} \boldsymbol{n}^d, \quad \mathring{\boldsymbol{n}}^s = \boldsymbol{\chi}_{n^s} \boldsymbol{n}^s, \quad \mathring{\boldsymbol{x}} = \boldsymbol{\chi}_x \boldsymbol{x} = \boldsymbol{\xi} \boldsymbol{y}, \quad \boldsymbol{\xi} = \boldsymbol{\chi}_x \boldsymbol{\phi}_x.$$

Note that

$$\mathbf{1}_c\boldsymbol{\chi}_c = \boldsymbol{I}_J,\ \mathbf{1}_{n^s}\boldsymbol{\chi}_{n^s} = \boldsymbol{I}_F,\ \mathbf{1}_{n^d}\boldsymbol{\chi}_{n^d} = \boldsymbol{I}_F,\ \mathbf{1}_x\boldsymbol{\chi}_x = \boldsymbol{I}_J,\ \mathbf{1}_x\boldsymbol{\xi} = \boldsymbol{\phi}_x,$$

where $\boldsymbol{I}_J$ and $\boldsymbol{I}_F$ denote identity matrices of dimensions J and F respectively.

Marginal products/technology change. We collect marginal products of intermediates in a $J \times JL$ matrix $\boldsymbol{G}_x$, marginal products of factors in a $J \times JF$ matrix $\boldsymbol{G}_n$, and technology changes in a $J \times 1$ vector $\boldsymbol{G}_\theta$, given by

$$\boldsymbol{G}_x = \begin{pmatrix} \frac{\partial G^1}{\partial x^{11}} & 0 & 0 & \frac{\partial G^1}{\partial x^{12}} & 0 & 0 & \frac{\partial G^1}{\partial x^{13}} & 0 & 0 \\ 0 & \frac{\partial G^2}{\partial x^{21}} & 0 & 0 & \frac{\partial G^2}{\partial x^{22}} & 0 & 0 & \frac{\partial G^2}{\partial x^{23}} & 0 \\ 0 & 0 & \frac{\partial G^3}{\partial x^{31}} & 0 & 0 & \frac{\partial G^3}{\partial x^{32}} & 0 & 0 & \frac{\partial G^3}{\partial x^{33}} \end{pmatrix}_{J \times JL}$$

$$\boldsymbol{G}_n = \begin{pmatrix} \frac{\partial G^1}{\partial n^{11,d}} & 0 & 0 & \frac{\partial G^1}{\partial n^{12,d}} & 0 & 0 \\ 0 & \frac{\partial G^2}{\partial n^{21,d}} & 0 & 0 & \frac{\partial G^2}{\partial n^{22,d}} & 0 \\ 0 & 0 & \frac{\partial G^3}{\partial n^{31,d}} & 0 & 0 & \frac{\partial G^3}{\partial n^{32,d}} \end{pmatrix}_{J \times JF}, \quad \boldsymbol{G}_\theta = \begin{pmatrix} \frac{\partial G^1}{\partial \theta} \\ \frac{\partial G^2}{\partial \theta} \\ \frac{\partial G^3}{\partial \theta} \end{pmatrix}_{J \times 1}.$$

Marginal rates of substitution. We collect marginal rates of substitution in $1 \times IJ$ and $1 \times IF$ vectors $\boldsymbol{MRS}_c$ and $\boldsymbol{MRS}_n$, given by

$$\boldsymbol{MRS}_c = \begin{pmatrix} MRS_c^{11} & MRS_c^{21} & MRS_c^{12} & MRS_c^{22} & MRS_c^{13} & MRS_c^{23} \end{pmatrix}_{1 \times IJ}$$
$$\boldsymbol{MRS}_n = \begin{pmatrix} MRS_n^{11} & MRS_n^{21} & MRS_n^{12} & MRS_n^{22} \end{pmatrix}_{1 \times IF}.$$

The $1 \times J$ and $1 \times F$ vectors of aggregate marginal rates of substitution, $\boldsymbol{AMRS}_c$ and $\boldsymbol{AMRS}_n$, can be written as

$$\boldsymbol{AMRS}_c = \boldsymbol{MRS}_c\boldsymbol{\chi}_c \quad \text{and} \quad \boldsymbol{AMRS}_n = \boldsymbol{MRS}_n\boldsymbol{\chi}_{n^s}.$$

Marginal social value of goods. We collect the marginal social value of goods in $1 \times J$ vector $\boldsymbol{MSV}_y$, given by

$$\boldsymbol{MSV}_y = \begin{pmatrix} MSV_y^1 & MSV_y^2 & MSV_y^3 \end{pmatrix}_{1 \times J}, \quad \text{where} \quad \boldsymbol{MSV}_y = \boldsymbol{AMRS}_c\boldsymbol{\phi}_c\boldsymbol{\Psi}_y.$$

Marginal welfare products. We collect marginal welfare products in $1 \times JL$ and $1 \times JF$ vectors $\boldsymbol{MWP}_x$ and $\boldsymbol{MWP}_n$, given by

$$\boldsymbol{MWP}_x = \boldsymbol{MSV}_y\boldsymbol{G}_x, \quad \text{and} \quad \boldsymbol{MWP}_n = \boldsymbol{MSV}_y\boldsymbol{G}_n.$$

The $1 \times J$ and $1 \times F$ vectors of aggregate marginal welfare products, $\boldsymbol{AMWP}_x$ and $\boldsymbol{AMWP}_n$, can be written as

$$\boldsymbol{AMWP}_x = \boldsymbol{MWP}_x\boldsymbol{\chi}_x \quad \text{and} \quad \boldsymbol{AMWP}_n = \boldsymbol{MWP}_n\boldsymbol{\chi}_{n^d}.$$

Goods inverse matrix. We define the elements of the $J \times J$ goods inverse $\boldsymbol{\Psi}_y$ as follows:

$$\boldsymbol{\Psi}_y = \begin{pmatrix} \psi_y^{11} & \psi_y^{12} & \psi_y^{13} \\ \psi_y^{21} & \psi_y^{22} & \psi_y^{23} \\ \psi_y^{31} & \psi_y^{32} & \psi_y^{33} \end{pmatrix}_{J \times J}, \quad \text{where} \quad \boldsymbol{\Psi}_y = (\boldsymbol{I}_J - \boldsymbol{G}_x\boldsymbol{\xi})^{-1}.$$

Competitive economies. In competitive economies, we collect prices p^j in the $1 \times J$ vector $\boldsymbol{p}$ and wages w^f in $1 \times F$ vector $\boldsymbol{w}$, given by

$$\boldsymbol{p} = \begin{pmatrix} p^1 & p^2 & p^3 \end{pmatrix}_{1 \times J}, \quad \boldsymbol{w} = \begin{pmatrix} w^1 & w^2 \end{pmatrix}_{1 \times F}.$$

We also collect consumption wedges in a $J \times IJ$ vector, $\boldsymbol{\tau}_c$, factor supply wedges in a $F \times IF$ vector, $\boldsymbol{\tau}_{n^s}$, intermediate use wedges in a $J \times JL$ vector, $\boldsymbol{\tau}_x$, and factor demand wedges in a $F \times JF$, $\boldsymbol{\tau}_{n^d}$, given by

$$\boldsymbol{\tau}_c = \begin{pmatrix} \tau_c^{11} & \tau_c^{21} & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau_c^{12} & \tau_c^{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & \tau_c^{13} & \tau_c^{23} \end{pmatrix}_{J \times IJ}, \quad \boldsymbol{\tau}_{n^s} = \begin{pmatrix} \tau_n^{11,s} & \tau_n^{21,s} & 0 & 0 \\ 0 & 0 & \tau_n^{12,s} & \tau_n^{22,s} \end{pmatrix}_{F \times IF},$$

$$\boldsymbol{\tau}_x = \begin{pmatrix} \frac{\tau_x^{11}-\tau_y^1}{1+\tau_y^1} & \frac{\tau_x^{21}-\tau_y^2}{1+\tau_y^2} & \frac{\tau_x^{31}-\tau_y^3}{1+\tau_y^3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\tau_x^{12}-\tau_y^1}{1+\tau_y^1} & \frac{\tau_x^{22}-\tau_y^2}{1+\tau_y^2} & \frac{\tau_x^{32}-\tau_y^3}{1+\tau_y^3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\tau_x^{13}-\tau_y^1}{1+\tau_y^1} & \frac{\tau_x^{23}-\tau_y^2}{1+\tau_y^2} & \frac{\tau_x^{33}-\tau_y^3}{1+\tau_y^3} \end{pmatrix}_{J \times JL},$$

$$\boldsymbol{\tau}_{n^d} = \begin{pmatrix} \frac{\tau_n^{11,d}-\tau_y^1}{1+\tau_y^1} & \frac{\tau_n^{12,d}-\tau_y^1}{1+\tau_y^1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\tau_n^{21,d}-\tau_y^2}{1+\tau_y^2} & \frac{\tau_n^{22,d}-\tau_y^2}{1+\tau_y^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\tau_n^{31,d}-\tau_y^3}{1+\tau_y^3} & \frac{\tau_n^{32,d}-\tau_y^3}{1+\tau_y^3} \end{pmatrix}_{F \times JF}.$$

We use $\boldsymbol{I}_c$ and $\boldsymbol{I}_x$ to denote the following $J \times J$ indicator matrices:

$$\boldsymbol{I}_c = \begin{pmatrix} 1\left[c^1 > 0\right] & 0 & 0 \\ 0 & 1\left[c^2 > 0\right] & 0 \\ 0 & 0 & 1\left[c^3 > 0\right] \end{pmatrix}_{J \times J}, \quad \boldsymbol{I}_x = \begin{pmatrix} 1\left[x^1 > 0\right] & 0 & 0 \\ 0 & 1\left[x^2 > 0\right] & 0 \\ 0 & 0 & 1\left[x^3 > 0\right] \end{pmatrix}_{J \times J}.$$

We use and $\boldsymbol{I}_{n^s}$ to denote the following $F \times F$ indicator matrix:

$$\boldsymbol{I}_{n^s} = \begin{pmatrix} 1\left[n^{1,s} > 0\right] & 0 \\ 0 & 1\left[n^{2,s} > 0\right] \end{pmatrix}_{F \times F}.$$

We collect aggregate supply, aggregate consumption, and prices in $J \times J$ diagonal matrices $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{c}}$, and $\hat{\boldsymbol{p}}$, given by

$$\hat{\boldsymbol{y}} = \operatorname{diag}(\boldsymbol{y}) = \begin{pmatrix} y^1 & 0 & 0 \\ 0 & y^2 & 0 \\ 0 & 0 & y^3 \end{pmatrix}_{J \times J}, \quad \hat{\boldsymbol{c}} = \operatorname{diag}(\boldsymbol{c}) = \begin{pmatrix} c^1 & 0 & 0 \\ 0 & c^2 & 0 \\ 0 & 0 & c^3 \end{pmatrix}_{J \times J}, \quad \hat{\boldsymbol{p}} = \operatorname{diag}(\boldsymbol{p}) = \begin{pmatrix} p^1 & 0 & 0 \\ 0 & p^2 & 0 \\ 0 & 0 & p^3 \end{pmatrix}_{J \times J}.$$

B Proofs and Derivations

B.1 Section 2

Proof of Lemma 1. (Efficiency/Redistribution Decomposition)

Proof. For any welfarist planner with social welfare function $\mathcal{W}(\cdot)$, we can express $\frac{dW}{d\theta}$ as

$$\frac{dW}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \frac{dV^i}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \lambda^i \frac{\frac{dV^i}{d\theta}}{\lambda^i},$$

where λ^i is an individual normalizing factor with units $\dim\left(\lambda^i\right)=\frac{\text{utils of individual } i}{\text{units of numeraire}}$ that allows us to express individual welfare assessments into a common unit/numeraire. We can therefore write

$$\frac{dW^\lambda}{d\theta}=\frac{\frac{dW}{d\theta}}{\frac{1}{I}\sum_i\frac{\partial\mathcal{W}}{\partial V^i}\lambda^i}=\sum_i\omega^i\frac{\frac{dV^i}{d\theta}}{\lambda^i}=\underbrace{\frac{1}{I}\sum_i\omega^i}_{=1}\sum_i\frac{\frac{dV^i}{d\theta}}{\lambda^i}+I\mathbb{C}ov_i^{\Sigma}\left[\omega^i,\frac{\frac{dV^i}{d\theta}}{\lambda^i}\right]=\underbrace{\sum_i\frac{\frac{dV^i}{d\theta}}{\lambda^i}}_{=\Xi^E}+\underbrace{\mathbb{C}ov_i^{\Sigma}\left[\omega^i,\frac{\frac{dV^i}{d\theta}}{\lambda^i}\right]}_{=\Xi^{RD}},$$

where $\omega^i=\frac{\frac{\partial\mathcal{W}}{\partial V^i}\lambda^i}{\frac{1}{I}\sum_i\frac{\partial\mathcal{W}}{\partial V^i}\lambda^i}$, which implies that $\frac{1}{I}\sum_i\omega^i=1$. Since this economy is static, $\Xi^E=\Xi^{AE}$, as shown in Dávila and Schaab (2022). □

B.2 Section 3

Proof of Theorem 1. (Exchange Efficiency)

Proof. Given the definition of V^i in equation (1), we can express $\frac{\frac{dV^i}{d\theta}}{\lambda^i}$ as

$$\frac{\frac{dV^i}{d\theta}}{\lambda^i}=\sum_j\frac{\frac{\partial u^i}{\partial c^{ij}}}{\lambda^i}\frac{dc^{ij}}{d\theta}+\sum_f\frac{\frac{\partial u^i}{\partial n^{if,s}}}{\lambda^i}\frac{dn^{if,s}}{d\theta}=\sum_j MRS_c^{ij}\frac{dc^{ij}}{d\theta}-\sum_f MRS_n^{if}\frac{dn^{if,s}}{d\theta}.$$

Hence, from Lemma 1, it follows that

$$\Xi^E=\Xi^{AE}=\sum_i\frac{\frac{dV^i}{d\theta}}{\lambda^i}=\sum_j\sum_i MRS_c^{ij}\frac{dc^{ij}}{d\theta}-\sum_f\sum_i MRS_n^{if}\frac{dn^{if,s}}{d\theta}.$$

Given (10), we can write

$$\sum_i MRS_c^{ij}\frac{dc^{ij}}{d\theta}=\mathbb{C}ov_i^{\Sigma}\left[MRS_c^{ij},\frac{d\chi_c^{ij}}{d\theta}\right]c^j+AMRS_c^j\frac{dc^j}{d\theta},$$

where $AMRS_c^j$ is defined in (19). Similarly, we can write

$$\sum_i MRS_n^{if,s}\frac{dn^{if,s}}{d\theta}=\mathbb{C}ov_i^{\Sigma}\left[MRS_n^{if,s},\frac{d\chi_n^{if,s}}{d\theta}\right]n^{f,s}+AMRS_n^f\frac{dn^{f,s}}{d\theta},$$

where $AMRS_n^f$ is also defined in (19). Hence, exchange efficiency, $\Xi^{AE,X}$, can be expressed as

$$\Xi^{AE,X}=\underbrace{\mathbb{C}ov_i^{\Sigma}\left[MRS_c^{ij},\frac{d\chi_c^{ij}}{d\theta}\right]c^j}_{\substack{\text{Cross-Sectional}\\\text{Consumption Efficiency}}}\underbrace{-\mathbb{C}ov_i^{\Sigma}\left[MRS_n^{if,s},\frac{d\chi_n^{if,s}}{d\theta}\right]n^{f,s}}_{\substack{\text{Cross-Sectional}\\\text{Factor Supply Efficiency}}},$$

while production efficiency corresponds to

$$\Xi^{AE,P}=\sum_j AMRS_c^j\frac{dc^j}{d\theta}-\sum_f AMRS_n^f\frac{dn^{f,s}}{d\theta}.$$

Alternatively, in matrix form, we can write

$$\Xi^E=\Xi^{AE}=\sum_i\frac{\frac{dV^i}{d\theta}}{\lambda^i}=\boldsymbol{MRS}_c\frac{d\mathring{\boldsymbol{c}}}{d\theta}-\boldsymbol{MRS}_n\frac{d\mathring{\boldsymbol{n}}^s}{d\theta},$$

where (10) can be expressed as

$$\frac{d\mathring{\boldsymbol{c}}}{d\theta}=\frac{d\boldsymbol{\chi}_c}{d\theta}\boldsymbol{c}+\boldsymbol{\chi}_c\frac{d\boldsymbol{c}}{d\theta}\quad\text{and}\quad\frac{d\mathring{\boldsymbol{n}}^s}{d\theta}=\frac{d\boldsymbol{\chi}_{n^s}}{d\theta}\boldsymbol{n}^s+\boldsymbol{\chi}_{n^s}\frac{d\boldsymbol{n}^s}{d\theta}.$$

Hence,

$$\boldsymbol{MRS}_c\frac{d\mathring{\boldsymbol{c}}}{d\theta}=\boldsymbol{MRS}_c\frac{d\boldsymbol{\chi}_c}{d\theta}\boldsymbol{c}+\boldsymbol{AMRS}_c\frac{d\boldsymbol{c}}{d\theta}\quad\text{and}\quad\boldsymbol{MRS}_n\frac{d\mathring{\boldsymbol{n}}^s}{d\theta}=\boldsymbol{MRS}_n\frac{d\boldsymbol{\chi}_{n^s}}{d\theta}\boldsymbol{n}^s+\boldsymbol{AMRS}_n\frac{d\boldsymbol{n}^s}{d\theta},$$

where $\boldsymbol{AMRS}_c=\boldsymbol{MRS}_c\boldsymbol{\chi}_c$ and $\boldsymbol{AMRS}_n=\boldsymbol{MRS}_n\boldsymbol{\chi}_{n^s}$. We can thus write

$$\Xi^{AE}=\underbrace{\underbrace{\boldsymbol{MRS}_c\frac{d\boldsymbol{\chi}_c}{d\theta}\boldsymbol{c}}_{\substack{\text{Cross-Sectional}\\\text{Consumption Efficiency}}}\underbrace{-\boldsymbol{MRS}_n\frac{d\boldsymbol{\chi}_{n^s}}{d\theta}\boldsymbol{n}^s}_{\substack{\text{Cross-Sectional}\\\text{Factor Supply Efficiency}}}}_{\Xi^{AE,X}\text{ (Exchange Efficiency)}}\underbrace{+\boldsymbol{AMRS}_c\frac{d\boldsymbol{c}}{d\theta}-\boldsymbol{AMRS}_n\frac{d\boldsymbol{n}^s}{d\theta}}_{\Xi^{AE,P}\text{ (Production Efficiency)}}$$

□

Proof of Corollary 1. (Properties of Exchange Efficiency)

Proof. Proceeding item-by-item:

(a) When $I=1$, $\mathbb{C}ov_i^{\Sigma}\left[MRS_c^{ij},\frac{dc^{ij}}{d\theta}\right]=\mathbb{C}ov_i^{\Sigma}\left[MRS_n^{if,s},\frac{dn^{if,s}}{d\theta}\right]=0$, $\forall j$ and $\forall f$.

