Welfare Assessments with Heterogeneous Individuals*

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Abstract

This paper develops a new approach to make welfare assessments based on the notion of Dynamic Stochastic Generalized Social Marginal Welfare Weights (DS-weights). For a large class of dynamic stochastic economies with heterogeneous individuals, we show that the aggregate welfare assessment of a DS-planner can be decomposed into four components: i) aggregate efficiency, ii) risk-sharing, iii) intertemporal-sharing, and iv) redistribution. By using DS-weights, we formalize new welfare criteria that are exclusively based on one or several of the components that we identify, and revisit how a utilitarian planner makes interpersonal welfare comparisons. We illustrate our results in several applications.

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1 Introduction

Assessing whether a policy change is desirable in dynamic stochastic economies with rich individual heterogeneity and imperfect insurance is far from trivial. One significant challenge is to understand the channels — such as aggregate efficiency, redistribution, or risk-sharing — through which a particular normative criterion finds a policy change desirable. Another challenge is how to define a specific welfare criteria that exclusively value a particular channel — or set of channels — but not others. For instance, one may want to understand whether the welfare assessment of a particular policy is driven by aggregate efficiency or redistributional considerations. Alternatively, one may want to define a mandate for a policymaker that exclusively focuses on aggregates, disregarding other channels, or perhaps one that finds risk-sharing among individuals desirable but does not want to engage in redistribution.[1]

In this paper, we develop a new approach to make welfare assessments in dynamic stochastic economies based on the notion of Dynamic Stochastic Generalized Social Marginal Welfare Weights (Dynamic Stochastic weights or DS-weights, for short). The introduction of DS-weights accomplishes two main objectives. First, for a given welfare criterion, DS-weights allow us to decompose aggregate welfare assessments of policy changes into four distinct components: aggregate efficiency, intertemporal-sharing, risk-sharing, and redistribution, each capturing different normative considerations. Second, DS-weights allow us to systematically formalize a new, larger set of welfare criteria that society may find appealing. In particular, we are able to define normative criteria that are exclusively based on one or several of the four normative considerations that we identify, potentially disregarding the others.

We introduce our results in a canonical environment with heterogeneous individuals that encompasses a wide variety of dynamic stochastic models. Initially, as a benchmark, we explicitly define in our environment i) Pareto improving policies and ii) desirable policies for a utilitarian planner. Pareto-improving policies are desirable but rare. The utilitarian criterion is popular and widely applicable, but it is hard to understand how interpersonal welfare comparisons are exactly made, because it is based on aggregating utilities of potentially different individuals.[2] By reviewing these well-understood approaches and treating them as benchmarks, we set the stage for the introduction of DS-weights.

In our approach, there is no social welfare objective that a planner maximizes. Instead, the

[1]In recent times, the Federal Reserve seems to have explicitly included inequality considerations in their policy-making process — see e.g., https://www.nytimes.com/2021/04/19/business/economy/federal-reserve-politics.html. The approach that we develop in this paper can plausibly be used to define a mandate for a monetary authority or other policymaker that explicitly incorporates or removes inequality concerns based on redistribution or intertemporal/risk-sharing motives from policy assessments. The current goals of the Federal Reserve, "promote maximum employment, production, and price stability", which date to the Federal Reserve Reform Act of 1977, have been often interpreted as maximizing aggregate measures of employment, production, and price stability.

[2]Throughout the paper, to simplify the exposition, we mostly relate our approach based on DS-weights to the utilitarian welfare criterion. In Section A of the Appendix, we further contrast our approach to other commonly used welfare criteria, e.g., isoelastic (Atkinson, 1970) or maximin/Rawlsian (Rawls, 1971, 1974).

primitives to make welfare assessments are DS-weights, which represent the value that society places on an additional dollar of consumption by a particular individual i, at a particular time t, and along a particular history s^t. Equipped with these weights, we define a policy to be desirable when a weighted sum — using DS-weights — across all individuals, dates, and histories of the normalized instantaneous utility effect of a policy, measured in consumption units of the relevant history, is positive. By defining DS-weights marginally, we are able to define normative criteria that cannot be captured by the utilitarian framework.

In order to understand how a DS-planner, that is, a planner who adopts DS-weights, carries out welfare assessments, we introduce two different decompositions. First, we introduce an individual multiplicative decomposition of DS-weights. We show that, in general, the DS-weights assigned to a given individual can be decomposed into i) an individual normalization component, which is measurable with respect to the initial history in which the welfare assessment takes place; ii) a dynamic component, which is measurable with respect time t; and iii) a stochastic component, which is measurable with respect to history s^t. This decomposition, which is unique once the units of each of its components are determined, will become helpful to understand how a particular planner trades off gains and losses across histories, dates, and individuals.

Next, leveraging the individual multiplicative decomposition of DS-weights, we introduce an aggregate additive decomposition of welfare assessments. Formally, we show that, in dynamic stochastic environments with potentially incomplete markets, aggregate welfare assessments for a broad class of welfare criteria can be decomposed into four components: i) an aggregate efficiency component,[3] ii) a risk-sharing component, iii) an intertemporal-sharing component, and iv) a redistribution component. The aggregate efficiency component accounts for the change in aggregate consumption net of compensation per hours worked across all individuals. The remaining three components of the decomposition are driven by the cross-sectional variation of each of the three elements (individual, dynamic, stochastic) of the individual multiplicative decomposition. In particular, the risk-sharing component adds up across all dates and histories the cross-sectional covariances between the stochastic component of the individual multiplicative decomposition and the change in normalized instantaneous utility at each date and history. Similarly, the intertemporal-sharing component adds up across all dates the covariances between the dynamic component of the individual multiplicative decomposition and the change in normalized net utility at each date. Finally, the redistribution component consists of a single cross-sectional covariance between the individual components of the individual multiplicative decomposition and the change in individual lifetime marginal utility from the perspective of a DS-planner.

Our next set of results fleshes out the properties of the aggregate additive welfare decomposition in different contexts. First, we characterize conditions under which a planner can make welfare

[3] The aggregate efficiency component is shaped by aggregate valuation considerations, which allows us to further decompose the aggregate efficiency component into an expected aggregate efficiency component and an aggregate insurance component.

assessments purely based on aggregate efficiency considerations. This results connects with the literature that seeks to find conditions under which a normative representative consumer exists. We formally show that a DS-planner who assigns DS-weights that do not vary across individuals is able to make welfare assessments purely based on aggregate changes in (net) consumption. We also show that different components of the aggregate additive decomposition may vanish depending on which specific components of the individual multiplicative decomposition of DS-weights are invariant across individuals: if the individual multiplicative component is constant across individuals, then the redistribution component of the aggregate decomposition is zero; if the dynamic multiplicative component is constant across individuals, then the intertemporal-sharing component of the aggregate decomposition is zero; if the stochastic multiplicative component is constant across individuals, then the risk-sharing component of the aggregate decomposition is zero. Second, we show that under the natural normalization that the dynamic and stochastic components of DS-weights add up to 1, the risk-sharing, intertemporal-sharing, and redistribution components are zero whenever a given policy change impacts all individuals identically. Third, we show that the aggregate efficiency component of the aggregate welfare decomposition is zero in endowment economies.

Our aggregate additive decomposition provides direct insights into which particular forms of inequality matter for the determination of aggregate welfare assessments and each of their components. Formally, we provide bounds for the risk-sharing component, the intertemporal-sharing component, and the redistribution component based on the cross-sectional dispersion of each of the elements in the individual multiplicative decomposition of DS-weights. This result shows that choosing a normative criterion with highly dispersed DS-weights has the potential to generate a large welfare effect of policies via risk/intertemporal-sharing and redistribution. It also shows that, by computing the cross-sectional dispersion of the different components of DS-weights for a given welfare criterion (e.g., utilitarian), it is possible to understand the potential scope that inequality may play when determining the risk/intertemporal-sharing and redistribution components of aggregate welfare assessments.

Next, we show how DS-weights allow us to systematically formalize different welfare criteria. First, we introduce a novel individual multiplicative decomposition of DS-weights for a utilitarian planner. This decomposition, which we say defines a "normalized utilitarian planner", allows us to meaningfully describe how such a planner makes welfare comparisons i) across periods and histories for a given individual, and ii) across individuals. Leveraging the notion of normalized utilitarian planner, we are able to clearly describe how a utilitarian planner makes interpersonal welfare comparisons in terms of consumption units, not utils.

We then establish several desirable properties. We formally show that the normalized utilitarian planner is the only utilitarian planner for whom each of the elements of the individual multiplicative decomposition of DS-weights for a utilitarian planner defines proper weights, which add up to 1 along each relevant dimension. We also show that the normalized utilitarian planner is the only utilitarian planner who satisfies the following two properties. First, this planner finds that the risk-sharing and intertemporal-sharing components of the aggregate decomposition are zero for policy changes with with normalized instantaneous utility effects that are i) identical across individuals at all histories on a date, for all dates. Second, this planner finds that, in addition of the risk- and intertemporal-sharing components, the redistribution component is also zero for policies identical across individuals at all dates and histories.

It seems natural to conjecture that the intertemporal- and risk-sharing components of the aggregate additive decomposition of a welfare assessment will depend critically on the ability of individuals to smooth consumption intertemporally and across states. Our next result shows that a normalized utilitarian planner finds that the intertemporal-sharing and the risk-sharing components of the aggregate additive welfare decomposition are zero when markets are complete, that is, when the marginal rates of substitution across all dates and histories are equalized across agents.[4]

Next, using the normalized utilitarian planner as a reference, we systematically introduce three different DS-planners that capture new normative criteria beyond the typical utilitarian/non-paternalistic Paretian framework: an aggregate efficiency DS-planner, a no-sharing DS-planner, and a no-redistribution DS-planner. The welfare assessments made by these planners purposefully set to zero particular components of the aggregate additive decomposition and are meant to capture particular normative objectives that society may find appealing. Our ultimate goal in introducing these new planners is to be able to formalize welfare criteria that isolate aggregate efficiency and stabilization objectives from those related to interpersonal smoothing or redistribution, and vice versa. These new criteria may be of actual practical import in the context of designing independent technocratic institutions, like central banks and regulators. Such institutions must be given a "mandate", much like defining a set of DS-weights. And society may consider forcing these institutions to behave as a particular DS-planner that values some normative objectives, but not others.[5] Moreover, each of these welfare criteria can be a useful analytical and quantitative device. They may be helpful to sharply characterize and compute desirable policies when valuing some normative considerations but not others. Finally, we provide a brief discussion of paternalism and relate our results to the normative literature in environments with heterogeneous beliefs.

Before presenting the applications, we explain how welfare assessments made under DS-weights can be i) used in recursive environments, ii) used to assess non-marginal policy changes, and iii) formulated in terms of an instantaneous Social Welfare Function with endogenous welfare weights. In the Online Appendix, we include additional results and extensions, in particular a detailed comparison of how our results relate to those in Lucas (1987) and Alvarez and Jermann (2004).

Finally, we illustrate how one can employ DS-weights to reach normative conclusions in three

[4] This result applies more generally to any non-paternalistic Paretian planner.

[5] For instance, the current "dual mandate" (stable prices and maximum employment) of the Federal Reserve (as defined by the 1977 Federal Reserve Act) seems to be better described by an aggregate efficiency DS-planner, rather than a normalized utilitarian planner, which would care about redistribution, intertemporal-sharing, and risk-sharing considerations.

different applications. In our first application in Section 6, we illustrate the mechanics of our approach by conducting welfare assessments of policies in a single good economy with no financial markets. In this application, we explore two specific scenarios in the body of the paper, and explore two additional scenarios in the Appendix. Scenario 1 corresponds to an economy in which individuals with identical preferences face idiosyncratic shocks. We consider transfer policies that can potentially provide full insurance and carefully explain how a normalized utilitarian planner finds such policies desirable mostly due to risk-sharing but also to intertemporal-sharing and redistribution. Scenario 2 corresponds to an economy in which individuals with different preferences towards risk/intertemporal-sharing face aggregate shocks. We consider transfer policies that shift aggregate risk to the more risk tolerant investors and carefully explain how a normalized utilitarian planner finds such policies desirable for different reasons depending on the state in which welfare assessments take place. Scenario 3 considers an environment analogous to the one considered in representative-agent models that explore the cost of business cycles. Scenario 4 explores the role of full insurance transfers in the same environment as Scenario 2.

In our second application in Section 7, we assess the welfare consequences of savings taxes in the standard incomplete market economy. As in Huggett (1993), households face idiosyncratic earnings risk and can trade a risk-free bond to partially insure themselves against uncertainty. We abstract from capital and there is no aggregate uncertainty. We characterize the implications of tax policy for risk-sharing, intertemporal-sharing, and redistribution. We show that, from the perspective of a no-sharing DS-planner who weights individuals by their permanent marginal utilities but does not value insurance against future employment risk, efficient tax policy implies an infinite subsidy on saving. By subsidizing saving, this planner reduces inequality through two channels. In partial equilibrium for a given level of income, the subsidy increases the effective rate at which borrowers accumulate wealth and escape indebtedness. In general equilibrium, the savings subsidy is accompanied by a fall in the real interest rate. This implies that, for a given wealth profile, the interest burden of indebted households falls. We show numerically that the no-redistribution DS-planner, on the other hand, finds a positive savings tax rate of 46% efficient. The unnormalized utilitarian planner combines both normative motives and therefore targets an intermediate tax rate of -12%. This application illustrates how to use our approach to decompose — both analytically and numerically — the welfare assessments made by a utilitarian planner.

Related Literature This paper contributes to several literatures, specifically those on i) interpersonal welfare comparisons, ii) welfare decompositions, and iii) welfare evaluation of policy changes in dynamic environments.

Interpersonal welfare comparisons. The question of how to make interpersonal welfare comparisons to make aggregate welfare assessments has a long history in economics — see, among many others, Kaldor (1939), Hicks (1939), Bergson (1938), Samuelson (1947), Harsanyi (1955), Sen (1970) or, more recently, Kaplow (2004), Saez and Stantcheva (2016), Hendren (2020), Tsyvinski and Werquin (2020), and Hendren and Sprung-Keyser (2020). Our approach based on endogenous welfare weights is most closely related to the work of Saez and Stantcheva (2016), who introduce Generalized Social Marginal Welfare Weights. Building on their terminology, in this paper we introduce the notion of Dynamic Stochastic Generalized Social Marginal Welfare Weights (Dynamic Stochastic weights or DS-weights, for short). In static environments, our approach collapses to theirs.[6] In dynamic stochastic environments, the use of DS-weights allows us to formalize a new, larger set of welfare criteria and to understand the normative implications for aggregate efficiency, risk/intertemporal-sharing, and redistribution of different welfare criteria, including the widely used utilitarian criterion. In particular, Proposition 4 leverages the use of DS-weights to provide a novel interpretation of how a utilitarian planner exactly trades off welfare gains and losses across individuals, a result at the heart of the question of how to make interpersonal welfare comparisons.

Welfare decompositions. Our results, in particular the aggregate additive decomposition introduced in Proposition 1, contribute to the body of work that seeks to decompose welfare changes in models with heterogeneous agents. Our approach is most closely related to the recent work by Bhandari et al. (2021), who propose a decomposition of welfare changes when switching from a given policy to another that is more general than the seminal contributions of Benabou (2002) and Floden (2001). A fundamental difference between these papers and ours is that, besides decomposing the aggregate welfare effects of a policy change, our approach allows us to define a new set of normative criteria that can potentially be used to endow a planner/policymaker with a specific mandate. There are many other significant differences between the approach of Bhandari et al. (2021) and ours. For instance, their decomposition is based on a particular social welfare function (utilitarian), while ours critically hinges on the choice of welfare weights. Also, their welfare decomposition, which is defined for non-marginal changes, relies on a Taylor expansion and is hence based on an approximation. Our welfare decomposition is defined for marginal policy changes, which makes it locally exact everywhere and suitable to assess non-marginal changes when used as described in Section 5.2.[7]

Welfare assessments in dynamic stochastic models. Finally, our results are also related to the approach to making welfare assessments in dynamic environments introduced in Lucas (1987), in particular to the marginal formulation of such approach introduced in Alvarez and Jermann (2004). Formally, as we show in Section E of the Online Appendix, the marginal approach to making welfare assessment of Alvarez and Jermann (2004) corresponds to choosing a particular set of DS-weights. While both Lucas (1987) and Alvarez and Jermann (2004) study representative-agent environments, others have used a similar approach in environments with rich heterogeneity, e.g. Atkeson and Phelan (1994), Krusell and Smith (1999), or Krusell et al. (2009), among others.

[6] Saez and Stantcheva (2016) show how their approach can nest alternatives to welfarism, such as the equality of opportunity, Libertarianism, Rawlsianism, among others. It should be evident that our approach can also accommodate those.

[7] In Section E.3 of the Online Appendix, we explain how the use of DS-weights to assess non-marginal policy changes resembles the use of consumer surplus in classical demand theory.

However, as highlighted by these papers, a well-known downside of Lucas's approach is that it does not aggregate meaningfully, because individual welfare assessments are reported as a constant share of individual consumption. As we explain in Section 4 and in the Online Appendix, the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner that we present in Proposition 4 provides a novel characterizations of how a utilitarian planner meaningfully aggregates welfare assessments among heterogeneous individuals.

More broadly, we hope that our approach informs ongoing and future discussions on policy-making mandates, in particular when trading off aggregate stabilization objectives against individual insurance and redistribution objectives.

Outline Section 2 introduces our environment and describes conventional approaches used to make welfare assessments. Section 3 introduces the notion of DS-weights, defines an individual multiplicative decomposition of DS-weights and an aggregate additive decomposition of welfare assessments, explores alternative welfare criteria under DS-weights, and explains how to make use of DS-weights in recursive environments, how to make nonlocal welfare assessments, and how to define DS-weights from a instantaneous Social Welfare Function. Section 6 illustrates how to carry out welfare assessments using DS-weights in a tractable model with incomplete markets. Section 7 presents an application to capital taxation in a canonical model with heterogeneous agents and incomplete markets. All proofs and derivations are in the Appendix. The Appendix also includes several extensions and additional results.

2 Environment and Benchmarks

In this section, we first describe a canonical environment that is sufficiently general to encompass a wide variety of dynamic stochastic models with heterogeneous individuals. Subsequently, we present several approaches that are commonly used to make welfare assessments. In particular, we explicitly define in our environment i) Pareto improving policies and ii) desirable policies for a utilitarian planner. By reviewing these well-understood approaches and treating them as benchmarks, we set the stage for the introduction of DS-weights in Section 3.

2.1 Environment

Our notation closely follows that of Ljungqvist and Sargent (2018). We consider an economy populated by a finite number of types of individuals (individuals, for short). Individual types are indexed by $i \in I$, where the measure of each type is $dG(i)$, with $\int_{i \in I} dG(i) = 1$. At each date $t \in \{0, \ldots, T\}$, where $T \leq \infty$, there is a realization of a stochastic event $s_t \in S$. We denote the history of events up to and until date t by $s^t = (s_0, s_1, \ldots, s_t)$. We denote the unconditional probability of observing a particular sequence of events s^t by $\pi_t(s^t | s_0)$. We assume that the initial value of s_0 is known, so $\pi_0(s_0|s_0) = 1$.

We consider economies with a single consumption good (dollar), which serves as numeraire.[8] Each individual i derives utility from consumption and (dis)utility from working, with a lifetime utility representation, starting from s_0, given by

$$V_i(s_0) = \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right), \tag{1}$$

where $c_t^i(s^t)$ and $n_t^i(s^t)$ respectively denote the consumption and hours worked by individual i at date t given a history s^t, $u_i(\cdot)$ corresponds to individual i's instantaneous utility (assumed to be well-behaved), and $\beta_i \in [0,1)$ is a discount factor.[9] Equation (1) corresponds to the conventional time-separable expected utility preferences with homogeneous beliefs used in most of dynamic macroeconomics and finance. In Section 4, we describe how to account for heterogeneous beliefs. In the Online Appendix, we show how our approach extends naturally to richer preference specifications, for instance the widely used Epstein-Zin preferences. In the Online Appendix, we describe how to account for births, deaths, bequest motives, and related intergenerational considerations.

For now, we directly impose that the values of $c_t^i(s^t)$ and $n_t^i(s^t)$ are smooth functions of a parameter $\theta \in [0,1]$. We interpret changes in θ as changes in (government) policies, although at this level of generality our approach is valid for any change in primitives.[10] This approach allows us to consider a wide range of policies, as we illustrate in our applications in Sections 6 and 7. In those applications — and more generally — the mapping between outcomes, $c_t^i(s^t)$ and $n_t^i(s^t)$, and policy, θ, emerges endogenously. We focus throughout on marginal policy changes, relegating the discussion of how to account for non-marginal policy changes to Section 5.2.

2.2 Benchmarks: conventional welfare approaches

Lifetime utility effect of policy change. Given its central role for conventional welfare approaches, it is useful to first characterize the change in the lifetime utility of an individual i induced by a marginal policy change. Starting from Equation (1), the welfare effect of such a policy change, measured in lifetime utils (utility units), can be expressed as

$$\frac{dV_i(s_0)}{d\theta} = \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i} \frac{du_{i|c}(s^t)}{d\theta}, \tag{2}$$

[8] For simplicity, we exclusively consider economies with a single consumption good, although our results can be extended to economies with multiple consumption goods.

[9] We consistently refer to $V_i(\cdot)$ as lifetime utility and to $u_i(\cdot)$ as instantaneous utility, following Acemoglu (2009). As in Ljungqvist and Sargent (2018), we use a subscript i to refer to $V_i(\cdot)$, β_i, and $u_i(\cdot)$, and a superscript i to refer to other individual variables indexed by time or histories.

[10] We assume throughout that individual preferences and probabilities are invariant to changes in θ. In Section E.5 of the Online Appendix, we describe how to extend our approach to environments in which policy changes can affect probabilities.

where we respectively denote individual i's marginal utilities of consumption and hours worked at history s^t by

$$\frac{\partial u_i\left(s^t\right)}{\partial c_t^i} \equiv \frac{\partial u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right)}{\partial c_t^i\left(s^t\right)} \quad \text{and} \quad \frac{\partial u_i\left(s^t\right)}{\partial n_t^i} \equiv \frac{\partial u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right)}{\partial n_t^i\left(s^t\right)}.$$

We denote the normalized instantaneous utility effect of the policy at date t given a history s^t, expressed in consumption units, by $\frac{du_{i|c}\left(s^t\right)}{d\theta}$, where

$$\frac{du_{i|c}\left(s^t\right)}{d\theta} \equiv \frac{\frac{du_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right)}{d\theta}}{\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}} = \frac{dc_t^i\left(s^t\right)}{d\theta} + \frac{\frac{\partial u_i\left(s^t\right)}{\partial n_t^i}}{\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}} \frac{dn_t^i\left(s^t\right)}{d\theta}. \tag{3}$$

Equation (2) shows that the impact of a policy change on the lifetime utility of individual i is given by a particular combination of normalized instantaneous utility effects, which are expressed in consumption units at a specific history. The importance of each of these effects for $\frac{dV_i(s_0)}{d\theta}$ is determined by $\left(\beta_i\right)^t \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i}$, that is, by how far in the future and how likely a given history is, and by how much individual i values a marginal unit of consumption at that particular history.

Equation (3) highlights that while the direct impact of a policy on consumption partially determines the normalized instantaneous utility effect, it is necessary to understand how hours worked change, as well as the individual marginal rate of substitution between consumption and hours worked.[11] Note that the definition $\frac{du_{i|c}\left(s^t\right)}{d\theta}$ does not make use of individual optimality (i.e., the envelope theorem). However, in specific applications, exploiting individual optimality conditions can potentially yield simple expressions for $\frac{du_{i|c}\left(s^t\right)}{d\theta}$ — see, for instance, Application 2. Finally, by making s_0 an explicit argument of $\frac{dV_i(s_0)}{d\theta}$, we emphasize that any welfare assessment made in dynamic stochastic environments needs to specify the date and state in which the assessment takes place. This observation becomes of first-order importance when we discuss the recursive formulation of our approach in Section 5.1.

Pareto improving policy change. Equation (2) is useful because it allows us to determine whether a given individual i perceives to be better off or worse off after a policy change. That is, when $\frac{dV_i(s_0)}{d\theta} > 0$, individual i perceives to better off after a policy change, while when $\frac{dV_i(s_0)}{d\theta} < 0$, individual i perceives to be worse off. Moreover, it is possible to define a Pareto improving policy change in terms of $\frac{dV_i(s_0)}{d\theta}$ as follows.

[11] More generally, the normalized instantaneous utility effect of a policy change depends on how other variables besides consumption that directly enter the instantaneous utility of an individual change with policy, as well as the marginal rates of substitution between these variables and consumption. In particular, in a frictionless labor market, the marginal rate of substitution between consumption and hours worked for individual i equals the marginal product of labor and the wage. In that case, the normalized instantaneous utility effect of the policy, $\frac{du_{i|c}(s^t)}{d\theta}$, simply corresponds to the increase in consumption induced by the policy change net of the wage increase due to a change in hours worked. In general, $\frac{du_{i|c}(s^t)}{d\theta}$ accounts for labor wedges, when these exist.

Definition 1. *(Pareto improving policy change) A policy is (strictly) Pareto improving if every individual i perceives to be better off after the policy change. Hence, a necessary and sufficient condition for a policy change to be Pareto improving is that $\frac{dV_i(s_0)}{d\theta} > 0$, $\forall i$.*

Note that one could define a policy change to be weakly Pareto improving when $\frac{dV_i}{d\theta} \geq 0$, $\forall i$, with strict inequality for at least one individual. Note also that the notion of Pareto improving policy change does not involve any interpersonal welfare comparisons, and simply exploits the ordinal nature of utility.

Desirable policy change for a utilitarian planner. Pareto improving policies are desirable but rare to find, which forces planners/policymakers to make interpersonal welfare comparisons. A widely used approach when a planner must decide how to balance welfare gains and losses among different individuals is to adopt a utilitarian criterion. In this section, as is common in the existing literature, we use the term utilitarian planner to refer to a planner who computes the aggregate welfare effect of a policy change by computing a weighted sum of the (unnormalized) individual lifetime utility effects.[12] In Section 4, we draw a distinction between unnormalized and normalized utilitarian planners. Next, we formally define when a policy change is desirable for a utilitarian planner.

Definition 2. *(Desirable policy change for a utilitarian planner) A utilitarian planner finds a policy change desirable if and only if $\frac{dW^U(s_0)}{d\theta} > 0$, where*

$$\frac{dW^U(s_0)}{d\theta} = \int \lambda_i \frac{dV_i(s_0)}{d\theta} dG(i) = \int \lambda_i \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i} \frac{du_{i|c}(s^t)}{d\theta} dG(i), \tag{4}$$

where $\lambda_i > 0$, $\forall i \in I$, is a collection of welfare weights assigned to each individual and where $\frac{dV_i(s_0)}{d\theta}$ is defined in Equation (2).

In Section A of the Appendix, we define when a policy change is desirable for a general non-paternalistic Paretian planner.

Remarks on conventional welfare approaches. The utilitarian criterion is widely used and its properties are well understood — see e.g., Chapter 22 of Mas-Colell, Whinston and Green (1995) for a textbook treatment. In particular, a utilitarian planner is non-paternalistic, since aggregate welfare assessments are based on individual welfare assessments, and Paretian, since every Pareto improving policy is desirable. Moreover, when individuals are ex-ante homogeneous, for instance when they have identical preferences and face an identical environment from the perspective of s_0, the (equal-weighted) utilitarian criterion is a natural welfare criterion.

[12]By varying the weights assigned to every individual, a planner who maximizes a utilitarian social welfare function can fully trace the Pareto frontier whenever a utility possibility set is convex, and partially when it is not – see e.g., Chapter 16 of Mas-Colell, Whinston and Green (1995) or Chapter 8 of Ljungqvist and Sargent (2018). DS-weights do not speak to the issue of how to trace Pareto frontiers.

However, when individuals are ex-ante heterogeneous along some dimension, e.g., preferences, endowments, shocks, etc., it is not easy to understand how a utilitarian planner exactly trades off welfare gains and losses among different individuals. For instance, if one were to multiply the utility of an individual by a positive constant factor, leaving all allocations in the economy unchanged, a utilitarian planner of the form considered in Definition 2 would mechanically put more weight on the gains and losses of such individual when making aggregate welfare assessments, for given social welfare weights. Relatedly, it is not clear how a utilitarian planner weights the welfare gains and losses of individuals with different preferences, for instance, risk aversion coefficients.[13] Similar concerns emerge when individuals have different life-cycle or intergenerational profiles, face different shocks, or have access to different insurance opportunities.

By introducing Dynamic Stochastic weights, we are able to i) define new welfare criteria outside of the non-paternalistic Paretian class that capture normative objectives that society may find appealing, and ii) provide a new transparent interpretation of how a given planner (for example, a utilitarian planner) implicitly makes aggregate welfare assessments weighting gains and losses across individuals, dates, and histories.

3 Dynamic Stochastic Weights: Definition and Decompositions

In this section, we introduce a new approach to assess the desirability of policy changes. Our approach is based on the notion of Dynamic Stochastic Generalized Social Marginal Welfare Weights (Dynamic Stochastic weights or DS-weights, for short). We begin by formally defining when a policy change is desirable for a planner who adopts DS-weights, a "DS-planner". The use of DS-weights is helpful for two main reasons. First, DS-weights can be used to formalize new, more general normative objectives that society may find appealing. Second, when combined with the individual multiplicative decomposition and the aggregate additive decomposition that we introduce in this section, DS-weights are useful to interpret how a particular planner implicitly conducts interpersonal welfare comparisons. In particular, we are able to provide a new, transparent interpretation of how a conventional utilitarian planner compares welfare across individuals, dates, and histories.

3.1 Definition of DS-weights: Desirable policy change for a DS-planner

Here, we formally define when a policy change is desirable for a planner who adopts DS-weights.

Definition 3. *(Desirable policy change for a DS-planner/Definition of DS-weights) A DS-planner, that is, a planner who adopts DS-weights, finds a policy change desirable if and only if* $\frac{dW^{DS}(s_0)}{d\theta} > 0$,

[13] A stark example emerges when considering isoelastic/CRRA preferences, which are bounded above for risk aversion coefficients less than 1, but unbounded above for risk aversion coefficients greater than or equal to 1. This observation highlights that adding up the utilities of individuals with heterogeneous risk aversion coefficients may yield very different conclusions relative to adding up consumption-based welfare measures.

where

$$\frac{dW^{DS}(s_0)}{d\theta} = \int \sum_{t=0}^{T} \sum_{s^t} \omega_t^i \left(s^t \middle| s_0 \right) \frac{du_{i|c}(s^t)}{d\theta} dG(i), \tag{5}$$

where $\frac{du_{i|c}(s^t)}{d\theta}$ *denotes the normalized instantaneous utility effect of the policy at date* t *given a history* s^t*, defined in Equation (3), and where* $\omega_t^i(s^t|\, s_0) \geq 0$ *denotes the DS-weight assigned to individual* i *at date* t *given a history* s^t *for a welfare assessment that takes place at date* 0.

