# Multi-Source Domain Adaptation through Dataset Dictionary Learning in Wasserstein Space

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## Abstract

We seek to solve **Multi-Source Domain Adaptation (MSDA)**, which aims to mitigate data distribution shifts when transferring knowledge from multiple labeled source domains to an unlabeled target domain. We propose a novel MSDA framework based on dictionary learning and optimal transport. We interpret each domain in MSDA as an empirical distribution. As such, we express each domain as a Wasserstein barycenter of dictionary atoms, which are empirical distributions. We propose a novel algorithm, **Dataset Dictionary Learning (DaDiL)**, for learning via mini-batches: (i) atom distributions; (ii) a matrix of barycentric coordinates. Based on our dictionary, we propose two novel methods for MSDA: **DaDil-R**, based on the reconstruction of labeled samples in the target domain, and **DaDiL-E**, based on the ensembling of classifiers learned on atom distributions. We evaluate our methods in 3 benchmarks: **Caltech-Office**, **Refurbished-Office 31**, and **CRWU**, where we improved previous state-of-the-art by **3.15%**, **2.29%**, and **7.71%** in classification performance. Finally, we show that interpolations in the Wasserstein hull of learned atoms provide data that can generalize to the target domain.

## Methodology

### Wasserstein Barycenters of Labeled Distributions

When calculating Optimal Transport between labeled distributions, one needs to integrate labels in the ground-cost. Let $\mathbf{y}_i^{(P)} \in \Delta_{n_c}$ denote the soft-labels of sample $\mathbf{x}_i$. We use,

$$C_{i,j} = \|\mathbf{x}_i^{(P)} - \mathbf{x}_j^{(Q)}\|_2^2 + \beta\|\mathbf{y}_i^{(P)} - \mathbf{y}_j^{(Q)}\|_2^2, \quad (1)$$

where $\beta > 0$ controls the importance of label discrepancy. While simple, this choice allows us to motivate the barycentric projection of [1], and the label propagation of [2] as first-order optimality conditions of $W_c(\hat{P}, \hat{Q})$,

$$\begin{cases} \hat{\mathbf{x}}_i^{(P)} = T_\pi(\mathbf{x}_i^{(P)}) = n_P \sum_{j=1}^{n_Q} \pi_{i,j}\mathbf{x}_j^{(Q)}, \\ \hat{\mathbf{y}}_i^{(P)} = T_\pi(\mathbf{y}_i^{(P)}) = n_P \sum_{j=1}^{n_Q} \pi_{i,j}\mathbf{y}_j^{(Q)}. \end{cases} \quad (2)$$

As a consequence, we can interpolate between two point clouds, since $\hat{\mathbf{y}}_i^{(P)}$ corresponds to a soft-label (i.e., probabilities). We use equations 1 and 2 for proposing a new barycenter strategy between labeled point clouds, shown in algorithm 1.

**Algorithm 1** Labeled Wasserstein Barycenter

**Input:** $\{\mathbf{X}^{(P_k)}, \mathbf{Y}^{(P_k)}\}_{k=1}^K$, $\alpha \in \Delta_K$, $\tau > 0$, $N_{itb}$.

```
for i = 1, ..., n_B do
    x_i^(B) ~ N(0, I_d), y_i^(B) = randint(n_c)
end for
while |J_it - J_it-1| >= tau and it <= N_itb do
    for k = 1, ... K do
        pi^(k,it) = OT((X^(P_k), Y^(P_k)); (X_it^(B), Y_it^(B)))
    end for
    J_it = sum_{k=1}^K alpha_k <pi^(k,it), C^(k)>_F
    X_{it+1}^(B) = sum_{k=1}^K alpha_k T_{pi^(k,it)}(X_it^(B))
    Y_{it+1}^(B) = sum_{k=1}^K alpha_k T_{pi^(k,it)}(Y_it^(B))
end while
```