(b) When $n^{f,s}=0$, $\mathbb{C}ov_i^{\Sigma}\left[MRS_n^{if,s},\frac{d\chi_n^{if,s}}{d\theta}\right]n^{f,s}=0$, $\forall f$.

(c) When MRS_c^{ij} is identical for all i, $\mathbb{C}ov_i^{\Sigma}\left[MRS_c^{ij},\frac{de^{ij}}{d\theta}\right]=0$. When MRS_n^{if} is identical for all f, $\mathbb{C}ov_i^{\Sigma}\left[MRS_n^{if},\frac{dn^{if,s}}{d\theta}\right]=0$.

□

Proof of Lemma 2. (Goods Inverse Matrix)

Proof. Given (13) and (16) we can write $\frac{dy^j}{d\theta}$ and $\frac{dx^{j\ell}}{d\theta}$ in matrix form, as

$$\frac{d\boldsymbol{y}^s}{d\theta}=\boldsymbol{G}_x\frac{d\mathring{\boldsymbol{x}}}{d\theta}+\boldsymbol{G}_n\frac{d\mathring{\boldsymbol{n}}^d}{d\theta}+\boldsymbol{G}_\theta\quad\text{and}\quad\frac{d\mathring{\boldsymbol{x}}}{d\theta}=\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y}+\boldsymbol{\xi}\frac{d\boldsymbol{y}}{d\theta},\tag{OA1}$$

where

$$\frac{d\boldsymbol{y}}{d\theta}=\frac{d\boldsymbol{y}^s}{d\theta}+\frac{d\bar{\boldsymbol{y}}^s}{d\theta}.$$

Combining these expressions, we can express $\frac{d\boldsymbol{y}^s}{d\theta}$ as

$$\begin{aligned}\frac{d\boldsymbol{y}^s}{d\theta}&=\boldsymbol{G}_x\left(\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y}+\boldsymbol{\xi}\left(\frac{d\boldsymbol{y}^s}{d\theta}+\frac{d\bar{\boldsymbol{y}}^s}{d\theta}\right)\right)+\boldsymbol{G}_n\frac{d\mathring{\boldsymbol{n}}^d}{d\theta}+\boldsymbol{G}_\theta\\&=\boldsymbol{\Psi}_y\left(\boldsymbol{G}_x\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y}+\boldsymbol{G}_n\frac{d\mathring{\boldsymbol{n}}^d}{d\theta}+\boldsymbol{G}_x\boldsymbol{\xi}\frac{d\bar{\boldsymbol{y}}^s}{d\theta}+\boldsymbol{G}_\theta\right),\end{aligned}$$

where $\boldsymbol{\Psi}_y=(\boldsymbol{I}_J-\boldsymbol{G}_x\boldsymbol{\xi})^{-1}$. Finally, we use the fact that $\boldsymbol{\Psi}_y=\boldsymbol{I}_J+\boldsymbol{\Psi}_y\boldsymbol{G}_x\boldsymbol{\xi}$, so that we can express $\frac{d\boldsymbol{y}}{d\theta}$ as

$$\frac{d\boldsymbol{y}}{d\theta}=\frac{d\boldsymbol{y}^s}{d\theta}+\frac{d\bar{\boldsymbol{y}}^s}{d\theta}=\boldsymbol{\Psi}_y\left(\boldsymbol{G}_x\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y}+\boldsymbol{G}_n\frac{d\mathring{\boldsymbol{n}}^d}{d\theta}+\frac{d\bar{\boldsymbol{y}}^s}{d\theta}+\boldsymbol{G}_\theta\right),$$

which corresponds to equation (17) in the text. □

Proof of Theorem 2. (Production Efficiency)

Proof. As shown above, we can express $\Xi^{AE,P}$ in matrix form as

$$\Xi^{AE,P} = \boldsymbol{AMRS}_c \frac{d\boldsymbol{c}}{d\theta} - \boldsymbol{AMRS}_n \frac{d\boldsymbol{n}^s}{d\theta}.$$

First, note that we can express the change in aggregate consumption, $\frac{d\boldsymbol{c}}{d\theta}$, as

$$\frac{d\boldsymbol{c}}{d\theta} = \frac{d\boldsymbol{y}}{d\theta} - \frac{d\boldsymbol{x}}{d\theta} = \frac{d\boldsymbol{y}}{d\theta} - \left(\boldsymbol{\phi}_x \frac{d\boldsymbol{y}}{d\theta} + \frac{d\boldsymbol{\phi}_x}{d\theta}\boldsymbol{y}\right) = \boldsymbol{\phi}_c \frac{d\boldsymbol{y}}{d\theta} - \frac{d\boldsymbol{\phi}_x}{d\theta}\boldsymbol{y},$$

where we use the fact that $\frac{d\boldsymbol{x}}{d\theta} = \boldsymbol{\phi}_x \frac{d\boldsymbol{y}}{d\theta} + \frac{d\boldsymbol{\phi}_x}{d\theta}\boldsymbol{y}$ and $\boldsymbol{\phi}_c = \boldsymbol{I}_J - \boldsymbol{\phi}_x$.

Next, note that $\frac{d\boldsymbol{y}}{d\theta}$ can be written as

$$\frac{d\boldsymbol{y}}{d\theta} = \boldsymbol{\Psi}_y \left(\boldsymbol{G}_x \frac{d\boldsymbol{\chi}_x}{d\theta}\boldsymbol{x} + \boldsymbol{G}_x\boldsymbol{\chi}_x \frac{d\boldsymbol{\phi}_x}{d\theta}\boldsymbol{y} + \boldsymbol{G}_n \frac{d\boldsymbol{\chi}_{n^d}}{d\theta}\boldsymbol{n}^d + \boldsymbol{G}_n\boldsymbol{\chi}_{n^d}\frac{d\boldsymbol{n}^d}{d\theta} + \frac{d\bar{\boldsymbol{y}}}{d\theta} + \boldsymbol{G}_\theta\right),$$

where we use the fact that

$$\frac{d\boldsymbol{\xi}}{d\theta} = \frac{d\boldsymbol{\chi}_x}{d\theta}\boldsymbol{x} + \boldsymbol{\chi}_x \frac{d\boldsymbol{\phi}_x}{d\theta}\boldsymbol{y} \quad \text{and} \quad \frac{d\mathring{\boldsymbol{n}}^d}{d\theta} = \frac{d\boldsymbol{\chi}_{n^d}}{d\theta}\boldsymbol{n}^d + \boldsymbol{\chi}_{n^d}\frac{d\boldsymbol{n}^d}{d\theta}.$$

This result allows us to express $\frac{d\boldsymbol{c}}{d\theta}$ as

$$\frac{d\boldsymbol{c}}{d\theta} = \boldsymbol{\phi}_c\boldsymbol{\Psi}_y\boldsymbol{G}_x \frac{d\boldsymbol{\chi}_x}{d\theta}\boldsymbol{x} + (\boldsymbol{\phi}_c\boldsymbol{\Psi}_y\boldsymbol{G}_x\boldsymbol{\chi}_x - \boldsymbol{I}_J)\frac{d\boldsymbol{\phi}_x}{d\theta}\boldsymbol{y} + \boldsymbol{\phi}_c\boldsymbol{\Psi}_y\boldsymbol{G}_n\frac{d\boldsymbol{\chi}_{n^d}}{d\theta}\boldsymbol{n}^d + \boldsymbol{\phi}_c\boldsymbol{\Psi}_y\boldsymbol{G}_n\boldsymbol{\chi}_{n^d}\frac{d\boldsymbol{n}^d}{d\theta} + \boldsymbol{\phi}_c\boldsymbol{\Psi}_y\left(\frac{d\bar{\boldsymbol{y}}^s}{d\theta} + \boldsymbol{G}_\theta\right)$$

Hence, combining this expression for $\frac{d\boldsymbol{c}}{d\theta}$ with the resource constraint for factors, which implies that $\frac{d\boldsymbol{n}^d}{d\theta} = \frac{d\boldsymbol{n}^s}{d\theta} + \frac{d\bar{\boldsymbol{n}}^s}{d\theta}$, we can express production efficiency exactly as in text, as follows:

$$\begin{aligned}
\Xi^{AE,P} = \quad & \underbrace{\boldsymbol{MWP}_x \frac{d\boldsymbol{\chi}_x}{d\theta}\boldsymbol{x}}_{\substack{\text{Cross-Sectional}\\ \text{Intermediate Input Efficiency}}} + \underbrace{(\boldsymbol{AMWP}_x - \boldsymbol{AMRS}_c)\frac{d\boldsymbol{\phi}_x}{d\theta}\boldsymbol{y}}_{\substack{\text{Aggregate}\\ \text{Intermediate Input Efficiency}}} \\
& + \underbrace{\boldsymbol{MWP}_n \frac{d\boldsymbol{\chi}_{n^d}}{d\theta}\boldsymbol{n}^d}_{\substack{\text{Cross-Sectional}\\ \text{Factor Efficiency}}} + \underbrace{(\boldsymbol{AMWP}_n - \boldsymbol{AMRS}_n)\frac{d\boldsymbol{n}^s}{d\theta}}_{\substack{\text{Aggregate}\\ \text{Factor Efficiency}}} \\
& + \underbrace{\boldsymbol{MSV}_y \frac{d\bar{\boldsymbol{y}}^s}{d\theta}}_{\substack{\text{Technology}\\ \text{Change}}} + \underbrace{\boldsymbol{MSV}_y\boldsymbol{G}_\theta}_{\substack{\text{Good Endowment}\\ \text{Change}}} + \underbrace{\boldsymbol{AMRS}_n \frac{d\bar{\boldsymbol{n}}^s}{d\theta}}_{\substack{\text{Factor Endowment}\\ \text{Change}}},
\end{aligned}$$

where

$$\boldsymbol{MWP}_x = \boldsymbol{MSV}_y\boldsymbol{G}_x,\ \boldsymbol{MWP}_n = \boldsymbol{MSV}_y\boldsymbol{G}_n,\ \boldsymbol{MSV}_y = \boldsymbol{AMRS}_c\boldsymbol{\phi}_c\boldsymbol{\Psi}_y.$$

□

Proof of Corollary 2. (Properties of Production Efficiency Decomposition)

Proof. Proceeding item-by-item:

(a) When $J = 1$, $\mathbb{C}ov_j^{\Sigma}\left[MWP_x^{j\ell}, \frac{d\chi_x^{j\ell}}{d\theta}\right] = 0$, $\forall \ell$.

(b) With no intermediate goods, $x^{\ell} = \frac{d\phi_x^{\ell}}{d\theta} = 0$, $\forall \ell$.

(c) If all factors are in fixed supply, $\frac{dn^{f,s}}{d\theta} = 0$, $\forall s$.

(d) If all intermediate inputs are specialized: $\frac{dx_x^{j\ell}}{d\theta} = 0$, $\forall j$, $\forall \ell$. If all factors are specialized, $\frac{dx_n^{jf,d}}{d\theta} = 0$, $\forall j$, $\forall f$.

(e) When marginal welfare products are equalized for intermediates: $\mathbb{C}ov_j^{\Sigma}\left[MWP_x^{j\ell}, \frac{d\chi_x^{j\ell}}{d\theta}\right] = 0$, $\forall \ell$; for factors: $\mathbb{C}ov_j^{\Sigma}\left[MWP_n^{jf}, \frac{d\chi_n^{jf,d}}{d\theta}\right] = 0$, $\forall f$.

□

B.3 Section 4

Proof of Theorem 3. (Efficiency Conditions: Exchange Efficiency)

Proof. If MRS_c^{ij} is different across any two individuals with $\chi_c^{ij} > 0$ for good j with $c^j > 0$, then there exists a perturbation of consumption shares in which cross-sectional consumption efficiency is positive. If MRS_c^{ij} is less than $AMRS_c^j$ when $\chi_c^{ij} = 0$, then there is no feasible perturbation that reduces the share of consumption for individual i. The same logic applies to cross-sectional factor supply efficiency. □

Proof of Theorem 4. (Efficiency Conditions: Production Efficiency)

Proof. If $MWP_x^{j\ell}$ is different across any two intermediate uses of good ℓ two individuals with $\chi_x^{j\ell} > 0$, then there exists a perturbation of intermediate use shares in which cross-sectional intermediate input efficiency is positive. The same logic applies to cross-sectional factor use efficiency.

When $\phi_x^{\ell} \in (0,1)$, then there exists a perturbation of ϕ_x^{ℓ} such that aggregate intermediate input efficiency is positive unless $AMWP_x^{\ell} = AMRS_c^{\ell}$. If $\phi_x^{\ell} = 0$, it must be that $AMWP_x^{\ell} \leq AMRS_c^{\ell}$ for the best possible combination of intermediate use shares, which is the one that allocates good ℓ to its highest marginal welfare product intermediate use. If $\phi_x^{\ell} = 1$, it must be that $AMWP_x^{\ell} \geq AMRS_c^{\ell}$ for the possible combinations of consumption shares, which is the one that allocates the consumption of good j to the individual with the highest $MRS_c^{i\ell}$.

When $n^{f,s} > 0$ (and $n^{f,d} > 0$), then there exists a perturbation of $n^{f,s}$ such that aggregate factor supply efficiency is positive unless $AMWP_n^f = AMRS_n^f$. If $n^{f,s} = 0$, it must be that $AMWP_n^f \leq AMRS_n^f$ for the best possible combination of factor supply shares, which is the one that allocates the consumption of good j to the individual with the lowest MRS_n^{if}. If $n^{f,s} = n^{f,d} = 0$, then it must be that the most costly way of supplying a factor is higher than the highest marginal welfare product of doing so, formally: $\max_j \left\{MWP_n^{jf}\right\} \leq \min_i \left\{MRS_n^{if}\right\}$.[44] □

Proof of Theorem 5. (MSV under Efficiency)

Proof. In matrix form, it follows from Equation (23) that

$$\boldsymbol{MSV}_y = \boldsymbol{AMRS}_c\boldsymbol{\phi}_c + \boldsymbol{MSV}_y\boldsymbol{G}_x\boldsymbol{\xi} = \boldsymbol{AMRS}_c\boldsymbol{\phi}_c + \boldsymbol{AMWP}_x\boldsymbol{\phi}_x,$$

where $\boldsymbol{\xi} = \boldsymbol{\chi}_x\boldsymbol{\phi}_x$ and $\boldsymbol{AMWP}_x = \boldsymbol{MSV}_y\boldsymbol{G}_x\boldsymbol{\chi}_x$. Therefore, equation (30) follows immediately when aggregate intermediate input efficiency holds. Equation (31) follows directly from the cross-sectional efficiency conditions. □

[44] When $n^{f,d} = 0$, the value of a marginal unit of endowment of factor f is simply $\max_j \left\{MWP_n^{jf}\right\}$.

