Corrective Regulation with Imperfect Instruments*

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Abstract

This paper studies optimal second-best corrective regulation when some decisions cannot be perfectly regulated. We show that policy elasticities and Pigouvian wedges are sufficient statistics to characterize the marginal welfare impact of regulatory changes. We show that leakage elasticities — a subset of policy elasticities — and Pigouvian wedges jointly determine optimal second-best corrective policy. We further characterize the marginal value of reforms that relax regulatory constraints. In an application to financial regulation with shadow banks, we show that empirical estimates of leakage elasticities can be used to directly determine the desirability of adjusting regulations and to quantitatively determine optimal policy.

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1 Introduction

Many economic policies are motivated by the desire to correct externalities. However, the instruments available to policymakers are often imperfect. Financial regulation is a prime example of this phenomenon. In particular, in the aftermath of the 2008 financial crisis and guided by theories of corrective policy in the presence of a diverse set of market failures — including fire-sale externalities and distortive government subsidies (e.g., Lorenzoni, 2008; Bianchi, 2016; Farhi and Werning, 2016; Dávila and Korinek, 2018) — most economies have expanded the set and scope of regulations faced by the financial sector. At the same time, many agents and decisions in the financial system are still imperfectly regulated. These imperfections are often viewed as generating "unintended consequences", typically in the form of regulatory arbitrage (e.g., Adrian and Ashcraft, 2016; Hachem, 2018). Hence, a natural normative question is how regulators should proceed once aware of such imperfections. The associated second-best policy problem appears daunting because, as we have outlined, there are many possible market failures to consider and many seemingly disparate imperfections in policy instruments.

In this paper, we aim to identify the unifying principles that determine optimal second-best corrective policy, and to enable their use for quantitative welfare analysis. We proceed in two steps. First, on the theoretical side, we analyze a canonical economy in which agents impose externalities on each other. In this environment, we derive principles for optimal second-best policy in a world where corrective regulation is costly or subject to constraints. These results unify and extend existing insights on corrective taxation. Second, to demonstrate the quantitative usefulness of our approach, we combine direct measurement and quantitative modeling to derive new insights for optimal banking regulation in the presence of shadow banks.

We initially consider a competitive exchange economy in which heterogeneous agents make consumption decisions. To introduce a role for corrective regulation, we allow for direct consumption externalities among agents. We saturate the model with corrective taxes/regulations that are chosen by a planner and can be agent- and decision-specific. However, these policy instruments are subject to a general set of constraints that capture regulatory imperfections. For example, the planner might i) be unable to regulate certain agents or decisions, ii) be forced to set the same regulation across different agents or decisions, or iii) simply face costs that are increasing in the size of the regulations, perhaps capturing political economy limits to regulation.

We first characterize the marginal welfare effects of arbitrary policy changes. We show that these effects are determined by two sets of statistics: i) *Pigouvian wedges* and ii) *policy elasticities*. Pigouvian wedges correspond to the difference between the existing corrective regulation associated with a particular decision and the marginal distortion (externality) generated by that decision. A positive (negative) wedge implies that a decision is overregulated (underregulated), in the sense that the regulation imposed on it is greater (smaller) than the associated marginal distortion. Policy elasticities capture the equilibrium responses of different decisions to changes in regulation. Intuitively, our characterization shows that policy changes that discourage underregulated decisions or encourage overregulated decisions are welfare-improving. As a benchmark, we show that the first-best policy, in which a planner faces no regulatory constraints, is chosen so that all Pigouvian wedges are zero – the classical Pigouvian principle. Importantly, policy elasticities do not form part of the first-best policy. They only matter for corrective regulation in second-best scenarios.

We use the characterization of marginal welfare effects to study optimal second-best policy. To do so, it is useful to distinguish between perfectly and imperfectly regulated decisions. A decision is perfectly regulated if its associated corrective regulation does not enter in any binding constraint faced by the planner, and imperfectly regulated otherwise. We then derive three main insights on the optimal second-best policy.

First, we characterize the optimal second-best regulation of *perfectly regulated* decisions. In the first-best benchmark, the Pigouvian principle implies that the corrective regulation of each decision equals its marginal distortion. By contrast, with imperfect instruments, the optimal corrective regulation is given by the sum of its associated marginal distortion, and a second-best correction. The second-best correction depends on two sufficient statistics: i) Pigouvian wedges associated with imperfectly regulated decisions, and ii) *leakage elasticities*, which are a subset of policy elasticities that measure the equilibrium response of imperfectly regulated decisions to changes in the regulation of perfectly regulated decisions. For instance, if unregulated decisions are underregulated and complements to perfectly regulated decisions, then it is optimal to impose a regulation above the Pigouvian level. The opposite conclusion, that is, a second-best regulation below the Pigouvian level, arises in the cases of underregulated substitutes or overregulated complements.

While over- and underregulation relative to the first-best are both possible, these results show that there is significant structure on how to determine the optimal second-best policy. This finding contrasts with common "anything goes" second-best arguments (Lipsey and Lancaster, 1956). We also connect our results to the Tinbergen (1952) targeting rule, by demonstrating the precise role of the number of targets and instruments.

Second, we characterize the optimal second-best regulation of *imperfectly regulated* decisions. By definition, these regulations are subject to binding constraints, but the planner may nonetheless have degrees of freedom in choosing them — for instance, if a constraint dictates that two decisions must be taxed at a uniform rate, while the level of the uniform tax can be chosen freely. When choosing regulations on imperfectly regulated decisions, the planner must not only consider direct effects, but also more nuanced feedback effects. In general, the optimal second-best regulation depends both on leakage and *reverse leakage* elasticities, which capture how perfectly regulated decisions adjust to regulating imperfectly regulated ones. Whenever perfectly and imperfectly regulated decisions are *either* complements or substitutes, reverse leakage attenuates the welfare effect of regulating imperfectly regulated decisions. This result is reminiscent of the Le Chatelier principle (Samuelson, 1948; Milgrom and Roberts, 1996). However, while our results also describe how the direct effect of a parameter change is augmented by feedback in a system, we find attenuation, rather than amplification, for welfare effects.

Building on these insights, we derive optimal second-best regulation of imperfectly regulated decisions in two common scenarios. On one hand, if taxes are constrained to be *uniform* across heterogeneous agents or decisions, then the optimal regulation is a weighted average of distortions, where the appropriate weights are augmented to incorporate reverse leakage elasticities. These results generalize the uniform corrective taxation result of Diamond (1973), which follows as a special case in the absence of reverse leakage — that is, when there are no perfectly regulated decisions. On the other hand, if a subset of regulations is subject to convex costs, then the optimal regulation is given by an attenuated version of the first-best policy. In the presence of perfectly regulated decisions, reverse leakage is a force that contributes to further attenuating the optimal regulation.

Finally, we characterize the social value of relaxing the constraints faced by a planner who is implementing the optimal second-best policy. This is an informative exercise, for instance, for a planner that considers an institutional reform, such as allowing to regulate previously unregulated decisions. Once again, the Le Chatelier/reverse leakage effects are a force towards attenuating the welfare benefits of reforms both in the substitutes and the complements case. Intuitively, if perfectly and imperfectly regulated decisions are substitutes, tightening the regulation on imperfectly regulated decisions increases perfectly regulated decisions through reverse leakage. But this is welfare-reducing since perfectly regulated decisions are underregulated at the second-best. Conversely, if perfectly and imperfectly regulated decisions are complements, tightening the regulation on imperfectly regulated decisions reduces perfectly regulated decisions through reverse leakage, which is again welfare-reducing.

In our quantitative application, which translates the general principles into quantitative insights for optimal financial regulation with imperfect instruments, we consider an environment in which banks make leverage and investment decisions. First, in Section 4.1, we present empirical measures of the leakage elasticities that, when combined with our characterization or marginal welfare effects, directly inform whether it is desirable to adjust leverage regulation in the presence of unregulated shadow banks. Empirically, we focus on the market for US residential mortgages, in which unregulated shadow banks (and, relatedly, "FinTech" entrants) account for more than half of overall activity (Jiang, 2023). In this context, we adapt measurements of leakage elasticities from the recent empirical literature. When translated into our framework, estimates in Buchak, Matvos, Piskorski and Seru (2018) imply that a one percentage-point increase in traditional banks' leverage regulation, for example via standard capital requirements, reduces traditional bank investment in mortgage loans by 5%, while increasing shadow bank investment by 4%. Without the need to fully specify a model, these estimates on their own allow us to quantify the welfare effects of stricter bank regulation as a function of Pigouvian wedges. In particular, they show that marginal increases in leverage regulation are welfare-increasing despite the presence of shadow banks, unless a) traditional banks are already severely over-regulated, or b) shadow banks impose quantitatively larger externalities than traditional banks' investment.

Next, in Section 4.2, we present a fully specified equilibrium model that allows us to explore how regulatory constraints quantitatively impact the optimal second-best regulation. Our model is designed to be the simplest that allows us to draw quantitative conclusions in a framework that features three distinct distortions central to the design of financial regulation: i) bailouts, ii) pecuniary externalities, and iii) distorted beliefs. Using the measured elasticities as key calibration targets, we initially study and compare different scenarios that illustrate how the nature of constraints faced by the planner determines the optimal policy.

We study three scenarios, which impose increasingly tight constraints on the planner. First, as a benchmark, we study the first-best scenario, in which all leverage and investment decisions are perfectly regulated. Second, we study an unconstrained-leverage-regulation scenario, in which the planner can freely set leverage regulation on both traditional and shadow banks, but the regulation on the scale of banks' investments is restricted to be zero — this is a common feature of financial regulations, such as the Basel Accords, which concentrate on ratios and leaves scale as a free variable. Third, we study a constrained-leverage-regulation scenario in which the planner can exclusively regulate the leverage decision of traditional banks. The latter scenario maps most closely to the modern regulatory system, in which a subset of banks are not subject to direct regulation. For that reason, we use it as the reference for the model calibration.

In the unconstrained leverage scenario, the optimal regulation overregulates the leverage decisions of traditional banks but underregulates the leverage decisions of shadow banks relative to the first best. As implied by our theoretical results, both results are due to contrasting leakage elasticities with different signs. Given our calibration, investment and leverage decisions within a bank are gross complements, which calls for overregulating leverage. However, leverage and investment decisions across banks are substitutes, which calls for underregulating leverage. While the complementarity force dominates for traditional banks, optimally overregulating leverage, the substitutability dominates for shadow banks, optimally underregulating leverage.

Next, we turn to the constrained leverage scenario, in which the planner is no longer able to regulate shadow banks. In this scenario, the optimal regulation underregulates the leverage decisions of traditional banks relative to the first-best and the unconstrained-leverage-regulation scenarios. Once again, we can make use of our theoretical results to clarify the role of three relevant leakage elasticities at play. Given our calibration, the strong substitutability between traditional banks' leverage and shadow banks' investment dominates, justifying the optimal underregulation of shadow banks.

Finally, we also illustrate the form of the optimal uniform regulation and consider a welfare analysis of the hypothetical scenario in which, starting from the constrained-leverage-regulation scenario, we relax the regulatory constraint on shadow bank leverage. The latter exercise is informative for potential regulatory reforms that mandate some shadow banks to become regulated. In the Online Appendix, we explore an application to financial regulation with environmental externalities and four other minimal applications.

Overall, our quantitative results highlight that leakage elasticities featuring both substitutability and complementarity naturally emerge and nontrivially interact in common regulatory scenarios. A quantitative analysis, which incorporates different banks' different decisions is typically necessary to optimally determine optimal second-best policy. A useful contribution of our theoretical results is to provide a clear and interpretable quantitative analysis of the associated welfare implications, despite the complexity of the relevant effects.

Related Literature. Our theoretical results are directly related to existing work — mostly in public economics — that studies imperfect corrective regulation. In fact, we show that several classic results that have been treated as independent can be derived and expanded upon using our approach. For instance, the optimal tax formulae in Diamond (1973) are seemingly distinct from the characterization of second-best policy in Lipsey and Lancaster (1956) or the Tinbergen (1952) Rule, but these results can all be derived as corollaries of our main results. We explicitly compare and contrast our results to existing work in the text. Other contributions in this literature, often comparing indirect and direct regulation in particular scenarios, include Baumol (1972), Sandmo (1975), Green and Sheshinski (1976), Balcer (1980), Wijkander (1985), and Cremer, Gahvari and Ladoux (1998).[1] Textbook-level treatments are available in Myles (1995), Salanié (2011), or Werning (2012). In common with Hendren (2016), we adopt the terminology policy elasticity, identifying the special role played by leakage elasticities in determining optimal second-best regulation. Second-best corrective regulation is often discussed in the context of environmental policy and congestion (e.g., Bovenberg and Goulder, 2002), as well as rent-seeking (e.g., Rothschild and Scheuer, 2014, 2016). Our results in Section 3.4 characterizing the value of relaxing constraints on regulation provide a novel manifestation of the Le Chatelier principle, introduced by Samuelson (1948), and further studied in Milgrom and Roberts (1996), Acemoglu (2007), and Dekel, Quah and Sinander (2023) among others.

Within the theoretical literature on financial regulation, Plantin (2015), Huang (2018), Martinez-Miera and Repullo (2019), and Farhi and Tirole (2021), study the impact of bank regulation on banking activity and financial stability. Hachem and Song (2021) explore how increased liquidity requirements can generate credit booms when banks are heterogeneous. Grochulski and Zhang (2019) show, in an environment in which regulation is motivated by a pecuniary externality as in Farhi, Golosov and Tsyvinski (2009), how regulation is constrained by the presence of shadow banks. Gennaioli, Shleifer and Vishny (2013) and Moreira and Savov (2017) develop theories that highlight the fragile nature of shadow banking arrangements. Ordoñez (2018) shows how shadow banking enables better-informed banks to avoid blunt regulations. Bengui and Bianchi (2022), building on Bianchi (2011), provide a theoretical and quantitative analysis of macroprudential policy with imperfect instruments based on a collateral pecuniary externality. Dávila and Korinek (2018) briefly discuss the impact of specific regulatory constraints on policy in a setup with pecuniary externalities, while Korinek (2017) provides a systematic study of optimal corrective policy in environments with multiple regulators. Clayton and Schaab (2021) study regulatory policy in the presence of shadow banks when there are pecuniary externalities. Korinek, Montecino and Stiglitz (2022) study the role of technological innovation as regulatory arbitrage. Begenau and Landvoigt (2022) provide a quantitative general equilibrium assessment of regulating traditional banks for financial stability and macroeconomic outcomes in the presence of ex-post subsidies — see Dempsey (2020) for a related quantitative assessment. In addition to the work of Buchak et al. (2018; 2024) and Jiang (2023) on which we base our quantitative assessment, there is

[1] Corlett and Hague (1953) is the seminal study on optimal commodity taxation with incomplete taxes in economies without externalities. See e.g. Myles (1995) for a survey of related results.

a growing empirical literature on regulatory arbitrage and shadow banking that includes Acharya, Schnabl and Suarez (2013) and Demyanyk and Loutskina (2016), among others.

2 Model

This section describes a canonical exchange economy with externalities. We use this model to introduce the principles that determine optimal corrective regulation with imperfect instruments in Section 3.[2] Section 4 shows how such principles can be used to quantitatively determine optimal constrained policy in a rich application to financial regulation, an environment in which regulatory imperfections abound.

2.1 Environment and Equilibrium Definition

We consider an economy with a finite number $I \geq 1$ of agents (equivalently, agent types in unit measure), indexed by $i, j \in \mathcal{I}$, where $\mathcal{I} = \{1, \ldots, I\}$. There are $N \geq 1$ goods (commodities) indexed by $n \in \mathcal{N}$, where $\mathcal{N} = \{1, \ldots, N\}$.

Agent i's preferences are represented by

$$u^i\left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}\right), \tag{1}$$

where $\boldsymbol{x}^i \in \mathbb{R}_+^N$ denotes agent i's consumption bundle, and $\bar{\boldsymbol{x}} = \{\bar{\boldsymbol{x}}^j\}_{j \in \mathcal{I}}$ denotes the collection of bundles of all agents.[3] Each agent takes $\bar{\boldsymbol{x}}$ as given, so the second argument in $u^i(\cdot)$ captures externalities across agents.

Each agent i faces a budget constraint

$$\boldsymbol{p} \cdot \left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) + \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i = T^i, \tag{2}$$

where $\boldsymbol{e}^i \in \mathbb{R}_+^N$ denotes agent i's endowment of goods; $\boldsymbol{p} \in \mathbb{R}_+^N$ is a price vector; $\boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$ introduces a set taxes/subsidies (regulations) specific to each agent and commodity, where $\boldsymbol{\tau}^i \in \mathbb{R}^N$; and $T^i \in \mathbb{R}$ denotes the lump-sum transfer or tax that agent i receives or faces to ensure that the planner runs a balanced budget. We denote the elements of $\boldsymbol{x}^i$, $\bar{\boldsymbol{x}}^i$, and $\boldsymbol{\tau}^i$ by x_n^i, $\bar{x}_n^i$ and τ_n^i, respectively.

For a given set of regulations $\{\boldsymbol{\tau}^i\}_{i \in \mathcal{I}}$ and transfers $\{T^i\}_{i \in \mathcal{I}}$, an *equilibrium* consists of bundles $\{\boldsymbol{x}^i\}_{i \in \mathcal{I}}$ and a price vector $\boldsymbol{p}$ such that i) agents choose $\boldsymbol{x}^i$ to maximize utility (1) taking $\boldsymbol{p}$ and $\bar{\boldsymbol{x}}$ as given, subject to the budget constraint (2); ii) the planner's budget is balanced, so that $\sum_i T^i = \sum_i \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$; iii) agent allocations are consistent in the aggregate, so $\bar{\boldsymbol{x}}^i = \boldsymbol{x}^i$, $\forall i$; and iv) markets clear, so $\sum_i \boldsymbol{x}^i = \sum_i \boldsymbol{e}^i$. We assume at all times that the model is well-behaved.

[2] Sections C.1 and C.2 of the Online Appendix respectively show that the same conclusions apply to i) production and ii) game-theoretic economies.

[3] Section B of the Online Appendix includes explicit definitions of all vectors and matrices used in the paper.

2.2 Imperfect Policy Instruments

As explained below, a planner who can freely adjust all policy instruments $\{\boldsymbol{\tau}^i\}_{i\in\mathcal{I}}$ is able to achieve a first-best outcome. However, our focus is on optimal corrective policy with *imperfect* policy instruments. We formalize such imperfections by assuming that a planner chooses regulations subject to a vector-valued constraint

$$\boldsymbol{\Phi}(\boldsymbol{\tau}) \leq 0, \tag{3}$$

where $\boldsymbol{\tau} \in \mathbb{R}^{IN}$ denotes the stacked vector of agent-specific regulations $\boldsymbol{\tau}^i$. The function $\boldsymbol{\Phi} : \mathbb{R}^{IN} \to \mathbb{R}^M$, where M is the number of constraints, flexibly defines the set of feasible regulations. Appealing to the duality between constraints and costs, we can also interpret $\boldsymbol{\Phi}(\boldsymbol{\tau})$ as defining the cost of setting regulations.

For example, an unconstrained planner, who can achieve the first-best (Pigouvian) solution, corresponds to setting $\boldsymbol{\Phi}(\boldsymbol{\tau}) \equiv 0$ for all $\boldsymbol{\tau} \in \mathbb{R}^{IN}$. Alternatively, a linear constraint

$$\boldsymbol{\Phi}(\boldsymbol{\tau}) \equiv \boldsymbol{A}\boldsymbol{\tau} - \boldsymbol{c}, \tag{4}$$

for appropriate matrices $\boldsymbol{A}$ and vectors $\boldsymbol{c}$, can be used to model planners who i) are able to regulate only particular subsets of agents or commodities, leaving others unregulated, or ii) must impose uniform regulations across different agents or commodities. A quadratic constraint

$$\boldsymbol{\Phi}(\boldsymbol{\tau}) \equiv \frac{1}{2}\boldsymbol{\tau}'\boldsymbol{B}\boldsymbol{\tau} + \boldsymbol{d}, \tag{5}$$

for a given matrix $\boldsymbol{B}$ and a vector $\boldsymbol{d}$, can instead represent convex costs of regulation. In addition, if the costs of regulation induce sparsity (Tibshirani, 1996; Gabaix, 2014) — for instance, when based on the L^1 norm of $\boldsymbol{\tau}$ — the set of unregulated agents or commodities arises endogenously. Our main results — Propositions 1 and 2 — are valid for a general $\boldsymbol{\Phi}(\cdot)$.

2.3 Remarks

We conclude the description of the environment with three remarks. First, the insights of this paper do not hinge on the price-theoretic formulation of the model. For instance, our model has a game-theoretic interpretation if a subset of x_n^i is interpreted as actions or decisions made by agents, as shown in Section C.2 of the Online Appendix.[4] In this case, rather than trading in a competitive market, agents make decisions as the best response to others.

Second, assuming that agents' utilities directly depend on others' decisions is the simplest formulation that justifies corrective regulation. However, the insights of this paper apply to any environment in which a planner wants to correct individual agents' decisions (e.g., consumption or production externalities, public goods, lack of commitment, behavioral distortions, etc.), regardless of the exact rationale justifying such regulation. For instance, our application to financial regulation

[4] Having a budget constraint as in Equation (2) — at least for a subset of commodities — is useful to i) make aggregate welfare assessments and ii) express marginal distortions in a common unit.

in Section 4 features three widely studied rationales — bailouts, pecuniary externalities, and internalities/belief distortions — that do not arise directly from consumption externalities of the form modeled in (1). We further elaborate on this point after introducing Lemma 1 below.

Finally, note that by assuming that $T^i = \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$, $\forall i$ — instead of the less restrictive condition $\sum_i T^i = \sum_i \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$ — all results can be interpreted as quantity regulation. In this case, $\boldsymbol{\tau}^i$ represents implied shadow prices of quantity regulations, instead of actual taxes or subsidies.

3 Optimal Policy with Imperfect Instruments

We study the problem of a planner who optimally sets corrective regulation subject to constraints on the set of regulatory instruments. We abstract from redistributional considerations, and focus on the corrective nature of the regulation. Therefore, we assess the aggregate welfare gains/losses of a marginal policy change by aggregating money-metric welfare changes across agents.[5] That is, the planner evaluates the desirability of a marginal change in a given variable (or vector) z, denoted by $\frac{dW}{dz}$, according to

$$\frac{dW}{dz} \equiv \sum_{i \in \mathcal{I}} \frac{\frac{dV^i}{dz}}{\lambda^i},$$

where $\frac{dV^i}{dz}$ denotes the change in agent i's indirect utility in equilibrium and $\lambda^i > 0$ denotes agent i's marginal value of wealth.

3.1 Marginal Welfare Effects and Pigouvian Principle

To characterize the marginal welfare effect of adjusting regulations, it is useful to first define the *marginal distortion/externality* δ_n^i associated with decision n by agent i:

$$\delta_n^i = -\sum_{j \in \mathcal{I}} \frac{1}{\lambda^j} \frac{\partial u^j}{\partial \bar{x}_n^i}. \qquad \text{(Marginal Distortion)}$$

The marginal distortion δ_n^i measures the direct welfare impact that a change in $\bar{x}_n^i$ has on all agents. We define the distortion as the negative of marginal utility, so that δ_n^i measures the damage caused by $\bar{x}_n^i$. Concretely, when $\frac{\partial u^j}{\partial \bar{x}_n^i}$ is negative (positive), an increase in $\bar{x}_n^i$ generates a negative (positive) externality on agent j, contributing to making δ_n^i positive (negative). We use $\boldsymbol{\delta}^i \in \mathbb{R}^N$ to denote the vector of marginal distortions associated with agent i's decisions and $\boldsymbol{\delta} \in \mathbb{R}^{IN}$ to denote the stacked vector of $\boldsymbol{\delta}^i$'s for all agents.

It is also useful to define the *Pigouvian wedge* ω_n^i between the regulation τ_n^i and the marginal distortion δ_n^i associated with decision n by agent i:

$$\omega_n^i = \tau_n^i - \delta_n^i, \qquad \text{(Pigouvian Wedge)}$$

[5]This approach imposes equal generalized social marginal welfare weights, in the sense of Saez and Stantcheva (2016), and is akin to maximizing Kaldor-Hicks efficiency — see Davila and Schaab (2022). Section C.3 of the Online Appendix explains how to consider traditional welfare weights and redistributional considerations.

where we can again write $\boldsymbol{\omega}^i = \boldsymbol{\tau}^i - \boldsymbol{\delta}^i \in \mathbb{R}^N$ or $\boldsymbol{\omega} = \boldsymbol{\tau} - \boldsymbol{\delta} \in \mathbb{R}^{IN}$ in vector form. As explained below, Pigouvian wedges are zero at the first-best. A positive (negative) ω_n^i indicates that decision n by agent i is overregulated (underregulated), in the sense that increasing (decreasing) $\bar{x}_n^i$ is welfare-improving.

Lemma 1 presents two useful intermediate results. Part a) highlights the role of Pigouvian wedges and leakage elasticities as sufficient statistics when evaluating corrective policy. Part b) characterizes the first-best policy, which provides a benchmark when studying second-best policy.

Lemma 1. *a) (Marginal Welfare Effects of Regulation) The marginal welfare effects of varying regulations $\boldsymbol{\tau}$, $\frac{dW}{d\boldsymbol{\tau}}$, are given by*

$$\frac{dW}{d\boldsymbol{\tau}} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} (\boldsymbol{\tau} - \boldsymbol{\delta}) = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} \boldsymbol{\omega}, \tag{6}$$

where $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ is the Jacobian matrix of policy elasticities, of dimension $IN \times IN$.

b) (First-Best Policy/Pigouvian Principle) The optimal (first-best) policy for a planner who can freely choose regulations — when $\boldsymbol{\Phi}(\boldsymbol{\tau}) \equiv 0$ — is characterized by

$$\boldsymbol{\omega} = 0 \iff \boldsymbol{\tau}^\star = \boldsymbol{\delta}. \tag{7}$$

Lemma 1a) highlights that the welfare impact of changes in regulation can always be characterized in terms of two sets of sufficient statistics: policy elasticities and Pigouvian wedges. The matrix $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ of *policy elasticities* — borrowing the terminology of Hendren (2016) — captures the full equilibrium response that a particular change in regulation has on all agents' decisions. The vector $\boldsymbol{\omega}$ of Pigouvian wedges captures the extent to which an agent's decision is underregulated or overregulated. The overall welfare effect of varying a particular regulation corresponds to the sum of the product of the relevant leakage elasticities and Pigouvian wedges, where regulations that i) decrease underregulated decisions (with $\omega_n^i < 0$) or ii) increase overregulated decisions (with $\omega_n^i > 0$) in equilibrium are welfare-improving.

Lemma 1b) characterizes the well-understood Pigouvian principle in our model, i.e., the "polluter pays" (Pigou, 1920; Sandmo, 1975), also referred to as the principle of targeting (e.g., Dixit, 1985; Rothschild and Scheuer, 2016). The first-best policy perfectly aligns private and social incentives by setting taxes equal to marginal distortions for each decision.[6] Note that the first-best policy is independent of the magnitude of the policy elasticities, being exclusively a function of the Pigouvian wedges. This result contrasts with the optimal second-best policy, as we describe next.

In line with our second remark above, our conclusions for second-best policy will be valid whenever marginal welfare effects take the form of Equation (6), regardless of the exact nature of the marginal distortions in $\boldsymbol{\delta}$. For example, in our quantitative application in Section 4.2, we consider a class of economies with incomplete markets and different sources of externalities, and

[6]Note that Equation (7) does not provide a solution for optimal regulations in terms of primitives unless marginal distortions are invariant to the level of regulation. Whenever marginal distortions are endogenous to the level of the regulation, our statements pertain to the form of the optimal policy formulas. The same caveat applies to Propositions 1 and 2. In our application in Section 4, marginal distortions are largely insensitive to the level of the regulation.

derive marginal welfare effects that mirror Equation (6). In that setting, Lemma 2 will become the exact counterpart of Lemma 1a).

3.2 Second-Best Policy: Perfectly Regulated Decisions

To characterize the optimal second-best policy, it is useful to distinguish between perfectly and imperfectly regulated decisions. We say that decision x_n^i is *perfectly regulated* when its associated policy instrument τ_n^i does not enter any binding constraint in the planner's problem, and is *imperfectly regulated* when it does enter in at least one binding constraint. Formally, if the vector $\boldsymbol{\mu} \in \mathbb{R}^M$ denotes the vector of Lagrange multipliers associated with the regulatory constraints in Equation (3), the n^{th} element of the vector $\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}}\boldsymbol{\mu}$ is zero (non-zero) for perfectly (imperfectly) regulated decisions.

Accordingly, we collect the R perfectly regulated decisions and U imperfectly regulated decisions — where $R + U = IN$ — in the vectors $\boldsymbol{x}^R$ and $\boldsymbol{x}^U$. We apply the same partition to the associated regulations $\boldsymbol{\tau} = \{\boldsymbol{\tau}^R, \boldsymbol{\tau}^U\}$, marginal distortions $\boldsymbol{\delta} = \{\boldsymbol{\delta}^R, \boldsymbol{\delta}^U\}$, and Pigouvian wedges $\boldsymbol{\omega} = \{\boldsymbol{\omega}^R, \boldsymbol{\omega}^U\}$.[7] We also partition the matrix of policy elasticities $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ into four smaller matrices. Two are matrices of leakage and reverse leakage elasticities: $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$ and $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}$, respectively, of dimensions $R \times U$ and $U \times R$. The other two are matrices — invertible in a well-behaved model — of own-perfectly regulated and own-imperfectly regulated elasticities: $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ and $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}$, respectively, of dimensions $R \times R$ and $U \times U$. In particular, the matrix of leakage elasticities $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$ captures how imperfectly regulated decisions respond to changes in the regulations that can be freely adjusted. We further describe these matrices in Section B of the Online Appendix.

Proposition 1 shows that leakage elasticities are a key determinant of the second-best policy.

Proposition 1. *(Second-Best Policy: Perfectly Regulated Decisions) The optimal second-best regulation of perfectly regulated decisions satisfies*

$$\boldsymbol{\tau}^R = \boldsymbol{\delta}^R + \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\omega}^U, \tag{8}$$

where $\boldsymbol{\delta}^R$ is a vector of marginal distortions, $\boldsymbol{\omega}^U = \boldsymbol{\tau}^U - \boldsymbol{\delta}^U$ is a vector of Pigouvian wedges, $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ is a matrix of own-perfectly regulated elasticities, and $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$ is a matrix of leakage elasticities.

Proposition 1 shows that the optimal regulation for a planner who can freely adjust the regulation of a subset of decisions is the sum of two components. The first component is the marginal distortion imposed by the perfectly regulated decisions, $\boldsymbol{\delta}^R$, as in the first-best case — see Lemma 1b).

The second component is a correction for regulatory imperfections that depends on two sets of sufficient statistics: leakage elasticities and Pigouvian wedges of imperfectly regulated decisions. First, the leakage elasticities $-\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$ — normalized by the own-perfectly regulated elasticities $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ —

[7] We denote the set of imperfectly regulated decisions by U since "unregulated" decisions are a leading case of imperfectly regulated decisions. Note that the sets of perfectly regulated and unregulated decisions can vary with the regulation itself, although this is not common in most applications.

capture how tightening the regulation of perfectly regulated decisions affects imperfectly regulated decisions. Heuristically, $\left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} \equiv -\frac{d\boldsymbol{x}^U}{d\boldsymbol{x}^R}$ is positive (negative) if regulated and unregulated decisions are gross substitutes (complements). Second, the Pigouvian wedge $\boldsymbol{\omega}^U$ measure whether imperfectly regulated decisions are regulated above or below their marginal distortions.

Consider the case where imperfectly regulated decisions are underregulated, with $\boldsymbol{\omega}^U < 0$. In the gross substitutes case, the second component of Equation (8) is negative, so a planner finds it optimal to adjust $\boldsymbol{\tau}^R$ downwards relative to the first-best. Hence, the second-best policy underregulates perfectly regulated decisions. By a parallel argument, in the gross complements case, the second-best policy overregulates perfectly regulated decisions. When imperfectly regulated decisions are overregulated, with $\boldsymbol{\omega}^U > 0$, those conclusions in the substitutes/complements cases are reversed. We summarize this logic in the following corollary to Proposition 1.

Corollary. *Whenever imperfectly regulated decisions are underregulated, it is optimal to underregulate (overregulate) perfectly regulated decisions when perfectly and imperfectly regulated decisions are substitutes (complements). These conclusions are reversed when imperfectly regulated decisions are overregulated.*

Before progressing to our next result, we illustrate Proposition 1 in a special practical scenario, and connect it to two classical results, namely, the general theory of the second best (Lipsey and Lancaster, 1956), and the Tinbergen (1952) rule.

Practical Scenario: Unregulated Decisions. A common scenario in which Proposition 1 is relevant in practice is when some decisions cannot be regulated at all. Formally, in this case the planner faces a constraint $\boldsymbol{\Phi}(\boldsymbol{\tau}) = \boldsymbol{\tau}^U = 0$, so Equation (8) specializes to

$$\boldsymbol{\tau}^R = \boldsymbol{\delta}^R - \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\delta}^U, \tag{9}$$

where unregulated decisions associated with negative (positive) externalities are automatically underregulated (overregulated). As in Proposition 1, whether the regulated and unregulated decisions are gross complements or substitutes is critical for the determination of the optimal second-best policy.

Connection to the General Theory of the Second Best. Lipsey and Lancaster (1956) argue that once one of the conditions required to achieve a first-best outcome is not satisfied, it is typically optimal to distort all other decisions. This insight is consistent with Equation (8): whenever $\boldsymbol{\omega}^U \neq 0$ (and $\frac{d\boldsymbol{x}^U}{d\boldsymbol{x}^R} \neq 0$), it is optimal to set $\boldsymbol{\omega}^R \neq 0$. However, while over- and underregulation relative to the first-best are possible, Equation (8) shows that there is significant structure on how to determine the optimal second-best policy: leakage elasticities and Pigouvian wedges for imperfectly regulated decisions unambiguously determine the optimal second-best regulation.[8]

[8]Proposition 1 thus achieves a different conclusion than Lipsey and Lancaster (1956), who write:

Connection to the Tinbergen Rule. The Tinbergen (1952) rule states that first-best policy requires the same number of instruments as it has targets. A concordant interpretation of Equation (9) is that a second-best planner uses the R instruments $\boldsymbol{\tau}^R$ (on the left-hand side) to target the $R+U$ distortions contained in $\boldsymbol{\delta}^R$ and $\boldsymbol{\delta}^U$ (on the right-hand side). Only when $\boldsymbol{\delta}^U = 0$ a first-best outcome emerges, consistent with the Tinbergen rule. Equation (9) offers a further refinement of the Tinbergen rule: with insufficient policy instruments, the optimal regulation equals a weighted sum of all distortions in the economy, with weights shaped by leakage elasticities.

3.3 Second-Best Policy: Imperfectly Regulated Decisions

Proposition 2 characterizes the marginal welfare impact of adjusting the regulation of imperfectly regulated decisions under the optimal second-best regulation. This result allows us to i) characterize the optimal regulation of imperfectly regulated decisions in specific scenarios and ii) determine the welfare impact of relaxing constraints on regulation — in Section 3.4.

Proposition 2. *(Second-Best Policy: Imperfectly Regulated Decisions) The optimal second-best regulation of imperfectly regulated decisions satisfies* $\frac{dW}{d\boldsymbol{\tau}^U} = \frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}^U}\boldsymbol{\mu}$*, where* $\boldsymbol{\mu}$ *is the vector of Lagrange multipliers associated with the constraints on policy instruments, and where*

$$\frac{dW}{d\boldsymbol{\tau}^U} = \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \boldsymbol{L}\right)\boldsymbol{\omega}^U, \tag{10}$$

where $\boldsymbol{I}$ *denotes an identity matrix* $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}$ *is a matrix of own-imperfectly regulated elasticities,* $\boldsymbol{\omega}^U$ *is the vector of Pigouvian wedges associated with imperfectly regulated decisions, and where we define a (Le Chatelier) matrix* $\boldsymbol{L}$ *by:*

$$\boldsymbol{L} = \left(\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right)^{-1}\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}.$$

Equation (10) decomposes the marginal value of adjusting $\boldsymbol{\tau}^U$ into two parts. First, there is the direct effect on imperfectly regulated decisions, given by $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\boldsymbol{\omega}^U$, as implied by Lemma 1a). Second, there is the indirect equilibrium effect on perfectly regulated decisions, given by $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\boldsymbol{\omega}^R = -\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\omega}^U$, as implied by combining Lemma 1a) and Proposition 1. This is a form of *reverse leakage*, which attenuates the welfare effect of regulating imperfectly regulated decisions whenever regulated and imperfectly regulated decisions are either complements or substitutes. The magnitude of the attenuating effect is proportional to the matrix $\boldsymbol{L}$ defined in the proposition. Heuristically, one can express this matrix as $\boldsymbol{L} = \frac{d\boldsymbol{x}^R}{d\boldsymbol{x}^U}\frac{d\boldsymbol{x}^U}{d\boldsymbol{x}^R}$, so that it measures the (matrix) product of leakages and reverse leakages. We refer to $\boldsymbol{L}$ as the "Le Chatelier" matrix, for reasons that will become clear in Section 3.4.