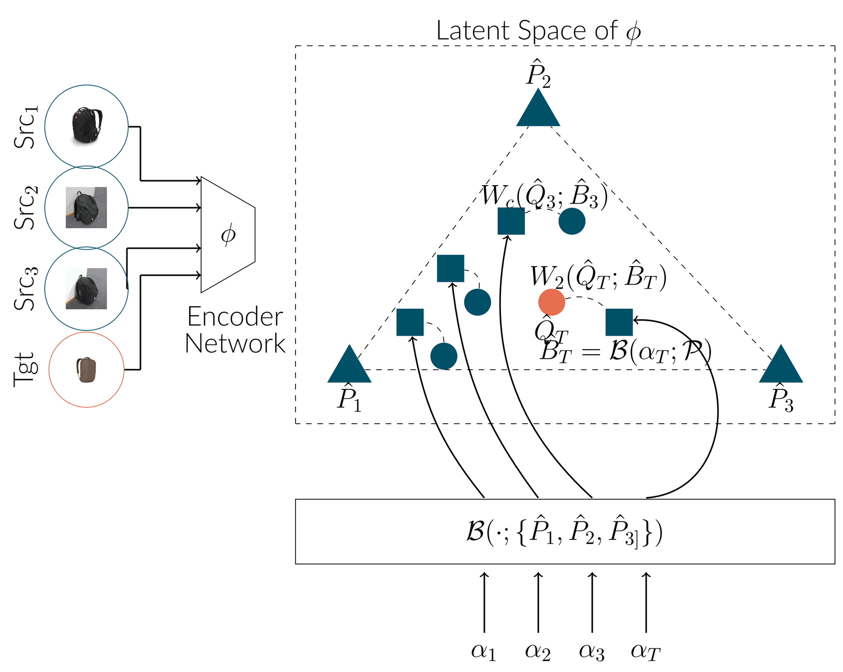
**Output:** Labeled barycenter support $(\mathbf{X}^{(B)}, \mathbf{Y}^{(B)})$.

### Dataset Dictionary Learning (DaDiL)

Let $\mathcal{Q} = \{\hat{Q}_{S_\ell}\}_{\ell=1}^{N_S} \cup \{\hat{Q}_T\}$ correspond to $N_S$ labeled sources and an unlabeled target. Let $\mathcal{A} = [\alpha_1, \cdots, \alpha_{N_S}, \alpha_{N_S+1}]$, and $\mathcal{P} = \{\hat{P}_k\}_{k=1}^K$. The $\hat{P}_k$'s are an empirical approximation of the point clouds that interpolate distributional shift and $\alpha_T := \alpha_{N_S+1}$. For $N = N_S + 1$, DaDiL consists on minimizing,

$$(\mathcal{P}^\star, \mathcal{A}^\star) = \underset{\mathcal{P}, \mathcal{A} \in (\Delta_K)^N}{\text{argmin}} \frac{1}{N}\sum_{\ell=1}^N \mathcal{L}(\hat{Q}_\ell, \mathcal{B}(\alpha_\ell; \mathcal{P})),$$

where, $\mathcal{L}(\hat{Q}_\ell, \hat{B}_\ell) = W_c(\hat{Q}_\ell, \hat{B}_\ell)$ for the sources, and $\mathcal{L}(\hat{Q}_T, \hat{B}_T) = W_2(\hat{Q}_T, \hat{B}_T)$, for the target.