Proof of Corollary 3. (Revisiting Lange (1942) and Mas-Colell et al. (1995))

Proof. Follows from derivations in footnote 27. □

Proof of Corollary 4. (Classic Efficiency Conditions Hold for Interior Links)

Proof. Proceeding item-by-item: □

(a) At an interior link, Theorems 4 and 5 imply that both equations in (32) hold.

(b) The result follows then from the same logic as in Corollary 3.

Proof of Corollary 5. (Scenarios in which Classic Efficiency Conditions Do Not Hold)

Proof. Proceeding item-by-item:

(a) If good j is a pure intermediate, then $MSV_y^j \neq AMRS_c^j$, which implies that the classic efficiency conditions cannot hold, since efficiency requires that $MSV_y^j \frac{\partial G^j}{\partial x^{j\ell}} = MRS_c^{ij}$.

(b) If good ℓ is a pure intermediate, then last condition of equation (27) already implies that the classic efficiency conditions cannot hold.

(c) As in (a), $MSV_y^j \neq AMRS_c^j$, which implies that the classic efficiency conditions cannot hold, since efficiency requires that $MSV_y^j \frac{\partial G^j}{\partial n^{jf,d}} = MRS_n^{if}$.

□

B.4 Section 5

Proof of Theorem 6. (MSV in Competitive Economies with Wedges)

Proof. In a competitive equilibrium with wedges, we can express aggregate marginal rates of substitution as

$$\boldsymbol{AMRS}_c = \boldsymbol{MRS}_c \boldsymbol{\chi}_c = \boldsymbol{p}\left(\boldsymbol{I}_c + \bar{\boldsymbol{\tau}}_c\right),$$

where $\boldsymbol{I}_c$ is $J \times J$ diagonal matrix in which the j'th element is 1 when $c^j > 0$ and 0 if $c^j = 0$, and where we define a $J \times J$ matrix of aggregate consumption wedges as $\bar{\boldsymbol{\tau}}_c = \boldsymbol{\tau}_c \boldsymbol{\chi}_c$. It is also the case that

$$\boldsymbol{p}\boldsymbol{G}_x \boldsymbol{\chi}_x = \boldsymbol{p}\left(\boldsymbol{I}_x + \boldsymbol{\tau}_x \boldsymbol{\chi}_x\right) = \boldsymbol{p}\left(\boldsymbol{I}_x + \bar{\boldsymbol{\tau}}_x\right),$$

where $\boldsymbol{I}_x$ is $J \times J$ diagonal matrix in which the j'th element is 1 when $x^j > 0$ and 0 if $x^j = 0$, and where we define a $J \times J$ matrix of aggregate intermediate use wedges as $\bar{\boldsymbol{\tau}}_x = \boldsymbol{\tau}_x \boldsymbol{\chi}_x$. Hence, we can express the marginal social value of goods as

$$\begin{aligned}\boldsymbol{MSV}_y &= \boldsymbol{AMRS}_c \boldsymbol{\phi}_c \boldsymbol{\Psi}_y = \boldsymbol{p}\left(\boldsymbol{I}_c + \bar{\boldsymbol{\tau}}_c\right)\boldsymbol{\phi}_c \boldsymbol{\Psi}_y = \boldsymbol{p}\boldsymbol{\phi}_c \boldsymbol{\Psi}_y + \boldsymbol{p}\bar{\boldsymbol{\tau}}_c \boldsymbol{\phi}_c \boldsymbol{\Psi}_y \\ &= \boldsymbol{p} + \boldsymbol{p}\left(\bar{\boldsymbol{\tau}}_x \boldsymbol{\phi}_x + \bar{\boldsymbol{\tau}}_c \boldsymbol{\phi}_c\right)\boldsymbol{\Psi}_y,\end{aligned}$$

where we use the fact that $\boldsymbol{I}_c \boldsymbol{\phi}_c = \boldsymbol{\phi}_c$ and that

$$\begin{aligned}\boldsymbol{p}\boldsymbol{\phi}_c \boldsymbol{\Psi}_y &= \boldsymbol{p}\left(\left(\boldsymbol{G}_x - \boldsymbol{1}_x\right)\boldsymbol{\xi}\boldsymbol{\Psi}_y + \boldsymbol{I}_J\right) = \left(\boldsymbol{p}\boldsymbol{G}_x - \boldsymbol{p}\boldsymbol{1}_x\right)\boldsymbol{\chi}_x \boldsymbol{\phi}_x \boldsymbol{\Psi}_y + \boldsymbol{p} \\ &= \left(\boldsymbol{p}\boldsymbol{G}_x \boldsymbol{\chi}_x - \boldsymbol{p}\right)\boldsymbol{\phi}_x \boldsymbol{\Psi}_y + \boldsymbol{p} = \left(\boldsymbol{p}\left(\boldsymbol{I}_x + \bar{\boldsymbol{\tau}}_x\right) - \boldsymbol{p}\right)\boldsymbol{\phi}_x \boldsymbol{\Psi}_y + \boldsymbol{p} = \boldsymbol{p}\bar{\boldsymbol{\tau}}_x \boldsymbol{\phi}_x \boldsymbol{\Psi}_y + \boldsymbol{p}.\end{aligned}$$

□

Proof of Corollary 6. (Hulten's Theorem Revisited)

Proof. Since frictionless competitive economies are efficient, Ξ^{AE} simply equals technology change. When $\bar{\boldsymbol{\tau}}_c = \bar{\boldsymbol{\tau}}_x = 0$, it follows from Theorem 6 that $MSV_y^j = p^j$. Hence, $\Xi^{AE} = p^j \frac{\partial G^j}{\partial \theta} = p^j y^j$, where we use the fact that $\frac{\partial G^j}{\partial \theta} = y^j$ for proportional Hicks-neutral technology changes. Simply dividing by $\sum_j p^j c^j$ yields equation (38) in the text. □

Proof of Theorem 7. (Production Efficiency in Competitive Economies)

Proof. It follows from the optimality conditions for production and the definition of $\Xi^{AE,P}$. □

Proof of Theorem 8. (Exchange Efficiency in Competitive Economies)

Proof. It follows from the individual optimality conditions for consumption and factor supply and the definition of $\Xi^{AE,X}$. □

B.5 Section 6

Proof of Theorem 9. (Redistribution Decomposition)

Proof. Note that

$$\Xi^{RD} = \mathbb{C}ov_i^{\Sigma}\left[\omega^i, \frac{\frac{dV^i}{d\theta}}{\lambda^i}\right],$$

where

$$\frac{\frac{dV^i}{d\theta}}{\lambda^i} = \sum_j MRS_c^{ij} \frac{dc^{ij}}{d\theta} - \sum_f MRS_n^{if} \frac{dn^{if,s}}{d\theta}.$$

Hence, using the fact that

$$\frac{dc^{ij}}{d\theta} = \frac{d\chi_c^{ij}}{d\theta} c^j + \chi_c^{ij} \frac{dc^j}{d\theta} \quad \text{and} \quad \frac{dn^{if,s}}{d\theta} = \frac{d\chi_n^{if,s}}{d\theta} n^{f,s} + \chi_n^{if,s} \frac{dn^{f,s}}{d\theta},$$

we can express Ξ^{RD} as

$$\begin{aligned}
\Xi^{RD} = & \overbrace{\sum_j \mathbb{C}ov_i^{\Sigma}\left[\omega^i, MRS_c^{ij} \frac{d\chi_c^{ij}}{d\theta}\right] c^j}^{\substack{\text{Cross-Sectional} \\ \text{Consumption Redistribution}}} + \overbrace{\sum_j \mathbb{C}ov_i^{\Sigma}\left[\omega^i, MRS_c^{ij} \chi_c^{ij}\right] \frac{dc^j}{d\theta}}^{\substack{\text{Aggregate} \\ \text{Consumption Redistribution}}} \\
& \underbrace{- \sum_f \mathbb{C}ov_i^{\Sigma}\left[\omega^i, MRS_n^{if} \frac{d\chi_n^{if,s}}{d\theta}\right] n^{f,s}}_{\substack{\text{Cross-Sectional} \\ \text{Factor Supply Redistribution}}} \underbrace{- \sum_f \mathbb{C}ov_i^{\Sigma}\left[\omega^i, MRS_n^{if} \chi_n^{if,s}\right] \frac{dn^{f,s}}{d\theta}}_{\substack{\text{Aggregate} \\ \text{Factor Supply Redistribution}}}.
\end{aligned}$$

□

Note that, in economies that satisfy exchange efficiency, Theorem 9 simplifies to

$$\begin{aligned}
\Xi^{RD} = & \sum_j AMRS_c^j \left(\mathbb{C}ov_i^{\Sigma}\left[\omega^i, \frac{d\chi_c^{ij}}{d\theta}\right] c^j + \mathbb{C}ov_i^{\Sigma}\left[\omega^i, \chi_c^{ij}\right] \frac{dc^j}{d\theta}\right) \\
& + \sum_f AMRS_n^f \left(\mathbb{C}ov_i^{\Sigma}\left[\omega^i, \frac{d\chi_n^{if,s}}{d\theta}\right] n^{f,s} + \mathbb{C}ov_i^{\Sigma}\left[\omega^i, \chi_n^{if,s}\right] \frac{dn^{f,s}}{d\theta}\right),
\end{aligned}$$

where $AMRS_c^j = MRS_c^{ij}$ and $AMRS_n^f = MRS_n^{if}$, $\forall i$.

C Additional Results

C.1 Dynamic Stochastic Environment

Here we consider a general dynamic stochastic economy in which individuals have preferences of the form

$$V^i = \sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t\left(s^t\right) u_t^i\left(\left\{c_t^{ij}\left(s^t\right)\right\}_{j\in\mathcal{J}}, \left\{n_t^{if,s}\left(s^t\right)\right\}_{f\in\mathcal{F}}; s^t\right),$$

and in which the production structure introduced in Section 2 repeats history by history. Figure OA-1 illustrates the results

For any welfarist planner with social welfare function $\mathcal{W}\left(V^1,\ldots,V^i,\ldots,V^I\right)$, we can express $\frac{dW}{d\theta}$ as

$$\frac{dW}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i}\frac{dV^i}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i \frac{\frac{dV^i}{d\theta}}{\lambda^i},$$

where λ^i is an individual normalizing factor that allows us to express individual welfare assessments into a common unit/numeraire. We can therefore write

$$\frac{dW^\lambda}{d\theta} = \frac{\frac{dW}{d\theta}}{\frac{\sum_i \alpha^i\lambda^i}{I}} = \sum_i \omega^i \frac{\frac{dV^i}{d\theta}}{\lambda^i} = \underbrace{\frac{\sum_i \omega^i}{I}}_{=1}\sum_i \frac{\frac{dV^i}{d\theta}}{\lambda^i} + I\mathbb{C}ov_i^\Sigma\left[\omega^i, \frac{\frac{dV^i}{d\theta}}{\lambda^i}\right] = \underbrace{\sum_i \frac{\frac{dV^i}{d\theta}}{\lambda^i}}_{=\Xi^E} + \underbrace{\mathbb{C}ov_i^\Sigma\left[\omega^i, \frac{\frac{dV^i}{d\theta}}{\lambda^i}\right]}_{=\Xi^{RD}},$$

where $\omega^i = \frac{\alpha^i\lambda^i}{\frac{1}{I}\sum_i \alpha^i\lambda^i}$, which implies that $\frac{1}{I}\sum_i \omega^i = 1$.

We can express individual i's lifetime welfare gains in units of the lifetime welfare numeraire as

$$\frac{dV^{i|\lambda}}{d\theta} = \frac{\frac{dV^i}{d\theta}}{\lambda^i} = \sum_t \frac{\lambda_t^i}{\lambda^i}\sum_{s^t} \frac{\left(\beta^i\right)^t \pi_t\left(s^t\right)\lambda_t^i\left(s^t\right)}{\lambda_t^i}\frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} = \sum_t \omega_t^i \sum_{s^t}\omega_t^i\left(s^t\right)\frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta},$$

where λ^i and $\lambda_t^i\left(s^t\right)$ are normalizing factors to express welfare gains at particular dates or histories across individuals in a common unit. In this case, $\omega_t^i = \frac{\lambda_t^i}{\lambda^i}$ and $\omega_t^i\left(s^t\right) = \frac{\left(\beta^i\right)^t\pi_t\left(s^t\right)\lambda_t^i\left(s^t\right)}{\lambda_t^i}$, where

$$\begin{aligned}\frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} &= \sum_j \frac{\frac{\partial u_t^i\left(s^t\right)}{\partial c_t^{ij}}}{\lambda_t^i\left(s^t\right)}\frac{dc_t^{ij}\left(s^t\right)}{d\theta} + \sum_f \frac{\frac{\partial u_t^i\left(s^t\right)}{\partial n_t^{if,s}}}{\lambda_t^i\left(s^t\right)}\frac{dn_t^{if,s}\left(s^t\right)}{d\theta},\\ &= \sum_j MRS_{t,c}^{ij}\left(s^t\right)\frac{dc_t^{ij}\left(s^t\right)}{d\theta} - \sum_f MRS_{t,n}^{if}\left(s^t\right)\frac{dn_t^{if,s}\left(s^t\right)}{d\theta}\end{aligned}$$

where $MRS_{t,c}^{ij}\left(s^t\right) = \frac{\frac{\partial u_t^i\left(s^t\right)}{\partial c_t^{ij}}}{\lambda_t^i\left(s^t\right)}$ and $MRS_{t,n}^{if}\left(s^t\right) = -\frac{\frac{\partial u_t^i\left(s^t\right)}{\partial n_t^{if,s}}}{\lambda_t^i\left(s^t\right)}$.

Following Dávila and Schaab (2022), note that the efficiency component can be decomposed into aggregate

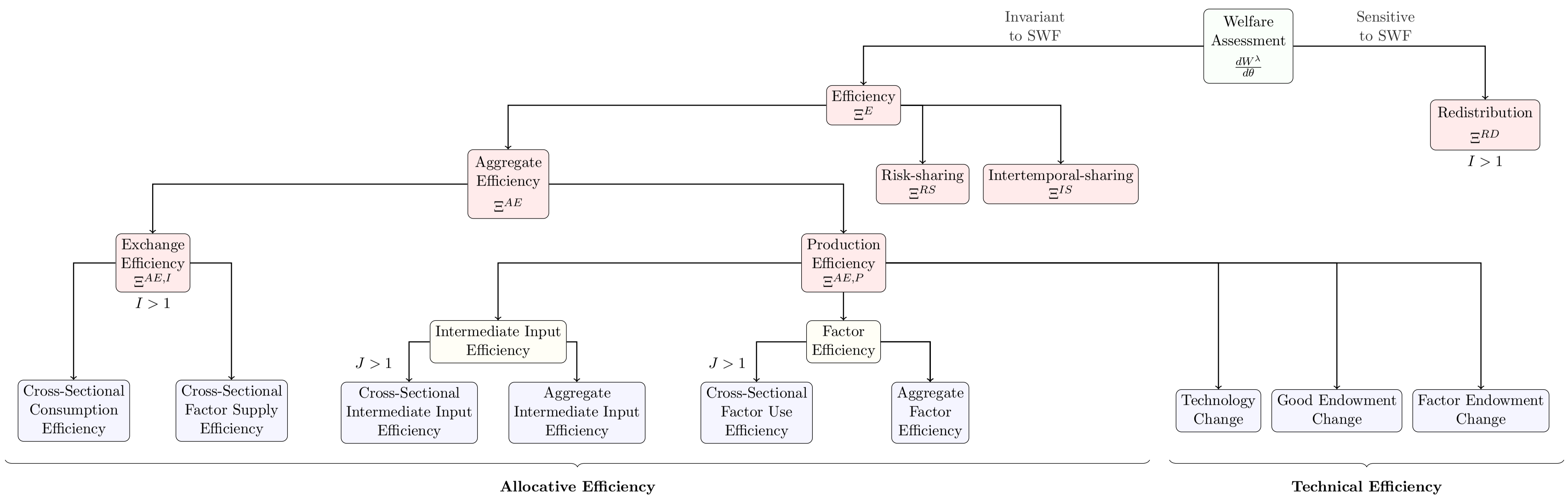


Figure OA-1: Welfare Accounting Decomposition for Dynamic Stochastic Economies

Note: This diagram illustrates the welfare accounting decomposition for dynamic stochastic economies without accumulation technologies.

efficiency, risk-sharing, and intertemporal-sharing:

$$\Xi^E = \sum_i \frac{\frac{dV^i}{d\theta}}{\lambda^i} = \underbrace{\sum_t \omega_t \sum_{s^t} \omega_t\left(s^t\right) \sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}}_{\Xi^{AE}} + \underbrace{\sum_t \omega_t \sum_{s^t} \omega_t\left(s^t\right) \mathbb{C}ov_i^{\Sigma}\left[\frac{\omega_t^i\left(s^t\right)}{\omega_t\left(s^t\right)}, \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right]}_{\Xi^{RS}},$$

$$+ \underbrace{\sum_t \omega_t \mathbb{C}ov_i^{\Sigma}\left[\frac{\omega_t^i}{\omega_t}, \frac{dV_t^{i|\lambda}}{d\theta}\right]}_{\Xi^{IS}},$$

where $\Xi^{AE} = \sum_t \omega_t \sum_{s^t} \omega_t\left(s^t\right) \Xi_t^{AE}\left(s^t\right)$, with

$$\Xi_t^{AE}\left(s^t\right) = \sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} = \sum_j \sum_i MRS_{t,c}^{ij}\left(s^t\right) \frac{dc_t^{ij}\left(s^t\right)}{d\theta} - \sum_f \sum_i MRS_{t,n}^{if}\left(s^t\right) \frac{dn_t^{if,s}\left(s^t\right)}{d\theta},$$

Hence, it follows that

$$\sum_i MRS_{t,c}^{ij}\left(s^t\right) \frac{dc_t^{ij}\left(s^t\right)}{d\theta} = \mathbb{C}ov_i^{\Sigma}\left[MRS_{t,c}^{ij}\left(s^t\right), \frac{d\chi_c^{ij}\left(s^t\right)}{d\theta}\right] c_t^j\left(s^t\right) + AMRS_{t,c}^j\left(s^t\right) \frac{dc_t^j\left(s^t\right)}{d\theta}$$

$$\sum_i MRS_{t,n}^{if}\left(s^t\right) \frac{dn_t^{if,s}\left(s^t\right)}{d\theta} = \mathbb{C}ov_i^{\Sigma}\left[MRS_{t,n}^{if}\left(s^t\right), \frac{d\chi_n^{if,s}\left(s^t\right)}{d\theta}\right] n_t^{f,s}\left(s^t\right) + AMRS_{t,n}^f\left(s^t\right) \frac{dn_t^{f,s}\left(s^t\right)}{d\theta},$$

so from this stage onwards it is possible to follow the steps in the proof of Theorems 1 and 2.

