

Time Series Forecasting

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Typical Time Series

$$\begin{aligned}\hat{y}_{t+1} = & f(y_t, y_{t-1}, y_{t-2} \dots) \\ & + f(x_1, x_2, x_3 \dots)\end{aligned}$$

f can be linear or nonlinear

Important Concept

In time series, the order of observation is of primary importance. So is autocorrelation. They play a very important role in identifying the models and their characteristics

Autocorrelation (ACF) and Partial ACF (PACF)

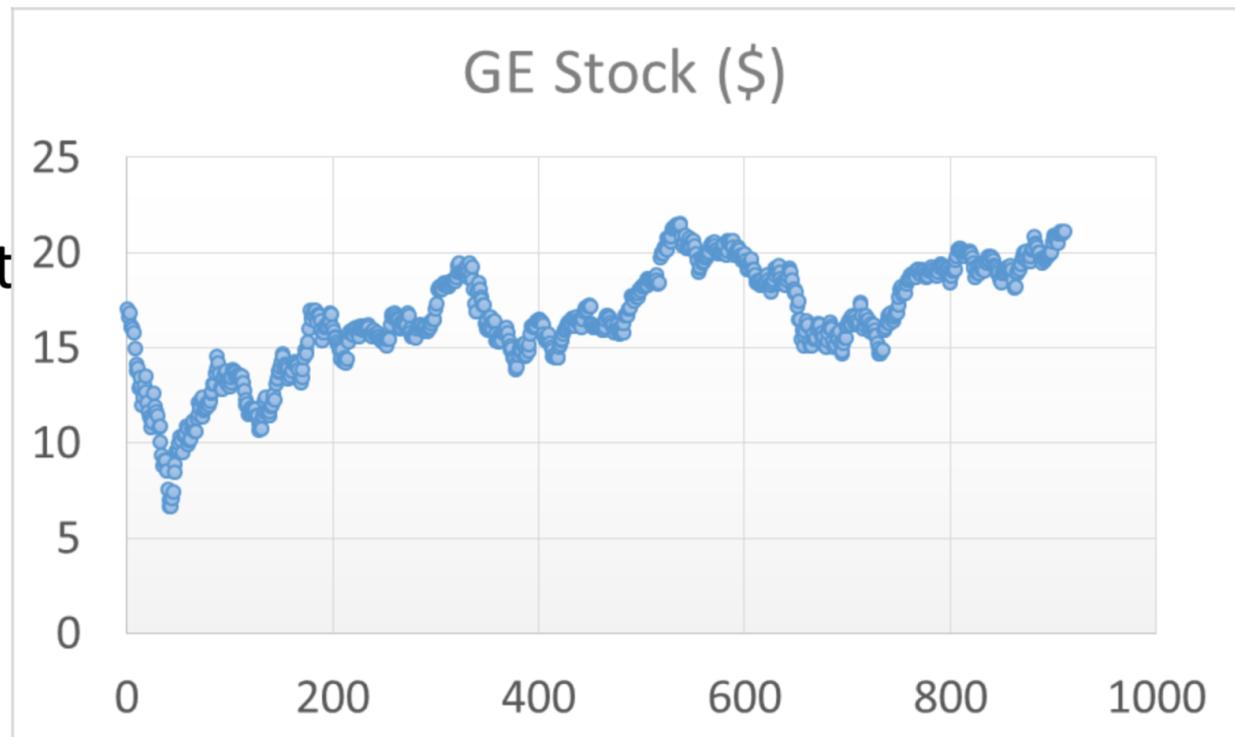
- ACF: n^{th} lag of ACF is the correlation between a day and n days before that.
- PACF: The same as ACF with all intermediate correlations removed. It is the k_{th} coefficient of the ordinary least squares regression.

$$[y_t] = \beta_0 + \sum_{i=1}^k \beta_i [y_{t-i}] \text{ where}$$

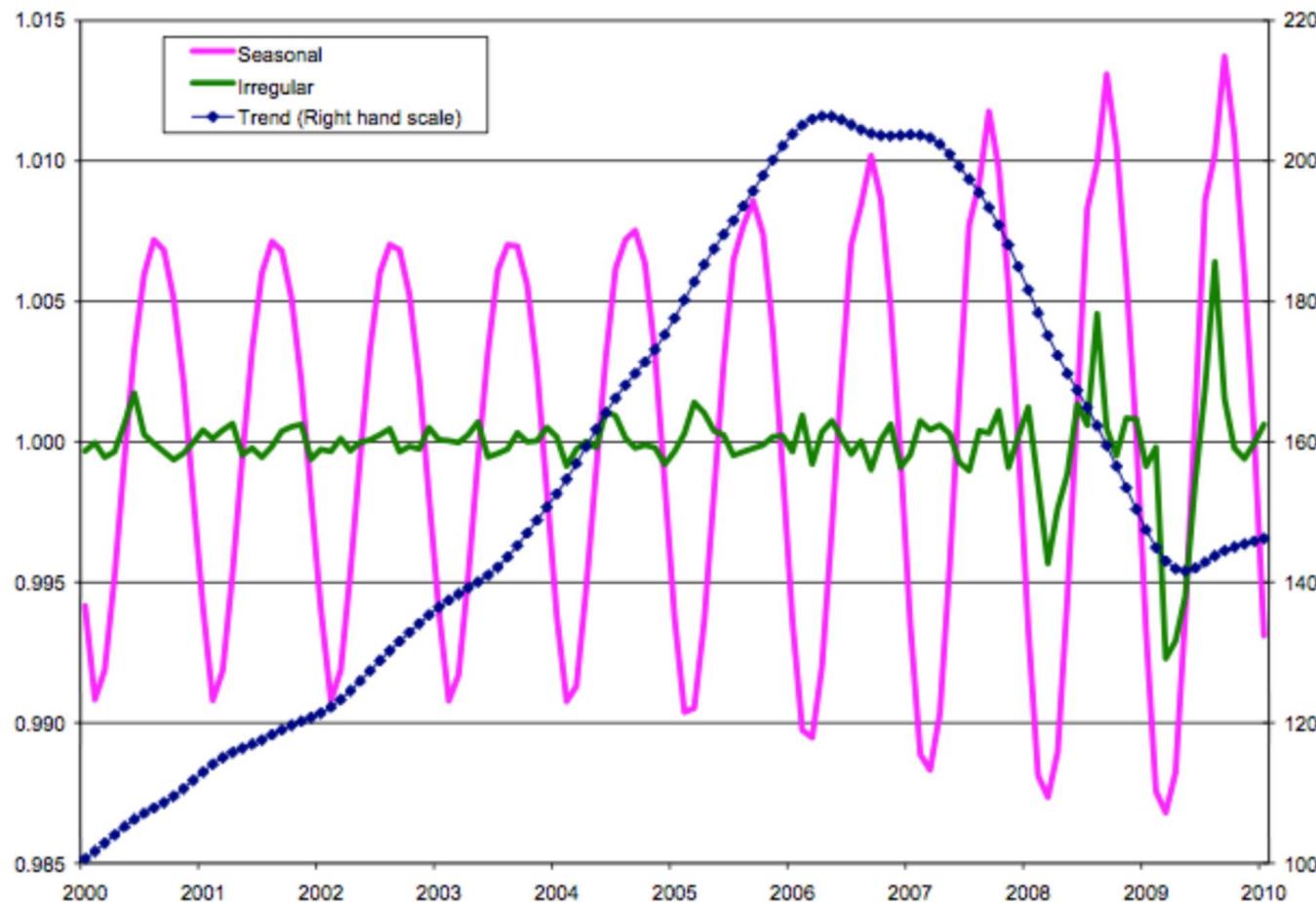
$[y_t]$ is the input time series, k is the lag order and β_i is the i_{th} coefficient of the linear multiple regression.

Components of Time Series

- Trend
- Seasonality
- Random component



Trend, Seasonality and Randomness



US Air Carrier Traffic – Revenue Passenger Miles ('000)

RPM

```
> milestimeseries <- ts(miles, frequency = 12, start = c(1996,1))
> milestimeseries
```

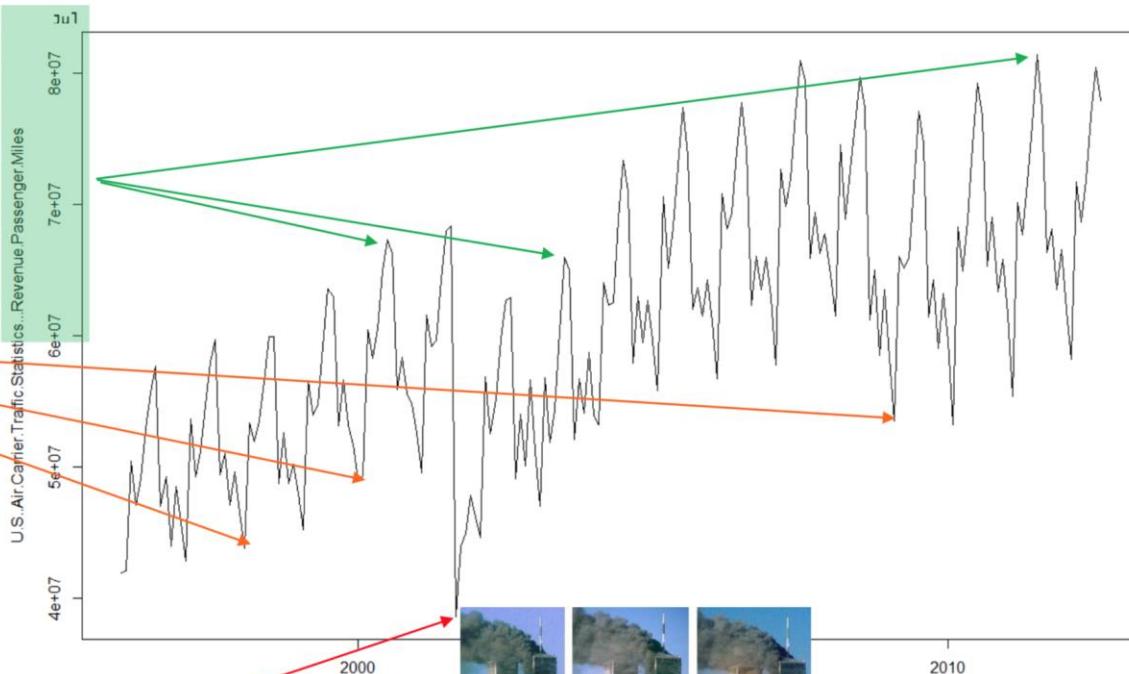
	Jan	Feb	Mar	Apr	May	Jun
1996	41972194	42054796	50443045	47112397	49118248	52880510
1997	45850623	42838949	53620994	49282817	51191842	54707221
1998	46514139	43769273	53361926	51968480	53515798	56460422
1999	47988560	45241211	56555731	53920853	54674958	59213000
2000	49045412	49306303	60443541	58286680	60533783	64903295
2001	52634354	49532578	61575055	59151645	59662416	64353323
2002	46224031	44615129	56897729	52542164	55116060	59745343
2003	51197175	47040806	56766580	51857453	54335598	60272900
2004	53979786	53179693	64035864	62340117	62530704	68866398
2005	59629608	55795165	70595861	65145552	68268899	72952959
2006	61035027	56729212	70799794	68120559	69352606	74099239
2007	63016013	57793832	72700241	69836156	71933109	76926452
2008	64667106	61504426	74575531	68906882	72725750	76162105
2009	58373786	53506580	66027341	65166300	65868254	71350227
2010	59651061	53240066	68307090	64953250	68850904	74474550
2011	61630362	55391206	70158268	67683558	71711448	76057910
2012	61940180	58243763	71696039	68669228	71887523	76760759
	Aug	Sep	Oct	Nov	Dec	
1996	57723208	47035464	49263120	43937074	48539606	
1997	59715433	49418190	51058879	47056048	49654209	

Data sources:

http://www.bts.gov/xml/air_traffic/src/index.xml

and <https://datamarket.com/data/set/281x/us-air-carrier-traffic-statistics-revenue-passenger-miles>

