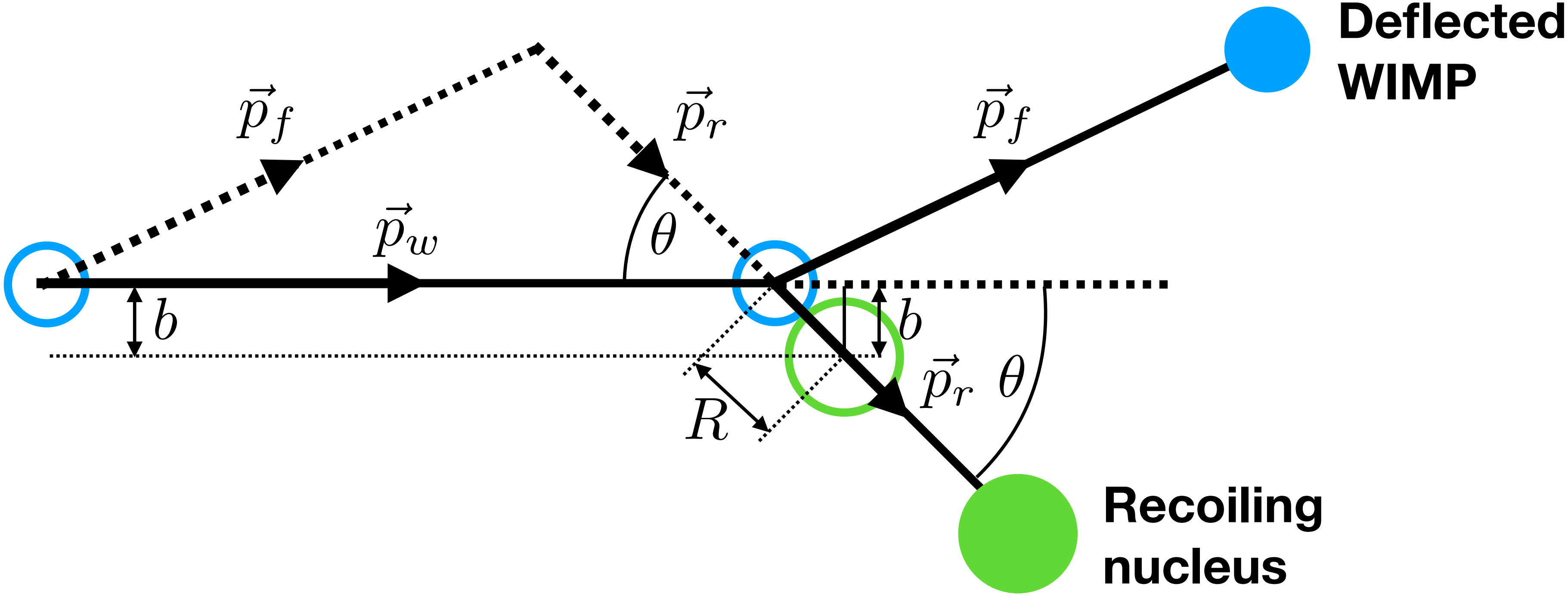
Dark Matter – Illustrations for Lecture 3.