C.2 Planning Problem

The Lagrangian of the planning problem can be expressed as

$$\begin{aligned}
\mathcal{L} &= \mathcal{W}\left(V_1, \ldots, V^i, \ldots, V^I\right) \\
&- \sum_j \zeta_y^j \left(\sum_i c^{ij} + \sum_\ell x^{\ell j} - G^j\left(\left\{x^{j\ell}\right\}_\ell, \left\{n^{jf,d}\right\}_f\right)\right) - \sum_f \zeta_n^f \left(\sum_j n^{jf,d} - \sum_i n^{if,s} - \sum_i \bar{n}^{if,s}\right) \\
&+ \sum_i \sum_j \kappa_c^{ij} c^{ij} + \sum_i \sum_f \kappa_n^{if,s} n^{if,s} + \sum_j \sum_\ell \kappa_x^{j\ell} x^{j\ell} + \sum_j \sum_f \kappa_n^{jf,d} n^{jf,d},
\end{aligned}$$

where V^i is defined in (1). Hence, the first-order conditions can be derived from a perturbation of the form

$$\begin{aligned}
d\mathcal{L} &= \sum_j \sum_i \left(\alpha^i \frac{\partial u^i}{\partial c^{ij}} - \zeta_y^j + \kappa_c^{ij}\right) dc^{ij} + \sum_i \sum_f \left(\alpha^i \frac{\partial u^i}{\partial n^{if,s}} + \zeta_n^f + \kappa_n^{if,s}\right) dn^{if,s} \\
&+ \sum_j \sum_\ell \left(\zeta_y^j \frac{\partial G^j}{\partial x^{j\ell}} - \zeta_y^\ell + \kappa_x^{j\ell}\right) dx^{j\ell} + \sum_j \sum_f \left(\zeta_y^j \frac{\partial G^j}{\partial n^{jf,d}} - \zeta_n^f + \kappa_n^{jf,d}\right) dn^{jf,d},
\end{aligned}$$

where we take good j' as numeraire, which allows us to substitute α^i for $\alpha^i \frac{\partial u^i}{\partial c^{ij'}} = \zeta_y^{j'} \Rightarrow \alpha^i = \left(\frac{\frac{\partial u^i}{\partial c^{j'}}}{\zeta_y^{j'}}\right)^{-1}$, and where we define $MWP_x^{j\ell} = \zeta_y^j \frac{\partial G^j}{\partial x^{j\ell}}$ and $MWP_n^{jf} = \zeta_y^j \frac{\partial G^j}{\partial n^{jf,d}}$. Formally, the Kuhn-Tucker conditions are

i) $\kappa_c^{ij} c^{ij} = 0 \Rightarrow \left(\zeta_y^j - MRS_c^{ij}\right) c^{ij} = 0$, with generically one of the two terms > 0;

ii) $\kappa_n^{if,s} n^{if,s} = 0 \Rightarrow \left(\zeta_n^f + MRS_n^{ij}\right) n^{if,s} = 0$, with generically one of the two terms > 0;

iii) $\kappa_x^{j\ell} x^{j\ell} = 0 \Rightarrow \left(\zeta_y^\ell - MWP_x^{j\ell}\right) x^{j\ell} = 0$, with generically one of the two terms > 0;

iv) $\kappa_n^{if,d} n^{jf,d} = 0 \Rightarrow \left(\zeta_n^f - MWP_n^{jf}\right) n^{jf,d} = 0$, with generically one of the two terms > 0.

By adding up the consumption optimality conditions for all individuals for good j :

$$\sum_i \left(\zeta_y^j - MRS_c^{ij}\right) c^{ij} = 0 \Rightarrow \sum_i MRS_c^{ij} c^{ij} - \zeta_y^j \sum_i c^{ij} \Rightarrow \sum_i MRS_c^{ij} c^{ij} = \zeta_y^j c^j.$$

If $c^j > 0$ (as long as one agent is consuming the good, so good j is final):

$$\zeta_y^j = \sum_i MRS_c^{ij} \frac{c^{ij}}{\sum_i c^{ij}} = \sum_i \chi_c^{ij} MRS_c^{ij} = AMRS_c^\ell.$$

If $c^j = 0$, we must have $\zeta_y^j > MRS_c^{ij}$, for all i, which means that $\zeta_y^j > \max_i \left\{MRS_c^{ij}\right\}$. By adding up the intermediate good optimality conditions for all uses j of good ℓ:

$$\sum_j \left(MWP_x^{j\ell} - \zeta_y^\ell\right) x^{j\ell} = 0 \Rightarrow \sum_j MWP_x^{j\ell} x^{j\ell} - \zeta_y^\ell \sum_j x^{j\ell} \Rightarrow \sum_j MWP_x^{j\ell} x^{j\ell} = \zeta_y^\ell x^\ell.$$

If $x^\ell > 0$ (as long as one good j uses good ℓ as input, so good ℓ is intermediate):

$$\zeta_y^\ell = \sum_j MWP_x^{j\ell} \frac{x^{j\ell}}{\sum_j x^{j\ell}} = \sum_j \chi_x^{j\ell} MWP_x^{j\ell} = AMWP_x^\ell.$$

If $x^\ell = 0$, we must have $\zeta_y^\ell > MWP_x^{j\ell}$, for all j, which means that $\zeta_y^\ell > \max_j \left\{MWP_x^{j\ell}\right\}$. Combining consumption and intermediate good optimality:

$$\sum_i MRS_c^{ij} c^{i\ell} + \sum_j MWP_x^{j\ell} x^{j\ell} = \zeta_y^\ell y^\ell,$$

so if $y^\ell > 0$, it must be that $\zeta_y^\ell = AMRS_c^\ell \phi_c^\ell + \sum_j \zeta_y^j \frac{\partial G^j}{\partial x^{j\ell}} \xi^{j\ell}$, which can be written in matrix form as

$$\boldsymbol{\zeta}_y = \boldsymbol{AMRS}_c \boldsymbol{\phi}_c \boldsymbol{\Psi}_y, \quad \text{where} \quad \boldsymbol{\Psi}_y = \left(\boldsymbol{I}_J - \boldsymbol{G}_x \boldsymbol{\xi}\right)^{-1}.$$

Similarly, for factors, if $n^{f,s} > 0$ (as long as one agent is supplying factor f):

$$\zeta_n^f = \sum_i \frac{n^{if,s}}{n^{f,s}} MRS_n^{ij} = \sum_i \chi_n^{if,s} MRS_n^{ij} = AMRS_n^f$$

If $n^{f,s} = 0$, we must have $\zeta_n^f < MRS_n^{ij}$, for all i, which means that $\zeta_n^f < \max_i \left\{MRS_n^{ij}\right\}$. If $n^{jf,d} > 0$ (as long as factor f is used to produce a good j):

$$\zeta_n^f = \sum_j MWP_n^{jf} \frac{n^{jf,d}}{n^{f,d}} = \sum_j MWP_n^{jf} \chi_n^{jf,d} = AMWP_n^{jf}$$

If $n^{jf,d} = 0$, we must have $\zeta_n^f > \sum_j MWP_n^{jf} \chi_n^{jf,d}$, for all j, which means that $\zeta_n^f > \max_j \left\{MWP_n^{jf}\right\}$. If $n^{f,s} > 0$ and $n^{f,d} > 0$: $AMWP_n^f = AMRS_n^f$. If $n^{f,s} = 0$, it must be that $\zeta_n^f < MRS_n^{if}$, or $\zeta_n^f < \min_i \left\{MRS_n^{if}\right\}$. If $n^{f,d} = 0$, it must be that $MWP_n^{jf} < \zeta_n^f$, or $\max_j \left\{MWP_n^{jf}\right\} < \zeta_n^f$. Hence, for $n^{f,s} = 0 = n^{f,d}$, we must have that $\max_j \left\{MWP_n^{jf}\right\} < \min_i \left\{MRS_n^{if}\right\}$. Finally, for $y^j = 0$ to be optimal, it must be that $c^j = x^{\ell j} = 0$ on the use side and $x^{j\ell} = n^{jf,d} = 0$ on the input side. This condition can be written as

$$\max\left\{\max_i\left\{\frac{\partial u^i}{\partial c^{ij}}\right\},\max_\ell\left\{\zeta_y^\ell\frac{\partial G^\ell}{\partial x^{\ell j}}\right\}\right\}<\zeta_y^j<\min\left\{\min_f\left\{\left(\frac{\partial G^j}{\partial n^{jf,d}}\right)^{-1}\zeta_n^f\right\},\min_\ell\left\{\left(\frac{\partial G^j}{\partial x^{j\ell}}\right)^{-1}\zeta_y^\ell\right\}\right\}.$$

C.3 Propagation Matrices

Intermediate inverse matrix. Following similar steps as in the Proof of Lemma 2, we can express changes in intermediate input use as follows. Using both equations in (OA1), we can instead solve for $\frac{dx}{d\theta}$ as follows

$$\frac{d\boldsymbol{x}}{d\theta}=\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y}+\boldsymbol{\xi}\left(\boldsymbol{G}_x\frac{d\mathring{\boldsymbol{x}}}{d\theta}+\boldsymbol{G}_n\frac{d\mathring{\boldsymbol{n}}^d}{d\theta}+\frac{d\bar{\boldsymbol{y}}^s}{d\theta}+\boldsymbol{G}_\theta\right),$$

so we can define a $JL\times JL$ propagation matrix in the space of intermediate links $\boldsymbol{\Psi}_x$:

$$\frac{d\boldsymbol{x}}{d\theta}=\underbrace{\boldsymbol{\Psi}_x}_{\text{Propagation}}\underbrace{\left(\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y}+\boldsymbol{\xi}\left(\boldsymbol{G}_n\frac{d\mathring{\boldsymbol{n}}^d}{d\theta}+\frac{d\bar{\boldsymbol{y}}^s}{d\theta}+\boldsymbol{G}_\theta\right)\right)}_{\text{Impulse}},\quad\text{where}\quad\boldsymbol{\Psi}_x=\left(\boldsymbol{I}_{JL}-\boldsymbol{\xi}\boldsymbol{G}_x\right)^{-1}.\tag{OA2}$$

Propagation in the space of goods and the space of intermediate links is connected. In particular, Woodbury's identity implies that

$$\boldsymbol{\Psi}_x=\boldsymbol{I}_{JL}+\boldsymbol{\xi}\boldsymbol{\Psi}_y\boldsymbol{G}_x,$$

and it is also the case that

$$\boldsymbol{\Psi}_x\boldsymbol{\xi}=\boldsymbol{\xi}\boldsymbol{\Psi}_y,$$

connecting propagation in the space of goods and the space of intermediate links. Leveraging (OA2), it is possible to solve for changes in consumption as

$$\begin{aligned}\frac{d\boldsymbol{c}}{d\theta}&=\frac{d\boldsymbol{y}}{d\theta}-\frac{d\boldsymbol{x}}{d\theta}=\boldsymbol{G}_x\frac{d\boldsymbol{x}}{d\theta}+\boldsymbol{G}_n\frac{d\mathring{\boldsymbol{n}}^d}{d\theta}+\boldsymbol{G}_\theta-\frac{d\boldsymbol{x}}{d\theta}\\&=\left(\boldsymbol{G}_x-\boldsymbol{1}_x\right)\boldsymbol{\Psi}_x\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y}+\left(\left(\boldsymbol{G}_x-\boldsymbol{1}_x\right)\boldsymbol{\Psi}_x\boldsymbol{\xi}+\boldsymbol{I}_J\right)\boldsymbol{G}_n\frac{d\mathring{\boldsymbol{n}}^d}{d\theta}+\left(\left(\boldsymbol{G}_x-\boldsymbol{1}_x\right)\boldsymbol{\Psi}_x\boldsymbol{\xi}+\boldsymbol{I}_J\right)\boldsymbol{G}_\theta.\end{aligned}$$

Proportional goods inverse matrix. While the goods inverse is expressed in levels, at times, it may be useful to work with proportional propagation matrix. Starting from the definition of $\frac{dy}{d\theta}$, it follows that

$$\begin{aligned}\hat{\boldsymbol{y}}^{-1}\frac{d\boldsymbol{y}}{d\theta}&=\hat{\boldsymbol{y}}^{-1}\boldsymbol{\Psi}_y\hat{\boldsymbol{y}}\left(\hat{\boldsymbol{y}}^{-1}\boldsymbol{G}_x\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y}+\hat{\boldsymbol{y}}^{-1}\boldsymbol{G}_n\frac{d\mathring{\boldsymbol{n}}^d}{d\theta}+\hat{\boldsymbol{y}}^{-1}\boldsymbol{G}_\theta\right)\\&=\tilde{\boldsymbol{\Psi}}_y\left(\hat{\boldsymbol{y}}^{-1}\boldsymbol{G}_x\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y}+\hat{\boldsymbol{y}}^{-1}\boldsymbol{G}_n\frac{d\boldsymbol{n}^d}{d\theta}+\hat{\boldsymbol{y}}^{-1}\boldsymbol{G}_\theta\right),\end{aligned}$$

where

$$\tilde{\boldsymbol{\Psi}}_y=\hat{\boldsymbol{y}}^{-1}\boldsymbol{\Psi}_y\hat{\boldsymbol{y}}.$$

In the competitive case, $\boldsymbol{\Psi}_y=\hat{\boldsymbol{y}}\left(\hat{\boldsymbol{p}}\hat{\boldsymbol{y}}-\left(\mathbb{I}_x+\tilde{\boldsymbol{\tau}}_x\right)\check{\boldsymbol{p}}\check{\boldsymbol{x}}\right)^{-1}\hat{\boldsymbol{p}}$ and $\tilde{\boldsymbol{\Psi}}_y=\left(\hat{\boldsymbol{p}}\hat{\boldsymbol{y}}-\left(\mathbb{I}_x+\tilde{\boldsymbol{\tau}}_x\right)\check{\boldsymbol{p}}\check{\boldsymbol{x}}\right)^{-1}\hat{\boldsymbol{p}}\hat{\boldsymbol{y}}$, where we define a $JL\times JL$ matrix of prices as $\check{\boldsymbol{p}}=\hat{\boldsymbol{p}}\otimes\boldsymbol{I}_J$, where $\tilde{\boldsymbol{\tau}}_x$ is a $J\times JL$ matrix analogous to $\bar{\boldsymbol{\tau}}_x$, but with the same ordering as the $J \times JL$ matrix $\mathbb{I}_x$, given by

$$\mathbb{I}_x = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}_{J \times JL},$$

and where we define an alternative $JL \times J$ matrix of intermediates uses $\check{\boldsymbol{x}}$, given by

$$\check{\boldsymbol{x}} = \begin{pmatrix} x^{11} & 0 & 0 \\ x^{21} & 0 & 0 \\ x^{31} & 0 & 0 \\ 0 & x^{12} & 0 \\ 0 & x^{22} & 0 \\ 0 & x^{32} & 0 \\ 0 & 0 & x^{13} \\ 0 & 0 & x^{23} \\ 0 & 0 & x^{33} \end{pmatrix}_{JL \times J}.$$

Regularity conditions for goods inverse matrix. In order to provide conditions under which the inversion step to define the goods inverse is valid, we can appeal to the Perron–Frobenius theory of non-negative matrices. If production functions have constant returns to scale, then by the homogeneous function theorem, we have that

$$y^{j,s} = \sum_k \frac{\partial G^j}{\partial x^{jk}} x^{jk} + \sum_f \frac{\partial G^j}{\partial n^{jf,d}} n^{jf,d} \quad \Rightarrow \quad 1 = \sum_k \frac{\partial \log G^j}{\partial \log x^{jk}} + \sum_f \frac{\partial \log G^j}{\partial \log n^{jf,d}}.$$

This implies that the matrix (here represented for $J = 2$ case)

$$\hat{\boldsymbol{y}}^{-1} \boldsymbol{G}_x \boldsymbol{\xi} \hat{\boldsymbol{y}} = \begin{pmatrix} \frac{\partial \log G^1}{\partial \log x^{11}} & \frac{\partial \log G^1}{\partial \log x^{12}} \\ \frac{\partial \log G^2}{\partial \log x^{21}} & \frac{\partial \log G^2}{\partial \log x^{22}} \end{pmatrix},$$

features rows whose sum can be written as

$$r^j = \sum_k \frac{\partial \log G^j}{\partial \log x^{jk}} < 1.$$

Hence, this result implies that the spectral radius (maximum of the absolute value of eigenvalues) of $\hat{\boldsymbol{y}}^{-1} \boldsymbol{G}_x \boldsymbol{\xi} \hat{\boldsymbol{y}}$ is less than 1, so the Neumann series lemma concludes that the proportional goods inverse is well defined (Meyer, 2023). It is possible to derive bounds of convergence, so that the sectors with lowest and highest intermediate shares drive the speed of convergence. Convergence of the proportional goods inverse is sufficient for convergence of the the goods inverse. Hence, the goods inverse exists in economies with constant or decreasing returns to scale.

