

# Optimal Corporate Taxation Under Financial Frictions\*

Eduardo Dávila<sup>†</sup>

Benjamin Hébert<sup>‡</sup>

April 2020

## Abstract

This paper studies the optimal design of corporate taxes when firms are financially constrained. We identify a corporate taxation principle: taxes should be levied on unconstrained firms, which value resources inside the firm less than constrained firms. Under complete information, this principle fully characterizes the optimal corporate tax policy. Under incomplete information about firms' future investment opportunities, the government uses firms' payout decisions to elicit whether a firm is constrained or not, setting taxes accordingly. We show that a constant corporate payout tax, levied on both dividend payments and share repurchases, is optimal in static and dynamic environments. Quantitatively, we find that a revenue-neutral switch to a payout tax would increase the overall value of existing firms and new entrants by 7%.

**JEL Codes:** G38, H21, H25

**Keywords:** corporate taxation, payout taxes, dividend taxes, financial frictions

---

\*We would like to thank our discussant Vish Viswanathan, Anat Admati, Vladimir Asriyan, Andy Atkeson, Juliane Begenau, John Campbell, V. V. Chari, John Cochrane, Peter DeMarzo, Sebastian Di Tella, Peter Diamond, Darrell Duffie, Emmanuel Farhi, Michael Faulkender, Xavier Gabaix, Piero Gottardi, Oleg Itskhoki, Anton Korinek, Hanno Lustig, Tim McQuade, Holger Mueller, Jonathan Parker, Adriano Rampini, Martin Schneider, Andy Skrzypacz, Jeremy Stein, Ludwig Straub, Aleh Tsyvinski, Felipe Varas, Iván Werning, as well as seminar and conference participants for helpful comments. All remaining errors are our own.

<sup>†</sup>Yale University and NBER. Email: eduardo.davila@yale.edu

<sup>‡</sup>Stanford University and NBER. Email: bhebert@stanford.edu

# 1 Introduction

Virtually every developed country collects taxes from corporations. In this paper, we take as given that governments will tax firms, and ask how firms should be taxed. We study economies with financial frictions, in which, for some firms who are financially constrained, the marginal value of funds is higher inside the firm than outside the firm.

By studying the problem of a government that sets taxes to maximize the total value of the corporate sector subject to a revenue target, we identify an elementary principle that shapes the optimal design of corporate taxes. We refer to it as the *corporate taxation principle*: corporate taxes should be designed to minimize the tax burden faced by financially constrained firms. In other words, whenever possible, corporate taxes should be levied on unconstrained firms. If the government had full information about firms' investment opportunities, this principle would be sufficient to determine optimal corporate taxes: only unconstrained firms should be taxed.

However, it is not easy for the government to determine whether a firm is financially constrained or not.<sup>1</sup> When firms have private information about their future investment opportunities, the government must design a tax mechanism that levies taxes primarily on unconstrained firms while ensuring that it is incentive compatible for these firms to reveal that they are unconstrained. The set of feasible corporate taxes is rich and includes both standard corporate profit taxes and more complex policies.<sup>2</sup> We show that the optimal tax policy features a simple implementation: a *corporate payout tax*, which is proportionally levied on dividends, share repurchases, and other payouts to firms' shareholders.

We begin by studying a full information single-date model that illustrates the corporate taxation principle. We next study two-date and infinite-horizon models in which firms have private information about their future investment opportunities. We demonstrate the optimality of the corporate payout tax in these models. We then quantify the benefits of a revenue-neutral switch from standard corporate profit taxes to a corporate payout tax. In a calibrated version of our infinite-horizon model, we find that a permanent switch from profit taxes to payout taxes increases the total value of incumbent firms and future entrants by 7%.

In our model, firms make investment and financing decisions at the beginning of each date, before production materializes. As in [Rampini and Viswanathan \(2010\)](#), we consider an environment with limited enforcement and no exclusion after default. This constrains the ability of firms to raise financing and the ability of the government to raise taxes. After production occurs, firms make a payout decision, face taxes, and contemplate the possibility of defaulting.

In the single-date full information model, the optimal tax policy of a government that can tax (but not subsidize) firms involves taxing only unconstrained firms. This is the corporate taxation principle. Intuitively, the government, valuing the welfare of all firms equally, wishes to collect taxes from the firms for whom paying

---

<sup>1</sup>The existence of a large literature (e.g. [Fazzari, Hubbard and Petersen \(1988\)](#), [Kaplan and Zingales \(1997\)](#)) that seeks to identify financially constrained firms supports the premise that it is difficult for the government to determine whether a firm is financially constrained.

<sup>2</sup>Throughout the paper, we use the term corporate taxes to mean taxes collected from corporations. Our framework allows for conventional corporate profit taxes, which in practice take the form of a tax on firms' profits, adjusted for various credits and deductions. The optimal policy in our model is a corporate tax levied on payouts, as opposed to profits or other variables.

taxes is least costly.<sup>3</sup>

In the two-date model, in which the second date corresponds to the solution to the single-date full-information model, firms privately learn their second-date productivity on the first date. This forces the government to solve a mechanism design problem. Our main result shows that the optimal mechanism can be implemented with a corporate payout tax. The key intuition is that the desire to pay dividends separates firms that will be unconstrained in the second date from firms that will be financially constrained. Firms that will be unconstrained in the second date anticipate that they will have low marginal products and will be taxed, and therefore prefer making payouts on the first date to retaining earnings. Firms that will be financially constrained in the second date are in the opposite situation. They will have high marginal products and will not be taxed, and therefore prefer to keep funds inside the firm. This difference between constrained and unconstrained firms allows the government during the first date to raise taxes in an incentive-compatible way by taxing payouts. Other firms' choices (e.g., investment or production) during the first date are not distorted at the optimum because they are determined by firms' current productivity, not their future productivity. As a result, they are imperfect ways of distinguishing between firms that have good or bad investment opportunities in the future.

Our most general results arise in an infinite-horizon model in which the productivity of a given firm is time-varying, and at each date the firm — but not the government — knows its next date productivity. A dynamic model is necessary to capture the evolving nature of firms' payout and investment policies. This model allows for entry and exit of firms, accounts for the evolution of the distribution of firms, and gives the government the ability to borrow and save. We consider stationary Markov sub-game perfect equilibria, in which the government's policies are a function of the distribution of firms and the level of government debt. We purposely assume that the government cannot commit to future policies to prevent the use of its commitment ability to trivially overcome the financial frictions. Although solving dynamic models with asymmetric information is challenging, our results from the single-date and two-date models allow us to guess and verify the optimal policies in the infinite-horizon setup. We show that the optimal sequence of mechanisms that solves the government's problem can be implemented by a corporate payout tax.

Since most countries use profits-based corporate taxes, we also explore quantitatively the benefits of switching from profit taxes to payout taxes. To that end, we calibrate our model to the [Li, Whited and Wu \(2016\)](#) estimation of firms' productivity dynamics and financial frictions, and estimate the parameters relating to entry and exit to match several key moments documented by [Lee and Mukoyama \(2015\)](#) and [Djankov et al. \(2010\)](#). We find that a revenue-neutral switch from a profit tax to a payout tax increases the overall value of existing firms and future entrants by 7%. This switch redistributes the tax burden from financially constrained firms, who do not make payouts, to unconstrained firms, who do. It encourages entry, because many potential entrants would enter as constrained firms, but exacerbates distortions relating to the choice of debt vs. equity financing.

Before concluding, we describe how our results extend to alternative formulations of firms' financial frictions and discuss several conceptual and practical issues related to our results. Two practical implications of our results are worth highlighting. First, any corporate payout, and in particular dividends and share repurchases, should be taxed at the same rate.<sup>4</sup> Second, the optimal payout tax could be implemented in our current tax

---

<sup>3</sup>If the government could tax and subsidize firms, the optimal policy would trivially undo all financial frictions by taxing unconstrained firms and subsidizing constrained firms.

<sup>4</sup>A corporate payout tax is different from a tax levied on dividends through the personal income tax system, both because the payout

system by making all retained earnings fully deductible. That is, our results rationalize the tax deductibility of interest on debt as part of an optimal corporate tax policy.<sup>5</sup>

This paper is related to several literatures. There is an extensive literature on corporate taxation, surveyed by [Auerbach and Hines Jr \(2002\)](#) and [Graham \(2013\)](#). This literature, which studies the impact of corporate taxes on firms' decisions, largely takes as given the existing structure of corporate taxes. The literature closest to our work studies dividend taxation in the personal income tax system. The "old view" (e.g., [Poterba and Summers \(1984\)](#)) is that dividend taxes raise the cost of equity financing, distorting investment decisions. The "new view" (e.g., [Korinek and Stiglitz \(2009\)](#)) is that firms, except at the beginning of their life-cycle, do not actively issue equity, and as a result dividend taxes are not distortionary for existing firms. Our model embeds this second perspective into a setting with financial frictions and asymmetric information.

A related literature argues that corporate taxes can be used to correct managerial distortions. This is the "agency view", recently analyzed in [Chetty and Saez \(2010\)](#). By assuming that firms maximize the expected value of dividends, we study optimal revenue-raising policies in an environment in which there is no role for corrective policies.<sup>6</sup> We find it useful to separately study corrective and revenue raising taxes, given their additive nature ([Sandmo, 1975](#); [Kopczuk, 2003](#)).

The corporate taxation principle has a strong analogy to the personal taxation principle in [Mirrlees \(1971\)](#), with three key differences.<sup>7</sup> First, in our paper, the government has no particular desire to redistribute resources across firms. In the personal income taxation literature, the government wants to transfer resources from low to high marginal utility individuals. In contrast, our results are not driven by the "incentives vs. equality" tradeoff, but rather by "plucking the goose as to obtain the largest amount of feathers with the least possible amount of hissing." In particular, when there are no revenue needs, optimal corporate taxes are zero. Second, the financial frictions that firms face, which arise from their ability to default or restructure and cannot be circumvented by the government, endogenously restrict the revenue-raising capacity of the government. Third, the curvature of value functions in our model arises endogenously from financial frictions, instead of preferences.

The mechanism design approach we adopt builds on the optimal non-linear taxation literature that follows [Mirrlees \(1971\)](#) and was recently surveyed in [Golosov, Tsyvinski and Werquin \(2016\)](#). The key difference between our paper and this body of work is our focus on firms and financial frictions. While dynamic Mirrleesian models focus on the behavior of households and treat firms as a veil, we emphasize how financial frictions create a meaningful distinction between corporate and personal taxes. Relatedly, even though we study a dynamic private information environment, an Inverse Euler Equation does not arise in our paper. Capital wedges in dynamic Mirrleesian models arise through a Jensen's inequality effect, which requires uncertainty about the household's future type or other relevant variables. Firms in our static model are perfectly informed about their future type (productivity).

---

tax treats all payouts symmetrically (e.g., share repurchases and dividends) and because some shareholders do not pay personal income taxes (e.g., endowments). The clientele effects generated by dividend taxes in the personal income tax code as a result of this second difference ([Allen and Gale, 2000](#)) do not arise with a corporate payout tax.

<sup>5</sup>To our knowledge, only [He and Matvos \(2015\)](#) have provided a rationale for deducting interest on a firm's debt. In their model, deducting interest on debt is desirable because the laissez-faire level of firms' debt is too low to begin with, due to competitive distortions at the industry level.

<sup>6</sup>The simplest case in which this is true is when the manager is paid in proportion to the dividends shareholders receive.

<sup>7</sup>[Salanie \(2011\)](#) traces back the personal taxation principle to [Edgeworth \(1897\)](#), who refers to it as the principle of equimarginal sacrifice: "The richer should be taxed for the benefit of the poorer up to the point where complete equality of fortunes is attained."

Our formulation of financial frictions builds on the work of [Kehoe and Levine \(1993\)](#), [Alvarez and Jermann \(2000\)](#), and, most closely, [Rampini and Viswanathan \(2010\)](#), as we describe in the paper. Like [Li, Whited and Wu \(2016\)](#), we study corporate taxation using a financial frictions model of the firm. Our results also relate to the literature on dynamic contracting under financial frictions. For instance, in [Albuquerque and Hopenhayn \(2004\)](#), the optimal contract for an entrepreneur who faces financial frictions only features payments to the entrepreneur (dividends) once the project operates at the optimal level of capital. The optimal tax policy in our model generates a similar outcome. There are two major differences between our paper and this body of work. First, we focus on government policy and a population of firms, as opposed to a single firm and its outside investors.<sup>8</sup> Second, our results are derived assuming that the government lacks commitment (ensuring that the government does not circumvent the financial frictions), whereas [Albuquerque and Hopenhayn \(2004\)](#) assume that the outside investors can commit to a long-term contract. In considering a government that chooses taxes, spending, and debt optimally, but lacks commitment, our approach follows [Debortoli, Nunes and Yared \(2017\)](#). Our model also emphasizes asymmetric information, a feature it shares with [Clementi and Hopenhayn \(2006\)](#). However, the asymmetric information in our model is about investment opportunities, as opposed to profit realizations. This distinction is important when studying taxation — the government in our model can measure realized profits perfectly, and as a result profit taxes in our model are feasible, but the government has difficulty predicting future profitability. Policy in our model focuses on the allocation of the tax burden, not minimizing tax evasion, although both issues are surely important real-world concerns.

Because we assume that the government must raise revenues by taxing firms, we cannot address the question of whether taxing firms is optimal if there are other sources of revenue. Most, but not all ([Straub and Werning, 2020](#)), of the work on capital taxation under full information ([Judd, 1985](#); [Chamley, 1986](#); [Chari and Kehoe, 1999](#)) finds that long-run capital taxes should be zero. With asymmetric information, taxing capital is optimal, but the welfare gains of capital taxation might be small ([Farhi and Werning, 2012](#)). To our knowledge, there are no results on whether corporate taxes are optimal in a general equilibrium environment with asymmetric information and financial frictions. There is also a large literature on the incidence of corporate taxes, going back to [Harberger \(1962\)](#), and on the related issue of the choice of organizational form — see most recently [Barro and Wheaton \(2019\)](#). Our results can be thought of as a building block towards addressing the more general questions of whether corporate taxation is desirable at all, its incidence in general equilibrium, and its interactions with the taxation of households.

Lastly, the key insight behind the optimality of payout taxation — that constrained firms dislike paying dividends, while unconstrained firms do not — has been discussed at least implicitly in the literature on financial frictions. For example, [Fazzari, Hubbard and Petersen \(1988\)](#) argue that firms that consistently pay large dividends are not likely to be financially constrained, and [Kaplan and Zingales \(1997\)](#) provide direct evidence relating dividend payments to financial constraints. They show that firms that pay more dividends are less likely to report being financially constrained. Consistent with these ideas, the optimal corporate tax uses the payout policy of a firm to determine whether it should be taxed or not.

Section 2 describes the single-date environment common to all results in this paper. Sections 3 and 4 study

---

<sup>8</sup>[Cooley, Marimon and Quadrini \(2004\)](#) extend the framework of [Albuquerque and Hopenhayn \(2004\)](#) to study firm and business cycle dynamics in general equilibrium with a population of firms. Our paper also studies a population of firms, but differs by focusing on optimal taxation.

optimal taxation with full and private information in one- and two-date models. Section 5 presents our main results in an infinite-horizon environment, while Section 6 quantitatively assesses the benefits of switching from profit taxes to payout taxes. Section 7 discusses extensions of the model relating to the nature of the financial friction and to equity issuance, and Section 8 discusses policy implications. Section 9 concludes. The Appendix contains all proofs and derivations, and more details on our quantitative analysis.

## 2 Common Environment

We begin by describing the common structure of a single date that applies to all of the static and dynamic environments studied in this paper. The economy is populated by firms and outside investors. There is also a government, who optimally sets taxes to fulfill a revenue-raising goal.

There is a single consumption good (dollars), which serves as numeraire. Both firms and outside investors are risk-neutral, and discount cash flows at a predetermined gross real interest rate of  $R > 1$ . Figure 1 illustrates the timeline of events within a date. At the beginning of the date, before production occurs, firms make financing and investment decisions. After production and depreciation materialize, firms make payout decisions, pay taxes, repay outside investors, and consider the possibility of defaulting.

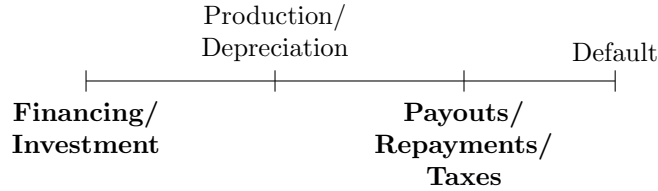


Figure 1: Single Date Timeline

**Financing/Investment stage** Firms are initially endowed with resources  $w_t$  and can raise additional funds  $m_t \geq 0$  from outside investors, as described below.<sup>9</sup> Firms invest these resources in capital  $k_t$ , broadly defined, satisfying the budget constraint

$$k_t \leq w_t + m_t. \quad (1)$$

An investment of  $k_t$  dollars at the beginning of date  $t$  yields  $f(k_t, \theta_t)$  dollars when production occurs, where  $\theta_t \in [0, 1]$  denotes the date  $t$  productivity of a firm. The function  $f(k_t, \theta_t)$  is differentiable in both arguments, and increasing and concave in  $k_t$ . The marginal product of capital is increasing in the firm's productivity  $\theta_t$ , and positive capital is essential for production. That is,  $f_k(k_t, \theta_t)$  is weakly positive, decreasing in  $k_t$ , increasing in  $\theta_t$ , and  $f(0, \theta_t) = 0$ . Capital depreciates neo-classically at a rate  $\delta \in [0, 1]$ .

There exists a first-best level of capital,  $k^*(\theta_t)$ , which is the smallest level of capital such that

$$f_k(k^*(\theta_t), \theta_t) + 1 - \delta = R. \quad (2)$$

<sup>9</sup>See our remarks on page 10 on how to interpret firms' financing decisions.

That is, the first-best level of capital for a firm of productivity  $\theta_t$  at date  $t$  corresponds to the smallest solution to the problem:  $\max_k f(k, \theta_t) + (1 - \delta)k - Rk$ . Once capital exceeds the first-best level, we further assume that  $f(k_t, \theta_t)$  is such that the marginal product of capital remains constant and satisfies

$$f_k(k, \theta_t) + 1 - \delta = R, \quad \forall k > k^*(\theta_t).$$

This assumption mimics the ability of a firm, after reaching its optimal scale, to invest at the risk-free rate. Firms lose nothing by accumulating wealth (because they earn exactly the risk-free rate), but their productivity  $\theta_t$  is bounded above. As a result, once firms accumulate sufficient resources to ensure first-best production, they will be willing to make positive payouts.<sup>10</sup>

The following example provides an explicit illustration of firms' production technology. In this example, the production function is a standard decreasing returns to scale production function, augmented with the ability to earn the risk-free rate once the optimal scale has been reached.

**Example.** (Production function) When numerically solving the model, we assume the following functional form for firms' production technology:

$$f(k, \theta) - \delta k = \begin{cases} \theta A k^\alpha - \delta k & k \leq k^*(\theta) \\ \theta A k^*(\theta) - \delta k^*(\theta) + (R - 1)(k - k^*(\theta)) & k > k^*(\theta), \end{cases} \quad (3)$$

where  $\alpha \in (0, 1)$ ,  $A > 0$ , and the first-best level of capital  $k^*(\theta)$  is given by  $k^*(\theta) = \left(\frac{R-1+\delta}{\alpha\theta A}\right)^{\frac{1}{\alpha-1}}$ . Figure 2 illustrates this example production function for three different productivity levels.

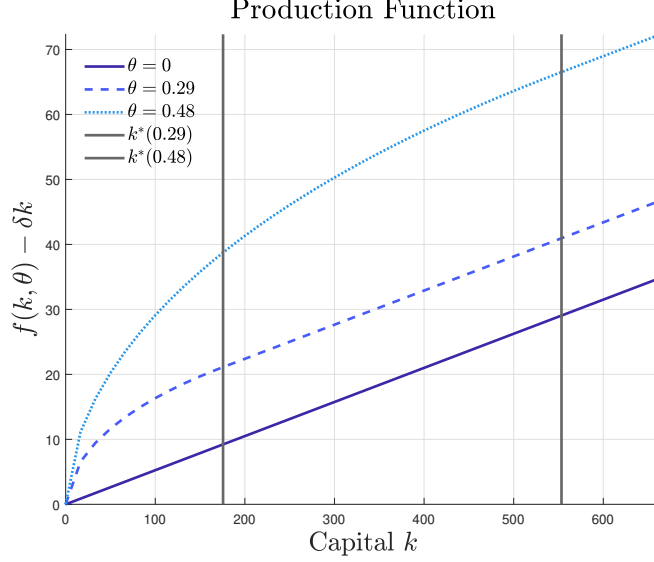
Both outside investors and the government observe firms' initial wealth, financing and investment choices, and production outcomes, as well as firms' date  $t$  productivity. When studying asymmetric information environments, we assume that each firm privately knows their own *future* investment opportunities. That is, at the beginning of date  $t$ , firms privately learn their future productivity,  $\theta_{t+1}$ . However, outside investors and the government do not learn  $\theta_{t+1}$  until the beginning of date  $t + 1$ . Because firms learn  $\theta_{t+1}$  at the beginning of date  $t$ , they can condition all their date  $t$  choices on this information. Because repayments to outside investors and taxes depend on firms' decisions, these can be conditioned on  $\theta_{t+1}$ , subject to incentive-compatibility conditions. That is, the future productivity parameter  $\theta_{t+1}$  corresponds to a firm's "type" about which there is asymmetric information.

**Taxes/Payouts stage** After production and depreciation take place, firms declare a weakly positive payout,  $d_t \geq 0$ . To simplify the exposition, we at times refer to this payout as a dividend. However, as discussed in the introduction,  $d_t$  also encompasses share repurchases and other discretionary transfers of funds to firms' owners. After the dividend is declared, firms must pay back  $b_t \geq 0$  to outside investors and pay taxes  $\tau_t \geq 0$  to the government.<sup>11</sup> At that point, the government/outside investors can block the proposed dividend, which

<sup>10</sup>This formulation contrasts with other models, e.g., DeMarzo and Sannikov (2006), in which firms always earn an above-risk-free rate of return but pay dividends because they discount cash flows at a higher rate than their creditors.

<sup>11</sup>The non-negativity constraint on  $\tau_t$  implies that the government cannot subsidize firms, as discussed in more detail below.





**Note:** Figure 2 shows the production function defined in Equation (3), using the parameters of our quantitative exercise (see Section 6 for details). The plot shows the production functions for firms with productivity  $\theta = 0$ ,  $\theta = 0.29$ , and  $\theta = 0.48$ , productivity levels used in our quantitative exercise, along with the first-best capital levels  $k^*(0.29)$  and  $k^*(0.48)$ .

Figure 2: Production Functions

prevents funds from leaving the firm, although it cannot prevent default. Firms then optimally decide whether or not to default.

As in [Rampini and Viswanathan \(2010\)](#), we consider an environment with i) limited enforcement and ii) no exclusion after default. If a firm pays its obligations, its continuation wealth is given by  $w_{t+1}$ , where

$$w_{t+1} = f(k_t, \theta_t) + (1 - \delta)k_t - d_t - b_t - \tau_t. \quad (4)$$

If  $w_{t+1}$  is not weakly positive, repayment is not feasible and the firm is forced to default. If a firm defaults, its continuation wealth is given by its flow output and a fraction  $1 - \varphi$  of the depreciated capital stock. The continuation wealth of a firm that defaults is  $w^D(k_t, \theta_t)$ , which can be expressed as a function of the firm's capital choice,  $k_t$ , and its date  $t$  productivity,  $\theta_t$ , as follows:

$$w^D(k_t, \theta_t) = f(k_t, \theta_t) + (1 - \varphi)(1 - \delta)k_t, \quad (5)$$

where  $\varphi \in (0, 1]$ . The value of  $\varphi$  captures a form of limited enforcement that restricts the amount of funds that outside investors and the government can receive from a firm. To prevent a firm from continuing with negative wealth, its declared dividend must be no larger than its continuation wealth in the event of default, so the constraint  $d_t \leq w^D(k_t, \theta_t)$  must hold.

After defaulting, a firm cannot be excluded from starting a new firm with its same productivity. To ensure that the government cannot circumvent this friction, the government will not be able to condition its tax policy on firms' histories, only on firms' current wealth level and productivity. Consistent with this assumption, in dynamic environments, we assume that the government lacks commitment. Otherwise, the government might



treat new firms that have defaulted in the past differently, discouraging default. Formally, a firm will not default if

$$d_t + V_{t+1}(w_{t+1}, \theta_{t+1}) \geq \max \{d_t + V_{t+1}(w^D(k_t, \theta_t) - d_t, \theta_{t+1}), V_{t+1}(w^D(k_t, \theta_t), \theta_{t+1})\}, \quad (6)$$

where  $V_{t+1}(w, \theta)$  denotes the continuation value for a firm that starts date  $t + 1$  with resources  $w$  and productivity  $\theta$ . The left-hand side of Equation (6) corresponds to the flow and continuation value of a firm that does not default. The maximization operator in the right-hand side of Equation (6) reflects the ability of the government/outside investors to block a proposed dividend. If the firm proposes an acceptable dividend, the first term in the maximization is the relevant constraint. If the firm's proposed dividend is blocked, the second term in the maximization becomes the relevant constraint. In order not to default, the firm must find both of these options less desirable than repaying its obligations. In this environment, it is without loss of generality to avoid default, and strictly optimal in the presence of deadweight losses associated with default. We therefore treat Equation (6) as a no-default constraint that must be satisfied.

Firms' initial funding,  $m_t$ , and the associated repayment,  $b_t$ , can be contingent on a firm's capital,  $k_t$ , initial wealth  $w_t$ , and current productivity,  $\theta_t$ , but not on the firm's dividend payment. As a result,

$$m_t = R^{-1}b_t. \quad (7)$$

The simplest interpretation of this restriction is that firms only issue one-period debt. An alternative interpretation is that outside investors also know the firms' next date productivity, and hence must break-even for each productivity type.<sup>12</sup>

**Remarks on the environment** The environment considered here is meant to be the simplest one that allows for non-trivial financing, investment, and payout decisions — all of which are necessary to study corporate taxation — while incorporating financial constraints. The following remarks discuss our modeling choices in further detail.

*Remark 1. Additional uncertainty.* Our environment features no uncertainty, with the exception of the process that determines firms' productivity. Introducing observable shocks, under the assumption that both outside investors and the government can condition their payments/taxes on these shocks, does not affect our conclusions, as in [Rampini and Viswanathan \(2010\)](#). Including these shocks would allow us to discuss issues like security design in more detail, at the expense of additional notation.

*Remark 2. Trade-off Theory.* Our environment can be thought of as the zero-uncertainty limit of a model in which firms trade off the tax benefits of issuing debt against the deadweight costs of bankruptcy (trade-off theory) in the presence of non-contractable shocks. As non-contractable uncertainty vanishes, creditors will know ex-ante whether or not the firm will repay its debt, leading to a borrowing constraint along the lines of Equation (6). Focusing on this limit greatly simplifies our analysis. In this limit, the social cost of having too little wealth inside the firm is under-investment as opposed to the deadweight costs of bankruptcy.

---

<sup>12</sup>In this case, there is no uncertainty for the firm or outside investors, and the claims of outsiders can be interpreted as either debt or equity. This interpretation requires that outside investors have an informational advantage over the government, but the government may be able to extract this information almost costlessly, along the lines of [Cr mer and McLean \(1988\)](#).

*Remark 3. Marginal vs. average products.* The productivity parameter  $\theta$  should be interpreted as controlling the marginal product of capital, not the average, since we assume that  $f_{k\theta} \geq 0$  but make no assumption on  $f_\theta$ . For simplicity, we consider a one-dimensional parameter space for  $\theta$ , which links marginal and average products of capital, as in our example production function. Our results could be readily extended to, e.g., two-dimensional parameter spaces capable of distinguishing between average and marginal products.

*Remark 4. Payout interpretation.* In our environment, the agent that receives a firm's payouts is the one who controls the decisions made by that firm. Therefore, if firms maximize shareholder value, payouts in the model exactly correspond to payments to shareholders in reality, which include dividends and share repurchases. This is our preferred interpretation. Alternatively, if one assumes that firms are controlled by managers who maximize the value of their compensation independently of shareholder's value, one could interpret firms' payouts in our model as managerial compensation, which may include wages or in-kind benefits, and payments to outside investors as debt or equity. We discuss this issue in more detail in Section 8. The critical assumption under both interpretations is that the government can distinguish between payments to the agents controlling the firm and payments to outside investors.

*Remark 5. No equity issuance.* Throughout most of the paper, we assume that firms' payouts cannot be negative, so firms' shareholders cannot inject additional funds into the firm. In Section 7.2, we extend our results to a model in which negative dividends (equity issuance) are feasible but costly. If such issuance is feasible and not costly, firms will never be financially constrained.

*Remark 6. Symmetry between taxes and repayments to outside investors.* Taxes and repayments to outside investors enter symmetrically in the model, operating entirely through firms' continuation wealth  $w_{t+1}$ . This formulation ensures that the government cannot circumvent the financial frictions by assessing taxes that are not limited by the possibility of default. Higher taxes will tighten firms' financing constraint in the same way that higher promised repayments would, as implied by Equations (4) through (6).

*Remark 7. No government subsidies.* Since the government's objective is to find the best revenue-raising policy, we purposely restrict the ability of the government to subsidize firms by making tax payments non-negative. This restriction addresses the concern that, in models with financial frictions, the government may find optimal to circumvent all financial frictions by taxing unconstrained firms and subsidizing constrained firms, even without a revenue-raising goal.

Our modeling choices are designed to ensure that the government cannot use taxes to circumvent financial frictions or correct some other distortion in the economy. The only purpose of taxation in our model is to raise revenue. Therefore, if the government did not need to raise any revenue, the optimal policy would involve no taxation of any kind. Setting up our model in this way allows us to focus on the tradeoff between raising revenue and exacerbating financial frictions.