It follows from Equation (5) that, in order to carry out a welfare assessment, a DS-planner must i) know the normalized instantaneous utility effect of a policy for each individual at all dates and histories, that is, $\frac{du_{i|c}(s^t)}{d\theta}$, $\forall i$, $\forall t$, $\forall s^t$, which is measured in consumption units; and ii) specify weights $\omega_t^i(s^t|\, s_0)$ for each individual at all dates and histories, that is, $\omega_t^i(s^t|\, s_0)$, $\forall i$, $\forall t$, $\forall s^t$. Hence, different assumptions on the set of weights $\omega_t^i(s^t|\, s_0)$ will have different normative implications, as we illustrate in detail in the rest of this section. If the set of DS-weights is judiciously chosen, Equation (5) can also be used to quantify the magnitude of the welfare gains and losses associated with a policy change, as we describe in detail in Section E.1 of the Online Appendix.[14] Equation (5) allows us to define a local optimum for a DS-planner as a value of θ for which $\frac{dW^{DS}(s_0)}{d\theta} = 0$.

In general, DS-weights can be expressed in terms of endogenous variables at particular dates and histories. For instance, by comparing Equation (4) with Equation (5), it is straightforward to establish that a utilitarian planner of the form described in Equation (2) can be interpreted as a DS-planner who uses a particular set of DS-weights, specifically

$$\omega_t^i \left(s^t \middle| s_0 \right) = \lambda_i (\beta_i)^t \pi_t \left(s^t \middle| s_0 \right) \frac{\partial u_i(s^t)}{\partial c_t^i}. \tag{6}$$

Importantly, note that DS-weights, and consequently the welfare assessments of a DS-planner, are defined in marginal form. In Section 5.3, we show how to define endogenous social welfare weights in terms of a Social Welfare Function based on instantaneous utilities.

3.2 Individual multiplicative decomposition of DS-weights

In order to understand how a DS-planner carries out welfare assessments, in Lemma 1 we introduce an individual multiplicative decomposition of DS-weights into individual, dynamic, and stochastic components.[15] This decomposition of DS-weights is useful for three reasons. First, the individual multiplicative decomposition is central to define the aggregate additive decomposition of welfare assessments that we present next and to characterize its properties. Second, the individual

[14] Since $\frac{dW^{DS}(s_0)}{d\theta}$ is defined for marginal policy changes, it follows from our assumptions on differentiability that $\frac{dW^{DS}(s_0)}{d\theta}$ has the same magnitude but opposite sign for marginal increases and decreases in θ. In a related context, Bhandari et al. (2021) refer to this property as reflexivity.

[15] Our individual multiplicative decomposition is inspired by the work of Alvarez and Jermann (2005) and Hansen and Scheinkman (2009), who make use of multiplicative decompositions to decompose pricing kernels into permanent and transitory components.

multiplicative decomposition allows us to formalize welfare criteria by defining DS-weights in terms of each of their components, as we illustrate in Section 4. Third, given a set of DS-weights, the individual multiplicative decomposition can be be used to provide a meaningful economic interpretation of how a planner trades off the welfare consequences of policy changes across individuals, dates, and histories, as we describe below.

Lemma 1. *(DS-weights: individual multiplicative decomposition) The DS-weights that a DS-planner assigns to an individual i can be multiplicatively decomposed into three different components, as follows:*

$$\omega_t^i\left(s^t\middle|\, s_0\right) = \underbrace{\tilde{\omega}^i\left(s_0\right)}_{individual}\underbrace{\tilde{\omega}_t^i\left(s_0\right)}_{dynamic}\underbrace{\tilde{\omega}_t^i(s^t|s_0)}_{stochastic}, \qquad where \tag{7}$$

i) $\tilde{\omega}^i\left(s_0\right)$ corresponds to an individual normalization component, which is measurable with respect to the initial history s_0 in which the welfare assessment takes place;
ii) $\tilde{\omega}_t^i\left(s_0\right)$ corresponds to a dynamic component, which is measurable with respect to time t; and
iii) $\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)$ corresponds to a stochastic component, which is measurable with respect to history s^t.

Each of the components in Lemma 1 captures different forces. The stochastic component, $\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)$, can capture i) the probability of a particular history taking place, as well as ii) the valuation assigned to a unit of consumption in that particular history. Similarly, the dynamic term, $\tilde{\omega}_t^i\left(s_0\right)$, will be related to time-discounting, while the individual term, $\tilde{\omega}^i\left(s_0\right)$, captures the weight that a DS-planner assigns to the lifetime utility of individual i, as perceived by the planner.[16]

We would like to make two important remarks in connection to Lemma 1. First, exploiting Lemma (1), we can reformulate the definition of a desirable policy change for a DS-planner as

$$\frac{dW^{DS}\left(s_0\right)}{d\theta} = \int \tilde{\omega}^i\left(s_0\right)\frac{dV_i^{DS}\left(s_0\right)}{d\theta}dG\left(i\right) \quad \text{where} \quad \frac{dV_i^{DS}\left(s_0\right)}{d\theta} = \sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}, \tag{8}$$

where, in this case, $\tilde{\omega}^i\left(s_0\right)\frac{dV_i^{DS}(s_0)}{d\theta}$ denotes the contribution of the welfare change of individual i to aggregate welfare from the perspective of a DS-planner. We purposefully represent Equation (8) so as to parallel the formulation of the aggregate welfare effect of policy changes for a utilitarian planner, Equation (4), and the non-paternalistic Paretian formulation, Equation (35) in the Appendix. A comparison of Equation (8) with Equations (4) and (35) highlights that a DS-planner has additional degrees of freedom, including $\tilde{\omega}_t^i\left(s_0\right)$ and $\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)$, to make welfare assessments relative to a conventional planner, who can exclusively vary $\tilde{\omega}^i\left(s_0\right)$.

Second, from the perspective of making directional aggregate welfare assessments, that is, whether a marginal policy change is desirable or not, the decomposition introduced in Equation (7) is not unique. However, once the units of $\omega_t^i\left(s^t\middle|\, s_0\right)$ and each of its components are determined,

[16] At this stage, we do not insist that each of the components of the DS-weights add up to 1 along specific dimensions. We revisit this possibility below.

the individual multiplicative decomposition introduced in Lemma 1 becomes unique, as we explain in detail in Section E.1 of the Online Appendix. For instance, a utilitarian planner can be seen as a DS-planner whose DS-weights are respectively given by

$$\tilde{\omega}^{i}\left(s_{0}\right)=\lambda_{i}, \quad \tilde{\omega}_{t}^{i}\left(s_{0}\right)=\left(\beta_{i}\right)^{t}, \quad \text{and} \quad \tilde{\omega}_{t}^{i}\left(s^{t} \middle| s_{0}\right)=\pi_{t}\left(s^{t} \middle| s_{0}\right) \frac{\partial u_{i}\left(s^{t}\right)}{\partial c_{t}^{i}}. \tag{9}$$

Yet, in Section 4.1, we provide an alternative individual multiplicative decomposition of DS-weights for a utilitarian planner (we say that that decomposition defines a normalized utilitarian planner). Since the sign of $\frac{dW^{DS}(s_0)}{d\theta}$ — and hence whether a policy change is desirable or not — is the same for both decompositions of the utilitarian welfare criterion, one could say that the individual multiplicative decomposition is not unique. However, the units, and hence the economic interpretation of each component of $\omega_t^i\left(s^t\middle|s_0\right)$ as well as $\frac{dW^{DS}(s_0)}{d\theta}$, will be different for both decompositions, with nontrivial consequences for the purposes of making interpersonal utility comparisons, as we illustrate in Section 4.1. Once the units of $\omega_t^i\left(s^t\middle|s_0\right)$ and its components are defined, the individual multiplicative decomposition is unique.

3.3 Aggregate additive decomposition of welfare assessments under DS-weights

Armed with the multiplicative decomposition of DS-weights for each individual and the reformulation of $\frac{dW^{DS}(s_0)}{d\theta}$ in terms of the components of such a decomposition, in Proposition 1 we introduce an exact additive decomposition of the aggregate welfare assessments made by a DS-planner. This decomposition describes to what extent a particular welfare assessment of a policy change is driven by four considerations: aggregate efficiency, risk-sharing, intertemporal-sharing, and redistribution.[17]

Proposition 1. *(Welfare assessments: aggregate additive decomposition) The aggregate welfare assessment of a DS-planner,* $\frac{dW^{DS}(s_0)}{d\theta}$*, can be decomposed into four components: i) an aggregate efficiency component, ii) a risk-sharing component, iii) an intertemporal-sharing component, and iv) a redistribution component, as follows:*

[17] We have chosen the term risk-sharing and the (somewhat unconventional) term intertemporal-sharing to highlight that both components of the aggregate additive decomposition are driven by cross-sectional differences, via interpersonal sharing. Alternative terms, such as insurance, consumption smoothing, or intertemporal smoothing, do not have such connotation, since they are applicable to a single individual.

$$
\begin{aligned}
\frac{dW^{DS}(s_0)}{d\theta} = & \underbrace{\sum_{t=0}^{T} \mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right] \sum_{s^t} \mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t \middle| s_0\right)\right] \mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_1 \text{ (Aggregate Efficiency)}} \\
& + \underbrace{\sum_{t=0}^{T} \mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right] \sum_{s^t} \mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t \middle| s_0\right), \frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_2 \text{ (Risk-sharing)}} \\
& + \underbrace{\sum_{t=0}^{T} \mathbb{C}ov_i\left[\tilde{\omega}_t^i(s_0), \sum_{s^t} \tilde{\omega}_t^i\left(s^t \middle| s_0\right) \frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_3 \text{ (Intertemporal-sharing)}} \\
& + \underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \sum_{t=0}^{T} \tilde{\omega}_t^i(s_0) \sum_{s^t} \tilde{\omega}_t^i\left(s^t \middle| s_0\right) \frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_4 \text{ (Redistribution)}},
\end{aligned} \tag{10}
$$

where $\mathbb{E}_i[\cdot]$ and $\mathbb{C}ov_i[\cdot,\cdot]$ respectively denote cross-sectional expectations and covariances, where the history-specific term that determines the aggregate efficiency component, $\mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right]$, is given by

$$
\mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right] = \int \frac{dc_t^i(s^t)}{d\theta} dG(i) + \int \frac{\frac{\partial u_i(s^t)}{\partial n_t^i}}{\frac{\partial u_i(s^t)}{\partial c_t^i}} \frac{dn_t^i(s^t)}{d\theta} dG(i), \tag{11}
$$

and where, without loss of generality, we have assumed that $\mathbb{E}_i\left[\tilde{\omega}^i(s_0)\right] = \int \tilde{\omega}^i(s_0)\, dG(i) = 1$.

Proposition 1 allows us to attribute the welfare gains and losses associated with any marginal policy change to four different components. First, the aggregate efficiency component, Ξ_1, accounts for the normalized (in consumption units) instantaneous utility effect of the policy aggregated across all individuals, as shown in Equation (11). That is, Ξ_1 adds up the changes in (net) aggregate consumption resulting from the marginal policy change across all dates and histories. Because Ξ_1 can be computed using exclusively aggregate variables, that is, cross-sectional averages of $\tilde{\omega}_t^i(s_0)$, $\tilde{\omega}_t^i\left(s^t \middle| s_0\right)$, and $\frac{du_{i|c}(s^t)}{d\theta}$, we refer to the this term as aggregate efficiency.

The remaining three components of the decomposition are driven by the cross-sectional variation of each of the three elements (individual, dynamic, stochastic) of the individual multiplicative decomposition of DS-weights. In particular, the risk-sharing component, Ξ_2, adds up across all dates and histories the covariances between the stochastic component of the individual multiplicative decomposition and the normalized instantaneous utility effect at each date and history. Similarly, the intertemporal-sharing component, Ξ_3, adds up across all dates the covariances between the dynamic component of the individual multiplicative decomposition and the change in (expected, interpreting the stochastic weights as probabilities) normalized instantaneous utility effect at each date. Finally, the redistribution component, Ξ_4, consists of a single cross-sectional covariance between the individual components of the individual multiplicative decomposition of DS-weights and the change in individual lifetime utility from the perspective of a DS-planner.

We would like to make two important remarks regarding Proposition 1. First, note that the aggregate additive decomposition is exact for any marginal policy change and does not rely on any approximations. Relatedly, for a given set of primitives, the decomposition can be computed using exclusively DS-weights — typically based on allocations — and normalized instantaneous utility effects. Due to this, the decomposition is immune to concerns associated with welfare assessments based on transfers, which are sensitive to income effects (e.g., distinctions between compensating and equivalent variation).

Second, the aggregate additive decomposition is based on cross-sectional averages and covariances, and does not include covariances over future periods or histories. We further decompose some of the components along those lines below — see Propositions 6 and 8. While it is possible to conceive alternative aggregate welfare decompositions, the one presented in Proposition 1 is systematic in the way it treats the components of the multiplicative decomposition of DS-weights. While there are no natural alternatives for the aggregate efficiency and redistribution components, in Proposition 7, we provide an alternative decomposition of the risk-sharing and intertemporal-sharing components based on cross-sectional averages and covariances.

3.4 Properties of the aggregate additive decomposition

The merits of the decomposition introduced in Proposition 1 lie in its properties. Similarly, the names attributed to each of the components, Ξ_1 through Ξ_4, are only meaningful if they satisfy desirable properties. Hence, the remainder of this section can be seen as fleshing out the properties of the aggregate additive decomposition and its components in different contexts.

First, we characterize conditions on the set of DS-weights under which a planner can make welfare assessments purely based on aggregate efficiency considerations. This approach is similar to the question of finding a normative representative consumer — see e.g., Chapter 4 of Mas-Colell, Whinston and Green (1995) — and may be relevant in specific applications, as shown in Section 4.1.

Proposition 2. *(Properties of aggregate additive decomposition: Individual-invariant DS-weights)*
a) If DS-weights $\omega_t^i\left(s^t \middle| s_0\right)$ are constant across all individuals at all dates and histories, then the welfare assessment of a DS-planner is exclusively based on aggregate efficiency considerations, i.e., $\Xi_2 = \Xi_3 = \Xi_4 = 0$.
b) If the stochastic component of DS-weights is constant across all individuals at all dates and histories, then $\Xi_2 = 0$.
c) If the dynamic component of DS-weights is constant across all individuals at all dates, then $\Xi_3 = 0$.

d) If the individual component of DS-weights is constant across all individuals, then $\Xi_4 = 0$.

Proposition 2 formally shows that a DS-planner who assigns DS-weights that do not vary across individuals at all dates and histories is able to make welfare assessments purely based on aggregate efficiency considerations. It also shows that depending on which specific components of the individual multiplicative decomposition of DS-weights are invariant across individuals, it may be that $\Xi_2 = 0$, $\Xi_3 = 0$, or $\Xi_4 = 0$. These results highlight the cross-sectional nature of the risk-sharing, intertemporal-sharing, and redistribution components. Proposition 2 can be used to establish that the risk-sharing, intertemporal-sharing, and redistribution components are zero in representative-agent economies under reasonable conditions. For instance, in a representative-agent economy in which all individuals have the same DS-weights, Proposition 2a) applies directly.

Second, we characterize conditions on the set of policy changes under which a planner can make welfare assessments purely based on aggregate efficiency considerations. Generically, any policy change will affect all components of the aggregate additive decomposition: for instance, a policy that grants one dollar to one individual in a single history will impact all four components. Proposition 3 leverages policies that identically impact all individuals.

Proposition 3. *(Properties of aggregate additive decomposition: Individual-invariant policies) Suppose that the dynamic and stochastic components of DS-weights of a given individual are proper weights, i.e., add up to 1, for all individuals and dates, that is, $\sum_{s^t} \tilde{\omega}_t^i \left(s^t \middle| s_0 \right) = 1$, $\forall t$, $\forall i$, and $\sum_{t=0}^{T} \tilde{\omega}_t^i \left(s_0 \right) = 1$, $\forall i$.*
a) If the normalized instantaneous utility effect of a policy change, $\frac{du_{i|c}\left(s^t\right)}{d\theta}$, is identical across individuals at all dates and histories, then the welfare assessment of a DS-planner is exclusively based on aggregate efficiency considerations, i.e., $\Xi_2 = \Xi_3 = \Xi_4 = 0$.
b) If the normalized instantaneous utility effect of a policy change, $\frac{du_{i|c}\left(s^t\right)}{d\theta}$, is identical across individuals at all histories on a date, for all dates, $\Xi_2 = \Xi_3 = 0$.
c) If the normalized instantaneous utility effect of a policy change, $\frac{du_{i|c}\left(s^t\right)}{d\theta}$, is identical across individuals conditional on a date and history, for all dates and histories, $\Xi_2 = 0$.

Proposition 2 formally shows that any DS-planner who uses proper DS-weights will find that policies that do not vary across individuals in certain dimensions cannot load on particular components of the aggregate decomposition. Proposition 3a) shows that a policy change that affects all individuals identically across all dates and states can only affect aggregate welfare via aggregate efficiency considerations. Proposition 3b) shows that a policy change that varies over time but that affects all agents at a given date identically cannot affect the intertemporal-smoothing component, although it can affect the redistribution component. Proposition 3c) shows that a policy change that affects all individuals identically in each state but that can vary across dates and histories will have no risk-sharing component. For generic DS-weights, the converse of these results also holds. That is, policy changes must affect different individuals differently if they load on the risk-sharing, intertemporal-sharing, or redistribution components of the aggregate additive decomposition.

The need to normalize DS-weights to add up to 1 highlights that the aggregate additive decomposition introduced in Proposition 1 depends on the particular individual multiplicative decomposition presented in Lemma 1, which, as explained above, is only unique up to a set of units. The adding-up restriction imposed in Proposition 3 implicitly restricts the set of units. In Section 4.1 and Section E.1 of the Online Appendix, we illustrate the importance of the units of $\omega_t^i\left(s^t\middle|\, s_0\right)$ and its components for the interpretation of the aggregate additive decomposition.

Third, we show that, in an endowment economy, aggregate efficiency considerations play no role for a DS-planner when making normative assessments. We use the term endowment economy to refer to economies in which all consumption comes from predetermined endowments of the consumption good at each date and history, and individuals' instantaneous utility exclusively depends on consumption. If individual utility depended on other variables, the second term in Equation (11) would have to add up to zero too for Proposition 4 to be valid.

Proposition 4. *(Properties of aggregate additive decomposition: Endowment economy) In an endowment economy in which the aggregate endowment of the consumption good is invariant to policy, the aggregate efficiency component of the welfare assessment of a DS-planner is zero for any set of DS-weights,* $\omega_t^i\left(s^t\middle|\, s_0\right)$*. That is,* $\Xi_1 = 0$*.*

Proposition 4 highlights that the aggregate efficiency component of the aggregate additive decomposition captures the impact of policies on the production side of the economy. We further discuss properties of the decomposition in endowment economies below — see Proposition 6.

3.5 Inequality and welfare assessments

Concerns related to inequality often take a prominent role when assessing policies. Our aggregate additive decomposition provides direct insights into which particular forms of inequality matter for the determination of aggregate welfare assessments and each of their components. Formally, in Proposition 5, we provide bounds for the risk-sharing component, the intertemporal-sharing component, and the redistribution component defined in Proposition 1 based on the cross-sectional dispersion of DS-weights and policy effects.[18] These bounds are helpful in practice because they can be computed using univariate statistics, i.e., cross-sectional standard deviations, and do not require the joint distribution of DS-weights and normalized utility effects, which are necessary to compute cross-sectional covariances (a multivariate statistic).

Proposition 5. *(Cross-sectional dispersion bounds) The value of the risk-sharing, the intertemporal-*

[18] It should be clear that cross-sectional variances and standard deviations can only bound the welfare effect of policies. Equation (10) shows that cross-sectional covariances exactly determine each of the components of the aggregate additive decomposition.

sharing, and the redistribution components defined in Proposition 1 satisfy the following bounds:

$$\left|\Xi_2\right| \leq \sum_{t=0}^{T} \mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^0\right)\right] \sum_{s^t} \mathbb{SD}_i\left[\tilde{\omega}_t^i\left(s^t \middle| s_0\right)\right] \times \mathbb{SD}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right] \tag{12}$$

$$\left|\Xi_3\right| \leq \sum_{t=0}^{T} \mathbb{SD}_i\left[\tilde{\omega}_t^i\left(s^0\right)\right] \times \mathbb{SD}_i\left[\sum_{s^t} \tilde{\omega}_t^i\left(s^t \middle| s_0\right) \frac{du_{i|c}\left(s^t\right)}{d\theta}\right] \tag{13}$$

$$\left|\Xi_4\right| \leq \mathbb{SD}_i\left[\tilde{\omega}^i\left(s^0\right)\right] \times \mathbb{SD}_i\left[\sum_{t=0}^{T} \tilde{\omega}_t^i\left(s^0\right) \sum_{s^t} \tilde{\omega}_t^i\left(s^t \middle| s_0\right) \frac{du_{i|c}\left(s^t\right)}{d\theta}\right], \tag{14}$$

where $\mathbb{SD}_i\left[\cdot\right]$ denotes a cross-sectional standard deviation.

Proposition 5 shows that the magnitude of each of the three components considered here is determined (bounded above) by i) the cross-sectional dispersion of the different components of DS-weights, $\mathbb{SD}_i\left[\tilde{\omega}_t^i\left(s^t \middle| s^0\right)\right]$, $\mathbb{SD}_i\left[\tilde{\omega}_t^i\left(s^0\right)\right]$, and $\mathbb{SD}_i\left[\tilde{\omega}^i\left(s^0\right)\right]$, as well as ii) the cross-sectional dispersion of the normalized instantaneous utility effect of the policy, effectively $\mathbb{SD}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]$. Consequently, inequality considerations do matter for the aggregate assessments of policies via the cross-sectional dispersion of DS-weights or the impact of a policy by itself.

Proposition 5 is helpful for three reasons. First, it shows that choosing a normative criterion with highly dispersed DS-weights has the potential to generate a large welfare effect of policies via risk-sharing, intertemporal-sharing and redistribution. Second, by computing the cross-sectional dispersion of the different components of DS-weights for a given criterion (e.g., normalized utilitarian, as defined in Equation (18) through (20) below), it shows that it is possible to understand the potential scope that inequality may play when determining the risk/intertemporal-sharing and redistribution components of aggregate welfare assessments. Finally, Proposition 5 shows that the risk-sharing, intertemporal-sharing and redistribution components depend on the extent to which which policies impact different individuals differently. That is, the more $\frac{du_{i|c}\left(s^t\right)}{d\theta}$ varies across individuals, dates, and/or histories, the more likely dispersion in DS-weights matters for welfare assessments.

3.6 Inspecting the components of the aggregate additive decomposition

Here, we decompose and provide additional insights into the four components of the aggregate additive decomposition.

Aggregate efficiency (Ξ_1). It is important to highlight that the aggregate efficiency component Ξ_1 is also shaped by aggregate valuation considerations. We formalize this insight by further decomposing the aggregate efficiency component of the aggregate additive decomposition into an expected aggregate efficiency component and an aggregate insurance component.

Proposition 6. *(Aggregate efficiency component: stochastic decomposition) The aggregate efficiency component of the aggregate additive decomposition defined in Proposition 1, Ξ_1, can be decomposed into i) an expected aggregate efficiency component, Ξ_1^{EAE}, and ii) an aggregate insurance component, Ξ_1^{AI}, as follows:*

$$\Xi_1 = \underbrace{\sum_{t=0}^{T} \overline{\omega}_t (s_0) \mathbb{E}_0 \left[\overline{\omega}_t^{\pi} \left(s^t \middle| s_0 \right) \right] \mathbb{E}_0 \left[\frac{d\overline{u}_{i|c} \left(s^t \right)}{d\theta} \right]}_{=\Xi_1^{EAE} \text{ (Expected Aggregate Efficiency)}} + \underbrace{\sum_{t=0}^{T} \overline{\omega}_t (s_0) \mathbb{C}ov_0 \left[\overline{\omega}_t^{\pi} \left(s^t \middle| s_0 \right), \frac{d\overline{u}_{i|c} \left(s^t \right)}{d\theta} \right]}_{=\Xi_1^{AI} \text{ (Aggregate Insurance)}}, \tag{15}$$

where we define $\overline{\omega}_t (s_0) = \mathbb{E}_i \left[\tilde{\omega}_t^i (s_0) \right]$, $\overline{\omega}_t^{\pi} \left(s^t \middle| s_0 \right) = \frac{\mathbb{E}_i \left[\tilde{\omega}_t^i \left(s^t \middle| s_0 \right) \right]}{\pi_t \left(s^t \middle| s_0 \right)}$, and $\frac{d\overline{u}_{i|c}(s^t)}{d\theta} = \mathbb{E}_i \left[\frac{du_{i|c}(s^t)}{d\theta} \right]$, and where $\mathbb{E}_0 [\cdot]$ and $\mathbb{C}ov_0 [\cdot, \cdot]$ denote expectations and covariances conditional on s_0.

The expected aggregate efficiency component, Ξ_1^{EAE}, captures the discounted expectation over time and histories of the aggregate normalized instantaneous utility effect of the policy change. The aggregate insurance component, Ξ_1^{AI}, captures whether aggregate efficiency gains take place in states that a DS-planner values more in aggregate terms, and vice versa. It should be evident that aggregate insurance, Ξ_1^{AI}, based on aggregate covariances over histories, is logically different from the risk-sharing and intertemporal-sharing components, Ξ_2 and Ξ_3, based on cross-sectional covariances.

In practical terms, the welfare gains associated with eliminating aggregate business cycles in a representative-agent economy, as in the policy experiment of Lucas (1987), fully arise from aggregate insurance considerations, that is, Ξ_1^{EAE}, as we show in Scenario 3 of Application 1 — see Section D of the Online Appendix. Note that both the expected aggregate efficiency and the aggregate insurance components incorporate discounting via $\overline{\omega}_t (s_0)$, so policy changes that front-load gains from expected aggregate efficiency or aggregate insurance are more desirable.

Risk-sharing and intertemporal-sharing components (Ξ_2 and Ξ_3). While Propositions 2 through 4 establish desirable properties of the aggregate additive decomposition, there is scope to provide alternative formulations of the risk-sharing and intertemporal-sharing components. In Proposition 7 we further decompose the intertemporal-sharing component into a pure intertemporal-sharing component, a weight concentration component, and a policy-weights coskewness component. We also show a new identity that the sum of the risk-sharing and intertemporal-sharing components, $\Xi_2 + \Xi_3$, must satisfy.

Proposition 7. *(Risk-sharing/intertemporal-sharing components: alternative cross-sectional decompositions)* *a) The intertemporal-sharing component of the aggregate additive decomposition defined in Proposition 1, Ξ_3, can be decomposed into i) a pure intertemporal-sharing component, Ξ_3^{PI}, ii) a weight concentration component, Ξ_3^{WC} and iii) a policy-weights coskewness component, Ξ_3^{PC} as follows:*

$$
\begin{aligned}
\Xi_3 = & \underbrace{\sum_{t=0}^{T}\sum_{s^t}\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)\right]\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right),\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_3^{PI}\text{ (Pure Intertemporal-sharing)}} \\
& \underbrace{+\sum_{t=0}^{T}\sum_{s^t}\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right),\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)\right]\mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_3^{WC}\text{ (Weight Concentration)}} \\
& \underbrace{+\sum_{t=0}^{T}\sum_{s^t}\mathbb{E}_i\left[\left(\frac{du_{i|c}\left(s^t\right)}{d\theta}-\mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\right)\left(\tilde{\omega}_t^i\left(s_0\right)-\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s_0\right)\right]\right)\left(\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)-\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)\right]\right)\right]}_{=\Xi_3^{PC}\text{ (Policy-weights Coskewness)}}.
\end{aligned}
\tag{16}
$$

b) The sum of the risk-sharing and the intertemporal-sharing components, $\Xi_2+\Xi_3$, can be decomposed into i) a weight concentration component, Ξ_{23}^{C} and ii) an interpersonal-sharing component, Ξ_{23}^{I} as follows:

$$
\begin{aligned}
\Xi_2+\Xi_3 = & \underbrace{\sum_{t=0}^{T}\sum_{s^t}\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right),\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)\right]\mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{23}^{WC}\text{ (Weight Concentration)}} \\
& \underbrace{+\sum_{t=0}^{T}\sum_{s^t}\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right)\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right),\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{23}^{I}\text{ (Interpersonal-sharing)}},
\end{aligned}
\tag{17}
$$

where $\Xi_{23}^{WC}=\Xi_3^{WC}$, defined above, and where $\Xi_{23}^{I}=\Xi_2+\Xi_3^{PI}+\Xi_3^{C}$.

The first component of Ξ_3 introduced in Proposition 7a), Ξ_3^{PI}, can be interpreted as capturing pure intertemporal-sharing considerations. The major difference between Ξ_3 and Ξ_3^{PI} is that the former is based on cross-sectional covariances of the dynamic component of DS-weights with the expected — interpreting the stochastic weights as probabilities — normalized instantaneous utility effect of the policy at a given date. The latter, on the other hand, is based on the expectation of cross-sectional covariances of the dynamic component of DS-weights with the actual normalized instantaneous utility effect of the policy. Formally, the difference between Ξ_3 and Ξ_3^{PI} is captured by the remaining two components, which we describe next.

The second component of Ξ_3 introduced in Proposition 7a), Ξ_3^{WC}, can be interpreted as capturing the welfare gain (loss) associated with policies that increase aggregate normalized instantaneous utility when the dynamic and stochastic components of DS-weights are positively (negatively) correlated across individuals. One may argue that Ξ_3^{WC} should be included in the aggregate efficiency component. There are two good reasons not to do so. First, it would require knowledge of the cross-section of the dynamic and stochastic components of DS-weights, which goes against expressing the aggregate efficiency component exclusively as a function of aggregate statistics. Second, as shown in Section 4.1, for the case of utilitarian (and non-paternalistic Paretian) planners, $\Xi_3^{WC} = 0$ when markets are complete. This fact highlights that Ξ_3^{WC} necessarily relies on imperfect insurance across individuals, which makes this term unsuitable to capture aggregate efficiency considerations.

The third component of Ξ_3 introduced in Proposition 7a), Ξ_3^{PC}, is exactly based on the coskewness between i) the dynamic component of DS-weights, ii) the stochastic component of DS-weights, and iii) the normalized instantaneous utility effect of a policy. Coskewness is a measure of how much three random variables jointly change together. For instance, note that Ξ_3^{PC} could be non-zero even when $\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right), \tilde{\omega}_t^i\left(s^t \middle| s_0\right)\right] = 0$ and, consequently, $\Xi_{23}^{WC} = 0$. Also, coskewness is zero when the random variables are multivariate normal, so it relies on higher-order moments. Note also that if one of $\tilde{\omega}_t^i\left(s_0\right)$, $\tilde{\omega}_t^i\left(s^t \middle| s_0\right)$, and $\frac{du_{i|c}\left(s^t\right)}{d\theta}$ is constant across all individuals, then $\Xi_{23}^{WC} = 0$.