C.4 Alternative Redistribution Decomposition

Theorem 10. *(Redistribution Decomposition in Competitive Economies). In competitive economies with wedges, the redistribution component of the welfare accounting decomposition, Ξ^{RD}, can be decomposed into* *distributive pecuniary and distortionary redistribution components, given by*

$$
\begin{aligned}
\Xi^{RD} = {} & \overbrace{\mathbb{C}ov_i^{\Sigma}\left[\omega^i, -\sum_j \frac{dp^j}{d\theta} c^{ij} + \sum_f \frac{dw^f}{d\theta}\left(n^{if,s} + \bar{n}^{if,s}\right) + \sum_j \nu^{ij} \frac{d\pi^j}{d\theta}\right]}^{\text{Distributive Pecuniary Redistribution}} \\
& \underbrace{+\,\mathbb{C}ov_i^{\Sigma}\left[\omega^i, \sum_j p^j \tau_c^{ij} \frac{dc^{ij}}{d\theta} - \sum_f w^f \tau_n^{if,s}\left(\frac{dn^{if,s}}{d\theta} + \frac{d\bar{n}^{if,s}}{d\theta}\right)\right]}_{\text{Distortionary Redistribution}}.
\end{aligned}
$$

The distributive pecuniary redistribution component captures the differential impact of changes in prices and profits on individual welfare - these are the distributive pecuniary effects present in any competitive economy. Intuitively, if a perturbation reduces the prices of goods consumed or increases the income earned by individuals with high ω^i, the distributive pecuniary redistribution component will be positive. Importantly, in the absence of technology and endowment changes, the sum across individuals of distributive pecuniary effects is zero (see e.g. Dávila and Korinek, 2018).

The distortionary redistribution component captures the differential impact on individual welfare of changes in the allocation of goods and factors that are distorted by individual wedges. This component is positive if a perturbation reallocates consumption (factor supply) towards individuals with higher consumption (lower factor supply) wedges. When $\tau_c^{ij} > 0$, for instance, individual i consumes too little of good j. An increase in c^{ij} when individual i is relatively favored by the planner contributes positively to the distortionary redistribution component. In contrast to the distributive effects, the sum across individuals of distortionary redistribution effects will typically not be zero.

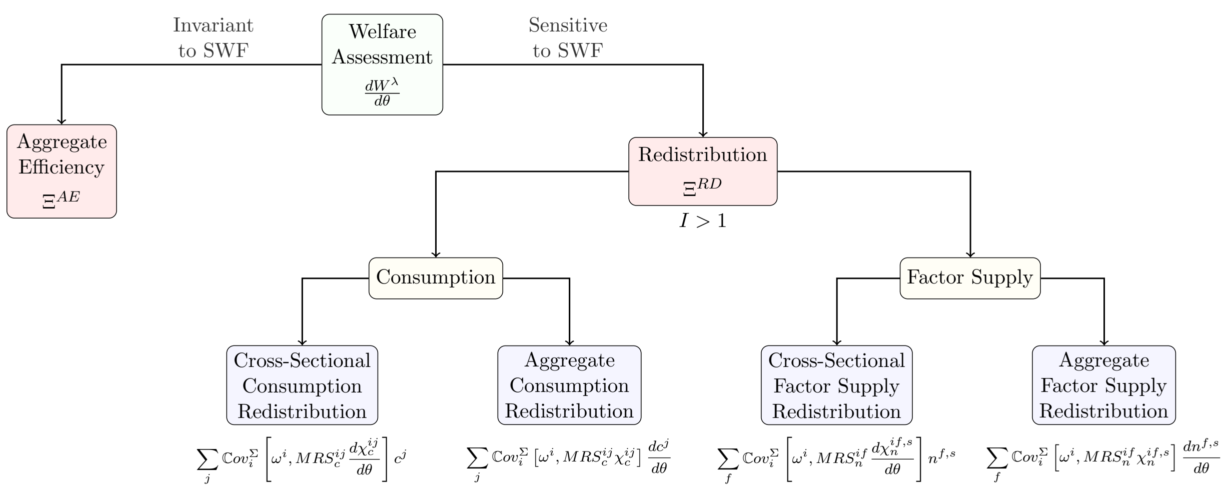


Figure OA-2: Welfare Accounting Decomposition: Redistribution

C.5 Welfare Accounting vs. Growth Accounting

Here we discuss the relation between welfare accounting, as developed in this paper, and the well-established approach of growth accounting. Growth accounting measures the contribution of different inputs to final output (i.e. aggregate consumption), indirectly computing technological growth as a residual. Instead, welfare accounting attributes aggregate welfare gains to different sources, which brings it closer to the "beyond GDP" literature (Fleurbaey, 2009; Jones and Klenow, 2016).

Heuristically, the welfare accounting decomposition can be expressed as

$$\text{Welfare} = \text{Exchange Efficiency} + \underbrace{\text{Final Output} - \text{Factor Supply Cost}}_{\text{Production Efficiency}} + \text{Redistribution},$$

where the goal is to compute welfare changes by computing or measuring all right-hand side elements. Instead, growth accounting abstracts from exchange efficiency, factor supply costs, and redistribution, and exploits a relation of the form

$$\text{Final Output} = \text{Intermediate Inputs} + \text{Factors} + \text{Technology}, \tag{OA3}$$

where the goal is to measure both final output (left-hand side) and the intermediate input and factor components (part of the right-hand side) to back out the technology component. These are distinct exercises which are nonetheless related. For instance, when $I = 1$, exchange efficiency and redistribution are zero, and when factors are not supplied by individuals, the welfare cost of factor supply is also zero. In that case, welfare and final output are identical.

Moreover, when directly measuring the components of the welfare accounting decomposition, growth accounting can be used to measure technology growth. Through the lens of the welfare accounting decomposition, the adequate counterpart of the growth accounting relation in (OA3), solving for the technology change component, is

$$\underbrace{\boldsymbol{MSV}_y \boldsymbol{G}_\theta}_{\text{Technology Change}} = \underbrace{\boldsymbol{AMRS}_c \frac{d\boldsymbol{c}}{d\theta}}_{\text{Final Output}} - \underbrace{\left(\boldsymbol{AMWP}_x - \boldsymbol{AMRS}_c\right) \frac{d\boldsymbol{\phi}_x}{d\theta} \boldsymbol{y}}_{\text{Intermediate Input Use}} - \underbrace{\boldsymbol{MWP}_n \frac{d\boldsymbol{n}^d}{d\theta}}_{\text{Factor Use}}, \tag{OA4}$$

where $\boldsymbol{AMRS}_c \frac{dc}{d\theta}$ becomes the welfare-relevant change in final output, which is a welfare-analog of GDP. Equation (OA4) is stated exclusively in terms of preferences and technologies. Additional assumptions about market structure would make it possible to conduct a growth accounting exercise by measuring all right-hand side components of (OA4), a task we leave for future work.

D Minimal Welfare Accounting Economy: Special Cases

For simplicity we assume that good endowments are zero.

D.1 Minimal Welfare Accounting Economy

The minimal welfare accounting economy features two individuals, two goods, and single factor in elastic supply: $I = 2$, $J = 2$, and $F = 1$. Individual preferences take the form $V^1 = u^1\left(c^{11}, c^{12}, n^{11,s}\right)$ and $V^2 = u^2\left(c^{21}, c^{22}, n^{21,s}\right)$ and technologies are given by $y^1 = G^1\left(x^{11}, x^{12}, n^{11,d}; \theta\right)$ and $y^2 = G^2\left(x^{21}, x^{22}, n^{21,d}; \theta\right)$. Finally, resource constraints are simply given by $y^1 = c^{11} + c^{21} + x^{11} + x^{21}$ and $y^2 = c^{12} + c^{22} + x^{12} + x^{22}$ and $n^{11,s} + n^{21,s} + \bar{n}^{11,s} + \bar{n}^{21,s} = n^{11,d} + n^{21,d}$. In this economy, all of the components of aggregate efficiency can be non-zero, as we illustrate in a series of special cases.[45]

D.2 Vertical Economy

This minimal vertical economy is a special case of the minimal welfare accounting economy. In this economy, there is a single individual who consumes a final good produced using an intermediate good, which is in turn produced by a single factor in fixed supply, so $I = 1$, $J = 2$, and $F = 1$. This is the simplest economy that illustrates the role played by pure intermediate goods. In this economy, individual preferences are given by $V^1 = u^1\left(c^{11}\right)$, technologies by $y^1 = G^1\left(x^{12}; \theta\right)$ and $y^2 = G^2\left(n^{21,d}; \theta\right)$, and resource constraints by $y^1 = c^{11}$, $y^2 = x^{12}$, and $\bar{n}^{1,s} = n^{21,d}$. By construction, all allocative efficiency components of the welfare accounting decomposition are zero, so this economy exclusively features technology and endowment change components.

Aggregate and production efficiency are given by

$$\Xi^{AE} = \Xi^{AE,P} = MSV_y^1 \frac{G^1}{\partial \theta} + MSV_y^2 \frac{G^2}{\partial \theta} + MSV_y^1 \frac{\partial G^1}{\partial n^{11,d}} \frac{d\bar{n}^{1,s}}{d\theta},$$

where $MSV_y^1 = MRS_c^{11}$ and $MSV_y^2 = MRS_c^{11} \frac{\partial G^1}{\partial x^{12}}$. In this economy, an efficient allocation must satisfy $MRS_c^{11} > 0$ and $MRS_c^{11} \frac{\partial G^1}{\partial x^{12}} > 0$.

D.3 Robinson Crusoe Economy

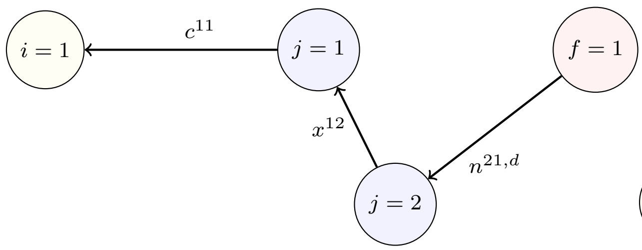
One-producer one-consumer economies (i.e., Robinson Crusoe economies) are the simplest to study production - see Section 15.C of Mas-Colell et al. (1995). In these economies, a single individual consumes a single good and elastically supplies a single factor of production. A single production technology uses the supplied factor to produce the good, so $I = 1$, $J = 1$, and $F = 1$. Formally, preferences, technology, and resource constraints are respectively given by $V^1 = u^1\left(c^{11}, n^{11,s}\right)$, $y^1 = G^1\left(n^{11,d}; \theta\right)$, $y^1 = c^{11}$, and $n^{11,s} = n^{11,d}$. This economy exclusively features aggregate factor efficiency and technology change components.

The production efficiency decomposition takes the form

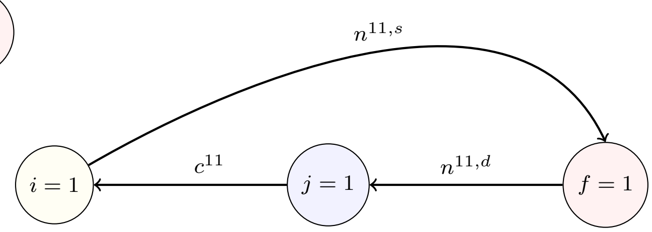
$$\Xi^{AE,P} = \left(\underbrace{MSV_y^1 \frac{\partial G^1}{\partial n^{11,d}}}_{AMWP_n^1} - \underbrace{MRS_n^{11}}_{AMRS_n^1} \right) \frac{dn^{1,s}}{d\theta} + MSV_y^1 \frac{\partial G^1}{\partial \theta},$$

where the marginal social value of good 1 is given by $MSV_y^1 = MRS_c^{11}$. In this economy, an efficient allocation must satisfy $MSV_y^1 \frac{\partial G^1}{\partial n^{11,d}} = MRS_n^{11}$.

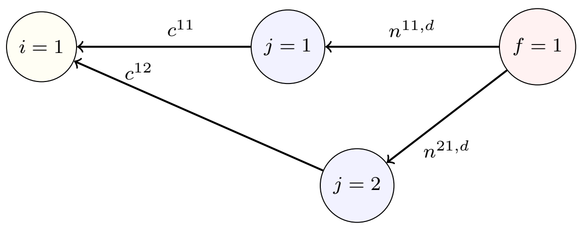
[45] At times, it is necessary to have $J = 3$ goods to represent some phenomena in production networks. For instance, three goods are necessary to have a pure intermediate good being used to produce another pure intermediate good. This is a relevant case in which classic efficiency conditions do not apply, as illustrated in examples 1 and 2.