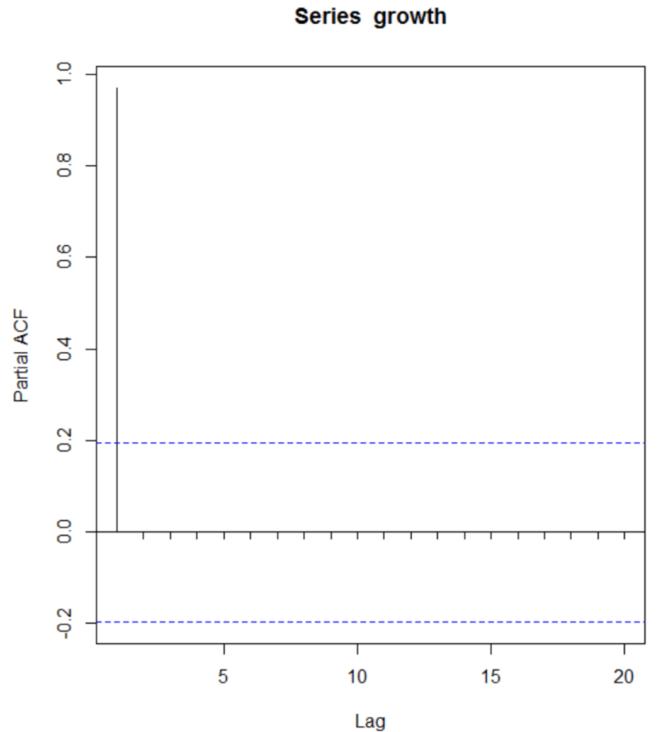
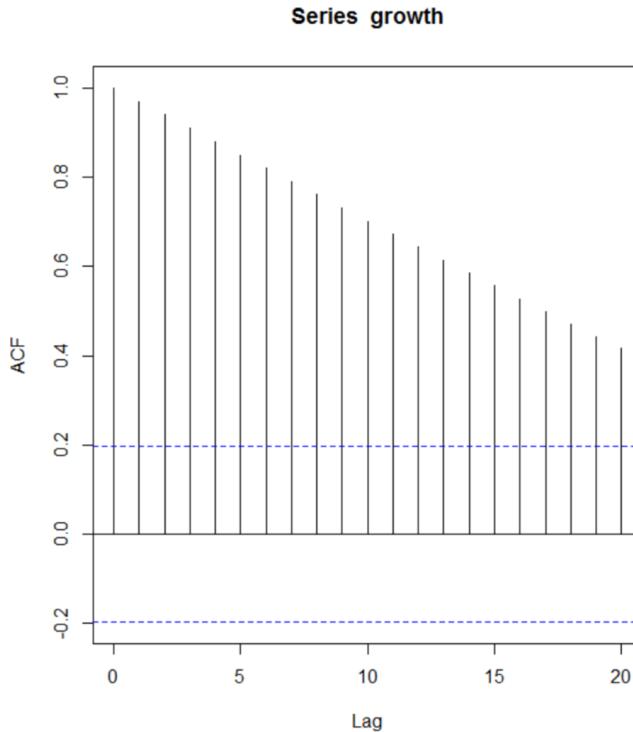
Last accessed: 31-Mar-2016



	Aug	Sep	Oct	Nov	Dec
1996	57723208	47035464	49263120	43937074	48539606
1997	59715433	49418190	51058879	47056048	49654209
1998	59927214	48751280	52578217	48734375	50208641
1999	63003663	53131972	56653901	53215500	51746821
2000	66256804	55900504	58373996	55590325	54822970
2001	68377080	38601868	43964788	44915764	47836501
2002	62944816	49096035	54019748	50106814	56656594
2003	64989766	52121480	56724551	54128776	58739845
2004	70961522	57881042	63021142	59453943	62680310



ACF and PACF – Idealized Trend

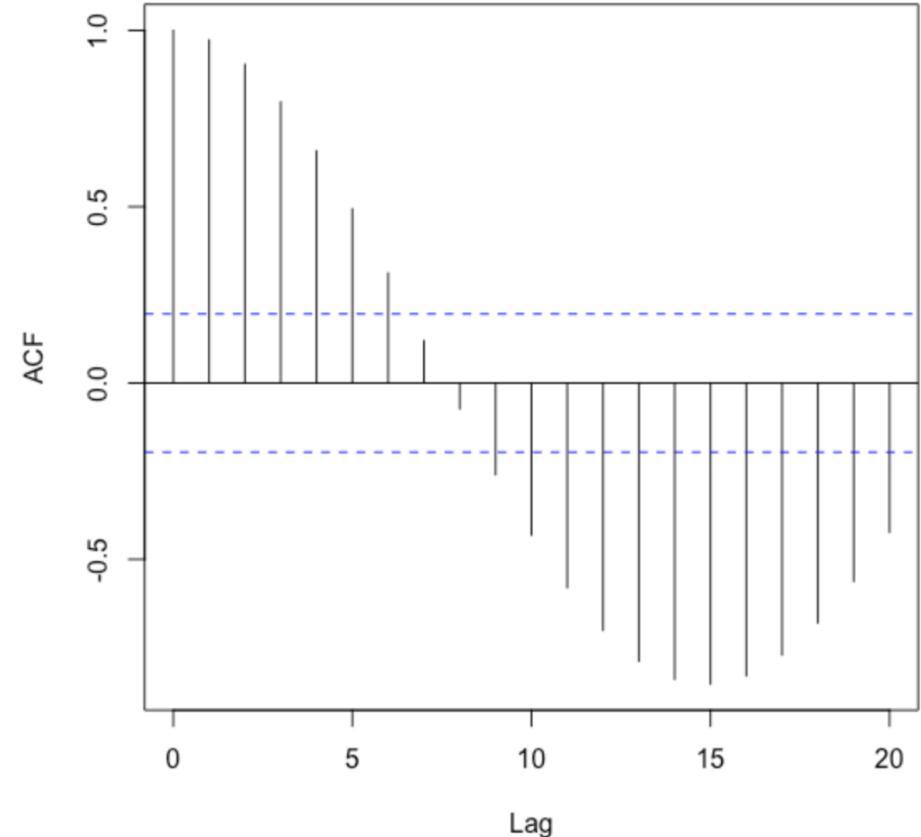


$$95\% \text{ CI: } 0 \pm \frac{1.96}{\sqrt{n}}$$

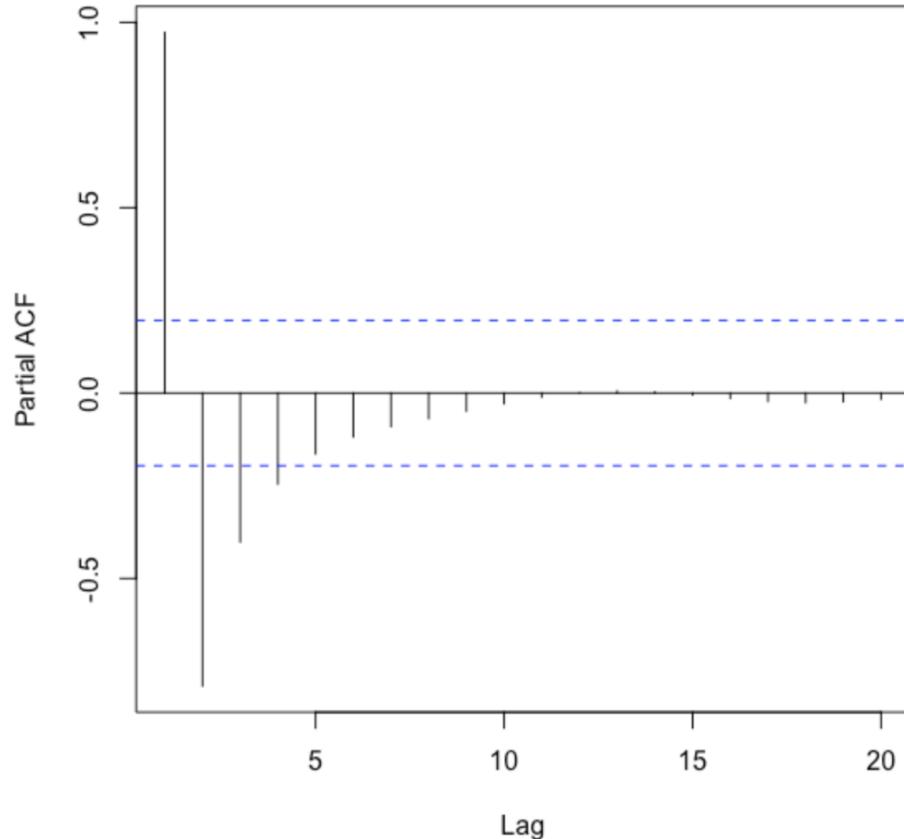
- ACF is a bar chart of correlation coefficients of the time series and its lags.
- PACF is a plot of the partial correlation coefficients of the time series and its lags.

ACF and PACF – Idealized Seasonality

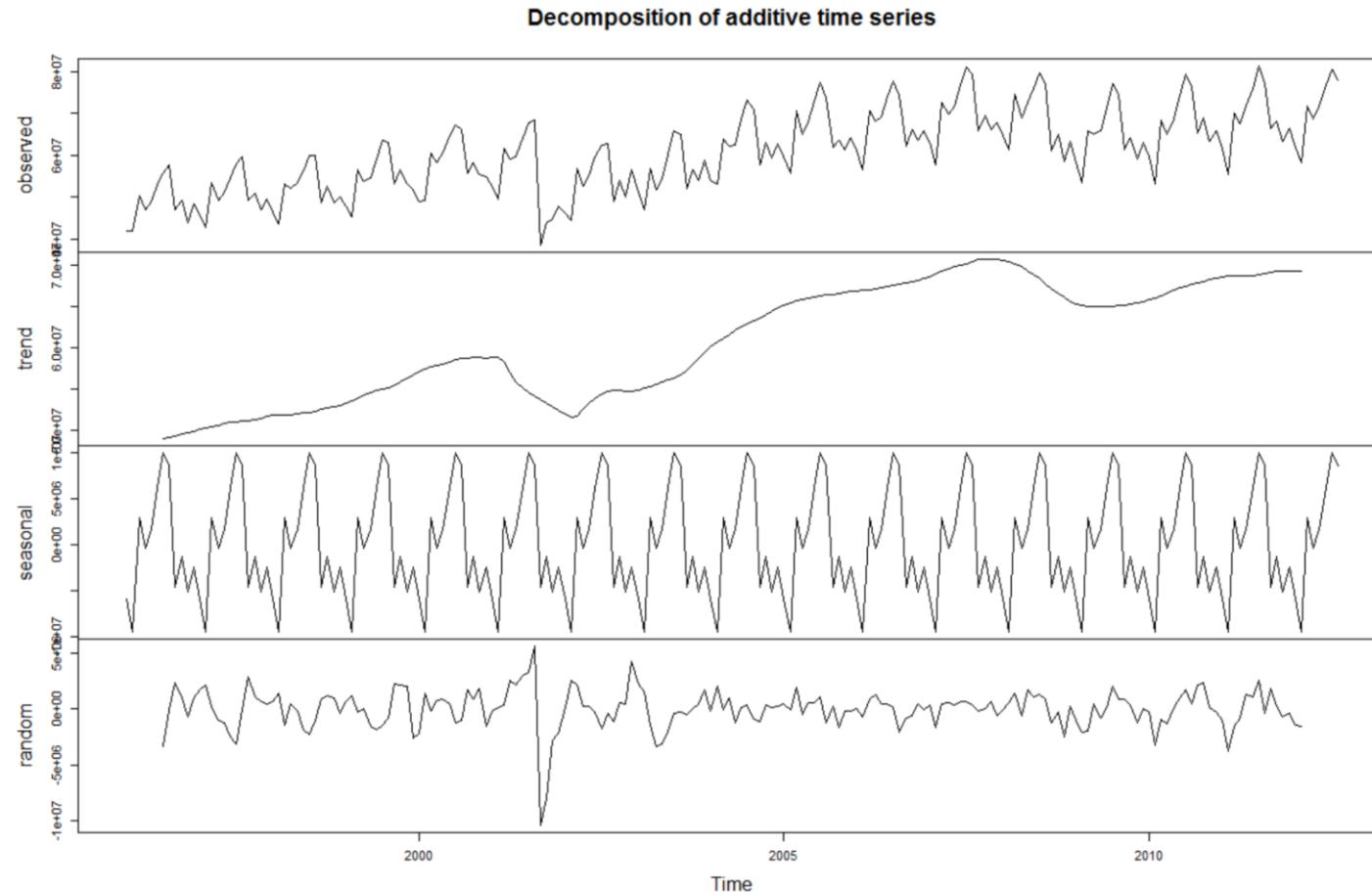
Series growth



Series growth



ACF and PACF (Real-world): Decomposing Time Series into the 3 Components – Revenue Passenger Miles (RPM)



Stationary and Non-Stationary

- Stationary data has constant statistical properties – mean, variance, autocorrelation, etc. – over time
- If the data is stationary, forecasting is easier!