### 3 Full Information

In this section, we study a single-date model in which there is no private information about firms' future productivity. We refer to the date in this single-date model as  $t = 1$ . Subsequently, in Section 4, we study a two-date model in which firms' future productivity is private information.

**Government's problem** To be consistent with later sections, we describe the government's optimal policy problem using a mechanism design approach, even though without private information there are no incentive compatibility constraints. In the rest of the paper, we will introduce private information and exploit direct revelation mechanisms to characterize the optimal policy. Therefore, we proceed as if the government directly determines firms' choice variables  $\{k_1, b_1, d_1, w_2\}$  and taxes  $\tau_1$  for each firm, for a given initial wealth level  $w_1$  and a productivity  $\theta_1$ , which are observable. That is, the solution to the government's problem is here characterized by the non-negative functions  $\{k_1(w_1, \theta_1), b_1(w_1, \theta_1), d_1(w_1, \theta_1), w_2(w_1, \theta_1), \tau_1(w_1, \theta_1)\}$ .

Assuming that firms consume any remaining wealth inside the firm at the end of date one, we can express a firm's continuation value as follows

$$V_2(w_2, \theta_2) = w_2,$$

which implies that the solution to the government's problem is independent of firms' future types. Given this continuation value function, the no-default constraint defined in Equation (6) simplifies to

$$d_1(w_1, \theta_1) + w_2(w_1, \theta_1) \geq w^D(k_1(w_1, \theta_1), \theta_1),$$

where  $w^D(\cdot)$  is defined in Equation (5). This no-default constraint arises from the possibility of a firm complying with the government's mechanism, but then defaulting instead of paying its obligations. It can be interpreted as an interim (after learning its productivity) participation constraint. A second interim participation constraint arises from the possibility of a firm disregarding the government's mechanism entirely. The government can assign infinite taxes to a firm that does this, mechanically inducing default and preventing outside borrowing. As a result, the firm would be limited to investing its initial wealth in capital and then defaulting. This second interim participation constraint simplifies to  $d_1(w_1, \theta_1) + w_2(w_1, \theta_1) \geq w^D(w_1, \theta_1)$ , and will therefore be satisfied if the no-default constraint is satisfied and  $k_1(w_1, \theta_1) \geq w_1$ .

The government's revenue-raising constraint can be expressed as follows. Let  $\mu_1(w_1, \theta_1)$  denote the measure of firms with wealth  $w_1 \in [0, \infty)$  and type  $\theta_1 \in [0, 1]$ . Any tax policy must satisfy

$$R^{-1} \int_0^\infty \int_0^1 \tau_1(w_1, \theta_1) \mu_1(w_1, \theta_1) d\theta_1 dw_1 \geq G + B_1,$$

where  $G > 0$  denotes the required expenditure level and  $B_1 \geq 0$  denotes a predetermined level of government debt that must be repaid. In this single-date model,  $G$  and  $B_1$  play the same role, but we introduce government debt here to connect this single-date model to the multiple-date models studied later. Moreover, the government must respect firms' initial budget constraints, their law of motion for wealth, and the funding terms offered by outside investors, introduced in Equations (1), (4), and (7).

The government's utilitarian objective corresponds to the sum of all firm values:

$$R^{-1} \int_0^\infty \int_0^1 \{d_1(w_1, \theta_1) + w_2(w_1, \theta_1)\} \mu_1(w_1, \theta_1) d\theta_1 dw_1.$$

Anticipating the analysis of multiple-date models, we adopt the measure of firms  $\mu_1$  and the initial government debt  $B_1$  as state variables of the government problem. Given  $\mu_1$  and  $B_1$ , the government chooses an optimal

policy that induces a firm value  $V_1(\cdot)$  denoted by

$$V_1(w_1, \theta_1; \mu_1, B_1) = R^{-1} \{d_1(w_1, \theta_1; \mu_1, B_1) + w_2(w_1, \theta_1; \mu_1, B_1)\},$$

where  $d_1(w_1, \theta_1; \mu_1, B_1)$  and  $w_2(w_1, \theta_1; \mu_1, B_1)$  correspond to the government's optimal dividend and continuation wealth allocations.

Although the government's problem features numerous constraints, it can be simplified as follows. First, a firm's payout policy is irrelevant in this single-date problem, since any remaining wealth will be consumed. Second, firms' initial budget constraints always bind, so the government effectively chooses only investment and taxes for each firm. Finally, since firms only interact through the government's revenue-raising constraint, the government's problem can be studied firm by firm given a Lagrange multiplier  $\chi_1$ , which corresponds to the marginal cost of raising a marginal dollar through taxes. Lemma 1 exploits these observations and formally introduces the government's problem.

**Lemma 1.** (*Government's problem*) *The government's mechanism design problem can be written as*

$$J_1(\mu_1, B_1) = \min_{\chi_1 \geq 0} \left\{ \int_0^\infty \int_0^1 U_1(w_1, \theta_1; \chi_1) \mu_1(w_1, \theta_1) d\theta_1 dw_1 - \chi_1 (G + B_1) \right\},$$

where  $\chi_1$  denotes the Lagrange multiplier associated with the government's revenue-raising constraint and where  $U_1(w_1, \theta_1; \chi_1)$  is given by

$$U_1(w_1, \theta_1; \chi_1) = \max_{k_1 \geq 0, \tau_1 \geq 0} \{R^{-1} \{f(k_1, \theta_1) + (1 - \delta)k_1 - Rk_1\} + w_1 + R^{-1}(\chi_1 - 1)\tau_1\},$$

subject to the constraint

$$w_1 \leq k_1 \leq \frac{w_1 - R^{-1}\tau_1}{1 - R^{-1}\varphi(1 - \delta)}. \quad (8)$$

*Proof.* See the Appendix, Section C.1. □

We refer to  $U_1(\cdot)$  as the social value of a firm, as it combines the private value  $V_1(\cdot)$  with the value of the taxes raised by the government.<sup>13</sup> The optimization problem that defines  $U_1(\cdot)$  can be understood as follows. If the constraints on capital defined in Equation (8) do not bind, the government chooses the first-best level of capital  $k^*(\theta_1)$ , defined in Equation (2). If a firm's wealth exceeds the first-best level of capital without borrowing, that is,  $w_1 > k^*(\theta)$ , the government without loss of generality sets the level of capital to equal the level of wealth, since the the marginal product of capital equals the risk-free rate when  $k > k^*(\theta)$ . If instead the upper bound on capital binds, the marginal product of capital for that firm will be greater than the risk-free rate. These firms are financially constrained.

**Optimal policy** The marginal social benefit of increasing taxes, given by  $R^{-1}(\chi_1 - 1)$ , is the same for all firms. When  $\chi_1 = 1$ , the government is indifferent about which unconstrained firms to tax.<sup>14</sup> The optimal

<sup>13</sup>Exploiting the fact that the multiplier  $\chi_1$  is endogenous and a function of  $\mu_1$  and  $B_1$ , we at times express  $U_1(\cdot)$  as a function of those variables, that is, we write  $U_1(w_1, \theta_1; \mu_1, B_1) = U_1(w_1, \theta_1; \chi_1^*(\mu_1, B_1))$ , where  $\chi_1^*(\mu_1, B_1)$  is the endogenous multiplier.

<sup>14</sup>When  $\chi_1 > 1$ , the government collects taxes on every initially unconstrained firm until the upper bound on capital in Lemma 1 binds. In this case, the optimal policy is uniquely determined for a given  $\chi_1$ . Our analysis focuses on the  $\chi_1 = 1$  case, for reasons that

policy is indeterminate along this dimension. However, the government will never tax a constrained firm if an unconstrained firm could be taxed instead. Therefore, if the government can raise enough revenue from unconstrained firms, it will not tax financially constrained firms. Proposition 1 formalizes these results. Note that the constraints in Equation (8) require that  $\tau_1 \leq \varphi(1 - \delta)w_1$ , so that taxes do not induce default.

**Proposition 1.** (*Optimal tax policy*) For a given measure of firms,  $\mu_1$ , and debt repayment,  $B_1$ , if there exists a positive mass of firms for which  $k^*(\theta_1) < \frac{w_1}{1 - R^{-1}\varphi(1 - \delta)}$  and the government's financing need  $B_1 + G$  is sufficiently small, there exists an optimal policy in which  $\chi_1^*(\mu_1, B_1) = 1$ . If  $\chi_1^*(\mu_1, B_1) = 1$ , there exists an optimal policy in which the government sets

$$\tau_1(w_1, \theta_1; \mu_1, B_1) = \min \left\{ \varphi(1 - \delta)w_1, \frac{\tau_d}{1 + \tau_d} R \max \{w_1 - \bar{w}_1(\theta_1), 0\} \right\},$$

for some  $\tau_d \geq 0$ , where  $\bar{w}_1(\theta)$  is defined as the level of wealth required to achieve the first-best level of capital in the absence of taxes, that is,

$$\bar{w}_1(\theta) = (1 - R^{-1}\varphi(1 - \delta))k^*(\theta). \quad (9)$$

*Proof.* See the Appendix, Section C.2. □

Proposition 1 focuses on a particular policy — a linear tax on excess wealth capped to avoid default — because of its simplicity and because it induces certain properties in the government and firms' value functions that parallel those that emerge in the dynamic model. In particular, under the optimal policy described in Proposition 1, all of the dependence of the firms' value function  $V_1(w_1, \theta_1; \mu_1, B_1)$  on  $(\mu_1, B_1)$  operates through the multiplier  $\chi_1^*(\mu_1, B_1)$  and the tax rate  $\tau_d^*(\mu_1, B_1)$ .

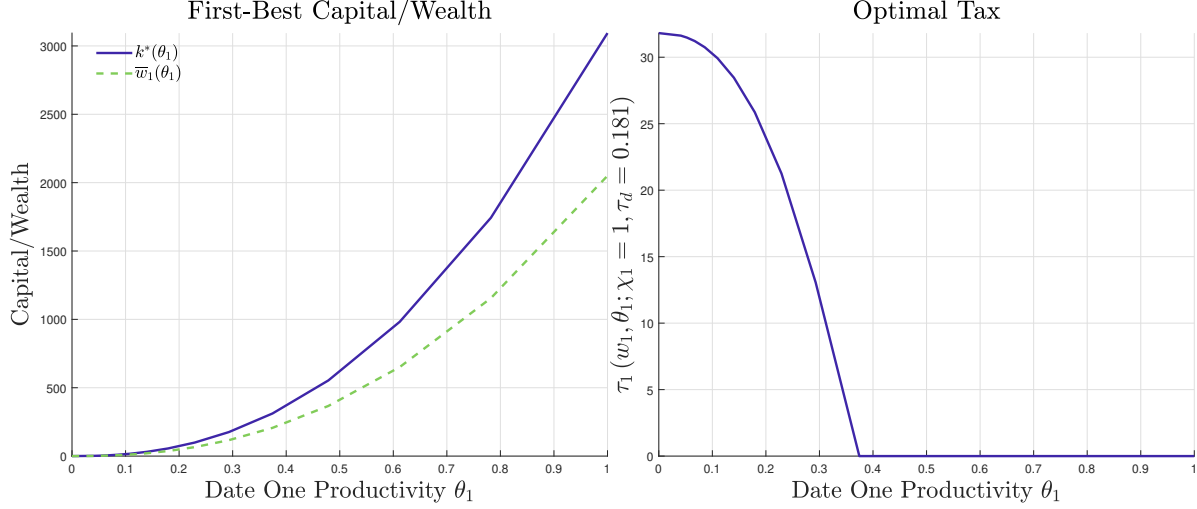
Define  $\tau_1^*(w, \theta; \chi_1 = 1, \tau_d)$  as the taxes raised from a firm with date one wealth  $w$  and productivity  $\theta$  under the policies described in Proposition 1, which are fully characterized by  $(\chi_1 = 1, \tau_d)$ . Figure 3 illustrates the first-best level of capital function  $k^*(\theta_1)$  and the level of wealth required to achieve the first-best level of capital in the absence of taxes,  $\bar{w}_1(\theta_1)$ . It also shows the optimal tax policy,  $\tau_1(w_1, \theta_1; \chi_1 = 1, \tau_d)$ , as a function of firms' current productivity, for a specific  $w_1$  and  $\tau_d$ .

*Remark. (Corporate Taxation Principle)* Proposition 1 illustrates the principle that optimal corporate taxation under financial frictions implies taxing unconstrained firms, which have a low marginal value of funds inside the firm, and leaving untaxed financially constrained firms, which have a high marginal value of funds inside the firm.

Focusing on the  $\chi_1^*(\mu_1, B_1) = 1$  case, we define a collection of properties that  $V_1(\cdot)$  and  $U_1(\cdot)$  must satisfy to be “consistent with a constant payout tax.” This terminology indicates that these properties are satisfied in our dynamic model — described below — if the government implements a payout tax with a tax rate that is constant over time, and no other taxes. Before introducing the dynamic model, Definition 1 is simply a useful construction to study the two-date model, which we do next. Anticipating the two-date and the dynamic models, Definition 1 uses general time  $t$  subscripts. We use the notation  $V_{t,w^+}(\cdot)$  and  $U_{t,w^+}(\cdot)$  to refer to the right derivative of those functions with respect to  $w$ .<sup>15</sup>

are explained below.

<sup>15</sup>Recall that all concave functions are directionally differentiable.



**Note:** The left panel in Figure 3 shows the first-best level of capital,  $k^*(\theta_1)$ , and the level of wealth required to achieve the first-best level of investment in the absence of taxes,  $\bar{w}_1(\theta_1)$ , for firms of different productivity levels. The right panel in Figure 3 shows the optimal tax policy characterized in Proposition 1. Both plots are based on the production function defined in Equation (3) and use the parameters of our quantitative exercise (see Section 6 for details). The right panel uses a dividend tax rate of  $\tau_d = 0.181$  and a value of  $w_1$  equal to the median wealth of firms in the steady state of our quantitative exercise.

Figure 3: Single-Date Optimal Tax

**Definition 1.** (Consistency with a payout tax) Given some  $(\mu_t, B_t)$ , the functions  $V_t(w, \theta; \mu_t, B_t)$  and  $U_t(w, \theta; \mu_t, B_t)$  are *consistent with a constant payout tax* if there exists a tax rate  $\tau_d \geq 0$  and a weakly positive, continuous function  $\bar{w}(\theta)$  such that  $V_t(\cdot)$  and  $U_t(\cdot)$

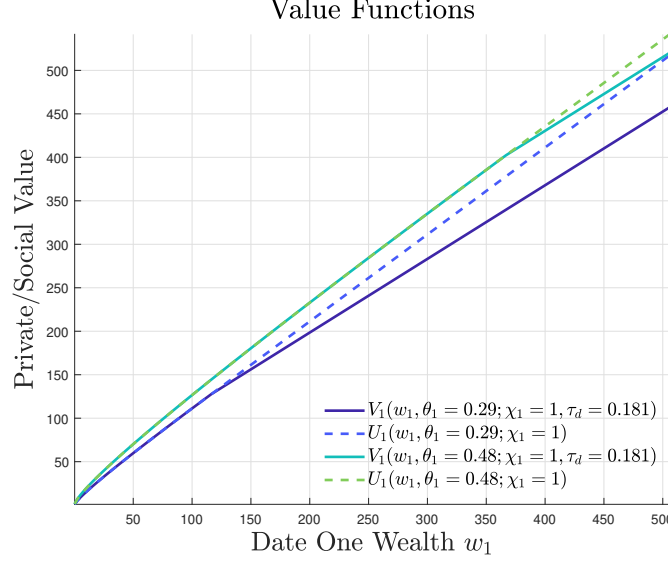
- i) are concave in wealth  $w$ ,
- ii) satisfy, for all  $w > \bar{w}(\theta)$ ,  $V_{t,w^+}(w, \theta; \mu_t, B_t) = \frac{1}{1+\tau_d}$  and  $U_{t,w^+}(w, \theta; \mu_t, B_t) = 1$ , and
- iii) satisfy, for all  $w < \bar{w}(\theta)$ ,  $V_{t,w^+}(w, \theta) > \frac{1}{1+\tau_d}$  and  $U_{t,w^+}(w, \theta; \mu_t, B_t) > 1$ .

This definition can be understood in terms of the “payout boundary”  $\bar{w}(\theta)$ . Consider a firm whose continuation value function  $V_t(\cdot)$  is consistent with a constant payout tax, and suppose that firm faces a tax rate on payouts equal to  $\tau_d$ , so that getting one dollar in payouts would cost  $1 + \tau_d$  dollars in continuation wealth. Such a firm would be willing to pay dividends once the firm’s continuation wealth exceeded  $\bar{w}(\theta)$ , but would prefer not to pay dividends if the continuation wealth was less than  $\bar{w}(\theta)$ .

Corollary 1 makes use of this definition to show that  $V_1(\cdot)$  and  $U_1(\cdot)$  are consistent with a constant payout tax, provided that  $\chi_1^*(\mu_1, B_1) = 1$  and  $\tau_d^*(\mu_1, B_1) \leq \frac{\varphi(1-\delta)}{R-\varphi(1-\delta)}$ .<sup>16</sup>

**Corollary 1.** (Properties of value functions) If  $\chi_1^*(\mu_1, B_1) = 1$  and  $\tau_d^*(\mu_1, B_1) \in \left[0, \frac{\varphi(1-\delta)}{R-\varphi(1-\delta)}\right]$ , the functions  $V_1(w, \theta; \mu_1, B_1)$  and  $U_1(w, \theta; \mu_1, B_1)$  are consistent with a constant payout tax (Definition 1) with  $\tau_d = \tau_d^*(\mu_1, B_1)$  and  $\bar{w}(\theta) = \bar{w}_1(\theta)$  as defined in Equation (9).

<sup>16</sup>If  $\chi_1^*(\mu_1, B_1) = 1$  and  $\tau^*(\mu_1, B_1) > \frac{\varphi(1-\delta)}{R-\varphi(1-\delta)}$ , the functions  $V_1$  and  $U_1$  satisfy the properties of Definition 1 on some interval of wealth  $[0, \tilde{w}]$  but not for larger values of wealth. Our results in the next section could be extended to this case under parameters that ensure wealth remains in this interval.



**Note:** Figure 4 shows the private and social value functions,  $V_1(w_1, \theta_1; \chi_1 = 1, \tau_d = 0.181)$  and  $U_1(w_1, \theta_1; \chi_1 = 1)$ , consistent with a constant payout tax for different wealth levels. This Figure is based on the production function defined in Equation (3) and the parameters used in our quantitative exercise, with  $\tau_d = 0.181$  (see Section 6 for details). The plot shows the private and social value functions for firms with productivity  $\theta_1 = 0.29$  and  $\theta_1 = 0.48$ , two productivity levels used in our quantitative exercise (see Section 6 for details).

Figure 4: Value Functions

*Proof.* See the Appendix, Section C.3. □

These properties summarize the intuitive idea that firms with wealth levels below  $\bar{w}_1(\theta)$  are constrained and untaxed, whereas firms with wealth levels above  $\bar{w}_1(\theta)$  pay a linear tax rate  $\frac{\tau_d}{1+\tau_d}$  on excess wealth. Figure 4 illustrates the properties of functions  $V_1(\cdot)$  and  $U_1(\cdot)$  that are consistent with a constant payout tax.

In the next section, we use this single-date model without private information as the second date in a two-date model with asymmetric information. The functions  $V_1(\cdot)$  and  $U_1(\cdot)$  just characterized become the continuation value functions from the perspective of the initial date. When these continuation value functions are consistent with a constant payout tax, we show that the optimal mechanism at the initial date will be a constant payout tax.

## 4 Private Information: Two-Date Environment

In this section, we study a two-date model in which firms have private information about their *future* productivity,  $\theta_1$ . We refer to the initial date in this two-date model as  $t = 0$ . For simplicity, we assume that all firms start with the same initial wealth  $w_0$  and current productivity  $\theta_0$ . It is straightforward to introduce observable heterogeneity along both dimensions, as we show when studying the dynamic model in the next section. The final date in this two-date model corresponds to the single-date full-information model analyzed in the previous section, in which the government sets an optimal policy given some funding need.



**Feasible and incentive compatible mechanisms** In this section, we study the mechanism design problem of the government at the initial date, which takes into account how the optimal government policy will be set at the final date. The government faces incentive compatibility constraints, because the government must induce firms to truthfully reveal their future productivity parameter,  $\theta_1$ . We consider incentive-compatible direct revelation mechanisms with incentive-compatibility (IC) constraints at the financing/investment stage and the payout stage. That is, we allow for the possibility of a double deviation in the mechanism, in which a firm reports some type  $\theta'_1$  at the investment/financing stage, and then reports a potentially different type  $\theta''_1$  at the payout stage. The dividend allocated to a firm that reports  $\theta'_1$  at the investment/financing stage and then reports  $\theta''_1$  at the payout stage is  $d_0(\theta'_1, \theta''_1)$ . We use the same two-argument notation for other variables determined at the payout stage. For the variables  $k_0$  and  $b_0$ , which are determined at the financing/investment stage and exclusively depend on the first report, we use a single argument notation,  $k_0(\theta'_1)$  and  $b_0(\theta'_1)$ .

Definition 2 describes the set of feasible and incentive-compatible mechanisms. This definition takes a given initial wealth level  $w_t$  and a type  $\theta_t$  as inputs, as well as a continuation value function  $V_{t+1}(\cdot)$ . Anticipating the dynamic model, Definition 2 uses general time  $t$  subscripts.

**Definition 2.** (Feasible and incentive compatible mechanisms) Given an observable initial wealth  $w_t$  and initial productivity  $\theta_t$ , and continuation value function  $V_{t+1}(\cdot)$  for firms values, a feasible and incentive compatible direct revelation mechanism  $m \in \mathcal{M}(w_t, \theta_t, V_{t+1}(\cdot))$  is a collection of non-negative functions  $\{b_t(\theta'), k_t(\theta'), w_{t+1}(\theta', \theta''), d_t(\theta', \theta''), \tau_t(\theta', \theta'')\}$  such that the following constraints are satisfied:

Upper Limit on Dividends:

$$d_t(\theta'_{t+1}, \theta''_{t+1}) \leq w^D(k_t(\theta'_{t+1}), \theta_t), \forall \theta'_{t+1}, \theta''_{t+1}, \quad (10)$$

Budget Constraint:

$$k_t(\theta'_{t+1}) \leq w_t + R^{-1}b_t(\theta'_{t+1}), \forall \theta'_{t+1}, \quad (11)$$

Wealth Accumulation:

$$\begin{aligned} w_{t+1}(\theta'_{t+1}, \theta''_{t+1}) &\leq f(k_t(\theta'_{t+1}), \theta_t) + (1 - \delta)k_t(\theta'_{t+1}) \\ &\quad - d_t(\theta'_{t+1}, \theta''_{t+1}) - b_t(\theta'_{t+1}) - \tau_t(\theta'_{t+1}, \theta''_{t+1}), \forall \theta'_{t+1}, \theta''_{t+1}, \end{aligned} \quad (12)$$

Post-Dividend No-Default:

$$\begin{aligned} d_t(\theta'_{t+1}, \theta''_{t+1}) + V_{t+1}(w^D(k_t(\theta'_{t+1}), \theta_t) - d_t(\theta'_{t+1}, \theta''_{t+1}), \theta_{t+1}) &\leq \\ d_t(\theta'_{t+1}, \theta_{t+1}) + V_{t+1}(w_{t+1}(\theta'_{t+1}, \theta_{t+1}), \theta_{t+1}), \forall \theta_{t+1}, \theta'_{t+1}, \theta''_{t+1}, \end{aligned} \quad (13)$$

Blocked Dividend No-Default:

$$V_{t+1}(w^D(k_t(\theta'_{t+1}), \theta_t), \theta_{t+1}) \leq d_t(\theta'_{t+1}, \theta_{t+1}) + V_{t+1}(w_{t+1}(\theta'_{t+1}, \theta_{t+1}), \theta_{t+1}), \forall \theta_{t+1}, \theta'_{t+1}, \quad (14)$$

Financing/Investment IC:

$$d_t(\theta'_{t+1}, \theta_{t+1}) + V_{t+1}(w_{t+1}(\theta'_{t+1}, \theta_{t+1}), \theta_{t+1}) \leq d_t(\theta_{t+1}, \theta_{t+1}) + V_{t+1}(w_{t+1}(\theta_{t+1}, \theta_{t+1}), \theta_{t+1}), \forall \theta_{t+1}, \theta'_{t+1}, \quad (15)$$

Dividend/Taxes IC:

$$d_t(\theta'_{t+1}, \theta''_{t+1}) + V_{t+1}(w_{t+1}(\theta'_{t+1}, \theta''_{t+1}), \theta_{t+1}) \leq d_t(\theta'_{t+1}, \theta_{t+1}) + V_{t+1}(w_{t+1}(\theta'_{t+1}, \theta_{t+1}), \theta_{t+1}), \forall \theta_{t+1}, \theta'_{t+1}, \theta''_{t+1}, \quad (16)$$

Interim Participation Constraint:

$$V_{t+1}(w^D(w_t, \theta_t), \theta_{t+1}) \leq d_t(\theta_{t+1}, \theta_{t+1}) + V_{t+1}(w_{t+1}(\theta_{t+1}, \theta_{t+1}), \theta_{t+1}), \forall \theta_{t+1}, \quad (17)$$

where  $\mathcal{M}(w_t, \theta_t, V_{t+1})$  denotes set of all such mechanisms.<sup>17</sup>

First, note that the post-dividend no-default constraint combines a no-default constraint and an incentive compatibility constraint. That is, a firm with future productivity  $\theta_{t+1}$  must avoid default when truthfully reporting ( $\theta''_{t+1} = \theta_{t+1}$  in Equation (13)) and also must not want to falsely report a different future productivity and then default. In contrast, the blocked-dividend no-default constraint can be interpreted as an interim participation constraint, which requires that no firm attempts to exit the mechanism after production occurs. Second, note that the government must account for two sets of incentive constraints. The first set of IC constraints applies to the financing/investment stage. These constraints guarantee that firms find it optimal not to deviate when investment and financing from outside investors is determined. The second set of IC constraints applies to the payout stage. These constraints prevent firms from deviating when payouts are determined and taxes are collected. Third, an interim participation constraint arises from the possibility of the firm disregarding the mechanism entirely. As mentioned in the previous section, the government can exclude the firm from markets in this case and induce default, generating the constraint described by (17).

Lemma 2 shows that it is possible to simplify the no-default constraint. It implies that we can restrict our attention to mechanisms that satisfy Equation (18) below and the constraints in Definition 2 excluding the post-dividend no-default constraint.

**Lemma 2.** (*Post-dividend no-default constraint simplified*) Suppose that the value function  $V_{t+1}(\cdot)$  is strictly increasing in wealth and that the Dividend/Taxes IC constraint defined in Equation (16) holds. Then the post-dividend no-default constraint (13) is satisfied if and only if

$$w^D(k_t(\theta'_{t+1}), \theta_t) - d_t(\theta'_{t+1}, \theta''_{t+1}) \leq w_{t+1}(\theta'_{t+1}, \theta''_{t+1}), \forall \theta'_{t+1}, \theta''_{t+1}. \quad (18)$$

*Proof.* See the Appendix, C.4. □

Using the definition of  $w^D(\cdot)$  and the fact that the wealth accumulation constraint holds with equality, Equation (18) can be further simplified to

$$b_0(\theta'_{t+1}) + \tau_0(\theta'_{t+1}, \theta''_{t+1}) \leq \varphi(1 - \delta)k_0(\theta'_{t+1}), \forall \theta'_{t+1}, \theta''_{t+1}.$$

This constraint, which limits the amount of capital the firm can obtain, augments the constraint derived in Rampini and Viswanathan (2010) by treating taxes as additional debt payments.

**Government's problem** Definition 2, which introduces the set of feasible mechanisms, does not include the revenue-raising constraint that the date-zero government faces. The government's budget constraint at date zero determines the level of government debt outstanding at date one,

$$B_1 = R(B_0 + G) - \int_0^1 \tau_0(\theta', \theta') \Pi(\theta' | \theta_0) d\theta', \quad (19)$$

---

<sup>17</sup>In Equations (10) through (17),  $\theta_t$  denotes the initial-date productivity, assumed to be identical for all firms,  $\theta_{t+1}$  denotes the next-date productivity, which is private information to the firms, while  $\theta'_{t+1}$  and  $\theta''_{t+1}$  respectively denote the reports at the financing/investment and payout stages.

where  $\tau_0(\theta, \theta)$  denotes the taxes collected from each type in the government's date-zero mechanism and  $\Pi(\theta'|\theta_0)$  is the likelihood of a firm with date zero productivity  $\theta_0$  having date one productivity  $\theta'$ . The government's date-zero mechanism also induces a date-one distribution of firms' wealth and type:

$$\mu_1(w', \theta') = \int_0^\infty \delta_{\text{dirac}}(w' - w_1(\theta', \theta')) \Pi(\theta'|\theta_0) dw', \quad (20)$$

where  $\delta_{\text{dirac}}(\cdot)$  denotes the Dirac delta function. Subject to the constraints in Equations (19) and (20), the government chooses an optimal mechanism and date-one debt level

$$J_0(w_0, \theta_0, B_0) = \max_{B_1, m \in \mathcal{M}(w_0, \theta_0, V_1(\cdot))} R^{-1} \int_0^1 d_0(\theta', \theta') \Pi(\theta'|\theta_0) d\theta' + R^{-1} J_1(\mu_1, B_1), \quad (21)$$

where  $J_1(\cdot)$ , defined in Equation (9), denotes the government's date one continuation value function, which depends on the induced distribution of types  $\mu_1$  and on the level of outstanding debt  $B_1$ . Since we have assumed for simplicity in this section that all firms have the same initial wealth and type,  $J_0$  is a function of  $w_0$  and  $\theta_0$ , rather than the date-zero joint distribution of firms' wealth and productivity.