Proposition 7b) simply provides an alternative decomposition of the sum of risk-sharing and intertemporal-sharing. Its first component is exactly the weight concentration component just described, $\Xi_{23}^{WC} = \Xi_3^{WC}$, while the second component corresponds to the sum of risk-sharing, Ξ_2, pure intertemporal-sharing, Ξ_3^{PI}, and policy-weights coskewness, Ξ_3^{C}. At times, this alternative decomposition may provide additional insights relative to the one in Proposition 1.

Redistribution component (Ξ_4). Similarly to the aggregate efficiency component, the redistribution component Ξ_4, which can also be written as

$$\Xi_4 = \mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \frac{dV_i^{DS}\left(s_0\right)}{d\theta}\right],$$

where $\frac{dV_i^{DS}(s_0)}{d\theta}$ is defined in Equation (7), is also shaped by individual valuation considerations. Here, we decompose the redistribution component of the aggregate additive decomposition into an expected redistribution component and a redistributive insurance component.

Proposition 8. *(Redistribution component: stochastic decomposition) The redistribution component of the aggregate additive decomposition defined in Proposition 1, Ξ_4, can be decomposed into i) an expected redistribution component, Ξ_4^{ER}, and ii) a redistributive insurance component, Ξ_4^{RI},* *as follows:*

$$\Xi_4 = \underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\mathbb{E}_0\left[\tilde{\omega}_t^{i,\pi}\left(s^t\middle|\,s_0\right)\right]\mathbb{E}_0\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\right]}_{=\Xi_4^{ER}\ \textit{(Expected Redistribution)}}$$
$$+\underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\mathbb{C}ov_0\left[\tilde{\omega}_t^{i,\pi}\left(s^t\middle|\,s_0\right), \frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\right]}_{=\Xi_4^{RI}\ \textit{(Redistributive Insurance)}},$$

where we define $\overline{\omega}_t^{i,\pi}\left(s^t\middle|\,s_0\right) = \frac{\tilde{\omega}_t^i\left(s^t|s_0\right)}{\pi_t\left(s^t|s_0\right)}$*, and where* $\mathbb{E}_0\left[\cdot\right]$ *and* $\mathbb{C}ov_0\left[\cdot,\cdot\right]$ *denote expectations and covariances conditional on* s_0*.*

The expected redistribution component, Ξ_4^{ER}, captures the perceived gains for a DS-planner from changes in the expected normalized instantaneous utility effect of the policy change. When individuals with a high individual component of DS-weights, $\tilde{\omega}^i\left(s^0\right)$, have higher expected normalized instantaneous utility effect, a planner attributes this to the redistribution component. The redistributive insurance component, Ξ_4^{RI}, captures whether individual gains from the policy change take place in states that are more desirable for individuals with higher individual component of DS-weights, $\tilde{\omega}^i\left(s^0\right)$. In practical terms, the redistributive insurance component will be non-zero when a policy improves individual insurance for individuals with a higher individual component of DS-weights. Finally, note that it is possible for the redistribution component, Ξ_4, to be positive even for Pareto improving policies, provided that different individuals are differentially affected by the policy and that a DS-planner has different individual multiplicative components for different individuals.

4 Welfare criteria under DS-weights

In this section, we describe how to use DS-weights to formalize different welfare criteria. First, we introduce a novel individual multiplicative decomposition of DS-weights for a utilitarian planner. This decomposition, which we say defines a "normalized utilitarian planner", allows us to describe how such a planner makes meaningful welfare comparisons i) across periods and histories for a given individual, and ii) across individuals.

Next, using the normalized utilitarian planner as a reference, we introduce three different DS-planners that capture new normative criteria: an aggregate efficiency DS-planner, a no-sharing DS-planner, and a no-redistribution DS-planner. The welfare assessments made by these planners purposefully set to zero particular components of the aggregate additive decomposition introduced in Proposition 1, and are meant to capture particular normative objectives that society may find appealing. Our ultimate goal in introducing these new planners is to be able to define welfare criteria that isolate aggregate efficiency and stabilization objectives from those related to interpersonal smoothing or redistribution, and vice versa. Finally, we provide a brief discussion of paternalism and relate our results to the normative literature in environments with heterogeneous beliefs.

4.1 Normalized utilitarian planner

Equation (9) already introduced an individual multiplicative decomposition of DS-weights for a conventional (unnormalized) utilitarian planner. However, such a decomposition, by virtue of expressing $\omega_t^i\left(s^t \middle| s_0\right)$ and each of its components in utility units, is not helpful to understand how a planner makes welfare comparisons across periods, histories and individuals. As we describe next, these concerns become particularly clear in the context of the aggregate additive decomposition introduced in Proposition 1.

For instance, if we set $\lambda_i = 1$, $\forall i$, in the decomposition presented in Equation (9), the redistribution component of the aggregate additive decomposition would be zero, $\Xi_4 = 0$. This result captures the fact that an unnormalized utilitarian planner is indifferent between redistribution across individuals in utility terms. If we directly added up utils, we would fail to capture the idea that a utilitarian planner does desire to redistribute resources (in consumption units) towards individuals with low marginal utility — see e.g., Salanie (2011). Similarly, if individual discount factors are identical, that is, $\beta_i = \beta$, $\forall i$, a utilitarian planner under the decomposition presented in Equation (9) will conclude that intertemporal-sharing is zero, that is, $\Xi_3 = 0$, regardless of the form of the policy under consideration. Equally important, the dynamic and stochastic weights for a utilitarian planner defined as in Equation (9) need not add up to 1. Hence, according to Proposition 3, even when the normalized instantaneous utility effect of a policy change is identical across individuals at all dates and histories, an unnormalized utilitarian planner would typically find non-zero intertemporal-sharing components and redistribution components of the aggregate additive decomposition. This is is another undesirable property of the unnormalized utilitarian welfare criterion.

In order to avoid these concerns, we introduce the normalized utilitarian planner.

Definition 4. *(Normalized utilitarian planner) A normalized utilitarian (DS-)planner is a utilitarian planner with DS-weights* $\omega_t^{i,NU}\left(s^t \middle| s_0\right)$ *defined by:*

$$\tilde{\omega}_t^{i,NU}\left(s^t \middle| s_0\right) = \frac{\left(\beta_i\right)^t \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}{\sum_{s^t} \left(\beta_i\right)^t \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i}} = \frac{\pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}{\sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i}} \tag{18}$$

$$\tilde{\omega}_t^{i,NU}\left(s_0\right) = \frac{\left(\beta_i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}{\sum_{t=0}^{T} \left(\beta_i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i}} \tag{19}$$

$$\tilde{\omega}^{i,NU}\left(s_0\right) = \frac{\lambda_i \sum_{t=0}^{T} \left(\beta_i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}{\int \lambda_i \sum_{t=0}^{T} \left(\beta_i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i} dG\left(i\right)}. \tag{20}$$

As shown in Section of C the Online Appendix, it is straightforward to verify that an unnormalized and a normalized utilitarian planner agree on whether a policy is desirable or not.

Properties of normalized utilitarian planner. Propositions 9 and 10 lend support to the definition of the normalized utilitarian planner by characterizing several desirable properties.

Proposition 9. *(Properties of normalized utilitarian planner: Proper weights)*
a) The normalized utilitarian planner is the only utilitarian planner for whom i) stochastic weights add up to 1 *at every date, that is,* $\sum_{s^t} \tilde{\omega}_t^{i,NU}\left(s^t \middle| s_0\right) = 1$, $\forall t$, $\forall i$, *ii) dynamic weights add up to* 1 *across all dates, that is,* $\sum_{t=0}^{T} \tilde{\omega}_t^{i,NU}\left(s_0\right) = 1$, $\forall i$, *and iii) individual weights add up to* 1 *across individuals, that is,* $\int \tilde{\omega}^i\left(s_0\right) dG\left(i\right) = 1$.
b) The normalized utilitarian planner is the only utilitarian planner for whom policy changes with normalized instantaneous utility effects that are i) identical across individuals at all histories on a date, for all dates, imply that $\Xi_2 = \Xi_3 = 0$, *and ii) identical across individuals at all dates and histories imply that* $\Xi_2 = \Xi_3 = \Xi_4 = 0$.

Since the dynamic and stochastic components of DS-weights for the normalized utilitarian planner add up to 1 and are nonnegative, $\tilde{\omega}_t^{i,NU}\left(s_0\right)$ and $\tilde{\omega}_t^{i,NU}\left(s^t \middle| s_0\right)$ are proper weights that lie between 0 and 1. Importantly, the components of the individual multiplicative decomposition that defines a normalized utilitarian planner have an intuitive interpretation based on dollar comparisons, as we described next.

First, note that the normalized instantaneous utility effect of the policy at date t given a history s^t, given by $\frac{du_{i|c}\left(s^t\right)}{d\theta}$, is expressed in dollars (units of the consumption good) at such a history. Consequently, the stochastic component, $\tilde{\omega}_t^{i,NU}\left(s^t \middle| s_0\right)$, can be interpreted as providing a marginal rate of substitution between a dollar in history s^t and a dollar across all possible histories at date t for an individual i from the planner's perspective. Formally, the denominator of Equation (18) corresponds to the marginal value of transferring one dollar across all possible histories at date t. For instance, if the stochastic component is 0.4 for a given individual, history, and date, a normalized utilitarian planner values equally — for that individual — a one-dollar transfer at that particularly history and a transfer of 0.4 dollars across all histories in the same date.

Similarly, the dynamic component, $\tilde{\omega}_t^{i,NU}\left(s_0\right)$, can be interpreted as providing a marginal rate of substitution between a dollar across all possible histories at date t and a dollar at date 0 for an individual i from the planner's perspective. Formally, the denominator of Equation (19) corresponds to the marginal value of permanently transferring one dollar across all dates and histories. Note that both the stochastic and the dynamic components are useful because they allow the planner to meaningfully compare the impact of policy changes across dates and histories for a given individual i. For instance, if the dynamic component is 0.3 for a given individual and date, a normalized utilitarian planner values equally — for that individual — a one-dollar permanent transfer across all histories at that particular date and a transfer of 0.3 dollars at date 0.

Finally, the individual component, $\tilde{\omega}_t^{i,NU}(s_0)$, can be interpreted as the weight that a planner assigns to welfare changes for a given individual, expressed in terms of a permanent dollar transfer across dates and states. Formally, the denominator of Equation (20) corresponds to the marginal value of permanently transferring one dollar to each individual in the economy across all dates and histories. For instance, if the individual component is 0.2 for a given individual, a normalized utilitarian planner values equally a one-dollar permanent transfer to that individual across all dates and histories and a permanent transfer of 0.2 dollars to all individuals across all dates and histories.[19]

In Section E.1 of the Online Appendix we provide a detailed dimensional analysis of the aggregate additive decomposition. There, we show that marginal welfare assessments, $\frac{dW^{NU}}{d\theta}$, as well as all the components of the aggregate additive decomposition, have a cardinal interpretation, since they are measured in dollars at all dates and histories for all individuals. In other words, if $\frac{dW^{NU}}{d\theta} = 0.1$, a normalized utilitarian planner concludes that a marginal policy change is equivalent to a permanent transfer to all individuals at all dates and histories of 0.1 dollars.

It seems natural to conjecture that the intertemporal- and risk-sharing components of the aggregate additive decomposition depend critically on the ability of individuals to smooth consumption intertemporally and across states. In Proposition 10, we show that a normalized utilitarian planner finds that both components are zero when markets are complete. For the purposes of Proposition 10, we say that markets are complete when the marginal rates of substitution across all dates and histories are equalized across agents — this condition is endogenously satisfied in any equilibrium model in which individuals can freely trade claims that span all possible contingencies.

Proposition 10. *(Properties of normalized utilitarian planner: Complete markets) When markets are complete, that is, when the marginal rates of substitution across all dates and histories are equalized across agents, the intertemporal-sharing and the risk-sharing components of the aggregate welfare decomposition for a normalized utilitarian planner introduced in Proposition 1 are zero, that is,* $\Xi_2 = \Xi_3 = 0$*, so any welfare assessment made by a normalized utilitarian planner is driven by aggregate efficiency or redistributional considerations.*

When markets are complete, $\tilde{\omega}_t^{i,NU}(s^t | s_0)$ and $\tilde{\omega}_t^{i,NU}(s_0)$, as defined in Equations (18) and (19), are identical across individuals, which combined with Proposition 2b) implies the result of Proposition 10. Intuitively, a normalized utilitarian planner perceives no gains or losses from improving intertemporal- or risk-sharing among individuals, since they have already exhausted

[19] From the perspective of aggregation of lifetime utilities, which takes places through the individual component $\tilde{\omega}^i(s_0)$, a utilitarian planner has access to $|I| + 1$ normalizations: the planner can normalize each of the $|I|$ individual assessments, and it can further normalize the units of aggregate welfare. If one is willing to use DS-weights whose individual components do not add up to 1 across individuals, there are alternative valid normalizations for a utilitarian planner. For instance, one may consider normalizing the individual welfare effect of a policy change by date-0 marginal utility, or by marginal utility at a different date/history. We discuss these possibilities further in Section E.4 of the Online Appendix.

privately all gains from doing so.[20]

Normalized non-paternalistic Paretian planner. At last, it is worth highlighting that Propositions 9 and 10 can be extended to the more general class of normalized non-paternalistic Paretian planners. Importantly, other conventional planners, e.g., isoelastic (Atkinson, 1970) or maximin/Rawlsian (Rawls, 1971, 1974), exclusively modify the form of the individual component of DS-weights, which in turn will only impact the redistribution component of the aggregate additive decomposition.

4.2 Aggregate efficiency/no-sharing/no-redistribution planners

A central objective of this paper is to be able to operationalize new welfare criteria to conduct policy. We now define three different planners in terms of DS-weights: an aggregate efficiency planner, a no-sharing planner, and a no-redistribution planner. To facilitate comparisons, we use the normalized utilitarian planner defined in Proposition 4 as reference to define the DS-weights for these new planners (hence the pseudo-utilitarian qualification). Using the normalized utilitarian planner as reference justifies using the components of the aggregate additive decomposition for the normalized utilitarian planner as the exact objective of a particular — aggregate efficiency, no-sharing, or no-redistribution — planner. In Proposition 11, we formally show that each of the planners assesses policies so that particular components of the aggregate additive welfare decomposition presented in Proposition 1 are zero.

Definition 5. *(Definitions of aggregate efficiency, no-sharing, and no-redistribution planners)*
a) (Aggregate efficiency DS-planners) An aggregate efficiency (pseudo-utilitarian) DS-planner, that is, a planner who values aggregate efficiency as a normalized utilitarian planner, disregarding intertemporal-sharing, risk-sharing, and redistribution motives, has DS-weights $\omega_t^{i,AE}\left(s^t\middle|\, s_0\right)$ *defined by*

$$\tilde{\omega}^{i,AE}\left(s_0\right)=1,\quad \tilde{\omega}_t^{i,AE}\left(s_0\right)=\mathbb{E}_i\left[\tilde{\omega}_t^{i,NU}\left(s_0\right)\right],\quad \text{and}\quad \tilde{\omega}_t^{i,AE}\left(s^t\middle|\, s_0\right)=\mathbb{E}_i\left[\tilde{\omega}_t^{i,NU}\left(s^t\middle|\, s_0\right)\right]. \tag{21}$$

b) (No-sharing DS-planners) A no-risk-sharing/no-intertemporal-sharing (pseudo-utilitarian) DS-planner, no-sharing DS-planner for short, that is, a planner who modifies the normalized utilitarian criterion to disregard risk-sharing and intertemporal-sharing motives but who is willing to redistribute towards individuals with a higher marginal valuation of a permanent transfer, has DS-weights defined by

$$\tilde{\omega}^{i,NS}\left(s_0\right)=\tilde{\omega}^{i,NU}\left(s_0\right),\quad \tilde{\omega}_t^{i,NS}\left(s_0\right)=\mathbb{E}_i\left[\tilde{\omega}_t^{i,NU}\left(s_0\right)\right]\quad \text{and}\quad \tilde{\omega}_t^{i,NS}\left(s^t\middle|\, s_0\right)=\mathbb{E}_i\left[\tilde{\omega}_t^{i,NU}\left(s^t\middle|\, s_0\right)\right]. \tag{22}$$

[20] Proposition 10 suggests that the cross-sectional dispersions of the dynamic and stochastic components of DS-weights, $\mathbb{SD}_i\left[\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)\right]$ and $\mathbb{SD}_i\left[\tilde{\omega}_t^i\left(s^0\right)\right]$, introduced in Proposition 5, may be natural candidates to measure the potential welfare gains from completing markets for a normalized utilitarian planner. It may be worth pursuing this idea further.

c) (No-redistribution DS-planners) A no-redistribution (pseudo-utilitarian) DS-planner, that is, a planner who modifies the normalized utilitarian criterion to be indifferent to permanent transfers, has DS-weights defined by

$$\tilde{\omega}^{i,NR}(s_0) = 1, \quad \tilde{\omega}_t^{i,NR}(s_0) = \tilde{\omega}_t^{i,NU}(s_0), \quad \text{and} \quad \tilde{\omega}_t^{i,NR}\left(s^t \middle| s_0\right) = \tilde{\omega}_t^{i,NU}\left(s^t \middle| s_0\right). \tag{23}$$

Intuitively, an aggregate efficiency DS-planner adopts components of the individual multiplicative decomposition of DS-weights that are individual invariant and that equal the cross-sectional average of those used by the normalized utilitarian planner. A no-sharing DS-planner adopts dynamic and stochastic components of the individual multiplicative decomposition of DS-weights that are individual invariant, but preserves the individual component used by the normalized utilitarian planner. A no-redistribution DS-planner uses the same dynamic and stochastic components of the individual multiplicative decomposition of DS-weights that a normalized utilitarian planner uses, but adopts the cross-sectional average individual component, which is simply 1.

Intuitively, a no-sharing DS-planner assigns instead the same weights to all individuals for a given date and history, but then gives more weight to those individuals for whom the marginal value of a permanent transfer across all dates and histories is higher.[21] Instead, a no-redistribution DS-planner measures the welfare change for each individual in terms of transfers of permanent dollars across all dates and histories, and aggregates across individuals on a one-to-one basis. An aggregate efficiency DS-planner can be seen as a combination of the no-sharing DS-planner, since the planner values changes in (net) consumption identically across all individuals in all dates and histories, and the no-redistribution DS-planner, since all individual welfare changes are given equal weight in terms of transfers of permanent dollars. Alternatively, an aggregate efficiency DS-planner assesses welfare as if individuals were risk-neutral, even though they are not. We formalize the implications of these choices of DS-weights for the aggregate additive decomposition of $\frac{dW^{DS}(s_0)}{d\theta}$ in Proposition 11.

Proposition 11. *(Properties of aggregate efficiency/no-sharing/no-redistribution planners)*
a) For an aggregate efficiency DS-planner, the risk-sharing, intertemporal-sharing, and redistribution components of the aggregate additive decomposition introduced in Proposition 1, are zero, that is, $\Xi_2 = \Xi_3 = \Xi_4 = 0$.
b) For a no-sharing DS-planner, the risk-sharing and intertemporal-sharing components of the aggregate additive decomposition introduced in Proposition 1 are zero, that is, $\Xi_2 = \Xi_3 = 0$.
c) For a no-redistribution DS-planner, the redistribution component of the aggregate additive decomposition introduced in Proposition 1, is zero, that is, $\Xi_4 = 0$.

The results of Proposition 11 can be seen as a direct implication of Proposition 3. Note that it is possible to define alternative DS-planners that exclusively consider aggregate motives, although

[21] Note that it is straightforward to separately define a no-intertemporal-smoothing DS-planner and a no-risk-smoothing DS-planner. We focus on the no-sharing DS-planner only to simplify the exposition.

the one in Definition 5a) is the only one for whom the aggregate efficiency component Ξ_1 coincides with the one of the normalized utilitarian planner introduced in Proposition 4. For instance, a DS-planner with DS-weights defined by

$$\tilde{\omega}^{i,AE}(s_0) = 1, \quad \tilde{\omega}_t^{i,AE}(s_0) = \overline{\beta}^t, \quad \text{and} \quad \tilde{\omega}_t^{i,AE}\left(s^t \middle| s_0\right) = \pi_t\left(s^t \middle| s_0\right), \tag{24}$$

for some $\overline{\beta}$, also perceives that $\Xi_2 = \Xi_3 = \Xi_4 = 0$. Similarly, a DS-planner with DS-weights defined by

$$\tilde{\omega}^{i,NS}(s_0) = \tilde{\omega}^{i,NU}(s_0), \quad \tilde{\omega}_t^{i,NS}(s_0) = \overline{\beta}^t, \quad \text{and} \quad \tilde{\omega}_t^{i,NS}\left(s^t \middle| s_0\right) = \pi_t\left(s^t \middle| s_0\right), \tag{25}$$

also perceives that $\Xi_2 = \Xi_3 = \Xi_4 = 0$, as the no-sharing pseudo-utilitarian planner. This is helpful because, in some applications, it may be easier to operationalize Equations (24) and (25) instead of Equations (21) and (22). Finally, note that we have focused on polar cases for which some of the components of the aggregate additive decomposition are zero. It is straightforward to define DS-planners that under- or overweight particular components of the aggregate additive decomposition.

Implications for policy mandates and institutional design. These new welfare criteria serve a dual objective. First, the welfare criteria may be of actual practical import in the context of designing independent technocratic institutions. Such institutions must be given a "mandate", much like defining a set of DS-weights. And society may want to consider designing independent technocratic institutions that have some normative considerations in their mandate but not others.

For instance, the current "dual mandate" (stable prices and maximum employment) of the Federal Reserve (as defined by the 1977 Federal Reserve Act) seems to be better described by an aggregate efficiency DS-planner, rather than a normalized utilitarian planner, which would care about redistribution, intertemporal-sharing, and risk-sharing considerations. Alternatively, an institution like the Federal Emergency Management Agency (FEMA) has as part of its mandate to "support the Nation in a risk-based, comprehensive emergency management system", which unavoidably involves risk-sharing and redistributional considerations.

Second, each of these welfare criteria can be a useful analytical and quantitative device. At times, they may be helpful to sharply characterize and compute desirable policies when valuing some normative considerations but not others. This may be of practical relevance when characterizing policy prescriptions in specific scenarios.

4.3 Paternalistic DS-planners/heterogeneous beliefs

Note that both the no-sharing and the aggregate efficiency DS-planners are paternalistic, in the sense that the planner and an individual may have different assessments of whether a policy change is welfare improving or not for that individual.[22] Formally, as we show in Section C of the Online Appendix, for any non-paternalistic planner, it must be that $\tilde{\omega}_t^i\left(s^t \middle| s_0\right) \propto \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i}$ and $\tilde{\omega}_t^i\left(s_0\right) \propto \left(\beta_i\right)^t$. The choice of the individual multiplicative weight $\tilde{\omega}^i\left(s_0\right)$ has no connection to whether a planner is paternalistic or not.

Intuitively, the welfare assessments of any planner who respects individual preferences must value intertemporal-sharing and risk-sharing considerations as long individuals do, as long as markets are not complete. Redistributional concerns are independent of whether a planner respects individual preferences. Therefore, if a planner wants to make welfare assessments that do not value intertemporal-sharing or risk sharing, a planner must necessarily be paternalistic.

Finally, as discussed in Section C of the Online Appendix, we would like to highlight that a DS-planner can easily account for non-paternalistic and paternalistic welfare criteria in environments with heterogeneous beliefs.[23] This is another environment in which DS-weights are useful. For instance, we could have assumed instead that individual preferences take the form

$$V_i\left(s_0\right) = \sum_{t=0}^{T} \left(\beta_i\right)^t \sum_{s^t} \pi_t^i\left(s^t \middle| s_0\right) u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right), \tag{26}$$

where $\pi_t^i\left(s^t \middle| s_0\right)$ varies across individuals. In this case, a non-paternalistic planner would substitute $\pi_t^i\left(s^t \middle| s_0\right)$ for $\pi_t\left(s^t \middle| s_0\right)$ whenever it appears in Equations (6) through (23). Alternatively, a single-belief paternalistic planner would substitute some planner's belief, $\pi_t^P\left(s^t \middle| s_0\right)$, which is invariant across individuals, for $\pi_t\left(s^t \middle| s_0\right)$ whenever it appears in Equations (6) through (23).

5 Extensions and Additional Results

Before presenting the applications, it is worth explaining how welfare assessments made under DS-weights can be i) used in recursive environments, ii) used to assess non-marginal policy changes, and iii) formulated in terms of an instantaneous Social Welfare Function.

5.1 Recursive formulation

So far, we have defined DS-weights in the context of a sequence formulation of a dynamic stochastic economy. In practice, it is often more convenient to work with a recursive formulation. As one would expect, the set of DS-weights that admit a recursive representation is smaller than the set of DS-weights that can be expressed non-recursively.

[22] It is not surprising that a DS-planner who disregards consumption smoothing must be paternalistic, since individuals do value intertemporal- and risk-sharing.

[23] A recent literature has explored how to make normative assessments in environments with heterogeneous beliefs. See, among others, Brunnermeier, Simsek and Xiong (2014), Gilboa, Samuelson and Schmeidler (2014), Simsek (2013), Dávila (2020), Blume et al. (2018), Heyerdahl-Larsen and Walden (2021), Caballero and Simsek (2019), and Dávila and Walther (2021).

Formally, we say that DS-weights admit a recursive representation when they can be expressed as $\omega_t^i\left(s^t \middle| s\right) = \tilde{\omega}^i\left(s\right)\tilde{\omega}_t^i\left(s\right)\tilde{\omega}_t^i\left(s^t \middle| s\right)$ for a given state s, and when $\frac{dV_i^{DS}(s)}{d\theta}$, introduced in Equation (8), can be expressed recursively. It should be clear that the individual component of the multiplicative decomposition introduced in Lemma 1, $\tilde{\omega}^i\left(s_0\right)$, which captures the desire to redistributive across individuals, must be factored out when building a recursive representation, hence we only insist on $\frac{dV_i^{DS}(s)}{d\theta}$ to be expressed recursively. This is related to the broader idea that once time goes by, insurance motives translate into redistribution.[24]

In Section C of the Online Appendix, we identify the conditions under which $\frac{dV_i^{DS}(s_0)}{d\theta}$ can be expressed recursively. For instance, in the utilitarian case, it is possible to express $\frac{dV_i^{DS}(s)}{d\theta}$ as follows:

$$\frac{dV_i^{DS}\left(s\right)}{d\theta} = u_i'\left(c^i\left(s\right)\right)\frac{du_{i|c}\left(s\right)}{d\theta} + \beta\sum_{s'}\pi\left(s'|s\right)\frac{dV_i^{DS}\left(s'\right)}{d\theta}. \tag{27}$$

In our applications, all of which have a recursive structure, we further illustrate how to use DS-weights in recursive environments in practice.

5.2 Global welfare assessments

So far, we have exclusively focused on marginal welfare assessments. However, one may be interested in exploring the impact of non-marginal welfare assessments. To do so, we assume that any policy change can be scaled by $\theta \in [0,1]$, where θ corresponds to no policy change, while $\theta = 1$ corresponds to a global non-marginal change.

Formally, we can define a non-marginal welfare change as follows:

$$W^{DS}\left(s_0;\theta=1\right) - W^{DS}\left(s_0;\theta=0\right) = \int_0^1 \frac{dW^{DS}\left(s_0;\theta\right)}{d\theta}d\theta, \tag{28}$$

where we explicitly make θ an argument of $\frac{dW^{DS}(s_0;\theta)}{d\theta}$, as follows:

$$\frac{dW^{DS}\left(s_0;\theta\right)}{d\theta} = \int\sum_{t=0}^{T}\sum_{s^t}\omega_t^i\left(s^t \middle| s_0;\theta\right)\frac{du_{i|c}\left(s^t;\theta\right)}{d\theta}dG\left(i;\theta\right). \tag{29}$$

Equation (29) points towards three different issues that need to be dealt with when using DS-weights to make global assessments. First, it is important to understand how $\frac{du_{i|c}\left(s^t;\theta\right)}{d\theta}$ changes with θ. Computing this term is straightforward, involves no normative assessment, and can be easily operationalized by solving for the equilibrium of an economy for different values of θ.

[24]Throughout the paper, we formally assume that $\pi_0\left(s_0|s_0\right) = 1$, which implies that any welfare assessment must specify an original history. However, at times, one may want to account for uncertainty regarding s_0. There are two ways to proceed. First, one could simply argue that there is a predecessor state, say s_{-1}, and carry out welfare assessments starting at s_{-1}. In that case, all the results of the paper remain valid without modification. Alternatively, one may want to compute $\frac{dW(s_0)}{d\theta}$ for different values of s_0 and then weight these according to some given weights that may depend (or not) on the likelihood of each s_0 taking place. Both approaches are reasonable but the interpretation of the welfare assessments in terms of the aggregate additive decomposition will be different — see Woodford (2010) for a discussion of similar issues.

Second, it is important to understand how the DS-weights and its components (individual, dynamic, stochastic), vary with θ. For instance, a normalized utilitarian planner could select DS-weights normalized at $\theta = 0$, or normalized along the path of θ. This issue is closely related to the distinction between consumer surplus, equivalent variation, and compensating variation in classic demand theory. In our applications, we assume that the DS-weights change with θ, which is akin to adopting a consumer surplus approach, as we discuss in Section E of the Online Appendix.

Third, it is important to understand how the distribution of individuals, $dG(i;\theta)$, varies with θ. This is critical, for instance, in environments in which entire cross-sectional distributions act as state variables, as we explain in detail in the context of Application 2. In this case, there are two reasonable assumptions. We can either assume that the distribution of individuals is fixed at the initial state, so we aggregate individuals using $dG(i;\theta = 0)$, which can be interpreted as a transition path assessment. Or we can assume that $dG(i;\theta)$ is computed in a new stationary equilibrium reached after changing θ, which can be interpreted as a stationary equilibrium assessment. In the first case, an initial set of individuals is accounted for when assessing local welfare changes; then, as θ moves, these individuals continue to be accounted when assessing whether the next local change is desirable or not. In the second case, both $\omega_t^i\left(s^t \middle| s_0;\theta\right)$ and $dG(i;\theta)$ are computed using their values at a stationary equilibrium, so welfare changes in these cases capture comparisons across stationary equilibria. This is helpful because there is a sense in which $\frac{dW^{DS}(s_0;\theta)}{d\theta} = 0$ implies that a given stationary distribution is locally optimal. We discuss both approaches, the one that makes path assessments and the one that makes stationary assessments, in the context of our applications.

