UCL
Université catholique de Louvain

INGI2145: CLOUD COMPUTING (Fall 2015)

Algorithms in MapReduce

15 October 2015

Lecture slides adapted from Upenn NETS212 by A. Haeberlen, Z. Ives
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What we have seen so far

- We saw how the MapReduce model could be used to filter, collect, and aggregate values
- This is useful for data with limited structure
  - We could extract pieces of input data items and collect them to run various reduce operations
  - We could "join" two different data sets on a common key
- But that's not enough…

Beyond average/sum/count

- Much of the world is a network of relationships and shared features
  - Members of a social network can be friends, and may have shared interests / memberships / etc.
  - Customers might view similar movies, and might even be clustered by interest groups
  - The Web consists of documents with links
  - Documents are also related by topics, words, authors, etc.

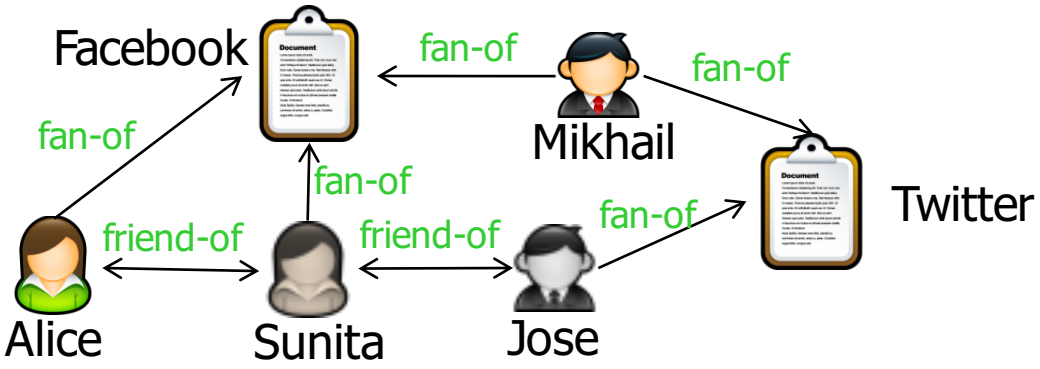
- We need a toolbox of algorithms for analyzing data that has both relationships and properties

Plan for today

- Representing data in graphs 

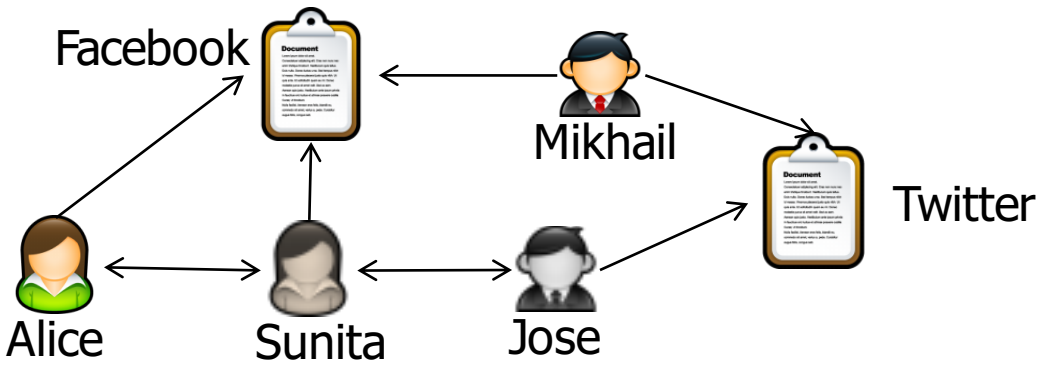
- Graph algorithms in MapReduce
  - Computation model
  - Iterative MapReduce
- A toolbox of algorithms
  - Single-source shortest path (SSSP)
  - k-means clustering
  - Classification with Naïve Bayes
  - PageRank

Thinking about related objects



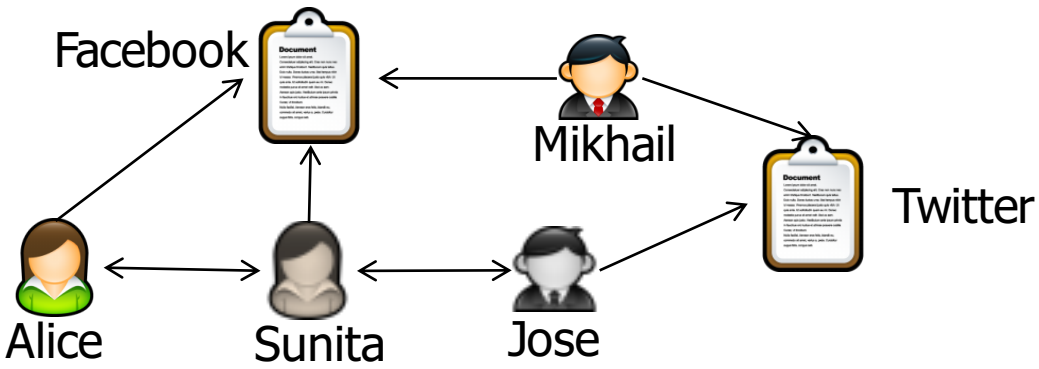
- We can represent related objects as a labeled, directed graph
- Entities are typically represented as nodes; relationships are typically edges
  - Nodes all have IDs, and possibly other properties
  - Edges typically have values, possibly IDs and other properties

Encoding the data in a graph



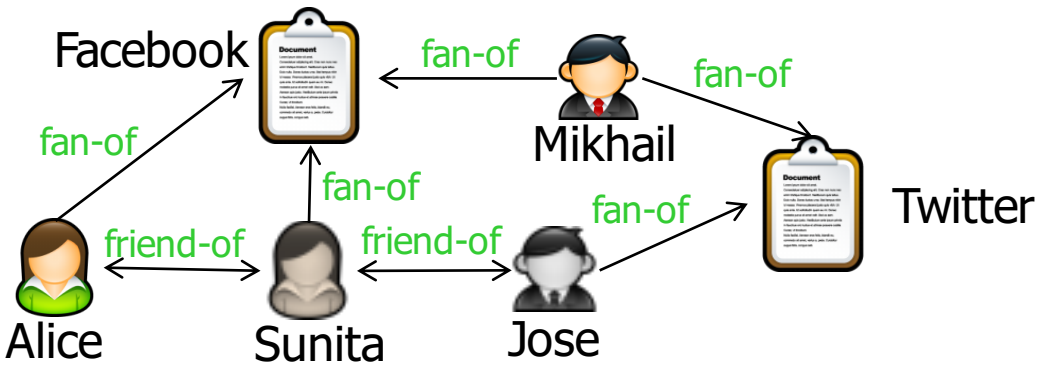
- Recall basic definition of a graph:
  - G = (V, E) where V denotes vertices, E denotes edges of the form (v_1,v_2) where $v_1,v_2 \in V$
- Assume we only care about connected vertices
  - Then we can capture a graph simply as a set of edges
  - … or as an adjacency list: v_i goes to [v_j, v_{j+1}, …]

Graph encodings: Set of edges



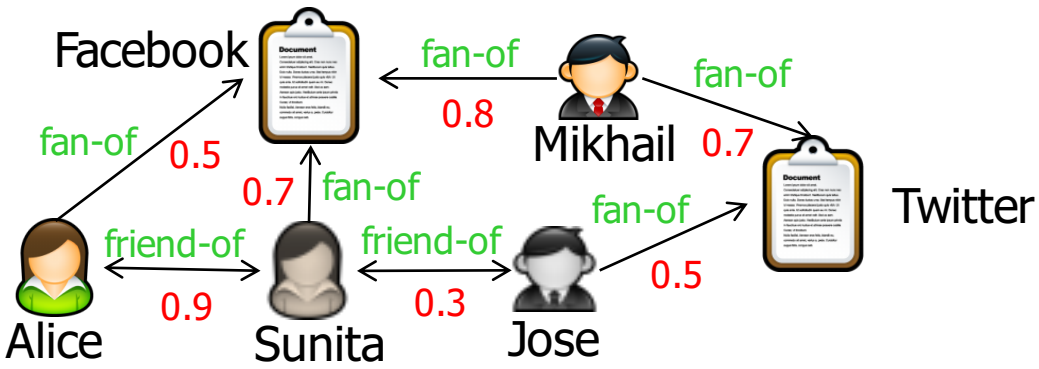
(Alice, Facebook)
(Alice, Sunita)
(Jose, Twitter)
(Jose, Sunita)
(Mikhail, Facebook)
(Mikhail, Twitter)
(Sunita, Facebook)
(Sunita, Alice)
(Sunita, Jose)

Graph encodings: Adding edge types



(Alice, fan-of, Facebook)
(Alice, friend-of, Sunita)
(Jose, fan-of, Twitter)
(Jose, friend-of, Sunita)
(Mikhail, fan-of, Facebook)
(Mikhail, fan-of, Twitter)
(Sunita, fan-of, Facebook)
(Sunita, friend-of, Alice)
(Sunita, friend-of, Jose)

Graph encodings: Adding weights



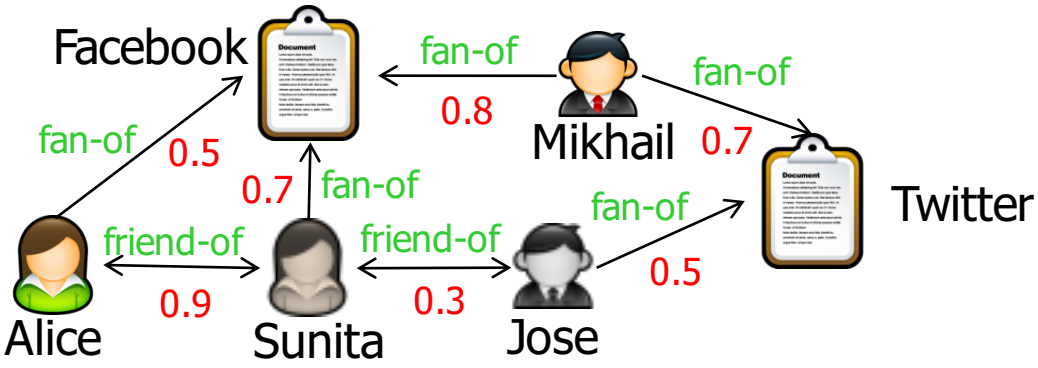
(Alice, fan-of, 0.5, Facebook)
(Alice, friend-of, 0.9, Sunita)
(Jose, fan-of, 0.5, Twitter)
(Jose, friend-of, 0.3, Sunita)
(Mikhail, fan-of, 0.8, Facebook)
(Mikhail, fan-of, 0.7, Twitter)
(Sunita, fan-of, 0.7, Facebook)
(Sunita, friend-of, 0.9, Alice)
(Sunita, friend-of, 0.3, Jose)

Plan for today

- Representing data in graphs 
- Graph algorithms in MapReduce 

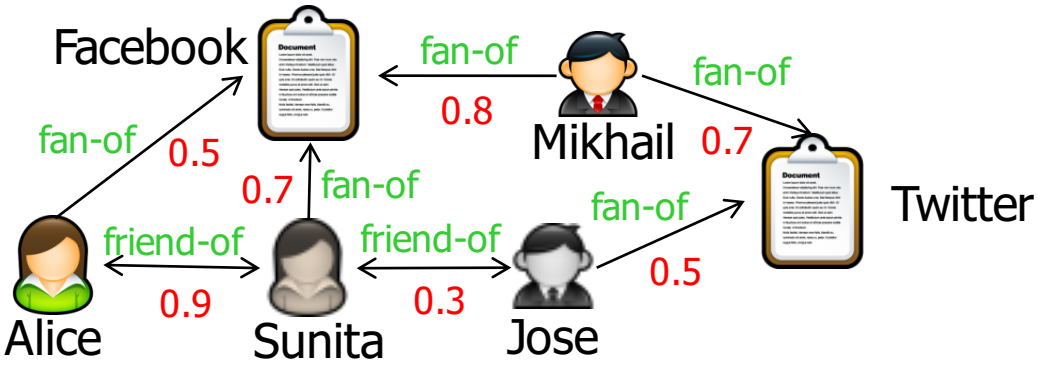
  - Computation model
  - Iterative MapReduce
- A toolbox of algorithms
  - Single-source shortest path (SSSP)
  - k-means clustering
  - Classification with Naïve Bayes
  - PageRank

A computation model for graphs



- Once the data is encoded in this way, we can perform various computations on it
  - Simple example: Which users are their friends' best friend?
- This is often done by
  - annotating the vertices with additional information, and
  - propagating the information along the edges
  - "Think like a vertex"!