The variables  $\chi_1 \geq 1$  and  $\tau_d \geq 0$  fully summarize the policies of the date one government (see Proposition 1), and are themselves determined by  $\mu_1$  and  $B_1$ . Note that the parameter  $\tau_d$  influences the date-one government's policies only if  $\chi_1 = 1$ , which is the case we will focus on in what follows. In this case,  $\tau_1^*(w, \theta; \chi_1 = 1, \tau_d)$  is described by Proposition 1.<sup>18</sup>

The date zero government anticipates that the date one government will implement an optimal policy that satisfies its budget constraint,

$$B_1 + G = R^{-1} \int_0^\infty \int_0^1 \tau_1^*(w, \theta; \chi_1, \tau_d) \mu_1(w, \theta) d\theta dw.$$

From the perspective of the date zero government, this constraint can be thought of as an implementation constraint. Consider the problem of a date zero government who wants to ensure that the date one government chooses policies characterized by  $(\chi_1, \tau_d)$ . To ensure the date one government will do this, it is sufficient<sup>19</sup> for the date zero government to ensure that  $\mu_1$  and  $B_1$  are such that this constraint is satisfied. Combining this constraint with the date zero budget constraint (19), the implementation constraint is

$$B_0 + G + R^{-1}G = R^{-1} \int_0^1 \tau_0(\theta, \theta) \Pi(\theta|\theta_0) d\theta + R^{-2} \int_0^\infty \int_0^1 \tau_1^*(w, \theta; \chi_1, \tau_d) \mu_1(w, \theta) d\theta dw. \quad (22)$$

Let us therefore consider the problem of the government at date zero, assuming that the government first chooses a  $(\chi_1, \tau_d)$  pair for the date one government to implement, and then chooses an optimal mechanism

<sup>18</sup>We have not explicitly described the date one policies in the  $\chi_1 > 1$  case; these are uniquely determined given  $\chi_1$ , but the details do not matter for our argument.

<sup>19</sup>As we show below, the date zero government will choose  $\chi_1 = 1$  if possible. The date one government also prefers to implement  $\chi_1 = 1$  if possible, and is indifferent between all values of  $\tau_d$  that raise the same amount of revenue. Consequently, if some pair  $(\chi_1 = 1, \tau_d)$  satisfies the implementation constraint, the date one government will be willing to implement it.

subject to this implementation constraint. Using Lemma 1 to substitute out the function  $J_1(\cdot)$ , we have

$$\begin{aligned} J_0(w_0, \theta_0, B_0) = & \max_{\chi_1 \geq 1, \tau_d \geq 0} \max_{m \in \mathcal{M}(w_0, \theta_0, V_1(\cdot))} R^{-1} \int_0^1 \{d_0(\theta, \theta) + \chi_1 \tau_0(\theta, \theta)\} \Pi(\theta|\theta_0) d\theta \\ & + R^{-1} \int_0^1 U_1(w_1(\theta, \theta), \theta; \chi_1) \Pi(\theta|\theta_0) d\theta \\ & - \chi_1 (B_0 + G + R^{-1}G) \end{aligned} \quad (23)$$

subject to the implementation constraint (22).

Note that the private continuation value function  $V_1$  enters the definition of the set of feasible mechanisms  $\mathcal{M}$ . In principle, this makes the problem quite complicated, because the value function  $V_1$  depends on the future government policy, which is itself determined by current government policy, and endogenously defines the set of feasible mechanisms. However, because date one government policies are completely characterized by  $(\chi_1, \tau_d)$ , the private continuation value function  $V_1$  is also determined by those variables. Consequently, the set of feasible mechanisms  $\mathcal{M}(w_0, \theta_0, V_1(\cdot))$  is determined by  $(\chi_1, \tau_d)$ , and the inner maximization problem in Equation (23) (i.e. the mechanism design problem) is well-defined. This is the key advantage of studying the date zero government's problem in Equation (23) as opposed to the original problem in Equation (21).

**Firms' problem** Before characterizing the optimal policy, we study the problem that firms face and how the government's optimal mechanism can be implemented via taxes. Anticipating the dynamic model, we use general time  $t$  subscripts when describing the firms' problem. Firms must take the government's tax policies as given; these taxes could be an arbitrary function of the observable state variables  $(w_t, \theta_t)$  and observables firm choices  $(b_t, k_t, d_t, w_{t+1})$ . Subject to these taxes, the firm must raise funds from outside investors, produce, and either pay its obligations and taxes or default. Firms have information (about the future productivity,  $\theta_{t+1}$ ) that the outside investors do not; at least in theory, this complicates the problem between a firm and its outside investors.

However, if the government implements a payout tax in the current date that is consistent with future payout taxes, unconstrained firms will be indifferent between paying dividends in the current date or retaining wealth for the future. Constrained firms will always prefer not to pay any dividends. As a result, both groups of firms' decisions to default will be as-if they paid no dividends (and therefore no taxes). This observation can be used to show that the default decision does not depend on the future productivity  $\theta_{t+1}$ .

**Lemma 3.** (*Default invariance*) Fix some  $w_t > 0$  and  $\theta_t \in [0, 1]$ , and suppose that  $V_{t+1}(\cdot)$  is consistent with a constant payout tax (Definition 1) for some  $\tau_d \geq 0$ , and that the government implements the payout tax

$$\tau_t(w_t, \theta_t, b_t, k_t, d_t, w_{t+1}) = \tau_d d_t.$$

Then a firm will not default if and only if it will not default with  $d_t = \tau_t = 0$ ,

$$f(k_t, \theta_t) + (1 - \delta)k_t - b_t \geq w^D(k_t, \theta_t).$$

*Proof.* See the Appendix, C.5. □

Lemma 3 implies that outside investors have no particular reason to care about the firms' future type  $\theta_{t+1}$  when the government implements this particular payout tax. Because a firm and its outside investors can contract on the level of capital, they will both know with certainty whether or not the firm will default, and the outside investors will limit the firms' borrowing to avoid default. That is, even though there is asymmetric information between firms and outside investors, it has no economic implications provided the government implements this particular form of taxation. Profit taxation, the usual form of corporate taxation, also exhibits this property in our model; see Section 6 and Appendix Section A for details.

Using this insight, we write the problem of a single firm facing a borrowing constraint in Definition 3. Note that we write the no-default constraint as  $w_{t+1} + d_t \geq w^D(k_t, \theta_t)$ , taking advantage of the result above and the observation that, if this constraint binds in the firm problem as we have written it, the firm will not pay dividends. This observation is part of the reason a constant payout tax is optimal: firms that are constrained will not pay dividends in the presence of a constant payout tax, whereas firms that are not constrained are willing to pay dividends in the presence of such a tax.

**Definition 3.** (Firms' problem) Fix some  $w_t > 0$  and  $\theta_t \in [0, 1]$ , and suppose that  $V_{t+1}$  is consistent with a constant payout tax for some  $\tau_d \geq 0$ , and that the government implements the payout tax  $\tau_t(w_t, \theta_t, b_t, k_t, d_t, w_{t+1}) = \tau_d d_t$ . Then the current-date problem of a firm with future type  $\theta_{t+1}$  is

$$\bar{V}_t(w_t, \theta_t, \theta_{t+1}) = \max_{b_t \geq 0, k_t \geq 0, w_{t+1} \geq 0, d_t \geq 0} R^{-1} \{d_t + V_{t+1}(w_{t+1}, \theta_{t+1}; \mu_{t+1}, B_{t+1})\}$$

subject to

$$\begin{aligned} w_{t+1} &\leq f(k_t, \theta_t) + (1 - \delta)k_t - (1 + \tau_d)d_t - b_t, \\ k_t &\leq w_t + R^{-1}b_t, \\ w_{t+1} + d_t &\geq w^D(k_t, \theta_t), \\ d_t &\leq w^D(k_t, \theta_t). \end{aligned}$$

**Optimal policy** We are now in a position to introduce our main result. As we have discussed in context of the firm problem, under a constant payout tax, an unconstrained firm is indifferent between paying dividends and retaining wealth inside the firm. If the date zero government is also indifferent to the timing of dividends (assuming the future government implements policies consistent with a constant payout tax), then the incentive compatibility constraints will not bind in the government's problem.

Formally, we conjecture that there is some set of allocations  $\mathcal{M}^+(w_0, \theta_0)$  that is a superset of  $\mathcal{M}(w_0, \theta_0, V_1(\cdot))$  and such that the optimal allocation  $m^*$  in the government's problem (23) is also the optimal allocation of a relaxed problem in which the government at date zero chooses a mechanism from  $\mathcal{M}^+$ . In particular, let  $\mathcal{M}^+(w_0, \theta_0)$  be the set of allocations satisfying the simplified no-default constraint (18) and the dividend limit (10), initial budget constraint (11), and wealth accumulation constraint (12). Note that these are the exact analogs of the constraints facing a firm with a constant payout tax (Definition 3), and that  $\mathcal{M}^+$  does not depend on the continuation value function  $V_1(\cdot)$ .

It follows almost immediately in the relaxed problem that  $\chi_1 = 1$  (this maximizes firm welfare) and that the multiplier on the implementation constraint is zero ( $\tau_d$  now only enters the implementation constraint). Let us therefore consider the problem, taking the tax rate  $\tau_d$  as given,

$$m^* \in \arg \max_{m \in \mathcal{M}^+(w_0, \theta_0)} R^{-1} \int_0^1 \{d_0(\theta, \theta) + \tau_0(\theta, \theta) + U_1(w_1(\theta, \theta), \theta; 1)\} \Pi(\theta|\theta_0) d\theta_1.$$

Our main result studies a generalized version of this mechanism design problem. We show that the optimal mechanism  $m^*$  is equivalent to a payout tax at rate  $\tau_d$  and is a member of  $\mathcal{M}(w_0, \theta_0, V_1)$ , provided that functions  $V_1$  and  $U_1$  are consistent with a constant payout tax (Definition 1) for that value of  $\tau_d$ .

**Proposition 2.** (*Constant payout tax implementation*) *For any  $w_t > 0$  and  $\theta_t \in [0, 1]$ , if the continuation value functions  $V_{t+1}(\cdot)$  and  $U_{t+1}(\cdot, \cdot; 1)$  are consistent with a constant payout tax for some  $\tau_d \geq 0$  (Definition 1), then there exists an optimal mechanism*

$$m^* \in \arg \max_{m \in \mathcal{M}^+(w_t, \theta_t)} R^{-1} \int_0^1 \{d_t(\theta, \theta) + \tau_t(\theta, \theta) + U_{t+1}(w_{t+1}(\theta, \theta), \theta; 1)\} \Pi(\theta|\theta_t) d\theta$$

such that  $m^* \in \mathcal{M}(w_t, \theta_t, V_{t+1}(\cdot))$  and that can be implemented by a constant payout tax at rate  $\tau_d$ , meaning that  $\tau_t(\theta, \theta') = \tau_d d_t(\theta, \theta') \forall \theta, \theta'$  and that  $(b_t(\theta), k_t(\theta), d_t(\theta, \theta), w_{t+1}(\theta, \theta))$  are, for each type  $\theta$ , a solution to the firm's problem (Definition 3).

*Proof.* See the Appendix, Section C.6. □

Proposition 2 shows that a constant payout tax can raise a dollar of revenue while reducing firm values by exactly one dollar, that is, without creating any additional distortions in the economy. From this result, it is a small step to demonstrate that such a tax is optimal in this two-date environment, provided that it raises sufficient revenue. We simply verify that, if the government's net funding need  $B_0 + G + R^{-1}G$  is not too large, the government can raise enough revenue using a tax rate  $\tau_d$ , and that the tax rate  $\tau_d$  is small enough that the functions  $V_1$  and  $U_1$  are consistent with a constant payout tax.

**Proposition 3.** (*Optimal tax policy*) *If the government financing needs  $B_0 + G(1 + R^{-1})$  are strictly positive and a payout tax  $\tau_0 = \tau_d d_0$  raises a sufficient amount of funds for some  $\tau_d \in \left(0, \frac{\varphi(1-\delta)}{R-\varphi(1-\delta)}\right]$ , then such a tax implements the optimal mechanism.*

*Proof.* See the Appendix, Section C.7. □

Proposition 3 shows that the principle of taxing only unconstrained firms remains valid even when the government has asymmetric information about firms' future investment opportunities. The constant payout tax allows the government to cleanly separate financially constrained and unconstrained firms while raising revenue, precisely because paying dividends is costly for financially constrained firms but not for unconstrained firms. To ensure by Corollary 1 that  $V_1(\cdot)$  and  $U_1(\cdot)$  are consistent with a constant payout tax, Proposition 3 requires that the payout tax rate  $\tau_d$  be less than  $\frac{\varphi(1-\delta)}{R-\varphi(1-\delta)}$ . Depending on primitives, they may continue to be consistent with a constant payout tax over the relevant wealth interval at higher tax rates; we have chosen for

expositional purposes not to make Proposition 3 as general as possible. We make this choice in part because the reason that the functions  $V_1(\cdot)$  and  $U_1(\cdot)$  might cease to be consistent with a constant payout tax at high tax rates is related to the model ending after two periods. In our dynamic model, described in the next section, this issue will not arise. For the same reason, we do not discuss in the two-date environment what happens when the government needs more funds than can be raised by a constant payout tax, deferring discussion of this issue to our dynamic model.

## 5 Private Information: Infinite-Horizon Environment

In this section, we extend the results of the previous section to an infinite-horizon context. Formally, we study stationary Markov sub-game perfect equilibria, in which the government's policies are a function of the measure of firms  $\mu_t(w, \theta)$  and the level of government debt  $B_t$ . As in the two-date model, at each date  $t$ , the government inherits a measure of firms  $\mu_t(w, \theta)$  and a level of government debt  $B_t$ , and takes as given the continuation value functions  $J_{t+1}(\mu_{t+1}(\cdot), B_{t+1})$  and  $V_{t+1}(w, \theta; \mu_{t+1}(\cdot), B_{t+1})$ . Firm productivity follows an exogenous Markov process that allows firms to exit, as described below. The transition probability  $\Pi(\theta_{t+1}|\theta_t)$  denotes the probability that a firm with productivity  $\theta_t$  at date  $t$  will have productivity  $\theta_{t+1}$  at date  $t+1$ . As in the previous section, firms — but not the government — learn  $\theta_{t+1}$  at the start of date  $t$ . This is the key form of private information in the model.

For each firm with observable wealth and productivity  $(w, \theta)$  in the support of  $\mu_t(w, \theta)$ , and for a given  $V_{t+1}(\cdot)$ , the government designs a mechanism  $m(w, \theta) \in \mathcal{M}(w, \theta, V_{t+1}(\cdot))$ , as described in Definition 2. As in the two-date model, the function  $V_{t+1}(\cdot)$  depends on  $\mu_{t+1}(\cdot)$  and  $B_{t+1}$ , whose evolution we describe below. Because the government now designs many mechanisms, one for each observable  $(w, \theta)$ , we use the notation  $\tau_t(\theta', \theta''; w, \theta)$  to describe the taxes assigned to a firm with observable characteristics  $(w, \theta)$  that reports  $\theta'$  at the financing/investment stage and  $\theta''$  at the payout stage. We use the same notation for dividends and continuation wealth. For capital and outside funding, which depend only on the initial report, we use the notation  $k_t(\theta'; w, \theta)$  and  $b_t(\theta'; w, \theta)$ .

First, we describe how we introduce in the infinite-horizon environment i) firm exit, ii) firm entry, and iii) a government spending decision. Introducing entry and exit allows us to study a model with a steady state population of firms, to incorporate a debt/equity tradeoff, to study how corporate taxation affects firms' entry decisions, and to describe the revenue Laffer curve. Introducing a government spending decision allows us to discuss the behavior of the government when it cannot raise a given amount of revenue. Subsequently, we formally describe the government's problem. Our main result shows again that, under some assumptions, an equilibrium exists in which the private and social value functions are consistent with a constant payout tax, and the government chooses each date to implement a constant payout tax.

**Firm Exit** Firms with the lowest type,  $\theta = 0$ , are “exiting.” Exit is not default: it is possible for a firm to shut down by liquidating and paying off its taxes and repaying outside investors. Because default does not occur in equilibrium in our model, exit is the only way in which firms leave the economy.

Each date, some firms learn that they will become exiting at the next date. The optimal scale for these firms



is zero ( $k^*(0) = 0$ ), and they earn a return on wealth equal to the risk-free rate. Exiting is an absorbing state: once a firm becomes exiting, it remains exiting until it reaches zero continuation wealth, at which point it truly exits. Formally,  $\Pi(\theta_{t+1}|0) = \delta_{\text{dirac}}(\theta_{t+1})$ , and

$$V_t(w, 0; \mu_t(\cdot), B_t) = R^{-1}(d_t(w, 0) + V_{t+1}(w_{t+1}(w, 0), 0; \mu_{t+1}(\cdot), B_{t+1})).$$

Because the government observes a firm's current type, it knows whether or not a firm is currently exiting. Moreover, because exiting is an absorbing state, there is no asymmetric information between the government and an exiting firm. For this reason, we use the notation  $d_t(w, 0)$  and  $w_{t+1}(w, 0)$  to denote the dividend and continuation wealth of an exiting firm. A firm with productivity  $\theta_t$  at date  $t$  will be forced to become exiting at date  $t + 1$  with probability  $\Pi(0|\theta_t)$ .

As in the two-date model just studied, firms privately know their current and next date productivity, including whether or not they are exiting, at the beginning of the current date, but the government only observes firms' current productivity. As a result, the government can identify which firms are exiting, but cannot directly observe which firms will become exiting next date. We allow firms to voluntarily become exiting next date, that is, to set their next date productivity  $\theta_{t+1}$  to zero, instead of or in addition to defaulting. If the government does not want a firm to voluntarily exit (which it will not), the government must ensure that the firm has an incentive to continue if it does not default,

$$V_{t+1}(w_{t+1}, \theta_{t+1}; \mu_{t+1}(\cdot), B_{t+1}) \geq V_{t+1}(w_{t+1}, 0; \mu_{t+1}(\cdot), B_{t+1}), \quad (24)$$

and that it has no incentive to deviate by both defaulting and exiting,

$$d_t + V_{t+1}(w_{t+1}, \theta_{t+1}; \mu_{t+1}(\cdot), B_{t+1}) \geq \max \{d_t + V_{t+1}(w_t^D - d_t, 0; \mu_{t+1}(\cdot), B_{t+1}), V_{t+1}(w_t^D, 0; \mu_{t+1}(\cdot), B_{t+1})\}. \quad (25)$$

This exit decision is made at the end of date  $t$ , when the firm knows  $\theta_{t+1}$  but not  $\theta_{t+2}$ . In the solution to the government's problem, these constraints will be satisfied in any allocation satisfying the no-default constraint, and the government does not want any firm to exit prematurely.<sup>20</sup>

**Firm Entry** We next describe firm entry. At the beginning of date  $t$ , before the date  $t$  government designs its mechanism, a measure of potential entrants,  $e(\hat{w}, \theta')$ , enter the economy with initial resources  $F + \hat{w} > 0$  and next date type  $\theta'$ .<sup>21</sup> Each potential entrant faces the same fixed cost of entry,  $F > 0$ , and can choose how much resources to put into the firm,  $w_E \leq \hat{w}$ . If a potential entrant chooses to enter, it begins to produce next date with type  $\theta'$  and entry wealth  $w_E$ .

Because a potential entrant begins operation in the next date, it makes its entry decision based on the continuation value  $V_{t+1}(\cdot)$ . This in turn implies that the firm's entry decision depends on the firm's expectations

<sup>20</sup>The form of exit considered here involves shutting down the firm. One could also consider the possibility that firms may shift their activities to a different tax jurisdiction. This possibility may limit the taxes the government could collect but otherwise leaves the problem unchanged.

<sup>21</sup>Assuming that the mass of potential entrants  $e(\hat{w}, \theta')$  is the same each date simplifies the exposition but it is not necessary.

of  $\mu_{t+1}(\cdot)$  and  $B_{t+1}$ . Because the entry decision occurs before the date  $t$  government designs its mechanism, these expectations are a function of the date  $t$  state variables  $\mu_t(\cdot)$  and  $B_t$ . We define  $w_E(\hat{w}, \theta; \mu_t(\cdot), B_t)$  as the level of entry wealth that maximizes a potential entrant's expected value conditional on entry,

$$w_E(\hat{w}, \theta'; \mu_t(\cdot), B_t) \in \arg \max_{w \in [0, \hat{w}]} \mathbb{E}[V_{t+1}(w, \theta'; \mu_{t+1}(\cdot), B_{t+1}) | \mu_t(\cdot), B_t] - w.$$

In the equilibria that we consider, there is a single optimal level of entry wealth  $w_E$  for each  $(\hat{w}, \theta')$ . A potential entrant will decide to enter if the benefits of entry exceed the fixed cost,

$$\mathbb{E}[V_{t+1}(w_E(\hat{w}, \theta; \mu_t(\cdot), B_t), \theta; \mu_{t+1}(\cdot), B_{t+1}) | \mu_t(\cdot), B_t] - w_E(\hat{w}, \theta; \mu_t(\cdot), B_t) \geq F.$$

The measure of firms entering the economy during date  $t$  (i.e., starting operations at date  $t+1$ ) with wealth  $w$  and type  $\theta$  therefore is

$$e_t(w, \theta; \mu_t, B_t) = \int_0^\infty \int_0^\infty \mathbf{1}\{\mathbb{E}[V_{t+1}(w_E(\hat{w}, \theta; \mu_t(\cdot), B_t), \theta; \mu_{t+1}(\cdot), B_{t+1}) | \mu_t(\cdot), B_t] - w_E(\hat{w}, \theta; \mu_t(\cdot), B_t) \geq F\} \\ \times \delta_{\text{dirac}}(w_E(\hat{w}, \theta; \mu_t(\cdot), B_t) - w) e(\hat{w}, \theta) d\hat{w}dw, \quad (26)$$

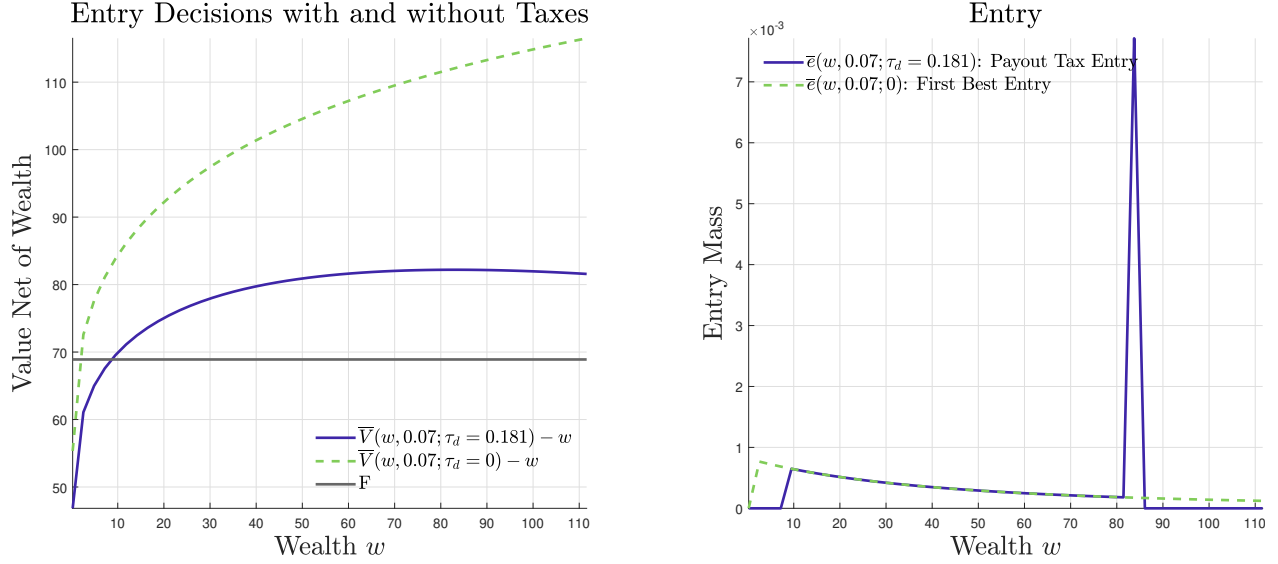
where  $\delta_{\text{dirac}}(\cdot)$  denotes the Dirac delta function and  $\mathbf{1}\{\cdot\}$  denotes the indicator function.

Entry adds a new channel through which future government policies matter. The date  $t$  government cannot affect entry at date  $t$ , because potential entrants have already made their decisions. However, through the impact of its policies on  $\mu_{t+1}(\cdot)$  and  $B_{t+1}$ , the date  $t$  government might be able to affect the equilibrium values of  $\mu_{t+2}(\cdot)$  and  $B_{t+2}$ , which will in turn affect the entry decisions made at date  $t+1$ . Intuitively, the date  $t$  government recognizes that by setting a policy that increases the debt burden of future governments, those governments will have to increase taxes in the future, which deters  $t+1$  firms from entering.

Taxes distort entry in our model on both the extensive and intensive margins. Potential entrants with relatively low outside wealth  $\hat{w}$  and productivity  $\theta'$  might choose not to enter at all, if taxes lower the continuation value function  $V_{t+1}$  to the point that entry no longer justifies the fixed costs. More productive entrants, particularly those with high levels of outside wealth  $\hat{w}$ , will still choose to enter, but enter with lower levels of wealth  $w_E$  than they would have chosen in the absence of taxes (assuming the marginal tax rate is positive). Our model is designed to tractably capture both of these margins.

The left panel of Figure 5 below plots  $V_{t+1}(w, \theta; \cdot) - w$  for one particular value of  $\theta$  (a low one) from our quantitative analysis, assuming a constant payout tax rate ( $\tau_d = 0$  or  $\tau_d = 0.181$ ). The right panel of Figure 5 shows the corresponding distribution of entrants.

We can observe both extensive and intensive margin distortions in Figure 5. At low levels of outside wealth  $\hat{w}$ , entry is deterred by taxation, meaning that some firms will choose not to enter due to taxation. This is the extensive margin distortion visible for low wealth in the right panel of Figure 5. At high levels of outside wealth, firms will choose to enter with less wealth than they would enter with in the absence of taxation. This intensive margin distortion leads to the bunching visible at higher levels of wealth in the right panel of Figure 5. This effect captures the idea that taxing payouts to shareholders but not debt holders leads to debt crowding out equity.



**Note:** Figure 5 illustrates the extensive and intensive margins of entry distortion under payout taxation for one particular productivity level ( $\theta = 0.07$ ) from our quantitative analysis (see Section 6 for details). The left panel plots the private value functions net of wealth,  $\bar{V}(w, \theta; \tau_d = 0.181) - w$  and  $\bar{V}(w, \theta; \tau_d = 0) - w$ . The right panel plots the entering mass of firms under payout taxation and in the absence of taxation (first-best).

Figure 5: Entry with and without Payout Taxation

**The Population of Firms** Having described the entry and exit of firms, we next discuss the evolution of the population of firms. The date  $t$  government inherits a measure of firms with wealth  $w$  and productivity  $\theta$  given by  $\mu_t(w, \theta)$  and, through its policies, influences the measure  $\mu_{t+1}(w, \theta)$  that carries over to the next date. We denote by  $w_{t+1}(\theta', \theta'; w, \theta)$  the continuation wealth of a firm with current wealth  $w$ , current productivity  $\theta$ , and future productivity  $\theta'$ , assuming truthful reporting. The law of motion for the measure of firms with different levels of  $w'$  and  $\theta'$  can be expressed as

$$\begin{aligned} \mu_{t+1}(w', \theta') &= e_t(w', \theta'; \mu_t(\cdot), B_t) + \int_0^\infty \int_{0^+}^1 \delta(w_{t+1}(\theta', \theta'; w, \theta) - w') \Pi(\theta' | \theta) \mu_t(w, \theta) d\theta dw \\ &\quad + \mathbf{1}\{\theta' = 0\} \int_{0^+}^\infty \delta_{\text{dirac}}(w_{t+1}(w, 0) - w') \mu_t(w, 0) dw, \end{aligned} \quad (27)$$

where the last element of Equation (27) guarantees that exiting firms remain exiting until they reach zero wealth. Relative to Equation (20) — its counterpart in the two-date model — Equation (27) includes entry, incorporates heterogeneity in the initial distribution of  $(w, \theta)$ , and treats exiting firms as a special case. One benefit of our assumptions on the timing of firm entry is that Equation (27) is not a fixed-point equation, because entry decisions are a function of  $\mu_t(\cdot)$  and  $B_t$  instead of  $\mu_{t+1}(\cdot)$  and  $B_{t+1}$ .

**Government Spending and Debt Dynamics** We next describe the evolution of government debt  $B_t$ , which is the other state variable in the government's problem. The government can borrow and save freely, and choose a level of spending, without commitment. If the government chooses to spend  $G_t$  this date, the next date's debt

level is

$$B_{t+1} = R(B_t + G_t) - \int_0^\infty \int_{0^+}^1 \int_0^1 \tau_t(\theta', \theta'; w, \theta) \Pi(\theta' | \theta) \mu_t(w, \theta) d\theta' d\theta dw - \int_{0^+}^\infty \tau_t(w, 0) \mu_t(w, 0) dw, \quad (28)$$

where  $\tau_t(\theta', \theta'; w, \theta)$  corresponds to the tax revenue raised under truthful reporting from a firm with wealth  $w$ , current productivity  $\theta > 0$ , and next date's productivity  $\theta'$ , and  $\tau_t(w, 0)$  corresponds to the revenue raised from exiting firms. Relative to Equation (19) — its counterpart in the two-date model — Equation (19) allows for heterogeneity in the initial distribution of  $(w, \theta)$  and treats exiting firms as a special case.