Finally, note that it is possible to use DS-weights to globally rank local optima defined by the condition $\frac{dW^{DS}(s_0)}{d\theta} = 0$.

5.3 Instantaneous Social Welfare Function formulation

The conventional approach to making aggregate welfare assessments relies on a global welfare objective based on a Social Welfare Function that takes individual instantaneous utilities as arguments. In this paper, by formulating welfare assessments under DS-weights in marginal form, we are able to consider a larger class of normative objectives. As in Saez and Stantcheva (2016), who also treat marginal welfare weights as primitives, it is possible to interpret $\frac{dW^{DS}(s_0)}{d\theta}$, defined in Equation (5), as the first-order condition of a planner with a particular objective function. However, in the case of DS-weights, we must use a Social Welfare Function that takes as arguments individuals' instantaneous utilities, not lifetime utilities.

Formally, we define a linear Instantaneous Social Welfare Function $ISWF(\theta)$ as

$$ISWF(\theta) = \int \sum_{t=0}^{T} \sum_{s^t} \lambda_t^i\left(s^t\right) u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right) dG(i), \tag{30}$$

where the instantaneous Pareto weights $\lambda_t^i\left(s^t\right)$ define scalars that are individual-, time-, and history-specific. Proposition 12 shows that welfare assessments made under DS-weights correspond to the first-order condition of a planner whose objective function is given by a particular linear ISWF. It also shows that any local optimum can be found as the first-order condition of a planner who maximizes a linear ISWF, where DS-weights are evaluated at the optimum.

Proposition 12. *(Instantaneous Social Welfare Function formulation) For any nonnegative set of DS-weights, there exist instantaneous Pareto weights* $\{\lambda_t^i(s^t)\}_{i,t,s^t}$ *such that* $\frac{dW^{DS}(s_0)}{d\theta}$*, defined in Equation (5), corresponds to the first-order condition of a planner who maximizes a linear Instantaneous Social Welfare Function, with instantaneous Pareto weights* $\lambda_t^i(s^t) = \omega_t^i(s^t;\theta) / \frac{\partial u_i(s^t;\theta)}{\partial c_t^i}$*. Moreover, at a local optimum, in which* $\frac{dW^{DS}(s_0)}{d\theta} = 0$*, there exist instantaneous Pareto weights* $\{\lambda_t^i(s^t)\}_{i,t,s^t}$ *such that the optimal policy satisfies the first-order condition formula of a linear* $ISWF(\theta)$*, defined in Equation (30). The instantaneous Pareto weights in that case are evaluated at the optimum, so* $\lambda_t^i(s^t) = \omega_t^i(s^t;\theta^\star) / \frac{\partial u_i(s^t;\theta^\star)}{\partial c_t^i}$*, where* $\theta^\star$ *denotes the value of* θ *at the local optimum.*

Proposition 12 is helpful because it shows how to reverse-engineer instantaneous Pareto weights of a linear ISWF from DS-weights. Proposition 12 also guarantees that any local optimum can be interpreted as the solution to the maximization of a particular ISWF. Importantly, because the instantaneous Pareto weights $\lambda_t^i(s^t)$ are evaluated at the optimum $\theta^\star$, they are taken as fixed in the maximization of an ISWF. In practice, it is impossible to define the instantaneous Pareto weights $\lambda_t^i(s^t)$ without first having solved for the optimum using our approach that starts with DS-weights as primitives. Relatedly, it is typically impossible to translate DS-weights into instantaneous Pareto weights that are invariant to θ and the rest of the environment.[25]

6 Application I: Assessing Transfer Policies under Incomplete Markets

In Sections 6 and 7, we illustrate how one can employ DS-weights to reach normative conclusions in different applications. First, in Application 1 in Section 6, we illustrate the mechanics of our approach by conducting welfare assessments of policies in a single good economy with no financial markets. In this application, we explore two specific scenarios in the body of the paper, and explore two additional scenarios in the Appendix.

Table 1 summarizes the four scenarios that we study in Application 1. Scenario 1 corresponds to an economy in which individuals with identical preferences face idiosyncratic shocks. We consider transfer policies that can potentially provide full insurance and carefully explained how a normalized utilitarian planner finds such policies desirable mostly due to risk-sharing but also to intertemporal-sharing and redistribution. Scenario 2 corresponds to an economy in which individuals with different

[25]This discussion does not limit the applicability of DS-weights in practice. It is straightforward to show that DS-weights can be used to solve optimal policy problems using both primal and dual methods.

preferences towards risk/intertemporal-sharing face aggregate shocks. We consider transfer policies that shift aggregate risk to the more risk tolerant investors and carefully explained how a normalized utilitarian planner finds such policies desirable for different reasons depending on the state in which welfare assessments take place. Scenario 3 considers an environment analogous to the one consider in representative-agent models that explore the cost of business cycles. Scenario 4 explores the role of full insurance transfers in the same environment as Scenario 2.

Table 1: Summary of scenarios (Application 1)

	Uncertainty	Preferences	Endowment $y^i(s)$		Policy $T^i(s)$		Consumption $c^i(s) = y^i(s) + \theta T^i(s)$	
			$y^1(s)$	$y^2(s)$	$T^1(s)$	$T^2(s)$	$c^1(s)$	$c^2(s)$
#1	Idiosyncratic	Common	$\overline{y}+\varepsilon(s)$	$\overline{y}-\varepsilon(s)$	$-\varepsilon(s)$	$\varepsilon(s)$	$\overline{y}+\varepsilon(s)(1-\theta)$	$\overline{y}-\varepsilon(s)(1-\theta)$
#2	Aggregate	Heterogeneous	$\overline{y}+\varepsilon(s)$	$\overline{y}+\varepsilon(s)$	$-\varepsilon(s)$	$\varepsilon(s)$	$\overline{y}+\varepsilon(s)(1-\theta)$	$\overline{y}+\varepsilon(s)(1+\theta)$
#3*	Aggregate	Common	$\overline{y}+\varepsilon(s)$	$\overline{y}+\varepsilon(s)$	$-\varepsilon(s)$	$-\varepsilon(s)$	$\overline{y}+\varepsilon(s)(1-\theta)$	$\overline{y}+\varepsilon(s)(1-\theta)$
#4*	Aggregate	Heterogeneous						

Note: Instantaneous utility for both investors is given by $u_i(c) = \frac{c^{1-\gamma_i}}{1-\gamma_i}$. Our benchmark parameterization is given by $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, and $\rho = 0.95$. If preferences are common, $\gamma_1 = \gamma_2 = 2$. If preferences are heterogeneous, we assume that individual 1 is more risk averse, so $\gamma_1 > \gamma_2$, where $\gamma_1 = 5$ and $\gamma_2 = 2$. Scenarios 1 and 2 are described in the text. Scenarios 3 and 4 are described in the Appendix.

Common environment across scenarios. In each scenario we consider an economy with two types of individuals (individuals, for short), with each corresponding to half of the population. Hence, $I = \{1, 2\}$. Both individuals have time-separable constant relative risk aversion (CRRA) preferences with exponential discounting.[26] We formulate individual lifetime utility recursively as follows:

$$V_i(s) = u_i\left(c^i(s)\right) + \beta \sum_{s'} \pi(s'|s) V_i(s'), \quad \text{where} \quad u_i(c) = \frac{c^{1-\gamma_i}}{1-\gamma_i},$$

where $V_i(s)$ and $c^i(s)$ respectively denote the lifetime utility and the consumption of individual i in a given state s; s and s' denotes possible states, and $\pi(s'|s)$ is a Markov transition matrix, both described below; β is a discount factor, equal for both individuals; and $u_i(c)$ denotes the instantaneous utility function of an individual i.[27]

There is a single consumption good (dollar), which serves as numeraire. We consider an economy with no financial markets. Therefore, in the absence of transfers, individuals consume their endowments. The consumption of individual i at state s is given by their endowment $y^i(s)$, and a transfer, $\theta T^i(s) \gtreqless 0$, where $\theta \in [0, 1]$ scales the size of the transfers at all dates/states. Hence, the budget constraint of individual i in state s is given by

$$c^i(s) = y^i(s) + \theta T^i(s), \tag{31}$$

[26]In sequence form: $V_i(s_0) = \sum_{t=0}^{T} (\beta)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u_i\left(c_t^i\left(s^t\right)\right)$. By formulating the environment recursively, we highlight how DS-weights can be easily used in recursive environments.

[27]A higher CRRA coefficient γ_i is mechanically associated with a lower willingness to substitute consumption intertemporally. In Section D of the Online Appendix, we illustrate how to extend our results to environments with more general preferences of the Epstein-Zin form.

where the form of $y^i(s)$ and $T^i(s)$ varies in each scenario considered. Given the lack of financial markets, the equilibrium definition is trivial, and Equation (31) also defines equilibrium consumption for individual i. Uncertainty in this economy is captured by a two-state Markov chain, with states denoted by $s = \{L, H\}$, standing for a low (L) and a high (H) realization of $y^1(s)$ (for individual 1) and a transition matrix given by

$$\Pi = \begin{pmatrix} \rho & 1-\rho \\ 1-\rho & \rho \end{pmatrix}.$$

In the Online Appendix, we provide explicit formulae to compute $\frac{dW^{DS}(s)}{d\theta}$ and its determinants. Since $\frac{du_{i|c}(s^t)}{d\theta} = T^i(s)$, $\frac{dW^{DS}(s_0)}{d\theta}$ is simply given by

$$\frac{dW^{DS}(s_0)}{d\theta} = \int \sum_{t=0}^{T} \sum_{s^t} \omega_t^i \left(s^t \middle| s_0 \right) T^i(s)\, dG(i)\,. \tag{32}$$

Our goal in each of the following four scenarios is to clearly illustrate how DS-weights can be used to make welfare assessments of different policies in different environments.

6.1 Scenario 1: Common preferences with idiosyncratic uncertainty

Environment. In our first scenario, we assume i) that both individuals have identical preferences, so $\gamma_1 = \gamma_2 = \gamma$, and ii) that they exclusively face idiosyncratic risk. Formally, we assume that

$$y^1(s) = \overline{y} + \varepsilon(s) \quad \text{and} \quad y^2(s) = \overline{y} - \varepsilon(s)\,,$$

where $\overline{y} \geq 0$, and and where $\varepsilon(L) = -\varepsilon(H)$. We consider the welfare assessment of a full-insurance transfer policy. Formally, we set $T^1(s) = -\varepsilon(s)$ and $T^2(s) = \varepsilon(s)$, so individual consumption takes the form

$$c^1(s) = \overline{y} + \varepsilon(s)(1-\theta) \quad \text{and} \quad c^2(s) = \overline{y} - \varepsilon(s)(1-\theta)\,.$$

Under this policy, when $\theta = 1$, both individuals are fully insured. Note that aggregate consumption does not depend on s or θ since $\int c^i(s)\, dG(i) = \overline{y}$.

Results. We adopt the following parameters: $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, and $\gamma_1 = \gamma_2 = 2$. Importantly, we make the endowment processes persistent, by setting $\rho = 0.95$. In the Appendix, we compare our results to scenarios in which endowment processes are extremely persistent ($\rho = 0.999$) and fully transitory ($\rho = 0.5$). We start by describing the DS-weights used by a normalized utilitarian planner with $\lambda_i = 1$, as defined in Proposition 4.

Individual multiplicative decomposition of DS-weights. In Figure 1, we show the components of

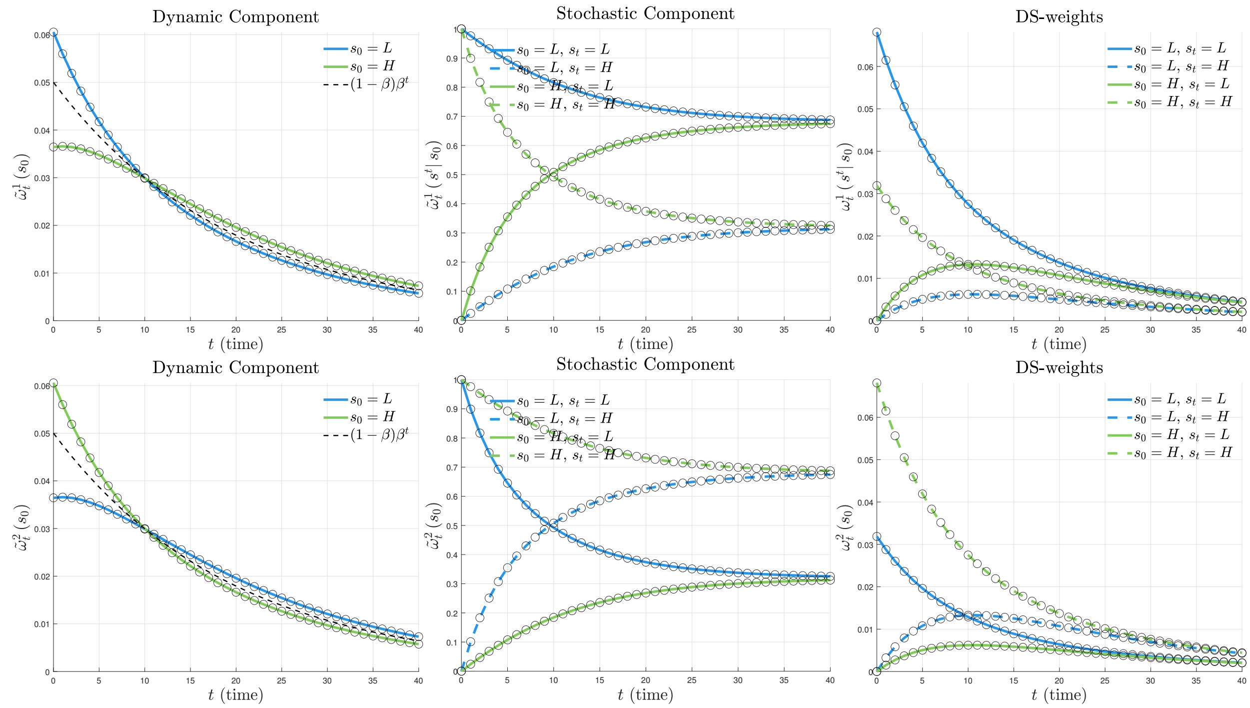

Figure 1: Individual multiplicative decomposition of DS-weights (Scenario 1)

Note: Figure 1 shows the components of the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner, defined in Proposition 4. We assume that $\theta = 0.25$, although all figures are qualitatively similar when $\theta \in [0, 1)$. The top row shows each of the components for individual 1, while the bottom row shows them for individual 2. The left plots show the dynamic component, $\tilde{\omega}_t^i(s_0)$, for different values of t for different initial states, $s_0 = \{H, L\}$. For reference, we also show the dynamic weight for a hypothetical individual with linear marginal utility, given by $(1-\beta)\beta^t = \beta^t / \sum_t \beta^t$. Note that the sum under each of the curves adds up to 1. The middle plots show the stochastic component, $\tilde{\omega}_t^i\left(s^t \middle| s_0\right)$, for different values of t, for different initial states, $s_0 = \{H, L\}$, and final states, $s_t = \{H, L\}$. The right plots show the actual DS-weights, $\omega_t^i\left(s^t \middle| s_0\right)$, also for different values of t, and different initial and final states: $s_0 = \{H, L\}$ and $s_t = \{H, L\}$. The parameters are $\theta = 0.25$, $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, $\rho = 0.95$, and $\gamma_1 = \gamma_2 = 2$. The individual component of DS-weights are $\tilde{\omega}^1(s_0 = L) = 1.125$ and $\tilde{\omega}^2(s_0 = L) = 0.875$ when an assessment takes place at $s_0 = L$; and $\tilde{\omega}^1(s_0 = H) = 0.875$ and $\tilde{\omega}^2(s_0 = H) = 1.125$ when the assessment takes place at $s_0 = H$.

the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner for each of the individuals when $\theta = 0.25$. Several insights emerge.

First, the plots of the dynamic components shows that a normalized utilitarian planner overweights earlier periods for those individuals that initially have a low endowment/high marginal utility. As reference we include the value of $(1-\beta)\beta^t = \frac{\beta^t}{\sum_t \beta^t}$, which corresponds to the dynamic weight for a hypothetical individual with linear marginal utility, i.e., when $u_i'(c^i(s)) = 1$. Importantly, since dynamic weights must add up to 1 over time, overweighting initial periods for individuals with low endowment/higher marginal utility necessarily implies underweighting periods later in the future.

Second, the plots of the stochastic components show that a normalized utilitarian planner initially overweights those states which are more likely, given the initial state, although eventually

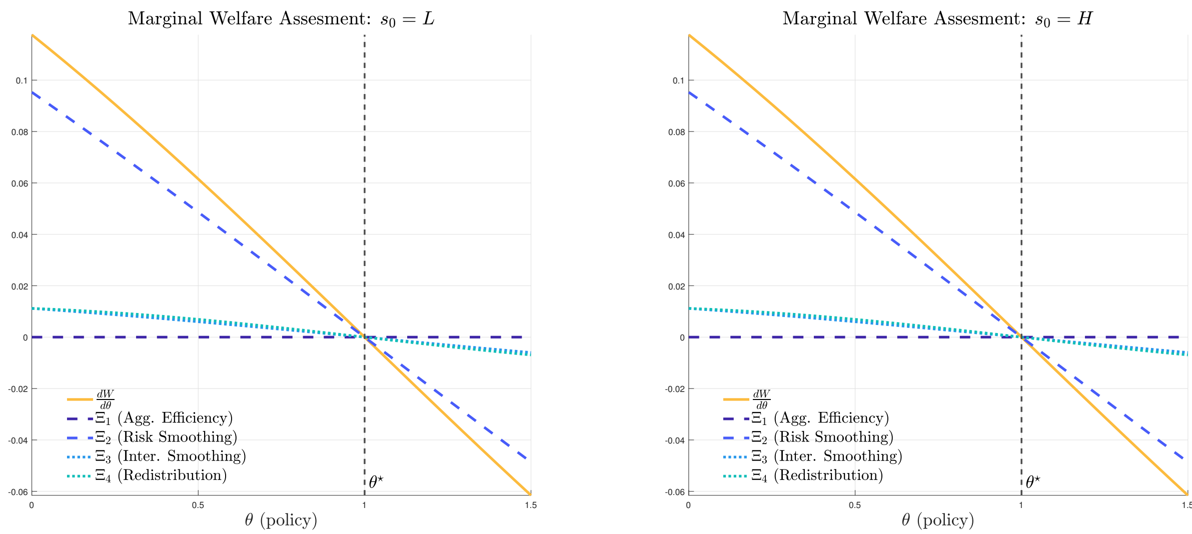

Figure 2: Aggregate additive decomposition of welfare assessments (Scenario 1)

Note: Figure 2 shows the marginal welfare assessment of a normalized utilitarian planner, $\frac{dW}{d\theta}$, and the components of its aggregate additive decomposition, as defined in Proposition 4. The left plot corresponds to the assessment when $s_0 = L$, while the right plot corresponds to the assessments when $s_0 = H$. Due to the symmetry of the model, both aggregate assessments are identical regardless of the state in which they are made. The solid line is computed as described in Equation (32). The dashed and dotted lines are computed as described in Equation (10), where the DS-weights are given by in Equations (18), (19), and (20). Note that $\frac{dW}{d\theta} = \Xi_1 + \Xi_2 + \Xi_3 + \Xi_4$.

the impact of the initial state dissipates. More importantly, in the long run (although also in the short run), regardless of the initial state, the stochastic components are higher for those states in which an individual has a lower endowment/higher marginal utility.

Third, the individual components of the DS-weights further capture the differences in the marginal valuation of transfers among individuals for different initial states. A normalized utilitarian planner values a hypothetical permanent transfer at all dates/states towards the individual with low endowment at s_0 at 1.125, and towards the individual with a high endowment at 0.875. The plot of DS-weights simply combines multiplicatively the dynamic, stochastic, and individual components just discussed.

*Aggregate additive decomposition of welfare assessment*s. In Figure 1, we show the components of the aggregate additive decomposition of welfare assessments for a normalized utilitarian planner. To facilitate the comparison with other scenarios, we show the decomposition for different initial states, $s_0 = \{L, H\}$. However, given the symmetry of the model, the aggregate welfare assessments are identical in both states. A different set of insights emerge from the aggregate additive decomposition.

First, as formally shown in Proposition 3, the aggregate efficiency component is zero, that is, $\Xi_1 = 0$. This occurs because we study an endowment economy for which aggregate consumption is invariant to the policy.

Second, we show that a normalized utilitarian finds it optimal to increase transfers until $\theta = 1$, which corresponds to full insurance. Such a planner finds welfare gains mostly due to risk-sharing (Ξ_2), and to a lesser degree to intertemporal-sharing (Ξ_3), and redistribution (Ξ_4). This finding is consistent with Figure 1, which shows that there is more cross-sectional dispersion in the stochastic component of DS-weights, relative to the dynamic and individual components.

Third, as formally shown in Proposition 10, when $\theta = 1$, markets are effectively complete, which implies that both risk and intertemporal-sharing components are zero, that is, $\Xi_2 = \Xi_3 = 0$. The symmetry of the model further implies that redistribution component is also zero in this particular scenario, that is, $\Xi_4 = 0$. Coincidentally, in this scenario, both a no-sharing and a no-redistribution (pseudo-utilitarian) planner would find $\theta = 1$ to be the optimal policy, as the normalized utilitarian planner.

Sensitivity to persistence of shocks. The nature of endowment shocks, in particular whether such shocks are transitory or permanent, significantly impacts the aggregate additive decomposition welfare assessments. In the Online Appendix, we illustrate this fact by including versions of Figures 1 and 2 when $\rho = 0.999$ (almost permanent shocks) and $\rho = 0.5$ (transitory endowment shocks). As one could anticipate, when endowments are almost permanent, almost all welfare gains are attributed to the redistribution component Ξ_4. Alternatively, when shocks are purely transitory

6.2 Scenario 2: Heterogeneous preferences with aggregate uncertainty

Environment. In our second scenario, we assume i) that some individuals are more risk-averse/unwilling to substitute intertemporally than others, and ii) that all endowment risk is aggregate. In particular, we assume that individual 1 is more risk averse than individual 2, so $\gamma_1 > \gamma_2$. Formally, we assume that

$$y^1(s) = \overline{y} + \varepsilon(s) \quad \text{and} \quad y^2(s) = \overline{y} + \varepsilon(s),$$

where $\overline{y} \geq 0$, and and where $\varepsilon(L) = -\varepsilon(H)$. We consider the welfare assessment of a transfer policy that shifts the amount of risk borne by individual 1 to individual 2. Formally, we set $T^1(s) = -\varepsilon(s)$ and $T^2(s) = \varepsilon(s)$, so individual consumption takes the form

$$c^1(s) = \overline{y} + \varepsilon(s)(1-\theta) \quad \text{and} \quad c^2(s) = \overline{y} + \varepsilon(s)(1+\theta).$$

Under this policy, when $\theta = 1$, individual 1 is fully insured, at the expense of increasing the consumption fluctuations of individual 2 in response to aggregate shocks. In this scenario, aggregate consumption varies with the aggregate state, but not with θ, since $\int c^i(s)\, dG(i) = \overline{y} + \varepsilon(s)$.

Results. With the exception of risk aversion, set to $\gamma_1 = 5$ and $\gamma_2 = 2$, we use the same parameters as in Scenario 1: $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$. As in the benchmark parameterization of Scenario 1, we set $\rho = 0.95$, so endowment shocks are moderately persistent.

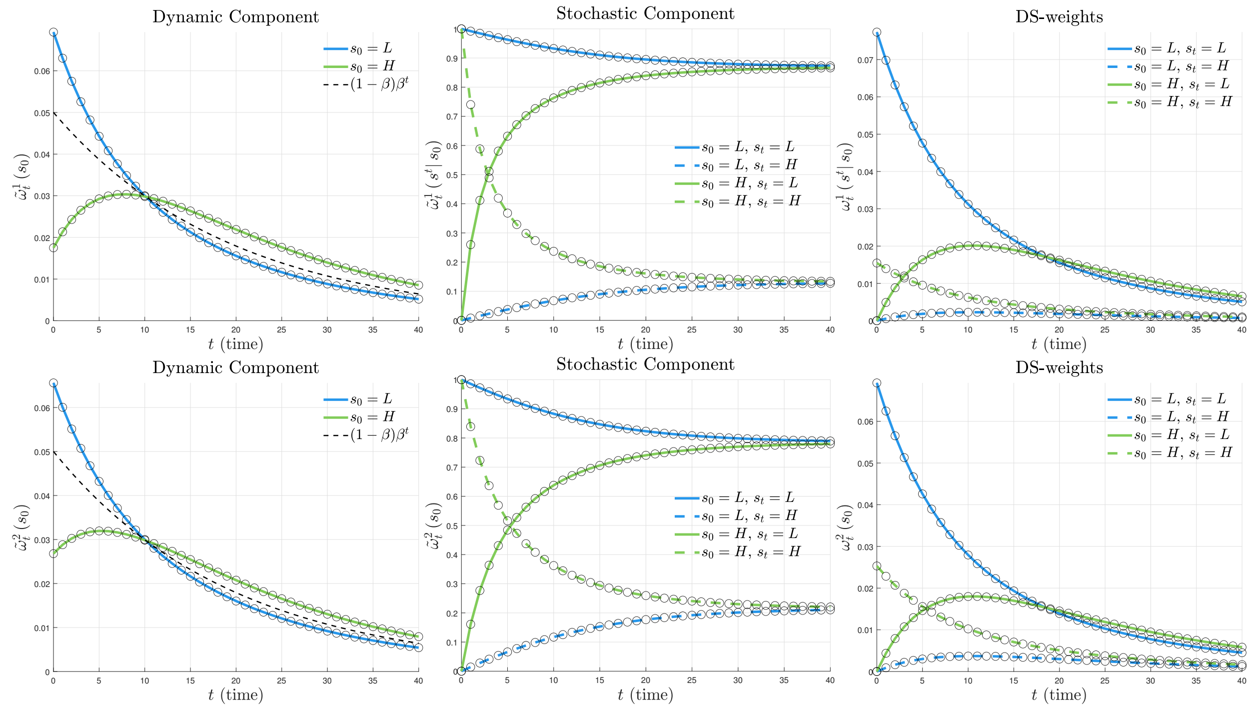

Figure 3: Individual multiplicative decomposition of DS-weights (Scenario 2)

Note: Figure 3 shows the components of the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner, defined in Proposition 4. We assume that $\theta = 0.25$, although all figures are qualitatively similar when $\theta \in [0, 1)$. The top row shows each of the components for individual 1, while the bottom row shows them for individual 2. The left plots show the dynamic component, $\tilde{\omega}_t^i(s_0)$, for different values of t for different initial states, $s_0 = \{H, L\}$. For reference, we also show the dynamic weight for a hypothetical individual with linear marginal utility, given by $(1-\beta)\beta^t = \beta^t / \sum_t \beta^t$. Note that the sum under each of the curves adds up to 1. The middle plots show the stochastic component, $\tilde{\omega}_t^i\left(s^t \middle| s_0\right)$, for different values of t, for different initial states, $s_0 = \{H, L\}$, and final states, $s_t = \{H, L\}$. The right plots show the actual DS-weights, $\omega_t^i\left(s^t \middle| s_0\right)$, also for different values of t, and different initial and final states: $s_0 = \{H, L\}$ and $s_t = \{H, L\}$. The parameters are $\theta = 0.5$, $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, $\rho = 0.95$, $\gamma_1 = 5$, and$\gamma_2 = 2$. The individual component of DS-weights are $\tilde{\omega}^1(s_0 = L) = 1.117$ and $\tilde{\omega}^2(s_0 = L) = 0.883$ when an assessment takes place at $s_0 = L$; and $\tilde{\omega}^1(s_0 = H) = 1.055$ and $\tilde{\omega}^2(s_0 = H) = 0.945$ when the assessment takes place at $s_0 = H$.

Once again, we describe the DS-weights used by a normalized utilitarian planner with $\lambda_i = 1$, as defined in Proposition 4.

Individual multiplicative decomposition of DS-weights. In Figure 3, we show the components of the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner for each of the individuals when $\theta = 0.25$. This new scenario is associated with new insights.

First, the plots of the dynamic components show that a normalized utilitarian planner overweights earlier periods for *all* individuals when the aggregate endowment is low/individual marginal utility is relative high. Importantly, it does so more for individual 1, with the highest curvature coefficient $\gamma_1 = 5$. Note, for instance, that $\tilde{\omega}_0^1(s_0 = L) > \tilde{\omega}_0^2(s_0 = L)$ and that $\tilde{\omega}_0^1(s_0 = H) < \tilde{\omega}_0^2(s_0 = H)$.

Second, as in Scenario 1, the plots of the stochastic components show that a normalized utilitarian planner overweights those states which are more likely, given the initial state. More importantly, in the long run (although also in the short run), regardless of the initial state, the stochastic components give relatively more weight to those states in which an individual has a lower endowment/higher marginal utility, but differentially more for the individual 1, with the highest curvature coefficient $\gamma_1 = 5$. Note, for instance, that $\tilde{\omega}^1_\infty (s_t = L) > \tilde{\omega}^2_\infty (s_t = L)$ and that $\tilde{\omega}^1_\infty (s_t = H) < \tilde{\omega}^2_\infty (s_t = H)$.

Third, the individual components of the DS-weights still capture differences in the marginal valuation of permanent transfers among individuals for different initial states. However, in this scenario this differences are mostly driven by the differences in preferences between individuals. Unlike in scenario 1, a normalized utilitarian planner gives more value to a hypothetical permanent transfer towards individual 1 at all states, since $\tilde{\omega}^1 (s_0 = L) > \tilde{\omega}^2 (s_0 = L)$ and $\tilde{\omega}^1 (s_0 = H) > \tilde{\omega}^2 (s_0 = H)$. This result illustrates how our approach is able to clearly compute the implicit desire for redistribution of a utilitarian planner.

*Aggregate additive decomposition of welfare assessment*s. In Figure 4, we show the components of the aggregate additive decomposition of welfare assessments for a normalized utilitarian planner. As in Scenario 1, and as shown in Proposition 3, the aggregate efficiency component is zero, that is, $\Xi_1 = 0$. Once again, this occurs because we study an endowment economy for which aggregate consumption is invariant to the policy. There is a new set of insights.

