Sufficient Statistics and Normative Finance
2025 FTG Summer School

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Slides posted at https://www.eduardodavila.com/teaching.html

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Introduction

- $\boxed{\text{Normative Macro-Finance}}$: study of policy/welfare in macro and finance environments

$$\text{Finance} + \text{Macro} + \text{Public Finance} \iff \text{GE} + \text{WE}$$

- See $\boxed{\text{here}}$ for a high-level summary

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- Fact: share of theoretical work in Finance (and Economics) continues to shrink
- Question: how can theorists have impact in 2025 and beyond?
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- Part I: Sufficient Statistics $\Rightarrow$ $\boxed{\text{Theory of Measurement}}$

Theory determines what to measure

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- Part I: Sufficient Statistics $\Rightarrow$ $\boxed{\text{Theory of Measurement}}$

Theory determines what to measure

- Part II: Normative Finance $\Rightarrow$ $\boxed{\text{Theory of Policy}}$

Theory is necessary to make normative assessments

This Lecture

- I conjecture that many of you have not
  i) thought about sufficient statistics
  ii) thought about normative/policy questions
- My goal today is to get you excited about both!
  - <u>Many</u> interesting open questions

$\boxed{\text{Part I: Sufficient Statistics}}$

Some of these ideas build on (unfinished!) survey: Auclert and Dávila (2025)

Sufficient Statistics: Definition

- Influential paper: Chetty (2009) on Public Finance
  - Traditional distinction: reduced-form vs. structural work
    Burns and Mitchell (1946) vs. Koopmans (1947)
  - Sufficient statistics: intermediate approach
- Chetty's definition:

  "The central concept of the sufficient-statistic approach is to derive formulas for the welfare consequences of policies that are functions of high-level elasticities rather than deep primitives. Even though there are multiple combinations of primitives consistent with the inputs to the formulas, all such combinations have the same welfare implications."

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- Broader definition:

  "Sufficient statistics are variables whose knowledge is sufficient to directly answer a question of interest"

- This definition may seem too broad: the word directly is key
  - A parameter is not a sufficient statistic

$$\begin{array}{c}\text{(potentially different)}\\ \text{primitives}\end{array} \Rightarrow \text{sufficient statistics} \Rightarrow \begin{array}{c}\text{(positive or normative)}\\ \text{outcomes}\end{array}$$

  - Different from "testing" a model prediction

Sufficient Statistics: Characteristics

- Sufficient statistics are *typically*
  1. high-level, **endogenous** variables that
  2. are valid in a **large class** of economic environments,
  3. and are (ideally) **measurable** directly or through inference

Sufficient Statistics in Finance

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But central contributions in Finance take the form of sufficient statistics!

Sufficient Statistics in Finance

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But central contributions in Finance take the form of sufficient statistics!

- **Asset Pricing**
  1. Fundamental AP equation: $p = \mathbb{E}[mx]$
     - "SDF and payoffs are sufficient statistics for asset prices"
     - Endogenous, measurable, applies broadly (frictionless markets)
  2. Lucas (1978): $m = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}$
     - "aggregate consumption is a sufficient statistic for pricing assets in representative agent economies"
  3. CAPM's security market line: $\mathbb{E}[R_i] - R_f = \beta_i \mathbb{E}[R_m - R_f]$
     - "β's are sufficient statistics to determine excess returns"

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- **Corporate Finance**
  1. Q-Theory: $I/K = f(Q)$
     - "Tobin's Q is a sufficient statistic for investment"
     - Investment sensitivity regressions
       Fazzari, Hubbard, and Petersen (1988), Kaplan and Zingales (1997)

Sufficient Statistics: Observations

1. Direct Measurement vs. Structural Modeling
   - Finance/Macro rely more on structural models than Public
   - Sufficient statistics can be used for *calibrating* or *discriminating among* structural models
     - Not only for direct measurement
   - Examples:
     i) "Net trades, prices changes and valuation differences" are sufficient statistics for distributive pecuniary externalities
        Dávila and Korinek (2018), Lanteri and Rampini (2023)
     ii) "Pigouvian wedges and leakage elasticities" are sufficient statistics for second-best regulation
        Dávila and Walther (2025)
2. Positive vs. Normative Uses
   - Positive: Euler equations
   - Normative: envelope theorems
3. Sufficient statistics as *dimensionality reduction*
   - Real world is very complicated
   - Yet we want to make general claims

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- Two illustrations of these ideas in *normative finance*
  i) Bankruptcy exemptions
  ii) Deposit insurance

Paper #1: Sufficient Statistics for Bankruptcy Exemptions

Based on "Using Elasticities to Derive Optimal Bankruptcy Exemptions" (Dávila, 2020)

Title homage to Saez (2001): sufficient statistics for optimal income taxes

Sufficient Statistics for Bankruptcy Exemptions

- **Question**: How large should bankruptcy exemptions be?
  - *Exemption*: dollar amount borrower gets to keep after default
  - Substantial variation on exemptions across regions/time
  - Household bankruptcy
- **Main Result**: Test to determine whether to increase/decrease exemptions
  - Knowledge of four variables is sufficient (*sufficient statistics*)
  - Empirical implementation for US states (see paper)
    - Increasing exemption levels is welfare-improving
    - Substantial variation across states and income quintiles

Baseline Environment

- Two dates $t = \{0, 1\}$

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- Risk averse borrower

$$W(m) = \max_{b_1} u(c_0) + \beta \mathbb{E}\left[\max\left\{u\left(c_1^{\mathcal{N}}(s)\right), u\left(c_1^{\mathcal{D}}(s)\right)\right\}\right]$$

- Budget constraints

$$c_0 = n_0 + \overbrace{Q_0(b_1, m)}^{q_0(b_1,m)b_1} \qquad \begin{aligned} c_1^{\mathcal{N}}(s) &= n(s) - b_1 \\ c_1^{\mathcal{D}}(s) &= \min\{n(s), m\} \end{aligned}$$

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- Risk neutral competitive lenders $\Rightarrow$ zero profit

$$Q_0(b_1, m) = \frac{\delta \int_{\mathcal{D}} \max\{n(s) - m, 0\} dF(s) + b_1 \int_{\mathcal{N}} dF(s)}{1 + r^{\ell}}$$

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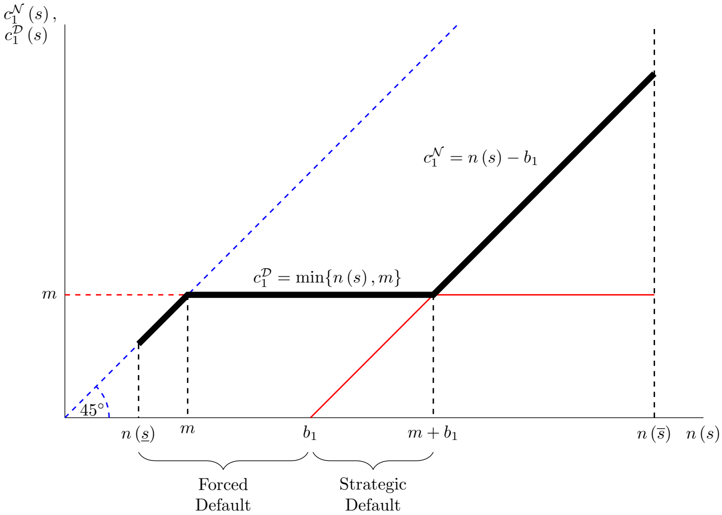
- Remarks
  - Fixed repayment debt b_1 (contract as primitive, GEI)
  - Constant exemption m
  - Regularity conditions to guarantee $b_1 > 0$
  - *Equilibrium notion*: borrowers internalize $Q_0(b_1, m)$

Borrower's Problem: Default

- Two economic decisions
  - Default (given b_1)
  - Borrowing b_1



Borrower's Problem: Borrowing

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$$u'\left(c_0\right)\frac{\partial Q_0}{\partial b_1}=\beta\int_{\hat{s}}^{\overline{s}}u'\left(c_1^{\mathcal{N}}\left(s\right)\right)dF\left(s\right)\Rightarrow\boxed{b_1\left(m\right)}$$

Borrower's Problem: Borrowing

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- Sign of $\frac{db_1}{dm}$ ambiguous

$$\text{sign}\left(\frac{db_1}{dm}\right) = \text{sign}\left(\underbrace{u''(c_0)\frac{\partial Q_0}{\partial m}\frac{\partial Q_0}{\partial b_1}}_{\text{Income effect }(>0)} + \underbrace{u'(c_0)\frac{\partial^2 Q_0}{\partial b_1 \partial m}}_{\text{Substitution effect }(<0)} + \underbrace{\beta u'(m) f(m)}_{\text{Direct effect }(>0)}\right)$$

Main Results

- Social welfare $W(m)$ is given by borrowers indirect utility

$$W = V^b + \underbrace{V^\ell}_{=0}$$

- $\boxed{\text{Directional test}}$ for change in exemption level m:

$$\frac{\frac{dW}{dm}}{u'(c_0)} = \underbrace{\frac{\partial q_0}{\partial m} b_1}_{=\text{Mg. Cost}} + \underbrace{\pi_m \mathbb{E}_m \left[\frac{\beta u'\left(c_1^{\mathcal{D}}\right)}{u'(c_0)} \right]}_{=\text{Mg. Benefit}}$$