" (...) in general, nothing can be said about the direction or the magnitude of the secondary departures from optimum conditions made necessary by the original non-fulfillment of one condition".

We now discuss the insights derived from Proposition 2 in two practical scenarios: uniform regulation and convex costs of regulation. We also show how Proposition 2 generalizes the classical results on uniform regulation in Diamond (1973).

Practical Scenario: Uniform Regulation. A second common scenario of regulatory imperfections arises when a planner is forced to set the same regulation across a subset of decisions associated with different marginal distortions. Formally, we consider a planner who faces a constraint of the form $\tau_n^i = \overline{\tau}^U$ for a subset of agents i and decisions n. In this case, Proposition 2 implies that the optimal second-best regulation of imperfectly regulated decisions specializes to

$$\overline{\tau}^U = \frac{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} \left(\boldsymbol{I} - \boldsymbol{L}\right) \boldsymbol{\delta}^U}{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} \left(\boldsymbol{I} - \boldsymbol{L}\right) \boldsymbol{\iota}}, \tag{11}$$

where $\boldsymbol{\iota}$ denotes a U-dimensional vector of ones. Equation (11) shows that the optimal second-best uniform regulation $\overline{\tau}^U$ over a subset of decisions is simply a weighted sum/average of their marginal distortions $\boldsymbol{\delta}^U$, which can be equivalently written as $\overline{\tau}^U = \sum_i \sum_n w_n^i \delta_n^i$, for some weights w_n^i, which need not be strictly positive. Decisions with stronger equilibrium responses to a change in regulation — those more responsive to the regulation — carry a higher weight. As expected, when marginal distortions are symmetric (so $\boldsymbol{\delta}^U = \boldsymbol{\iota}\overline{\delta}$), Equation (11) implies that the first-best regulation $\overline{\tau}^U = \overline{\delta}$ is optimal. However, when marginal distortions are heterogeneous the first-best cannot be achieved with uniform regulation.

Practical Scenario: Convex Costs of Regulation. A third common scenario of regulatory imperfections is when increasing regulations becomes increasingly costly. Formally, we consider a planner who faces quadratic costs of regulation over a subset of decisions, with $\boldsymbol{\Phi}\left(\boldsymbol{\tau}\right) = \frac{1}{2}\boldsymbol{\tau}^{U\prime}\boldsymbol{B}\boldsymbol{\tau}^U$, for some positive definite matrix $\boldsymbol{B}$. In this case, Proposition 2 implies that the optimal second-best regulation of imperfectly regulated decisions specializes to

$$\boldsymbol{\tau}^U = \left(\boldsymbol{B} + \boldsymbol{K}\right)^{-1} \boldsymbol{K}\boldsymbol{\delta}^U, \tag{12}$$

where $\boldsymbol{K} = \left(-\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right)\left(\boldsymbol{I} - \boldsymbol{L}\right)$ is once again a key input for the optimal second-best policy.

Equation (12) shows that the optimal policy in the presence of quadratic adjustment costs is given by an attenuated version of the first-best policy. As expected, as costs vanish and $\boldsymbol{B} \to 0$, the optimal policy approaches to the first-best, so $\boldsymbol{\tau}^U \to \boldsymbol{\delta}^U$. But as costs grow relative to $\boldsymbol{K}$, the optimal policy approaches zero, so $\boldsymbol{\tau}^U \to \boldsymbol{\delta}^U$. As explained in Section 3.4, the presence of perfectly regulated decisions — by making $\boldsymbol{L}$ larger — is a force that contributes to attenuating the optimal choice of $\boldsymbol{\tau}^U$.

Connection to Diamond (1973). The insight that uniform regulation of heterogeneous externalities is characterized by a weighted sum/average of distortions can be traced back to Diamond (1973). Indeed, when *all* decisions are uniformly regulated, Equation (11) corresponds to

Diamond's result. Equation (11) generalizes his results by allowing for a subset of decisions to be perfectly regulated. In this more general case, the optimal weights account for the reverse leakage of imperfectly regulated decisions on perfectly regulated ones through the Le Chatelier matrix $\boldsymbol{L}$, as further explained in Section 3.4 below.

3.4 The Value of Relaxing Constraints on Regulation

The characterization of $\frac{dW}{d\boldsymbol{\tau}^U}$ in Proposition 2 provides the marginal welfare gain of relaxing constraints on regulation for a planner who is implementing the optimal second-best policy. Proposition 2 shows that accounting for the equilibrium welfare effects on perfectly regulated decisions boils down to adjusting the direct welfare effect by in proportion to the factor $-\boldsymbol{L}$.

Interestingly, accounting for equilibrium effects on perfectly regulated decisions *attenuates* the direct welfare effect both in the substitutes and complements cases. Heuristically, in the well-behaved case in which $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R} < 0$ and $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} < 0$, the Le Chatelier correction — via $\boldsymbol{L}$ — is positive both when perfectly and imperfectly regulated decisions are gross substitutes ($\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U} < 0$ and $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} < 0$), and gross complements ($\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U} > 0$ and $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} > 0$).

The economic intuition is as follows: If perfectly and imperfectly regulated decisions are substitutes, tightening the regulation on imperfectly regulated decisions increases perfectly regulated decisions through reverse leakage. But Proposition 1 shows that perfectly regulated decisions are underregulated at the second-best, with $\boldsymbol{\omega}^R < 0$, so this increase reduces welfare. Conversely, if perfectly and imperfectly regulated decisions are complements, tightening the regulation on imperfectly regulated decisions reduces perfectly regulated decisions through reverse leakage. But Proposition 1 shows that perfectly regulated decisions are overregulated at the second-best, with $\boldsymbol{\omega}^R < 0$, so this reduction reduces welfare. We summarize this logic in the following Corollary to Proposition 2.

Corollary. *Whenever perfectly and imperfectly regulated decisions are complements or substitutes, the welfare gains associated with relaxing constraints on regulation are attenuated relative to their direct effect.*

To illustrate this effect most clearly, consider an environment with two agents ($I = 2$) who make a single decision each ($N = 1$, so we drop the n index), and where only agent 1 is regulated, so $\tau^2 = 0$. In this case, the welfare effect of marginally increasing τ^2 above zero is

$$\frac{dW}{d\tau^2} = -\frac{dx^2}{d\tau^2}\left(1 - \underbrace{\frac{\frac{dx^2}{d\tau^1}\frac{dx^1}{d\tau^2}}{\frac{dx^1}{d\tau^1}\frac{dx^2}{d\tau^2}}}_{=L}\right)\delta^2. \tag{13}$$

Suppose that $\delta^2 > 0$ and consider the well-behaved scenario in which $\frac{dx^1}{d\tau^1} < 0$ and $\frac{dx^2}{d\tau^2} < 0$. In the substitutes case, $\frac{dx^2}{d\tau^1} < 0$ and $\frac{dx^1}{d\tau^2} < 0$, so $L > 0$ and the overall welfare gain from increasing τ^2 is smaller than its direct effect. In the complements case, $\frac{dx^2}{d\tau^1} > 0$ and $\frac{dx^1}{d\tau^2} > 0$, $L > 0$ once again.

However, when decisions are neither global complements nor substitutes — that is, when $\frac{dx^1}{d\tau^2}$ and $\frac{dx^2}{d\tau^1}$ have opposite signs — $L < 0$ and the direct effect of relaxing constraints on regulation can be amplified.

The corollary above also implies a connection between our results and the Le Chatelier principle, which we now discuss in more detail.

Connection to the Le Chatelier Principle. The Le Chatelier principle states that whenever decisions are either complements or substitutes, the long-run response of a system is larger than its short-term response — see Samuelson (1948), Milgrom and Roberts (1996) for a modern treatment, and Acemoglu (2007) and Dekel, Quah and Sinander (2023) for recent related work. As noted by Milgrom (2006), the Le Chatelier principle more generally explains how the direct effect of a parameter change is augmented by feedback in a system. While existing versions of the principle point towards *amplification* by feedback in a system, we find the opposite implication, namely *attenuation*, in terms of welfare effects. When we let our system adjust further by accounting for the welfare impact of relaxing a constraint on the perfectly regulated decisions under the second-best policy, the welfare gains from regulation are typically dampened, not amplified.

It is worth making two final observations. First, note that when $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\boldsymbol{\omega}^U > 0$, it is possible to finds scenarios in which $\frac{dW}{d\boldsymbol{\tau}^U} < 0$. That is, it is possible that adjusting the regulation of imperfectly regulated decisions towards their first-best value turns out to be welfare decreasing — in Equation (13), this occurs when $L > 1$. Proposition 2 shows that this type reversal is necessarily explained by the Le Chatelier matrix $\boldsymbol{L}$, and requires strong complementarity or substitutability. Second, note that the imperfectly regulated decisions more significantly attenuated in the presence of perfectly regulated decisions (through $\boldsymbol{L}$) have a smaller weight on the optimal uniform regulation, in Equation (11), or face a further attenuated regulation in the presence of convex costs of regulation, in Equation (12).

In summary, our theoretical results shed light on the nature of second-best corrective regulation by characterizing the optimal regulation of perfectly regulated decisions (Proposition 1) and imperfectly regulated decisions (Proposition 2), for a general class of regulatory imperfections. These results facilitate the analysis of a variety of practical scenarios. In addition, they unify and extend a suite of classical insights that have traditionally been viewed as distinct. Our results emphasize leakage elasticities and Pigouvian wedges as sufficient statistics for second-best regulation, which makes them promising for quantitative analysis of constrained optimal regulation in practice. In the next section, we develop a full quantitative application that highlights the usefulness of our approach in the context of financial regulation, a context in which regulation is notoriously imperfect.

4 Quantitative Application: Imperfect Financial Regulation

In this section, we show how to quantitatively determine second best policy in an application to financial regulation with imperfect instruments. This application provides a roadmap for translating the general principles derived in Section 3 into quantitative insights.

Initially, in Section 4.1, we present empirical measures of the leakage elasticities that — when combined with values for Pigouvian wedges — directly inform whether it is desirable to adjust traditional bank regulation in the presence of unregulated "shadow" banks. Next, in Section 4.2, we specify a full quantitative equilibrium model. Using the measured elasticities as calibration targets, we explore different scenarios that illustrate how the nature of regulatory constraints that the planner faces determines the optimal policy in this environment.

4.1 Direct Measurement

Even though we will describe a fully specified model of financial regulation in Section 4.2, we begin here by introducing only the elements of the model needed for the direct measurement of marginal welfare effects: the set of regulated and unregulated agents, their decisions, and the form of the regulation. This form of analysis – akin to a "sufficient statistics" approach (e.g., Chetty, 2009) – is helpful since it abstracts from modeling details, and delivers insights that do not require the calibration of a full equilibrium model.

We consider an economy with two bank types (banks, for short), indexed by $i \in \{R, U\}$, where R stands for regulated, traditional banks, while U represents unregulated, shadow banks. Banks make decisions $\boldsymbol{x}^i = \{k_0^i, b^i\}$, where k_0^i stands for banks' initial capital investments (e.g., investments in loans to firms or households), and b^i measures banks' debt/asset ratio or leverage, which expresses bank borrowing per unit of investment. As we discuss below, there is a variety of well-understood rationales for corrective regulation in the banking system, such as the distortions generated by government guarantees/bailouts, fire sales, and behavioral biases. We model financial regulation as a corrective tax/regulation τ_b^R on traditional banks' leverage decisions.[9] Regulation in this setting is imperfect because i) the investment/scale decision k_0^R of traditional banks, and ii) the leverage and investment decisions of shadow banks remain unregulated.

The examine the effects of a marginal increase in τ_b^R. The associated welfare effect takes the same shape as as in Lemma 1a):

$$\frac{dW}{d\tau_b^R} = \sum_{i \in \{R,U\}} \omega_k^i \frac{dk_0^i}{d\tau_b^R} + \omega_b^i \frac{db^i}{d\tau_b^R} k_0^i,$$

where $\omega_k^i = \tau_k^i - \delta_k^i$ and $\omega_b^i = \tau_b^i - \delta_b^i$ stand for the Pigouvian wedges associated with banks'

[9] As above, we proceed as if this tax is reimbursed as a lump sum directly to traditional banks, so that the regulation is equivalent to a quantity constraint

$$\underbrace{k_0^R \left(1 - b^R\right)}_{\text{bank equity/capital}} \geq \theta^R \underbrace{k_0^R}_{\text{assets}}$$

where θ^R denotes a minimum equity/capital to asset ratio. Therefore, the regulation we consider here closely reflects financial regulation in practice, whose headline tool is a capital requirement for traditional banks. Risk-weights are used in practice to adjust the right-hand side of this constraint for different types of risky investments. We abstract from risk weights because we focus on a single type of risky investment, which we interpret as residential mortgage lending in the empirical application below.

investment and leverage decisions. It is convenient to rescale the leverage wedge in this expression so that it is proportional to k_0^i. For empirical measurement, it is useful to divide by aggregate investment $K = \sum_i k_0^i$ to obtain the scale-invariant expression:

$$\frac{1}{K}\frac{dW}{d\tau_b^R} = \sum_{i\in\{R,U\}} \varphi^i \left(\omega_k^i \frac{d\log k_0^i}{d\tau_b^R} + \omega_b^i \frac{db^i}{d\tau_b^R}\right), \tag{14}$$

where $\varphi^R = \frac{k_0^R}{K}$ and $\varphi^U = \frac{k_0^U}{K}$ respectively denote traditional and shadow banks' share of aggregate investment. Our general theoretical analysis, as well as the equilibrium model with financial frictions detailed below, show that this equation characterizes marginal welfare effects in a large class of economies.

In order to quantify the welfare effects in Equation (14), we obtain direct measurement of leakage elasticities and market shares in the context of the US market for residential mortgages. Table 1 summarizes available empirical estimates. First, we use data from Jiang (2023), which shows that the unregulated share is $\varphi^U \simeq 50\%$ in residential mortgage lending.

Second, for the leakage elasticities, we employ estimates from Buchak, Matvos, Piskorski and Seru (2018), who measure the responses of traditional and shadow banks' mortgage origination to changes in traditional banks' required equity/asset ratio. The empirical setting is based on quantity constraints. The regulatory change is equivalent to a tax change $d\tau_b^R$ that increases the equity/asset ratio $\theta^R = 1-b^R$ of traditional banks by one marginal unit or, conversely, reduces b^R by one marginal unit. We therefore assume that $\frac{db^R}{d\tau_b^R} = -1$ in Equation (14).[10] The leakage elasticities $\frac{d\log k_0^i}{d\tau_b^R}$ estimated by Buchak, Matvos, Piskorski and Seru (2018) imply that this tax change is associated with an approximate 4% increase in shadow bank investment in loans, and with an approximate 5% decrease in traditional bank investment.[11] We further require an estimate for the leakage elasticity $\frac{db^U}{d\tau_b^R}$, which is challenging because shadow bank leverage b^U is not recorded in standard administrative datasets. Using recently collected data on b^U, Jiang, Matvos, Piskorski and Seru (2020) emphasize that there is no clear time trend in unregulated lenders' leverage between 2011 and 2017 — a period during which traditional bank regulation became much tighter, and shadow banks dramatically grew their market share — and that shadow banks' leverage decision is mostly explained by firm fixed effects such as size, rather than interactions with traditional bank regulation.[12] We therefore proceed as if the associated leakage is negligible, using $\frac{db^U}{d\tau_b^R} \simeq 0$ as a our

[10] More rigorously, we apply a local normalization (or, equivalently, change of numeraire). Let τ_b^R be the per-unit tax on leverage in units of date 0 consumption, and define a normalized tax $\tilde{\tau}_b^R = \chi\tau_b^R$. We can set $\chi = -\frac{db^R}{d\tau_b^R}$ to impose (locally) that $\frac{db^R}{d\tilde{\tau}_b^R} = -1$. All empirical leakage elasticities reported in this section are in terms of normalized taxes. When using these values to calibrate our quantitative model in Section E, we apply the same normalization: Given a solution to the model, we generate $\frac{d\log k_0^i}{d\tau_b^R}$ in terms of per-unit taxes, and then use $\frac{d\log k_0^i}{d\tilde{\tau}_b^R} = \frac{1}{\chi}\frac{d\log k_0^i}{d\tau_b^R}$ as the target to match empirical estimates.

[11] We derive an exact mapping between the regression results in Buchak, Matvos, Piskorski and Seru (2018) and these elasticities in Appendix D.

[12] In addition, the typical need for leverage among shadow banks in the US mortgage market arises from warehouse lines of credit, which provide short-term bridge financing for loans that shadow banks plan to sell in secondary

Table 1: Direct Measurement

Statistic	Variable	Value
Shadow Banks' Market Share	φ^U	0.5
Leakage Elasticities	$\frac{d\log k_0^U}{d\tau_b^R}$	3.9860
	$\frac{d\log k_0^R}{d\tau_b^R}$	-5.3577
	$\frac{db^U}{d\tau_b^R}$	$\simeq 0$

Note: A detailed description our calculations and assumptions is given in the text. Market shares and leakage elasticities are based on estimates in Buchak, Matvos, Piskorski and Seru (2018) and Jiang, Matvos, Piskorski and Seru (2020).

baseline estimate, and report a sensitivity analysis in Appendix D.2.

With these estimates in hand, we report welfare effects as a function of three Pigouvian wedges ω_k^R, ω_b^R and ω_k^U.[13] We parametrize $\omega_k^U = \lambda\omega_k^R$, so that λ is the relative distortions associated with shadow and traditional banks. In Figure 1, we plot the contours of the scale-free welfare effect in Equation (14), as a function of ω_k^R (horizontal axis) and ω_b^R (vertical axis), and for different values of λ (across the three panels). The thick dashed line in each panel delineates the points where $\frac{dW}{d\tau_b^R} = 0$. Lighter shaded areas delineate the region where $\frac{dW}{d\tau_b^R}$ is positive, implying that stricter leverage regulation would locally improve welfare, while looser regulation is welfare-improving in the darker areas.

The left panel in Figure 1 is a benchmark in which shadow banks impose no relevant distortions ($\lambda = 0$). In this case, the planner is not concerned with leakage from traditional to shadow banks, but does consider leakage from traditional bank leverage regulation to traditional bank investment, which remains unregulated. Since the leakage elasticity $\frac{d\log k_0^R}{d\tau_b^R} < 0$ in the data, traditional bank leverage and investment are complements. Our theoretical results suggest that it is optimal to overregulate traditional banks' leverage, as long as the planner considers investment to be underregulated ($\omega_k^R < 0$). Consistently with this intuition, the plot shows that tighter leverage regulation tends to improves welfare in this region. However, this conclusion changes when traditional bank leverage is already significantly overregulated relative to first-best ($\omega_b^R \gg 0$).

In the middle panel in Figure 1, traditional and shadow banks impose symmetric distortions ($\lambda = 1$). Welfare effects become more nuanced, as the planner trades off i) the complementarity captured by $\frac{d\log k_0^R}{d\tau_b^R} < 0$, which pushes for overregulation, against ii) the substitutability between traditional bank leverage and shadow bank investment via $\frac{d\log k_0^U}{d\tau_b^R} > 0$, which pushes for underregulation. The plot shows that, given our empirical estimates, the first effect tends to dominate. As before, tighter

markets — see Jiang (2023). The proportion of shadow bank originations that is sold is also roughly constant over time, again suggesting that their leverage decisions are independent of traditional banks' leverage regulation.

[13]The numerical range for Pigouvian wedges we consider is an interval around the range of plausible quantitative estimates from our equilibrium model below. In our baseline scenario, the wedge ω_b^U associated with shadow banks' leverage does not affect welfare.

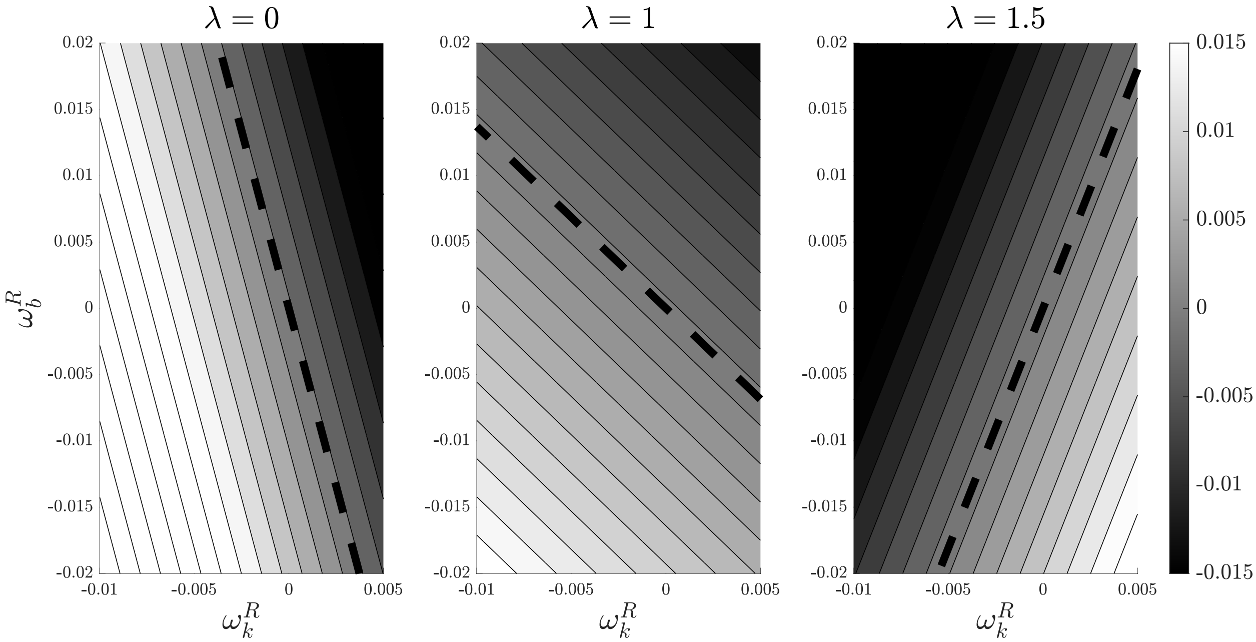

Figure 1: Marginal Welfare Effects Based on Direct Measurement

Note: This figure plots $\frac{dW}{d\tau_b^R}$, the marginal welfare effect of tighter leverage regulation, as a function of Pigouvian wedges $\left\{\omega_k^R, \omega_k^U, \omega_b^R\right\}$ using the empirical estimates for leakage elasticities from Table 1, setting $\omega_k^U = \lambda\omega_k^R$. The thick dashed lines show combinations of wedges for which $\frac{dW}{d\tau_b^R} = 0$.

leverage regulation tends to improves welfare when investment is underregulated.

In the right panel in Figure 1, shadow banks impose larger distortions than traditional banks ($\lambda = 1.5$). The conclusions of the previous two cases are reversed. Indeed, leakage to shadow bank investment, i.e., the substitutes case, becomes quantitatively more important. Thus, the plot shows that tighter leverage regulation now tends to reduce welfare (and, by symmetry, looser regulation would improve welfare), unless leverage is already significantly underregulated.

All panels highlight the role of the investment wedge ω_k^R and how it interacts with λ. When $\lambda = 0$, stricter leverage regulation improves welfare if ω_k^R is *large* in absolute value, by crowding out investment since $\omega_k^R < 0$ means that investment is currently excessive. When $\lambda = 1$, the welfare conclusions are less sensitive to the value of ω_k^R. When shadow banks impose large marginal distortion — with $\lambda = 1.5$ — tighter leverage regulation improves welfare if ω_k^R is *small* in absolute value. This is because the benefits from tightening traditional banks' leverage are offset by an investment increase by shadow banks, which is associated with a large distortion.

Our analysis so far demonstrates the role of Pigouvian wedges in driving second-best regulation, given a set of empirically observed leakage elasticities. In principle, one could attempt to measure the relevant Pigouvian wedges directly, for example, by leveraging statistical measures of systemic risk in banking (e.g., Adrian and Brunnermeier, 2016). However, our preferred approach is to combine direct measurement of leakage elasticities with a quantitative model that generates Pigouvian wedges. We turn to this approach in the next section.

4.2 Model Quantification

We now present a fully specified equilibrium model that allows us to explore how regulatory constraints quantitatively impact the optimal constrained regulation. Our model is designed to be the simplest that allows us to draw quantitative conclusions, while capturing three distinct distortions central to the design of financial regulation: i) bailouts, ii) pecuniary externalities, and iii) distorted beliefs.

The study of the equilibrium model complements our direct measurement results in Section 4.1 along three dimensions. First, the calibrated model provides quantitative estimates of the different marginal distortions and wedges, which are difficult to observe directly. Second, while direct measurement allowed us to characterize local welfare effects, the quantitative model extends these results to characterizing optimal policies. Finally, the model is also designed to illustrate further how the theoretical insights from Section 3 emerge in a quantitative framework, in particular, i) the role of complementarity vs. substitutability in perfectly and imperfectly regulated decisions, ii) the presence of unregulated or uniformly regulated decisions, and iii) the value of relaxing constraints on regulation.

4.2.1 Environment

There are three dates $t \in \{0, 1, 2\}$ and two goods: a nonstorable consumption good, which serves as numeraire, and a productive investment good/capital.[14] The state of the world has two components, denoted by (z, s). The component $z \in Z$ corresponds to an interim shock realized at date 1. We assume that $Z = \{0, 1\}$ is binary, capturing normal ($z = 0$) and crisis times ($z = 1$). The second component $s \in S \equiv [\underline{s}, \bar{s}]$, which has continuous support, is realized at date 2 and determines final payoffs.

There are four groups of agents, each in unit measure, denoted $i \in \{R, U, C, O\}$. There are two types of banks, indexed by $i \in \{R, U\}$, where R stands for traditional, regulated banks, while U represents unregulated shadow banks in our benchmark scenario. This is the same terminology used in Section 4.1. There are also creditors $i = C$ and outsiders $i = O$. All agents are risk-neutral, with preferences given by

$$c_0^i + \beta^i \mathbb{E}^i \left[c_1^i (z) + c_2^i (z, s) \right], \tag{15}$$

where $c_0^i \geq 0$, $c_1^i (z) \geq 0$, and $c_2^i (z, s) \geq 0$ denote state-contingent consumption and β^i is a discount factor — for simplicity, agents do not discount between dates 1 and 2. The operator $\mathbb{E}^i [\cdot]$ denotes subjective expectations over z and s, with the associated probabilities defined below. We assume that creditors are more patient than other agents, with $\beta^C > \beta^i \equiv \beta$ for all $i \in \{R, U, O\}$. This assumption generates gains from trade whereby creditors provide financing for investments at date 0.

[14] In the rest of this section, we refrain from calling this good "capital" because, in the context of banks, this term is reserved in practice for describing equity/asset ratios.

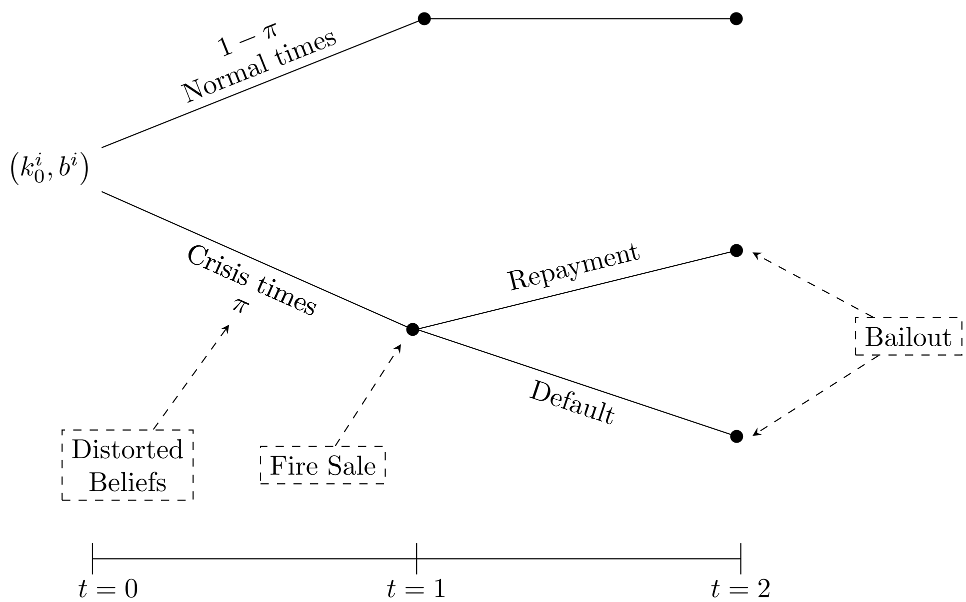

Figure 2: Model Timing

Note: This figure illustrates the timing of the equilibrium model used in Section 4.2. Crisis times occur with probability π at date 1, after banks have made investment and leverage decisions, k_0^i and b^i respectively, at date 0. Distorted beliefs, fire sales, and bailouts are the three rationales for corrective regulation in this model.

Banks. At date 0, banks decide how much to invest and how much to leverage to take. Type i banks purchase k_0^i units of durable investment/capital goods from outsiders at a competitive price p_0, and incur a convex adjustment cost $\Psi^i\left(k_0^i\right)$. Moreover, they issue debt with face value $b^i k_0^i$ that allows them to raise $Q^i\left(b^i\right)k_0^i$ at date 0, where the schedule $Q^i\left(\cdot\right)$ defines the market value of debt, i.e., a credit surface as in Dubey, Geanakoplos and Shubik (2005), for different levels of leverage b^i. Formally, type i banks face the following date 0 budget constraint:

$$c_0^i = n_0^i + Q^i\left(b^i\right)k_0^i - p_0 k_0^i - \Psi^i\left(k_0^i\right) - \tau_b^i b^i k_0^i - \hat{\tau}_k^i k_0^i + T_0^i, \tag{16}$$

where n_0^i denotes banks' initial endowment, $\tau_b^i b^i k_0^i$ and $\hat{\tau}_k^i k_0^i$ are leverage and capital investment regulations, and T_0^i corresponds to a lump-sum transfer. To preserve the scale invariance of banks' leverage decisions, we scale taxes/wedges associated with b^i by the level of investment k_0^i, so τ_b^i corresponds to the marginal tax on leverage per unit of investment. This normalization implies that, for a given b^i, the marginal investment tax is effectively given by $\tau_k^i = \tau_b^i b^i + \hat{\tau}_k^i$.[15]

At date 1 in state z, banks' initial investment yields $\rho_1^i\left(z\right)$ units of consumption good, which is used to pay back a (predetermined) fraction ζ^i of the face value of the debt. Banks can also sell part of their capital investments back to outsiders at a competitive price $p_1\left(z\right)$, by choosing

[15] As in the rest of the paper, we exploit the equivalence between quantity- and tax-based interventions, which in this case requires that $T_0^i = \tau_b^i b^i k_0^i + \hat{\tau}_k^i k_0^i$ for all i.

$k_1^i(z) \in [0, k_0^i]$. Formally, type i banks face the following date 1 budget constraint:

$$c_1^i(z) = \left(\rho_1^i(z) - \zeta^i b^i\right) k_0^i + p_1(z)\left(k_0^i - k_1^i(z)\right), \tag{17}$$

where we assume throughout that banks are able to make the early repayment $\zeta^i b^i k_0^i$ at date 1 without defaulting.

Finally, at date 2 in state s, banks make a default decision. If they do not default, they consume the payoff $\rho_2^i(s)$ from their final holding $k_1^i(z)$ of capital investments, augmented by a (predetermined) bailout $t^i(b^i, s, z) k_0^i$ provided by the government, after paying back the remaining fraction $1 - \zeta^i$ of the debt. The bailout, whose generosity scales with the initial size of the banks' investment, can be type- and state-contingent, and can depend on the level of leverage b^i. This form of modeling bailouts captures ex-post interventions without commitment. If banks default, their consumption is zero. Formally, type i banks face the following date 2 budget constraint:

$$c_2^i(z,s) = \begin{cases} \rho_2^i(s) k_1^i(z) - \left(\left(1-\zeta^i\right) b^i - t^i\left(b^i, s, z\right)\right) k_0^i, & (z,s) \in \mathcal{N}^i \\ 0, & (z,s) \in \mathcal{D}^i, \end{cases} \tag{18}$$

where $\mathcal{N}^i$ and $\mathcal{D}^i$ respectively denote the no-default and default regions, which are determined in equilibrium.

Creditors. At date 0, creditors purchase/fund a share h_i^C of the debt of type i banks. Notice that market clearing will require $h_i^C = 1$ in equilibrium, as explained below. Hence, creditors' date 0 and date 1 budget constraints are given by

$$c_0^C = n_0^C - \sum_i h_i^C Q^i\left(b^i\right) k_0^i \quad \text{and} \quad c_1^C(z) = \sum_i h_i^C \zeta^i b^i k_0^i, \tag{19}$$

where creditors take banks' decisions b^i and k_0^i as given.

At date 2, if banks default, creditors seize their investments and extract a fraction ϕ of the final payoff. The remaining fraction $1-\phi$ captures the deadweight losses associated with default. Creditors appropriate any bailout transfer in case of default. We write $\mathcal{R}^i(z,s)$ for the repayment that creditors receive from i banks in state (z,s) per unit of k_0^i, which is given by

$$\mathcal{R}^i(z,s) = \begin{cases} \left(1-\zeta^i\right) b^i & (z,s) \in \mathcal{N}^i \\ \phi \rho_2^i(s) k_1^i(z) / k_0^i + t^i\left(b^i, s, z\right), & (z,s) \in \mathcal{D}^i. \end{cases} \tag{20}$$

We assume that bailouts are funded by a distortionary tax $(1+\kappa) t^i(b^i, s, z) k_0^i$ on creditors, where $\kappa \geq 0$ represents the marginal (deadweight) cost of public funds. Hence, creditors' date 2 budget constraint is given by

$$c_2^C(z,s) = \sum_i h_i^C \mathcal{R}^i(z,s) k_0^i - (1+\kappa) \sum_i t^i\left(b^i, s, z\right) k_0^i. \tag{21}$$

Outsiders. Outsiders O have technologies that convert between the consumption good and the investment good/capital at each date, but cannot hold investment goods across dates. At date 0, outsiders produce k_0^O units of capital — which they sell to banks — at a convex cost $\Upsilon\left(k_0^O\right)$. At date 1 in state z, outsiders purchase investments $k_1^O(z)$ from banks, which yields a liquidation payoff $H\left(k_1^O(z)\right)$. Assuming that $H(\cdot)$ is concave generates a downward sloping demand curve for sold investments. Hence, outsiders' budget constraints are given by

$$c_0^O = p_0 k_0^O - \Psi^O\left(k_0^O\right) \quad \text{and} \quad c_1^O(z) = H\left(k_1^O(z)\right) - p_1(z)\, k_1^O(z), \tag{22}$$

where $c_2^O(z,s) = 0$ without loss of generality. Note that k_0^O denotes capital sold by outsiders while $k_1^O(z)$ denotes capital purchased.