Ed Daw

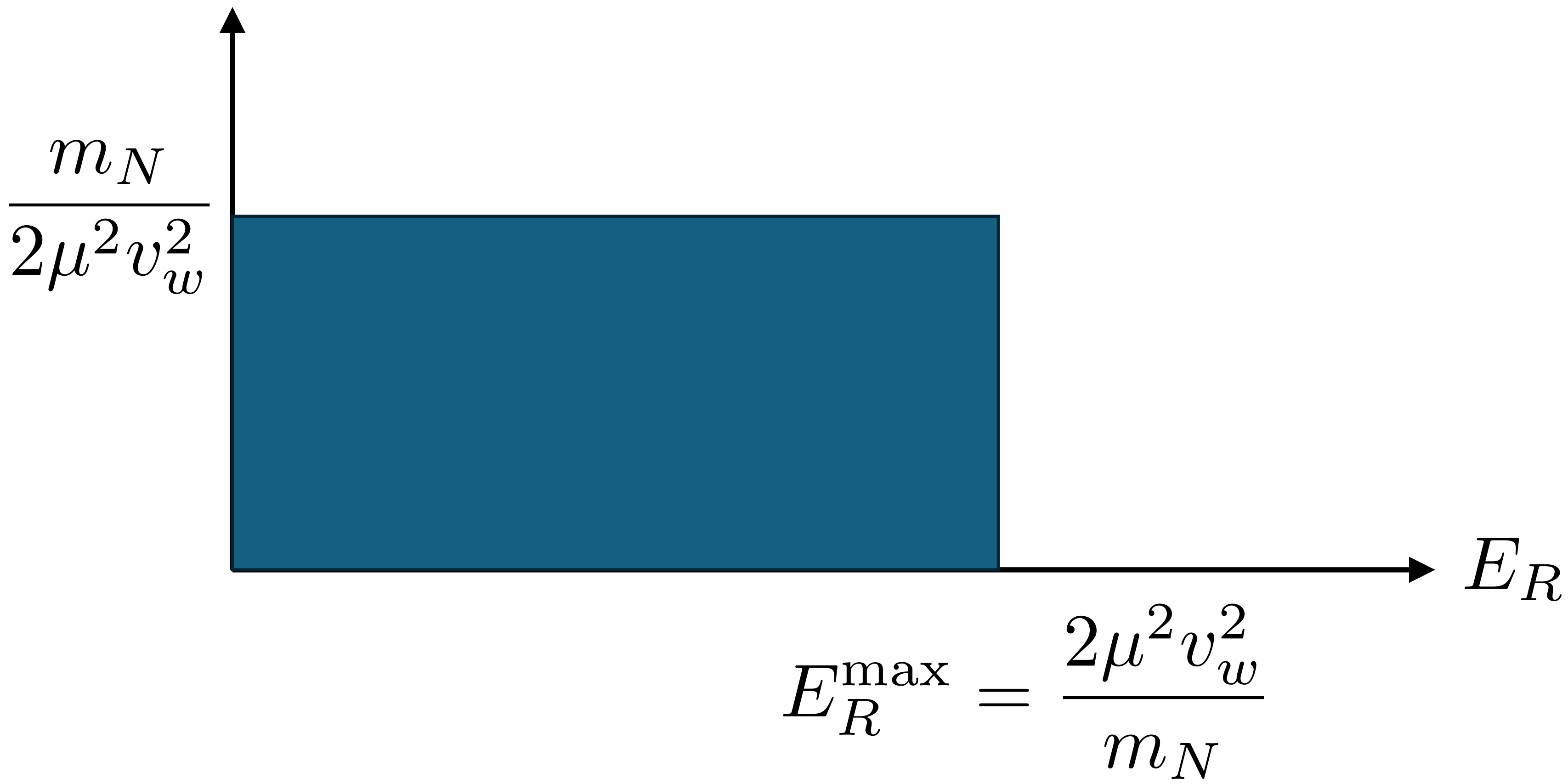
Some notes about my teaching style and response to feedback.

1. *The slides are not notes* – they are a set of drawings and other material to supplement what I write on the board. My 'style' is chalk and talk, not powerpoints. I find that when powerpoints are used, the pace tends to be totally unmanageable, particularly when going through mathematics.
2. *The notes that you should read if you miss a lecture and for understanding* and revision are the ones linked two from the blackboard site: https://eddaw1.github.io/dark_matter_course/dark_matter_detection.html
3. *I shall try and speak more slowly*, as some of you fed back that you are having trouble following me.

Last time, WIMP nuclear scattering modelled as colliding hard classical spheres



Review of the probability distribution of nuclear recoil energies based on hard sphere scattering, from lecture 2

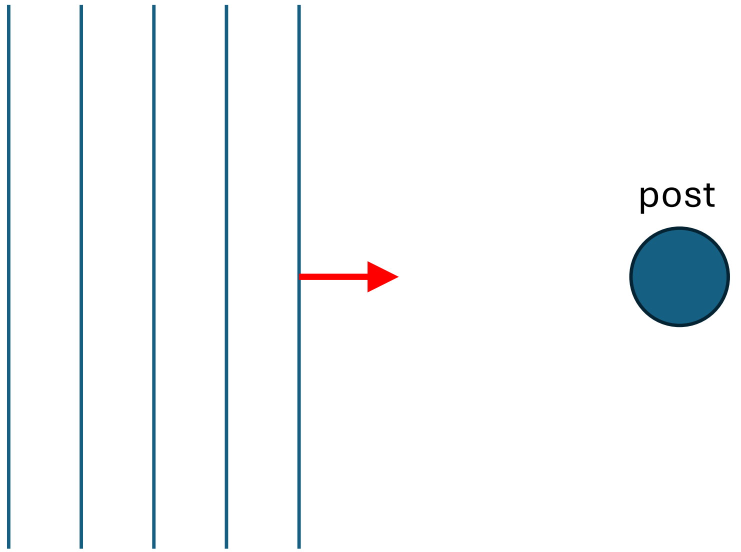


Classical scattering of waves off particles

Last time we did classical scattering of particles. What happens when a plane wave wave scatters off a post sticking out of the water?

