Motion
in
Different Reference Frames

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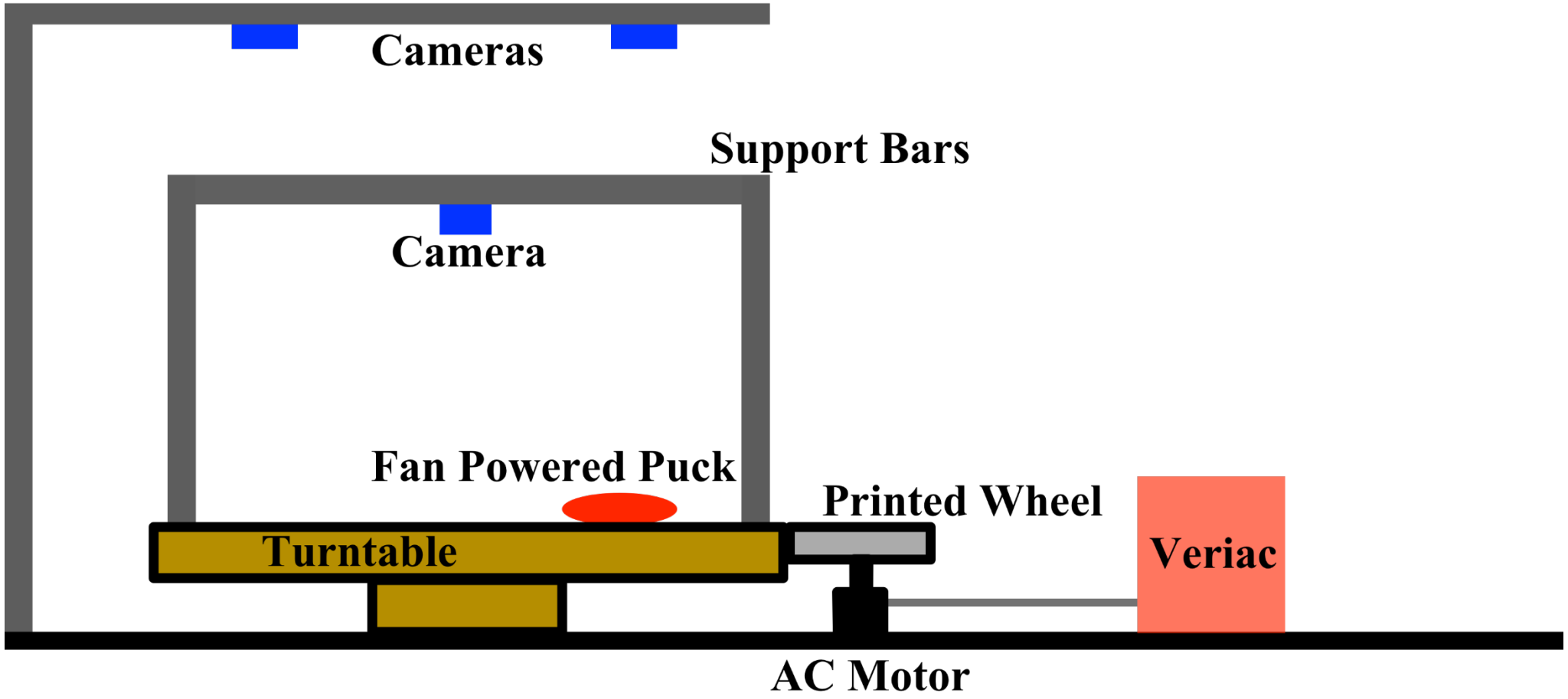
Introduction

- Same angular velocity ω in different reference frames
- Rotating Reference Frame: Non-Inertial Reference Frame (NIRF)
- Stationary Reference Frame: Inertial Reference Frame (IRF)
- Fictitious/Pseudo Forces in NIRF
- Astronomical measurements