First, we show that a normalized utilitarian finds it optimal to increase transfers until some value of $\theta^\star$, regardless of whether the optimal policy is determined from $s_0 = L$ or $s_0 = H$. This should not be surprising, since transferring aggregate risk to the individual most willing to bear such a risk seems desirable. Interestingly, the reason for why a planner finds desirable to increase θ until $\theta^\star$ differs with the initial state of the economy. When $s_0 = L$, we show that a normalized utilitarian planner mostly attributes welfare gains to redistribution (Ξ_4), followed by risk-sharing (Ξ_2), with intertemporal-sharing (Ξ_3) barely playing a role. Instead, when $s_0 = H$, we show that a normalized utilitarian planner mostly attributes welfare gains to risk-sharing (Ξ_2), followed by redistribution (Ξ_4) and intertemporal-sharing (Ξ_3). Building on the insights of Proposition 5, one can trace these results to the cross-sectional dispersion of the different components of DS-weights. In particular, Figure 4 illustrates how the cross-sectional dispersion of the individual component is significantly higher when $s_0 = L$, which explains why the redistribution component is more important when $s_0 = L$. Alternatively, Figure 4 illustrates how the cross-sectional dispersion of the dynamic and the stochastic components is higher when $s_0 = H$.

Second, note that at the optimal $\theta^\star$ for both $s_0 = L$ and $s_0 = H$, the normalized utilitarian planner perceives Ξ_4 to be negative, while $\Xi_2 + \Xi_3$ are positive. This implies that a no-sharing (pseudo-utilitarian) DS-planner would choose a lower level of $\theta^\star$ than the normalized utilitarian planner, while a no-redistribution (pseudo-utilitarian) DS-planner would choose a higher level.

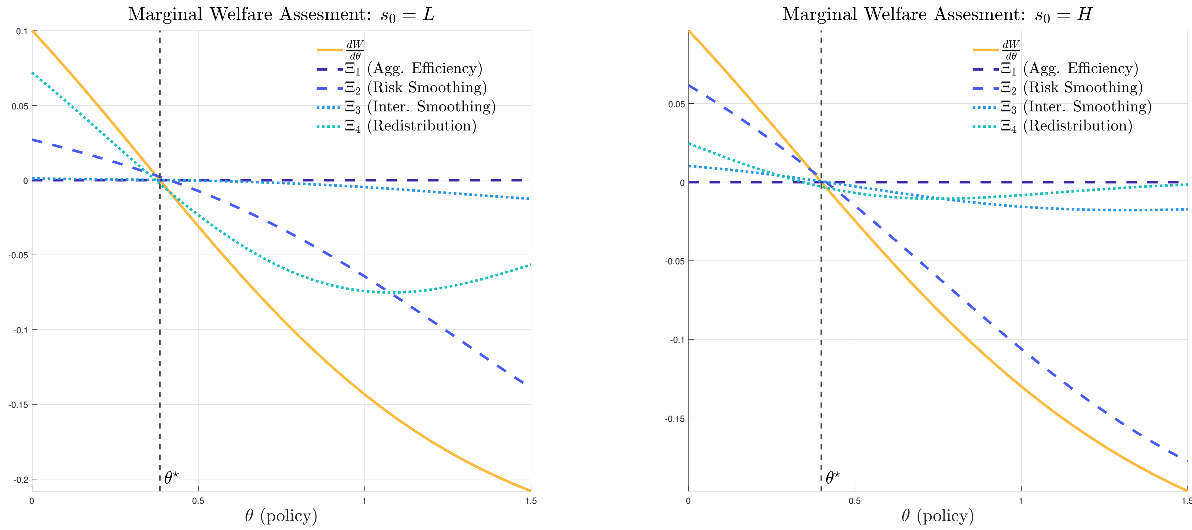

Figure 4: Aggregate additive decomposition of welfare assessments (Scenario 2)

Note: Figure 4 shows the marginal welfare assessment of a normalized utilitarian planner, $\frac{dW}{d\theta}$, and the components of its aggregate additive decomposition, as defined in Proposition 4. The left plot corresponds to the assessment when $s_0 = L$, while the right plot corresponds to the assessments when $s_0 = H$. The dashed and dotted lines are computed as described in Equation (10), where the DS-weights are computed as in Equations (18), (19), and (20). Note that $\frac{dW}{d\theta} = \Xi_1 + \Xi_2 + \Xi_3 + \Xi_4$.

7 Application II: Taxation under Incomplete Markets

Finally, we study taxation in a workhorse incomplete markets model, which prominently features individual heterogeneity.[28] We provide a formal welfare assessment of savings taxes in a version of Huggett (1993). In this model, households, who face idiosyncratic earnings risk but no aggregate uncertainty, can self-insure using a risk-free asset in an environment without capital.

7.1 Environment

Our notation closely follows the notation used in earlier sections of the paper.

Households. There is a unit measure of households, indexed by i, with preferences given by

$$V_i(s_0) = \sum_{t=0}^{T} \beta^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u\left(c_t^i\left(s^t\right)\right), \quad \text{where} \quad u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

Individual i faces the budget constraint

$$c_t^i\left(s^t\right) + \frac{1}{1-\theta}\left(a_{t+1}^i\left(s^t\right) - a_t^i\left(s^{t-1}\right)\right) = r_t\left(s^{t-1}\right) a_t^i\left(s^{t-1}\right) + \left(1-\tau^{\text{lab}}\right) z_t^i\left(s^t\right) + T_t\left(s^t\right),$$

[28] There is a rich literature that explores different forms of taxation in dynamic environments with imperfect insurance, including, among others, Aiyagari (1995), Benabou (2002), Domeij and Heathcote (2004), Conesa, Kitao and Krueger (2009), Davila et al. (2012), Gottardi, Kajii and Nakajima (2015), and Heathcote, Storesletten and Violante (2017).

where $a_t^i(s^{t-1})$ denotes beginning-of-period assets and $z_t^i(s^t)$ denotes idiosyncratic labor productivity. Households provide labor inelastically, so that $z_t^i(s^t)$ equals labor income. Households face a real interest rate $r_t(s^{t-1})$, pay a uniform labor income tax rate $\tau^{\text{lab}} \geq 0$, and receive a uniform lump-sum transfer $T_t(s^t)$ from the government. Finally, households face a borrowing constraint of the form $a_{t+1}^i(s^t) \geq \underline{a}$.

In this section, we focus on the taxation of savings, which we denote here by θ. Letting $s_t^i(s^t) = a_{t+1}^i(s^t) - a_t^i(s^{t-1})$ denote per-period savings, the above budget constraint can be rearranged as

$$s_t^i\left(s^t\right) = (1-\theta)\left[r_t\left(s^{t-1}\right)a_t^i\left(s^{t-1}\right) + \left(1-\tau^{\text{lab}}\right)z_t^i\left(s^t\right) + T_t\left(s^t\right) - c_t^i\left(s^t\right)\right]$$

underscoring the interpretation of θ as a tax on savings.

Idiosyncratic labor productivity follows a two-state Markov process, with $z_t^i(s^t) \in \left\{z^L, z^H\right\}$. In particular, we assume that transition probabilities only depend on the current earnings state, and take the form λ^j for $j \in \{L, H\}$. As a result of the Markov assumption, the economy can be characterized recursively, with wealth, $a_t^i(s^{t-1})$, and earnings, $z_t^i(s^t)$, defining the pertinent state variables.

Firms. On the supply side, output is produced using labor by a representative firm according to

$$Y_t\left(s^t\right) = L_t\left(s^t\right),$$

where $L_t(s^t) = \sum_i z_t^i(s^t)$. Since idiosyncratic productivity (and wages), z_t^i, evolves exogenously an is invariant to time and the tax policy, a law of large numbers implies that $L_t(s^t) = L$ at all times. We normalize $L = 1$, which implies that $Y_t(s^t) = 1$. Under perfect competition, the real wage rate must also be equal to 1, a fact we implicitly use when defining the household budget constraint.

Government. Government policy in this economy is defined by $\left\{\theta, B_{t+1}(s^t), T_t(s^t), \tau^{\text{lab}}\right\}$. The government's budget constraint is given by

$$B_{t+1}\left(s^t\right) = \left(1 + r_t\left(s^{t-1}\right)\right)B_t\left(s^{t-1}\right) + \frac{\theta}{1-\theta}\sum_i s_t^i\left(s^t\right) - T_t\left(s^t\right),$$

where $-B_t(s^{t-1})$ denotes the government's debt position, and $\sum_i s_t^i(s^t) = A_{t+1}(s^t) - A_t(s^t)$ denotes the change in aggregate household wealth. For a given θ and government debt policy $\{B_t(s^t)\}$, the lump-sum transfers $\{T_t(s^t)\}$ adjust to clear the government budget constraint.

Equilibrium. Given a tax θ, an exogenous sequence for government debt $\{B_t(s^{t-1})\}$ as well as an initial cross-sectional asset distribution, a competitive equilibrium is a sequence of consumption and asset allocations $\{c_t^i(s^t), a_{t+1}^i(s^t)\}$, and prices $\{r_{t+1}(s^t)\}$, such that:

(i) The decision rules for consumption and savings solve the household problem subject to the budget constraint, given government policies, interest rates, and earnings shocks.

(ii) The goods market and asset market clear, that is:

$$1 = C_t\left(s^t\right) = \sum_i c_t^i\left(s^t\right),$$
$$0 = A_{t+1}\left(s^t\right) + B_{t+1}\left(s^t\right) = \sum_i a_{t+1}^i\left(s^t\right) + B_{t+1}\left(s^t\right),$$

where $-B_{t+1}\left(s^t\right)$ is the government's outstanding debt position and $A_{t+1}\left(s^t\right)$ is the household's aggregate wealth at the end of period t.

(iii) The government budget constraint holds.

(iv) The sequence of cross-sectional distributions is consistent with household behavior.

7.2 Analytical Results

We initially study the local efficiency of stationary equilibria, parametrized by θ, for a given DS-planner. In any given stationary equilibrium, macroeconomic aggregates are constant and the only source of uncertainty that households face is earnings risk. In particular, government debt must be constant, $B_t\left(s^t\right) = B$, which implies that aggregate savings must be 0, $\sum_i s_t^i\left(s^t\right) = 0$. Furthermore, all macroeconomic aggregates may depend on the tax rate θ, which we denote, in the case of the real interest rate for example, by $r_{t+1}\left(s^t\right) = r\left(\theta\right)$.

We now characterize the welfare impact across different stationary equilibria of a marginal tax change for a given DS-planner. Our results here determine whether a DS-planner prefers one stationary equilibrium over another — see our discussion in Section 5.2.[29] Here, a DS-planner finds a tax change desirable if $\frac{dW^{DS}}{d\theta} > 0$, where

$$\frac{dW^{DS}\left(\theta\right)}{d\theta} = \int \tilde{\omega}^i\left(s_0\right) \sum_{t=0}^{T} \tilde{\omega}_t^i\left(s_0\right) \sum_{s^t} \tilde{\omega}_t^i\left(s^t \middle| s_0\right) \frac{dc_t^i\left(s^t\right)}{d\theta} dG\left(i\right), \tag{33}$$

and where $\frac{dc_t^i\left(s^t\right)}{d\theta}$ satisfies

$$\frac{dc_t^i\left(s^t\right)}{d\theta} = -\frac{s_t^i\left(s^t\right)}{\left(1-\theta\right)^2} + \frac{dr}{d\theta}\left(a_t^i\left(s^{t-1}\right) - A\right) + r\frac{da_t^i\left(s^{t-1}\right)}{d\theta} - \frac{1}{1-\theta}\frac{ds_t^i\left(s^t\right)}{d\theta}. \tag{34}$$

A stationary equilibrium is locally efficient at θ^* under DS-weights if $\frac{dW^{DS}\left(\theta^*\right)}{d\theta} = 0$.

Equation (34) highlights that the tax change affects household i's consumption in three ways.[30]

[29] Instead, we could have initialized the economy at a particular θ, and then study a transition path to a new stationary equilibrium. We study this alternative experiment in the Appendix.

[30] Note that, in any stationary equilibrium with $\sum_i s_t^i = 0$ and $B_t = B$, we have $\tau\left(\theta\right) = r\left(\theta\right) B$ and therefore $\frac{d\tau}{d\theta} = \frac{dr}{d\theta} B = -A\frac{dr}{d\theta}$.

First, for a given level of wealth and income, and for a given savings policy, a change in the tax rate affects the effective rate of savings and wealth accumulation that household i achieves. This is a partial equilibrium effect. In particular, a tax (subsidy) on savings makes it harder (easier) for debtors to repay their debt and accumulate wealth. This force turns out to be a key determinant in our welfare analysis.

The second effect of the tax change works through the general equilibrium response of the real interest rate. When the real interest rate increases, then, for a given wealth profile, financial income rises. Conversely, the effective debt burden of indebted households also increases. On the other hand, when the government has a positive supply of debt in steady state so that $A > 0$, then an increase in the real interest rate also implies larger government interest payments that are financed through lump-sum taxation. The net effect of a change in the real interest rate on household i's income therefore depends on i's *relative* wealth position, $a_t^i\left(s^{t-1}\right) - A$.

Finally, the tax change affects household i's savings behavior, which is captured by the last two terms of Equation (34). This effect does not impact welfare assessments when evaluated using the individual's private marginal utilities. However, when making aggregate welfare assessments, this effect may not vanish for paternalistic planners such as the No-Sharing DS-planner we consider below.

The aggregate welfare assessment depends on the DS-weights used by the planner. In the following, we compare the welfare assessments of the DS-planners we introduced in Section 4: the (unnormalized) utilitarian (U) planner, the No-Redistribution (NR) planner, and the No-Sharing (NS) planner. Consistent with Proposition 3, aggregate efficiency plays no role in the welfare assessments of tax policy in this endowment economy. A pure-AE DS-planner would be indifferent between any of the tax rates, with $\frac{dW^{AE}(\theta)}{d\theta} = 0$ for all θ.

As we explain in Section 4, all three planners adopt the same individual component of DS-weights, $\tilde{\omega}_t^i\left(s_0\right) = \beta^t$, but use different dynamic and stochastic components, $\tilde{\omega}^i(s_0)$ and $\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)$. In particular, the UU and NR DS-planners are non-paternalistic and therefore use $\tilde{\omega}_t^{i,UU}\left(s^t\middle|\, s_0\right) = \tilde{\omega}_t^{i,NR}\left(s^t\middle|\, s_0\right) = \pi_t\left(s^t\middle|\, s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}$, consistent with household i's private assessment. The paternalistic NS DS-planner, on the other hand, uses $\tilde{\omega}_t^{i,NS}\left(s^t\middle|\, s_0\right) = \pi_t\left(s^t\middle|\, s_0\right)$. We report the individual components in the Appendix.

Before presenting our numerical results, it is helpful to analytically explore how the NS planner, who only values redistribution, makes welfare assessments. Exploiting the fact that, in any stationary equilibrium, $\int s_t^i\left(s^t\right) dG(i) = \mathbb{E}_i[s_t^i\left(s^t\right)] = 0$ and $\int a_t^i\left(s^{t-1}\right) dG(i) = \mathbb{E}_i[a_t^i\left(s^{t-1}\right)] = A$, we can express $\frac{dW^{NS}(\theta)}{d\theta}$ for the No-Sharing DS-planner as follows:

$$\frac{dW^{NS}(\theta)}{d\theta} = \sum_t \beta^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right)\Bigg[-\frac{\mathbb{C}ov_i\left[\tilde{\omega}^{i,NS}(s_0), s_t^i\left(s^t\right)\right]}{(1-\theta)^2} + \frac{dr}{d\theta}\mathbb{C}ov_i\left[\tilde{\omega}^{i,NS}(s_0), a_t^i\left(s^{t-1}\right)\right]$$
$$+ \mathbb{C}ov_i\left(\tilde{\omega}^{i,NS}(s_0), r\frac{da_t^i\left(s^{t-1}\right)}{d\theta} - \frac{1}{1-\bar{\theta}}\frac{ds_t^i\left(s^t\right)}{d\theta}\right)\Bigg].$$

The only dimensions of heterogeneity in this economy, that is the only differences between households i and i', are wealth and earnings. By construction, a DS-planner that only values redistribution uses DS-weights that are strictly decreasing in wealth. At the same time, the savings policy function is also strictly decreasing in wealth.[31] Therefore, we find that

$$\mathbb{C}ov_i\left(\tilde{\omega}^{i,NS}(s_0), s_t^i\left(s^t\right)\right) > 0 \quad \text{and} \quad \mathbb{C}ov_i\left(\tilde{\omega}^{i,NS}(s_0), a_t^i\left(s^{t-1}\right)\right) < 0.$$

Intuitively, a No-Sharing planner who only values redistribution puts large DS-weights on indebted households. A subsidy on savings helps debtors for two distinct reasons. First, for a given level of income, the effective savings rate is higher, which helps borrowers accumulate wealth and escape indebtedness more quickly. Second, we have that

$$\frac{dr}{d\theta} > 0$$

because, when taxes are large, a higher real interest rate is required in general equilibrium to clear the bond market. Therefore, a subsidy on savings also lowers the real interest rate in general equilibrium, which reduces the effective debt burden of indebted households by decreasing their interest payments.

7.3 Quantitative results

To characterize and compare the welfare assessments of the three DS-planners, we solve the model numerically. We initially discuss our calibration and present the details of our solution method in the Appendix.

We calibrate a standard discount rate of $-\log\beta = 0.05$. Household's constant relative risk aversion coefficient is $\gamma = 2$. Fiscal policy is calibrated so that, in steady state, government expenditures account for 20% of output, and the outstanding supply of government debt is equal to 100% of GDP. We set a labor income tax rate of 30%. Finally, we calibrate the earnings process to match unemployment transitions. We let $z^H = 1$ represent the employment state and $z^L = 0.30$ the

[31] While the savings policy function is decreasing in wealth, it is increasing in earnings. In all of our numerical experiments, $\mathbb{C}ov_i\left(\tilde{\omega}^{i,NS}, s_t^i\left(s^t\right)\right)$ is positive. To state this result formally, we would have to assume that earnings heterogeneity is bounded, so that its contribution to the covariance between permanent marginal utilities and savings does not dominate the effect of wealth.

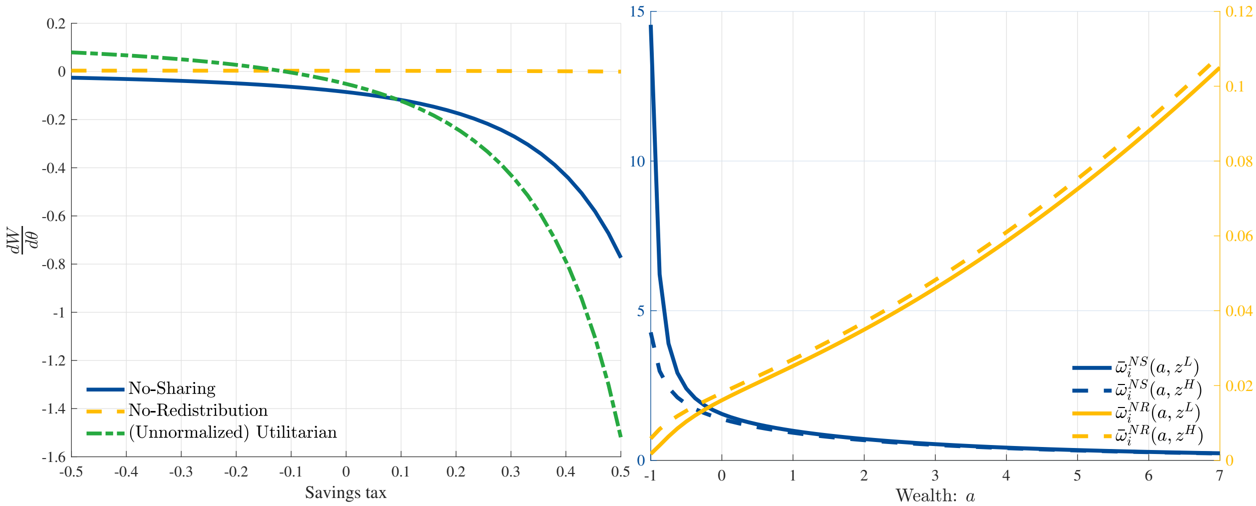

Figure 5: Welfare Assessments of Savings Taxation and DS-weights

Note: The left panel of Figure 5 shows the aggregate welfare assessment $\frac{dW(\theta)}{d\theta}$ for three DS-planners: the utilitarian (U), the No-Redistribution (NR), and the No-Sharing (NS) planners. Optimal tax rates are implied by $\frac{dW(\theta^*)}{d\theta} = 0$. The right panel of Figure 5 shows the individual components of the DS-weights for the NS and NR planners, $\tilde{\omega}^{i,NS}(a, z)$ (blue, LHS) and $\tilde{\omega}^{i,NR}(a, z)$ (yellow, RHS), across wealth a. The solid lines correspond to the DS-weight for unemployed agents and the dashed lines for the employed.

unemployment state, where we account for unemployment insurance. We calibrate the transition probabilities so that 8.49% of households are unemployed in steady state, which is the long-run average unemployment rate.

Figure 5 shows the welfare assessments of the unnormalized-utilitarian, the pure-redistribution, and the pure-insurance planners, illustrating how the optimal tax rate is determined. We show that the No-Sharing DS-planner's welfare assessment yields $\frac{dW^{NS}(\theta)}{d\theta} < 0$ for all θ. The optimal tax rate from a pure redistribution perspective is therefore the corner solution with an infinite subsidy on savings. The No-Redistribution planner who only values insurance, on the other hand, prefers a positive tax on savings of 46%, which is where $\frac{dW^{NR}(\theta)}{d\theta}$ crosses 0. The utilitarian planner trades off both of these normative motives. The implied optimal tax rate is therefore intermediate between the two extremes. The utilitarian planner prefers an optimal tax on savings of -12%.

8 Conclusion

In this paper, we have introduced the notion of Dynamic Stochastic Generalized Social Marginal Welfare Weights (Dynamic Stochastic weights or DS-weights, for short) and explored their properties. First, we have shown that DS-weights are useful to decompose aggregate welfare assessments of policy changes into four distinct components: aggregate efficiency, intertemporal-sharing, risk-sharing, and redistribution. Second, we have shown that by using DS-weights it is possible to formalize a new, larger set of welfare criteria that society may find appealing. In particular, we have been able to define normative criteria that are exclusively based on one or several of the components that we identify, potentially disregarding the others.

Looking backward, our definition of normalized utilitarian planner — based on DS-weights — opens the door to revisiting the exact rationales that have justified particular welfare assessments in existing work. Looking forward, we hope that our approach informs ongoing and future discussions on policy-making mandates, in particular when trading off aggregate stabilization objectives against interpersonal insurance and redistribution objectives.

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Appendix

A Proofs and Derivations: Section 2

The derivation of Equations (2) and (3) in the text is straightforward. While we use the utilitarian planner as a benchmark throughout the paper, it is straightforward to adapt Definition 2 to more general non-paternalistic Paretian planners, as follows:

Definition 6. *(Desirable policy change for a non-paternalistic Paretian planner)* A non-paternalistic Paretian planner finds a policy change desirable if and only if $\frac{dW^{NPP}(s_0)}{d\theta} > 0$, where

$$\frac{dW^{NPP}(s_0)}{d\theta} = \int h_i(\cdot) \frac{dV_i(s_0)}{d\theta} dG(i) = \int h_i(\cdot) \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0 \right) \frac{\partial u_i(s^t)}{\partial c_t^i} \frac{du_{i|c}(s^t)}{d\theta} dG(i), \tag{35}$$

where $h_i(\cdot) > 0$, $\forall i \in I$, are a collection of individual-specific strictly positive functions, where $\frac{dV_i(s_0)}{d\theta}$ is defined in Equation (2).

Since the aggregate welfare assessment of a policy change is based on individual lifetime welfare effects, $\frac{dV_i(s_0)}{d\theta}$, this welfare criterion is non-paternalistic. Moreover, the Paretian property is immediately implied by the fact that $h_i(\cdot)$ is a strictly positive function.

While the utilitarian planner of Definition 2 is a particular case of a non-paternalistic Paretian planner, there are other non-paternalistic Paretian planner that are not utilitarian. Two approaches are particularly prominent. First, Definition 6 is consistent with the use of nonlinear social welfare functions, e.g., isoelastic (Atkinson, 1970) or maximin/Rawlsian (Rawls, 1971, 1974); see Mas-Colell, Whinston and Green (1995) for a textbook treatment. In these cases, $h_i(\cdot)$ equals $\frac{\partial W(s_0)}{\partial V_i}$, where $W(\{V_i\}_i)$ defines a lifetime Social Welfare Function — see Section 5.3 in the paper for more details. Second, Definition 6 is consistent with the use of Generalized Social Marginal Welfare Weights (Saez and Stantcheva, 2016; Fleurbaey and Maniquet, 2018), in which the functions $h_i(\cdot)$ are endogenous and can capture alternatives to Utilitarianism, such as Libertarianism or Equality of Opportunity.

B Proofs and Derivations: Section 3

Proof of Lemma 1. (DS-weights: individual multiplicative decomposition)

Proof. We offer a constructive proof. Let us start with DS-weights $\omega_t^i(s^t | s_0)$, defined for each individual, date, and history. The stochastic component, defined for each history, and the dynamic component, defined for each date, can be built as follows:

$$\tilde{\omega}_t^i \left(s^t \middle| s_0 \right) \equiv \frac{\omega_t^i(s^t | s_0)}{\sum_{s^t} \omega_t^i(s^t | s_0)} \quad \text{and} \quad \tilde{\omega}_t^i(s_0) \equiv \frac{\sum_{s^t} \omega_t^i(s^t | s_0)}{\sum_{t=0}^{T} \sum_{s^t} \omega_t^i(s^t | s_0)}.$$

The individual component can be defined as $\tilde{\omega}^i(s_0) \equiv \sum_{t=0}^{T}\sum_{s^t}\omega_t^i\left(s^t\middle| s_0\right)$. As explained in the text, even though this decomposition is not unique, it becomes unique once units are specified. □

Proof of Proposition 1. (Welfare assessments: aggregate additive decomposition)

Proof. Starting from Equation (8), we can express $\frac{dW^{DS}(s_0)}{d\theta}$ as follows:

$$\frac{dW^{DS}(s_0)}{d\theta} = \mathbb{E}_i\left[\tilde{\omega}^i\left(s^0\right)\frac{dV_i^{DS}(s_0)}{d\theta}\right] \quad \text{where} \quad \frac{dV_i^{DS}(s_0)}{d\theta} = \sum_{t=0}^{T}\tilde{\omega}_t^i(s_0)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle| s_0\right)\frac{du_{i|c}(s^t)}{d\theta}, \tag{36}$$

and where $\int dG(i) = 1$. Hence, we can decompose $\frac{dW^{DS}(s_0)}{d\theta}$ as follows:

$$\begin{aligned}\frac{dW^{DS}(s_0)}{d\theta} &= \mathbb{E}_i\left[\tilde{\omega}^i\left(s^0\right)\right]\mathbb{E}_i\left[\frac{dV_i^{DS}(s_0)}{d\theta}\right] + \underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \frac{dV_i^{DS}(s_0)}{d\theta}\right]}_{=\Xi_4} \\ &= \mathbb{E}_i\left[\frac{dV_i^{DS}(s_0)}{d\theta}\right] + \Xi_4,\end{aligned} \tag{37}$$

where we use the fact that $\mathbb{E}_i\left[\tilde{\omega}^i\left(s^0\right)\right] = \int \tilde{\omega}^i\left(s^0\right)dG(i) = 1$, and where we define Ξ_4 as follows:

$$\Xi_4 = \mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \sum_{t=0}^{T}\tilde{\omega}_t^i(s_0)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle| s_0\right)\frac{du_{i|c}(s^t)}{d\theta}\right].$$

Next, we can decompose $\mathbb{E}_i\left[\frac{dV_i^{DS}(s_0)}{d\theta}\right]$ as follows:

$$\begin{aligned}\mathbb{E}_i\left[\frac{dV_i^{DS}(s_0)}{d\theta}\right] &= \mathbb{E}_i\left[\sum_{t=0}^{T}\tilde{\omega}_t^i(s_0)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle| s_0\right)\frac{du_{i|c}(s^t)}{d\theta}\right] \\ &= \sum_{t=0}^{T}\mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right]\mathbb{E}_i\left[\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle| s_0\right)\frac{du_{i|c}(s^t)}{d\theta}\right] + \underbrace{\sum_{t=0}^{T}\mathbb{C}ov_i\left[\tilde{\omega}_t^i(s_0), \sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle| s_0\right)\frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_3} \\ &= \sum_{t=0}^{T}\mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right]\sum_{s^t}\left(\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle| s_0\right)\right]\mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right] + \mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t\middle| s_0\right), \frac{du_{i|c}(s^t)}{d\theta}\right]\right) + \Xi_3 \\ &= \underbrace{\sum_{t=0}^{T}\mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right]\sum_{s^t}\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle| s_0\right)\right]\mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_1} \\ &\quad + \underbrace{\sum_{t=0}^{T}\mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right]\sum_{s^t}\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t\middle| s_0\right), \frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_2} + \Xi_3 \\ &= \Xi_1 + \Xi_2 + \Xi_3.\end{aligned} \tag{38}$$

Combining Equations (37) and (38) immediately yields the results in Proposition 1. $\square$

Proof of Proposition 2. (Properties of aggregate additive decomposition: Individual-invariant DS-weights)

Proof. a) If DS-weights $\omega_t^i\left(s^t \middle| s_0\right)$ do not vary across individuals, b), c), and d) below are valid.
b) If the stochastic components, $\tilde{\omega}_t^i\left(s^t \middle| s_0\right)$, do not vary across individuals, then

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t \middle| s_0\right), \frac{du_{i|c}\left(s^t\right)}{d\theta}\right] = 0 \Rightarrow \Xi_2 = 0.$$

c) If the dynamic components, $\tilde{\omega}_t^i\left(s_0\right)$, do not vary across individuals, then

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right), \sum_{s^t}\tilde{\omega}_t^i\left(s^t \middle| s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right] = 0 \Rightarrow \Xi_3 = 0.$$

d) If the individual components, $\tilde{\omega}^i\left(s_0\right)$, do not vary across individuals, then

$$\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\sum_{s^t}\tilde{\omega}_t^i\left(s^t \middle| s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right] = 0 \Rightarrow \Xi_4 = 0.$$

$\square$

Proof of Proposition 3. (Properties of aggregate additive decomposition: Individual-invariant policies)

Proof. Note that $\sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)$ and $\sum_{s^t}\tilde{\omega}_t^i\left(s^t \middle| s_0\right) = 1$ imply that $\sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\sum_{s^t}\tilde{\omega}_t^i\left(s^t \middle| s_0\right) = 1$.
a) If $\frac{du_{i|c}\left(s^t\right)}{d\theta} = g\left(\cdot\right)$, where $g\left(\cdot\right)$ does not depend on i, t, or s^t, then

$$\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\sum_{s^t}\tilde{\omega}_t^i\left(s^t \middle| s_0\right)\right]\frac{du_{i|c}\left(s^t\right)}{d\theta} = 0 \Rightarrow \Xi_4 = 0.$$

And the results from b) and c) also apply.
b) If $\frac{du_{i|c}\left(s^t\right)}{d\theta} = g\left(t\right)$, where $g\left(t\right)$ may depend on t, but not on i or s^t, then

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right), \sum_{s^t}\tilde{\omega}_t^i\left(s^t \middle| s_0\right)\right]\frac{du_{i|c}\left(s^t\right)}{d\theta} = 0 \Rightarrow \Xi_3 = 0.$$

And the result from c) also applies.
c) If $\frac{du_{i|c}\left(s^t\right)}{d\theta} = g\left(t, s^t\right)$, where $g\left(t, s^t\right)$ may depend on t and s^t, but not on i, then

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t \middle| s_0\right), \frac{du_{i|c}\left(s^t\right)}{d\theta}\right] = 0 \Rightarrow \Xi_2 = 0.$$

$\square$

Proof of Proposition 4. (Properties of aggregate additive decomposition: Endowment economy)

Proof. In an endowment economy, Equation (10) simply corresponds to

$$\mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]=\int\frac{dc_t^i\left(s^t\right)}{d\theta}dG\left(i\right)=0, \tag{39}$$

where the last equality follows from the fact that aggregate consumption is fixed and invariant to θ, that is, $\frac{d\int c_t^i\left(s^t\right)dG(i)}{d\theta}=0$. $\square$

Proof of Proposition 5. (Cross-sectional dispersion bounds) Equations (12) through (14) are an immediate application of the Cauchy–Schwarz inequality, which states that $\left|\mathbb{C}ov\left[X,Y\right]\right|\leq\sqrt{\mathbb{V}ar\left[X\right]}\sqrt{\mathbb{V}ar\left[Y\right]}$ for any pair of random variables X and Y. When applied to the relevant elements of Ξ_2, Ξ_3, and Ξ_4, we find that:

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t\middle|s_0\right),\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\leq\sqrt{\mathbb{V}ar_i\left[\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\right]}\sqrt{\mathbb{V}ar_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}$$

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right),\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\leq\sqrt{\mathbb{V}ar_i\left[\tilde{\omega}_t^i\right]}\sqrt{\mathbb{V}ar_i\left[\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}$$

$$\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right),\sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\leq\sqrt{\mathbb{V}ar_i\left[\tilde{\omega}^i\left(s^0\right)\right]}\sqrt{\mathbb{V}ar_i\left[\sum_{t=0}^{T}\sum_{s^t}\tilde{\omega}_t^i\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}.$$

These three inequalities, when combined with the definitions of Ξ_2, Ξ_3, and Ξ_4 in Equation (10), immediately imply Equations (12) through (14) in the text.