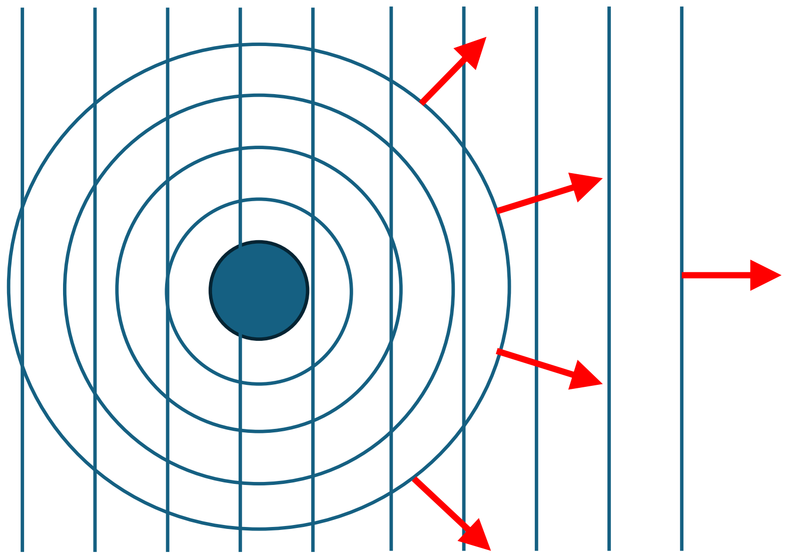
BEFORE

Incoming plane wavefronts



AFTER

Plane wavefronts if unaffected by the post



Scattered waves from the post moving outwards in circles (in 2 dimensions) or spheres (3 dimensions). The plane and circular waves add causing interference patterns on the water.

Differences and similarities of classical and quantum scattering

In classical wave physics, you are solving the wave equation

The solutions are the amplitude of the wave as a function of position and time. These amplitudes are observables.

In quantum mechanics you are solving the Schrodinger equation instead of the wave equation.

The solutions are wave functions, which are scalar functions of position. Wave functions are not observable.

The interpretation of the solutions is as probability densities for finding the WIMP there.

The Schrodinger equation is not exactly soluble for most problems (exceptions are the infinite square well, the harmonic oscillator, the free particle, and the central coulomb potential – hydrogen atom)