4.2.2 Equilibrium and Welfare Effects

We first define and subsequently characterize an equilibrium. For a given set of regulations $\{\tau_b^i, \hat{\tau}_k^i\}$ and transfers $\{T_0^i\}$ for $i \in \{R, U\}$, an *equilibrium* consists of banks' investment and leverage decisions $\{k_0^i, k_1^i(z), b^i\}$ for $i \in \{R, U\}$, outsiders' investment decisions $\left\{k_0^O, k_1^O(z)\right\}$, creditors' debt holdings $\left\{h_R^C, h_U^C\right\}$, and consumption decisions $\{c_0^i, c_1^i(z), c_2^i(z,s)\}$ for all agents $i \in \{R, U, O, C\}$, as well as capital prices $\{p_0, p_1(z)\}$, and banks' credit surfaces $Q^i(\cdot)$ such that i) all agents maximize utility (15) taking prices and the credit surface as given subject to their budget constraints: (16), (17), and (18) for banks, (19) and (21) for creditors, and (22) for outsiders; ii) the government budget is balanced, so $\sum_i T_0^i = \sum_i \left(\tau_b^i b^i k_0^i + \hat{\tau}_k^i k_0^i\right)$;[16] iii) capital investment markets clear, with $k_0^R + k_0^U = k_0^O$ and $k_0^R + k_0^U - k_1^R(z) - k_1^U(z) = k_1^O(z)$; and iii) debt markets clear, with $h_U^C = h_R^C = 1$.

In Section E.1 of the Online Appendix, we provide a step-by-step characterization of the equilibrium and a detailed discussion of the required regularity conditions. Here, we first summarize the key equations that characterize the equilibrium. Then, in Lemma 2, we present the marginal welfare effects and marginal distortions that become inputs for the optimal policy.

Equilibrium Characterization. We characterize the solution to the model by backward induction. At date 2, banks default whenever their consumption defaulting is higher than their consumption repaying. This decision defines the no-default and default regions $\mathcal{N}^i$ and $\mathcal{D}^i$, as shown in Online Appendix E.1.[17]

At date 1, type i banks have a cash flow $\rho_1^i(z)$ per unit of investment, but need to repay $\zeta^i b^i$. Hence, if $\rho_1^i(z) < \zeta^i b^i$, banks are forced to sell a fraction $\frac{\zeta^i b^i - \rho_1^i(z)}{p_1(z)}$ of their investments to meet debt repayments. We refer to this as a *fire sale* event. If instead $\rho_1^i(z) \geq \zeta^i b^i$, banks do not sell investments and consume the remaining funds. Therefore, the amount of investments sold by type

[16] The specification of creditors' budget constraint ensures that the government also runs a balanced budget when conducting bank bailouts.

[17] Under mild regularity conditions, type i banks default whenever $s < \hat{s}^i\left(b^i, z\right)$. The default threshold $\hat{s}^i(\cdot)$ is endogenously determined and can depend on all leverage decisions.

i banks at date 1 is

$$k_0^i - k_1^i(z) = \frac{\max\{\zeta^i b^i - \rho_1^i(z), 0\}}{p_1(z)} k_0^i. \tag{23}$$

Outsiders provide a downward sloping demand for capital investments given by $p_1(z) = H'\left(k_1^O(z)\right)$. When combined with market clearing and Equation (23), it yields an equilibrium pricing function for date 1 investments in terms of z and the initial investment and leverage decisions.

At date 0, accounting for banks' default and investment sale decisions at date 1, creditors offer banks a credit surface of the form:

$$Q^i\left(b^i\right) = \beta^C\left(\zeta^i b^i + \mathbb{E}^C\left[\mathcal{R}^i(z,s)\right]\right),$$

where $\mathcal{R}^i(z,s)$ was introduced in (20), and where $\mathbb{E}^{i,C}[\cdot]$ denotes the creditors' expectation over type-i banks' payoffs. Given $Q^i(b^i)$, we show in the Appendix that banks' date 0 investment and leverage decision problem reduces to

$$V^i = \max_{k_0^i, b^i}\left\{\left(M^i\left(b^i\right) - p_0 - \tau_k^i\right)k_0^i - \Psi^i\left(k_0^i\right)\right\},$$

where the function $M^i(b^i)$ corresponds to the joint valuation of the (inside) equity and debt claims issued by type i banks. Therefore, banks' optimality conditions for optimal leverage b^i and investment k_0^i are given by

$$M^i\left(b^i\right) = p_0 + \Psi^{i\prime}\left(k_0^i\right) + \tau_k^i \tag{24}$$

$$\frac{\partial M^i\left(b^i\right)}{\partial b^i} = \tau_b^i. \tag{25}$$

Outsiders supply investment goods at date 0 according to $p_0 = \Psi^{O\prime}\left(k_0^O\right)$. When combined with market clearing and Equation (25), it yields an equilibrium pricing function for date 0 investments, closing the model. It is worth highlighting that different bank types in our model are exclusively linked through their impact on prices. At date 1, both sell investments, with sales by one type reducing the price that the other type obtains — since $H(\cdot)$ is concave. At date 0, both compete for investments, with purchases by one type increasing the price that the other type must pay.[18]

Marginal Welfare Effects and Marginal Distortions. We consider an equal-weighted utilitarian planner who, given the linearity of preferences, maximizes money-metric welfare changes. Since our model allows for heterogeneous beliefs, we assume that the planner evaluates agent utilities using a common set of probabilities. Belief distortions, along with bailouts and pecuniary externalities, are the three rationales that motivate government intervention in this model, as we

[18]It is possible to introduce further linkages via creditors' demand for debt, for example, by making creditors risk-averse — as in an earlier version of this paper — nonpecuniary preferences for debt holdings, or direct linkages, for example banks providing funding to shadow banks. These extensions do not change the fundamental structure of welfare effects characterized in Lemma 2.

explain next.

In order to connect our results to those in Section 3, we now introduce Lemma 2, which formally defines the marginal distortions and acts as the counterpart of Lemma 1a).

Lemma 2. *(Marginal Welfare Effects and Marginal Distortions) The marginal welfare effect of varying regulations* $\boldsymbol{\tau} \in \left\{\left\{\tau_b^i\right\}_i, \left\{\hat{\tau}_k^i\right\}_i\right\}$ *is given by*

$$\frac{dW}{d\boldsymbol{\tau}} = \sum_{i\in\{R,U\}} \left(\left(\tau_k^i - \delta_k^i\right)\frac{dk_0^i}{d\boldsymbol{\tau}} + \left(\tau_b^i - \delta_b^i\right)\frac{db^i}{d\boldsymbol{\tau}}k_0^i\right), \tag{26}$$

where investment and leverage marginal distortions are respectively defined by

$$\delta_k^i = \underbrace{(1+\kappa)\,\beta^C\mathbb{E}^P\left[t^i\left(b^i,s,z\right)\right]}_{=\delta_k^{i,bailout}} + \underbrace{\sum_z \pi^P(z)\,\delta_p^i(z)\,\frac{\partial p_1(z)}{\partial b_0^i}}_{=\delta_k^{i,pecuniary}} + \underbrace{M^i\left(b^i\right) - M^{i,P}\left(b^i\right)}_{=\delta_k^{i,beliefs}} \tag{27}$$

$$\delta_b^i = \underbrace{(1+\kappa)\,\beta^C\mathbb{E}^P\left[\frac{\partial t^i\left(b^i,s,z\right)}{\partial b^i}\right]}_{=\delta_b^{i,bailout}} + \underbrace{\sum_z \pi^P(z)\,\frac{\delta_p^i(z)}{k_0^i}\frac{\partial p_1(z)}{\partial k_0^i}}_{=\delta_b^{i,pecuniary}} + \underbrace{\frac{\partial M^i\left(b^i\right)}{\partial b^i} - \frac{\partial M^{i,P}\left(b^i\right)}{\partial b^i}}_{=\delta_b^{i,beliefs}}, \tag{28}$$

where $\mathbb{E}^P[\cdot]$ *and* $\pi^P(z)$ *respectively denote the planner's expectation over* (z,s) *and the planner's probability over* z*;* $\frac{\partial p_1(z)}{\partial b_0^i}$ *and* $\frac{\partial p_1(z)}{\partial k_0^i}$ *respectively correspond to the price sensitivities to changes in leverage and investment;* $\delta_p^i(z)$*, defined in (29), corresponds to the marginal pecuniary distortion, and* $M^{i,P}$ *corresponds to the joint valuation of the equity and debt claims issued by type* i *banks from the planner's perspective.*

Equation (26), which is the counterpart of Equation (6), shows that policy in this environment affects welfare through the product of Pigouvian wedges, $\omega_k^i = \tau_k^i - \delta_k^i$ and $\omega_b^i = \tau_b^i - \delta_b^i$, and policy elasticities, $\frac{dk_0^i}{d\tau}$ and $\frac{db^i}{d\tau}$. Equations (27) and (28) highlight the three sources of inefficiency in the model.

The first term in Equations (27) and (28) captures the *bailout*-induced externality that banks' decisions impose on creditors, who ultimately fund the bailout. The distortion in Equation (27) is equal to the value of the expected bailout for creditors, augmented by the fiscal cost κ. In (28), the distortion is given by the expected marginal change in the bailout induced by additional leverage.[19]

The second term in Equations (27) and (28) accounts for the *pecuniary* externalities that banks' decisions impose on other banks and outsiders via prices in fire sale events. Formally, the marginal distortion associated with a change in the price $p_1(z)$ in state z has two components, and is given by

$$\delta_p^i(z) = \underbrace{\sum_{i\in\{R,U\}}\left(\lambda_1^i(z) - \beta\right)\left(k_1^i(z) - k_0^i\right)}_{\text{distributive pecuniary externality}} + \underbrace{\beta^C(1-\phi)\sum_{i\in\{R,U\}}\rho_2^i\left(\hat{s}^i\right)k_1^i(z)\,\frac{\partial\mathcal{F}^i(z)}{\partial p_1(z)}}_{\text{frictional pecuniary externality}}, \tag{29}$$

[19] These distortions are equal to zero when there is no bailout, i.e., when $t^i\left(b^i,s,z\right)\equiv 0$.

where $\lambda_1^i(z)$ denotes marginal value of wealth for type i banks at date 1 in state s, $\rho_2^i(\hat{s}^i)$ is the date-2 payoff at the default boundary, and $\frac{\partial \mathcal{F}^i(z)}{\partial p_1(z)}$ denotes the sensitivity of the default probability to a price change in state z, explicitly defined in the Appendix. This model features two types of *pecuniary externalities.* First, an increase (decrease) in $p_1(z)$ redistributes resources from (towards) banks, who are net sellers of investments at date 1, and whose marginal utility of wealth is high, to outsiders, who are net buyers of investments and whose marginal utility is lower. This is a *distributive* pecuniary externality, using the terminology of Dávila and Korinek (2018), which is due to the lack of insurance markets, as in Lorenzoni (2008). Second, an increase (decrease) in $p_1(z)$ lowers the default threshold $\hat{s}^i$ at date 2, which changes the payoffs of creditors who suffer a deadweight loss in default. We refer to this term as a *frictional* pecuniary externality, which is akin to collateral externalities in Dávila and Korinek (2018).

The third term in Equations (27) and (28) account for the planner's desire to correct perceived *belief distortions*, or internalities. In our setting, the planner finds it desirable to regulate decisions whenever agent's beliefs and planner's beliefs differ. Notice that beliefs distortions depend on the beliefs of both banks and creditors, since they are driven by $M^i(b^i)$, as explained in Dávila and Walther (2023).

4.2.3 Calibration

In order to solve the model, we make specific functional form assumptions and parametrize the model, as we describe next.

Functional Forms. First, we describe our assumptions about the process for uncertainty and the form of the payoffs from investment. We denote the probability of crisis times ($z = 1$) assigned by banks by $\pi \in [0, 1]$. In normal times, the payoff from capital investment at dates 1 and 2 is respectively given by

$$\rho_1^i(z = 0) = \eta^i v^i \quad \text{and} \quad \rho_2^i(z = 0) = \left(1 - \eta^i\right) v^i,$$

where $v^i > 0$ captures expected payoff to investment and $\eta^i \in [0, 1]$ modulates the timing of payoffs. In crisis times, the payoffs from investments at dates 1 and 2 are respectively given by

$$\rho_1^i(z = 1) = 0 \quad \text{and} \quad \rho_2^i(z = 1) = \left(1 - \eta^i\right) v^i + \varepsilon^i,$$

where ε^i is normally distributed with mean zero and standard deviation $\sigma^i > 0$. Assuming that the date 1 payoff in crisis times is zero ensures that there is a fire sale for any $b^i > 0$ in that case.[20]

Next, we assume that banks' adjustment cost functions are isoelastic, given by $\Psi^i(k_0^i) = \psi^i \frac{(k_0^i)^{\nu^i}}{\nu^i}$, and that the outsiders cost function at date 0 is quadratic, with $\Upsilon(k_0^O) = \frac{1}{2}(k_0^O)^2$. We also assume that the outsiders date 1 technology takes a shifted isoelastic form $H(k_1^O) = \frac{(k_1^O + \varsigma)^\gamma}{\gamma}$, with

[20]Our parameterization is such that the probability of having negative realizations of payoffs is negligible. All results remain unchanged when explicitly truncating the distribution of payoffs to be non-negative.

$\gamma \leq 1$ and $\varsigma > 0$. Finally, we assume that the bailout transfer that banks receive can be written as a linear function of leverage, and is given by

$$t^i(b^i, s, z) = \alpha_0^i(z) + \alpha_b^i(z) b^i,$$

where $\alpha_0^i(z) > 0$ and $\alpha_b^i(z) > 0$ in crisis times but $\alpha_0^i(z) = \alpha_b^i(z) = 0$ in normal times. This functional form concisely captures the notion that banks expect higher bailouts when they are more levered in a crisis, reflecting a higher likelihood of eventual failure.

Parameter Values. Table 2 summarizes the parameter choices in our baseline parametrization. As in the direct measurement exercise, we select parameters consistent with US banking markets over the last decade, combining externally chosen parameters with internally calibrated targets. As we explain below, one of the regulatory scenarios we consider is constrained leverage regulation. In this regime, the planner can regulate only the leverage decision of traditional banks, while the leverage decision of shadow banks, as well as investment decisions, remain unregulated. Since the constrained leverage regulation is close to current regulatory practice, we use it as the reference case for calibration.

Interpreting the time between date 0 and dates 1 (and 2) as a year, we set a discount factor for creditors of 0.98, consistent with a 2% risk-free interest rate, and set $\beta = 0.97$ for other agents, targeting an average leverage (debt to assets) of 0.82. We assume that all agents perceive that the crisis state takes place with probability $\pi = 0.03$, or roughly every 33 years. We instead assume that the planner uses a more conservative assessment, in which a crisis state occurs every twenty years, so $\pi^P = 0.05$, consistently with Reinhart and Rogoff (2009).

The banks' adjustment cost parameters ψ^R, ψ^U, ν^R, and ν^U are jointly chosen to match the leakage elasticities reported in Table 1 — and used in our direct measurement exercise — and to ensure that the market share of shadow banks is 57%. By choosing $\varsigma = 1$, we normalize the price of investments in good times at date 1 to one, and by setting the curvature parameter of banks' date 1 technology to $\gamma = 0.15$, we can target a sizable drop in prices in a fire sale of 22%. We set the recovery parameter $\phi = 0.72$ to match a 28% average default deadweight loss, consistent with estimates from Granja, Matvos and Seru (2017). By setting $v = 1.11$, the model matches a return on average equity of around 8%, while a value of the return dispersion $\sigma = 0.10$ ensures that failure probabilities for traditional banks are 5%, slightly larger than historical averages but consistent with implied default probabilities extracted from CDS spreads.

We set $\zeta^R = 0.2$ and $\zeta^U = 0.3$ to target differences in leverage ratios between regulated and shadow banks: 0.86 and 0.79, respectively, within the range reported in Jiang, Matvos, Piskorski and Seru (2020). This choice implies that a quarter of liabilities are on average due at date 1. By imposing that $\eta^i = \zeta^i$ we ensure that the timing of investment payoffs and liabilities is aligned in normal times, ensuring that no fire sales occur. To capture differences in government guarantees between traditional and shadow banks, we impose that bailout parameters are larger for traditional banks by a factor of 2.5. Setting $\alpha_0^R = 0.033$ and $\alpha_b^R = 0.0625$ and that the expected implied bailout

Table 2: Parameter Values

	Description	Parameter(s)	Value(s)
Preferences	Discount Factor Banks/Outsiders	β	0.97
	Discount Factor Creditors	β^C	0.98
Technology (Profitability)	Probability of Crisis Times	π, π^P	$0.03, 0.05$
	Share of Early Payoff	η^R, η^U	$0.2, 0.3$
	Investment Payoff	$\upsilon^R = \upsilon^U$	1.11
	Payoff Dispersion	$\sigma^R = \sigma^U$	0.1
Technology (Costs)	Traditional Banks' Adjustment Cost	ψ^R, ν^R	$0.0058, 1.19$
	Shadow Banks' Adjustment Cost	ψ^U, ν^U	$0.0005, 2$
	Outsiders' Technology	γ, ς	$0.15, 1$
Financing	Recovery after Default	ϕ	0.72
	Share of Early Debt	ζ^R, ζ^U	$0.2, 0.3$
Bailout	Bailout Generosity	$\alpha_0^R, \alpha_0^U, \alpha_b^R, \alpha_b^U$	$0.033, 0.022, 0.1095, 0.073$
	Marginal Cost of Public Funds	κ	0.15

Note: This table summarizes the choice of parameter value used in Section 4.

to equity ratio is of 2% for traditional banks, where 70% of the bailout is untargeted, while 30% is conditional on b. Based on Dahlby (2008), we set the net marginal cost of public funds to $\kappa = 0.15$.

4.2.4 Optimal Policy: Quantitative Analysis

We can now use the calibrated model to explore the quantitative impact of different regulatory constraints on the optimal regulation. Table 3 summarizes our quantitative results, which we describe next.

We initially contrast three regulatory scenarios. First, we consider the *first-best* scenario, in which all leverage and investment decisions are perfectly regulated. In this case the regulator can independently choose τ_b^i and τ_k^i for both traditional and shadow banks. This scenario illustrates the Pigouvian principle described in Lemma 2b). Second, we consider an *unconstrained-leverage-regulation* scenario, in which the regulator can freely set leverage regulation on both types of banks, but investment regulation is restricted to be zero. This scenario captures the fact that, as we discussed in Section 4.1, banking regulations are typically expressed in the form of leverage ratios, leaving scale of banks' investments unregulated. Third, we consider the *constrained-leverage-regulation* scenario, in which the planner can exclusively regulate directly the leverage decision of traditional banks. This is the scenario that maps more closely to a modern regulatory system, in which traditional banks are subject to leverage regulation via capital requirements, while shadow banks are not subject to direct regulation. For that reason, we use it as the reference for the model calibration.

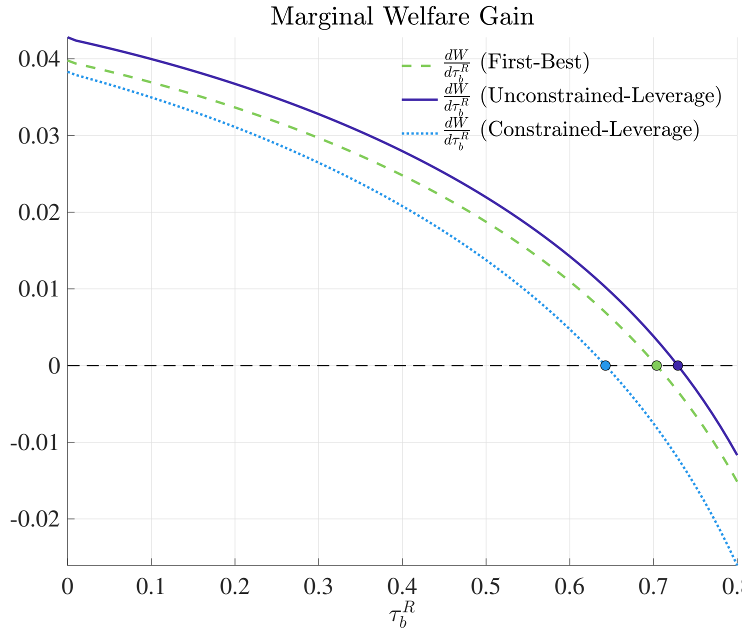

Figure 3: First-Best vs. Unconstrained-Leverage-Regulation vs. Constrained-Leverage-Regulation

Note: This figure shows the marginal welfare gain of adjusting the leverage regulation on R banks for three different scenarios. The green dashed line corresponds to the first-best scenario, in which all decisions are perfectly regulated. The solid blue line corresponds to the unconstrained-leverage-regulation scenario, in which investment regulations are constrained to be zero. The dotted light blue line corresponds to the constrained leverage scenario, in which the only available regulation is on the leverage decision of R banks. In the first-best and unconstrained-leverage-regulation scenarios, the remaining unconstrained regulations are held constant at their optimal value. The regulation τ_b^R is reported in percentage points.

First Best. In the first-best scenario, as expected, the optimal regulation discourages the leverage and investment decisions of both traditional and shadow banks, perfectly targeting the (strictly positive) marginal distortions characterized in Lemma 2. Traditional banks' leverage decisions are more tightly regulated since in our calibration they take on substantially more leverage, which makes their bailout-induced marginal distortions larger.

Unconstrained Leverage Regulation. In the unconstrained-leverage-regulation scenario, the planner can impose leverage regulation on both traditional and shadow banks, but is not able to regulate investment directly. The optimal regulation *overregulates* the leverage decisions of traditional banks but *underregulates* the leverage decisions of shadow banks relative to the first best. As implied by Proposition 1, both results are due to contrasting leakage elasticities with different signs. In this case, the second-best optimal Pigouvian wedges $\tau_b^R - \delta_b^R$ and $\tau_b^U - \delta_b^U$ in this case are — treating $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ in Equation (8)as an identity matrix — proportional to

$$\tau_b^R - \delta_b^R \propto -\delta_k^R \underbrace{\frac{dk_0^R}{d\tau_b^R}}_{<0} - \delta_k^U \underbrace{\frac{dk_0^U}{d\tau_b^R}}_{>0} \quad \text{and} \quad \tau_b^U - \delta_b^U \propto -\delta_k^R \underbrace{\frac{dk_0^R}{d\tau_b^U}}_{>0} - \delta_k^U \underbrace{\frac{dk_0^U}{d\tau_b^U}}_{<0}.$$

Table 3: Optimal Policies

	Optimal Regulation				Leverage		Investment	
Regulatory Scenario	τ_b^R	τ_b^U	τ_k^R	τ_k^U	b^R	b^U	k_0^R	k_0^U
First Best	0.704	0.588	0.959	0.949	0.855	0.749	0.454	0.614
Unconstrained Leverage	0.729	0.577	0	0	0.851	0.750	0.481	0.596
Constrained Leverage	0.643	0	0	0	0.860	0.790	0.460	0.618
Uniform Leverage	0.650		0	0	0.862	0.743	0.528	0.550

Note: All regulations, τ_b^R, τ_b^R, τ_b^R, and τ_b^R, are reported in percentage points.

In this model, the investment and leverage decisions within a bank are gross complements, that is, policies that discourage leverage for a bank reduce that bank's investment in equilibrium, so $\frac{dk_0^R}{d\tau_b^R}$ and $\frac{dk_0^U}{d\tau_b^U}$ are negative. This occurs because a tighter leverage regulation decreases the overall marginal valuation of investment, captured by $M^i\left(b^i\right)$. This form of complementarity calls for overregulating leverage. However, our model, calibrated to match the substitutability between leverage and investment decisions across banks, so $\frac{dk_0^U}{d\tau_b^R}$ and $\frac{dk_0^R}{d\tau_b^U}$ are positive. With δ_k^R and δ_k^U roughly equal, the complementarity force dominates for traditional banks, optimally overregulating leverage, while the substitutability dominates for shadow banks, optimally underregulating leverage. The solid blue line in Figure 3, to the right of the green dashed first-best line, illustrates this phenomenon for regulated banks, for which the optimal τ_b^R with unconstrained leverage regulation is greater than at the first-best.

Constrained Leverage Regulation. In the constrained-leverage-regulation scenario, the planner is also unable to regulate the leverage decisions of shadow banks. The optimal regulation *underregulates* the leverage decisions of traditional banks relative to the first-best and the constrained-leverage-regulation scenarios. Once again, we can make use of Proposition 1 to trace back this result to the three relevant leakage elasticities. In this case, similar to our discussion in Section 4.1, the Pigouvian wedge $\tau_b^R - \delta_b^R$ is exactly proportional to

$$\tau_b^R - \delta_b^R \propto -\delta_k^R \underbrace{\frac{dk_0^R}{d\tau_b^R}}_{<0} - \delta_b^U \underbrace{\frac{db^U}{d\tau_b^R}}_{\approx 0} k_0^U - \delta_k^U \underbrace{\frac{dk_0^U}{d\tau_b^R}}_{>0}.$$

First, as in the unconstrained-leverage-regulation scenario, the own-investment leakage elasticity $\frac{dk_0^R}{d\tau_b^R}$ among traditional banks features gross complementarity, pushing towards overregulation. Second, the planner considers the leakage to shadow banks' leverage, captured by $\frac{db^U}{d\tau_b^R}$. This effect is quantitatively negligible, since our calibration is designed to match the relevant leakage elasticity in Table 1. Finally, the planner considers the cross-investment leakage $\frac{dk_0^U}{d\tau_b^R}$. This effect is also a case of substitutes, largely because traditional and shadow banks compete for investments at $t = 0$ and, therefore, a tighter constraint on traditional banks lowers the initial price of capital investments and encourages shadow banks to increase investment. Given our calibration, the third force quantitatively dominates, justifying the optimally underregulation of shadow banks. The dotted light blue line in Figure 3, to the right of the green dashed first-best line, illustrates this phenomenon, with the optimal policy in the constrained-leverage to the left of the unconstrained-leverage and the first-best scenarios.

Comparison. Our results in this application highlight that leakage elasticities featuring both substitutes and complements naturally emerge in common regulatory scenarios, so a quantitative analysis that incorporates different banks' different decisions is necessary. Quantitatively, Table 3 shows that the leverage ratio of shadow banks in the constrained-leverage-regulation scenario is 0.79, while it would be roughly 0.75 in the first-best and unconstrained-leverage-regulation scenarios, a substantial difference. The difference in shadow bank investment is somewhat smaller but also significant, going from 0.767 to 0.735, or roughly a 4% reduction. This reduction is partly compensated by a roughly 3% increase in traditional banks' investment, from 0.581 to 0.601.

Uniform Leverage Regulation. We also consider an alternative regulatory scenario in which the planner imposes the same leverage regulation on traditional and shadow banks. This is illustrated in Figure 4. This figure shows the marginal welfare gain of adjusting different regulatory instruments to compare the unconstrained-leverage-regulation scenario with the uniform-leverage-regulation scenario. The green dashed line and the dotted blue line correspond to the marginal welfare gain associated with independently varying τ_b^R and τ_b^U, respectively, while the other is held at the optimal value in the unconstrained-leverage-regulation scenario. The solid blue line corresponds to the marginal welfare gain associated with adjusting the uniform regulation $\tau_b^R = \tau_b^U = \bar{\tau}_b$. Consistent with Equation (11), the optimal uniform regulation is a weighted average of the regulation the planner would set if chosen freely.

Relaxing Constraints on Regulation. Finally, we consider a welfare analysis of the hypothetical scenario in which, starting from the optimal regulation in the constrained-leverage-regulation scenario, we relax the regulatory constraint on shadow bank leverage until it approaches its optimal value in the unconstrained-leverage-regulation scenario. This exercise illustrates the effects of relaxing regulatory and demonstrates the role played by leakage and reverse leakage elasticities. Figure 5 provides an illustration of our results.

The solid yellow line shows the overall marginal welfare effect, which is initially positive, but approaches zero as τ_b^U approaches its optimum value. However, this well-behaved overall welfare gain masks multiple effects, as we explain next. Formally, we show in the appendix the marginal welfare gain of increasing τ_b^U can be expressed as

$$\frac{dW}{d\tau_b^U} = \left(\tau_b^U - \delta_b^U\right) \underbrace{\frac{db^U}{d\tau_b^U}}_{<0} k_0^U + \left(\tau_b^R - \delta_b^R\right) \underbrace{\frac{db^R}{d\tau_b^U}}_{\gtrsim 0} k_0^R - \delta_k^U \underbrace{\frac{dk_0^U}{d\tau_b^U}}_{<0} - \delta_k^R \underbrace{\frac{dk_0^R}{d\tau_b^U}}_{>0} .$$

This marginal gain consists of four terms. First, there is the immediate direct effect of tightening

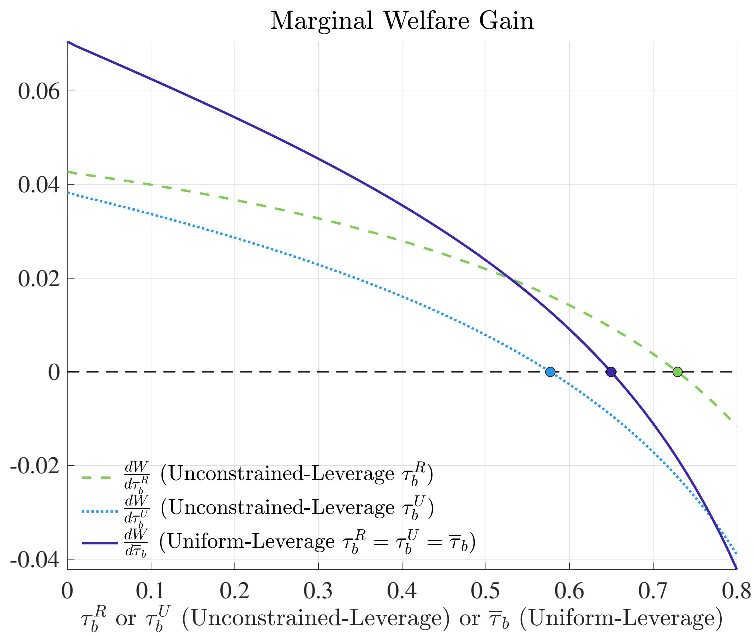

Figure 4: Uniform-Leverage-Regulation

Note: This figure shows the marginal welfare gain of adjusting different regulatory instruments to compare the the unconstrained-leverage-regulation scenario with the uniform-leverage-regulation scenario. The green dashed line and the dotted blue line correspond to the marginal welfare gain associated with independently varying τ_b^R and τ_b^U, respectively, while the other is held at the optimal value in the unconstrained-leverage-regulation scenario. The solid blue line corresponds to the marginal welfare gain associated with adjusting the uniform regulation $\tau_b^R = \tau_b^U = \overline{\tau}_b$. Consistent with Equation (11), the optimal uniform regulation is a weighted average of the regulation the planner would set if chosen freely.

leverage regulation on shadow banks, reducing their leverage ($\frac{db^U}{d\tau_b^U} < 0$). Since shadow banks are underregulated, this direct effect contributes positively to $\frac{dW}{d\tau_b^U}$, and is quantitatively important. Second, there is the reverse leakage effect on the leverage of traditional banks. In our calibration, it turns out that $\frac{db^R}{d\tau_b^U}$ is slightly positive but almost negligible, which explains why we plot using a different vertical axis. Since the Pigouvian wedge $\tau_b^R - \delta_b^R$ switches from negative (underregulation) to positive (overregulation) as the economy transitions from the constrained-leverage-regulation scenario to the unconstrained-leverage-regulation scenario, this term initially contributes positively to $\frac{dW}{d\tau_b^U}$ but eventually becomes negative.[21] Third, there is the impact on shadow banks' investment, which is a gross complement as explained above, so $\frac{dk_0^U}{d\tau_b^U} < 0$. Since shadow banks' investment is underregulated, this effect also contributes positively to $\frac{dW}{d\tau_b^U}$. Finally, there is the impact on traditional banks' investment, which in this model is the key source of substitutability among banks decisions, so $\frac{dk_0^R}{d\tau_b^U}$. Since traditional banks' investment is underregulated, this effect contributes negatively to $\frac{dW}{d\tau_b^U}$. Quantitatively, the last two effects working on opposite directions roughly cancel.

[21]In the Online Appendix, we show how to express $\frac{dW}{d\tau_b^U}$ in terms of elements of the Le Chatelier matrix $\boldsymbol{L}$ introduced in Section 3.

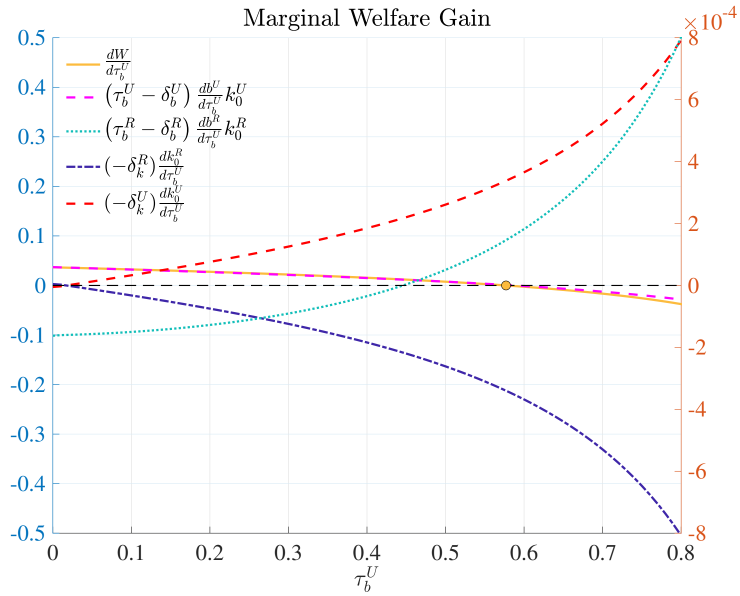

Figure 5: Relaxing Constraint on Leverage Regulation

Note: This figure shows the marginal welfare gains of increasing the leverage regulation on shadow banks starting from the constrained-leverage-regulation scenario, in which $\tau_b^U = 0$. The welfare gains of increasing τ_b^U are positive and decreasing. The direct effect of increasing τ_b^U is larger than the overall, consistently with our theoretical results. The reverse leakage effect is negative, attenuating the direct gains from regulation. The dotted green line $\left(\tau_b^R - \delta_b^R\right) \frac{db^R}{d\tau_b^U} k_0^R$ is plotted on the right axis.

This analysis illustrates the direct effects of relaxing constraints on regulation, as well as the attenuation of these effects due to reverse leakage, both of which are part of our general characterization in Proposition 2. As in our general theory, reverse leakage effects attenuate $\frac{dW}{d\tau_b^U}$ at the margin, because regulating shadow bank pushes distortive activities back into the regulated system, which is underregulated at the second-best optimum. Quantitatively, however, the attenuation is relatively weak in this application, so that our model suggests substantial local gains from imposing regulatory constraints on shadow banks.

5 Further Applications

In the Online Appendix, we present several additional applications, which we briefly describe here.

Financial Regulation with Environmental Externalities. First, we leverage our results to provide new insights into the question of financial regulation in the presence of environmental externalities, which has only recently received interest in academic and policy circles, and remains underexplored. We develop a model in which investors choose the scale of their risky investment, the composition of their portfolios, and their leverage. The planner controls a risk-weighted capital requirement, which effectively regulates investors' leverage and portfolio composition, but not the scale of their investments. Since leverage and the investment scale are gross complements, we find that overregulating leverage is optimal. We compare optimal policy under a narrow/financial mandate that only considers financial stability externalities, and a broad mandate that accounts for environmental externalities. We demonstrate that the optimal regulation is substantially different once we account for the imperfections inherent in current regulatory regimes. One implication is that it is natural to adjust risk weights, as opposed to leverage caps, when regulators become concerned with broader environmental mandates.

Minimal Applications. Finally, we study four minimal applications. Each application is designed to be the simplest one that illustrates the form of the second-best policy under a particular regulatory constraint. Table 4 provides a summary. Applications 1, 2 and 4 provide further insights on forces that are at work in our general quantitative model. Application 1 is an alternative model of shadow banking in which traditional and shadow banks compete for funding from outside investors. Application 2 isolates macro-prudential leverage regulation motivated by beliefs distortions, as in Dávila and Walther (2023), when investment scale is unregulated. Application 4 isolates fire sales externalities and analyzes the optimal uniform regulation of heterogeneous investors. In Application 3, we introduce a new feature relative to our quantitative model, namely, investors who make a portfolio choice across different investments. Regulation is constrained to be uniform across different investments, which captures asset substitution problems (Jensen and Meckling, 1976). The optimal regulation is a weighted average of the downside distortions imposed by different types of investment, with weights proportional to policy elasticities. Our general formula reveals that optimal weights are closely related to the elasticity of the probability of receiving government support.