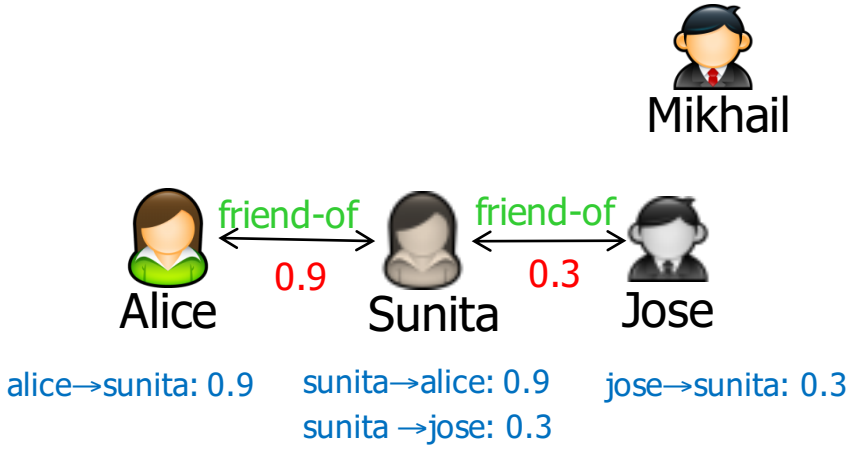
A computation model for graphs



All my friends have me as their best friend (highest strength edge)

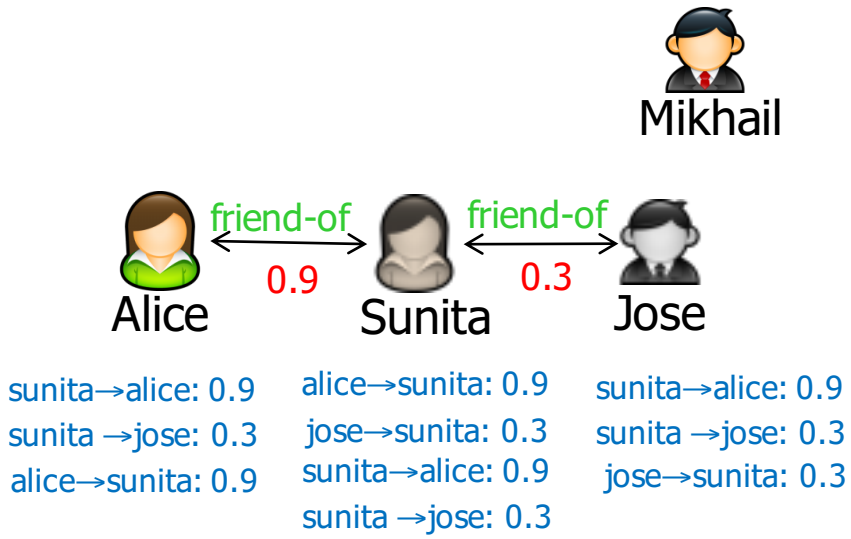
- Example: Am I my friends' best friend?
  - Step #1: Discard irrelevant vertices and edges

A computation model for graphs



- Example: Am I my friends' best friend?
  - Step #1: Discard irrelevant vertices and edges
  - Step #2: Annotate each vertex with list of friends
  - Step #3: Push annotations along each edge

A computation model for graphs



- Example: Am I my friends' best friend?
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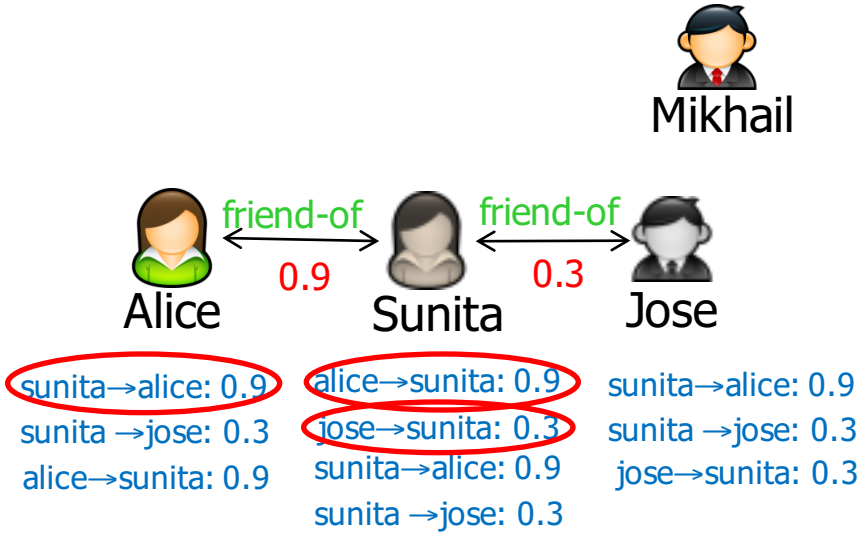
A computation model for graphs



- Example: Am I my friends' best friend?
  - Step #1: Discard irrelevant vertices and edges
  - Step #2: Annotate each vertex with list of friends
  - Step #3: Push annotations along each edge
  - Step #4: Determine result at each vertex

Can we do this in MapReduce?

```
map(key: node, value: [<otherNode, relType, strength>])
{
  for each otherNode, relType, strength in value:
    for each n in list of otherNodes:
      emit(n, <node, otherNode, relType, strength>)
}
reduce(key: ________, values: list of <src, dst, relType, strength>)
{
  for each <src,dst> find the dst where strength is highest:
    if (node != dst) emit(node, "NO"); return
  emit(node, "YES")
}
```

- Using adjacency list representation?

Can we do this in MapReduce?

```
map(key: node, value: <otherNode, relType, strength>)
{


}
reduce(key: _________, values: list of __________)
{


}
```

- Using single-edge data representation?

A real-world use case

- A variant that is actually used in social networks today: "Who are the friends of multiple of my friends?"
  - Where have you seen this before?
- Friend recommendation!
  - Maybe these people should be my friends too!

Generalizing…

- Now suppose we want to go beyond direct friend relationships
  - Example: How many of my friends' friends (distance-2 neighbors) have me as their best friend's best friend?
  - What do we need to do?
- How about distance k>2?
- To compute the answer, we need to run multiple iterations of MapReduce!

Iterative MapReduce

- The basic model:

```
copy files from input dir → staging dir N = 1
(optional: do some preprocessing)

while (!terminating condition) {
  map from staging dir N
  reduce into staging dir N+1
  N = N + 1 (optimization: use only 2 staging dirs alternately)
}

(optional: postprocessing)
move or process files from staging dir N+1 → output dir
```

- Note that reduce output must be compatible with the map input!
  - What can happen if we filter out some information in the mapper or in the reducer?

Graph algorithms and MapReduce

- A centralized algorithm typically traverses a tree or a graph one item at a time (there's only one "cursor")
  - You've learned breadth-first and depth-first traversals
- Most algorithms that are based on graphs make use of multiple map/reduce stages processing one "wave" at a time

Recap: MapReduce on graphs

- Suppose we want to:
  - compute a function for each vertex in a graph…
  - … using data from vertices at most k hops away
- We can do this as follows:
  - "Push" information along the edges
    - "Think like a vertex"
  - Finally, perform the computation at each vertex
- May need more than one MapReduce phase
  - Iterative MapReduce: Outputs of stage i → inputs of stage i+1

Plan for today

- Representing data in graphs ✓
- Graph algorithms in MapReduce ✓
  - Computation model ✓
  - Iterative MapReduce ✓
- A toolbox of algorithms 

  - Single-source shortest path (SSSP)
  - k-means clustering
  - Classification with Naïve Bayes
  - PageRank

Path-based algorithms

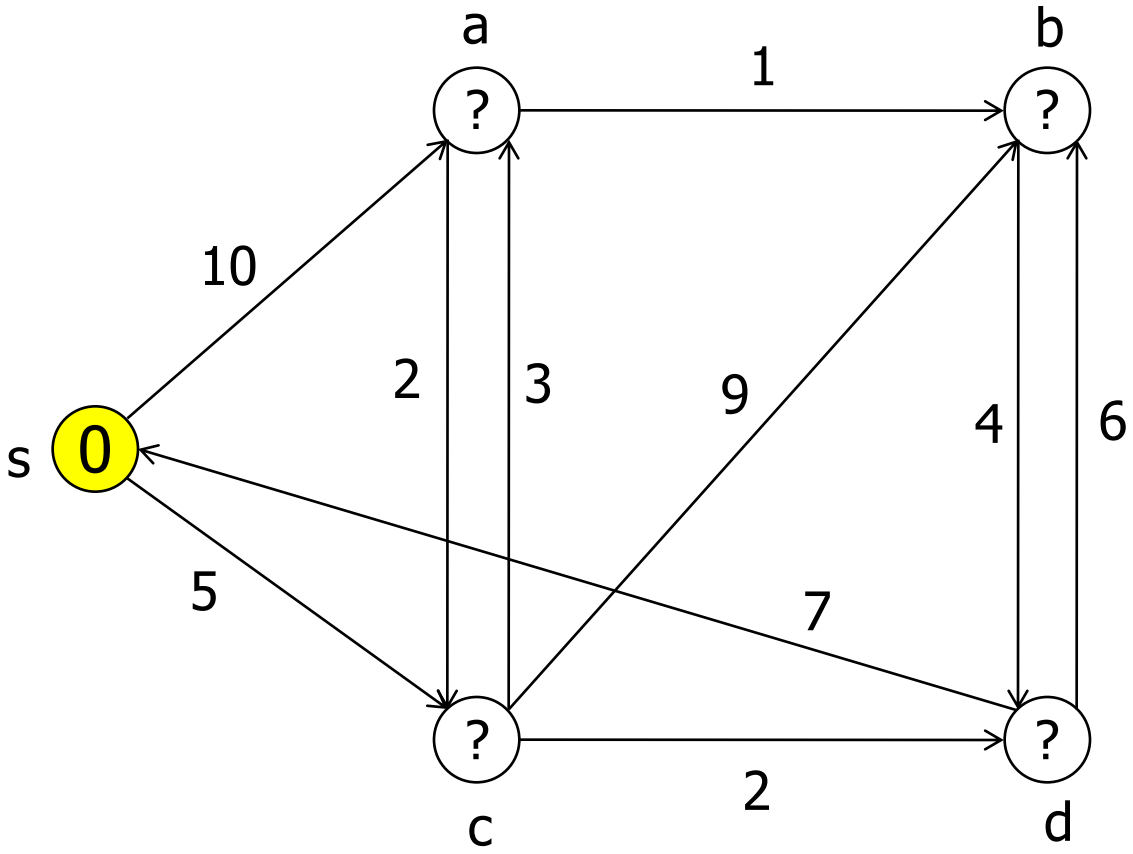
- Sometimes our goal is to compute information about the paths (sets of paths) between nodes
  - Edges may be annotated with cost, distance, or similarity

- Examples of such problems:
  - Shortest path from one node to another
  - Minimum spanning tree (minimal-cost tree connecting all vertices in a graph)
  - Steiner tree (minimal-cost tree connecting certain nodes)
  - Topological sort (node in a DAG comes before all nodes it points to)

Single-Source Shortest Path (SSSP)

Given a directed graph G = (V, E) in which each edge e has a cost c(e):

- Compute the cost of reaching each node from the source node s in the **most efficient way** (potentially after multiple 'hops')



SSSP: Intuition

- We can formulate the problem using induction
  - The shortest path follows the principle of optimality: the last step (u,v) makes use of the shortest path to u

- We can express this as follows:

```
bestDistanceAndPath(v) {
  if (v == source) then {
    return <distance 0, path[v]>
  } else {
    find argmin_u (bestDistanceAndPath[u] + dist[u,v])
    return <bestDistanceAndPath[u] + dist[u,v], path[u] + v>
  }
}
```