ACF and PACF of Stationary and Non-Stationary

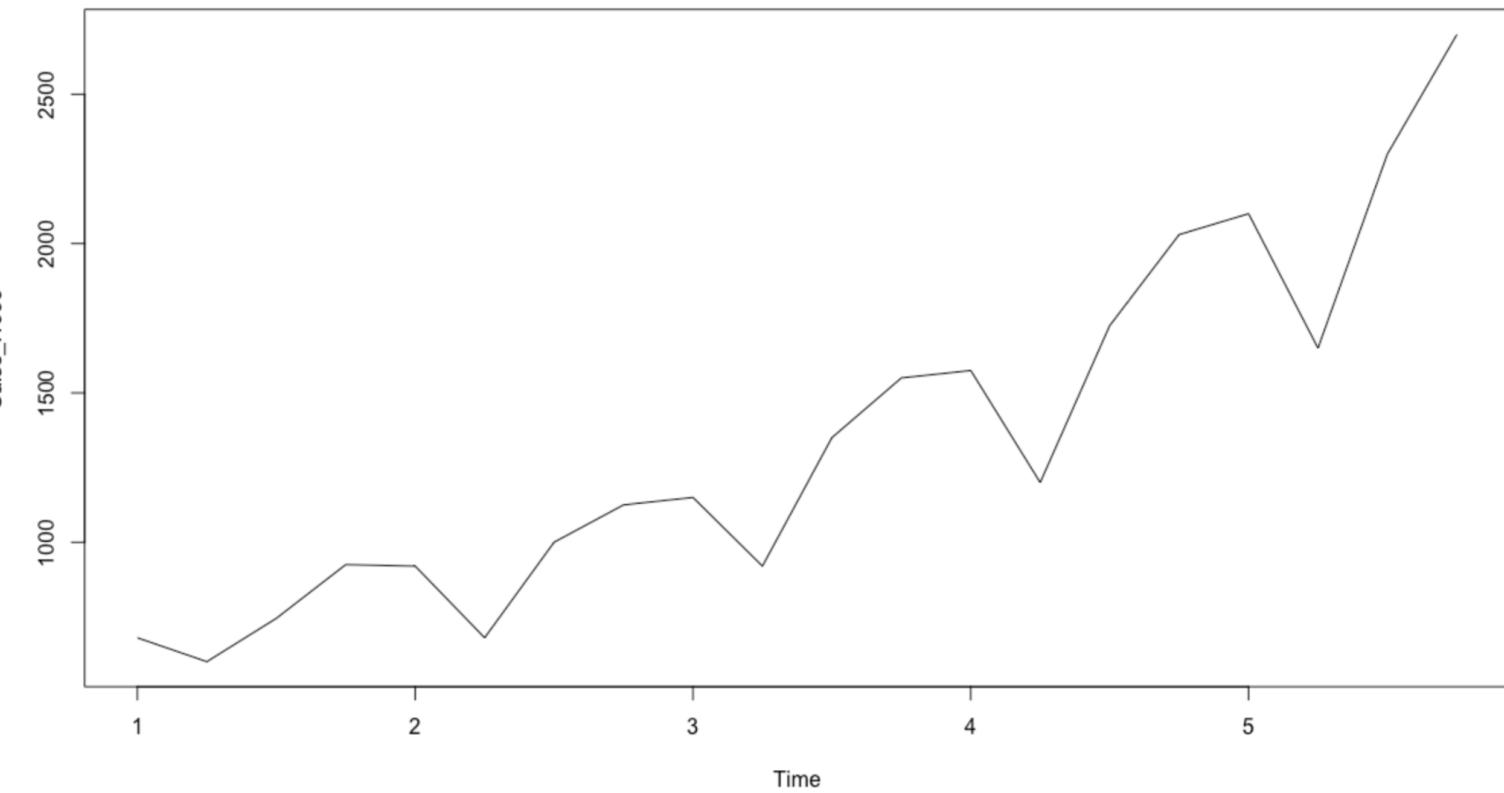
- Non-stationary series have an ACF that remains significant for half a dozen or more lags, rather than quickly declining to zero.
- You must difference such a series until it is stationary before you can identify the process.

CURVE FITTING / REGRESSION ON TIME METHODS

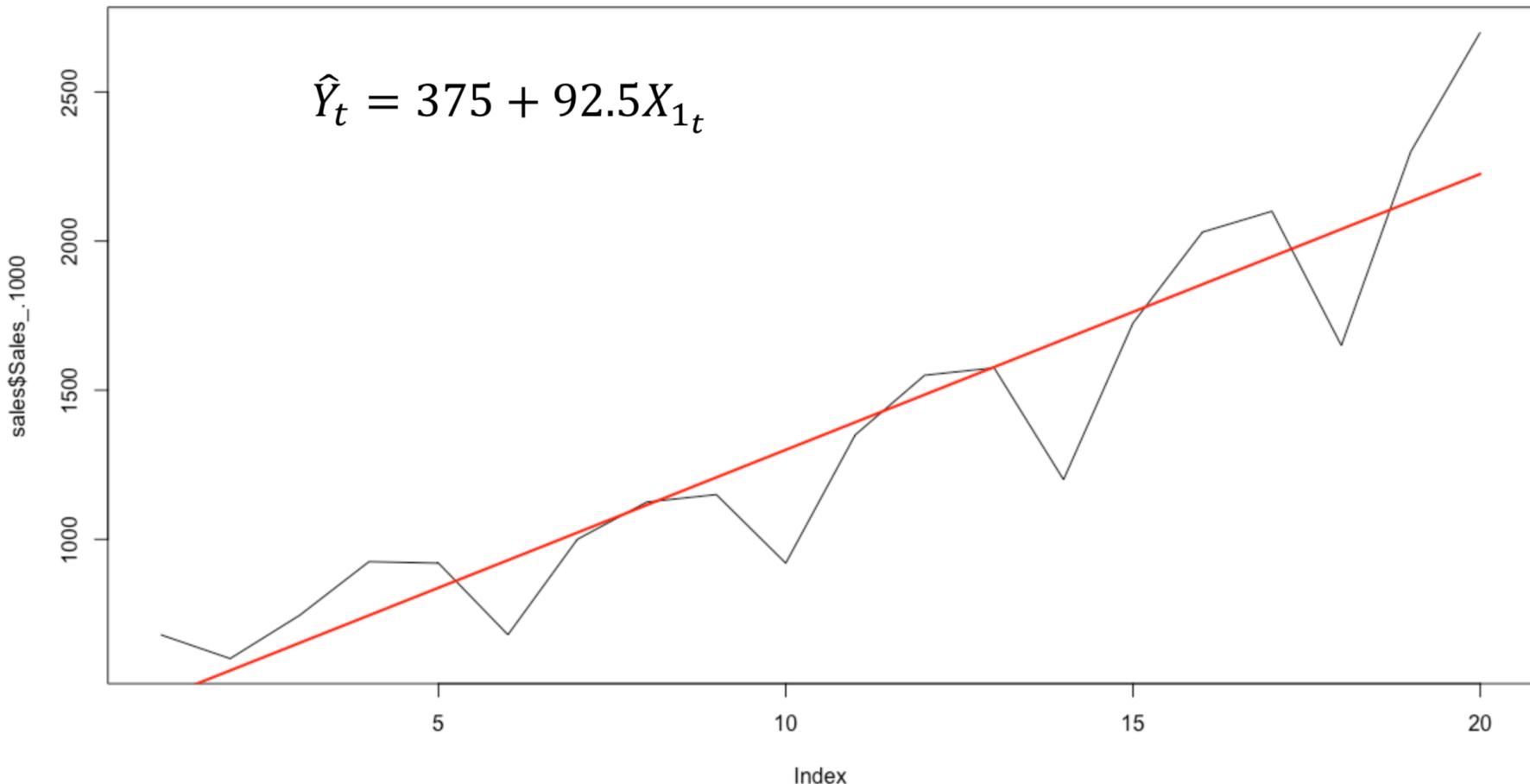
Regression on Time

- Use when trend is the most pronounced
- ACF decays exponentially and PACF has very few spikes

Quarter	Sales_ \$1000
1	680
2	600
3	745
4	925
5	920
6	680
7	1000
8	1125
9	1150
10	920
11	1350
12	1550
13	1575
14	1200
15	1725
16	2030
17	2100
18	1650



Regression Analysis

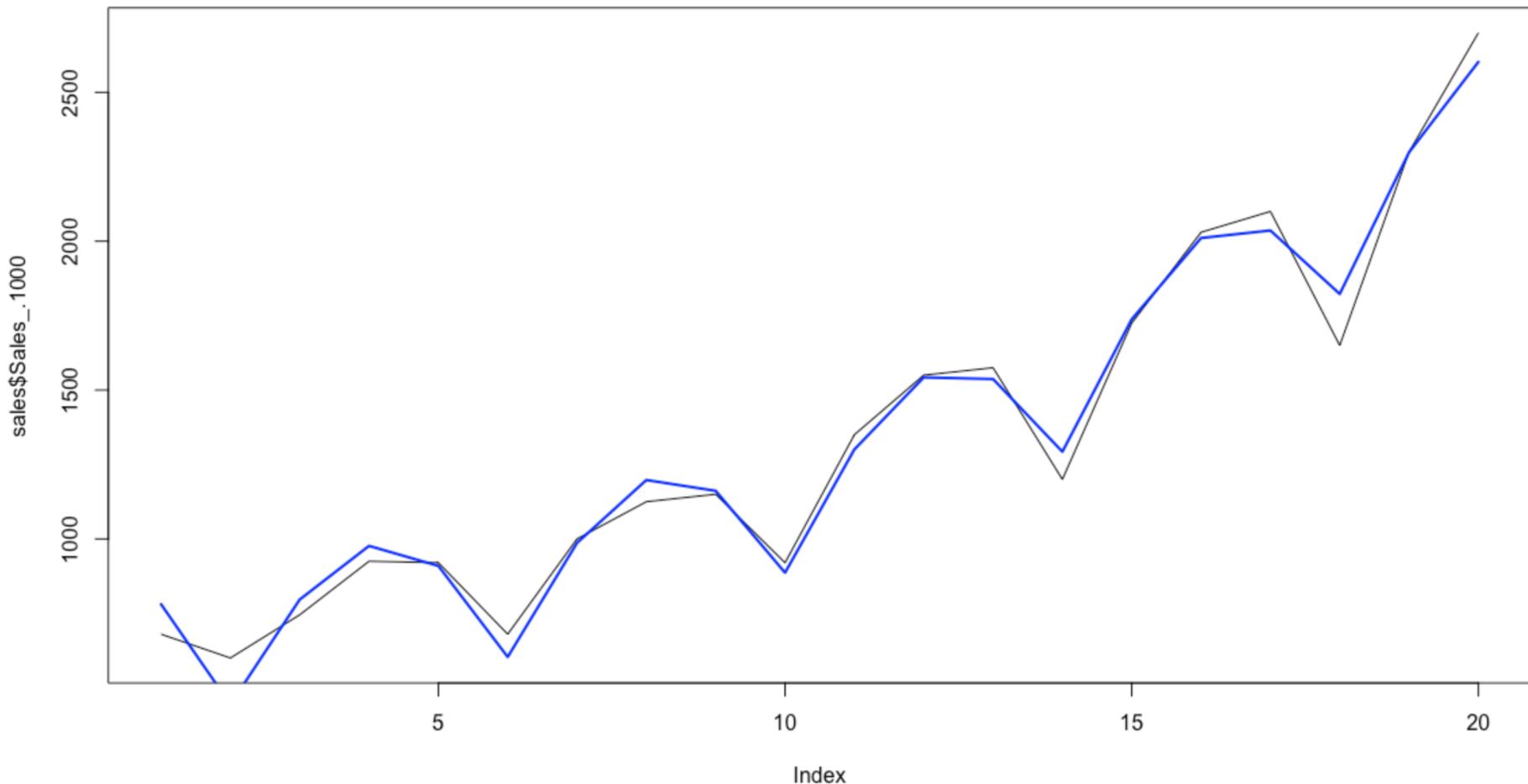


Seasonal Regression Models

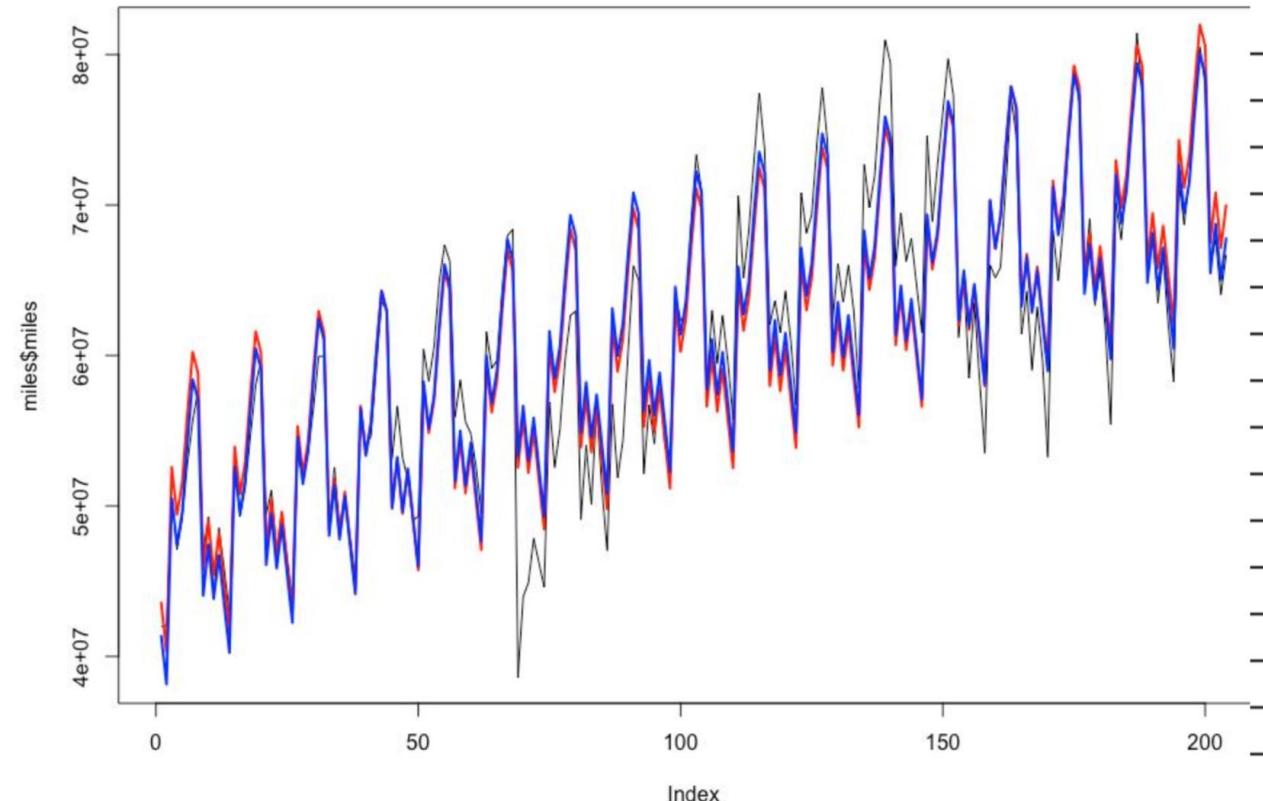
Quarter	Value of		
	X_{3t}	X_{4t}	X_{5t}
1	1	0	0
2	0	1	0
3	0	0	1
4	0	0	0

$$\hat{Y}_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 X_{3t} + \beta_4 X_{4t} + \beta_5 X_{5t} + \varepsilon_t$$