Proof of Proposition 6. (Aggregate efficiency component: stochastic decomposition) Starting from the definition of the aggregate efficiency component in Equation (10), we can express Ξ_1 as follows:

$$\begin{aligned}\Xi_1&=\sum_{t=0}^{T}\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s_0\right)\right]\sum_{s^t}\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\right]\mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\\&=\sum_{t=0}^{T}\overline{\omega}_t\sum_{s^t}\overline{\omega}_t\left(s^t\middle|s_0\right)\frac{d\overline{u}_{i|c}\left(s^t\right)}{d\theta},\end{aligned}$$

where we define $\overline{\omega}_t\left(s_0\right)=\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s_0\right)\right]$, $\overline{\omega}_t\left(s^t\middle|s_0\right)=\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\right]$, and $\frac{d\overline{u}_{i|c}\left(s^t\right)}{d\theta}=\mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]$. Multiplying and dividing by $\pi_t\left(s^t\middle|s_0\right)$ at every history, we can express and decompose Ξ_1 as follows:

$$\begin{aligned}\Xi_1 &= \sum_{t=0}^{T} \overline{\omega}_t(s_0) \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\overline{\omega}_t\left(s^t \middle| s_0\right)}{\pi_t\left(s^t \middle| s_0\right)} \frac{d\overline{u}_{i|c}\left(s^t\right)}{d\theta} = \sum_{t=0}^{T} \overline{\omega}_t(s_0)\, \mathbb{E}_0\left[\overline{\omega}_t^{\pi}\left(s^t \middle| s_0\right) \frac{d\overline{u}_{i|c}\left(s^t\right)}{d\theta}\right] \\ &= \sum_{t=0}^{T} \overline{\omega}_t(s_0) \left(\mathbb{E}_0\left[\overline{\omega}_t^{\pi}\left(s^t \middle| s_0\right)\right] \mathbb{E}_0\left[\frac{d\overline{u}_{i|c}\left(s^t\right)}{d\theta}\right] + \mathbb{C}ov_0\left[\overline{\omega}_t^{\pi}\left(s^t \middle| s_0\right), \frac{d\overline{u}_{i|c}\left(s^t\right)}{d\theta}\right]\right),\end{aligned}$$

which corresponds to Equation (15) in the text.

Proof of Proposition 7. (Risk-sharing/intertemporal-sharing components: alternative cross-sectional de- compositions) Here we make use of the following property of covariances:

$$Cov\left[X, YZ\right] = \mathbb{E}\left[y\right] \mathbb{C}ov\left[X, Z\right] + \mathbb{E}\left[z\right] \mathbb{C}ov\left[X, Y\right] + \mathbb{E}\left[\left(X - \mathbb{E}\left[X\right]\right)\left(Y - \mathbb{E}\left[Y\right]\right)\left(Z - \mathbb{E}\left[Z\right]\right)\right],$$

where X, Y, and Z denote random variables.

Applying this property to the definition of Ξ_3 immediately yields Equation (16) in the text. Equation (17) follows immediately after using once again the some property of covariances.

Proof of Proposition 8. (Redistribution component: stochastic decomposition) Note that $\frac{dV_i^{DS}(s_0)}{d\theta}$ can be written as

$$\begin{aligned}\frac{dV_i^{DS}(s_0)}{d\theta} &= \sum_{t=0}^{T} \tilde{\omega}_t^i(s_0)\, \mathbb{E}_0\left[\frac{\tilde{\omega}_t^i\left(s^t \middle| s_0\right)}{\pi_t\left(s^t \middle| s_0\right)} \frac{du_{i|c}\left(s^t\right)}{d\theta}\right] \\ &= \underbrace{\sum_{t=0}^{T} \tilde{\omega}_t^i(s_0)\, \mathbb{E}_0\left[\tilde{\omega}_t^{i,\pi}\left(s^t \middle| s_0\right)\right] \mathbb{E}_0\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\frac{dV_i^{DS,IE}(s_0)}{d\theta}} + \underbrace{\sum_{t=0}^{T} \tilde{\omega}_t^i(s_0)\, \mathbb{C}ov_0\left[\left[\tilde{\omega}_t^{i,\pi}\left(s^t \middle| s_0\right)\right], \frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\frac{dV_i^{DS,II}(s_0)}{d\theta}}.\end{aligned}$$

Hence, we can express Ξ_4 as follows:

$$\begin{aligned}\Xi_4 &= \mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \frac{dV_i^{DS}(s_0)}{d\theta}\right] \\ &= \underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \frac{dV_i^{DS,IE}(s_0)}{d\theta}\right]}_{\Xi_4^{EIE}} + \underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \frac{dV_i^{DS,II}(s_0)}{d\theta}\right]}_{\Xi_4^{II}},\end{aligned}$$

which corresponds to Equation (20) in the text.

Online Appendix

Section C of this Online Appendix includes additional proofs and derivations from Section 3 of the paper. Sections D of this Online Appendix includes additional results of Scenario 1, as well as Scenarios 3 and 4 of Application 1. It also includes individual marginal welfare effects for all scenarios. Section E includes several extensions and additional results. These include a detailed treatment of dimensional analysis for the normalized utilitarian planner, a detailed discussion of the relation between our results and those in Lucas (1987) and Alvarez and Jermann (2004), a discussion of how global welfare assessments are connected to consumer surplus measures in classical consumer theory, a discussion of the role of normalizations in the utilitarian case, and a description of how to extend our results to environments in which probabilities vary with θ.

C Proofs and Derivations: Section 4

Proof of Proposition 9. (Properties of normalized utilitarian planner: Proper weights)

Proof. a) Starting from Equation (4), we can first multiply and divide $\frac{dV_i^U(s_0)}{d\theta}$ by $\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t(s^t|s_0)\frac{\partial u_i(s^t)}{\partial c_t^i}$ and then multiply and divide at each date t by $(\beta_i)^t\sum_{s^t}\pi_t(s^t|s_0)\frac{\partial u_i(s^t)}{\partial c_t^i}$. Hence, we can express $\frac{dV_i^U(s_0)}{d\theta}$ as follows:

$$\begin{aligned}
\frac{dV_i^U(s_0)}{d\theta} &= \sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\frac{du_{i|c}\left(s^t\right)}{d\theta}\\
&= \sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\sum_{t=0}^{T}\frac{(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\frac{du_{i|c}\left(s^t\right)}{d\theta}}{\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t(s^t|s_0)\frac{\partial u_i(s^t)}{\partial c_t^i}}\\
&= \sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\sum_{t=0}^{T}\frac{(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}{\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t(s^t|s_0)\frac{\partial u_i(s^t)}{\partial c_t^i}}\frac{(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\frac{du_{i|c}\left(s^t\right)}{d\theta}}{(\beta_i)^t\sum_{s^t}\pi_t(s^t|s_0)\frac{\partial u_i(s^t)}{\partial c_t^i}}\\
&= \sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\sum_{t=0}^{T}\tilde{\omega}_t^i(s_0)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta},
\end{aligned}\tag{OA1}$$

where we define dynamic and stochastic components of DS-weights as follows:

$$\tilde{\omega}_t^i(s_0)=\frac{(\beta_i)^t\sum_{s^t}\pi_t(s^t|s_0)\frac{\partial u_i(s^t)}{\partial c_t^i}}{\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t(s^t|s_0)\frac{\partial u_i(s^t)}{\partial c_t^i}}\quad\text{and}\quad\tilde{\omega}_t^i\left(s^t\middle|s_0\right)=\frac{\pi_t(s^t|s_0)\frac{\partial u_i(s^t)}{\partial c_t^i}}{\sum_{s^t}\pi_t(s^t|s_0)\frac{\partial u_i(s^t)}{\partial c_t^i}}.$$

Hence, after multiplying by λ_i, and multiplying and dividing by $\int\lambda_i\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t(s^t|s_0)\frac{\partial u_i(s^t)}{\partial c_t^i}dG(i)$, it follows that

$$\frac{\lambda_i\frac{dV_i^U(s_0)}{d\theta}}{\int\lambda_i\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t(s^t|s_0)\frac{\partial u_i(s^t)}{\partial c_t^i}dG(i)}=\tilde{\omega}^i(s_0)\sum_{t=0}^{T}\tilde{\omega}_t^i(s_0)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta},$$

where we define the individual component of DS-weights as

$$\tilde{\omega}^{i}\left(s_{0}\right)=\frac{\lambda_{i} \sum_{t=0}^{T}\left(\beta_{i}\right)^{t} \sum_{s^{t}} \pi_{t}\left(s^{t} \mid s_{0}\right) \frac{\partial u_{i}\left(s^{t}\right)}{\partial c_{t}^{i}}}{\int \lambda_{i} \sum_{t=0}^{T}\left(\beta_{i}\right)^{t} \sum_{s^{t}} \pi_{t}\left(s^{t} \mid s_{0}\right) \frac{\partial u_{i}\left(s^{t}\right)}{\partial c_{t}^{i}} d G(i)}.$$

It is straightforward to verify that $\sum_{s^{t}} \tilde{\omega}_{t}^{i}\left(s^{t} \mid s_{0}\right)=1$, that $\sum_{t=0}^{T} \tilde{\omega}_{t}^{i}\left(s_{0}\right)=1$, and that $\int \tilde{\omega}^{i}\left(s_{0}\right) d G(i)=1$. Finally, note that the welfare assessment of a normalized utilitarian planner can be computed, respecting the units of $\frac{dW^{NU}(s_0)}{d\theta}$, as follows:

$$\begin{aligned}
\frac{d W^{N U}\left(s_{0}\right)}{d \theta} & =\frac{\frac{d W^{U}\left(s_{0}\right)}{d \theta}}{\int \lambda_{i} \sum_{t=0}^{T}\left(\beta_{i}\right)^{t} \sum_{s^{t}} \pi_{t}\left(s^{t} \mid s_{0}\right) \frac{\partial u_{i}\left(s^{t}\right)}{\partial c_{t}^{i}} d G(i)} \\
& =\frac{\int \lambda_{i} \frac{d V_{i}^{U}\left(s_{0}\right)}{d \theta} d G(i)}{\int \lambda_{i} \sum_{t=0}^{T}\left(\beta_{i}\right)^{t} \sum_{s^{t}} \pi_{t}\left(s^{t} \mid s_{0}\right) \frac{\partial u_{i}\left(s^{t}\right)}{\partial c_{t}^{i}} d G(i)},
\end{aligned}$$

where this expression is valid regardless of the assumed individual multiplicative decomposition of DS-weights. This concludes the proof.

b) Follows from Proposition 3. □

Proof of Proposition 10. (Properties of normalized utilitarian planner: Complete markets)

Proof. In any economy in which markets are complete, marginal rates of substitutions across all dates and histories are equalized across agents. Formally, in that case

$$\frac{\left(\beta_{i}\right)^{t} \pi_{t}\left(s^{t} \mid s_{0}\right) \frac{\partial u_{i}\left(s^{t}\right)}{\partial c_{t}^{i}}}{\frac{\partial u_{i}\left(s^{0}\right)}{\partial c_{0}^{i}}}=\frac{\left(\beta_{j}\right)^{t} \pi_{t}\left(s^{t} \mid s_{0}\right) \frac{\partial u_{j}\left(s^{t}\right)}{\partial c_{t}^{j}}}{\frac{\partial u_{j}\left(s^{0}\right)}{\partial c_{0}^{j}}}, \forall t, s^{t} ; \forall i, j. \tag{OA2}$$

Equation (OA2) implies that

$$\frac{\left(\beta_{i}\right)^{t} \pi_{t}\left(s^{t} \mid s_{0}\right) \frac{\partial u_{i}\left(s^{t}\right)}{\partial c_{t}^{i}}}{\left(\beta_{j}\right)^{t} \pi_{t}\left(s^{t} \mid s_{0}\right) \frac{\partial u_{j}\left(s^{t}\right)}{\partial c_{t}^{j}}}=\underbrace{\frac{\frac{\partial u_{i}\left(s^{0}\right)}{\partial c_{0}^{i}}}{\frac{\partial u_{j}\left(s^{0}\right)}{\partial c_{0}^{j}}}}_{=\bar{\alpha}\left(s^{0}\right)}, \forall t, s^{t} ; \forall i, j,$$

which in turn implies that $\left(\beta_{i}\right)^{t} \pi_{t}\left(s^{t} \mid s_{0}\right) \frac{\partial u_{i}\left(s^{t}\right)}{\partial c_{t}^{i}}$ is proportional across histories for all individuals. This fact implies that $\tilde{\omega}_{t}^{i}\left(s^{t} \mid s_{0}\right)$ and $\tilde{\omega}_{t}^{i}\left(s_{0}\right)$ do not vary across individuals, since

$$\begin{aligned}
\tilde{\omega}_{t}^{i}\left(s^{t} \mid s_{0}\right) & =\frac{\left(\beta_{i}\right)^{t} \pi_{t}\left(s^{t} \mid s_{0}\right) \frac{\partial u_{j}\left(s^{t}\right)}{\partial c_{t}^{j}}}{\left(\beta_{i}\right)^{t} \sum_{s^{t}} \pi_{t}\left(s^{t} \mid s_{0}\right) \frac{\partial u_{j}\left(s^{t}\right)}{\partial c_{t}^{j}}}=\frac{\bar{\alpha}\left(s^{0}\right) \pi_{t}\left(s^{t} \mid s_{0}\right)\left(\beta_{j}\right)^{t} \frac{\partial u_{j}\left(s^{t}\right)}{\partial c_{t}^{j}}}{\sum_{s^{t}} \pi_{t}\left(s^{t} \mid s_{0}\right) \bar{\alpha}\left(s^{0}\right)\left(\beta_{j}\right)^{t} \frac{\partial u_{j}\left(s^{t}\right)}{\partial c_{t}^{j}}}=\tilde{\omega}_{t}^{j}\left(s^{t} \mid s_{0}\right) \\
\tilde{\omega}_{t}^{i}\left(s_{0}\right) & =\frac{\left(\beta_{i}\right)^{t} \sum_{s^{t}} \pi_{t}\left(s^{t} \mid s_{0}\right) \frac{\partial u_{i}\left(s^{t}\right)}{\partial c_{t}^{i}}}{\sum_{t=0}^{T}\left(\beta_{i}\right)^{t} \sum_{s^{t}} \pi_{t}\left(s^{t} \mid s_{0}\right) \frac{\partial u_{i}\left(s^{t}\right)}{\partial c_{t}^{i}}}=\frac{\sum_{s^{t}} \bar{\alpha}\left(s^{0}\right) \pi_{t}\left(s^{t} \mid s_{0}\right)\left(\beta_{j}\right)^{t} \frac{\partial u_{j}\left(s^{t}\right)}{\partial c_{t}^{j}}}{\sum_{t=0}^{T}\left(\beta_{i}\right)^{t} \sum_{s^{t}} \pi_{t}\left(s^{t} \mid s_{0}\right) \bar{\alpha}\left(s^{0}\right)\left(\beta_{j}\right)^{t} \frac{\partial u_{j}\left(s^{t}\right)}{\partial c_{t}^{j}}}=\tilde{\omega}_{t}^{j}\left(s_{0}\right).
\end{aligned}$$

The fact that $\tilde{\omega}_t^i\left(s^t\middle|s_0\right)$ and $\tilde{\omega}_t^i\left(s_0\right)$ do not vary across individuals, combined with Proposition 3, generates the result. Note that it follows immediately that every single term in Equations (16) and (17) is also zero. □

Proof of Proposition 11 (Properties of aggregate efficiency/no-sharing/no-redistribution planners)

Proof. a) This part follows from Proposition 3, since $\tilde{\omega}_t^i\left(s^t\middle|s_0\right)$, $\tilde{\omega}_t^i\left(s_0\right)$, $\tilde{\omega}^i\left(s_0\right)$ and do not vary across individuals.
b) This part follows from Proposition 3, since $\tilde{\omega}_t^i\left(s^t\middle|s_0\right)$ and $\tilde{\omega}_t^i\left(s_0\right)$ do not vary across individuals.
c) This part follows from Proposition 3, since the individual components $\tilde{\omega}^i\left(s_0\right)$ do not vary across individuals. □

Note that we can explicitly express $\mathbb{E}_i\left[\tilde{\omega}_t^{i,NU}\left(s_0\right)\right]$ and $\mathbb{E}_i\left[\tilde{\omega}_t^{i,NU}\left(s^t\middle|s_0\right)\right]$ as follows

$$\mathbb{E}_i\left[\tilde{\omega}_t^{i,NU}\left(s_0\right)\right]=\int\frac{\left(\beta_i\right)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}{\sum_{t=0}^T\left(\beta_i\right)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}dG\left(i\right)$$

$$\mathbb{E}_i\left[\tilde{\omega}_t^{i,NU}\left(s^t\middle|s_0\right)\right]=\int\frac{\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}{\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}dG\left(i\right),$$

which are relevant for the definitions of the aggregate efficiency and the no-sharing DS-planners.

Paternalistic DS-planners/Heterogeneous beliefs From Equation (35), the claim that for any non-paternalistic planner it must be that $\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\propto\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}$ and $\tilde{\omega}_t^i\left(s_0\right)\propto\left(\beta_i\right)^t$ follows immediately.

Note that if we start from Equation (26), a non-paternalistic utilitarian planner finds a policy change desirable if and only if $\frac{dW^U\left(s_0\right)}{d\theta}>0$, where

$$\frac{dW^U\left(s_0\right)}{d\theta}=\int\lambda_i\frac{dV_i\left(s_0\right)}{d\theta}dG\left(i\right)=\int\lambda_i\sum_{t=0}^T\left(\beta_i\right)^t\sum_{s^t}\pi_t^i\left(s^t\middle|s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\frac{du_{i|c}\left(s^t\right)}{d\theta}dG\left(i\right),\tag{OA3}$$

which is the counterpart of Equation (4) for the common beliefs case.

Recursive formulation We can express $\frac{dV_i^{DS}\left(s_0\right)}{d\theta}$, defined in Equation (8), as follows:

$$\begin{aligned}\frac{dV_i^{DS}\left(s_0\right)}{d\theta}&=\sum_{t=0}^T\sum_{s^t}\tilde{\omega}_t^i\left(s_0\right)\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\\&=\tilde{\omega}_0^i\left(s_0\right)\tilde{\omega}_0^i\left(s^0\middle|s_0\right)\frac{du_{i|c}\left(s^0\right)}{d\theta}+\sum_{t=1}^T\sum_{s^t}\tilde{\omega}_t^i\left(s_0\right)\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\\&=\tilde{\omega}_0^i\left(s_0\right)\tilde{\omega}_0^i\left(s^0\middle|s_0\right)\frac{du_{i|c}\left(s^0\right)}{d\theta}\\&+\tilde{\omega}_1^i\left(s_0\right)\left(\sum_{s^1}\pi_1^i\left(s^t\middle|s_0\right)\frac{\tilde{\omega}_1^i\left(s^1\middle|s_0\right)}{\pi_1\left(s^t\middle|s_0\right)}\frac{du_{i|c}\left(s^1\right)}{d\theta}+\sum_{t=2}^T\sum_{s^t|s^1}\frac{\tilde{\omega}_t^i\left(s_0\right)}{\tilde{\omega}_1^i\left(s_0\right)}\frac{\pi_t\left(s^t\middle|s_0\right)}{\pi_1\left(s^1\middle|s_0\right)}\frac{\tilde{\omega}_t^i\left(s^t\middle|s_0\right)}{\pi_t\left(s^t\middle|s_0\right)}\frac{du_{i|c}\left(s^t\right)}{d\theta}\right)\end{aligned}$$

Hence, from this equation it is possible to derive sufficient conditions on DS-weights and $\pi_t^i\left(s^t\middle|\,s_0\right)$ to generate a recursive representation. In particular, note that whenever: i) $\tilde{\omega}_0^i\left(s_0\right)=1$ and $\pi_0\left(s^0|s_0\right)=1$, $\forall s_0$; ii) $\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\propto\pi_t\left(s^t\middle|\,s_0\right)$, so $\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)=\pi_t\left(s^t\middle|\,s_0\right)\tilde{\tilde{\omega}}_t^i\left(s^t\middle|\,s_0\right)$, where $\tilde{\tilde{\omega}}_t^i\left(s^t\middle|\,s_0\right)$ is not a function of s_0, only of s_t; iii) $\frac{\tilde{\omega}_t^i(s_0)}{\tilde{\omega}_1^i(s_0)}$ are independent of s_0; and iv) $\pi_t\left(s^t\middle|\,s_0\right)$ has a Markov structure, then $\frac{dV_i^{DS}(s)}{d\theta}$ satisfies a recursive functional equation:

$$\frac{dV_i^{DS}\left(s\right)}{d\theta}=\tilde{\tilde{\omega}}_0^i\left(s\right)\frac{du_{i|c}\left(s\right)}{d\theta}+\tilde{\omega}_1^i\left(s\right)\sum_{s'}\pi\left(s'|s\right)\frac{dV_i^{DS}\left(s'\right)}{d\theta}.$$

Social Welfare Function representation For completeness, we provide here formal definitions of i) a conventional linear social welfare function, which we refer to as a lifetime social welfare function, and ii) an instantaneous social welfare function.[32]

Definition 7. *(Lifetime Social Welfare Function) A linear lifetime Social Welfare Function, $SWF\left(\theta\right)$, is a function that takes individual lifetime utilities as inputs, with the following form:*

$$SWF\left(\theta\right)=\int\lambda_iV_i\left(\theta\right)dG\left(i\right),\tag{OA4}$$

where $V_i\left(\theta\right)$ is defined in Equation (1).

Definition 8. *(Instantaneous Social Welfare Function) A linear Instantaneous Social Welfare Function. $ISWF\left(\theta\right)$, is a function that takes individual instantaneous utilities as inputs, with the following form:*

$$ISWF\left(\theta\right)=\int\sum_{t=0}^{T}\sum_{s^t}\lambda_t^i\left(s^t\right)u_i\left(c_t^i\left(s^t\right),n_t^i\left(s^t\right)\right)dG\left(i\right),\tag{OA5}$$

where the instantaneous Pareto weight $\lambda_t^i\left(s^t\right)$ defines a scalar that is individual, time, and history-specific, and where $u_i\left(c_t^i\left(s^t\right),n_t^i\left(s^t\right)\right)$ is defined in Equation (1).

Proof of Proposition 12 (Instantaneous Social Welfare Function formulation) Proposition 12 in the paper can seen as a generalization of Proposition 4 in Saez and Stantcheva (2016). Note that

$$\frac{dISWF\left(\theta\right)}{d\theta}=\int\sum_{t=0}^{T}\sum_{s^t}\lambda_t^i\left(s^t\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\frac{du_{i|c}\left(s^t\right)}{d\theta}dG\left(i\right),\tag{OA6}$$

where $\frac{du_{i|c}\left(s^t\right)}{d\theta}$ is defined in Equation (3). The result for both the marginal welfare assessment and the optimum follows immediately by comparing Equation (5) to Equation (OA6), where the following relation must be satisfied:

$$\lambda_t^i\left(s^t\right)=\frac{\omega_t^i\left(s^t\right)}{\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}.$$

[32] One could define a Social Welfare Function that takes $c_t^i\left(s^t\right)$ and $n_t^i\left(s^t\right)$ directly as arguments, removing the dependence of the instantaneous utility function. It may be worth exploring that approach further.

D Application 1: Proofs, Derivations, and Additional Results

D.1 Proofs and derivations: Common environment

The results derived here are valid across all scenarios. At all times, we use $\odot$ to denote a Hadamard product and $\oslash$ to denote a Hadamard division.

Welfare assessments It is be useful to define the marginal value of a permanent transfer to individual i by $\mathrm{pmu}_i(s)$. Formally, $\mathrm{pmu}_i(s)$ is given by

$$\mathrm{pmu}_i(s) = \sum_{t=0}^{T} \sum_{s^t} \beta^t \pi_t\left(s^t | s\right) u_i'\left(c_t^i\left(s^t\right)\right) = u_i'\left(c^i(s)\right) + \beta \sum_{s'} \pi\left(s' | s\right) \mathrm{pmu}_i\left(s'\right).$$

In vector form, we can write a $S \times 1$ vector of $\mathrm{pmu}_i(s)$ as $\boldsymbol{pmu}_i = \begin{pmatrix} \vdots \\ \mathrm{pmu}_i(s) \\ \vdots \end{pmatrix}$, where

$$\boldsymbol{pmu}_i = \left(I_S - \beta \Pi\right)^{-1} \boldsymbol{u}_i'.$$

For any DS-planner, we can reformulate Equation (8) recursively, starting from a state s, as follows:

$$\frac{dW^{DS}(s)}{d\theta} = \int \tilde{\omega}^i(s) \underbrace{\sum_{t=0}^{T} \tilde{\omega}_t^i(s) \sum_{s^t} \tilde{\omega}_t^i\left(s^t \middle| s\right) \frac{du_{i|c}\left(s^t\right)}{d\theta}}_{= \frac{dV_i^{DS}(s)}{d\theta}} dG(i),$$

where $\frac{du_{i|c}(s)}{d\theta} = T^i(s)$. In the case of the normalized utilitarian planner, it is easy to see (following Equations (27) and (OA1)) that $\frac{dV_i^{DS}(s)}{d\theta}$ can be computed in vector form as follows:

$$\frac{d\boldsymbol{V}_i^{DS}(s)}{d\theta} = \left(\left(I_S - \beta \Pi\right)^{-1} \left(\boldsymbol{u}_i' \odot \boldsymbol{T}^i\right)\right) \oslash \boldsymbol{pmu}_i, \tag{OA7}$$

where $\frac{d\boldsymbol{V}_i^{DS}(s)}{d\theta} = \begin{pmatrix} \vdots \\ \frac{dV_i^{DS}(s)}{d\theta} \\ \vdots \end{pmatrix}$ denotes a $S \times 1$ vector, I_S is an S-dimensional identity matrix, $\boldsymbol{u}_i' = \begin{pmatrix} \vdots \\ u_i'\left(c^i(s)\right) \\ \vdots \end{pmatrix}$ denotes a $S \times 1$ vector of marginal utilities across states, and $\boldsymbol{T}^i = \begin{pmatrix} \vdots \\ T^i(s) \\ \vdots \end{pmatrix}$ denotes a $S \times 1$ vector of transfers across states. The discount factor β and the transition matrix Π are defined in the text. For given individual components, expressed in vector form $\tilde{\boldsymbol{\omega}}^i(s)$, as described below, we can then write

$$\frac{dW^{DS}(s)}{d\theta} = \int \tilde{\boldsymbol{\omega}}^i(s) \odot \frac{d\boldsymbol{V}_i^{DS}(s)}{d\theta} dG(i).$$

Multiplicative decomposition We can now compute each of the elements of the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner. In that case, the individual component $\tilde{\omega}^i(s)$, expressed as an $S \times 1$ vector depending on the state in which the welfare assessment takes place, is simply given by

$$\tilde{\omega}^i(s) = \frac{\sum_{t=0}^{T} \sum_{s^t} \beta^t \pi_t(s^t|s) u_i'(c_t^i(s^t))}{\int \sum_{t=0}^{T} \sum_{s^t} \beta^t \pi_t(s^t|s) u_i'(c_t^i(s^t)) dG(i)} \Rightarrow \tilde{\omega}^i(s) = \boldsymbol{pmu}_i \oslash \int \boldsymbol{pmu}_i dG(i),$$

where use $\oslash$ to denote a Hadamard division and where $\int \boldsymbol{pmu}_i dG(i)$ simple denotes a sum of the vector $\boldsymbol{pmu}_i$ across agents.

Next, the dynamic component for time t, $\tilde{\boldsymbol{\omega}}_t^i(s)$, expressed as an $S \times 1$ vector depending on the state in which the welfare assessment takes place, is given by

$$\tilde{\omega}_t^i(s) = \frac{\sum_{s^t} \beta^t \pi_t(s^t|s) u_i'(c_t^i(s^t))}{\sum_{t=0}^{T} \sum_{s^t} \beta^t \pi_t(s^t|s) u_i'(c_t^i(s^t))} \Rightarrow \tilde{\boldsymbol{\omega}}_t^i(s) = \left(\beta^t \Pi^t \boldsymbol{u}_i'\right) \oslash \boldsymbol{pmu}_i.$$

Finally, the stochastic component for time t, $\tilde{\boldsymbol{\omega}}_t^i(s^t|s_0)$, expressed as an $S \times S$ matrix, with the rows indicating the initial state and the columns indicating the final state at time t, is given by

$$\tilde{\boldsymbol{\omega}}_t^i(s^t|s) = \frac{\pi_t(s^t|s) u_i'(c_t^i(s^t))}{\sum_{s^t} \pi_t(s^t|s) u_i'(c_t^i(s^t))} \Rightarrow \tilde{\boldsymbol{\omega}}_t^i(s^t|s) = \left(\Pi^t \odot \boldsymbol{\iota}_S (\boldsymbol{u}_i')^{tr}\right) \oslash \left(\Pi^t \boldsymbol{u}_i' (\boldsymbol{\iota}_S)^{tr}\right),$$

where $\boldsymbol{\iota}_S$ is a unit vector of dimension $S \times 1$.