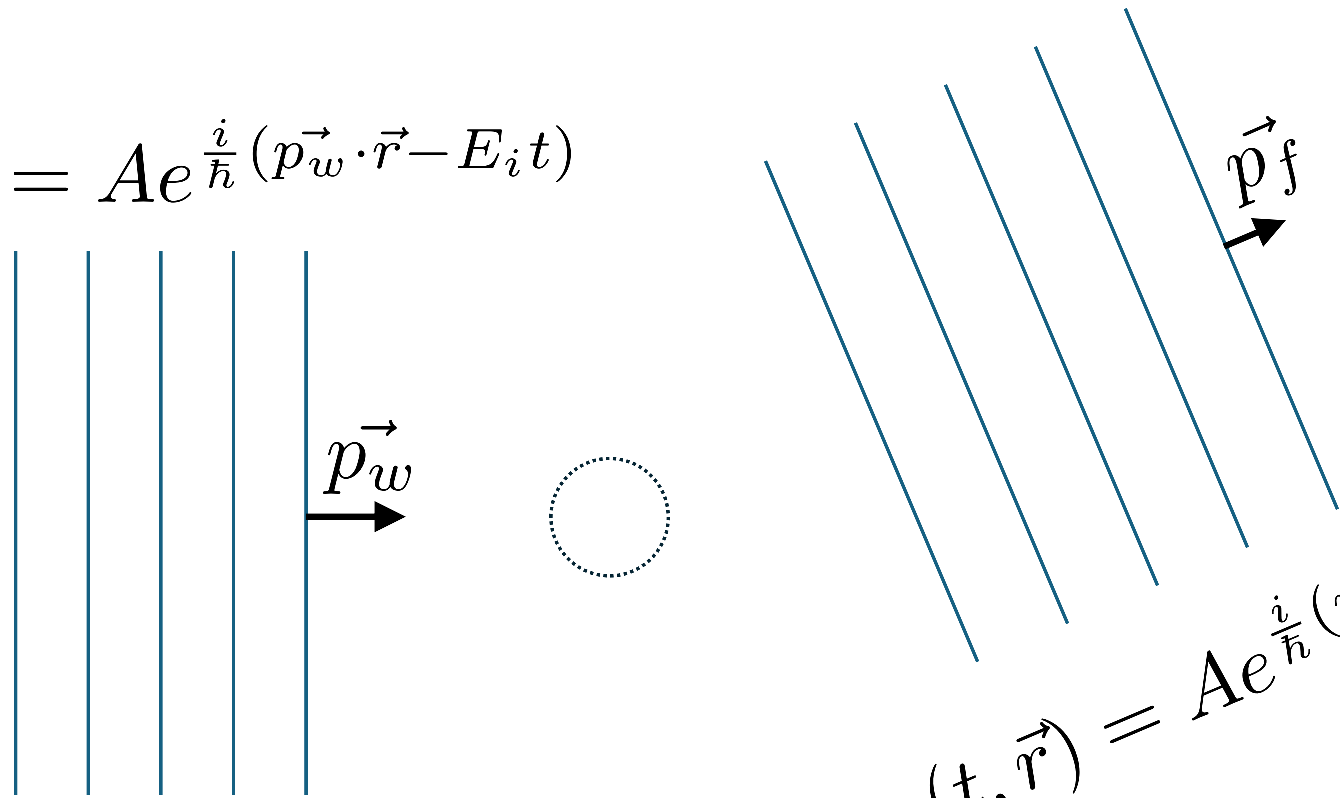
Other problems are solved approximately.

One method of solving approximately is a technique called perturbation theory that you learned a bit about in year 2 quantum mechanics.

Today we figure out the consequences of quantum mechanics for WIMP nuclear scattering.

Quantum scattering in first order perturbation theory

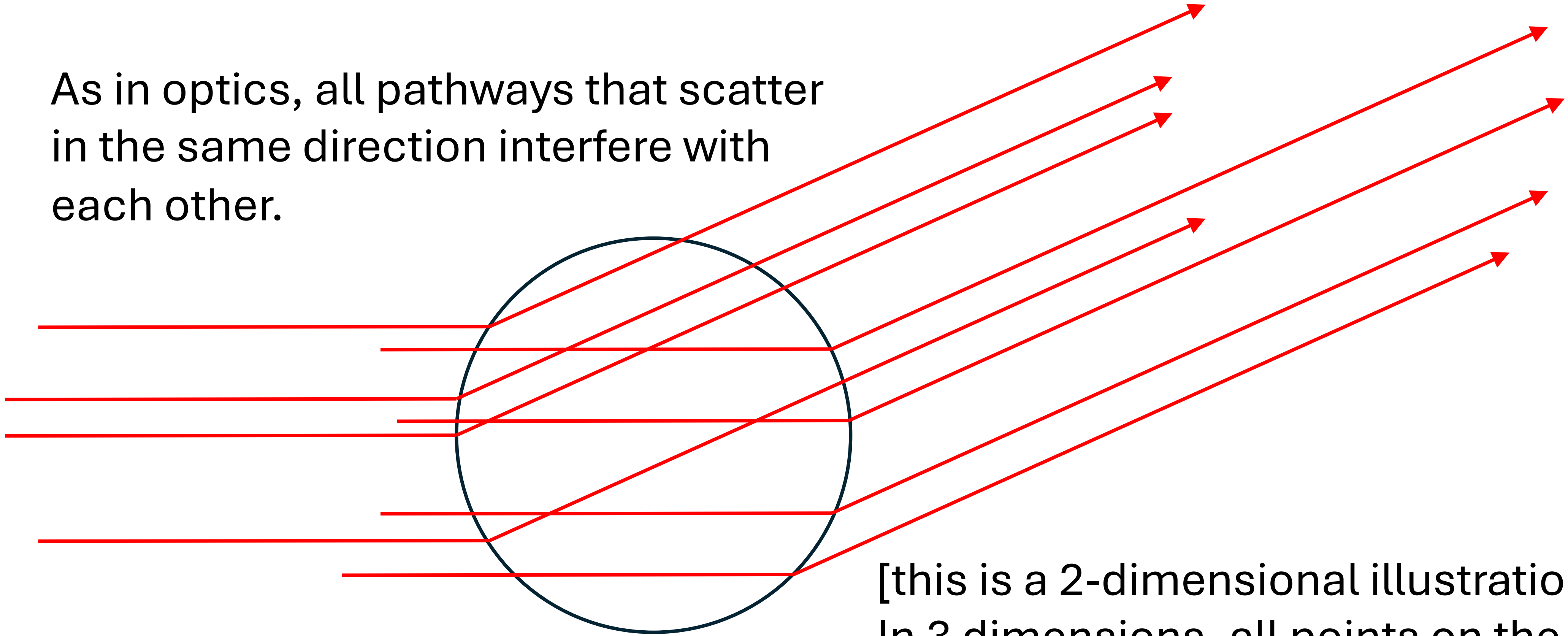
$$\Psi_i(t,\vec{r}) = Ae^{\frac{i}{\hbar}(\vec{p_w}\cdot\vec{r}-E_i t)}$$