Seasonal Regression Models



Seasonal Regression Models - RPM



	miles	time	seasonal
1	41972194	1	1
2	42054796	2	2
3	50443045	3	3
4	47112397	4	4
5	49118248	5	5
6	52880510	6	6
7	55664750	7	7
8	57723208	8	8
9	47035464	9	9
10	49263120	10	10
11	43937074	11	11
12	48539606	12	12
13	45850623	13	1
14	42838949	14	2
15	53620994	15	3

Another Way of Incorporating Seasonality

- Take the trend prediction and actual prediction.
- Depending on additive or multiplicative model compute the deviation and map it as seasonality effect for each prediction.
- Take averages of the seasonality value. Use this to make future predictions.

Year	Quarter	Time variable (this is created)	Revenues (in \$M)
2008	I	1	10.2
	II	2	12.4
	III	3	14.8
	IV	4	15
2009	I	5	11.2
	II	6	14.3
	III	7	18.4
	IV	8	18

Call:
lm(formula = y ~ x)

What is the Regression equation?

Residuals:
 $y = 10.0393 + 0.9440x$

Min	1Q	Median	3Q	Max
-3.5595	-0.9384	0.4405	1.3265	1.9286

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)						
(Intercept)	10.0393	1.5531	6.464	0.00065	***					
x	0.9440	0.3076	3.069	0.02196	*					

Signif. codes:	0	'****'	0.001	'**'	0.01	'*' 0.05	'.'	0.1	' '	1

Residual standard error: 1.993 on 6 degrees of freedom
Multiple R-squared: 0.6109, Adjusted R-squared: 0.5461
F-statistic: 9.422 on 1 and 6 DF, p-value: 0.02196

Seasonality: Multiplicative

Time	Observed values TSI* (assuming no impact of cyclicality)	Predicted values (per the regression) T*	SI* = TSI/T
1	10.2	10.983	0.929
2	12.4	11.927	1.040
3	14.8	12.871	1.150
4	15.0	13.815	1.086
5	11.2	14.759	0.759
6	14.3	15.703	0.911
7	18.4	16.647	1.105
8	18.0	17.591	1.023

* T: Trend; S: Seasonal; I: Irregular

Quarterly Seasonality

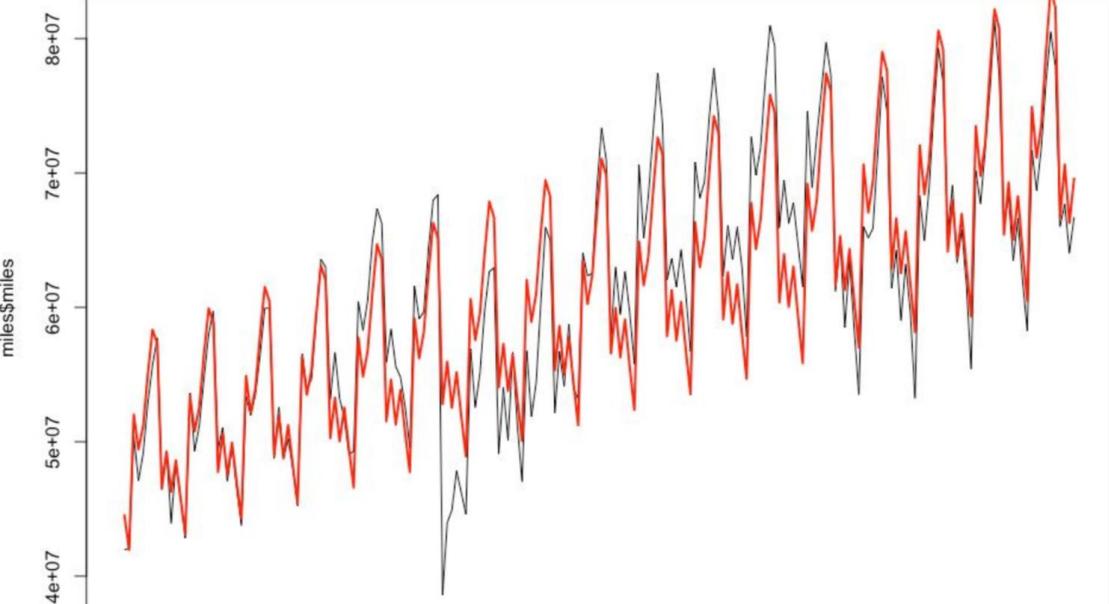
Time	Average seasonality factor
Q1	$0.844 \left(= \frac{0.929+0.759}{2} \right)$
Q2	0.975
Q3	1.127
Q4	1.054

Time	Observed values	Predicted values (per the regression)	$SI^* = TSI/T$
	TSI* (assuming no impact of cyclicalty)	T^*	
1	10.2	10.983	0.929
2	12.4	11.927	1.040
3	14.8	12.871	1.150
4	15.0	13.815	1.086
5	11.2	14.759	0.759
6	14.3	15.703	0.911
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Computations

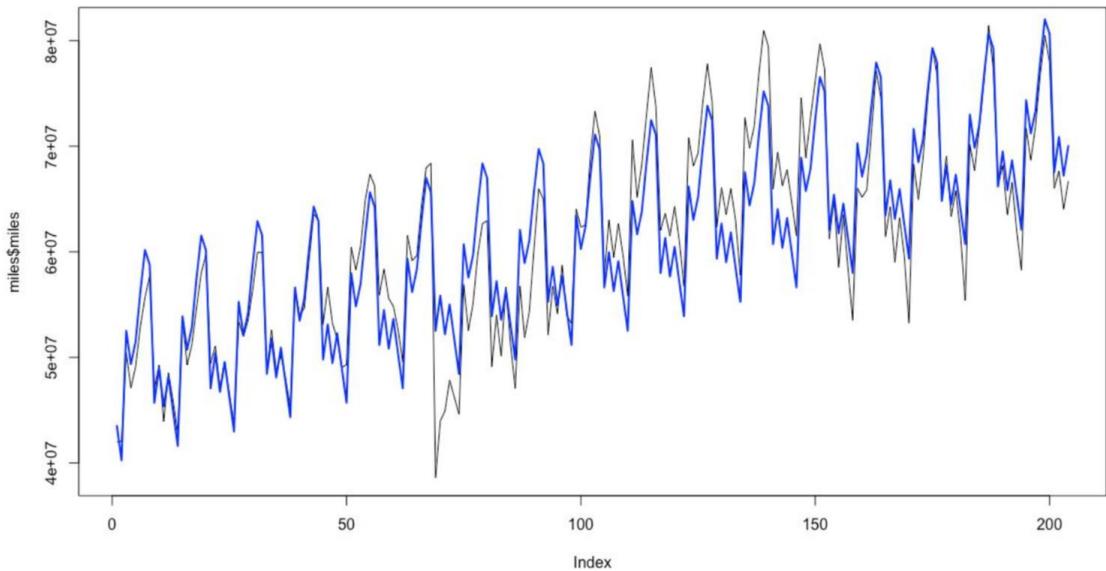
- Trend $Y_9 = 10.039 + 0.944(9) = 18.535$
- Corrected for seasonality and randomness: $18.535 * 0.844 = 15.643$

Seasonality: Multiplicative



	miles	time	seasonal	mae
1	41972194	1	1	0.849386
2	42054796	2	2	0.8491019
3	50443045	3	3	1.016129
4	47112397	4	4	0.946865
5	49118248	5	5	0.9849257
6	52880510	6	6	1.057953
7	55664750	7	7	1.111125
8	57723208	8	8	1.149602
9	47035464	9	9	0.9346292
10	49263120	10	10	0.9766855
11	43937074	11	11	0.8691307
12	48539606	12	12	0.9580177
13	45850623	13	1	0.9029174
14	42838949	14	2	0.8417232
15	53620994	15	3	1.051224

Seasonality: Additive



	miles	time	seasonal	mae
1	41972194	1	1	-7442550
2	42054796	2	2	-7473763
3	50443045	3	3	800670.5
4	47112397	4	4	-2643793
5	49118248	5	5	-751757.1
6	52880510	6	6	2896690
7	55664750	7	7	5567114
8	57723208	8	8	7511757
9	47035464	9	9	-3289802
10	49263120	10	10	-1175962
11	43937074	11	11	-6615823
12	48539606	12	12	-2127106
13	45850623	13	1	-4929905
14	42838949	14	2	-8055394
15	53620994	15	3	2612836
.

Issues with Regressing on Time

- If there is no trend or if seasonality and fluctuations are more important than trend, then the coefficients behave weirdly

Goodness of Fit

- MAE (Mean absolute error)

$$\frac{\sum |y_i - \hat{y}_i|}{n}$$

- MSE (Mean square error)

$$\frac{\sum (y_i - \hat{y}_i)^2}{n}$$

- RMSE (Root mean square error)

$$\sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n}}$$

- MAPE (Mean absolute percent error)

$$\frac{1}{n} \left(\frac{\sum |y_i - \hat{y}_i|}{y_i} \right) * 100$$

SIMPLE FORECASTING / BENCHMARKING METHODS

Average Method

$$\hat{y}_{t+h} = \bar{y} = \frac{y_1 + \cdots + y_t}{t}$$

- Forecasts of all future values are equal to the **mean of the historical data**

Naïve Method

$$\hat{y}_{t+h} = y_t$$

- Forecasts of all future values are equal to the last **observed** value
- A very useful method in many economic and financial time series

Seasonal Naïve Method

$$\hat{y}_{t+h} = y_{t+h-km}$$

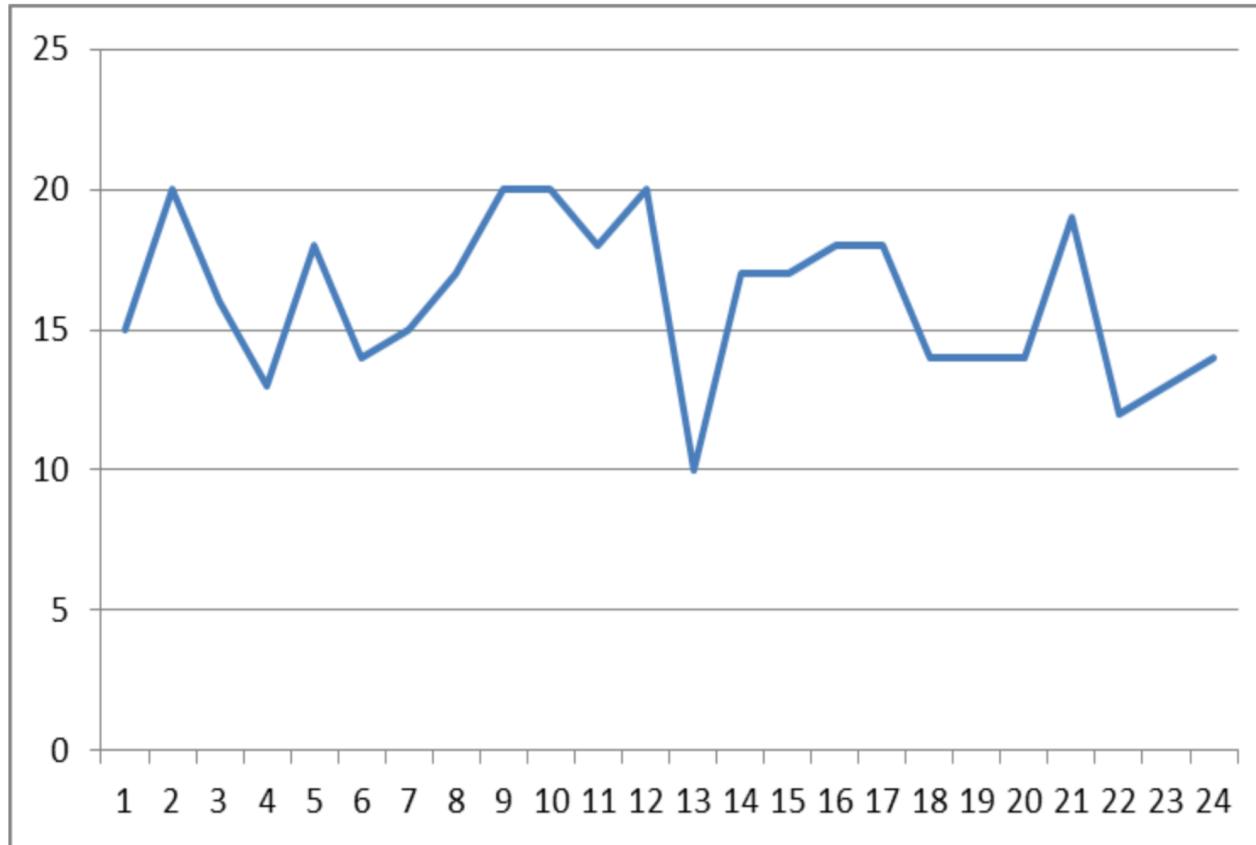
Integer part

where m is the seasonal period and $k = \left\lfloor \frac{h-1}{m} \right\rfloor + 1$

