Corrective Regulation with Imperfect Instruments[*]

Eduardo Dávila[†] Ansgar Walther[‡]

April 2025

Abstract

This paper studies optimal second-best corrective regulation, when some decisions cannot be perfectly regulated. We show that leakage elasticities from perfectly to imperfectly regulated decisions, along with Pigouvian wedges, are sufficient statistics to determine the optimal regulation of perfectly regulated decisions. Notably, the optimal second-best policy hinges on whether perfectly and imperfectly regulated decisions are gross substitutes or complements. Reverse leakage elasticities from imperfectly to perfectly regulated decisions influence the optimal regulation and determine the social value of relaxing constraints on regulation — a novel instance of the Le Chatelier principle. We explicitly characterize the optimal second-best policy in three practical scenarios: unregulated decisions, uniform regulation, and convex costs of regulation. Finally, we illustrate our results in applications to i) financial regulation with environmental externalities, ii) shadow banking, iii) behavioral distortions, iv) asset substitution, and v) fire sales.

JEL Codes: G28, H23

Keywords: corrective regulation, second-best policy, Pigouvian taxation, policy elasticities, leakage elasticities, reverse leakage, shadow banking, regulatory arbitrage, financial regulation

[*]We would like to thank Javier Bianchi, Dean Corbae, Doug Diamond, Kinda Hachem, Anil Kashyap, and Anton Korinek for detailed comments. We would also like to thank our discussants Thomas Geelen, Kinda Hachem, Narayana Kocherlakota, Alexander Kopytov, John Kuong, Guillaume Plantin, Eva Schliephake, and Jing Zeng, as well as Franklin Allen, Peter Diamond, Emmanuel Farhi, Borys Grochulski, Piero Gottardi, Nathan Hendren, Raj Iyer, Arvind Krishnamurthy, Luc Laeven, Guido Lorenzoni, Gregor Matvos, José Luis Peydró, Adriano Rampini, Jeremy Stein, Joseph Stiglitz, Dimitrios Tsomocos, Harald Uhlig, Wei Xiong, and Iván Werning for useful discussions and conversations. We would also like to thank the audiences at various conferences and seminars for their comments. Abdelrahman Hassanein, Alex Kane, Jack Kelly, Flint O'Neil, and Ran Wang provided excellent research assistance. Eduardo Dávila gratefully acknowledges financial support from the European Central Bank's Lamfalussy Fellowship. This material is based upon work supported by the National Science Foundation under Grant DGE-2237790.

[†]Yale University and NBER. Email: `eduardo.davila@yale.edu`

[‡]Imperial College London and CEPR. Email: `a.walther@imperial.ac.uk`

1 Introduction

Many economic policies are motivated by the desire to correct externalities. However, the instruments available to policymakers are often imperfect. Financial regulation is a prime example of this phenomenon. In particular, in the aftermath of the 2008 financial crisis and guided by theories of corrective policy in the presence of a diverse set of market failures — including fire-sale externalities and distortive government subsidies (e.g., Lorenzoni, 2008; Bianchi, 2011, 2016; Farhi and Werning, 2016; Dávila and Korinek, 2018) — most economies have expanded the set and scope of regulations faced by the financial sector. At the same time, many agents and decisions in the financial system are still imperfectly regulated. These imperfections are often viewed as generating "unintended consequences", typically in the form of regulatory arbitrage (e.g., Adrian and Ashcraft, 2016; Hachem, 2018). Hence, a natural normative question is how regulators should proceed once aware of such imperfections. The associated second-best policy problem appears daunting because, as we have outlined, there are many possible market failures to consider and many seemingly disparate imperfections in policy instruments.

This paper characterizes how the presence of imperfect regulatory instruments affects the design of optimal corrective regulation. We study a broad class of economies, including economies with financial frictions. Our goal is to identify a set of unifying economic principles for regulation in a second-best world in which regulation is costly and/or subject to constraints. Therefore, our results build on and complement the existing theoretical literature on corrective taxation, which focuses on the properties of particular types of market failures and regulatory imperfections.

General Principles. We initially consider a general model in which many agents make decisions that may induce externalities. Since our study is partly motivated by applications to financial regulation, we model these agents as investors who make investment and financing decisions.[1] We saturate the model with corrective taxes/regulations that are chosen by a planner and can in principle be investor- and decision-specific. However, these policy instruments are subject to a general set of constraints that capture regulatory imperfections. For example, the planner might i) be unable to regulate certain agents or decisions, ii) be forced to set the same regulation across different agents or decisions, or iii) simply face costs that are increasing in the size of the regulations, perhaps capturing political economy limits to regulation.

We first characterize the marginal welfare effects of arbitrary policy changes. We show that these effects are determined by two sets of statistics: *Pigouvian wedges* and *policy elasticities*. Pigouvian wedges correspond to the difference between the existing corrective regulation associated with a particular decision and the marginal distortion (externality) generated by that decision. A positive (negative) wedge implies that a decision is overregulated (underregulated), in the sense

[1] This leading case is the most challenging to analyze, because it includes economies with various sources of market incompleteness, including commonly analyzed sources of financial frictions and default. Focusing on financial regulation is natural, since financial activity is inherently hard to regulate (Arseneau et al., 2023). However, our results also apply to classical price-theoretic environments with consumption/production decisions, as well as game-theoretic environments, as we illustrate in the Online Appendix.

that the regulation imposed on it is greater (smaller) than the associated marginal distortion. Policy elasticities capture the equilibrium responses of different decisions to changes in regulation. Intuitively, our characterization shows that policy changes that discourage underregulated decisions or encourage overregulated decisions are welfare-improving. As a benchmark, we show that the first-best policy, in which a planner faces no regulatory constraints, is chosen so that all Pigouvian wedges are zero — the classical Pigouvian principle. Importantly, policy elasticities do not form part of the first-best policy, and only matter for corrective regulation in second-best scenarios.

We use the characterization of marginal welfare effects to study optimal second-best policy. To do so, it is useful to distinguish between perfectly and imperfectly regulated decisions. A decision is perfectly regulated if its associated corrective regulation does not enter in any binding constraint faced by the planner, and imperfectly regulated otherwise. We then derive three main insights on the optimal second-best policy.

First, we characterize the optimal second-best regulation of *perfectly regulated* decisions. In contrast to the Pigouvian principle, with imperfect instruments, the optimal corrective regulation is given by the sum of its associated marginal distortions, and a second-best correction. The second-best correction depends on i) Pigouvian wedges associated with imperfectly regulated decisions, and ii) *leakage elasticities*, which are a subset of policy elasticities that measure the equilibrium response of imperfectly regulated decisions to changes in the regulation of perfectly regulated decisions. For instance, if unregulated decisions are underregulated and complements to perfectly regulated decisions, then it is optimal to impose a regulation above the Pigouvian level. The opposite conclusion, that is, a second-best regulation below the Pigouvian level, arises in the cases of underregulated substitutes or overregulated complements.

While over- and underregulation relative to the first-best are both possible, these results show that there is significant structure on how to determine the optimal second-best policy, which is especially relevant in settings where the sign of leakage elasticities can be inferred empirically.[2] This finding contrasts with common "anything goes" second-best arguments (Lipsey and Lancaster, 1956). We also connect our results to the Tinbergen (1952) targeting rule, by demonstrating the precise role of the number of targets and instruments.

Second, we characterize the optimal second-best regulation of *imperfectly regulated* decisions. By definition, these regulations are subject to binding constraints, but the planner may nonetheless have degrees of freedom in choosing them — for instance, if a constraint dictates that two decisions must be taxed at a uniform rate, while the level of the uniform tax can be chosen freely. When choosing regulations on imperfectly regulated decisions, the planner must not only consider direct effects, but also more nuanced feedback effects. In general, the optimal second-best regulation depends both on leakage and *reverse leakage* elasticities, which capture how perfectly regulated decisions adjust to regulating imperfectly regulated ones. Whenever perfectly and imperfectly regulated decisions are *either* complements or substitutes, reverse leakage attenuates the welfare effect of regulating

[2] For instance, in the case of financial regulation, a growing body of empirical studies provides guidance on the response of unregulated activities to policy changes (e.g., Demyanyk and Loutskina, 2016; Buchak, Matvos, Piskorski and Seru, 2018; Xiao, 2020; Irani, Iyer, Meisenzahl and Peydro, 2021).

imperfectly regulated decisions. This result is reminiscent of the Le Chatelier principle (Samuelson, 1948; Milgrom and Roberts, 1996). However, while our results also describe how the direct effect of a parameter change is augmented by feedback in a system, we find attenuation, rather than amplification, for welfare effects.

Building on these insights, we derive optimal second-best regulation of imperfectly regulated decisions in two common scenarios. On one hand, if taxes are constrained to be *uniform* across heterogeneous agents or decisions, then the optimal regulation is a weighted average of distortions, where the appropriate weights are augmented to incorporate reverse leakage elasticities. These results generalize the uniform corrective taxation result of Diamond (1973), which follows as a special case in the absence of reverse leakage — that is, when there are no perfectly regulated decisions. On the other hand, if a subset of regulations is subject to convex costs, then the optimal regulation is given by an attenuated version of the first-best policy. In the presence of perfectly regulated decisions, reverse leakage is a force that contributes to further attenuating the optimal regulation.

Finally, we characterize the social value of relaxing the constraints faced by a planner who is implementing the optimal second-best policy. This is an informative exercise, for instance, for a planner that considers an institutional reform, such as allowing to regulate previously unregulated agents or decisions. Once again, the Le Chatelier/reverse leakage effects are a force towards attenuating the welfare benefits of reforms both in the substitutes and the complements case. Intuitively, if perfectly and imperfectly regulated decisions are substitutes, tightening the regulation on imperfectly regulated decisions increases perfectly regulated decisions through reverse leakage. But this is welfare-reducing since perfectly regulated decisions are underregulated at the second-best. Conversely, if perfectly and imperfectly regulated decisions are complements, tightening the regulation on imperfectly regulated decisions reduces perfectly regulated decisions through reverse leakage, which is again welfare-reducing.

Financial Regulation with Environmental Externalities. To demonstrate the usefulness of the general principles, we consider a suite of applications. First, we leverage our results to provide new insights into the question of financial regulation in the presence of environmental externalities, which has only recently received interest in academic and policy circles, and remains underexplored. In this application, we develop a model in which investors choose the scale of their risky investment, the composition of their portfolios, and their leverage. The planner controls a risk-weighted capital requirement. This is an imperfect instrument, which effectively regulates investors' leverage and portfolio composition, but not the scale of their investments. Since leverage and the investment scale are gross complements, we find that overregulating leverage is optimal, consistent with our general results.

Following the current policy debate on climate finance, we compare optimal policy under a narrow/financial mandate that only considers externalities related to financial stability, and a broad mandate that considers the impact of financial regulation on environmental externalities. We demonstrate that the nature of optimal regulation is substantially different once we account for the imperfections inherent in current regulatory regimes. One implication of our approach is that it is natural to adjust risk weights, as opposed to leverage caps, when regulators become concerned with broader environmental mandates.

Further Applications. In four final applications, we show how our results can be employed in common regulatory scenarios, each with different kinds of regulatory instruments and constraints. When possible, we discuss how the existing empirical findings can be used to guide the optimal policy.

Application 1 studies a model of *shadow banking* in which traditional and shadow banks compete for funding from outside investors. Regulation is imperfect because shadow banks cannot be directly regulated. We derive optimal second-best leverage regulation when the government provides ex-post bailouts without commitment. We find that the optimal policy underregulates traditional banks when leverage choices between traditional and shadow banks are gross substitutes — the empirically relevant case (Irani, Iyer, Meisenzahl and Peydro, 2021). Our results further clarify how optimal second-best policy responds to potential changes in marginal distortions that arise from unregulated activities in general equilibrium. We also illustrate how Le Chatelier/reverse leakage effects impact the welfare gains of being able to regulate unregulated investors.

Application 2 illustrates how our results can be employed to analyze economies with behavioral distortions. We consider a model in which macro-prudential regulation is motivated by distortions in investors' and creditors' beliefs about investment returns. We derive optimal policy under the assumption that the planner can regulate investors' leverage, i.e., the ratio of borrowing to risky investment, but not the overall scale of investment. In this situation, regulated and unregulated activities (e.g., leverage and the scale of risky investment) are gross complements, and the second-best optimal policy overregulates leverage.

Application 3 considers an environment where investors choose between two types of risky investment, but where regulation is imperfect in that the regulator imposes a uniform regulation across both types of investments. Regulation in this application is motivated by the fact that investors are "too big to fail" and enjoy an implicit government subsidy. This case leads to novel insights into the classical "asset substitution" problem in financial economics (e.g., Jensen and Meckling, 1976). The optimal second-best regulation is a weighted average of the downside distortions imposed by different types of investment, with weights proportional to the policy elasticities of investment. Our general formula also leads us to a deeper characterization of the optimal weights, which reveals that they are closely related to the elasticity of the probability of receiving government support.

Finally, Application 4 studies a model of excessive credit booms along the lines of Lorenzoni (2008) in which the investment decisions of investors/entrepreneurs are associated with distributive pecuniary/fire-sale externalities. While most of the related literature focuses on characterizing constrained-efficient allocations, often assuming that a planner has access to investor-specific regulations, we assume that all investors must face the same regulation. Consistent with our general results, we show that the optimal second-best regulation is a weighted average of the induced distortions (pecuniary externalities), which in this case are given by differences in marginal valuations, net trade positions, and price sensitivities. This application is of independent interest, since it shows that even when a planner does not have access to investor-specific regulations, it may still be desirable to set corrective regulation to address pecuniary externalities.

Related Literature. Our paper is directly related to the literature on imperfect regulation. In particular, the issue of regulatory arbitrage in the financial system has been widely studied in recent years. Within the theoretical literature, Plantin (2015), Huang (2018), Martinez-Miera and Repullo (2019), and Farhi and Tirole (2021) study the impact of capital requirements on banking activity and financial stability. Hachem and Song (2021) explore how increased liquidity requirements can generate credit booms when banks are heterogeneous. Grochulski and Zhang (2019) show how regulation is constrained by the presence of shadow banks in an environment in which regulation is motivated by a pecuniary externality. Gennaioli, Shleifer and Vishny (2013) and Moreira and Savov (2017) develop theories that highlight the fragile nature of shadow banking arrangements. Ordoñez (2018) shows how shadow banking enables better-informed banks to avoid blunt regulations. Bengui and Bianchi (2022), building on Bianchi (2011), provide a theoretical and quantitative analysis of macroprudential policy with imperfect instruments based on a collateral pecuniary externality. Dávila and Korinek (2018) briefly discuss the impact of specific regulatory constraints on policy in a setup with pecuniary externalities, while Korinek (2017) provides a systematic study of optimal corrective policy in environments with multiple regulators. Clayton and Schaab (2021) study regulatory policy in the presence of shadow banks when there are pecuniary externalities. Korinek, Montecino and Stiglitz (2022) study the role of technological innovation as regulatory arbitrage. Begenau and Landvoigt (2022) provide a quantitative general equilibrium assessment of regulating commercial banks for financial stability and macroeconomic outcomes in the presence of ex-post subsidies — see Dempsey (2020) for a related quantitative assessment. Hachem and Kuncl (2025) study prudential regulation with shadow banks. There is also a growing empirical literature on regulatory arbitrage and shadow banking, that includes Acharya, Schnabl and Suarez (2013), Demyanyk and Loutskina (2016), and Buchak, Matvos, Piskorski and Seru (2018, 2024), among others.

More broadly, our results are connected to the public economics literature that studies imperfect corrective regulation. In fact, we show that several results that have been treated as independent can be derived and expanded upon using our approach. For instance, the optimal tax formulae in Diamond (1973) are seemingly distinct from the characterization of second-best policy in Lipsey and Lancaster (1956) or the Tinbergen (1952) Rule, but these results can all be derived as corollaries of our general results. We explicitly compare and contrast our results to existing work in the text. Other contributions in this literature, often comparing indirect and direct regulation in particular scenarios, include Baumol (1972), Sandmo (1975), Green and Sheshinski (1976), Balcer (1980), Wijkander (1985), and Cremer, Gahvari and Ladoux (1998).[3] Textbook treatments are available

[3] Corlett and Hague (1953) is the seminal study on optimal commodity taxation with incomplete taxes in economies *without* externalities.

in Myles (1995), Salanié (2011), or Werning (2012). In common with Hendren (2016), we adopt the terminology "policy elasticity", identifying the special role played by leakage elasticities in determining optimal second-best regulation. Second-best corrective regulation is often discussed in the context of environmental policy and congestion (e.g., Bovenberg and Goulder, 2002), as well as rent-seeking (e.g., Rothschild and Scheuer, 2016). Our results in Section 3.4 characterizing the value of relaxing constraints on regulation provide a novel manifestation of the Le Chatelier principle, introduced by Samuelson (1948), and further studied in Milgrom and Roberts (1996) and Acemoglu (2007), among others.

2 General Framework

This section lays out our general framework, which is broad enough to capture a wide range of scenarios but sufficiently tractable to yield precise insights. We use this framework to present the principles that determine optimal corrective regulation with imperfect instruments in Section 3. In Sections 4 and 5, we present concrete applications, illustrating how our general formulation encompasses various rationales for regulation.

2.1 Environment

We consider an economy with two dates, $t \in \{0, 1\}$, and a single consumption good. At date 1, there is a continuum of possible states $s \in S$, with a cumulative distribution function $F(s)$.

There are two groups of agents: investors and creditors. There is a finite number of investor types in unit measure (investors, for short), indexed by $i, j \in \mathcal{I}$, where $\mathcal{I} = \{1, 2, \ldots, I\}$. There is also a unit measure of creditors, indexed by C.

Investors. Each investor i makes $N \geq 0$ financing and investment decisions, which we collect in a vector $\boldsymbol{x}^i \in \mathbb{R}^N$.[4] Investor i's preferences are represented by

$$u^i \left(c_0^i, \left\{ c_1^i(s) \right\}_{s \in S}, \left\{ \overline{\boldsymbol{x}}^j \right\}_{j \in \mathcal{I}} \right), \tag{1}$$

where c_0^i and $c_1^i(s)$ denote the investor i's consumption at date 0 and date 1 in state s, while $\{\bar{\boldsymbol{x}}^j\}_{j \in \mathcal{I}}$ denotes the collection of decisions made by all investors. Since each investor takes $\overline{\boldsymbol{x}}^j$ as given, the last argument in $u^i(\cdot)$ captures externalities across agents. In equilibrium, as explained below, $\boldsymbol{x}^j = \overline{\boldsymbol{x}}^j$, $\forall j \in \mathcal{I}$.

Investor i faces the following budget constraints:

$$c_0^i = n_0^i + Q^i\left(\boldsymbol{x}^i\right) - \Upsilon^i\left(\boldsymbol{x}^i\right) - \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i + T^i \tag{2}$$

$$c_1^i(s) = n_1^i(s) + \rho^i\left(\boldsymbol{x}^i, s\right), \quad \forall s, \tag{3}$$

[4] In our applications, we explicitly split $\boldsymbol{x}^i$ into financing ($\boldsymbol{b}^i$) and investment ($\boldsymbol{k}^i$) decisions. Section A of the Online Appendix includes explicit definitions of all vectors and matrices used.

where n_0^i and $n_1^i(s)$ denote investor endowments. Total financing raised by investor i at date 0 is denoted by $Q^i(\boldsymbol{x}^i)$, while $\rho^i(\boldsymbol{x}^i, s)$ denotes investor i's final return to investments net of any financing obligations. This general formulation of $Q^i(\cdot)$ and $\rho^i(\cdot)$ — whose equilibrium determination is described below — accommodates the possibility of investor default, as we show in the Appendix and illustrate in our applications. Moreover, investor i's decisions face a cost $\Upsilon^i(\boldsymbol{x}^i) \geq 0$. This term can, for instance, capture technological adjustment costs or represent financing frictions.

The term $\boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$ introduces a set of taxes/subsidies/regulations in principle specific to each choice made by each investor, where $\boldsymbol{\tau}^i \in \mathbb{R}^N$, and $T^i \in \mathbb{R}$ denotes the lump-sum transfer or tax that investor i receives or faces to ensure that the planner runs a balanced budget. We denote the elements of $\boldsymbol{x}^i$, $\bar{\boldsymbol{x}}^i$, and $\boldsymbol{\tau}^i$ by x_n^i, $\bar{x}_n^i$ and τ_n^i, respectively.

Creditors. Creditors, who close the model, have preferences given by

$$u^C\left(c_0^C, \left\{c_1^C(s)\right\}_{s\in S}, \left\{\overline{\boldsymbol{x}}^j\right\}_{j\in\mathcal{I}}\right), \tag{4}$$

where c_0^C and $c_1^C(s)$ denote creditors' consumption at date 0 and at date 1 in state s. As above, $\overline{\boldsymbol{x}}^j$ denotes the collection of decisions made by all investors.

Creditors face the following budget constraints:

$$c_0^C = n_0^C - \sum_i h_i^C Q^i\left(\overline{\boldsymbol{x}}^i\right) \tag{5}$$

$$c_1^C(s) = n_1^C(s) + \sum_i h_i^C \rho_i^C\left(\overline{\boldsymbol{x}}^i, s\right), \quad \forall s. \tag{6}$$

where n_0^i and $n_1^i(s)$ denote creditors' endowments. At date 0, creditors choose to fund a share h_i^C of each investor i's financing needs $Q^i(\cdot)$, although, in equilibrium, $h_i^C = 1$, as explained below. At date 1, creditors receive repayments $\rho_i^C(\overline{\boldsymbol{x}}^i, s)$ from investor i. This general formulation of $\rho_i^C(\cdot)$ accommodates deadweight losses associated with investors' default.

Equilibrium. For given corrective taxes/subsidies/regulations $\{\boldsymbol{\tau}^i\}_i$ and lump-sum transfers $\{T^i\}_i$, an *equilibrium* consists of consumption allocations $\{c_0^i, c_1^i(s)\}_i$ and $\{c_0^C, c_1^C(s)\}$, investors decisions $\{\overline{\boldsymbol{x}}^i\}_i$, creditors' funding decisions $\{h_i^C\}_i$, financing schedules $\{Q^i(\cdot)\}_i$, investors' net investment returns $\{\rho^i(\cdot)\}_i$, and creditors' repayments $\{\rho_i^C(\cdot)\}_i$ given investors' default decisions such that i) investors maximize utility, (1), subject to budget constraints (2) and (3); ii) creditors maximize utility, (4), subject to budget constraints (5) and (6); iii) the planner's budget is balanced, so that $\sum_i T^i = \sum_i \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$; iv) investors' decisions are consistent in the aggregate, that is, $\boldsymbol{x}^i = \overline{\boldsymbol{x}}^i$, $\forall i$; v) financing decisions satisfy market clearing, that is, $h_i^C = 1$, $\forall i$.

This equilibrium notion is standard in models that allow for default (e.g., Dubey, Geanakoplos and Shubik, 2005). We proceed as if the model is well-behaved, discussing the necessary regularity conditions in each application. Assuming $T^i = \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$, $\forall i$ — instead of the less restrictive

$\sum_i T^i = \sum_i \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$ — allows us to interpret the results as quantity regulation.

2.2 Imperfect Policy Instruments

As explained below, a planner who can freely adjust all policy instruments $\{\boldsymbol{\tau}^i\}_{i\in\mathcal{I}}$ is able to achieve a first-best outcome. However, our focus is on optimal corrective policy with *imperfect* policy instruments. We formalize such imperfections by assuming that a planner chooses regulations subject to a vector-valued constraint

$$\boldsymbol{\Phi}(\boldsymbol{\tau}) \leq 0, \tag{7}$$

where $\boldsymbol{\tau} \in \mathbb{R}^{IN}$ denotes the stacked vector of agent-specific regulations $\boldsymbol{\tau}^i$. The function $\boldsymbol{\Phi}: \mathbb{R}^{IN} \to \mathbb{R}^M$, where M is the number of constraints, flexibly defines the set of feasible regulations. Appealing to the duality between constraints and costs, we can also interpret $\boldsymbol{\Phi}(\boldsymbol{\tau})$ as defining the cost of setting regulations.

For example, an unconstrained planner, who can achieve the first-best (Pigouvian) solution, corresponds to setting $\boldsymbol{\Phi}(\boldsymbol{\tau}) \equiv 0$ for all $\boldsymbol{\tau} \in \mathbb{R}^{IN}$. Alternatively, a linear constraint

$$\boldsymbol{\Phi}(\boldsymbol{\tau}) \equiv \boldsymbol{A}\boldsymbol{\tau} - \boldsymbol{c}, \tag{8}$$

for appropriate matrices $\boldsymbol{A}$ and vectors $\boldsymbol{c}$, can be used to model planners who i) are able to regulate only particular subsets of agents or commodities, leaving others unregulated, or ii) must impose uniform regulations across different agents or commodities. A quadratic constraint

$$\boldsymbol{\Phi}(\boldsymbol{\tau}) \equiv \frac{1}{2}\boldsymbol{\tau}'\boldsymbol{B}\boldsymbol{\tau} + \boldsymbol{d}, \tag{9}$$

for a given matrix $\boldsymbol{B}$ and a vector $\boldsymbol{d}$, can instead represent convex costs of regulation. In addition, if the costs of regulation induce sparsity (Tibshirani, 1996; Gabaix, 2014) — for instance, when based on the L^1 norm of $\boldsymbol{\tau}$ — the set of unregulated agents or commodities arises endogenously. Our main results — Propositions 1 and 2 — are valid for a general $\boldsymbol{\Phi}(\cdot)$.

2.3 Remarks

We conclude the description of the environment with three remarks. First, assuming that agents' utilities directly depend on others' decisions is the simplest formulation that justifies corrective regulation. However, the insights of this paper apply to any environment in which a planner wants to correct agents' decisions (e.g., consumption or production externalities, public goods, lack of commitment, behavioral distortions, etc.), regardless of the exact rationale justifying such regulation. In particular, our applications in Sections 4 and 5 feature three widely studied rationales — bailouts, pecuniary externalities, and internalities/belief distortions — that do not arise directly from consumption externalities of the form modeled in (1) and (4). We further elaborate on this point after introducing Lemma 1 below.

Second, we model investors and creditors as distinct groups to better connect our general results to the applications. Creditors can be interpreted as a type of investor who is only allowed to fund other investors.[5] By suitably interpreting creditors' preferences, our model can capture non-pecuniary benefits/convenience yields from particular forms of financing, as in e.g. Stein (2012) or Sunderam (2015).

Finally, note that our general results do not depend on the financing/investment model studied here, and would also apply to price-theoretic or game-theoretic environments. We illustrate this possibility in Section E of the Online Appendix.

3 Optimal Corrective Policy with Imperfect Instruments

We now study the problem of a planner who optimally sets corrective regulation subject to constraints on the set of regulatory instruments. We abstract from redistributional considerations, and focus on the corrective nature of the regulation. Therefore, we assess the aggregate gains/losses of a marginal policy change by aggregating money-metric welfare changes across agents.[6] That is, the planner evaluates the desirability of a marginal change in a given variable (or vector) z, denoted by $\frac{dW}{dz}$, according to

$$\frac{dW}{dz} \equiv \sum_{i \in \mathcal{I}} \frac{\frac{dV^i}{dz}}{\lambda_0^i} + \frac{\frac{dV^C}{dz}}{\lambda_0^C},$$

where $\frac{dV^i}{dz}$ ($\frac{dV^C}{dz}$) denotes the change in investor i's (creditors) indirect utility in equilibrium and $\lambda_0^i > 0$ ($\lambda_0^C > 0$) denotes investor i's (creditors) date 0 marginal value of consumption.

3.1 Marginal Welfare Effects and Pigouvian Principle

To characterize the marginal welfare effect of adjusting regulations, it is useful to first define the *marginal distortion/externality* δ_n^i associated with decision n by investor i:

$$\delta_n^i = -\left(\sum_{j \in \mathcal{I}} \frac{1}{\lambda_0^j} \frac{\partial u^j}{\partial \bar{x}_n^i} + \frac{1}{\lambda_0^C} \frac{\partial u^C}{\partial \bar{x}_n^i} \right). \quad \text{(Marginal Distortion)}$$

The marginal distortion δ_n^i measures the direct welfare impact that a change in $\bar{x}_n^i$ has on all agents. We define the distortion as the negative of marginal utility, so that δ_n^i measures the damage caused by $\bar{x}_n^i$. Concretely, when $\frac{\partial u^j}{\partial \bar{x}_n^i}$ is negative (positive), an increase in $\bar{x}_n^i$ generates a negative (positive) externality on agent j, contributing to making δ_n^i positive (negative). We use $\boldsymbol{\delta}^i \in \mathbb{R}^N$ to denote the vector of marginal distortions associated with agent i's decisions and $\boldsymbol{\delta} \in \mathbb{R}^{IN}$ to denote the stacked vector of $\boldsymbol{\delta}^i$'s for all agents.

It is also useful to define the *Pigouvian wedge* ω_n^i between the regulation τ_n^i and the marginal

[5] In earlier versions of this paper, we allowed for investors to invest in each other's liabilities and for creditors' decisions to also be associated with welfare-relevant externalities. Since the main insights are identical in both formulations, we adopt the current formulation, which substantially simplifies the notation.

[6] This approach is akin to maximizing Kaldor-Hicks efficiency — see Dávila and Schaab (2025). Section E.3 of the Online Appendix explains how to consider traditional welfare functions and redistributional considerations.

distortion δ_n^i associated with decision n by investor i:

$$\omega_n^i = \tau_n^i - \delta_n^i, \qquad \text{(Pigouvian Wedge)}$$

where we can again write $\boldsymbol{\omega}^i = \boldsymbol{\tau}^i - \boldsymbol{\delta}^i \in \mathbb{R}^N$ or $\boldsymbol{\omega} = \boldsymbol{\tau} - \boldsymbol{\delta} \in \mathbb{R}^{IN}$ in vector form. As explained below, Pigouvian wedges are zero at the first-best. A positive (negative) ω_n^i indicates that decision n by agent i is overregulated (underregulated), in the sense that increasing (decreasing) $\bar{x}_n^i$ is welfare-improving.

Lemma 1 presents two useful intermediate results. Part a) highlights the role of Pigouvian wedges and leakage elasticities as sufficient statistics when evaluating corrective policy. Part b) characterizes the first-best policy, which states the classical Pigouvian principle in our context, and provides a benchmark when studying second-best policy.

Lemma 1. *a) (Marginal Welfare Effects of Regulation) The marginal welfare effects of varying regulations $\boldsymbol{\tau}$, $\frac{dW}{d\boldsymbol{\tau}}$, are given by*

$$\frac{dW}{d\boldsymbol{\tau}} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} (\boldsymbol{\tau} - \boldsymbol{\delta}) = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} \boldsymbol{\omega}, \tag{10}$$

where $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ is the Jacobian matrix of policy elasticities, of dimension $IN \times IN$.

b) (First-Best Policy/Pigouvian Principle) The optimal (first-best) policy for a planner who can freely choose regulations — when $\boldsymbol{\Phi}(\boldsymbol{\tau}) \equiv 0$ — is characterized by

$$\boldsymbol{\omega} = 0 \iff \boldsymbol{\tau}^\star = \boldsymbol{\delta}. \tag{11}$$

Lemma 1a) highlights that the welfare impact of changes in regulation can always be characterized in terms of i) policy elasticities and ii) Pigouvian wedges. The matrix $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ of *policy elasticities* — borrowing the terminology of Hendren (2016) — captures the full equilibrium response that a particular change in regulation has on all agents' decisions. The vector $\boldsymbol{\omega}$ of Pigouvian wedges captures the extent to which an agent's decision is underregulated or overregulated. The overall welfare effect of varying a particular regulation corresponds to the sum of the product of the relevant leakage elasticities and Pigouvian wedges, where regulations that i) decrease underregulated decisions (with $\omega_n^i < 0$) or ii) increase overregulated decisions (with $\omega_n^i > 0$) in equilibrium are welfare-improving.

Lemma 1b) characterizes the well-understood Pigouvian principle in our model, i.e., the "polluter pays" (Pigou, 1920; Sandmo, 1975), also referred to as the principle of targeting (e.g., Dixit, 1985; Rothschild and Scheuer, 2016). The first-best policy perfectly aligns private and social incentives by setting taxes equal to marginal distortions for each decision.[7] Note that the first-best policy is independent of the magnitude of the policy elasticities, being exclusively a function of the Pigouvian

[7]Note that Equation (11) does not provide a solution for optimal regulations in terms of primitives unless marginal distortions are invariant to the level of regulation. Whenever marginal distortions are endogenous to the level of the regulation, our statements pertain to the form of the optimal policy formulas. The same caveat applies to Propositions 1 and 2. In our application in Sections 4 and 5, marginal distortions are largely insensitive to the level of the regulation.

wedges. This result contrasts with the optimal second-best policy, as we describe next.