Allowing the government to adjust its spending level simplifies the analysis of off-equilibrium behavior and allows us to discuss what happens when a constant payout tax cannot raise sufficient revenue. We assume that the government derives the following per date flow benefit/cost from spending an amount  $G_t$ :

$$u(G_t) = \begin{cases} (G_t - \bar{G}), & \text{if } G_t \geq \bar{G} \\ -\bar{\chi}(\bar{G} - G_t), & \text{if } G_t < \bar{G}, \end{cases}$$

where  $\bar{\chi} > 1$  and  $\bar{G} > 0$ . We interpret  $\bar{G}$  as the level of socially useful government spending. Any spending beyond this target ( $G_t > \bar{G}$ ) has a marginal value that is equal to the marginal value of firm payouts in the government's objective function. That is, government spending above  $\bar{G}$  can be thought of as equivalent to lump sum transfers to firm owners, which explains why the government values this extra spending and payouts to the firms equally.<sup>22</sup> In contrast, the marginal value of government spending below  $\bar{G}$  is  $\bar{\chi} > 1$ , meaning that this spending generates a higher social value than firm payouts. In most of the equilibria we study, the government sets  $G$  equal to  $\bar{G}$  at all times, because it is socially useful for the government to tax firms so that it can spend  $\bar{G}$ , but serves no social purpose to increase taxes beyond this point.<sup>23</sup>

**Government's Problem** Following the notation of the two-date model, we denote the government's value function by  $J_t(\mu_t(\cdot), B_t)$  and the firms' value function, which corresponds to the net present value of dividends under the current and future optimal policies of the government, by  $V_t(w, \theta; \mu_t(\cdot), B_t)$ . As mentioned above, the Markov nature of the equilibrium we consider implies that the government takes the continuation value functions  $J_{t+1}(\mu_{t+1}(\cdot), B_{t+1})$  and  $V_{t+1}(w', \theta'; \mu_{t+1}(\cdot), B_{t+1})$  as given. Given these, it designs each date for each observable type  $(w, \theta)$  a mechanism of the form described in Definition 2, with the additional non-voluntary-exit constraints, introduced in Equations (24) and (25). Formally, we denote by  $\mathcal{M}_D(w, \theta, V_{t+1}(\cdot))$  the set of mechanisms that satisfy Definition 2 and Equations (24) and (25).

The Markov sub-game perfect equilibrium of the game between the current government and future govern-

<sup>22</sup>Note that this is not the same as making a lump sum transfer to the firm itself, which would circumvent financial frictions.

<sup>23</sup>The value of  $\bar{\chi}$  also acts as an upper bound on the marginal cost to firms of raising a marginal dollar of tax revenue (the multiplier on Equation (28)), which maps to  $\chi_1$  in the two-date model. If the marginal cost of taxation reaches  $\bar{\chi}$ , the government will decide to spend less than  $\bar{G}$  instead of increasing taxes.

ments can be expressed recursively as

$$\begin{aligned}
J_t(\mu_t(\cdot), B_t) = & \max_{B_{t+1}, G_t, \{m_t(w, \theta) \in \mathcal{M}_D(w, \theta, V_{t+1}(\cdot))\}_{w \in \mathbb{R}_+, \theta \in [0,1]}} u(G_t) + \\
& + R^{-1} \int_0^\infty \int_{0^+}^1 \int_0^1 d_t(\theta', \theta'; w, \theta) \Pi(\theta' | \theta) \mu_t(w, \theta) d\theta' d\theta dw \\
& + R^{-1} \int_{0^+}^\infty d_t(w, 0) \mu_t(w, 0) dw + R^{-1} J_{t+1}(\mu_{t+1}(\cdot), B_{t+1}),
\end{aligned} \tag{29}$$

subject to Equations (27) and (28), which define the evolution of the population firms and the level of debt.

Relative to Equation (21) (its counterpart in the two-date model), the infinite-horizon government problem incorporates heterogeneity in the initial distribution of  $(w, \theta)$ , treats exiting firms as a special case, allows for firm entry, and includes the no-voluntary-exit constraints. The equilibrium must also satisfy a no-Ponzi and transversality-type condition, which we describe below.

The solution to this problem induces a value function for the non-exiting firms,

$$V_t(w, \theta; \mu_t(\cdot), B_t) = R^{-1} \int_0^1 \{d_t(\theta', \theta'; w, \theta, \mu_t(\cdot), B_t) + V_{t+1}(w_{t+1}(\theta', \theta'; w, \theta, \mu_t(\cdot), B_t), \theta'; B_{t+1}, \mu_{t+1}(\cdot))\} \Pi(\theta' | \theta) d\theta'$$

and for exiting firms,

$$V_t(w, 0; B_t, \mu_t(\cdot)) = R^{-1} \{d_t(w, 0; \mu_t(\cdot), B_t) + V_{t+1}(w_{t+1}(w, 0; \mu_t(\cdot), B_t), 0; B_{t+1}, \mu_{t+1}(\cdot))\},$$

where  $d_t(\cdot, \mu_t(\cdot), B_t)$  and  $w_{t+1}(\cdot, \mu_t(\cdot), B_t)$  denote policies under the government's optimal mechanism given the state variables  $\mu_t(\cdot)$  and  $B_t$ .

Sub-game perfection requires that each government optimizes, meaning that  $J_t(\mu(\cdot), B) = J_{t+1}(\mu(\cdot), B) = J(\mu(\cdot), B)$ . We study stationary equilibrium in which the value function is also stationary,  $V_t(w, \theta; \mu(\cdot), B) = V_{t+1}(w, \theta; \mu(\cdot), B) = V(w, \theta; \mu(\cdot), B)$ .<sup>24</sup> The equilibrium must satisfy the transversality conditions

$$\begin{aligned}
\lim_{s \rightarrow \infty} R^{-s} \mathbb{E}[J(\mu_{t+s}(\cdot), B_{t+s}) | \mu_t, B_t] &= 0 \\
\lim_{s \rightarrow \infty} R^{-s} \mathbb{E}[V(w_{t+s}, \theta_{t+s}; \mu_{t+s}(\cdot), B_{t+s}) | w_t, \theta_t, \mu_t, B_t] &= 0,
\end{aligned}$$

which ensure that the value functions  $J(\cdot)$  and  $V(\cdot)$  are solutions to the corresponding sequence problems, and a no-Ponzi condition for the government,

$$\lim_{s \rightarrow \infty} R^{-s} \mathbb{E}[B_{t+s} | \mu_t(\cdot), B_t] \leq 0,$$

which arises from the requirement that the government repay its creditors. To sum up, we study equilibria in which Equations (27), (28), and (29) hold, the Bellman equation for the value function  $V$  and the stationarity restrictions are satisfied, the transversality-type equations and the no-Ponzi condition hold, policies are Markov, and expectations are consistent with those Markov policies. We will show next that there exists such

<sup>24</sup>This restriction is mild, but rules out equilibria in which the government oscillates between different policies that achieve the same value for the government but have different continuation values for specific firms.

an equilibrium in which the government implements a constant payout tax.

Before characterizing the optimal policy, we make the following two remarks.

*Remark 8.* (Markov structure/lack of commitment) The problem that we study here is not equivalent to a single-agent optimization problem. Because the government lacks commitment, it does not internalize the effects that the current date mechanism has on previous governments, via the influence of  $V$  on the feasible set of mechanisms and firms' entry decisions. A government with commitment would internalize these effects and in particular be able to circumvent financial frictions by punishing firms that attempt to default and re-enter.<sup>25</sup> We will demonstrate that, despite this lack of commitment, the government is able to refrain from attempting to completely expropriate firms.

*Remark 9.* (Government's smoothing ability) Allowing the government to borrow, save, and adjust spending is essential to guarantee that a constant payout tax is optimal whenever the population of firms does not start in a steady state. If the government were required to raise a particular amount of tax revenue each date, with no ability to smooth distortions across dates, the government would need to impose different payout tax rates in each date. Because firms can defer dividend payments at no cost by investing at the risk-free rate, if firms expect payout taxes to be lower in the future, the government will not be able to collect any payout taxes today. Consequently, a payout tax could not be optimal in this case. Because the government can borrow and save, and off equilibrium adjust spending, today's government can leave tomorrow's government with a debt level that ensures tomorrow's government will implement the same payout tax rate that today's government implements. Consequently, it is possible for the government to maintain a constant payout tax rate. Relatedly, it should be clear that assuming that the government target spending  $\bar{G}$  is time invariant is not essential, since only the net present value of the stream of government spending matters.

**Optimal policy** First, we describe the notion of implementing a constant payout tax in the infinite-horizon model. We denote by  $\bar{V}(w, \theta; \tau_d)$  the firm value function under a constant payout tax  $\tau_d$ , before it learns its next date type, which corresponds to

$$\bar{V}(w, \theta; \tau_d) = \int_0^1 \bar{V}_t(w, \theta, \theta') \Pi(\theta' | \theta) d\theta', \quad (30)$$

where  $\bar{V}_t(w, \theta, \theta')$ , introduced in Definition 3 for a given a tax rate  $\tau_d$  and a continuation value function  $V_{t+1}(w', \theta'; \mu'(\cdot), B')$ , is defined using  $V_{t+1}(w', \theta'; \mu'(\cdot), B') = \bar{V}(w', \theta'; \tau_d)$ . That is, Equation (30) characterizes  $\bar{V}(w, \theta; \tau_d)$  as functional equation. As shown in Lemma 3 in the context of the two-date model, if the government implements a constant payout tax, firm default decisions will not depend on their future productivity. The firm problem can therefore be analyzed separately for each firm and the firm's value function is the solution to this fixed point problem.

Under a constant payout tax, the government raises taxes in expected net present value terms from a firm equal to  $\tau_d$  times the firm value function  $\bar{V}(w, \theta; \tau_d)$ , because that value function is the net present value of

<sup>25</sup>Even if the government with commitment could not observe the default history of an individual firm, it would be able to statistically discriminate between firms that had previously defaulted and firms that had not. We leave open the question of whether there are non-Markov "sustainable" equilibria (Chari and Kehoe, 1990) in which a government without commitment nevertheless punishes defaulting firms and circumvents the financial frictions. We thank Felipe Varas for pointing out this possibility.

dividends, and taxes are proportional to dividends. Moreover, because firm policies are identical in the presence or absence of a constant payout tax once the firm enters the economy, except that dividends are scaled down,  $\bar{V}(w, \theta; \tau_d) = \frac{1}{1+\tau_d} \bar{V}(w, \theta; 0)$ . Combining these two observations, we can observe that the social value of a firm (the sum of its dividends and taxes, which is the analog of  $U(w, \theta; \chi_1 = 1)$  in the two-date environment) is equal to  $\bar{V}(w, \theta; 0)$ . Lemma 4 shows that these private and social value functions are consistent with a constant payout tax (Definition 1), justifying our terminology.

**Lemma 4.** *The functions  $V_{t+1}(w, \theta; \mu(\cdot), B) = \bar{V}(w, \theta; \tau_d)$  and  $U_{t+1}(w, \theta; \mu(\cdot), B) = \bar{V}(w, \theta; 0)$  are consistent with a constant payout tax (Definition 1) for all  $\tau_d \geq 0$ , for some non-negative function  $\bar{w}(\theta)$ .*

*Proof.* See the Appendix, Section C.8. □

The function  $\bar{w}(\theta)$  again characterizes the wealth level at which a firm chooses to make positive payouts under a constant payout tax. Here, the function  $\bar{w}(\theta)$  is not equal to the function  $\bar{w}_1(\theta)$  that describes the wealth necessary to achieve first-best production in the current date (defined in Proposition 1). In the infinite-horizon model, the relevant level of wealth at which a firm knows that can achieve first-best production regardless of the future evolution of its type satisfies the following functional equation:

$$\bar{w}(\theta) = \max \left\{ \bar{w}_1(\theta), R^{-1} \left( -f(k^*(\theta), \theta) - (1 - \delta - R)k^*(\theta) + \max_{\theta' \in [0,1]; \mu(\theta'|\theta) > 0} \bar{w}(\theta') \right) \right\}. \quad (31)$$

Intuitively, financially unconstrained firms must have enough wealth to achieve first-best production in the current date — the first term in Equation (31) — and also have enough wealth to achieve first-best production in the future after including profits from the current date — the second term in Equation (31). For exiting types,  $\bar{w}(0) = 0$ , for the most productive type,  $\bar{w}(1) = \bar{w}_1(1)$ , and for all other types  $\bar{w}(\theta) \in (0, \bar{w}_1(1))$ . In our numerical example, because it is always possible (although perhaps very unlikely) for non-exiting firms to reach the highest productivity level in the next date, this expression becomes

$$\bar{w}(\theta) = \max \left\{ \bar{w}_1(\theta), R^{-1} (-f(k^*(\theta), \theta) - (1 - \delta - R)k^*(\theta) + \bar{w}_1(1)) \right\}.$$

Let  $\bar{e}(w, \theta; \tau_d)$  be the mass of entering firms under a constant payout tax, defined by (26) with the value function  $\bar{V}(w, \theta; \tau_d)$  in the place of the continuation value. Under the assumption that the government spends exactly its target  $\bar{G}$  each date, and raises revenue from a constant payout tax, the net violation of the government's inter-temporal budget constraint is

$$\begin{aligned} N(\mu, B, \tau_d) &= \frac{\tau_d}{1 + \tau_d} \int_0^\infty \int_0^1 \bar{V}(w, \theta; 0) \mu(w, \theta) d\theta dw \\ &\quad + \frac{\tau_d}{1 + \tau_d} \frac{1}{R - 1} \int_0^\infty \int_0^1 \bar{V}(w, \theta; 0) \bar{e}(w, \theta; \tau_d) d\theta dw \\ &\quad - B - \frac{\bar{G}}{1 - R^{-1}}. \end{aligned} \quad (32)$$

This equation illustrates the Laffer curve present in our model. The first term in this equation concerns the taxation of the existing stock of firms. The equity in these firms is “trapped,” and taxing these firms more



(increasing  $\tau_d$ ) always raises revenue. The second term in this equation is the net tax revenue from firms that will enter in the future. Increasing taxes raises more revenue from these firms holding entry constant, but will reduce entry. In particular, increasing the tax rate  $\tau_d$  both reduces the entering mass  $\bar{e}$  (the extensive margin) and reduces wealth conditional on entry, shifting mass to lower wealth levels (the intensive margin). The last terms in this equation represent the net present value of the current debt and future government (target) spending. The government's inter-temporal budget constraint is satisfied when  $N(\mu, B, \tau_d) = 0$ . Appendix Figure A1 illustrates the function  $N$  using the parameters of our quantitative analysis.

A constant payout tax can satisfy the inter-temporal budget constraint if, given the initial measure of firms  $\mu_0$  and initial debt  $B_0$ , there exists a  $\bar{\tau}_d \geq 0$  such that  $N(\mu_0, B_0, \bar{\tau}_d) \geq 0$ . We consider this case in Proposition 4 below, and discuss what happens when a constant payout tax is not feasible below. To simplify our exposition, we assume that the function  $N(\mu_0, B_0, \tau_d)$  is continuous in  $\tau_d$  in the interval  $[0, \bar{\tau}_d]$ .<sup>26</sup>

We can now introduce our results for the infinite-horizon setting. Formally, we characterize equilibria, in which, on the equilibrium path, i) the government implements a constant payout tax equal to  $\tau_0 \in [0, \bar{\tau}_d]$ , and ii) sets government spending equal to  $\bar{G}$  in all dates after the initial date.<sup>27</sup>

**Proposition 4.** *If there exists a  $\bar{\tau}_d \geq 0$  such that  $N(\mu_0, B_0, \bar{\tau}_d) \geq 0$ , and  $N(\mu_0, B_0, \tau_d)$  is continuous on  $\tau_d \in [0, \bar{\tau}_d]$ , then there exists an equilibrium characterized by a tax rate  $\tau_d \in [0, \bar{\tau}_d]$  in which, on the equilibrium path, the government implements a constant payout tax rate equal to  $\tau_d$ , and chooses a level of spending equal to its target,  $G_t = \bar{G}$ ,  $\forall t > 0$ . If  $B_0 + \frac{\bar{G}}{1-R} > 0$ , then  $\tau_d > 0$ ,  $G_0 = \bar{G}$ , and  $N(\mu_0, B_0, \tau_d) = 0$ .*

*Proof.* See the Appendix, Section C.9. □

Proposition 4 shows that there exists an equilibrium in which the optimal policy sets a constant payout tax rate. Provided that the government needs to raise some revenue ( $B_0 + \frac{\bar{G}}{1-R} > 0$ ), this tax rate will be strictly positive and exactly satisfy the government's inter-temporal budget constraint along the equilibrium path. The intuition behind this equilibrium is essentially identical to the intuition in our static model. In this equilibrium, both the private value function  $\bar{V}(w, \theta; \tau_d)$  and the social value function  $\bar{V}(w, \theta; 0)$  are consistent with a constant payout tax (Definition 1). As a result, constrained firms prefer not to pay dividends, and it is socially optimal for them not to, whereas both unconstrained firms and the government are indifferent about whether the unconstrained firms pay dividends. By Proposition 2, a payout tax in the current date can efficiently separate constrained and unconstrained firms while raising revenue, and is therefore optimal if feasible. Appendix Figure A2 illustrates the optimal policies of firms in such an equilibrium, using the parameters of our quantitative analysis (Section 6).

To sustain this equilibrium, if the date  $t$  government improved the date  $t + 1$  government's fiscal position ( $N(\mu_{t+1}(\cdot), B_{t+1}, \tau_d) > 0$ ), the date  $t + 1$  government would respond by allowing firms to make higher payouts, or equivalently increasing government spending. In contrast, if the date  $t$  government worsened the  $t + 1$  fiscal

<sup>26</sup>This assumption could be derived from more primitive assumptions (i.e. finiteness and continuity in the appropriate sense) on the population of initial firms and of potential entrants.

<sup>27</sup>These equilibria use the intuition of our static model. In particular, we observe that if the Lagrangian version of the government's problem values dividends and taxes equally (the analog of  $\chi_1 = 1$  in our static model), the social value of a firm is equal to its value in the absence of taxes. That is,  $U(w, \theta; \mu(\cdot), B) = \bar{V}(w, \theta; 0)$ , and therefore by Lemma 4, both the private and social value functions in this case are consistent with a constant payout tax. This allow us to invoke Proposition 2, which is the key step of the proof.

position ( $N(\mu_{t+1}(\cdot), B_{t+1}, \tau_d) < 0$ ), “something bad” happens, but our proof of Proposition 4 is silent about exactly what occurs in this case.<sup>28</sup> We discuss this situation heuristically below. Note that this discussion also applies if a constant payout tax cannot satisfy the inter-temporal budget constraint given the initial conditions, meaning that  $N(\mu_0, B_0, \tau_d) < 0 \forall \tau_d \in [0, \infty)$ .

Let  $\chi_t$  denote the multiplier on the constraint (28) in the government’s problem (29) (the analog of  $\chi_1$  from the static problem). Assuming differentiability, the FOC for government debt and optimal government spending yields

$$\bar{\chi} \geq \chi_t = -J_B(\mu_{t+1}(\cdot), B_{t+1}) \geq 1.$$

In the equilibrium we describe in Proposition 4,  $\chi_t = -J_B(\mu_t(\cdot), B_t) = 1$  along the equilibrium path. Now consider what happens when  $N(\mu_0, B_0, \tau_0)$  is close to but less than zero. In this case, it may be possible for the government at date zero to raise additional taxes from unconstrained firms ( $w > \bar{w}(\theta)$ ) in such a way that creates no distortions and satisfies  $N(\mu_1, B_1, \tau_0) = 0$ . If this holds, the equilibrium will be the one described in Proposition 4 from date one onwards, with additional taxation at date zero and with  $\chi_t = 1$ . If this outcome is not possible, then  $\chi_t > 1$ , which leads to the government engaging in distortionary taxation, meaning that it will tax firms that are financially constrained. We speculate in this case that the government will use distortionary taxation for one or more dates, with  $\chi_t$  decreasing over time, until it is able to reach  $\chi_t = 1$ . At that point, it would use a constant payout tax as described in Proposition 4. What makes this case difficult to characterize is that no simplification along the lines of Proposition 2 is possible. With distortionary taxation, the incentive compatibility constraints bind, which means the set of feasible mechanisms is influenced by the value function  $V$ , which is in turn influenced by whatever distortionary taxes will occur in the future. There is, however, an upper bound to the amount of distortionary taxation the government will implement. The multiplier  $\chi_t$  is bounded above by  $\bar{\chi}$ , due to the government’s ability to cut spending. If the cost of additional distortionary taxation is high enough, the government will choose cut spending and then use distortionary taxation to eventually reach  $\chi_t = 1$ . Summarizing, we conjecture that, regardless of initial conditions, the conclusions of Proposition 4 will hold starting from some date  $t \geq 0$ .

These equilibria capture the key results of the static model, which is that if future governments will implement a constant payout tax, the current government can use the same constant payout tax rate to efficiently separate constrained and unconstrained firms. In the next section, we attempt to quantify the benefits of switching from the usual form of corporate taxation to payout taxation. In the subsequent section, we discuss the connections between two key assumptions of the model (the form of the financial friction and the limit on equity issuance) and our results.

## 6 Quantitative Analysis

Our theoretical results show that payout taxation emerges as an optimal policy in our model. In practice, most countries use profits-based corporate taxation (henceforth, profit taxation). In this section we explore quantitatively the magnitude of the gains of switching from profit taxation to payout taxation, as well as the effects on entry (which the government in our theoretical model does not internalize). We find that a revenue-

---

<sup>28</sup>The proof uses a bound on the value function on this domain to show that the government would never choose to enter it.

neutral switch from a profit tax to a payout tax would increase the overall value of existing firms and future entrants by 7%.

**Comparing Profit and Payout Taxes** A profits-based corporate tax in our model takes the form

$$\tau_t(w_t, \theta_t, b_t, k_t, d_t, w_{t+1}) = \tau_c [f(k_t, \theta_t) - \delta k_t - (1 - R^{-1}) b_t],$$

where  $\tau_c$  is the tax rate. This tax applies to profits net of depreciation and includes a deduction for interest paid on debt. Deductions for depreciation and interest payments are features of the existing tax code in the United States and many other countries. In Appendix Section A we explicitly describe the problem of a firm in our model facing this profits-based corporate tax.

Let us compare a profit tax at rate  $\tau_c$  with a payout tax at rate  $\tau_d$  that raises the same revenue. It is useful to separately consider the impact of switching from a profit tax to a payout tax on incumbent firms and on entry decisions. Beginning with incumbent firms, the value of each incumbent firm can be thought of as consisting of two parts: the value of the wealth  $w$  inside the firm and the value of the future profits. A payout tax taxes these two components at the same rate, when they are paid out. A profit tax taxes the value of future profits more than the value of wealth inside the firm.<sup>29</sup> As a result, firms whose value consists mostly of future profits (constrained firms) would prefer the payout tax, whereas firms whose value comes more from wealth inside the firm (unconstrained firms) would prefer the profit tax.

If the tax rates  $\tau_c$  and  $\tau_d$  raise the same amount of tax revenue, switching from a profit tax to a payout tax creates winners (constrained firms) and losers (unconstrained firms). But because the constrained firms that benefit from this switch endogenously have better investment opportunities than the unconstrained firms, switching to a payout tax increases overall output and the total value of all firms in the economy.

Switching to a payout tax also affects entry. Marginal entrants — those for whom the benefits of entry are close to the fixed costs of entry — will enter the economy as constrained firms and hence benefit from a switch to payout taxation. Switching to payout taxation therefore reduces the extensive margin distortions caused by taxing firms. However, payout taxation can induce well-capitalized potential entrants to reduce their wealth on entry. Switching to payout taxation can therefore exacerbate the intensive margin distortions caused by taxation. In total, a switch from profit taxation to payout taxation will increase the number of firms entering but reduce their average size, and have ambiguous effects on the total value of entering firms.

**Description of quantitative exercise** We study these tradeoffs in our quantitative exercise. First, we calibrate our model of firms to the estimated parameters of Li, Whited and Wu (2016), using a corporate tax rate of  $\tau_c = 20\%$ . We then estimate the parameters of our model that govern entry and exit dynamics to match relevant facts documented by Lee and Mukoyama (2015) and the elasticity of entry with respect to corporate tax rates estimated by Djankov et al. (2010).

Armed with our quantitative model, we can calculate how much revenue the corporate profit tax  $\tau_c = 20\%$  raises, and find the steady state population of firms,  $\mu_c^*$ . We can then compute the payout tax rate  $\tau_d$  that

---

<sup>29</sup>The value of wealth inside the firm is taxed by a profit tax due to the taxation of the risk-free net return  $R - 1$ , but this effect is smaller than the effect on the value of future profits.

would be required to raise the same amount of revenue (in a net-present-value sense), assuming that the initial population of firms is equal to  $\mu_c^*$ . The payout tax rate  $\tau_d$  is the tax rate required to implement a revenue-neutral switch from a profit tax to a payout tax in our calibrated model. Comparing the two taxes, we study how the total value of entering and continuing firms is affected by the switch to payout taxation.

We start by describing our functional form assumptions, then discuss our calibration, and lastly present the results of our quantitative analysis. Appendix Section B contains a more detailed description of our functional form assumptions and calibration procedure.

**Functional Forms** Our model requires functional forms for the production function  $f(k, \theta)$ , transition probabilities  $\Pi(\theta_{t+1}|\theta_t)$ , and potential entry mass  $e(\hat{w}, \theta')$ . For  $f(k, \theta)$ , we use a decreasing returns to scale production function, augmented with the cash-like investment option, described in Equation (3). For the transition probabilities, we assume that log productivity (conditional on not exiting) follows a discrete approximation of an AR(1) process. We use the [Tauchen \(1986\)](#) approximation, with 14 non-zero log productivity levels spanning two standard deviations in both directions. The parameters  $\rho$  (persistence) and  $\sigma$  (standard deviation of shocks) describe the properties of the AR(1) process being discretized.

We define the exit probabilities  $\Pi(0|\theta_t)$  using the parameters  $\Pi(0|1)$  and  $\Pi(0|\theta_L)$ , where  $\theta_L$  is the lowest non-zero productivity level. We log-linearly interpolate the between the log values of these two parameters. That is,  $\ln(\Pi(0|\theta_t))$  is linear in  $\ln(\theta_t)$  and varies between  $\ln(\Pi(0|\theta_L))$  and  $\ln(\Pi(0|1))$ . These exit probabilities, plus the discretized AR(1) process, fully determine the transition probabilities  $\Pi(\theta_{t+1}|\theta_t)$ .

The total potential entry mass will scale the steady state mass  $\mu_c^*$  up and down, but will not affect any of our calculations. We therefore normalize the total potential entry mass to one. We assume that the outside wealth  $\hat{w}$  and productivity  $\theta'$  are independently distributed. For productivity  $\theta'$ , we assume that entrant productivity is drawn from the steady state of the AR(1) process<sup>30</sup>, with the mean shifted by the parameter  $\mu_z^e$ . This parameter controls the relative productivity of entrants and existing firms. For outside wealth  $\hat{w}$ , we use a shifted Pareto (Lomax) distribution with a tail parameter of  $\frac{3}{2}$ .<sup>31</sup> We use the scale parameter  $\xi_w$  to control the fraction of potential entrants with low wealth as opposed to high wealth, which influences the relative size of entrants and existing firms.

**Calibration** The starting point for our calibration is the estimated results of [Li, Whited and Wu \(2016\)](#). The model employed by [Li, Whited and Wu \(2016\)](#) is essentially identical to ours (and in particular uses the same [Rampini and Viswanathan \(2010\)](#) financial friction), with a few differences. [Li, Whited and Wu \(2016\)](#) estimate the parameters of their model using simulated method of moments on the population of Compustat firms, with a sample period of 1965-2012. Because our model and theirs are so similar, we are able to calibrate all of the parameters relating to firms (as opposed to entry/exit) using their results.

<sup>30</sup>Note that this is not the same as the steady state distribution of our productivity parameter, because of the different rates of exit across productivity levels.

<sup>31</sup>This tail parameter is close to the value of the tail parameter for the 2005 wealth distribution in the United States estimated by [Atkinson, Piketty and Saez \(2011\)](#), although it is not obvious that the distribution of potential entrant wealth should be similar to the distribution of total wealth. Fixing the tail parameter and controlling the scale parameter allows us to vary between distributions that are close to uniform and distributions that are highly concentrated at low wealth levels, and our results are not very sensitive to the choice of tail parameter.

However, we should note that our model differs from theirs in ways that are important for our theoretical exercise. We assume that future productivity is the privately known to the firm and that firms finance themselves using debt, whereas Li, Whited and Wu (2016) assume productivity shocks are contractible. We also assume that taxes are subject to default, whereas Li, Whited and Wu (2016) assume taxes are not subject to default.<sup>32</sup> We calibrate our parameters to their estimation results, acknowledging that estimating our model on their dataset would likely result in a somewhat different set of parameters. Appendix Table A1 lists both the parameters we calibrate from Li, Whited and Wu (2016) and the parameters relating to entry and exit that we estimate, described next.

There are five parameters relating to entry and exit that we estimate,  $(\Pi(0|1), \Pi(0|\theta_L), \mu_z^e, \xi_w, F)$ . We follow Clementi and Palazzo (2016) and target several moments regarding the entry and exit of manufacturing plants documented by Lee and Mukoyama (2015). Specifically, we attempt to match the exit rate, relative TFP of exiting and continuing firms, relative TFP of entering and continuing firms, and relative size of entering and continuing firms, under the steady state distribution  $\mu_c^*$ . These four moments help us pin down the five parameters above.

The fifth moment we use is the semi-elasticity of entry to corporate tax rates estimated by Djankov et al. (2010) using a cross-section of countries. Calibrating our model to this elasticity ensures that the extensive margin response to tax changes in the model is consistent with empirical evidence. As discussed above, switching from profit taxation to payout taxation will affect the extensive margin of entry, which is why we target this particular moment. Note that this moment is about changes in the rate of profit taxation, and not directly about what would happen under a switch to payout taxation. We are using the structure of our model to translate what is known about how profit taxation affects entry into a prediction about how a switch to payout taxation would affect entry. Appendix Table A2 summarizes the targeted moments, our interpretation of these moments within the model, and our model fit of those moments.