- Marginal cost: change in interest rate
- Marginal benefit: appropriately valued cash flow

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- Marginal cost: change in interest rate
- Marginal benefit: appropriately valued cash flow
- Key insight: borrowing and default decisions are optimal
  - Logic extends broadly ("envelope theorem")
  - Not always: belief distortions, market power, GE externalities
    See Section 5 of the paper: simply more terms (not not fewer!)
- Note that $\frac{\partial q_0}{\partial m}$ is partial, not total derivative

Main Results

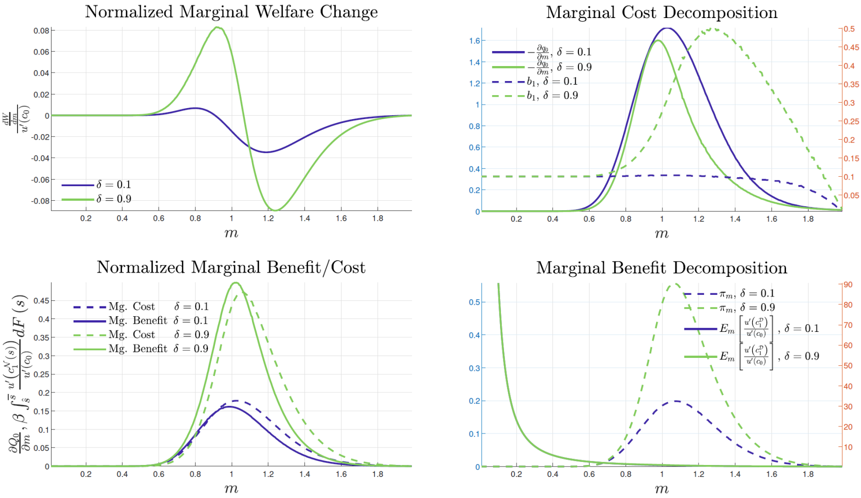
- **Sufficient Statistics**
  1. Debt position: b_1
  2. Credit supply sensitivity to a change in the exemption level: $\frac{\partial q_0}{\partial m}$
  3. Probability of bankruptcy (claiming full exemption): π_m
  4. Value of exemption dollar when claimed: $\mathbb{E}_m \left[\frac{\beta u'\left(c_1^{\mathcal{D}}\right)}{u'(c_0)} \right]$
- $\frac{db_1}{dm}$ or $\frac{d\pi_m}{dm}$ are <u>not</u> sufficient statistics in this case
  - Prices/pricing schedules typically encapsulate important information
- Possible to construct <u>measurable</u> counterparts
  - Look at the paper!

Local/Marginal vs. Global

- You may think sufficient statistics are only *local*
- Not quite true
  1. Measurement is always local to status-quo
  2. But we can use the model to extrapolate
- Even better: globally estimate your sufficient-statistics (typically not possible)

Illustration

- Compare default DWL $\delta = 0.1$ and $\delta = 0.9$



- Everything is recomputed as m varies
- Nature of sufficient statistics invariant $\Rightarrow$ Values obviously change

Perspective on Related Work

- Bankruptcy literature had been
  i) Purely Theoretical: Dubey, Geanakoplos, and Shubik (2005), Zame (1993); DGS05 written in 1980's
  ii) Structural/Quantitative: Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007); Livshits, MacGee, and Tertilt (2007)
  iii) Purely Empirical: Fay, Hurst, and White (2002)
- Paper I just explained connects all three strands
- **Recent advances** using sufficient statistics:
  i) Bankruptcy and Aggregate Demand: Auclert and Mitman (2022)
    - Nicely connecting to structural/quantitative work
  ii) Evictions: Collinson, Humphries, Kestelman, Nelson, van Dijk, and Waldinger (2024)
    - Nicely connecting to purely empirical work
- Lots to do
  - Click here to see my discussion of Acharya, Anshuman, and Viswanathan (2024) on Repo Exemptions
  - Corporate vs. Consumer Bankruptcy

Paper #2: Sufficient Statistics for Deposit Insurance Coverage

Based on "Optimal Deposit Insurance" (Dávila and Goldstein, 2023)

Sufficient Statistics for Deposit Insurance

- Deposit insurance: main explicit financial guarantee
- Significant effects in the US
  - 4,000 bank failures only in 1933
  - 4,000 bank failures between 1934 and 2014
- Ongoing debate in many countries

Sufficient Statistics for Deposit Insurance

- Deposit insurance: main explicit financial guarantee
- Significant effects in the US
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  - 4,000 bank failures between 1934 and 2014
- Ongoing debate in many countries
- **Question**: What is the optimal level of deposit insurance?
  - Are existing coverage levels optimal?
- **Main Result**: Characterize welfare impact of changes in the level of DI coverage $\frac{dW}{d\delta}$
  - Applies broadly
  - As a function of a small number of sufficient statistics

Perspective on Related Work

- Earlier work on deposit insurance
  i) Purely Theoretical: hundreds (thousands?) of papers based on Diamond and Dybvig (1983)
    - Theoretical innovation of Dávila and Goldstein (2023) is to have heterogeneous depositors

  ii) Purely Empirical: scattered and largely disconnected from theory

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- Policymakers didn't know which variables to measure to quantitatively determine exemption limits
  - Lots of interest from deposit insurers around the world

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- Policymakers didn't know which variables to measure to quantitatively determine exemption limits
  - Lots of interest from deposit insurers around the world
- **Recent advances:**
  - Empirical work measuring sufficient statistics
    - De Roux and Limodio (2023), Quintero-Valdivieso (2025)
  - Regulators computing/reporting key variables

Main Results

1. Welfare impact of change in level of coverage

$$\frac{dW}{d\delta} = \boxed{A} \times \boxed{B} - \boxed{C} \times \boxed{D}$$

- Marginal benefit
  - $\boxed{A}$ -Sensitivity of bank failure probability to DI change
  - $\boxed{B}$ Gain of preventing marginal failure
- Marginal cost
  - $\boxed{C}$ Probability of bank failure
  - $\boxed{D}$ Expected marginal social cost of intervention in case of bank failure
- More in the paper
  - Ex-ante regulation
  - Quantification

Environment

- $t = \{0, 1, 2\}$
- Aggregate state (profitability) $s \in [\underline{s}, \bar{s}]$, known at date 1, cdf $F(\cdot)$
- **Depositors**
  - Double continuum of depositors, mass $D_{0i} \sim G(\cdot)$ (cdf)
  - Fraction λ of early types
  - Endowments $Y_{1i}(s)$ (early), $Y_{2i}(s)$ (late)
  - Ex-ante expected utility

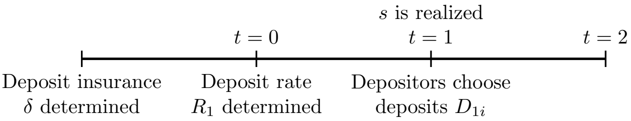
$$\mathbb{E}_s \left[\lambda U(C_{1i}(s)) + (1-\lambda)U(C_{2i}(s))\right]$$

  - Depositors choose $D_{1i}(s) \in [0, R_1 D_{0i}]$
- **Banks technology**
  - $-1 \to \rho_1(s)$ (date 1) $\to \rho_2(s) > 0$ (date 2)
  - Returns $\rho_1(s) > 0$ and $\rho_2(s) > 0$, increasing in s
- **Deposit contract**
  - Banks offer noncontingent deposit rate R_1
  - Pro-rata distribution after failure
- **Deposit insurance**
  - Government guarantees δ dollars
  - Fiscal shortfall is $T(s)$; Cost of public funds $\kappa(T(s))$
  - DWL $1 - \chi(s)$ after bank failure

Environment

- **Taxpayers**

$$V_\tau(\delta, R_1) = \mathbb{E}_s\left[U\left(Y_\tau(s) - T(s) - \kappa(T(s))\right)\right]$$



- Two possibilities at date 1: failure or no failure

$$C_{1i}(s) = \begin{cases} \min\{D_{0i}R_1, \delta\} + \alpha_F(s)\max\{D_{0i}R_1 - \delta, 0\} + Y_{1i}(s), & \text{Bank Failure} \\ D_{0i}R_1 + Y_{1i}(s), & \text{No Failure} \end{cases}$$

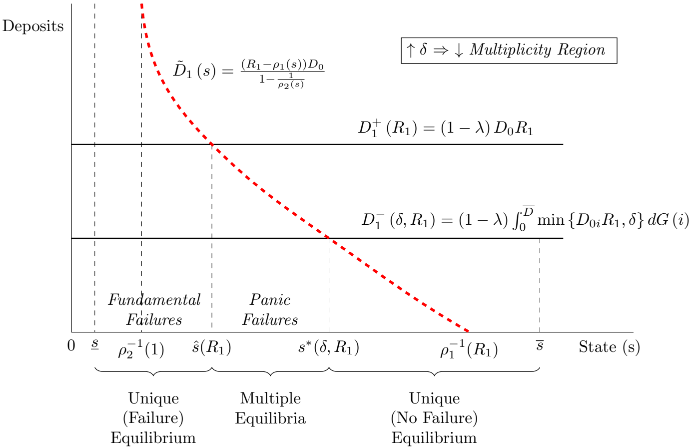
$$C_{2i}(s) = \begin{cases} \min\{D_{0i}R_1, \delta\} + \alpha_F(s)\max\{D_{0i}R_1 - \delta, 0\} + Y_{2i}(s), & \text{Bank Failure} \\ \alpha_N(s)D_{0i}R_1 + Y_{2i}(s), & \text{No Failure} \end{cases}$$