Goal

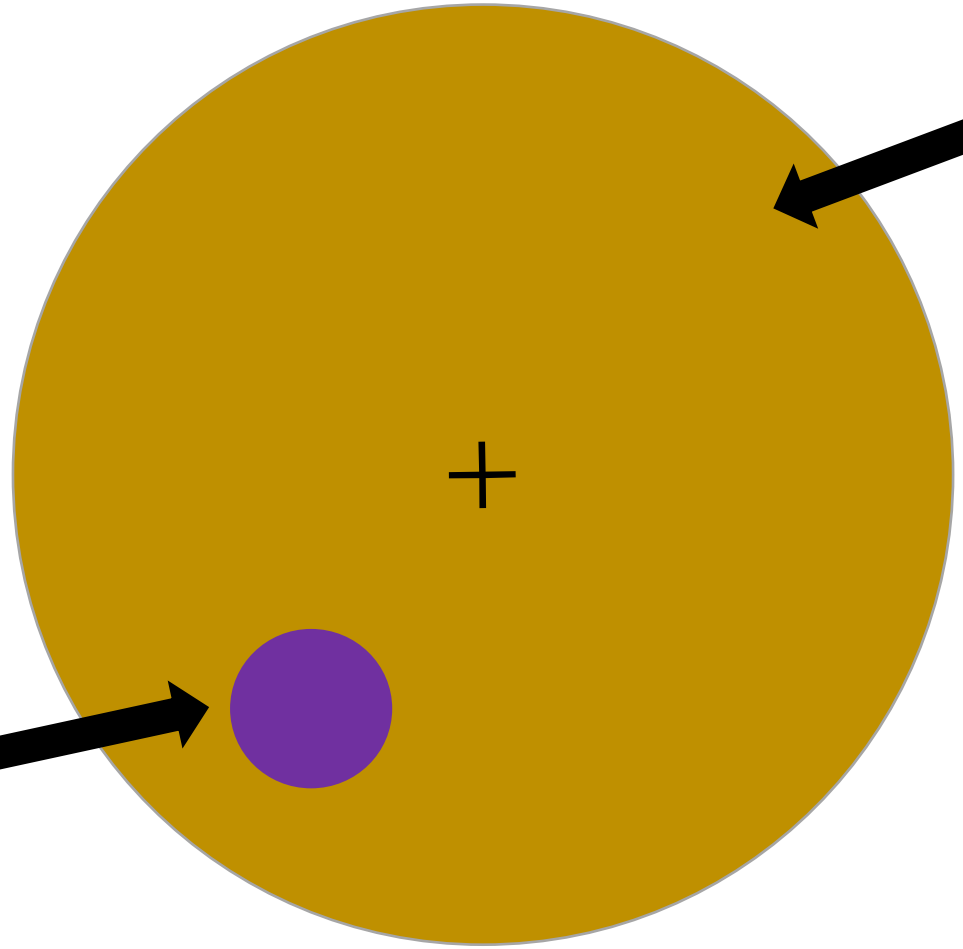
- Have a rotating and non-rotating frame of reference
- Rotate at different angular velocities
- Find angular velocity ω in different reference frames
- Find v_0 in IRF and NIRF, then:
  - Transform v_0 in NIRF to IRF
  - Compare transformed v_0 and IRF v_0
- Compare NIRF theory and data paths

Setup


Cameras
Support Bars
Camera
Fan Powered Puck
Printed Wheel
Turntable
Veriac
AC Motor