$$\Psi_f(t,\vec{r}) = Ae^{\frac{i}{\hbar}(\vec{p_f}\cdot\vec{r}-E_f t)}$$

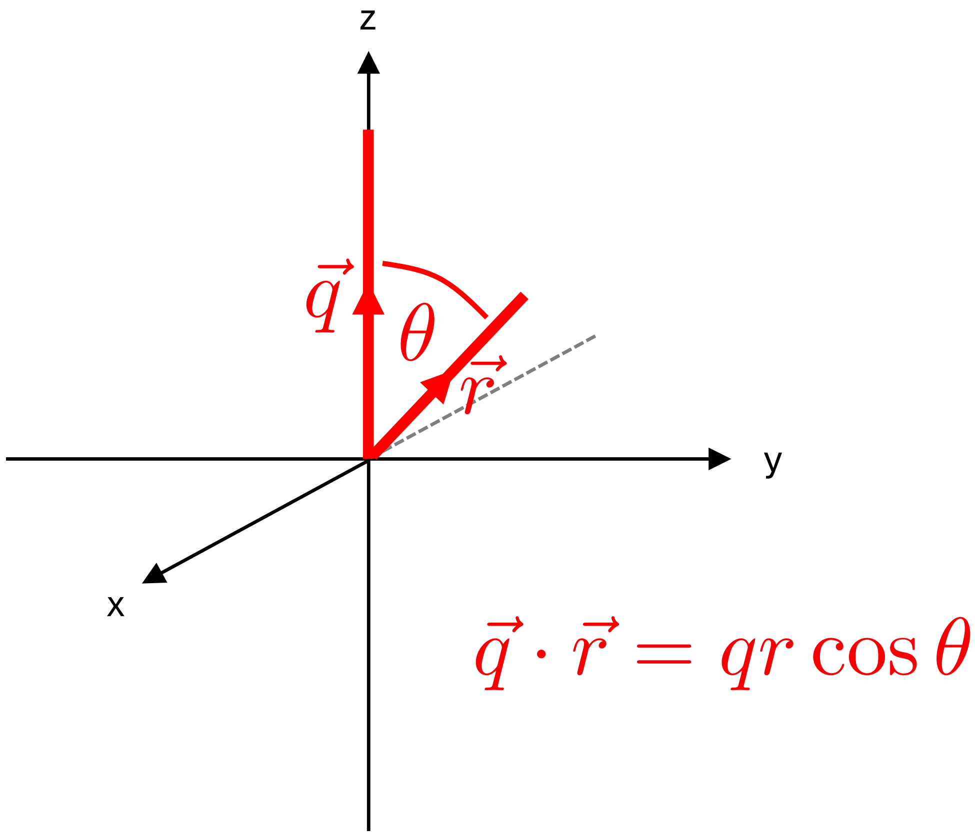
Scattering off all points on the surface of the nucleus at once

As in optics, all pathways that scatter in the same direction interfere with each other.