D.2 Scenario 1: Varying the persistence of endowment process

Figures OA-1 and OA-2 are the counterparts of Figures 1 and 2 in the text when $\rho = 1$. In this case, there is no uncertainty, so the components of the individual multiplicative decompositions are time-invariant. Given the lack of uncertainty, all of the welfare gains from increasing θ arise from redistribution (Ξ_4).

Figures OA-3 and OA-4 are the counterparts of Figures 1 and 2 in the text when $\rho = 0.5$. In this case, endowments shocks are fully temporary is no uncertainty, and all welfare gains arise from redistribution, so the initial state exclusively determines the $t = 0$ components of the individual multiplicative decompositions, although qualitatively the results are the same as in the benchmark case with $\rho = 0.95$. In this case, most of the welfare gains from increasing θ arise from smoothing considerations, mostly risk-sharing but also intertemporal-sharing to a lesser degree. The gains from redistribution are nonzero, but very small, since they are only driven by marginal utility differences at $t = 0$.

D.3 Scenario 3: Common preferences with aggregate uncertainty

Environment In this scenario, we assume i) that both individuals have identical preferences, so $\gamma_1 = \gamma_2 = \gamma$, and ii) that they exclusively face aggregate risk. Formally, we assume that

$$y^1(s) = \overline{y} + \varepsilon(s) \quad \text{and} \quad y^2(s) = \overline{y} + \varepsilon(s),$$

where $\overline{y} \geq 0$, and where $\varepsilon(L) = -\varepsilon(H)$. We consider the welfare assessment of a policy that reduces the size of aggregate fluctuations, so this scenario is the counterpart of the one in Lucas (1987) and Alvarez and Jermann (2004). Formally, we set $T^1(s) = -\varepsilon(s)$ and $T^2(s) = -\varepsilon(s)$, so individual consumption takes the

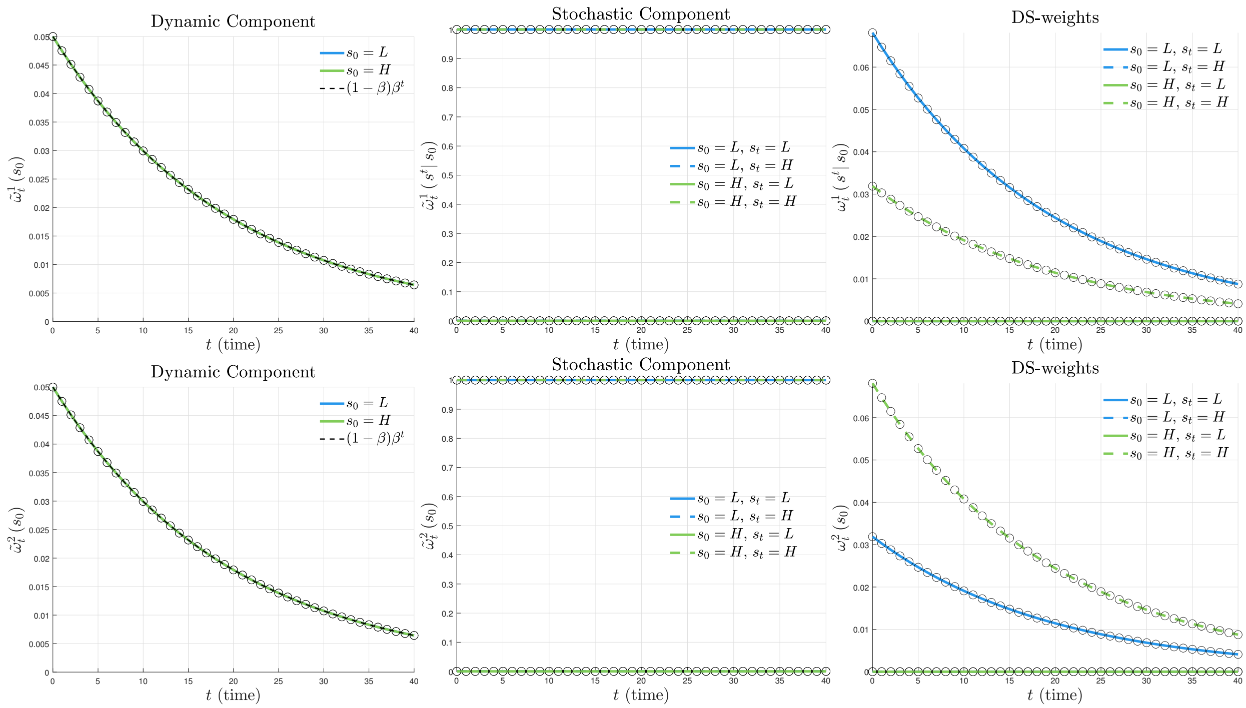

Figure OA-1: Individual multiplicative decomposition of DS-weights (Scenario 1; $\rho = 1$)

Note: Figure OA-1 is the counterpart of Figure 1 in the text when endowment shocks are fully persistent ($\rho = 1$). The individual component of DS-weights in this case are $\tilde{\omega}^1(s_0 = L) = 1.362$ and $\tilde{\omega}^2(s_0 = L) = 0.638$ when an assessment takes place at $s_0 = L$; and $\tilde{\omega}^1(s_0 = H) = 0.638$ and $\tilde{\omega}^2(s_0 = H) = 1.362$ when the assessment takes place at $s_0 = H$.

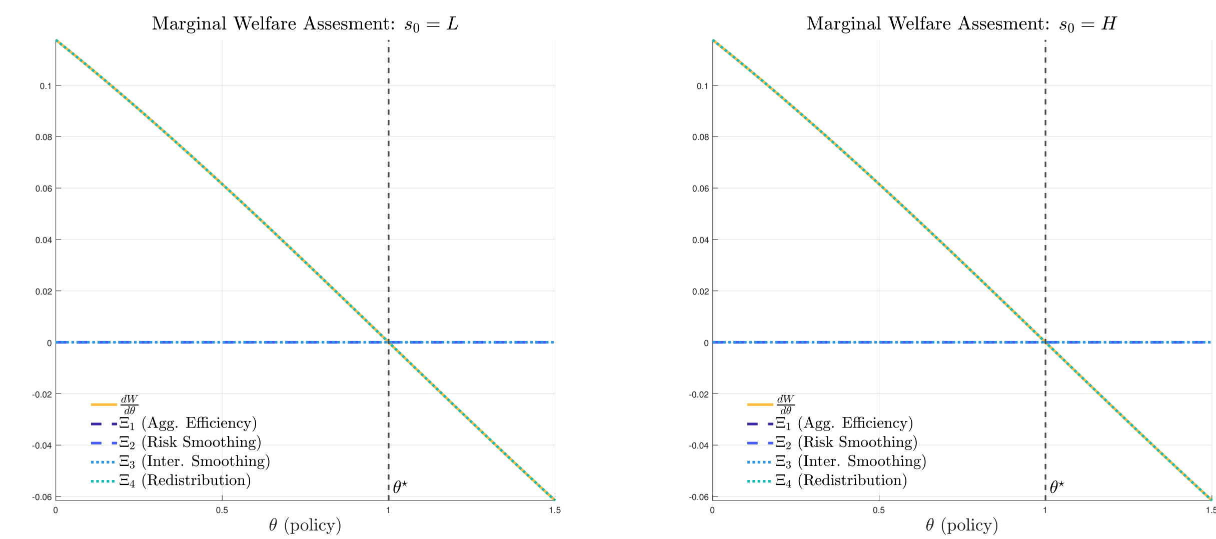

Figure OA-2: Aggregate additive decomposition of welfare assessments (Scenario 1; $\rho = 1$)

Note: Figure OA-2 is the counterpart of Figure 1 in the text when endowment shocks are fully persistent ($\rho = 1$). Note that $\Xi_1 = \Xi_2 = \Xi_3 = 0$, and that $\frac{dW}{d\theta} = \Xi_4$.

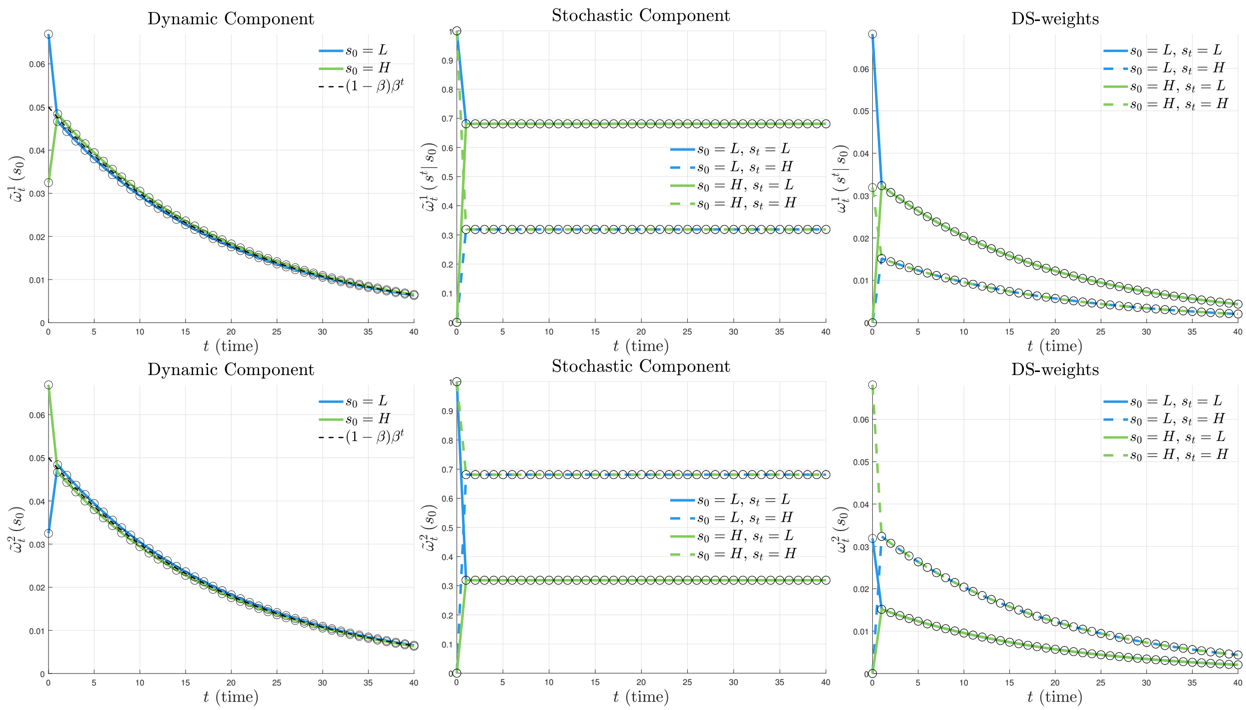

Figure OA-3: Individual multiplicative decomposition of DS-weights (Scenario 1; $\rho = 0.5$)

Note: Figure OA-3 is the counterpart of Figure 1 in the text when endowment shocks are fully temporary ($\rho = 0.5$). The individual component of DS-weights in this case are $\tilde{\omega}^1(s_0 = L) = 1.362$ and $\tilde{\omega}^2(s_0 = L) = 0.638$ when an assessment takes place at $s_0 = L$; and $\tilde{\omega}^1(s_0 = H) = 0.638$ and $\tilde{\omega}^2(s_0 = H) = 1.362$ when the assessment takes place at $s_0 = H$.

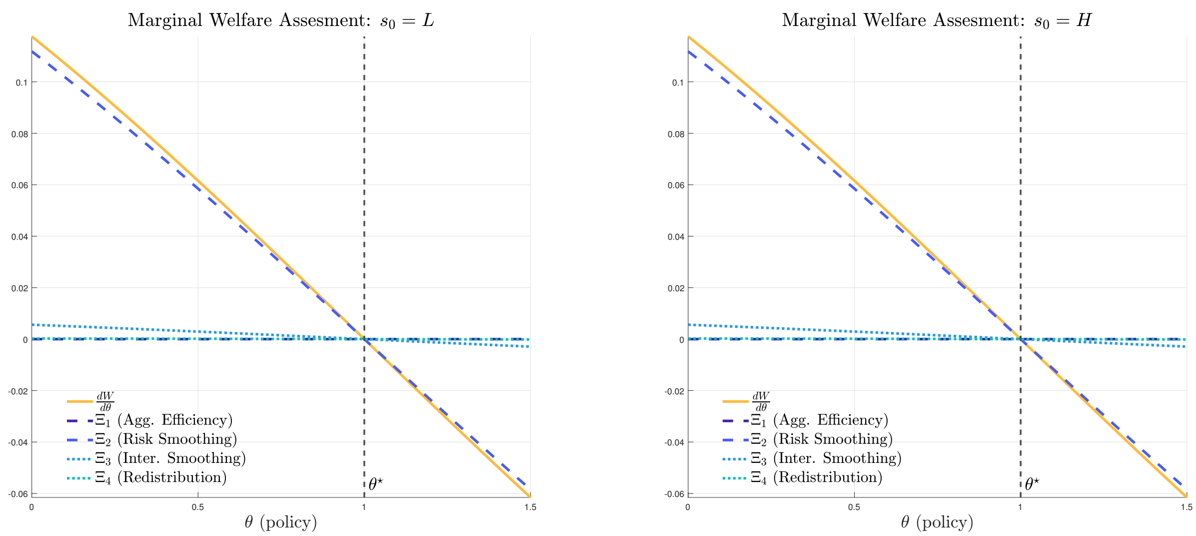

Figure OA-4: Aggregate additive decomposition of welfare assessments (Scenario 1; $\rho = 0.5$)

Note: Figure OA-4 is the counterpart of Figure 1 in the text when endowments shocks are fully temporary ($\rho = 0.5$).

form

$$c^1(s) = \overline{y} + \varepsilon(s)(1-\theta) \quad \text{and} \quad c^2(s) = \overline{y} + \varepsilon(s)(1-\theta).$$

Under this policy, when $\theta = 1$, there is no risk in the economy, and individual (and aggregate) consumption are fixed. Note that aggregate consumption in this scenario does depend on both s and θ since

$$\int c^i(s)\, dG(i) = \overline{y} + \varepsilon(s)(1-\theta).$$

Results We use the same parameters as in the benchmark parameterization of Scenario 1: $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, $\gamma_1 = \gamma_2 = 2$, and $\rho = 0.95$, so endowment shocks are moderately persistent. Once again, we start by describing the DS-weights used by a normalized utilitarian planner with $\lambda_i = 1$, as defined in Proposition 4.

Individual multiplicative decomposition of DS-weights. In Figure OA-5, we show the components of the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner for each of the individuals when $\theta = 0.25$. The central insights in this scenario is that the individual multiplicative decomposition of DS-weights for both individuals in this applications is identical to that for individual 1 in Scenario 1. This should not be surprising since the consumption path of individual 1 in Scenario 1 is identical to the consumption for both individuals in this scenario. As in Scenario 1 for individual 1, a normalized utilitarian planner i) overweights early periods in state $s_0 = L$ relative to $s_0 = H$, ii) overweights final periods $s_t = L$ for both individuals in this case. However, in this case the individuals components are 1 by construction, since both individuals are identical.

Aggregate additive decomposition of welfare assessments. In Figure OA-6, we show the components of the aggregate additive decomposition of welfare assessments for a normalized utilitarian planner. In this scenario, since all individuals have the same preferences, face aggregate uncertainty, and the policy consider affects them identical, it should be evident that all normative assessments are driven by aggregate efficiency considerations. It is worth highlighting that the optimal policy for a normalized utilitarian planner is very sensitive to the initial state. When $s_0 = L$, individuals in this economy are much poorer (and persistently so), so their willingness to pay for increasing θ captures the fact that this policy persistently increases their consumption. Alternatively, when they make welfare assessments starting at $s_0 = H$, they understand that increase θ permanently reduces their consumption in H states, which they find costly, despite the fact that an increase θ also reduces fluctuations.[33]

D.4 Scenario 4: Heterogeneous preferences with aggregate uncertainty

Environment In our fourth and final scenario, we consider a physical environment identical to the one in scenario 2, in which i) some individuals are more risk-averse/unwilling to substitute intertemporally than others, as in Scenario 2 and ii) all endowment risk is aggregate. Formally, we assume that we assume that individual 1 is more risk averse than individual 2, so $\gamma_1 > \gamma_2$, and that

$$y^1(s) = \overline{y} + \varepsilon(s) \quad \text{and} \quad y^2(s) = \overline{y} + \varepsilon(s),$$

[33] See our discussion in Footnote 24 for how one may want to trade off marginal assessments starting from different states.

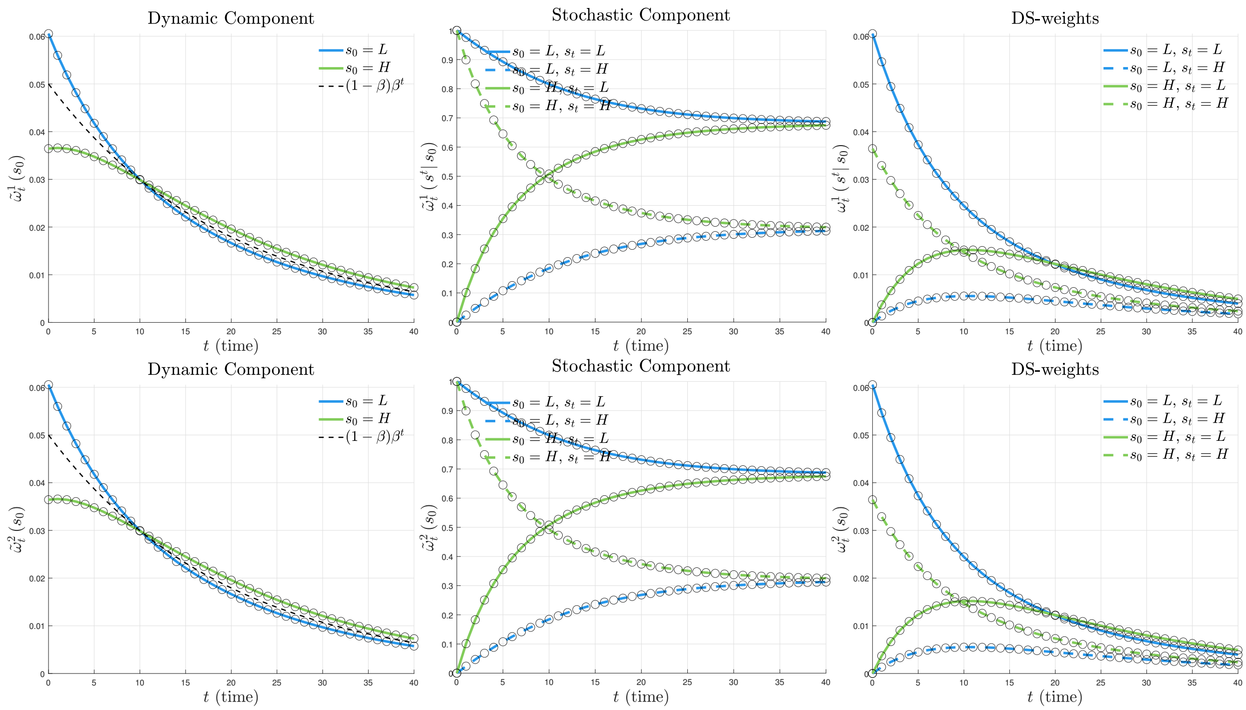

Figure OA-5: Individual multiplicative decomposition of DS-weights (Scenario 3)

Note: Figure OA-5 shows the components of the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner, defined in Proposition 4. We assume that $\theta = 0.25$, although all figures are qualitatively similar when $\theta \in [0, 1)$. The top row shows each of the components for individual 1, while the bottom row shows them for individual 2. The left plots show the dynamic component, $\tilde{\omega}_t^i(s_0)$, for different values of t for different initial states, $s_0 = \{H, L\}$. For reference, we also show the dynamic weight for a hypothetical individual with linear marginal utility, given by $(1-\beta)\beta^t = \beta^t / \sum_t \beta^t$. Note that the sum under each of the curves adds up to 1. The middle plots show the stochastic component, $\tilde{\omega}_t^i\left(s^t \middle| s_0\right)$, for different values of t, for different initial states, $s_0 = \{H, L\}$, and final states, $s_t = \{H, L\}$. The right plots show the actual DS-weights, $\omega_t^i\left(s^t \middle| s_0\right)$, also for different values of t, and different initial and final states: $s_0 = \{H, L\}$ and $s_t = \{H, L\}$. The parameters are $\theta = 0.25$, $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, $\rho = 0.95$, and $\gamma_1 = \gamma_2 = 2$. The individual component of DS-weights are $\tilde{\omega}^1(s_0 = L) = \tilde{\omega}^2(s_0 = L) = 1$ when an assessment takes place at $s_0 = L$; and $\tilde{\omega}^1(s_0 = H) = \tilde{\omega}^2(s_0 = H) = 1$ when the assessment takes place at $s_0 = H$.

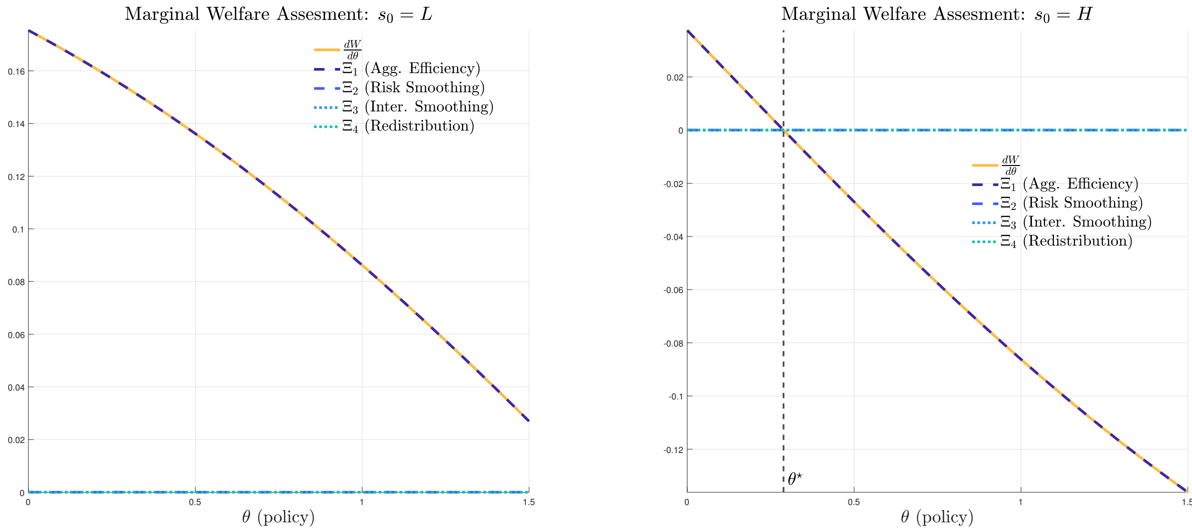

Figure OA-6: Aggregate additive decomposition of welfare assessments (Scenario 3)

Note: Figure 4 shows the marginal welfare assessment of a normalized utilitarian planner, $\frac{dW}{d\theta}$, and the components of its aggregate additive decomposition, as defined in Proposition 4. The left plot corresponds to the assessment when $s_0 = L$, while the right plot corresponds to the assessments when $s_0 = H$. The dashed and dotted lines are computed as described in Equation (10), where the DS-weights are computed as in Equations (18), (19), and (20). In this scenario, $\frac{dW}{d\theta} = \Xi_1$, while $\Xi_2 = \Xi_3 = \Xi_4 = 0$.

where $\overline{y} \geq 0$, and and where $\varepsilon(L) = -\varepsilon(H)$. Unlike Scenario 2, now we consider a policy that reduces the size of aggregate fluctuations for both agents, as in Scenario 3. Hence, this scenario can be seen as a version of Lucas (1987) and Alvarez and Jermann (2004) with heterogeneous individuals.[34] Formally, we set $T^1(s) = -\varepsilon(s)$ and $T^2(s) = \varepsilon(s)$, so individual consumption takes the form

$$c^1(s) = \overline{y} + \varepsilon(s)(1-\theta) \quad \text{and} \quad c^2(s) = \overline{y} + \varepsilon(s)(1-\theta).$$

Under this policy, when $\theta = 1$, both individuals experience no consumption fluctuations. As in Scenario 3, aggregate consumption does depend on both s and θ since

$$\int c^i(s)\, dG(i) = \overline{y} + \varepsilon(s)(1-\theta).$$

Results We use the same parameters as in the benchmark parameterization of Scenario 1: $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, $\gamma_1 = \gamma_2 = 2$, and $\rho = 0.95$, so endowment shocks are moderately persistent. Once again, we start by describing the DS-weights used by a normalized utilitarian planner with $\lambda_i = 1$, as defined in Proposition 4.

Individual multiplicative decomposition of DS-weights. In Figure OA-5, we show the components of the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner for each of the individuals when $\theta = 0.25$. In this scenario, the shape of the components of the individual multiplicative decomposition of DS-weights is very similar to Scenario 2, with the insights discussed there carrying over to this scenario.

[34]There is scope to apply our approach to revisit the results of Atkeson and Phelan (1994), Krusell and Smith (1999), and Krusell et al. (2009).

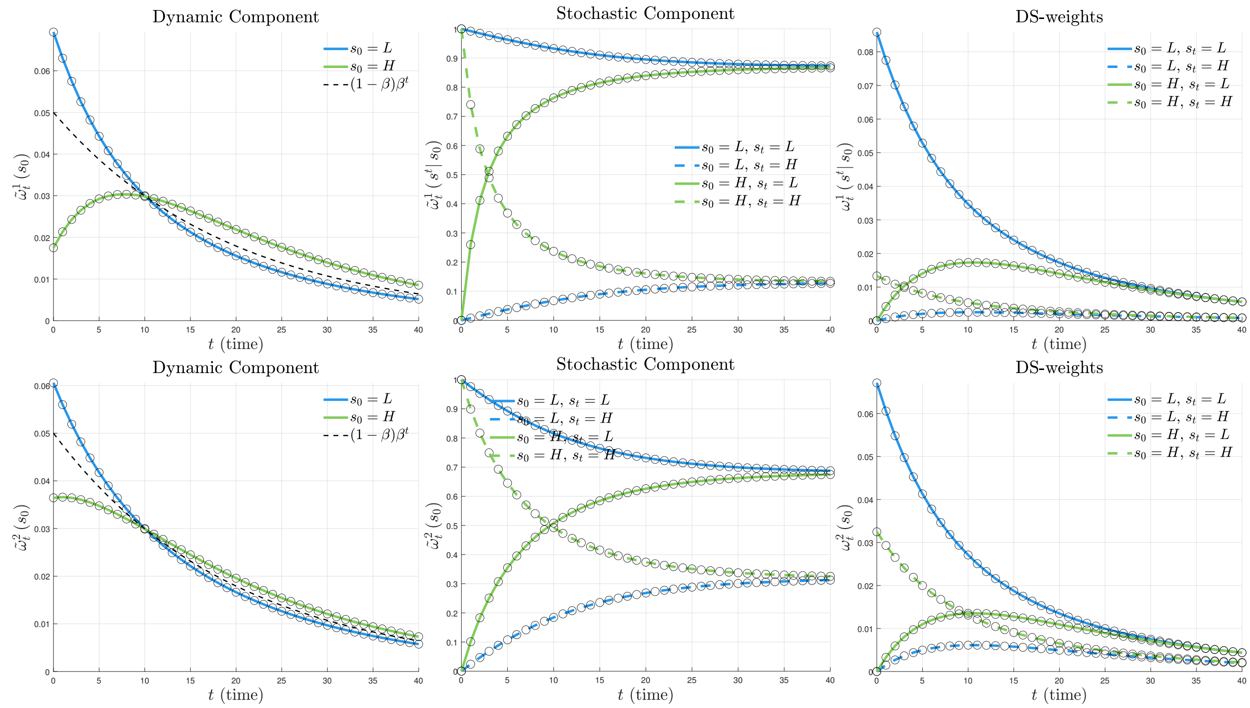

Figure OA-7: Individual multiplicative decomposition of DS-weights (Scenario 4)

Note: Figure OA-7 shows the components of the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner, defined in Proposition 4. We assume that $\theta = 0.25$, although all figures are qualitatively similar when $\theta \in [0, 1)$. The top row shows each of the components for individual 1, while the bottom row shows them for individual 2. The left plots show the dynamic component, $\tilde{\omega}_t^i(s_0)$, for different values of t for different initial states, $s_0 = \{H, L\}$. For reference, we also show the dynamic weight for a hypothetical individual with linear marginal utility, given by $(1-\beta)\beta^t = \beta^t / \sum_t \beta^t$. Note that the sum under each of the curves adds up to 1. The middle plots show the stochastic component, $\tilde{\omega}_t^i\left(s^t \middle| s_0\right)$, for different values of t, for different initial states, $s_0 = \{H, L\}$, and final states, $s_t = \{H, L\}$. The right plots show the actual DS-weights, $\omega_t^i\left(s^t \middle| s_0\right)$, also for different values of t, and different initial and final states: $s_0 = \{H, L\}$ and $s_t = \{H, L\}$. The parameters are $\theta = 0.25$, $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, $\rho = 0.95$, and $\gamma_1 = \gamma_2 = 2$. The individual component of DS-weights are $\tilde{\omega}^1(s_0 = L) = 1.239$ and $\tilde{\omega}^2(s_0 = L) = 0.761$ when an assessment takes place at $s_0 = L$; and $\tilde{\omega}^1(s_0 = H) = 1.108$ and $\tilde{\omega}^2(s_0 = H) = 0.892$ when the assessment takes place at $s_0 = H$.

Aggregate additive decomposition of welfare assessments. In Figure OA-8, we show the components of the aggregate additive decomposition of welfare assessments for a normalized utilitarian planner. Several insights emerge. First, from Equation (12), it is straightforward to conclude that the risk-sharing component is zero, that is, $\Xi_2 = 0$. This occurs because the policy consider is identical across individuals regardless of the aggregate state. Second, we find that the aggregate efficiency, the intertemporal smoothing, and the redistribution components are nonzero, although the intertemporal smoothing component is very small, as one would expect from Equation (13). Perhaps as expected, the aggregate efficiency component is the most important, since we are considering a policy that reduces aggregate risk. Interestingly, this scenario shows that the it is possible to attribute welfare assessments to a redistribution motive in models with purely aggregate shocks and policies that affect all agents symmetrically, since the valuation of such policies differs across individuals. In this case, the normalized planner gives more values to gains more risk averse agents (note that $\tilde{\omega}^1(s_0 = L) > 1$ and $\tilde{\omega}^1(s_0 = H) > 1$), so an increase in θ increases the utility of the more risk averse individuals by more, generating this result.