Equilibrium: Definition

- **Equilibrium**: depositors choose $D_{1i}(s)$ optimally, given other depositors' choices and given values of R_1 and δ
  - Symmetric equilibria
  - Sunspot $\pi \in [0, 1]$
- **Key assumptions**
  1. Restriction to deposit contract (noncontingent and demandable)
  2. Single policy instrument (noncontingent deposit insurance with full commitment)
- **Three scenarios**
  1. R_1 predetermined (baseline)
  2. R_1 chosen by competitive banks
  3. R_1 chosen by the planner (perfect regulation)

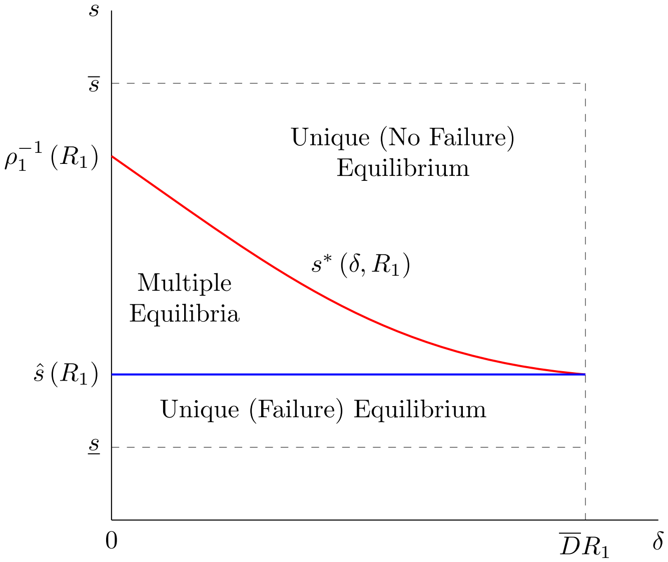
Equilibrium: Depositor's Behavior

- Three types of depositor
  1. Early depositors: withdraw all deposits
  2. Full insured late depositors: leave all deposits
  3. Partially insured late depositors: leave δ deposits (indeterminacy)



Equilibrium: Regions

- Failure probabilty



Welfare

$$W(\delta) = \int V_j\left(\delta, R_1\right) dj = \underbrace{\int V_i\left(\delta, R_1\right) dG(i)}_{\text{Depositors}} + \underbrace{V_\tau\left(\delta, R_1\right)}_{\text{Taxpayers}}$$

- Utilitarian welfare: not trivial (see below)

Main result: Directional Test for level of DI

- Marginal change in DI

$$\frac{dW}{d\delta} = - \underbrace{\frac{\partial q^F}{\partial \delta}}_{\substack{\text{Change in} \\ \text{Failure Probability}}} \underbrace{\int \left[U\left(C_j^N\left(s^*\right)\right) - U\left(C_j^F\left(s^*\right)\right)\right] dj}_{\text{Aggregate Consumption Drop}} +$$

$$+ \underbrace{q^F}_{\substack{\text{Failure} \\ \text{Probability}}} \mathbb{E}_s^F \left[\int U'\left(C_j^F\right) \frac{\partial C_j^F}{\partial \delta} dj \right]$$

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- Aggregate changes
  - $\int \frac{\partial C_j^F}{\partial \delta} dj = - \overbrace{\kappa'(T(s))}^{\substack{\text{Mg. Cost}\\ \text{of Public Funds}}} \overbrace{\int_{\frac{\delta}{R_1}}^{\bar{D}} dG(i)}^{\substack{\text{Mass of}\\ \text{Partially Insured}}}$
  - $\int \left[C_j^N\left(s^*\right) - C_j^F\left(s^*\right)\right] dj =$
    $\underbrace{\left(\rho_2\left(s^*\right) - 1\right)\left(\rho_1\left(s^*\right) - \lambda R_1\right) D_0}_{\text{Net Return Loss}} + \underbrace{\left(1 - \chi\left(s^*\right)\right)\rho_1\left(s^*\right) D_0}_{\substack{\text{Bank Failure}\\ \text{Deadweight Loss}}} + \underbrace{\kappa\left(T\left(s^*\right)\right)}_{\substack{\text{Total Net Cost}\\ \text{of Public Funds}}}$

Quantitative Implications

$$\text{Model Primitives} \underbrace{\Rightarrow}_{(2)} \text{Sufficient Statistics} \underbrace{\Rightarrow}_{(1)} \text{Welfare}$$

1. Direct measurement
   - Local test for whether to change δ
   - No need to specify primitives
2. Model simulation
   - Global comparative statics/counterfactuals

- See paper for both

Quantitative Implications: Direct Measurement

- Normalizing welfare change:

$$\frac{\frac{dW_k}{d\delta}}{\bar{G}_k} \approx q_k^F \left(-\frac{\partial \log q_k^F}{\partial \delta} \frac{\int \left[C_{j,k}^N(s^*) - C_{j,k}^F(s^*) \right] dj}{\bar{G}_k} - \mathbb{E}_s^F \left[\kappa'(\cdot) \right] \frac{\int_{\frac{\delta}{R_1}}^{\bar{D}} dG_k(i)}{\bar{G}_k} \right)$$

Table 1: Direct Measurement: Sufficient Statistics (Baseline)

Variable	Description	Value
	Marginal benefit	
$\frac{\partial \log q_k^F}{\partial \delta}$	Sensitivity of log failure probability to change in DI limit	$-\frac{0.129}{150{,}000}$
$\int \left[C_{j,k}^N(s^*) - C_{j,k}^F(s^*) \right] dj / \bar{G}_k$	Resource losses per account after failure	\$7,840
	Marginal cost	
$\kappa'(\cdot)$	Net marginal cost of funds	13%
$\int_{\frac{\delta}{R_1}}^{\bar{D}} dG_k(i) / \bar{G}_k$	Fraction of partially insured depositors	6.4%
q_k^F	Probability of bank failure	0.75%

Note: Table 1 includes the baseline measures of the relevant sufficient statistics. The sensitivity of the probability of bank failure to a change in the coverage limit is computed using CDS data from Markit through WRDS. The measure of resource losses combines information from Martin, Puri and Ulfier (2017) with estimates from Granja, Matvos and Seru (2017) and Bennett and Unal (2015). The cost of public funds is consistent with Dahlby (2008). The fraction of partially insured depositors comes from Martin, Puri and Ulfier (2017). The probability of bank failure, as discussed in the text, combines FDIC's historical banking statistics with the CDS data.

Closing Thoughts on Sufficient Statistics

- Sufficient statistics
  1. Bridge theory and measurement
  2. Allow us to make statements that apply broadly
  3. Can be used with structural models (calibration/model selection)
  4. And to guide reduced-form measurement

Closing Thoughts on Sufficient Statistics

- Sufficient statistics
  1. Bridge theory and measurement
  2. Allow us to make statements that apply broadly
  3. Can be used with structural models (calibration/model selection)
  4. And to guide reduced-form measurement
- Already used in finance very successfully
  - But only in a narrow set of topics
  - And were not called sufficient statistics!

Closing Thoughts on Sufficient Statistics

- Sufficient statistics
  1. Bridge theory and measurement
  2. Allow us to make statements that apply broadly
  3. Can be used with structural models (calibration/model selection)
  4. And to guide reduced-form measurement

- Practical challenges:
  1. Not easy to judiciously choose question and environment
  2. Publishing these papers is <u>hard</u>: critiques from both theorists and empiricists!

Part II: Normative Finance

Normative vs. Positive Economics

- Normative Economics: “What is desirable”
- Positive Economics: “What happens”

Normative vs. Positive Economics

- Normative Economics: "What is desirable"
- Positive Economics: "What happens"
- Normative questions <u>always matter</u>
  - $\boxed{\text{Welfare Primacy}}$ $\Rightarrow$ we should study questions that matter for well-being
  - Demand from policymakers $\Rightarrow$ Policy jobs

Normative vs. Positive Economics

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- Positive Economics: "What happens"
- Normative questions always matter
  - $\boxed{\text{Welfare Primacy}}$ $\Rightarrow$ we should study questions that matter for well-being
  - Demand from policymakers $\Rightarrow$ Policy jobs
- Normative assessments
  - are exclusive to Economics (not in physics!)
  - and require theory

Normative Finance vs. Other Fields

- Normative work varies widely across fields
- Finance ⇒ very small
  Mostly in intermediation, banking, market design
  - Big questions are positive
    - AP: cross-section and time-series of returns/prices
    - CF: capital structure, payout policy
  - Gaps became evident during and after 2008 crisis
- Normative work much larger in other fields
  - Macro: Monetary and fiscal policy
  - Trade: Welfare gains from trade, trade policy
  - IO: Merger analysis

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  - Trade: Welfare gains from trade, trade policy
  - IO: Merger analysis
- Why not more normative finance work? Three hypothesis
  1. Selection into the field
  2. Lack of training ⇒ fixable
  3. Harder than other areas ⇒ fixable

Normative Finance has Unique Challenges

- Study of policy in Finance is hard because we deal with
  Same applies to heterogeneous-agents macro
  - dynamic stochastic environments
  - heterogeneous agents
- Significant conceptual challenges
  1. **Units**: From ordinal utilities to cardinal welfare assessments
  2. **Aggregation**: Interpersonal welfare comparisons
  3. **Sources**: Why do welfare gains emerge?