- Forecasts of all future values for a particular period (e.g., July) are equal to the last **observed** value for the same period (e.g., last July)

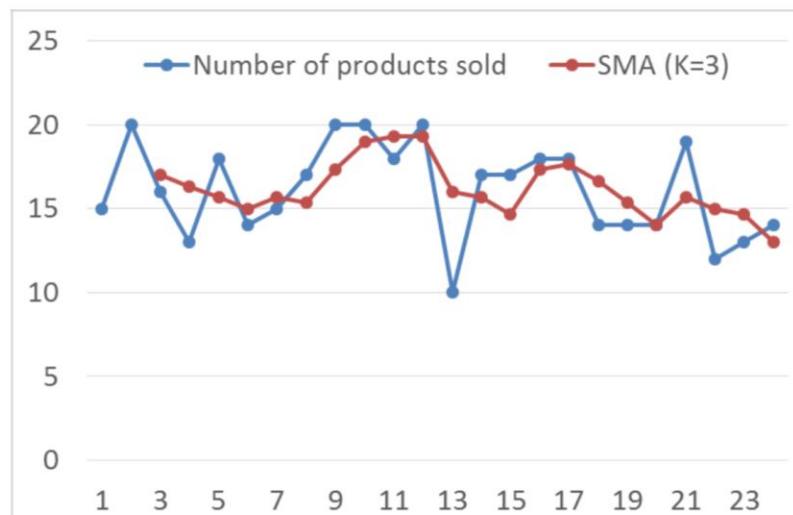
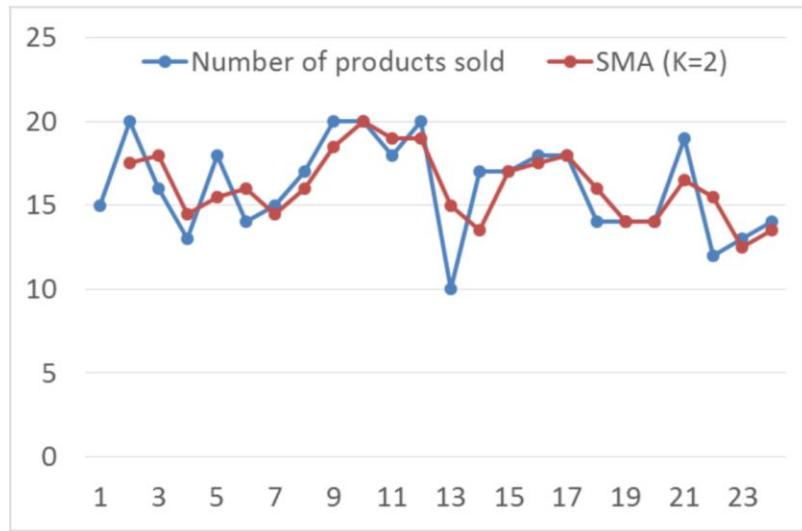
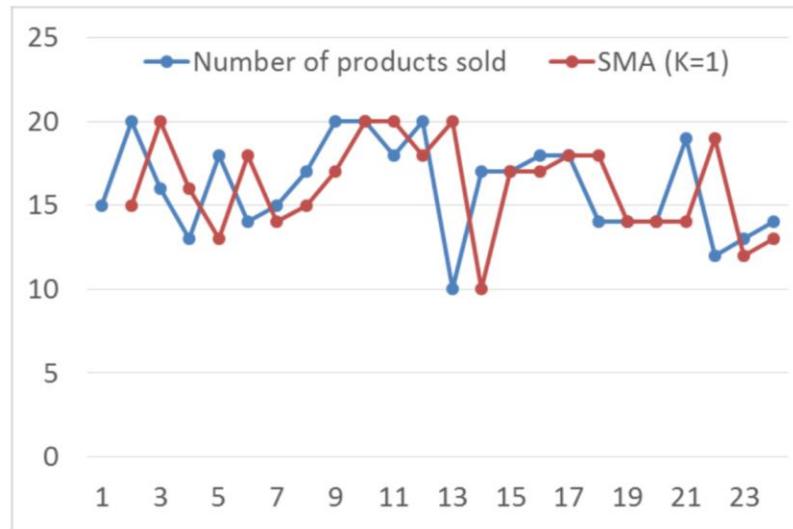
SMOOTHING METHODS

Stationary Model: Moving Averages



Stationary Model: Case 1 – Simple Moving Averages

	Number of products sold	SMA (K=1)	Error	SMA (K=2)	Error	SMA (K=3)	Error
1							
2	15						
3	20	15	5	17.5	2.5		
4	16	20	4	18	2	17	1
5	13	16	3	14.5	1.5	16.333333	3.33333
6	18	13	5	15.5	2.5	15.666667	2.33333
7	14	18	4	16	2	15	1
8	15	14	1	14.5	0.5	15.666667	0.66667
9	17	15	2	16	1	15.333333	1.66667
10	20	17	3	18.5	1.5	17.333333	2.66667
11	20	20	0	20	0	19	1
12	18	20	2	19	1	19.333333	1.33333
13	20	18	2	19	1	19.333333	0.66667
14	10	20	10	15	5	16	6
15	17	10	7	13.5	3.5	15.666667	1.33333
16	17	17	0	17	0	14.666667	2.33333
17	18	17	1	17.5	0.5	17.333333	0.66667
18	18	18	0	18	0	17.666667	0.33333
19	14	18	4	16	2	16.666667	2.66667
20	14	14	0	14	0	15.333333	1.33333
21	14	14	0	14	0	14	0
22	19	14	5	16.5	2.5	15.666667	3.33333
23	12	19	7	15.5	3.5	15	3
24	13	12	1	12.5	0.5	14.666667	1.66667
25	14	13	1	13.5	0.5	13	1
26			2.91304		1.45652		1.78788



Only decision point is K

Stationary Model: Case 2 – Weighted Moving Averages

$$\hat{Y}_{t+1} = w_1 Y_t + w_2 Y_{t-1} + \cdots + w_k Y_{t-k+1}$$

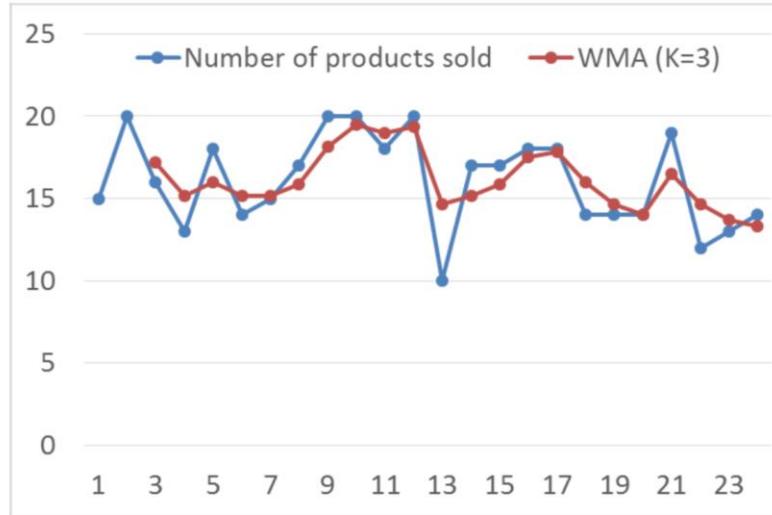
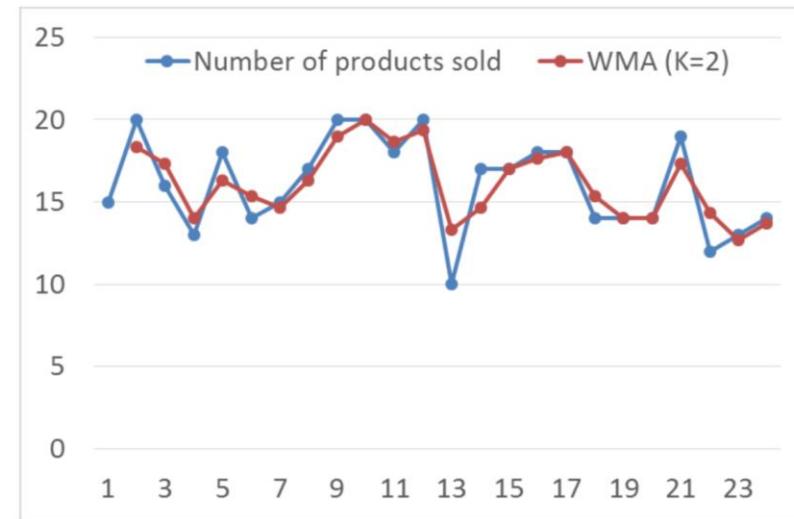
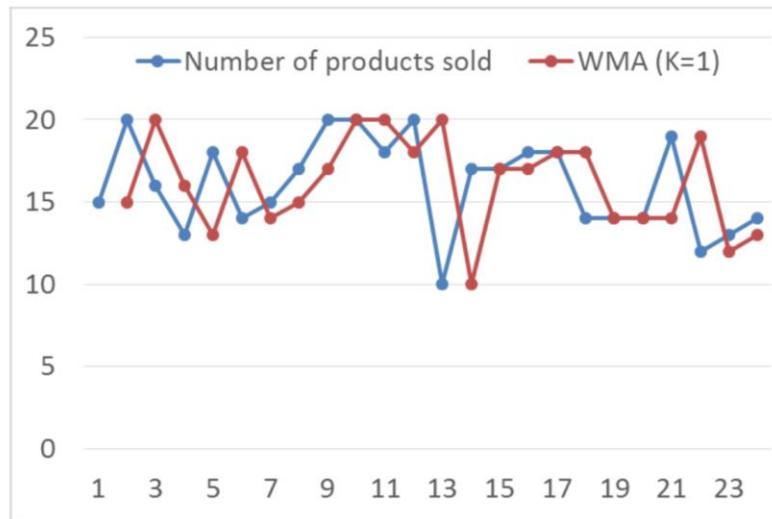
- Typically we choose a time period of moving average and weights are chosen such that the error is minimized