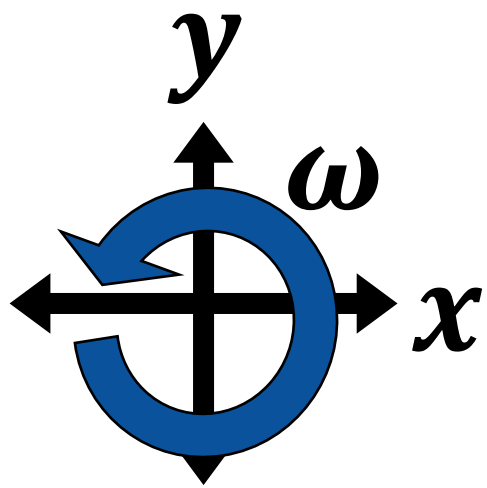
Setup

NIRF Camera View

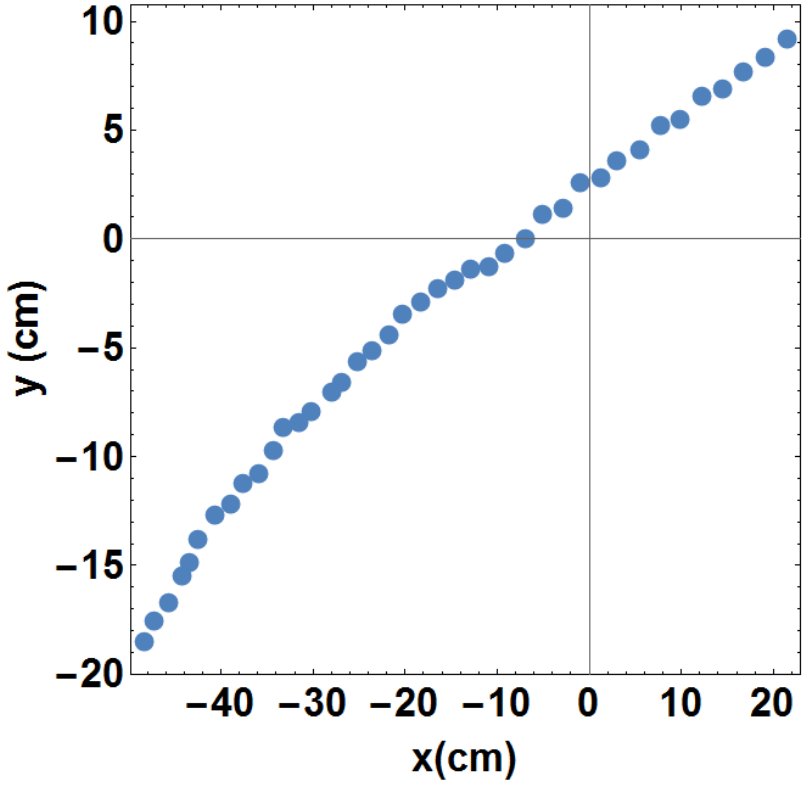


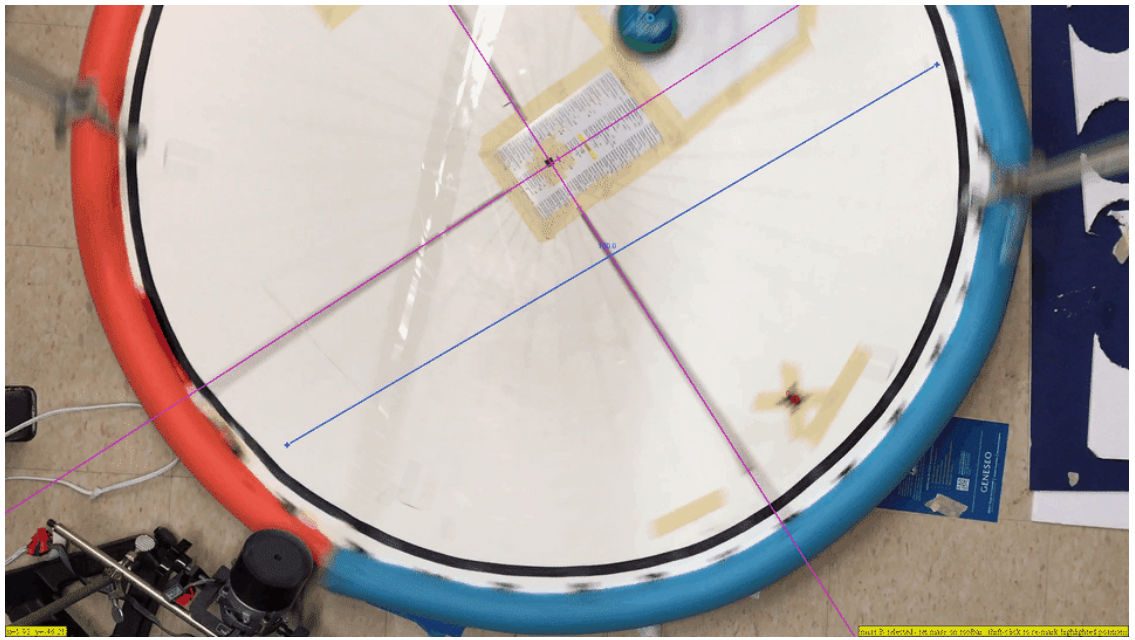
Rotating Table

Hover Puck



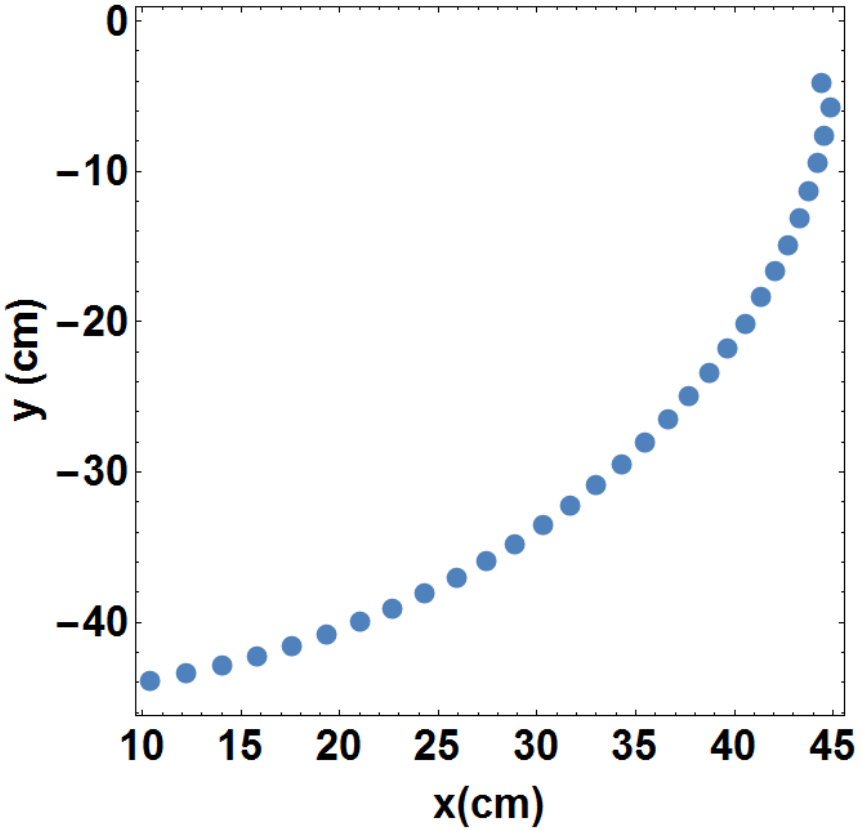