Table 4: Summary of Minimal Applications

Application		Instrument	I	N
#1	Shadow Banking	Unregulated Investors	2	1
#2	Behavioral Distortions	Unregulated Decisions	1	2
#3	Asset Substitution	Uniform Decision Regulation	1	2
#4	Pecuniary Externalities	Uniform Investor Regulation	2	1

Note: The column I denotes the number of investors and the column N denotes the number of decisions.

6 Conclusion

This paper has systematically studied optimal corrective regulation with imperfect instruments. We have shown that leakage elasticities from perfectly regulated to imperfectly regulated decisions, along with Pigouvian wedges, are sufficient statistics to determine the optimal regulation of perfectly regulated decisions. Notably, the optimal second-best policy hinges on whether perfectly and imperfectly regulated decisions are gross substitutes or complements. Reverse leakage elasticities from imperfectly to perfectly regulated decisions influence the optimal regulation of imperfectly regulated decisions and determine the social value of relaxing constraints on regulation — a novel instance of the Le Chatelier principle. We have explicitly characterized the optimal second-best policy in three practical scenarios: unregulated decisions, uniform regulation, and convex costs of regulation.

The quantitative application of the paper demonstrates the value of our approach in the context of optimal financial regulation, an environment with notoriously imperfect regulatory instruments. In a direct measurement exercise, we present empirical counterparts of the leakage elasticities that directly inform whether it is desirable to adjust leverage regulation in the presence of unregulated shadow banks.[22] Within a quantitative model of financial regulation that encompasses multiple rationales for policy intervention, we explore how different regulatory constraints quantitatively impact the optimal regulation.

Appendix

A Proofs and Derivations: Section 3

Proof of Lemma 1a). (Marginal Welfare Effects of Regulation)

Proof. First, we characterize the change in agent i's indirect utility, denoted by V^i, when varying a specific regulation τ_n^j. Making use of the envelope theorem, we have

$$\frac{dV^i}{d\tau_n^j} = \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} \cdot \frac{d\bar{\boldsymbol{x}}}{d\tau_n^j} - \lambda^i \left(\frac{d\boldsymbol{p}}{d\tau_n^j} \cdot \left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) + \frac{d\boldsymbol{\tau}^i}{d\tau_n^j} \cdot \boldsymbol{x}^i - \frac{dT^i}{d\tau_n^j} \right),$$

where λ^i denotes the Lagrange multiplier associated with the budget constraint. We use x and $\bar{x}$ equivalently going forward, since the are equal in equilibrium.

Expressing this welfare change in money-metric terms by normalizing by agent i's marginal value of wealth, λ^i, and aggregating across agents, we have

$$\begin{aligned}
\frac{dW}{d\tau_n^j} = \sum_i \frac{\frac{dV^i}{d\tau_n^j}}{\lambda^i} &= \sum_i \frac{1}{\lambda^i} \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} \cdot \frac{d\boldsymbol{x}}{d\tau_n^j} - \frac{d\boldsymbol{p}}{d\tau_n^j} \cdot \sum_i \left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) - \sum_i \left(\frac{d\boldsymbol{\tau}^i}{d\tau_n^j} \cdot \boldsymbol{x}^i - \frac{dT^i}{d\tau_n^j} \right) \\
&= \sum_i \frac{1}{\lambda^i} \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} \cdot \frac{d\boldsymbol{x}}{d\tau_n^j} + \sum_i \frac{d\boldsymbol{x}^i}{d\tau_n^j} \cdot \boldsymbol{\tau}^i,
\end{aligned}$$

where the last line follows from i) market clearing, $\sum_i \left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) = 0$, and ii) the fact that the planner's budget is balanced, so

$$\sum_i \frac{dT^i}{d\tau_n^j} = \sum_i \frac{d\boldsymbol{\tau}^i}{d\tau_n^j} \cdot \boldsymbol{x}^i + \sum_i \frac{d\boldsymbol{x}^i}{d\tau_n^j} \cdot \boldsymbol{\tau}^i.$$

[22] A growing empirical literature in financial economics and beyond studies leakages and unintended consequences of regulation. A broader implication of our direct measurement approach is that that reduced-form estimates from this literature can be directly used to make welfare-relevant decisions for optimal constrained corrective policy independently of model details. This insight not only highlights the usefulness of existing empirical estimates for rigorous normative analysis, but also fosters a closer link between the empirical and theoretical literatures on financial intermediation and crises, which are often perceived as quite disjoint.

Note that $\frac{dW}{d\tau_n^j}$ can be equivalently expressed as

$$\frac{dW}{d\tau_n^j} = \sum_i \frac{1}{\lambda^i} \sum_\ell \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}^\ell} \cdot \frac{d\boldsymbol{x}^\ell}{d\tau_n^j} + \sum_\ell \frac{d\boldsymbol{x}^\ell}{d\tau_n^j} \cdot \boldsymbol{\tau}^\ell = \sum_\ell \frac{d\boldsymbol{x}^\ell}{d\tau_n^j} \left(\boldsymbol{\tau}^\ell - \boldsymbol{\delta}^\ell\right),$$

where $\boldsymbol{\delta}^\ell = -\sum_i \frac{1}{\lambda^i} \frac{\partial u^i}{\partial \boldsymbol{x}^\ell}$, so by switching indexes and stacking we find that

$$\frac{dW}{d\boldsymbol{\tau}} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} (\boldsymbol{\tau} - \boldsymbol{\delta}) = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} \boldsymbol{\omega},$$

as in Equation (6) in the text. □

Proof of Lemma 1b). (First-Best Policy/Pigouvian Principle)

Proof. The optimal first-best regulation is characterized by

$$\frac{dW}{d\boldsymbol{\tau}} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} \boldsymbol{\omega} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} (\boldsymbol{\tau} - \boldsymbol{\delta}) = 0,$$

which defines a system of homogeneous linear equations in $\boldsymbol{\omega}$. If the matrix of policy elasticities $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ is invertible (i.e., has full rank), the only solution to this system is the trivial solution, in which $\boldsymbol{\omega} = 0$ and $\boldsymbol{\tau}^\star = \boldsymbol{\delta}$. □

Proof of Proposition 1. (Second-Best Policy: Perfectly Regulated Decisions)

Proof. At the second-best optimum, it must be that the marginal welfare effects of adjusting the regulation of perfectly regulated decisions (those for which the planning constraints do not bind), satisfy $\frac{dW}{d\boldsymbol{\tau}^R} = 0$. Leveraging Lemma 1a), we can express these optimality conditions as

$$\frac{dW}{d\boldsymbol{\tau}^R} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}^R} (\boldsymbol{\tau} - \boldsymbol{\delta}) = \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} \left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) + \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R} \left(\boldsymbol{\tau}^R - \boldsymbol{\delta}^R\right) = 0.$$

Assuming that the matrix $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ is invertible, we rearrange this expression as follows:

$$\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R} \left(\boldsymbol{\tau}^R - \boldsymbol{\delta}^R\right) = -\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} \left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) \iff \boldsymbol{\tau}^R = \boldsymbol{\delta}^R - \left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} \left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right),$$

which corresponds to Equation (8) in the text. □

Proof of Proposition 2. (Second-Best Policy: Imperfectly Regulated Decisions)

Proof. Leveraging Lemma 1a), we can express the marginal welfare effects of adjusting the regulation of imperfectly regulated decisions (those for which the planning constraints bind) as

$$\frac{dW}{d\boldsymbol{\tau}^U} = \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} \left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) + \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U} \left(\boldsymbol{\tau}^R - \boldsymbol{\delta}^R\right).$$

So defining the vector of Lagrange multipliers associated with the regulatory constraints by $\boldsymbol{\mu}$, the optimality conditions for such decisions are given by $\frac{dW}{d\boldsymbol{\tau}^U} = \frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}^U} \boldsymbol{\mu}$, along with $\boldsymbol{\Phi}\left(\boldsymbol{\tau}^U\right) = 0$ when $\boldsymbol{\Phi}(\cdot)$ captures constraints.

From Proposition 1, we have that

$$\boldsymbol{\tau}^R - \boldsymbol{\delta}^R = \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} \left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right),$$

so combining the last two equations we obtain

$$\begin{aligned}
\frac{dW}{d\boldsymbol{\tau}^U} &= \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} \left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) - \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U} \left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} \left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) \\
&= \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} \left(\boldsymbol{I} - \underbrace{\left(\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right)^{-1} \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U} \left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}}_{\equiv \boldsymbol{L}}\right) \left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) = \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} \left(\boldsymbol{I} - \boldsymbol{L}\right) \boldsymbol{\omega}^U,
\end{aligned}$$

which corresponds to Equation (10) in the text. □

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Online Appendix

Section B of this Online Appendix provides explicit definitions of the vectors and matrices used in the paper. Section C presents additional proofs and derivations. Sections E and D include additional material related to the quantitative application in Section 4.

B Matrix Definitions

The consumption bundle, the endowment, the set of regulations and marginal distortions that pertain agent i, as well as commodity prices, can be expressed as $N \times 1$ vectors $\boldsymbol{x}^i$, $\boldsymbol{e}^i$, $\boldsymbol{\tau}^i$, $\boldsymbol{\delta}^i$, and $\boldsymbol{p}$, as follows:

$$\boldsymbol{x}^i = \begin{pmatrix} x_1^i \\ \vdots \\ x_n^i \\ \vdots \\ x_N^i \end{pmatrix}_{N\times 1}, \quad \boldsymbol{e}^i = \begin{pmatrix} e_1^i \\ \vdots \\ e_n^i \\ \vdots \\ e_N^i \end{pmatrix}_{N\times 1}, \quad \boldsymbol{\tau}^i = \begin{pmatrix} \tau_1^i \\ \vdots \\ \tau_n^i \\ \vdots \\ \tau_N^i \end{pmatrix}_{N\times 1}, \quad \boldsymbol{\delta}^i = \begin{pmatrix} \delta_1^i \\ \vdots \\ \delta_n^i \\ \vdots \\ \delta_N^i \end{pmatrix}_{N\times 1}, \text{ and } \boldsymbol{p} = \begin{pmatrix} p_1 \\ \vdots \\ p_n \\ \vdots \\ p_N \end{pmatrix}_{N\times 1}.$$

We collect consumption bundles, regulations, and marginal distortions associated with decisions for all agents in $IN \times 1$ vectors, as follows:

$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}^1 \\ \vdots \\ \boldsymbol{x}^i \\ \vdots \\ \boldsymbol{x}^I \end{pmatrix}_{IN\times 1}, \quad \boldsymbol{\tau} = \begin{pmatrix} \boldsymbol{\tau}^1 \\ \vdots \\ \boldsymbol{\tau}^i \\ \vdots \\ \boldsymbol{\tau}^I \end{pmatrix}_{IN\times 1}, \quad \text{and} \quad \boldsymbol{\delta} = \begin{pmatrix} \boldsymbol{\delta}^1 \\ \vdots \\ \boldsymbol{\delta}^i \\ \vdots \\ \boldsymbol{\delta}^I \end{pmatrix}_{IN\times 1},$$

where the vectors $\bar{\boldsymbol{x}}^i$ and $\bar{\boldsymbol{x}}$ are defined analogously to $\boldsymbol{x}^i$ and $\bar{\boldsymbol{x}}$.

The marginal welfare effects of varying all regulations, $\frac{dW}{d\boldsymbol{\tau}}$, is given by

$$\frac{dW}{d\boldsymbol{\tau}} = \begin{pmatrix} \frac{dW}{d\boldsymbol{\tau}^1} \\ \vdots \\ \frac{dW}{d\boldsymbol{\tau}^j} \\ \vdots \\ \frac{dW}{d\boldsymbol{\tau}^I} \end{pmatrix}_{IN\times 1}, \quad \text{where} \quad \frac{dW}{d\boldsymbol{\tau}^j} = \begin{pmatrix} \frac{dW}{d\tau_1^j} \\ \vdots \\ \frac{dW}{d\tau_n^j} \\ \vdots \\ \frac{dW}{d\tau_N^j} \end{pmatrix}_{N\times 1},$$

and where $\frac{dW}{d\tau_n^j}$ denotes the marginal welfare effect of varying the regulation associated with decision n by agent j. Note that

$$\frac{dW}{d\boldsymbol{\tau}^j} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}^j}\boldsymbol{\omega} = \sum_i \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\boldsymbol{\omega}^i = \sum_i \sum_n \frac{dx_n^i}{d\boldsymbol{\tau}^j}\left(\tau_n^i - \delta_n^i\right).$$

The Jacobian matrix of decisions $\boldsymbol{x}$ with respect to $\boldsymbol{\tau}$, of dimension $IN \times IN$, is given by

$$\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} = \begin{pmatrix} \frac{d\boldsymbol{x}^1}{d\boldsymbol{\tau}^1} & \cdots & \frac{d\boldsymbol{x}^I}{d\boldsymbol{\tau}^1} \\ \vdots & \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} & \vdots \\ \frac{d\boldsymbol{x}^1}{d\boldsymbol{\tau}^I} & \cdots & \frac{d\boldsymbol{x}^{\mathcal{I}}}{d\boldsymbol{\tau}^I} \end{pmatrix}_{IN\times IN}, \quad \text{where} \quad \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} = \begin{pmatrix} \frac{dx_1^i}{d\tau_1^j} & \cdots & \frac{dx_N^i}{d\tau_1^j} \\ \vdots & \frac{dx_n^i}{d\tau_{n'}^j} & \vdots \\ \frac{dx_1^i}{d\tau_N^j} & \cdots & \frac{dx_N^i}{d\tau_N^j} \end{pmatrix}_{N\times N}.$$

In particular, the Jacobian matrix $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$, of dimensions $R \times U$, can be written as

$$\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} = \begin{pmatrix} & \cdots & \\ \vdots & \frac{dx_n^i}{d\tau_{n'}^j} & \vdots \\ & \cdots & \end{pmatrix}_{U\times R},$$

where the decisions are such that (i, n) are imperfectly regulated and (j, n') are perfectly regulated. One can similarly define $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}$, $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}$, and $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$, with dimensions $U \times U$, $U \times R$, and $R \times R$ respectively, by switching the sets of coefficients. This allows us to express $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ when needed as

$$\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} = \begin{pmatrix} \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R} & \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} \\ \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U} & \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} \end{pmatrix}_{IN\times IN}.$$

We express the Jacobian of the constraints, $\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}}$, a matrix of dimension $IN \times M$, as follows

$$\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}} = \begin{pmatrix} \frac{d\Phi^1}{d\boldsymbol{\tau}^1} & \cdots & \frac{d\Phi^M}{d\boldsymbol{\tau}^1} \\ \vdots & \frac{d\Phi^m}{d\boldsymbol{\tau}^j} & \vdots \\ \frac{d\Phi^1}{d\boldsymbol{\tau}^I} & \cdots & \frac{d\Phi^M}{d\boldsymbol{\tau}^I} \end{pmatrix}_{IN\times M}, \quad \text{where} \quad \frac{d\Phi^m}{d\boldsymbol{\tau}^j} = \begin{pmatrix} \frac{d\Phi^m}{d\boldsymbol{\tau}_1^j} \\ \vdots \\ \frac{d\Phi^m}{d\boldsymbol{\tau}_N^j} \end{pmatrix}_{N\times 1},$$

and the vector of Lagrange multipliers associated with the constraints, $\boldsymbol{\mu}$, by

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_M \end{pmatrix}_{M\times 1}.$$

Finally, we define the vector $\frac{\partial u^i}{\partial \bar{\boldsymbol{x}}}$ (analogously for $\frac{\partial u^i}{\partial \boldsymbol{x}}$) as

$$\frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} = \begin{pmatrix} \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}^1} \\ \vdots \\ \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}^j} \\ \vdots \\ \frac{\partial u^i}{\partial \bar{x}^I} \end{pmatrix}_{IN\times 1}, \quad \text{where} \quad \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}^j} = \begin{pmatrix} \frac{\partial u^i}{\partial \bar{x}_1^j} \\ \vdots \\ \frac{dW}{d\bar{x}_n^j} \\ \vdots \\ \frac{dW}{d\bar{x}_N^j} \end{pmatrix}_{N\times 1}.$$

C Additional Proofs and Derivations

C.1 Production Economies

In this appendix, we extend our results to production economies. This formulation allows for arbitrary consumption and production externalities across firms and agents/consumers. There is a finite number of agent types $i \in \mathcal{I}$, each in unit measure, a finite number of firm types $j \in \mathcal{J}$, and there are ℓ goods. Agent i is endowed with a vector $\boldsymbol{e}^i$ of goods. Her preferences are represented by

$$u^i\left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}\right),$$

where $\boldsymbol{x}^i$ is the agent's consumption bundle, $\bar{\boldsymbol{x}} = \left(\bar{\boldsymbol{x}}^i\right)_{i \in \mathcal{I}}$ is a vector collecting the overall decisions of all agent types, and $\bar{\boldsymbol{y}} = \left(\bar{\boldsymbol{y}}^j\right)_{j \in \mathcal{J}}$ is a vector collecting the overall production decisions of firms. Firms production possibilities are given by

$$F^j\left(\boldsymbol{y}^j, \bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}\right) \leq 0.$$

Agent i owns a share α_{ij} of each firm j and obtains the associated share of its profits, with $\sum_{i \in \mathcal{I}} \alpha_{ij} = 1$ for all j.

The government imposes a corrective tax $\boldsymbol{\tau}_x^i \cdot \boldsymbol{x}^i$ on each agent type $i \in \mathcal{I}$, a corrective tax $\boldsymbol{\tau}_y^j \cdot \boldsymbol{y}^j$ on each firm type $j \in \mathcal{J}$, and reimburses tax receipts as lump sums T^i to agents and T^j to firms. The government runs a balanced budget, so that

$$\sum_{i \in \mathcal{I}} \left(T^i - \boldsymbol{\tau}_x^i \cdot \boldsymbol{x}^i\right) + \sum_{j \in \mathcal{J}} \left(T^j - \boldsymbol{\tau}_y^j \cdot \boldsymbol{y}^j\right) = 0.$$

The equilibrium definition is standard, with the market clearing condition given by

$$\sum_{i \in \mathcal{I}} \left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) = \sum_{j \in \mathcal{J}} \boldsymbol{y}^j.$$

An equilibrium with taxes is a price vector $\boldsymbol{p}$, agents' decisions $\boldsymbol{x}^i = \bar{\boldsymbol{x}}^i$ for all $i \in \mathcal{I}$, and production $\boldsymbol{y}$, such that agents maximize their utility, firms maximize profits $\boldsymbol{p} \cdot \boldsymbol{y}$ subject to feasibility, and markets clear with $\sum_i \left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) \leq 0$.We write $\boldsymbol{\tau} = (\boldsymbol{\tau}_x, \boldsymbol{\tau}_y)$. We also write $\boldsymbol{z} = (\boldsymbol{x}, \boldsymbol{y})$ for all consumption and production decisions in this economy.

We now establish that, modulo a change in notation, the welfare effects of an arbitrary tax reform take the same shape as in Lemma 1. The relevant vectors and matrices are analogous to those defined for the baseline model. After this change, our remaining results, which are derived from Lemma 1, also go through.

Lemma. *(Marginal Welfare Effects of Corrective Regulation / Production Economy) The marginal welfare effect of any marginal policy variation is*

$$\frac{dW}{d\boldsymbol{\tau}} = \boldsymbol{\omega} \cdot \frac{d\boldsymbol{z}}{d\boldsymbol{\tau}}, \tag{OA1}$$

where $\frac{d\boldsymbol{z}}{d\boldsymbol{\tau}} = \frac{(d\boldsymbol{x}, d\boldsymbol{y})}{d\boldsymbol{\tau}}$ *is the vector of marginal equilibrium responses, and where* $\boldsymbol{\omega} = \boldsymbol{\tau} - \boldsymbol{\delta}$ *is the vector of*

Pigouvian wedges. The vector of distortions is now defined by $\boldsymbol{\delta} = (\boldsymbol{\delta}_x, \boldsymbol{\delta}_y)$*, and*

$$\boldsymbol{\delta}_x = -\sum_i \frac{1}{\lambda^i}\frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} + \sum_j \mu^j \frac{\partial F^j}{\partial \bar{\boldsymbol{x}}}$$
$$\boldsymbol{\delta}_y = -\sum_i \frac{1}{\lambda^i}\frac{\partial u^i}{\partial \bar{\boldsymbol{y}}} + \sum_j \mu^j \frac{\partial F^j}{\partial \bar{\boldsymbol{y}}},$$

where μ^j *denotes the Lagrange multiplier on firm* j*'s feasibility constraint.*

Proof. Agents' indirect utility in equilibrium is

$$V^i\left(\boldsymbol{p}, \bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}; \boldsymbol{\tau}, T\right) = \max_{\boldsymbol{x}^i} \left\{ u^i\left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}\right) \text{ subject to } \boldsymbol{p} \cdot \left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) + \boldsymbol{\tau}_x^i \cdot \boldsymbol{x}^i \leq T^i + \sum_{j \in \mathcal{J}} \alpha_{ij} \Pi^j \right\}$$

and firms' profit is

$$\Pi^j\left(\boldsymbol{p}, \bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}; \boldsymbol{\tau}, T\right) = \max_{\boldsymbol{y}^j} \left\{ \boldsymbol{p} \cdot \boldsymbol{y}^j - \boldsymbol{\tau}_y^j \cdot \boldsymbol{y}^j + T^j \text{ subject to } F^j\left(\boldsymbol{y}^j, \bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}\right) \leq 0 \right\}.$$

We consider an arbitrary marginal policy perturbation $d\theta$, where θ can represent, for example, a single element of the tax vector $\boldsymbol{\tau}$. We obtain

$$\frac{dV^i}{d\theta} = \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}}\frac{d\bar{\boldsymbol{x}}}{d\theta} + \frac{\partial u^i}{\partial \bar{\boldsymbol{y}}}\frac{d\bar{\boldsymbol{y}}}{d\theta} + \lambda^i \left(\frac{dT^i}{d\theta} + \sum_{j \in \mathcal{J}} \alpha_{ij} \frac{d\Pi^j}{d\theta} - \frac{d\boldsymbol{\tau}_x^i}{d\theta} \cdot \boldsymbol{x}^i - \left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) \cdot \frac{d\boldsymbol{p}}{d\theta} \right),$$

so that

$$\frac{1}{\lambda^i}\frac{dV^i}{d\theta} = \frac{1}{\lambda^i}\left(\frac{\partial u^i}{\partial \bar{\boldsymbol{x}}}\frac{d\bar{\boldsymbol{x}}}{d\theta} + \frac{\partial u^i}{\partial \bar{\boldsymbol{y}}}\frac{d\bar{\boldsymbol{y}}}{d\theta} \right) + \frac{dT^i}{d\theta} + \sum_{j \in \mathcal{J}} \alpha_{ij} \frac{d\Pi^j}{d\theta} - \frac{d\boldsymbol{\tau}_x^i}{d\theta} \cdot \boldsymbol{x}^i - \left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) \cdot \frac{d\boldsymbol{p}}{d\theta}.$$

Adding up across agents, we obtain

$$\frac{dW}{d\theta} = \sum_{i \in \mathcal{I}} \frac{1}{\lambda^i}\left(\frac{\partial u^i}{\partial \bar{\boldsymbol{x}}}\frac{d\bar{\boldsymbol{x}}}{d\theta} + \frac{\partial u^i}{\partial \bar{\boldsymbol{y}}}\frac{d\bar{\boldsymbol{y}}}{d\theta} \right) - \sum_{i \in \mathcal{I}} \left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) \frac{d\boldsymbol{p}}{d\theta} + \sum_{j \in \mathcal{J}} \frac{d\Pi^j}{d\theta} + \sum_{i \in \mathcal{I}} \left(\frac{dT^i}{d\theta} - \frac{d\boldsymbol{\tau}_x^i}{d\theta} \cdot \boldsymbol{x}^i \right).$$

Marginal changes in profit satisfy

$$\frac{d\Pi^j}{d\theta} = \boldsymbol{y}^j \frac{d\boldsymbol{p}}{d\theta} + \frac{dT^j}{d\theta} - \frac{d\boldsymbol{\tau}_y^j}{d\theta}\boldsymbol{y}^j - \mu^j \left(\frac{\partial F^j}{\partial \bar{\boldsymbol{x}}}\frac{d\bar{\boldsymbol{x}}}{d\theta} + \frac{\partial F^j}{\partial \bar{\boldsymbol{y}}}\frac{d\bar{\boldsymbol{y}}}{d\theta} \right),$$

where μ^j is the multiplier on firm j's feasibility constraint. Combining, we obtain

$$\begin{aligned}\frac{dW}{d\theta} = {} & \underbrace{\left(\sum_{i \in \mathcal{I}} \frac{1}{\lambda^i}\frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} - \sum_{j \in \mathcal{J}} \mu^j \frac{\partial F^j}{\partial \bar{\boldsymbol{x}}} \right)}_{\equiv -\boldsymbol{\delta}_x} \frac{d\bar{\boldsymbol{x}}}{d\theta} + \underbrace{\left(\sum_{i \in \mathcal{I}} \frac{1}{\lambda^i}\frac{\partial u^i}{\partial \bar{\boldsymbol{y}}} - \sum_{j \in \mathcal{J}} \mu^j \frac{\partial F^j}{\partial \bar{\boldsymbol{y}}} \right)}_{\equiv -\boldsymbol{\delta}_y} \frac{d\bar{\boldsymbol{y}}}{d\theta} \\ & - \underbrace{\left(\sum_{i \in \mathcal{I}} \left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) + \sum_{j \in \mathcal{J}} \boldsymbol{y}^j \right)}_{=0 \text{ (market clearing)}} \frac{d\boldsymbol{p}}{d\theta} + \sum_{i \in \mathcal{I}} \left(\frac{dT^i}{d\theta} - \frac{d\boldsymbol{\tau}_x^i}{d\theta} \cdot \boldsymbol{x}^i \right) + \sum_{j \in \mathcal{J}} \left(\frac{dT^j}{d\theta} - \frac{d\boldsymbol{\tau}_y^j}{d\theta} \cdot \boldsymbol{y}^j \right).\end{aligned}$$

Budget balance requires that

$$
\begin{aligned}
0 &= \frac{d}{d\theta}\left(\sum_{i\in\mathcal{I}}\left(T^i - \boldsymbol{\tau}_x^i\cdot\boldsymbol{x}^i\right) + \sum_{j\in\mathcal{J}}\left(T^j - \boldsymbol{\tau}_y^j\cdot\boldsymbol{y}^j\right)\right) \\
&= \sum_{i\in\mathcal{I}}\left(\frac{dT^i}{d\theta} - \boldsymbol{\tau}_x^i\cdot\frac{d\boldsymbol{x}^i}{d\theta} - \frac{d\boldsymbol{\tau}_x^i}{d\theta}\cdot\boldsymbol{x}^i\right) + \sum_{j\in\mathcal{J}}\left(\frac{dT^j}{d\theta} - \boldsymbol{\tau}_y^j\cdot\frac{d\boldsymbol{y}^j}{d\theta} - \frac{d\boldsymbol{\tau}_y^j}{d\theta}\cdot\boldsymbol{y}^j\right),
\end{aligned}
$$

which is equivalent to

$$
\sum_{i\in\mathcal{I}}\left(\frac{dT^i}{d\theta} - \frac{d\boldsymbol{\tau}_x^i}{d\theta}\cdot\boldsymbol{x}^i\right) + \sum_{j\in\mathcal{J}}\left(\frac{dT^j}{d\theta} - \frac{d\boldsymbol{\tau}_y^j}{d\theta}\cdot\boldsymbol{y}^j\right) = \sum_{i\in\mathcal{I}}\boldsymbol{\tau}_x^i\cdot\frac{d\bar{\boldsymbol{x}}^i}{d\theta} + \sum_{j\in\mathcal{J}}\boldsymbol{\tau}_y^j\cdot\frac{d\bar{\boldsymbol{y}}^j}{d\theta}.
$$

Substituting, we obtain

$$
\frac{dW}{d\theta} = \left(\boldsymbol{\tau}_x - \boldsymbol{\delta}_x\right)\cdot\frac{d\bar{\boldsymbol{x}}}{d\theta} + \left(\boldsymbol{\tau}_y - \boldsymbol{\delta}_y\right)\cdot\frac{d\bar{\boldsymbol{y}}}{d\theta} = \boldsymbol{\omega}\cdot\frac{d\boldsymbol{z}}{d\theta},
$$

which directly implies the result. □

C.2 Game Theoretic Formulation

In this appendix, we show that it is possible to extend our results to a game theoretic environment as follows. Suppose that agents have preferences of the form

$$
u^i\left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}\right)
$$

and that they face a constraint given by

$$
\Psi^i\left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}; \theta\right) = 0,
$$

where θ is a parameter that indexes a general perturbation. In the competitive model in Section 2, the function $\Psi^i\left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}; \theta\right)$ takes the simple form

$$
\Psi^i\left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}; \theta\right) = p\left(\bar{x}\right)\left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) + \boldsymbol{\tau}^i\left(\theta\right)\boldsymbol{x}^i - T^i.
$$

We consider a Walras-Nash equilibrium with taxes, in which optimality requires that $\frac{\partial u^i}{\partial \boldsymbol{x}^i} = \lambda^i\frac{\partial \Psi^i}{\partial \boldsymbol{x}^i}$. In general, we can express the marginal welfare change in the utility of agent i induced by a general perturbation as

$$
\frac{dV^i}{d\theta} = \frac{\partial u^i}{\partial \boldsymbol{x}^i}\cdot\frac{d\boldsymbol{x}^i}{d\theta} + \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}}\cdot\frac{d\bar{\boldsymbol{x}}}{d\theta} - \lambda^i\left(\frac{\partial \Psi^i}{\partial \boldsymbol{x}^i}\cdot\frac{d\boldsymbol{x}^i}{d\theta} + \frac{\partial \Psi^i}{\partial \bar{\boldsymbol{x}}}\cdot\frac{d\bar{\boldsymbol{x}}}{d\theta} + \frac{\partial \Psi^i}{\partial \theta}\right),
$$

and normalizing by λ^i to express the marginal welfare change in units of the constraint

$$
\frac{dV^i}{\lambda^i} = \left(\frac{1}{\lambda^i}\frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} - \frac{\partial \Psi^i}{\partial \bar{\boldsymbol{x}}}\right)\cdot\frac{d\bar{\boldsymbol{x}}}{d\theta} - \frac{\partial \Psi^i}{\partial \theta}.
$$

This normalization allows us to express aggregate welfare gains as

$$
\frac{dW}{d\theta} = \sum_i \frac{dV^i}{\lambda^i} = \sum_i\left(\frac{1}{\lambda^i}\frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} - \frac{\partial \Psi^i}{\partial \bar{\boldsymbol{x}}}\right)\cdot\frac{d\bar{\boldsymbol{x}}}{d\theta} - \sum_i\frac{\partial \Psi^i}{\partial \theta}. \tag{OA2}
$$

This expression is the generalization of Equation (6) in Lemma 1a), where the term multiplying $\frac{d\bar{\boldsymbol{x}}}{d\theta}$ exactly defines marginal distortions, and the term $\sum_i \frac{\partial \Psi^i}{\partial \theta}$ captures the direct impact of any policy perturbation. When the policy perturbation takes the form $\boldsymbol{\tau}^i(\theta)\, \boldsymbol{x}^i$, then Equation (OA2) is exactly a generalized version of Lemma 1a), with a slightly redefined marginal distortion, which is now augmented to capture interactions among agents via constraints.

C.3 Redistributional Concerns

We can express the marginal welfare effects of varying $\boldsymbol{\tau}^j$, for any set of generalized social marginal welfare weights (Saez and Stantcheva, 2016; Davila and Schaab, 2022), ω^i for $i \in \mathcal{I}$, as follows:

$$\frac{dW}{d\boldsymbol{\tau}^j} = \sum_i \omega^i \frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^i} = \underbrace{\sum_i \frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^i}}_{\text{Efficiency}} + \underbrace{I \cdot \mathbb{C}ov_i\left[\omega^i, \frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^x}\right]}_{\text{Redistribution}}, \tag{OA3}$$

where we assume, without loss of generality that the weights add up to one, that is, $\sum_i \omega^i = 1$. When $\omega^i = 1$, then the redistribution term in Equation (OA3) is zero. This is the case studied in the body of the paper. Equation (OA3) shows that redistributional concerns enter additively to the marginal welfare effects of varying $\boldsymbol{\tau}^j$. A utilitarian planner simply corresponds to setting marginal welfare weights of the form $\omega^i = \lambda_0^i$, where λ_0^i equals marginal utility of consumption. Note that a utilitarian planner with access to lump-sum taxes/transfers finds it optimal to endogenously set policies so that $\omega^i = 1$, $\forall i$.

C.4 Practical Scenarios

C.4.1 Unregulated Decisions

Equation (9) follows directly from Proposition 1 since at the second-best optimum, the constraints are binding with $\boldsymbol{\tau}^U = 0$. Concretely, we have

$$\boldsymbol{\tau}^R = \boldsymbol{\delta}^R + \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\underbrace{\boldsymbol{\tau}^U}_{=0} - \boldsymbol{\delta}^U\right) = \boldsymbol{\delta}^R - \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\delta}^U,$$

as required. It is useful to consider the simple scenario in which there are two agents, $I = 2$, and each agent has a single decision, $N = 1$. Assume that only agent 1 can be regulated, with regulatory constraints dictating that $\tau^2 \equiv 0$. In that case, it follows from (9) that the optimal regulation for the regulated is simply given by

$$\tau^1 = \delta^1 - \left(-\frac{dx^1}{d\tau^1}\right)^{-1} \frac{dx^2}{d\tau^1}\delta^2.$$

The optimal regulation on agent 1 is equal to the first-best equivalent δ^1 minus a correction that accounts for the distortion imposed by the other unregulated agent. Assume, for instance, that the distortion by the unregulated agent satisfies $\delta^2 > 0$. The weight on the distortion by the unregulated agent is negative, implying that it pushes τ^1 towards underregulation, whenever i) the regulated agent responds negatively to increased regulation (the “regular” case with $\frac{dx^1}{d\tau^1} < 0$), and ii) the associated leakage elasticity indicates gross substitutes with $\frac{dx^2}{d\tau^1} > 0$.

While this is the simplest case for building intuition, note that the same insight extends to any economy with a single regulated decision and with an arbitrary set of unregulated decisions for which taxes/subsidies are forced to be zero. In this more general case, the optimal policy formula becomes

$$\tau^R = \delta^R - \sum_{(j,n)\in U} \left(-\frac{dx^R}{d\tau^R}\right)^{-1} \frac{dx_n^j}{d\tau^R}\delta_n^j,$$

where U denotes the set of imperfectly regulated decisions.