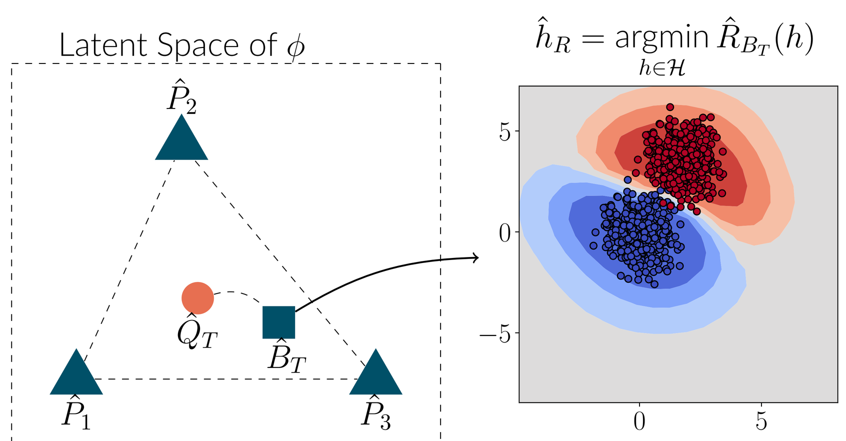

## Multi-Source Domain Adaptation Strategies

**DaDiL-Reconstruction.** Relies on the reconstruction of distributions through Wasserstein barycenters.

$$\mathbf{X}^{(B_T)} = \sum_{k=1}^K \alpha_{T,k}\pi^{(k)}\mathbf{X}^{(P_k)}$$

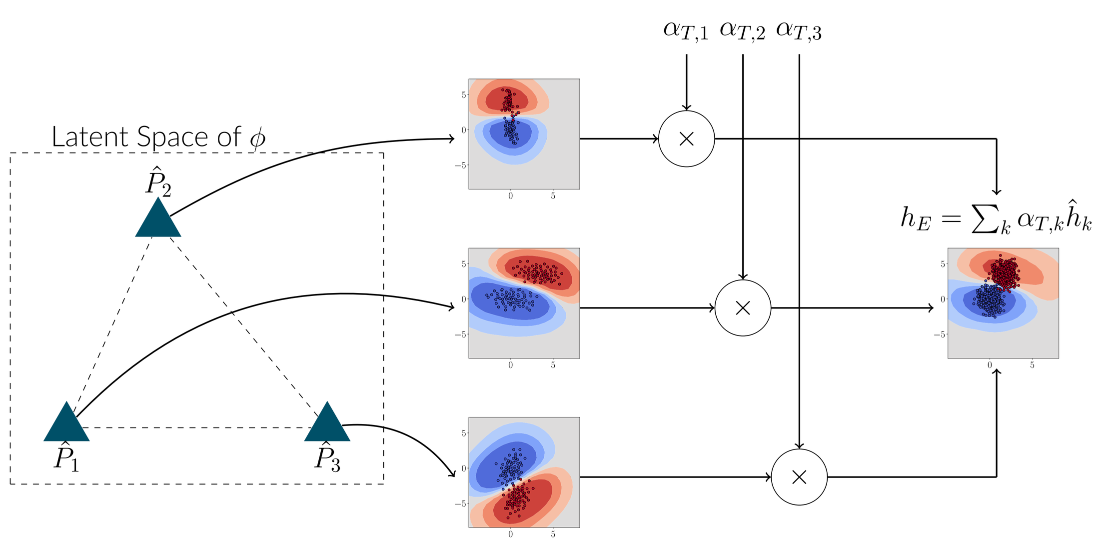
$$\mathbf{Y}^{(B_T)} = \sum_{k=1}^K \alpha_{T,k}\pi^{(k)}\mathbf{Y}^{(P_k)}$$

$$\hat{h}_R = \underset{h \in \mathcal{H}}{\text{argmin}} \frac{1}{n}\sum_{i=1}^n \mathcal{L}(h(\mathbf{x}_i^{(B_T)}), y_i^{(B_T)})$$




$$\mathcal{R}_{Q_T}(h) \leq \mathcal{R}_{B_T}(h) + \underbrace{W_2(\hat{Q}_T, \hat{B}_T)}_{\text{Reconstruction Error}} + \underbrace{\sqrt{2(\log 1/\delta)/\xi'}\left(\sqrt{1/n_P} + \sqrt{1/n_Q}\right)}_{\text{Sample Complexity } \mathcal{O}(n^{-1/2})} + \underbrace{\min_{h \in \mathcal{H}} \mathcal{R}_{Q_T}(h) + \mathcal{R}_{B_T}(h)}_{\text{Adaptation Complexity}},$$