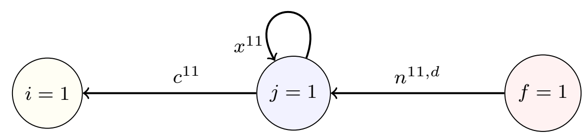
(a) Minimal Vertical



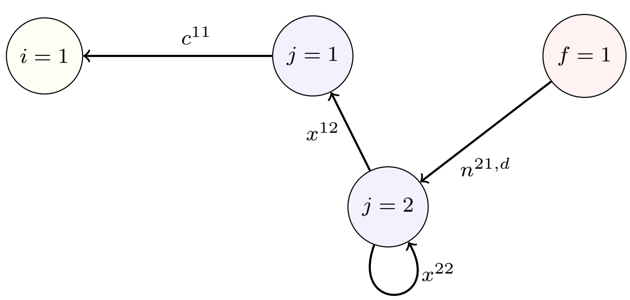
(b) Robinson Crusoe



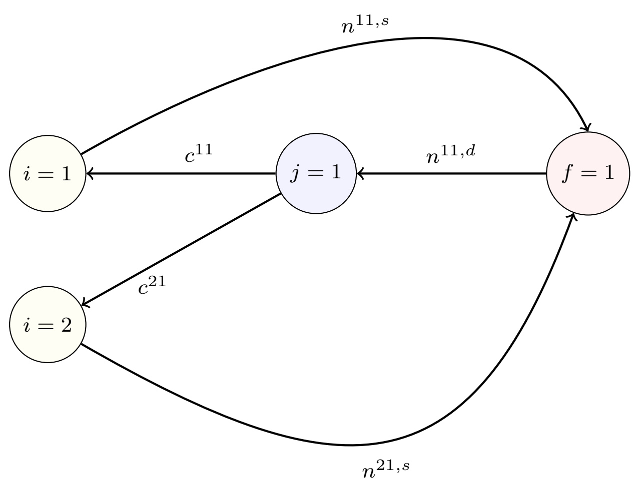
(c) Minimal Horizontal



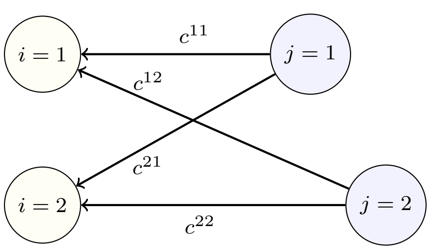
(d) Minimal Roundabout



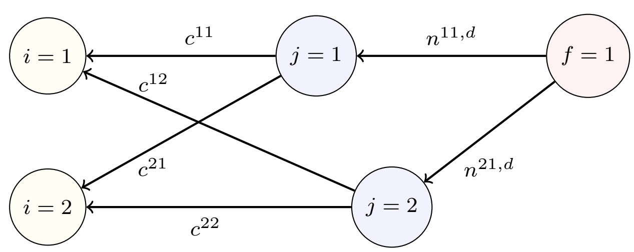
(e) Minimal Diversified Intermediate



(f) Minimal Multiple Factor Suppliers



(g) Edgeworth Box



(h) Edgeworth Box (alternative)

Figure OA-3: Minimal Welfare Accounting Economy: Special Cases

D.4 Horizontal Economy

This minimal horizontal economy is the simplest to illustrate the role played by the possibility of reallocating factors across different uses. This economy generalizes to many well-known frameworks, including Heckscher-Ohlin, Armington (1969), and Hsieh and Klenow (2009). In this economy, a single individual consumes two different goods that can be produced using the same factor, which we assume to be in fixed supply, so $I = 1$, $J = 2$, and $F = 1$. Formally, preferences, technology, and resource constraints are given by $V^1 = u^1\left(c^{11}, c^{12}\right)$, $y^1 = G^1\left(n^{11,d}; \theta\right)$, $y^2 = G^2\left(n^{21,d}; \theta\right)$, $y^1 = c^{11}$, $y^2 = c^{12}$, and $\bar{n}^{1,s} = n^{11,d} + n^{21,d}$. This economy exclusively features cross-sectional factor efficiency and technology and endowment change components

The production efficiency decomposition takes the form

$$\Xi^{AE,P} = \mathbb{C}ov_j^{\Sigma}\left[\underbrace{MSV_y^j \frac{\partial G^j}{\partial n^{j1,d}}}_{MWP_n^{j1}}, \frac{d\chi_n^{j1,d}}{d\theta}\right] n^{1,d} + MSV_y^1 \frac{\partial G^1}{\partial \theta} + MSV_y^2 \frac{\partial G^2}{\partial \theta} + AMWP_n^1 \frac{d\bar{n}^{1,s}}{d\theta},$$

where $AMWP_n^1 = \chi_n^{11,d} MSV_y^1 \frac{\partial G^1}{\partial n^{11,d}} + \chi_n^{21,d} MSV_y^2 \frac{\partial G^2}{\partial n^{21,d}}$, and where $MSV_y^1 = MRS_c^{11}$ and $MSV_y^2 = MRS_c^{12}$. In this economy, an efficient allocation must satisfy $MSV_y^1 \frac{\partial G^1}{\partial n^{11,d}} = MSV_y^2 \frac{\partial G^2}{\partial n^{21,d}}$.

D.5 Minimal Roundabout Economy

Roundabout economies have been used to illustrate the impact of intermediate goods on production—see e.g., Jones (2011). The minimal roundabout economy is the simplest economy in which aggregate intermediate input efficiency can exist. In this economy a single individual consumes a single mixed good, which is at the same time final and intermediate to itself, so $I = 1$, $J = 1$, and $F = 1$. Formally, preferences, technology, and resource constraints are given by $V^1 = u^1\left(c^{11}\right)$, $y^1 = G^1\left(x^{11}, n^{11,d}; \theta\right)$, $y^1 = c^{11} + x^{11}$, and $\bar{n}^{1,s} = n^{11,d}$. This economy only features aggregate intermediate input efficiency, and technology and endowment change components.

The production efficiency decomposition takes the following form

$$\Xi^{AE,P} = \left(\underbrace{MSV_y^1 \frac{\partial G^1}{\partial x^{11}}}_{AMWP_x^1} - \underbrace{MRS_c^{11}}_{AMRS_c^1}\right) \frac{d\phi_x^1}{d\theta} y^1 + MSV_y^1 \frac{\partial G^1}{\partial \theta} + AMWP_n^1 \frac{d\bar{n}^{1,s}}{d\theta},$$

where $AMWP_n^1 = MSV_y^1 \frac{\partial G^1}{\partial n^{11,d}}$ and $MSV_y^1 = \frac{MRS_c^{11}}{1 - \xi^{11} \frac{\partial G^1}{\partial x^{11}}}$. In this economy, an efficient allocation must satisfy $MSV_y^1 = MSV_y^1 \frac{\partial G^1}{\partial x^{11}} = MRS_c^{11}$.

D.6 Diversified Intermediate

This minimal diversified intermediate economy is the simplest economy in which cross-sectional intermediate input efficiency can exist. In this economy, a single individual consumes a final good, which is exclusively produced by a pure intermediate that can be also used for roundabout production. This pure intermediate is produced using a single factor in fixed supply, so $I = 1$, $J = 2$, and $F = 1$. Formally, preferences, technology, and resource constraints are given by $V^1 = u^1\left(c^{11}\right)$, $y^1 = G^1\left(x^{12}; \theta\right)$, $y^2 = G^2\left(x^{22}, n^{21,d}; \theta\right)$, $y^1 = c^{11}$, $y^2 = c^{12} + x^{12} + x^{22}$, and $\bar{n}^{1,s} = n^{21,d}$. This economy features cross-sectional intermediate input efficiency, aggregate intermediate input efficiency, and technology and endowment change components.

The production efficiency decomposition takes the form

$$\Xi^{AE,P} = \mathbb{C}ov_j^{\Sigma}\left[\underbrace{MSV^j \frac{\partial G^j}{\partial x^{j2}}}_{MWP_x^{j2}}, \frac{d\chi_n^{j2}}{d\theta}\right] x^2 + \underbrace{\left(\chi_x^{12} MSV_y^1 \frac{\partial G^1}{\partial x^{12}} + \chi_x^{22} MSV_y^2 \frac{\partial G^2}{\partial x^{22}}\right)}_{AMWP_x^2} \frac{d\phi_x^2}{d\theta} y^2$$
$$+ MSV_y^1 \frac{\partial G^1}{\partial \theta} + MSV_y^2 \frac{\partial G^2}{\partial \theta} + \underbrace{\left(MSV_y^2 \frac{\partial G^2}{\partial n^{21,d}}\right)}_{AMWP_n^1} \frac{d\bar{n}^{1,s}}{d\theta},$$

where $MSV_y^1 = MRS_c^{11}$ and $MSV_y^2 = MRS_c^{11} \frac{\frac{\partial G^1}{\partial x^{12}} \xi^{12}}{1 - \xi^{22} \frac{\partial G^2}{\partial x^{22}}}$.

D.7 Two Factor Supplier Economy

This minimal two factor supplier economy (we could also call it Robinson Crusoe and Friday economy) is the simplest economy in which cross-sectional factor supply efficiency can exist. In this economy, we assume that two individuals have identical linear preferences for consumption of a single produced good, which we use as numeraire. This eliminates potential gains from cross-sectional consumption efficiency, since $MRS_c^{11} = MRS_c^{21} = 1$. We also assume that there is a single production technology that uses a single factor that can be supplied either of the two individuals, with in principle different disutility, so $I = 2$, $J = 1$, and $F = 1$. Formally, preferences, technology, and resource constraints are given by $V^1 = c^{11} + u^1\left(n^{11,s}\right)$, $V^2 = c^{21} + u^2\left(n^{21,s}\right)$, $y^1 = G^1\left(n^{11,d}; \theta\right)$, $y^1 = c^{11} + c^{21}$, and $n^{11,s} + n^{21,s} = n^{31,d}$. This economy features cross-sectional factor supply efficiency, aggregate factor efficiency, and technology change components.

The exchange efficiency decomposition takes the form

$$\Xi^{AE,X} = -\mathbb{C}ov_i^{\Sigma}\left[MRS_n^{i1}, \frac{d\chi_n^{i1,s}}{d\theta}\right] n^{1,s}.$$

The production efficiency decomposition takes the form

$$\Xi^{AE,P} = \left(\underbrace{MSV_y^1 \frac{\partial G^1}{\partial n^{11,d}}}_{AMWP_n^1} - \underbrace{\left(\chi_n^{11,s} MRS_n^{11} + \chi_n^{21,s} MRS_n^{21}\right)}_{AMRS_n^1}\right) \frac{dn^{1,s}}{d\theta} + MSV_y^1 \frac{\partial G^1}{\partial \theta} + AMWP_n^1 \frac{d\bar{n}^{1,s}}{d\theta},$$

where $AMWP_n^1 = MSV_y^1 \frac{\partial G^1}{\partial n^{11,d}}$ where the marginal social value of good 1 is

$$MSV_y^1 = \chi_c^{11} MRS_c^{11} + \chi_c^{21} MRS_c^{21} = 1.$$

D.8 Edgeworth Box Economy

Pure exchange economies (i.e., Edgeworth Box economies) are the simplest to study most phenomena in general equilibrium and welfare economics. In this economy, two individuals consume two different goods, which appear as endowments. It is possible to formalize endowments by assuming that there is a single factor in fixed supply and that factor uses are predetermined, so $I = 2$, $J = 2$, and $F = 1$. Formally, preferences, technologies, and resource constraints are respectively given by $V^1 = u^1\left(c^{11}, c^{12}\right)$, $V^2 = u^2\left(c^{21}, c^{22}\right)$, $y^1 = G^1\left(n^{11,d}; \theta\right)$, $y^2 = G^2\left(n^{21,d}; \theta\right)$, $y^1 = c^{11} + c^{21}$, $y^2 = c^{12} + c^{22}$, and $\bar{n}^{1,s} = n^{11,d} + n^{12,d}$. This economy features cross-sectional consumption efficiency, and technology and endowment change components, where the last two can be interpreted as changes in endowments. Alternatively, we could simply model endowments of the goods.

The exchange efficiency component takes the form

$$\Xi^{AE,X} = \mathbb{C}ov_i^{\Sigma}\left[MRS_c^{i1}, \frac{d\chi_c^{i1}}{d\theta}\right]c^1 + \mathbb{C}ov_i^{\Sigma}\left[MRS_c^{i2}, \frac{d\chi_c^{i2}}{d\theta}\right]c^2.$$

The production efficiency component takes the form

$$\Xi^{AE,P} = MSV_y^1\frac{\partial G^1}{\partial\theta} + MSV_y^2\frac{\partial G^2}{\partial\theta} + AMWP_n^1\frac{d\bar{n}^{1,s}}{d\theta} + AMWP_n^2\frac{d\bar{n}^{2,s}}{d\theta}.$$

where the marginal social value of goods is

$$\boldsymbol{MSV}_y = \begin{pmatrix}\chi_c^{11}MRS_c^{11} + \chi_c^{21}MRS_c^{21} & \chi_c^{12}MRS_c^{12} + \chi_c^{22}MRS_c^{22}\end{pmatrix}.$$

E Applications

E.1 Armington (1969) Model

Model Solution. First, note that country profits are given by $\pi^j = \left(p^j A^j - w^j\right) n^{jj,s}$, where we already impose that $j = f$. Hence, profit maximization requires that $p^j = \frac{w^j}{A^j}$. Without loss of generality, we normalize $p^1 = 1$, so $w^1 = A^1$. We also assume that $\tau^{ii} = 0$ and $\tau^{ij} = \tau^{ji} = \tau$.

Hence, exploiting Walras' law, an equilibrium of the model can be expressed as the solution to the system

$$
\begin{aligned}
\frac{c^{11}}{c^{12}} &= \left(\frac{1}{p^2\left(1+\tau\right)}\right)^{-\sigma} \quad \text{and} \quad \frac{c^{21}}{c^{22}} = \left(\frac{1+\tau}{p^2}\right)^{-\sigma} \\
A^1 &= c^{11} + c^{21} \quad \text{and} \quad A^2 = c^{12} + c^{22} \\
p^2 A^2 &= c^{21} + p^2 c^{22},
\end{aligned}
$$

for $\left\{c^{11}, c^{12}, c^{21}, c^{22}, p^2\right\}$. If instead we had assumed that countries have endowments of goods, then their budget constraints take the form

$$
\sum_j p^j \left(1+\tau^{ij}\right) c^{ij} = p^i \bar{y}^{i,s} + \sum_j T^{ij},
$$

which is equivalent to the formulation in the text when $A^i = \bar{y}^{i,s}$. Hence, our parameterization implies that country 2's good is 50 times more abundant than country 1's.

Welfare Accounting Decomposition. Country i's welfare gains induced by a perturbation take the form

$$
\frac{dV^{i|\lambda}}{d\tau} = \frac{\frac{dV^i}{d\tau}}{\lambda^i} = \sum_j \frac{\frac{\partial V^i}{\partial c^{ij}}}{\lambda^i} \frac{dc^{ij}}{d\tau} = \sum_j MRS_c^{ij} \frac{dc^{ij}}{d\tau} = \sum_j MRS_c^{ij} \frac{d\chi_c^{ij}}{d\tau} c^j,
$$

where $MRS_c^{ij} = \frac{\partial V^i}{\partial c^{ij}} / \lambda^i$, $\frac{dc^{ij}}{d\tau} = \frac{d\chi_c^{ij}}{d\tau} c^j + \chi_c^{ij} \frac{dc^j}{d\tau}$, and $\frac{dc^j}{d\tau} = 0$.

We can therefore expresses the normalized welfare gain as

$$
\begin{aligned}
\frac{dW^\lambda}{d\tau} = \frac{\frac{dW}{d\tau}}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i} &= \sum_i \omega^i \sum_j MRS_c^{ij} \frac{d\chi_c^{ij}}{d\tau} c^j \\
&= \underbrace{\sum_j \mathbb{C}ov_i^\Sigma \left[MRS_c^{ij}, \frac{d\chi_c^{ij}}{d\tau}\right] c^j}_{\text{Cross-Sectional Consumption Efficiency}} + \underbrace{\sum_j \mathbb{C}ov_i^\Sigma \left[\omega^i, MRS_c^{ij} \frac{d\chi_c^{ij}}{d\tau}\right] c^j}_{\text{Cross-Sectional Consumption Redistribution}},
\end{aligned}
$$

where $\omega^i = \frac{\lambda^i \frac{\partial \mathcal{W}}{\partial V^i}}{\frac{1}{I}\sum_i \lambda^i \frac{\partial \mathcal{W}}{\partial V^i}}$. We choose aggregate world consumption as welfare numeraire, which implies that $\lambda^i = \sum_j \frac{\partial V^i}{\partial c^{ij}} c^j$. Similar results obtain if we choose unit world consumption, which implies that $\lambda^i = \sum_j \frac{\partial V^i}{\partial c^{ij}}$. Even though country 2's consumption is substantially higher than country 1 in the absence of tariffs, as shown in the middle plots in Figure OA-4, the linear homogeneity of the preferences imply that $\frac{\partial V^1}{\partial c^{11}} = \frac{\partial V^2}{\partial c^{21}}$ and $\frac{\partial V^1}{\partial c^{12}} = \frac{\partial V^2}{\partial c^{22}}$. Hence, to ensure that the planner attaches a higher weight to the country that consumes less (country 1), we use a social welfare function of the form $\mathcal{W}\left(V^1, V^2\right) = \sum_i \left(V^i\right)^{\frac{\sigma-1}{\sigma}}$, which implies that $\omega^i = \frac{\lambda^i \left(V^i\right)^{-\frac{1}{\sigma}}}{\frac{1}{I}\sum_i \lambda^i \left(V^i\right)^{-\frac{1}{\sigma}}}$. This is equivalent to expressing country preferences as $V^i = \sum_j \left(c^{ij}\right)^{\frac{\sigma-1}{\sigma}}$ and assuming a utilitarian social welfare function. The bottom two plots in Figure OA-4 illustrate the equilibrium values of ω^i and λ^i.