“Origins of Welfare Gains” Agenda

- Ongoing agenda (with Andreas Schaab)

  What are the sources of welfare gains and losses?

- Core papers
  1. Welfare Assessments with Heterogeneous Individuals
     Incomplete Markets
  2. Welfare Accounting
     Production and Exchange
  3. Intergenerational Welfare Assessments (w/Barcons)
     Demographics/OLG
  4. The Inconsistency of Welfare Assessments with Heterogeneous Agents
     Time Consistency
  5. Dynamic Stochastic with Capital Accumulation (w/Hassanein)
     Capital Accumulation

- Monograph coming this Fall:

  “Welfare Assessments: Theory and Applications”

This Lecture

- I would love to tell you about the "origins" papers...
  - But I usually spend >6 hours only on paper #1
  - I teach a full semester on normative topics
- Today: two illustrations of *Normative Finance + Ten Rules*
  i) Value of Arbitrage
  ii) Probability Pricing
- Deep connections between Financial and Welfare Economics
  - Both are theories of valuation

$\boxed{\text{Paper \#3: The Value of Arbitrage}}$

Based on Dávila, Graves, and Parlatore (2024)

Motivation

- Absence of Arbitrage $\Rightarrow$ Pillar of modern finance
- Active (empirical) literature documents arbitrage violations
  - CIP, Swap spreads, ADR's, dual-listed stocks, etc.

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What is the (social) value of closing an arbitrage gap?

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  - CIP, Swap spreads, ADR's, dual-listed stocks, etc.
- Open normative question:

What is the (social) value of closing an arbitrage gap?

- **This paper** ⇒ Framework to understand the welfare costs of arbitrage violations
- Perspective on literature
  1. Most important question in trade is "welfare gains from trade"
  2. Little work on "welfare gains from financial markets"

"The welfare cost of the CIP deviations is beyond the scope of this paper as it would necessitate a general equilibrium model."

Du, Tepper, and Verdelhan (2018)

Results

- Theoretical
  1. Marginal social value of arbitrage is *exactly* the arbitrage gap
     - *Useful interpretation of existing evidence*

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  3. Total value of arbitrage linked to market liquidity/price impact
     - *Arbitrage gaps in liquid markets are costlier*
     - *CIP deviations can potentially have nontrivial welfare costs*

Results

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  3. Total value of arbitrage linked to market liquidity/price impact
     - *Arbitrage gaps in liquid markets are costlier*
     - *CIP deviations can potentially have nontrivial welfare costs*
- Empirical Application: Covered Interest Parity $\Rightarrow$ see paper
  - Directly measure price impact (in FX Futures market)
  - Welfare gains: $< \$300M$ outside yen-dollar
  - Why? CIP deviations are large when markets are illiquid

Baseline Environment: Investors

- Two dates, $t = \{0, 1\}$; no uncertainty
- Single good endowment economy
- Two markets: $i = \{A, B\}$
  - One risk-free asset in each market: payoff d_1, price p^i

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  - Arbitrageurs: trade across markets

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$$\max_{q_0^i} u_i\left(c_0^i, c_1^i\right)$$

subject to

$$c_0^i = n_0^i - p^i q_0^i + p^i q_{-1}^i$$
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- Preferences and endowments can differ across markets

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- $p^B > p^A$ in autarky (w.l.o.g)
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c_0^\alpha &= -\left(p^A q_0^{A\alpha} + p^B q_0^{B\alpha}\right) \\
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- Arbitrage strategy
  - Scale: $\boxed{m \equiv q_0^{A\alpha}}$
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- Arbitrage strategy
  - Scale: $\boxed{m \equiv q_0^{A\alpha}}$
  - Direction: $\boxed{x_0^\alpha \equiv \frac{q_0^{B\alpha}}{q_0^{A\alpha}}}$
- Arbitrageurs' ~~indirect~~ utility ($= c_0^\alpha$)

$$V^\alpha\left(m, p^A, p^B\right) = \left(p^B - p^A\right) m$$

Baseline Environment: Arbitrageurs

- No assumptions on the behavior of arbitrageurs
  - Unlike, e.g., Gromb and Vayanos (2002), Shleifer and Vishny (1997)
- **Scale** of arbitrage trade m **as** a **primitive**
  - Different frictions map to m
  - Relaxing frictions increases m
  - Tightening frictions decreases m
- Microfoundations in Appendix
  1. Trading costs
  2. Strategic trading
  3. Short sales/Borrowing constraints
  4. Collateral constraints
- Sufficient statistics logic

Equilibrium

- An *arbitrage equilibrium*, parametrized by m, is a set of allocations and prices $p^A(m)$ and $p^B(m)$ such that i) investors maximize utility, and ii) asset markets clear:

$$\Delta q_0^A + m = 0$$

$$\Delta q_0^B - m = 0$$

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- **Remarks**
  1. Smooth way of going from (differential methods)
     - Autarky equilibrium: $m = 0$
     - to Integrated equilibrium: $p^A = p^B$ and $m = m^\star$
  2. Hypothetical experiment
     - Lucas (1987)/Alvarez and Jermann (2004) on cost of business cycles

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- In well-behaved model

$$\frac{dp^A}{dm} > 0 \quad \text{and} \quad \frac{dp^B}{dm} < 0$$

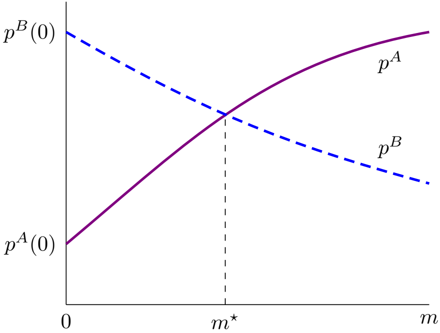
Definitions

- *Arbitrage gap*:

$$\mathcal{G}_{BA}(m) := p^B(m) - p^A(m)$$

- *Gap-closing trade*:

$$m^\star \quad \text{such that} \quad p^B(m^\star) = p^A(m^\star)$$



Marginal Value of Arbitrage

Lemma 1: The marginal individual value of arbitrage is

$$\frac{\frac{dV^A}{dm}}{\lambda_0^A} = \underbrace{\frac{dp^A(m)}{dm}}_{>0} \underbrace{\left(q_{-1}^A - q_0^A\right)}_{=m} > 0$$

$$\frac{\frac{dV^B}{dm}}{\lambda_0^B} = \underbrace{\frac{dp^B(m)}{dm}}_{<0} \underbrace{\left(q_{-1}^B - q_0^B\right)}_{=-m} > 0$$

$$\frac{\frac{dV^\alpha}{dm}}{\lambda_0^\alpha} = \underbrace{\left(\frac{dp^B(m)}{dm} - \frac{dp^A(m)}{dm}\right)}_{<0} m + \underbrace{p_B(m) - p_A(m)}_{>0}$$

Proposition 1: The marginal social value of arbitrage is

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- Direct effect + distributive pecuniary externalities (cancel out)
  Dávila and Korinek (2018)
- Arbitrage gap gives marginal social value of arbitrage

Social Value of Arbitrage

Proposition 2: The social value of arbitrage is given by

$$W(m^*) - W(m_0) = \int_{m_0}^{m^*} W'(m)\, dm = \int_{m_0}^{m^*} \mathcal{G}_{BA}(m)\, dm.$$

- Knowing
  i) the initial arbitrage gap, $\mathcal{G}_{BA}(m_0)$, and
  ii) measures of price impact in both markets A and B

  is sufficient to compute the social value of arbitrage:

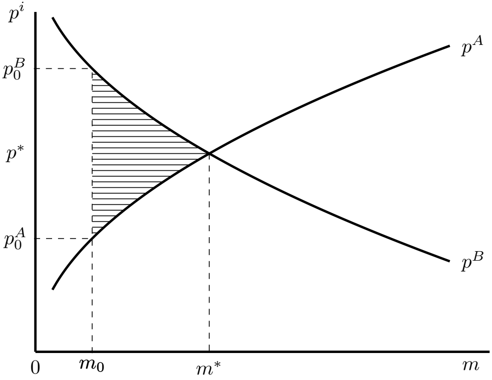
$$\mathcal{G}_{BA}(m) = \underbrace{p_B(m_0) - p_A(m_0)}_{=\mathcal{G}_{BA}(m_0)} + \int_{m_0}^{m} \left(\frac{dp_B(\tilde{m})}{dm} - \frac{dp_A(\tilde{m})}{dm} \right) d\tilde{m}.$$