SSSP: Traditional Solution

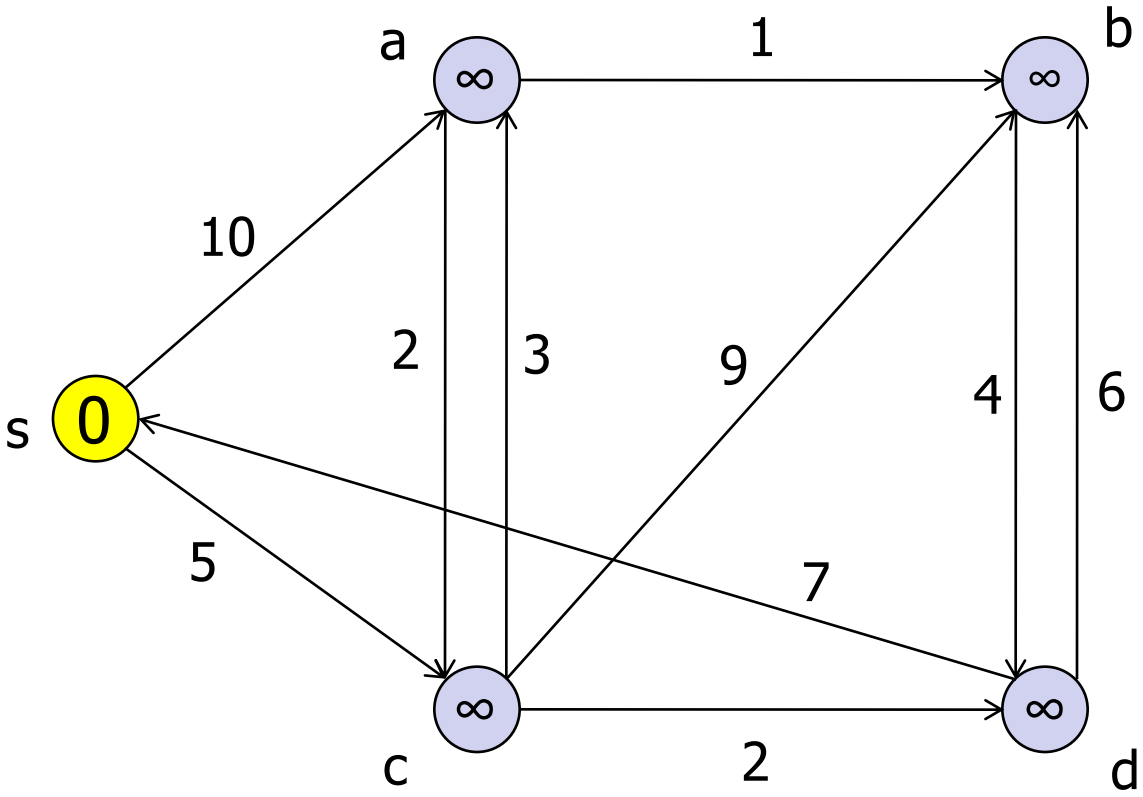
- Dijkstra's algorithm

```
V: vertices, E: edges, S: start node

foreach v in V
  dist_S_To[v] := infinity
  predecessor[v] = nil
dist_S_To[S] := 0
Q := V

while (Q not empty) do
  u := Q.removeNodeClosestTo(S)
  foreach v in V where (u,v) in E
    if (dist_S_To[v] > dist_S_To[u] + cost(u,v)) then
      dist_S_To[v] = dist_S_To[u] + cost(u,v)
      predecessor[v] = u
```

Initialize length and last step of path to default values

Update length and path based on edges radiating from u

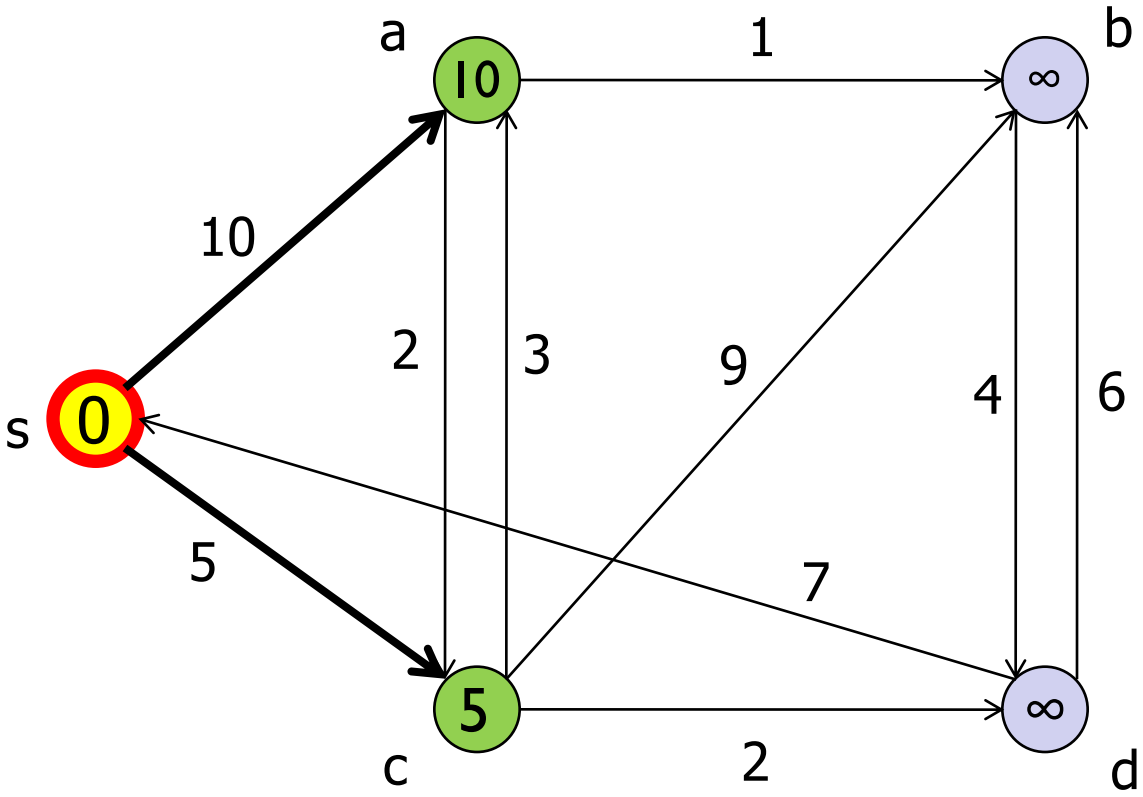
SSSP: Dijkstra in Action



Q = {s,a,b,c,d}
dist_S_To: {(a,∞), (b,∞), (c,∞), (d,∞)}
predecessor: {(a,nil), (b,nil), (c,nil), (d,nil)}

Example from CLR 2nd ed. p. 528

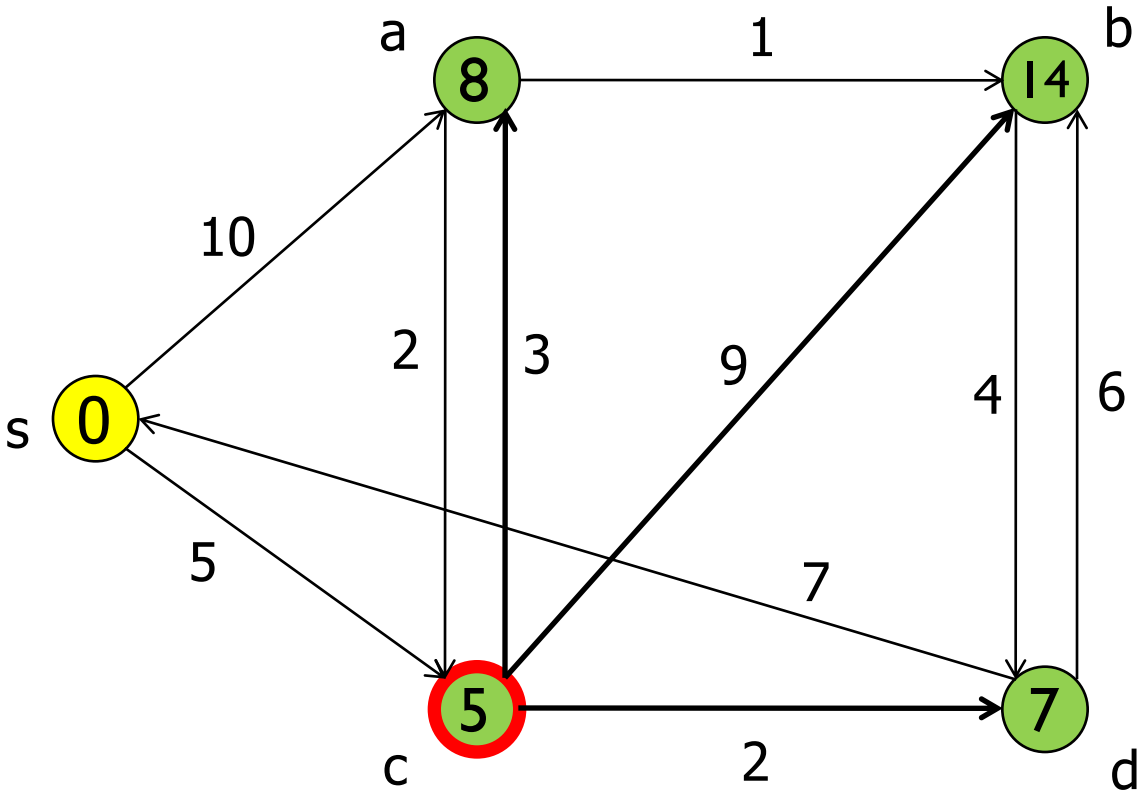
SSSP: Dijkstra in Action



Q = {a,b,c,d}
dist_S_To: {(a,10), (b,∞), (c,5), (d,∞)}
predecessor: {(a,s), (b,nil), (c,s), (d,nil)}

Example from CLR 2nd ed. p. 528

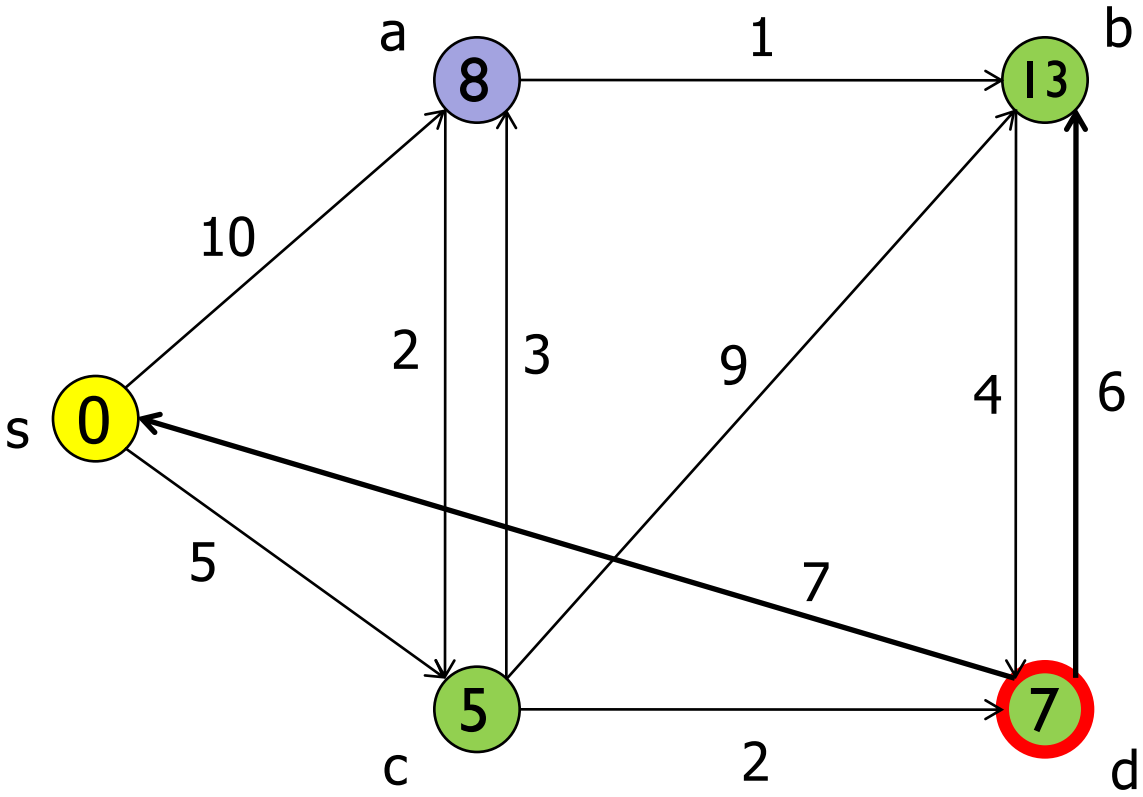
SSSP: Dijkstra in Action



Q = {a,b,d}
dist_S_To: {(a,8), (b,14), (c,5), (d,7)}
predecessor: {(a,c), (b,c), (c,s), (d,c)}