Stationary Model: Case 2 – Weighted Moving Averages

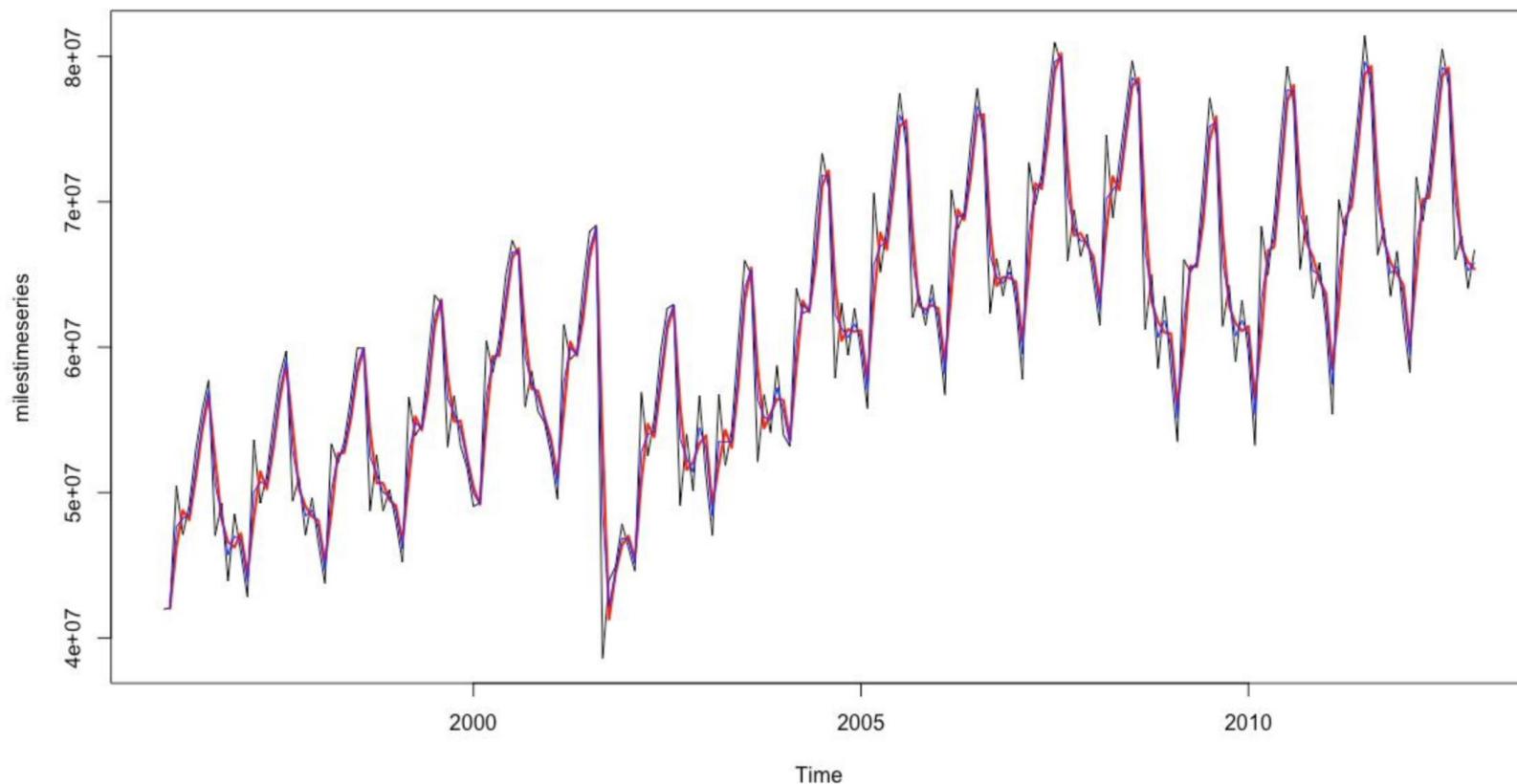
	Number of products sold	WMA (K=1)	Error	WMA (K=2)	Error	WMA (K=3)	Error
1							
2	15						
3	20	=A2*1	=ABS(B3-A3)	=(A3*2+A2*1)/3	=ABS(A3-D3)		
4	16	=A3*1	=ABS(B4-A4)	=(A4*2+A3*1)/3	=ABS(A4-D4)	=(A4*3+A3*2+A2*1)/6	=ABS(A4-F4)
5	13	=A4*1	=ABS(B5-A5)	=(A5*2+A4*1)/3	=ABS(A5-D5)	=(A5*3+A4*2+A3*1)/6	=ABS(A5-F5)
6	18	=A5*1	=ABS(B6-A6)	=(A6*2+A5*1)/3	=ABS(A6-D6)	=(A6*3+A5*2+A4*1)/6	=ABS(A6-F6)
7	14	=A6*1	=ABS(B7-A7)	=(A7*2+A6*1)/3	=ABS(A7-D7)	=(A7*3+A6*2+A5*1)/6	=ABS(A7-F7)
8	15	=A7*1	=ABS(B8-A8)	=(A8*2+A7*1)/3	=ABS(A8-D8)	=(A8*3+A7*2+A6*1)/6	=ABS(A8-F8)
9	17	=A8*1	=ABS(B9-A9)	=(A9*2+A8*1)/3	=ABS(A9-D9)	=(A9*3+A8*2+A7*1)/6	=ABS(A9-F9)
10	20	=A9*1	=ABS(B10-A10)	=(A10*2+A9*1)/3	=ABS(A10-D10)	=(A10*3+A9*2+A8*1)/6	=ABS(A10-F10)
11	20	=A10*1	=ABS(B11-A11)	=(A11*2+A10*1)/3	=ABS(A11-D11)	=(A11*3+A10*2+A9*1)/6	=ABS(A11-F11)
12	18	=A11*1	=ABS(B12-A12)	=(A12*2+A11*1)/3	=ABS(A12-D12)	=(A12*3+A11*2+A10*1)/6	=ABS(A12-F12)
13	20	=A12*1	=ABS(B13-A13)	=(A13*2+A12*1)/3	=ABS(A13-D13)	=(A13*3+A12*2+A11*1)/6	=ABS(A13-F13)
14	10	=A13*1	=ABS(B14-A14)	=(A14*2+A13*1)/3	=ABS(A14-D14)	=(A14*3+A13*2+A12*1)/6	=ABS(A14-F14)
15	17	=A14*1	=ABS(B15-A15)	=(A15*2+A14*1)/3	=ABS(A15-D15)	=(A15*3+A14*2+A13*1)/6	=ABS(A15-F15)
16	17	=A15*1	=ABS(B16-A16)	=(A16*2+A15*1)/3	=ABS(A16-D16)	=(A16*3+A15*2+A14*1)/6	=ABS(A16-F16)
17	18	=A16*1	=ABS(B17-A17)	=(A17*2+A16*1)/3	=ABS(A17-D17)	=(A17*3+A16*2+A15*1)/6	=ABS(A17-F17)
18	18	=A17*1	=ABS(B18-A18)	=(A18*2+A17*1)/3	=ABS(A18-D18)	=(A18*3+A17*2+A16*1)/6	=ABS(A18-F18)
19	14	=A18*1	=ABS(B19-A19)	=(A19*2+A18*1)/3	=ABS(A19-D19)	=(A19*3+A18*2+A17*1)/6	=ABS(A19-F19)
20	14	=A19*1	=ABS(B20-A20)	=(A20*2+A19*1)/3	=ABS(A20-D20)	=(A20*3+A19*2+A18*1)/6	=ABS(A20-F20)
21	14	=A20*1	=ABS(B21-A21)	=(A21*2+A20*1)/3	=ABS(A21-D21)	=(A21*3+A20*2+A19*1)/6	=ABS(A21-F21)
22	19	=A21*1	=ABS(B22-A22)	=(A22*2+A21*1)/3	=ABS(A22-D22)	=(A22*3+A21*2+A20*1)/6	=ABS(A22-F22)
23	12	=A22*1	=ABS(B23-A23)	=(A23*2+A22*1)/3	=ABS(A23-D23)	=(A23*3+A22*2+A21*1)/6	=ABS(A23-F23)
24	13	=A23*1	=ABS(B24-A24)	=(A24*2+A23*1)/3	=ABS(A24-D24)	=(A24*3+A23*2+A22*1)/6	=ABS(A24-F24)
25	14	=A24*1	=ABS(B25-A25)	=(A25*2+A24*1)/3	=ABS(A25-D25)	=(A25*3+A24*2+A23*1)/6	=ABS(A25-F25)
26			=AVERAGE(C3:C25)		=AVERAGE(E3:E25)		=AVERAGE(G3:G25)

Stationary Model: Case 2 – Weighted Moving Averages

1	Number of products sold	WMA (K=1)	Error	WMA (K=2)	Error	WMA (K=3)	Error
2	15						
3	20	15	5	18.3333333	1.66666667		
4	16	20	4	17.3333333	1.33333333	17.1666667	1.16666667
5	13	16	3	14	1	15.1666667	2.16666667
6	18	13	5	16.3333333	1.66666667	16	2
7	14	18	4	15.3333333	1.33333333	15.1666667	1.16666667
8	15	14	1	14.6666667	0.33333333	15.1666667	0.16666667
9	17	15	2	16.3333333	0.66666667	15.8333333	1.16666667
10	20	17	3	19	1	18.1666667	1.83333333
11	20	20	0	20	0	19.5	0.5
12	18	20	2	18.6666667	0.66666667	19	1
13	20	18	2	19.3333333	0.66666667	19.3333333	0.66666667
14	10	20	10	13.3333333	3.33333333	14.6666667	4.66666667
15	17	10	7	14.6666667	2.33333333	15.1666667	1.83333333
16	17	17	0	17	0	15.8333333	1.16666667
17	18	17	1	17.6666667	0.33333333	17.5	0.5
18	18	18	0	18	0	17.8333333	0.16666667
19	14	18	4	15.3333333	1.33333333	16	2
20	14	14	0	14	0	14.6666667	0.66666667
21	14	14	0	14	0	14	0
22	19	14	5	17.3333333	1.66666667	16.5	2.5
23	12	19	7	14.3333333	2.33333333	14.6666667	2.66666667
24	13	12	1	12.6666667	0.33333333	13.6666667	0.66666667
25	14	13	1	13.6666667	0.33333333	13.3333333	0.66666667
26				2.91304348		0.97101449	



SMA and WMA – Revenue Passenger Miles



> MAPE-SMA 4.093731 > MAPE-WMA 2.729154

Stationary Model: Case 3 – Exponential ~~Weighted Moving Averages~~ or Exponential Smoothing

Averaging over long periods dampens fluctuations, removing not only the noise but also trend and seasonality.

Moving averages over short recent periods maintains trend and seasonality but determining an optimum number for periods is tricky, even when using metrics like MAE. If averaged over too few periods, irregularities continue to remain and if averaged over long periods, dampening again becomes a problem.

Exponential smoothing **retains all older periods** while giving a greater weight to more recent periods (hence not a MOVING average).

Caution: It doesn't make any one method superior for all situations.

Stationary Model: Case 3 – Exponential Weighted Moving Averages or Exponential Smoothing

$$\hat{Y}_{t+1} = \hat{Y}_t + \alpha(Y_t - \hat{Y}_t)$$

Above equation indicates that the predicted value for time period $t+1$ (\hat{Y}_{t+1}) is equal to the predicted value for the previous period (\hat{Y}_t) plus an adjustment for the error made in predicting the previous period's value ($\alpha(Y_t - \hat{Y}_t)$).

The parameter α can assume any value between 0 and 1 ($0 \leq \alpha \leq 1$).

Various Ways of Understanding Exponential Smoothing

- Forecast
 - Interpolation between previous *forecast* and previous *observation*
$$= \alpha Y_t + (1 - \alpha) \hat{Y}_t$$
 - Previous *forecast* plus fraction of previous error
$$= \hat{Y}_t + \alpha(Y_t - \hat{Y}_t)$$
 - Previous *observation* minus fraction 1- of previous error
$$= Y_t - (1 - \alpha)(Y_t - \hat{Y}_t)$$
 - *Exponentially weighted (i.e., discounted) moving average*
$$= \alpha Y_t + \alpha(1 - \alpha)Y_{t-1} + \alpha(1 - \alpha)^2 Y_{t-2} + \cdots + \alpha(1 - \alpha)^n Y_{t-n} + \cdots$$

Exponential Smoothing

- Y at $t+1$
$$\hat{Y}_{t+1} = \hat{Y}_t + \alpha(Y_t - \hat{Y}_t)$$
- Y at $t+2$
- All future predictions are same! This is in accordance with stationary assumption.