C.4.2 Uniform Regulation

To build intuition for the uniform regulation results, it is useful to first consider the special case where *all* decisions are subject to uniform regulation ($\boldsymbol{x}^U = \boldsymbol{x}$). In that case, it follows from Proposition 2 that the optimal uniform regulation is given by

$$\overline{\tau}^U = \frac{\sum_i \sum_n \frac{dx_n^i}{d\overline{\tau}^U}\delta_n^i}{\sum_j \sum_n \frac{dx_n^i}{d\overline{\tau}^U}} = \frac{\sum_i \sum_n w_n^i \delta_n^i}{\sum_j \sum_n \frac{dx_n^i}{d\overline{\tau}^U}}, \tag{OA4}$$

where we have re-written the total response of decision x_n^j to the uniform regulation as

$$\frac{dx_n^j}{d\overline{\tau}^U} = \sum_{j'\in\mathcal{I}} \sum_{n'\in\mathcal{N}} \frac{dx_n^j}{d\tau_{n'}^{j'}}.$$

In general, note that the case of uniform regulation is a particular case of linear constraints $\boldsymbol{A}\boldsymbol{\tau}^U = 0$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & -1 & & \cdots & 0 \\ & 1 & -1 & & \vdots \\ \vdots & & \ddots & \ddots & \\ 0 & \cdots & & 1 & -1 \end{pmatrix},$$

where $\boldsymbol{A}$ has dimensions $(N_U - 1) \times N_U$. In this case, $\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}} = \boldsymbol{A}'$, so Proposition 2 implies that the regulator optimally sets

$$\frac{dW}{d\boldsymbol{\tau}^U} = \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \boldsymbol{L}\right)\boldsymbol{\omega}^U = \boldsymbol{A}'\boldsymbol{\mu},$$

which also implies that

$$\boldsymbol{\iota}'\frac{dW}{d\boldsymbol{\tau}^U} = \boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \boldsymbol{L}\right)\boldsymbol{\omega}^U = \boldsymbol{\iota}'\boldsymbol{A}'\boldsymbol{\mu} = 0,$$

since $\boldsymbol{A}\boldsymbol{\iota} = 0$, where $\boldsymbol{\iota}$ denotes a vector of ones. Rearranging, we obtain

$$\begin{aligned} 0 &= \boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \boldsymbol{L}\right)\boldsymbol{\omega}^U \\ &= \boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \boldsymbol{L}\right)\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) \\ &= \boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \boldsymbol{L}\right)\left(\overline{\tau}^U\boldsymbol{\iota} - \boldsymbol{\delta}^U\right), \end{aligned}$$

where the last line uses the fact that all elements of $\boldsymbol{\tau}^U$ must be equal to the same scalar, denoted $\overline{\tau}^U$, at the constrained solution. We solve as follows for the scalar $\overline{\tau}^U$ to complete the derivation of Equation (11):

$$\underbrace{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \boldsymbol{\iota} \overline{\tau}^U}_{\text{scalar}} = \underbrace{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \boldsymbol{\delta}^U}_{\text{scalar}} \iff \overline{\tau}^U = \frac{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \boldsymbol{\delta}^U}{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \boldsymbol{\iota}}.$$

C.4.3 Convex Costs of Regulation

In this case, we have $\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}} = \boldsymbol{B}\boldsymbol{\tau}^{\boldsymbol{U}}$, so Proposition 2 implies that the planner optimally sets

$$\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) = \boldsymbol{B}\boldsymbol{\tau}^U.$$

Solving for $\boldsymbol{\tau}^U$ yields

$$\boldsymbol{\tau}^U = \left(\boldsymbol{B} + \left(-\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right)(\boldsymbol{I} - \boldsymbol{L})\right)^{-1} \left(\left(-\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right)(\boldsymbol{I} - \boldsymbol{L}) \boldsymbol{\delta}^U\right). \tag{OA5}$$

which establishes Equation (12). In general, the correction relative to the first-best policy is given by $(\boldsymbol{B} + \boldsymbol{K})^{-1} \boldsymbol{K}$, which has the interpretation of an attenuation matrix. For instance, in a scenario with a single agent, $I = 1$, and a single decision, $N = 1$, Equation (OA5) becomes

$$\tau^U = \frac{\left(-\frac{dx^U}{d\tau^U}\right)}{b + \left(-\frac{dx^U}{d\tau^U}\right)} \delta^U, \tag{OA6}$$

where b is a non-negative scalar that modulates the cost. In the well-behaved case in which $\frac{dx}{d\tau} < 0$, it follows that the optimal regulation is simply a scaled down version of the first-best regulation.[23]

C.5 Diagonal Case

Finally, it is useful to discuss the case in which $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ is a diagonal matrix. In this case, the second-best regulation on a perfectly regulated decision (j, n) is

$$\tau_n^j = \delta_n^j + \left(-\frac{dx_n^j}{d\tau_n^j}\right)^{-1} \sum_{(j',n') \in \mathcal{U}} \frac{dx_{n'}^{j'}}{d\tau_n^j} \omega_{n'}^{j'}, \tag{OA7}$$

where $\mathcal{U}$ denotes the set of imperfectly regulated decisions.

The simplified formula again shows the importance of leakage elasticities, which are weighted by wedges and summed across all unregulated decisions $(j', n') \in \mathcal{U}$. It is clear in this case that it is optimal to underregulate the regulated ($\tau_n^j < \delta_n^j$) if each of the imperfectly regulated decisions is underregulated ($\omega_{n'}^{j'} < 0$) and is a gross substitute to the regulated decision ($\frac{dx_{n'}^{j'}}{d\tau_n^j} > 0$). In addition, the formula shows that, even when not every decision satisfies gross substitutes, it is optimal to underregulate the regulated when

[23] In this case, note that the perfectly regulated decisions in turn must satisfy:

$$\boldsymbol{\tau}^R = \boldsymbol{\delta}^R + \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} \left((\boldsymbol{B} + \boldsymbol{K})^{-1} \boldsymbol{K} - \boldsymbol{I}\right) \boldsymbol{\delta}^U.$$

a weighted average of leakage elasticities — with the weights proportional to the associated wedges — is positive.

Formally, when the own-regulatory policy elasticity matrix $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ is diagonal, we have

$$\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\omega}^U = \begin{pmatrix} \left(\frac{dx_1^R}{d\tau_1^R}\right)^{-1} & 0 & \\ 0 & \ddots & \\ & & \left(\frac{dx_R^R}{d\tau_R^R}\right)^{-1} \end{pmatrix} \begin{pmatrix} \frac{dx_1^U}{d\tau_1^R} & \frac{dx_2^U}{d\tau_1^R} & & \\ \frac{dx_1^U}{d\tau_2^R} & \frac{dx_2^U}{d\tau_2^R} & & \\ & & \ddots & \\ & & & \frac{dx_U^U}{d\tau_R^R} \end{pmatrix} \begin{pmatrix} \omega_1^U \\ \vdots \\ \omega_U^U \end{pmatrix}$$

$$= \begin{pmatrix} \left(\frac{dx_1^R}{d\tau_1^R}\right)^{-1}\frac{dx_1^U}{d\tau_1^R} & \left(\frac{dx_1^R}{d\tau_1^R}\right)^{-1}\frac{dx_2^U}{d\tau_1^R} & & \\ \left(\frac{dx_2^R}{d\tau_2^R}\right)^{-1}\frac{dx_1^U}{d\tau_2^R} & \left(\frac{dx_2^R}{d\tau_2^R}\right)^{-1}\frac{dx_2^U}{d\tau_2^R} & & \\ & & \ddots & \\ & & & \left(\frac{dx_R^R}{d\tau_R^R}\right)^{-1}\frac{dx_U^U}{d\tau_U^R} \end{pmatrix} \begin{pmatrix} \omega_1^U \\ \vdots \\ \omega_U^U \end{pmatrix}$$

$$= \begin{pmatrix} \left(\frac{dx_1^R}{d\tau_1^R}\right)^{-1}\left(\frac{dx_1^U}{d\tau_1^R}\omega_1^U + \frac{dx_2^U}{d\tau_1^R}\omega_2^U + \cdots\right) \\ \left(\frac{dx_2^R}{d\tau_2^R}\right)^{-1}\left(\frac{dx_1^U}{d\tau_2^R}\omega_1^U + \frac{dx_2^U}{d\tau_2^R}\omega_2^U + \cdots\right) \\ \vdots \\ \left(\frac{dx_R^R}{d\tau_R^R}\right)^{-1}\left(\frac{dx_1^U}{d\tau_R^R}\omega_1^U + \frac{dx_2^U}{d\tau_R^R}\omega_2^U + \cdots\right) \end{pmatrix}.$$

It follows that the second-best regulation on a perfectly regulated decision (j, n) is thus given by (OA7).

D Direct Measurement: Additional Results

Here we provide additional details on the direct measurement results in Section D.

D.1 Empirical Estimates of Leakage Elasticities

This appendix explains our measurement of leakage elasticities, reported in Table 1 in the paper. Our estimates are based on Buchak, Matvos, Piskorski and Seru (2018). Their estimates use geographic variation in changes in banks' regulatory capital ratios between 2008 and 2015. Increases in equity/asset ratios over this period capture stricter bank regulation mandated in the Dodd-Frank Act. County-level variation is driven by differences in the balance sheets of traditional banks across regions before Dodd-Frank. As an instrumental variable for these differences, Buchak, Matvos, Piskorski and Seru (2018) use changes in banks' capital ratios during the 2007-8 financial crisis. The instrument captures plausibly exogenous variation in exposure to the crisis, with greater exposure leading to a larger additional regulatory burdens under Dodd-Frank.

Buchak, Matvos, Piskorski and Seru (2018) present the following two regressions to assess the effect of changes in bank capital ratios $\Delta\text{Capital ratio}_c$ across counties, indexed by c:

$$\Delta\text{Shadow bank share}_c = \beta_0^{SS} + \beta_1^{SS}\Delta\text{Capital ratio}_c + X_c'\Gamma^{SS} + \epsilon_c$$
$$\Delta\text{Shadow bank lending}_c = \beta_0^{SL} + \beta_1^{SL}\Delta\text{Capital ratio}_c + X_c'\Gamma^{SL} + \epsilon_c,$$

where X_c is a vector of county-level controls, and where capital ratios are scaled by the standard deviation across counties, which is $\sigma = 0.004135$ (supplied to us by the authors). The outcome variables are defined as follows:

$$\Delta\text{Shadow bank share}_c = 100 \times \left(\frac{\text{Shadow bank originations}_{c,2015}}{\text{All originations}_{c,2015}} - \frac{\text{Shadow bank originations}_{c,2008}}{\text{All originations}_{c,2008}}\right)$$

and

$$\Delta\text{Shadow bank lending}_c = 100 \times \left(\frac{\text{Shadow bank originations}_{c,2015} - \text{Shadow bank originations}_{c,2008}}{\text{All originations}_{c,2015}}\right).$$

We can map the coefficients of these regressions to the variables in our model:

$$\beta_1^{SS} = 100\sigma \times \frac{d\varphi^U}{d\tau_b^R} = 100\sigma \times \varphi^U\left(1-\varphi^U\right)\left(\frac{d\log k_0^U}{d\tau_b^R} - \frac{d\log k_0^R}{d\tau_b^R}\right),$$

where $\varphi^U = \frac{k_0^U}{k_0^U + k_0^R}$ is the market share of shadow banks, and

$$\beta_1^{SL} = 100\sigma \times \frac{dk^U}{d\tau_b^R}\frac{1}{k^U + k^R} = 100\sigma \times \varphi^U \frac{d\log k^U}{d\tau_b^R}.$$

Using these expressions, we obtain formulae for the leakage elasticities $\frac{d\log k_0^U}{d\tau_b^R}$ and $\frac{d\log k_0^R}{d\tau_b^R}$ in terms of reduced-form estimates β_1^{SL} and β_1^{SS} and the shadow bank market share φ^U. First, we have

$$\frac{d\log k_0^U}{d\tau_b^R} = \frac{1}{100\sigma}\frac{\beta_1^{SL}}{\varphi^U}. \tag{OA8}$$

Second, we have

$$\frac{1}{100\sigma}\frac{\beta_1^{SS}}{\varphi^U\left(1-\varphi^U\right)}=\frac{d\log k_0^U}{d\tau_b^R}-\frac{d\log k_0^R}{d\tau_b^R}=\frac{1}{100\sigma}\frac{\beta_1^{SL}}{\varphi^U}-\frac{d\log k_0^R}{d\tau_b^R},$$

which we rearrange to obtain

$$\frac{d\log k_0^R}{d\tau_b^R}=\frac{1}{100\sigma}\left(\frac{\beta_1^{SL}}{\varphi^U}-\frac{\beta_1^{SS}}{\varphi^U\left(1-\varphi^U\right)}\right). \tag{OA9}$$

We derive the estimates for leakage reported in Table 1 by using a shadow bank share of $\varphi^U=0.5$, as well as the IV coefficient estimates reported by Buchak, Matvos, Piskorski and Seru (2018, Table 8B), which are

$$\hat{\beta}_1^{SL}=8.241 \quad \text{and} \quad \hat{\beta}_1^{SS}=9.634.$$

Substituting into (OA8) and (OA9), we obtain the calibration in Table 1:

$$\begin{aligned}\frac{d\log k_0^U}{d\tau_b^R}&=\frac{1}{100\sigma}\frac{8.241}{0.5}=3.9860\\ \frac{d\log k_0^R}{d\tau_b^R}&=\frac{1}{100\sigma}\left(\frac{8.241}{0.5}-\frac{9.634}{0.25}\right)=-5.3577.\end{aligned}$$

D.2 Supplementary Quantitative Results

Figures OA-1 and OA-2 present additional quantitative results using the empirical estimates outlined in Section 4.1. Figure OA-1 extends the results presented in Figure 1 in the paper to a finer grid of values for the distortion λ generated by shadow banks relative to traditional banks. These results are consistent with the discussion in the text, since welfare effects are linear in λ, and are presented for completeness.

Figure OA-2 repeats the analysis in Figure OA-1 for values of the leverage leakage elasticity $\frac{db^U}{d\tau_b^R}\neq 0$. In this case, we use the parametrization $\kappa=-\frac{\omega_b^U\frac{db^U}{d\tau_b^R}}{\omega_b^R\frac{db^R}{d\tau_b^R}}$, so that κ measures the fraction of the direct welfare benefit that is offset by leakage to shadow bank leverage. For the range of values for κ up to 60% that we consider, the presence of leverage leakage does not substantially change the conclusions we draw in the text.

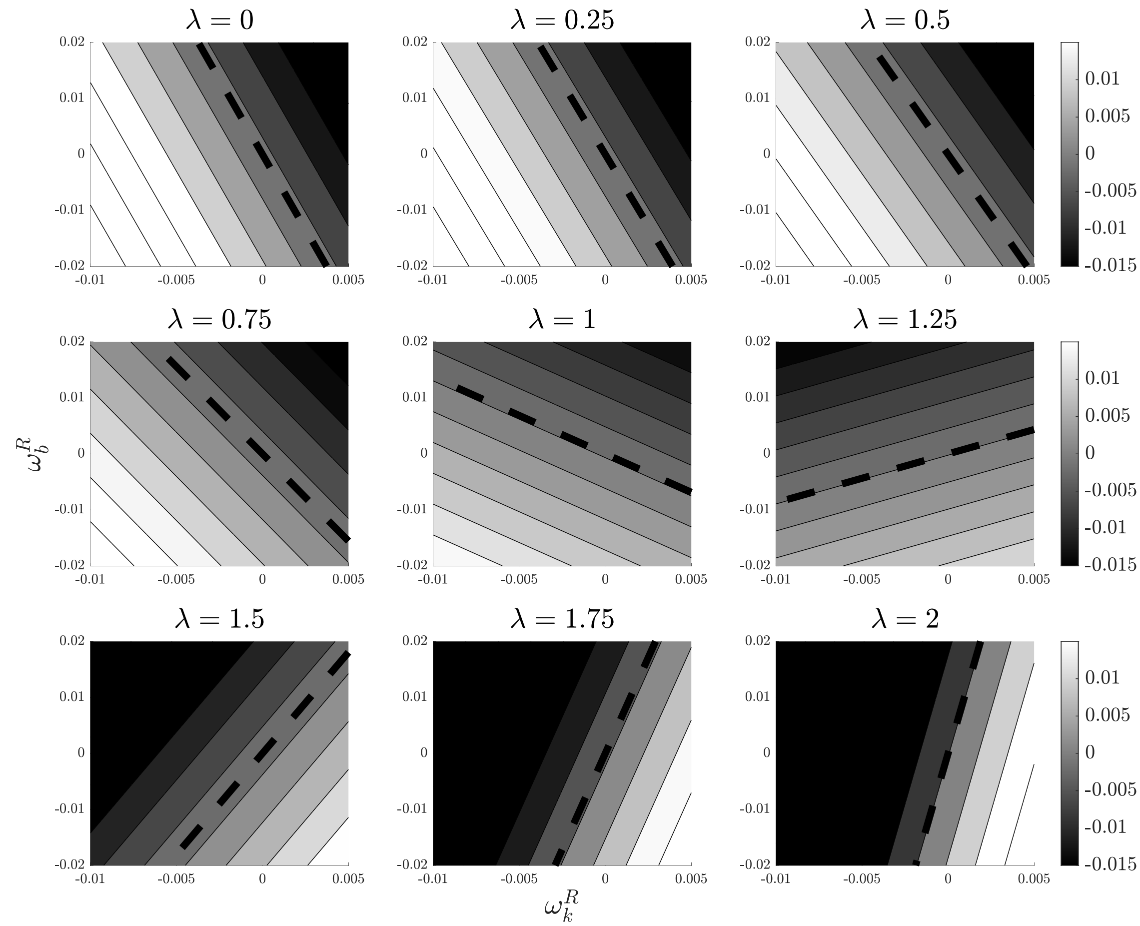

Figure OA-1: Marginal Welfare Effects with Finer Grid for λ

Note: This figure plots $\frac{dW}{d\tau_b^R}$, the marginal welfare effect of tighter leverage regulation, as a function of Pigouvian wedges $\left\{\omega_k^R, \omega_k^U, \omega_b^R\right\}$ using the empirical estimates for leakage elasticities from Table 1, setting $\omega_k^U = \lambda\omega_k^R$. The thick dashed lines show combinations of wedges for which $\frac{dW}{d\tau_b^R} = 0$. The figure is equivalent to Figure 1 in the text, but uses a finer grid of values for the relative externality parameter λ.

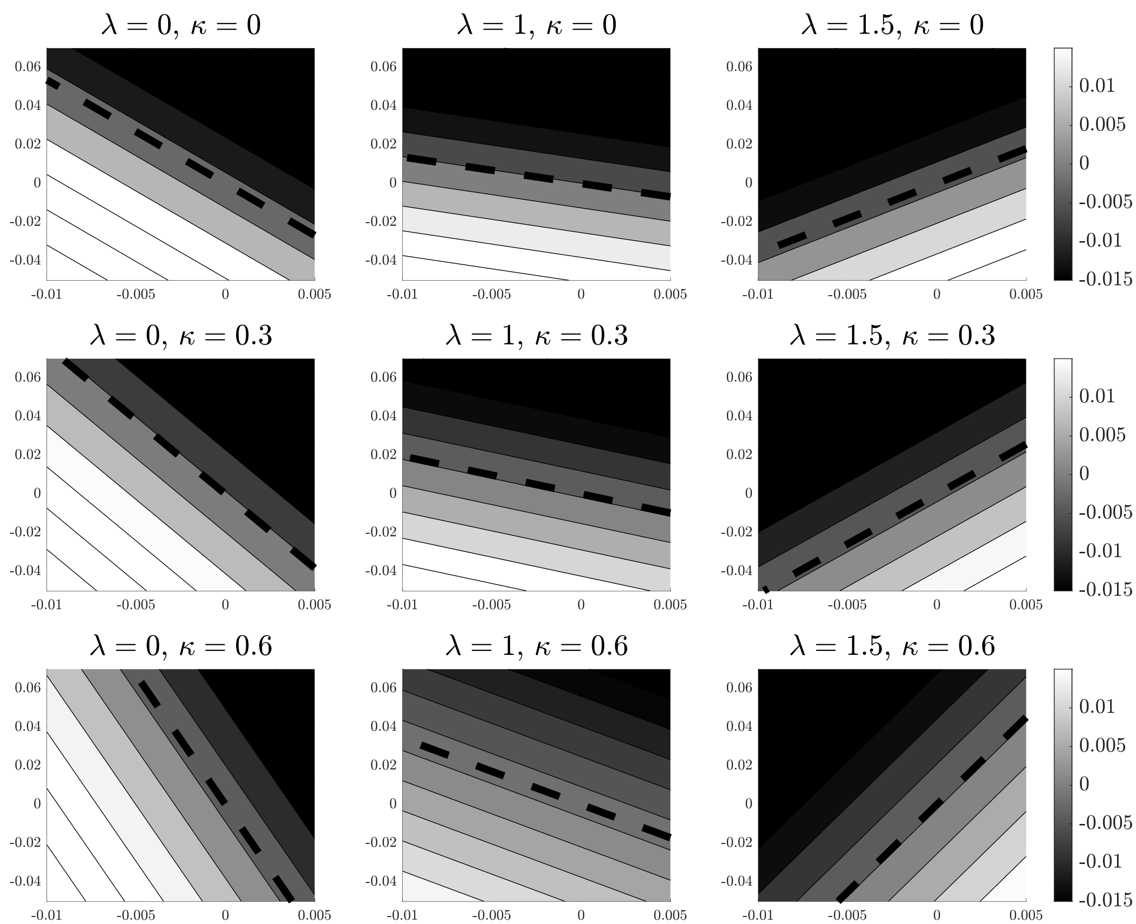

Figure OA-2: Marginal Welfare Effects with Leakage $\frac{db^U}{d\theta^R} \neq 0$

Note: This figure plots $\frac{dW}{d\tau_b^R}$, the marginal welfare effect of tighter leverage regulation, as a function of Pigouvian wedges $\left\{\omega_k^R, \omega_k^U, \omega_b^R, \omega_b^U\right\}$, in cases where $\frac{db^U}{d\tau_b^R} \neq 0$. We again use the empirical estimates for leakage elasticities from Table 1, setting $\omega_k^U = \lambda\omega_k^R$, and $\kappa = -\frac{\omega_b^U \frac{db^U}{d\tau_b^R}}{\omega_b^R \frac{db^R}{d\tau_b^R}}$. The thick dashed lines show combinations of wedges for which $\frac{dW}{d\tau_b^R} = 0$. The figure is equivalent to Figure 1 in the text, but relaxes our baseline assumption that leakage from traditional bank regulation to shadow bank leverage is close to zero.

E Model Quantification: Additional Results

In this section, we provide detailed derivations of how the equilibrium and the normative results of the model in the quantitative application in Section 4.2 are characterized and computed.

E.1 Equilibrium Characterization: Detailed Derivations

We denote the subjective probability that agent $i \in \{R, U, C, O\}$ attaches to state z by $\pi^i(z)$, and the subjective cumulative distribution of s conditional on observing z by $F^i(s|z)$. The planner computes welfare respectively using $\pi^P(z)$ and $F^P(s|z)$ for all agents.[24] Therefore, the agents' utility, introduced in Equation (15), can be written as

$$c_0^i + \beta \mathbb{E}^i \left[c_1^i(z) + c_2^i(z,s) \right] = c_0^i + \beta \sum_z \pi^i(z) \left(c_1^i(z) + \int c_2^i(z,s) \, dF^i(s|z) \right).$$

We assume that creditors' and outsiders' endowments are sufficiently large to ensure that their non-negativity constraints on consumption never bind. As in the paper, we characterize the solution of the model by backward induction, starting from date 2.

E.1.1 Banks' Default Decision (Date 2)

At date 2, after s is realized (z was realized at date 1), banks decide whether to default or not. It follows from (18) that banks default when

$$\rho_2^i(s) k_1^i(z) - \left(\left(1 - \zeta^i\right) b^i - t^i\left(b^i, s, z\right) \right) k_0^i < 0. \tag{OA10}$$

Hence, provided that the left-hand side of Equation (OA10) is increasing in s, there exist a unique default threshold for each bank type, $\hat{s}^i(z)$, which solves:

$$\rho_2^i\left(\hat{s}^i(z)\right) k_1^i(z) = \left(\left(1 - \zeta^i\right) b^i - t^i\left(b^i, \hat{s}^i(z), z\right) \right) k_0^i.$$

Given our functional form assumptions, $t^i\left(b^i, s, z\right) = \alpha_0^i(z) + \alpha_b^i(z) b^i$ and $\rho_2^i(s) = s$, we can express the default threshold in closed form as

$$\hat{s}^i(z) = \frac{\left(1 - \zeta^i - \alpha_b^i(z)\right) b^i - \alpha_0^i(z)}{k_1^i(z) / k_0^i}.$$

Banks do not default when $\hat{s}^i(z) < \underline{s}$, and default when $\hat{s}^i(z) > \underline{s}$. Hence, the default and no-default regions are defined as follows:

$$\begin{aligned} (z,s) \in \mathcal{D}^i &\iff s \leq \hat{s}^i(z) \\ (z,s) \in \mathcal{N}^i &\iff s > \hat{s}^i(z). \end{aligned}$$

Note that $\hat{s}^i(z)$ is increasing in b^i and decreasing in $k_1^i(z)/k_0^i$, b^i, $\alpha_0^i(z)$, and $\alpha_b^i(z)$.

[24] It is straightforward to allow the planner to compute welfare using different probability distributions for different agents — see e.g. Dávila and Walther (2023).

E.1.2 Banks' Investment Sale Decision (Date 1)

At date 1, once z is realized, banks choose $k_1^i(z)$, given b^i and k_0^i, determined at date 0, and given a price of investments $p_1(z)$, determined in equilibrium. The relevant continuation objective for banks at that point is given by

$$c_1^i(z) + \int c_2^i(z,s)\, dF(s|z) = \left(\int_{\hat{s}^i\left(z; k_1^i(z)\right)}^{\overline{s}} \rho_2^i(s)\, dF^i(s|z) - p_1(z) \right) k_1^i(z) + \dots .$$

Hence, under the sustained assumption that date 2 payoffs are sufficiently desirable, so $\int_{\hat{s}^i\left(z;k_1^i(z)\right)}^{\overline{s}} \rho_2^i(s)\, dF(s|z) > p_1(z)$, banks optimally choose $k_1^{i\star}(z) = k_0^i$, provided that $c_1^i(z) \geq 0$.

There are two possibilities. First, if $\rho_1^i(z) < \zeta^i b^i$, then $c_1^i(z) = 0$ is optimal and a fire sale event ensues. In this case, Equation (17) implies that

$$k_1^{i\star}(z) = \left(1 - \frac{\zeta^i b^i - \rho_1^i(z)}{p_1(z)}\right) k_0^i \iff k_0^i - k_1^{i\star}(z) = \frac{\zeta^i b^i - \rho_1^i(z)}{p_1(z)} k_0^i.$$

Second, if $\rho_1^i(z) > \zeta^i b^i$, then $c_1^i(z) > 0$ and $k_1^{i\star}(z) = k_0^i$, with date 1 consumption given by

$$c_1(z) = \left(\rho_1^i(z) - \zeta^i b^i\right) k_0^i > 0.$$

Therefore, we can express banks' date 1 decision as

$$k_1^{i\star}(z) = \left(1 - \frac{\max\left\{\zeta^i b^i - \rho_1^i(z), 0\right\}}{p_1(z)}\right) k_0^i \iff k_0^i - k_1^{i\star}(z) = \frac{\max\left\{\zeta^i b^i - \rho_1^i(z), 0\right\}}{p_1(z)} k_0^i.$$

E.1.3 Equilibrium Price (Date 1)

Outsiders' date 1 optimality condition is given by $p_1(z) = H'\left(k_1^O(z)\right)$. When combined with market clearing, we can express the date 1 equilibrium price $p_1^\star(z)$ as the following fixed point:

$$p_1^\star(z) = H'\left(\sum_{i\in\{R,U\}} \left(k_0^i - k_1^{i\star}(z)\right)\right) = H'\left(\frac{\sum_{i\in\{R,U\}} \max\left\{\zeta^i b^i - \rho_1^i(z), 0\right\} k_0^i}{p_1^\star(z)}\right) \Rightarrow p_1^\star(z; \boldsymbol{x}), \quad \text{(OA11)}$$

where the notation $p_1^\star(z;\boldsymbol{x})$ collects date 0 investment and leverage decisions in a set of "big-K" (Ljungqvist and Sargent, 2004) decisions $\boldsymbol{x} = \left\{\left\{k_0^i\right\}_i, \left\{b^i\right\}_i\right\}$ — see e.g. Dávila and Korinek (2018) in a similar context. Hence, the function $p_1^\star(z;\boldsymbol{x})$ determines the equilibrium price of investments at date 1 in state z for given date 0 the investment and leverage decisions of banks.

Given our functional form assumption, $H\left(k_1^O\right) = \frac{\left(k_1^O + \varsigma\right)^\gamma}{\gamma}$, we have that $H'\left(k_1^O\right) = \left(k_1^O + \varsigma\right)^{\gamma-1}$, so $p_1^\star(z;\boldsymbol{x})$ takes the form

$$p_1^\star(z;\boldsymbol{x}) = \left(\frac{\sum_i \max\left\{\zeta^i b^i - \rho_1^i(z), 0\right\} k_0^i}{p_1^\star(z;\boldsymbol{x})} + \varsigma\right)^{\gamma-1},$$

which yields a unique solution when $\gamma < 1$.

E.1.4 Banks' Continuation Indirect Utility (Date 1)

At this stage, it is convenient to define equilibrium investment sales at date 1 and default at date 2 as functions of $\boldsymbol{x} = \left\{\left\{k_0^i\right\}_i, \left\{b^i\right\}_i\right\}$. We define i) the equilibrium share of banks' retained investments, $\theta^i\left(b^i, z; \boldsymbol{x}\right) \in [0,1]$, as

$$\theta^{i}\left(b^{i},z;\boldsymbol{x}\right)=\frac{k_{1}^{i\star}\left(z\right)}{k_{0}^{i}}=1-\frac{\max\left\{\zeta^{i}b^{i}-\rho_{1}^{i}\left(z\right),0\right\}}{p_{1}^{\star}\left(z;\boldsymbol{x}\right)},$$

and ii) the equilibrium default threshold as

$$\hat{s}^{i}\left(b^{i},z;\boldsymbol{x}\right)=\frac{\left(1-\zeta^{i}-\alpha_{b}^{i}\left(z\right)\right)b^{i}-\alpha_{0}^{i}\left(z\right)}{\theta^{i}\left(b^{i},z;\boldsymbol{x}\right)}.$$

Note that $1-\theta^{i}\left(b^{i},z;\boldsymbol{x}\right)=\frac{k_{0}^{i}-k_{1}^{i\star}(z)}{k_{0}^{i}}$, which in turn implies that $k_{0}^{i}-k_{1}^{i\star}\left(z\right)=\left(1-\theta^{i}\left(b^{i},z;\boldsymbol{x}\right)\right)k_{0}^{i}$. Note also that even though we express θ^{i} and $\hat{s}^{i}$ in terms of $\boldsymbol{x}$, we will initially take derivatives in terms of $p_{1}^{\star}\left(z;\boldsymbol{x}\right)$.

Therefore, we can express the continuation indirect utility of type i banks from date 1 onwards as

$$\nu_{1}^{i}\left(k_{0}^{i},b^{i},z;\boldsymbol{x}\right)=c_{1}^{i}\left(z\right)+\int_{\hat{s}^{i}\left(b^{i},z;\boldsymbol{x}\right)}^{\overline{s}}c_{2}^{i}\left(z,s\right)dF^{i}\left(s|z\right)=e^{i}\left(b^{i},z;\boldsymbol{x}\right)k_{0}^{i},$$

where $e^{i}\left(b^{i},z;\boldsymbol{x}\right)$ denotes the payoff per unit of date 0 investment, given by

$$e^{i}\left(b^{i},z;\boldsymbol{x}\right)=\max\left\{\rho_{1}^{i}\left(z\right)-\zeta^{i}b^{i},0\right\}+\int_{\hat{s}^{i}\left(b^{i},z;\boldsymbol{x}\right)}^{\overline{s}}\left(\rho_{2}^{i}\left(s\right)\theta^{i}\left(b^{i},z;\boldsymbol{x}\right)-\left(\left(1-\zeta^{i}\right)b^{i}-t^{i}\left(b^{i},s,z\right)\right)\right)dF^{i}\left(s|z\right).\tag{OA12}$$

E.1.5 Debt Pricing Schedule/Credit Surface (Date 0)

At date 0, accounting for banks' default and investment sale decisions at date 1, which we have already characterized, competitive creditors price debt according to

$$Q^{i}\left(b^{i};\boldsymbol{x}\right)=\beta^{C}\left(\zeta^{i}b^{i}+\mathbb{E}^{C}\left[\mathcal{R}^{i}\left(z,s\right)\right]\right),\tag{OA13}$$

where the expected date 2 repayment (including bailout transfers) is given by

$$\begin{aligned}\mathbb{E}^{C}\left[\mathcal{R}^{i}\left(z,s\right)\right]&=\sum_{z}\pi^{C}\left(z\right)\int\mathcal{R}^{i}\left(z,s\right)dF\left(s|z\right)\\&=\sum_{z}\pi^{C}\left(z\right)\left(\begin{array}{c}\int_{\underline{s}}^{\hat{s}^{i}\left(b^{i},z;\boldsymbol{x}\right)}\left(\phi\rho_{2}^{i}\left(s\right)\theta^{i}\left(b^{i},z;\boldsymbol{x}\right)+t^{i}\left(b^{i},s,z\right)\right)dF^{C}\left(s|z\right)\\+\left(1-\zeta^{i}\right)b^{i}\int_{\hat{s}^{i}\left(b^{i},z;\boldsymbol{x}\right)}^{\overline{s}}dF^{C}\left(s|z\right)\end{array}\right).\end{aligned}$$

E.1.6 Banks' Problem (Date 0)

At date 0, since $c_{0}^{i}>0$, which we always assume, banks maximize

$$c_{0}^{i}+\beta\sum_{z}\pi\left(z\right)e^{i}\left(b^{i},z;\boldsymbol{x}\right)k_{0}^{i}=\left(M^{i}\left(b^{i};\boldsymbol{x}\right)-p_{0}\right)k_{0}^{i}-\Psi\left(k_{0}^{i}\right)-\tau_{b}^{i}b^{i}k_{0}^{i}-\hat{\tau}_{k}^{i}k_{0}^{i},\tag{OA14}$$

where we define the date 0 value of investment as the sum of payoffs that go to banks, $\mathcal{E}^{i}\left(b^{i};\boldsymbol{x}\right)$, and creditors, $Q^{i}\left(b^{i};\boldsymbol{x}\right)$, as follows:

$$M^{i}\left(b^{i};\boldsymbol{x}\right)=\mathcal{E}^{i}\left(b^{i};\boldsymbol{x}\right)+Q^{i}\left(b^{i};\boldsymbol{x}\right),\quad\text{where}\quad\mathcal{E}^{i}\left(b^{i};\boldsymbol{x}\right)=\beta\sum_{z}\pi^{i}\left(z\right)e^{i}\left(b^{i},z;\boldsymbol{x}\right),\tag{OA15}$$

with $e^{i}\left(b^{i},z;\boldsymbol{x}\right)$ is defined in (OA12), and where $Q^{i}\left(b^{i};\boldsymbol{x}\right)$ is defined in (OA13).

Leverage Decision. Given the banks' date 0 objective in (OA14), their optimal leverage decision is made independently of k_0^i, as follows:

$$\frac{\partial M^i\left(b^i;\boldsymbol{x}\right)}{\partial b^i}=\frac{\partial \mathcal{E}(b^i;\boldsymbol{x})}{\partial b^i}+\frac{\partial Q(b^i;\boldsymbol{x})}{\partial b^i}=\tau_b^i, \tag{OA16}$$

where $\frac{\partial \mathcal{E}(b^i;\boldsymbol{x})}{\partial b^i}$ and $\frac{\partial Q(b^i;\boldsymbol{x})}{\partial b^i}$ are characterized below. Note that our assumptions ensure that banks' leverage is independent of their investment decision, which simplifies the computation of the model.