**Quantitative Results** Armed with our calibrated model, we first calculate the steady state distribution  $\mu_c^*$  and the total value of the taxes raised each period in that steady state, which are by definition equal to  $\tau_c = 20\%$  of the profits after deductions for the population of firms. Because we have normalized the mass of entering firms each period to one, the total value of the taxes raised each period has by itself little meaning.

It becomes more meaningful when we consider a switch to payout taxation, and answer the question: what payout tax rate is required to raise this amount of revenue (in an NPV sense)? We estimate this value to be  $\tau_d = 18.1\%$ , and note that it is not a priori obvious whether this tax rate should be larger or smaller than the corresponding profit tax rate.

We next consider the effects of switching to payout taxation. We begin by discussing the effects on entry. With a profit tax of  $\tau_c = 20\%$ , the total mass of entrants is 74.6% of what entry would be in the absence of taxation (i.e. if  $\tau_c = \tau_d = 0$ ). Our model is calibrated to match this number (i.e. to match the semi-elasticity of Djankov et al. (2010)). With a payout tax rate of  $\tau_d = 18.1\%$ , the total mass of entrants is 98.0% of entry in the absence of taxation. That is, switching to payout taxation increases entry by 31%.

<sup>32</sup>There are a few other differences relating to timing, and Li, Whited and Wu (2016) estimate a productivity process that does not account for exit. Li, Whited and Wu (2016) also assume the corporate tax does not include a deduction for depreciation.

This increase occurs because marginal entrants are constrained, and payout taxation is preferable for constrained firms. However, because these entrants are marginal, they are not very valuable from a private perspective (this echoes a point made by Jaimovich and Rebelo (2017)). Moreover, because payout taxation is worse for firms with more wealth, and some potential entrants could choose to enter with high wealth, payout taxation can exacerbate intensive margin distortions. Nevertheless, we find that the total private value of entering firms increases by 10.2% when switching to payout taxation.

Figure 6 shows the value functions net of wealth and resulting entry measures for three different productivity levels, under profit taxation and payout taxation. It is immediately apparent from the left panel that switching from profit taxation to payout taxation benefits low-wealth firms and harms high-wealth firms. As a result, this switch reduces extensive margin distortions (the profit tax shuts down entry completely for the lowest productivity type) and exacerbates intensive margin distortions.

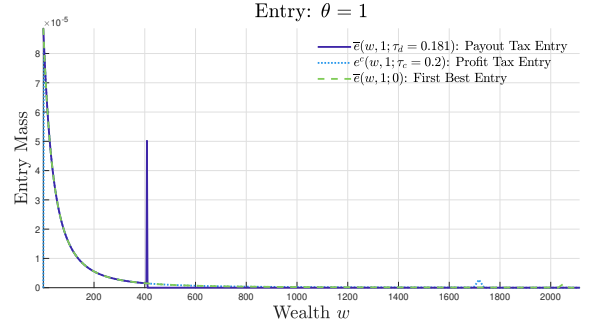
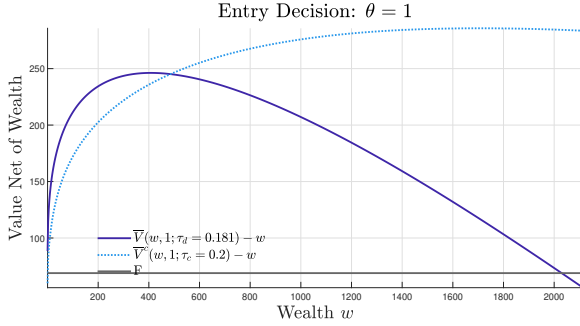
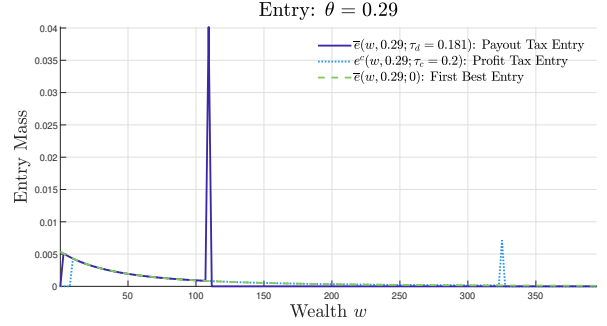
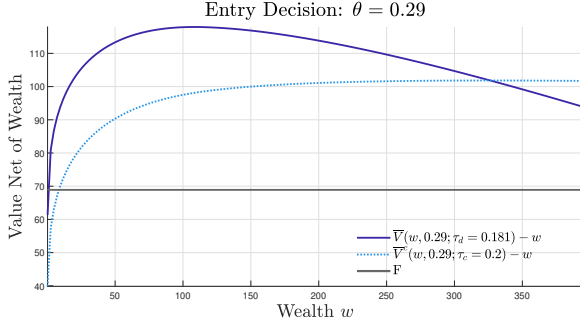
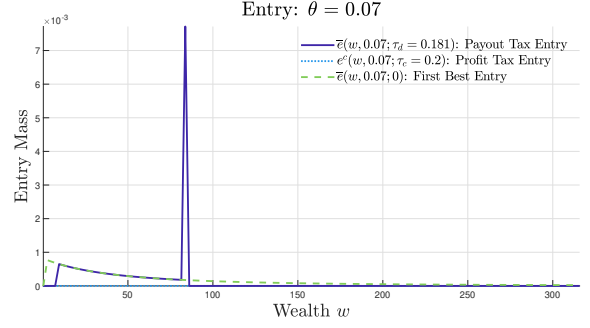
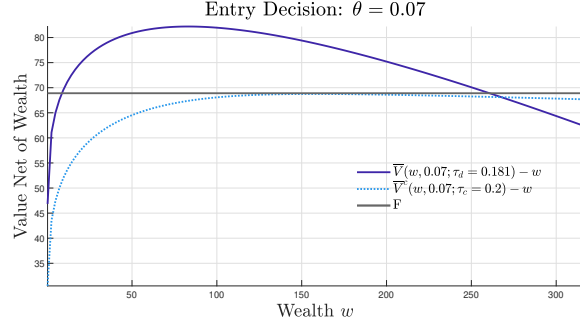
We can decompose the increase in entrant value into two parts: the increase due to changing entry patterns, holding fixed the value of each firm at its value with the profit tax  $\tau_c = 20\%$ , and the increase due to the change in value for each firm type  $(w, \theta)$  under the new entry patterns. The effect due to changing entry patterns, holding fixed the value function, is  $-4.0\%$ . That is, although 31% more firms enter, the increase in the intensive margin distortion is such that the overall value of entering firms would shrink, if the value of each firm were held fixed. However, this value is not fixed; in particular, constrained firms become much more valuable under a payout tax. This change increases the total value of firms by 14.7%. The 10.2% increase in firm value mentioned above is the product of these two effects.

Switching to payout taxation also increases the value of incumbent firms. This increase is smaller than for entrants, because entrants are more constrained than incumbents. We find that switching to payout taxation increases the value of incumbent firms by 4.7%. If we restrict attention to the set of incumbent firms in the top five percent of the firm size distribution (measured by capital under the profit tax regime), we observe a small decrease in firm value.

This might seem counter-intuitive, in light of our discussion about how payout taxation is worse for firms with high levels of wealth. To understand this result, observe that with profit taxation, firms will pay out wealth before they have enough wealth to achieve first-best production. Even though they are unconstrained in the sense that they have equated their internal and external value of funds, they are still constrained in the sense that they are not achieving production efficiency. As a result, in the steady state distribution under profit taxation, almost all firms will either benefit or be close to indifferent to a switch to payout taxation. Firms wealthy enough to be substantially harmed by a switch to payout taxation are rare in the steady state because they would have found it optimal to pay out their wealth earlier.

Combining the effects on entry with the effects on incumbent firms, we find that the total value of all firms (the value of incumbents plus the net present value of all future entrants) increases by 7.0%. This estimate is of course influenced by all of the parameters of our model, but is particularly sensitive to magnitude of the entry semi-elasticity. If we re-calibrate our model using an entry semi-elasticity twice as large, we estimate a total value increase for all firms of 16.9%. This increase is driven primarily by a larger increase in the value of new firms under payout taxation as opposed to profit taxation, and to a lesser extent by a reduction in the payout tax rate required from  $\tau_d = 18.1\%$  to  $\tau_d = 16.7\%$ . If instead we calibrate our model with zero entry





**Note:** Figure 6 illustrates the extensive and intensive margins of entry distortion under payout taxation and profit taxation from our quantitative analysis. Each row of the figure uses a different productivity level ( $\theta = 0.07, \theta = 0.29, \theta = 1$ ). The left panel plots the private value functions under payout and profit taxation net of wealth,  $\bar{V}(w, \theta; \tau_d = 0.181) - w$  and  $\bar{V}^c(w, \theta; \tau_c = 0.2) - w$  (for details on  $\bar{V}^c$ , see Appendix Section A). The right panel plots the entering mass of firms under payout taxation, profit taxation, and in the absence of taxation (first-best). Note that each row uses a different scale on both axes.

Figure 6: Entry under Payout Taxation and Profit Taxation



semi-elasticity, we find that switching to a payout tax reduces the total value of firms by 1.4%. The switch still increases the current value of the existing stock of firms (by slightly less, because the required payout tax rate is  $\tau_d = 19.9\%$ ), but this effect is overwhelmed by negative effects on the intensive margin of entry, and there are no offsetting positive effects on the extensive margin of entry in this case.

These results reinforce our core conclusions: switching to payout taxation increases the value of existing firms and increases entry by new firms, but reduces the size of new entrants. Overall, a revenue-neutral switch to payout taxation would substantially increase the total value of all firms in our calibrated model.

Note, however, that such a switch would require the government to borrow. Tax revenues will fall after the switch to payout taxation, as firms accumulate wealth to reach first-best production levels. The government can borrow to maintain its target spending levels, and eventually as firms reach optimal scale and make payouts, tax revenue will rise to cover both target spending and the interest payments on the government's extra debt.

## 7 Extensions

We next discuss how our results extend when relaxing two critical modeling assumptions. First, we describe how the functional form of the financial frictions that firms face affects our results. Second, we describe how to introduce equity issuance by existing firms.

### 7.1 Financial Frictions

As in [Rampini and Viswanathan \(2010\)](#), our model assumes that firms' financial contracts face limited enforcement and that there is no exclusion after default. While the option to re-enter after default is important, the exact formulation of limited enforcement, which determines the functional form of  $w^D(\cdot)$  in Equation (5), is not, as we explain here.

Let us begin by incorporating an option for the defaulting firm to re-enter, rather than explicitly requiring re-entry, which is implicit in Equation (6). In particular, suppose that a defaulting firm can liquidate and consume a fraction  $1 - \lambda$  of its wealth, instead of re-entering. This assumption, which makes liquidation less efficient than continuation, leads to an additional constraint

$$d_t + V_{t+1}(w_{t+1}, \theta_{t+1}; \mu_{t+1}(\cdot), B_{t+1}) \geq (1 - \lambda) w^D(k_t, \theta_t). \quad (33)$$

This constraint will not bind provided that the payout tax rate  $\tau_d$  is less than  $\lambda$ . Under a constant payout tax rate  $\tau_d$ , the left-hand side of this expression is (weakly) maximized with  $d_t = 0$ : this is the logic of Lemma 3. In this case, the continuation wealth  $w_{t+1}$  will be weakly greater than the wealth given default for any level of capital and debt that satisfy Equation (6). Moreover, for all firms,  $V_{t+1}(w_{t+1}, \theta_{t+1}; \mu_{t+1}(\cdot), B_{t+1}) \geq (1 - \tau_d) w_{t+1}$ . Consequently, Proposition 4 will hold for equilibrium tax rates  $\tau_d \in [0, \lambda]$ .

The conclusion is different when firms face the default-and-liquidate constraint (33) but cannot re-enter after default. In this case, which maps to [Kehoe and Levine \(1993\)](#), the constraint in Equation (33) depends on the private information of the firm,  $\theta_{t+1}$ . As a result, even in the absence of taxes, there is an asymmetric information problem between outside investors and the firm, and no simplification along the lines of Lemma 3

applies. Because of this asymmetric information problem, the government will find optimal to use corrective taxes to improve allocations, even if it has no revenue-raising objective (Greenwald and Stiglitz, 1986). Consequently, the optimal policy in this case will have both corrective and revenue-raising objectives, which will non-trivially depend on assumptions about the structure of the market between firms and creditors. We leave an exploration of this case to future work.

More generally, what is key to our analysis is that the financial frictions that a firm faces can be written as a function of a firm's current productivity, but not its future productivity. This restriction applies to the limited enforcement assumption of Rampini and Viswanathan (2010), but also to many other forms of financial constraints, including those that restrict repayment in terms of firms' cash flows.<sup>33</sup> In the context of our model, assuming that  $f(k_t, \theta_t)$  corresponds to a firm's date  $t$  EBITDA, we can require that

$$f(k_t, \theta_t) - \tau_t \geq \varphi b_t, \quad (34)$$

that is, that either the EBITDA to repayment ratio (net of taxes) exceeds a constant  $\varphi > 1$  or that the repayment is zero.<sup>34</sup> To incorporate this cash flow lending friction, we allow for the continuation wealth of a firm after default  $w^D(\cdot)$  to depend on current productivity  $\theta_t$ , capital  $k_t$ , and repayment  $b_t$  (the last of these does not enter into the Rampini and Viswanathan (2013) specification). Combining this constraint with Equation (4), we can rewrite Equation (34) as

$$\begin{aligned} w_{t+1} + d_t &\geq w^D(k_t, b_t, \theta_t), \\ w^D(k_t, b_t, \theta_t) &= (\varphi - 1)b_t + (1 - \delta)k_t. \end{aligned} \quad (35)$$

We can further rewrite this equation as requiring

$$d_t + V(w_{t+1}, \theta_{t+1}) \geq d_t + V(w^D(k_t, b_t, \theta_t) - d_t, \theta_{t+1}),$$

and interpret the constraint in the style of Rampini and Viswanathan (2010) as arising from firm owners' ability to re-enter with wealth  $w^D(k_t, b_t, \theta_t)$ . Under this specification, Equation (6) will also include a blocked dividend no default constraint, but our proof shows that this constraint will not bind. Our point here is not that the continuation wealth on default implied by this interpretation is sensible, only that this particular  $w^D(\cdot)$  generates an earnings-based constraint of the type that appears common in practice. The proof of Proposition 2, which is the key proposition for our results,<sup>35</sup> assumes only that  $\frac{\partial}{\partial k} w^D(k, b, \theta) < f_k(k, \theta) + 1 - \delta, \forall k, b, \theta$ , which ensures that the initial budget constraint binds. To have a binding financial friction under some circumstances, it suffices to have  $\frac{\partial}{\partial k} w^D(k, b, \theta) + R \frac{\partial}{\partial b} w^D(k, b, \theta) > 0$ , so that debt-financed capital accumulation increases the temptation

<sup>33</sup>A growing literature — see e.g., Lian and Ma (2018) — shows that earnings-based covenants are commonly used. We would like to thank Ludwig Straub for suggesting that we consider such constraints.

<sup>34</sup>Firms' covenants typically take the form of restrictions on the ratio of earnings (commonly EBITDA) to debt or to interest payments. Treating a constraint on the EBITDA/debt ratio as structural is problematic from the perspective of optimal taxation, since this constraint is meant to ensure that firms have enough earnings to service their obligations, including their taxes. For this reason, we use EBITDA net of taxes as the earnings measure in Equation (34).

<sup>35</sup>All but one of our results apply to more general specifications of  $w^D(\cdot)$ . Our proof of Lemma 4 uses the specific limited enforcement assumptions of Rampini and Viswanathan (2010), but could be adapted to other specifications.

to default. The Rampini and Viswanathan (2010) specification (5), the “earnings-based” specification (35), and many other constraints share these properties, and our results apply to all of these cases.

## 7.2 Equity Issuance

So far, firms have only been able to issue equity at entry. In this subsection, we discuss how to accommodate equity issuance by existing firms in a manner similar to how we treat entry. Introducing equity issuance in this way does not change our main results, in particular Proposition 4.

In the infinite-horizon environment, entering firms decide whether to enter and if so how much wealth to enter with in the beginning of the period, before the government designs its mechanism. We now assume that, at the same time and before learning their next-date productivity, existing firms can raise equity at a cost of  $1 + \kappa$  per unit of wealth raised. For simplicity, we assume that this cost is borne by the existing firms’ shareholders, as opposed to the firm itself.<sup>36</sup>

We denote the measure of existing firms before and after equity raising and entry by, respectively,  $\mu_t^{\text{pre}}$  and  $\mu_t^{\text{post}}$ . The law of motion of firms’ wealth can be split into two equations:

$$\begin{aligned}\mu_{t+1}^{\text{pre}}(w', \theta') &= e_t(w, \theta; \mu_t^{\text{pre}}(\cdot), B_t) \\ &\quad + \int_0^\infty \int_{0^+}^1 \delta(w_{t+1}(\theta', \theta'; w, \theta) - w') \Pi(\theta' | \theta) \mu_t^{\text{post}}(w, \theta) d\theta dw \\ &\quad + \mathbf{1}\{\theta' = 0\} \int_0^\infty \delta_{\text{dirac}}(w_{t+1}(w, 0) - w') \mu_t^{\text{post}}(w, 0) dw, \\ \mu_t^{\text{post}}(w', \theta) &= \int_0^\infty \int_0^\infty \int_0^1 \delta_{\text{dirac}}(w' - w - x(w, \theta; \mu_t^{\text{pre}}(\cdot), B_t)) \mu_t^{\text{pre}}(w, \theta) dw' dw d\theta.\end{aligned}$$

The first of these equations is the analog of Equation (27). Entering firms that decide to enter at time  $t$  appear as new firms at time  $t + 1$ , which is why entry appears in the first equation and why the relevant expectations are based on  $\mu_t^{\text{pre}}$ . The second equation describes the impact of equity issuance. Here,  $x(w, \theta; \mu_t^{\text{pre}}, B_t)$  is the equity issuance decision of a firm with wealth  $w$  and type  $\theta$  at the beginning of date  $t$ , before the firm knows  $\theta_{t+1}$  and before the government designs its mechanism. The firm will choose  $x(w, \theta; \mu_t^{\text{pre}}, B_t)$  to solve

$$x(w, \theta; \mu_t^{\text{pre}}(\cdot), B_t) \in \arg \max_{x \geq 0} \mathbb{E}[V(w + x, \theta; \mu_t^{\text{post}}(\cdot), B_t) | \mu_t^{\text{pre}}(\cdot)] - (1 + \kappa)x. \quad (36)$$

Because each firm is small, it treats  $\mu_t^{\text{post}}$  as exogenous when forming expectations. The measure  $\mu_t^{\text{post}}$  takes the place of  $\mu_t$  as the relevant state variable in the government’s objective and budget constraint, introduced in Equations (29) and (28). Combining these equations, the evolution of  $\mu_{t+1}^{\text{post}}$  depends on  $\mathbb{E}[V(\cdot; \mu_{t+1}^{\text{post}}, B_{t+1})]$ , which is based on the presumption of a constant payout tax in the equilibrium described in Proposition 4. As a result, the proof of Proposition 4 applies almost un-modified, except that the social value function  $U(w, \theta; \mu, B)$

<sup>36</sup>The most straightforward interpretation of the cost  $\kappa$  is as a transaction cost, and note that our results hold even if  $\kappa = 0$ . Transaction costs do not introduce new inefficiencies that can be addressed by government policy. If instead the cost  $\kappa$  is associated with distortions, for instance due to adverse selection problems, our approach should be interpreted as either assuming optimal regulation in the background or abstracting from the corrective benefits of the tax policy.

is no longer equal to  $\bar{V}(w, \theta; 0)$ , but instead to

$$U(w, \theta; \mu_t^{\text{pre}}(\cdot), B_t) = \bar{V}(w + x(w, \theta; \tau_0), \theta; 0), \quad \text{where} \quad x(w, \theta; \tau_0) \in \arg \max_{x \geq 0} \bar{V}(w + x, \theta; \tau_0) - (1 + \kappa)x.$$

This social value function is consistent with a constant payout tax (Definition 1), so Proposition 2, which is the critical result, still applies.

Because the government designs its mechanism after the equity issuance/entry decisions are made at a given date, the government does not internalize the effects of its mechanism on equity issuance and entry decisions. If instead equity issuance decisions were made after the mechanism is designed, in effect allowing the government to have a single period of commitment, the government would want firms to raise more equity than they would choose on their own. This follows from Equation (36), which involves the private, not social, value of the firm. By threatening to force firms to their outside option if they do not raise sufficient equity, the government could induce firms to raise more equity. The remainder of the problem would remain the same, so the optimal policy in this case would be a constant payout tax plus an equity-raising mandate.

## 8 Implications for Policy

Before concluding, we discuss several conceptual and practical issues related to our results. First, at various times in the United States and other countries, the tax code has treated dividends and share repurchases differently. In this paper, firms' payouts include all payments to the agents controlling the firm, and hence include both dividends and share repurchases under our preferred interpretation. Under the optimal policy characterized in this paper, dividends and share repurchases are taxed at the same rate.<sup>37</sup>

Second, in the United States and most other countries, corporate taxes are assessed on firms' earnings, net of various deductions. Our results imply that this is a desirable way to structure corporate taxation as long as these deductions account for all retained earnings, since earnings less retained earnings is, by definition, equal to dividends plus share repurchases.<sup>38</sup> This conclusion is reminiscent of the "new view" of dividend taxation (Auerbach and Hines Jr, 2002), but applied to a model with financial frictions. Perhaps more importantly, interpreting the payments to outside investors in the model as debt, our results justify exempting interest payments on debt. The key distinction between equity and debt financing, from the model's perspective, is control. A time-invariant tax on payments to the agents controlling the firm does not distort firms' inter-temporal decisions, except when issuing equity; see Korinek and Stiglitz (2009) for more on this point. Our results show that any other tax would both distort inter-temporal decisions and equity issuance decisions. Therefore, even though firms issue less equity when they face higher payout taxes, payout taxes are still optimal.

Third, the optimality of a payout tax is sensitive to how payout policies are determined. As discussed in

<sup>37</sup>The global rise in corporate saving (see, e.g., Chen, Karabarbounis and Neiman (2017)) makes this observation particularly relevant. For instance, Apple Inc., the world's largest company by market capitalization as of 2015, has experienced a large increase in profits, but not dividends, over the last few decades. At the same time, it has accumulated large holdings of cash and it has recently repurchased its equity. The optimal payout tax that we characterize implies that these equity repurchases should be taxed.

<sup>38</sup>One could argue that the current tax system, modified to have full expensing of investment, is not too far from this policy. In fact, Barro and Furman (2018) estimate that a switch to full expensing would increase output per worker by 8.1%, which is comparable to our estimate that a switch to payout taxation would increase the total value of firms in our model by 7%.

Section 2, if there is managerial entrenchment (Zwiebel, 1996) or conflicts of interest between shareholders and managers and the latter control the firm, our results can be reinterpreted to justify a tax on managerial compensation. If managers and shareholders can optimally contract, and there is no scope for corrective policies by the government, then it does not matter which agent is taxed and our results continue to hold. If instead there is scope for corrective policies, but these corrective policies can be implemented using non-tax instruments, our results should also still apply.<sup>39</sup> Similar points apply to issues related to signaling or catering through dividend policy, which may also be important for how payouts are determined.

Finally, we should highlight that France in 2012 implemented a 3% corporate dividend tax that resembles the optimal tax we have characterized, in addition to more traditional corporate taxes.<sup>40</sup> A recent analysis of other dividend tax changes in France (Boissel and Matray, 2019) finds, consistent with our model, that dividend tax changes induced some changes in payout policy but only small changes in real outcomes, consistent with patterns documented for the personal dividend tax in the United States (Yagan, 2015).

## 9 Conclusion

This paper provides a normative analysis of the design of corporate taxes when firms are financially constrained. We identify a corporate taxation principle that defines the optimal policy of a government whose goal is to efficiently raise a given amount of revenue from its corporate sector: taxes should be levied on unconstrained firms, which value resources inside the firm less than financially constrained firms. A government with full information about firms' productivity finds it preferable to exclusively tax financially unconstrained firms. When firms privately know their future productivity, the government must use a mechanism to elicit which firms are financially constrained in an incentive compatible way. A corporate payout tax (a tax on dividends and share repurchases) can both separate constrained and unconstrained firms and raise revenue, and is therefore optimal. Our quantitative analysis suggests that a revenue-neutral switch from profit taxation to payout taxation could increase the total value of current and future firms by 7%. This switch could be implemented in our current tax system by making retained earnings fully deductible.

Even though there are well-developed literatures that explore the optimal determination of personal taxation, including income, capital, and commodity taxation, to our knowledge, this paper is the first to address the question of how corporate taxes should be structured in an environment in which any feasible non-negative tax instrument can be employed. Future work taking into account manager-shareholder conflicts, security design, interactions between personal and corporate taxation, and general equilibrium considerations should result in a deeper understanding of optimal corporate taxation.

---

<sup>39</sup>Regardless of who controls the firm, it might also be possible for the agent controlling the firm to extract value from the firm through in-kind benefits, such as markups for goods or services provided, or disguised as payments to unrelated parties. If these payments destroy value relative to dividend payments (along the lines of, e.g., DeMarzo and Urošević (2006)), then the possibility of such payments will limit the payout tax rate the government can implement but otherwise leave our results unchanged.

<sup>40</sup>Court rulings eventually declared the tax to be unconstitutional, due to double taxation issues. It was repealed in 2017.

## References

- Albuquerque, Rui, and Hugo A. Hopenhayn.** 2004. "Optimal Lending Contracts and Firm Dynamics." *The Review of Economic Studies*, 71(2): 285–315.
- Allen, Franklin, and Douglas Gale.** 2000. "Financial Contagion." *Journal of Political Economy*, 108(1): 1–33.
- Alvarez, Fernando, and Urban J. Jermann.** 2000. "Efficiency, Equilibrium, and Asset Pricing with Risk of Default." *Econometrica*, 68(4): 775–797.
- Atkinson, Anthony B., Thomas Piketty, and Emmanuel Saez.** 2011. "Top Incomes in the Long Run of History." *Journal of Economic Literature*, 49(1): 3–71.
- Auerbach, Alan J, and James R Hines Jr.** 2002. "Taxation and Economic Efficiency." *Handbook of Public Economics*, 3: 1347–1421.
- Barro, Robert J., and Brian Wheaton.** 2019. "Taxes, Incorporation, and Productivity." National Bureau of Economic Research.
- Barro, Robert J., and Jason Furman.** 2018. "Macroeconomic Effects of the 2017 Tax Reform." *Brookings Papers on Economic Activity*, 2018(1): 257–345.
- Boissel, Charles, and Adrien Matray.** 2019. "Higher Dividend Taxes, No Problem! Evidence from Taxing Entrepreneurs in France." *Working Paper*.
- Chamley, Christophe.** 1986. "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives." *Econometrica: Journal of the Econometric Society*, 607–622.
- Chari, Varadarajan V., and Patrick J. Kehoe.** 1990. "Sustainable Plans." *Journal of Political Economy*, 98(4): 783–802.
- Chari, Varadarajan V., and Patrick J. Kehoe.** 1999. "Optimal Fiscal and Monetary Policy." *Handbook of Macroeconomics*, 1: 1671–1745.
- Chen, Peter, Loukas Karabarbounis, and Brent Neiman.** 2017. "The Global Rise of Corporate Saving." *Journal of Monetary Economics*, 89: 1–19.
- Chetty, Raj, and Emmanuel Saez.** 2010. "Optimal Taxation and Social Insurance with Endogenous Private Insurance." *American Economic Journal: Economic Policy*, 2(2): 85–114.
- Clementi, Gian Luca, and Berardino Palazzo.** 2016. "Entry, Exit, Firm Dynamics, and Aggregate Fluctuations." *American Economic Journal: Macroeconomics*, 8(3): 1–41.
- Clementi, Gian Luca, and Hugo A. Hopenhayn.** 2006. "A Theory of Financing Constraints and Firm Dynamics." *The Quarterly Journal of Economics*, 121(1): 229–265.
- Cooley, Thomas, Ramon Marimon, and Vincenzo Quadrini.** 2004. "Aggregate Consequences of Limited Contract Enforceability." *Journal of Political Economy*, 112(4): 817–847.
- Crémer, Jacques, and Richard P McLean.** 1988. "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions." *Econometrica: Journal of the Econometric Society*, 1247–1257.
- Debortoli, Davide, Ricardo Nunes, and Pierre Yared.** 2017. "Optimal Time-Consistent Government Debt Maturity." *The Quarterly Journal of Economics*, 132(1): 55–102.
- DeMarzo, Peter M., and Yuliy Sannikov.** 2006. "Optimal Security Design and Dynamic Capital Structure in a Continuous-Time Agency Model." *The Journal of Finance*, 61(6): 2681–2724.
- DeMarzo, P.M., and B. Urošević.** 2006. "Ownership Dynamics and Asset Pricing with a Large Shareholder." *Journal of Political Economy*, 114(4): 774–815.
- Djankov, Simeon, Tim Ganser, Caralee McLiesh, Rita Ramalho, and Andrei Shleifer.** 2010. "The Effect of Corporate Taxes on Investment and Entrepreneurship." *American Economic Journal: Macroeconomics*, 2(3): 31–64.
- Edgeworth, Francis Y.** 1897. "The Pure Theory of Taxation." *The Economic Journal*, 7(25): 46–70.