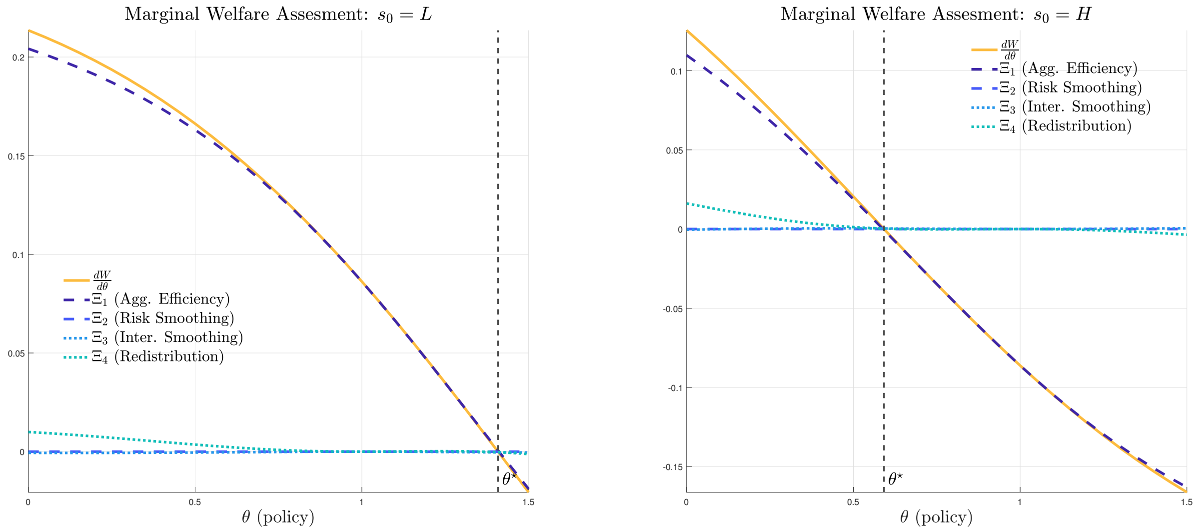

Figure OA-8: Aggregate additive decomposition of welfare assessments (Scenario 4)

Note: Figure OA-8 shows the marginal welfare assessment of a normalized utilitarian planner, $\frac{dW}{d\theta}$, and the components of its aggregate additive decomposition, as defined in Proposition 4. The left plot corresponds to the assessment when $s_0 = L$, while the right plot corresponds to the assessments when $s_0 = H$. The dashed and dotted lines are computed as described in Equation (10), where the DS-weights are computed as in Equations (18), (19), and (20). In this scenario, $\Xi_2 = 0$.

D.5 Scenarios 1 through 4: Marginal Welfare Effects

Figure OA-9 shows the individual marginal welfare effects of a policy change from the perspective of a normalized utilitarian planner. It highlights the fact that, with the exception of Scenario 3, in all other scenarios most policy changes are not Pareto efficient. For instance, in Scenario 1, individual 1 is better off when a policy change at $\theta = 0$ when $s = L$ but individual 2 is worse off.

E Additional Extensions and Results

E.1 Dimensional analysis

Normalized utilitarian planner As we discuss in the text, the units of our formulation of DS-weights for the case of the normalized utilitarian planner introduced in Section 4 have a clear interpretation in terms of dollars at different dates and histories. Here, we provide a systematic dimensional analysis (de Jong, 1967) of the welfare assessments made by a normalized utilitarian planner. We denote the units of a specific variable by $\dim(\cdot)$, where, for instance, $\dim\left(c_t^i\left(s^t\right)\right)$ = dollars at history s^t ,where we interchangeably use dollars and units of consumption good.

First, note that the units of $\tilde{\omega}_t^i\left(s^t \middle| s_0\right)$, $\tilde{\omega}_t^i\left(s_0\right)$, and $\tilde{\omega}^i\left(s_0\right)$, as defined in Equations (18), (19), and (20)

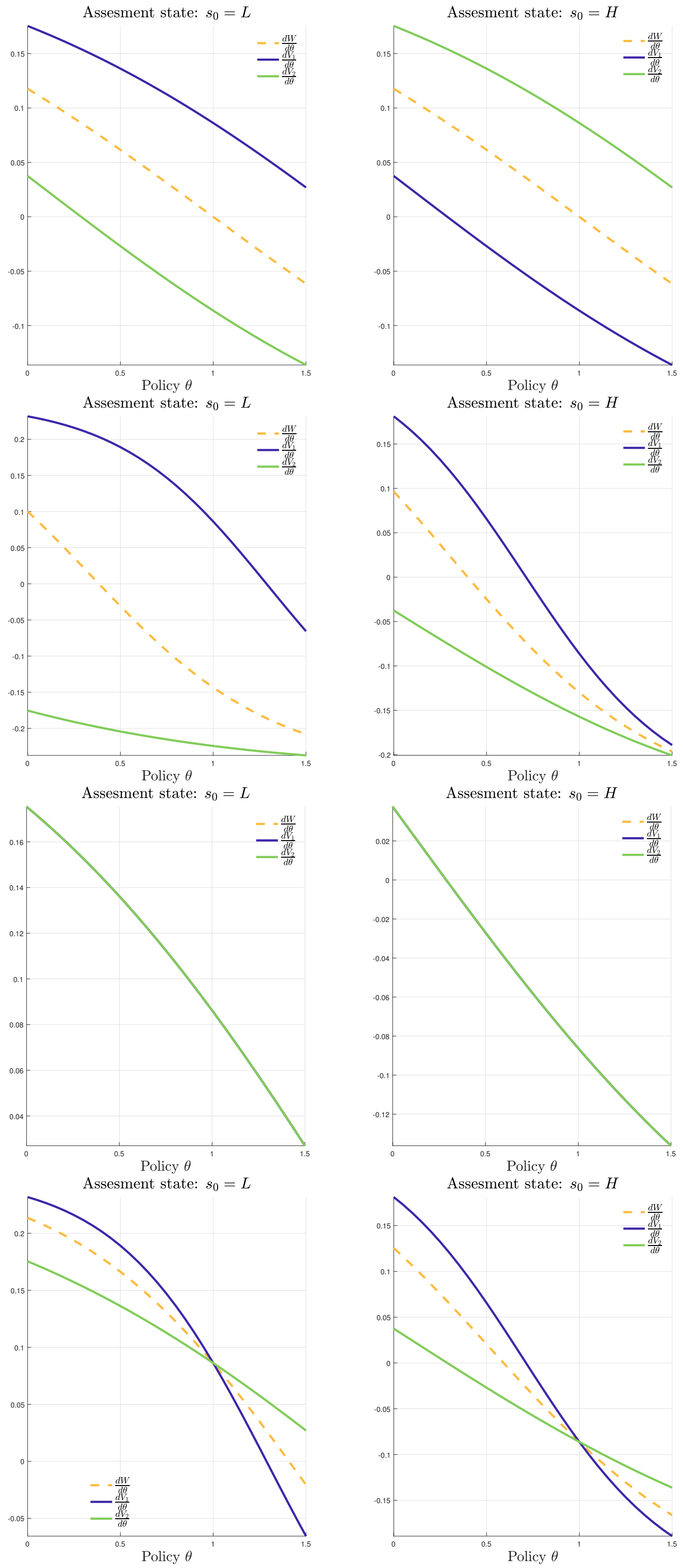

Figure OA-9: Individual marginal welfare effects of policies (all scenarios)

Note: Figure OA-9 shows individual marginal welfare assessments from the perspective of a normalized utilitarian planner for all four scenarios studied in Application 1. The left plots correspond to assessments in state $s_0 = L$, while the right plots correspond to the assessments in state $s_0 = H$. The top row corresponds to Scenario 1, the next row to Scenario 2, the next row to Scenario 3, and the bottom row to Scenario 4.

are respectively given by

$$\dim\left(\tilde{\omega}_t^{i,NU}\left(s^t\middle|\,s_0\right)\right)=\frac{\frac{\text{instantaneous utils at } s_0}{\text{dollars at history } s^t}}{\frac{\text{instantaneous utils at } s_0}{\text{dollars at date } t}}=\frac{\text{dollars at date } t}{\text{dollars at history } s^t};$$
$$\dim\left(\tilde{\omega}_t^{i,NU}\left(s_0\right)\right)=\frac{\frac{\text{instantaneous utils at } s_0}{\text{dollars at date } t}}{\frac{\text{instantaneous utils at } s_0}{\text{dollars at all dates and histories}}}=\frac{\text{dollars at all dates and histories}}{\text{dollars at date } t};$$
$$\begin{aligned}\dim\left(\tilde{\omega}^{i,NU}\left(s_0\right)\right)&=\frac{\frac{\text{instantaneous utils at } s_0}{\text{dollars at all dates and histories}}}{\frac{\text{instantaneous utils at } s_0}{\text{dollars at all dates and histories for all individuals}}}\\&=\frac{\text{dollars at all dates and histories for all individuals}}{\text{dollars at all dates and histories}}.\end{aligned}$$

In particular, the term $(\beta_i)^t\,\pi_t\left(s^t\middle|\,s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}$, which defines the numerator of $\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)$, is measured in instantaneous utils at s_0 per dollars at history s^t for individual i, since

$$\dim\left((\beta_i)^t\right)=\frac{\text{instantaneous utils at } s_0}{\text{instantaneous utils at history} s^t}$$
$$\dim\left(\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\right)=\frac{\text{instantaneous utils at history } s^t}{\text{dollars at history } s^t},$$

and probabilities, like $\pi_t\left(s^t\middle|\,s_0\right)$, are unitless, where the same logic applies to the remaining elements of $\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)$, $\tilde{\omega}_t^i\left(s_0\right)$, and $\tilde{\omega}^i\left(s_0\right)$.

Consequently, it follows that

$$\begin{aligned}\dim\left(\omega_t^i\left(s^t\middle|\,s_0\right)\right)&=\dim\left(\tilde{\omega}^i\left(s_0\right)\tilde{\omega}_t^i\left(s_0\right)\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\right)\\&=\frac{\text{dollars at all dates and histories for all individuals}}{\text{dollars at history } s^t \text{ for individual } i}.\end{aligned}\tag{OA8}$$

Second, note also that the units of $\frac{du_{i|c}(s^t)}{d\theta}$ are given by

$$\dim\left(\frac{du_{i|c}\left(s^t\right)}{d\theta}\right)=\frac{\frac{\text{instantaneous utils at history } s^t}{\text{unit of policy change}}}{\frac{\text{instantaneous utils at history } s^t}{\text{dollars at history } s^t \text{ for individual } i}}=\frac{\text{dollars at history } s^t \text{ for individual } i}{\text{unit of policy change}},\tag{OA9}$$

which follows directly from Equation (19).

Finally, combining Equations (OA8) and (OA9), it follows that

$$\dim\left(\frac{dW^{NU}\left(s_0\right)}{d\theta}\right)=\dim\left(\omega_t^i\left(s^t\middle|\,s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right)=\frac{\text{dollars at all dates and histories for all individuals}}{\text{unit of policy change}}.\tag{OA10}$$

Hence, the units of W^{NU} for a normalized utilitarian planner are dollars paid to all individuals at all dates and histories. That is, if $\frac{dW^{NU}}{d\theta}=537$ for a given policy change, the welfare gain associated with a marginal unit policy change is equivalent to paying 537 dollars at all dates and states to all individuals in the economy.

E.2 Relation to Lucas (1987)/Alvarez and Jermann (2004)

It is common in papers that study the welfare consequences of policies in dynamic and stochastic environments to compute welfare gains/losses of policies as in Lucas (1987), who measures the welfare gains associated with a policy change — specifically, the welfare gains associated with eliminating business cycles. Since our approach is built on marginal arguments, we connect instead our results to those in Alvarez and Jermann (2004), who provide a marginal formulation of the approach in Lucas (1987). While both Lucas (1987) and Alvarez and Jermann (2004) focus on aggregate consumption, others explore the cost of business cycles with rich heterogeneity, see e.g., Atkeson and Phelan (1994), Krusell and Smith (1999), or Krusell et al. (2009), among others. Here, we study a policy change for a given individual i, who could be a representative-agent or not.

Formally, here we consider a special case of the environment laid out in Section 3, in which an individual i has preferences given by

$$V_i\left(s_0\right)=\sum_{t=0}^{T}\left(\beta_i\right)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i\left(c_t^i\left(s^t\right)\right).$$

We suppose that the consumption of individual i at date t and history s^t can be written as

$$c_t^i\left(s^t\right)=\overline{c_t^i}\left(s^t\right)+\theta\overline{\Delta c_t^i}\left(s^t\right),$$

where both $\overline{c_t^i}\left(s^t\right)$ and $\overline{\Delta c_t^i}\left(s^t\right)$ are sequences measurable with respect to history s^t. The sequence $\overline{c_t^i}\left(s^t\right)$ can be interpreted as a given initial consumption path (when $\theta=0$) and the sequence $\overline{\Delta c_t^i}\left(s^t\right)$ can be interpreted as a final consumption path (when $\theta=1$). In the case of Lucas (1987), $\theta=1$ corresponds to eliminating business cycles.

First, we compute the marginal gains from marginally reducing business cycles, as in Alvarez and Jermann (2004). Next, we compute the marginal gains from marginally reducing business cycles using an additive compensation.

Multiplicative compensation Lucas (1987) proposes using a time-invariant compensating variation, expressed multiplicatively as a constant fraction of consumption at each date/history as follows

$$\sum_{t=0}^{T}\left(\beta_i\right)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i\left(c_t^i\left(s^t\right)\left(1+\lambda\left(\theta\right)\right)\right)=\sum_{t=0}^{T}\left(\beta_i\right)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i\left(\overline{c_t^i}\left(s^t\right)+\theta\overline{\Delta c_t^i}\left(s^t\right)\right),\quad\text{(OA11)}$$

where $\lambda\left(\theta\right)$ implicitly defines the welfare gains associated with a policy indexed by θ; the exact definition in Lucas (1987) exactly corresponds to solving for $\lambda\left(\theta=1\right)$.

Following Alvarez and Jermann (2004), we can compute the derivative of the RHS of Equation (OA11) as follows:

$$\frac{d\left(\left(\beta_i\right)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i\left(\overline{c_t^i}\left(s^t\right)+\theta\overline{\Delta c_t^i}\left(s^t\right)\right)\right)}{d\theta}=\sum_{t=0}^{T}\left(\beta_i\right)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i'\left(\overline{c_t^i}\left(s^t\right)+\theta\overline{\Delta c_t^i}\left(s^t\right)\right)\frac{dc_t^i\left(s^t\right)}{d\theta}\quad\text{(OA12)}$$

where here $\frac{dc_t^i\left(s^t\right)}{d\theta}=\overline{\Delta c_t^i}\left(s^t\right)$.

Analogously, we can also compute the derivative of the LHS of Equation (OA11) as follows:

$$\frac{d\left(\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i\left(c_t^i\left(s^t\right)(1+\lambda(\theta))\right)\right)}{d\theta}=\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i'\left(c_t^i\left(s^t\right)(1+\lambda(\theta))\right)c_t^i\left(s^t\right)\lambda'(\theta)\,. \tag{OA13}$$

Hence, combining Equations (OA12) and (OA13) and solving for $\frac{d\lambda}{d\theta}=\lambda'(\theta)$, yields the marginal cost of business cycles, as defined in Alvarez and Jermann (2004). Formally, we can express $\frac{d\lambda}{d\theta}$ as follows

$$\begin{aligned}\frac{d\lambda}{d\theta}=\lambda'(\theta)&=\frac{\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i'\left(\overline{c_t^i}\left(s^t\right)+\theta\overline{\Delta c_t^i}\left(s^t\right)\right)\frac{dc_t^i\left(s^t\right)}{d\theta}}{\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i'\left(c_t^i\left(s^t\right)(1+\lambda(\theta))\right)c_t^i\left(s^t\right)}\\&=\sum_{t=0}^{T}\sum_{s^t}\omega_t^i\left(s^t\middle|\,s_0\right)\frac{dc_t^i\left(s^t\right)}{d\theta},\end{aligned} \tag{OA14}$$

where the second line shows how to reformulate $\frac{d\lambda}{d\theta}$ in terms of DS-weights given by

$$\omega_t^i\left(s^t\middle|\,s_0\right)=\frac{(\beta_i)^t\pi_t\left(s^t\middle|\,s_0\right)u_i'\left(\overline{c_t^i}\left(s^t\right)+\theta\overline{\Delta c_t^i}\left(s^t\right)\right)}{\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i'\left(c_t^i\left(s^t\right)(1+\lambda(\theta))\right)c_t^i\left(s^t\right)}. \tag{OA15}$$

Additive compensation Here, we would like to contrast the approach in Lucas (1987), to one that relies on a time-invariant compensating variation, expressed additively in terms of consumption at each date/history as follows

$$\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i\left(c_t^i\left(s^t\right)+\lambda(\theta)\right)=\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i\left(\overline{c_t^i}\left(s^t\right)+\theta\overline{\Delta c_t^i}\left(s^t\right)\right). \tag{OA16}$$

In this case, we can follow the same steps as above to find the counterpart of Equation (OA14), which is given by

$$\begin{aligned}\frac{d\lambda}{d\theta}=\lambda'(\theta)&=\frac{\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i'\left(\overline{c_t^i}\left(s^t\right)+\theta\overline{\Delta c_t^i}\left(s^t\right)\right)\frac{dc_t^i\left(s^t\right)}{d\theta}}{\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i'\left(c_t^i\left(s^t\right)+\lambda(\theta)\right)c_t^i\left(s^t\right)}\\&=\sum_{t=0}^{T}\sum_{s^t}\omega_t^i\left(s^t\middle|\,s_0\right)\frac{dc_t^i\left(s^t\right)}{d\theta},\end{aligned} \tag{OA17}$$

where the second line shows how to reformulate $\frac{d\lambda}{d\theta}$ in terms of DS-weights given by

$$\omega_t^i\left(s^t\middle|\,s_0\right)=\frac{(\beta_i)^t\pi_t\left(s^t\middle|\,s_0\right)u_i'\left(\overline{c_t^i}\left(s^t\right)+\theta\overline{\Delta c_t^i}\left(s^t\right)\right)}{\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i'\left(c_t^i\left(s^t\right)+\lambda(\theta)\right)}. \tag{OA18}$$

Comparison and implications We focus on comparing Equations (OA14) and (OA17) in the case of $\theta = 0$ — similar insights emerge when $\theta \neq 0$. When $\theta = 0$, Equations (OA15) and (OA18) become

$$\omega_t^i\left(s^t \middle| s_0\right) = \frac{\left(\beta_i\right)^t \pi_t\left(s^t \middle| s_0\right) u_i'\left(\overline{c_t^i}\left(s^t\right)\right)}{\sum_{t=0}^{T}\left(\beta_i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u_i'\left(\overline{c_t^i}\left(s^t\right)\right) \overline{c_t^i}\left(s^t\right)} \quad \text{(multiplicative)} \tag{OA19}$$

$$\omega_t^i\left(s^t \middle| s_0\right) = \frac{\left(\beta_i\right)^t \pi_t\left(s^t \middle| s_0\right) u_i'\left(\overline{c_t^i}\left(s^t\right)\right)}{\sum_{t=0}^{T}\left(\beta_i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u_i'\left(\overline{c_t^i}\left(s^t\right)\right)}. \quad \text{(additive)} \tag{OA20}$$

Two major insights emerge from Equations (OA19) and (OA20). First, the DS-weights in the additive case exactly correspond to the DS-weights for a normalized utilitarian planner, as defined in Equations (18) and (19). Second, note that the denominator of the DS-weights in the multiplicative case includes $\overline{c_t^i}\left(s^t\right)$ at all dates and histories. This captures the fact that the welfare assessment is computed as a fraction of consumption at each date/history, not in dollars. The presence of $\overline{c_t^i}\left(s^t\right)$ in the denominator is what complicates the aggregation of welfare assessments that follow Lucas (1987), because the numerator of the multiplicative decomposition is expressed on units for permanent transfer of consumption for individual i across all dates and histories, not simply a transfer of dollars.

E.3 Global welfare assessments: Relation to consumer surplus

In this section we, seek to illustrate how the way in which we describe to conduct non-marginal welfare assessments when using DS-weights — see Section 5.2 — is analogous to the way in which Consumer Surplus can be used to conduct non-marginal welfare assessments in Classical Demand Theory.

Our environment and notation here closely follow the Hausman (1981). We consider the maximization by a consumer of a strictly quasi-concave utility function $u(x)$ defined over n goods, $x = (x_1, \ldots, x_n)$, who faces prices $p = (p_1, \ldots, p_n)$ subject to a budget constraint $p \cdot x \leq y$, where y represents some exogenous income. The indirect utility function and the expenditure function for this problem are respectively given by

$$v\left(p, y\right) \equiv \max_{x} u\left(x\right) \quad \text{s.t.} \quad p \cdot x \leq y.$$

$$e\left(p, \bar{u}\right) \equiv \min_{x} p \cdot x \quad \text{s.t.} \quad u\left(x\right) \geq \bar{u}.$$

For simplicity, let us consider the case in which only the first price changes from p_1^0 to p_1^1 with all other prices held constant. Let us formally define compensating variation (CV), equivalent variation (EV), and

Marshallian consume surplus (CS):[35]

$$CV\left(p^0,p^1,y^0\right)=e\left(p^1,u^0\right)-e\left(p^0,u^0\right)=\int_{p_1^0}^{p_1^1}\frac{\partial e\left(p,u^0\right)}{\partial p_1}dp_1=\int_{p_1^0}^{p_1^1}h_1\left(p,u^0\right)dp_1 \tag{OA21}$$

$$EV\left(p^0,p^1,y^0\right)=e\left(p^1,u^1\right)-e\left(p^0,u^1\right)=\int_{p_1^0}^{p_1^1}\frac{\partial e\left(p,u^1\right)}{\partial p_1}dp_1=\int_{p_1^0}^{p_1^1}h_1\left(p,u^1\right)dp_1 \tag{OA22}$$

$$CS\left(p^0,p^1,y^0\right)=\int_{p_1^0}^{p_1^1}x_1\left(p,y^0\right)dp_1=-\int_{p_1^0}^{p_1^1}\underbrace{\frac{1}{\frac{\partial v(p,y^0)}{\partial y}}}_{\sim\frac{1}{\frac{\partial u_i(s^t)}{\partial c_t^i}}}\underbrace{\frac{\partial v\left(p,y^0\right)}{\partial p_1}}_{\sim\frac{du_i\left(c_t^i(s^t),n_t^i(s^t)\right)}{d\theta}}dp_1, \tag{OA23}$$

where the last equality in the consumer surplus definition follows immediately from Roy's identity, and where u_0 and u_1 denote utility before and after the price change. By comparing Equation (OA23) to Equations (3) and (28) in the body of the paper, one can interpret the CS measures as one in which the DS-weight exactly equals 1. More importantly, Equation (OA23) can be interpreted as adding up utility changes along a path between p_1^0 to p_1^1 in a way in which an individual is compensated locally along the path. This is identical to the way in which we present the non-marginal use of DS-weights in Section 5.2.

E.4 Normalizations and welfare reversals

In this section, we illustrate the importance of normalizations in the context of an example.

Example. (Welfare reversal: utilitarian planner under alternative normalizations) We consider an endowment economy with two types of individuals, indexed by $i=\{A,B\}$; two dates, indexed by $t=\{0,1\}$; no uncertainty; and a single consumption good at each date. Utility is given by

$$V_i=u_i\left(c_0^i\right)+\beta_i u_i\left(c_1^i\right)$$

and budget constraints are $c_0^i=y_0^i+\theta T_0^i$ and $c_1^i=y_1^i+\theta T_1^i$, where y_0^i and y_1^i denote initial endowments, and θT_t^i denote transfers. The lifetime utility effect of a policy change in this environment is simply given by

$$\frac{dV_i}{d\theta}=u_i'\left(c_0^i\right)T_0^i+\beta_i u_i'\left(c_1^i\right)T_1^i.$$

To illustrate the possibility of welfare reversals more clearly, we further consider permanent transfers, that is, we assume that $T_0^A=T_1^A=T$ and that $T_0^B=T_1^B=-T$.

If we restrict the choice of DS-weights to be consistent with a utilitarian planner, there are three reasonable normalizations: date-0 marginal utility, date-1 marginal utility, and the marginal utility of a permanent transfer. Formally, we consider three different DS-planners who use DS-weights i) $\omega^i=u_i'\left(c_0^i\right)$,

[35] It is well-known that CV, EV, and CS are identical for marginal changes.

ii) $\omega^i = \beta u_i'\left(c_1^i\right)$, and iii) $\omega^i = u_i'\left(c_0^i\right) + \beta_i u_i'\left(c_1^i\right)$.[36] Their aggregate welfare assessments are given by

$$\frac{dW^{\text{date}-0}}{d\theta} = \left(T + \frac{\beta_A u_A'\left(c_1^A\right)}{u_A'\left(c_0^A\right)}T\right) - \left(T + \frac{\beta_B u_B'\left(c_1^B\right)}{u_B'\left(c_0^B\right)}T\right) = \left(\frac{\beta_A u_A'\left(c_1^A\right)}{u_A'\left(c_0^A\right)} - \frac{\beta_B u_B'\left(c_1^B\right)}{u_B'\left(c_0^B\right)}\right)T \tag{OA24}$$

$$\frac{dW^{\text{date}-1}}{d\theta} = \left(\frac{u_A'\left(c_0^A\right)}{\beta_A u_A'\left(c_1^A\right)}T + T\right) - \left(\frac{u_B'\left(c_0^B\right)}{\beta_B u_B'\left(c_1^B\right)}T + T\right) = \left(\frac{u_A'\left(c_0^A\right)}{\beta_A u_A'\left(c_1^A\right)} - \frac{u_B'\left(c_0^B\right)}{\beta_B u_B'\left(c_1^B\right)}\right)T \tag{OA25}$$

$$\begin{aligned}\frac{dW^{\text{permanent}}}{d\theta} &= \left(\frac{u_A'\left(c_0^A\right)}{u_A'\left(c_0^A\right) + \beta_A u_A'\left(c_1^A\right)} + \frac{\beta_A u_A'\left(c_1^A\right)}{u_A'\left(c_0^A\right) + \beta_A u_A'\left(c_1^A\right)}\right)T \\ &\quad - \left(\frac{u_B'\left(c_0^B\right)}{u_B'\left(c_0^B\right) + \beta_B u_B'\left(c_1^B\right)} + \frac{\beta_B u_B'\left(c_1^B\right)}{u_B'\left(c_0^B\right) + \beta_B u_B'\left(c_1^B\right)}\right)T = 0.\end{aligned} \tag{OA26}$$

Two main insights emerge by comparing Equations (OA24), (OA25), and (OA26). First, if a planner concludes that the transfer policy is welfare improving when normalizing by date-0 marginal utility, the planner will conclude that it is not welfare improving when normalizing by date-1 marginal utility. Formally, if $\frac{dW^0}{d\theta} > 0$, then $\frac{dW^1}{d\theta} < 0$, and vice versa, for any set of primitives (endowments, risk aversion, or discount factors). We refer to this result as a *welfare reversal.* Intuitively, when normalizing by date-0 marginal utility, the planner gives more weight to the individual with the lowest marginal utility at date 0 relative to date 1. If the planner normalizes instead by date-1 marginal utility, the planner automatically gives more weight to the individual with the lowest marginal utility at date 1 relative to date 0, generating the reversal. Second, Equation (OA26) shows that a permanent transfer is welfare neutral for the DS-planner who normalized by the marginal utility of permanent transfer. This results is an interesting observation that also applies in the general environment studied in this paper.

E.5 Policy changes that affect probabilities

In this section, we describe how to use our approach in environments in which policy changes affect probabilities. We begin by modifying Equation (5) for a given individual to include instantaneous Pareto weights, as in Section 5.3:

$$V_i\left(s_0\right) = \sum_{t=0}^{T} \left(\beta_i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \lambda_t^i\left(s^t\right) u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right). \tag{OA27}$$

We can now differentiate Equation (OA27), to find that

$$\begin{aligned}\frac{dV_i\left(s_0\right)}{d\theta} &= \sum_{t=0}^{T} \left(\beta_i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \lambda_t^i\left(s^t\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i} \frac{du_{i|c}\left(s^t\right)}{d\theta} \\ &\quad + \sum_{t=0}^{T} \left(\beta_i\right)^t \sum_{s^t} \frac{d\pi_t\left(s^t \middle| s_0\right)}{d\theta} \lambda_t^i\left(s^t\right) u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right).\end{aligned} \tag{OA28}$$

[36] In a more general dynamic stochastic setup, a DS-planner could potentially decide to normalize by marginal utility at any date/history. The normalized utilitarian planner introduced in Proposition 4 is the counterpart of the planner who assess welfare as $\frac{dW^{\text{permanent}}}{d\theta}$ in this example.

At this stage, we impose the consistency condition that relates DS-weights with instantaneous Pareto weights, and solve for the implied instantaneous Pareto weights in terms of the relevant DS-weight:

$$\omega_t^i\left(s^t\middle|\, s_0\right) = \left(\beta_i\right)^t \pi_t\left(s^t\middle|\, s_0\right) \lambda_t^i\left(s^t\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i} \Rightarrow \lambda_t^i\left(s^t\right) = \frac{\omega_t^i\left(s^t\middle|\, s_0\right)}{\left(\beta_i\right)^t \pi_t\left(s^t\middle|\, s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}. \tag{OA29}$$

Combining Equation (OA28) with Equation (OA29), we find the counterpart of Equation (8) in the text in environments in which policy changes affect probabilities.

Definition 9. *(Desirable policy change for a DS-planner/Definition of DS-weights) A DS-planner, that is, a planner who adopts DS-weights, finds a policy change desirable in an environment in which policies can also affect probabilities if and only if* $\frac{dW(s_0)}{d\theta} > 0$*, where*

$$\frac{dW^{DS}\left(s_0\right)}{d\theta} = \int \sum_{t=0}^{T} \sum_{s^t} \omega_t^i\left(s^t\middle|\, s_0\right) \left(\frac{du_{i|c}\left(s^t\right)}{d\theta} + \frac{d\ln \pi_t\left(s^t\middle|\, s_0\right)}{d\theta} \frac{u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right)}{\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}} \right) dG\left(i\right),$$

where $\frac{d\ln \pi_t\left(s^t\middle|s_0\right)}{d\theta} = \frac{\frac{d\pi_t\left(s^t\middle|s_0\right)}{d\theta}}{\pi_t\left(s^t\middle|s_0\right)}$.