In line with our first remark above, our conclusions for second-best policy will be valid whenever marginal welfare effects take the form of Equation (10), regardless of the exact nature of the marginal distortions in $\boldsymbol{\delta}$. Our applications explicitly illustrate how the marginal distortions in Equation (10) encompass multiple rationales for regulation.

3.2 Second-Best Policy: Perfectly Regulated Decisions

To characterize the optimal second-best policy, it is useful to distinguish between perfectly and imperfectly regulated decisions. We say that decision x_n^i is *perfectly regulated* when its associated policy instrument τ_n^i does not enter any binding constraint in the planner's problem, and is *imperfectly regulated* when it does enter in at least one binding constraint. Formally, if the vector $\boldsymbol{\mu} \in \mathbb{R}^M$ denotes the vector of Lagrange multipliers associated with the regulatory constraints in Equation (7), the n^{th} element of the vector $\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}}\boldsymbol{\mu}$ is zero (non-zero) for perfectly (imperfectly) regulated decisions.

Accordingly, we collect the R perfectly regulated decisions and U imperfectly regulated decisions — where $R + U = IN$ — in the vectors $\boldsymbol{x}^R$ and $\boldsymbol{x}^U$. We apply the same partition to the associated regulations $\boldsymbol{\tau} = \{\boldsymbol{\tau}^R, \boldsymbol{\tau}^U\}$, marginal distortions $\boldsymbol{\delta} = \{\boldsymbol{\delta}^R, \boldsymbol{\delta}^U\}$, and Pigouvian wedges $\boldsymbol{\omega} = \{\boldsymbol{\omega}^R, \boldsymbol{\omega}^U\}$.[8] We also partition the matrix of policy elasticities $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ into four smaller matrices. Two are matrices of leakage and reverse leakage elasticities: $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$ and $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}$, respectively, of dimensions $R \times U$ and $U \times R$. The other two are matrices — invertible in a well-behaved model — of own-perfectly regulated and own-imperfectly regulated elasticities: $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ and $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}$, respectively, of dimensions $R \times R$ and $U \times U$. In particular, the matrix of leakage elasticities $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$ captures how imperfectly regulated decisions respond to changes in the regulations that can be freely adjusted. We further describe these matrices in Section A of the Online Appendix.

Proposition 1 shows that leakage elasticities are a key determinant of the second-best policy.

Proposition 1. *(Second-Best Policy: Perfectly Regulated Decisions) The optimal second-best regulation of perfectly regulated decisions satisfies*

$$\boldsymbol{\tau}^R = \boldsymbol{\delta}^R + \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\omega}^U, \tag{12}$$

where $\boldsymbol{\delta}^R$ is a vector of marginal distortions, $\boldsymbol{\omega}^U = \boldsymbol{\tau}^U - \boldsymbol{\delta}^U$ is a vector of Pigouvian wedges, $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ is a matrix of own-perfectly regulated elasticities, and $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$ is a matrix of leakage elasticities.

Proposition 1 shows that the optimal regulation for a planner who can freely adjust the regulation of a subset of decisions is the sum of two components. The first component is the marginal distortion imposed by the perfectly regulated decisions, $\boldsymbol{\delta}^R$, as in the first-best case — see Lemma 1b).

[8] We denote the set of imperfectly regulated decisions by U since "unregulated" decisions are a leading case of imperfectly regulated decisions. Note that the sets of perfectly regulated and unregulated decisions can vary with the regulation itself, although this is not common in most applications.

The second component is a correction for regulatory imperfections that depends on i) leakage elasticities and ii) Pigouvian wedges of imperfectly regulated decisions. First, the leakage elasticities $-\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$ — normalized by the own-perfectly regulated elasticities $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ — capture how tightening the regulation of perfectly regulated decisions affects imperfectly regulated decisions. Heuristically, $\left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} \equiv -\frac{d\boldsymbol{x}^U}{d\boldsymbol{x}^R}$ is positive (negative) if regulated and unregulated decisions are gross substitutes (complements). Second, Pigouvian wedges $\boldsymbol{\omega}^U$ measure whether imperfectly regulated decisions are regulated above or below their marginal distortions.

Consider the case where imperfectly regulated decisions are underregulated, with $\boldsymbol{\omega}^U < 0$. In the gross substitutes case, the second component of Equation (12) is negative, so a planner finds it optimal to adjust $\boldsymbol{\tau}^R$ downwards relative to the first-best. Hence, the second-best policy underregulates perfectly regulated decisions. By a parallel argument, in the gross complements case, the second-best policy overregulates perfectly regulated decisions. When imperfectly regulated decisions are overregulated, with $\boldsymbol{\omega}^U > 0$, those conclusions in the substitutes/complements cases are reversed. We summarize this logic in the following corollary to Proposition 1.

Corollary. *Whenever imperfectly regulated decisions are underregulated, it is optimal to underregulate (overregulate) perfectly regulated decisions when perfectly and imperfectly regulated decisions are substitutes (complements). These conclusions are reversed when imperfectly regulated decisions are overregulated.*

Before progressing to our next result, we illustrate Proposition 1 in a special practical scenario, and connect it to two classical results, namely, the general theory of the second best (Lipsey and Lancaster, 1956), and the Tinbergen (1952) rule.

Practical Scenario: Unregulated Decisions. A common scenario in which Proposition 1 is relevant in practice is when some decisions cannot be regulated at all. Formally, in this case the planner faces a constraint $\boldsymbol{\Phi}(\boldsymbol{\tau}) = \boldsymbol{\tau}^U = 0$, so Equation (12) specializes to

$$\boldsymbol{\tau}^R = \boldsymbol{\delta}^R - \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\delta}^U, \tag{13}$$

where unregulated decisions associated with negative (positive) externalities are automatically underregulated (overregulated). As in Proposition 1, whether the regulated and unregulated decisions are gross complements or substitutes is critical for the determination of the optimal second-best policy.

Connection to the General Theory of the Second Best. Lipsey and Lancaster (1956) argue that once one of the conditions required to achieve a first-best outcome is not satisfied, it is typically optimal to distort all other decisions. This insight is consistent with Equation (12): whenever $\boldsymbol{\omega}^U \neq 0$ (and $\frac{d\boldsymbol{x}^U}{d\boldsymbol{x}^R} \neq 0$), it is optimal to set $\boldsymbol{\omega}^R \neq 0$. However, while over- and underregulation relative to the first-best are possible, Equation (12) shows that there is significant structure on how to determine the optimal second-best policy: leakage elasticities and Pigouvian wedges for imperfectly regulated decisions unambiguously determine the optimal second-best regulation.[9]

Connection to the Tinbergen Rule. The Tinbergen (1952) rule states that first-best policy requires the same number of instruments as it has targets. A concordant interpretation of Equation (13) is that a second-best planner uses the R instruments $\boldsymbol{\tau}^R$ (on the left-hand side) to target the $R+U$ distortions contained in $\boldsymbol{\delta}^R$ and $\boldsymbol{\delta}^U$ (on the right-hand side). Only when $\boldsymbol{\delta}^U=0$ a first-best outcome emerges, consistent with the Tinbergen rule. Equation (13) offers a further refinement of the Tinbergen rule: with insufficient policy instruments, the optimal regulation equals a weighted sum of all distortions in the economy, with weights shaped by leakage elasticities.

3.3 Second-Best Policy: Imperfectly Regulated Decisions

Proposition 2 characterizes the marginal welfare impact of adjusting the regulation of imperfectly regulated decisions under the optimal second-best regulation. This result allows us to i) characterize the optimal regulation of imperfectly regulated decisions in specific scenarios and ii) determine the welfare impact of relaxing constraints on regulation — in Section 3.4.

Proposition 2. *(Second-Best Policy: Imperfectly Regulated Decisions) The optimal second-best regulation of imperfectly regulated decisions satisfies $\frac{dW}{d\boldsymbol{\tau}^U}=\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}^U}\boldsymbol{\mu}$, where $\boldsymbol{\mu}$ is the vector of Lagrange multipliers associated with the constraints on policy instruments, and where*

$$\frac{dW}{d\boldsymbol{\tau}^U}=\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I}-\boldsymbol{L}\right)\boldsymbol{\omega}^U, \tag{14}$$

where $\boldsymbol{I}$ denotes an identity matrix, $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}$ is a matrix of own-imperfectly regulated elasticities, $\boldsymbol{\omega}^U$ is the vector of Pigouvian wedges associated with imperfectly regulated decisions, and where we define a (Le Chatelier) matrix $\boldsymbol{L}$ by:

$$\boldsymbol{L}=\left(\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right)^{-1}\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}.$$

Equation (14) decomposes the marginal value of adjusting $\boldsymbol{\tau}^U$ into two parts. First, there is the direct effect on imperfectly regulated decisions, given by $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\boldsymbol{\omega}^U$, as implied by Lemma 1a). Second, there is the indirect equilibrium effect on perfectly regulated decisions, given by $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\boldsymbol{\omega}^R=-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\omega}^U$, as implied by combining Lemma 1a) and Proposition 1. This is a form of *reverse leakage*, which attenuates the welfare effect of regulating imperfectly regulated decisions whenever regulated and imperfectly regulated decisions are either complements or substitutes. The magnitude of the attenuating effect is proportional to the matrix $\boldsymbol{L}$ defined in the proposition.

[9] Proposition 1 thus achieves a different conclusion than Lipsey and Lancaster (1956), who write:

> *" (...) in general, nothing can be said about the direction or the magnitude of the secondary departures from optimum conditions made necessary by the original non-fulfillment of one condition".*

Heuristically, one can express this matrix as $\boldsymbol{L} \equiv \frac{d\boldsymbol{x}^R}{d\boldsymbol{x}^U}\frac{d\boldsymbol{x}^U}{d\boldsymbol{x}^R}$, so that it measures the (matrix) product of leakages and reverse leakages. We refer to $\boldsymbol{L}$ as the "Le Chatelier" matrix, for reasons that will become clear in Section 3.4.

We now discuss the insights derived from Proposition 2 in two practical scenarios: uniform regulation and convex costs of regulation. We also show how Proposition 2 generalizes the classical results on uniform regulation in Diamond (1973).

Practical Scenario: Uniform Regulation. A second common scenario of regulatory imperfections arises when a planner is forced to set the same regulation across a subset of decisions associated with different marginal distortions. Formally, we consider a planner who faces a constraint of the form $\tau_n^i = \overline{\tau}^U$ for a subset of agents i and decisions n. In this case, Proposition 2 implies that the optimal second-best regulation of imperfectly regulated decisions specializes to

$$\overline{\tau}^U = \frac{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \boldsymbol{L}\right)\boldsymbol{\delta}^U}{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \boldsymbol{L}\right)\boldsymbol{\iota}}, \tag{15}$$

where $\boldsymbol{\iota}$ denotes a U-dimensional vector of ones. Equation (15) shows that the optimal second-best uniform regulation $\overline{\tau}^U$ over a subset of decisions is simply a weighted sum/average of their marginal distortions $\boldsymbol{\delta}^U$, which can be equivalently written as $\overline{\tau}^U = \sum_i \sum_n w_n^i \delta_n^i$, for some weights w_n^i, which need not be strictly positive. Decisions with stronger equilibrium responses to a change in regulation — those more responsive to the regulation — carry a higher weight. As expected, when marginal distortions are symmetric (so $\boldsymbol{\delta}^U = \boldsymbol{\iota}\overline{\delta}$), Equation (15) implies that the first-best regulation $\overline{\tau}^U = \overline{\delta}$ is optimal. However, when marginal distortions are heterogeneous, the first-best cannot be achieved with uniform regulation.

Practical Scenario: Convex Costs of Regulation. A third common scenario of regulatory imperfections is when increasing regulations becomes increasingly costly. Formally, we consider a planner who faces quadratic costs of regulation over a subset of decisions, with $\boldsymbol{\Phi}\left(\boldsymbol{\tau}\right) = \frac{1}{2}\boldsymbol{\tau}^{U\prime}\boldsymbol{B}\boldsymbol{\tau}^U$, for some positive definite matrix $\boldsymbol{B}$. In this case, Proposition 2 implies that the optimal second-best regulation of imperfectly regulated decisions specializes to

$$\boldsymbol{\tau}^U = \left(\boldsymbol{B} + \boldsymbol{K}\right)^{-1}\boldsymbol{K}\boldsymbol{\delta}^U, \tag{16}$$

where $\boldsymbol{K} = \left(-\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right)\left(\boldsymbol{I} - \boldsymbol{L}\right)$ is once again a key input for the optimal second-best policy.

Equation (16) shows that the optimal policy in the presence of quadratic adjustment costs is given by an attenuated version of the first-best policy. As expected, as costs vanish and $\boldsymbol{B} \to 0$, the optimal policy approaches to the first-best, so $\boldsymbol{\tau}^U \to \boldsymbol{\delta}^U$. But as costs grow relative to $\boldsymbol{K}$, the optimal policy approaches zero, so $\boldsymbol{\tau}^U \to 0$. As explained in Section 3.4, the presence of perfectly regulated decisions — by making $\boldsymbol{L}$ larger and hence $\boldsymbol{K}$ relatively smaller — is a force that contributes to attenuating the optimal choice of $\boldsymbol{\tau}^U$.

Connection to Diamond (1973). The insight that uniform regulation of heterogeneous externalities is characterized by a weighted sum/average of distortions can be traced back to Diamond (1973). Indeed, when *all* decisions are uniformly regulated, Equation (15) corresponds to Diamond's result. Equation (15) generalizes his results by allowing for a subset of decisions to be perfectly regulated. In this more general case, the optimal weights account for the reverse leakage of imperfectly regulated decisions on perfectly regulated ones through the Le Chatelier matrix $\boldsymbol{L}$, as further explained in Section 3.4 below.

3.4 The Value of Relaxing Constraints on Regulation

The characterization of $\frac{dW}{d\boldsymbol{\tau}^U}$ in Proposition 2 provides the marginal welfare gain of relaxing constraints on regulation for a planner who is implementing the optimal second-best policy. Proposition 2 shows that accounting for the equilibrium welfare effects on perfectly regulated decisions boils down to adjusting the direct welfare effect by in proportion to the factor $-\boldsymbol{L}$.

Interestingly, accounting for equilibrium effects on perfectly regulated decisions *attenuates* the direct welfare effect both in the substitutes and complements cases. Heuristically, in the well-behaved case in which $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R} < 0$ and $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} < 0$, the Le Chatelier correction — via $\boldsymbol{L}$ — is positive both when perfectly and imperfectly regulated decisions are gross substitutes ($\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U} < 0$ and $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} < 0$), and gross complements ($\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U} > 0$ and $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} > 0$).

The economic intuition is as follows: If perfectly and imperfectly regulated decisions are substitutes, tightening the regulation on imperfectly regulated decisions increases perfectly regulated decisions through reverse leakage. But Proposition 1 shows that perfectly regulated decisions are underregulated at the second-best, with $\boldsymbol{\omega}^R < 0$, so this increase reduces welfare. Conversely, if perfectly and imperfectly regulated decisions are complements, tightening the regulation on imperfectly regulated decisions reduces perfectly regulated decisions through reverse leakage. But Proposition 1 shows that perfectly regulated decisions are overregulated at the second-best, with $\boldsymbol{\omega}^R > 0$, so this decrease also reduces welfare. Intuitively, optimally regulating perfectly regulated decisions reduces the gains from further regulation. We summarize this logic in the following Corollary to Proposition 2.

Corollary. *Whenever perfectly and imperfectly regulated decisions are complements or substitutes, the welfare gains associated with relaxing constraints on regulation are attenuated relative to their direct effect.*

To illustrate this effect most clearly, consider an environment with two agents ($I = 2$) who make a single decision each ($N = 1$, so we drop the n index), and where only agent 1 is regulated, so $\tau^2 = 0$. In this case, the welfare effect of marginally increasing τ^2 above zero is

$$\frac{dW}{d\tau^2} = -\frac{dx^2}{d\tau^2}\left(1 - \underbrace{\frac{\frac{dx^2}{d\tau^1}\frac{dx^1}{d\tau^2}}{\frac{dx^1}{d\tau^1}\frac{dx^2}{d\tau^2}}}_{=L}\right)\delta^2. \tag{17}$$

Suppose that $\delta^2 > 0$ and consider the well-behaved scenario in which $\frac{dx^1}{d\tau^1} < 0$ and $\frac{dx^2}{d\tau^2} < 0$. In the substitutes case, $\frac{dx^2}{d\tau^1} < 0$ and $\frac{dx^1}{d\tau^2} < 0$, so $L > 0$ and the overall welfare gain from increasing τ^2 is smaller than its direct effect. In the complements case, $\frac{dx^2}{d\tau^1} > 0$ and $\frac{dx^1}{d\tau^2} > 0$, so $L > 0$ once again. However, when decisions are neither global complements nor substitutes — that is, when $\frac{dx^1}{d\tau^2}$ and $\frac{dx^2}{d\tau^1}$ have opposite signs — $L < 0$ and the direct effect of relaxing constraints on regulation can be amplified.

The corollary above also implies a connection between our results and the Le Chatelier principle, which we now discuss in more detail.

Connection to the Le Chatelier Principle. The Le Chatelier principle states that whenever decisions are either complements or substitutes, the long-run response of a system is larger than its short-term response — see Samuelson (1948), Milgrom and Roberts (1996) for a modern treatment, and Acemoglu (2007) for recent related work. As noted by Milgrom (2006), the Le Chatelier principle more generally explains how the direct effect of a parameter change is augmented by feedback in a system. While existing versions of the principle point towards *amplification* by feedback in a system, we find the opposite implication, namely *attenuation*, in terms of welfare effects. When we let our system adjust further by accounting for the welfare impact of relaxing a constraint on the perfectly regulated decisions under the second-best policy, the welfare gains from regulation are typically dampened, not amplified.

It is worth making two final observations. First, note that when $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\boldsymbol{\omega}^U > 0$, it is possible to find scenarios in which $\frac{dW}{d\boldsymbol{\tau}^U} < 0$. That is, it is possible that adjusting the regulation of imperfectly regulated decisions towards their first-best value turns out to be welfare decreasing — in Equation (17), this occurs when $L > 1$. Proposition 2 shows that this type reversal is necessarily explained by the Le Chatelier matrix $\boldsymbol{L}$, and requires strong complementarity or substitutability. Second, note that the imperfectly regulated decisions more significantly attenuated in the presence of perfectly regulated decisions (through $\boldsymbol{L}$) have a smaller weight on the optimal uniform regulation, in (15), or face a further attenuated regulation in the presence of convex costs of regulation, in (16).

In summary, our theoretical results shed light on the nature of second-best corrective regulation by characterizing the optimal regulation of perfectly regulated decisions (Proposition 1) and imperfectly regulated decisions (Proposition 2), for a general class of regulatory imperfections. These results facilitate the analysis of a variety of practical scenarios. In addition, they unify and extend a suite of classical insights that have traditionally been viewed as distinct. In the remainder of this paper, we illustrate the usefulness of our results within multiple applications in financial regulation, a context in which regulation is notoriously imperfect.

4 Application: Financial Regulation with Environmental Externalities

In this section, we present an application to financial regulation with environmental externalities, while in Section 5 we describe four additional minimal applications.

Central banks and macro-prudential regulators have increasingly become interested in accounting for environmental concerns. There are two possible motivations for this. First, there are links between the financial system, a primary target of macro-prudential regulation, and climate-related risks, as evidenced by a growing literature on climate finance (e.g., Giglio, Kelly and Stroebel, 2021). For instance, the safety and soundness of financial institutions may be at risk if they are heavily exposed to climate-related risks. Second, some believe that prudential regulation should take into account its effect on the broader societal goal of sustainable investment. For instance, the European Central Bank's bond purchase program has taken the latter motivation into account by introducing preferential treatment for bonds associated with "green" technologies (Piazessi, Papoutsi and Schneider, 2021).

A nascent academic literature studies the welfare implications of financial regulatory reforms when there are environmental concerns (e.g., Oehmke and Opp, 2022; Rola-Janicka and Döttling, 2022). We now use our general results to characterize optimal policy with environmental externalities and *imperfect macro-prudential regulation.* This is a particularly important question because regulators are already discussing potential imperfections and unintended consequences of policies in the presence of environmental externalities.[10]

In this application, we first show that imperfections are inherent to the primary mode of financial regulation in advanced economies, namely, risk-weighted leverage constraints. Indeed, these requirements constrain only relative quantities on institutions' balance sheets but leave the overall scale of investment as a free variable. Next, we analyze second-best optimal regulation in this setting. To capture the two motivations for policy discussed above, we pay special attention to contrasting the role of climate-related risks when regulation has a narrow/financial mandate versus a broad/environmental mandate. Our results, which directly leverage the formulae from the general model, yield new insights into the differences between these two cases, and into the way in which climate-conscious regulation should be adjusted for imperfections. Finally, we characterize the value of extending the set of policy tools in the face of environmental externalities, which relates to the Le Chatelier/reverse leakage adjustments that we have characterized in the general case.

Environment. We assume that there is a single type of investor, in unit measure and indexed by i, and a unit measure of creditors, indexed by C. Both investors and creditors have risk-neutral preferences given by

$$c_0^i + \beta^i \int c_1^i\left(s\right) dF\left(s\right) \quad \text{and} \quad c_0^C + \beta^C \int c_1^C\left(s\right) dF\left(s\right) - \Psi\left(\theta^i\right) k^i,$$

[10] For example, Andrew Bailey, the Governor of the Bank of England, has stated that

> *"any incorporation of climate change into regulatory capital requirements would need to be grounded in robust data and be designed to support safety and soundness while avoiding unintended consequences or compromising our other objectives".*

See https://www.bankofengland.co.uk/speech/2021/june/andrew-bailey-reuters-events-global-responsible-business-2021.

where the term $\Psi\left(\theta^i\right)k^i$ introduces an environmental externality, as described below.[11] The budget constraints of investors at date 0 and date 1 are given by

$$\begin{aligned} c_0^i &= n_0^i + Q^i\left(b^i,\theta^i\right)k^i - \Upsilon\left(k^i\right) - \Omega\left(\theta^i\right)k^i, \\ c_1^i\left(s\right) &= k^i \max\left\{d_1\left(s\right)\theta^i + d_2\left(s\right)\left(1-\theta^i\right) + t\left(b^i,\theta^i,s\right) - b^i, 0\right\}. \quad \forall s. \end{aligned}$$

At date 0, investors, endowed with n_0^i, make capital investments k^i in two sectors. A fraction θ^i is invested in sector 1, and the remaining $1-\theta^i$ in sector 2. Investors issue debt with face value $b^i k^i$ to creditors, so that b^i measures investors' leverage. The equilibrium price of debt can be written as $Q^i\left(b^i,\theta^i\right)k^i$, where $Q^i\left(b^i,k^i\right)$ denotes the market value of debt per unit of capital. Capital investments are subject to an adjustment cost $\Upsilon\left(k^i\right)$ and an additional cost $\Omega\left(\theta^i\right)k^i$ of adjusting the sectoral portfolio composition. At date 1, once a state s is realized, investor i receives $d_j\left(s\right)$ per unit of investment in sector $j \in \{1,2\}$, and a bailout transfer $t^i\left(b^i,\theta^i,s\right)$ per unit of capital that may depend on the amount of debt issued and portfolio weights. If the sum of these revenues exceeds the face value of debt, investors repay their debt and consume the residual claim. Otherwise, they optimally choose to default.[12]

In this application, motivated by the existing regulatory instruments, we assume that investors are subject to a *risk-weighted capital requirement*:[13]

$$b^i + \varphi\theta^i \leq \bar{b}. \tag{18}$$

Imposing this constraint on investors is equivalent to imposing corrective taxes. Therefore, this application also illustrates how our approach to imperfect regulation can be applied to quantity-based instruments often used in practice. Intuitively, the requirement in Equation (18) places an upper bound $\bar{b}$ on investors' leverage, which is adjusted in proportion to the share θ^i invested in sector 1. In the case with $\varphi > 0$, on which we will focus without loss of generality, the relative risk weight φ on sector 1 is positive, and the leverage cap becomes tighter when investors increase θ^i.

[11] The assumption that this distortion only impacts creditors and is linear in capital simplifies the exposition, but does not affect the qualitative insights of our analysis.

[12] This specification of bailouts corresponds to a model where the government has limited commitment, which connects our work to the treatment of bailouts in Farhi and Tirole (2012), Bianchi (2016), Chari and Kehoe (2016), Keister (2016), Gourinchas and Martin (2017), Cordella, Dell'Ariccia and Marquez (2018), Dávila and Walther (2020), and Dovis and Kirpalani (2020), among others.

[13] Risk-weighted capital requirements under the Basel accords ensure that the ratio of equity to risk-weighted assets in leveraged institutions (e.g., banks) is at least equal to a constant fraction C. In our context, equity is $\left(1-b^i\right)k^i$ and risk-weighted assets can be represented as $\left[w_1\theta^i + w_2\left(1-\theta^i\right)\right]k^i$, where w_j is the risk weight on sector j investments. Thus, we can express a risk-weighted capital requirement as

$$1 - b^i \geq C\left[w_1\theta^i + w_2\left(1-\theta^i\right)\right] \iff b^i + \underbrace{(w_1 - w_2)}_{\equiv\varphi}\theta^i \leq \underbrace{1 - Cw_2}_{\equiv\bar{b}},$$

which is equivalent to our formulation in (18), with φ denoting the relative risk weight on sector 1 investments.

The budget constraints of creditors at date 0 and date 1 are given by

$$c_0^C = n_0^C - h^i Q^i\left(b^i, \theta^i\right) k^i,$$
$$c_1^C(s) = n_1^C(s) - (1+\kappa)\, t\left(b^i, \theta^i, s\right) k^i + h^i \mathcal{P}^i\left(b^i, \theta^i, s\right) k^i.$$

At date 1, creditors are taxed $(1+\kappa)$ times the government bailout, where $\kappa > 0$ denotes the deadweight cost of fiscal intervention. Moreover, creditors who buy a fraction h^i of investors' debt pay the market price at date 0, and receive a payment $\mathcal{P}^i\left(b^i, \theta^i, s\right) k^i$ at date 1. This payment, which preemptively incorporates investors' optimal default decision, is defined as follows:

$$\mathcal{P}^i\left(b^i, \theta^i, s\right) = \begin{cases} b^i, & d_1(s)\,\theta^i + d_2(s)\left(1-\theta^i\right) + t\left(b^i, \theta^i, s\right) \geq b^i \\ \phi\left[d_1(s)\,\theta^i + d_2(s)\left(1-\theta^i\right) + t\left(b^i, \theta^i, s\right)\right], & \text{otherwise.} \end{cases}$$

Investors default when their assets are worth less than the promised repayment b^i per unit of capital, and repay b^i in full otherwise. In default, creditors recover a fraction $\phi < 1$ of their assets, so that $1-\phi$ can be interpreted as the deadweight cost of default. For simplicity, we assume that primitives are such that there exists a default threshold $s^\star\left(b^i, \theta^i\right)$, so that investors default when $s < s^\star\left(b^i, \theta^i\right)$ and repay otherwise.[14]

Finally, recall that creditors' preferences include a utility loss of $\Psi\left(\theta^i\right) k^i$ as a result of investors' choices. This term reflects an environmental externality. Investors' portfolio choices θ^i can affect this loss. For example, if $\frac{\partial \Psi'}{\partial \theta^i} > 0$, then the environmental externality is increasing in the investment share in sector 1, meaning that sector 1 is associated with more pollution than sector 2.

Equilibrium. For given regulatory parameters $\left\{\bar{b}, \varphi\right\}$ defining the constraint (18) and a given bailout policy $t\left(b^i, \theta^i, s\right)$, an *equilibrium* is defined by leverage, portfolio, and investment decisions $\left\{b^i, \theta^i, k^i\right\}$, a default decision rule, and a pricing schedule $Q^i\left(b^i, \theta^i\right)$ such that investors and creditors maximize their utility and the market for debt clears, i.e., $h^i = 1$.

We rely on the fact that equilibrium choices $\left\{b^i, \theta^i, k^i\right\}$ are given by the solution to the following reformulation of the problem faced by investors:

$$\max_{\left\{b^i, \theta^i, k^i\right\}} \left[M\left(b^i, \theta^i\right) - \Omega\left(\theta^i\right)\right] k^i - \Upsilon\left(k^i\right) \qquad \text{subject to} \qquad k^i\left(b^i + \varphi\theta^i\right) \leq k^i \bar{b}, \tag{19}$$

[14] The threshold $s^\star\left(b^i, \theta^i\right)$ is unique under the standard assumptions that i) $d_j(s)$, $j \in \{1,2\}$, is increasing in s (i.e., higher asset returns in good states), and ii) the bailout transfer $t\left(b^i, \theta^i, s\right)$ is decreasing in s and increasing in b^i (i.e., larger bailouts in bad states/for more levered investors).

where $M\left(b^i,\theta^i\right)$ is given by

$$M\left(b^i,\theta^i\right)=\underbrace{\beta^i\int_{s^\star(b^i,\theta^i)}^{\bar{s}}\left(d_1\left(s\right)\theta^i+d_2\left(s\right)\left(1-\theta^i\right)+t\left(b^i,\theta^i,s\right)-b^i\right)dF\left(s\right)}_{\text{equity}}$$
$$+\underbrace{\beta^C\left(\int_{s^\star(b^i,\theta^i)}^{\bar{s}}b^i dF\left(s\right)+\phi\int_{\underline{s}}^{s^\star\left(b^i,\theta^i\right)}\left(d_1\left(s\right)\theta^i+d_2\left(s\right)\left(1-\theta^i\right)+t\left(b^i,\theta^i,s\right)\right)dF\left(s\right)\right)}_{\text{debt }=Q^i(b^i,\theta^i)},\tag{20}$$

and $s^\star\left(b^i,\theta^i\right)$ solves

$$d_1\left(s^\star\right)\theta^i+d_2\left(s^\star\right)\left(1-\theta^i\right)+t\left(b^i,\theta^i,s^\star\right)=b^i.$$

Intuitively, the function $M\left(b^i,\theta^i\right)$ can be interpreted as the sum of the market values of equity (owned by investors) and debt (owned by creditors) per unit of investment. The second term in Equation (20) corresponds to the equilibrium price of debt $Q\left(b^i,\theta^i\right)$, which incorporates the fact that investors default in states $s<s^\star\left(b^i,\theta^i\right)$ in which the value of their assets is less than the promised repayment b^i. In problem (19), investors maximize the market value of investment net of costs. Without loss of generality, we scale the regulatory constraint by total investment $k^i\geq 0$.

An important aspect of this application is that the planner's instruments are imperfect. This can be seen by writing investors' first-order conditions as

$$\frac{\partial M\left(b^i,\theta^i\right)}{\partial b^i}=\mu\equiv\tau_b \tag{21}$$
$$\frac{\partial M\left(b^i,\theta^i\right)}{\partial \theta^i}-\Omega'\left(\theta^i\right)=\mu\varphi\equiv\tau_\theta \tag{22}$$
$$M\left(b^i,\theta^i\right)-\Omega\left(\theta^i\right)-\Upsilon'\left(k^i\right)=0, \tag{23}$$

where $\mu\geq 0$ is the Lagrange multiplier on the regulatory constraint. The first two conditions, which define optimal leverage and portfolio weights, show that the constraint in Equation (18) implies effective corrective taxes τ_b on leverage b^i and τ_θ on portfolios θ^i. The third condition, which defines optimal total investment k^i, does not contain a corrective tax. Intuitively, the capital requirement (18) constrains ratios but leaves the overall scale k^i of investors' balance sheet as a free, unregulated variable. By contrast, in a world with perfect instruments, the planner would be able to set a corrective tax τ_k on k^i in addition to τ_b and τ_θ. We return to the value of introducing such a tax below.

Optimal Corrective Policy. In this environment, we can express the marginal externalities $\{\delta_b,\delta_\theta,\delta_k\}$ associated with investors' choices and decompose them into a financial (i.e., bailout-related) and an environmental component as follows:

$$\delta_b = \underbrace{(1+\kappa)\,\beta^C \int_{\underline{s}}^{\bar{s}} \frac{\partial t\left(b^i,\theta^i,s\right)}{\partial b^i} dF(s)}_{\equiv \chi_b} \tag{24}$$

$$\delta_\theta = \underbrace{(1+\kappa)\,\beta^C \int_{\underline{s}}^{\bar{s}} \frac{\partial t\left(b^i,\theta^i,s\right)}{\partial \theta^i} dF(s)}_{\equiv \chi_\theta} + \underbrace{\frac{\partial \Psi\left(\theta^i\right)}{\partial \theta^i}}_{\equiv \psi_\theta} \tag{25}$$

$$\delta_k = \underbrace{(1+\kappa)\,\beta^C \int_{\underline{s}}^{\bar{s}} t\left(b^i,\theta^i,s\right) dF(s)}_{\equiv \chi_k} + \underbrace{\Psi\left(\theta^i\right)}_{\equiv \psi_k}. \tag{26}$$

The term χ_k in Equation (26) measures the marginal distortion in capital choices due to bailouts, while ψ_k is the distortion due to environmental externalities. Equations (24) and (25) define the distortions associated with leverage and portfolio choices per unit of capital. An important point is that leverage induces only a financial distortion, since environmental damage is determined by the technologies that are operated in this economy, and is independent of how these technologies are financed.