- Sufficient statistics

Liquidity and the Value of Arbitrage

- **Proposition 3**: For a given arbitrage gap $p^A(m) - p^B(m)$, the social value of arbitrage is
  - higher in liquid markets (small price impact)
  - lower in illiquid markets (large price impact)

Illustration



- Shaded areas measure total value of arbitrage
  - High price impact (steep curves) $\Rightarrow$ Small gains

- **Intuition**: Large gaps in illiquid markets are easy to close

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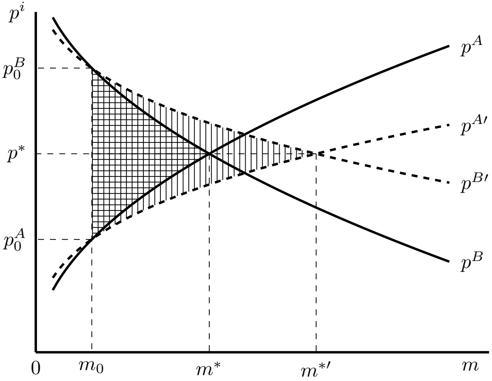
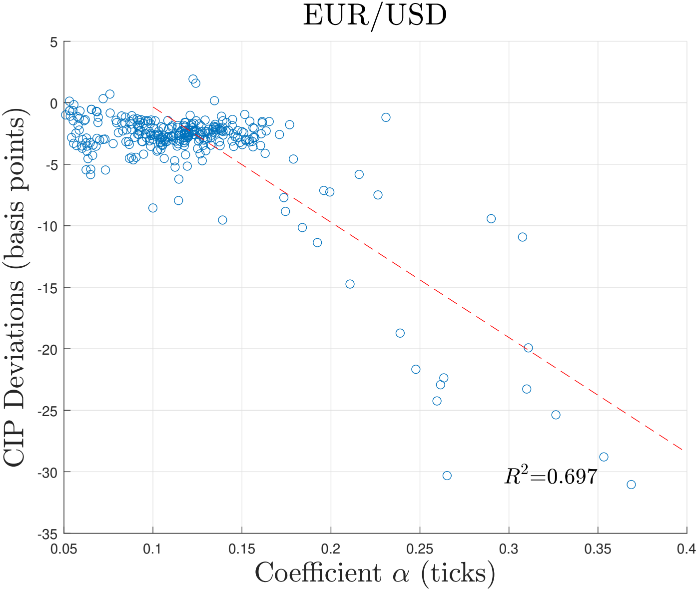


- Shaded areas measure total value of arbitrage
  - High price impact (steep curves) $\Rightarrow$ Small gains
  - Low price impact (flat curves) $\Rightarrow$ Large gains
- **Intuition**: Large gaps in illiquid markets are easy to close

Fact

- CIP Fact: gaps are larger when markets are illiquid



Empirical Results and Takeaways

- Paper measures price impact $\Rightarrow$ See paper for details
  - Uses formulas to compute welfare
  - Another sufficient-statistic application
- Illustration of Normative Finance exercise
  - Results and measurement driven by normative question
  - Theory is necessary

Paper #4: Probability Pricing

Based on Dávila, Parlatore, and Walther (2025)

Motivation

- What is the value of uncertain cash-flows?
  changes in consumption

Cash-Flow/Asset Pricing ⇒ Widely studied

- What is the value of changes in uncertainty itself?
  changes in probabilities

$\boxed{\text{Probability Pricing}}$ ⇒ This paper

- This paper illustrates how the tools of Asset Pricing are very useful in Normative Finance ⇒ Valuation

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- What is the value of changes in uncertainty itself?
  changes in probabilities

  $\boxed{\text{Probability Pricing}}$ ⇒ This paper

- Practical relevance: what is the value of

  i) reducing the probability of a disaster?
  how to hedge a change in disaster probabilities? (not the actual disaster!)

  ii) reducing consumption or output volatility?

  iii) more precise public or private information?
  social value (welfare) vs. private value ⇒ disclosure, aggregation, etc.

- This paper illustrates how the tools of Asset Pricing are very useful in Normative Finance ⇒ Valuation

Asset/Cash-Flow Pricing (reminder)

- Expected utility preferences
  easy to generalize: epstein-zin, ambiguity, habits, etc.

$$V = u(c_0) + \beta \int_{\underline{s}}^{\overline{s}} u(c_1(s)) f(s) \, ds$$

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$$V = u(c_0) + \beta \int_{\underline{s}}^{\overline{s}} u(c_1(s)) f(s) \, ds$$

- $\boxed{\text{Asset Price:}}$ willingness-to-pay p_x for marginal unit q of asset with cash flows $x(s)$

$$c_0 = \ldots - p_x q$$
$$c_1(s) = \ldots + x(s) q$$

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$$c_0 = \ldots - p_x q$$
$$c_1(s) = \ldots + x(s)\, q$$

- Asset (cash-flow) pricing:

$$\boxed{p_x = \int_{\underline{s}}^{\bar{s}} \omega(s)\, x(s)\, ds} \qquad \text{where} \qquad \omega(s) = \underbrace{\overbrace{\frac{\beta u'(c_1(s))}{u'(c_0)}}^{m(s)\ \text{(SDF)}} f(s)}_{\text{state-price}}$$

  - $p_x \uparrow$ if payoffs $x(s) \uparrow$, in particular in high $\omega(s)$ states

(Towards) Probability Pricing

- Expected utility preferences

$$V = u(c_0) + \beta \int_{\underline{s}}^{\overline{s}} u(c_1(s)) f(s; \theta) \, ds$$

s.t.

$$c_0 = \ldots - p_\theta \theta$$

(Towards) Probability Pricing

- Expected utility preferences

$$V = u(c_0) + \beta \int_{\underline{s}}^{\overline{s}} u(c_1(s)) f(s;\theta)\, ds$$

s.t.

$$c_0 = \ldots - p_\theta \theta$$

- $\boxed{\text{Probability Price:}}$ willingness-to-pay p_θ for marginal perturbation $d\theta$ to $f(s;\theta) \rightarrow \int_{\underline{s}}^{\overline{s}} \frac{df(s;\theta)}{d\theta} ds = 0$
  - Perturbation → Gateaux derivative → $\theta \underline{F}(s) + (1-\theta)\overline{F}(s)$
    Dávila and Walther (2023) → non-parametric comparative static (beliefs)

(Towards) Probability Pricing

- Expected utility preferences

$$V = u\left(c_0\right) + \beta \int_{\underline{s}}^{\overline{s}} u\left(c_1\left(s\right)\right) f\left(s; \theta\right) ds$$

s.t.

$$c_0 = \ldots - p_\theta \theta$$

- $\boxed{\text{Probability Price:}}$ willingness-to-pay p_θ for marginal perturbation $d\theta$ to $f\left(s; \theta\right) \rightarrow \int_{\underline{s}}^{\overline{s}} \frac{df(s;\theta)}{d\theta} ds = 0$
  - Perturbation $\rightarrow$ Gateaux derivative $\rightarrow \theta \underline{F}\left(s\right) + \left(1 - \theta\right) \overline{F}\left(s\right)$
    Dávila and Walther (2023) $\rightarrow$ non-parametric comparative static (beliefs)
- (Towards) probability pricing:
  consumption invariant to θ (relaxed later)

$$p_\theta = \int_{\underline{s}}^{\overline{s}} \frac{\beta u\left(c_1\left(s\right)\right)}{u'\left(c_0\right)} \frac{df\left(s; \theta\right)}{d\theta} ds$$

  - $p_\theta \uparrow$ if probability shifts $\frac{df(s)}{d\theta}$ to high $u\left(c_1\left(s\right)\right)$ states

(Towards) Probability Pricing

- Problems with

$$p_\theta = \int_{\underline{s}}^{\bar{s}} \frac{\beta u\left(c_1\left(s\right)\right)}{u'\left(c_0\right)} \frac{df\left(s;\theta\right)}{d\theta} ds$$

1. Hard to compare to cash-flow pricing
   different units
2. $\frac{\beta u(c_1(s))}{u'(c_0)}$ is problematic $\Rightarrow$ <u>not</u> a "SDF" for probabilities
   - e.g. $u\left(\cdot\right) \to u\left(\cdot\right) + a$

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- Solution $\Rightarrow$ integration-by-parts: $\int u dv = \overbrace{uv}^{=0} - \int v du$
  widely used in screening/mechanism design $\Rightarrow$ different focus

Probability Pricing

- **Proposition 1**: The probability price p_θ is

$$\boxed{p_\theta = \int_{\underline{s}}^{\bar{s}} \omega(s)\, x_\theta(s)\, ds} \quad \text{where} \quad \omega(s) = \underbrace{\overbrace{\frac{\beta u'(c_1(s))}{u'(c_0)}}^{m(s)\ \text{(SDF)}} f(s;\theta)}_{\text{state-price}}$$

with consumption-equivalent cash-flows

$$\boxed{x_\theta(s) = \underbrace{\frac{\frac{d(1-F(s;\theta))}{d\theta}}{f(s;\theta)}}_{\substack{\text{Normalized}\\ \text{Survival}\\ \text{Change}}} \underbrace{\frac{dc_1(s)}{ds}}_{\substack{\text{Consumption}\\ \text{Sensitivity}}}}$$