Example from CLR 2nd ed. p. 528

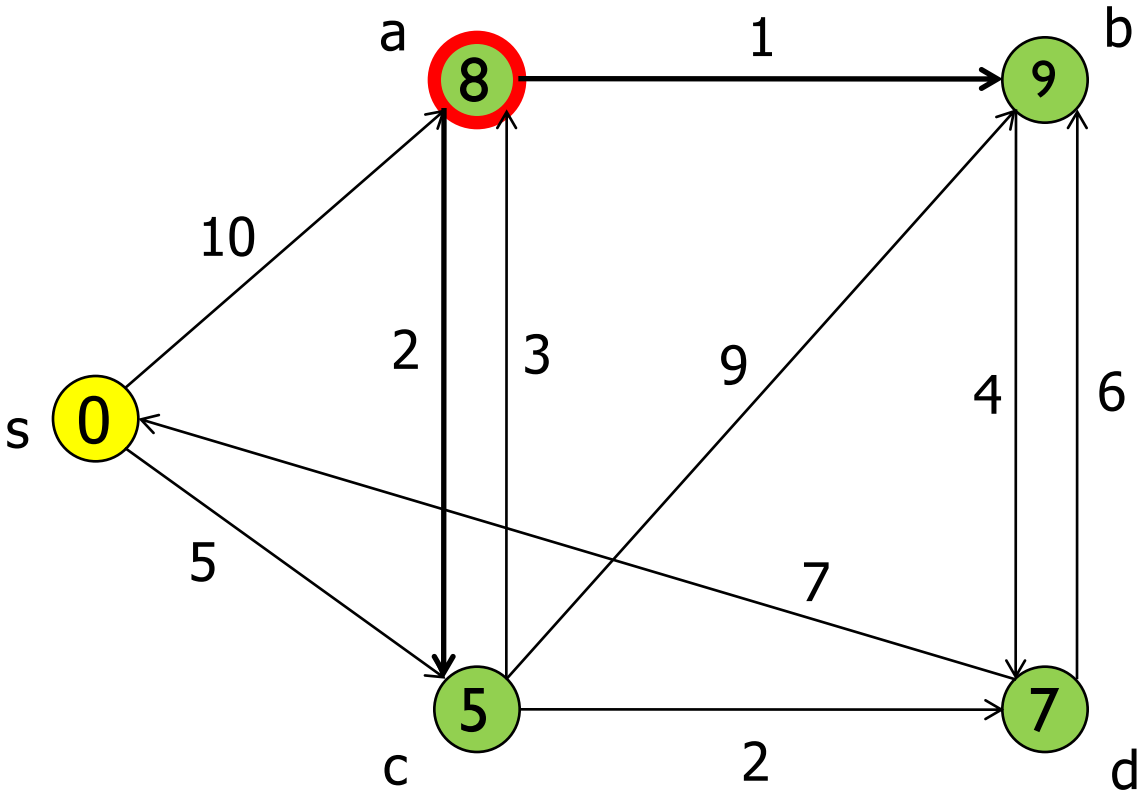
SSSP: Dijkstra in Action



Q = {a,b}
dist_S_To: {(a,8), (b,13), (c,5), (d,7)}
predecessor: {(a,c), (b,d), (c,s), (d,c)}

Example from CLR 2nd ed. p. 528

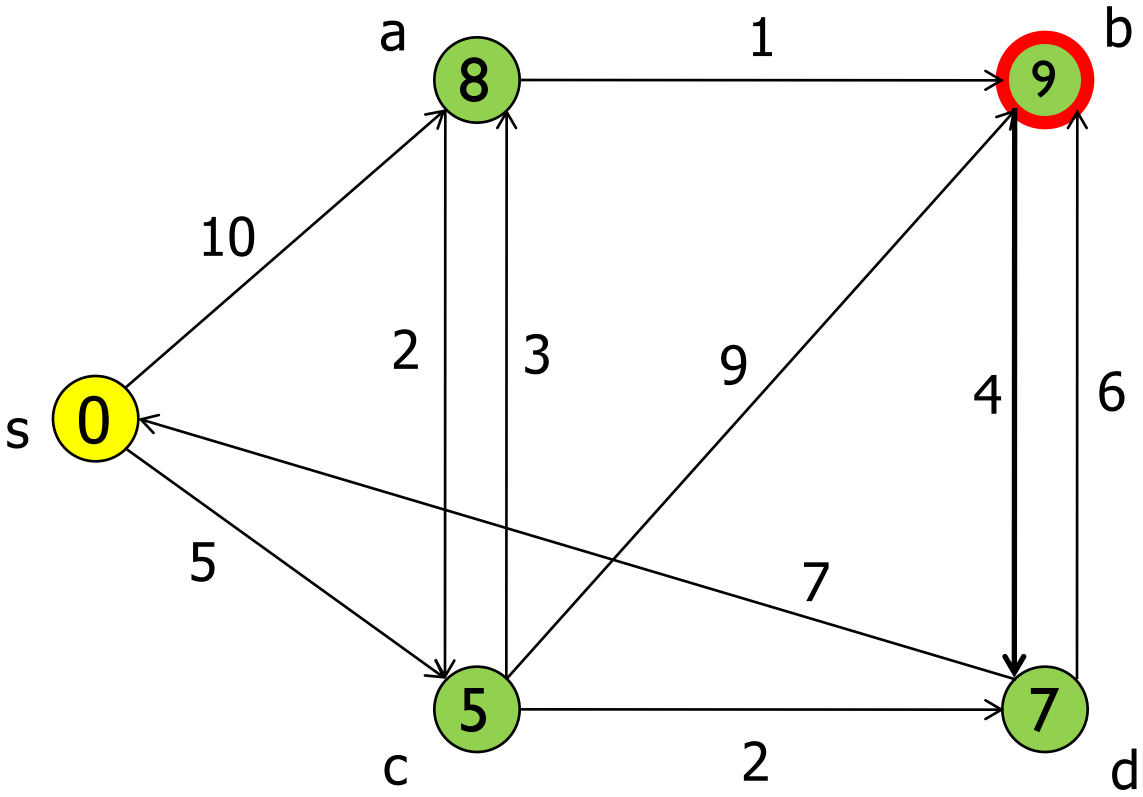
SSSP: Dijkstra in Action



Q = {b}
dist_S_To: {(a,8), (b,9), (c,5), (d,7)}
predecessor: {(a,c), (b,a), (c,s), (d,c)}

Example from CLR 2nd ed. p. 528

SSSP: Dijkstra in Action

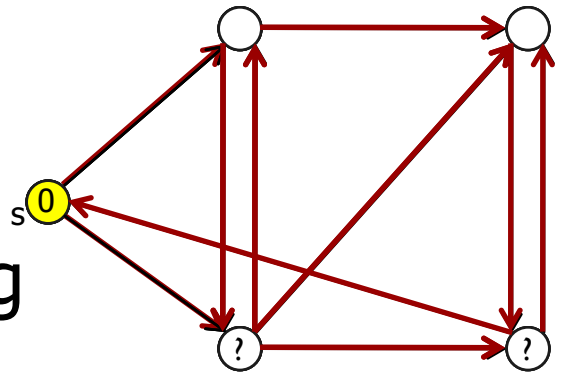


Q = {}
dist_S_To: {(a,8), (b,9), (c,5), (d,7)}
predecessor: {(a,c), (b,a), (c,s), (d,c)}

Example from CLR 2nd ed. p. 528

SSSP: How to parallelize?

- Dijkstra traverses the graph along a single route at a time, prioritizing its traversal to the next step based on total path length (and avoiding cycles)
  - No real parallelism to be had here!



- Intuitively, we want something that "radiates" from the origin, one "edge hop distance" at a time
  - Each step outwards can be done in parallel, before another iteration occurs - or we are done
  - Scalability depends on the algorithm, not (just) on the problem!

SSSP: Revisiting the inductive definition

```
bestDistanceAndPath(v) {
  if (v == source) then {
    return <distance 0, path [v]>
  } else {
    find argmin_u (bestDistanceAndPath[u] + dist[u,v])
    return <bestDistanceAndPath[u] + dist[u,v], path[u] + v>
  }
}
```

- Dijkstra's algorithm carefully considered each u in a way that allowed us to prune certain points
- Instead we can look at all potential u's for each v
  - Compute iteratively, by keeping a "frontier set" of u nodes located at *i* edge-hops from the source

SSSP: MapReduce formulation

- init:
  - For each node, node ID → <∞, -, {<succ-node-ID,edge-cost>}>

The shortest path we have found so far from the source to nodeID has length ∞… (this is 0 for the source)

… this is the next hop on that path…

… and here is the adjacency list for nodeID

- map:
  - take node ID → <distance, next, {<succ-node-ID,edge-cost>}>
  - For each succ-node-ID:
    - emit succ-node ID → {<node ID, distance+edge-cost>}
  - emit node ID → distance,{<succ-node-ID,edge-cost>}

This is a new path from the source to succ-node-ID that we just discovered (not necessarily shortest)

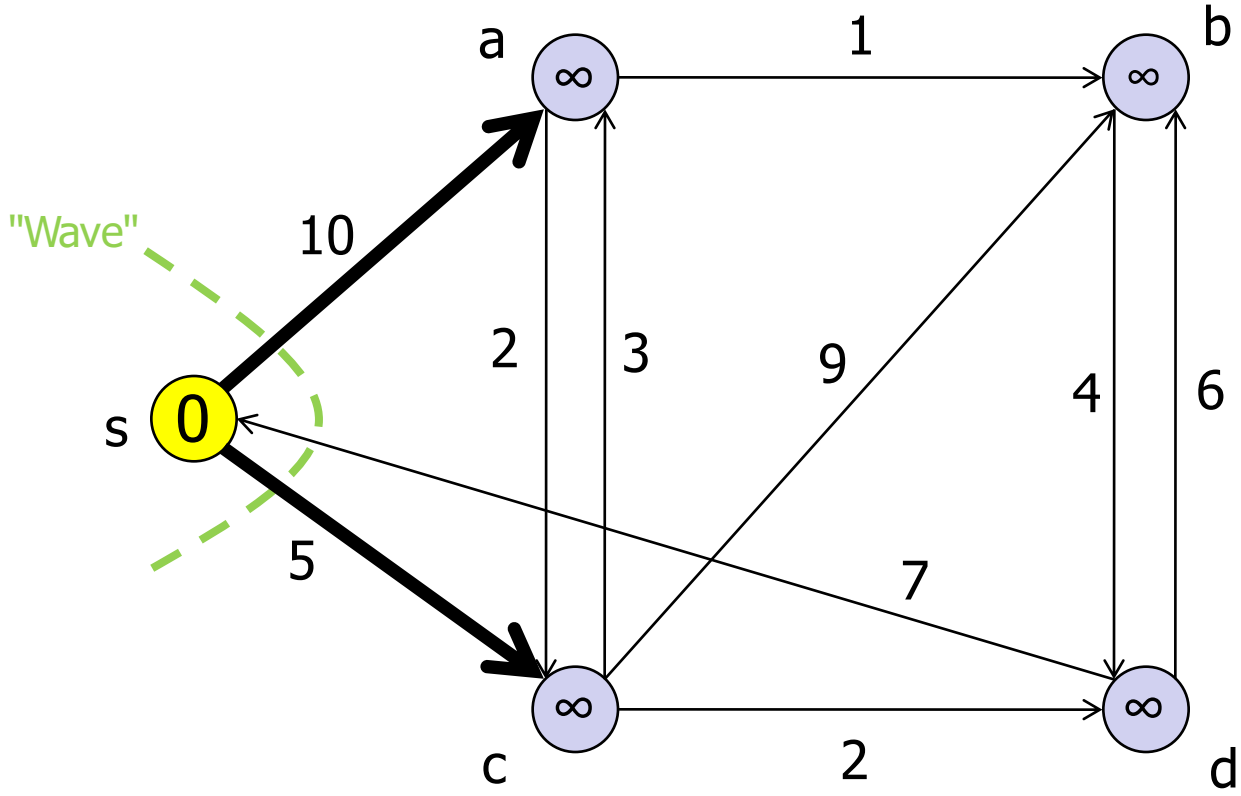
Why is this necessary?