Proposition 3 characterizes the form of the second-best policy.

Proposition 3. *(Financial Regulation with Environmental Externalities)*
a) The marginal welfare effects of varying the leverage cap $\bar{b}$ and the risk weight φ, respectively, are given by

$$\frac{dW}{d\bar{b}} = \frac{db^i}{d\bar{b}}\left(\tau_b - \delta_b\right)k^i + \frac{d\theta^i}{d\bar{b}}\left(\tau_\theta - \delta_\theta\right)k^i - \frac{dk^i}{d\bar{b}}\delta_k, \tag{27}$$

$$\frac{dW}{d\varphi} = \frac{db^i}{d\varphi}\left(\tau_b - \delta_b\right)k^i + \frac{d\theta^i}{d\varphi}\left(\tau_\theta - \delta_\theta\right)k^i - \frac{dk^i}{d\varphi}\delta_k. \tag{28}$$

b) The optimal regulation satisfies

$$\begin{pmatrix} \tau_b \\ \tau_\theta \end{pmatrix} = \begin{pmatrix} \delta_b \\ \delta_\theta \end{pmatrix} + \begin{pmatrix} \frac{db^i}{d\bar{b}} & \frac{d\theta^i}{d\bar{b}} \\ \frac{db^i}{d\varphi} & \frac{d\theta^i}{d\varphi} \end{pmatrix}^{-1} \begin{pmatrix} \frac{d\log k^i}{d\bar{b}} \\ \frac{d\log k^i}{d\varphi} \end{pmatrix} \delta_k. \tag{29}$$

Proposition 3a) characterizes the marginal welfare effects of adjusting the two instruments available to the planner and the optimal regulation in terms of the parameters of the risk-weighted capital constraint, which are the leverage cap $\bar{b}$ and the relative risk weight φ. Notice that, even though we are working in terms of a quantity constraint, our general characterization of welfare effects from Lemma 1 applies, after suitably adjusting for k^i. This feature highlights the usefulness of our approach for analyzing quantity-based regulation.

Specifically, Equations (27) and (28) show that marginal welfare effects depend on Pigouvian wedges — defined in terms of the equivalent taxes $\{\tau_b, \tau_\theta\}$ in Equations (21) and (22) — as well as policy elasticities. First-best regulation is prevented by the fact that the unregulated scale decision

k^i introduces an additional distortion δ_k. The optimal regulation, which we discuss in more detail below, takes into account this distortion along with the appropriate leakage elasticities $\frac{d\log k^i}{db}$ and $\frac{d\log k^i}{d\varphi}$.[15]

In the remainder of this section, we use this characterization to derive concrete insights into optimal regulation with environmental externalities. First, we analyze the distinction between optimal policy motivated by a narrow financial stability mandate and a broader mandates that accounts for environmental externalities. Importantly, we provide a novel treatment of these questions accounting for imperfections in policy instruments. Finally, we characterize the value of relaxing constraints on regulation by imposing corrective regulation on the total scale of investment.

4.1 Imperfect Regulation with Narrow/Financial Mandates.

We first consider a financial regulator who has a narrow mandate and is only concerned with financial externalities. In terms of our decomposition of distortions, we interpret a narrow mandate as meaning that the regulator acts as if the climate-related distortions $\{\psi_\theta, \psi_k\}$ are both equal to zero. In the background, one can interpret that the distribution of states, $F(s)$, and the payoffs of the different investments, $d_1(s)$ and $d_2(s)$, account for climate risks. Applying Proposition 3 and substituting Equations (24) through (26) yields the optimal policy in this case:

Corollary 1. *(Imperfect Regulation with Narrow/Financial Mandates) The optimal policy of a regulator with a narrow/financial mandate is given by*

$$\begin{pmatrix} \tau_b \\ \tau_\theta \end{pmatrix} = \begin{pmatrix} \chi_b \\ \chi_\theta \end{pmatrix} + \begin{pmatrix} \frac{db^i}{d\bar{b}} & \frac{d\theta^i}{d\bar{b}} \\ \frac{db^i}{d\varphi} & \frac{d\theta^i}{d\varphi} \end{pmatrix}^{-1} \begin{pmatrix} \frac{d\log k^i}{d\bar{b}} \\ \frac{d\log k^i}{d\varphi} \end{pmatrix} \chi_k, \tag{30}$$

where χ_b, χ_θ, and χ_k denote the financial component of the respective externalities, defined in Equations (24) through (26).

Equation (30) shows that the optimal leverage cap — represented by τ_b — and the optimal risk weight — represented by τ_θ — are set in response to two terms. The first term captures the marginal externality associated with a change in b or θ, which in this case corresponds to the marginal response of expected bailouts to more leverage or more investment in sector 1.

The second term, which arises only with imperfect instruments, is proportional to the leakage elasticities $\frac{d\log k^i}{db}$ and $\frac{d\log k^i}{d\varphi}$, and scales with the total expected bailout, via χ_k. Both these elasticities fall into the "complements" case: Stricter leverage regulation ($\downarrow \bar{b}$) or a stricter relative risk weight ($\uparrow \varphi$) both lead to increases in k^i in equilibrium. Therefore, Equation (30) generally calls for *overregulation* of leverage and risk. Finally, notice that the relevant leakage elasticities are modulated by an inverse matrix of policy elasticities between b^i and θ^i.

The implication for financial regulation with environmental externalities is that any adjustment for climate-related risk should be determined only by its impact on financial externalities (in this

[15]The appropriate leakage elasticities in this application are semi-elasticities, i.e., responses of log investment to policy reforms. This occurs because we have expressed leverage and portfolio choices per unit of capital.

particular case, this emerges from the presence of bailouts). For instance, the risk weight equivalent tax τ_θ should be increased if sector 1 is associated with climate-related tail risk that makes large bailouts more likely (i.e., if $\mathbb{E}_s\left[\frac{\partial t\left(b^i,\theta^i,s\right)}{\partial\theta^i}\right]>0$). In addition, the setting with imperfect instruments implies that taxes on *both* leverage and portfolio weights should increase if climate-related risk increases the magnitude of the total expected bailout. This prediction is unique to our analysis and directly leverages our general tools.

4.2 Imperfect Regulation with Broad/Environmental Mandates.

We now consider a financial regulator with a broad mandate who cares directly about mitigating environmental distortions. We will focus now on the case where the environmental distortions satisfy $\psi_\theta > 0$ and $\psi_k > 0$. In this case, increases in overall scale as well as concentrated investments in sector 1 are associated with greater environmental damage.

Corollary 2. *(Imperfect Regulation with Broad/Environmental Mandates)*

a) When policy has been set optimally according to a narrow/financial mandate, the welfare benefits of marginal policy changes are given by

$$\frac{dW}{d\bar{b}} = -\frac{d\theta^i}{d\bar{b}}\psi_\theta - \frac{dk^i}{d\bar{b}}\psi_k \tag{31}$$

$$\frac{dW}{d\varphi} = -\frac{d\theta^i}{d\varphi}\psi_\theta - \frac{dk^i}{d\varphi}\psi_k, \tag{32}$$

where ψ_θ and ψ_k denote the environmental component of the respective externalities, defined in Equations (25) and (26).

b) The optimal policy of a regulator with a broad/environmental mandate is given by

$$\begin{pmatrix}\tau_b\\ \tau_\theta\end{pmatrix} = \begin{pmatrix}\chi_b\\ \chi_\theta+\psi_\theta\end{pmatrix} + \begin{pmatrix}\frac{db^i}{d\bar{b}} & \frac{d\theta^i}{d\bar{b}}\\ \frac{db^i}{d\varphi} & \frac{d\theta^i}{d\varphi}\end{pmatrix}^{-1}\begin{pmatrix}\frac{d\log k^i}{d\bar{b}}\\ \frac{d\log k^i}{d\varphi}\end{pmatrix}(\chi_k+\psi_k)\,, \tag{33}$$

where χ_b, χ_θ, and χ_k denote the financial component and ψ_θ and ψ_k denote the environmental component of the respective externalities, defined in Equations (24) through (26).

Corollary 2 develops two insights into the distinction between narrow/financial and broad/environmental mandates. First, Equations (31) and (32) highlight the additional welfare effects, from the perspective of a broad mandate, of adjusting either the leverage cap and risk weights, when policy has previously been optimized according a narrow mandate. These equations are useful for deciding whether policy should be adjusted at the margin once a regulator decides to take environmental outcomes into account. The relevant marginal welfare effects are determined by the environmental distortions ψ_θ and ψ_k and the associated leakage elasticities. It is interesting to note that the leakage elasticities to leverage (i.e., $\frac{db^i}{d\bar{b}}$ and $\frac{db^i}{d\varphi}$) are irrelevant here, because the mode of financing has no marginal impact on environmental concerns.

Equation (31) shows that a regulator who adjusts the leverage cap $\bar{b}$ in response to environmental concerns faces a potential conflict of interest. Indeed, while it is natural that scale and leverage are generally complements, implying $\frac{dk^i}{d\bar{b}} > 0$,[16] the response of optimal portfolio choices $\frac{d\theta^i}{d\bar{b}}$ is ambiguous in theory, and depends on the functional form of returns to investment in each sector. Since the environmental distortions ψ_θ, ψ_b are assumed positive, the two terms in Equation (31) may have opposite signs. As a result, it is unclear whether leverage requirements should be relaxed or tightened in response to environmental concerns, and their impact on welfare may be offset by portfolio adjustments.

By contrast, Equation (32) demonstrates that risk weights are a natural tool for addressing environmental concerns. Both the portfolio share θ^i and total capital k^i are generally complements to the risk weight, implying that $\frac{d\theta^i}{d\varphi} < 0$ and $\frac{dk^i}{d\varphi} < 0$. Therefore, it is clear that risk weights ought to be tightened when regulators account for environmental externalities.

The second insight emerging from Corollary 2 is the characterization of optimal policy in Equation (33). There are two differences to the equivalent characterization with a narrow mandate in Equation (30). First, the marginal distortion on portfolio choices is augmented, which calls for greater relative risk weights on the polluting sector (sector 1). Second, the scale distortion is augmented by ψ_k. The latter point is particularly important for our analysis. The scale distortion matters purely due to imperfect regulation and leakage elasticities. Equation (33) demonstrates that adjustments for leakage elasticities become *more* important once the regulator cares about environmental effects.

Figure 1 illustrates the relation between the first-best and second-best solutions in both the narrow and the broad mandate cases. In particular, the left panel shows the marginal welfare effect of varying leverage regulation (in terms of τ_b), while the right panel shows the marginal welfare effect of varying risk-weights (in terms of τ_θ). As we have formally shown above, Figure 1 illustrates that the optimal second-best policy under a broad mandate overregulates both leverage and portfolio weights relative to the first-best. However, consistent with the insights discussed above, the relation between the first-best regulation and the second-best regulation for a regulator with a narrow mandate is more nuanced. In the case we illustrate, it turns out that a narrow regulator overregulates leverage relative to the first-best, but not portfolio weights. This is mainly due to the fact that the leakage elasticity with respect to capital is greater in magnitude for leverage. By contrast, a broad regulator overregulates both leverage and portfolio weights relative to first best, because she places a greater weight on all leakage elasticities to capital.

4.3 The Value of Regulating Scale

To close the analysis of this application, we consider a regulator who is able to impose a corrective tax $\tau_k k^i$ on investors in order to correct for the (previously unregulated) externalities associated with the scale decision k^i. The key economic insights can be obtained by considering the marginal

[16] Recall that $d\bar{b} > 0$ stands for a *looser* leverage cap, that is, a lower effective tax on leverage. Hence, $\frac{dk^i}{d\bar{b}} > 0$ is equivalent to $\frac{dk^i}{d\tau_b} < 0$, which corresponds to the complements case in our general, tax-based notation.

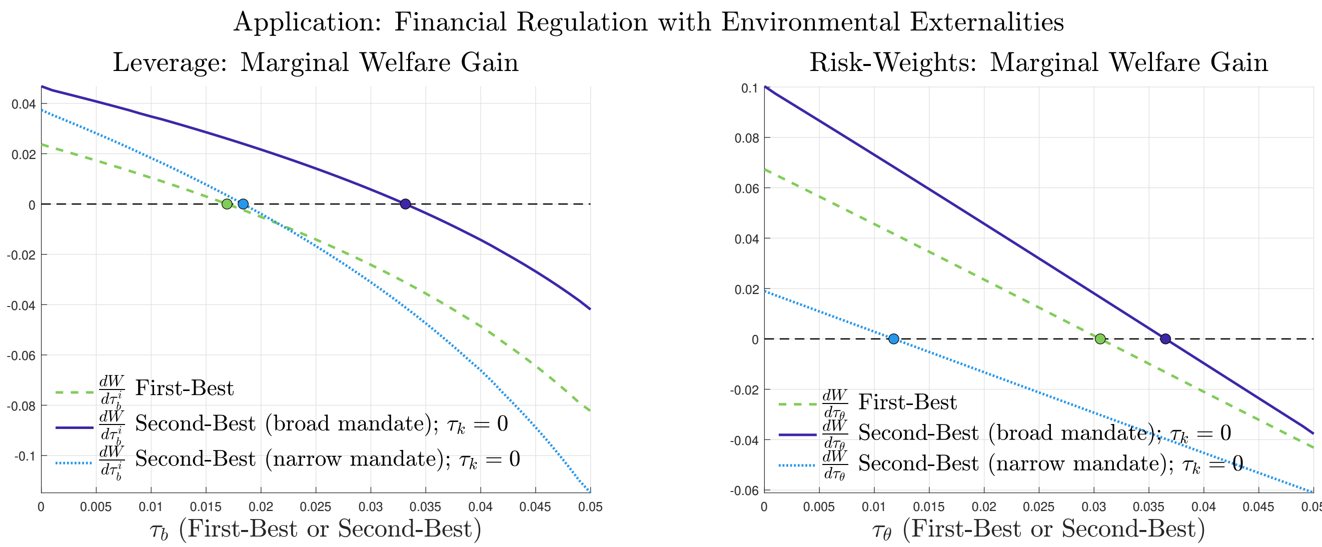

Figure 1: Financial Regulation with Environmental Externalities

Note: The left panel of Figure 1 compares the marginal welfare effects of varying corrective leverage regulation (τ_b) in three different scenarios. The green dashed line corresponds to the first-best scenario, in which τ_θ and τ_k are held fixed at their first-best levels (previously computed). The solid dark blue line corresponds to a second-best scenario in which the regulator has a broad mandate and cares about financial and environmental distortions. In this case, we compute welfare gains setting $\tau_k = 0$ and holding τ_θ fixed at the optimal second-best level for a broad mandate (previously computed). The light blue dotted line corresponds to a second-best scenario in which the regulator has a narrow mandate and cares exclusively about financial distortions. In this case, we compute welfare gains setting $\tau_k = 0$ and holding τ_θ fixed at the optimal second-best level for a narrow mandate (previously computed). The right panel of Figure 1 compares the analogous marginal welfare effects of varying corrective risk-weights regulation (τ_θ) in the same three scenarios.

This figure assumes that the bailout policy is linearly separable, $t^i\left(b^i, s\right) = \alpha_0^i + \alpha_b^i b^i + \alpha_\theta^i \theta^i - \alpha_s^i s$, that the adjustment cost is quadratic, $\Upsilon\left(k^i\right) = k^i + \frac{a}{2}\left(k^i\right)^2$, and that the functions $\Omega\left(\theta^i\right)$ and $\Psi\left(\theta^i\right)$ are of the CES (constant elasticity of substitution) form in terms of k_1^i and k_2^i, so $\Omega\left(\theta\right) = z_\Omega\left(a_\Omega\left(\theta^i\right)^{\eta_\Omega} + \left(1 - a_\Omega\right)\left(1 - \theta^i\right)^{\eta_\Omega}\right)^{\frac{1}{\eta_\Omega}}$ and $\Psi\left(\theta\right) = z_\Psi\left(a_\Psi\left(\theta^i\right)^{\eta_\Psi} + \left(1 - a_\Psi\right)\left(1 - \theta^i\right)^{\eta_\Psi}\right)^{\frac{1}{\eta_\Psi}}$. The parameters used to generate this figure are $\beta^i = 0.9$, $\beta^C = 0.98$ $\phi^i = 0.7$, $a = 1$, $\alpha_0^i = \alpha_s^i = 0$, $\alpha_b^i = 0.015$, $\alpha_\theta^i = 0.01$, $\kappa = 0.15$, $d_1\left(s\right) = d_1 s$ with $d_1 = 1.01$, $d_2\left(s\right) = d_2 s$ with $d_2 = 1$, $z_\Omega = 0.25$, $a_\Omega = 0.5$, $\eta_\Omega = 1.5$, $z_\Psi = 0.25$, $a_\Psi = 0.55$, $\eta_\Psi = 1.5$, $n_0^C = 50$, and $n_1^C\left(s\right) = 50 + 0.1s$, where s is normally distributed with mean 1.7 and standard deviation 0.8, truncated to the interval $[0, 3]$. For reference, the optimal first-best regulation is $\tau_b = 1.69\%$, $\tau_\theta = 3.05\%$, and $\tau_k = 14.22\%$, the optimal second-best regulation with a broad mandate is $\tau_b = 3.33\%$, $\tau_\theta = 3.65\%$, and $\tau_k = 0$, while the optimal second-best regulation with a narrow mandate is $\tau_b = 1.83\%$, $\tau_\theta = 1.11\%$, and $\tau_k = 0$.

welfare effect of increasing τ_k.

Corollary 3. *(Environmental Externalities/Regulating Unregulated Decision)*

When the planner can impose a corrective tax τ_k on the total scale of investment k^i, the marginal welfare effect of varying τ_k is given by

$$\frac{dW}{d\tau_k} = \underbrace{\frac{db^i}{d\tau_k}}_{=0} (\tau_b - \delta_b) k^i + \underbrace{\frac{d\theta^i}{d\tau_k}}_{=0} (\tau_\theta - \delta_\theta) k^i - \frac{dk^i}{d\tau_k} (\tau_k - \delta_k) = -\frac{dk^i}{d\tau_k} \omega_k. \tag{34}$$

An interesting property of this environment is that there are no reverse leakage effects from regulating scale onto leverage and portfolio decisions. Intuitively, the investors' problem in (19) can be approached in two steps. First, investors choose leverage and portfolios to maximize market values $M(b^i, \theta^i)$ per unit of total capital. Second, they set the marginal cost of capital equal to its maximized market value. Since the first step does not depend on the cost/tax of capital, b^i and θ^i are independent of τ_k in equilibrium.

This fact has two novel economic implications. First, we note that the case for regulating scale here is much stronger than in other applications. In particular, the capital-specific elements of the Le Chatelier/reverse leakage adjustment matrix $\boldsymbol{L}$, which usually dampens the welfare impact of regulating unregulated decisions, are zero. Moreover, the case for regulating scale is clearly *stronger* when the regulator has a *broad/environmental mandate*, other things equal, since this mandate takes into account the full marginal distortion $\delta_k = \chi_k + \psi_k$.

Second, we see from Equation (34) that the optimal level of the tax on capital is always given by $\tau_k = \delta_k$, which corresponds to the first best or Pigouvian correction. The absence of reverse leakage implies that there is no incentive to over- or underregulate scale, once the regulator is allowed to do so. This is true even when the regulation of leverage and portfolio decisions is imperfect (with $\tau_b \neq \delta_b$ and/or $\tau_\theta \neq \delta_\theta$).

5 Further Applications

This section, which presents four minimal applications of our general results, has several purposes. First, these applications show how our general results can be employed to determine the optimal second-best policy in several scenarios of practical relevance. Second, these applications illustrate how our general results encompass widely studied rationales for regulation, including bailouts, pecuniary externalities, and internalities. Third, by studying specific applications, we can connect leakage (and policy) elasticities and Pigouvian wedges to model primitives. Finally, we discuss how to use our general results to interpret empirical findings and guide measurement efforts in the context of each application.

Table 1 provides a schematic summary of our applications. Each application is designed to be the simplest one that illustrates the form of the optimal second-best policy in a particular second-best scenario.[17] In the Online Appendix, we provide detailed derivations for each application. In Application 1, we study a model in which some investors are unregulated and regulation is motivated by the presence of implicit government subsidies. In Application 2, we consider an environment where regulation constrains the ratio of investors' risky investments to borrowing. In this application, a behavioral distortion (distorted beliefs) provides the rationale for intervention. In Application 3, regulation is constrained to be uniform across different investment, with intervention motivated by government bailouts, which yields new insights into asset substitution problems. In the final application, we analyze a model of fire-sale externalities — along the lines of Lorenzoni (2008) — with regulation constrained to be uniform across different investors.

Table 1: Summary of Minimal Applications

Application		Instrument	I	N
#1	Shadow Banking	Unregulated Investors	2	1
#2	Behavioral Distortions	Unregulated Decisions	1	2
#3	Asset Substitution	Uniform Decision Regulation	1	2
#4	Pecuniary Externalities	Uniform Investor Regulation	2	1

Note: The column I denotes the number of investors and the column N denotes the number of decisions.

5.1 Application 1: Shadow Banking/Unregulated Investors

The notion of shadow banking is typically used to describe the financial activities that take place outside of the regulated financial sector.[18] In this application, we consider an environment with two types of investors, in which only one type of investor can be regulated (the traditional sector), while the other is outside of the scope of the regulation (the shadow sector).

Environment. We assume that there are two types of investors $i \in \{1,2\}$. In this application, investors should be broadly interpreted as financial intermediaries or banks. Investors have risk-neutral preferences given by

$$c_0^i + \beta^i \int c_1^i(s)\, dF(s),$$

with budget constraints

$$\begin{aligned} c_0^i &= n_0^i + Q^i\left(b^i\right) - \tau_b^i b^i + T^i \\ c_1^i(s) &= n_1^i(s) + \max\left\{v^i s + t^i\left(b^i, s\right) - b^i, 0\right\}, \quad \forall s. \end{aligned}$$

At date 0, an investor i endowed with n_0^i chooses the face value of its debt, b^i, which determines the amount of financing obtained at date 0, $Q^i\left(b^i\right)$, determined in equilibrium by creditors, as described

[17] One could also study the role of imperfect corrective regulation in models of strategic behavior and imperfect competition, as in Corbae and D'Erasmo (2021, 2025) and Corbae and Levine (2018, 2019), or in the context of regulation of asset markets, as in Cai, He, Jiang and Xiong (2021) or Dávila (2023).

[18] Pozsar, Adrian, Ashcraft and Boesky (2010), Gorton, Metrick, Shleifer and Tarullo (2010), and Claessens, Pozsar, Ratnovski and Singh (2012) provide a detailed overview of shadow banking institutions, activities, and regulations.

below. Investor i faces a corrective tax τ_b^i per unit of b^i due at date 0. At date 1 in state s, investor i receives $v^i s$, as well as a bailout transfer $t^i\left(b^i, s\right)$.

Creditors are risk-averse, with preferences given by

$$u\left(c_0^C\right)+\beta^C \int u\left(c_1^C(s)\right) dF(s).$$

Their budget constraints are given by

$$\begin{aligned}
c_0^C &= n_0^C-\sum_{i \in \mathcal{I}} h^i Q^i\left(b^i\right), \\
c_1^C(s) &= n_1^C(s)+\sum_{i \in \mathcal{I}} h^i \mathcal{P}^i\left(b^i, s\right)-(1+\kappa) \sum_{i \in \mathcal{I}} t^i\left(b^i, s\right), \quad \forall s,
\end{aligned}$$

where h^i is the fraction of bonds purchased from investor i, and $\mathcal{P}^i\left(b^i, s\right)$ denotes the repayment received by creditors from investor i in state s. At date 1, all bailout funds are raised from creditors, with a constant net marginal cost of public funds $\kappa \geq 0$. Note that investors only interact in this application through changes in the price of credit, i.e., through the creditors' stochastic discount factor: $m^C(s)=\frac{\beta^C u'\left(c_1^C(s)\right)}{u'\left(c_0^C\right)}$.

Equilibrium. For given corrective taxes/subsidies $\left\{\tau_b^1, \tau_b^2\right\}$, lump-sum transfers $\left\{T_0^1, T_0^2\right\}$, and bailout transfers $\left\{t^1\left(b^1, s\right), t^2\left(b^2, s\right)\right\}$, an *equilibrium* is fully determined by investors' borrowing decisions, $\left\{b^1, b^2\right\}$, and financing schedules, $\left\{Q^1\left(b^1\right), Q^2\left(b^2\right)\right\}$, such that investors maximize their utility, given the financing schedules, and creditors set the schedules optimally, so that $h^1=h^2=1$.

In the first-best scenario, the planner is able to set τ_b^1 and τ_b^2 freely. However, we are interested in scenarios in which the planner cannot regulate type 2 investors, so

$$\tau_b^2=0,$$

which makes the problem of choosing the optimal τ_b^1 a second-best problem.

Optimal Corrective Policy/Simulation. First, Proposition 4 characterizes the form of the optimal second-best policy. Next, we explore a numerical simulation of this application.

Proposition 4. *(Shadow Banking/Unregulated Investors)*
a) The marginal welfare effect of varying the corrective regulation of regulated investors, τ_b^1, is given by

$$\frac{dW}{d\tau_b^1}=\frac{db^1}{d\tau_b^1}\left(\tau_b^1-\delta_b^1\right)-\frac{db^2}{d\tau_b^1}\delta_b^2,$$

where the marginal distortions in this application are defined by

$$\delta_b^i=(1+\kappa) \int m^C(s) \frac{\partial t^i\left(b^i, s\right)}{\partial b^i} dF(s), \tag{35}$$

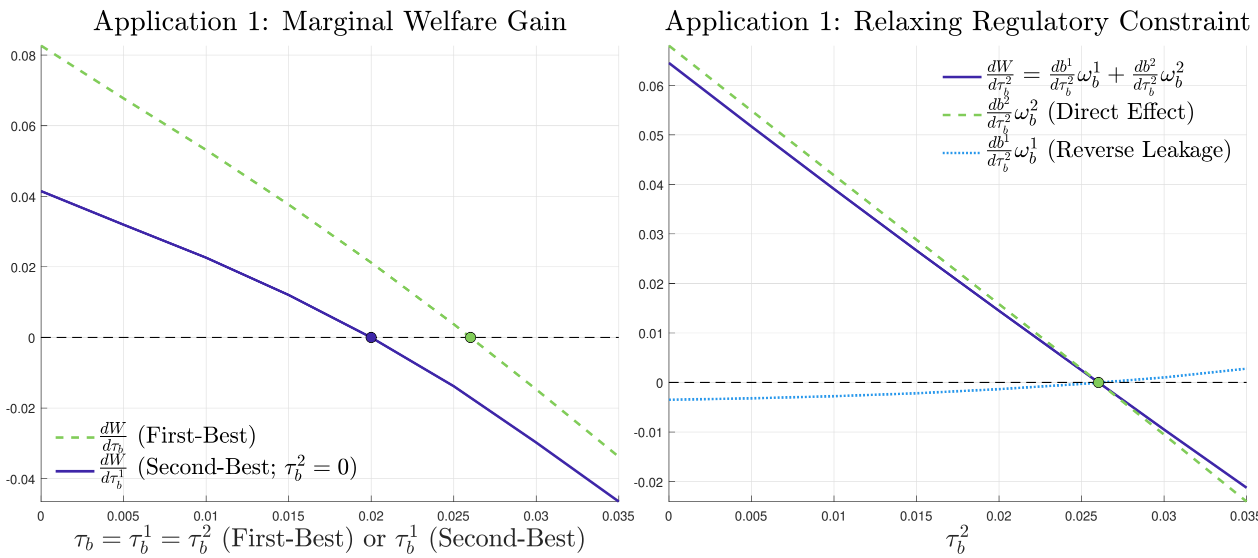

Figure 2: Shadow Banking/Unregulated Investors (Application 1)

Note: The left panel of this figure compares the marginal welfare effects of varying corrective regulations in two different scenarios. The green dashed line corresponds to the first-best scenario in which the horizontal axis corresponds to $\tau_b = \tau_b^1 = \tau_b^2$. The solid blue line corresponds to a second-best scenario in which $\tau_b^2 = 0$ and the horizontal axis corresponds to τ_b^1. Since we assume that both types of investors are symmetric, the value of τ_b that makes the first-best marginal welfare effect zero defines the first-best regulation. The value of τ_b^1 that makes the second-best marginal welfare effect zero defines the second-best regulation.
The right panel of this figure illustrates Proposition 2 by showing the marginal value of being able to regulate the shadow sector. The solid dark blue line corresponds to the total marginal welfare gain of increasing τ_b^2, while τ_b^1 is continually adjusted to be at the optimal second-best value given τ_b^2. The total gain can be decomposed into a direct effect, which corresponds to $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\boldsymbol{\omega}^U$ in Equation (13), and a reverse leakage effect, which corresponds to $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\boldsymbol{L}\boldsymbol{\omega}^U$ in Equation (13). The green dashed line corresponds to the direct effect of relaxing the regulatory constraint, while the light blue dotted line corresponds to the reverse leakage effect. Note that both the direct effect and the reverse leakage effect are zero at the first-best, when $\tau_b = \tau_b^1 = \tau_b^2 = 2.60\%$, but have opposite signs otherwise.
To generate this figure, we assume that the bailout policy is linearly separable: $t^i\left(b^i, s\right) = \alpha_0^i - \alpha_s^i s + \alpha_b^i b^i$, and that creditors' utility is isoelastic: $u\left(c\right) = \frac{c^{1-\gamma}}{1-\gamma}$. The parameters used to generate this figure are $\beta^i = 0.7$, $\phi^i = 0.25$, $v^i = 1$, $\alpha_0^i = \alpha_s^i = 0$, $\alpha_b^i = 0.01$, for $i \in \{1, 2\}$. Also $\kappa = 0.13$, $\gamma = 6$, $\beta^C = 0.98$, $n_0^C = 50$, and $n_1^C\left(s\right) = 50 + 0.1s$, where s is normally distributed with mean 1.3 and standard deviation 0.3, truncated to the interval $[0, 3]$. For reference, the optimal first-best regulation is $\tau_b^1 = \tau_b^2 = 2.60\%$, while the optimal second-best regulation, when the second type of investors cannot be regulated, is $\tau_b^1 = 1.99\%$. Since borrowing decisions are gross substitutes in this application, the optimal second-best policy is *sub-Pigouvian* ($\tau_b^1 < \delta_b^1$).

where $m^C(s)$ denotes the stochastic discount factor of creditors.
b) The optimal corrective regulation satisfies

$$\tau_b^1 = \delta_b^1 - \left(-\frac{db^1}{d\tau_b^1}\right)^{-1} \frac{db^2}{d\tau_b^1}\delta_b^2.$$

Proposition 4 is an application of Lemma 1 and Proposition 1 and exploits the structure of this application to extract further insights. In this application, the marginal distortions associated with borrowing, δ_b^i, are determined by the expected marginal bailout $\frac{\partial t^i(b^i,s)}{\partial b^i}$, augmented by default deadweight losses κ if present, valued using the creditors' stochastic discount factor. The departure of the optimal regulation from the first-best critically depends on the leakage elasticity $\frac{db^2}{d\tau_b^1}$ and the unregulated distortion δ_b^2. A number of recent studies provide direct measurements of the relevant leakage elasticity (e.g., Irani, Iyer, Meisenzahl and Peydro, 2021; Buchak, Matvos, Piskorski and Seru, 2024).[19] In this application, consistent with the empirical literature, we find that tighter regulation on the regulated sector (higher τ_b^1) increases the activities carried out by the unregulated/shadow sector ($\frac{db^2}{d\tau_b^1} > 0$), so leverage choices are gross substitutes. Therefore, we expect the optimal second-best policy to be sub-Pigouvian ($\tau_b^1 < \delta_b^1$).[20]

Moreover, the presence of unregulated investors may exacerbate the welfare distortion δ_b^1 associated with regulated investors. Concretely, when unregulated investors receive bailouts in state s, the marginal utility of creditors increases in this state due to taxation. In Equation (35), this increases the distortion associated with marginal increases in regulated investors' leverage. In this sense, our results reconcile two common narratives. On the one hand, leakage to the shadow banking system motivates sub-Pigouvian regulation. On the other hand, the optimal corrective policy must also adjust to increases in overall leverage, which raise marginal distortions δ_b^1 in general equilibrium.