Data

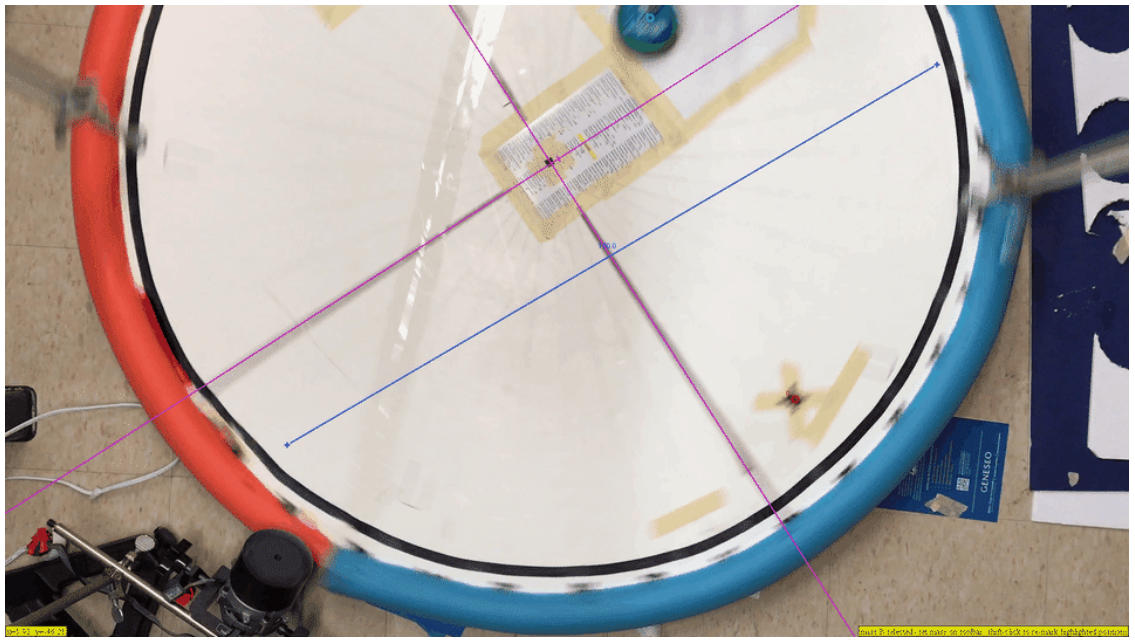




Tracking Puck - Inertial Reference Frame

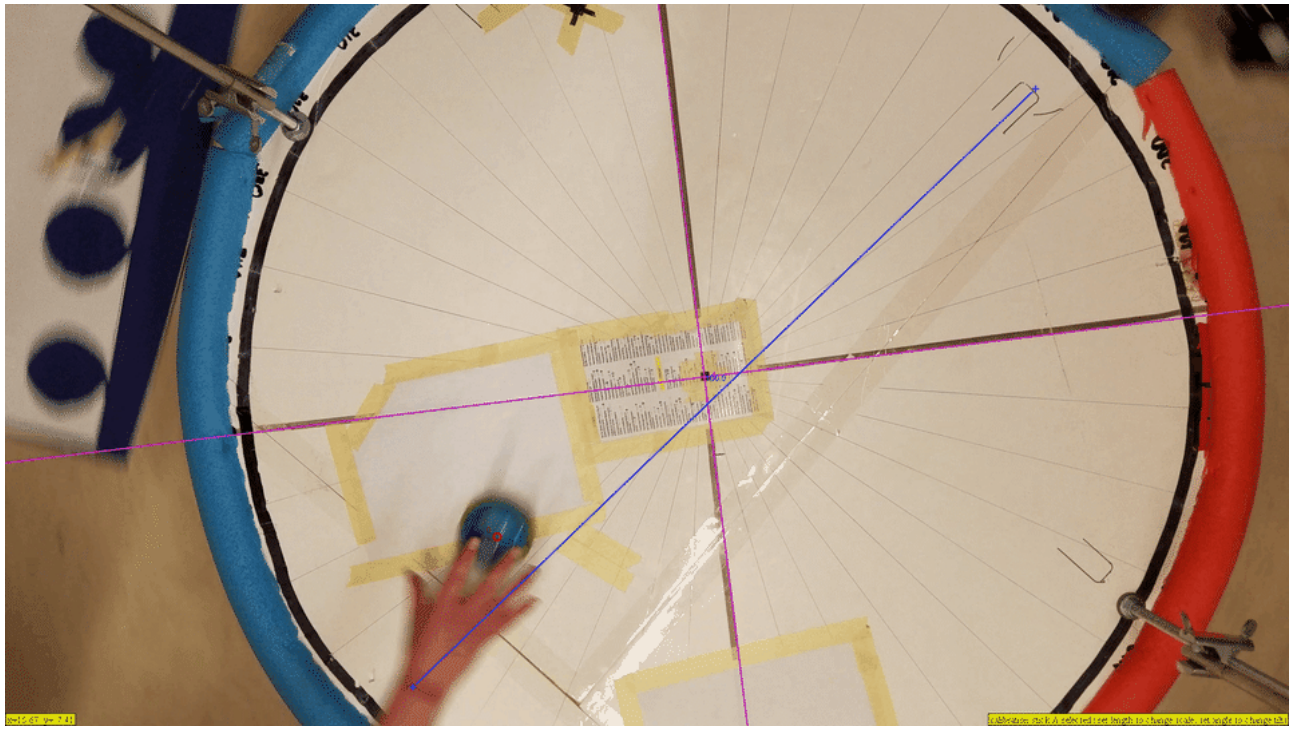
Data

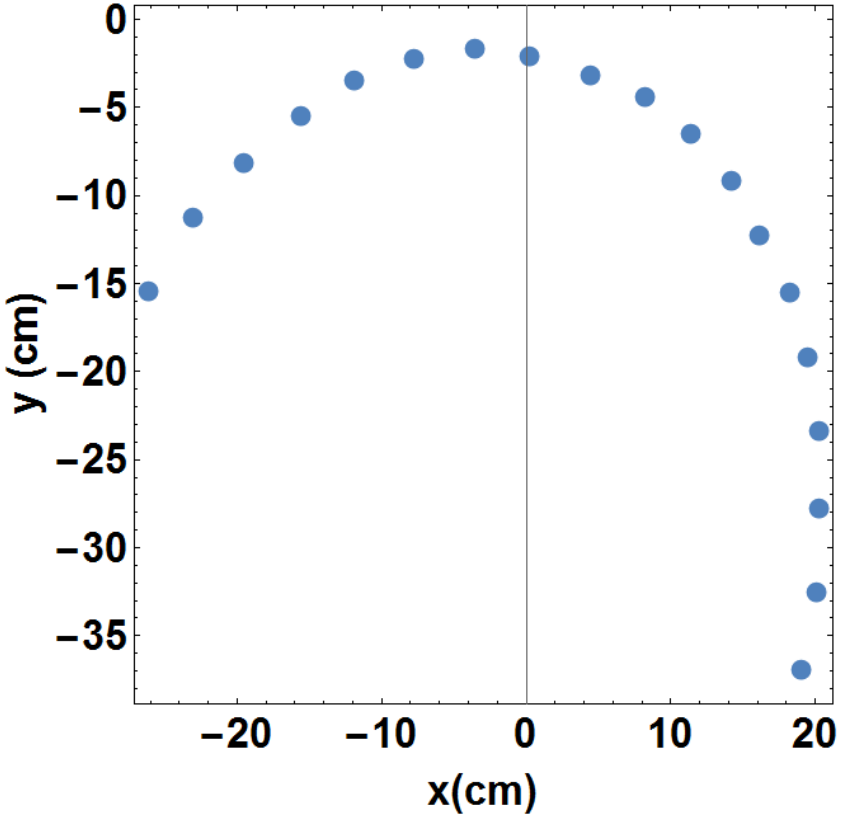