- Farhi, Emmanuel, and Iván Werning.** 2012. "Capital Taxation: Quantitative Explorations of the Inverse Euler Equation." *Journal of Political Economy*, 120(3): 398–445.
- Fazzari, Steven M., R. Glenn Hubbard, and Bruce C. Petersen.** 1988. "Financing Constraints and Corporate Investment." *Brookings Papers on Economic Activity*, (1): 141–206.
- Golosov, Mikhail, Aleh Tsyvinski, and Nicolas Werquin.** 2016. "Recursive Contracts and Endogenously Incomplete Markets." *Handbook of Macroeconomics*, 2: 725–841.
- Graham, John R.** 2013. "Do Taxes Affect Corporate Decisions? A Review." *Handbook of the Economics of Finance*, 2: 123–210.
- Greenwald, Bruce C., and Joseph E. Stiglitz.** 1986. "Externalities in Economies with Imperfect Information and Incomplete Markets." *The Quarterly Journal of Economics*, 101(2): 229–264.
- Harberger, Arnold C.** 1962. "The Incidence of the Corporation Income Tax." *Journal of Political Economy*, 70(3): 215–240.
- He, Zhiguo, and Gregor Matvos.** 2015. "Debt and creative destruction: Why could subsidizing corporate debt be optimal?" *Management Science*, 62(2): 303–325.
- Jaimovich, Nir, and Sergio Rebelo.** 2017. "Nonlinear Effects of Taxation on Growth." *Journal of Political Economy*, 125(1): 265–291.
- Judd, Kenneth L.** 1985. "Redistributive Taxation in a Simple Perfect Foresight Model." *Journal of Public Economics*, 28(1): 59–83.
- Kaplan, Steven N., and Luigi Zingales.** 1997. "Do Investment-Cash Flow Sensitivities Provide Useful Measures of Financing Constraints?" *The Quarterly Journal of Economics*, 112(1): 169–215.
- Kehoe, Timothy J., and David K. Levine.** 1993. "Debt-Constrained Asset Markets." *The Review of Economic Studies*, 60(4): 865–888.
- Kopczuk, Wojciech.** 2003. "A Note on Optimal Taxation in the Presence of Externalities." *Economics Letters*, 80(1): 81–86.
- Korinek, Anton, and Joseph E. Stiglitz.** 2009. "Dividend Taxation and Intertemporal Tax Arbitrage." *Journal of Public Economics*, 93(1-2): 142–159.
- Lee, Yoonsoo, and Toshihiko Mukoyama.** 2015. "Entry and Exit of Manufacturing Plants over the Business Cycle." *European Economic Review*, 77: 20–27.
- Lian, Chen, and Yueran Ma.** 2018. "Anatomy of Corporate Borrowing Constraints." *Working Paper*.
- Li, Shaojin, Toni M. Whited, and Yufeng Wu.** 2016. "Collateral, Taxes, and Leverage." *The Review of Financial Studies*, 29(6): 1453–1500.
- Mirrlees, James A.** 1971. "An Exploration in the Theory of Optimum Income Taxation." *The Review of Economic Studies*, 38(2): 175–208.
- Poterba, James M, and Lawrence H Summers.** 1984. "The Economic Effects of Dividend Taxation." National Bureau of Economic Research.
- Rampini, Adriano A., and S. Viswanathan.** 2010. "Collateral, Risk Management, and the Distribution of Debt Capacity." *The Journal of Finance*, 65(6): 2293–2322.
- Rampini, Adriano A., and S Viswanathan.** 2013. "Collateral and capital structure." *Journal of Financial Economics*, 109(2): 466–492.
- Rockafellar, R. Tyrrell.** 1970. *Convex Analysis*. Princeton University Press.
- Salanie, Bernard.** 2011. *The Economics of Taxation*. MIT Press.
- Sandmo, Agnar.** 1975. "Optimal Taxation in the Presence of Externalities." *The Swedish Journal of Economics*, 86–98.



- Stokey, Nancy, Robert Lucas, and Edward Prescott.** 1989. *Recursive Methods in Economic Dynamics*. Harvard University Press.
- Straub, Ludwig, and Iván Werning.** 2020. “Positive Long-Run Capital Taxation: Chamley-Judd Revisited.” *American Economic Review*, 110(1): 86–119.
- Tauchen, George.** 1986. “Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions.” *Economics Letters*, 20(2): 177–181.
- Yagan, Danny.** 2015. “Capital Tax Reform and the Real Economy: The Effects of the 2003 Dividend Tax Cut.” *American Economic Review*, 105(12): 3531–63.
- Zwiebel, Jeffrey.** 1996. “Dynamic Capital Structure under Managerial Entrenchment.” *The American Economic Review*, 1197–1215.

# APPENDIX

The Appendix contains additional information about our quantitative exercise and all proofs referenced in the paper.

## A The Firm's Problem with a Profit Tax

In this section, we describe the firm's problem in the presence of a corporate tax

$$\tau_t(w_t, \theta_t, b_t, k_t, d_t, w_{t+1}) = \tau_c [f(k_t, \theta_t) - \delta k_t - (1 - R^{-1})b_t].$$

In our model, because taxes are subject to default, the profit tax can induce default. Defaults will occur for firms that are productive but have very little wealth. These defaults are predictable (because taxes depend on capital and debt and not dividends), in the sense that lenders will know with certainty whether a firm will default or not. As a result, lenders will not lend to defaulting firms.

Let  $x_t \in \{0, 1\}$  be an indicator for whether the firm chooses to default. We will assume in the event of default that the government collects the maximum taxes possible (that is, there is no deadweight loss due to default). This can also be interpreted as a cap on the profit tax so as to avoid default. This assumption (as opposed to assuming positive deadweight loss due to default) makes the profit tax look better relative to the payout tax in our quantitative exercise.

The firm's problem (echoing Definition 3) is given below.

**Definition 4.** (Firms' problem with a profit tax) Fix some  $w_t > 0$  and  $\theta_t \in [0, 1]$ , and suppose that the government implements a constant profit tax with interest and depreciation deductibility,

$$\tau_{t+j}(w_t, \theta_t, b_t, k_t, d_t, w_{t+1}) = \tau_c [f(k_t, \theta_t) - \delta k_t - (1 - R^{-1})b_t] \forall j \geq 0.$$

Then the current-date problem of a firm with future type  $\theta_{t+1}$  is

$$\begin{aligned} V^c(w_t, \theta_t, \theta_{t+1}) = & \max_{b_t \geq 0, k_t \geq 0, w_{t+1} \geq 0, d_t \geq 0, x_t \in \{0, 1\}} R^{-1} x_t \left\{ d_t + \int_0^1 V^c(w_{t+1}, \theta_{t+1}, \theta_{t+2}) \Pi(\theta_{t+2} | \theta_{t+1}) d\theta_{t+2} \right\} \\ & + (1 - x_t) R^{-1} \left\{ d_t + \int_0^1 V^c(w^D(k_t, \theta_t) - d_t, \theta_{t+1}, \theta_{t+2}) \Pi(\theta_{t+2} | \theta_{t+1}) d\theta_{t+2} \right\} \end{aligned}$$

subject to

$$\begin{aligned} w_{t+1} & \leq f(k_t, \theta_t) + (1 - \delta)k_t - d_t - b_t - \tau_c [f(k_t, \theta_t) - \delta k_t - (1 - R^{-1})b_t], \\ k_t & \leq w_t + R^{-1}x_t b_t, \\ d_t & \leq w^D(k_t, \theta_t). \end{aligned}$$

We can solve for the optimal debt, capital, and default choices in this problem without explicitly characterizing the value function, and simplify the firm's problem of choosing dividends. The following lemma summarizes these results. We assume that the firm does not default if it is indifferent between defaulting and not defaulting.

**Lemma 5.** *In the firm's problem with a profit tax (Definition 4), the firm will default if and only if*

$$\tau_c [f(w_t, \theta_t) - \delta w_t] > \varphi(1 - \delta) w_t.$$

*If the firm defaults,  $k_t = w_t$  and  $b_t = 0$ . If the firm does not default,  $b_t = R(k_t - w_t)$  and  $k_t \geq w_t$  is maximal and solves*

$$(R - \varphi(1 - \delta))k_t + \tau_c [f(k_t, \theta_t) + (1 - \delta)k_t - Rk_t] = R(1 - \tau_c(1 - R^{-1}))w_t.$$

*In both cases, the firm will choose  $d_t \in [0, w^D(k_t, \theta_t)]$  to maximize*

$$d_t + \int_0^1 V^c(w^D(k_t, \theta_t) - d_t, \theta_{t+1}, \theta_{t+2}) \Pi(\theta_{t+2} | \theta_{t+1}) d\theta_{t+2}.$$

*Proof.* See the appendix, C.10. □

This lemma simplifies the firm's problem in several ways. First, it shows that the firm will not default unless forced to by the profit tax. This will occur for low-wealth, high-productivity firms. Second, it shows that either the firm will default or that capital will be maximal, meaning that the firm exhausts its borrowing capacity. This implies that the firm's dividend  $d_t$  plus continuation wealth  $w_{t+1}$  will be equal to wealth given default  $w^D(k_t, \theta_t)$ , hence our simplification of the choice of dividend. We use these simplifications when computing the firm's value function and optimal payout policy under a profit tax in our quantitative exercise.

Because the choice of capital and default decision does not depend on the future type  $\theta_{t+1}$ , we can rewrite the problem in terms of the value function

$$\bar{V}^c(w_t, \theta_t) = \int_0^1 V^c(w_t, \theta_t, \theta_{t+1}) \Pi(\theta_{t+1} | \theta_t) d\theta_{t+1}$$

as

$$\bar{V}^c(w_t, \theta_t) = \max_{d_t \in [0, w^D(k(w_t, \theta_t), \theta_t)]} R^{-1} \left\{ d_t + \int_0^1 \bar{V}^c(w^D(k(w_t, \theta_t), \theta_t) - d_t, \theta_{t+1}) \Pi(\theta_{t+1} | \theta_t) d\theta_{t+1} \right\},$$

where  $k(w_t, \theta_t)$  is the optimal policy described in Lemma 5. We use this formulation in the numerical procedures underlying our quantitative exercise.

## B Details on Quantitative Exercise

### B.1 Functional Forms

In this subsection, we provide more details on the functional forms used in our quantitative exercise. The productivity parameter transition matrix  $\Pi(\theta_{t+1} | \theta_t)$  is defined using the Tauchen (1986) approximation of an AR(1) process for log productivity. That is, define an AR(1) process

$$z_{t+1} = \rho z_t + \sigma \varepsilon_{t+1},$$

with  $\varepsilon_{t+1} \sim N(0, 1)$ . Let  $\Pi_z(z_{t+1}|z_t)$  be the transition matrix associated with the discrete approximation of this process. We define  $\theta_t = A^{-1} \exp(z_t)$  for all productive types ( $\theta_t > 0$ ), scaling the parameter  $A$  to ensure the highest productivity level is  $\theta_t = 1$ . The production function we employ and the AR(1) approximation for productivity both follow Li, Whited and Wu (2016).

We incorporate the exiting type,  $\theta_t = 0$ , by assuming that the probability of exiting next period is

$$\Pi(0|\theta_t) = \Pi(0|1) \times \theta_t^{\frac{\ln(\Pi(0|\theta_L)) - \ln(\Pi(0|1))}{\ln(\theta_L)}},$$

where  $\theta_L$  is the smallest non-zero productivity level. This can also be written as

$$\ln(\Pi(0|\theta_t)) = \ln(\Pi(0|1)) + (\ln(\Pi(0|\theta_L)) - \ln(\Pi(0|1))) \times \frac{\ln(1) - \ln(\theta_t)}{\ln(1) - \ln(\theta_L)},$$

which is to say  $\ln(\Pi(0|\theta_t))$  is linear in  $\ln(\theta_t)$ . The parameters  $\Pi(0|1)$  and  $\Pi(0|\theta_L)$  control the exit probabilities of the highest and lowest productivity types; this functional form smoothly interpolates the transition probability between them for other values of  $\theta_t$ . The final transition matrix is

$$\Pi(\theta_{t+1}|\theta_t) = \begin{cases} \Pi(0|\theta_t) & \theta_{t+1} = 0, \\ (1 - \Pi(0|\theta_t))\Pi_z(\ln(A\theta_{t+1})|\ln(A\theta_t)) & \theta_{t+1} \neq 0. \end{cases}$$

This final transition matrix is a function of the parameters  $(\rho, \sigma, \Pi(0|\theta_L), \Pi(0|1))$ .

The last functional form required is the distribution of potential entrant wealth and type. We assume that outside wealth and productivity are independent. For productivity, we use  $\ln(A\theta') \sim N\left(\mu_z^e, \frac{\sigma^2}{1-\rho^2}\right)$ . For potential entrant wealth, the exact functional form of the shifted Pareto (Lomax) distribution is

$$f(\hat{w}|\xi_w) = \frac{3}{2\xi_w} \left(1 + \frac{\hat{w}}{\xi_w}\right)^{-\frac{5}{2}}.$$

## B.2 Calibration

Table A1 below lists both the parameters that we calibrate to the results of Li, Whited and Wu (2016) and the parameters that we estimate to match the moments described in Lee and Mukoyama (2015) and Djankov et al. (2010). The estimates from Li, Whited and Wu (2016) come from Panel B of Table 1 in that paper, under the assumption of a 20% tax rate.

Table A1: Calibrated Parameter Values

Parameter Name	Symbol	Value	Source
Risk-Free Rate	$R$	1.0525	1965-2012 Avg. 3-mo T-bill rate (FRED), following Li, Whited and Wu (2016)
Depreciation Rate	$\delta$	0.038	Li, Whited and Wu (2016) Estimation, $\tau_c = 20\%$
Decreasing Returns	$\alpha$	0.572	Li, Whited and Wu (2016) Estimation, $\tau_c = 20\%$
Collateral Haircut	$\varphi$	0.369	Li, Whited and Wu (2016) Estimation, $\tau_c = 20\%$
Productivity Persistence	$\rho$	0.530	Li, Whited and Wu (2016) Estimation, $\tau_c = 20\%$
Productivity Volatility	$\sigma$	0.458	Li, Whited and Wu (2016) Estimation, $\tau_c = 20\%$
Low-Prod. Exit Rate	$\Pi(0 \theta_L)$	8.3%	Estimated, Table A2
High-Prod. Exit Rate	$\Pi(0 1)$	3.55%	Estimated, Table A2
Potential Entrant Prod. Mean	$\mu_z^e$	0.006	Estimated, Table A2
Potential Entrant Wealth Shape	$\xi_w$	0.042	Estimated, Table A2
Fixed Cost of Entry	$F$	68.9	Estimated, Table A2

**Note:** The time interval assumed in the calibration is one year.

Table A2 below describes the moments from Lee and Mukoyama (2015) and Djankov et al. (2010) that we use to estimate our other parameters. Because our model is exactly identified by these moments, the (very small) differences between the target moments and fitted values are due to the limitations of our numerical procedures. We use the notation  $e_c^*(w, \theta)$  to denote the entering mass under the tax rate  $\tau_c = 20\%$ , and  $k_c(w, \theta)$  to denote the optimal capital choice of a firm facing that tax rate. Note that, by definition, the total entering mass is equal to the exit rate times the steady state mass of firms, which is to say

$$\int_{0^+}^1 \int_0^\infty e_c^*(w, \theta) dw d\theta = \int_{0^+}^1 \int_0^\infty \Pi(0|\theta) \mu_c^*(w, \theta) dw d\theta.$$

Table A2: Target Moments and Fit

Moment	Model Formula	Calib. Value	Target Value	Source
Exit Rate	$\frac{\int_{0+}^1 \int_0^\infty \Pi(0 \theta) \mu_c^*(w, \theta) dw d\theta}{\int_{0+}^1 \int_0^\infty \mu_c^*(w, \theta) dw d\theta}$	0.0550	0.0550	Lee and Mukoyama (2015), Table 2
Relative TFP, Exiters vs. Stayers	$\frac{(1 - \text{ExitRate}) \int_{0+}^1 \int_0^\infty \theta \Pi(0 \theta) \mu_c^*(w, \theta) dw d\theta}{\text{ExitRate} \int_{0+}^1 \int_0^\infty \theta (1 - \Pi(0 \theta)) \mu_c^*(w, \theta) dw d\theta}$	0.8596	0.8600	Lee and Mukoyama (2015), Table 1
Relative TFP, Entrants vs. Existing Firms	$\frac{(1 - \text{ExitRate}) \int_{0+}^1 \int_0^\infty \theta e_c^*(w, \theta) dw d\theta}{\text{ExitRate} \int_{0+}^1 \int_0^\infty \theta (\mu_c^*(w, \theta) - e_c^*(w, \theta)) dw d\theta}$	0.9590	0.9600	Lee and Mukoyama (2015), Table 1
Relative Size, Entrants vs. Existing Firms	$\frac{(1 - \text{ExitRate}) \int_{0+}^1 \int_0^\infty \min\{k_c(w, \theta), k^*(\theta)\} e_c^*(w, \theta) dw d\theta}{\text{ExitRate} \int_{0+}^1 \int_0^\infty \min\{k_c(w, \theta), k^*(\theta)\} (\mu_c^*(w, \theta) - e_c^*(w, \theta)) dw d\theta}$	0.6000	0.6000	Lee and Mukoyama (2015), Table 1
Semi-Elasticity of Entry to Tax Rates	$\left( \frac{\int_{0+}^1 \int_0^\infty \bar{e}(w, \theta; 0) dw d\theta}{\int_{0+}^1 \int_0^\infty e_c^*(w, \theta) dw d\theta} - 1 \right) \frac{10\%}{\tau_c}$	0.1700	0.1700	Djankov et al. (2010), Table 5

Note that we define the relative TFP of exiters vs. stayers based on the prior period's TFP. In our model, currently exiting firms each the risk-free rate on their investments. In the data, after a plant exits its TFP is not observed. We therefore compare the prior period's TFP of a firm that subsequently exits to the prior period's TFP of firms that do not exit.

For entrants, the timing is different. Entrant TFP is not observed in data until after entry, which is to say after the first period of operation. We therefore compare the TFP of firms that began operations at the current date to firms that began operations at a prior date (and are not currently exiting). For both of these measures, we use the relative TFP moments from Table 1 of Lee and Mukoyama (2015), using those authors' TFP measure (as opposed to their average labor productivity measure).

Entrant relative size is defined like entrant TFP, but with capital in the place of productivity. We exclude cash from this capital measure, which is why we use the minimum of the broadly-defined capital  $k_c(w, \theta)$  and the first-best capital level  $k^*(\theta)$ . We compare this to the relative size measure in Table 1 of Lee and Mukoyama (2015), which is based on the number of workers at a manufacturing plant (those authors do not provide a size comparison based on capital).

For the entry semi-elasticity, we solve the model under both a 20% profit tax rate and under zero taxes, and then calculate the increase in the mass of entrants. Recall that  $\bar{e}(w, \theta; 0)$  is entry under a payout tax of zero, which is identical to entry under a profit tax of zero, hence the notation in Table A2. We compare this value to the point estimate in Table 5 of Djankov et al. (2010) for the effect of the five-year effective tax rate on the average entry rate in their sample period. We use the average entry rate (8%) described by those authors in the text below their Table 5 to convert their point estimate of a 1.36 percent increase in the entry rate given a 10% change in the tax rate to an elasticity ( $0.17 = \frac{1.36}{8}$ ).

### B.3 Numerical Methods

Our numerical procedure uses a grid of 15 productivity levels by 200 wealth levels to solve for the value function under a profit tax. Our wealth grid is size based on the first best capital level of the most productive type,  $k^*(1)$ . The first 100 wealth levels span from  $k^*(1)/1000$  to  $k^*(1)/5$  in equally spaced intervals, and the second 100 span from  $k^*(1)/5$  to  $2k^*(1)$  in equally spaced intervals. We oversample low wealth levels to more accurately capture the curvature of the value function in that region.

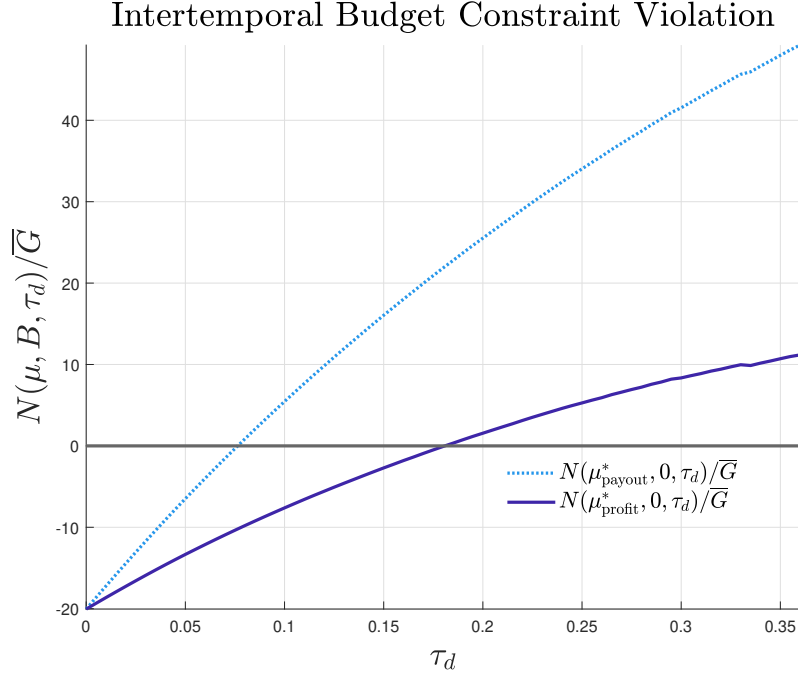
We pre-compute many variables in the firm's problem (in particular,  $b, k$ ) that can be solved for analytically without knowing the value function or the optimal payout boundary in the firm's problem. We then use value function iteration to solve for the value function and for the optimal payout policy.

Note that solving for the value function requires knowing the transition matrix  $\Pi(\theta_{t+1}|\theta_t)$ , and therefore the value function depends on the parameters  $\Pi(0|\theta_L)$  and  $\Pi(0|1)$ . However, the value function does not depend on the parameters  $(\mu_z^e, \xi_w, F)$  which affect entry. After computing the value function, we solve for the values of  $(\mu_z^e, \xi_w, F)$  that minimize the percentage deviation between our model moments and their target values.

We repeat this procedure (solve for the value function, solve for  $(\mu_z^e, \xi_w, F)$ ) over a grid of possible values for  $\Pi(0|\theta_L), \Pi(0|1)$ , choosing the values for those two parameters that result in the best fit.

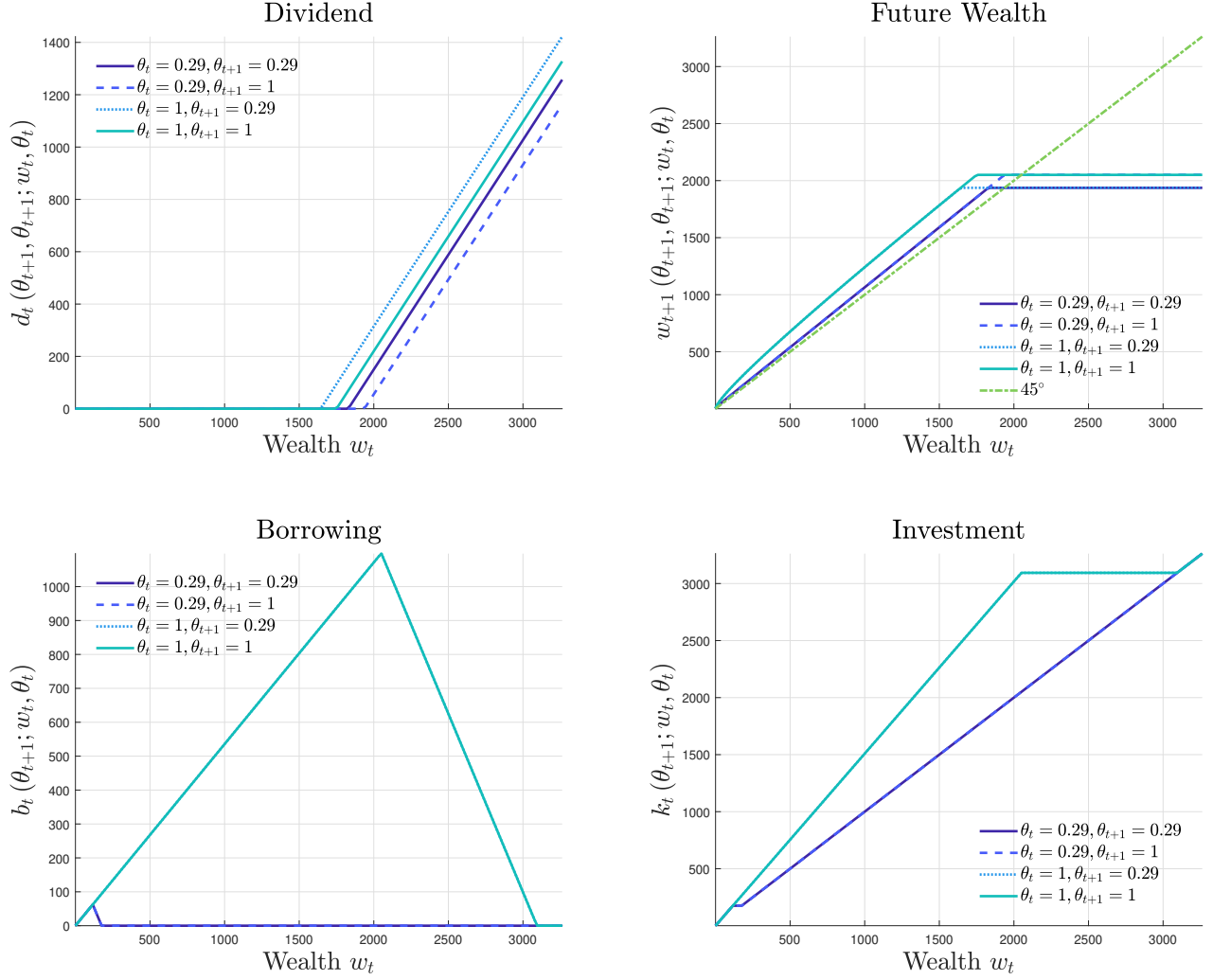


## B.4 Additional Figures



**Note:** Figure A1 shows the Laffer curve present in our model, plotting the net violation of the inter-temporal budget violation, normalized by the target spending  $\bar{G}$ , from our quantitative exercise (see Section 6 for details). The horizontal axis varies the rate of payout taxation (assumed to be constant from date zero onward), and the vertical axis shows  $N(\mu, 0, \tau_d)$  as defined in Equation (32). The figure plots these curves for two values of  $\mu$ , the steady-state under a constant payout tax rate of  $\tau_d = 0.181$ ,  $\mu_{\text{payout}}^*$ , and the steady-state under a constant rate of profit taxation  $\tau_c = 0.2$ ,  $\mu_{\text{profit}}^*$ . In our quantitative exercise, the latter is the initial measure of firms in the economy, and the former is the eventual steady state after switching from profit taxation to payout taxation. In that exercise,  $\bar{G}$  is assumed to be equal to the steady state annual revenue from a profit tax of  $\tau_c = 0.2$ . The units of the vertical axis can therefore be interpreted as multiples of annual tax revenue.

Figure A1: Inter-temporal Budget Constraint Violation vs. Tax Rate



**Note:** Figure A2 illustrates the optimal policies for  $(d_t, w_{t+1}, b_t, k_t)$  in our quantitative exercise (see Section 6 for details), as a function of  $w_t$ , under four different assumptions about current and future productivity:  $(\theta_t, \theta_{t+1}) \in \{(0.29, 0.29), (0.29, 1), (1, 0.29), (1, 1)\}$ . Note that  $b_t$  and  $k_t$  do not depend on the firm's future productivity under the optimal policies.

Figure A2: Optimal Policies in the Dynamic Model

## C Proofs

### C.1 Proof of Lemma 1

The government solves the problem

$$\max \int_0^\infty \int_0^1 R^{-1} \{d_1(w_1, \theta_1) + w_2(w_1, \theta_1)\} \mu_1(w_1, \theta_1) dw_1 d\theta_1,$$

subject to the following set of constraints that apply to each level of initial wealth  $w_1$  and type  $\theta_1$

$$k_1(w_1, \theta_1) \leq R^{-1}b(w_1, \theta_1) + w_1, \forall w_1, \forall \theta_1 \quad (\text{Financing/Investment})$$

$$w_2(w_1, \theta_1) \leq f(k_1(w_1, \theta_1), \theta_1) + (1 - \delta)k_1(w_1, \theta_1) - d_1(w_1, \theta_1) - b_1(w_1, \theta_1) - \tau_1(w_1, \theta_1), \forall w_1, \forall \theta_1 \quad (\text{Production Function})$$

$$w_1^D(k_1(w_1, \theta_1), \theta_1) \leq d_1(w_1, \theta_1) + w_2(w_1, \theta_1), \forall w_1, \forall \theta_1 \quad (\text{No Default})$$

$$d_1(w_1, \theta_1) \leq w_1^D(k_1(w_1, \theta_1), \theta_1), \forall w_1, \forall \theta_1 \quad (\text{Upper Limit on Dividends})$$

as well as the revenue-raising constraint,

$$B_1 + G \leq \int_0^\infty \int_0^1 R^{-1} \tau_1(w_1, \theta_1) \mu_1(w_1, \theta_1) dw_1 d\theta_1, \quad (\text{Revenue Raising})$$

and the non-negativity constraints for  $k_1$ ,  $d_1$ ,  $b_1$ , and  $\tau_1$ .

Observe that firms' production constraint must bind. Therefore, we can express  $d_1(w_1, \theta_1) + w_2(w_1, \theta_1)$  as follows:

$$d_1(w_1, \theta_1) + w_2(w_1, \theta_1) = f(k_1(w_1, \theta_1), \theta_1) + (1 - \delta)k_1(w_1, \theta_1) - b_1(w_1, \theta_1) - \tau_1(w_1, \theta_1).$$

When combined with the definition of continuation wealth after default in Equation (5), the no-default constraint simplifies to

$$b_1(w_1, \theta_1) + \tau_1(w_1, \theta_1) \leq \varphi(1 - \delta)k_1(w_1, \theta_1).$$

As a result,  $d_1(w_1, \theta_1)$  enters only in upper limit on dividends, and therefore it is without loss of generality to assume  $d_1(w_1, \theta_1) = 0$  and ignore the limit on dividends. Moreover, if the financing/investment budget constraint does not bind, the government can increase  $k_1(w_1, \theta_1)$ , increasing the objective and relaxing the no-default constraint, which implies that financing/investment constraint must bind at the optimum.