An instructive special case, which we use to solve the model numerically, is obtained by using a linearly separable bailout policy: $t^i(b^i, s) = \alpha_0^i - \alpha_s^i s + \alpha_b^i b^i$, where $\alpha_s^i, \alpha_b^i \geq 0$. In this case, marginal distortions $\delta_b^i = \frac{1+\kappa}{R^f}\alpha_b^i$ are invariant to policy, and the optimal corrective regulation is

$$\tau_b^1 = \frac{1+\kappa}{R^f}\left[\alpha_b^1 - \left(-\frac{db^1}{d\tau_b^1}\right)^{-1} \frac{db^2}{d\tau_b^1}\alpha_b^2\right],$$

[19] This work focuses on the elasticity of substitution between the market share of regulated and unregulated investments. While we have held the scale of investment fixed in this application, but one could easily extend the framework to account for both leverage and investment choices, in which case the measured elasticities of substitution in those papers become directly relevant. In addition, our application highlights that the elasticity of substitution between regulated and unregulated leverage is a key statistic for second-best regulation.

[20] Note that one can also use this model to analyze quantity-based policies, such as capital requirements. For instance, suppose that regulated investors are subject to a binding quantity regulation $b^1 \leq \bar{b}^1$, where the regulator chooses the upper bound $\bar{b}^1$. In our model, a marginal change $d\bar{b}^1$ is equivalent to the local tax reform $d\tau_b^1 = \left(\frac{db^1}{d\tau_b^1}\right)^{-1} d\bar{b}^1$. The associated leakage elasticity is $\frac{db^2}{d\bar{b}^1} = \left(\frac{db^1}{d\tau_b^1}\right)^{-1} \frac{db^2}{d\tau_b^1}$, and the optimal corrective regulation in Proposition 4 can be alternatively expressed as

$$\tau_b^1 = \delta_b^1 + \frac{db^2}{d\bar{b}^1}\delta_b^2.$$

where $R^f = \left(\int m^C(s)\, dF(s)\right)^{-1}$ denotes the creditors' riskless discount rate.

The left panel of Figure 2 illustrates the comparison between the first-best and second-best policy when simulating this model. To more clearly illustrate the insights that we present in this paper, in Figure 2 we assume that both types of investors are ex-ante identical, and that the only difference between the two is that investor 2 cannot be regulated. Given this symmetry assumption, it is possible to represent the marginal value of varying the regulation $\tau_b = \tau_b^1 = \tau_b^2$ for both investors, which yields the first-best regulation when $\frac{dW}{d\tau_b} = 0$. In contrast, the solid line in Figure 2 shows the marginal value of varying the regulation that investor 1 faces (the traditional sector), when investor 2 (the shadow sector) is unregulated, that is, when $\tau_b^2 = 0$. As implied by our theoretical results, since $\frac{db^2}{d\tau_b^1} > 0$ and $\frac{db^1}{d\tau_b^1} < 0$, we find that the optimal second-best policy is sub-Pigouvian, so the optimal second-best regulation that investor 1 faces is lower than the first-best regulation. In this particular simulation, the optimal first-best regulation is $\tau_b^1 = \tau_b^2 = 2.60\%$, while the second-best regulation (when $\tau_b^2 = 0$) is $\tau_b^1 = 1.99\%$.

The right panel of Figure 2 illustrates Proposition 2 by showing the marginal value of being able to regulate the shadow sector. This panel provides a clear illustration of the Le Chatelier/reverse leakage adjustment discussed above. Regardless of whether the shadow sector is underregulated (when τ_b^2 is below first-best) or overregulated (when τ_b^2 is above first-best), the reverse leakage effect has the opposite sign of the direct effect of adjusting the regulation of the shadow sector, attenuate welfare gains/losses. This illustrates how the presence of perfectly regulated decisions contributes to attenuating the welfare gains of relaxing constraints on regulation.

5.2 Application 2: Behavioral Distortions/Unregulated Decisions

In this application, we characterize the form of the optimal scale-invariant policy in a model in which regulation is motivated by belief distortions.

Environment. We assume that there is a single type of investor, in unit measure and indexed by i, and a unit measure of creditors, indexed by C. Both investors and creditors have risk-neutral preferences given by

$$c_0^i + \beta^i \int c_1^i(s)\, dF^i(s) \quad \text{and} \quad c_0^C + \beta^C \int c_1^C(s)\, dF^C(s)\,,$$

where $F^i(s)$ and $F^C(s)$ denote the beliefs (cumulative distribution functions) of investors and creditors over the possible states. Endowments and technologies are specified as in Section 4, with the simplification that investors do not choose the composition of their capital portfolio. Accordingly, the budget constraints of investors at date 0 and date 1 are given by

$$\begin{aligned} c_0^i &= n_0^i + Q^i\left(b^i\right) k^i - \Upsilon\left(k^i\right) \\ c_1^i(s) &= n_1^i(s) + \max\left\{s - b^i, 0\right\} k^i, \quad \forall s. \end{aligned}$$

Creditors' budget constraints are given by

$$c_0^C = n_0^C - h^i Q^i \left(b^i\right) k^i$$
$$c_1^C(s) = n_1^C(s) + h^i \mathcal{P}^i \left(b^i, s\right) k^i, \quad \forall s,$$

where $\mathcal{P}^i(b^i, s)$ denotes the repayment received by creditors from investors in state s per unit of investment.

As in Section 4, we consider regulation via a capital requirement

$$b^i \leq \bar{b}.$$

We show below that this is equivalent to a corrective tax on leverage choices b^i.

We assume that the planner computes welfare using different probability assessments than those used by investors and creditors to make decisions. This provides a corrective rationale for intervention. As highlighted in Dávila and Walther (2023) and Proposition 5 below, the rationale for regulation is determined by the difference between private agents' and the planner's valuations per unit of risky investment, which represent a levered version of Tobin's q. These valuations are, respectively, given by

$$M\left(b^i\right) = \beta^i \int \max\left\{s - b^i, 0\right\} dF^i(s) + \beta^C \int \mathcal{P}^i\left(b^i, s\right) dF^C(s)$$
$$M^P\left(b^i\right) = \beta^i \int \max\left\{s - b^i, 0\right\} dF^P(s) + \beta^C \int \mathcal{P}^i\left(b^i, s\right) dF^P(s),$$

where $F^P(s)$ denotes the probability distribution used by the planner to calculate welfare.

Equilibrium. For a given leverage cap $\bar{b}$, an *equilibrium* is defined by an investment decision, k^i, a leverage decision, b^i, and a default decision rule such that i) investors maximize their utility given $Q^i(\cdot)$, and ii) creditors set the schedule $Q^i(\cdot)$ optimally, so that $h^i = 1$.

In the first-best scenario, the planner is able to set corrective taxes on both leverage and investment. In this application, the planner's only instrument is the leverage cap $\bar{b}$, which is imperfect. This can be seen by writing investors' first-order conditions as

$$\frac{\partial M(b^i)}{\partial b^i} = \mu \equiv \tau_b \quad \text{and} \quad M\left(b^i, \theta^i\right) - \Upsilon'\left(k^i\right) = 0.$$

As in Section 4, the planner can therefore impose an effective tax on leverage via $\bar{b}$, but cannot affect investors' marginal incentive to create investment capital k^i.

Optimal Corrective Policy/Simulation. In Proposition 5, we characterize the form of the optimal second-best policy, which we discuss along with a numerical simulation.

Proposition 5. *(Behavioral Distortions/Unregulated Activities)*

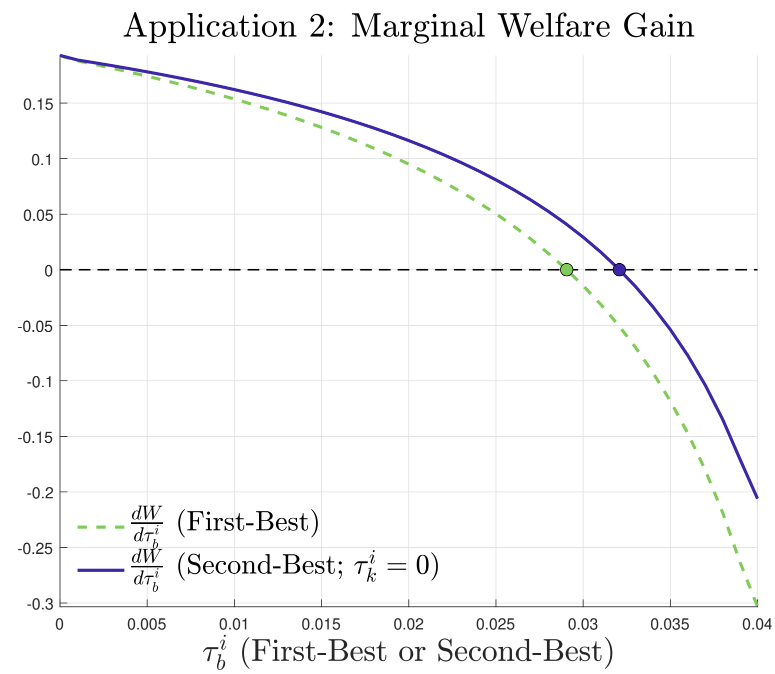

Figure 3: Behavioral Distortions/Unregulated Activities (Application 2)

Note: This figure compares the marginal welfare effects of varying the corrective regulation in two different scenarios. The green dashed line corresponds to a scenario in which τ_k^i is set at the first-best level. The solid blue line corresponds to a second-best scenario in which $\tau_k^i = 0$. Therefore, the value of τ_b^i that makes the first-best marginal welfare effect zero defines the first-best leverage regulation, since τ_k^i is already set at the first-best level. The value of τ_b^i that makes the second-best marginal welfare effect zero defines the second-best regulation. To generate this figure, we assume that the adjustment cost is quadratic: $\Upsilon\left(k^i\right) = k^i + \frac{a}{2}\left(k^i\right)^2$. The parameters used to generate this figure are $\beta^i = 0.9$, $\beta^C = 0.95$, $\phi^i = 0.8$, and $a = 1$. We assume that investors and creditors perceive s to be normally distributed with mean 1.5 and standard deviation 0.4, and the planner perceives the mean to be 1.3 instead. For reference, the optimal first-best regulation is given by $\tau_b^i = 2.91\%$ and $\tau_k^i = 18.45\%$, while the second-best regulation, when investment cannot be regulated, is $\tau_b^i = 3.21\%$. Since leverage and investment decisions are gross complements in this application, the optimal second-best policy is *super-Pigouvian* $\left(\tau_b^i > \delta_b^i\right)$.

a) The marginal welfare effect of varying the regulation of investors' leverage, τ_b^i, *is given by*

$$\frac{dW}{d\tau_b^i} = \frac{db^i}{d\tau_b^i}\left(\tau_b^i - \delta_b^i\right) - \frac{dk^i}{d\tau_b^i}\delta_k^i,$$

where the marginal distortions in this application are defined by

$$\delta_b^i = \frac{dM\left(b^i\right)}{db^i} - \frac{dM^P\left(b^i\right)}{db^i} \quad \textit{and} \quad \delta_k^i = M\left(b^i\right) - M^P\left(b^i\right).$$

b) The optimal corrective regulation satisfies

$$\tau_b^i = \delta_b^i - \left(-\frac{db^i}{d\tau_b^i}\right)^{-1}\frac{dk^i}{d\tau_b^i}\delta_k^i.$$

Proposition 5 is the counterpart of Lemma 1 and Proposition 1, and it identifies the distortions associated with leverage and investment the planner perceives. In this application, the welfare distortion associated with leverage, δ_b^i, is driven by the difference in marginal valuations, while the distortion associated with investment, δ_k^i, is driven by the difference in the level of valuations. In this application we have $\frac{db^i}{d\tau_b^i} < 0$ and, critically, the leakage elasticity from leverage to investment is negative, that is, $\frac{dk^i}{d\tau_b^i} < 0$, implying that leverage and investment are gross complements. As implied by our results in Section 3, the optimal second-best regulation on leverage is super-Pigouvian ($\tau_b^i > \delta_b^i$).

Importantly, a comparison between this application with the previous one (shadow banking) highlights that both leakage elasticities featuring substitutes and those featuring complements are important in common regulatory scenarios. A number of recent empirical studies confirm that the leakage elasticity from leverage to risky investments is negative, in the sense that banks with lower capital ratios originate a larger volume of risky loans (e.g., Jiménez, Ongena, Peydró and Saurina, 2014; Dell'Ariccia, Laeven and Suarez, 2017; Acharya, Eisert, Eufinger and Hirsch, 2018).

Figure 3 compares the marginal welfare effects of varying regulation in the first-best and second-best scenarios when simulating this model. To illustrate the first-best solution for leverage, we fix τ_k^i to its first-best value when showing the marginal welfare associated with varying τ_b^i. The second-best marginal welfare gain simply sets $\tau_k^i = 0$. As implied by our theoretical results, the optimal second-best policy is super-Pigouvian, so it is optimal for the planner to overregulate leverage relative to the first-best scenario. In this particular simulation, the optimal first-best regulation is $\tau_b^i = 2.91\%$ and $\tau_k^i = 18.45\%$, while the second-best regulation (when $\tau_k^i = 0$) is $\tau_b^i = 3.21\%$.

5.3 Application 3: Asset Substitution/Uniform Decision Regulation

A common concern in financial regulation is that corrective policy instruments are somewhat coarse in practice. For example, when imposing capital requirements on banks, financial regulators tend to set risk weights for wide classes of risky investments (e.g., mortgage loans), but within the class, banks can freely optimize their portfolios (e.g., among loans to borrowers with different credit scores) without any change in the associated capital charge. In our model, this situation corresponds to a uniform regulation across different capital investments. In this application, we consider uniform corrective policy in a model where investors enjoy government guarantees. We use the properties of uniform regulation to derive new insights into the classical asset substitution problem (e.g., Jensen and Meckling, 1976), and characterize the optimal second-best policy.

Environment. We assume that there is a single type of investor, in unit measure and indexed by i, and a unit measure of creditors, indexed by C. Both investors and creditors have risk-neutral preferences given by

$$c_0^i + \beta^i \int c_1^i(s)\, dF(s) \quad \text{and} \quad c_0^C + \beta^C \int c_1^C(s)\, dF(s).$$

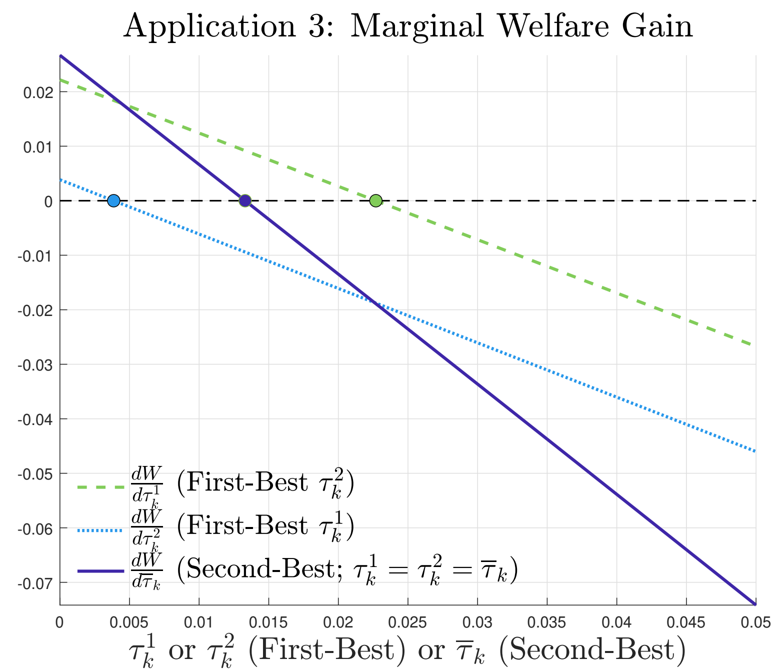

Figure 4: Asset Substitution/Uniform Activity Regulation (Application 3)

Note: This figure compares the marginal welfare effects of varying the corrective regulation in two different scenarios. The green dashed line and the light blue dotted line illustrate the first-best regulation. The green dashed line corresponds to a scenario in which τ_k^2 is set at the first-best level (previously computed), while the light blue dotted line corresponds to a scenario in which τ_k^1 is set at the first-best level (previously computed). Therefore, the values of τ_k^1 and τ_k^2 that respectively make each line zero define the first-best regulation. The solid dark blue line corresponds to a second-best scenario in which $\overline{\tau}_k = \tau_k^1 = \tau_k^2$, so its zero defines the second-best regulation. To generate this figure, we assume that the adjustment cost is quadratic: $\Upsilon\left(k_1^i, k_2^i\right) = \frac{z_1}{2}\left(k_1^i\right)^2 + \frac{z_2}{2}\left(k_2^i\right)^2$. We also assume that $d_1(s) = \mu_1 + \sigma_1 s$ and $d_2(s) = \mu_2 + \sigma_2 s$ when s is distributed as a standard normal. The parameters used to generate this figure are $\beta^i = 0.8$, $\beta^C = 1$, $\kappa = 0.1$, $z_1 = z_2 = 1$, $b^i = 1.4$, $\mu_1 = 1.5$, $\mu_2 = 1.3$, $\sigma_1 = 0.3$, and $\sigma_2 = 0.5$. For reference, the optimal first-best regulation is given by $\tau_k^1 = 2.27\%$ and $\tau_k^2 = 0.39\%$, while the second-best regulation, when the regulation is uniform, is $\overline{\tau}_k = 1.33\%$.

The budget constraints of investors at date 0 and date 1 are given by

$$c_0^i = n_0^i - \Upsilon\left(k_1^i, k_2^i\right) - \tau_k^1 k_1^i - \tau_k^2 k_2^i + T^i$$
$$c_1^i(s) = \max\left\{d_1(s) k_1^i + d_2(s) k_2^i + t\left(k_1^i, k_2^i, b^i, s\right) - b^i, 0\right\}, \quad \forall s.$$

At date 0, investors, endowed with n_0^i, choose the scale of two risky capital investments k_1^i and k_2^i, which are subject to an adjustment cost of $\Upsilon\left(k_1^i, k_2^i\right)$. Hence, investors make $|\mathcal{X}| = 2$ free choices regarding their balance-sheet.

At date 1, investors earn the realized returns on capital investments k_1^i and k_2^i, which are given by $d_1(s)$ and $d_2(s)$ and are increasing in s. In addition, they receive a bailout transfer $t\left(k_1^i, k_2^i, b^i, s\right)$ from the government. We further assume that investors have legacy debt (i.e., debt issued before the start of the model) with face value b^i. Hence, investors owe a predetermined repayment of b^i to creditors at date 1. We make this simplifying assumption in order to sharpen our focus on *asset substitution*, which describes investors' choice between different risky investments, as opposed to leverage choices. At date 1, investors consume the difference between i) the cash flow from investments augmented by the bailout transfer and ii) the debt owed, if this difference is positive. Otherwise, they default and consume zero.

For simplicity, we focus on a particular form of bailout that fully prevents default — this may correspond to an investor that is "too big to fail". Concretely, we assume that the government bailout is equal to the minimum amount required to avoid default

$$t\left(k_1^i, k_2^i, b^i, s\right) = \max\left\{b^i - d_1(s) k_1^i - d_2(s) k_2^i, 0\right\}. \tag{36}$$

Given this form of bailout policy, creditors are guaranteed a repayment of b^i at date 1. We write $s^\star\left(k_1^i, k_2^i\right)$ for the threshold state below which bailouts are positive.[21]

Hence, the budget constraints of creditors at date 0 and date 1 are given by

$$c_0^C = n_0^C \quad \text{and} \quad c_1^C(s) = n_1^C(s) + b^i - (1+\kappa)\, t\left(k_1^i, k_2^i, b^i, s\right), \ \forall s.$$

Even though creditors are always repaid b^i in every state, we assume that in order to finance the bailout, the government imposes a tax of $(1+\kappa)$ per dollar of bailout on creditors, where $\kappa > 0$ measures the deadweight fiscal cost of bailout transfers. The rationale for regulation in this environment is a classical "moral hazard" argument. Investors, whose debt is implicitly guaranteed by the government, do not internalize the impact of their risky capital investments on fiscal costs, which ultimately reduces the consumption of creditors.

Equilibrium. For given corrective taxes/subsidies $\left\{\tau_k^1, \tau_k^2\right\}$, lump-sum transfers $T^i = \tau_k^1 k_1^i + \tau_k^2 k_2^i$, bailout policy $t\left(k_1^i, k_2^i, b^i, s\right)$, and legacy debt b^i, an *equilibrium* is defined by investment decisions such that investors maximize their utility. In the first-best scenario, the planner is able to set τ_k^1 and τ_k^2 freely. However, we are interested in a scenario in which the planner is unable to treat investments differentially for regulation purposes. Thus, the planner chooses $\tau_k^1 \geq 0$ and $\tau_k^2 \geq 0$ subject to the uniform regulation constraint:

$$\overline{\tau}_k = \tau_k^1 = \tau_k^2.$$

Optimal Corrective Policy/Simulation. In Proposition 6 we characterize the form of the second-best policy, which we discuss along with a numerical simulation.

Proposition 6. (Asset Substitution/Uniform Activity Regulation)
a) The marginal welfare effect of varying the uniform corrective regulation of capital investments, $\overline{\tau}_k = \tau_k^1 = \tau_k^2$, is given by

$$\frac{dW}{d\overline{\tau}_k} = \frac{dk_1^i}{d\overline{\tau}_k}\left(\overline{\tau}_k - \delta_1\right) + \frac{dk_2^i}{d\overline{\tau}_k}\left(\overline{\tau}_k - \delta_2\right),$$

[21] Formally, for a fixed value b^i of legacy debt, this threshold is the unique solution to $b^i - d_1(s) k_1^i - d_2(s) k_2^i = 0$.

where the marginal distortions in this application are defined by

$$\delta_j = (1+\kappa)\,\beta^C \int_{\underline{s}}^{s^\star\left(k_1^i,k_2^i\right)} d_j(s)\,dF(s)\,.$$

b) The optimal corrective regulation satisfies

$$\overline{\tau}_k = \frac{\frac{dk_1^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k}+\frac{dk_2^i}{d\overline{\tau}_k}}\delta_1 + \frac{\frac{dk_2^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k}+\frac{dk_2^i}{d\overline{\tau}_k}}\delta_2.$$

Proposition 6 identifies the distortions associated with the different types of investment decisions in this application. The shape of the distortions δ_j highlights the nature of the asset substitution problem: investors' private incentives are driven by the returns to investment in "upside" states $s \geq s^\star\left(k_1^i,k_2^i\right)$, while the planner's concern about bailouts focuses on "downside" states $s < s^\star\left(k_1^i,k_2^i\right)$. The optimal uniform regulation is a weighted average of the downside distortions imposed by both types of capital. As implied by our general results in Section 3, the appropriate weight assigned by the planner to each of the distortions in the optimal second-best policy is given by how sensitive each capital decision is to changes in the regulation, $\frac{\frac{dk_1^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k}+\frac{dk_2^i}{d\overline{\tau}_k}}$ and $\frac{\frac{dk_2^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k}+\frac{dk_2^i}{d\overline{\tau}_k}}$. Figure 4 illustrates this intuition by comparing the marginal welfare effects of varying regulation in the first-best and second-best scenarios.

Assuming that investment costs are quadratic, it is possible to provide further intuition on how the weights $\frac{dk_1^i}{d\overline{\tau}_k}$ and $\frac{dk_2^i}{d\overline{\tau}_k}$ are determined. The optimal weights depend on i) the sensitivity of the probability of receiving a bailout to the uniform regulation, and ii) the marginal contribution $d_n(s^\star)$ of each asset class at the bailout boundary. Intuitively, a large ratio $\frac{d_2(s^\star)}{d_1(s^\star)}$ means that changes in the default boundary affect mostly returns to k_2^i, which makes investors' optimal investment in k_2^i more sensitive to the uniform regulation.

5.4 Application 4: Pecuniary Externalities/Uniform Investor Regulation

Pecuniary/fire-sale externalities coupled with incomplete markets and/or collateral constraints provide a well-studied rationale for corrective macro-prudential regulation. The natural notion of efficiency in those environments, constrained efficiency, typically requires agent-specific regulations, which can be mapped to our first-best benchmark. In this application, we study the form of the second-best policy in an environment in which it would be optimal to set investor-specific regulations, but the planner is constrained to set the same corrective regulation for all investors.

Environment. We assume that there are two types of investors/entrepreneurs, indexed by $i \in \{1,2\}$, and households, indexed by H — who in a richer model would also play the role of creditors. There are three dates, $t \in \{0,1,2\}$ and no uncertainty. Investors, who for simplicity do not discount the future, have preferences of the form:

$$u^i = c_0^i + c_1^i + c_2^i,$$

subject to non-negativity constraints, $c_0^i \geq 0$, $c_1^i \geq 0$, $c_2^i \geq 0$, where their budget constraints are given by

$$\begin{aligned}
c_0^i &= n_0^i - \Upsilon^i\left(k_0^i\right) - \tau_k^i k_0^i + T^i \\
c_1^i &= q\left(k_0^i - k_1^i\right) - \xi^i k_0^i \\
c_2^i &= z^i k_1^i.
\end{aligned}$$

At date 0, an investor i endowed with n_0^i chooses how much to produce, k_0^i, given a technology $\Upsilon^i\left(k_0^i\right)$. Investor i also faces a tax τ_k^i per unit invested at date 0. At date 1, an investor i must reinvest $\xi^i > 0$ per unit of invested capital at date 0, which needs to be satisfied by selling $k_0^i - k_1^i$ units of capital at a market price q — this is a simple way to generate a fire-sale. At the final date, any capital left yields an output $z^i k_1^i$. For simplicity, we assume that, in equilibrium, $T^i = \tau_k^i k_0^i$, $\forall i$.

Households, who exclusively consume at date 1, have access to a decreasing returns to scale technology to transform capital into output at date 1. Formally, the utility of households is

$$u^H = c_1^H = F\left(k_1^H\right) - q k_1^H,$$

where $F(\cdot)$ is a well-behaved concave function and k_1^H denotes the amount of capital purchased by households at date 1. The solution to the households' problem will define a downward sloping demand curve for sold capital at date 1.

Equilibrium. For given corrective taxes/subsidies $\{\tau_k^1, \tau_k^2\}$ and lump-sum transfers $\{T_0^1, T_0^2\} = \{\tau_k^1 k_0^1, \tau_k^2 k_0^2\}$, an *equilibrium* is fully determined by investors/entrepreneurs' investment decisions $\{k_0^i, k_1^i\}$ at dates 0 and 1, households' capital allocation k_1^H at date 1, and an equilibrium price q, such that investors' and households' utilities are maximized, subject to constraints, and the capital market clears, that is, $\sum_i \left(k_0^i - k_1^i\right) = k_1^H$.

In the first-best scenario, the planner is able to set τ_k^1 and τ_k^2 freely. However, we are interested in scenarios in which the planner must regulate both investors equally, so

$$\overline{\tau}_k = \tau_k^1 = \tau_k^2,$$

which makes the problem of choosing the optimal $\overline{\tau}_k$ a second-best problem.

At date 1, the non-negativity constraint of investors' consumption will necessarily bind, so the amount sold by investor i at date 1 will be proportional to date 0 investment: $k_0^i - k_1^i = \frac{\xi^i}{q} k_0^i$. The households' optimality condition is given by $q = F'\left(k_1^H\right)$. When combined with market clearing and the optimal investment decision at date 0 , we show that the equilibrium price can be characterized

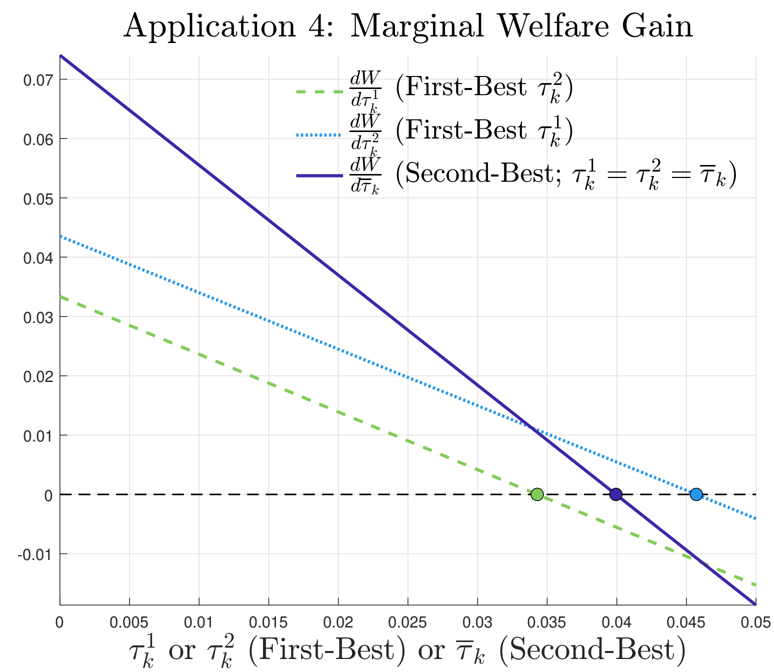

Figure 5: Pecuniary Externalities/Uniform Investor Regulation (Application 4)

Note: This figure compares the marginal welfare effects of varying the corrective regulation in two different scenarios. The green dashed line and the light blue dotted line illustrate the first-best regulation. The green dashed line corresponds to a scenario in which τ_k^2 is set at the first-best level (previously computed), while the light blue dotted line corresponds to a scenario in which τ_k^1 is set at the first-best level (previously computed). Therefore, the values of τ_k^1 and τ_k^2 that respectively make each line zero define the first-best regulation. The solid dark blue line corresponds to a second-best scenario in which $\tau_k^1 = \tau_k^2 = \overline{\tau}_k$, so its zero defines the second-best regulation. To generate this figure, we assume that the adjustment cost of investment is quadratic: $\Upsilon^i\left(k_0^i\right) = \frac{a^i}{2}\left(k_0^i\right)^2$, and that the technology of households is isoelastic: $F\left(k_1^H\right) = \frac{\left(k_1^H\right)^\alpha}{\alpha}$. The parameters used to generate this figure are $\alpha = 0.5$, $a^1 = a^2 = 1$, $z^1 = z^2 = 1.5$, $\xi^1 = 0.3$, and $\xi^2 = 0.4$. For reference, the optimal first-best regulation is given by $\tau_k^1 = 3.43\%$ and $\tau_k^2 = 4.57\%$, while the second-best regulation, when the regulation is uniform, is $\overline{\tau}_k = 3.99\%$.

in terms of primitives as the solution to

$$q = \left(\sum_i \frac{\xi^i}{a^i}\left(z^i\left(1 - \frac{\xi^i}{q}\right) - \tau_k^i\right)\right)^{\frac{\alpha-1}{\alpha}},$$

where we have assumed quadratic adjustment costs $\Upsilon^i\left(k_0^i\right) = \frac{a^i}{2}\left(k_0^i\right)^2$ and the isoelastic production function $F\left(k_1^H\right) = \frac{\left(k_1^H\right)^\alpha}{\alpha}$.

Optimal Corrective Policy/Simulation. In Proposition 7 we characterize the form of the second-best policy, which we discuss along with a numerical simulation.

Proposition 7. *(Pecuniary Externalities/Uniform Investor Regulation)*
a) The marginal welfare effect of varying the uniform corrective regulation of investments, $\overline{\tau}_k = \tau_k^1 = \tau_k^2$*, is given by*

$$\frac{dW}{d\overline{\tau}_k} = \frac{dk_0^1}{d\overline{\tau}_k}\left(\overline{\tau}_k - \delta_k^1\right) + \frac{dk_0^2}{d\overline{\tau}_k}\left(\overline{\tau}_k - \delta_k^2\right),$$

where

$$\delta_k^i = -\frac{\partial q}{\partial k_0^i} \sum_{\ell=1}^{2} \left(\frac{z^\ell}{q} - 1 \right) \left(k_0^\ell - k_1^\ell \right).$$

b) The optimal corrective regulation satisfies

$$\overline{\tau}_k = \frac{\frac{dk_0^1}{d\overline{\tau}_k}}{\frac{dk_0^1}{d\overline{\tau}_k} + \frac{dk_0^2}{d\overline{\tau}_k}} \delta_k^1 + \frac{\frac{dk_0^2}{d\overline{\tau}_k}}{\frac{dk_0^1}{d\overline{\tau}_k} + \frac{dk_0^2}{d\overline{\tau}_k}} \delta_k^2.$$

Proposition 7 identifies the distortions associated with the investment choices of investors/entrepreneurs. In this application, the distortion is generated by a distributive pecuniary externality, using the terminology of Dávila and Korinek (2018). Consistent with the results in that paper, this type of externality is determined by price sensitivities, differences in marginal valuations, and net trade positions. In this case, these three statistics are given by $\frac{\partial q}{\partial k_0^i}$, $\frac{z^\ell}{q} - 1$, and $k_0^\ell - k_1^\ell$. Note that δ_k^i includes the sum of the latter two terms across both types of investors, since a given investor does not internalize how his own investment decision affects prices and consequently the welfare of other investors of the same and different types.