- reduce:
  - distance := min cost from a predecessor; next := that predec.
  - emit node ID → <distance, next, {<succ-node-ID,edge-cost>}>
- Repeat until no changes
- Postprocessing: Remove adjacency lists

Iteration 0: Base case

mapper: (a,<s,10>) (c,<s,5>) edges

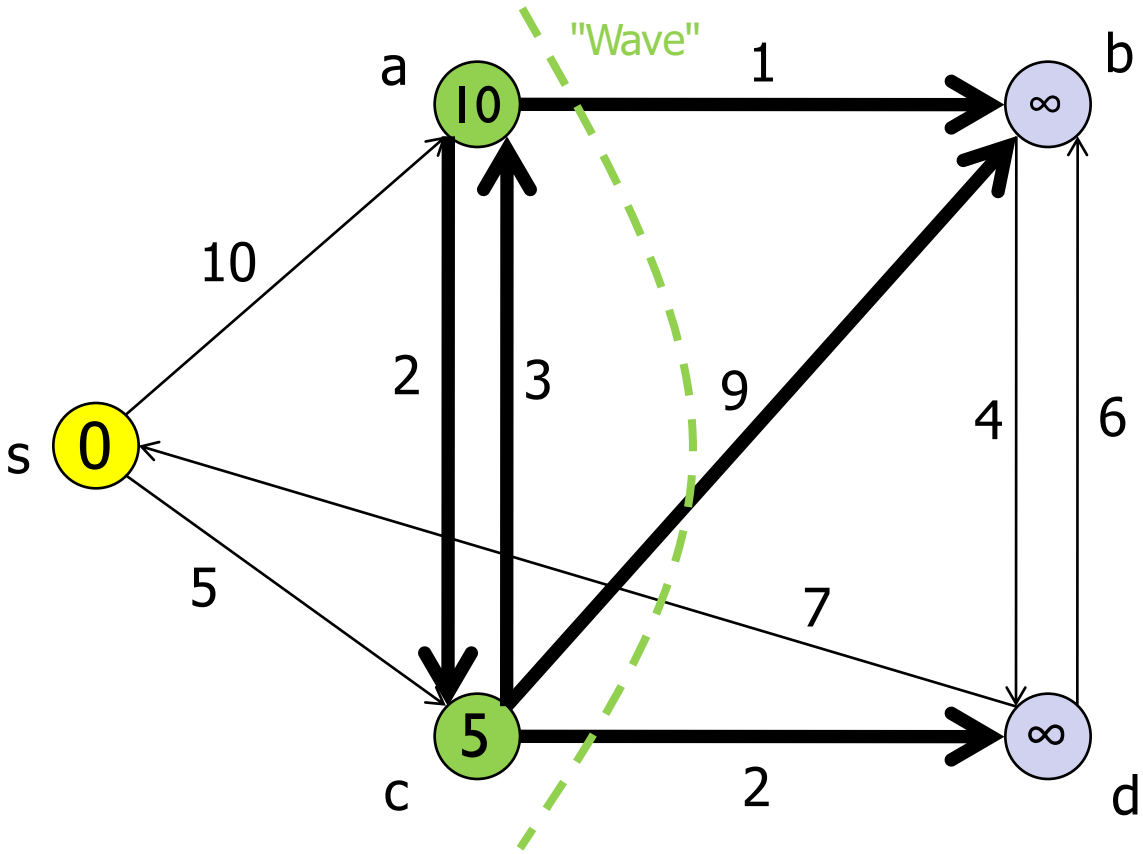
reducer: (a,<10, ...>) (c,<5, ...>)



Iteration 1

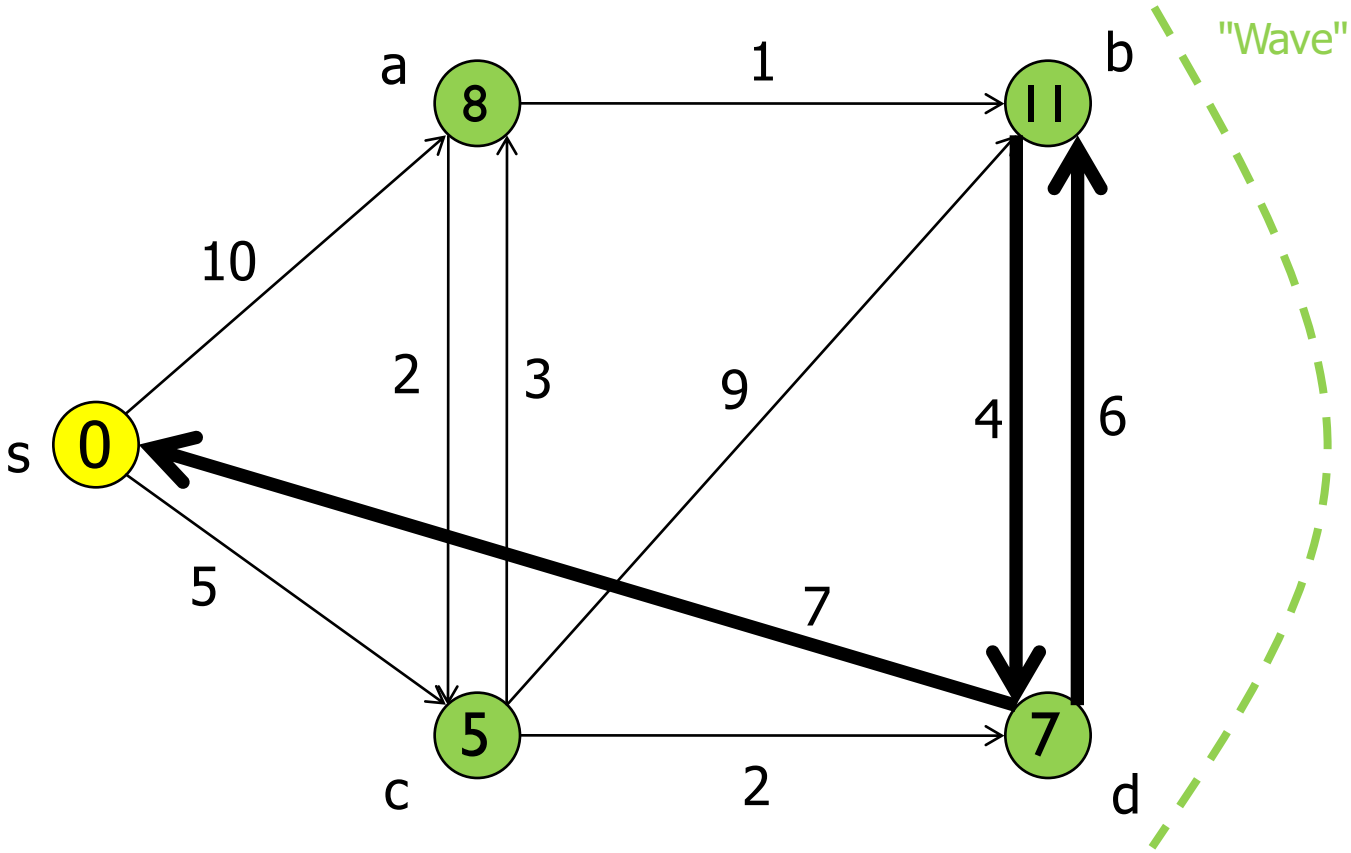
mapper: (a,<s,10>) (c,<s,5>) (a,<c,8>) (c,<a,9>) (b,<a,11>) (b,<c,14>) (d,<c,7>) edges

reducer: (a,<8, ...>) (c,<5, ...>) (b,<11, ...>) (d,<7, ...>)



Iteration 2

mapper:	(a,<s,10>) (c,<s,5>) (a,<c,8>) (c,<a,9>) (b,<a,11>) (b,<c,14>) (d,<c,7>) (b,<d,13>) (d,<b,15>) edges
reducer:	(a,<8>) (c,<5>) (b,<11>) (d,<7>)

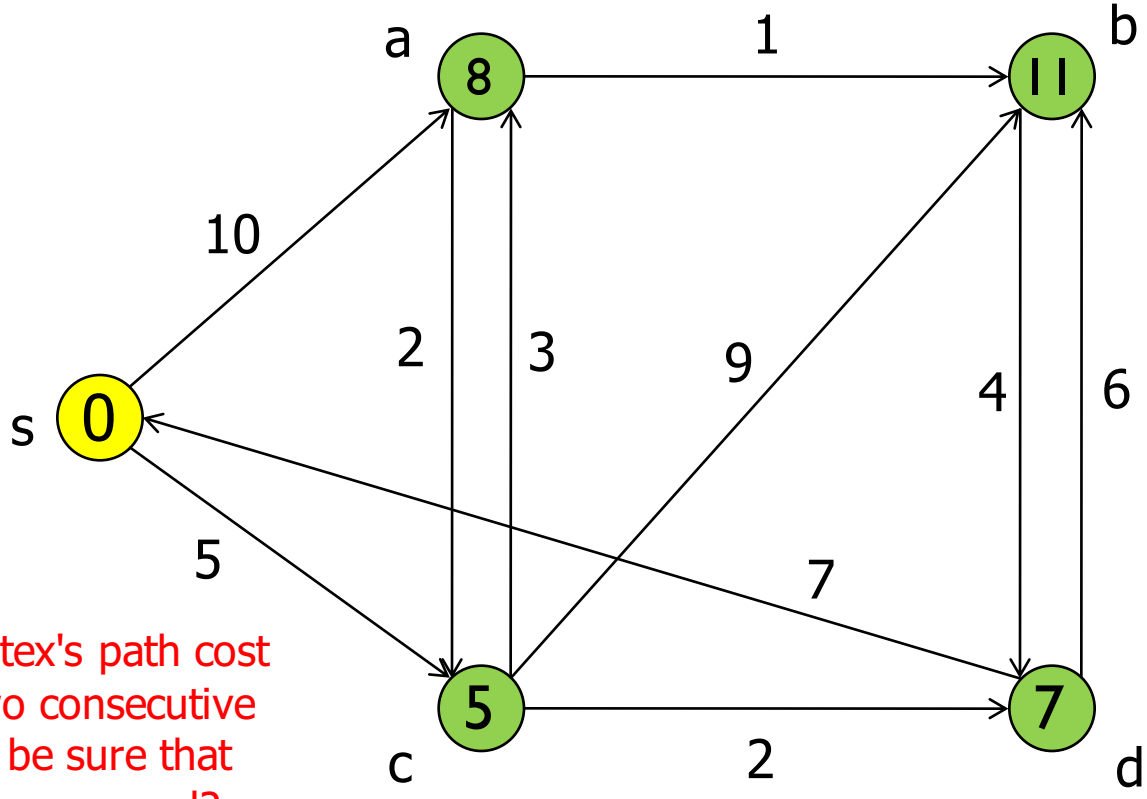


Iteration 3

No change!
Convergence!

mapper: (a,<s,10>) (c,<s,5>) (a,<c,8>) (c,<a,9>) (b,<a,11>) (b,<c,14>) (d,<c,7>) (b,<d,13>) (d,<b,15>) edges

reducer: (a,<8>) (c,<5>) (b,<11>) (d,<7>)



Question: If a vertex's path cost is the same in two consecutive rounds, can we be sure that this vertex has converged?

Summary: SSSP

- Path-based algorithms typically involve iterative map/reduce
- They are typically formulated in a way that traverses in “waves” or “stages”, like breadth-first search
  - This allows for parallelism
  - They need a way to test for convergence
- Example: Single-source shortest path (SSSP)
  - Original Dijkstra formulation is hard to parallelize
  - But we can make it work with the "wave" approach

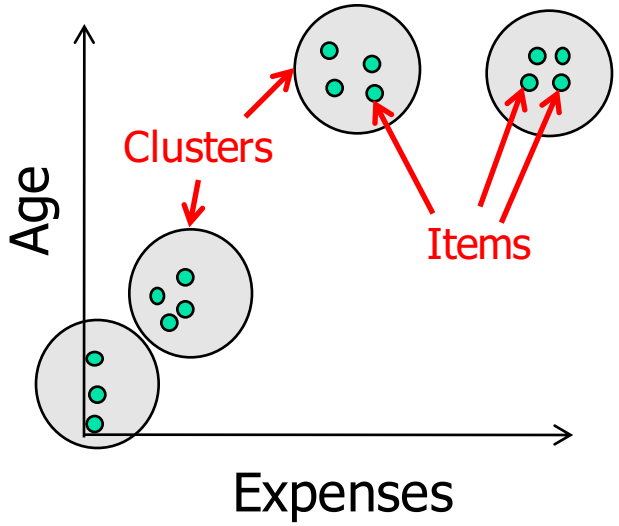
Plan for today

- Representing data in graphs ✓
- Graph algorithms in MapReduce ✓
  - Computation model ✓
  - Iterative MapReduce ✓
- A toolbox of algorithms
  - Single-source shortest path (SSSP) ✓
  - k-means clustering ← NEXT
  - Classification with Naïve Bayes
  - PageRank

Learning (clustering / classification)

- Sometimes our goal is to take a set of entities, possibly related, and group them
  - If the groups are based on similarity, we call this clustering
  - If the groups are based on putting them into a semantically meaningful class, we call this classification
- Both are instances of machine learning

The k-clustering Problem



- Given: A set of items in a n-dimensional feature space
  - Example: data points from survey, people in a social network
- Goal: Group the items into k "clusters"
  - What would be a 'good' set of clusters?