Investment Decision. Given their optimal leverage decision, banks optimally choose their date 0 investment holdings as follows:

$$M^i\left(b^i;\boldsymbol{x}\right)=p_0+\Psi^{i\prime}\left(k_0^i\right)+\tau_k^i, \tag{OA17}$$

where $\tau_k^i=\hat{\tau}_k^i+\tau_b^i b^i$. Given our functional form assumption, $\Psi^i\left(k_0^i\right)=\psi^i\frac{\left(k_0^i\right)^{\nu^i}}{\nu^i}$, banks' date 0 investment demand can be expressed as

$$k_0^i=\left(\frac{1}{\psi^i}\left(M^i\left(b^i;\boldsymbol{x}\right)-p_0-\tau_k^i\right)\right)^{\frac{1}{\nu^i-1}}. \tag{OA18}$$

Continuation Indirect Utility Derivatives. In order to characterize $\frac{\partial \mathcal{E}(b^i;\boldsymbol{x})}{\partial b^i}$ and $\frac{\partial Q(b^i;\boldsymbol{x})}{\partial b^i}$, it is useful to first compute $\frac{\partial \theta^i(b^i,z;\boldsymbol{x})}{\partial b^i}$ and $\frac{\partial \hat{s}^i(b^i,z;\boldsymbol{x})}{\partial b^i}$, which are given by

$$\frac{\partial \theta^i(b^i,z;\boldsymbol{x})}{\partial b^i}=\begin{cases}0, & \text{if } z=0\\ -\frac{\zeta^i}{p_1^\star(z;\boldsymbol{x})}, & \text{if } z=1,\end{cases}$$

and

$$\frac{\partial \hat{s}^i\left(b^i,z;\boldsymbol{x}\right)}{\partial b^i}=\begin{cases}\frac{1-\zeta^i-\alpha_b^i(z)}{\theta^i(b^i,z;\boldsymbol{x})} & \text{if } z=0,\\ \frac{1-\zeta^i-\alpha_b^i(z)}{\theta^i(b^i,z;\boldsymbol{x})}-\frac{\hat{s}^i\left(b^i,z;\boldsymbol{x}\right)}{\theta^i(b^i,z;\boldsymbol{x})}\frac{\partial\theta^i\left(b^i,z;\boldsymbol{x}\right)}{\partial b^i}, & \text{if } z=1.\end{cases}$$

Hence, we can compute $\frac{\partial \mathcal{E}(b^i;\boldsymbol{x})}{\partial b^i}$ as follows:

$$\frac{\partial \mathcal{E}(b^i;\boldsymbol{x})}{\partial b^i}=\beta\sum_z \pi^i(z)\left(\begin{array}{c}-\zeta^i\mathbb{I}\left[\rho_1^i(z)>\zeta^i b^i\right]\\ +\int_{\hat{s}^i(b^i,z;\boldsymbol{x})}^{\bar{s}}\left(\rho_2^i(s)\frac{\partial\theta^i\left(b^i,z;\boldsymbol{x}\right)}{\partial b^i}-\left(1-\zeta^i\right)+\frac{\partial t^i\left(b^i,s,z\right)}{\partial b^i}\right)dF^i(s|z)\end{array}\right),$$

where the boundary term of the Leibniz rule is zero due to the optimality of the default decision. Similarly for $\frac{\partial Q(b^i;\boldsymbol{x})}{\partial b^i}$, we find that

$$\begin{aligned}\frac{\partial Q(b^i;\boldsymbol{x})}{\partial b^i}=\beta^C\Bigg[\zeta^i+\sum_z\pi^C(z)\Bigg(&\int_{\underline{s}}^{\hat{s}^i\left(b^i,z;\boldsymbol{x}\right)}\left(\phi\rho_2^i(s)\frac{\partial\theta^i\left(b^i,z;\boldsymbol{x}\right)}{\partial b^i}+\frac{\partial t^i(b^i,s,z)}{\partial b^i}\right)dF^C(s|z)\\ &+\left(\left(\phi-1\right)\rho_2^i\left(\hat{s}^i\left(b^i,z;\boldsymbol{x}\right)\right)\theta^i\left(b^i,z;\boldsymbol{x}\right)\right)f^C\left(\hat{s}^i\left(b^i,z;\boldsymbol{x}\right)|z\right)\frac{\partial\hat{s}^i\left(b^i,z;\boldsymbol{x}\right)}{\partial b^i}\\ &+\left(1-\zeta^i\right)\left[1-F^C\left(\hat{s}^i\left(b^i,z;\boldsymbol{x}\right)|z\right)\right]\Bigg)\Bigg],\end{aligned}$$

where $\frac{\partial t^i(b^i,s,z)}{\partial b^i}=\alpha_b^i(z)$. Note that these derivations imply that leverage is driven by i) differences in discount factors, ii) differences in beliefs, iii) default deadweight losses, and iv) the form of the bailout.

E.1.7 Equilibrium Price (Date 0)

Outsiders' date 0 optimality condition is given by $p_0 = \Upsilon'\left(k_0^O\right)$. When combined with market clearing, the date 0 equilibrium price $p_0^\star$ simply solves the following fixed point:

$$p_0^\star = \Upsilon'\left(\sum_i k_0^i\left(p_0^\star\right)\right),$$

where $k_0^i\left(p_0^\star\right)$ is determined as in Equation (OA18).

E.1.8 Model Computation

Given our functional form assumptions, and the preceding derivations, for given regulations, computing a model solution boils down to solving a nonlinear system of 5 equations, given by

$$\begin{aligned}
\frac{\partial M^R\left(b^R;\boldsymbol{x}\right)}{\partial b^R} &= \tau_b^R \\
\frac{\partial M^U\left(b^U;\boldsymbol{x}\right)}{\partial b^U} &= \tau_b^U \\
k_0^R &= \left(\frac{1}{\psi^R}\left(M^R\left(b^R;\boldsymbol{x}\right) - p_0 - \tau_k^R\right)\right)^{\frac{1}{\nu^R-1}} \\
k_0^U &= \left(\frac{1}{\psi^U}\left(M^U\left(b^U;\boldsymbol{x}\right) - p_0 - \tau_k^U\right)\right)^{\frac{1}{\nu^U-1}} \\
p_1\left(z=1;\boldsymbol{x}\right) &= \left(\zeta^R b^R k_0^R + \zeta^U b^U k_0^U + p_1\left(z=1;\boldsymbol{x}\right)\varsigma\right)^{\frac{\gamma-1}{\gamma}},
\end{aligned}$$

and 5 unknowns: b^R, b^U, k_0^R, k_0^U, and $p_1\left(z=1\right)$.

E.2 Normative Results: Detailed Derivations

E.2.1 Welfare Effects

We first characterize the marginal welfare impact on banks, creditors, and outsiders of a change in the regulations $\boldsymbol{\tau} \in \left\{\left\{\tau_b^i\right\}_i, \left\{\hat{\tau}_k^i\right\}_i\right\}$.

Banks. Leaving aside endowments, the indirect utility of banks — from the perspective of a planner with beliefs indexed by P — can be expressed as

$$V^{i,P} = \max_{b^i,k_0^i}\left\{\left(\mathcal{E}^{i,P}\left(b^i;\boldsymbol{x}\right) + Q^i\left(b^i;\boldsymbol{x}\right) - p_0\right)k_0^i - \Psi^i\left(k_0^i\right) - \tau_b^i b^i k_0^i - \hat{\tau}_k^i k_0^i + T_0^i\right\},$$

where $\mathcal{E}^{i,P}\left(b^i;\boldsymbol{x}\right)$ is defined as in (OA15), but taking expectations under the planner's beliefs. Note that $Q^i\left(b^i;\boldsymbol{x}\right)$ is the credit surface determined in equilibrium, by the creditors' beliefs, not the planner's beliefs. The marginal welfare effect of a change in regulation is thus given by

$$\begin{aligned}
\frac{dV^{i,P}}{d\tau} = &\left(\frac{d\mathcal{E}^{i,P}\left(b^i;\boldsymbol{x}\right)}{d\tau} + \frac{dQ^i\left(b^i;\boldsymbol{x}\right)}{d\tau} - \frac{dp_0}{d\tau}\right)k_0^i + \left(\mathcal{E}^{i,P}\left(b^i;\boldsymbol{x}\right) + Q^i\left(b^i;\boldsymbol{x}\right) - p_0 - \Psi^{i\prime}\left(k_0^i\right)\right)\frac{dk_0^i}{d\tau} \\
&- \left(\tau_b^i b^i + \hat{\tau}_k^i\right)\frac{dk_0^i}{d\tau} - \tau_b^i\frac{db^i}{d\tau}k_0^i - \left(\frac{d\tau_b^i}{d\tau}b^i + \frac{d\hat{\tau}_k^i}{d\tau}\right)k_0^i - \frac{dT_0^i}{d\tau}.
\end{aligned}$$

So we can express the marginal welfare effect aggregated across banks as follows:[25]

$$\sum_{i\in\{R,U\}}\frac{dV^{i,P}}{d\tau}=\sum_{i\in\{R,U\}}\left(\frac{d\mathcal{E}^{i,P}\left(b^i;\boldsymbol{x}\right)}{d\tau}+\frac{dQ^i\left(b^i;\boldsymbol{x}\right)}{d\tau}-\frac{dp_0}{d\tau}\right)k_0^i$$
$$+\sum_{i\in\{R,U\}}\left(\mathcal{E}^{i,P}\left(b^i;\boldsymbol{x}\right)+Q^i\left(b^i;\boldsymbol{x}\right)-p_0-\Psi^{i\prime}\left(k_0^i\right)\right)\frac{dk_0^i}{d\tau}.$$

Outsiders. The indirect utility of outsiders — from the perspective of a planner with beliefs indexed by P — is given by

$$V^{O,P}=\max_{k_0^O,k_1^O(z)}\left\{p_0k_0^O-\Upsilon\left(k_0^O\right)+\beta\sum_z\pi^P\left(z\right)\left(H\left(k_1^O\left(z\right)\right)-p_1\left(z;\boldsymbol{x}\right)k_1^O\left(z\right)\right)\right\}.$$

The marginal welfare effect of a change in regulation is — using optimality/envelope theorem — given by

$$\frac{dV^{O,P}}{d\tau}=\frac{dp_0}{d\tau}k_0^u-\beta\sum_z\pi^P\left(z\right)\frac{dp_1\left(z;\boldsymbol{x}\right)}{d\tau}k_1^O\left(z\right).$$

Creditors. Leaving aside endowments, the indirect utility of creditors — from the perspective of a planner with beliefs indexed by P — can be expressed as

$$V^{C,P}=\sum_{i\in\{R,U\}}\left(Q^{i,P}\left(b^i;\boldsymbol{x}\right)-Q^i\left(b^i;\boldsymbol{x}\right)-\left(1+\kappa\right)\mathcal{T}^i\left(b^i\right)\right)k_0^i,$$

where $Q^i\left(b^i;\boldsymbol{x}\right)$ is defined in Equation (OA13), and where $Q^{i,P}\left(b^i;\boldsymbol{x}\right)$ is given by

$$Q^{i,P}\left(b^i;\boldsymbol{x}\right)=\beta^C\left(\zeta^ib^i+\sum_z\pi^P\left(z\right)\left(\begin{array}{c}\int_{\underline{s}}^{s^\star\left(b^i,z;\boldsymbol{x}\right)}\left(\phi\rho_2^i\left(s\right)\theta^i\left(b^i,z;\boldsymbol{x}\right)+t^i\left(b^i,s,z\right)\right)dF^P\left(s|z\right)\\+\left(1-\zeta^i\right)b^i\int_{s^\star\left(b^i,z;\boldsymbol{x}\right)}^{\overline{s}}dF^P\left(s|z\right)\end{array}\right)\right),$$

where we denote the net present value for creditors of the bailout transfer per unit of investment to type i banks by[26]

$$\mathcal{T}^i\left(b^i\right)=\beta^C\sum_z\pi^P\left(z\right)\int_{\underline{s}}^{\overline{s}}t^i\left(b^i,s,z\right)dF^{C,P}\left(s|z\right).$$

The marginal welfare effect of a change in regulation is thus given by

$$\frac{dV^{C,P}}{d\tau}=\sum_{i\in\{R,U\}}\left(\frac{dQ^{i,P}\left(b^i;\boldsymbol{x}\right)}{d\tau}-\frac{dQ^i\left(b^i;\boldsymbol{x}\right)}{d\tau}-\left(1+\kappa\right)\frac{d\mathcal{T}^i\left(b^i\right)}{d\tau}\right)k_0^i$$
$$+\sum_{i\in\{R,U\}}\left(Q^{i,P}\left(b^i;\boldsymbol{x}\right)-Q^i\left(b^i;\boldsymbol{x}\right)-\left(1+\kappa\right)\mathcal{T}^i\left(b^i\right)\right)\frac{dk_0^i}{d\tau}.$$

[25] This result uses the fact that $\sum_i T_0^i=\sum_i\tau_b^ib^ik_0^i+\sum_i\hat{\tau}_k^ik_0^i$, which implies that

$$\sum_i\frac{dT_0^i}{d\tau}=\sum_i\left[\left(\tau_b^ib^i+\hat{\tau}_k^i\right)\frac{dk_0^i}{d\tau}+\tau_b^i\frac{db^i}{d\tau}k_0^i+\left(\frac{d\tau_b^i}{d\tau}b^i+\frac{d\hat{\tau}_k^i}{d\tau}\right)k_0^i\right].$$

[26] It is straightforward to allow for the bailout to be a function of the decisions of all banks, as in Dávila and Walther (2020). In that case, $\mathcal{T}^i\left(b^i\right)$ also depends on $\boldsymbol{x}$, introducing additional marginal distortions.

E.2.2 Aggregate Welfare Effects

After aggregating, we can express $dW = \sum_i dV^{i,P}$ as

$$\frac{dW}{d\tau} = \sum_{i\in\{R,U\}} \left(\frac{dM^{i,P}\left(b^i;\boldsymbol{x}\right)}{d\tau} - (1+\kappa)\frac{d\mathcal{T}^i\left(b^i\right)}{d\tau} \right) k_0^i - \beta \sum_z \pi^P(z) \frac{dp_1(z;\boldsymbol{x})}{d\tau} k_1^O(z)$$
$$+ \sum_{i\in\{R,U\}} \left(M^{i,P}\left(b^i;\boldsymbol{x}\right) - p_0 - \Psi^{i\prime}\left(k_0^i\right) - (1+\kappa)\mathcal{T}^i\left(b^i\right) \right) \frac{dk_0^i}{d\tau},$$

where $M^{i,P}\left(b^i;\boldsymbol{x}\right) = \mathcal{E}^{i,P}\left(b^i;\boldsymbol{x}\right) + Q^{i,P}\left(b^i;\boldsymbol{x}\right)$ and $\frac{dM^{i,P}\left(b^i;\boldsymbol{x}\right)}{d\tau} = \frac{d\mathcal{E}^{i,P}\left(b^i;\boldsymbol{x}\right)}{d\tau} + \frac{dQ^{i,P}\left(b^i;\boldsymbol{x}\right)}{d\tau}$. Note that we can express $\frac{d\mathcal{T}^i\left(b^i\right)}{d\tau}$ and $\frac{dM^{i,P}\left(b^i;\boldsymbol{x}\right)}{d\tau}$ as follows:

$$\frac{d\mathcal{T}^i\left(b^i\right)}{d\tau} = \frac{\partial\mathcal{T}^i\left(b^i\right)}{\partial b^i}\frac{db^i}{d\tau}$$
$$\frac{dM^{i,P}\left(b^i;\boldsymbol{x}\right)}{d\tau} = \frac{\partial M^{i,P}\left(b^i;\boldsymbol{x}\right)}{\partial b^i}\frac{db^i}{d\tau} + \sum_z \frac{\partial M^{i,P}\left(b^i;\boldsymbol{x}\right)}{\partial p_1(z)}\frac{dp_1(z;\boldsymbol{x})}{d\tau},$$

so we can express $\frac{dW}{d\tau}$ — also exploiting Equations (OA16) and (OA17) — as

$$\frac{dW}{d\tau} = \sum_{i\in\{R,U\}} \left(\tau_b^i - \left(\frac{\partial M^{i}\left(b^i;\boldsymbol{x}\right)}{\partial b^i} - \frac{\partial M^{i,P}\left(b^i;\boldsymbol{x}\right)}{\partial b^i} \right) - (1+\kappa)\frac{\partial\mathcal{T}^i\left(b^i\right)}{\partial b^i} \right) k_0^i \frac{db^i}{d\tau}$$
$$+ \sum_{i\in\{R,U\}} \left(\tau_k^i - \left(M^i\left(b^i;\boldsymbol{x}\right) - M^{i,P}\left(b^i;\boldsymbol{x}\right) \right) - (1+\kappa)\mathcal{T}^i\left(b^i\right) \right) \frac{dk_0^i}{d\tau},$$
$$- \sum_z \left(\sum_{i\in\{R,U\}} \frac{\partial M^{i,P}\left(b^i;\boldsymbol{x}\right)}{\partial p_1(z)} k_0^i - \beta\pi^P(z) k_1^O(z) \right) \frac{dp_1(z;\boldsymbol{x})}{d\tau}$$

where $\tau_k^i = \hat{\tau}_k^i + \tau_b^i b^i$ and where we can write

$$\frac{dp_1(z;\boldsymbol{x})}{d\tau} = \sum_i \frac{\partial p_1^\star(z;\boldsymbol{x})}{\partial k_0^i}\frac{dk_0^i}{d\tau} + \sum_i \frac{1}{k_0^i}\frac{\partial p_1^\star(z;\boldsymbol{x})}{\partial b^i}\frac{db^i}{d\tau},$$

where $\frac{\partial p_1^\star(z;\boldsymbol{x})}{\partial k_0^i}$ and $\frac{1}{k_0^i}\frac{\partial p_1^\star(z;\boldsymbol{x})}{\partial b^i}$ can be explicitly computed from Equation (OA11).

Hence, $\frac{dW}{d\tau}$ can be expressed as

$$\frac{dW}{d\tau} = \sum_{i\in\{R,U\}} \left(\tau_b^i - \left(\frac{\partial M^{i}\left(b^i;\boldsymbol{x}\right)}{\partial b^i} - \frac{\partial M^{i,P}\left(b^i;\boldsymbol{x}\right)}{\partial b^i} \right) - (1+\kappa)\frac{\partial\mathcal{T}^i\left(b^i\right)}{\partial b^i} - \sum_z \pi^P(z)\frac{\delta_p^i(z)}{k_0^i}\frac{\partial p_1^\star(z;\boldsymbol{x})}{\partial b_0^i} \right) k_0^i \frac{db^i}{d\tau}$$
$$+ \sum_{i\in\{R,U\}} \left(\tau_k^i - \left(M^i\left(b^i;\boldsymbol{x}\right) - M^{i,P}\left(b^i;\boldsymbol{x}\right) \right) - (1+\kappa)\mathcal{T}^i\left(b^i\right) - \sum_z \pi^P(z)\delta_p^i(z)\frac{\partial p_1^\star(z;\boldsymbol{x})}{\partial k_0^i} \right) \frac{dk_0^i}{d\tau},$$

where $\delta_p^i(z)$ is defined below. In a more compact form, we can express $\frac{dW}{d\tau}$ as

$$dW = \sum_i \left(\tau_b^i - \delta_b^i\right) k_0^i \frac{db^i}{d\tau} + \sum_i \left(\tau_k^i - \delta_k^i\right) \frac{dk_0^i}{d\tau},$$

where we define

$$\delta_b^i = \underbrace{\left(\frac{\partial M^i\left(b^i;\boldsymbol{x}\right)}{\partial b^i} - \frac{\partial M^{i,P}\left(b^i;\boldsymbol{x}\right)}{\partial b^i}\right)}_{=\delta^i_{b,\text{beliefs}}} + \underbrace{(1+\kappa)\frac{\partial \mathcal{T}^i\left(b^i\right)}{\partial \tau}}_{=\delta^i_{b,\text{bailout}}} + \underbrace{\sum_z \pi^P(z)\frac{\delta_p^i(z)}{k_0^i}\frac{\partial p_1^\star(z;\boldsymbol{x})}{\partial b_0^i}}_{=\delta^i_{b,\text{pecuniary}}}$$

$$\delta_k^i = \underbrace{\left(M^i\left(b^i;\boldsymbol{x}\right) - M^{i,P}\left(b^i;\boldsymbol{x}\right)\right)}_{=\delta^i_{k,\text{beliefs}}} + \underbrace{(1+\kappa)\mathcal{T}^i\left(b^i\right)}_{=\delta^i_{k,\text{bailout}}} + \underbrace{\sum_z \pi^P(z)\,\delta_p^i(z)\frac{\partial p_1^\star(z;\boldsymbol{x})}{\partial k_0^i}}_{=\delta^i_{k,\text{pecuniary}}}.$$

To complete the characterization of marginal welfare effects, we must compute $\delta_p^i(z)$, and to do so $\frac{\partial M^{i,P}\left(b^i;\boldsymbol{x}\right)}{\partial p_1(z)}$, which is given by

$$\frac{\partial M^{i,P}\left(b^i;\boldsymbol{x}\right)}{\partial p_1(z)} = \frac{\partial \mathcal{E}^{i,P}\left(b^i;\boldsymbol{x}\right)}{\partial p_1(z)} + \frac{\partial Q^{i,P}\left(b^i;\boldsymbol{x}\right)}{\partial p_1(z)}.$$

Using banks' optimal default decisions, we can express $\frac{\partial \mathcal{E}^{i,P}\left(b^i;\boldsymbol{x}\right)}{\partial p_1(z)}$ and $\frac{\partial Q^{i,P}\left(b^i;\boldsymbol{x}\right)}{\partial p_1(z)}$ as follows:

$$\frac{\partial \mathcal{E}^{i,P}\left(b^i;\boldsymbol{x}\right)}{\partial p_1(z)} = \beta\pi^P(z)\left(1-\theta^i\left(b^i,z;\boldsymbol{x}\right)\right)\frac{\int_{\hat{s}^i(b^i,z;\boldsymbol{x})}^{\overline{s}}\rho_2^i(s)\,dF^{i,P}(s|z)}{p_1^\star(z)}$$

$$\frac{\partial Q^{i,P}\left(b^i;\boldsymbol{x}\right)}{\partial p_1(z)} = \beta^C\pi^P(z)\left(\begin{array}{c}\phi\left(1-\theta^i\left(b^i,z;\boldsymbol{x}\right)\right)\frac{\int_{\underline{s}}^{\hat{s}^i\left(b^i,z;\boldsymbol{x}\right)}\rho_2^i(s)dF^{C,P}(s|z)}{p_1^\star(z)}\\ +(\phi-1)\,\rho_2^i\left(\hat{s}^i\left(b^i,z;\boldsymbol{x}\right)\right)\theta^i\left(b^i,z;\boldsymbol{x}\right)f^{C,P}\left(\hat{s}^i\left(b^i,z;\boldsymbol{x}\right)|z\right)\frac{\partial \hat{s}^i\left(b^i,z;\boldsymbol{x}\right)}{\partial p_1(z)}\end{array}\right),$$

where we use the fact that $\frac{\partial\theta^i(\cdot)}{\partial p_1^\star(z)} = \frac{1-\theta^i(\cdot)}{p_1^\star(z)}$ and where $1-\theta^i\left(b^i,z;\boldsymbol{x}\right) = \frac{k_0^i - k_1^{i\star}(z)}{k_0^i}$. Hence, we can write $\delta_p^i(z)$ as

$$\delta_p^i(z) = \sum_{i\in\{R,U\}}\left(\underbrace{\beta\frac{\int_{\hat{s}^i(b^i,z;\boldsymbol{x})}^{\overline{s}}\rho_2^i(s)\,dF^{i,P}(s|z)}{p_1^\star(z)} + \beta^C\frac{\phi\int_{\underline{s}}^{\hat{s}^i\left(b^i,z;\boldsymbol{x}\right)}\rho_2^i(s)\,dF^{C,P}(s|z)}{p_1^\star(z)}}_{=\lambda_1^i(z)} - \beta\right)\left(k_1^{i\star}(z) - k_0^i\right)$$
$$+\,\beta^C(1-\phi)\sum_{i\in\{R,U\}}\rho_2^i\left(\hat{s}^i\left(b^i,z;\boldsymbol{x}\right)\right)k_1^{i\star}(z)\underbrace{f^{C,P}\left(\hat{s}^i\left(b^i,z;\boldsymbol{x}\right)|z\right)\frac{\partial\hat{s}^i\left(b^i,z;\boldsymbol{x}\right)}{\partial p_1(z)}}_{=\frac{\partial\mathcal{F}^i(z)}{\partial p_1(z)}}.$$

E.3 Scenarios

Here we provide explicit formulae for the different regulatory scenarios and experiments consider in Section 4.2. Note that the first-best scenario is straightforward, and simply characterized by $\tau_b^i = \delta_b^i$ and $\tau_k^i = \delta_k^i$, as implied by Lemma 1a).

E.3.1 Unconstrained-Leverage-Regulation Scenario

In the unconstrained-leverage-regulation scenario, the optimal regulation satisfies

$$\frac{dW}{d\tau_b^R} = \left(\tau_b^R - \delta_b^R\right) \frac{db^R}{d\tau_b^R} k_0^R + \left(\tau_b^U - \delta_b^U\right) \frac{db^U}{d\tau_b^R} k_0^U - \delta_k^R \frac{dk_0^R}{d\tau_b^R} - \delta_k^U \frac{dk_0^U}{d\tau_b^R} = 0$$

$$\frac{dW}{d\tau_b^U} = \left(\tau_b^R - \delta_b^R\right) \frac{db^R}{d\tau_b^U} k_0^R + \left(\tau_b^U - \delta_b^U\right) \frac{db^U}{d\tau_b^U} k_0^U - \delta_k^R \frac{dk_0^R}{d\tau_b^U} - \delta_k^U \frac{dk_0^U}{d\tau_b^U} = 0,$$

or solving for the optimal leverage regulations $\tau_b^R - \delta_b^R$ and $\tau_b^U - \delta_b^U$ in matrix form,

$$\begin{pmatrix} \tau_b^R \\ \tau_b^U \end{pmatrix} = \begin{pmatrix} \delta_b^R \\ \tau_b^U \end{pmatrix} + \begin{pmatrix} \frac{db^R}{d\tau_b^R} k_0^R & \frac{db^U}{d\tau_b^R} k_0^U \\ \frac{db^R}{d\tau_b^U} k_0^R & \frac{db^U}{d\tau_b^U} k_0^U \end{pmatrix}^{-1} \begin{pmatrix} \delta_k^R \frac{dk_0^R}{d\tau_b^R} + \delta_k^U \frac{dk_0^U}{d\tau_b^R} \\ \delta_k^R \frac{dk_0^R}{d\tau_b^U} + \delta_k^U \frac{dk_0^U}{d\tau_b^U} \end{pmatrix},$$

as implied by Proposition 1.

E.3.2 Constrained-Leverage-Regulation Scenario

In the constrained-leverage-regulation scenario, the optimal leverage regulation of regulated banks satisfies

$$\frac{dW}{d\tau_b^R} = \left(\tau_b^R - \delta_b^R\right) \frac{db^R}{d\tau_b^R} k_0^R - \delta_b^U \frac{db^U}{d\tau_b^R} k_0^U - \delta_k^U \frac{dk_0^U}{d\tau_b^R} - \delta_k^R \frac{dk_0^R}{d\tau_b^R} = 0,$$

or solving for τ_b^R,

$$\tau_b^R = \delta_b^R - \left(-\frac{db^R}{d\tau_b^R}\right)^{-1} \left(\delta_k^U \frac{d \log k_0^U}{d\tau_b^R} \frac{k_0^U}{k_0^R} + \delta_k^R \frac{d \log k_0^R}{d\tau_b^R} + \delta_b^U \frac{db^U}{d\tau_b^R} \frac{k_0^U}{k_0^R}\right).$$

This expression highlights the role played by the three estimated leakage elasticities in Table 1: $\frac{d \log k_0^U}{d\tau_b^R} / \frac{db^R}{d\tau_b^R}$, $\frac{d \log k_0^R}{d\tau_b^R} / \frac{db^R}{d\tau_b^R}$, and $\frac{db^U}{d\tau_b^R} / \frac{db^R}{d\tau_b^R}$ (Footnote 10 explains the normalization by $\frac{db^R}{d\tau_b^R}$ that we use to compute tax elasticities). In this case, the marginal welfare change associated with changing the leverage regulation on shadow banks in the constrained-leverage-regulation scenario is given by

$$\frac{dW}{d\tau_b^U} = \left(\tau_b^U - \delta_b^U\right) \frac{db^U}{d\tau_b^U} k_0^U - \delta_k^U \frac{dk_0^U}{d\tau_b^U} - \delta_k^R \frac{dk_0^R}{d\tau_b^U} + \left(\tau_b^R - \delta_b^R\right) \frac{db^R}{d\tau_b^U} k_0^R.$$

Note that we can express $\frac{dW}{d\tau_b^U}$ using the elements of the Le Chatelier matrix as follows:

$$\frac{dW}{d\tau_b^U} = \left(\tau_b^U - \delta_b^U\right) \frac{db^U}{d\tau_b^U} k_0^U \left(1 - L_b^U\right) + \left(-\delta_k^U\right) \frac{dk_0^U}{d\tau_b^U} \left(1 - L_k^U\right) + \left(-\delta_k^R\right) \frac{dk_0^R}{d\tau_b^U} \left(1 - L_k^R\right),$$

where

$$L_b^U = \left(\frac{db^U}{d\tau_b^U}\right)^{-1} \frac{db^R}{d\tau_b^U} \left(\frac{db^R}{d\tau_b^R}\right)^{-1} \frac{db^U}{d\tau_b^R}, \quad L_k^U = \left(\frac{dk_0^U}{d\tau_b^U}\right)^{-1} \frac{db^R}{d\tau_b^U} \left(\frac{db^R}{d\tau_b^R}\right)^{-1} \frac{db^U}{d\tau_b^R}, \quad \text{and}$$

$$L_k^R = \left(\frac{dk_0^R}{d\tau_b^U}\right)^{-1} \frac{db^R}{d\tau_b^U} \left(\frac{db^R}{d\tau_b^R}\right)^{-1} \frac{dk_0^R}{d\tau_b^R}.$$

E.3.3 Uniform-Leverage-Regulation Scenario

In the uniform regulation scenario with unconstrained-leverage, the regulator sets $\tau_b^U = \tau_b^R = \bar{\tau}_b$ to maximize $\frac{dW}{d\bar{\tau}_b}$, given by

$$\frac{dW}{d\bar{\tau}_b} = \sum_i \left(\bar{\tau}_b - \delta_b^i\right) \frac{db^i}{d\bar{\tau}_b} k_0^i + \sum_i \left(-\delta_k^i\right) \frac{dk_0^i}{d\bar{\tau}_b},$$

which yields the formula for the optimal uniform leverage regulation

$$\bar{\tau}_b = \frac{\sum_i \frac{db^i}{d\bar{\tau}_b} k_0^i \delta_b^i + \sum_i \frac{dk_0^i}{d\bar{\tau}_b} \delta_k^i}{\sum_i \frac{db^i}{d\bar{\tau}_b} k_0^i}.$$

F Further Applications

In Section F.1 we describe our application to financial regulation with environmental externalities, while in Section F.2 we describe four additional minimal applications. These applications are not exhaustive. For instance, one could explore the role of imperfect corrective regulation in models of strategic behavior and imperfect competition, as in Corbae and D'Erasmo (2010), Corbae and Levine (2018, 2019), or Dávila and Walther (2020), or in the context of regulation of asset markets, as in Cai, He, Jiang and Xiong (2020) or Dávila (2023).

F.1 Financial Regulation with Environmental Externalities

Central banks and macro-prudential regulators have increasingly become interested in accounting for environmental concerns. There are two possible motivations for this. First, there are links between the financial system, a primary target of macro-prudential regulation, and climate-related risks, as evidenced by a growing literature on climate finance (e.g., Giglio, Kelly and Stroebel, 2021). For instance, the safety and soundness of financial institutions may be at risk if they are heavily exposed to climate-related risks. Second, some believe that prudential regulation should take into account its effect on the broader societal goal of sustainable investment.[27] For instance, the European Central Bank's bond purchase program has taken the latter motivation into account by introducing preferential treatment for bonds associated with "green" technologies (Piazessi, Papoutsi and Schneider, 2021).

A nascent academic literature studies the welfare implications of financial regulatory reforms when there are environmental concerns (e.g., Oehmke and Opp, 2022; Rola-Janicka and Döttling, 2022). In this section, we use our general results to characterize optimal policy with environmental externalities and *imperfect macro-prudential regulation*. This is a particularly important question because regulators are already discussing potential imperfections and unintended consequences of policies in the presence of environmental externalities.[28]

In this application, we first show that imperfections are inherent to the primary mode of financial regulation in advanced economies, namely, risk-weighted leverage constraints. Indeed, these requirements constrain only relative quantities on institutions' balance sheets but leave the overall scale of investment as a free variable. Next, we analyze second-best optimal regulation in this setting. To capture the two motivations for policy discussed above, we pay special attention to contrasting the role of climate-related risks when regulation has a narrow/financial mandate versus a broad/environmental mandate. Our results, which directly leverage the formulae from the general model, yield new insights into the differences between these two cases, and into the way in which climate-conscious regulation should be adjusted for imperfections. Finally, we characterize the value of extending the set of policy tools in the face of environmental externalities, which relates to the Le Chatelier/reverse leakage adjustments that we have characterized in the general case.

[27] The Bank for International Settlements has recently summarized these two concerns as follows: "Given the impact of climate change on traditional risk categories, the speech makes the case that prudential policy needs to be adjusted to account for the impact of climate-related risks on the safety and soundness of financial institutions as part of the core mandate of supervisory authorities (what we could call the financial motivation for regulatory action). Moreover, this adjustment has often been presented as a contribution by prudential regulation to facilitate the transition to a more sustainable economy by providing incentives for a more climate-friendly allocation of financial resources (that would be the economic motivation)."

[28] For example, Andrew Bailey, the Governor of the Bank of England, has recently commented that "any incorporation of climate change into regulatory capital requirements would need to be grounded in robust data and be designed to support safety and soundness while avoiding unintended consequences or compromising our other objectives". See: https://www.bankofengland.co.uk/speech/2021/june/andrew-bailey-reuters-events-global-responsible-business-2021.

Environment. We assume that there is a single type of investor, in unit measure and indexed by i, and a unit measure of creditors, indexed by C. Both investors and creditors have risk-neutral preferences given by

$$c_0^i + \beta^i \int c_1^i(s)\, dF(s) \quad \text{and} \quad c_0^C + \beta^C \int c_1^C(s)\, dF(s) - \Psi\left(\theta^i\right) k^i,$$

where the term $\Psi\left(\theta^i\right) k^i$ introduces an environmental externality, as described below.[29] The budget constraints of investors at date 0 and date 1 are given by

$$c_0^i = n_0^i + Q^i\left(b^i, \theta^i\right) k^i - \Upsilon\left(k^i\right) - \Omega\left(\theta^i\right) k^i,$$
$$c_1^i(s) = k^i \max\left\{d_1(s)\,\theta^i + d_2(s)\left(1-\theta^i\right) + t\left(b^i, \theta^i, s\right) - b^i, 0\right\}. \quad \forall s.$$

At date 0, investors, endowed with n_0^i dollars, make capital investments k^i in two sectors of the economy. A fraction θ^i is invested in sector 1, and the remaining $1-\theta^i$ in in sector 2. Investors issue debt with face value $b^i k^i$ to creditors, so that b^i measures investors' leverage. We conjecture and verify that the equilibrium price of debt can be written as $Q^i\left(b^i, \theta^i\right) k^i$, where $Q^i\left(b^i, k^i\right)$ denotes the market value per unit of capital. Capital investments are subject to an adjustment cost $\Upsilon\left(k^i\right)$ and an additional cost $\Omega\left(\theta^i\right) k^i$ of adjusting the sectoral composition of investors' portfolios.[30] At date 1, once a state s is realized, investor i receives $d_j(s)$ dollars for each unit of investment in sector $j \in \{1, 2\}$ and a bailout transfer $t^i\left(b^i, \theta^i, s\right)$ per unit of capital that potentially depends on the amount of debt issued by the investor and portfolio weights. If the sum of these revenues exceeds the face value of debt, then investors repay their debt and consume the residual claim. Otherwise, as discussed below, they optimally choose to default and consume zero.[31]

In this application, motivated by the existing regulatory instruments, we assume that investors are subject to a *risk-weighted capital requirement*:[32]

$$b^i + \varphi\theta^i \leq \bar{b}. \tag{OA19}$$

As we show below, imposing this constraint on investors is equivalent to imposing corrective taxes. Therefore, this application also serves to illustrate how our approach to imperfect regulation can be applied to quantity-based instruments that are often used in practice. Intuitively, the requirement in Equation (OA19) places an upper bound $\bar{b}$ on investors' leverage, which is adjusted in proportion to the share θ^i invested in sector 1. In the case with $\varphi > 0$, on which we will focus without loss of generality, the relative risk weight φ on

[29]The assumption that this distortion only impacts creditors and is linear in capital simplifies the exposition, but does not affect the qualitative insights of our analysis.

[30]Alternatively, the investors' problem can be formulated in terms of the total capital investments, namely, $k_1^i = \theta^i k^i$ and $k_2^i = \left(1-\theta^i\right) k^i$. Our formulation holds as long as portfolio adjustment costs are homogeneous of degree 1 in capital investments.

[31]This specification of bailouts corresponds to a model where the government has limited commitment, which connects our work to the treatment of bailouts in Farhi and Tirole (2012), Bianchi (2016), Chari and Kehoe (2016), Keister (2016), Gourinchas and Martin (2017), Cordella, Dell'Ariccia and Marquez (2018), Dávila and Walther (2020), and Dovis and Kirpalani (2020), among others.