**DaDiL-Ensembling:** relies on the ensembling of classifiers fit on atom data.

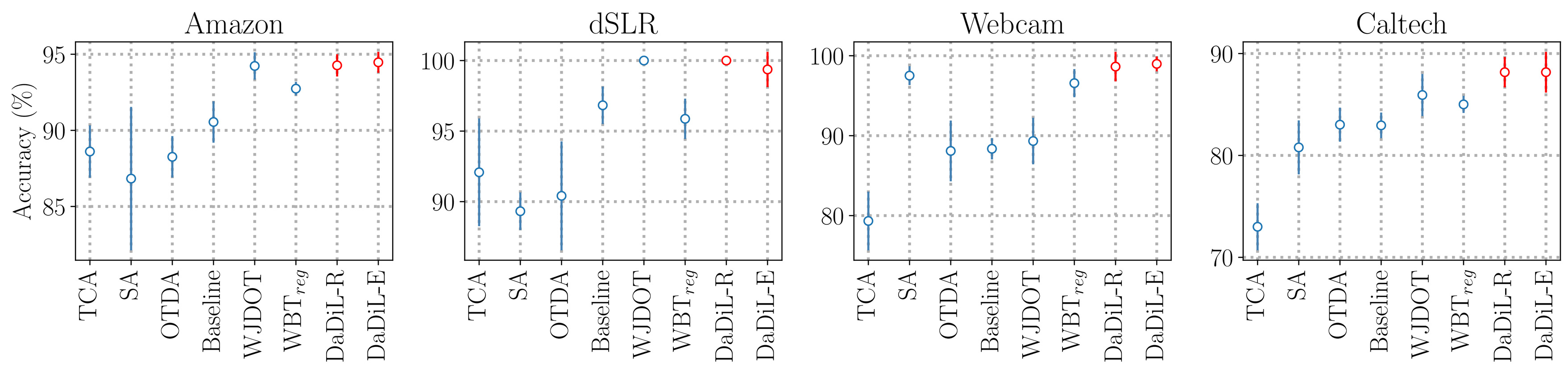



$$\mathcal{R}_{Q_T}(\hat{h}_\alpha) \leq \mathcal{R}_\alpha(\hat{h}_\alpha) + \underbrace{W_2(\mathcal{B}(\alpha; \mathcal{P}), \hat{Q}_T)}_{\text{Reconstruction Error}} + \underbrace{\sum_{k=1}^K \alpha_k W_2(\hat{P}_k, \mathcal{B}(\alpha; \mathcal{P}))}_{\text{Dictionary Geometry}}$$

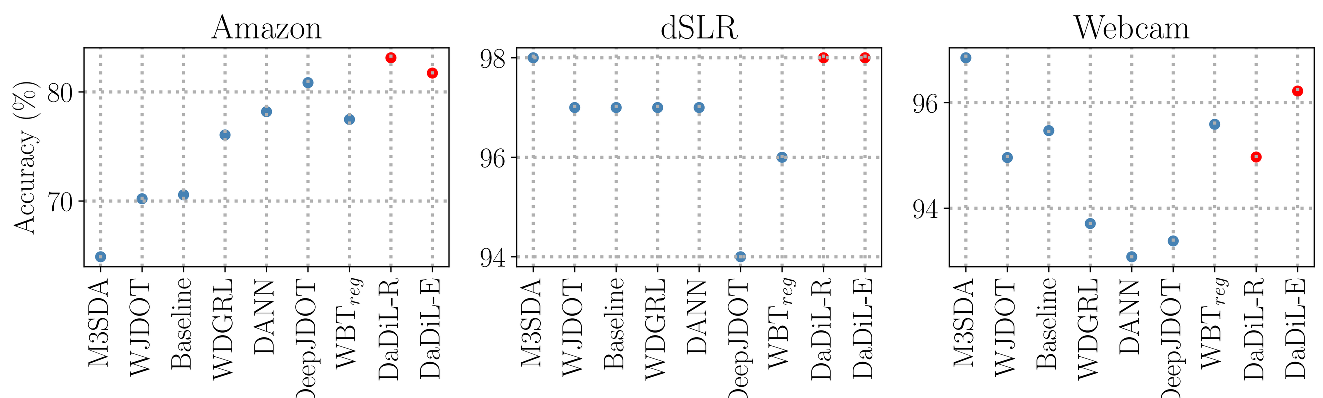
$$+ \underbrace{\sum_{k=1}^K \alpha_k \sqrt{2\log 1/\delta/\xi'}\left(\sqrt{1/n_k} + \sqrt{1/n_T}\right)}_{\text{Sample Complexity}} + \underbrace{\sum_{k=1}^K \alpha_k\left(\min_{h \in \mathcal{H}} \mathcal{R}_{P_k}(h) + \mathcal{R}_{Q_T}(h)\right)}_{\text{Adaptation Complexity}},$$