Finding ω - Inertial Reference Frame

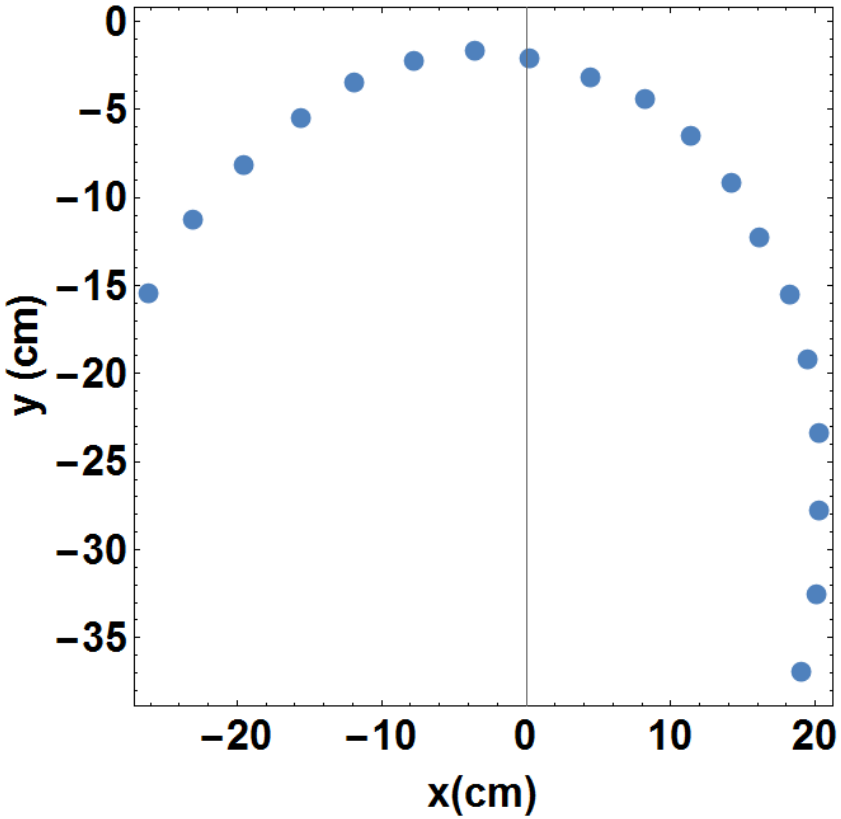
Data



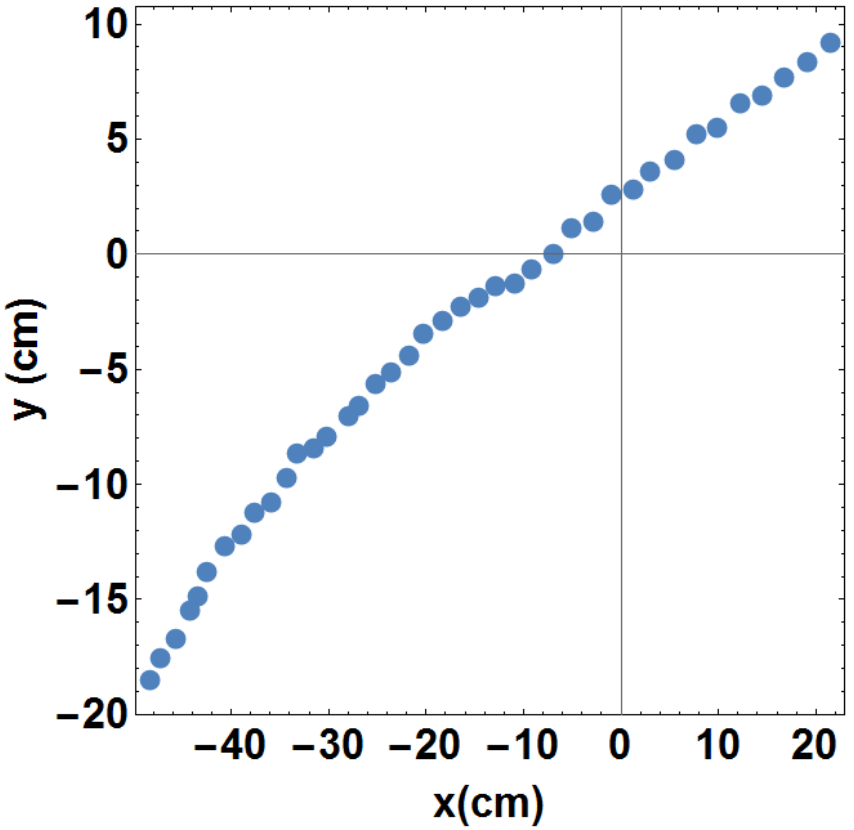


Tracking Puck - Non-Inertial Reference Frame

Data



Non-Inertial Reference Frame