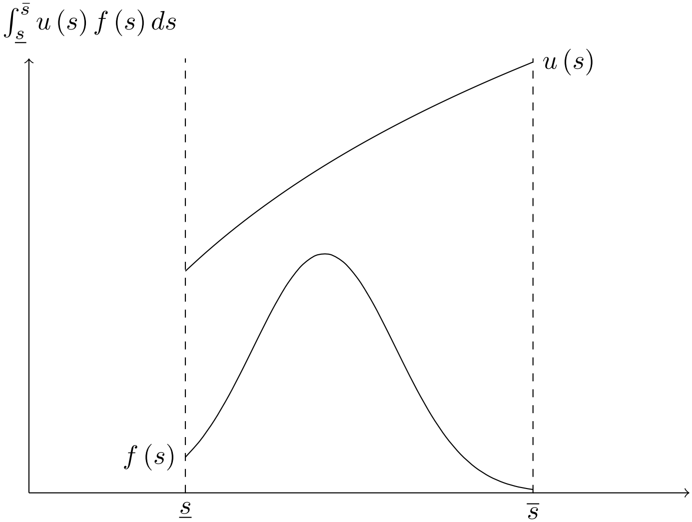
i) Probability price p_θ is the price of an asset with payoffs $x_\theta(s)$

ii) Changes in probabilities $\Rightarrow$ consumption-equivalents

Discrete States | $p_\theta \neq$ Comparative Static of p_x

Economic Intuition

assumption: $c_1(s) = s \Rightarrow \frac{dc_1(s)}{ds} = 1$



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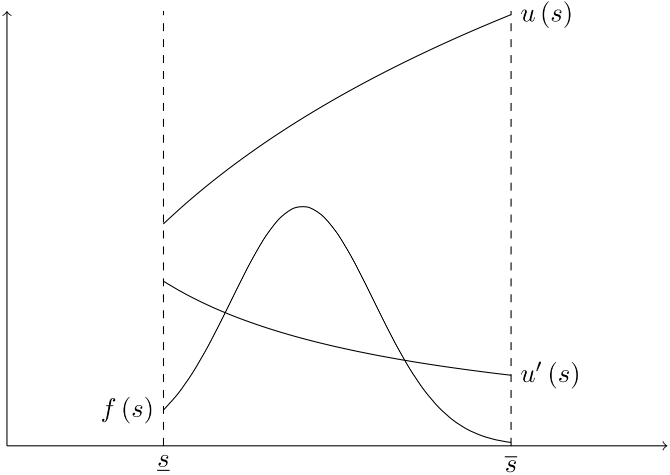

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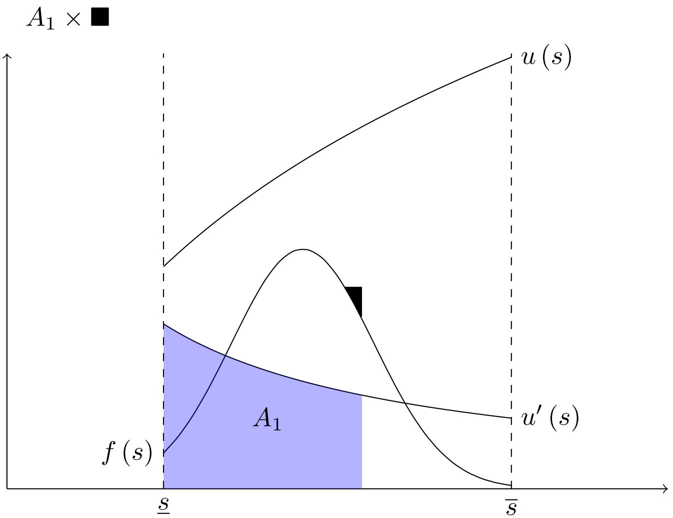

- Perturb pdf at a point: $u(s)\, df(s) = \left(u(\underline{s}) + \int_{\underline{s}}^{s} u'(t)\, dt \right) df(s)$

  "gaining $u(s)$ is equivalent to gaining all $u'(s)$ to the left"

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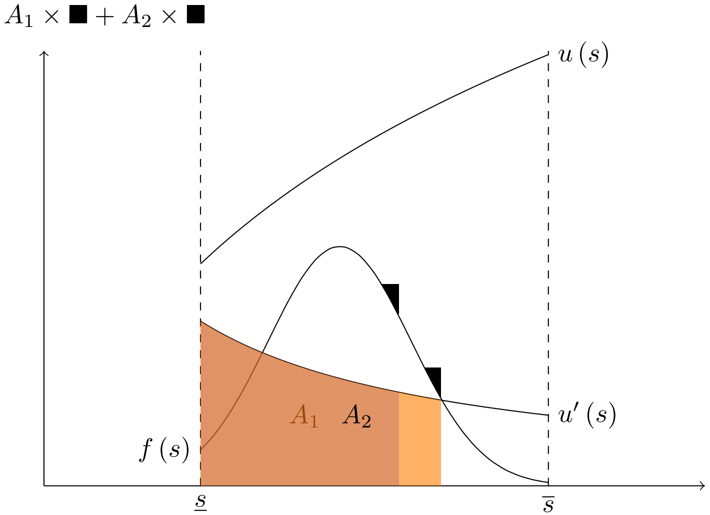

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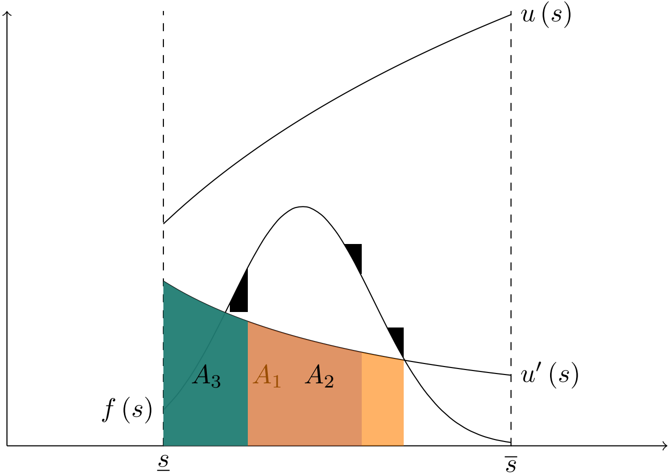
$A_1 \times \blacksquare + A_2 \times \blacksquare - A_3 \times \blacksquare$



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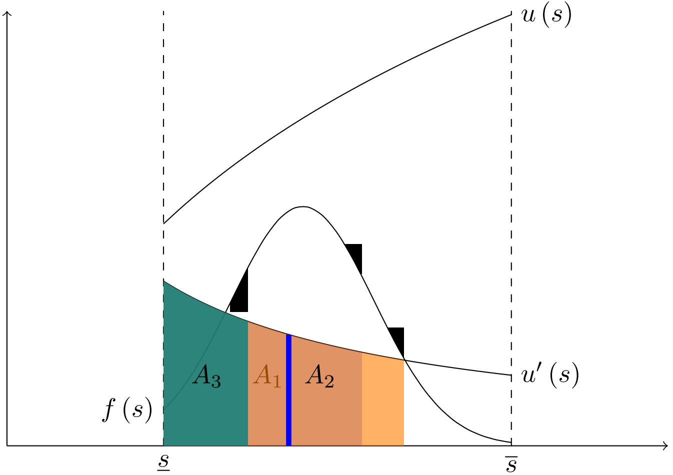
Economic Intuition assumption: $c_1(s) = s \Rightarrow \frac{dc_1(s)}{ds} = 1$

$$A_1 \times \blacksquare + A_2 \times \blacksquare - A_3 \times \blacksquare = \int_{\underline{s}}^{\bar{s}} u'(s) \frac{d(1-F(s;\theta))}{d\theta} ds$$



- Alternative: $u'(s) \times \underbrace{\text{sum of density changes to the right}}_{\frac{d(1-F(s;\theta))}{d\theta}}, \forall s$

Probability Pricing

$$p_\theta = \int_{\underline{s}}^{\bar{s}} \omega(s)\, x_\theta(s)\, ds \quad \text{where} \quad x_\theta(s) = \underbrace{\frac{\frac{d(1-F(s;\theta))}{d\theta}}{f(s;\theta)}}_{\substack{\text{Normalized}\\ \text{Survival}\\ \text{Change}}} \underbrace{\frac{dc_1(s)}{ds}}_{\substack{\text{Consumption}\\ \text{Sensitivity}}}$$

arbitrary preferences + distributions + perturbations

Why is Probability Pricing Useful?

$$x_\theta(s) = \frac{\frac{d(1-F(s;\theta))}{d\theta}}{f(s;\theta)} \frac{dc_1(s)}{ds}$$

1. How to **hedge/immunize** against changes in probabilities?

2. Decompositions $\Leftarrow$ rely on consumption-equivalents

3. Cash-Flow + Probability Pricing

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   - $x_\theta(s) \Rightarrow$ cash-flows hedging/immunization strategy must replicate

     "Tell me your exposure $\Rightarrow$ I can build you a portfolio equivalent to reducing the chance of some disaster by x%"