Approach: k-Means

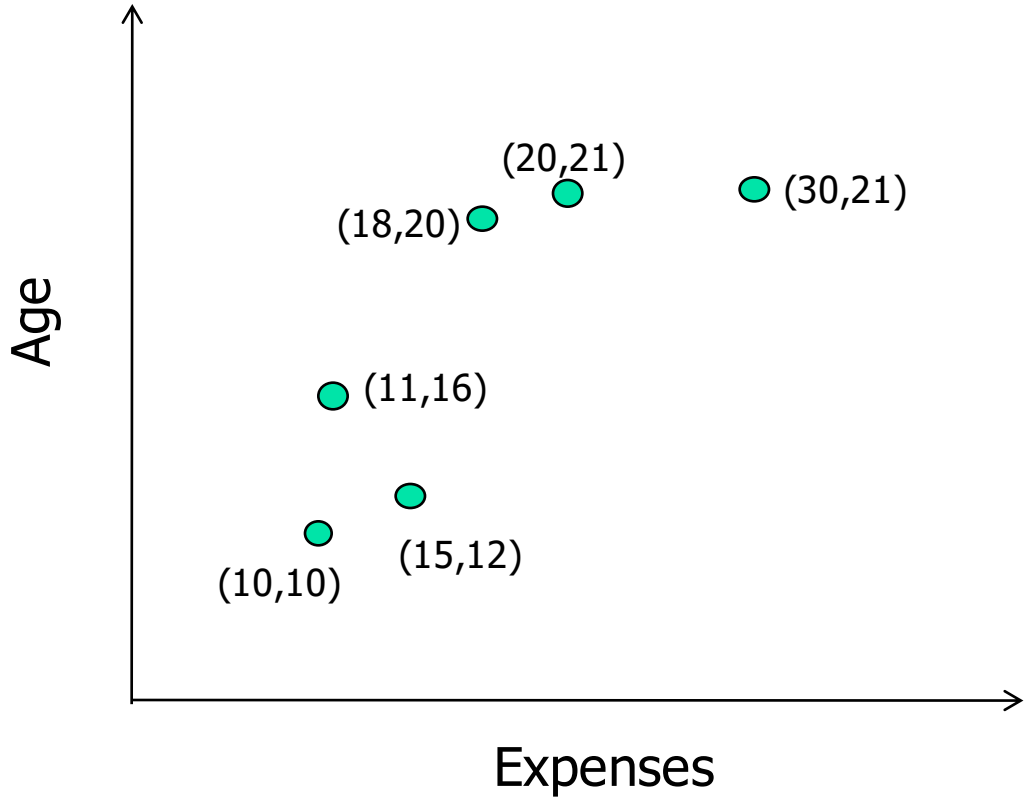
- Let m_1, m_2, …, m_k be representative points for each of our k clusters
  - Specifically: the centroid of the cluster
- Initialize m_1, m_2, …, m_k to random values in the data
- For t = 1, 2, …:
  - Assign each point to the closest centroid

$$S_i^{(t)} = \left\{ x_j : \left\| x_j - m_i^{(t)} \right\| \leq \left\| x_j - m_{i^*}^{(t)} \right\|, i^* = 1, \ldots, k \right\}$$

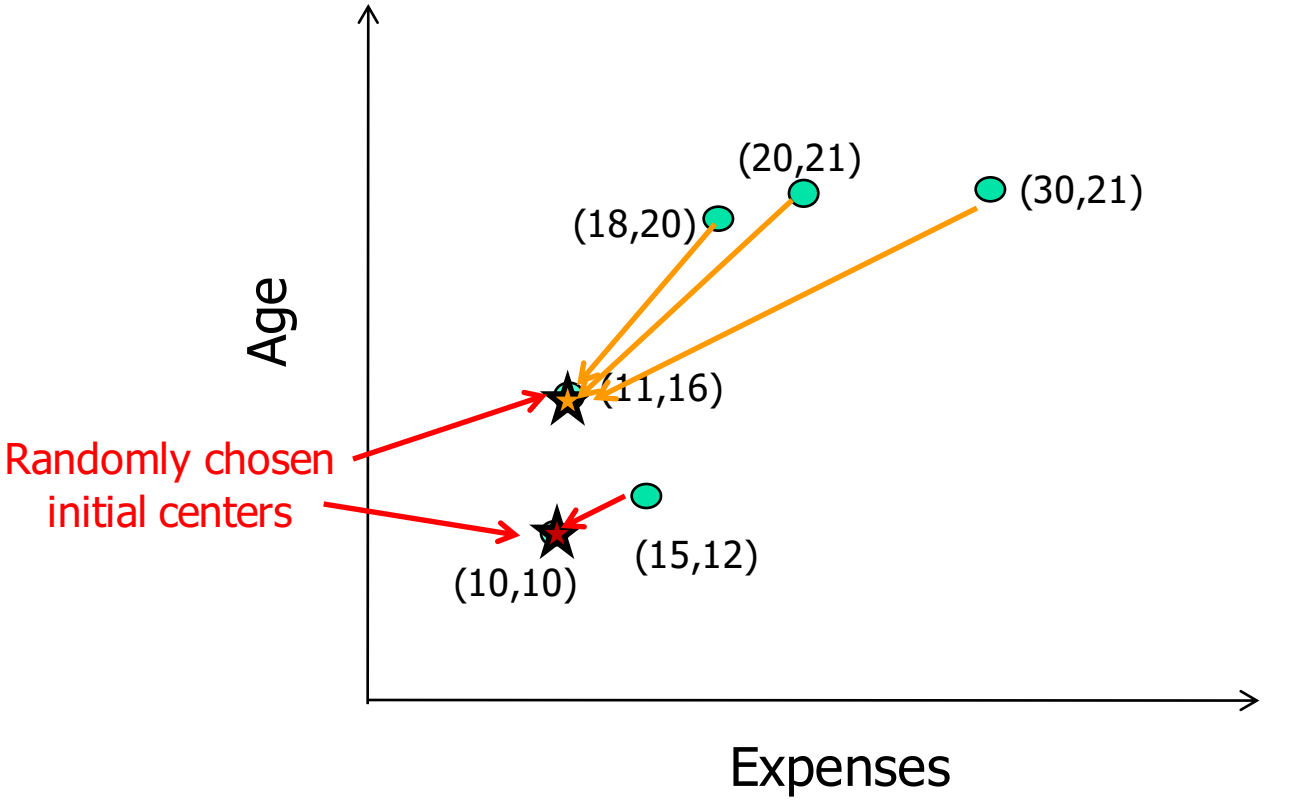
  - m_i becomes new centroid for its points

$$m_i^{(t+1)} = \frac{1}{\left| S_i^{(t)} \right|} \sum_{x_j \in S_i^{(t)}} x_j$$

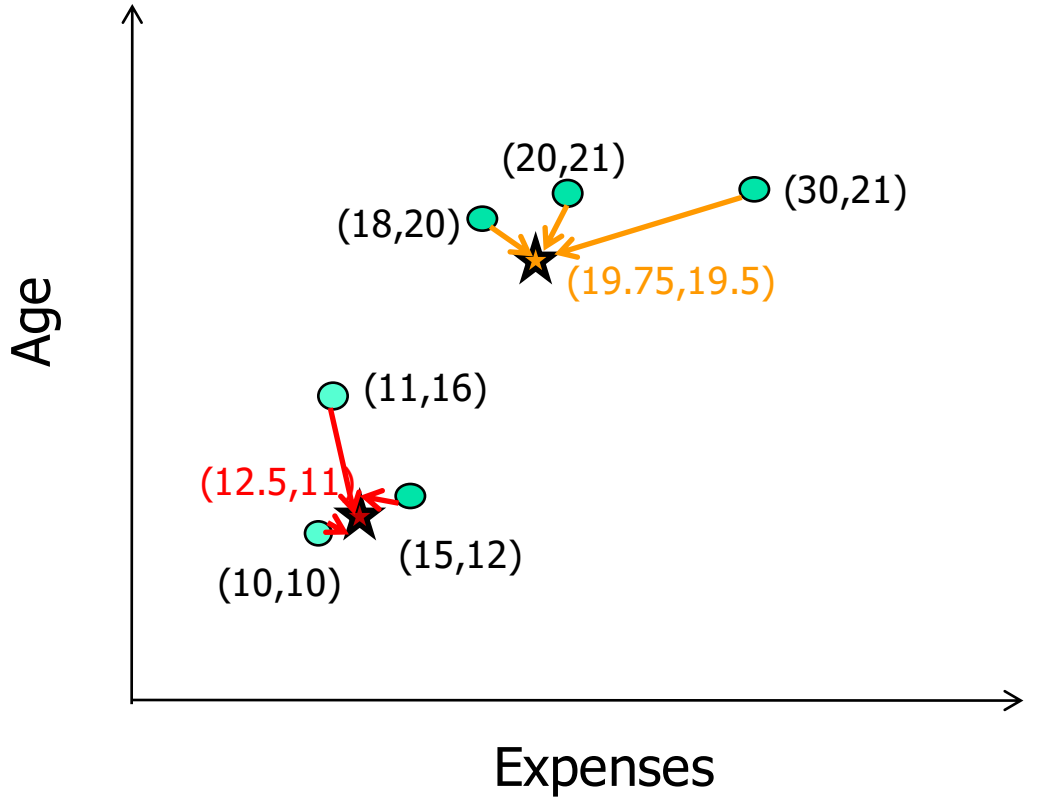
A simple example (1/4)



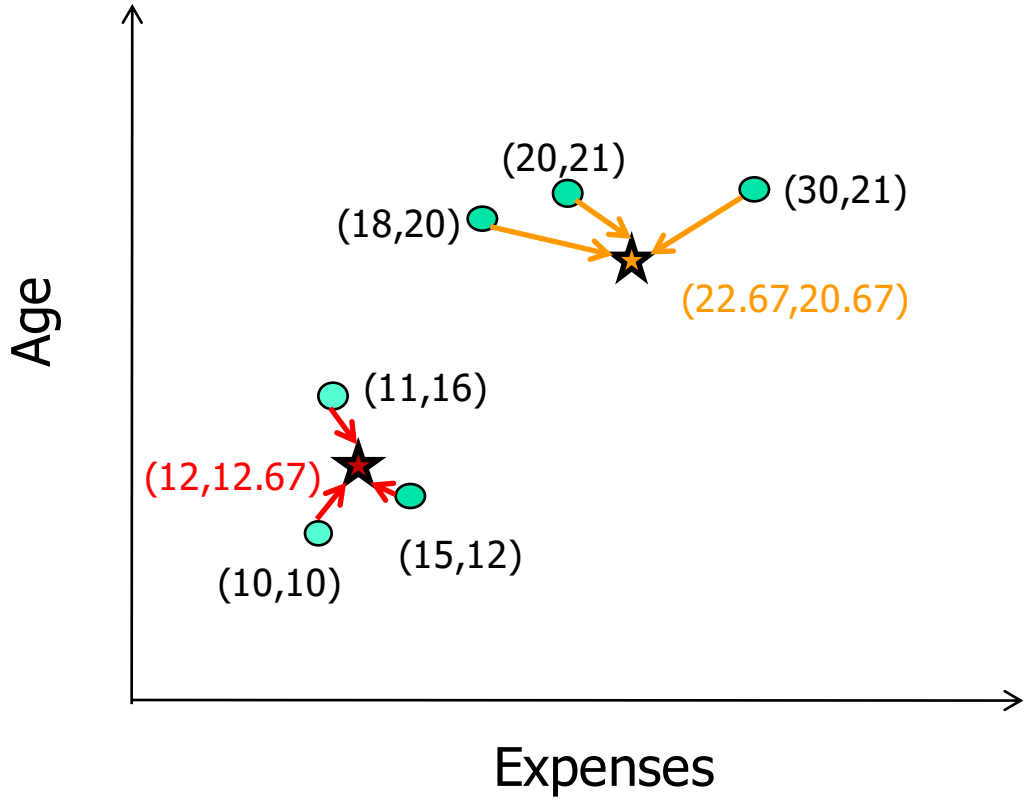
A simple example (2/4)



A simple example (3/4)



A simple example (4/4)



Stable!

k-Means in MapReduce

This is the Map phase

- Classify
  - Assign each point to the closest centroid

$$S_i^{(t)} = \left\{ x_j : \left\| x_j - m_i^{(t)} \right\| \le \left\| x_j - m_{i^*}^{(t)} \right\|, i^* = 1, \ldots, k \right\}$$

This is the Reduce phase

- Recenter
  - m_i becomes new centroid for its points

$$m_i^{(t+1)} = \frac{1}{\left| S_i^{(t)} \right|} \sum_{x_j \in S_i^{(t)}} x_j$$

- Repeat until no change
  - Centroids have converged

Classification Step as Map

- init:
  - Read in global var centroids from file
    - Initially k random points

How do we know the centroids?

- map(centroid, point):
  - Compute nearest centroid based on centroids
  - emit(nearest centroid, point)

Recenter Step as Reduce

- Initialize global var centroids = []
- reduce(centroid, points[])
  - Recompute centroid from points in it
  - Foreach point in points:
  - emit(centroid, point)
  - Add centroid to global centroids
- cleanup (after all calls to reduce are made):
  - Save global centroids to file

Practical Notes

- After reduce phase finishes, must check if any centroid has changed and, in such case, start another MapReduce iteration
- Our solution doesn't fit pure MapReduce model
  - It uses global variable to efficiently know current centroids
  - This state is read and written to a shared file
  - When more than 1 reducer is used, it needs to avoid file conflicts and merge before next iteration of MapReduce
- Can you think of solution without global state?
  - What data needs to be passed around?
  - How many MapReduce jobs?