As implied by our general results in Section 3, the weight assigned by the planner to each of the distortions in the optimal second-best policy is given by the sensitivity of each investment decision to changes in the regulation. Figure 5 illustrates this intuition by comparing the marginal welfare effects of varying regulation in the first- and second-best scenarios. Comparing Applications 3 and 4, it becomes evident that the principles that guide the second-best regulation when forced to be uniform across choices for a given agent or across agents for a given choice are identical.

6 Conclusion

This paper has systematically studied optimal corrective regulation with imperfect instruments. We have shown that leakage elasticities from perfectly regulated to imperfectly regulated decisions, along with Pigouvian wedges, are sufficient statistics to determine the optimal regulation of perfectly regulated decisions. Notably, the optimal second-best policy hinges on whether perfectly and imperfectly regulated decisions are gross substitutes or complements. Reverse leakage elasticities from imperfectly to perfectly regulated decisions influence the optimal regulation of imperfectly regulated decisions and determine the social value of relaxing constraints on regulation — a novel instance of the Le Chatelier principle. We have explicitly characterized the optimal second-best policy in three practical scenarios: unregulated decisions, uniform regulation, and convex costs of regulation.

We have leveraged the general methodology to highlight the common fundamental economic principles in a number of practical scenarios, such as financial regulation with environmental externalities, shadow banking, behavioral distortions, asset substitution, and fire-sale externalities with heterogeneous investors. We hope that our general results spur the development of future measurement efforts and new applications of practical interest.

References

Acemoglu, D. 2007. "Equilibrium Bias of Technology." *Econometrica*, 1371–1409.

Acharya, Viral V, Philipp Schnabl, and Gustavo Suarez. 2013. "Securitization without Risk Transfer." *Journal of Financial Economics*, 107(3): 515–536.

Acharya, Viral V, Tim Eisert, Christian Eufinger, and Christian Hirsch. 2018. "Real Effects of the Sovereign Debt Crisis in Europe: Evidence from Syndicated Loans." *The Review of Financial Studies*, 31(8): 2855–2896.

Adrian, Tobias, and Adam B Ashcraft. 2016. "Shadow Banking: A Review of the Literature." In *Banking Crises.* , ed. Garett Jones, 282–315. Springer.

Arseneau, David M, Grace Brang, Matt Darst, Jacob M Faber, David E Rappoport, and Alexandros P Vardoulakis. 2023. "A Macroprudential Perspective on the Regulatory Boundaries of US Financial Assets." *Journal of Financial Crises*, 5(1): 1–24.

Balcer, Yves. 1980. "Taxation of Externalities: Direct versus Indirect." *Journal of Public Economics*, 13(1): 121–129.

Baumol, William J. 1972. "On Taxation and the Control of Externalities." *American Economic Review*, 62(3): 307–322.

Begenau, Juliane, and Tim Landvoigt. 2022. "Financial regulation in a quantitative model of the modern banking system." *The Review of Economic Studies*, 89(4): 1748–1784.

Bengui, Julien, and Javier Bianchi. 2022. "Macroprudential policy with leakages." *Journal of International Economics*, 139: 103659.

Bianchi, Javier. 2011. "Overborrowing and Systemic Externalities in the Business Cycle." *American Economic Review*, 101(7): 3400–3426.

Bianchi, Javier. 2016. "Efficient Bailouts?" *American Economic Review*, 106(12): 3607–3659.

Bovenberg, A Lans, and Lawrence H Goulder. 2002. "Environmental Taxation and Regulation." *Handbook of Public Economics*, 3: 1471–1545.

Buchak, Greg, Gregor Matvos, Tomasz Piskorski, and Amit Seru. 2018. "Fintech, Regulatory Arbitrage, and the Rise of Shadow Banks." *Journal of Financial Economics*, 130(3): 453–483.

Buchak, Greg, Gregor Matvos, Tomasz Piskorski, and Amit Seru. 2024. "Beyond the Balance Sheet Model of Banking: Implications for Bank Regulation and Monetary Policy." *Journal of Political Economy*, 132(2): 616–693.

Cai, Jinghan, Jibao He, Wenxi Jiang, and Wei Xiong. 2021. "The whack-a-mole game: Tobin taxes and trading frenzy." *The Review of Financial Studies*, 34(12): 5723–5755.

Chari, VV, and Patrick J Kehoe. 2016. "Bailouts, Time Inconsistency, and Optimal Regulation: A Macroeconomic View." *American Economic Review*, 106(9): 2458–2493.

Claessens, Stijn, Zoltan Pozsar, Lev Ratnovski, and Manmohan Singh. 2012. "Shadow Banking: Economics and Policy." *IMF Staff Discussion Note.*

Clayton, Cristopher, and Andreas Schaab. 2021. "Shadow Banks and Optimal Regulation." *Working Paper.*

Corbae, Dean, and Pablo D'Erasmo. 2021. "Capital buffers in a quantitative model of banking industry dynamics." *Econometrica*, 89(6): 2975–3023.

Corbae, Dean, and Pablo D'Erasmo. 2025. "A Quantitative Model of Banking Industry Dynamics." *Working Paper.*

Corbae, Dean, and Ross Levine. 2018. "Competition, Stability, and Efficiency in Financial Markets." *Jackson Hole Symposium: Changing Market Structure and Implications for Monetary Policy, Kansas City Federal Reserve.*

Corbae, Dean, and Ross Levine. 2019. "Competition, Stability, and Efficiency in the Banking Industry."

Working Paper.
Cordella, T., G. Dell'Ariccia, and R. Marquez. 2018. "Government Guarantees, Transparency, and Bank Risk Taking." *IMF Economic Review*, 66: 116–143.
Corlett, Wilfred J., and Douglas C. Hague. 1953. "Complementarity and the Excess Burden of Taxation." *The Review of Economic Studies*, 21(1): 21–30.
Cremer, Helmuth, Firouz Gahvari, and Norbert Ladoux. 1998. "Externalities and Optimal Taxation." *Journal of Public Economics*, 70(3): 343–364.
Dávila, Eduardo. 2023. "Optimal Financial Transaction Taxes." *The Journal of Finance*, 78(1): 5–61.
Dávila, Eduardo, and Andreas Schaab. 2025. "Welfare Assessments with Heterogeneous Individuals." *Journal of Political Economy (Forthcoming)*, 133(9).
Dávila, Eduardo, and Ansgar Walther. 2020. "Does Size Matter? Bailouts with Large and Small Banks." *Journal of Financial Economics*, 136(1): 1–22.
Dávila, Eduardo, and Ansgar Walther. 2023. "Prudential Policy with Distorted Beliefs." *American Economic Review*, 113(7): 1967–2006.
Dávila, Eduardo, and Anton Korinek. 2018. "Pecuniary Externalities in Economies with Financial Frictions." *Review of Economic Studies*, 85(1): 352–395.
Dell'Ariccia, Giovanni, Luc Laeven, and Gustavo A Suarez. 2017. "Bank Leverage and Monetary Policy's Risk-Taking Channel: Evidence from the United States." *The Journal of Finance*, 72(2): 613–654.
Dempsey, Kyle. 2020. "Macroprudential Capital Requirements with Non-Bank Finance." *ECB Working Paper.*
Demyanyk, Yuliya, and Elena Loutskina. 2016. "Mortgage Companies and Regulatory Arbitrage." *Journal of Financial Economics*, 122(2): 328–351.
Diamond, Peter A. 1973. "Consumption Externalities and Imperfect Corrective Pricing." *The Bell Journal of Economics and Management Science*, 4(2): 526–538.
Dixit, Avinash. 1985. "Tax Policy in Open Economies." In *Handbook of Public Economics.* Vol. 1, 313–374. Elsevier.
Dovis, Alessandro, and Rishabh Kirpalani. 2020. "Fiscal Rules, Bailouts, and Reputation in Federal Governments." *American Economic Review*, 110(3): 860–888.
Dubey, P., J. Geanakoplos, and M. Shubik. 2005. "Default and Punishment in General Equilibrium." *Econometrica*, 73(1): 1–37.
Farhi, Emmanuel, and Iván Werning. 2016. "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities." *Econometrica*, 84(5): 1645–1704.
Farhi, Emmanuel, and Jean Tirole. 2012. "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts." *American Economic Review*, 102(1): 60–93.
Farhi, Emmanuel, and Jean Tirole. 2021. "Shadow banking and the four pillars of traditional financial intermediation." *The Review of Economic Studies*, 88(6): 2622–2653.
Gabaix, Xavier. 2014. "A sparsity-based model of bounded rationality." *The Quarterly Journal of Economics*, 129(4): 1661–1710.
Gennaioli, Nicola, Andrei Shleifer, and Robert W Vishny. 2013. "A Model of Shadow Banking." *The Journal of Finance*, 68(4): 1331–1363.
Giglio, Stefano, Bryan Kelly, and Johannes Stroebel. 2021. "Climate finance." *Annual Review of Financial Economics*, 13: 15–36.
Gorton, Gary, Andrew Metrick, Andrei Shleifer, and Daniel K Tarullo. 2010. "Regulating the Shadow Banking System." *Brookings Papers on Economic Activity*, 261–312.
Gourinchas, Pierre Olivier, and Philippe Martin. 2017. "Economics of Sovereign Debt, Bailouts and the Eurozone Crisis." *Working Paper.*
Green, Jerry, and Eytan Sheshinski. 1976. "Direct versus Indirect Remedies for Externalities." *Journal*

of Political Economy, 84(4, Part 1): 797–808.

Grochulski, Borys, and Yuzhe Zhang. 2019. "Optimal Liquidity Regulation with Shadow Banking." *Economic Theory*, 68(4): 967–1015.

Hachem, Kinda. 2018. "Shadow Banking in China." *Annual Review of Financial Economics*, 10(1): 287–308.

Hachem, Kinda, and Martin Kuncl. 2025. "The Prudential Toolkit with Shadow Banking." *Working Paper.*

Hachem, Kinda, and Zheng Song. 2021. "Liquidity Rules and Credit Booms." *Journal of Political Economy*, 129(10): 2721–2765.

Hendren, Nathaniel. 2016. "The Policy Elasticity." *Tax Policy and the Economy*, 30(1): 51–89.

Huang, Ji. 2018. "Banking and Shadow Banking." *Journal of Economic Theory*, 178: 124–152.

Irani, Rustom M, Rajkamal Iyer, Ralf R Meisenzahl, and Jose-Luis Peydro. 2021. "The Rise of Shadow Banking: Evidence from Capital Regulation." *The Review of Financial Studies*, 34(5).

Jensen, Michael C, and William H Meckling. 1976. "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." *Journal of Financial Economics*, 3(4): 305–360.

Jiménez, Gabriel, Steven Ongena, José-Luis Peydró, and Jesús Saurina. 2014. "Hazardous Times for Monetary Policy: What do Twenty-Three Million Bank Loans Say about the Effects of Monetary Policy on Credit Risk-Taking?" *Econometrica*, 82(2): 463–505.

Keister, Todd. 2016. "Bailouts and Financial Fragility." *The Review of Economic Studies*, 83(2): 704–736.

Korinek, Anton. 2017. "Currency Wars or Efficient Spillovers? A General Theory of International Policy Cooperation." *NBER Working Paper.*

Korinek, Anton, Juan A. Montecino, and Joseph E. Stiglitz. 2022. "Technological Innovation as Regulatory Arbitrage." *Working Paper.*

Lipsey, Richard G, and Kelvin Lancaster. 1956. "The General Theory of Second Best." *Review of Economic Studies*, 24(1): 11–32.

Lorenzoni, Guido. 2008. "Inefficient Credit Booms." *Review of Economic Studies*, 75(3): 809–833.

Martinez-Miera, David, and Rafael Repullo. 2019. "Markets, Banks, and Shadow Banks." *ECB Working Paper.*

Milgrom, Paul. 2006. "Multipliers and the LeChatelier principle." *Samuelsonian Economics and the Twenty-First Century*, 303–10.

Milgrom, Paul, and John Roberts. 1996. "The LeChatelier Principle." *American Economic Review*, 58(2): 173–179.

Moreira, Alan, and Alexi Savov. 2017. "The Macroeconomics of Shadow Banking." *The Journal of Finance*, 72(6): 2381–2432.

Myles, Gareth D. 1995. *Public Economics.* Cambridge University Press.

Oehmke, Martin, and Marcus M Opp. 2022. "Green Capital Requirements." *Working paper.*

Ordoñez, Guillermo. 2018. "Sustainable Shadow Banking." *American Economic Journal: Macroeconomics*, 10(1): 33–56.

Piazessi, Monika, Melina Papoutsi, and Martin Schneider. 2021. "How unconventional is green monetary policy?" Mimeo, Stanford University.

Pigou, Arthur Cecil. 1920. *The Economics of Welfare.* London, Macmillan and Co.

Plantin, Guillaume. 2015. "Shadow Banking and Bank Capital Regulation." *The Review of Financial Studies*, 28(1): 146–175.

Pozsar, Zoltan, Tobias Adrian, Adam Ashcraft, and Hayley Boesky. 2010. "Shadow Banking." *New York Fed Staff Report No. 458.*

Rola-Janicka, Magdalena, and Robin Döttling. 2022. "Too Levered for Pigou? A Model of Environmental and Financial Regulation."

Rothschild, Casey, and Florian Scheuer. 2016. "Optimal Taxation with Rent-Seeking." *Review of*

Economic Studies, 83(3): 1225–1262.

Saez, Emmanuel, and Stefanie Stantcheva. 2016. "Generalized Social Marginal Welfare Weights for Optimal Tax Theory." *American Economic Review*, 106(1): 24–45.

Salanié, Bernard. 2011. *The Economics of Taxation.* The MIT Press.

Samuelson, Paul Anthony. 1948. *Foundations of Economic Analysis.* Harvard University Press.

Sandmo, Agnar. 1975. "Optimal Taxation in the Presence of Externalities." *The Swedish Journal of Economics*, 77(1): 86–98.

Stein, Jeremy C. 2012. "Monetary Policy as Financial Stability Regulation." *The Quarterly Journal of Economics*, 127(1): 57–95.

Sunderam, Adi. 2015. "Money Creation and the Shadow Banking System." *The Review of Financial Studies*, 28(4): 939–977.

Tibshirani, Robert. 1996. "Regression shrinkage and selection via the lasso." *Journal of the Royal Statistical Society. Series B (Methodological)*, 267–288.

Tinbergen, Jan. 1952. *On the Theory of Economic Policy.* North-Holland Publishing Company, Amsterdam.

Werning, Ivan. 2012. "Lecture Notes on Corrective Taxation." *Unpublished.*

Wijkander, Hans. 1985. "Correcting Externalities through Taxes on/Subsidies to Related Goods." *Journal of Public Economics*, 28(1): 111–125.

Xiao, Kairong. 2020. "Monetary Transmission Through Shadow Banks." *The Review of Financial Studies*, 33(6): 2379–2420.

Online Appendix

Section A of this Online Appendix provides explicit definitions of the vectors and matrices used in the paper. Sections B, C, D provides proofs and derivations for Section 3, 4, 5 of the paper. Section E presents additional proofs and derivations.

A Matrix Definitions

Investor i's decisions, regulations, and marginal distortions can be expressed as $N \times 1$ vectors $\boldsymbol{x}^i$, $\boldsymbol{\tau}^i$, and $\boldsymbol{\delta}^i$, as follows:

$$\boldsymbol{x}^i = \begin{pmatrix} x_1^i \\ \vdots \\ x_n^i \\ \vdots \\ x_N^i \end{pmatrix}_{N\times 1}, \quad \boldsymbol{\tau}^i = \begin{pmatrix} \tau_1^i \\ \vdots \\ \tau_n^i \\ \vdots \\ \tau_N^i \end{pmatrix}_{N\times 1}, \quad \text{and} \quad \boldsymbol{\delta}^i = \begin{pmatrix} \delta_1^i \\ \vdots \\ \delta_n^i \\ \vdots \\ \delta_N^i \end{pmatrix}_{N\times 1}.$$

These can be collected for all agents in $IN \times 1$ stacked vectors, as follows:

$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}^1 \\ \vdots \\ \boldsymbol{x}^i \\ \vdots \\ \boldsymbol{x}^I \end{pmatrix}_{IN\times 1}, \quad \boldsymbol{\tau} = \begin{pmatrix} \boldsymbol{\tau}^1 \\ \vdots \\ \boldsymbol{\tau}^i \\ \vdots \\ \boldsymbol{\tau}^I \end{pmatrix}_{IN\times 1}, \quad \text{and} \quad \boldsymbol{\delta} = \begin{pmatrix} \boldsymbol{\delta}^1 \\ \vdots \\ \boldsymbol{\delta}^i \\ \vdots \\ \boldsymbol{\delta}^I \end{pmatrix}_{IN\times 1},$$

where the vectors $\bar{\boldsymbol{x}}^i$ and $\bar{\boldsymbol{x}}$ are defined analogously to $\boldsymbol{x}^i$ and $\boldsymbol{x}$.

The marginal welfare effects of varying all regulations, $\frac{dW}{d\boldsymbol{\tau}}$, are given by

$$\frac{dW}{d\boldsymbol{\tau}} = \begin{pmatrix} \frac{dW}{d\boldsymbol{\tau}^1} \\ \vdots \\ \frac{dW}{d\boldsymbol{\tau}^j} \\ \vdots \\ \frac{dW}{d\boldsymbol{\tau}^I} \end{pmatrix}_{IN\times 1}, \quad \text{where} \quad \frac{dW}{d\boldsymbol{\tau}^j} = \begin{pmatrix} \frac{dW}{d\tau_1^j} \\ \vdots \\ \frac{dW}{d\tau_n^j} \\ \vdots \\ \frac{dW}{d\tau_N^j} \end{pmatrix}_{N\times 1},$$

and where $\frac{dW}{d\tau_n^j}$ denotes the marginal welfare effect of varying the regulation associated with decision n by investor j. Note that

$$\frac{dW}{d\boldsymbol{\tau}^j} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}^j}\boldsymbol{\omega} = \sum_i \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\boldsymbol{\omega}^i = \sum_i \sum_n \frac{dx_n^i}{d\boldsymbol{\tau}^j}\left(\tau_n^i - \delta_n^i\right).$$

The Jacobian matrix of decisions $\boldsymbol{x}$ with respect to $\boldsymbol{\tau}$, of dimension $IN \times IN$, is given by

$$\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} = \begin{pmatrix} \frac{d\boldsymbol{x}^1}{d\boldsymbol{\tau}^1} & \cdots & \frac{d\boldsymbol{x}^I}{d\boldsymbol{\tau}^1} \\ \vdots & \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} & \vdots \\ \frac{d\boldsymbol{x}^1}{d\boldsymbol{\tau}^I} & \cdots & \frac{d\boldsymbol{x}^{\mathcal{I}}}{d\boldsymbol{\tau}^I} \end{pmatrix}_{IN\times IN}, \quad \text{where} \quad \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} = \begin{pmatrix} \frac{dx_1^i}{d\tau_1^j} & \cdots & \frac{dx_N^i}{d\tau_1^j} \\ \vdots & \frac{dx_n^i}{d\tau_{n'}^j} & \vdots \\ \frac{dx_1^i}{d\tau_N^j} & \cdots & \frac{dx_N^i}{d\tau_N^j} \end{pmatrix}_{N\times N}.$$

In particular, the Jacobian matrix $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$, of dimensions $R \times U$, can be written as

$$\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} = \begin{pmatrix} & \cdots & \\ \vdots & \frac{dx_n^i}{d\tau_{n'}^j} & \vdots \\ & \cdots & \end{pmatrix}_{R\times U},$$

where the decisions are such that (i,n) are imperfectly regulated and (j,n') are perfectly regulated. One can similarly define $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}$, $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}$, and $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$, with dimensions $U\times U$, $U\times R$, and $R\times R$ respectively, by switching the sets of coefficients. This allows us to express $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ when needed as

$$\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} = \begin{pmatrix} \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R} & \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} \\ \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U} & \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} \end{pmatrix}_{IN\times IN}.$$

We express the Jacobian of the constraints, $\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}}$, a matrix of dimension $IN\times M$, as follows

$$\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}} = \begin{pmatrix} \frac{d\Phi^1}{d\boldsymbol{\tau}^1} & \cdots & \frac{d\Phi^M}{d\boldsymbol{\tau}^1} \\ \vdots & \frac{d\Phi^m}{d\boldsymbol{\tau}^j} & \vdots \\ \frac{d\Phi^1}{d\boldsymbol{\tau}^I} & \cdots & \frac{d\Phi^M}{d\boldsymbol{\tau}^I} \end{pmatrix}_{IN\times M}, \quad \text{where} \quad \frac{d\Phi^m}{d\boldsymbol{\tau}^j} = \begin{pmatrix} \frac{d\Phi^m}{d\tau_1^j} \\ \vdots \\ \frac{d\Phi^m}{d\tau_N^j} \end{pmatrix}_{N\times 1},$$

and the vector of Lagrange multipliers associated with the constraints, $\boldsymbol{\mu}$, by

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_M \end{pmatrix}_{M\times 1}.$$

B Proofs and Derivations: Section 3

Investors' problem: The problem solved by investor i in Lagrangian form is

$$\max_{c_0^i, \{c_1^i(s)\}, \boldsymbol{x}^i} \mathcal{L}^i,$$

where $\mathcal{L}^i$ is given by

$$\begin{aligned} \mathcal{L}^i &= u^i\left(c_0^i, \left\{c_1^i(s)\right\}_{s\in S}, \left\{\overline{\boldsymbol{x}}^j\right\}_{j\in\mathcal{I}}\right) \\ &\quad - \lambda_0^i\left(c_0^i - n_0^i - Q^i\left(\boldsymbol{x}^i\right) + \Upsilon^i\left(\boldsymbol{x}^i\right) + \boldsymbol{\tau}^i\cdot\boldsymbol{x}^i - T^i\right) \\ &\quad - \int \lambda_1^i(s)\left(c_1^i(s) - n_1^i(s) - \rho_i\left(\boldsymbol{x}^i, s\right)\right) dF(s), \end{aligned}$$

where λ_0^i and $\lambda_1^i(s)$ denote the Lagrange multipliers that correspond to investor i's budget constraints.[22] The consumption optimality conditions imply that $\lambda_0^i = \frac{\partial u^i}{\partial c_0^i}$ and $\lambda_1^i(s)\,dF(s) = \frac{\partial u^i}{\partial c_1^i(s)}$. The optimality conditions for investor i are given by

$$-\lambda_0^i\left(-\frac{\partial Q^i\left(\boldsymbol{x}^i\right)}{\partial \boldsymbol{x}^i} + \frac{\partial \Upsilon^i\left(\boldsymbol{x}^i\right)}{\partial \boldsymbol{x}^i} + \boldsymbol{\tau}^i\right) + \int \lambda_1^i(s)\frac{\partial \rho_i\left(\boldsymbol{x}^i, s\right)}{\partial \boldsymbol{x}^i} dF(s) = 0,\ \forall i, \tag{OA1}$$

[22] Without loss of generality, we define the state s multipliers $\lambda_1^i(s)$ inside the expectation.

where Equation (OA1) can be expressed as

$$-\frac{\partial Q^{i}\left(\boldsymbol{x}^{i}\right)}{\partial \boldsymbol{x}^{i}}+\frac{\partial \Upsilon^{i}\left(\boldsymbol{x}^{i}\right)}{\partial \boldsymbol{x}^{i}}+\boldsymbol{\tau}^{i}=\int m^{i}\left(s\right)\frac{\partial \rho^{i}\left(\boldsymbol{x}^{i},s\right)}{\partial \boldsymbol{x}^{i}}dF\left(s\right),\ \forall i, \tag{OA2}$$

where $m^{i}\left(s\right)=\frac{\lambda_{1}^{i}(s)}{\lambda_{0}^{i}}$ denotes the stochastic discount factor of investor i.[23] These conditions are Euler equations for both financing and investment.

Formally, the $N\times 1$ vectors $\frac{\partial Q^{i}\left(\boldsymbol{x}^{i}\right)}{\partial \boldsymbol{x}^{i}}$, $\frac{\partial \Upsilon^{i}\left(\boldsymbol{x}^{i}\right)}{\partial \boldsymbol{x}^{i}}$, and $\boldsymbol{\tau}^{i}$ are given by:

$$\frac{\partial Q^{i}}{\partial \boldsymbol{x}^{i}}=\begin{pmatrix}\frac{\partial Q^{i}}{\partial x_{1}^{i}}\\ \vdots\\ \frac{\partial Q^{i}}{\partial x_{N}^{i}}\end{pmatrix},\quad \frac{\partial \Upsilon^{i}}{\partial \boldsymbol{x}^{i}}=\begin{pmatrix}\frac{\partial \Upsilon^{i}}{\partial x_{1}^{i}}\\ \vdots\\ \frac{\partial \Upsilon^{i}}{\partial x_{N}^{i}}\end{pmatrix},\quad \text{and}\quad \boldsymbol{\tau}^{i}=\begin{pmatrix}\tau_{1}^{i}\\ \vdots\\ \tau_{N}^{i}\end{pmatrix}.$$

Similarly, we define the $N\times 1$ vector $\int \lambda_{1}^{i}\left(s\right)\frac{\partial \rho_{i}\left(\boldsymbol{x}^{i},s\right)}{\partial \boldsymbol{x}^{i}}dF\left(s\right)$ as follows:

$$\int \lambda_{1}^{i}\left(s\right)\frac{\partial \rho_{i}\left(\boldsymbol{x}^{i},s\right)}{\partial \boldsymbol{x}^{i}}dF\left(s\right)=\begin{pmatrix}\int \lambda_{1}^{i}\left(s\right)\frac{\partial \rho_{i}\left(\boldsymbol{x}^{i},s\right)}{\partial x_{1}^{i}}dF\left(s\right)\\ \vdots\\ \int \lambda_{1}^{i}\left(s\right)\frac{\partial \rho_{i}\left(\boldsymbol{x}^{i},s\right)}{\partial x_{N}^{i}}dF\left(s\right)\end{pmatrix}.$$

Creditors' problem: The problem solved by creditors in Lagrangian form is

$$\max_{c_{0}^{C},\left\{c_{1}^{C}(s)\right\},\left\{h_{i}^{C}\right\}}\mathcal{L}^{C},$$

where $\mathcal{L}^{C}$ is given by

$$\begin{aligned}\mathcal{L}^{C}&=u^{C}\left(c_{0}^{C},\left\{c_{1}^{C}\left(s\right)\right\}_{s\in S},\left\{\overline{\boldsymbol{x}}^{j}\right\}_{j\in\mathcal{I}}\right)-\lambda_{0}^{C}\left(c_{0}^{C}-n_{0}^{C}+\sum_{i\in\mathcal{I}}h_{i}^{C}Q^{i}\left(\overline{\boldsymbol{x}}^{i}\right)\right)\\&-\int \lambda_{1}^{C}\left(s\right)\left(c_{1}^{C}\left(s\right)-n_{1}^{C}\left(s\right)-\sum_{i\in\mathcal{I}}h_{i}^{C}\rho_{i}^{C}\left(\overline{\boldsymbol{x}}^{i},s\right)\right)dF\left(s\right),\end{aligned}$$

where λ_{0}^{C} and $\lambda_{1}^{C}\left(s\right)$ denote the Lagrange multipliers that correspond to the creditors' budget constraints. The consumption optimality conditions imply that $\lambda_{0}^{C}=\frac{\partial u^{C}}{\partial c_{0}^{C}}$ and $\lambda_{1}^{C}\left(s\right)dF\left(s\right)=\frac{\partial u^{C}}{\partial c_{1}^{C}(s)}$. The optimality conditions for creditors regarding $\left\{h_{i}^{C}\right\}$ are

$$-\lambda_{0}^{C}Q^{i}\left(\boldsymbol{x}^{i}\right)+\int \lambda_{1}^{C}\left(s\right)\rho_{i}^{C}\left(\boldsymbol{x}^{i},s\right)dF\left(s\right)=0,\ \forall i, \tag{OA3}$$

where we use the fact that $\boldsymbol{x}^{i}=\overline{\boldsymbol{x}}^{i}$ in equilibrium. Equation (OA3) can be expressed as

$$Q^{i}\left(\boldsymbol{x}^{i}\right)=\int m^{C}\left(s\right)\rho_{i}^{C}\left(\boldsymbol{x}^{i},s\right)dF\left(s\right),\ \forall i, \tag{OA4}$$

[23] Note that a sufficient regularity condition for the second term of Equation (OA1) to be valid is that $\rho_{i}\left(\boldsymbol{x}^{i},s\right)$ is continuous. Otherwise, all results follow when the second term is $\frac{\partial}{\partial \boldsymbol{x}^{i}}\left[\int \lambda_{1}^{i}\left(s\right)\rho_{i}\left(\boldsymbol{x}^{i},s\right)dF\left(s\right)\right]$.

where we define $m^C(s) = \frac{\lambda_1^C(s)}{\lambda_0^C}$, characterizing the financing schedules $Q^i(\boldsymbol{x}^i)$ that investors face. Note that the stochastic discount factor $m^C(s)$ is an equilibrium object, which depends on the choices of all investors in the model and the policy.

Proof of Lemma 1a). (Marginal Welfare Effects of Regulation)

Proof. We initially characterize the $N \times 1$ vectors $\frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^j}$ and $\frac{\frac{dV^C}{d\boldsymbol{\tau}^j}}{\lambda_0^C}$, which correspond to the money-metric welfare changes of type i investors and creditors when $\boldsymbol{\tau}^j$ changes.

Investors: We express the financing schedules faced by investors as a function of the creditors' stochastic discount factor $m^C(s)$, which is in turn in equilibrium a function of the consumption of creditors in all dates and states. This allows us to separately account for any general equilibrium pecuniary effects. Formally, we represent the equilibrium financing schedules in Equation (OA4) for an investor i as follows:

$$Q^i\left(\boldsymbol{x}^i; m^C(s)\right) = \int m^C(s)\,\rho_i^C\left(\boldsymbol{x}^i, s\right) dF(s),$$

where we make explicit the dependence on $m^C(s)$. The money-metric change in indirect utility for investor i when varying the regulation that investor j faces is given by the following $N \times 1$ vector:

$$\begin{aligned}\frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^i} = {} & \frac{dc_0^i}{d\boldsymbol{\tau}^j}\overbrace{\left(\frac{\frac{\partial u^i}{\partial c_0^i} - \lambda_0^i}{\lambda_0^i}\right)}^{=0} + \int \frac{dc_1^i(s)}{d\boldsymbol{\tau}^j}\overbrace{\left(\frac{\frac{\frac{\partial u^i}{\partial c_1^i(s)}}{dF(s)} - \lambda_1^i(s)}{\lambda_0^i}\right)}^{=0} dF(s) \\ & + \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\overbrace{\left(\frac{\partial Q^i\left(\boldsymbol{x}^i; m^C(s)\right)}{\partial \boldsymbol{x}^i} - \frac{\partial \Upsilon^i\left(\boldsymbol{x}^i\right)}{\partial \boldsymbol{x}^i} - \boldsymbol{\tau}^i + \int m^i(s)\frac{\partial \rho_i\left(\boldsymbol{x}^i, s\right)}{\partial \boldsymbol{x}^i} dF(s)\right)}^{=0} \\ & + \frac{dT^i}{d\boldsymbol{\tau}^j} - \frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j}\boldsymbol{x}^i + \frac{\partial Q^i\left(\boldsymbol{x}^i; m^C(s)\right)}{\partial m^C(s)}\frac{dm^C(s)}{d\boldsymbol{\tau}^j} + \sum_{\ell \in \mathcal{I}} \frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j}\frac{1}{\lambda_0^i}\frac{\partial u^i}{\partial \overline{\boldsymbol{x}}^\ell},\end{aligned} \tag{OA5}$$

where the $N \times 1$ vectors $\frac{dT^i}{d\boldsymbol{\tau}^j}$ and $\boldsymbol{x}^j$ are given by

$$\frac{dT^i}{d\boldsymbol{\tau}^j} = \begin{pmatrix} \frac{dT^i}{d\tau_1^j} \\ \vdots \\ \frac{dT^i}{d\tau_N^j} \end{pmatrix} \quad \text{and} \quad \boldsymbol{x}^i = \begin{pmatrix} x_1^i \\ \vdots \\ x_n^i \\ \vdots \\ x_N^i \end{pmatrix},$$

and where the matrix $\frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j}$, of dimension $N \times N$, is given by

$$\frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j} = \begin{cases} I_N, & \text{if } i = j \\ 0_N, & \text{if } i \neq j, \end{cases}$$

which is either a N-dimensional identity matrix, I_N, when $i = j$, or a $N \times N$ matrix of zeros, 0_N, when $i \neq j$.