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2. Decompositions $\Leftarrow$ rely on consumption-equivalents
   - i) Stochastic: $p_\theta = \underbrace{\frac{1}{1+r^f}\frac{d\mathbb{E}[c_1(s)]}{d\theta}}_{\text{Expected Payoff}} + \underbrace{\mathbb{C}ov[m(s), x_\theta(s)]}_{\text{Risk Compensation}}$
   - ii) Cross-sectional:

$$\sum_i p^i_\theta = \underbrace{\int \bar{\omega}(s) \sum_i x^i_\theta(s)\, ds}_{\text{Aggregate gains/efficiency}} + \underbrace{\int \mathbb{C}ov_i\left[\omega^i(s), x^i_\theta(s)\right] ds}_{\text{Risk-sharing (re-shuffling to high MU)}}$$

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3. Cash-Flow + Probability Pricing $\Leftarrow$ $\boxed{\textbf{Welfare in Equilibrium Models}}$

$$\underbrace{\frac{\frac{dV}{d\theta}}{u'(c_0)}}_{\text{Valuation/Welfare}} = \int_{\underline{s}}^{\bar{s}} \omega(s) \left(\underbrace{\frac{\partial c_1(s;\theta)}{\partial\theta}}_{\text{Consumption}} + \underbrace{\frac{\frac{d(1-F(s;\theta))}{d\theta}}{f(s;\theta)} \frac{\partial c_1(s;\theta)}{\partial s}}_{\text{Probability}} \right) f(s;\theta)\, ds$$

Particular Perturbations

$x_\theta(s) = \frac{\frac{d(1-F(s;\theta))}{d\theta}}{f(s;\theta)} \frac{dc_1(s)}{ds}$

1. Mean/Variance perturbations: $s = \mu + \sigma n$
   $\mathbb{E}[n] = 0$ and $\mathbb{V}\mathrm{ar}[n] = 1$

   - Change in $\mu \Rightarrow \frac{\frac{d(1-F(s))}{d\mu}}{f(s)} = 1$ (sanity check)

$$c_1(s) = s \quad \Longrightarrow \quad p_\mu = \int_{\underline{s}}^{\bar{s}} \omega(s)\, ds \text{ (risk-free asset)}$$

   Uniform mass shift to the right $\Longleftrightarrow$ uncontingent consumption

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   - Change in h: $\frac{\frac{d(1-F(s))}{dh}}{f(s)} = \frac{\overline{F}(s)-\underline{F}(s)}{(1-h)\overline{f}(s)+h\underline{f}(s)}$ e.g. disaster

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3. Stochastic Dominance $\Rightarrow$ money lotteries $c_1(s) = s$
   Arrow, Pratt, Rothschild-Stiglitz, ...
   - FOSD: $\frac{d(1-F(s))}{d\theta} \geq 0 \Rightarrow p_\theta \geq 0$
   - SOSD: $\frac{d\mathbb{E}[s]}{d\theta} = 0$ and $\int_{\underline{s}}^{s} \frac{dF(t)}{d\theta} dt \geq 0 \Rightarrow p_\theta \leq 0$ if risk-averse

Perspective on Related Work

- Classic Work: Arrow (1971) Pratt (1964), Rothschild and Stiglitz (1970)
- Textbook: Gollier (2001)
- Classic work largely focused on defining "stochastic orders"
  - Orders are typically incomplete
  - Imagine we only price assets that we can rank!!
- Probability Pricing ⇒ Value of marginal methods

$$\boxed{\text{Application: Principal-Agent Problem}}$$

Application

- Consider a classic principal-agent problem
- Suppose that the distribution of output compresses (agent's output is less noisy)
- **Questions**: Is this good or bad? Why?

Application

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- Suppose that the distribution of output compresses (agent's output is less noisy)
- **Questions**: Is this good or bad? Why?
- This application illustrates the connection between
  - Welfare
  - Valuations
  - Willingness-to-pay
- More *normative* applications in the paper
  - What is the value of public information?
  - What is the value of private information?

Principal-Agent: Environment

- Principal $i = B$ (boss) contracts with $i = A$ (agent)

$$V^B = \int c^B(s)\, f(s)\, ds \quad \text{and} \quad V^A = \int \underbrace{u\left(c^A(s)\right)}_{-e^{-\eta c}} f(s)\, ds$$

- Output: $y(s) = \underbrace{e}_{\text{effort}} + s$, with $s \sim \mathcal{N}\left(0, \sigma^2\right)$ and $\tau = \frac{1}{\sigma^2}$ (precision)

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- Principal: $c^B(s) = y(s) - w(s)$
- Agent: $c^A(s) = w(s) - \psi(e)$, where $\psi(e) = \frac{\kappa}{2} e^2$
- Linear compensation: $w(s) = t + \alpha y(s)$

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- Linear compensation: $w(s) = t + \alpha y(s)$
- Optimal contract

$$\max_{\{e,t,\alpha\}} \int c^B(s) f(s)\, ds$$

subject to

$$\int u\left(c^A(s)\right) f(s)\, ds = \overline{V} \qquad (PC)$$

$$e \in \arg\max \int u\left(c^A(s)\right) f(s)\, ds \qquad (IC)$$

Principal-Agent: Solution

- Equilibrium without IC: $e = \frac{1}{\kappa}$ and $\alpha = 1$
- Equilibrium with IC:
  i) Effort: $e = \frac{\alpha}{\kappa}$
  ii) Incentives: $\alpha = \frac{\tau}{\tau + \eta\kappa}$; as $\tau \to \infty$ (no uncertainty) $\Rightarrow \alpha \to 1$

- Probability Pricing: what is the welfare impact of a change in output uncertainty τ?

Definitions

- (K-H) Efficiency: sum of willingness-to-pay

$$\Xi^E = \sum_i \frac{\frac{dV^i}{d\tau}}{\lambda^i} \quad \text{where} \quad \lambda^i = \int \frac{\partial u^i\left(c^i(s)\right)}{\partial c^i(s)} f(s)\, ds$$

where

$$\frac{\frac{dV^i}{d\tau}}{\lambda^i} = \int \omega^i(s) \left(\frac{\partial c^i(s)}{\partial \tau} + \frac{\frac{d(1-F(s))}{d\tau}}{f(s)} \frac{\partial c^i(s)}{\partial s} \right) ds$$

Definitions

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- (K-H) Efficiency = Aggregate-Efficiency + Risk-Sharing

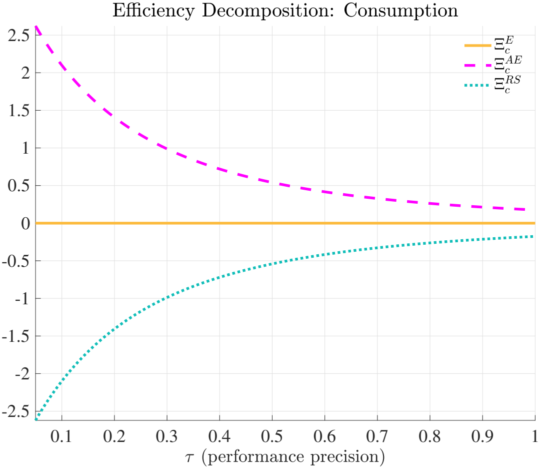
definitions as in Dávila and Schaab (2025 forthcoming)

$$\Xi^E = \underbrace{\Xi_c^{AE} + \Xi_s^{AE}}_{\Xi^{AE}} + \underbrace{\Xi_c^{RS} + \Xi_s^{RS}}_{\Xi^{RS}}$$

- Two sources of efficiency gains:
  i) Ξ^{AE}: changes in PV of aggregate net consumption Aggregate-Efficiency
  ii) Ξ^{RS}: reshuffling consumption towards high $\omega^i\left(s\right)$ Risk-Sharing

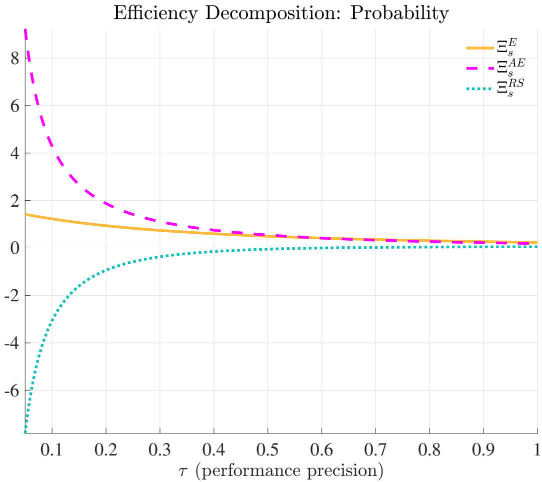
Efficiency Decomposition

Efficiency Decomposition: Consumption



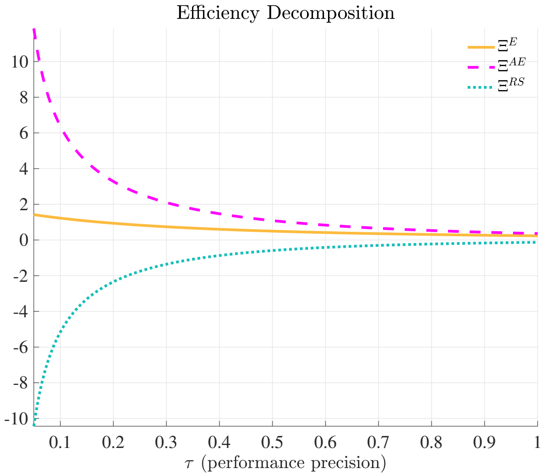
- Consumption: $\Xi_c^E = \Xi_c^{AE} + \Xi_c^{RS} = 0$ (contract adjusts)
  - $\uparrow \tau$ (less volatile output) $\Rightarrow \uparrow \alpha$ (more sensitive contract) $\Rightarrow \uparrow e \Rightarrow$ production efficiency gains $\boxed{\Xi_c^{AE} > 0}$ risk-sharing losses $\boxed{\Xi_c^{RS} < 0}$
  - Why $\Xi_c^E = 0$? constrained efficiency $\Rightarrow \Xi_c^E = 0$