E.2 DMP Model

Model Solution. We consider the standard search equilibrium definition (Pissarides, 2000), whose notation we mostly follow. Job-filling and job-finding rates, respectively denoted by $q_0(\theta_0)$ and $p_0(\theta_0)$, are given by

$$q_0(\theta_0) = \frac{m\left(\chi^1_{0,n}, v_0\right)}{v_0} = \mu\theta_0^{-\alpha} \quad \text{and} \quad p_0(\theta_0) = \theta_0 q_0(\theta_0) = \frac{m\left(\chi^1_{0,n}, v_0\right)}{\chi^1_{0,n}} = \mu\theta_0^{1-\alpha}.$$

The value of an occupied job, denoted by J_0, is given by

$$J_0 = z^2 - w_0 + \beta\left[(1-\varphi) J_1 + \varphi V_1\right] \quad \text{and} \quad J_1 = z^2 - w_1,$$

where w_t denotes the wage. The value of a vacant job is given by

$$V_0 = -\kappa_0 + \beta\left[q_0(\theta_0) J_1 + (1 - q_t(\theta_t)) V_1\right] \quad \text{and} \quad V_1 = 0.$$

At an equilibrium with free-entry, $V_0 = 0$, so

$$J_1 = \frac{\kappa_0}{\beta q_0(\theta_0)} \quad \text{and} \quad J_0 = z^2 - w_0 + (1-\varphi)\frac{\kappa_0}{q_0(\theta_0)}.$$

The value of employed and unemployed workers, respectively denoted by E_t and U_t, are given by

$$\begin{aligned} E_0 &= w_0 + \beta\left[\varphi U_1 + (1-\varphi) E_1\right] && \text{and} \quad E_1 = w_1 \\ U_0 &= b_0 + \beta\left[p_t(\theta_t) E_1 + (1 - p_t(\theta_t)) U_1\right] && \text{and} \quad U_1 = b_1. \end{aligned}$$

The wage is determined by Nash bargaining, with

$$w_t = \arg\max_{w_t} (E_t - U_t)^{\eta} (J_t - V_t)^{1-\eta}.$$

The solution to this problem is

$$E_t - U_t = \eta(E_t - U_t + J_t - V_t) \quad \text{and} \quad J_t - V_t = (1-\eta)(E_t - U_t + J_t - V_t).$$

Given our parametrization, we have that $U_1 = V_1 = 0$, which means that $w_1 = E_1 = \eta(E_1 + J_1) = \eta z^2$ and that $J_1 = (1-\eta) z^2$.

Hence, the condition

$$(1-\eta) z^2 = \frac{\kappa_0}{\beta q_0(\theta_0)}$$

pins down equilibrium tightness θ_0. Given θ_0 and $\chi^1_{0,n}$, we can compute equilibrium vacancies, which is sufficient to compute the welfare accounting decomposition. Figure OA-5 illustrates how an improvement in the matching technology translates in higher vacancies posted at date 0, which in turn translates into lower unemployment at date 1.

Welfare Accounting. The welfare gain of a marginal change in μ can be written as

$$\frac{dW}{d\mu} = \frac{dc_0}{d\mu} + \beta\frac{dc_1}{d\mu}$$

Using unit perpetual consumption as lifetime welfare numeraire, we can express the normalized welfare gain as

$$\frac{dW^{\lambda}}{d\mu} = \frac{\frac{dW}{d\mu}}{\lambda} = \omega_0 \frac{dc_0}{d\mu} + \omega_1 \frac{dc_1}{d\mu},$$

where $\lambda = 1 + \beta$ and where $\omega_0 = \frac{1}{1+\beta}$ and $\omega_1 = \frac{\beta}{1+\beta}$. Note that

$$\begin{aligned} \frac{dc_0}{d\mu} &= \phi_{0,c} \left(\frac{dy_0^1}{d\mu} + \frac{dy_0^2}{d\mu} \right) - \frac{d\phi_{x,0}}{d\mu} \left(y_0^1 + y_0^2 \right) \\ \frac{dc_1}{d\mu} &= \frac{dy_1^1}{d\mu} + \frac{dy_1^2}{d\mu}, \end{aligned}$$

where $\frac{dy_t^j}{d\mu} = z^j \frac{d\chi_{t,n}}{d\mu}$, which allows us to write

$$\begin{aligned} \frac{dW^{\lambda}}{d\mu} &= \omega_1 \sum_j \phi_{1,c} z^j \frac{d\chi_{1,n}^j}{d\mu} - \omega_0 \frac{d\phi_{x,0}}{d\mu} \left(y_0^1 + y_0^2 \right) \\ &= \underbrace{\omega_1 \mathbb{C}ov_j^{\Sigma} \left[MWP_{1,n}^j, \frac{d\chi_{1,n}^j}{d\mu} \right]}_{\text{Cross-Sectional Factor Efficiency}} - \underbrace{\omega_0 \frac{d\phi_{x,0}}{d\mu} \left(y_0^1 + y_0^2 \right)}_{\text{Aggregate Intermediate Input Efficiency}}, \end{aligned}$$

where marginal welfare products are given by $MWP_{1,n}^j = \phi_{1,c} z^j$.

E.3 Hsieh and Klenow (2009) Model

Welfare Accounting. Since the solution of the model is completely standard, we exclusively describe here how to characterize the welfare accounting decomposition. We consider a perturbation in σ_τ, which is associated with a welfare change given by

$$\frac{dW}{d\sigma_\tau} = \frac{\partial u}{\partial c^1} \frac{dc^1}{d\sigma_\tau}$$

Using good 1 as numeraire, $\lambda = \frac{\partial u}{\partial c^1}$, so

$$\frac{dW^{\lambda}}{d\sigma_\tau} = \frac{dy^1}{d\sigma_\tau} = \sum_{j=2}^{J} \frac{\partial y^1}{\partial y^j} A^j \frac{d\chi_n^{j,d}}{d\sigma_\tau} = \sum_{j=2}^{J} MWP_n^j \frac{d\chi_n^{j,d}}{d\sigma_\tau} = \underbrace{\mathbb{C}ov_j^{\Sigma} \left[MWP_n^j, \frac{d\chi_n^{j,d}}{d\sigma_\tau} \right]}_{\text{Cross-Sectional Factor Efficiency}},$$

where the marginal welfare product of factor use $\chi_n^{j,d}$ is $MWP_n^j = \frac{\partial y^1}{\partial y^j} A^j$ and where $\frac{\partial y^1}{\partial y^j} = \left(\frac{y^j}{y^1} \right)^{-\frac{1}{\epsilon}}$.

E.4 New Keynesian Model

This Appendix presents additional model details in E.4.1, competitive equilibrium in E.4.2, a self-contained quantitative calibration in E.4.3, and additional numerical results in E.4.4.

E.4.1 Additional Model Details

Households.

Household preferences (50) give rise to the usual CES demand functions

$$c^{ij} = \Gamma_c^{ij} \left(\frac{p^j}{P^i}\right)^{-\eta_c} c^i \qquad \text{and} \qquad c^{ij\ell} = \left(\frac{p^{j\ell}}{p^j}\right)^{-\epsilon^j} c^{ij}.$$

Under homothetic CES consumption preferences, each household i faces an ideal price index

$$P^i = \left[\sum_j \Gamma_c^{ij} (p^j)^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}}.$$

Production. The production function (51) features three nests of CES aggregates. Taking as given prices and wages, firms choose inputs to minimize cost

$$\mathcal{C}^{j\ell} = \min_{\{x^{j\ell\ell\ell'}\}_{\ell\ell'}, \{n^{j\ell i}\}_i} \sum_\ell \int_0^1 p^{\ell\ell'} x^{j\ell\ell\ell'} d\ell' + \sum_i W^i n^{j\ell i},$$

subject to the CES production structure in (51). This problem gives rise to labor demand

$$n^{j\ell} = (A^j)^{\eta-1}(1-\vartheta^j)\left(\frac{W^{j\ell}}{mc^j}\right)^{-\eta} y^{j\ell} \qquad \text{and} \qquad n^{j\ell i} = \Gamma_w^{ji}\left(\frac{W^i}{W^{j\ell}}\right)^{-\eta^w} n^{j\ell}$$

and intermediate input demand

$$x^{j\ell} = (A^j)^{\eta-1}\vartheta^j \left(\frac{p_x^{j\ell}}{mc^j}\right)^{-\eta} y^{j\ell}, \qquad x^{j\ell\ell} = \Gamma_x^{j\ell}\left(\frac{p^\ell}{p_x^{j\ell}}\right)^{-\eta_x} x^{j\ell} \qquad \text{and} \qquad x^{j\ell\ell\ell'} = \left(\frac{p^{\ell\ell'}}{p^\ell}\right)^{-\epsilon_\ell} x^{j\ell\ell}.$$

Nominal marginal cost is given by

$$mc^j = \frac{1}{A^j}\left[\left(1-\vartheta^j\right)\left(W^j\right)^{1-\eta} + \vartheta^j \left(p_x^j\right)^{1-\eta}\right]^{\frac{1}{1-\eta}},$$

which is symmetric across firms ℓ in sector j. Marginal cost is not affected by the revenue tax, which is the only wedge in this application. Finally, the cost indices are given by

$$W^j = \left[\sum_\ell \Gamma_w^{ji}(W^i)^{1-\eta_w}\right]^{\frac{1}{1-\eta_w}} \qquad \text{and} \qquad p_x^j = \left[\sum_\ell \Gamma_x^{j\ell}(p^\ell)^{1-\eta_x}\right]^{\frac{1}{1-\eta_x}}.$$

Since production functions are homogeneous of degree one, total cost is given by $\boldsymbol{C}^{j\ell} = mc^j y^{j\ell}$.

Sectoral aggregation. Firms set prices according to (52). Aggregating to the sectoral level, the price of sector j 's good is

$$\begin{aligned}
p^j &= \left(\int_0^1 (p^{j\ell})^{1-\epsilon^j} d\ell\right)^{\frac{1}{1-\epsilon^j}} = \left[\int_0^{\delta^j} \left(\frac{\epsilon^j}{\epsilon^j-1}\frac{1}{1-\tau^j} mc^j\right)^{1-\epsilon^j} d\ell + \int_{\delta^j}^1 (\bar{p}^j)^{1-\epsilon_j} d\ell\right]^{\frac{1}{1-\epsilon^j}} \\
&= \left[\delta^j\left(\frac{\epsilon^j}{\epsilon^j-1}\frac{1}{1-\tau^j} mc^j\right)^{1-\epsilon^j} + (1-\delta^j)(\bar{p}^j)^{1-\epsilon^j}\right]^{\frac{1}{1-\epsilon^j}} \\
&= \frac{\epsilon^j}{\epsilon^j-1}\frac{1}{1-\tau^j}\left[\delta^j(mc^j)^{1-\epsilon^j} + (1-\delta^j)(\bar{mc}^j)^{1-\epsilon^j}\right]^{\frac{1}{1-\epsilon^j}},
\end{aligned}$$

where the very first equality follows since

$$p^j c^{ij} = \int_0^1 p^{j\ell} c^{ij\ell} d\ell = \int_0^1 p^{j\ell} \left(\frac{p^{j\ell}}{p^j}\right)^{-\epsilon^j} c^{ij} d\ell \implies p^j = \left(\int_0^1 (p^{j\ell})^{1-\epsilon^j} d\ell\right)^{\frac{1}{1-\epsilon^j}}.$$

Aggregating the goods market clearing condition, we have

$$p^j y^j \equiv \int_0^1 p^{j\ell} y^{j\ell} d\ell = \sum_i \mu^i \int_0^1 p^{j\ell} c^{ij\ell} d\ell + \sum_\ell \int_0^1 \int_0^1 p^{j\ell} x^{\ell\ell' j\ell} d\ell' d\ell,$$

where $\int_0^1 p^{j\ell} y^{j\ell} d\ell$ denotes total nominal expenditures on sectoral good j. This also implies a resource constraint at the sectoral level, given by $y^j = \sum_i \mu^i c^{ij} + \sum_\ell \int_0^1 x^{\ell\ell j} d\ell$. All this relies on our assumption that all agents buying in sector j share the same homothetic demand aggregators over varieties ℓ. In particular, it implies that we also have

$$y^{j\ell} = \left(\frac{p^{j\ell}}{p^j}\right)^{-\epsilon^j} y^j \qquad \text{and} \qquad y^j = \left(\int_0^1 (y^{j\ell})^{\frac{\epsilon^j - 1}{\epsilon^j}} d\ell\right)^{\frac{\epsilon^j}{\epsilon^j - 1}}.$$

Fiscal rebates. In the absence of fiscal policy, the rebate T^i that household i receives simply corresponds to total corporate profits plus the proceeds from the revenue tax. That is,

$$\sum_i \mu^i T^i = \sum_j \int_0^1 \Pi^{j\ell} d\ell + \sum_j \int_0^1 \tau^j p^{j\ell} y^{j\ell} d\ell = \sum_j \int_0^1 \left(p^{j\ell} - mc^j\right) y^{j\ell} d\ell$$

Assuming a uniform rebate, we simply have $T^i = \sum_j \int_0^1 \left(p^j - mc^j\right) y^{j\ell} d\ell$.

E.4.2 Equilibrium

Definition 9. (Competitive Equilibrium). Taking as given an initial price distribution $\left\{\bar{p}^{j\ell}\right\}_{j\ell}$, a realization of technology shocks $\left\{A^j\right\}_j$, revenue taxes $\left\{\tau^j\right\}_j$, and monetary policy M, a competitive equilibrium comprises an allocation $\left\{c^{ij\ell}, n^i, x^{i\ell\ell\ell'}, y^{j\ell}\right\}_{i,j\ell,\ell\ell'}$ and prices $\left\{p^{j\ell}, W^i\right\}_{i,j\ell}$ such that (i) households optimize consumption and labor supply, (ii) firms $\ell \in \left[0, \delta^j\right)$ in sector j reset their prices optimally, and (iii) markets for goods and factors clear

$$y^{j\ell} = \sum_i \mu^i c^{ij\ell} + \sum_\ell \int_0^1 x^{\ell\ell' j\ell} d\ell' \qquad \text{and} \qquad \mu^i n^i = \sum_j \int_0^1 n^{j\ell i} d\ell.$$

Notice that each sector features two representative firms ex post since all firms are symmetric ex ante and those firms that reset prices all choose the same reset price. At the sector level, there is consequently a representative price-adjusting firm and a representative fixed-price firm.

Computing competitive equilibrium requires an initial price distribution $\left\{\bar{p}^{j\ell}\right\}_{j\ell}$. We assume that initial prices are given by

$$\bar{p}^{j\ell} = \bar{p}^j = \frac{\epsilon^j}{\epsilon^j - 1} \frac{1}{1 - \tau^j} \bar{mc}^j = \frac{\epsilon^j}{\epsilon^j - 1} \frac{1}{1 - \tau^j} mc^j \left(1, \left\{\bar{p}^{\ell\ell'}\right\}_{\ell\ell'}, \left\{\bar{W}^i\right\}_i\right).$$

That is, $\bar{p}^j$ corresponds to the price firms in sector j would set if all technologies remain at their default level $A^j = \bar{A}^j$. This initialization is heuristically consistent with the zero-inflation steady state of a dynamic New Keynesian model. In the absence of technology shocks, therefore, no firm faces an incentive to adjust prices. If $A^j \neq \bar{A}^j$, a fraction δ^j of firms in each sector reset their price.

Numeraire. We take as our numeraire total nominal expenditures in the absence of shocks, i.e., $\bar{M} = \sum_j p^j y^j = 1$. Therefore, $\bar{M} = 1$ provides a benchmark stance for monetary policy. In the absence of technology shocks, setting $M = \bar{M} = 1$ implies production efficiency and therefore aggregate efficiency since all firms are symmetric.

Macro block. To compute this model, it is particularly convenient to characterize a macro block by aggregating to the sectoral level. To that end, we aggregate several key equilibrium conditions. The aggregate labor market clearing condition (aggregated to the level of household type) is

$$
\begin{aligned}
\mu^i n^i &= \sum_j \int_0^1 n^{j\ell i} d\ell = \sum_j \int_0^1 \Gamma_w^{ji} \left(\frac{W^i}{W^j}\right)^{-\eta_w} n^{j\ell} d\ell \\
&= \sum_j \Gamma_w^{ji} \left(\frac{W^i}{W^j}\right)^{-\eta_w} (A^j)^{\eta-1}(1-\vartheta^j) \left(\frac{W^j}{mc^j}\right)^{-\eta} \int_0^1 y^{j\ell} d\ell \\
&= \sum_j \Gamma_w^{ji} \left(\frac{W^i}{W^j}\right)^{-\eta_w} (A^j)^{\eta-1}(1-\vartheta^j) \left(\frac{W^j}{mc^j}\right)^{-\eta} D^j y^j,
\end{aligned}
$$

where $D^j = \int_0^1 \left(\frac{p^{j\ell}}{p^j}\right)^{-\epsilon^j} d\ell$ is a measure of sectoral price dispersion. Aggregating the goods market clearing condition yields

$$
y^{j\ell} = \sum_i \mu^i c^{ij\ell} + \left(\frac{p^{j\ell}}{p^j}\right)^{-\epsilon^j} \sum_\ell \Gamma_x^{\ell j} \left(\frac{p^j}{p_x^\ell}\right)^{-\eta_x} \int_0^1 (A^\ell)^{\eta-1} \vartheta^\ell \left(\frac{p_x^\ell}{mc^\ell}\right)^{-\eta} y^{\ell\ell'} d\ell'.
$$

And plugging in for CES demand functions implies

$$
y^j = \sum_i \mu^i c^{ij} + \sum_\ell \Gamma_x^{\ell j} \left(\frac{p^j}{p_x^\ell}\right)^{-\eta_x} (A^\ell)^{\eta-1} \vartheta^\ell \left(\frac{p_x^\ell}{mc^\ell}\right)^{-\eta} y^\ell D^\ell,
$$

yielding sectoral goods market clearing conditions written as a fixed point in y^j.