Therefore, the government's problem can be reformulated in simplified form as follows:

$$\max \int_0^\infty \int_0^1 R^{-1} \{f(k_1(w_1, \theta_1), \theta_1) + (1 - \delta)k_1(w_1, \theta_1) - b_1(w_1, \theta_1) - \tau_1(w_1, \theta_1)\} \mu_1(w_1, \theta_1) dw_1 d\theta_1$$

subject to

$$k_1(w_1, \theta_1) = R^{-1}b_1(w_1, \theta_1) + w_1, \forall w_1, \forall \theta_1 \quad (\text{Financing/Investment})$$

$$b_1(w_1, \theta_1) + \tau_1(w_1, \theta_1) \leq \varphi(1 - \delta)k_1(w_1, \theta_1), \forall w_1, \forall \theta_1 \quad (\text{No Default})$$

$$B_1 + G \leq \int_0^\infty \int_0^1 R^{-1} \tau_1(w_1, \theta_1) \mu_1(w_1, \theta_1) dw_1 d\theta_1, \quad (\text{Revenue Raising})$$

in addition to the the non-negativity constraints. This problem has affine constraints and a concave objective function, and therefore the infinite dimensional analog of the Karush-Kuhn-Tucker conditions are necessary and sufficient to find an optimum. Solving for  $b_1(w_1, \theta_1)$  in the financing/investment constraint and substituting in,

allows us to rewrite the objective function as follows:

$$\int_0^\infty \int_0^1 \{R^{-1}(f(k_1(w_1, \theta_1), \theta_1) + (1 - \delta - R)k_1(w_1, \theta_1) - \tau_1(w_1, \theta_1)) + w_1\} \mu_1(w_1, \theta_1) dw_1 d\theta_1.$$

Solving for  $b_1(w_1, \theta_1)$  in the financing/investment constraint and substituting in, we can rewrite the no default constraint as follows,

$$k_1(w_1, \theta_1) \leq \frac{w_1 - R^{-1}\tau_1}{1 - R^{-1}\varphi(1 - \delta)}.$$

The non-negativity constraint on  $b_1$  implies that  $w_1 \leq k_1$ . By defining the Lagrange multiplier on the revenue-raising constraint by  $\chi_1$ , we can express the new objective function as

$$\int_0^\infty \int_0^1 \{R^{-1}(f(k_1(w_1, \theta_1), \theta_1) + (1 - \delta - R)k_1(w_1, \theta_1) + (\chi_1 - 1)\tau_1(w_1, \theta_1)) + w_1\} \mu_1(w_1, \theta_1) dw_1 d\theta_1 - \chi_1(G + B_1).$$

We can simplify the formulation of the government's problem by defining  $U_1(w_1, \theta_1; \chi_1)$  as follows:

$$U_1(w_1, \theta_1; \chi_1) = \max_{k_1 \geq 0, \tau_1 \geq 0} \{R^{-1}\{f(k_1, \theta_1) + (1 - \delta - R)k_1\} + w_1 + R^{-1}(\chi_1 - 1)\tau_1\},$$

subject to the constraint that

$$w_1 \leq k_1 \leq \frac{w_1 - R^{-1}\tau_1}{1 - R^{-1}\varphi(1 - \delta)}.$$

So the government's problem can be expressed as

$$J_1(\mu_1, B_1) = \min_{\chi_1 \geq 0} \left\{ \int_0^\infty \int_0^1 U_1(w_1, \theta_1; \chi_1) \mu_1(w_1, \theta_1) d\theta_1 dw_1 - \chi_1(G + B_1) \right\},$$

which concludes the proof. Note that we use the measure of types  $\mu_1$  and the level of debt  $B_1$  as arguments of the government's value function to be consistent with our formulation of the dynamic model. Since  $G$  and  $B_1$  enter symmetrically, it is without loss of generality to only include  $B_1$  as an argument of the government's value function.

## C.2 Proof of Proposition 1

First, note that if  $\chi_1 < 1$ , it will be optimal to set  $\tau_1 = 0$  always, and therefore raise no revenue. Therefore, it must be that  $\chi_1 \geq 1$ , and that, if  $\chi_1 = 1$  is feasible, it will be optimal.

If  $\chi_1 = 1$ , the government's problem corresponds to

$$U_1(w_1, \theta_1; 1) = \max_{k_1 \geq 0, \tau_1 \geq 0} R^{-1}\{f(k_1, \theta_1) + (1 - \delta)k_1 - Rk_1\} + w_1,$$

subject to the following constraint on capital allocations

$$w_1 \leq k_1 \leq \frac{w_1 - R^{-1}\tau_1}{1 - R^{-1}\varphi(1 - \delta)},$$

and non-negativity constraint for  $k_1$  and  $\tau_1$ . Let  $\mu$  and  $\phi$  be the multipliers on the upper and lower bounds for capital, and let  $v$  be the multiplier on the constraint that  $\tau_1 \geq 0$ . We can therefore define the following

Lagrangian:

$$\mathcal{L} = R^{-1} \{f(k_1, \theta_1) + (1 - \delta)k_1 - Rk_1\} + w_1 - \mu \left( k_1 - \frac{w_1 - R^{-1}\tau_1}{1 - R^{-1}\varphi(1 - \delta)} \right) + \phi(k_1 - w_1) + \nu\tau_1.$$

Note that, under the assumption that  $w_1 \geq 0$  for all firms, the capital non-negativity constraint becomes redundant. The optimality conditions for  $k_1$  and  $\tau_1$  respectively are

$$R^{-1} \{f_k(k_1, \theta_1) + (1 - \delta) - R\} - \mu + \phi = 0 \Rightarrow R^{-1} \{f_k(k_1, \theta_1) + (1 - \delta) - R\} = \mu - \phi \quad (\text{A1})$$

$$-\frac{1}{R - \varphi(1 - \delta)}\mu + \nu = 0 \Rightarrow \nu = \frac{1}{R - \varphi(1 - \delta)}\mu. \quad (\text{A2})$$

Equation (A2) implies that if  $\mu > 0$ , then  $\nu > 0$  and  $\tau_1 = 0$ . In general,  $\mu > 0$  and  $\phi > 0$  are mutually exclusive. Therefore, if  $\tau_1 = 0$ , that is,  $\nu > 0$ , it must be that  $\mu > 0$  and  $\phi = 0$ .

Because  $f_k(k_1, \theta_1) + (1 - \delta) - R \geq 0$ ,  $\forall k_1$ , with equality if and only if  $k_1 \geq k^*(\theta_1)$ , it follows that  $k_1 < k^*(\theta_1)$  implies that  $\mu > 0$  and  $\phi = 0$ , and also  $\nu > 0$  and  $\tau_1 = 0$ . This case requires that

$$k^*(\theta_1) > \frac{w_1}{1 - R^{-1}\varphi(1 - \delta)}.$$

If  $k_1 \geq k^*(\theta_1)$ , then we must have  $\mu = \phi = 0$  and  $\nu = 0$ . This case requires that

$$k^*(\theta_1) \leq \frac{w_1}{1 - R^{-1}\varphi(1 - \delta)}.$$

In this case, the tax is indeterminate, but must necessarily satisfy

$$0 \leq \tau_1 \leq \varphi(1 - \delta)w_1,$$

so that the feasible set for capital is non-empty. In the statement of Proposition 1, we propose the following tax function

$$\tau_1(w_1, \theta_1) = \min \left\{ \varphi(1 - \delta)w_1, \frac{\tau_d}{1 + \tau_d} R \max \{w_1 - (1 - R^{-1}\varphi(1 - \delta))k^*(\theta_1), 0\} \right\},$$

where  $\tau_d \geq 0$ . The proposed functional form satisfies the restrictions on  $\tau_1$  in both cases, and raises positive revenue. Moreover, in the limit as  $\tau_d \rightarrow \infty$ , this policy raises the maximum possible revenue. Consequently, if  $\chi_1 = 1$ , this policy is optimal for some  $\tau_d$ . It follows that, as long as the revenue raised by the proposed policy satisfies the revenue-raising constraint,  $\chi_1 = 1$ .

### C.3 Proof of Corollary 1

We begin by observing that if  $\tau_d \in \left[0, \frac{\varphi(1 - \delta)}{R - \varphi(1 - \delta)}\right]$ , then  $R \frac{\tau_d}{1 + \tau_d} \leq \varphi(1 - \delta)$ , and

$$\varphi(1 - \delta)w_1 \geq \frac{\tau_d}{1 + \tau_d} R w_1 \geq \frac{\tau_d}{1 + \tau_d} R \max \{w_1 - \bar{w}_1(\theta), 0\}$$

for all positive values of wealth. Consequently,

$$\tau_1(w_1, \theta_1) = \frac{\tau_d}{1 + \tau_d} R \max\{w_1 - \bar{w}_1(\theta), 0\}$$

in this case.

### Properties of $V_1$

The function  $V_1(w, \theta; \mu_1, B_1) = V_1(w, \theta; \chi_1 = 1, \tau_d) = R^{-1} \{d_1(w_1, \theta_1) + w_2(w_1, \theta_1)\}$  corresponds to an indirect utility function for a firm with wealth  $w_1$  and type  $\theta_1$ . Note that

$$\begin{aligned} d_1(w_1, \theta_1) + w_2(w_1, \theta_1) &= f(k_1(w_1, \theta_1), \theta_1) + (1 - \delta)k_1(w_1, \theta_1) - \tau_1(w_1, \theta_1) - Rk_1(w_1, \theta_1) + Rw_1, \\ &= f(k_1(w_1, \theta_1), \theta_1) + (1 - \delta)k_1(w_1, \theta_1) - \frac{\tau_d}{1 + \tau_d} R \max\{w_1 - (1 - R^{-1}\varphi(1 - \delta))k^*(\theta_1), 0\} \\ &\quad - Rk_1(w_1, \theta_1) + Rw_1, \end{aligned}$$

where we use the fact that the optimal policy satisfies  $\tau_1(w_1, \theta_1) = \frac{\tau_d}{1 + \tau_d} R \max\{w_1 - (1 - R^{-1}\varphi(1 - \delta))k^*(\theta_1), 0\}$ , and where the optimal choice of capital is given by

$$k_1(w_1, \theta_1) = \min\left\{k^*(\theta_1), \frac{w_1}{1 - R^{-1}\varphi(1 - \delta)}\right\}.$$

Note first that  $V_1(w, \theta; \chi_1 = 1, \tau_d)$  is continuous. Anywhere  $w_1 > \bar{w}_1(\theta)$ ,

$$V_{1,w}(w_1, \theta_1; \chi_1 = 1, \tau_d) = 1 - \frac{\tau_d}{1 + \tau_d} = \frac{1}{1 + \tau_d}.$$

Anywhere  $w_1 < \bar{w}_1(\theta)$ ,

$$V_{1,w}(w_1, \theta_1; \chi_1 = 1, \tau_d) = R^{-1} \frac{f_k\left(\frac{w_1}{1 - R^{-1}\varphi(1 - \delta)}, \theta_1\right) + (1 - \delta) - R}{1 - R^{-1}\varphi(1 - \delta)} + 1 > 1 > \frac{1}{1 + \tau_d}$$

by the assumption that if  $w_1 < \bar{w}_1(\theta)$ ,  $\frac{w_1}{1 - R^{-1}\varphi(1 - \delta)} < k^*(\theta_1)$ . Concavity follows from the concavity of the production function.

### Properties of $U_1$

The function  $U_1(w, \theta; \mu_1, B_1) = U_1(w_1, \theta_1; \chi_1 = 1)$ , which corresponds to an indirect utility function for firms from the perspective of the government, is given by

$$U_1(w_1, \theta_1; \chi_1 = 1) = f(k_1(w_1, \theta_1), \theta_1) + (1 - \delta)k_1(w_1, \theta_1) - Rk_1(w_1, \theta_1) + Rw_1,$$

where  $k_1(w_1, \theta_1)$  corresponds to

$$k_1(w_1, \theta_1) = \min\left\{k^*(\theta_1), \frac{w_1}{1 - R^{-1}\varphi(1 - \delta)}\right\}.$$

The argument above for  $V_1$  applies from this point, essentially unmodified.

### C.4 Proof of Lemma 2

The post-dividend no-default constraint with truthful reporting in the payout stage ( $\theta''_{t+1} = \theta_{t+1}$ ) requires, assuming a strictly increasing in wealth value function  $V_{t+1}$ , that

$$w^D(k_t(\theta'_{t+1}), \theta''_{t+1}) - d_t(\theta'_{t+1}, \theta''_{t+1}) \leq w_{t+1}(\theta', \theta''_{t+1}), \forall \theta'_{t+1}, \theta''_{t+1}.$$

This proves the “only if” part of the claim. To prove the “if” part, suppose Equation (18) holds. It follows that  $d_t(\theta'_{t+1}, \theta''_{t+1}) + V_{t+1}^{def}(w^D(k_t(\theta'_{t+1}), \theta_{t+1}) - d_t(\theta'_{t+1}, \theta''_{t+1}), \theta_{t+1}) \leq d_t(\theta'_{t+1}, \theta''_{t+1}) + V_{t+1}(w_{t+1}(\theta'_{t+1}, \theta''_{t+1}), \theta_{t+1}), \forall \theta_{t+1}, \theta'_{t+1}, \theta''_{t+1}$ , and by the dividend-taxes IC constraint,

$$d_t(\theta'_{t+1}, \theta''_{t+1}) + V_{t+1}(w_{t+1}(\theta'_{t+1}, \theta''_{t+1}), \theta_{t+1}) \leq d_t(\theta'_{t+1}, \theta_{t+1}) + V_{t+1}(w_{t+1}(\theta'_{t+1}, \theta_{t+1}), \theta_{t+1}), \forall \theta_{t+1}, \theta'_{t+1}, \theta''_{t+1}.$$

Therefore, the constraint in Equation (18) and the Dividend/Taxes IC constraint together imply the post-dividend no-default constraint.

### C.5 Proof of Lemma 3

To avoid default, we must have, for the firm's optimal choice of  $d_t$ ,

$$d_t + V_{t+1}(w_{t+1}, \theta_{t+1}) \geq d_t + V_{t+1}(w^D(k_t, \theta_t) - d_t, \theta_{t+1})$$

and

$$d_t + V_{t+1}(w_{t+1}, \theta_{t+1}) \geq V_{t+1}(w^D(k_t, \theta_t), \theta_{t+1}),$$

taking as given an arbitrary  $b_t, k_t, \theta_t, \theta_{t+1}$ . By the wealth accumulation constraint, which will bind, if the government implements a payout tax rate  $\tau_t = \tau d_t$  for some  $\tau \geq 0$ , then

$$w_{t+1} = f(k_t, \theta_t) + (1 - \delta)k_t - (1 + \tau)d_t - b_t.$$

If  $V_{t+1}$  is consistent with that constant payout tax rate  $\tau$ , then

$$\frac{\partial}{\partial d}[d + V_{t+1}(f(k_t, \theta_t) + (1 - \delta)k_t - (1 + \tau)d - b_t, \theta_{t+1})] \leq 0,$$

and it is without loss of generality to suppose that the firm will choose  $d_t = 0$ . In this case, the firm will not default if and only if  $w_{t+1} \geq w^D(k_t, \theta_t)$ , which is

$$f(k_t, \theta_t) + (1 - \delta)k_t - b_t \geq w^D(k_t, \theta_t).$$

### C.6 Proof of Proposition 2

We begin by defining the set of allocations  $\mathcal{M}^+(w_t, \theta_t)$ .

**Definition 5.** Given an observable initial wealth  $w_t$  and an initial type  $\theta_t$ , a relaxed-feasible direct revelation mechanism  $m$  is a collection of weakly positive functions  $\{b_t(\theta'), k_t(\theta'), w_{t+1}(\theta', \theta''), d_t(\theta', \theta''), \tau_t(\theta', \theta'')\}$



such that the following constraints are satisfied:

Upper Limit on Dividends:

$$d_t(\theta', \theta'') \leq w^D(k_t(\theta'), \theta_t), \forall \theta', \theta'',$$

Initial Budget Constraint (with free disposal):

$$k_t(\theta') \leq w_t + R^{-1}b_t(\theta'), \forall \theta',$$

Production Function (with free disposal):

$$\begin{aligned} w_{t+1}(\theta', \theta'') &\leq f(k_t(\theta'), \theta_t) + (1 - \delta)k_t(\theta') \\ &\quad - d_t(\theta', \theta'') - b_t(\theta') - \tau_t(\theta', \theta''), \forall \theta', \theta'', \end{aligned}$$

Simplified No-Default:

$$w^D(k_t(\theta'), \theta_t) - d_t(\theta', \theta'') \leq w_{t+1}(\theta', \theta''), \forall \theta', \theta'',$$

Let  $\mathcal{M}^+(w_t, \theta_t)$  be the set of all such mechanisms.

By Lemma 2, for all value functions  $V_{t+1}$  that are strictly increasing in wealth,  $\mathcal{M}(w_t, \theta_t, V_{t+1}) \subseteq \mathcal{M}^+(w_t, \theta_t)$ .

Let us now consider the mechanism design problem

$$m^* \in \arg \max_{m \in \mathcal{M}^+(w_t, \theta_t)} R^{-1} \int_0^1 \{d_t(\theta, \theta) + \tau_t(\theta, \theta) + U_{t+1}(w_{t+1}(\theta, \theta), \theta; 1)\} \Pi(\theta | \theta_t) d\theta.$$

Observing that there are no interactions across firms of different types. We can therefore solve this problem firm-by-firm.

We begin by observing that the production function constraint must bind with equality because  $U_{t+1}(w_{t+1}(\theta, \theta), \theta; 1)$  is strictly increasing in wealth. By similar logic, because

$$0 < \frac{\partial}{\partial k} w^D(k, \theta_t) \leq f_k(k, \theta_t) + (1 - \delta),$$

increasing capital holding debt fixed increases the objective and relaxes all constraints. Therefore, the initial budget constraint must bind. Note by the non-negativity of debt that this implies  $k_t(\theta) \geq w$ .

Because these constraints bind, we can rewrite the no-default constraint under truth-telling as

$$w^D(k_t(\theta), \theta_t) \leq f(k_t(\theta), \theta_t) + (1 - \delta - R)k_t(\theta) + R w_t - \tau_t(\theta, \theta),$$

and the objective for this firm as

$$d_t(\theta, \theta) + \tau_t(\theta, \theta) + U_{t+1}(f(k_t(\theta), \theta_t) + (1 - \delta - R)k_t(\theta) + R w_t - \tau_t(\theta, \theta) - d_t(\theta, \theta), \theta; 1).$$

It follows immediately that if  $d_t(\theta, \theta) > 0$  only if  $U_{t+1, w^-} = 1$  ( $U_{t+1, w^-}$  is the directional derivative of  $U_{t+1}$  with respect to reducing wealth, which exists by concavity). This requires, by the definition of consistency with a constant payout tax,

$$f(k_t(\theta), \theta_t) + (1 - \delta - R)k_t(\theta) + R w_t > \bar{w}(\theta).$$

The same logic applies to  $\tau_t(\theta, \theta) > 0$ , as reducing taxes might also relax the no-default constraint. Moreover,

if the no-default constraint binds, we must have  $\tau_t(\theta, \theta) = 0$  regardless of whether the above condition is satisfied. Therefore, if  $f(k_t(\theta), \theta_t) + (1 - \delta - R)k_t(\theta) + w_t \leq \bar{w}(\theta)$ , we must have  $d_t(\theta, \theta) = \tau_t(\theta, \theta) = 0$ . If  $f(k_t(\theta), \theta_t) + (1 - \delta - R)k_t(\theta) + w_t > \bar{w}(\theta)$ , any choice of taxes and dividends are optimal provided that

$$f(k_t(\theta), \theta_t) + (1 - \delta - R)k_t(\theta) + R w_t - \tau_t(\theta, \theta) - d_t(\theta, \theta) \geq \bar{w}(\theta).$$

We are therefore free to suppose that

$$\tau_t(\theta, \theta) = \tau d_t(\theta, \theta),$$

where  $\tau$  is defined by the assumption of consistency with a constant payout tax (Definition 1). Under this assumption, the no-default constraint requires

$$\tau d_t(\theta, \theta) \leq f(k_t(\theta), \theta_t) + (1 - \delta - R)k_t(\theta) + R w_t - w^D(k_t(\theta), \theta_t).$$

Capital will be chosen to maximize  $f(k_t(\theta), \theta_t) + (1 - \delta - R)k_t(\theta)$  if this is feasible, which is to say achieving at least  $k_t(\theta) \geq k^*(\theta_t)$ , and otherwise the no-default constraint will bind and capital will be maximal.

Define  $k_{max}(w_t, \theta_t)$  by

$$f(k_{max}(w_t, \theta_t), \theta_t) + (1 - \delta - R)k_{max}(w_t, \theta_t) + R w_t = w^D(k_{max}(w_t, \theta_t), \theta_t).$$

Observe under the functional form of (5) that this value exists and is strictly greater than  $w_t$ , and that for all  $k < k_{max}(w_t, \theta_t)$ ,

$$f(k, \theta_t) + (1 - \delta - R)k + R w_t \geq w^D(k, \theta_t).$$

It remains to select a level of  $d_t(\theta, \theta)$  so that the no-default and dividend limits are satisfied, and  $d_t(\theta, \theta) = 0$  if this is strictly optimal. The following allocation  $m^* \in \mathcal{M}^+(w_t, \theta_t)$  chooses the maximum possible dividend satisfying these constraints, and ignores the report on the first date by assigning all types the same level of capital:

$$\begin{aligned} k_t(\theta) &= \max\{w_t, \min\{k^*(\theta_t), k_{max}(w_t, \theta_t)\}\}, \\ d_t(\theta, \theta') &= \max\{0, \min\{\frac{1}{1+\tau}(f(k_t(\theta), \theta_t) + (1 - \delta - R)k_t(\theta) + R w_t - \bar{w}(\theta')), \\ &\quad w^D(k_t(\theta), \theta_t), \frac{1}{\tau}(f(k_t(\theta), \theta_t) + (1 - \delta - R)k_t(\theta) + R w_t - w^D(k_t(\theta), \theta_t))\}\}, \\ \tau_t(\theta, \theta') &= \tau d_t(\theta, \theta'), \\ b_t(\theta) &= R(k_t(\theta) - w_t), \\ w_{t+1}(\theta, \theta') &= f(k_t(\theta), \theta_t) + (1 - \delta)k_t(\theta) - d_t(\theta, \theta') - b_t(\theta) - \tau_t(\theta, \theta'). \end{aligned}$$

If  $\tau = 0$ , we will treat  $\tau^{-1}$  as infinite and ignore the term with  $\tau^{-1}$  in the min function that defines dividends.

Let us next consider the firm problem (Definition 3), and observe that the constraints are identical to the constraints of the relaxed mechanism design problem. Because  $V_{t+1}$  is strictly increasing in wealth, our arguments that the initial budget constraint and production function bind continue to hold.

Consequently, the firm objective is

$$d_t + V_{t+1} (f(k_t, \theta_t) + (1 - \delta - R)k_t + R w_t - (1 + \tau)d_t, \theta),$$

subject to the constraints  $k_t \geq w_t$ ,

$$0 \leq d_t \leq w^D(k_t(\theta), \theta_t),$$

and

$$\tau d_t \leq f(k_t, \theta_t) + (1 - \delta - R)k_t + R w_t - w^D(k_t, \theta_t).$$

It follows immediately that  $d_t > 0$  implies

$$f(k_t, \theta_t) + (1 - \delta - R)k_t + R w_t > \bar{w}(\theta),$$

and that the firm is indifferent to all dividend levels provided this condition holds. If this condition is violated, the firm will set  $d_t = 0$ .

This are the exact same conditions as the ones in the government's mechanism design problem, and therefore the allocation that solves the mechanism design problem described above is also a solution to the firm's problem.

To complete the proof, we must show that these allocations are contained in  $\mathcal{M}(w_t, \theta_t, V_{t+1})$  under the assumption that  $V_{t+1}$  and  $U_{t+1}$  are consistent with a constant payout tax. Let us observe first that in the proposed allocation,  $k_t \geq w_t$ , and therefore the interim participation constraint (17) will be satisfied provided that the no-default constraint is satisfied. By Lemma 2, if the dividend/investment IC constraint (16) is satisfied, the allocation will satisfy the no-default constraint (13) because it satisfies (18) by construction. Moreover, the financing/investment IC constraint (15) is automatically satisfied, as there is only a single capital/debt level shared by all firms with the observable type  $(w_t, \theta_t)$ .

Consider the blocked dividend no-default constraint (14). Note that the initial report is irrelevant, and therefore it suffices to consider the constraint under truth-telling. Observe first that if  $d_t(\theta, \theta) = 0$ , the constraint is satisfied because  $k_t(\theta) \leq k_{max}(w_t, \theta_t)$ , implying

$$f(k_t(\theta), \theta_t) + (1 - \delta - R)k_t(\theta) + R w_t \geq w^D(k_t(\theta), \theta_t).$$

But  $d_t(\theta, \theta) > 0$  only if  $w_{t+1}(\theta, \theta) \geq \bar{w}(\theta)$ , and therefore

$$\begin{aligned} d_t(\theta, \theta) + V_{t+1}(w_{t+1}(\theta, \theta), \theta) &= V_{t+1}(w_{t+1}(\theta, \theta) + (1 + \tau)d_t(\theta, \theta), \theta) \\ &= V_{t+1}(f(k_t(\theta), \theta_t) + (1 - \delta - R)k_t(\theta) + w_t, \theta) \\ &\geq V_{t+1}(w^D(k_t(\theta), \theta_t), \theta), \end{aligned}$$

and therefore the constraint is satisfied.

To conclude, we show that the dividend/investment IC constraint (16) is satisfied. Observe that in the proposed allocation,

$$w_{t+1}(\theta, \theta') = f(k_t(\theta), \theta_t) + (1 - \delta - R)k_t(\theta) + R w_t - (1 + \tau)d_t(\theta, \theta').$$

Let us suppose first that type  $\theta'$  has a lower dividend,  $d_t(\theta, \theta') < d_t(\theta, \theta)$ . This implies  $d_t(\theta, \theta) > 0$ , implying  $w_{t+1}(\theta, \theta') > w_{t+1}(\theta, \theta) \geq \bar{w}(\theta)$ , and therefore

$$d_t(\theta, \theta) + V_{t+1}(w_{t+1}(\theta, \theta), \theta) = d_t(\theta, \theta') + V_{t+1}(w_{t+1}(\theta, \theta'), \theta),$$

and hence the dividend/investment IC constraint (16) is satisfied in this case (noting that the initial report does not affect allocations).

Now suppose that type  $\theta'$  has a higher dividend,  $d_t(\theta, \theta') > d_t(\theta, \theta)$ . In this case,  $w_{t+1}(\theta, \theta') < w_{t+1}(\theta, \theta)$ . By  $V_{t,w^-} \geq 1 - \tau$  (the directional derivative again exists by concavity),

$$V_{t+1}(w_{t+1}(\theta, \theta), \theta) - (1 - \tau)(w_{t+1}(\theta, \theta) - w_{t+1}(\theta, \theta')) \geq V_{t+1}(w_{t+1}(\theta, \theta'), \theta),$$

which immediately implies

$$V_{t+1}(w_{t+1}(\theta, \theta), \theta) + (d_t(\theta, \theta) - d_t(\theta, \theta')) \geq V_{t+1}(w_{t+1}(\theta, \theta'), \theta),$$

and therefore the dividend/investment IC constraint (16) is satisfied in this case as well.

We conclude that the proposed allocation  $m^*$  is contained in  $\mathcal{M}(w_t, \theta_t, V_{t+1})$ .

### C.7 Proof of Proposition 3

Consider the problem defined by (23) and the implementation constraint. Let us now relax the problem by allowing the mechanism  $m$  to be chosen from the set  $\mathcal{M}^+(w_0, \theta_0)$  instead of  $\mathcal{M}(w_0, \theta_0, V_1)$ , and ignore the implementation constraint. The resulting problem is

$$\begin{aligned} J_0(w_0, \theta_0, B_0) = & \max_{\chi_1 \geq 1, \tau_d \geq 0} \max_{m \in \mathcal{M}^+(w_0, \theta_0)} R^{-1} \int_0^1 \{d_0(\theta, \theta) + \chi_1 \tau_0(\theta, \theta)\} \Pi(\theta | \theta_0) d\theta \\ & + R^{-1} \int_0^1 U_1(w_1(\theta, \theta), \theta; \chi_1) \Pi(\theta | \theta_0) d\theta \\ & - \chi_1(B_0 + G + R^{-1}G). \end{aligned}$$

Observing by the envelope theorem that

$$\frac{\partial}{\partial \chi_1} U_1(w_1(\theta, \theta), \theta; \chi_1) = R^{-1} \tau_1^*(w_1(\theta, \theta), \theta; \chi_1, \tau),$$

and that  $\tau_1^*(w_1(\theta, \theta), \theta; \chi_1, \tau_d)$  is weakly increasing in  $\chi_1$ , the problem is convex in  $\chi_1$ , and therefore either  $\chi_1 = 1$  or  $\chi_1$  is infinite.

Recall from Lemma 1 that

$$w_1(\theta, \theta) \leq k_1^*(w_1(\theta, \theta), \theta; \chi_1, \tau) \leq \frac{w_1(\theta, \theta) - R^{-1} \tau_1^*(w_1(\theta, \theta), \theta; \chi_1, \tau_d)}{1 - R^{-1} \varphi(1 - \delta)}.$$

If  $\chi_1$  is infinite,  $\tau_1$  is maximal, implying

$$\lim_{\chi_1 \rightarrow \infty} \tau_1^*(w_1(\theta, \theta), \theta; \chi_1, \tau) = \varphi(1 - \delta) w_1(\theta, \theta)$$

and

$$k_1^*(w_1(\theta, \theta), \theta; \chi_1, \tau) = w_1(\theta, \theta).$$

The optimal policy at date zero will also maximize taxes, setting

$$\tau_0(\theta, \theta) = \varphi(1 - \delta) k_0(\theta_0) + R(w_0 - k_0(\theta_0)).$$

Since this tax level is decreasing in capital, the optimal policy will be  $k_0(\theta_0) = w_0$ . Consequently, this policy will achieve the minimum feasible utility for firms, and is therefore dominated by the policy associated with  $\chi_1 = 1$  if that policy is feasible.