Efficiency Decomposition: Probability



- Probability: $\Xi_s^E = \Xi_s^{AE} + \Xi_s^{RS} > 0$ (contract given)
  - $\Xi_s^{AE} > 0$: $\uparrow \tau \Rightarrow$ aggregate consumption is smoother
  - $\Xi_s^{RS} \gtreqless 0$: $\uparrow \tau \Rightarrow$ smoother aggregate consumption *relatively*
    - i) benefits principal: if $\alpha < \frac{1}{2} \Rightarrow \Xi_s^{RS} < 0$
    - ii) benefits agent: if $\alpha > \frac{1}{2} \Rightarrow \Xi_s^{RS} > 0$

Efficiency Decomposition: AE vs. RS



- $\Xi^{AE} > 0$: production efficiency + smoother agg. consumption
- $\Xi^{RS} \gtreqless 0$: increased incentives (< 0) + relative gain from smoother agg. consumption ($\gtreqless 0$)

Cross-Sectional decomposition $\boxed{\text{requires}}$ probability pricing

$\boxed{\text{What if I want learn more Normative Macro-Finance?}}$

“Origins of Welfare Gains” Agenda (w/Schaab)

- Read
  1. Welfare Assessments with Heterogeneous Individuals
     Incomplete Markets
  2. Welfare Accounting
     Production and Exchange
  3. Intergenerational Welfare Assessments (w/Barcons)
     Demographics/OLG
  4. The Inconsistency of Welfare Assessments with Heterogeneous Agents
     Time Consistency
  5. Dynamic Stochastic with Capital Accumulation (w/Hassanein)
     Capital Accumulation
- Monograph coming this Fall:

  “Welfare Assessments: Theory and Applications”

Applied Papers

1. What is the optimal $\boxed{\text{financial transaction tax}}$ that limits speculative trading?
   Optimal Financial Transaction Taxes (JF, 2023)
2. What is the optimal $\boxed{\text{leverage regulation}}$ when agents have $\boxed{\text{distorted beliefs}}$?
   Prudential Policy with Distorted Beliefs (AER, 2023)
3. What is the optimal $\boxed{\text{second-best regulation}}$ when regulation is imperfect?
   Corrective Regulation with Imperfect Instruments (2025)
4. What is the best way to design $\boxed{\text{corporate taxes}}$?
   Corporate Taxation under Financial Frictions (RESTUD, 2023)
5. When do $\boxed{\text{prices}}$ changes generate $\boxed{\text{externalities}}$?
   Pecuniary Externalities in Economies with Financial Frictions (RESTUD, 2018)
6. How to conduct $\boxed{\text{optimal monetary policy}}$ when agents are heterogeneous?
   Optimal Monetary Policy with Heterogeneous Agents: Discretion, Commitment, and Timeless Policy (AER, R&R)

Extra Materials

- Click here or use the QR code below to download:
  1. Slides for my 1st-Year PhD course on General Equilibrium and Welfare Economics
  2. Slides for my 2nd-Year PhD course on Normative Macro-Finance
  3. Slides for several normative finance papers
  4. (very rough) Book manuscript for the first-year course $\Rightarrow$ Please do not share!

$$\boxed{\text{Ten Rules of Normative Macro-Finance}}$$

Ten Rules

- These are my $\boxed{\text{ten rules}}$ on how to do $\boxed{\text{normative research}}$
  - They are particularly useful in macro-finance environments, but can be applied more broadly
  - Some of the rules also apply to positive research

- **Rule 0:** *Carefully define the environment*
  - Define the physical environment: preferences, technologies, resource constraints, accumulation equations, information structure, etc.
  - Define the economic environment: how do agents (and firms) behave? What is the equilibrium notion (e.g., competitive, strategic, search, matching, REE, etc.)
  - What is a primitive? What is predetermined? What is exogenous vs. endogenous?
  - Avoid non-microfounded elements
  - This applies to any model, positive and normative

Rules 1 and 2

- **Rule 1:** *Normative analysis is most valued in well-accepted environments*
  - Writing a crazy, ad-hoc model and then trying to make welfare statements is typically a bad idea
    - e.g., the Mirrlees optimal taxation literature features the most basic consumption-leisure tradeoff
  - It is possible to do normative analysis with behavioral agents, but it should be done judiciously
    - We learned in class how
- **Rule 2:** *Find the best (e.g., easiest, most insightful, ...) way to pose your problem*
  - Are you working in the primal (choosing allocations or prices directly)? Or in the dual (choosing instruments)? Are you working in a sequence problem or recursively?
    - Different formulations may yield different insights
    - Finding the best formulation is an art, not a science
  - How many degrees of freedom do you have? Is it useful to drop particular constraints? Or to reformulate constraints? How about changes of variables?

Rules 3 and 4

- **Rule 3:** *Define the policy objective*
  - Are you looking for Pareto improvements? Constrained Pareto improvements? Pareto improvements with transfers?
  - Are you simply doing policy evaluation/welfare assessments?
  - Are you solving an optimal policy problem?
  - Are you using a Social Welfare Function? Are you utilitarian? Are you paternalistic? Do you value redistribution? Can you find results that are invariant to the answer to these questions?
    - Welfare assessments framework can be helpful here
- **Rule 4:** *Define the policy instruments available to the planner*
  - Do you have a "chicken paper"? Can the planner do more than the agents? This may be OK, but be clear about it
  - Do you have perfect or imperfect instruments? What type of instrument imperfection are you considering?
  - Are you allowing for transfers? (connected to rule #3)
    - You probably shouldn't
  - How much are your results subject to the "Lucas Critique"? (i.e., what is and what is not policy invariant in your model?)
    - Imperfect instruments paper can be helpful here

Rules 5 and 6

- **Rule 5:** *Identify the distortions/wedges in the economy*
  - What is the rationale for intervention? An externality? Pecuniary or not? An internality? Lack-of-commitment? Public good features (non-rivalry/non-excludability)?
  - What is the Pigouvian benchmark? (connected to rules #4 and #6)
  - Is your positive model good at modeling/explaining these distortions/wedges?
  - Perhaps there are no distortions/wedges, and the objective is simply to redistributive or raise revenue
- **Rule 6:** *Always use benchmarks*
  - First-best, Second-best, Third-best, etc.
  - Examples
    - Static vs. Dynamic vs. Stochastic
    - Complete vs. Incomplete Markets
    - Flexible vs. Sticky Prices
    - Perfect vs. Imperfect Information
    - Commitment vs. Discretion
    - Representative Agent vs. Heterogeneous Agents
    - Rational vs. Behavioral

Rules 7 and 8

- **Rule 7:** *Abstraction is valuable: avoid substituting functional forms or constraints and avoid approximations*
  - Avoid using functional forms until it is necessary
    - Functional forms are eventually unavoidable, in particular if we hope to solve the model in a computer
  - Avoid the temptation to substitute in constraints
    - Lagrange multipliers are your friends! Make sure you know how to interpret them
  - Avoid approximations (linear, log-linear, etc.)
    - Approximations can be misleading in particular in models with risk and heterogeneity
      e.g., tails and disasters are gone
  - The later we specialize the results the better: we can make more general claims
- **Rule 8:** *Understand the units*
  - Are welfare comparisons done in comparable units? What are the units of every variable or multiplier?
  - What are the magnitudes of welfare/gains and losses?

Rules 9 and 10

- **Rule 9:** *Try to write results in terms of observables*
  - Sufficient statistics, targeting rules, optimal tax formulas, etc.
    - e.g., effort is not observable, consumption is
  - Everything is endogenous, but some things are easier to measure than others
- **Rule 10:** *Check that your problem is well-behaved*
  - This is typically impossible analytically: use the computer
  - Be careful: there are deep forces that make normative problems ill-behaved
  - Make sure that there exists a well-behaved version of your problem (at least for some plausible parameters)

Bonus Rule: *If you have to break one of the ten rules, have a good reason to do so!*

Conclusion

- **Sufficient statistics** ⇒ What to measure?
  - Theory of Measurement
- **Normative finance** ⇒ What is good and bad?
  - Theory of Policy

Two natural areas to <u>grow Finance Theory</u>

- I'm always happy to talk about these topics
- Don't hesitate to reach out at `eduardo.davila@yale.edu`

Conclusion

- **Sufficient statistics** $\Rightarrow$ What to measure?
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Thank you for your attention!

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