[32]Risk-weighted capital requirements under the Basel accords ensure that the ratio of equity to risk-weighted assets in leveraged institutions (e.g., banks) is at least equal to a constant fraction C. In our context, equity is $\left(1-b^i\right) k^i$ and risk-weighted assets can be represented as $\left[w_1\theta^i + w_2\left(1-\theta^i\right)\right] k^i$, where w_j is the risk weight on sector j investments. Thus, we can express a risk-weighted capital requirement as

$$1 - b^i \geq C\left[w_1\theta^i + w_2\left(1-\theta^i\right)\right] \iff b^i + \underbrace{(w_1 - w_2)}_{\equiv\varphi}\theta^i \leq \underbrace{1 - Cw_2}_{\equiv\bar{b}},$$

which is equivalent to our formulation in (OA19), with φ denoting the relative risk weight on sector 1 investments.

sector 1 is positive, and the leverage cap becomes tighter when investors increase θ^i.

The budget constraints of creditors at date 0 and date 1 are given by

$$c_0^C = n_0^C - h^i Q^i\left(b^i,\theta^i\right)k^i,$$
$$c_1^C(s) = n_1^C(s) - (1+\kappa)\,t\left(b^i,\theta^i,s\right)k^i + h^i\mathcal{P}^i\left(b^i,\theta^i,s\right)k^i.$$

At date 1, creditors are taxed $(1+\kappa)$ times the government bailout, where $\kappa > 0$ denotes the deadweight cost of fiscal intervention. Moreover, creditors who buy a fraction h^i of investors' debt pay the market price at date 0, and receive a payment $\mathcal{P}^i\left(b^i,\theta^i,s\right)k^i$ at date 1. This payment, which preemptively incorporates investors' optimal default decision, is defined as follows:

$$\mathcal{P}^i\left(b^i,\theta^i,s\right) = \begin{cases} b^i, & d_1(s)\,\theta^i + d_2(s)\left(1-\theta^i\right) + t\left(b^i,\theta^i,s\right) \geq b^i \\ \phi\left[d_1(s)\,\theta^i + d_2(s)\left(1-\theta^i\right)\right], & \text{otherwise.} \end{cases}$$

Investors default when their assets are worth less than the promised repayment b^i per unit of capital, and repay b^i in full otherwise. In default, creditors recover a fraction $\phi < 1$ of their assets, so that $1-\phi$ can be interpreted as the deadweight cost of default. For simplicity, we assume that primitives are such that there exists a default threshold $s^\star\left(b^i,\theta^i\right)$, so that investors default when $s < s^\star\left(b^i,\theta^i\right)$ and repay otherwise.[33]

Finally, recall that creditors' preferences include a utility loss of $\Psi\left(\theta^i\right)k^i$ as a result of investors' choices. This term reflects an environmental externality. Investors' portfolio choices θ^i can affect this loss. For example, if $\frac{\partial\Psi'}{\partial\theta^i} > 0$, then the environmental externality is increasing in the investment share in sector 1, meaning that sector 1 is associated with more pollution than sector 2.

Equilibrium. For given regulatory parameters $\left\{\bar{b},\varphi\right\}$ defining the constraint (OA19) and a given bailout policy $t\left(b^i,\theta^i,s\right)$, an *equilibrium* is defined by leverage, portfolio, and investment decisions $\left\{b^i,\theta^i,k^i\right\}$, a default decision rule, and a pricing schedule $Q\left(b^i,\theta^i\right)$ such that investors and creditors maximize their utility and the market for debt clears, i.e., $h^i = 1$.

We rely on the following characterization of the equilibrium.

Lemma 3. [Equilibrium characterization] *Equilibrium choices $\left\{b^i,\theta^i,k^i\right\}$ are given by the solution to the following reformulation of the problem faced by investors:*

$$\max_{\left\{b^i,\theta^i,k^i\right\}} \left[M\left(b^i,\theta^i\right) - \Omega\left(\theta^i\right) - 1\right]k^i - \Upsilon\left(k^i\right) \;\; \textit{subject to } k^i\left(b^i+\varphi\theta^i\right) \leq k^i\bar{b}, \tag{OA20}$$

where $M\left(b^i,\theta^i\right)$ is given by

$$M\left(b^i,\theta^i\right) = \underbrace{\beta^i\int_{s^\star\left(b^i,\theta^i\right)}^{\bar{s}}\left(d_1(s)\,\theta^i + d_2(s)\left(1-\theta^i\right) + t\left(b^i,\theta^i,s\right) - b^i\right)dF(s)}_{equity}$$
$$+\underbrace{\beta^C\left(\int_{s^\star\left(b^i,\theta^i\right)}^{\bar{s}} b^i dF(s) + \phi\int_{\underline{s}}^{s^\star\left(b^i,\theta^i\right)}\left[d_1(s)\,\theta^i + d_2(s)\left(1-\theta^i\right) + t\left(b^i,\theta^i,s\right)\right]dF(s)\right)}_{debt\;=Q\left(b^i,\theta^i\right)}, \tag{OA21}$$

[33] The uniqueness of this threshold $s^\star\left(b^i,\theta^i\right)$ is guaranteed under the standard assumptions that i) $d_j(s)$, $j \in \{1,2\}$, is increasing in s (i.e., higher asset returns in good states), and ii) the bailout transfer $t\left(b^i,\theta^i,s\right)$ is decreasing in s and increasing in b^i (i.e., larger bailouts in bad states/for more levered investors).

and $s^\star\left(b^i,\theta^i\right)$ *solves the equation*

$$d_1\left(s^\star\right)\theta^i+d_2\left(s^\star\right)\left(1-\theta^i\right)+t\left(b^i,\theta^i,s^\star\right)=b^i.$$

Intuitively, we characterize the equilibrium by incorporating the pricing of debt into the investors' problem at date 1. The function $M\left(b^i,\theta^i\right)$ can be interpreted as the sum of the market values of equity (owned by investors) and debt (owned by creditors) per unit of investment. Notice that the second term in Equation (OA21) corresponds to the equilibrium price of debt $Q\left(b^i,\theta^i\right)$, which incorporates the fact that investors default in states $s<s^\star\left(b^i,\theta^i\right)$ in which the value of their assets is less than the promised repayment b^i. In problem (OA20), investors maximize the market value of investment net of costs. For convenience, and without loss of generality, we have scaled the regulatory constraint in this problem by total investment $k^i\geq 0$.

An important aspect of this application is that the planner's instruments are imperfect. This can be seen by writing investors' first-order conditions as

$$\frac{\partial M\left(b^i,\theta^i\right)}{\partial b^i}=\mu\equiv\tau_b \tag{OA22}$$

$$\frac{\partial M\left(b^i,\theta^i\right)}{\partial \theta^i}-\Omega'\left(\theta^i\right)=\mu\varphi\equiv\tau_\theta \tag{OA23}$$

$$M\left(b^i,\theta^i\right)-\Omega\left(\theta^i\right)-1-\Upsilon'\left(k^i\right)=0, \tag{OA24}$$

where $\mu\geq 0$ is the Lagrange multiplier on the regulatory constraint. The first two conditions, which define optimal leverage and portfolio weights, show that the constraint in Equation (OA19) implies effective corrective taxes τ_b on leverage b^i and τ_θ on portfolios θ^i. The third condition, which defines optimal total investment k^i, does not contain a corrective tax. Intuitively, the capital requirement (OA19) constrains ratios but leaves the overall scale k^i of investors' balance sheet as a free, unregulated variable. By contrast, in a world with perfect instruments, the planner would be able to set a corrective tax τ_k on k^i in addition to τ_b and τ_θ. We return to the value of introducing such a tax below.

Optimal Corrective Policy. In this environment, we can express the marginal externalities $\{\delta_k,\delta_b,\delta_\theta\}$ associated with investors' choices and decompose them into a financial (i.e., bailout-related) and an environmental component as follows:

$$\delta_b=\underbrace{\left(1+\kappa\right)\beta^C\int_{\underline{s}}^{\bar{s}}\frac{\partial t\left(b^i,\theta^i,s\right)}{\partial b^i}dF\left(s\right)}_{\equiv\chi_b} \tag{OA25}$$

$$\delta_\theta=\underbrace{\left(1+\kappa\right)\beta^C\int_{\underline{s}}^{\bar{s}}\frac{\partial t\left(b^i,\theta^i,s\right)}{\partial \theta^i}dF\left(s\right)}_{\equiv\chi_\theta}+\underbrace{\frac{\partial\Psi\left(\theta^i\right)}{\partial\theta^i}}_{\equiv\psi_\theta} \tag{OA26}$$

$$\delta_k=\underbrace{\left(1+\kappa\right)\beta^C\int_{\underline{s}}^{\bar{s}}t\left(b^i,\theta^i,s\right)dF\left(s\right)}_{\equiv\chi_k}+\underbrace{\Psi\left(\theta^i\right)}_{\equiv\psi_k}. \tag{OA27}$$

For instance, χ_k in Equation (OA27) measures the marginal distortion in capital choices due to bailouts, while ψ_k is the distortion due to environmental externalities. Equations (OA25) and (OA26) define the distortions associated with leverage and portfolio choices per unit of capital. An important point is that leverage induces only a financial distortion, since environmental damage is determined by the technologies that are operated in this economy, and is independent of how these technologies are financed.

In Proposition 3, we characterize the form of the second-best policy.

Proposition 3. *(Financial Regulation with Environmental Externalities)*
a) The marginal welfare effects of varying the leverage cap $\bar{b}$ and the risk weight φ, respectively, are given by

$$\frac{dW}{d\bar{b}} = \frac{db^i}{d\bar{b}}\left(\tau_b - \delta_b\right)k^i + \frac{d\theta^i}{d\bar{b}}\left(\tau_\theta - \delta_\theta\right)k^i - \frac{dk^i}{d\bar{b}}\delta_k, \tag{OA28}$$

$$\frac{dW}{d\varphi} = \frac{db^i}{d\varphi}\left(\tau_b - \delta_b\right)k^i + \frac{d\theta^i}{d\varphi}\left(\tau_\theta - \delta_\theta\right)k^i - \frac{dk^i}{d\varphi}\delta_k. \tag{OA29}$$

b) The optimal regulation satisfies

$$\begin{pmatrix} \tau_b \\ \tau_\theta \end{pmatrix} = \begin{pmatrix} \delta_b \\ \delta_\theta \end{pmatrix} + \begin{pmatrix} \frac{db^i}{d\bar{b}} & \frac{d\theta^i}{d\bar{b}} \\ \frac{db^i}{d\varphi} & \frac{d\theta^i}{d\varphi} \end{pmatrix}^{-1} \begin{pmatrix} \frac{d\log k^i}{d\bar{b}} \\ \frac{d\log k^i}{d\varphi} \end{pmatrix} \delta_k. \tag{OA30}$$

Proposition 3 characterizes the marginal welfare effects of adjusting the two instruments available to the planner and the optimal regulation in terms of the parameters of the risk-weighted capital constraint, which are the leverage cap $\bar{b}$ and the relative risk weight φ. Notice that, even though we are working in terms of a quantity constraint, our general characterization of welfare effects from Lemma 1 applies, after suitably adjusting for k^i. This feature highlights the usefulness of our approach for analyzing quantity-based regulation.

Specifically, Equations (OA28) and (OA29) show that marginal welfare effects depend on Pigouvian wedges — defined in terms of the equivalent taxes $\{\tau_b, \tau_\theta\}$ in Equations (OA22) and (OA23) — as well as policy elasticities. First-best regulation is prevented by the fact that the unregulated scale decision k^i introduces an additional distortion δ_k. The optimal regulation, which we discuss in more detail below, takes into account this distortion along with the appropriate leakage elasticities $\frac{d\log k^i}{d\bar{b}}$ and $\frac{d\log k^i}{d\varphi}$.[34]

In the remainder of this section, we use this characterization to derive several concrete insights into optimal regulation with environmental externalities. First, we analyze the distinction between optimal policy motivated by narrow financial stability mandates and broader mandates that take environmental externalities into account. Importantly, we provide a novel treatment of these questions taking into account imperfections in policy instruments. Finally, we characterize the value of relaxing constraints on regulation by imposing corrective regulation on the total scale of investment.

Imperfect Regulation with Narrow/Financial Mandates. We first consider a financial regulator who has a narrow mandate and is only concerned with financial externalities. In terms of our decomposition of distortions, we interpret a narrow mandate as meaning that the regulator acts as if the climate-related distortions $\{\psi_\theta, \psi_k\}$ are both equal to zero. In the background, one can interpret that the distribution of states, $F(s)$, and the payoffs of the different investments, $d_1(s)$ and $d_2(s)$, account for climate risks. Applying Proposition 3 and substituting Equations (OA25) through (OA27) yields the optimal policy in this case:

Corollary 1. *(Imperfect Regulation with Narrow/Financial Mandates) The optimal policy of a regulator*

[34] The appropriate leakage elasticities in this application are semi-elasticities, i.e., responses of log investment to policy reforms. Compared to our general model, this formulation arises because we have expressed leverage and portfolio choices per unit of capital.

with a narrow/financial mandate is given by

$$\left(\begin{array}{c}\tau_b\\ \tau_\theta\end{array}\right)=\left(\begin{array}{c}\chi_b\\ \chi_\theta\end{array}\right)+\left(\begin{array}{cc}\frac{db^i}{d\bar{b}} & \frac{d\theta^i}{d\bar{b}}\\ \frac{db^i}{d\varphi} & \frac{d\theta^i}{d\varphi}\end{array}\right)^{-1}\left(\begin{array}{c}\frac{d\log k^i}{d\bar{b}}\\ \frac{d\log k^i}{d\varphi}\end{array}\right)\chi_k, \tag{OA31}$$

where χ_b, χ_θ, and χ_k denote the financial component of the respective externalities, defined in Equations (OA25) *through* (OA27).

Equation (OA31) shows that the optimal leverage cap — represented by τ_b — and the optimal risk weight — represented by τ_θ — are set in response to two terms. The first term captures the marginal externality associated with a change in b or θ, which in this case corresponds to the marginal response of expected bailouts to more leverage or more investment in sector 1.

The second term, which arises only with imperfect instruments, is proportional to the leakage elasticities $\frac{d\log k^i}{d\bar{b}}$ and $\frac{d\log k^i}{d\varphi}$, and scales with the total expected bailout, via χ_k. In the Appendix, we prove that both these elasticities fall into the "complements" case: Stricter leverage regulation ($\downarrow\bar{b}$) or a stricter relative risk weight ($\uparrow\varphi$) both lead to increases in k^i in equilibrium. Therefore, Equation (OA31) generally calls for *overregulation* of leverage and risk. Finally, notice that the relevant leakage elasticities are modulated by an inverse matrix of policy elasticities between b^i and θ^i.

The implication for financial regulation with environmental externalities is that any adjustment for climate-related risk should be determined only by its impact on financial externalities (in this particular case, this emerges from the presence of bailouts). For instance, the risk weight equivalent tax τ_θ should be increased if sector 1 is associated with climate-related tail risk that makes large bailouts more likely (i.e., if $\mathbb{E}_s\left[\frac{\partial t\left(b^i,\theta^i,s\right)}{\partial\theta^i}\right]>0$). In addition, the setting with imperfect instruments implies that taxes on *both* leverage and portfolio weights should increase if climate-related risk increases the magnitude of the total expected bailout. This prediction is unique to our analysis and directly leverages our general tools.

Imperfect Regulation with Broad/Environmental Mandates. We now consider a financial regulator with a broad mandate who cares directly about mitigating environmental distortions. We will focus now on the case where the environmental distortions satisfy $\psi_\theta>0$ and $\psi_k>0$. In this case, increases in overall scale as well as concentrated investments in sector 1 are associated with greater environmental damage.

Corollary 2. (Imperfect Regulation with Broad/Environmental Mandates)

a) When policy has been set optimally according to a narrow/financial mandate, the welfare benefits of marginal policy changes are given by

$$\frac{dW}{d\bar{b}}=-\frac{d\theta^i}{d\bar{b}}\psi_\theta-\frac{dk^i}{d\bar{b}}\psi_k \tag{OA32}$$

$$\frac{dW}{d\varphi}=-\frac{d\theta^i}{d\varphi}\psi_\theta-\frac{dk^i}{d\varphi}\psi_k, \tag{OA33}$$

where ψ_θ and ψ_k denote the environmental component of the respective externalities, defined in Equations (OA26) *and* (OA27).

b) The optimal policy of a regulator with a broad/environmental mandate is given by

$$\left(\begin{array}{c}\tau_b\\ \tau_\theta\end{array}\right)=\left(\begin{array}{c}\chi_b\\ \chi_\theta+\psi_\theta\end{array}\right)+\left(\begin{array}{cc}\frac{db^i}{d\bar{b}} & \frac{d\theta^i}{d\bar{b}}\\ \frac{db^i}{d\varphi} & \frac{d\theta^i}{d\varphi}\end{array}\right)^{-1}\left(\begin{array}{c}\frac{d\log k^i}{d\bar{b}}\\ \frac{d\log k^i}{d\varphi}\end{array}\right)(\chi_k+\psi_k), \tag{OA34}$$

where χ_b, χ_θ, and χ_k denote the financial component and ψ_θ and ψ_k denote the environmental component of the respective externalities, defined in Equations (OA25) *through* (OA27)*.*

Corollary 2 develops two insights into the distinction between narrow/financial and broad/environmental mandates. First, Equations (OA32) and (OA33) highlight the additional welfare effects, from the perspective of a broad mandate, of adjusting either the leverage cap and risk weights, when policy has previously been optimized according a narrow mandate. These equations are useful for deciding whether policy should be adjusted at the margin once a regulator decides to take environmental outcomes into account. The relevant marginal welfare effects are determined by the environmental distortions ψ_θ and ψ_k and the associated leakage elasticities. It is interesting to note that the leakage elasticities to leverage (i.e., $\frac{db^i}{d\bar{b}}$ and $\frac{db^i}{d\varphi}$) are irrelevant here, because the mode of financing has no marginal impact on environmental concerns.

Equation (OA32) shows that a regulator who adjusts the leverage cap $\bar{b}$ in response to environmental concerns faces a potential conflict of interest. Indeed, while it is natural that scale and leverage are generally complements, implying $\frac{dk^i}{d\bar{b}} > 0$,[35] the response of optimal portfolio choices $\frac{d\theta^i}{d\bar{b}}$ is ambiguous in theory, and depends on the functional form of returns to investment in each sector. Since the environmental distortions ψ_θ, ψ_b are assumed positive, the two terms in Equation (OA32) may have opposite signs. As a result, it is unclear whether leverage requirements should be relaxed or tightened in response to environmental concerns, and their impact on welfare may be offset by portfolio adjustments.

By contrast, Equation (OA33) demonstrates that risk weights are a natural tool for addressing environmental concerns. Both the portfolio share θ^i and total capital k^i are generally complements to the risk weight, implying that $\frac{d\theta^i}{d\varphi} < 0$ and $\frac{dk^i}{d\varphi} < 0$. Therefore, it is clear that risk weights ought to be tightened when regulators account for environmental externalities.

The second insight emerging from Corollary 2 is the characterization of optimal policy in Equation (OA34). There are two differences to the equivalent characterization with a narrow mandate in Equation (OA31). First, the marginal distortion on portfolio choices is augmented, which calls for greater relative risk weights on the polluting sector (sector 1). Second, the scale distortion is augmented by ψ_k. The latter point is particularly important for our analysis. The scale distortion matters purely due to imperfect regulation and leakage elasticities. Equation (OA34) demonstrates that adjustments for leakage elasticities become *more* important once the regulator cares about environmental effects.

Figure OA-3 illustrates the relation between the first-best and second-best solutions in both the narrow and the broad mandate cases. In particular, the left panel shows the marginal welfare effect of varying leverage regulation (in terms of τ_b), while the right panel shows the marginal welfare effect of varying risk-weights (in terms of τ_θ). As we have formally shown above, Figure OA-3 illustrates that the optimal second-best policy under a broad mandate overregulates both leverage and portfolio weights relative to the first-best. However, consistent with the insights discussed above, the relation between the first-best regulation and the second-best regulation for a regulator with a narrow mandate is more nuanced. In the case we illustrate, it turns out that a narrow regulator overregulates leverage relative to the first-best, but not portfolio weights. This is mainly due to the fact that the leakage elasticity with respect to capital is greater in magnitude for leverage. By contrast, a broad regulator overregulates both leverage and portfolio weights relative to first best, because she places a greater weight on all leakage elasticities to capital.

The Value of Regulating Scale To close the analysis of this application, we consider a regulator who is able to impose a corrective tax $\tau_k k^i$ on investors in order to correct for the (previously unregulated)

[35] Recall that $d\bar{b} > 0$ stands for a *looser* leverage cap, that is, a lower effective tax on leverage. Hence, $\frac{dk^i}{d\bar{b}} > 0$ is equivalent to $\frac{dk^i}{d\tau_b} < 0$, which corresponds to the complements case in our general, tax-based notation.

Application: Financial Regulation with Environmental Externalities

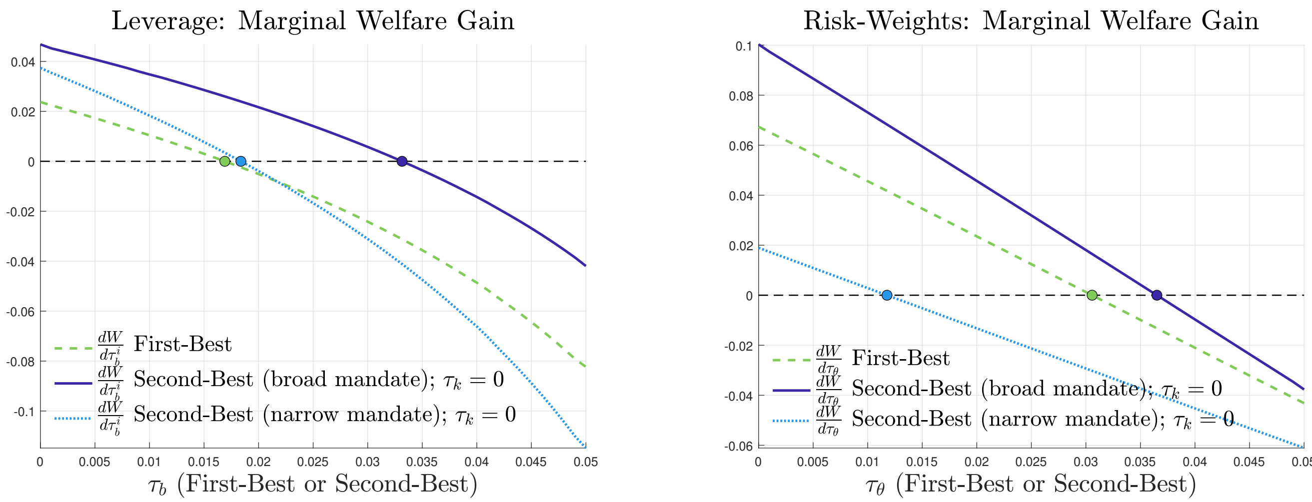

Figure OA-3: Financial Regulation with Environmental Externalities

Note: The left panel of Figure OA-3 compares the marginal welfare effects of varying corrective leverage regulation (τ_b) in three different scenarios. The green dashed line corresponds to the first-best scenario, in which τ_θ and τ_k are held fixed at their first-best levels (previously computed). The solid dark blue line corresponds to a second-best scenario in which the regulator has a broad mandate and cares about financial and environmental distortions. In this case, we compute welfare gains setting $\tau_k = 0$ and holding τ_θ fixed at the optimal second-best level for a broad mandate (previously computed). The light blue dotted line corresponds to a second-best scenario in which the regulator has a narrow mandate and cares exclusively about financial distortions. In this case, we compute welfare gains setting $\tau_k = 0$ and holding τ_θ fixed at the optimal second-best level for a narrow mandate (previously computed). The right panel of Figure OA-3 compares the analogous marginal welfare effects of varying corrective risk-weights regulation (τ_θ) in the same three scenarios.
To generate this figure, we assume that the bailout policy is linearly separable, $t^i\left(b^i, s\right) = \alpha_0^i + \alpha_b^i b^i + \alpha_\theta^i \theta^i - \alpha_s^i s$, that the adjustment cost is quadratic, $\Upsilon\left(k^i\right) = \frac{a}{2}\left(k^i\right)^2$, and that the functions $\Omega\left(\theta^i\right)$ and $\Psi\left(\theta^i\right)$ are of the CES (constant elasticity of substitution) form in terms of k_1^i and k_2^i, so $\Omega(\theta) = z_\Omega\left(a_\Omega\left(\theta^i\right)^{\eta_\Omega} + (1-a_\Omega)\left(1-\theta^i\right)^{\eta_\Omega}\right)^{\frac{1}{\eta_\Omega}}$ and $\Psi(\theta) = z_\Psi\left(a_\Psi\left(\theta^i\right)^{\eta_\Psi} + (1-a_\Psi)\left(1-\theta^i\right)^{\eta_\Psi}\right)^{\frac{1}{\eta_\Psi}}$. The parameters used to generate this figure are $\beta^i = 0.9$, $\beta^C = 0.98$ $\phi^i = 0.7$, $a = 1$, $\alpha_0^i = \alpha_s^i = 0$, $\alpha_b^i = 0.015$, $\alpha_\theta^i = 0.01$, $\kappa = 0.15$, $d_1(s) = d_1 s$ with $d_1 = 1.01$, $d_2(s) = d_2 s$ with $d_2 = 1$, $z_\Omega = 0.25$, $a_\Omega = 1.5$, $\eta_\Omega = 1.5$, $z_\Psi = 0.25$, $a_\Psi = 0.55$, $\eta_\Psi = 1.5$, $n_0^C = 50$, and $n_1^C(s) = 50 + 0.1s$, where s is normally distributed with mean 1.3 and standard deviation 0.8, truncated to the interval $[0, 3]$. For reference, the optimal first-best regulation is $\tau_b = 1.69\%$, $\tau_\theta = 3.05\%$, and $\tau_k = 14.22\%$, the optimal second-best regulation with a broad mandate is $\tau_b = 3.33\%$, $\tau_\theta = 3.65\%$, and $\tau_k = 0$, while the optimal second-best regulation with a narrow mandate is $\tau_b = 1.83\%$, $\tau_\theta = 1.11\%$, and $\tau_k = 0$.

externalities associated with the scale decision k^i. The key economic insights can be obtained by considering the marginal welfare effect of increasing τ_k.

Corollary 3. ***(Environmental Externalities/Regulating Unregulated Decision)***

When the planner can impose a corrective tax τ_k on the total scale of investment k^i, the marginal welfare effect of varying τ_k is given by

$$\begin{aligned} \frac{dW}{d\tau_k} &= \underbrace{\frac{db^i}{d\tau_k}}_{=0} \left(\tau_b - \delta_b\right) k^i + \underbrace{\frac{d\theta^i}{d\tau_k}}_{=0} \left(\tau_\theta - \delta_\theta\right) k^i - \frac{dk^i}{d\tau_k} \left(\tau_k - \delta_k\right) \\ &= -\frac{dk^i}{d\tau_k} \omega_k. \end{aligned} \tag{OA35}$$

An interesting property of this environment is that there are no reverse leakage effects from regulating scale onto leverage and portfolio decisions. Intuitively, the investors' problem — see Lemma 3 — can be broken down into a two-step procedure. First, investors choose leverage and portfolios to maximize market values $M\left(b^i, \theta^i\right)$ per unit of total capital. Second, they set the marginal cost of capital equal to its maximized market value. Since the first step does not depend on the cost/tax of capital, b^i and θ^i are independent of τ_k in equilibrium.

This fact has two novel economic implications. First, we note that the case for regulating scale here is much stronger than in other applications. In particular, the capital-specific elements of the Le Chatelier/reverse leakage adjustment matrix $\boldsymbol{L}$, which usually dampens the welfare impact of regulating unregulated decisions, are zero. Moreover, the case for regulating scale is clearly *stronger* when the regulator has a *broad/environmental mandate*, other things equal, since this mandate takes into account the full marginal distortion $\delta_k = \chi_k + \psi_k$.

Second, we see from Equation (OA35) that the optimal level of the tax on capital is always given by $\tau_k = \delta_k$, which corresponds to the first best or Pigouvian correction. The absence of reverse leakage implies that there is no incentive to over- or underregulate scale, once the regulator is allowed to do so. This is true even when the regulation of leverage and portfolio decisions is imperfect (with $\tau_b \neq \delta_b$ and/or $\tau_\theta \neq \delta_\theta$).

F.2 Minimal Applications

F.2.1 Application 1: Shadow Banking/Unregulated Investors

The notion of shadow banking is typically used to describe the financial activities that take place outside of the regulated financial sector.[36] In this application, we consider an environment with two types of investors, in which only one type of investor can be regulated (the traditional sector), while the other is outside of the scope of the regulation (the shadow sector).

Environment. We assume that there are two types of investors $i \in \{1, 2\}$. In this application, investors should be broadly interpreted as financial intermediaries or banks. Investors have risk-neutral preferences of the form:

$$c_0^i + \beta^i \int c_1^i(s)\, dF(s),$$

[36] Pozsar, Adrian, Ashcraft and Boesky (2010), Gorton, Metrick, Shleifer and Tarullo (2010), and Claessens, Pozsar, Ratnovski and Singh (2012) provide a detailed overview of shadow banking institutions, activities, and regulations.

with budget constraints given by

$$c_0^i = n_0^i + Q^i\left(b^i\right) - \tau_b^i b^i + T_0^i$$
$$c_1^i(s) = n_1^i(s) + \max\left\{v^i s + t^i\left(b^i, s\right) - b^i, 0\right\}, \quad \forall s.$$

At date 0, an investor i endowed with n_0^i dollars chooses the face value of its debt, b^i, which determines the amount of financing obtained at date 0, $Q^i\left(b^i\right)$, determined in equilibrium by creditors, as described below. Investor i faces a corrective tax τ_b^i per unit of b^i due at date 0. At date 1 in state s, investor i receives $v^i s$ dollars, as well as a bailout transfer $t^i\left(b^i, s\right)$.

Creditors are risk-averse, with preferences of the form

$$u\left(c_0^C\right) + \beta^C \int u\left(c_1^C(s)\right) dF(s).$$

Their budget constraints are given by

$$c_0^C = n_0^C - \sum_{i \in \mathcal{I}} h^i Q^i\left(b^i\right),$$
$$c_1^C(s) = n_1^C(s) + \sum_{i \in \mathcal{I}} h^i \mathcal{P}^i\left(b^i, s\right) - (1+\kappa) \sum_{i \in \mathcal{I}} t^i\left(b^i, s\right), \quad \forall s,$$

where h^i is the fraction of bonds purchased from investor i, and $\mathcal{P}^i\left(b^i, s\right)$ denotes the repayment received by creditors from investor i in state s, as we explicitly describe in the Online Appendix. At date 1, all bailout funds are raised from creditors, with a constant net marginal cost of public funds $\kappa \geq 0$. Note that investors only interact in this application through changes in the price of credit, i.e., through the stochastic discount factor of creditors: $m^C(s) = \frac{\beta^C u'\left(c_1^C(s)\right)}{u'\left(c_0^C\right)}$.

Equilibrium. In this application, for given corrective taxes/subsidies $\left\{\tau_b^1, \tau_b^2\right\}$, lump-sum transfers $\left\{T_0^1, T_0^2\right\}$, and bailout transfers $\left\{t^1\left(b^1, s\right), t^2\left(b^2, s\right)\right\}$, an *equilibrium* is fully determined by investors' borrowing decisions, $\left\{b^1, b^2\right\}$, and financing schedules, $\left\{Q^1\left(b^1\right), Q^2\left(b^2\right)\right\}$, such that investors maximize their utility, given the financing schedules, and creditors set the schedules optimally, so that $h^1 = h^2 = 1$.

In the first-best scenario, the planner is able to set τ_b^1 and τ_b^2 freely. However, we are interested in scenarios in which the planner cannot regulate type 2 investors, so

$$\tau_b^2 = 0,$$

which makes the problem of choosing the optimal τ_b^1 a second-best problem.

Optimal Corrective Policy/Simulation. First, in Proposition 4, we characterize the form of the optimal second-best policy. Next, we explore a numerical simulation of this application.

Proposition 4. *(Shadow Banking/Unregulated Investors)*
a) The marginal welfare effect of varying the corrective regulation of regulated investors, τ_b^1, is given by

$$\frac{dW}{d\tau_b^1} = \frac{db^1}{d\tau_b^1}\left(\tau_b^1 - \delta_b^1\right) - \frac{db^2}{d\tau_b^1}\delta_b^2,$$

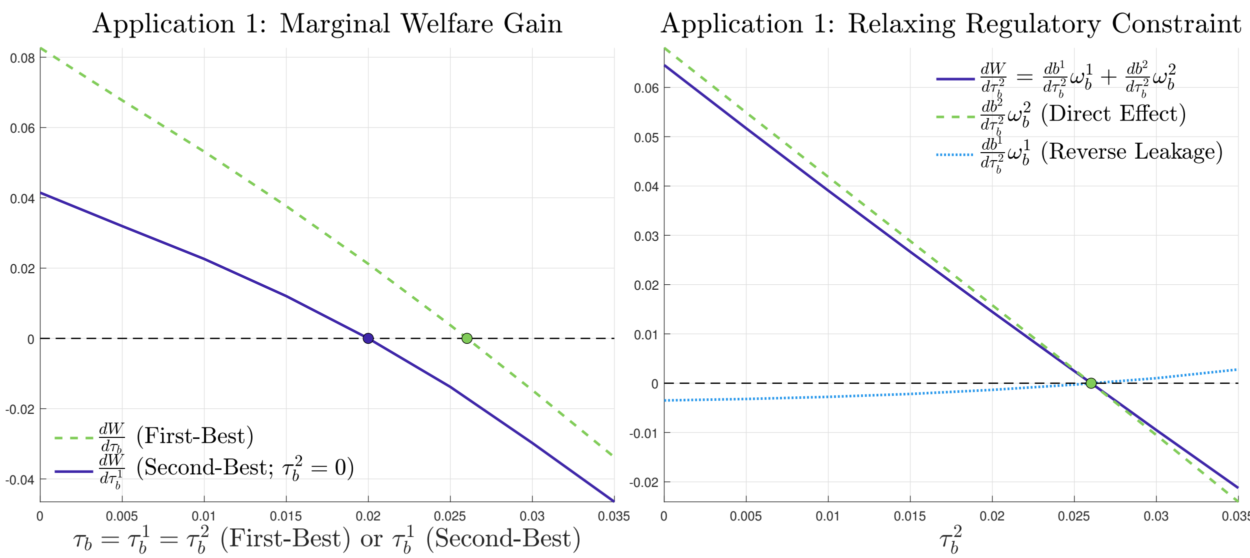

Figure OA-4: Shadow Banking/Unregulated Investors (Application 1)

Note: The left panel of this figure compares the marginal welfare effects of varying corrective regulations in two different scenarios. The green dashed line corresponds to the first-best scenario in which the horizontal axis corresponds to $\tau_b = \tau_b^1 = \tau_b^2$. The solid blue line corresponds to a second-best scenario in which $\tau_b^2 = 0$ and the horizontal axis corresponds to τ_b^1. Since we assume that both types of investors are symmetric, the value of τ_b that makes the first-best marginal welfare effect zero defines the first-best regulation. The value of τ_b^1 that makes the second-best marginal welfare effect zero defines the second-best regulation.

The right panel of this figure illustrates Proposition 2 by showing the marginal value of being able to regulate the shadow sector. The solid dark blue line corresponds to the total marginal welfare gain of increasing τ_b^2, while τ_b^1 is continually adjusted to be at the optimal second-best value given τ_b^2. The total gain can be decomposed into a direct effect, which corresponds to $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\boldsymbol{\omega}^U$ in Equation (9), and a reverse leakage effect, which corresponds to $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\boldsymbol{L}\boldsymbol{\omega}^U$ in Equation (9). The green dashed line corresponds to the direct effect of relaxing the regulatory constraint, while the light blue dotted line corresponds to the reverse leakage effect. Note that both the direct effect and the reverse leakage effect are zero at the first-best, when $\tau_b = \tau_b^1 = \tau_b^2 = 2.60\%$, but have opposite signs otherwise.