## Empirical Results

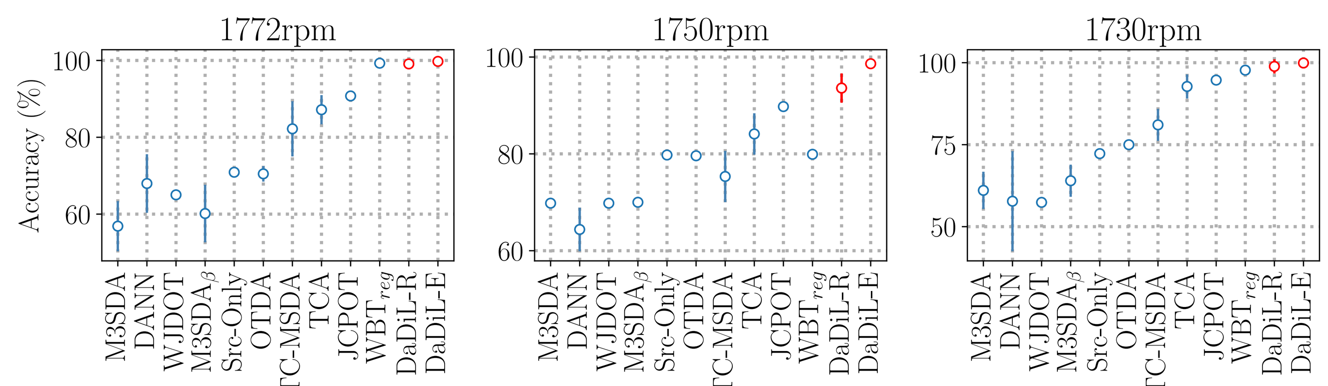
### Caltech-Office 10



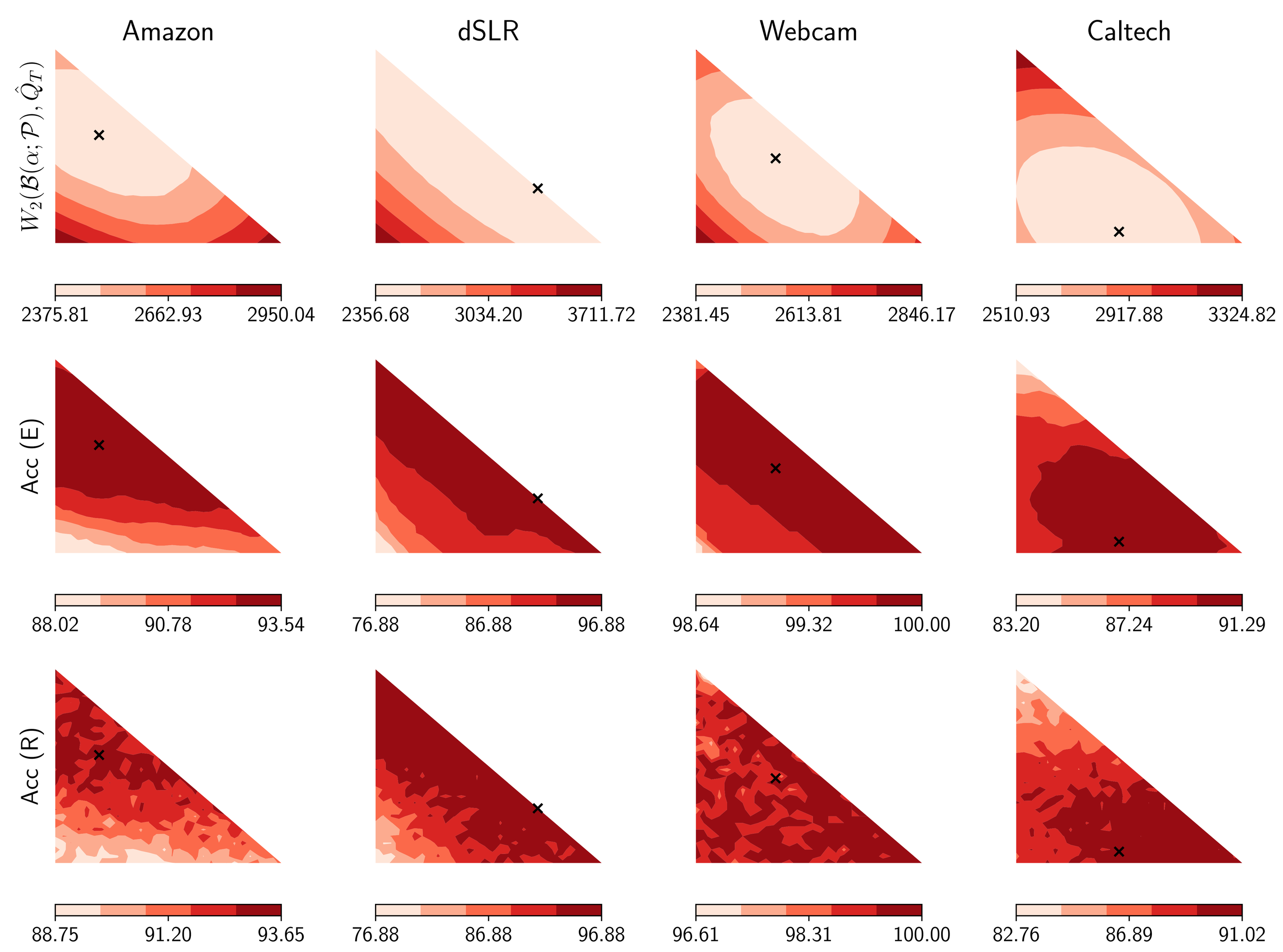

### Refurbished Office 31




### CWRU




### Atom Interpolations




## Conclusions

- We propose a **novel dictionary learning** method, called **DaDiL**
- **DaDiL learns to model distributional shift** between distributions
- DaDiL has **state-of-the-art performance** on various **domain adaptation** benchmarks.
- **DaDiL** defines a rich **interpolation space** between atoms.

## Future Works

Federated Learning [4]   Dataset Distillation [5]   Cross-Domain Fault Diagnosis [6].

## References

[1] Nicolas Courty, Rémi Flamary, Devis Tuia, and Alain Rakotomamonjy. Optimal transport for domain adaptation. IEEE transactions on pattern analysis and machine intelligence, 39(9):1853–1865, 2016.

[2] Ievgen Redko, Emilie Morvant, Amaury Habrard, Marc Sebban, and Younes Bennani. Advances in domain adaptation theory. Elsevier, 2019.

[3] Eduardo Fernandes Montesuma and Fred Ngolè Mboula. Wasserstein barycenter for multi-source domain adaptation. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR) June 2021.

[4] Eduardo Fernandes Montesuma, Michela Mulas, Fred Ngolè Mboula, Francesco Corona, and Antoine Souloumiac. Multi-Source Domain Adaptation for Cross-Domain Fault Diagnosis of Chemical Processes. arXiv:2308.11247. August 2023

[5] Fabiola Espinoza Castellon, Eduardo Fernandes Montesuma, Fred Ngolè Mboula, Aurélien Mayoue, Antoine Souloumiac, Cdric Gouy-Pailler. Federated Dataset Dictionary Learning for Multi-Source Domain Adaptation. arXiv:2309.07670. September 2023

[6] Eduardo Fernandes Montesuma, Fred Ngolè Mboula, Antoine Souloumiac. Multi-Source Domain Adaptation meets Dataset Distillation through Dataset Dictionary Learning. arXiv:2309.07666. September 2023

## Learn more about DaDiL!

DaDiL Paper   DaDiL Demo   Portfolio

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