Exponential Smoothing

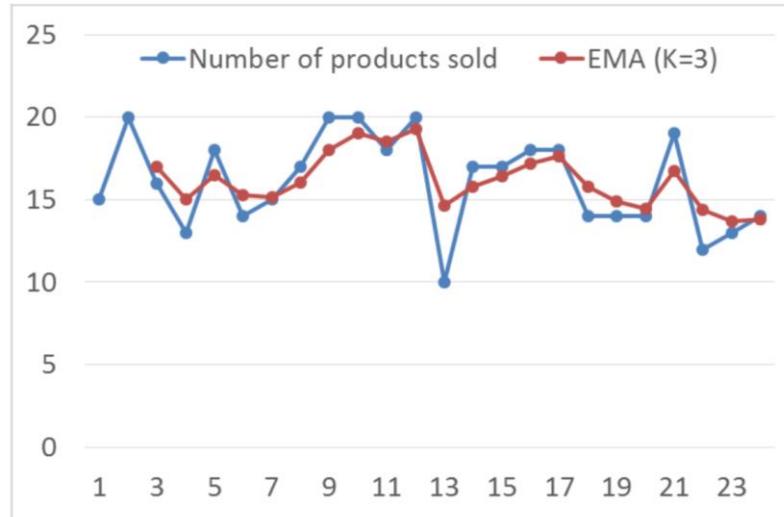
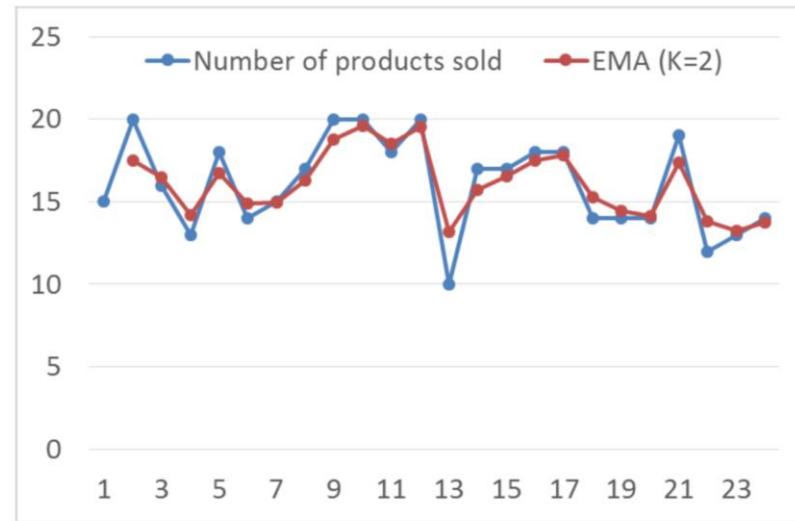
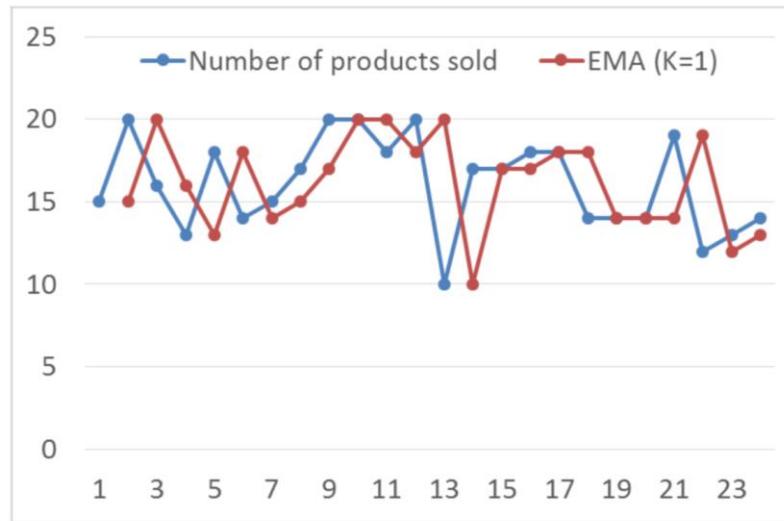
	A	B	C	D	E	F	G	H	I
1	Number of products sold	EMA (K=1)	Error	EMA (K=2)	Error	EMA (K=3)	Error	EMA (K=4)	Error
2	15								
3	20	=A2*1	=ABS(B3-A3)	=AVERAGE(A2:A3)					
4	16	=A3*\$K\$2+B3*\$L\$2	=ABS(B4-A4)	=A4*\$K\$3+D3*\$L\$3	=ABS(A4-D4)	=AVERAGE(A2:A4)	=ABS(A4-F4)		
5	13	=A4*\$K\$2+B4*\$L\$2	=ABS(B5-A5)	=A5*\$K\$3+D4*\$L\$3	=ABS(A5-D5)	=A5*\$K\$4+F4*\$L\$4	=ABS(A5-F5)	=AVERAGE(A2:A5)	=ABS(H5-A5)
6	18	=A5*\$K\$2+B5*\$L\$2	=ABS(B6-A6)	=A6*\$K\$3+D5*\$L\$3	=ABS(A6-D6)	=A6*\$K\$4+F5*\$L\$4	=ABS(A6-F6)	=A6*\$K\$5+H5*\$L\$5	=ABS(H6-A6)
7	14	=A6*\$K\$2+B6*\$L\$2	=ABS(B7-A7)	=A7*\$K\$3+D6*\$L\$3	=ABS(A7-D7)	=A7*\$K\$4+F6*\$L\$4	=ABS(A7-F7)	=A7*\$K\$5+H6*\$L\$5	=ABS(H7-A7)
8	15	=A7*\$K\$2+B7*\$L\$2	=ABS(B8-A8)	=A8*\$K\$3+D7*\$L\$3	=ABS(A8-D8)	=A8*\$K\$4+F7*\$L\$4	=ABS(A8-F8)	=A8*\$K\$5+H7*\$L\$5	=ABS(H8-A8)
9	17	=A8*\$K\$2+B8*\$L\$2	=ABS(B9-A9)	=A9*\$K\$3+D8*\$L\$3	=ABS(A9-D9)	=A9*\$K\$4+F8*\$L\$4	=ABS(A9-F9)	=A9*\$K\$5+H8*\$L\$5	=ABS(H9-A9)
10	20	=A9*\$K\$2+B9*\$L\$2	=ABS(B10-A10)	=A10*\$K\$3+D9*\$L\$3	=ABS(A10-D10)	=A10*\$K\$4+F9*\$L\$4	=ABS(A10-F10)	=A10*\$K\$5+H9*\$L\$5	=ABS(H10-A10)
11	20	=A10*\$K\$2+B10*\$L\$2	=ABS(B11-A11)	=A11*\$K\$3+D10*\$L\$3	=ABS(A11-D11)	=A11*\$K\$4+F10*\$L\$4	=ABS(A11-F11)	=A11*\$K\$5+H10*\$L\$5	=ABS(H11-A11)
12	18	=A11*\$K\$2+B11*\$L\$2	=ABS(B12-A12)	=A12*\$K\$3+D11*\$L\$3	=ABS(A12-D12)	=A12*\$K\$4+F11*\$L\$4	=ABS(A12-F12)	=A12*\$K\$5+H11*\$L\$5	=ABS(H12-A12)
13	20	=A12*\$K\$2+B12*\$L\$2	=ABS(B13-A13)	=A13*\$K\$3+D12*\$L\$3	=ABS(A13-D13)	=A13*\$K\$4+F12*\$L\$4	=ABS(A13-F13)	=A13*\$K\$5+H12*\$L\$5	=ABS(H13-A13)
14	10	=A13*\$K\$2+B13*\$L\$2	=ABS(B14-A14)	=A14*\$K\$3+D13*\$L\$3	=ABS(A14-D14)	=A14*\$K\$4+F13*\$L\$4	=ABS(A14-F14)	=A14*\$K\$5+H13*\$L\$5	=ABS(H14-A14)
15	17	=A14*\$K\$2+B14*\$L\$2	=ABS(B15-A15)	=A15*\$K\$3+D14*\$L\$3	=ABS(A15-D15)	=A15*\$K\$4+F14*\$L\$4	=ABS(A15-F15)	=A15*\$K\$5+H14*\$L\$5	=ABS(H15-A15)
16	17	=A15*\$K\$2+B15*\$L\$2	=ABS(B16-A16)	=A16*\$K\$3+D15*\$L\$3	=ABS(A16-D16)	=A16*\$K\$4+F15*\$L\$4	=ABS(A16-F16)	=A16*\$K\$5+H15*\$L\$5	=ABS(H16-A16)
17	18	=A16*\$K\$2+B16*\$L\$2	=ABS(B17-A17)	=A17*\$K\$3+D16*\$L\$3	=ABS(A17-D17)	=A17*\$K\$4+F16*\$L\$4	=ABS(A17-F17)	=A17*\$K\$5+H16*\$L\$5	=ABS(H17-A17)
18	18	=A17*\$K\$2+B17*\$L\$2	=ABS(B18-A18)	=A18*\$K\$3+D17*\$L\$3	=ABS(A18-D18)	=A18*\$K\$4+F17*\$L\$4	=ABS(A18-F18)	=A18*\$K\$5+H17*\$L\$5	=ABS(H18-A18)
19	14	=A18*\$K\$2+B18*\$L\$2	=ABS(B19-A19)	=A19*\$K\$3+D18*\$L\$3	=ABS(A19-D19)	=A19*\$K\$4+F18*\$L\$4	=ABS(A19-F19)	=A19*\$K\$5+H18*\$L\$5	=ABS(H19-A19)
20	14	=A19*\$K\$2+B19*\$L\$2	=ABS(B20-A20)	=A20*\$K\$3+D19*\$L\$3	=ABS(A20-D20)	=A20*\$K\$4+F19*\$L\$4	=ABS(A20-F20)	=A20*\$K\$5+H19*\$L\$5	=ABS(H20-A20)
21	14	=A20*\$K\$2+B20*\$L\$2	=ABS(B21-A21)	=A21*\$K\$3+D20*\$L\$3	=ABS(A21-D21)	=A21*\$K\$4+F20*\$L\$4	=ABS(A21-F21)	=A21*\$K\$5+H20*\$L\$5	=ABS(H21-A21)
22	19	=A21*\$K\$2+B21*\$L\$2	=ABS(B22-A22)	=A22*\$K\$3+D21*\$L\$3	=ABS(A22-D22)	=A22*\$K\$4+F21*\$L\$4	=ABS(A22-F22)	=A22*\$K\$5+H21*\$L\$5	=ABS(H22-A22)
23	12	=A22*\$K\$2+B22*\$L\$2	=ABS(B23-A23)	=A23*\$K\$3+D22*\$L\$3	=ABS(A23-D23)	=A23*\$K\$4+F22*\$L\$4	=ABS(A23-F23)	=A23*\$K\$5+H22*\$L\$5	=ABS(H23-A23)
24	13	=A23*\$K\$2+B23*\$L\$2	=ABS(B24-A24)	=A24*\$K\$3+D23*\$L\$3	=ABS(A24-D24)	=A24*\$K\$4+F23*\$L\$4	=ABS(A24-F24)	=A24*\$K\$5+H23*\$L\$5	=ABS(H24-A24)
25	14	=A24*\$K\$2+B24*\$L\$2	=ABS(B25-A25)	=A25*\$K\$3+D24*\$L\$3	=ABS(A25-D25)	=A25*\$K\$4+F24*\$L\$4	=ABS(A25-F25)	=A25*\$K\$5+H24*\$L\$5	=ABS(H25-A25)
26			=AVERAGE(C3:C25)		=AVERAGE(E3:E25)		=AVERAGE(G3:G25)		=AVERAGE(I3:I25)

J	K	L
K	2/(K+1)	1-[2/(K+1)]
1	1	=1-K2
2	=2/(J3+1)	=1-K3
3	=2/(J4+1)	=1-K4
4	=2/(J5+1)	=1-K5

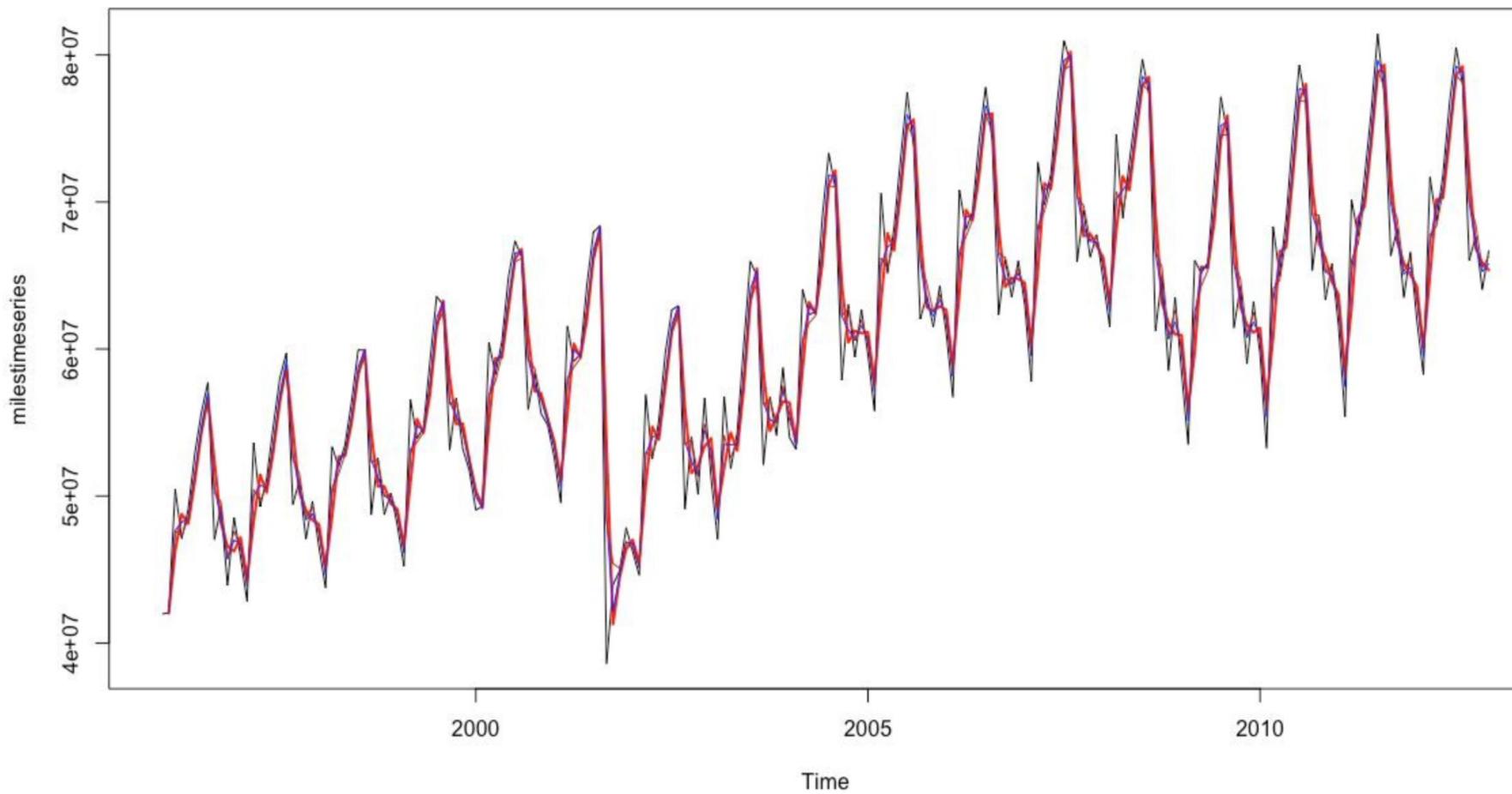
$$\hat{Y}_{t+1} = \alpha Y_t + (1 - \alpha) \hat{Y}_t$$