Finally, the budget constraint can be written as

$$
P^i c^i = W^i n^i + \sum_j (p^j - D^j mc^j) y^j.
$$

Computationally, it is now easiest to solve the macro block as a separate system of equations. Firm-level allocations can then be obtained from CES demand functions.

E.4.3 Calibration

Our calibration broadly follows Schaab and Tan (2023) and is summarized in Table OA-1. It is based on 66 production sectors and 10 household types, which we associate with deciles of the household income distribution.

For household preferences, we set the coefficient of relative risk aversion to $\gamma = 2$ and the inverse Frisch elasticity to $\varphi = 2$. We use an elasticity of substitution of $\eta_c = 1$, so the consumption aggregator is Cobb-Douglas, and we calibrate the consumption weights Γ_c^{ij} to match consumption expenditure shares across household types in the CEX.

On the production side, we set the elasticity of substitution between the labor and intermediate input bundles to $\eta = 1$. Therefore, ϑ^j and $1-\vartheta^j$ correspond respectively to the input and labor shares in production,

	Parameters	Value / Target	Source
	Household preferences		
γ	Relative risk aversion	2	Standard
φ	Inverse Frisch elasticity	2	Standard
η_c	Elasticity of substitution across goods	1	Cobb-Douglas
Γ_c^{ij}	CES consumption weights	Consumption expenditure shares	CEX
	Production and nominal rigidities		
η	Elasticity of substitution across inputs and labor	1	Cobb-Douglas
ϑ^j	CES input bundle weight	Sectoral input share	BEA
η_x	Elasticity of substitution across inputs	1	Cobb-Douglas
η_w	Elasticity of substitution across factors	1	Cobb-Douglas
Γ_x^{ij}	CES input use weights	Input-output network	BEA I-O
Γ_w^{ij}	CES factor use weights	Payroll shares	ACS
ϵ^j	Elasticities of substitution across varieties	Sectoral markups	Baqaee and Farhi (2020)
δ^j	Sectoral price adjustment probabilities	Price adjustment frequencies	Pasten et al. (2017)

Table OA-1: List of Calibrated Parameters

which we obtain from the BEA GDP-by-Industry data. We compute the input share ϑ^j as input expenditures relative to gross output, averaged between 1997 and 2015, and treat the labor share as its complement. We set the elasticities of substitution across intermediate inputs and factors to $\eta_x = \eta_w = 1$. We calibrate Γ_x^{ij} and Γ_w^{ij} to match data on input-output linkages and payroll shares. For the former, we use data from the BEA Input Output "Use" Table to compute input shares as a sector j's expenditures on goods from sector ℓ as a share of j's total expenditures on inputs, averaged between 1997 and 2015. We obtain payroll shares from a linked ACS-IO dataset as type i 's earnings from sector j as a share of total earnings, averaged between 1997 and 2015.

We use data from Baqaee and Farhi (2020) on sectoral markups to calibrate the elasticity of substitution across sectoral varieties ϵ^j. Sectoral markups are computed as $\mu^j = \frac{\epsilon^j}{\epsilon^j - 1}$.

Finally, we use data from Pasten et al. (2017) on price adjustment frequencies to calibrate δ^j. They estimate monthly price adjustment frequencies using the data underlying the Bureau of Labor Statistics' Producer Price Index for 754 industries from 2005 to 2011. First, we link these estimates to the 66 sectors in our data. Second, we obtain quarterly adjustment probabilities as $1 - \left(1 - \frac{\text{monthly adjustment frequency}}{100}\right)^3$. Finally, we bin these estimates into quintiles. This allows us to solve our model assuming that each of the 66 sectors consists of 5 firms.

E.4.4 Additional Results

In this subsection, we present additional numerical results that are referenced in the main text.

Divine coincidence. Consider an alternative calibration where households and sectors are symmetric, so there exist a representative household and a representative sector. Our model then collapses to the standard, one-sector New Keynesian model, albeit with roundabout production.

Divine Coincidence holds in this model. That is, the optimal monetary policy response to an aggregate technology shock closes both output and inflation gaps.

Figure OA-6 illustrates this benchmark from the perspective of our welfare accounting decomposition. In that context, Divine Coincidence implies that each allocative efficiency component is 0, indicating that optimal policy can attain an efficient allocation. Moreover, since households are symmetric, there is no scope for redistribution gains, so welfare and aggregate efficiency coincide.

Importance of markup distortions. Figure 5 in the main text corresponds to a calibration of the model that assumes revenue taxes are available to eliminate initial markups. We reproduce our main experiment in Figure OA-7 below, assuming that revenue taxes are not available.

It is well known from the New Keynesian literature that monopolistic competition implies inefficiently low steady state employment. In that context, optimal monetary policy under discretion, which is heuristically comparable to the static optimization problem we consider, seeks to raise employment via expansionary monetary policy. We revisit this result from the perspective of our welfare accounting decomposition. Figure OA-7 demonstrates that, in the presence of initial markup distortions, aggregate factor and input use efficiency considerations push optimal monetary policy towards a more expansionary stance. In the one-sector New Keynesian model (without roundabout production), aggregate factor efficiency corresponds to the standard labor wedge. In this multi-sector variant, aggregate factor and input use efficiency formally capture that aggregate employment and aggregate activity are inefficiently low.

Cross-sectional factor and input use efficiency, on the other hand, push monetary policy towards a relatively more contractionary stance. Optimal policy therefore trades off the gains from stimulating aggregate activity in the presence of markup distortions against the cost of creating misallocation in the form of price dispersion, captured by cross-sectional factor and input use efficiency.

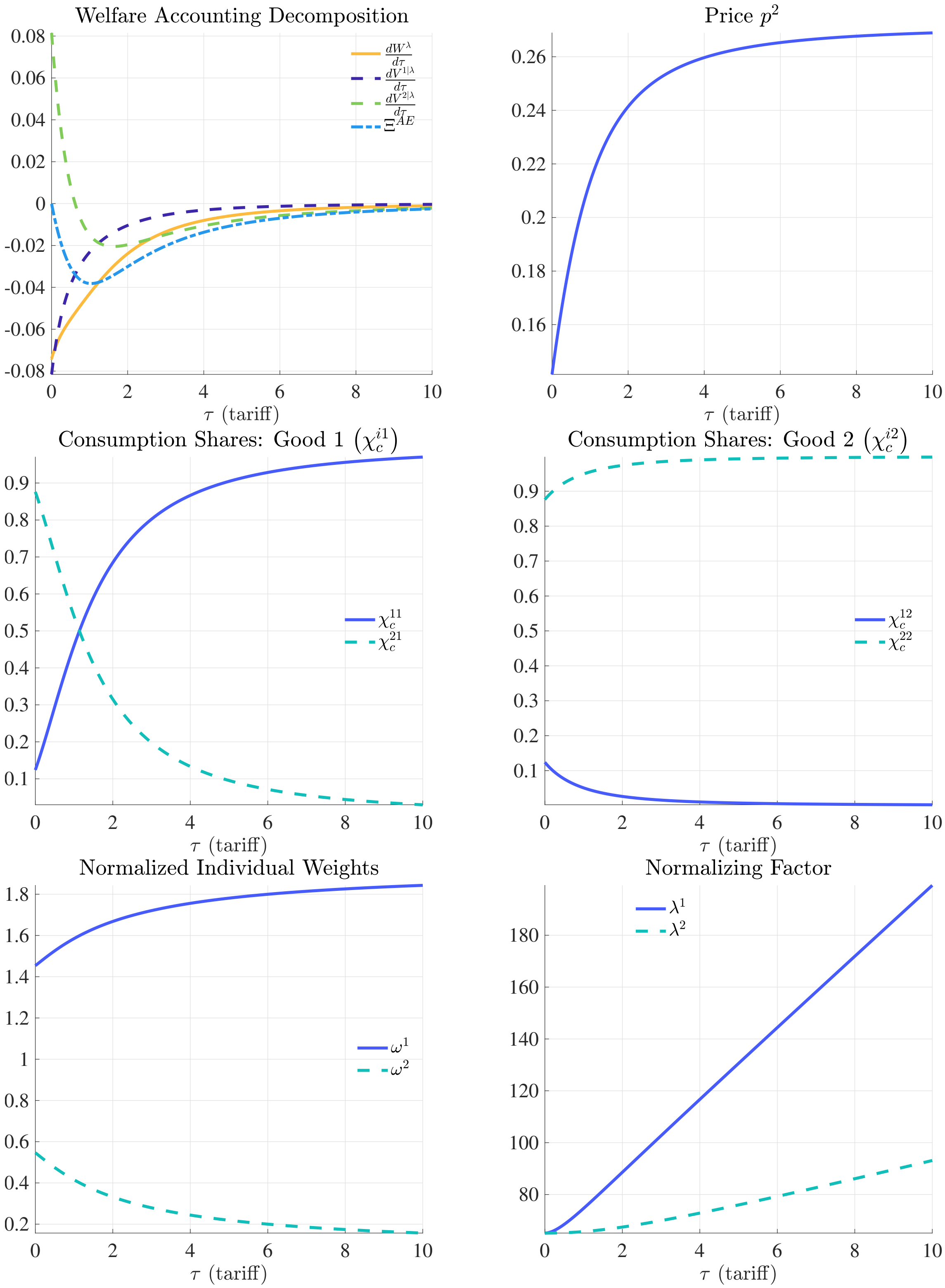


Figure OA-4: Armington Model

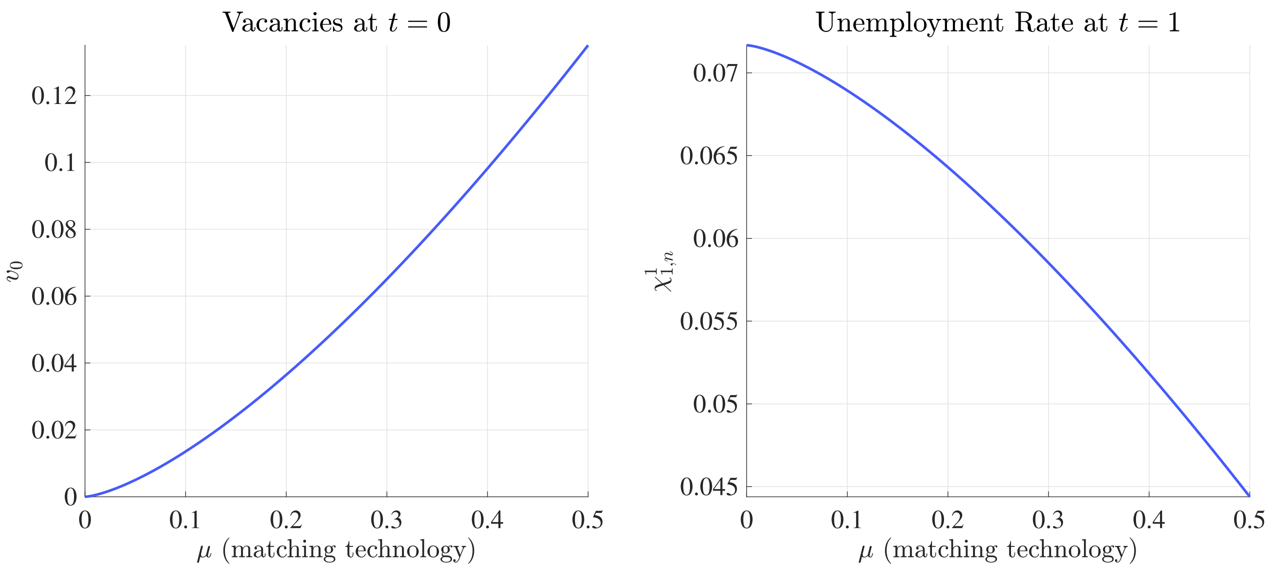
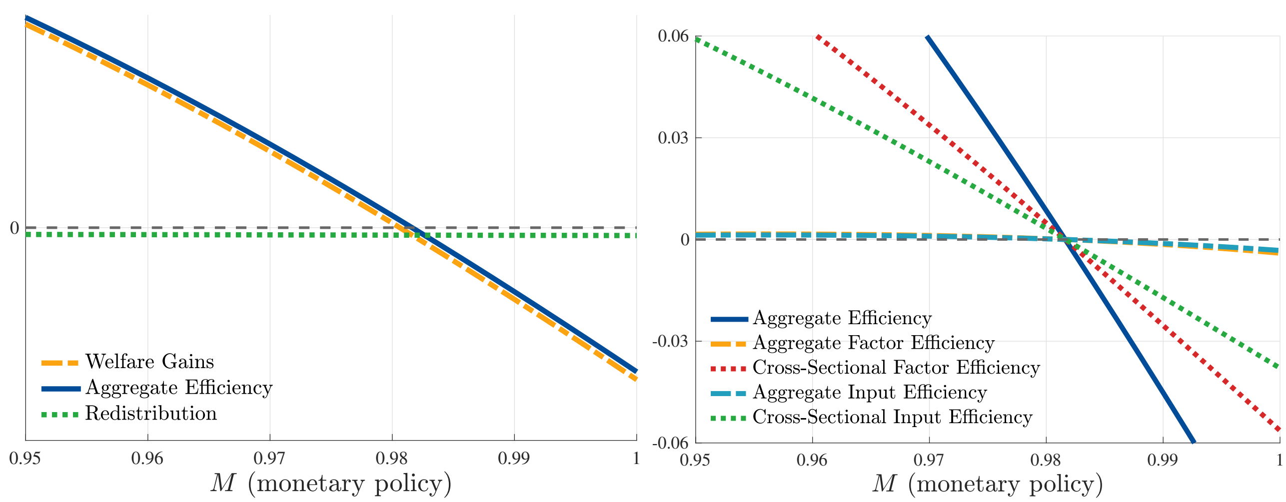


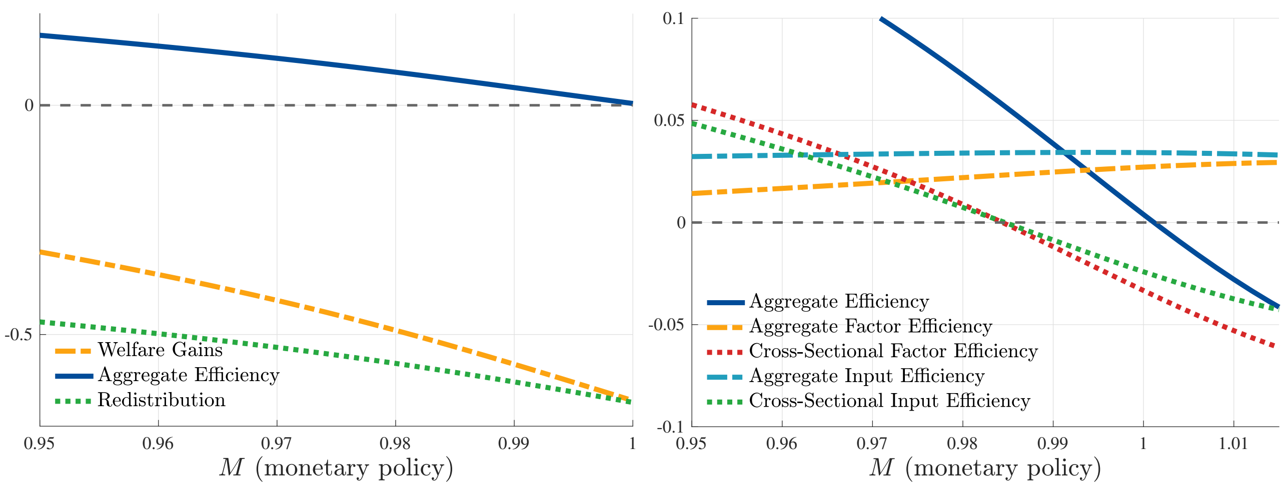
Figure OA-5: DMP Model



(a) Welfare Gains

(b) Aggregate Efficiency Gain

Figure OA-6: Optimal Monetary Policy under Divine Coincidence



(a) Welfare Gains

(b) Aggregate Efficiency Gain

Figure OA-7: Optimal Monetary Policy with Markup Distortions