

$$\frac{\partial}{\partial \theta} \left\{ \frac{\left(\frac{1}{\theta}\right)^{\frac{1}{\theta}} x^{\frac{1}{\theta}-1} \exp\left(\frac{-x}{\theta}\right)}{\Gamma\left(\frac{1}{\theta}\right)} \right\} = \frac{\left(\frac{x}{\theta}\right)^{\frac{1}{\theta}-1} \exp\left(\frac{-x}{\theta}\right) \left\{ \ln\left(\frac{\theta}{x}\right) + \psi^{(0)}\left(\frac{1}{\theta}\right) + x - 1 \right\}}{\theta^3 \Gamma\left(\frac{1}{\theta}\right)}$$