We also define the $N\times 1$ vector $\frac{\partial Q^i\left(\boldsymbol{x}^i;m^C(s)\right)}{\partial m^C(s)}\frac{dm^C(s)}{d\boldsymbol{\tau}^j}$ as

$$\frac{\partial Q^i\left(\boldsymbol{x}^i;m^C(s)\right)}{\partial m^C(s)}\frac{dm^C(s)}{d\boldsymbol{\tau}^j}=\begin{pmatrix}\int\frac{dm^C(s)}{d\tau_1^j}\rho_i^C\left(\boldsymbol{x}^i,s\right)dF(s)\\ \vdots\\ \int\frac{dm^C(s)}{d\tau_N^j}\rho_i^C\left(\boldsymbol{x}^i,s\right)dF(s)\end{pmatrix}.$$

Note that we use the fact that

$$\frac{dQ^i\left(\boldsymbol{x}^i;m^C(s)\right)}{d\boldsymbol{\tau}^j}=\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\frac{\partial Q^i}{\partial\boldsymbol{x}^i}+\frac{\partial Q^i}{\partial m^C(s)}\frac{dm^C(s)}{d\boldsymbol{\tau}^j},$$

as well as

$$\frac{d\left(\boldsymbol{\tau}^i\cdot\boldsymbol{x}^i-T^i\right)}{d\boldsymbol{\tau}^j}=\frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j}\boldsymbol{x}^i+\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\boldsymbol{\tau}^i-\frac{dT^i}{d\boldsymbol{\tau}^j}.$$

Note that $\frac{\partial u^i}{\partial\overline{\boldsymbol{x}}^\ell}$ denotes a $N\times 1$ gradient vector.

Creditors: In the case of creditors, we can express the $N\times 1$ vector $\frac{\frac{dV^C}{d\boldsymbol{\tau}^j}}{\lambda_0^C}$ as follows:

$$\begin{aligned}\frac{\frac{dV^C}{d\boldsymbol{\tau}^j}}{\lambda_0^C}&=\frac{dc_0^C}{d\boldsymbol{\tau}^j}\overbrace{\left(\frac{\frac{\partial u^C}{\partial c_0^C}-\lambda_0^C}{\lambda_0^C}\right)}^{=0}+\int\overbrace{\left(\frac{\frac{\frac{\partial u^C}{\partial c_1^C(s)}}{dF(s)}-\lambda_1^C(s)}{\lambda_0^C}\right)}^{=0}\frac{dc_1^C(s)}{d\boldsymbol{\tau}^j}dF(s)\\&\quad-\sum_{i\in\mathcal{I}}\frac{dh_i^C}{d\boldsymbol{\tau}^j}\overbrace{\left(Q^i\left(\boldsymbol{x}^i;m^C(s)\right)-\int m^C(s)\rho_i^C\left(\boldsymbol{x}^i,s\right)dF(s)\right)}^{=0}\\&\quad-\sum_{i\in\mathcal{I}}h_i^C\left(\frac{dQ^i\left(\boldsymbol{x}^i;m^C(s)\right)}{d\boldsymbol{\tau}^j}-\int m^C(s)\frac{d\rho_i^C\left(\boldsymbol{x}^i,s\right)}{d\boldsymbol{\tau}^j}dF(s)\right)+\sum_{\ell\in\mathcal{I}}\frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j}\frac{1}{\lambda_0^C}\frac{\partial u^C}{\partial\overline{\boldsymbol{x}}^\ell}\\&=-\sum_{i\in\mathcal{I}}\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\underbrace{\left(\frac{\partial Q^i}{\partial\boldsymbol{x}^i}-\int m^C(s)\frac{\partial\rho_i^C\left(\boldsymbol{x}^i,s\right)}{\partial\boldsymbol{x}^i}dF(s)\right)}_{=0}-\sum_{i\in\mathcal{I}}\frac{\partial Q^i}{\partial m^C(s)}\frac{dm^C(s)}{d\boldsymbol{\tau}^j}+\sum_{\ell\in\mathcal{I}}\frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j}\frac{1}{\lambda_0^C}\frac{\partial u^C}{\partial\overline{\boldsymbol{x}}^\ell},\end{aligned}\tag{OA6}$$

where in the second equality we use the fact that $h_i^C=1$ and the fact that $\frac{d\rho_i^C\left(\boldsymbol{x}^i,s\right)}{d\boldsymbol{\tau}^j}=\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\frac{\partial\rho_i^C}{\partial\boldsymbol{x}^i}$, and where the $N\times 1$ vector $\frac{\partial\rho_i^C}{\partial\boldsymbol{x}^i}$ is given by

$$\frac{\partial\rho_i^C}{\partial\boldsymbol{x}^i}=\begin{pmatrix}\frac{\partial\rho_i^C}{\partial x_1^i}\\ \vdots\\ \frac{\partial\rho_i^C}{\partial x_N^i}\end{pmatrix}.$$

Note that $\frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j}$ is defined above and $\frac{\partial u^C}{\partial\overline{\boldsymbol{x}}^\ell}$ denotes a $N\times 1$ gradient vector.

Aggregation: First, we can express the sum among investors of the change in money-metric indirect utilities as follows:

$$\sum_{i\in\mathcal{I}}\frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^i}=\sum_{i\in\mathcal{I}}\left(\frac{dT^i}{d\boldsymbol{\tau}^j}-\frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j}\boldsymbol{x}^i\right)+\sum_{i\in\mathcal{I}}\frac{\partial Q^i}{\partial m^C(s)}\frac{dm^C(s)}{d\boldsymbol{\tau}^j}+\sum_{i\in\mathcal{I}}\sum_{\ell\in\mathcal{I}}\frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j}\frac{1}{\lambda_0^i}\frac{\partial u^i}{\partial\overline{\boldsymbol{x}}^\ell}$$
$$=\sum_{i\in\mathcal{I}}\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\left(\boldsymbol{\tau}^i+\sum_{\ell\in\mathcal{I}}\frac{1}{\lambda_0^\ell}\frac{\partial u^\ell}{\partial\overline{\boldsymbol{x}}^i}\right)+\sum_{i\in\mathcal{I}}\frac{\partial Q^i}{\partial m^C(s)}\frac{dm^C(s)}{d\boldsymbol{\tau}^j},$$

where we use the fact that

$$\sum_{i\in\mathcal{I}}\left(\frac{dT^i}{d\boldsymbol{\tau}^j}-\frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j}\boldsymbol{x}^i\right)=\sum_{i\in\mathcal{I}}\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\boldsymbol{\tau}^i,$$

as well as the following identity:

$$\sum_{i\in\mathcal{I}}\sum_{\ell\in\mathcal{I}}\frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j}\frac{1}{\lambda_0^i}\frac{\partial u^i}{\partial\overline{\boldsymbol{x}}^\ell}=\sum_{i\in\mathcal{I}}\sum_{\ell\in\mathcal{I}}\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\frac{1}{\lambda_0^\ell}\frac{\partial u^\ell}{\partial\overline{\boldsymbol{x}}^i}.$$

Therefore, we can express $\frac{dW}{d\boldsymbol{\tau}^j}$ as follows:

$$\frac{dW}{d\boldsymbol{\tau}^j}=\sum_{i\in\mathcal{I}}\frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^i}+\frac{\frac{dV^C}{d\boldsymbol{\tau}^j}}{\lambda_0^C}$$
$$=\sum_{i\in\mathcal{I}}\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\left(\boldsymbol{\tau}^i+\sum_{\ell\in\mathcal{I}}\frac{1}{\lambda_0^\ell}\frac{\partial u^\ell}{\partial\overline{\boldsymbol{x}}^i}\right)+\sum_{i\in\mathcal{I}}\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\frac{1}{\lambda_0^C}\frac{\partial u^C}{\partial\overline{\boldsymbol{x}}^i}$$
$$=\sum_{i\in\mathcal{I}}\frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\left(\boldsymbol{\tau}^i-\underbrace{\left(-\left(\sum_{\ell\in\mathcal{I}}\frac{1}{\lambda_0^\ell}\frac{\partial u^\ell}{\partial\overline{\boldsymbol{x}}^i}+\frac{1}{\lambda_0^C}\frac{\partial u^C}{\partial\overline{\boldsymbol{x}}^i}\right)\right)}_{=\boldsymbol{\delta}^i}\right).$$

Hence, by switching indexes and stacking we find that

$$\frac{dW}{d\boldsymbol{\tau}}=\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}(\boldsymbol{\tau}-\boldsymbol{\delta})=\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}\boldsymbol{\omega},$$

as in Equation (10) in the text. □

Proof of Lemma 1b). (First-Best Policy/Pigouvian Principle)

Proof. The optimal first-best regulation is characterized by

$$\frac{dW}{d\boldsymbol{\tau}}=\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}\boldsymbol{\omega}=\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}(\boldsymbol{\tau}-\boldsymbol{\delta})=0,$$

which defines a system of homogeneous linear equations in $\boldsymbol{\omega}$. If the matrix of policy elasticities $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ is invertible (i.e., has full rank), the only solution to this system is the trivial solution, in which $\boldsymbol{\omega}=0$ and $\boldsymbol{\tau}^\star=\boldsymbol{\delta}$. □

Proof of Proposition 1. (Second-Best Policy: Perfectly Regulated Decisions)

Proof. At the second-best optimum, it must be that the marginal welfare effects of adjusting the regulation of perfectly regulated decisions (those for which the planning constraints do not bind), satisfy $\frac{dW}{d\boldsymbol{\tau}^R}=0$.

Leveraging Lemma 1a), we can express these optimality conditions as

$$\frac{dW}{d\boldsymbol{\tau}^R} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau} - \boldsymbol{\delta}\right) = \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) + \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^R - \boldsymbol{\delta}^R\right) = 0.$$

Assuming that the matrix $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ is invertible, we rearrange this expression as follows:

$$\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^R - \boldsymbol{\delta}^R\right) = -\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) \iff \boldsymbol{\tau}^R = \boldsymbol{\delta}^R - \left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right),$$

which corresponds to Equation (12) in the text. □

Proof of Proposition 2. (Second-Best Policy: Imperfectly Regulated Decisions)

Proof. Leveraging Lemma 1a), we can express the marginal welfare effects of adjusting the regulation of imperfectly regulated decisions (those for which the planning constraints bind) as

$$\frac{dW}{d\boldsymbol{\tau}^U} = \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) + \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\left(\boldsymbol{\tau}^R - \boldsymbol{\delta}^R\right).$$

So defining the vector of Lagrange multipliers associated with the regulatory constraints by $\boldsymbol{\mu}$, the optimality conditions for such decisions are given by $\frac{dW}{d\boldsymbol{\tau}^U} = \frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}^U}\boldsymbol{\mu}$, along with $\boldsymbol{\Phi}\left(\boldsymbol{\tau}^U\right) = 0$ when $\boldsymbol{\Phi}\left(\cdot\right)$ captures constraints.

From Proposition 1, we have that

$$\boldsymbol{\tau}^R - \boldsymbol{\delta}^R = \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right),$$

so combining the last two equations we obtain

$$\begin{aligned}
\frac{dW}{d\boldsymbol{\tau}^U} &= \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) - \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) \\
&= \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \underbrace{\left(\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right)^{-1}\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}}_{\equiv \boldsymbol{L}}\right)\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) = \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \boldsymbol{L}\right)\boldsymbol{\omega}^U,
\end{aligned}$$

which corresponds to Equation (14) in the text. □

C Proofs and Derivations: Section 4

Since creditors are risk-neutral, they must be indifferent between all quantities of debt purchased in equilibrium. Hence, the valuation of debt *per unit of capital* in equilibrium is

$$Q^i\left(b^i,\theta^i\right)=\beta^C\left(\int_{s^\star(b^i,\theta^i)}^{\bar{s}} b^i dF\left(s\right)+\phi\int_{\underline{s}}^{s^\star(b^i,\theta^i)}\left(d_1\left(s\right)\theta^i+d_2\left(s\right)\left(1-\theta^i\right)+t\left(b^i,\theta^i,s\right)\right)dF\left(s\right).\right)$$

Substituting the valuation of debt and the budget constraints into investors' objective function, and ignoring exogenous endowments, we obtain the simplified version of their maximization problem in Equation (19).

Proof of Proposition 3 [Financial Regulation With Environmental Externalities]: Adding the utilities of investors and creditors, imposing market clearing, and ignoring exogenous environments, we find that maximizing welfare is equivalent to maximizing

$$W\left(b^i,\theta^i,k^i\right)=\left(M\left(b^i,\theta^i\right)-\Omega\left(\theta^i\right)\right)k^i-\Upsilon\left(k^i\right)-\beta^C\left(1+\kappa\right)\int_{\underline{s}}^{\bar{s}} t\left(b^i,\theta^i,s\right)k^i dF\left(s\right)-\Psi\left(\theta^i\right)k^i.$$

We can now write $b^i\left(\bar{b},\varphi\right)$, $\theta^i\left(\bar{b},\varphi\right)$ and $k^i\left(\bar{b},\varphi\right)$ for optimal choices as a function of regulatory parameters, and totally differentiate the welfare function with respect to $\bar{b}$ to obtain

$$\begin{aligned}\frac{dW}{d\bar{b}}&=k^i\left(\frac{\partial M}{\partial b^i}-\delta_b\right)\frac{db^i}{d\bar{b}}+k^i\left(\frac{\partial M}{\partial\theta^i}-\Omega'\left(\theta^i\right)-\delta_\theta\right)\frac{d\theta^i}{d\bar{b}}-\delta_k\frac{dk^i}{d\bar{b}}\\&=k^i\left(\tau_b-\delta_b\right)\frac{db^i}{d\bar{b}}+k^i\left(\tau_\theta-\delta_\theta\right)\frac{d\theta^i}{d\bar{b}}-\delta_k\frac{dk^i}{d\bar{b}},\end{aligned}$$

where we have substituted the definitions of $\{\delta_b,\delta_\theta,\delta_k\}$ from Equations (24) through (26), as well as the first-order conditions (21), (22) and (23). Similarly, we differentiate with respect to φ to obtain

$$\begin{aligned}\frac{dW}{d\varphi}&=k^i\left(\frac{\partial M}{\partial b^i}-\delta_b\right)\frac{db^i}{d\varphi}+k^i\left(\frac{\partial M}{\partial\theta^i}-\Omega'\left(\theta^i\right)-\delta_\theta\right)\frac{d\theta^i}{d\varphi}-\delta_k\frac{dk^i}{d\varphi}\\&=k^i\left(\tau_b-\delta_b\right)\frac{db^i}{d\varphi}+k^i\left(\tau_\theta-\delta_\theta\right)\frac{d\theta^i}{d\varphi}-\delta\frac{dk^i}{d\varphi},\end{aligned}$$

as stated in Proposition 3, part a). Part b) of the proposition follows by solving for τ_b and τ_θ in the system $\frac{dW}{d\bar{b}}=0$, $\frac{dW}{d\varphi}=0$.

Characterization of complementarities: Investors' first-order condition for k^i can be written as

$$J\left(\bar{b},\varphi\right)=\Upsilon'\left(k^i\right), \tag{OA7}$$

where

$$J\left(\bar{b},\varphi\right)=\max_{b^i,\theta^i}\left\{M\left(b^i,\theta^i\right)-\Omega\left(\theta^i\right)\text{ subject to }b^i+\varphi\theta^i\leq\bar{b}\right\}. \tag{OA8}$$

By the envelope theorem, we have

$$\begin{aligned}\frac{\partial J}{\partial\bar{b}}&=\upsilon\geq 0\\\frac{\partial J}{\partial\varphi}&=-\upsilon\theta^i\leq 0,\end{aligned}$$

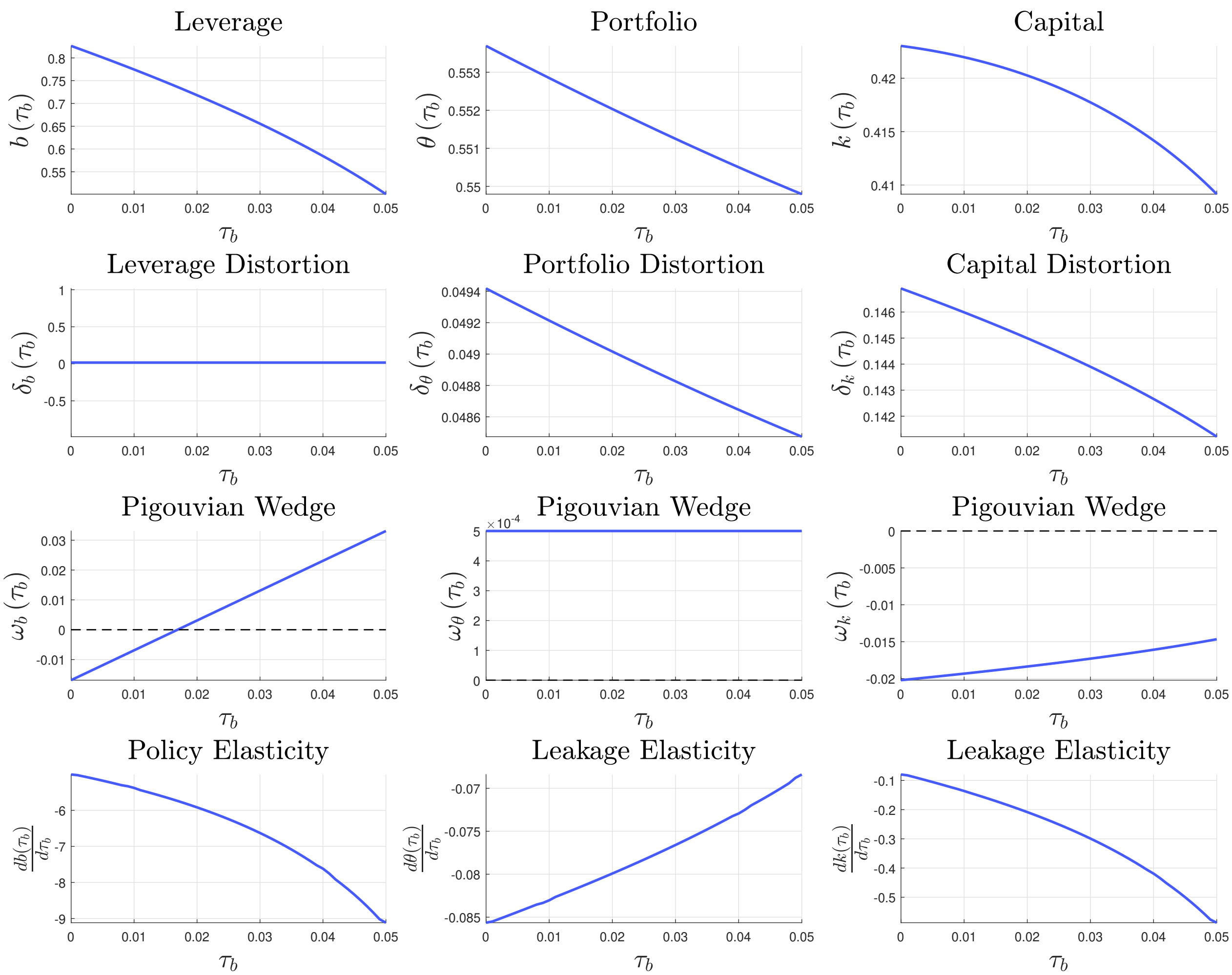

Figure OA-1: Financial Regulation with Environmental Externalities: Second-Best Comparative Statics, Leverage (τ_b)

Note: Figure OA-1 illustrates relevant comparative statics of our application on financial regulation with environmental externalities. In particular, we show how different variables vary with different values of τ_b, when $\tau_k = 0$ and when τ_θ is set at the second-best level (previously computed). The top row show equilibrium leverage b^i, portfolio allocations θ^i, and capital k^i. The second row shows leverage, portfolio, and capital distortions, defined in Equations (24) through (26), while the third row shows the associated Pigouvian wedges. The bottom row shows the policy elasticity $\frac{db}{d\tau_b}$ and the two leakage elasticities, $\frac{d\theta}{d\tau_b}$ and $\frac{dk}{d\tau_b}$ (since in this figure we are keeping τ_θ predetermined). The parameters used are described in Figure 1.

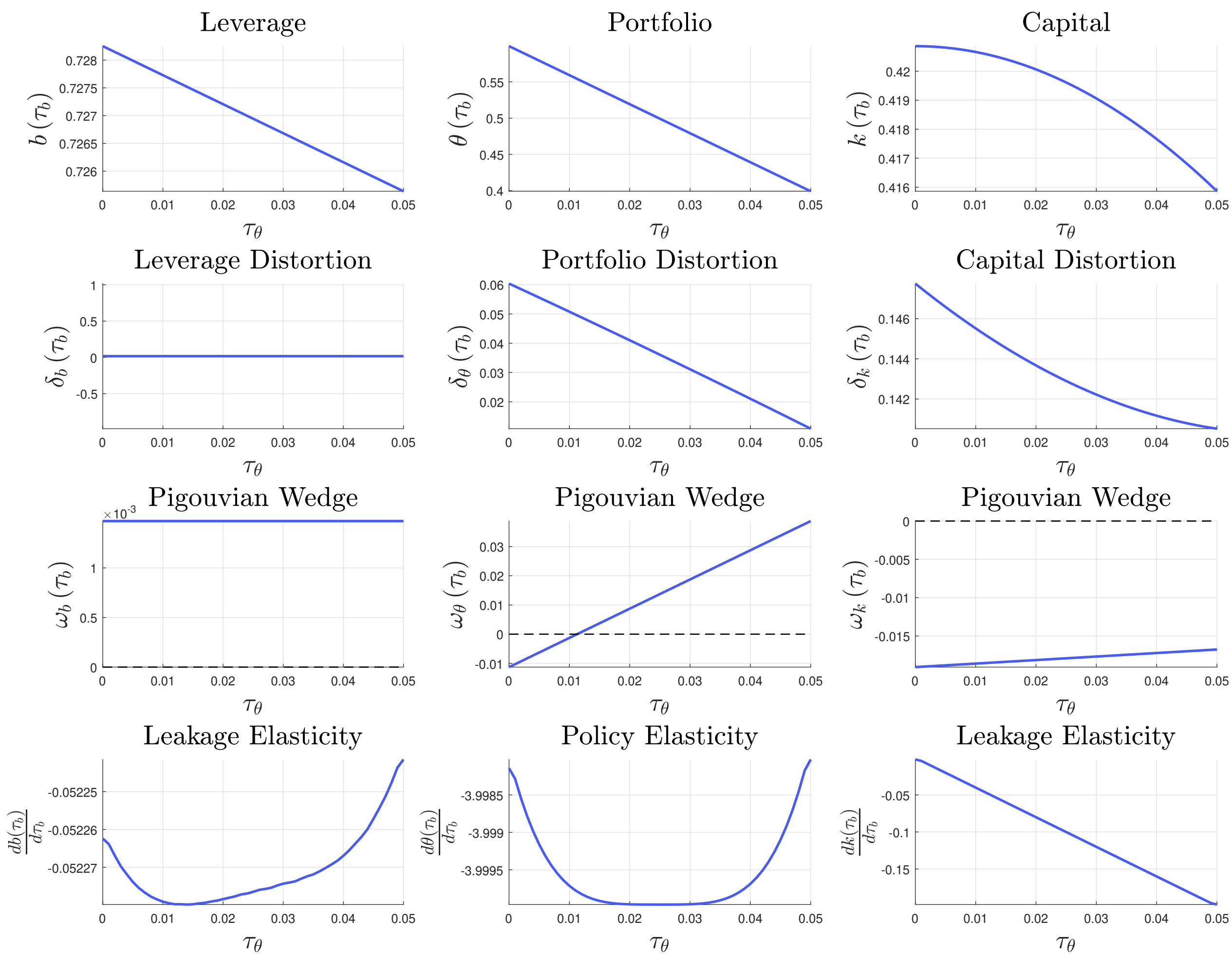

Figure OA-2: Financial Regulation with Environmental Externalities: Second-Best Comparative Statics, Risk Weights (τ_θ)

Note: Figure OA-2 illustrates relevant comparative statics of our application on regulation with environmental externalities. In particular, we show how different variables vary with different values of τ_b, when $\tau_k = 0$ and when τ_θ is set at the second-best level (previously computed). The top row show equilibrium leverage b^i, portfolio allocations θ^i, and capital k^i. The second row shows leverage, portfolio, and capital distortions, defined in Equations (24) through (26), while the third row shows the associated Pigouvian wedges. The bottom row shows the policy elasticity $\frac{db}{d\tau_b}$ and the two leakage elasticities, $\frac{d\theta}{d\tau_b}$ and $\frac{dk}{d\tau_b}$ (since in this figure we are keeping τ_θ predetermined). The parameters used are described in Figure 1.

where υ denotes the Lagrange multiplier on the constraint in (OA8). Totally differentiating (OA7), we now obtain

$$\frac{\partial J}{\partial \bar{b}} = \frac{dk^i}{d\bar{b}} \Upsilon'' (k^\star)$$
$$\frac{\partial J}{\partial \varphi} = \frac{dk^i}{d\varphi} \Upsilon'' (k^\star),$$

so that the convexity of $\Upsilon(\cdot)$ immediately implies

$$\frac{dk^i}{d\bar{b}} \geq 0, \frac{dk^i}{d\varphi} \leq 0,$$

as claimed in the text.

Further simulation results: Given our functional form assumptions for the simulation, note that we can express the default threshold $s^\star\left(b^i, \theta^i\right)$ as

$$s^\star\left(b^i, \theta^i\right) = \frac{b^i - \left(\alpha_0^i + \alpha_b^i b^i + \alpha_\theta^i \theta^i\right)}{d_1 \theta^i + d_2\left(1 - \theta^i\right) - \alpha_s^i}.$$

Relatedly, note that the marginal distortions in Equations (24) through (26) correspond then to

$$\delta_b = \underbrace{\beta^C (1+\kappa)\, \alpha_b}_{\equiv \chi_b}$$
$$\delta_\theta = \underbrace{\beta^C (1+\kappa)\, \alpha_\theta}_{\equiv \chi_\theta} + \underbrace{\frac{\partial \Psi\left(\theta^i\right)}{\partial \theta^i}}_{\equiv \psi_\theta}$$
$$\delta_k = \underbrace{\beta^C (1+\kappa) \left(\alpha_0^i + \alpha_b^i b^i + \alpha_\theta^i \theta^i - \alpha_s^i \int_{\underline{s}}^{\bar{s}} s dF(s)\right)}_{\equiv \chi_k} + \underbrace{\Psi\left(\theta^i\right)}_{\equiv \psi_k}.$$

Also, note that

$$\Omega'\left(\theta^i\right) = z^\eta \left(\Omega(\theta)\right)^{1-\eta} \left(a\left(\theta^i\right)^{\eta-1} - (1-a)\left(1-\theta^i\right)^{\eta-1}\right),$$

and similarly for $\Psi'\left(\theta^i\right) = \frac{\partial \Psi\left(\theta^i\right)}{\partial \theta^i}$.

D Proofs and Derivations: Section 5

D.1 Application 1

Default and repayments: Investor i optimally defaults at date 1 if $v^i s + t^i\left(b^i, s\right) - b^i < 0$.[24] Assuming that $v^i + \frac{\partial t^i\left(b^i, s\right)}{\partial s} > 0$, there exists a unique threshold $s^{i\star}\left(b^i\right)$ such that default occurs if and only if $s < s^{i\star}\left(b^i\right)$. Therefore, the definition of the repayment eventually received by creditors, $\mathcal{P}^i\left(b^i, s\right)$, is

$$\mathcal{P}^i\left(b^i, s\right) = \begin{cases} \phi^i v^i s + t^i\left(b^i, s\right) & s \in \left[\underline{s}, s^{i*}\left(b^i\right)\right) \\ b^i & s \in \left[s^{i*}\left(b^i\right), \overline{s}\right]. \end{cases}$$

In our simulation, we let $t^i\left(b^i, s\right) = \alpha_0^i - \alpha_s^i s + \alpha_b^i b^i$, with $\alpha_s^i < v^i$, so that we can solve explicitly for the default threshold

$$s^{i\star}\left(b^i\right) = \left(\frac{1-\alpha_b^i}{v^i - \alpha_s^i}\right) b^i - \frac{1}{v^i - \alpha_s^i}\alpha_0^i.$$

We further assume that creditors have constant relative risk aversion with coefficient γ.

Creditors' optimal choices and asset pricing: We conjecture and verify that the price $Q^i\left(b^i; m^C(s)\right)$ of investors' debt is a function of b^i and creditors' stochastic discount factor $m^C(s) = \beta^C \frac{u'\left(c_1^C(s)\right)}{u'\left(c_0^C\right)}$. Substituting creditors' budget constraints into their objective, we obtain the simplified version of their maximization problem:

$$\begin{aligned} V^C\left(b^i, m^C(s)\right) = \max_{\{h^i\}_{i\in\mathcal{I}}} u\left(n_0^C - \sum_{i\in\mathcal{I}} h^i Q^i\left(b^i; m^C(s)\right)\right) \\ + \beta^C \int u\left(n_1^C(s) + \sum_{i\in\mathcal{I}} h^i \mathcal{P}^i\left(b^i, s\right) - (1+\kappa)\sum_{i\in\mathcal{I}} t^i\left(b^i, s\right)\right) dF(s), \end{aligned}$$

where $V^C(\cdot)$ denotes creditors' indirect utility as a function of investors' debt choice and market prices. The first-order conditions for this problem, combined with market clearing ($h^i = 1$), yield the following debt-pricing equation:

$$Q^i\left(b^i; m^C(s)\right) = \int_{\underline{s}}^{s^{i\star}\left(b^i\right)} m^C(s)\left(\phi^i v^i s + t^i\left(b^i, s\right)\right) dF(s) + \int_{s^{i\star}\left(b^i\right)}^{\overline{s}} m^C(s)\, b^i dF(s).$$

Note that the stochastic discount factor in equilibrium must satisfy the fixed-point equation

$$m^C(s) = \beta^C \frac{u'\left(n_1^C(s) + \sum_{i\in\mathcal{I}} \mathcal{P}^i\left(b^i, s\right) - (1+\kappa)\sum_{i\in\mathcal{I}} t^i\left(b^i, s\right)\right)}{u'\left(n_0^C - \sum_{i\in\mathcal{I}} Q^i\left(b^i; m^C(s)\right)\right)}.$$

Investors' optimal choices: Substituting investors' budget constraints into their objective, and ignoring exogenous endowments, we obtain the simplified version of their maximization problem:

[24] It is straightforward to make bailouts depend on the decisions of all investors, as in, e.g., Farhi and Tirole (2012).