To generate this figure, we assume that the bailout policy is linearly separable: $t^i\left(b^i, s\right) = \alpha_0^i - \alpha_s^i s + \alpha_b^i b^i$, and that creditors' utility is isoelastic: $u\left(c\right) = \frac{c^{1-\gamma}}{1-\gamma}$. The parameters used to generate this figure are $\beta^i = 0.7$, $\phi^i = 0.25$, $v^i = 1$, $\alpha_0^i = \alpha_s^i = 0$, $\alpha_b^i = 0.01$, for $i \in \{1, 2\}$. Also $\kappa = 0.13$, $\gamma = 6$, $\beta^C = 0.98$, $n_0^C = 50$, and $n_1^C\left(s\right) = 50 + 0.1s$, where s is normally distributed with mean 1.3 and standard deviation 0.3, truncated to the interval $[0, 3]$. For reference, the optimal first-best regulation is $\tau_b^1 = \tau_b^2 = 2.60\%$, while the optimal second-best regulation, when the second type of investors cannot be regulated, is $\tau_b^1 = 1.99\%$. Since borrowing decisions are gross substitutes in this application, the optimal second-best policy is *sub-Pigouvian*.

where the marginal distortions in this application are defined by

$$\delta_b^i = (1+\kappa) \int m^C(s) \frac{\partial t^i(b^i, s)}{\partial b^i} dF(s), \tag{OA36}$$

where $m^C(s)$ denotes the stochastic discount factor of creditors.
b) The optimal corrective regulation satisfies

$$\tau_b^1 = \delta_b^1 - \left(-\frac{db^1}{d\tau_b^1}\right)^{-1} \frac{db^2}{d\tau_b^1} \delta_b^2.$$

Proposition 4 is an application of Lemma 1 and Proposition 1 and exploits the structure of this application to extract further insights. In this application, the marginal distortions associated with borrowing, δ_b^i, are determined by the expected marginal bailout $\frac{\partial t^i(b^i,s)}{\partial b^i}$, augmented by default deadweight losses κ if present, valued using the creditors' stochastic discount factor. The departure of the optimal regulation from the first-best critically depends on the leakage elasticity $\frac{db^2}{d\tau_b^1}$ and the unregulated distortion δ_b^2. A number of recent studies provide direct measurements of the relevant leakage elasticity (e.g., Buchak, Matvos, Piskorski and Seru, 2024; Irani, Iyer, Meisenzahl and Peydro, 2021).[37] As we show in the Online Appendix, in this application, consistent with the empirical literature, we find that tighter regulation on the regulated sector (higher τ_b^1) increases the activities carried out by the unregulated/shadow sector ($\frac{db^2}{d\tau_b^1} > 0$), so leverage choices are gross substitutes. Therefore, we expect the optimal second-best policy to be sub-Pigouvian.[38]

Moreover, the presence of unregulated investors may exacerbate the welfare distortion δ_b^1 associated with regulated investors. Concretely, when unregulated investors receive bailouts in state s, the marginal utility of creditors increases in this state due to taxation. In Equation (OA36), this increases the distortion associated with marginal increases in regulated investors' leverage. In this sense, our results reconcile two common narratives. On the one hand, leakage to the shadow banking system motivates sub-Pigouvian regulation. On the other hand, the optimal corrective policy must also adjust to increases in overall leverage, which raise marginal distortions δ_b^1 in general equilibrium.

An instructive special case, which we use to solve the model numerically, is obtained by using a linearly separable bailout policy: $t^i(b^i, s) = \alpha_0^i - \alpha_s^i s + \alpha_b^i b^i$, where $\alpha_s^i, \alpha_b^i \geq 0$. In this case, marginal distortions $\delta_b^i = \frac{1+\kappa}{R^f} \alpha_b^i$ are invariant to policy, and the optimal corrective regulation is

$$\tau_b^1 = \frac{1+\kappa}{R^f} \left[\alpha_b^1 - \left(-\frac{db^1}{d\tau_b^1}\right)^{-1} \frac{db^2}{d\tau_b^1} \alpha_b^2 \right],$$

[37] This work focuses on the elasticity of substitution between the market share of regulated and unregulated investments. While we have held the scale of investment fixed in this application, but one could easily extend the framework to account for both leverage and investment choices, in which case the measured elasticities of substitution in those papers become directly relevant. In addition, our application highlights that the elasticity of substitution between regulated and unregulated leverage is a key statistic for second-best regulation.

[38] Note that one can also use this model to analyze quantity-based policies, such as capital requirements. For instance, suppose that regulated investors are subject to a binding quantity regulation $b^1 \leq \bar{b}^1$, where the regulator chooses the upper bound $\bar{b}^1$. In our model, a marginal change $d\bar{b}^1$ is equivalent to the local tax reform $d\tau_b^1 = \left(\frac{db^1}{d\tau_b^1}\right)^{-1} d\bar{b}^1$. The associated leakage elasticity is $\frac{db^2}{d\bar{b}^1} = \left(\frac{db^1}{d\tau_b^1}\right)^{-1} \frac{db^2}{d\tau_b^1}$, and the optimal corrective regulation in Proposition 4 can be alternatively expressed as

$$\tau_b^1 = \delta_b^1 + \frac{db^2}{d\bar{b}^1} \delta_b^2.$$

where $R^f = \left(\int m^C(s)\,dF(s)\right)^{-1}$ denotes the creditors' riskless discount rate.

The left panel of Figure OA-4 illustrates the comparison between the first-best and second-best policy when simulating this model. To more clearly illustrate the insights that we present in this paper, in Figure OA-4 we assume that both types of investors are ex-ante identical, and that the only difference between the two is that investor 2 cannot be regulated. Given this symmetry assumption, it is possible to represent the marginal value of varying the regulation $\tau_b = \tau_b^1 = \tau_b^2$ for both investors, which yields the first-best regulation when $\frac{dW}{d\tau_b} = 0$. In contrast, the solid line in Figure OA-4 shows the marginal value of varying the regulation that investor 1 faces (the traditional sector), when investor 2 (the shadow sector) is unregulated, that is, when $\tau_b^2 = 0$. As implied by our theoretical results, since $\frac{db^2}{d\tau_b^1} > 0$ and $\frac{db^1}{d\tau_b^1} < 0$, we find that the optimal second-best policy is sub-Pigouvian, so the optimal second-best regulation that investor 1 faces is lower than the first-best regulation. In this particular simulation, the optimal first-best regulation is $\tau_b^1 = \tau_b^2 = 2.60\%$, while the second-best regulation (when $\tau_b^2 = 0$) is $\tau_b^1 = 1.99\%$.

The right panel of Figure OA-4 illustrates Proposition 2 by showing the marginal value of being able to regulate the shadow sector. This panel provides a clear illustration of the Le Chatelier/reverse leakage adjustment discussed above. Regardless of whether the shadow sector is underregulated (when τ_b^2 is below first-best) or overregulated when (when τ_b^2 is above first-best), the reverse leakage effect has the opposite sign of the direct effect of adjusting the regulation of the shadow sector, attenuate welfare gains/losses. This illustrates how the presence of perfectly regulated decisions contributes to attenuating the welfare gains of relaxing constraints on regulation.

F.2.2 Application 2: Behavioral Distortions/Unregulated Decisions

In this application, we characterize the form of the optimal scale-invariant policy in a model in which regulation is motivated by belief distortions.

Environment. We assume that there is a single type of investor, in unit measure and indexed by i, and a unit measure of creditors, indexed by C. Both investors and creditors have risk-neutral preferences given by

$$c_0^i + \beta^i \int c_1^i(s)\,dF^i(s) \quad \text{and} \quad c_0^C + \beta^C \int c_1^C(s)\,dF^C(s),$$

where $F^i(s)$ and $F^C(s)$ denote the beliefs (cumulative distribution functions) of investors and creditors over the possible states. Endowments and technologies are specified as in Section F.1, with the simplification that investors do not choose the composition of their capital portfolio. Accordingly, the budget constraints of investors at date 0 and date 1 are given by

$$\begin{aligned} c_0^i &= n_0^i + Q^i\left(b^i\right)k^i - \Upsilon\left(k^i\right) \\ c_1^i(s) &= n_1^i(s) + \max\left\{s - b^i, 0\right\}k^i, \quad \forall s. \end{aligned}$$

Creditors' budget constraints are given by

$$\begin{aligned} c_0^C &= n_0^C - h^i Q^i\left(b^i\right)k^i \\ c_1^C(s) &= n_1^C(s) + h^i \mathcal{P}^i\left(b^i, s\right)k^i, \quad \forall s, \end{aligned}$$

where $\mathcal{P}^i\left(b^i, s\right)$ denotes the repayment received by creditors from investors in state s per unit of investment, explicitly described in the Online Appendix.

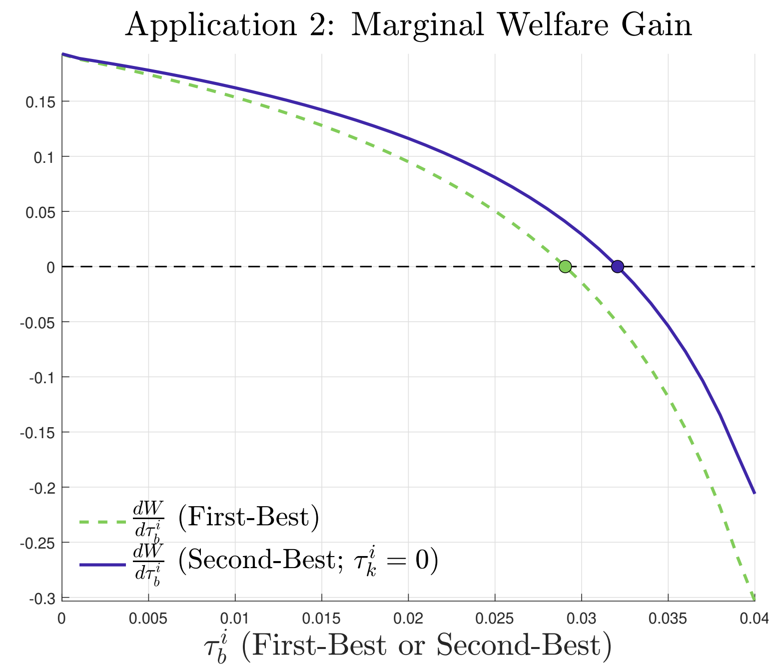

Figure OA-5: Behavioral Distortions/Unregulated Activities (Application 2)

Note: This figure compares the marginal welfare effects of varying the corrective regulation in two different scenarios. The green dashed line corresponds to a scenario in which τ_k^i is set at the first-best level. The solid blue line corresponds to a second-best scenario in which $\tau_k^i = 0$. Therefore, the value of τ_b^i that makes the first-best marginal welfare effect zero defines the first-best leverage regulation, since τ_k^i is already set at the first-best level. The value of τ_b^i that makes the second-best marginal welfare effect zero defines the second-best regulation. To generate this figure, we assume that the adjustment cost is quadratic: $\Upsilon\left(k^i\right) = \frac{a}{2}\left(k^i\right)^2$. The parameters used to generate this figure are $\beta^i = 0.9$, $\beta^C = 0.95$, $\phi^i = 0.8$, and $a = 1$. We assume that investors and creditors perceive s to be normally distributed with mean 1.5 and standard deviation 0.4, and the planner perceives the mean to be 1.3 instead. For reference, the optimal first-best regulation is given by $\tau_b^i = 2.91\%$ and $\tau_k^i = 18.45\%$, while the second-best regulation, when investment cannot be regulated, is $\tau_b^i = 3.21\%$. Since leverage and investment decisions are gross complements in this application, the optimal second-best policy is *super-Pigouvian*.

As in Section F.1, we consider regulation via a capital requirement

$$b^i \leq \bar{b}.$$

We show below that this is equivalent to a corrective tax on leverage choices b^i.

We assume that the planner computes welfare using different probability assessments than those used by investors and creditors to make decisions. This provides a corrective rationale for intervention. As highlighted in Dávila and Walther (2023) and Proposition 5 below, the rationale for regulation is determined by the difference between private agents' and the planner's valuations per unit of risky investment, which represent a levered version of Tobin's q. These valuations are, respectively, given by

$$M\left(b^i\right) = \beta^i \int \max\left\{s - b^i, 0\right\} dF^i\left(s\right) + \beta^C \int \mathcal{P}^i\left(b^i, s\right) dF^C\left(s\right)$$
$$M^P\left(b^i\right) = \beta^i \int \max\left\{s - b^i, 0\right\} dF^P\left(s\right) + \beta^C \int \mathcal{P}^i\left(b^i, s\right) dF^P\left(s\right),$$

where $F^P\left(s\right)$ denotes the probability distribution used by the planner to calculate welfare.

Equilibrium. In this application, for a given leverage cap $\bar{b}$, an *equilibrium* is defined by an investment decision, k^i, a leverage decision, b^i, and a default decision rule such that i) investors maximize their utility given $Q^i(\cdot)$, and ii) creditors set the schedule $Q^i(\cdot)$ optimally, so that $h^i = 1$.

In the first-best scenario, the planner is able to set corrective taxes on both leverage and investment. In this application, the planner's only instrument is the leverage cap $\bar{b}$, which is imperfect. This can be seen by writing investors' first-order conditions as

$$\frac{\partial M\left(b^i\right)}{\partial b^i} = \mu \equiv \tau_b$$
$$M\left(b^i, \theta^i\right) - 1 - \Upsilon'\left(k^i\right) = 0,$$

As in Section F.1, the planner can therefore impose an effective tax on leverage via $\bar{b}$, but cannot affect investors' marginal incentive to create investment capital k^i.

Optimal Corrective Policy/Simulation. In Proposition 5, we characterize the form of the optimal second-best policy, which we discuss along with a numerical simulation.

Proposition 5. *(Behavioral Distortions/Unregulated Activities)*
a) The marginal welfare effect of varying the regulation of investors' leverage, τ_b^i, is given by

$$\frac{dW}{d\tau_b^i} = \frac{db^i}{d\tau_b^i}\left(\tau_b^i - \delta_b^i\right) - \frac{dk^i}{d\tau_b^i}\delta_k^i,$$

where the marginal distortions in this application are defined by

$$\delta_b^i = \frac{dM\left(b^i\right)}{db^i} - \frac{dM^P\left(b^i\right)}{db^i}$$
$$\delta_k^i = M\left(b^i\right) - M^P\left(b^i\right).$$

b) The optimal corrective regulation satisfies

$$\tau_b^i = \delta_b^i - \left(-\frac{db^i}{d\tau_b^i}\right)^{-1}\frac{dk^i}{d\tau_b^i}\delta_k^i.$$

Proposition 5 is the counterpart of Lemma 1 and Proposition 1, and it identifies the distortions associated with leverage and investment the planner perceives. In this application, the welfare distortion associated with leverage, δ_b^i, is driven by the difference in marginal valuations, while the distortion associated with investment, δ_k^i, is driven by the difference in the level of valuations. In this application we have $\frac{db^i}{d\tau_b^i} < 0$ and, critically, the leakage elasticity from leverage to investment is negative, that is, $\frac{dk^i}{d\tau_b^i} < 0$, implying that leverage and investment are gross complements. As implied by our results in Section 3, the optimal second-best regulation on leverage is super-Pigouvian.

Importantly, a comparison between this application with the previous one (shadow banking) highlights that both leakage elasticities featuring substitutes and those featuring complements are important in common regulatory scenarios. A number of recent empirical studies confirm that the leakage elasticity from leverage to risky investments is negative, in the sense that banks with lower capital ratios originate a larger volume of risky loans (e.g., Jiménez, Ongena, Peydró and Saurina, 2014; Dell'Ariccia, Laeven and Suarez, 2017; Acharya, Eisert, Eufinger and Hirsch, 2018).

Figure OA-5 compares the marginal welfare effects of varying regulation in the first-best and second-best scenarios when simulating this model. To illustrate the first-best solution for leverage, we fix τ_k^i to its first-best value when showing the marginal welfare associated with varying τ_b^i. The second-best marginal welfare gain simply sets $\tau_k^i = 0$. As implied by our theoretical results, the optimal second-best policy is super-Pigouvian, so it is optimal for the planner to overregulate leverage relative to the first-best scenario. In this particular simulation, the optimal first-best regulation is $\tau_b^i = 2.91\%$ and $\tau_k^i = 18.45\%$, while the second-best regulation (when $\tau_k^i = 0$) is $\tau_b^i = 3.21\%$.

F.2.3 Application 3: Asset Substitution/Uniform Decision Regulation

A common concern in financial regulation is that corrective policy instruments are somewhat coarse in practice. For example, when imposing capital requirements on banks, financial regulators tend to set risk weights for wide classes of risky investments (e.g., mortgage loans), but within the class, banks can freely optimize their portfolios (e.g., among loans to borrowers with different credit scores) without any change in the associated capital charge. In our model, this situation corresponds to a uniform regulation across different capital investments. In this application, we consider uniform corrective policy in a model where investors enjoy government guarantees. We use the properties of uniform regulation to derive new insights into the classical asset substitution problem (e.g., Jensen and Meckling, 1976), and characterize the optimal second-best policy.

Environment. We assume that there is a single type of investor, in unit measure and indexed by i, and a unit measure of creditors, indexed by C. Both investors and creditors have risk-neutral preferences given by

$$c_0^i + \beta^i \int c_1^i (s)\, dF(s) \quad \text{and} \quad c_0^C + \beta^C \int c_1^C (s)\, dF(s).$$

The budget constraints of investors at date 0 and date 1 are given by

$$\begin{aligned} c_0^i &= n_0^i - \Upsilon\left(k_1^i, k_2^i\right) - \tau_k^1 k_1^i - \tau_k^2 k_2^i + T_0^i \\ c_1^i(s) &= \max\left\{d_1(s)\, k_1^i + d_2(s)\, k_2^i + t\left(k_1^i, k_2^i, b^i, s\right) - b^i, 0\right\}, \quad \forall s. \end{aligned}$$

At date 0, investors, endowed with n_0^i dollars, choose the scale of two risky capital investments k_1^i and k_2^i, which are subject to an adjustment cost of $\Upsilon\left(k_1^i, k_2^i\right)$. Hence, investors make $|\mathcal{X}| = 2$ free choices regarding their balance-sheet.

At date 1, investors earn the realized returns on capital investments k_1^i and k_2^i, which are given by $d_1(s)$ and $d_2(s)$ and are increasing in s. In addition, they receive a bailout transfer $t\left(k_1^i, k_2^i, b^i, s\right)$ from the government. We further assume that investors have legacy debt (i.e., debt issued before the start of the model) with face value b^i. Hence, investors owe a predetermined repayment of b^i to creditors at date 1. We make this simplifying assumption in order to sharpen our focus on *asset substitution*, which describes investors' choice between different risky investments, as opposed to leverage choices. At date 1, investors consume the difference between i) the cash flow from investments augmented by the bailout transfer and ii) the debt owed, if this difference is positive. Otherwise, they default and consume zero.

For simplicity, we focus on a particular form of bailout that fully prevents default — this may correspond to an investor that is "too big to fail". Concretely, we assume that the government bailout is equal to the minimum amount required to avoid default

$$t\left(k_1^i, k_2^i, b^i, s\right) = \max\left\{b^i - d_1(s)\, k_1^i - d_2(s)\, k_2^i, 0\right\}. \tag{OA37}$$

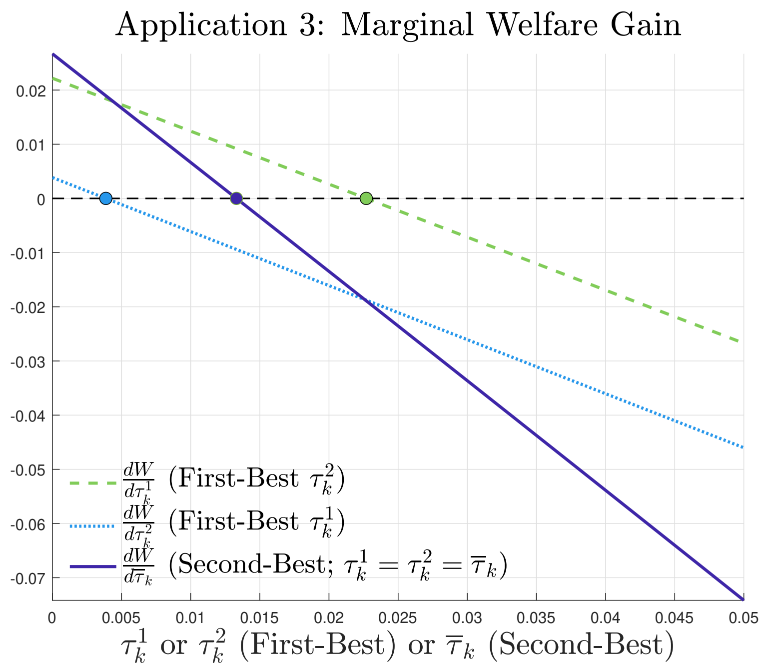

Figure OA-6: Asset Substitution/Uniform Activity Regulation (Application 3)

Note: This figure compares the marginal welfare effects of varying the corrective regulation in two different scenarios. The green dashed line and the light blue dotted line illustrate the first-best regulation. The green dashed line corresponds to a scenario in which τ_k^2 is set at the first-best level (previously computed), while the light blue dotted line corresponds to a scenario in which τ_k^1 is set at the first-best level (previously computed). Therefore, the values of τ_k^1 and τ_k^2 that respectively make each line zero define the first-best regulation. The solid dark blue line corresponds to a second-best scenario in which $\overline{\tau}_k = \tau_k^1 = \tau_k^2$, so its zero defines the second-best regulation. To generate this figure, we assume that the adjustment cost is quadratic: $\Upsilon\left(k_1^i, k_2^i\right) = \frac{z_1}{2}\left(k_1^i\right)^2 + \frac{z_2}{2}\left(k_2^i\right)^2$. We also assume that $d_1(s) = \mu_1 + \sigma_1 s$ and $d_2(s) = \mu_2 + \sigma_2 s$ when s is distributed as a standard normal. The parameters used to generate this figure are $\beta^i = 0.8$, $\beta^C = 1$, $\kappa = 0.1$, $z_1 = z_2 = 1$, $b^i = 1.4$, $\mu_1 = 1.5$, $\mu_2 = 1.3$, $\sigma_1 = 0.3$, and $\sigma_2 = 0.5$. For reference, the optimal first-best regulation is given by $\tau_k^1 = 2.27\%$ and $\tau_k^2 = 0.39\%$, while the second-best regulation, when the regulation is uniform, is $\overline{\tau}_k = 1.33\%$.

Given this form of bailout policy, creditors are guaranteed a repayment of b^i at date 1. We write $s^\star\left(k_1^i, k_2^i\right)$ for the threshold state below which bailouts are positive.[39]

Hence, the budget constraints of creditors at date 0 and date 1 are given by

$$
\begin{aligned}
c_0^C &= n_0^C \\
c_1^C(s) &= n_1^C(s) + b^i - (1+\kappa)\, t\left(k_1^i, k_2^i, b^i, s\right), \ \forall s.
\end{aligned}
$$

Even though creditors are always repaid b^i in every state, we assume that in order to finance the bailout, the government imposes a tax of $(1+\kappa)$ per dollar of bailout on creditors, where $\kappa > 0$ measures the deadweight fiscal cost of bailout transfers. The rationale for regulation in this environment is a classical "moral hazard" argument. Investors, whose debt is implicitly guaranteed by the government, do not internalize the impact of their risky capital investments on fiscal costs, which ultimately reduces the consumption of creditors.

[39]Formally, for a fixed value b^i of legacy debt, this threshold is the unique solution to $b^i - d_1(s)\, k_1^i - d_2(s)\, k_2^i = 0$.

Equilibrium. In this application, for given corrective taxes/subsidies $\left\{\tau_k^1, \tau_k^2\right\}$, lump-sum transfers $T_0^i = \tau_k^1 k_1^i + \tau_k^2 k_2^i$, bailout policy $t\left(k_1^i, k_2^i, b^i, s\right)$, and legacy debt b^i, an *equilibrium* is defined by investment decisions such that investors maximize their utility. In the first-best scenario, the planner is able to set τ_k^1 and τ_k^2 freely. However, we are interested in a scenario in which the planner is unable to treat investments differentially for regulation purposes. Thus, the planner chooses $\tau_k^1 \geq 0$ and $\tau_k^2 \geq 0$ subject to the uniform regulation constraint:

$$\overline{\tau}_k = \tau_k^1 = \tau_k^2.$$

Optimal Corrective Policy/Simulation. In Proposition 6 we characterize the form of the second-best policy, which we discuss along with a numerical simulation.

Proposition 6. (Asset Substitution/Uniform Activity Regulation)
a) The marginal welfare effect of varying the uniform corrective regulation of capital investments, $\overline{\tau}_k = \tau_k^1 = \tau_k^2$, is given by

$$\frac{dW}{d\overline{\tau}_k} = \frac{dk_1^i}{d\overline{\tau}_k}\left(\overline{\tau}_k - \delta_1\right) + \frac{dk_2^i}{d\overline{\tau}_k}\left(\overline{\tau}_k - \delta_2\right),$$

where the marginal distortions in this application are defined by

$$\delta_j = (1+\kappa)\,\beta^C \int_{\underline{s}}^{s^\star\left(k_1^i, k_2^i\right)} d_j(s)\, dF(s).$$

b) The optimal corrective regulation satisfies

$$\overline{\tau}_k = \frac{\frac{dk_1^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k} + \frac{dk_2^i}{d\overline{\tau}_k}}\delta_1 + \frac{\frac{dk_2^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k} + \frac{dk_2^i}{d\overline{\tau}_k}}\delta_2.$$

Proposition 6 identifies the distortions associated with the different types of investment decisions in this application. The shape of the distortions δ_j highlights the nature of the asset substitution problem: investors' private incentives are driven by the returns to investment in "upside" states $s \geq s^\star\left(k_1^i, k_2^i\right)$, while the planner's concern about bailouts focuses on "downside" states $s < s^\star\left(k_1^i, k_2^i\right)$. The optimal uniform regulation is a weighted average of the downside distortions imposed by both types of capital. As implied by our general results in Section 3, the appropriate weight assigned by the planner to each of the distortions in the optimal second-best policy is given by how sensitive each capital decision is to changes in the regulation, $\frac{\frac{dk_1^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k} + \frac{dk_2^i}{d\overline{\tau}_k}}$ and $\frac{\frac{dk_2^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k} + \frac{dk_2^i}{d\overline{\tau}_k}}$. Figure OA-6 illustrates this intuition by comparing the marginal welfare effects of varying regulation in the first-best and second-best scenarios.

In the Online Appendix, assuming that investment costs are quadratic, we provide further intuition on how the weights $\frac{dk_1^i}{d\overline{\tau}_k}$ and $\frac{dk_2^i}{d\overline{\tau}_k}$ are determined. We show that the sufficient statistics for the optimal weights are i) the sensitivity of the probability of receiving a bailout to the uniform regulation, and ii) the marginal contribution $d_n\left(s^\star\right)$ of each asset class at the bailout boundary. Intuitively, a large ratio $\frac{d_2(s^\star)}{d_1(s^\star)}$ means that changes in the default boundary affect mostly returns to k_2^i, which makes investors' optimal investment in k_2^i more sensitive to the uniform regulation.

F.2.4 Application 4: Pecuniary Externalities/Uniform Investor Regulation

Pecuniary/fire-sale externalities coupled with incomplete markets and/or collateral constraints provide a well-studied rationale for corrective macro-prudential regulation. The natural notion of efficiency in those environments, constrained efficiency, typically requires agent-specific regulations, which can be mapped to our first-best benchmark. In this application, we study the form of the second-best policy in an environment in which it would be optimal to set investor-specific regulations, but the planner is constrained to set the same corrective regulation for all investors.

Environment. We assume that there are two types of investors/entrepreneurs, indexed by $i \in \{1, 2\}$, and households, indexed by H — who in a richer model would also play the role of creditors. There are three dates, $t \in \{0, 1, 2\}$ and no uncertainty. Investors, who for simplicity do not discount the future, have preferences of the form:

$$u^i = c_0^i + c_1^i + c_2^i,$$

subject to non-negativity constraints, $c_0^i \geq 0$, $c_1^i \geq 0$, $c_2^i \geq 0$, where their budget constraints are given by

$$\begin{aligned} c_0^i &= n_0^i - \Upsilon^i \left(k_0^i\right) - \tau_k^i k_0^i + T_0^i \\ c_1^i &= q \left(k_0^i - k_1^i\right) - \xi^i k_0^i \\ c_2^i &= z^i k_1^i. \end{aligned}$$

At date 0, an investor i endowed with n_0^i dollars chooses how much to produce, k_0^i, given a technology $\Upsilon^i \left(k_0^i\right)$. Investor i also faces a corrective tax τ_k^i per unit invested at date 0. At date 1, an investor i must reinvest $\xi^i > 0$ per unit of invested capital at date 0, which needs to be satisfied by selling $k_0^i - k_1^i$ units of capital at a market price q — this is a simple way to generate a fire-sale. At the final date, whatever capital is left yields an output $z^i k_1^i$. For simplicity, we assume that, in equilibrium, $T_0^i = \tau_k^i k_0^i$, $\forall i$.

Households, who exclusively consume at date 1, have access to a decreasing returns to scale technology to transform capital into output at date 1. Formally, the utility of households is given by

$$u^H = c_1^H = F\left(k_1^H\right) - q k_1^H,$$

where $F(\cdot)$ is a well-behaved concave function and k_1^H denotes the amount of capital purchased by households at date 1. The solution to the households' problem will define a downward sloping demand curve for sold capital at date 1.

Equilibrium. In this application, for given corrective taxes/subsidies $\left\{\tau_k^1, \tau_k^2\right\}$ and lump-sum transfers $\left\{T_0^1, T_0^2\right\} = \left\{\tau_k^1 k_0^1, \tau_k^2 k_0^2\right\}$, an *equilibrium* is fully determined by investors/entrepreneurs' investment decisions $\left\{k_0^i, k_1^i\right\}$ at dates 0 and 1, households' capital allocation k_1^H at date 1, and an equilibrium price q, such that investors' and households' utilities are maximized, subject to constraints, and the capital market clears, that is, $\sum_i \left(k_0^i - k_1^i\right) = k_1^H$.

In the first-best scenario, the planner is able to set τ_k^1 and τ_k^2 freely. However, we are interested in scenarios in which the planner must regulate both investors equally, so

$$\overline{\tau}_k = \tau_k^1 = \tau_k^2,$$

which makes the problem of choosing the optimal $\overline{\tau}_k$ a second-best problem.

At date 1, the non-negativity constraint of investors' consumption will necessarily bind, so the amount sold by investor i at date 1 will be proportional to date 0 investment: $k_0^i - k_1^i = \frac{\xi^i}{q} k_0^i$. The households' optimality condition is given by $q = F'\left(k_1^H\right)$. When combined with market clearing and the characterization of optimal investment at date 0 that we present in the Online Appendix, we show that the equilibrium price

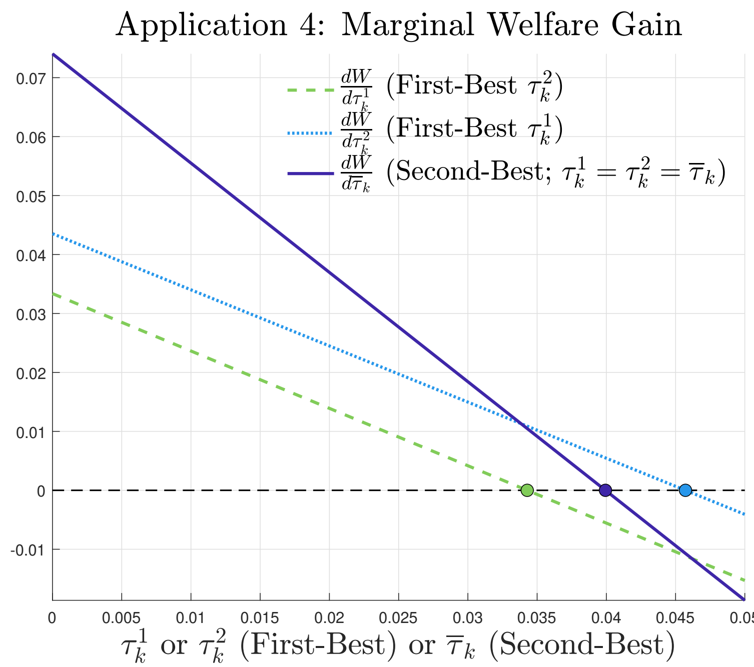

Figure OA-7: Pecuniary Externalities/Uniform Investor Regulation (Application 4)

Note: This figure compares the marginal welfare effects of varying the corrective regulation in two different scenarios. The green dashed line and the light blue dotted line illustrate the first-best regulation. The green dashed line corresponds to a scenario in which τ_k^2 is set at the first-best level (previously computed), while the light blue dotted line corresponds to a scenario in which τ_k^1 is set at the first-best level (previously computed). Therefore, the values of τ_k^1 and τ_k^2 that respectively make each line zero define the first-best regulation. The solid dark blue line corresponds to a second-best scenario in which $\tau_k^1 = \tau_k^2 = \overline{\tau}_k$, so its zero defines the second-best regulation. To generate this figure, we assume that the adjustment cost of investment is quadratic: $\Upsilon^i\left(k_0^i\right) = \frac{a^i}{2}\left(k_0^i\right)^2$, and that the technology of households is isoelastic: $F\left(k_1^H\right) = \frac{\left(k_1^H\right)^\alpha}{\alpha}$. The parameters used to generate this figure are $\alpha = 0.5$, $a^1 = a^2 = 1$, $z^1 = z^2 = 1.5$, $\xi^1 = 0.3$, and $\xi^2 = 0.4$. For reference, the optimal first-best regulation is given by $\tau_k^1 = 3.43\%$ and $\tau_k^2 = 4.57\%$, while the second-best regulation, when the regulation is uniform, is $\overline{\tau}_k = 3.99\%$.

can be characterized in terms of primitives as the solution to

$$q = \left(\sum_i \frac{\xi^i}{a^i}\left(z^i\left(1 - \frac{\xi^i}{q}\right) - \tau_k^i\right)\right)^{\frac{\alpha-1}{\alpha}},$$

where we have assumed quadratic adjustment costs $\Upsilon^i\left(k_0^i\right) = \frac{a^i}{2}\left(k_0^i\right)^2$ and the isoelastic production function $F\left(k_1^H\right) = \frac{\left(k_1^H\right)^\alpha}{\alpha}$.

Optimal Corrective Policy/Simulation. In Proposition 7 we characterize the form of the second-best policy, which we discuss along with a numerical simulation.

Proposition 7. *(Pecuniary Externalities/Uniform Investor Regulation)*
a) The marginal welfare effect of varying the uniform corrective regulation of investments, $\overline{\tau}_k = \tau_k^1 = \tau_k^2$*, is given by*

$$\frac{dW}{d\overline{\tau}_k} = \frac{dk_0^1}{d\overline{\tau}_k}\left(\overline{\tau}_k - \delta_k^1\right) + \frac{dk_0^2}{d\overline{\tau}_k}\left(\overline{\tau}_k - \delta_k^2\right),$$

where

$$\delta_k^i = -\frac{\partial q}{\partial k_0^i} \sum_{\ell=1}^{2} \left(\frac{z^\ell}{q} - 1 \right) \left(k_0^\ell - k_1^\ell \right).$$

b) The optimal corrective regulation satisfies

$$\overline{\tau}_k = \frac{\frac{dk_0^1}{d\overline{\tau}_k}}{\frac{dk_0^1}{d\overline{\tau}_k} + \frac{dk_0^2}{d\overline{\tau}_k}} \delta_k^1 + \frac{\frac{dk_0^2}{d\overline{\tau}_k}}{\frac{dk_0^1}{d\overline{\tau}_k} + \frac{dk_0^2}{d\overline{\tau}_k}} \delta_k^2.$$

Proposition 7 identifies the distortions associated with the investment choices of investors/entrepreneurs. In this application, the distortion is generated by a distributive pecuniary externality, using the terminology of Dávila and Korinek (2018). Consistent with the results in that paper, this type of externality is determined by price sensitivities, differences in marginal valuations, and net trade positions. In this case, these three statistics are given by $\frac{\partial q}{\partial k_0^i}$, $\frac{z^\ell}{q} - 1$, and $k_0^\ell - k_1^\ell$. Note that δ_k^i includes the sum of the latter two terms across both types of investors, since a given investor does not internalize how his individual investment decision affects prices and consequently the welfare of other investors of the same and different types.

As implied once again by our general results in Section 3, the appropriate weight assigned by the planner to each of the distortions in the optimal second-best policy is given by how sensitive each capital decision is to changes in the regulation. Figure OA-7 illustrates this intuition by comparing the marginal welfare effects of varying regulation in the first-best and second-best scenarios. By comparing Application 3 with Application 4, it becomes evident that the principles that guide the second-best regulation when it is forced to be uniform across choices for a given agent or across agents for a given choice are identical.