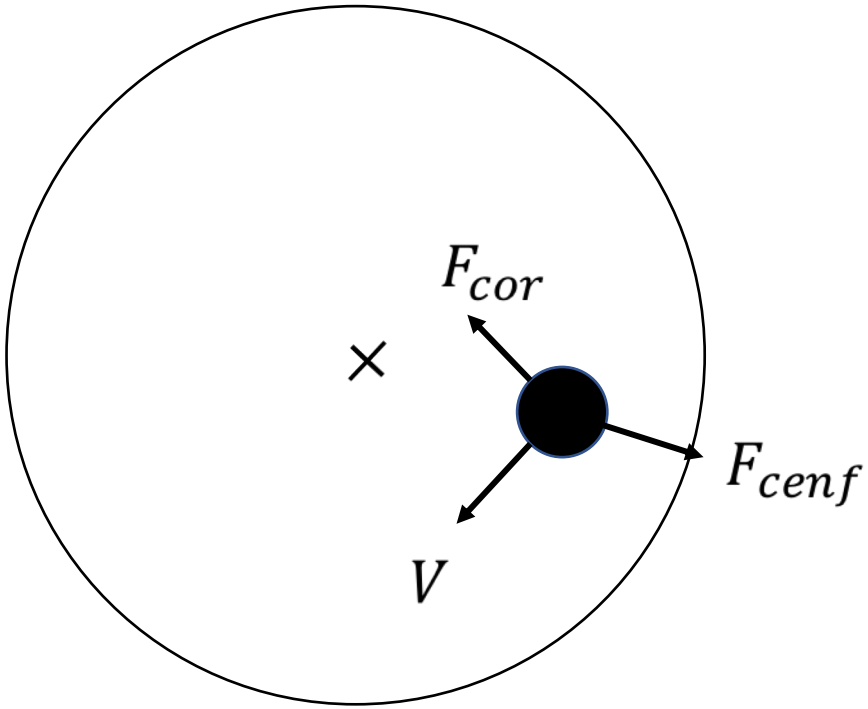


Inertial Reference Frame

Data Analysis

- Fit a frictionless pseudo force model to NIRF data
- Obtain best fit parameters: ω and v_0
- Compare best fit parameters with IRF