[this is a 2-dimensional illustration. In 3 dimensions, all points on the surface of the sphere contribute]

Geometry of the form factor integral



Form factor result, compared with hard sphere model

In the notes we calculated the form factor for WIMP-nuclear scattering in a simple model

$$F^2(E_R) = \frac{\sin^2\left(\frac{R\sqrt{2(m_N c^2)E_R}}{\hbar c}\right)}{\frac{2(m_N c^2)E_R R^2}{(\hbar c)^2}}$$

Assume the scattering nucleus is Xenon at A=131, nuclear radius 6.1fm

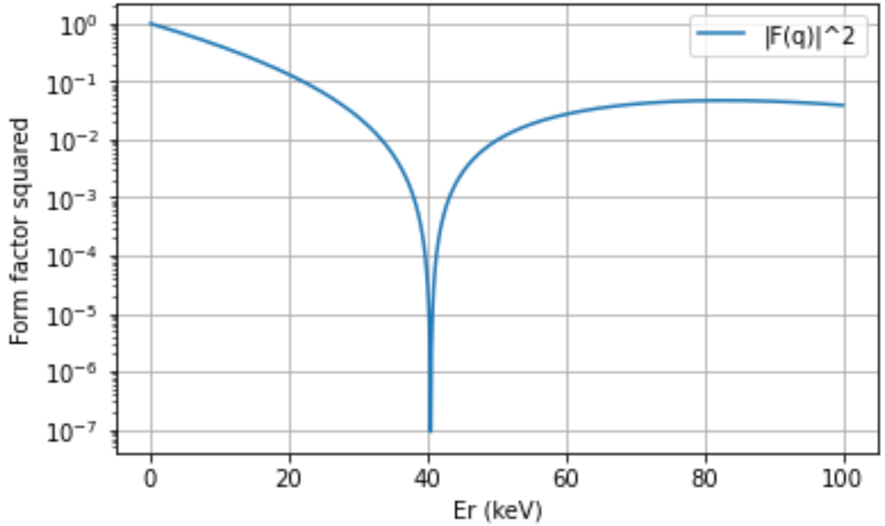
$$m_N c^2 = 131\,\mathrm{GeV}$$

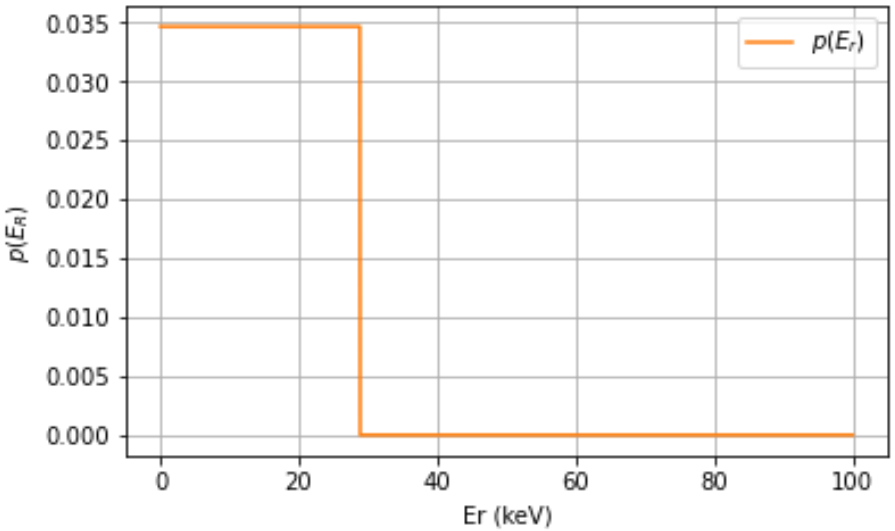
$$R = 6.1\,\mathrm{fm}$$

$$\hbar c = 0.2\,\mathrm{GeV\,fm}$$

From lecture 2 and the hard sphere scattering model the maximum attainable nuclear recoil energy is

$$E_R^{\max} = \frac{2\mu^2 v_w^2}{m_N}$$





Summary

- In the hard sphere scattering model, energy and momentum conservation lead to a maximum attainable recoil energy.
- Assuming a single point of impact equally likely anywhere on the disk of impact points relative to the incoming WIMP, leads to a prediction of a uniform probability density of WIMP recoil energies up to the maximum kinematically permitted.
- A quantum analysis contradicts this – interference predicts a form factor that is down by about a factor of 30 at the maximum recoil energy, for A=131.
- Therefore, to be sensitive, detectors need as low an energy threshold as possible, as event rates at low recoil energies are far more common than event rates at high recoil energies.