Plan for today

- Representing data in graphs ✓
- Graph algorithms in MapReduce ✓
  - Computation model ✓
  - Iterative MapReduce ✓
- A toolbox of algorithms
  - Single-source shortest path (SSSP) ✓
  - k-means clustering ✓
  - Classification with Naïve Bayes 

  - PageRank

Classification

- Suppose we want to learn what is spam (or interesting, or …)
  - Predefine a set of classes with semantic meaning
  - Train an algorithm to look at data and assign a class
    - Based on giving it some examples of data in each class
    - … and the sets of features they have
- Many probabilistic techniques exist
  - Each class has probabilistic relationships with others
    - e.g., p (spam | isSentLocally), p (isSentLocally | fromBob), …
  - But we'll focus on a simple, "flat" model: Naïve Bayes

A simple example

- Suppose we just look at the keywords in the email's title:

Message(1, "Won contract")
Message(2, "Won award")
Message(3, "Won the lottery")
Message(4, "Unsubscribe")
Message(5, "Millions of customers")
Message(6, "Millions of dollars")

- What is **probability** message "Won Millions" is 

?

$$p(spam \mid containsWon, containsMillions)$$
$$= \frac{p(spam)\, p(containsWon, containsMillions \mid spam)}{p(containsWon, containsMillions)}$$

Bayes' Theorem

Classification using Naïve Bayes

- Basic assumption: Probabilities of events are independent
  - This is why it is called 'naïve'
- Under this assumption,

$$\frac{p(spam)\, p(containsWon, containsMillions \mid spam)}{p(containsWon, containsMillions)}$$

$$= \frac{p(spam)\, p(containsWon \mid spam)\, p(containsMillions \mid spam)}{p(containsWon)\, p(containsMillions)}$$

$$= 0.5 * 0.67 * 0.33 \,/\, (0.5 * 0.33) = 0.67$$

- So how do we "train" a learner (compute the above probabilities) using MapReduce?

What do we need to train the learner?

- p(spam)
  - Count how many spam emails there are — Easy
  - Count total number of emails — Easy
- p(containsXYZ | spam)
  - Count how many spam emails contain XYZ — 1
  - Count how many emails contain XYZ overall — 2
- p(containsXYZ)
  - Count how many emails contain XYZ overall — 2
  - Count total number of emails — Easy

Training a Naïve Bayes Learner

- map 1:
  - takes messageId → <class, {words}>
  - emits <word, class> → 1
- reduce 1:
  - emits <word, class> → <count>

Count how many emails in the class contain the word (modified WordCount)

- map 2:
  - takes messageId → <class, {words}>
  - emits word → 1
- reduce 2:
  - emits word → <totalCount>

Count how many emails contain the word overall (WordCount)

Summary: Learning and MapReduce

- Clustering algorithms typically have multiple aggregation stages or iterations
  - k-means clustering repeatedly computes centroids, maps items to them
  - Fixpoint computation
- Classification algorithms can be quite complex
  - In general: need to capture conditional probabilities
  - Naïve Bayes assumes everything is independent
  - Training is a matter of computing probability distribution
    - Can be accomplished using two Map/Reduce passes

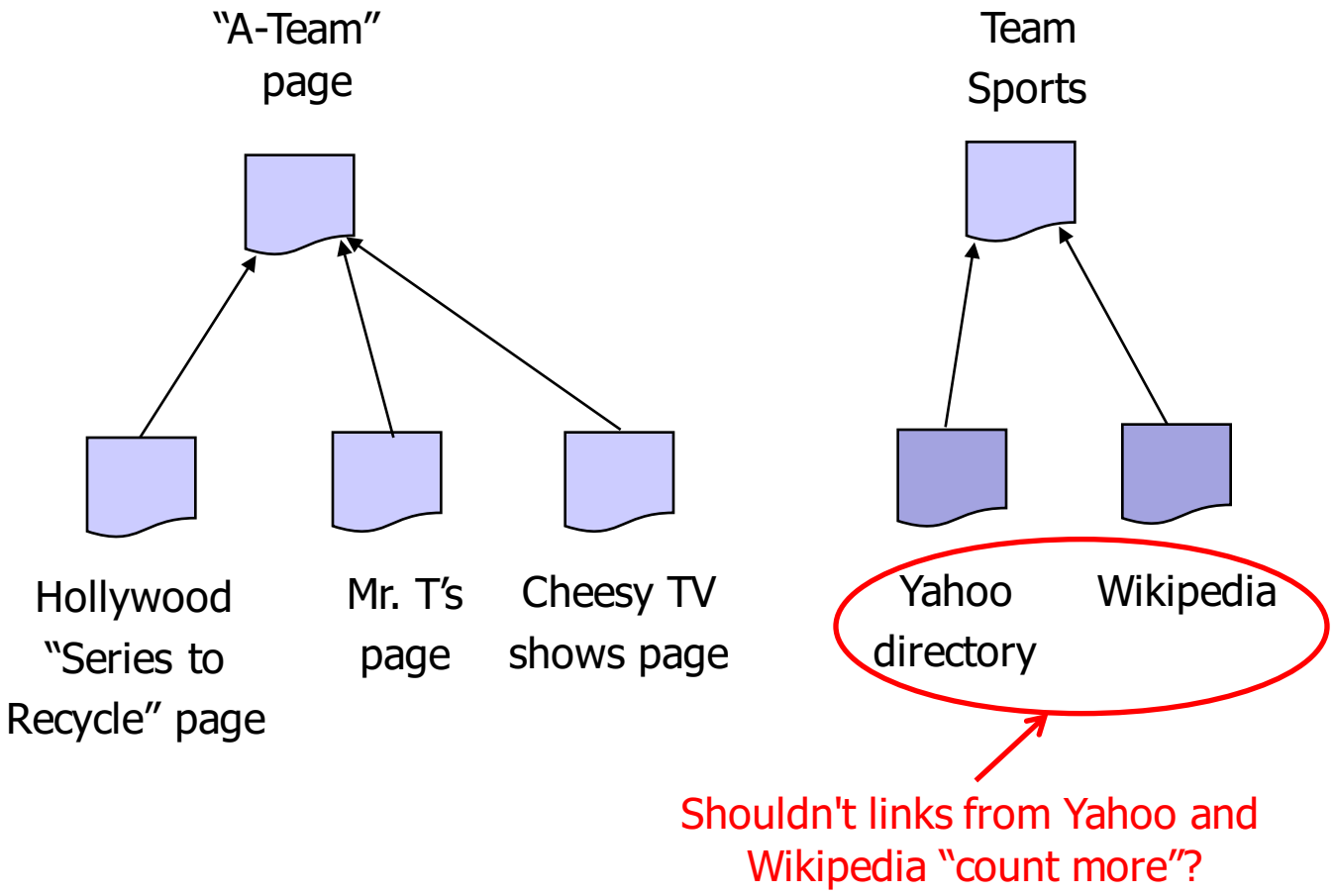
Plan for today

- Representing data in graphs ✓
- Graph algorithms in MapReduce ✓
  - Computation model ✓
  - Iterative MapReduce ✓
- A toolbox of algorithms
  - Single-source shortest path (SSSP) ✓
  - k-means clustering ✓
  - Classification with Naïve Bayes ✓
  - PageRank NEXT

Why link analysis?

- Suppose a search engine processes a query for "team sports"
  - Problem: Millions of pages contain these words!
  - Which ones should we return first?
- Idea: Hyperlinks encode a considerable amount of human judgment
  - What does it mean when a web page links another page?
  - Intra-domain links: Often created primarily for navigation
  - Inter-domain links: Confer some measure of authority
- So, can we simply boost the rank of pages with lots of inbound links?

Problem: Popularity ≠ relevance!



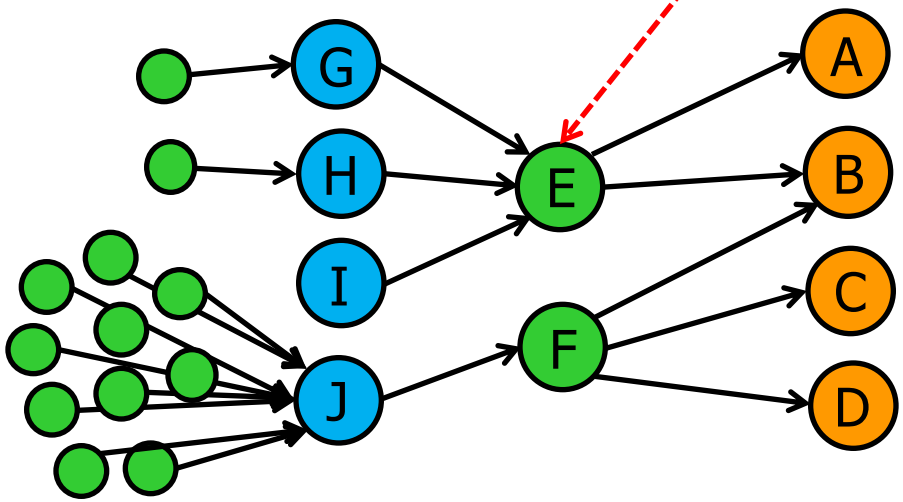
Other applications

- This question occurs in several other areas:
  - Who are the most "influential" individuals in a social network? (#friends?)
  - How do we measure the "impact" of a researcher? (#papers? #citations?)
  - Which programmers are writing the "best" code? (#uses?)
  - …

PageRank: Intuition

Shouldn't E's vote be worth more than F's?

How many levels should we consider?



- Imagine a contest for The Web's Best Page
  - Initially, each page has one vote
  - Each page votes for all the pages it has a link to
  - To ensure fairness, pages voting for more than one page must split their vote equally between them
  - Voting proceeds in rounds; in each round, each page has the number of votes it received in the previous round
  - In practice, it's a little more complicated - but not much!

PageRank

- Each page i is given a rank x_i
- Goal: Assign the x_i such that the rank of each page is governed by the ranks of the pages linking to it:

$$x_i = \sum_{j \in B_i} \frac{1}{N_j} x_j$$

Rank of page i

Rank of page j

Number of links out from page j

Every page j that links to i

How do we compute the rank values?

Random Surfer Model

- PageRank has an intuitive basis in random walks on graphs
- Imagine a random surfer, who starts on a random page and, in each step,
  - with probability d, clicks on a random link on the page
  - with probability 1-d, jumps to a random page (bored?)
- The PageRank of a page can be interpreted as the fraction of steps the surfer spends on the corresponding page
  - Transition matrix can be interpreted as a Markov Chain

Iterative PageRank (simplified)

Initialize all ranks to be equal, e.g.:

$$x_i^{(0)} = \frac{1}{n}$$

Iterate until convergence

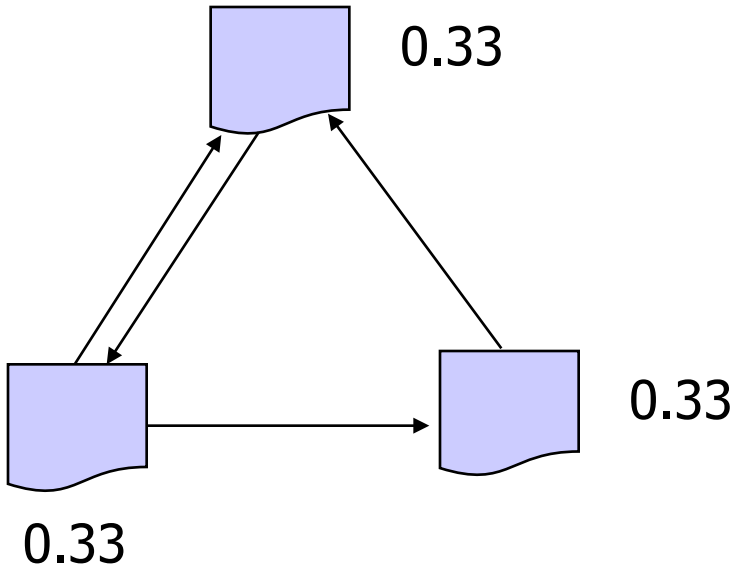
$$x_i^{(k+1)} = \sum_{j \in B_i} \frac{1}{N_j} x_j^{(k)}$$

No need to decide how many levels to consider!