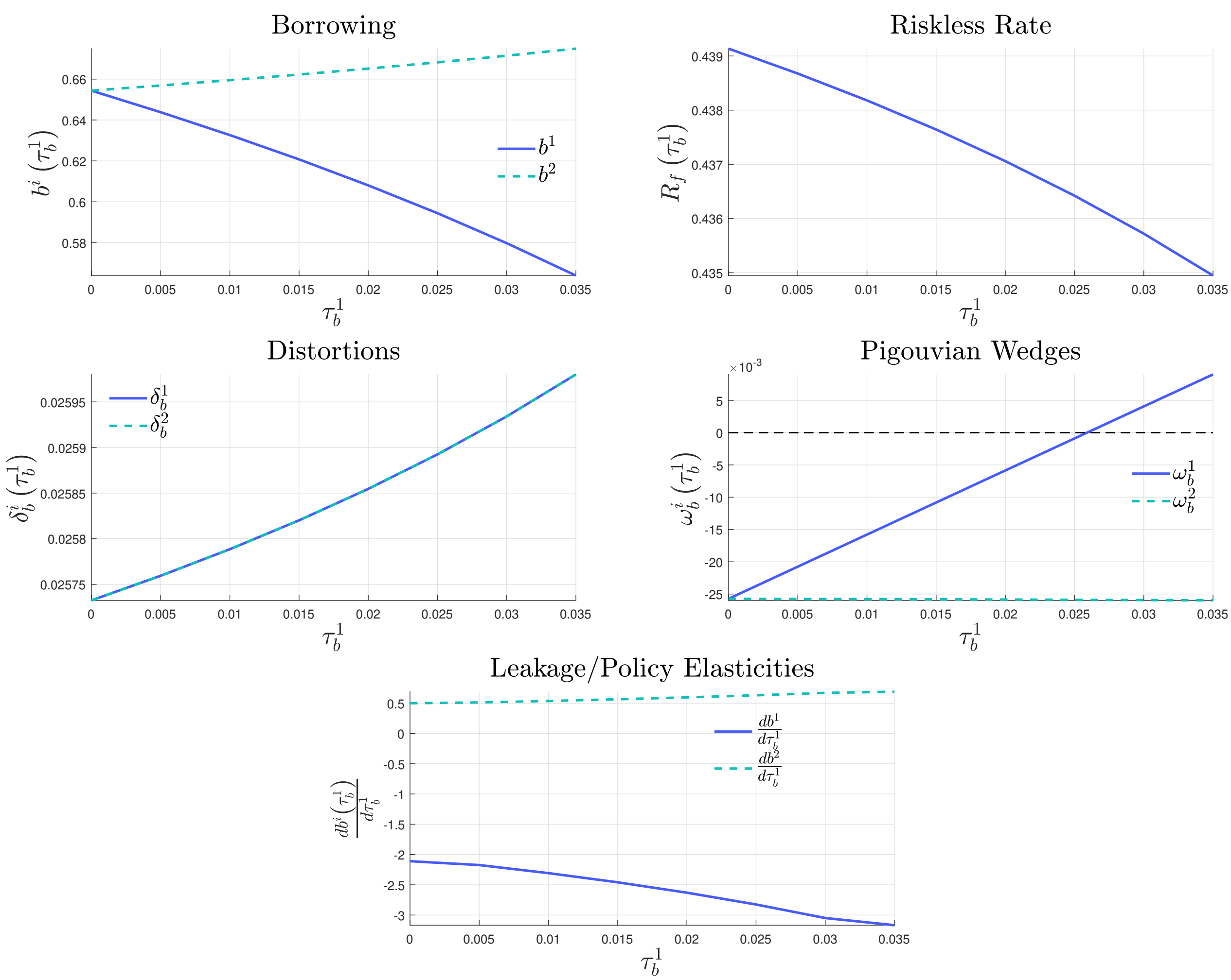

Figure OA-3: Application 1: Second-Best Comparative Statics

Note: Figure OA-3 illustrates relevant comparative statics of Application 1 for different values of τ_b^1, when $\tau_b^2 = 0$. The top left plot shows equilibrium borrowing b^i for both types of investors. The top right plot shows the equilibrium creditors' riskless rate, defined on page 31. The middle left plot shows the distortion associated with the borrowing choice of each investor, δ_b^1 and δ_b^2, defined in Equation (OA9) — note that the distortions move inversely with changes in the riskless rate R^f and quantitatively the changes are small. The middle right plot shows the Pigouvian wedge associated with the borrowing decision of each investor, ω_b^1 and ω_b^2. The bottom plot shows the policy elasticity $\frac{db^1}{d\tau_b^1}$ and the critical leakage elasticity $\frac{db^2}{d\tau_b^1} > 0$. The parameters used are described in Figure 2.

$$V^i\left(\tau_b^i, T^i, m^C(s)\right) = \max_{b^i} \beta^i \int_{s^{i\star}(b^i)}^{\overline{s}} \left(v^i s + t^i\left(b^i, s\right) - b^i\right) dF(s) \\ + Q^i\left(b^i; m^C(s)\right) - \tau_b^i b^i + T^i,$$

where $V^i(\cdot)$ denotes investors' indirect utility as a function of regulation and market prices. The first-order condition determining the optimal b^i is

$$-\beta^i \int_{s^{i\star}(b^i)}^{\overline{s}} \left(1 - \frac{\partial t^i}{\partial b}\left(b^i, s\right)\right) dF(s) + \frac{\partial Q^i\left(b^i; m^C(s)\right)}{\partial b^i} = \tau_b^i,$$

where

$$\frac{\partial Q^i\left(b^i; m^C(s)\right)}{\partial b^i} = \int_{s^{i\star}(b^i)}^{\overline{s}} m^C(s)\, dF(s) + \int_{\underline{s}}^{s^{i\star}(b^i)} \frac{\partial t^i}{\partial b}\left(b^i, s\right) m^C(s)\, dF(s) \\ - (1-\phi)\, m^C\left(s^{i\star}\left(b^i\right)\right) v^i s^{i\star}\left(b^i\right) f\left(s^{i\star}\left(b^i\right)\right).$$

Marginal welfare effects: The money-metric marginal welfare effects of changing the regulation τ_b^j of investor type $j \in \{1, 2\}$ are given by

$$\frac{dW}{d\tau_b^j} = \frac{1}{\lambda_0^C}\frac{dV^C}{d\tau_b^j} + \sum_{i\in\mathcal{I}} \frac{dV^i}{d\tau_b^j},$$

where $\lambda_0^C = u'\left(c_0^C\right)$, since $\lambda_0^i = 1$. Using an envelope argument parallel to our general results, we obtain, abstracting from pecuniary effects that cancel after aggregating,

$$\frac{dV^C}{d\tau_b^j} = -(1+\kappa)\,\beta^C \int u'\left(c_1(s)\right) \sum_{i\in\mathcal{I}} \frac{\partial t^i\left(b^i, s\right)}{\partial b^i}\frac{db^i}{d\tau_b^j} dF(s),$$

and

$$\frac{dV^i}{d\tau_b^j} = \tau_b^i \frac{db^i}{d\tau_b^j},$$

where we have used the assumption that $T^i = \tau_b^i b^i$. Thus, we obtain

$$\frac{dW}{d\tau_b^j} = \sum_{i\in\mathcal{I}} \frac{db^i}{d\tau_b^j}\left(\tau_b^i - \underbrace{(1+\kappa)\int m^C(s)\frac{\partial t^i\left(b^i, s\right)}{\partial b^i} dF(s)}_{=\delta_b^i}\right). \tag{OA9}$$

It follows that the first-best policy must satisfy $\tau_b^i = \delta_b^i$, $i \in \{1, 2\}$.

Proof of Proposition 4 [Shadow Banking/Unregulated Investors]:

Proof. The proposition follows directly by evaluating Equation (OA9) in the case where the planner is forced to set $\tau_b^2 \equiv 0$. □

Further simulation results: Figure OA-3 illustrates comparative statics of the model in the context of the second-best policy, in which $\tau_b^2 = 0$.

D.2 Application 2

Default and repayments: At date 1, investors optimally decide to default when $s < b^i$, and to repay otherwise. Therefore, the definition of the repayment eventually received by creditors per unit of capital k^i, $\mathcal{P}^i\left(b^i, s\right)$, is

$$\mathcal{P}^i\left(b^i, s\right) = \begin{cases} \phi^i s & s \in \left[\underline{s}, b^i\right) \\ b^i & s \in \left[b^i, \overline{s}\right]. \end{cases}$$

Creditors' optimal choices and asset pricing: Since creditors are risk-neutral, they must be indifferent between all quantities of debt purchase in equilibrium. Hence, the valuation of debt *per unit of capital* in equilibrium satisfies

$$Q^i\left(b^i\right) = \beta^C \left(\int_{b^i}^{\overline{s}} b^i dF^C(s) + \phi \int_{\underline{s}}^{b^i} s dF^C(s) \right).$$

Investors' optimal choices: Substituting the valuation of debt and the budget constraints into investors' objective function, and ignoring exogenous endowments, we obtain the simplified version of their maximization problem:

$$\max_{b^i, k^i} M\left(b^i\right) k^i - \Upsilon\left(k^i\right) - \tau_b^i b^i k^i - \tau_k^i k^i + T^i,$$

where $M\left(b^i\right)$ is given by

$$M\left(b^i\right) = \beta^i \int_{b^i}^{\overline{s}} \left(s - b^i\right) dF^i(s) + Q^i\left(b^i\right).$$

We assume that all corrective taxes/subsidies are reimbursed to investors with $T^i = \tau_b^i b^i k^i + \tau_k^i k^i$. The first-order conditions in this problem, which yield demand functions for credit and investment, are given by the solution to

$$\frac{dM\left(b^i\right)}{db^i} - \tau_b^i = 0 \tag{OA10}$$

$$M\left(b^i\right) - \Upsilon'\left(k^i\right) - \tau_k^i = 0, \tag{OA11}$$

where

$$\frac{dM\left(b^i\right)}{db^i} = \beta^C \int_{b^i}^{\overline{s}} dF^C(s) - \beta^i \int_{b^i}^{\overline{s}} dF^i(s) - (1 - \phi)\beta^C b^i f^C\left(b^i\right).$$

Assuming that $0 < \beta^i < \beta^C \leq 1$ and that ϕ is not too small guarantees an interior solution for leverage. Note that the equilibrium value of b^i is independent of k^i, and consequently of τ_k^i. In our simulation, we assume that investment adjustment costs are quadratic, i.e., $\Upsilon\left(k^i\right) = k^i + \frac{a}{2}\left(k^i\right)^2$, in which case Equation (OA11) takes the form

$$k^i = \frac{1}{a}\left(M\left(b^i\right) - 1 - \tau_k^i\right).$$

Marginal welfare effects: Social welfare for a planner who computes welfare using beliefs $F^{i,P}$ and $F^{C,P}$ is given by

$$W = M^P\left(b^i\right) k^i - \Upsilon\left(k^i\right),$$

where $M^P\left(b^i\right)$ denotes the present value of payoffs under the planner's beliefs

$$M^P\left(b^i\right)=\beta^i\int_{b^i}^{\overline{s}}\left(s-b^i\right)dF^{i,P}\left(s\right)+\beta^C\left(\int_{b^i}^{\overline{s}}b^i dF^{C,P}\left(s\right)+\phi\int_{\underline{s}}^{b^i}sdF^{C,P}\left(s\right)\right).$$

The marginal welfare effects of varying τ_b^i, after differentiating and substituting investors' first-order conditions, can be written as

$$\begin{aligned}\frac{dW}{d\tau_b^i}&=\frac{dM^P\left(b^i\right)}{db^i}\frac{db^i}{d\tau_b^i}+\left(M^P\left(b^i\right)-\Upsilon'\left(k^i\right)\right)\frac{dk^i}{d\tau_b^i}\\&=\left(\tau_b^i-\underbrace{\left(\frac{dM\left(b^i\right)}{db^i}-\frac{dM^P\left(b^i\right)}{db^i}\right)}_{\delta_b^i}\right)\frac{db^i}{d\tau_b^i}+\left(\tau_k^i-\underbrace{\left(M\left(b^i\right)-M^P\left(b^i\right)\right)}_{\delta_k^i}\right)\frac{dk^i}{d\tau_b^i}.\end{aligned}\tag{OA12}$$

Proof of Proposition 5 [Behavioral Distortions/Unregulated Activities]:

Proof. The proposition follows directly by evaluating Equation (OA12) in the case where the planner is forced to set $\tau_k^i\equiv 0$. □

Further simulation results: Figure OA-4 illustrates comparative statics of the model in the context of the second-best policy, in which $\tau_k^i=0$.

D.3 Application 3

Default and repayments: The bailout policy specified in Equation (36) implies that investors always (weakly) prefer not to default. Creditors are therefore guaranteed a repayment equal to the face value of legacy debt, b^i. We treat b^i as an exogenous constant throughout this application. The threshold state below which bailouts are positive, denoted $s^\star\left(k_1^i,k_2^i\right)$, is implicitly defined by

$$b^i=d_1\left(s^\star\left(k_1^i,k_2^i\right)\right)k_1^i+d_2\left(s^\star\left(k_1^i,k_2^i\right)\right)k_2^i.$$

Notice that this equation has a unique solution because we have assumed that the returns to investment, $d_1(s)$ and $d_2(s)$, are increasing in s.

Creditors' optimal choices and asset pricing: In this application, we assume for simplicity that investors' debt b^i is legacy debt, i.e., issued before the start of the model. Therefore, there is no market for debt, and no market price, at date 0. Creditors are passive agents who simply consume their endowments and debt repayments, and pay the taxes raised for bailouts. Creditors' indirect utility, as a function of investment choices, is then given by

$$\begin{aligned}V^C\left(k_1^i,k_2^i\right)&=\beta^C\left(b^i-(1+\kappa)\int_{\underline{s}}^{\overline{s}}t\left(k_1^i,k_2^i,b^i,s\right)dF\left(s\right)\right)\\&=\beta^C\left(b^i-(1+\kappa)\int_{\underline{s}}^{s^\star\left(k_1^i,k_2^i\right)}\left(b^i-d_1\left(s\right)k_1^i-d_2\left(s\right)k_2^i\right)dF\left(s\right)\right).\end{aligned}$$

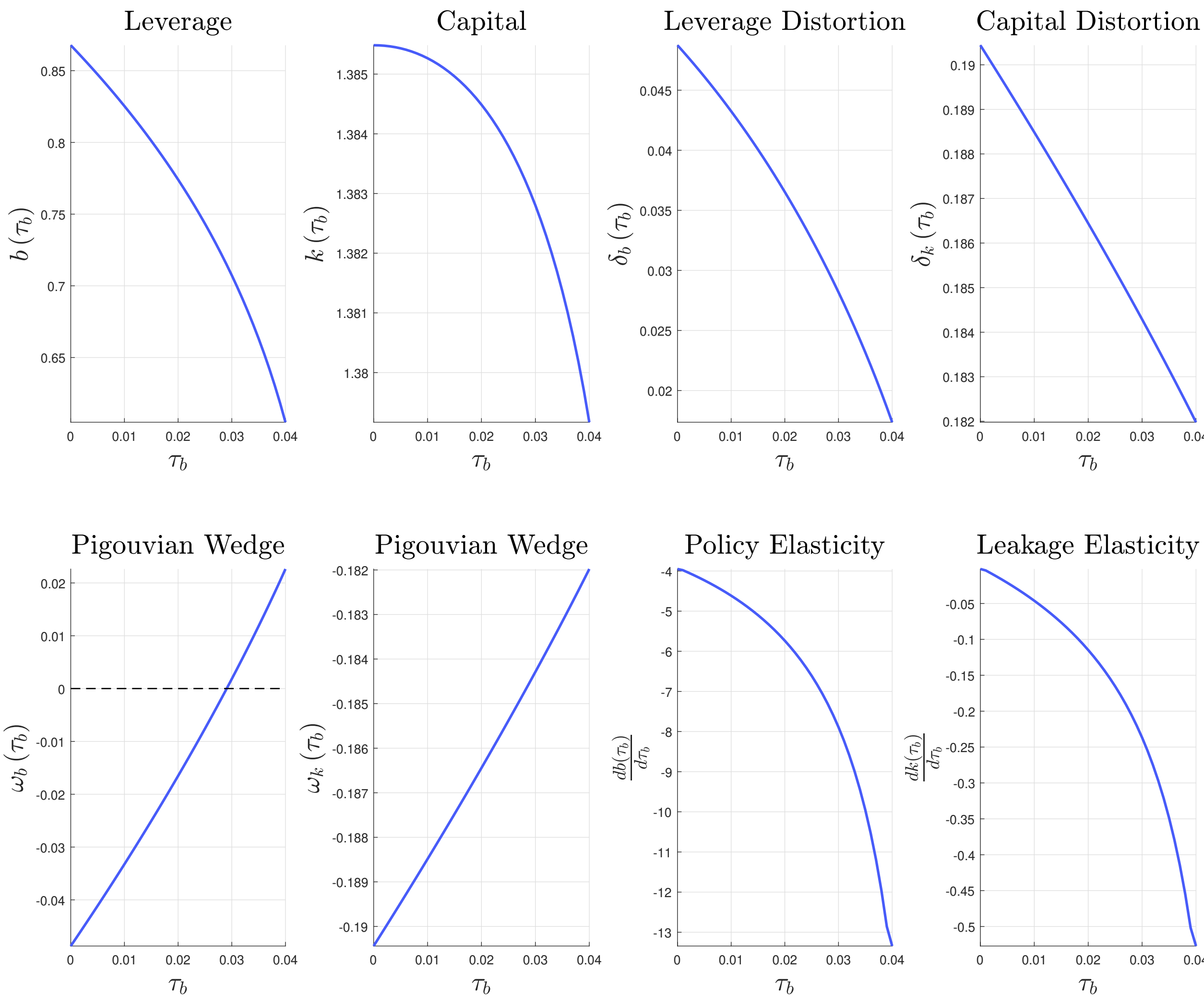

Figure OA-4: Application 2: Second-Best Comparative Statics

Note: Figure OA-4 illustrates relevant comparative statics of Application 2 for different values of τ_b, when $\tau_k = 0$. The top left plot and the top middle-left plot show equilibrium leverage b and investment k. The top middle-right and right plots show the leverage distortion δ_b and the capital distortion δ_k, respectively. The bottom left plot and the bottom middle-left plot show the associated Pigouvian wedges, ω_b and ω_k. The bottom middle-right plot and bottom right plot show the policy elasticity $\frac{db}{d\tau_b}$ and the leakage elasticity $\frac{dk}{d\tau_b}$. The parameters used are described in Figure 3.

Investors' optimal choices: Substituting investors' budget constraints into their objective, and ignoring exogenous endowments, we obtain the simplified version of their maximization problem:

$$V^i\left(\tau_k^1,\tau_k^2,T^i\right)=\max_{k_1^i,k_2^i}\beta^i\int_{s^\star\left(k_1^i,k_2^i\right)}^{\bar{s}}\left[d_1\left(s\right)k_1^i+d_2\left(s\right)k_2^i-b^i\right]dF\left(s\right)-\Upsilon\left(k_1^i,k_2^i\right)$$
$$-\tau_k^1k_1^i-\tau_k^2k_2^i+T^i,$$

where $V^i\left(\tau_k^1,\tau_k^2,T^i\right)$ denotes investors' indirect utility as a function of taxes/subsidies.

Investors' first-order conditions are given by

$$\beta^i\int_{s^\star\left(k_1^i,k_2^i\right)}^{\bar{s}}d_1\left(s\right)dF\left(s\right)-\frac{\partial\Upsilon\left(k_1^i,k_2^i\right)}{\partial k_1^i}-\tau_k^1=0$$
$$\beta^i\int_{s^\star\left(k_1^i,k_2^i\right)}^{\bar{s}}d_2\left(s\right)dF\left(s\right)-\frac{\partial\Upsilon\left(k_1^i,k_2^i\right)}{\partial k_2^i}-\tau_k^2=0.$$

Marginal welfare effects: The marginal welfare effect of changing the regulation τ_k^j of investment type $j\in\{1,2\}$ is given by

$$\frac{dW}{d\tau_k^j}=\frac{dV^C}{d\tau_k^j}+\frac{dV^i}{d\tau_k^j}.$$

Using the envelope theorem, parallel to our general results, we obtain

$$\frac{dV^C}{d\tau_k^j}=-\left(1+\kappa\right)\beta^C\sum_{m\in\{1,2\}}\int_{\underline{s}}^{s^\star\left(k_1^i,k_2^i\right)}d_m\left(s\right)dF\left(s\right)\frac{dk_m^i}{d\tau_k^j},$$

and

$$\frac{dV^i}{d\tau_k^j}=\frac{\partial V^i}{\partial\tau_k^j}+\frac{\partial V^i}{\partial T^i}\frac{dT^i}{d\tau_k^j}=\sum_{m\in\{1,2\}}\tau_m\frac{dk_m^i}{d\tau_k^j},$$

where we have used the assumption that $T^i=\tau_k^1k_1^i+\tau_k^2k_2^i$. Thus, we obtain

$$\frac{dW}{d\tau_k^j}=\sum_{m\in\{1,2\}}\frac{dk_m^i}{d\tau_k^j}\left(\tau_m-\underbrace{\left(1+\kappa\right)\beta^C\int_{\underline{s}}^{s^\star\left(k_1^i,k_2^i\right)}d_m\left(s\right)dF\left(s\right)}_{=\delta_m}\right).$$

Proof of Proposition 6 [Asset Substitution/Uniform Activity Regulation]:

Proof. To establish this proposition, we can use the general expression for optimal uniform regulation from Equation (15)

$$\tau^U=\frac{\boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I}-\boldsymbol{L}\right)\boldsymbol{\delta}^U}{\boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I}-\boldsymbol{L}\right)\boldsymbol{\iota}}.$$

We have $\boldsymbol{L} = 0$ in this application, because there is no perfectly regulated choice. Hence, we obtain

$$\begin{aligned}
\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} \boldsymbol{\delta}^U &= \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{dk_1^i}{d\tau_k^1} & \frac{dk_2^i}{d\tau_k^1} \\ \frac{dk_1^i}{d\tau_k^2} & \frac{dk_2^i}{d\tau_k^2} \end{pmatrix} \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} \\
&= \left(\frac{dk_1^i}{d\tau_k^1} + \frac{dk_1^i}{d\tau_k^2} \right) \delta_1 + \left(\frac{dk_2^i}{d\tau_k^1} + \frac{dk_2^i}{d\tau_k^2} \right) \delta_2,
\end{aligned}$$

and

$$\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} \boldsymbol{\iota} = \left(\frac{dk_1^i}{d\tau_k^1} + \frac{dk_1^i}{d\tau_k^2} \right) + \left(\frac{dk_2^i}{d\tau_k^1} + \frac{dk_2^i}{d\tau_k^2} \right).$$

Combining the last three expressions yields the required result, since

$$\begin{aligned}
\overline{\tau}_k &= \frac{\left(\frac{dk_1^i}{d\tau_k^1} + \frac{dk_1^i}{d\tau_k^2} \right) \delta_1 + \left(\frac{dk_2^i}{d\tau_k^1} + \frac{dk_2^i}{d\tau_k^2} \right) \delta_2}{\left(\frac{dk_1^i}{d\tau_k^1} + \frac{dk_1^i}{d\tau_k^2} \right) + \left(\frac{dk_2^i}{d\tau_k^1} + \frac{dk_2^i}{d\tau_k^2} \right)} \\
&= \frac{\frac{dk_1^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k} + \frac{dk_2^i}{d\overline{\tau}_k}} \delta_1 + \frac{\frac{dk_2^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k} + \frac{dk_2^i}{d\overline{\tau}_k}} \delta_2,
\end{aligned}$$

where we have defined the total response of k_m^i to a change in the uniform regulation as

$$\frac{dk_m^i}{d\overline{\tau}_k} = \frac{dk_m^i}{d\tau_k^1} + \frac{dk_m^i}{d\tau_k^2}.$$

$\square$

Derivation of leakage elasticities with separable costs: Assume that the adjustment cost takes the form $\Upsilon\left(k_1^i, k_2^i\right) = \frac{z_1}{2}\left(k_1^i\right)^2 + \frac{z_2}{2}\left(k_2^i\right)^2$. Investors' first-order conditions now become

$$\begin{aligned}
k_1^i &= \frac{1}{z_1} \left(\beta^i \int_{s^\star\left(k_1^i, k_2^i\right)}^{\bar{s}} d_1(s)\, dF(s) - \tau_k^1 \right) \\
k_2^i &= \frac{1}{z_2} \left(\beta^i \int_{s^\star\left(k_1^i, k_2^i\right)}^{\bar{s}} d_2(s)\, dF(s) - \tau_k^2 \right).
\end{aligned}$$

Applying the implicit function theorem and Leibniz rule to investors' first-order conditions, and imposing uniform regulation $\tau_k^1 = \tau_k^2 = \overline{\tau}_k$, we have

$$\frac{dk_n^i}{d\overline{\tau}_k} = \frac{1}{z_n} \left(-\beta^i d_n\left(s^\star\left(k_1^i, k_2^i\right)\right) f\left(s^\star\left(k_1^i, k_2^i\right)\right) \frac{ds^\star\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k} - 1 \right).$$

Notice that the probability of bailout is

$$\mathcal{P}\left(k_1^i, k_2^i\right) = F\left(s^\star\left(k_1^i, k_2^i\right)\right),$$

and has the property that

$$\frac{d\mathcal{P}\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k} = f\left(s^\star\left(k_1^i, k_2^i\right)\right) \frac{ds^\star\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k}.$$

Hence, we can write

$$\frac{dk_n^i}{d\overline{\tau}_k} = \frac{1}{z_n}\left(-\beta^i d_n\left(s^\star\left(k_1^i, k_2^i\right)\right)\frac{d\mathcal{P}\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k} - 1\right).$$

It follows that the sufficient statistics for leakage elasticities are i) the scaling factor z_n of the cost function, ii) the sensitivity of the probability of bailout to the regulation, and iii) the marginal contribution $d_n\left(s^\star\right)$ of each asset class at the bailout boundary. Notice that the weight on δ_1 in the optimal tax formula now becomes

$$\begin{aligned}\frac{\frac{dk_1^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k}+\frac{dk_2^i}{d\overline{\tau}_k}} &= \frac{\frac{1}{z_1}\left(-\beta^i d_1\left(s^\star\left(k_1^i, k_2^i\right)\right)\frac{d\mathcal{P}\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k} - 1\right)}{\frac{1}{z_1}\left(-\beta^i d_1\left(s^\star\left(k_1^i, k_2^i\right)\right)\frac{d\mathcal{P}\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k} - 1\right)+\frac{1}{z_2}\left(-\beta^i d_2\left(s^\star\left(k_1^i, k_2^i\right)\right)\frac{d\mathcal{P}\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k} - 1\right)}\\ &= \frac{1}{1+\xi_1},\end{aligned}$$

where

$$\xi_1 = \frac{z_1}{z_2}\frac{1+\beta^i d_2\left(s^\star\right)\frac{d\mathcal{P}\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k}}{1+\beta^i d_1\left(s^\star\right)\frac{d\mathcal{P}\left(k_1^i, k_2^i\right)}{d\overline{\tau}_k}}.$$

Further simulation results Figure OA-5 illustrates comparative statics of the model in the context of the second-best policy, in which $\overline{\tau}_k = \tau_k^1 = \tau_k^2$.

D.4 Application 4

Households' optimal choices and asset pricing: Households' optimization problem at date 1 can be expressed as

$$V^H(q) = \max_{k_1^H} F\left(k_1^H\right) - qk_1^H,$$

where $V^H(\cdot)$ denotes households' indirect utility as a function of market prices. The solution to the households' problem is characterized by $q = F'\left(k_1^H\right)$. When combined with market clearing, given by $\sum_i\left(k_0^i - k_1^i\right) = k_1^H$, we find the following equation, which the price q must satisfy:

$$q = F'\left(k_1^H\right) = F'\left(\sum_i\left(k_0^i - k_1^i\right)\right) = F'\left(\frac{1}{q}\sum_i \xi^i k_0^i\right).$$

Notice that this equation defines q as an implicit function of capital investments k_0^i. Below, we derive a solution for the equilibrium value of q in terms of primitives under standard functional forms.

Investors' optimal choices: We solve the investors' problem recursively. At date 1, the non-negativity constraint on consumption is necessarily binding. It follows that the investor optimally chooses $c_1^i = 0$ and

$$k_1^i = \left(1 - \frac{\xi^i}{q}\right)k_0^i.$$

Thus, investor i's maximized utility (i.e., value function) from date 1 onwards is

$$v_1^i\left(q, k_0^i\right) = z^i\left(1 - \frac{\xi^i}{q}\right)k_0^i.$$

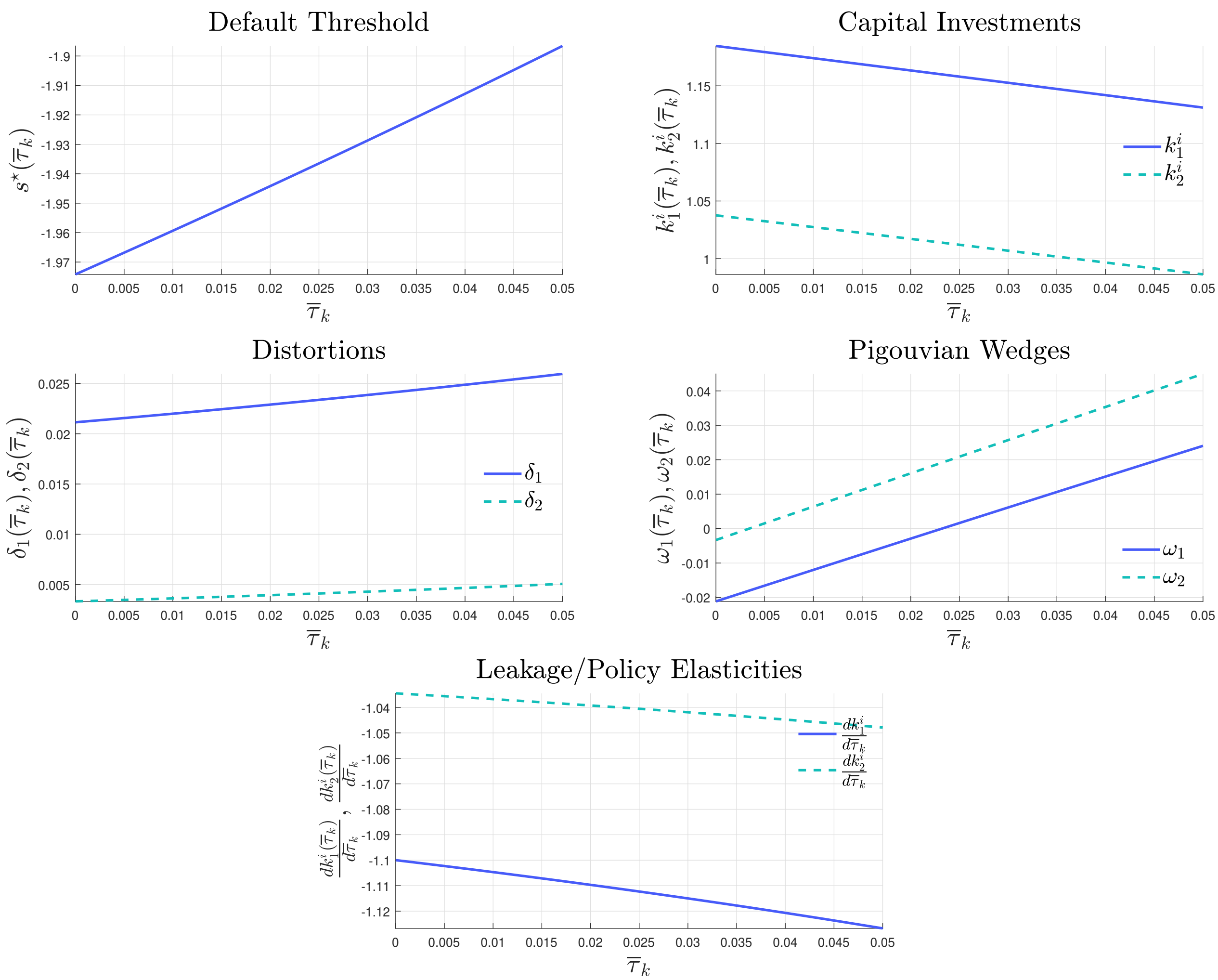

Figure OA-5: Application 3: Second-Best Comparative Statics

Note: Figure OA-5 illustrates relevant comparative statics of Application 3 for different values of $\overline{\tau}_k = \tau_k^1 = \tau_k^2$. The top left plot shows the default threshold $s^\star$. The top right plot shows risky capital investments k_1^i and k_2^i. The middle left plot shows the distortions associated with each investment decisions, δ_1 and δ_2, and the middle right plot shows the associated Pigouvian wedges, ω_1 and ω_2. The bottom plot shows the leakage/policy elasticities $\frac{dk_1^i}{d\overline{\tau}_k}$ and $\frac{dk_2^i}{d\overline{\tau}_k}$. The parameters used are described in Figure 4.