Theory



Centrifugal Force

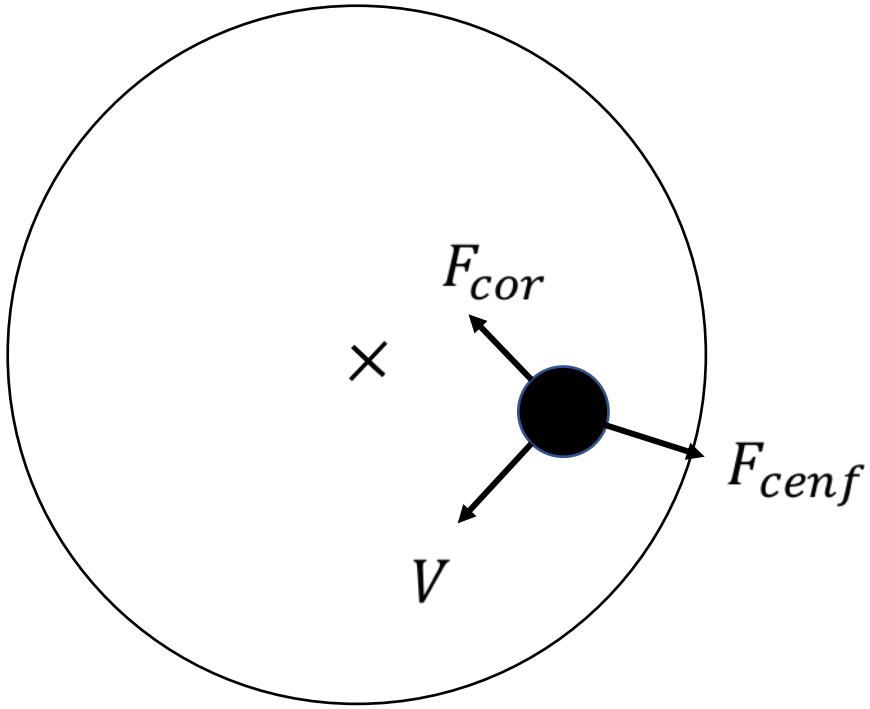
Coriolis Force

$$\boldsymbol{a}_r = \boldsymbol{a} - \ddot{\boldsymbol{R}}_r - \dot{\boldsymbol{\omega}}\times\boldsymbol{r} - \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}) - 2(\boldsymbol{\omega}\times\boldsymbol{v}_r)$$

Theory Model:

- $\boldsymbol{a}_r$ = Acceleration in Rotating Frame (NIRF)
- $\boldsymbol{a}$ = Acceleration in Non-Rotating Frame (IRF)
- $\ddot{\boldsymbol{R}}_r$ = Acceleration of Moving Origin
- $\boldsymbol{\omega}$ = Angular Velocity
- $\boldsymbol{r}$ = Position in Rotating Frame
- $\boldsymbol{v}_r$ = Velocity in Rotation Frame

Theory



Centrifugal Force

Coriolis Force

$$\boldsymbol{a}_r = \boldsymbol{a} - \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}) - 2(\boldsymbol{\omega}\times\boldsymbol{v}_r)$$

Theory Model:

- $\boldsymbol{a}_r$ = Acceleration in Rotating Frame (NIRF)
- $\boldsymbol{a}$ = Acceleration in Non-Rotating Frame (IRF)
- $\boldsymbol{\omega}$ = Angular Velocity
- $\boldsymbol{r}$ = Position in Rotating Frame
- $\boldsymbol{v}_r$ = Velocity in Rotation Frame

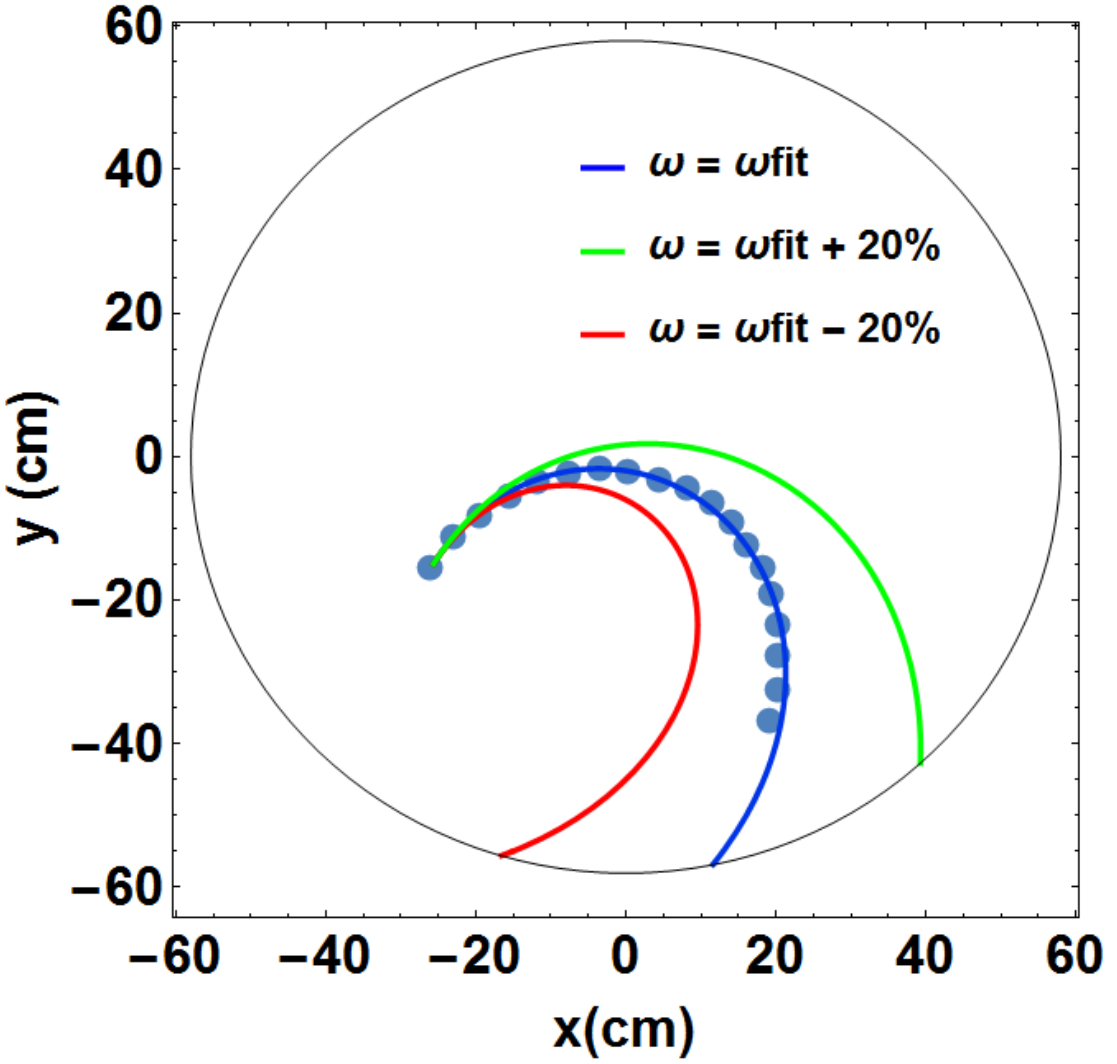
Theory

$$\left(\frac{d\vec{r}}{dt}\right)_{IRF} = \left(\frac{d\vec{r}}{dt}\right)_{NIRF} + \omega \times \vec{r}$$