Example: Step 0

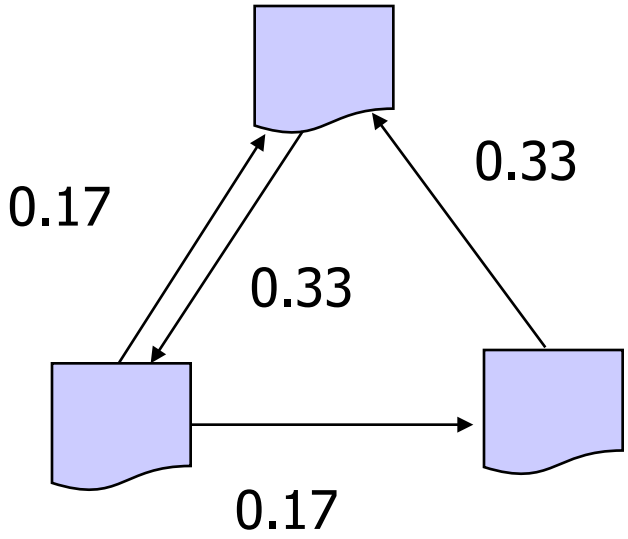
Initialize all ranks to be equal

$$x_i^{(0)} = \frac{1}{n}$$



Example: Step 1

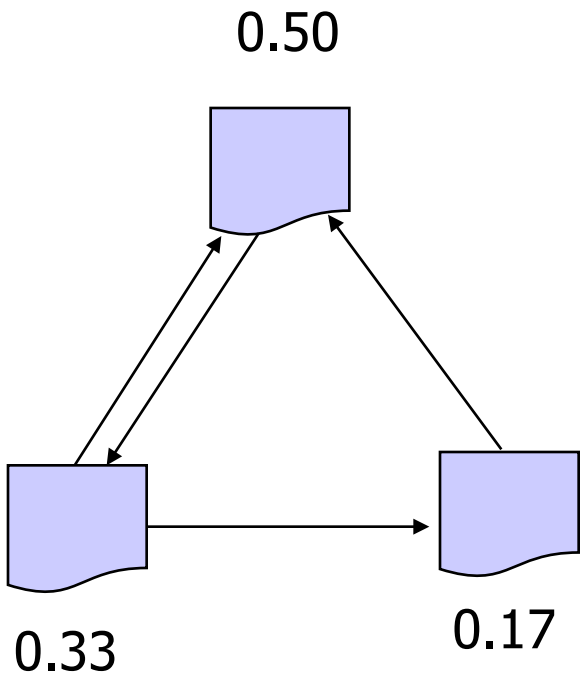
Propagate weights across out-edges



Example: Step 2

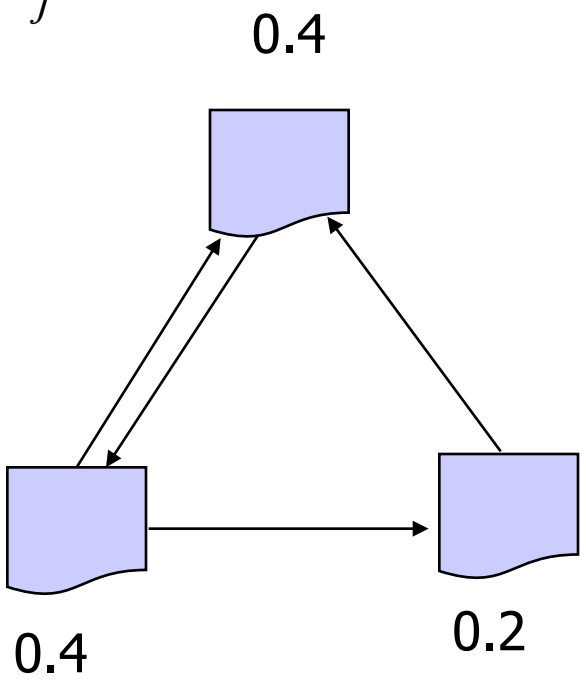
Compute weights based on in-edges

$$x_i^{(1)} = \sum_{j \in B_i} \frac{1}{N_j} x_j^{(0)}$$



Example: Convergence

$$x_i^{(k+1)} = \sum_{j \in B_i} \frac{1}{N_j} x_j^{(k)}$$



Naïve PageRank Algorithm Restated

- Let
  - N(p) = number outgoing links from page p
  - B(p) = number of back-links to page p

$$PageRank(p) = \sum_{b \in B(p)} \frac{1}{N(b)} PageRank(b)$$

  - Each page b distributes its importance to all of the pages it points to (so we scale by 1/N(b))
  - Page p's importance is increased by the importance of its back set

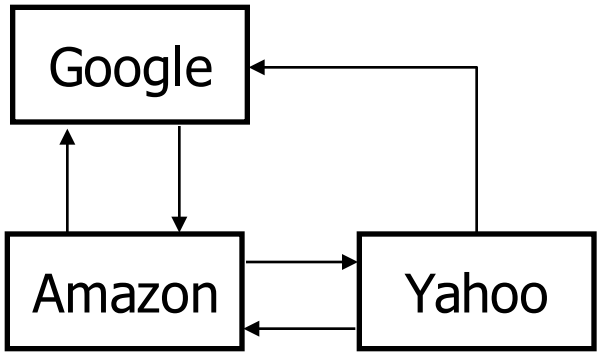
In Linear Algebra formulation

- Create an m x m matrix M to capture links:
  - M(i, j) = $1 / n_j$ if page i is pointed to by page j and page j has n_j outgoing links
    = 0 otherwise
  - Initialize all PageRanks to 1, multiply by M repeatedly until all values converge:

$$\begin{bmatrix} PageRank(p_1') \\ PageRank(p_2') \\ \dots \\ PageRank(p_m') \end{bmatrix} = M \begin{bmatrix} PageRank(p_1) \\ PageRank(p_2) \\ \dots \\ PageRank(p_m) \end{bmatrix}$$

  - Computes principal eigenvector via power iteration

A brief example



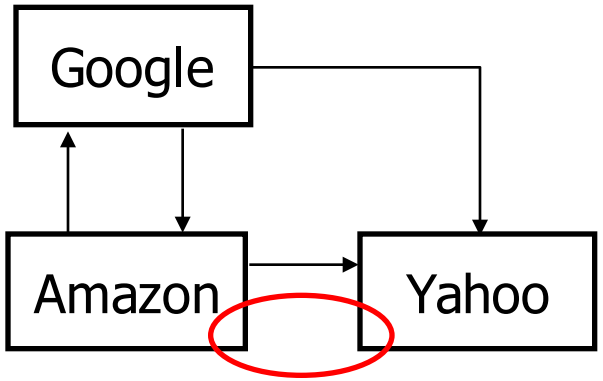
$$\begin{vmatrix} g' \\ y' \\ a' \end{vmatrix} = \begin{vmatrix} 0 & 0.5 & 0.5 \\ 0 & 0 & 0.5 \\ 1 & 0.5 & 0 \end{vmatrix} * \begin{vmatrix} g \\ y \\ a \end{vmatrix}$$

Running for multiple iterations:

$$\begin{vmatrix} g \\ y \\ a \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}, \begin{vmatrix} 1 \\ 0.5 \\ 1.5 \end{vmatrix}, \begin{vmatrix} 1 \\ 0.75 \\ 1.25 \end{vmatrix}, \ldots \begin{vmatrix} 1 \\ 0.67 \\ 1.33 \end{vmatrix}$$

Total rank sums to number of pages

Oops #1 – PageRank sinks



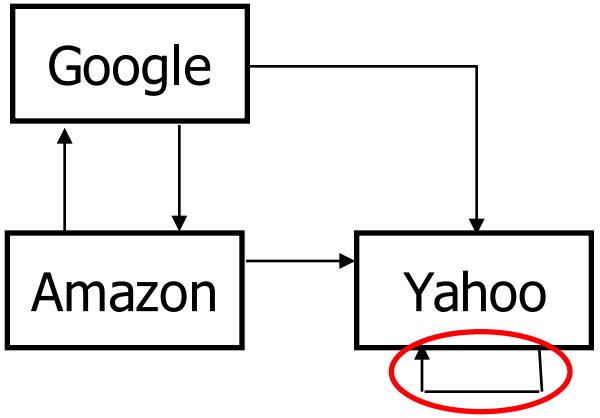
$$\begin{vmatrix} g' \\ y' \\ a' \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0 \end{vmatrix} * \begin{vmatrix} g \\ y \\ a \end{vmatrix}$$

'dead end' - PageRank is lost after each round

Running for multiple iterations:

$$\begin{vmatrix} g \\ y \\ a \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}, \begin{vmatrix} 0.5 \\ 1 \\ 0.5 \end{vmatrix}, \begin{vmatrix} 0.25 \\ 0.5 \\ 0.25 \end{vmatrix}, \ldots, \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

Oops #2 – PageRank hogs



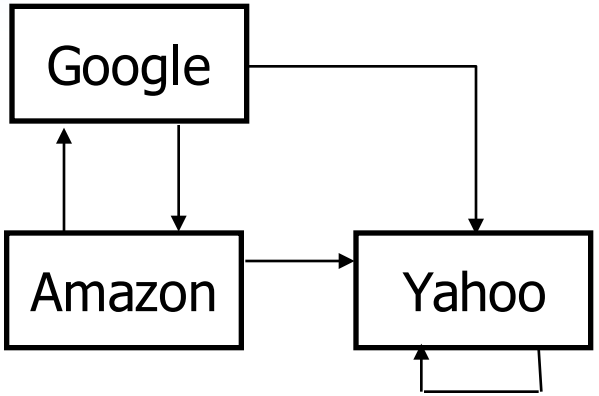
$$\begin{vmatrix} g' \\ y' \\ a' \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0.5 \\ 0.5 & 1 & 0.5 \\ 0.5 & 0 & 0 \end{vmatrix} * \begin{vmatrix} g \\ y \\ a \end{vmatrix}$$

PageRank cannot flow out and accumulates

Running for multiple iterations:

$$\begin{vmatrix} g \\ y \\ a \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}, \begin{vmatrix} 0.5 \\ 2 \\ 0.5 \end{vmatrix}, \begin{vmatrix} 0.25 \\ 2.5 \\ 0.25 \end{vmatrix}, \dots, \begin{vmatrix} 0 \\ 3 \\ 0 \end{vmatrix}$$

Stopping the Hog



$$\begin{vmatrix} g' \\ y' \\ a' \end{vmatrix} = 0.85 \begin{vmatrix} 0 & 0 & 0.5 \\ 0.5 & 1 & 0.5 \\ 0.5 & 0 & 0 \end{vmatrix} * \begin{vmatrix} g \\ y \\ a \end{vmatrix} + \begin{vmatrix} 0.15 \\ 0.15 \\ 0.15 \end{vmatrix}$$

Running for multiple iterations:

$$\begin{vmatrix} g \\ y \\ a \end{vmatrix} = \begin{vmatrix} 0.57 \\ 1.85 \\ 0.57 \end{vmatrix}, \begin{vmatrix} 0.39 \\ 2.21 \\ 0.39 \end{vmatrix}, \begin{vmatrix} 0.32 \\ 2.36 \\ 0.32 \end{vmatrix}, \cdots, \begin{vmatrix} 0.26 \\ 2.48 \\ 0.26 \end{vmatrix}$$

Improved PageRank

- Remove out-degree 0 nodes (or consider them to refer back to referrer)
- Add decay factor d to deal with sinks

$$PageRank(p) = (1-d) + d \sum_{b \in B_p} \frac{1}{N(b)} PageRank(b)$$

- Typical value: *d*=0.85

- Intuition in the idea of the "random surfer":
  - Surfer occasionally stops following link sequence and jumps to new random page, with probability 1 - *d*

PageRank on MapReduce

- Inputs
  - Of the form: page → (currentWeightOfPage, {adjacency list})
- Map
  - Page p "propagates" $1/N_p$ of its d * weight(p) to the destinations of its out-edges (think like a vertex!)
- Reduce
  - p-th page sums the incoming weights and adds (1-d), to get its weight'(p)
- Iterate until convergence
  - Common practice: run some fixed number of times, e.g., 25x
  - Alternatively: Test after each iteration with a second MapReduce job, to determine the maximum change between old and new weights

Recap and Take-aways

- We've had a whirlwind tour of common kinds of algorithms used on the Web
  - Path analysis: route planning, games, keyword search, etc.
  - Clustering and classification: mining, recommendations, spam filtering, context-sensitive search, ad placement, etc.
  - Link analysis: ranking
- Many such algorithms (though not all) have a reasonably straightforward, often iterative, MapReduce formulation

Additional references

Data-Intensive Text Processing with MapReduce

Jimmy Lin and Chris Dyer

Morgan & Claypool Publishers, 2010

http://lintool.github.io/MapReduceAlgorithms/

Stay tuned

Next time you will learn about:

Beyond MapReduce - Higher-level languages, Graphs