At date 0, ignoring exogenous endowments, we can express investors' optimization problem as

$$
\begin{aligned}
V^i\left(\tau_k^i, T^i, q\right) &= \max_{k_0^i} \left\{ v_1^i\left(q, k_0^i\right) - \Upsilon^i\left(k_0^i\right) - \tau_k^i k_0^i + T^i \right\}, \\
&= \max_{k_0^i} \left\{ z^i \left(1 - \frac{\xi^i}{q}\right) k_0^i - \Upsilon^i\left(k_0^i\right) - \tau_k^i k_0^i + T^i \right\},
\end{aligned}
$$

where $V^i(\cdot)$ denotes investors' indirect lifetime utility as a function of taxes and market prices. The first-order condition determining optimal investment k_0^i is given by

$$
z^i \left(1 - \frac{\xi^i}{q}\right) = \Upsilon^{i\prime}\left(k_0^i\right) + \tau_k^i.
$$

Assuming quadratic adjustment costs, we obtain the closed form solution

$$
k_0^i = \frac{1}{a^i}\left(z^i\left(1 - \frac{\xi^i}{q}\right) - \tau_k^i\right).
$$

Marginal welfare effects: The marginal welfare effect of changing the regulation τ_k^j of investor type j is given by

$$
\frac{dW}{d\tau_k^j} = \sum_{\ell \in \mathcal{I}} \frac{dV^\ell}{d\tau_k^j} + \frac{dV^H}{d\tau_k^j}.
$$

Using the envelope theorem, parallel to our general results, we obtain

$$
\frac{dV^H}{d\tau_k^j} = \frac{\partial V^H}{\partial q}\frac{dq}{d\tau_k^j}.
$$

Similarly, we have

$$
\begin{aligned}
\frac{dV^\ell}{d\tau_k^j} &= \frac{\partial V^\ell}{\partial \tau_k^j} + \frac{\partial V^\ell}{\partial T_0^\ell}\frac{dT_0^\ell}{d\tau_k^j} + \frac{\partial V^\ell}{\partial q}\frac{dq}{d\tau_k^j} \\
&= \tau_k^\ell \frac{dk_0^\ell}{d\tau_k^j} + \frac{\partial v_1^\ell}{\partial q}\frac{dq}{d\tau_k^j},
\end{aligned}
$$

where we have used the assumption that $T_0^\ell = \tau_k^\ell k_0^\ell$. Combining, we obtain

$$
\begin{aligned}
\frac{dW}{d\tau_k^j} &= -k_1^H \frac{dq}{d\tau_k^j} + \sum_{\ell \in \mathcal{I}} \left(\tau_k^\ell \frac{dk_0^\ell}{d\tau_k^j} + \frac{\partial v_1^\ell}{\partial q}\frac{dq}{d\tau_k^j}\right) \\
&= \sum_{i \in \mathcal{I}} \tau_k^i \frac{dk_0^i}{d\tau_k^j} + \left(\sum_{\ell \in \mathcal{I}} \frac{\partial v_1^\ell}{\partial q} - k_1^H\right)\frac{dq}{d\tau_k^j}.
\end{aligned} \tag{OA13}
$$

Since q in equilibrium is an implicit function of initial capital investments k_0^i, $i \in \{1, 2\}$, we can write

$$
\frac{dq}{d\tau_k^j} = \sum_{i \in \mathcal{I}} \frac{\partial q}{\partial k_0^i}\frac{dk_0^i}{d\tau_k^j}.
$$

Moreover, notice that

$$\sum_{\ell\in\mathcal{I}}\frac{\partial v_1^\ell}{\partial q}-k_1^H=\sum_{\ell\in\mathcal{I}}\frac{z^\ell}{q}\frac{\xi^\ell}{q}k_0^\ell-k_1^H=\sum_{\ell\in\mathcal{I}}\left(\frac{z^\ell}{q}-1\right)\left(k_0^\ell-k_1^\ell\right),$$

where the last equality follows from the market clearing condition $k_1^H=\sum_{\ell\in\mathcal{I}}\left(k_0^\ell-k_1^\ell\right)$. Substituting into (OA13) yields

$$\begin{aligned}\frac{dW}{d\tau_k^j}&=\sum_{i\in\mathcal{I}}\tau_k^i\frac{dk_0^i}{d\tau_k^j}+\sum_{\ell\in\mathcal{I}}\left(\frac{z^\ell}{q}-1\right)\left(k_0^\ell-k_1^\ell\right)\sum_{i\in\mathcal{I}}\frac{\partial q}{\partial k_0^i}\frac{dk_0^i}{d\tau_k^j}\\&=\sum_{i\in\mathcal{I}}\left(\tau_k^i-\underbrace{\left(-\frac{\partial q}{\partial k_0^i}\right)\sum_{\ell\in\mathcal{I}}\left(\frac{z^\ell}{q}-1\right)\left(k_0^\ell-k_1^\ell\right)}_{=\delta_k^i}\right)\frac{dk_0^i}{d\tau_k^j}.\end{aligned}$$

Proof of Proposition 7 [Fire-Sale Externalities/Uniform Investor Regulation]:

Proof. With uniform taxation, the planner is forced to set $\overline{\tau}_k=\tau_k^1=\tau_k^2$. The marginal welfare effect of changing the uniform tax is

$$\begin{aligned}\frac{dW}{d\overline{\tau}_k}&=\sum_{j\in\mathcal{I}}\frac{dW}{d\tau_k^j}\\&=\sum_{i\in\mathcal{I}}\left(\tau_k^i-\delta_k^i\right)\sum_{j\in\mathcal{I}}\frac{dk_0^i}{d\tau_k^j}\\&=\sum_{i\in\mathcal{I}}\left(\overline{\tau}_k-\delta_k^i\right)\frac{dk_0^i}{d\overline{\tau}_k},\end{aligned}$$

and solving for the optimal regulation $\frac{dW}{d\overline{\tau}_k}=0$, we obtain the required second-best solution:

$$\overline{\tau}_k=\frac{\sum_{i\in\mathcal{I}}\frac{dk_0^i}{d\overline{\tau}_k}\delta_k^i}{\sum_{i\in\mathcal{I}}\frac{dk_0^i}{d\overline{\tau}_k}}.$$

□

Closed-form solutions: Under the assumption that $F\left(k_1^H\right)=\frac{\left(k_1^H\right)^\alpha}{\alpha}$, which implies that $F'\left(k_1^H\right)=\left(k_1^H\right)^{\alpha-1}$, we can express the equilibrium price in closed form as

$$q=\left(\sum_i\xi^ik_0^i\right)^{\frac{\alpha-1}{\alpha}}. \tag{OA14}$$

With quadratic adjustment costs $\Upsilon^i\left(k_0^i\right)=\frac{a^i}{2}\left(k_0^i\right)^2$, investors' optimal choices at date 0 satisfy

$$k_0^i=\frac{1}{a^i}\left(z^i\left(1-\frac{\xi^i}{q}\right)-\tau_k^i\right).$$

Note that $\frac{\partial k_0^i}{\partial q} = \frac{z^i}{a^i}\frac{\xi^i}{q^2} > 0$. Note also that $z^i\left(1 - \frac{\xi^i}{q}\right) - \tau_k^i > 0$ is required for $k_0^i > 0$. Combining the optimal choice of k_0^i with the characterization of the price in Equation (OA14) yields a solution for q in terms of primitives:

$$q = \left(\sum_i \frac{\xi^i}{a^i}\left(z^i\left(1 - \frac{\xi^i}{q}\right) - \tau_k^i\right)\right)^{\frac{\alpha-1}{\alpha}}.$$

As expected, the same change in k_0^i has a stronger impact on the price at date 1 for those investors with a higher ξ^i, who are forced to sell more at date 1. Note that we can write $\frac{\partial q}{\partial k_0^i} = \xi^i \frac{\alpha-1}{\alpha} q^{\frac{1}{1-\alpha}}$, so $\frac{\partial q}{\partial k_0^i}$ is higher in absolute, when q is higher.

Further simulation results Figure OA-6 illustrates comparative statics of the model in the context of the second-best policy, in which $\overline{\tau}_k = \tau_k^1 = \tau_k^2$.

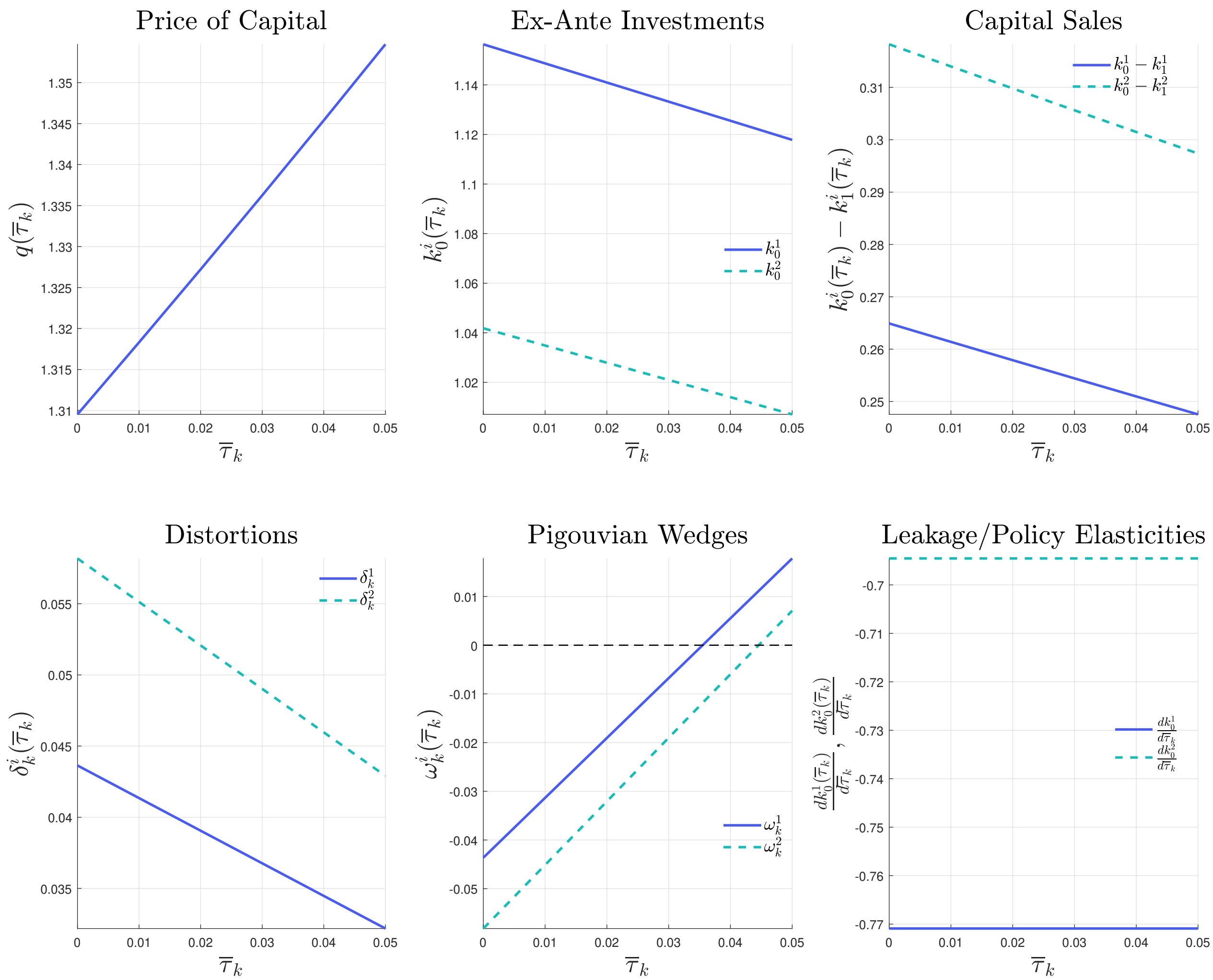

Figure OA-6: Application 4: Second-Best Comparative Statics

Note: Figure OA-6 illustrates relevant comparative statics of Application 4 for different values of $\tau_k^1 = \tau_k^2 = \overline{\tau}_k$. The top left plot shows the price of capital in equilibrium q. The top middle plot shows investment at date 0 for both investor types, k_0^1 and k_0^2. The top right plot shows the amount of capital sold at date 1 for both investor types, $k_0^1 - k_1^1$ and $k_0^2 - k_1^2$. The bottom left plot and the bottom middle plot show the distortions associated with the investment decisions of each investor, δ_k^1 and δ_k^2, and the associated Pigouvian wedges, ω_k^1 and ω_k^2. The bottom right plot shows the leakage/policy elasticities $\frac{dk_0^1}{d\overline{\tau}_k}$ and $\frac{dk_0^2}{d\overline{\tau}_k}$. The parameters used are described in Figure 5.

E Additional Proofs and Derivations

E.1 Price Theoretic Formulation

We now show that it is possible to extend our results to a price-theoretic environment as follows. We now consider an economy with a finite number $I \geq 1$ of agents (equivalently, agent types in unit measure), indexed by $i, j \in \mathcal{I}$, where $\mathcal{I} = \{1, \ldots, I\}$. There are $N \geq 1$ goods (commodities) indexed by $n \in \mathcal{N}$, where $\mathcal{N} = \{1, \ldots, N\}$. Agent i's preferences are represented by

$$u^i \left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}\right), \tag{OA15}$$

where $\boldsymbol{x}^i \in \mathbb{R}_+^N$ denotes agent i's consumption bundle, and $\bar{\boldsymbol{x}} = \left\{\bar{\boldsymbol{x}}^j\right\}_{j \in \mathcal{I}}$ denotes the collection of bundles of all agents.[25] Each agent takes $\bar{\boldsymbol{x}}$ as given, so the second argument in $u^i(\cdot)$ captures externalities across agents.

Each agent i faces a budget constraint

$$\boldsymbol{p} \cdot \left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) + \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i = T^i, \tag{OA16}$$

where $\boldsymbol{e}^i \in \mathbb{R}_+^N$ denotes agent i's endowment of goods; $\boldsymbol{p} \in \mathbb{R}_+^N$ is a price vector; $\boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$ introduces a set of taxes/subsidies (regulations) specific to each agent and commodity, where $\boldsymbol{\tau}^i \in \mathbb{R}^N$; and $T^i \in \mathbb{R}$ denotes the lump-sum transfer or tax that agent i receives or faces to ensure that the planner runs a balanced budget. We denote the elements of $\boldsymbol{x}^i$, $\bar{\boldsymbol{x}}^i$, and $\boldsymbol{\tau}^i$ by x_n^i, $\bar{x}_n^i$ and τ_n^i, respectively.

For a given set of regulations $\left\{\boldsymbol{\tau}^i\right\}_{i \in \mathcal{I}}$ and transfers $\left\{T^i\right\}_{i \in \mathcal{I}}$, an *equilibrium* consists of consumption allocations $\left\{\boldsymbol{x}^i\right\}_{i \in \mathcal{I}}$ and a price vector $\boldsymbol{p}$ such that i) agents choose $\boldsymbol{x}^i$ to maximize utility (OA15) taking $\boldsymbol{p}$ and $\bar{\boldsymbol{x}}$ as given, subject to the budget constraint (OA16); ii) the planner's budget is balanced, so that $\sum_i T^i = \sum_i \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$; iii) consumption allocations are consistent in the aggregate, so $\bar{\boldsymbol{x}}^i = \boldsymbol{x}^i$, $\forall i$; and iv) markets clear, so $\sum_i \boldsymbol{x}^i = \sum_i \boldsymbol{e}^i$. We assume at all times that the model is well-behaved.

First, we characterize the change in agent i's indirect utility, denoted by V^i, when varying a specific regulation τ_n^j. Making use of the envelope theorem, we have

$$\frac{dV^i}{d\tau_n^j} = \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} \cdot \frac{d\bar{\boldsymbol{x}}}{d\tau_n^j} - \lambda^i \left(\frac{d\boldsymbol{p}}{d\tau_n^j} \cdot \left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) + \frac{d\boldsymbol{\tau}^i}{d\tau_n^j} \cdot \boldsymbol{x}^i - \frac{dT^i}{d\tau_n^j}\right),$$

where λ^i denotes the Lagrange multiplier associated with the budget constraint. We use x and $\bar{x}$ equivalently going forward, since they are equal in equilibrium.

Expressing this welfare change in money-metric terms by normalizing by agent i's marginal value of wealth, λ^i, and aggregating across agents, we have

$$\begin{aligned}
\frac{dW}{d\tau_n^j} = \sum_i \frac{\frac{dV^i}{d\tau_n^j}}{\lambda^i} &= \sum_i \frac{1}{\lambda^i} \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} \cdot \frac{d\boldsymbol{x}}{d\tau_n^j} - \frac{d\boldsymbol{p}}{d\tau_n^j} \cdot \sum_i \left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) - \sum_i \left(\frac{d\boldsymbol{\tau}^i}{d\tau_n^j} \cdot \boldsymbol{x}^i - \frac{dT^i}{d\tau_n^j}\right) \\
&= \sum_i \frac{1}{\lambda^i} \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} \cdot \frac{d\boldsymbol{x}}{d\tau_n^j} + \sum_i \frac{d\boldsymbol{x}^i}{d\tau_n^j} \cdot \boldsymbol{\tau}^i,
\end{aligned}$$

where the last line follows from i) market clearing, $\sum_i \left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) = 0$, and ii) the fact that the planner's

[25] Section A of the Online Appendix includes explicit definitions of all vectors and matrices used in the paper.

budget is balanced, so

$$\sum_i \frac{dT^i}{d\tau_n^j} = \sum_i \frac{d\boldsymbol{\tau}^i}{d\tau_n^j} \cdot \boldsymbol{x}^i + \sum_i \frac{d\boldsymbol{x}^i}{d\tau_n^j} \cdot \boldsymbol{\tau}^i.$$

Note that $\frac{dW}{d\tau_n^j}$ can be equivalently expressed as

$$\frac{dW}{d\tau_n^j} = \sum_i \frac{1}{\lambda^i} \sum_\ell \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}^\ell} \cdot \frac{d\boldsymbol{x}^\ell}{d\tau_n^j} + \sum_\ell \frac{d\boldsymbol{x}^\ell}{d\tau_n^j} \cdot \boldsymbol{\tau}^\ell = \sum_\ell \frac{d\boldsymbol{x}^\ell}{d\tau_n^j} \left(\boldsymbol{\tau}^\ell - \boldsymbol{\delta}^\ell\right),$$

where $\boldsymbol{\delta}^\ell = -\sum_i \frac{1}{\lambda^i} \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}^\ell}$, so by switching indexes and stacking we find that

$$\frac{dW}{d\boldsymbol{\tau}} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} \left(\boldsymbol{\tau} - \boldsymbol{\delta}\right) = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} \boldsymbol{\omega},$$

as in Equation (10) in the text.

E.2 Game Theoretic Formulation

We now show that it is possible to extend our results to a game theoretic environment as follows. Suppose that agents have preferences of the form

$$u^i\left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}\right)$$

and that they face a constraint given by

$$\Psi^i\left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}; \theta\right) = 0,$$

where θ is a parameter that indexes a general perturbation. In the price theoretic formulation in Section E.1, the function $\Psi^i\left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}; \theta\right)$ takes the simple form

$$\Psi^i\left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}; \theta\right) = p\left(\bar{x}\right)\left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) + \boldsymbol{\tau}^i\left(\theta\right) \boldsymbol{x}^i - T^i.$$

We consider a Walras-Nash equilibrium with taxes, in which optimality requires that $\frac{\partial u^i}{\partial \boldsymbol{x}^i} = \lambda^i \frac{\partial \Psi^i}{\partial \boldsymbol{x}^i}$. In general, we can express the marginal welfare change in the utility of agent i induced by a general perturbation as

$$\frac{dV^i}{d\theta} = \frac{\partial u^i}{\partial \boldsymbol{x}^i} \cdot \frac{d\boldsymbol{x}^i}{d\theta} + \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} \cdot \frac{d\bar{\boldsymbol{x}}}{d\theta} - \lambda^i \left(\frac{\partial \Psi^i}{\partial \boldsymbol{x}^i} \cdot \frac{d\boldsymbol{x}^i}{d\theta} + \frac{\partial \Psi^i}{\partial \bar{\boldsymbol{x}}} \cdot \frac{d\bar{\boldsymbol{x}}}{d\theta} + \frac{\partial \Psi^i}{\partial \theta}\right),$$

and normalizing by λ^i to express the marginal welfare change in units of the constraint

$$\frac{dV^i}{\lambda^i} = \left(\frac{1}{\lambda^i} \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} - \frac{\partial \Psi^i}{\partial \bar{\boldsymbol{x}}}\right) \cdot \frac{d\bar{\boldsymbol{x}}}{d\theta} - \frac{\partial \Psi^i}{\partial \theta}.$$

This normalization allows us to express aggregate welfare gains as

$$\frac{dW}{d\theta} = \sum_i \frac{dV^i}{\lambda^i} = \sum_i \left(\frac{1}{\lambda^i} \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} - \frac{\partial \Psi^i}{\partial \bar{\boldsymbol{x}}}\right) \cdot \frac{d\bar{\boldsymbol{x}}}{d\theta} - \sum_i \frac{\partial \Psi^i}{\partial \theta}. \tag{OA17}$$

This expression is the generalization of Equation (10) in Lemma 1a), where the term multiplying $\frac{d\bar{\boldsymbol{x}}}{d\theta}$ exactly defines marginal distortions, and the term $\sum_i \frac{\partial \Psi^i}{\partial \theta}$ captures the direct impact of any policy perturbation. When the policy perturbation takes the form $\boldsymbol{\tau}^i\left(\theta\right) \boldsymbol{x}^i$, then Equation (OA17) is exactly a generalized version of Lemma 1a), with a slightly redefined marginal distortion, which is now augmented to capture interactions among agents via constraints.

E.3 Redistributional Concerns

Given the money-metric marginal welfare effects of varying $\boldsymbol{\tau}^j$, defined in Equations (OA5) and (OA6), we can express the marginal welfare effects of varying $\boldsymbol{\tau}^j$, for any set of generalized social marginal welfare weights (Saez and Stantcheva, 2016) or individual weights (Dávila and Schaab, 2025), as follows:

$$\frac{dW}{d\boldsymbol{\tau}^j} = \sum_i \omega^i \frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^i} + \omega^j \frac{\frac{dV^C}{d\boldsymbol{\tau}^j}}{\lambda_0^C} = \underbrace{\sum_i \frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^i} + \frac{\frac{dV^C}{d\boldsymbol{\tau}^j}}{\lambda_0^C}}_{\text{Efficiency}} + \underbrace{\mathbb{C}ov_{iC}^{\Sigma}\left[\omega^{iC}, \frac{\frac{dV^{iC}}{d\boldsymbol{\tau}^j}}{\lambda_0^x}\right]}_{\text{Redistribution}}, \tag{OA18}$$

where we assume, without loss of generality that the weights add up to one, that is, $\sum_i \omega^i + \omega^j = 1$, and where we use the index iC to refer to the set of investors and creditors.[26]

When $\omega^i = \omega^C = 1$, then the redistribution term in Equation (OA18) is zero. This case is the one studied in the body of the paper. When $\omega^i \neq \omega^C \neq 1$, Equation (OA18) shows that redistributional concerns enter additively to the marginal welfare effects of varying τ^j. A utilitarian planner simple corresponds to setting marginal welfare weights of the form $\omega^i = \lambda_0^i$, where λ_0^i typically equals marginal utility of consumption. Note that a utilitarian planner with access to lump-sum taxes/transfers finds it optimal to endogenously set $\omega^i = \omega^C = 1$.

E.4 Practical Scenarios

E.4.1 Unregulated Decisions

Equation (13) follows directly from Proposition 1 since at the second-best optimum, the constraints are binding with $\boldsymbol{\tau}^U = 0$. Concretely, we have

$$\boldsymbol{\tau}^R = \boldsymbol{\delta}^R + \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\underbrace{\boldsymbol{\tau}^U}_{=0} - \boldsymbol{\delta}^U\right) = \boldsymbol{\delta}^R - \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\delta}^U,$$

as required. It is useful to consider the simple scenario in which there are two agents, $I = 2$, and each agent has a single decision, $N = 1$. Assume that only agent 1 can be regulated, with regulatory constraints dictating that $\tau^2 \equiv 0$. In that case, it follows from (13) that the optimal regulation for the regulated is simply given by

$$\tau^1 = \delta^1 - \left(-\frac{dx^1}{d\tau^1}\right)^{-1} \frac{dx^2}{d\tau^1}\delta^2.$$

The optimal regulation on agent 1 is equal to the first-best equivalent δ^1 minus a correction that accounts for the distortion imposed by the other unregulated agent. Assume, for instance, that the distortion by the unregulated agent satisfies $\delta^2 > 0$. The weight on the distortion by the unregulated agent is negative, implying that it pushes τ^1 towards underregulation, whenever i) the regulated agent responds negatively to increased regulation (the "regular" case with $\frac{dx^1}{d\tau^1} < 0$), and ii) the associated leakage elasticity indicates gross substitutes with $\frac{dx^2}{d\tau^1} > 0$.

While this is the simplest case for building intuition, note that the same insight extends to any economy with a single regulated decision and with an arbitrary set of unregulated decisions for which taxes/subsidies

[26] In Equation (OA18), the covariance operator, indexed by iC, correspond to cross-sectional covariance-sum including investors and creditors.

are forced to be zero. In this more general case, the optimal policy formula becomes

$$\tau^R = \delta^R - \sum_{(j,n)\in U} \left(-\frac{dx^R}{d\tau^R}\right)^{-1} \frac{dx_n^j}{d\tau^R}\delta_n^j,$$

where U denotes the set of imperfectly regulated decisions.

E.4.2 Uniform Regulation

To build intuition for the uniform regulation results, it is useful to first consider the special case where *all* decisions are subject to uniform regulation ($\boldsymbol{x}^U = \boldsymbol{x}$). In that case, it follows from Proposition 2 that the optimal uniform regulation is given by

$$\overline{\tau}^U = \frac{\sum_i \sum_n \frac{dx_n^i}{d\overline{\tau}^U}\delta_n^i}{\sum_j \sum_n \frac{dx_n^i}{d\overline{\tau}^U}} = \frac{\sum_i \sum_n w_n^i \delta_n^i}{\sum_j \sum_n \frac{dx_n^i}{d\overline{\tau}^U}}, \tag{OA19}$$

where we have re-written the total response of decision x_n^j to the uniform regulation as

$$\frac{dx_n^j}{d\overline{\tau}^U} = \sum_{j'\in\mathcal{I}} \sum_{n'\in\mathcal{N}} \frac{dx_n^j}{d\tau_{n'}^{j'}}.$$

In general, note that the case of uniform regulation is a particular case of linear constraints $\boldsymbol{A}\boldsymbol{\tau}^U = 0$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & -1 & & \cdots & 0 \\ & 1 & -1 & & \vdots \\ \vdots & & \ddots & \ddots & \\ 0 & \cdots & & 1 & -1 \end{pmatrix},$$

where $\boldsymbol{A}$ has dimensions $(N_U - 1) \times N_U$. In this case, $\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}} = \boldsymbol{A}'$, so Proposition 2 implies that the regulator optimally sets

$$\frac{dW}{d\boldsymbol{\tau}^U} = \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{\omega}^U = \boldsymbol{A}'\boldsymbol{\mu},$$

which also implies that

$$\boldsymbol{\iota}'\frac{dW}{d\boldsymbol{\tau}^U} = \boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{\omega}^U = \boldsymbol{\iota}'\boldsymbol{A}'\boldsymbol{\mu} = 0,$$

since $\boldsymbol{A}\boldsymbol{\iota} = 0$, where $\boldsymbol{\iota}$ denotes a vector of ones. Rearranging, we obtain

$$\begin{aligned} 0 &= \boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{\omega}^U \\ &= \boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}(\boldsymbol{I}-\boldsymbol{L})\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) \\ &= \boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}(\boldsymbol{I}-\boldsymbol{L})\left(\overline{\tau}^U\boldsymbol{\iota} - \boldsymbol{\delta}^U\right), \end{aligned}$$

where the last line uses the fact that all elements of $\boldsymbol{\tau}^U$ must be equal to the same scalar, denoted $\overline{\tau}^U$, at the constrained solution. We solve as follows for the scalar $\overline{\tau}^U$ to complete the derivation of Equation (15):

$$\underbrace{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \boldsymbol{\iota}}_{\text{scalar}} \overline{\tau}^U = \underbrace{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \boldsymbol{\delta}^U}_{\text{scalar}} \iff \overline{\tau}^U = \frac{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \boldsymbol{\delta}^U}{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \boldsymbol{\iota}}.$$

E.4.3 Convex Costs of Regulation

In this case, we have $\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}} = \boldsymbol{B}\boldsymbol{\tau}^{\boldsymbol{U}}$, so Proposition 2 implies that the planner optimally sets

$$\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) = \boldsymbol{B}\boldsymbol{\tau}^U.$$

Solving for $\boldsymbol{\tau}^U$ yields

$$\boldsymbol{\tau}^U = \left(\boldsymbol{B} + \left(-\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right) (\boldsymbol{I} - \boldsymbol{L})\right)^{-1} \left(\left(-\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right) (\boldsymbol{I} - \boldsymbol{L}) \boldsymbol{\delta}^U\right). \tag{OA20}$$

which establishes Equation (16). In general, the correction relative to the first-best policy is given by $(\boldsymbol{B} + \boldsymbol{K})^{-1} \boldsymbol{K}$, which has the interpretation of an attenuation matrix. For instance, in a scenario with a single agent, $I = 1$, and a single decision, $N = 1$, Equation (OA20) becomes

$$\tau^U = \frac{\left(-\frac{dx^U}{d\tau^U}\right)}{b + \left(-\frac{dx^U}{d\tau^U}\right)} \delta^U, \tag{OA21}$$

where b is a non-negative scalar that modulates the cost. In the well-behaved case in which $\frac{dx}{d\tau} < 0$, it follows that the optimal regulation is simply a scaled down version of the first-best regulation.[27]

E.5 Diagonal Case

Finally, it is useful to discuss the case in which $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ is a diagonal matrix. In this case, the second-best regulation on a perfectly regulated decision (j, n) is

$$\tau_n^j = \delta_n^j + \left(-\frac{dx_n^j}{d\tau_n^j}\right)^{-1} \sum_{(j', n') \in \mathcal{U}} \frac{dx_{n'}^{j'}}{d\tau_n^j} \omega_{n'}^{j'}, \tag{OA22}$$

where $\mathcal{U}$ denotes the set of imperfectly regulated decisions.

The simplified formula again shows the importance of leakage elasticities, which are weighted by wedges and summed across all unregulated decisions $(j', n') \in \mathcal{U}$. It is clear in this case that it is optimal to underregulate the regulated ($\tau_n^j < \delta_n^j$) if each of the imperfectly regulated decisions is underregulated ($\omega_{n'}^{j'} < 0$) and is a gross substitute to the regulated decision ($\frac{dx_{n'}^{j'}}{d\tau_n^j} > 0$). In addition, the formula shows that, even when not every decision satisfies gross substitutes, it is optimal to underregulate the regulated when a weighted average of leakage elasticities — with the weights proportional to the associated wedges — is positive.

[27] In this case, note that the perfectly regulated decisions in turn must satisfy:

$$\boldsymbol{\tau}^R = \boldsymbol{\delta}^R + \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} \left((\boldsymbol{B} + \boldsymbol{K})^{-1} \boldsymbol{K} - \boldsymbol{I}\right) \boldsymbol{\delta}^U.$$

Formally, when the own-regulatory policy elasticity matrix $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ is diagonal, we have

$$\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\omega}^U = \begin{pmatrix} \left(\frac{dx_1^R}{d\tau_1^R}\right)^{-1} & 0 & \\ 0 & \ddots & \\ & & \left(\frac{dx_R^R}{d\tau_R^R}\right)^{-1} \end{pmatrix} \begin{pmatrix} \frac{dx_1^U}{d\tau_1^R} & \frac{dx_2^U}{d\tau_1^R} & & \\ \frac{dx_1^U}{d\tau_2^R} & \frac{dx_2^U}{d\tau_2^R} & & \\ & & \ddots & \\ & & & \frac{dx_U^U}{d\tau_R^R} \end{pmatrix} \begin{pmatrix} \omega_1^U \\ \vdots \\ \omega_U^U \end{pmatrix}$$

$$= \begin{pmatrix} \left(\frac{dx_1^R}{d\tau_1^R}\right)^{-1}\frac{dx_1^U}{d\tau_1^R} & \left(\frac{dx_1^R}{d\tau_1^R}\right)^{-1}\frac{dx_2^U}{d\tau_1^R} & & \\ \left(\frac{dx_2^R}{d\tau_2^R}\right)^{-1}\frac{dx_1^U}{d\tau_2^R} & \left(\frac{dx_2^R}{d\tau_2^R}\right)^{-1}\frac{dx_2^U}{d\tau_2^R} & & \\ & & \ddots & \\ & & & \left(\frac{dx_R^R}{d\tau_R^R}\right)^{-1}\frac{dx_U^U}{d\tau_U^R} \end{pmatrix} \begin{pmatrix} \omega_1^U \\ \vdots \\ \omega_U^U \end{pmatrix}$$

$$= \begin{pmatrix} \left(\frac{dx_1^R}{d\tau_1^R}\right)^{-1}\left(\frac{dx_1^U}{d\tau_1^R}\omega_1^U + \frac{dx_2^U}{d\tau_1^R}\omega_2^U + \cdots\right) \\ \left(\frac{dx_2^R}{d\tau_2^R}\right)^{-1}\left(\frac{dx_1^U}{d\tau_2^R}\omega_1^U + \frac{dx_2^U}{d\tau_2^R}\omega_2^U + \cdots\right) \\ \vdots \\ \left(\frac{dx_R^R}{d\tau_R^R}\right)^{-1}\left(\frac{dx_1^U}{d\tau_R^R}\omega_1^U + \frac{dx_2^U}{d\tau_R^R}\omega_2^U + \cdots\right) \end{pmatrix}.$$

It follows that the second-best regulation on a perfectly regulated decision (j, n) is thus given by (OA22).