Exponential Smoothing

1	Number of products sold	EMA (K=1)	Error	EMA (K=2)	Error	EMA (K=3)	Error	EMA (K=4)	Error	
2	15									
3	20	15	5	17.5						
4	16	20	4	16.5	0.5	17	1			
5	13	16	3	14.1666667	1.16666667	15	2	16	3	
6	18	13	5	16.7222222	1.27777778	16.5	1.5	16.8	1.2	
7	14	18	4	14.9074074	0.90740741	15.25	1.25	15.68	1.68	
8	15	14	1	14.9691358	0.0308642	15.125	0.125	15.408	0.408	
9	17	15	2	16.3230453	0.67695473	16.0625	0.9375	16.0448	0.9552	
10	20	17	3	18.7743484	1.22565158	18.03125	1.96875	17.62688	2.37312	
11	20	20	0	19.5914495	0.40855053	19.015625	0.984375	18.576128	1.423872	
12	18	20	2	18.5304832	0.53048316	18.5078125	0.5078125	18.3456768	0.3456768	
13	20	18	2	19.5101611	0.48983895	19.2539063	0.74609375	19.0074061	0.99259392	
14	10	20	10	13.1700537	3.17005368	14.6269531	4.62695313	15.4044436	5.40444365	
15	17	10	7	15.7233512	1.27664877	15.8134766	1.18652344	16.0426662	0.95733381	
16	17	17	0	16.5744504	0.42554959	16.4067383	0.59326172	16.4255997	0.57440029	
17	18	17	1	17.5248168	0.4751832	17.2033691	0.79663086	17.0553598	0.94464017	
18	18	18	0	17.8416056	0.1583944	17.6016846	0.39831543	17.4332159	0.5667841	
19	14	18	4	15.2805352	1.2805352	15.8008423	1.80084229	16.0599295	2.05992954	
20	14	14	0	14.4268451	0.42684507	14.9004211	0.90042114	15.2359577	1.23595772	
21	14	14	0	14.1422817	0.14228169	14.4502106	0.45021057	14.7415746	0.74157463	
22	19	14	5	17.3807606	1.61923944	16.7251053	2.27489471	16.4449448	2.55505522	
23	12	19	7	13.7935869	1.79358685	14.3625526	2.36255264	14.6669669	2.66696687	
24	13	12	1	13.264529	0.26452895	13.6812763	0.68127632	14.0001801	1.00018012	
25	14	13	1	13.754843	0.24515702	13.8406382	0.15936184	14.0001081	0.00010807	
26				2.91304348		0.84055449		1.23867161		1.48027795

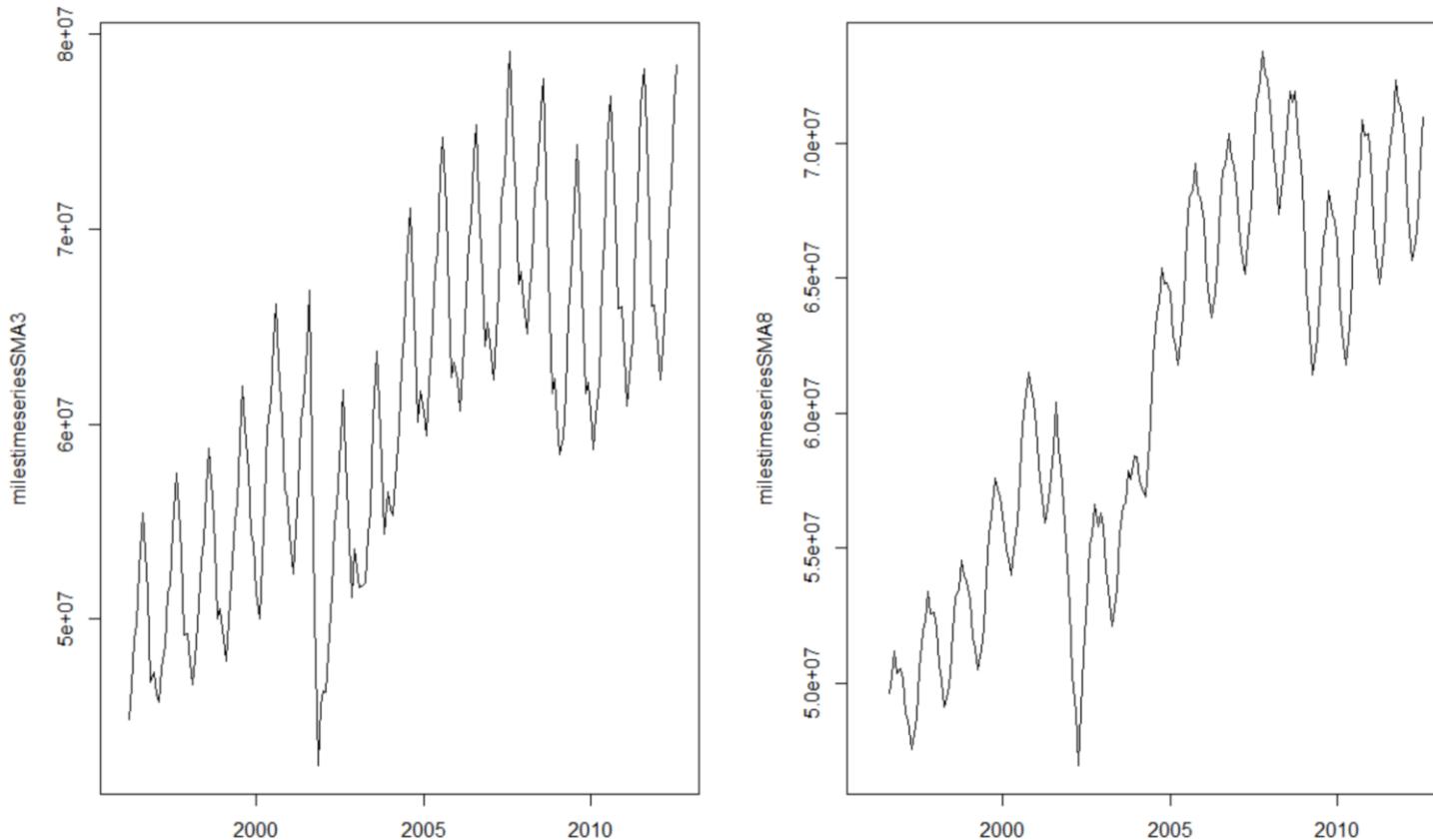


SMA, WMA and Exponential Smoothing – RPM



> MAPE-SMA 4.087519 > MAPE-WMA 2.725013 > MAPE-EMA 2.542173

Effect of k – Revenue Passenger Miles



SMA

K=3 vs K=8

Advanced Time Series Methods

ARIMA

AutoRegressive (AR) Models

- \hat{y}_t depends only on its own past values $y_{t-1}, y_{t-2}, y_{t-3}$, etc. Thus,

$$\hat{y}_t = f(y_{t-1}, y_{t-2}, y_{t-3}, \dots, \varepsilon_t)$$

- Autoregressive model where it depends on **p** of its **past values** (*lagged values having significant relationship with the most recent value*) is an **AR(p)** model and is represented as:

$$\hat{y}_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + e_t$$

Moving Average (MA) Models

- A moving average model is one where \hat{y}_t **depends** only on the **random error term**, i.e.,

$$\hat{y}_t = f(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \varepsilon_{t-3}, \dots)$$

- Moving average model where it depends on **q** of its **past values** (*residuals or lagged errors from earlier estimates*) is an **MA(q)** model and is represented as

$$\hat{y}_t = c + e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \dots + \theta_q e_{t-q}$$

AutoRegressive Moving Average (ARMA) Models

- ARMA(p,q)
 - Mix of both AR and MA
- General form of such a time-series model

$$\hat{y}_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} +$$

$$e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \dots + \theta_q e_{t-q}$$

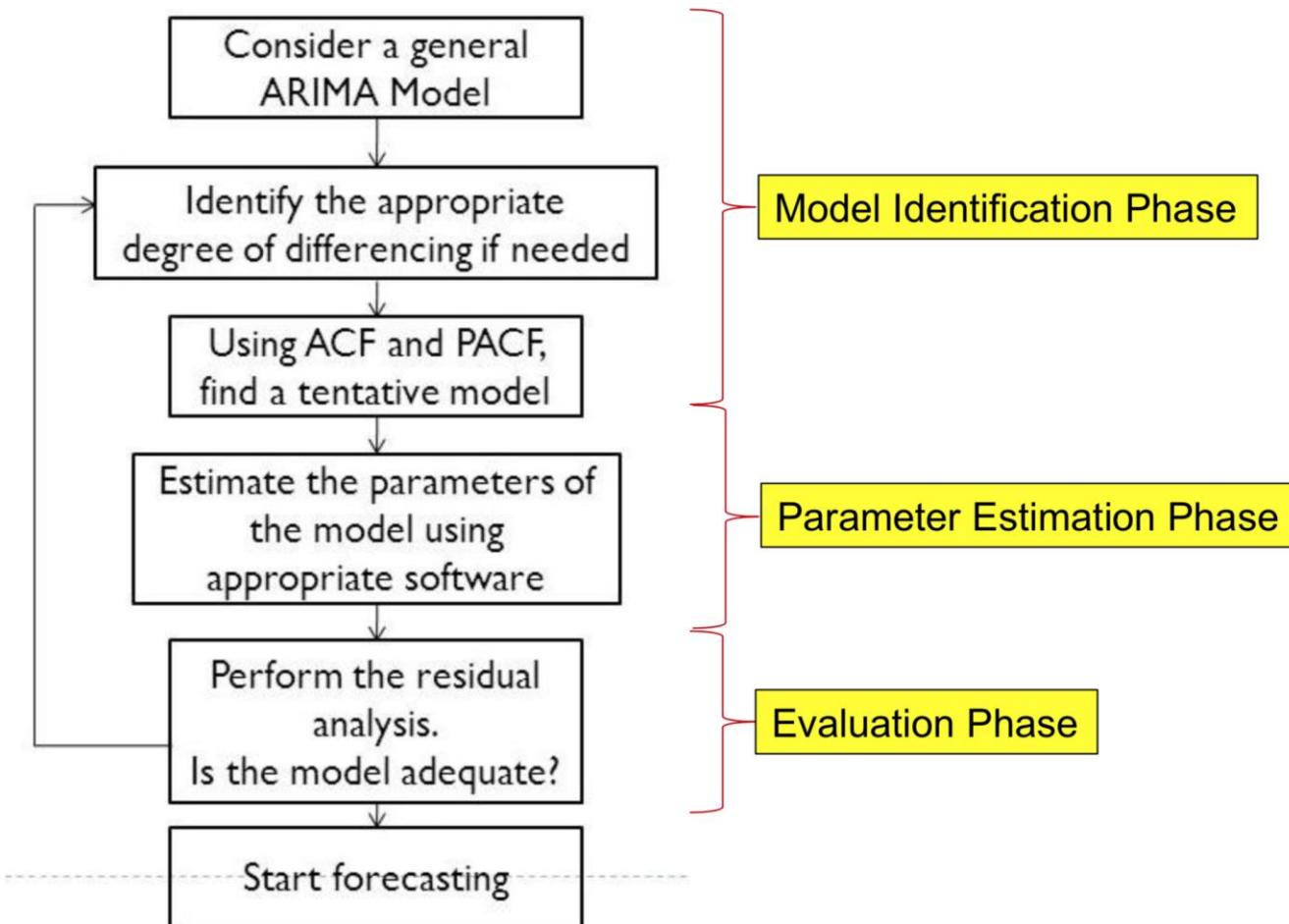
Integration of order d

- Non-stationary series can be made stationary series after differencing.
- Integration of order d
 - A series that is differentiated d times
 - Denoted as $I(d)$
 - If the series is differentiated once then it is $I(1)$.
 - $I(0)$ means series is stationary without differencing

ARIMA(p,d,q) Model

- p is the number of autoregressive [AR(p)] terms (a linear regression of the current value of the series against one or more prior values of the series)
 - Maximum lag beyond which PACF is 0
- d is the number of non-seasonal differences (order of the differencing) used to make the time series stationary [I(d)]
- q is the order of the moving average [MA(q)] model
 - Maximum lag beyond which the ACF is 0

Time Series Model Building Using ARIMA



THANK YOU