If  $\chi_1 = 1$ , the choice of  $\tau_d$  has no influence on the relaxed problem, and we are therefore free to choose some  $\tau_d \in \left[0, \frac{\varphi(1-\delta)}{R-\varphi(1-\delta)}\right]$ . In this case, by Corollary 1, the functions  $V_1$  and  $U_1$  are consistent with a constant payout tax (Definition 1). Therefore, by Proposition 2, a constant payout tax is optimal and the associated mechanism is feasible in the original problem.

In this case, the implementation constraint (which must hold if this solution is optimal) is

$$B_0 + G + R^{-1}G = R^{-1} \int_0^1 \tau_d d_0(\theta, \theta) \Pi(\theta|\theta_0) d\theta + R^{-2} \int_0^\infty \int_0^1 \frac{\tau_d}{1 + \tau_d} R \max\{w_1(\theta, \theta) - \bar{w}(\theta), 0\} \mu_1(w, \theta) d\theta dw.$$

Under the constant payout tax, production at date zero is optimal (this follows by arguments in the proof of Proposition 2, or directly from the fact that  $V_{t+1}$  is strictly increasing). Defining the capital level chosen as

$$k_0 = \max \left\{ w_0, \min \left\{ k^*(\theta_0), \frac{w_0}{1 - R^{-1}\varphi(1 - \delta)} \right\} \right\},$$

we have

$$w_1(\theta, \theta) = f(k_0, \theta_0) + (1 - \delta - R)k_0 + R w_0 - (1 + \tau_d)d_0(\theta, \theta)$$

and  $d_0(\theta, \theta) > 0$  only if  $w_1(\theta, \theta) \geq \bar{w}(\theta)$ . Therefore it is without loss of generality in the implementation constraint to suppose  $d_0(\theta, \theta) = 0$ , and the constraint is

$$B_0 + G + R^{-1}G = R^{-1} \int_0^1 \frac{\tau_d}{1 + \tau_d} \max\{f(k_0, \theta_0) + (1 - \delta - R)k_0 + R w_0 - \bar{w}(\theta), 0\} \Pi(\theta|\theta_0) d\theta.$$

If this holds for some  $\tau_d \in \left[0, \frac{\varphi(1-\delta)}{R-\varphi(1-\delta)}\right]$ , a payout tax at rate  $\tau_d$  is the optimal policy. Since  $B_0 + G + R^{-1}G$  is assumed to be strictly positive, this value of  $\tau_d$  must also be strictly positive.

### C.8 Proof of Lemma 4

The Bellman equation defining the firm's problem is

$$\begin{aligned}\bar{V}(w_t, \theta_t; \tau_d) = & \max_{b_t(\theta_{t+1}) \geq 0, k_t(\theta_{t+1}) \geq 0, w_{t+1}(\theta_{t+1}) \geq 0, d_t(\theta_{t+1}) \geq 0} \\ & R^{-1} \int_0^1 \{d_t(\theta_{t+1}) + \bar{V}(w_{t+1}(\theta_{t+1}), \theta_{t+1}; \tau_d)\} \mu(\theta_{t+1} | \theta_t) d\theta_{t+1}\end{aligned}$$

subject to

$$\begin{aligned}w_{t+1}(\theta_{t+1}) &\leq f(k_t(\theta_{t+1}), \theta_t) + (1 - \delta)k_t(\theta_{t+1}) - (1 + \tau)d_t(\theta_{t+1}) - b_t(\theta_{t+1}), \\ k_t(\theta_{t+1}) &\leq w_t + R^{-1}b_t(\theta_{t+1}), \\ w_{t+1}(\theta_{t+1}) + d_t(\theta_{t+1}) &\geq w^D(k_t(\theta_{t+1}), \theta_t), \\ d_t(\theta_{t+1}) &\leq w^D(k_t(\theta_{t+1}), \theta_t).\end{aligned}$$

It is immediately apparent the  $\bar{V}(w_t, \theta_t; \tau_d)$  is increasing in  $w_t$ , and hence that the initial budget constraint and production constraints bind. We can therefore rewrite the problem as

$$\begin{aligned}\bar{V}(w_t, \theta_t; \tau_d) = & \max_{k_t(\theta_{t+1}) \geq 0, d_t(\theta_{t+1}) \geq 0} \\ & R^{-1} \int_0^1 \{d_t(\theta_{t+1}) + \bar{V}(y(k_t(\theta_{t+1}), \theta_t) + R w_t - (1 + \tau)d_t(\theta_{t+1}), \theta_{t+1}; \tau_d)\} \mu(\theta_{t+1} | \theta_t) d\theta_{t+1}\end{aligned}$$

where

$$y(k, \theta_t) = f(k, \theta_t) + (1 - \delta)k - Rk$$

subject to

$$\begin{aligned}k_t(\theta_{t+1}) &\geq w_t, \\ y(k_t(\theta_{t+1}), \theta_t) + R w_t - \tau d_t(\theta_{t+1}) &\geq w^D(k_t(\theta_{t+1}), \theta_t), \\ d_t(\theta_{t+1}) &\leq w^D(k_t(\theta_{t+1}), \theta_t), \\ (1 + \tau)d_t(\theta_{t+1}) &\leq \pi(k_t(\theta_{t+1}), \theta_t) + R w_t.\end{aligned}$$

Note that the feasible set of policies is always non-empty ( $k_t(\theta_{t+1}) = w_t, d_t(\theta_{t+1}) = 0$  is always feasible), and that the flow payoff is bounded from below by zero.

Define the maximum possible profit

$$\bar{y} = \max_{\theta \in [0, 1]} y(k^*(\theta), \theta).$$

The firm's value function is also bounded from above via the inter-temporal budget constraint and the best possible outcome,

$$\bar{V}(w_t, \theta_t; \tau_d) = \mathbb{E}_t \left[ \sum_{j=0}^{\infty} R^{-j} d_{t+j}^* \right] \leq \frac{1}{1 + \tau_d} (w_t + \sum_{j=0}^{\infty} R^{-j} \bar{y}).$$

Let us conjecture and verify that a solution to the firm's problem exists and satisfies Definition 1. Define an upper bound on wealth

$$w^* = (1 - R^{-1}\varphi(1 - \delta))k^*(1),$$

and let  $\mathcal{V}$  be the set of bounded functions on  $w \in [0, w^*]$  and  $\theta \in [0, 1]$ . Define the Bellman operator  $T : \mathcal{V} \rightarrow \mathcal{V}$  by

$$\begin{aligned} T(V; \tau_d)(w_t, \theta_t) &= \max_{k_t(\theta_{t+1}) \geq 0, d_t(\theta_{t+1}) \geq 0} \\ &R^{-1} \int_0^1 \left\{ d_t(\theta_{t+1}) + \frac{1}{1 + \tau_d} \max\{y(k_t(\theta_{t+1}), \theta_t) + R w_t - (1 + \tau)d_t(\theta_{t+1}) - w^*, 0\} \right\} \mu(\theta_{t+1}|\theta_t) d\theta_{t+1} \\ &+ \int_0^1 \bar{V}(\min\{y(k_t(\theta_{t+1}), \theta_t) + R w_t - (1 + \tau)d_t(\theta_{t+1}), w^*\}, \theta_{t+1}) \mu(\theta_{t+1}|\theta_t) d\theta_{t+1} \end{aligned}$$

subject to the constraints above.

It is immediate that this Bellman operator satisfies Blackwell's sufficient conditions, and hence that the contraction mapping theorem holds and a unique fixed point exists. By the boundedness of the fixed point, the non-emptiness of the set of policies, and the fact that the flow payoff is bounded from below, [Stokey, Lucas and Prescott \(1989\)](#) theorem 9.2 applies. That is, letting  $V^*$  denote the fixed point,

$$\bar{V}(w_t, \theta_t; \tau_d) = \begin{cases} V^*(w_t, \theta_t) & w_t \leq w^* \\ V^*(w^*, \theta_t) + \frac{1}{1 + \tau_d}(w_t - w^*) & w_t > w^*. \end{cases}$$

Let us now suppose that the function  $V$  is consistent with a constant payout tax at rate  $\tau_d$ , with  $\bar{w}(\theta_t) \leq w^*$  for all  $\theta_t \in [0, 1]$ , and define  $\tilde{V} = T(V; \tau_d)$ . Conjecture that the only potentially binding constraint is the no-default constraint.

Observe immediately by the envelope theorem that the directional derivative exists and satisfies

$$\tilde{V}_{w^+}(w_t, \theta_t) \geq \frac{1}{1 + \tau_d}.$$

Note also that we must have either  $k_t(\theta_{t+1}) \geq k^*(\theta_t)$  or  $k_t(\theta_{t+1}) < k^*(\theta_t)$  and  $d_t(\theta_{t+1}) = 0$ . In the latter case,

$$\begin{aligned} \tilde{V}(w_t, \theta_t) &= R^{-1} \int_0^1 \left\{ \frac{1}{1 + \tau_d} \max\{y(k_{\max}(w_t, \theta_t), \theta_t) + R w_t - w^*, 0\} \right\} \mu(\theta_{t+1}|\theta_t) d\theta_{t+1} \\ &+ R^{-1} \int_0^1 \{V(\min\{y(k_{\max}(w_t, \theta_t), \theta_t) + R w_t, w^*\}, \theta_{t+1})\} \mu(\theta_{t+1}|\theta_t) d\theta_{t+1} \end{aligned}$$

where  $k_{\max}(w_t, \theta_t)$  is defined implicitly by

$$y(k_{\max}(w_t, \theta_t), \theta_t) + R w_t = w^D(k_{\max}(w_t, \theta_t), \theta_t),$$

and explicitly given our functional forms as

$$k_{\max}(w_t, \theta_t) = \frac{w_t}{1 - R^{-1}\varphi(1 - \delta)}.$$



It follows immediately by the concavity of  $\bar{V}$  and of the production function that  $\tilde{V}$  is concave in wealth on the domain  $\frac{w_t}{1-R^{-1}\varphi(1-\delta)} < k^*(\theta_t)$ , and its directional derivative satisfies

$$\tilde{V}_{w^+}(w_t, \theta_t) > \frac{1}{1 + \tau_d}.$$

Let us now consider the possibility that  $\frac{w_t}{1-R^{-1}\varphi(1-\delta)} \geq k^*(\theta_t)$ . In this case, it is without loss of generality to suppose that

$$k_t(\theta_{t+1}) = \max\{k^*(\theta_t), w_t\}$$

and therefore does not depend on  $\theta_{t+1}$ . It is always weakly optimal to suppose that  $d_t(\theta_{t+1}) = 0$ , and therefore

$$\begin{aligned} \tilde{V}_{w^+}(w_t, \theta_t) &= \int_0^1 \frac{1}{1 + \tau_d} \mathbf{1}\{y(k^*(\theta_t), \theta_t) + R w_t \geq w^*\} \mu(\theta_{t+1} | \theta_t) d\theta_{t+1} \\ &\quad + R^{-1} \int_0^1 \mathbf{1}\{y(k^*(\theta_t), \theta_t) + R w_t < w^*\} V_{w^+}(y(k^*(\theta_t), \theta_t) + R w_t, \theta_{t+1}) \mu(\theta_{t+1} | \theta_t) d\theta_{t+1} \end{aligned}$$

It follows immediately that  $\tilde{V}_{w^+}(w_t, \theta_t)$  is concave on this domain, and

$$\tilde{V}_{w^+}(w_t, \theta_t) = \frac{1}{1 + \tau_d}$$

only if

$$y(k^*(\theta_t), \theta_t) + R w_t \geq \max_{\theta \in [0,1]: \mu(\theta | \theta_t) > 0} \bar{w}(\theta),$$

where  $\bar{w}(\theta)$  is defined as in Definition 1 for  $\bar{V}$  (recalling the assumption that  $\bar{w}(1) \leq w^*$ ). Consequently, we can define

$$\tilde{w}(\theta_t) = \max \left\{ (1 - R^{-1}\varphi(1 - \delta)) k^*(\theta_t), R^{-1}(-y(k^*(\theta_t), \theta_t) + \max_{\theta \in [0,1]: \mu(\theta | \theta_t) > 0} \bar{w}(\theta)) \right\}$$

and observe that this also satisfies  $\tilde{w}(\theta_t) < w^*$  for all  $\theta_t \in [0, 1]$ . We also observe that proposed policies in this relaxed problem is feasible in the original problem, and hence optimal.

We conclude that if  $V$  is consistent with a constant payout tax rate  $\tau_d$  and satisfies  $\bar{w}(\theta) < w^*$  for all  $\theta$ ,  $T(V)$  will share these properties. By the contraction mapping theorem, the fixed point  $V^*$  must also share this properties (see e.g. [Rockafellar \(1970\)](#) theorem 24.5 regarding the convergence of the directional derivative). Repeating the argument for  $\tau_d = 0$ , and noting that the resulting  $\bar{w}$  function is identical in the two cases, we conclude that  $V_{t+1}(\cdot) = \bar{V}(\cdot; \tau_d)$  and  $U_{t+1}(\cdot) = \bar{V}(\cdot; 0)$  are consistent with a constant payout tax at rate  $\tau_d$ .

## C.9 Proof of Proposition 4

We begin by observing that it is without loss of generality to suppose that, for all  $\tau_d \in [0, \bar{\tau}_d]$ ,

$$N(\mu_0, B_0, \bar{\tau}_d) \geq N(\mu_0, B_0, \tau_d).$$

That is, if  $N(\mu_0, B_0, \tau_d) > N(\mu_0, B_0, \bar{\tau}_d) \geq 0$  for some  $\tau_d \in [0, \bar{\tau}_d]$ , we can redefine  $\bar{\tau}_d$  to be equal to this value.

By the positivity of the value function  $\bar{V}$  (which follows from the non-negativity of dividends), for all

$$\tau_d \in [0, \bar{\tau}_d],$$

$$N(\mu_0, B_0, \tau_d) \geq N(\mu_0, B_0, 0).$$

If  $N(\mu_0, B_0, 0) \geq 0$ , define  $\tau_d = 0$ . Otherwise, by our assumption that  $N(\mu_0, B_0, \tau_d)$  is continuous and the intermediate value theorem, we can define  $\tau_d \in [0, \bar{\tau}_d]$  as a root,

$$N(\mu_0, B_0, \tau_d) = 0.$$

Observe that  $N(\mu_0, B_0, 0) \geq 0$  requires

$$B_0 + \frac{\bar{G}}{1 - R^{-1}} \leq 0,$$

which is to say that the government has no net financing need. If this quantity is strictly negative, the government will set  $G_0 > \bar{G}$  and  $\tau_d = 0$  (we prove this below). If this quantity is strictly positive, then  $\tau_d > 0$ .

We conjecture and verify that, on the domain of  $(\mu, B)$  such that  $N(\mu, B, \tau_d) \geq 0$ ,  $V(w, \theta; \mu, B) = \bar{V}(w, \theta; \tau_d)$  and that

$$\begin{aligned} J(\mu, B) &= u(N(\mu, B, \tau_d) + \bar{G}) + \\ &+ \int_0^\infty \int_0^1 \bar{V}(w, \theta; \tau_d) \mu(w, \theta) d\theta dw \\ &+ \frac{1}{R-1} \int_0^\infty \int_0^1 \bar{V}(w, \theta; \tau_d) \bar{e}(w, \theta; \tau_d) d\theta dw, \end{aligned}$$

and most importantly that if  $N(\mu_t, B_t, \tau_d) \geq 0$ , then  $N(\mu_{t+1}, B_{t+1}, \tau_d) \geq 0$ . Since  $\tau_d$  is defined so that  $N(\mu_0, B_0, \tau_d) \geq 0$ , by induction  $N(\mu_t, B_t, \tau_d) \geq 0$  for all  $t \geq 0$ .

We will show that, under the policies associated with these value functions, the equilibrium tax rate and government spending are constant and equal to  $\tau_d$  and  $\bar{G}$  respectively (except that  $G_0 = N(\mu_0, B_0, \tau_d) + \bar{G}$ ). On this path, by construction, the no-Ponzi condition holds (due to the satisfaction of the inter-temporal budget constraint) and the transversality-type conditions hold due to the transversality condition associated with  $\bar{V}(w, \theta; \tau)$ . Stationarity holds by construction, the policies are Markov by construction, and expectations are consistent with a constant payout tax rate. Moreover, by definition, the value function  $V$  satisfies the Bellman equation, provided that the optimal mechanism is indeed a constant payout tax.

Therefore, to prove the existence of an equilibrium, it suffices to show that the Bellman equation of the government is satisfied under the evolution equations for the population of firms and for government debt (Equations (27), (28), and (29)), and that the optimal mechanism is implemented with a constant payout tax.

We will do this without explicitly characterizing the value function  $J(\mu, B)$  on the domain with  $N(\mu, B, \tau_d) < 0$ . Instead, we will bound from above  $J(\mu, B)$  on this domain, and then show that if  $N(\mu_t, B_t, \tau_d) \geq 0$ , the government will always choose to set  $N(\mu_{t+1}, B_{t+1}, \tau_d) \geq 0$  (that is, to not enter the domain  $N(\mu, B, \tau_d) < 0$ ).

Define  $B^*(\mu, \tau_d)$  as the solution to  $N(\mu, B^*(\mu, \tau_d), \tau_d) = 0$ , which exists by the linearity of  $N$  in  $B$ . For any  $B > B^*(\mu, \tau_d)$ , we must have  $N(\mu, B, \tau_d) < 0$ . In this case, the government with debt  $B^*(\mu, \tau_d)$  could always choose to spend  $B - B^*(\mu, \tau_d)$  more than the government with debt  $B$  would choose to spend, and otherwise replicate that government's policies. As a result, we must have

$$J(\mu, B^*(\mu, \tau_d)) \geq J(\mu, B) + B - B^*(\mu, \tau_d).$$

Therefore, using (29), the government's value function must satisfy

$$\begin{aligned}
J_t(\mu_t, B_t) \leq & \max_{B_{t+1}, G_t, \{m_t(w, \theta) \in \mathcal{M}_D(w, \theta, V_{t+1})\}_{w \in \mathbb{R}_+, \theta \in [0, 1]}} u(G_t) - R^{-1} \max\{B_{t+1} - B^*(\mu_{t+1}, \tau_d), 0\} \\
& + R^{-1} \int_0^\infty \int_{0^+}^1 \int_0^1 d_t(\theta', \theta'; w, \theta) \Pi(\theta' | \theta) \mu_t(w, \theta) d\theta' d\theta dw \\
& + R^{-1} \int_{0^+}^\infty d_t(w, 0) \mu_t(w, 0) dw + R^{-1} J_{t+1}(\mu_{t+1}, \min\{B_{t+1}, B^*(\mu_{t+1}, \tau_d)\}),
\end{aligned}$$

with equality if the optimal policy sets  $B_{t+1} \leq B^*(\mu_{t+1}, \tau_d)$  (which is to say,  $N(\mu_{t+1}, B_{t+1}, \tau_d) \geq 0$ ).

Let us now turn to our conjecture, and observe that, on the domain  $N(\mu, B, \tau_d) \geq 0$ ,

$$u(N(\mu, B, \tau_d) + \bar{G}) = N(\mu, B, \tau_d).$$

Using the definition of  $N$  and the fact that  $\tau_d \bar{V}(w, \theta; 0) + \bar{V}(w, \theta; \tau_d) = \bar{V}(w, \theta; 0)$ , our conjecture is

$$\begin{aligned}
J(\mu, B) = & -B - \frac{\bar{G}}{1 - R^{-1}} \\
& + \int_0^\infty \int_0^1 \bar{V}(w, \theta; 0) \mu(w, \theta) d\theta dw \\
& + \frac{1}{R - 1} \int_0^\infty \int_0^1 \bar{V}(w, \theta; 0) \bar{e}(w, \theta; \tau_d) d\theta dw.
\end{aligned}$$

Define the constant

$$J_E = \frac{1}{R - 1} \int_0^\infty \int_0^1 \bar{V}(w, \theta; 0) \bar{e}(w, \theta; \tau_d) d\theta dw$$

and note that

$$J_E = R^{-1} \int_0^\infty \int_0^1 \bar{V}(w, \theta; 0) \bar{e}(w, \theta; \tau_d) d\theta dw + R^{-1} J_E.$$

Now consider the Bellman inequality above using this conjectured function as the continuation value and supposing that firms conjecture a constant payout tax rate  $\tau_d$  beginning in the next date:

$$\begin{aligned}
J_t(\mu_t, B_t) = & \max_{B_{t+1}, G_t, \{m_t(w, \theta) \in \mathcal{M}_D(w, \theta, \bar{V}(\cdot; \tau_d))\}_{w \in \mathbb{R}_+, \theta \in [0, 1]}} u(G_t) \\
& - R^{-1} \max\{B_{t+1} - B^*(\mu_{t+1}, \tau_d), 0\} - R^{-1} \min\{B_{t+1}, B^*(\mu_{t+1}, \tau_d)\} - \frac{R^{-1} \bar{G}}{1 - R^{-1}} \\
& + R^{-1} \int_0^\infty \int_{0^+}^1 \int_0^1 \{d_t(\theta', \theta'; w, \theta) + \bar{V}(w_{t+1}(\theta', \theta'; w, \theta), \theta'; 0)\} \Pi(\theta' | \theta) \mu_t(w, \theta) d\theta' d\theta dw \\
& + R^{-1} \int_{0^+}^\infty \{d_t(w, 0) + \bar{V}(w_{t+1}(w, 0), \theta; 0)\} \mu_t(w, 0) dw \\
& + R^{-1} J_E + R^{-1} \int_0^\infty \int_0^1 \bar{V}(w, \theta; 0) \bar{e}(w, \theta; \tau_d) d\theta dw.
\end{aligned}$$

This simplifies to

$$\begin{aligned}
J_t(\mu_t, B_t) = & \max_{B_{t+1}, G_t, \{m_t(w, \theta) \in \mathcal{M}_D(w, \theta, \bar{V}(\cdot; \tau_d))\}_{w \in \mathbb{R}, \theta \in [0, 1]}} u(G_t) \\
& - R^{-1} B_{t+1} - \frac{R^{-1} \bar{G}}{1 - R^{-1}} + J_E \\
& + R^{-1} \int_0^\infty \int_{0^+}^1 \int_0^1 \{d_t(\theta', \theta'; w, \theta) + \bar{V}(w_{t+1}(\theta', \theta'; w, \theta), \theta'; 0)\} \Pi(\theta' | \theta) \mu_t(w, \theta) d\theta' d\theta dw \\
& + R^{-1} \int_{0^+}^\infty \{d_t(w, 0) + \bar{V}(w_{t+1}(w, 0), \theta; 0)\} \mu_t(w, 0) dw.
\end{aligned}$$

Now plug in the debt equation (28),

$$R^{-1} B_{t+1} = (B_t + G) - R^{-1} \int_0^\infty \int_{0^+}^1 \int_0^1 \tau_t(\theta', \theta'; w, \theta) \Pi(\theta' | \theta) \mu_t(w, \theta) d\theta' d\theta dw - R^{-1} \int_{0^+}^\infty \tau_t(w, 0) \mu_t(w, 0) dw,$$

to eliminate the choice variable  $B_{t+1}$  and simplify to

$$\begin{aligned}
J_t(\mu_t, B_t) = & \max_{G_t, \{m_t(w, \theta) \in \mathcal{M}_D(w, \theta, \bar{V}(\cdot; \tau_d))\}_{w \in \mathbb{R}, \theta \in [0, 1]}} u(G_t) - B_t - G_t - \frac{R^{-1} \bar{G}}{1 - R^{-1}} + J_E \\
& + R^{-1} \int_0^\infty \int_{0^+}^1 \int_0^1 \{d_t(\theta', \theta'; w, \theta) + \tau_t(\theta', \theta'; w, \theta) + \bar{V}(w_{t+1}(\theta', \theta'; w, \theta), \theta'; 0)\} \Pi(\theta' | \theta) \mu_t(w, \theta) d\theta' d\theta dw \\
& + R^{-1} \int_0^\infty \{d_t(w, 0) + \tau_t(w, 0) + \bar{V}(w_{t+1}(w, 0), \theta; 0)\} \mu_t(w, 0) dw.
\end{aligned}$$

Observe immediately that it is weakly optimal to set  $G_t = \bar{G}$ , and  $\bar{G} + \frac{R^{-1} \bar{G}}{1 - R^{-1}} = \frac{\bar{G}}{1 - R^{-1}}$ . Now observe that  $\bar{V}(w, \theta; 0)$  and  $\bar{V}(w, \theta; \tau_d)$  are consistent with a constant payout tax by Lemma 4, and that the deviation-to-exit constraints (24) and (25) are automatically satisfied provided that the no-default constraint (13) is satisfied. Therefore, we can replace the set  $\mathcal{M}_D(w, \theta, \bar{V}(\cdot; \tau_d))$  with the set  $\mathcal{M}(w, \theta, \bar{V}(\cdot; \tau_d))$ , and invoke Proposition 2 to show that a constant payout tax at rate  $\tau_d$  implements the optimal mechanism.

Observe in this case that

$$\begin{aligned}
d_t(\theta', \theta'; w, \theta) + \tau_t(\theta', \theta'; w, \theta) + \bar{V}(w_{t+1}(\theta', \theta'; w, \theta), \theta'; 0) = \\
(1 + \tau_d) d_t(\theta', \theta'; w, \theta) + \bar{V}(w_{t+1}(\theta', \theta'; w, \theta), \theta'; 0) = \\
(1 + \tau_d) d_t(\theta', \theta'; w, \theta) + (1 + \tau_d) \bar{V}(w_{t+1}(\theta', \theta'; w, \theta), \theta'; \tau_d),
\end{aligned}$$

and consequently

$$\begin{aligned}
\int_0^1 \{d_t(\theta', \theta'; w, \theta) + \tau_t(\theta', \theta'; w, \theta) + \bar{V}(w_{t+1}(\theta', \theta'; w, \theta), \theta'; 0)\} \mu(\theta' | \theta) = \\
(1 + \tau_d) \bar{V}(w, \theta; \tau_d) = \\
\bar{V}(w, \theta; 0).
\end{aligned}$$

A similar logic holds for exiting types, and it follows that the conjectured form of the value function  $J$  holds if

the Bellman inequality is an equality. By the definition of  $N$ , if  $N(\mu_t, B_t, \tau_d) \geq 0$ , under the constant payout tax rate  $\tau_d$ , the inter-temporal budget constraint will continue to hold,

$$N(\mu_{t+1}, B_{t+1}, \tau_d) \geq 0,$$

verifying our conjecture and concluding the proof.

### C.10 Proof of Lemma 5

It is immediate that the value function is non-decreasing in wealth, and therefore that the problem can be simplified by observing that the wealth accumulation and initial budget constraint bind. Therefore,

$$w_{t+1} + d_{t+1} = (1 - \tau_c)[f(k_t, \theta_t) + (1 - \delta)k_t - Rk_t] + R(1 - \tau_c(1 - R^{-1})w_t).$$

Default will only be beneficial if

$$w_{t+1} + d_t < w^D(k_t, \theta_t).$$

Consider first the case in which default is beneficial. In this case,  $b_t = 0$ , and therefore we must have

$$\tau_c[f(w_t, \theta_t) - \delta w_t] > \varphi(1 - \delta)w_t.$$

Now suppose not defaulting is optimal. In this case, we must have

$$(1 - \tau_c)[f(k_t, \theta_t) + (1 - \delta)k_t - Rk_t] + R(1 - \tau_c(1 - R^{-1})w_t) \geq f(k_t, \theta_t) + (1 - \varphi)(1 - \delta)k_t,$$

which simplifies to

$$(R - \varphi(1 - \delta))k_t + \tau_c[f(k_t, \theta_t) + (1 - \delta)k_t - Rk_t] \leq R(1 - \tau_c(1 - R^{-1})w_t).$$

The left-hand side is increasing in  $k_t$ , and therefore a necessary condition for no-default is

$$\tau_c(f(w_t, \theta_t) - \delta w_t) \leq \varphi(1 - \delta)w_t.$$

It follows the the firm will not default if and only if this condition is satisfied.

If the firm defaults,  $k_t = w_t$ , and it will set dividends solve

$$d_t \in \arg \max_{d \in [0, w^D(w_t, \theta_t)]} d_t + \int_0^1 V^c(w^D(k_t, \theta_t) - d_t, \theta_{t+1}, \theta_{t+2}) \Pi(\theta_{t+2} | \theta_{t+1}) d\theta_{t+2}.$$

If the firm does not default, it is weakly optimal to set  $k_t$  to its maximal value if possible (either because this increases production or because it relaxes the maximum dividend constraint). It follows in this case that

$$\tau_c(f(k_t, \theta_t) + (1 - \delta)k_t - R) + (R - \varphi(1 - \delta))k_t = w_t R(1 - \tau_c(1 - R^{-1})).$$

In this case, the firm sets dividends to solve

$$d_t \in \arg \max_{d \in [0, w^D(k_t, \theta_t)]} d_t + \int_0^1 V^c(w_{t+1}, \theta_{t+1}, \theta_{t+2}) \Pi(\theta_{t+2} | \theta_{t+1}) d\theta_{t+2},$$

where

$$\begin{aligned} w_{t+1} + d_{t+1} &= (1 - \tau_c)[f(k_t, \theta_t) + (1 - \delta)k_t - Rk_t] + R(1 - \tau_c(1 - R^{-1})w_t \\ &= w^D(k_t, \theta_t). \end{aligned}$$