Translation of NIRF Velocity

Results: Trial One

- Overall Trend
- 20% Increase and Decrease in $\boldsymbol{\omega}$
- $\boldsymbol{\omega_{NIRF}}$**= (2.514 ± 0.064)** $\frac{\boldsymbol{rad}}{\boldsymbol{s}}$



Results: Trial One

$\omega_{NIRF} = (2.514 \pm 0.064)\ \frac{rad}{s}$

$\omega_{IRF} = (2.501 \pm 0.057)\ \frac{rad}{s}$

$v_0 = (81.9 \pm 6.1)\ \frac{cm}{s}$

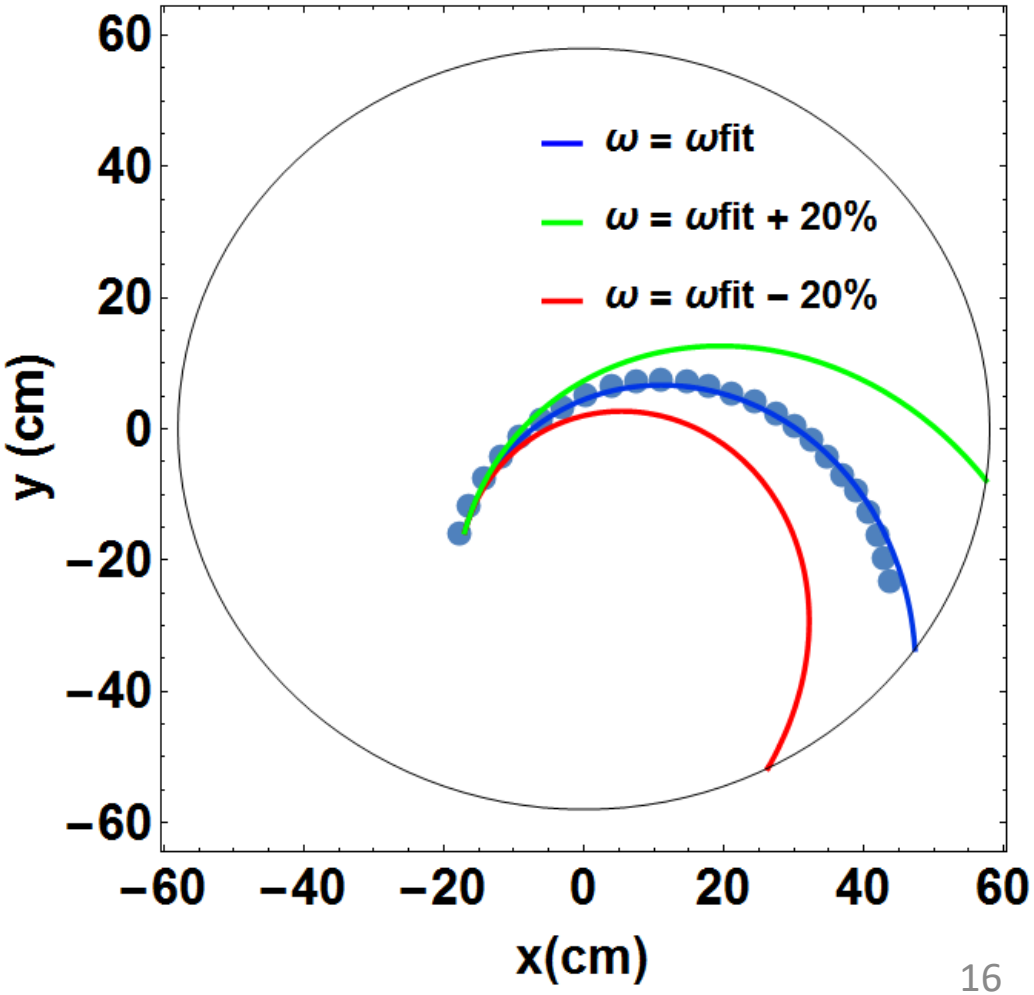
Transformation to IRF

$v_0 = (114.5 \pm 1.4)\ \frac{cm}{s}$

From IRF Data

Results: Trial Two

- Overall Trend
- 20% Increase and Decrease in $\boldsymbol{\omega}$
- $\boldsymbol{\omega_{NIRF}}$**= (1.807 ± 0.062)** $\frac{\boldsymbol{rad}}{\boldsymbol{s}}$



Results: Trial Two

ω_{NIRF}= (1.807 ± 0.062) $\frac{rad}{s}$

v_0= (74.7± 6.1) $\frac{cm}{s}$

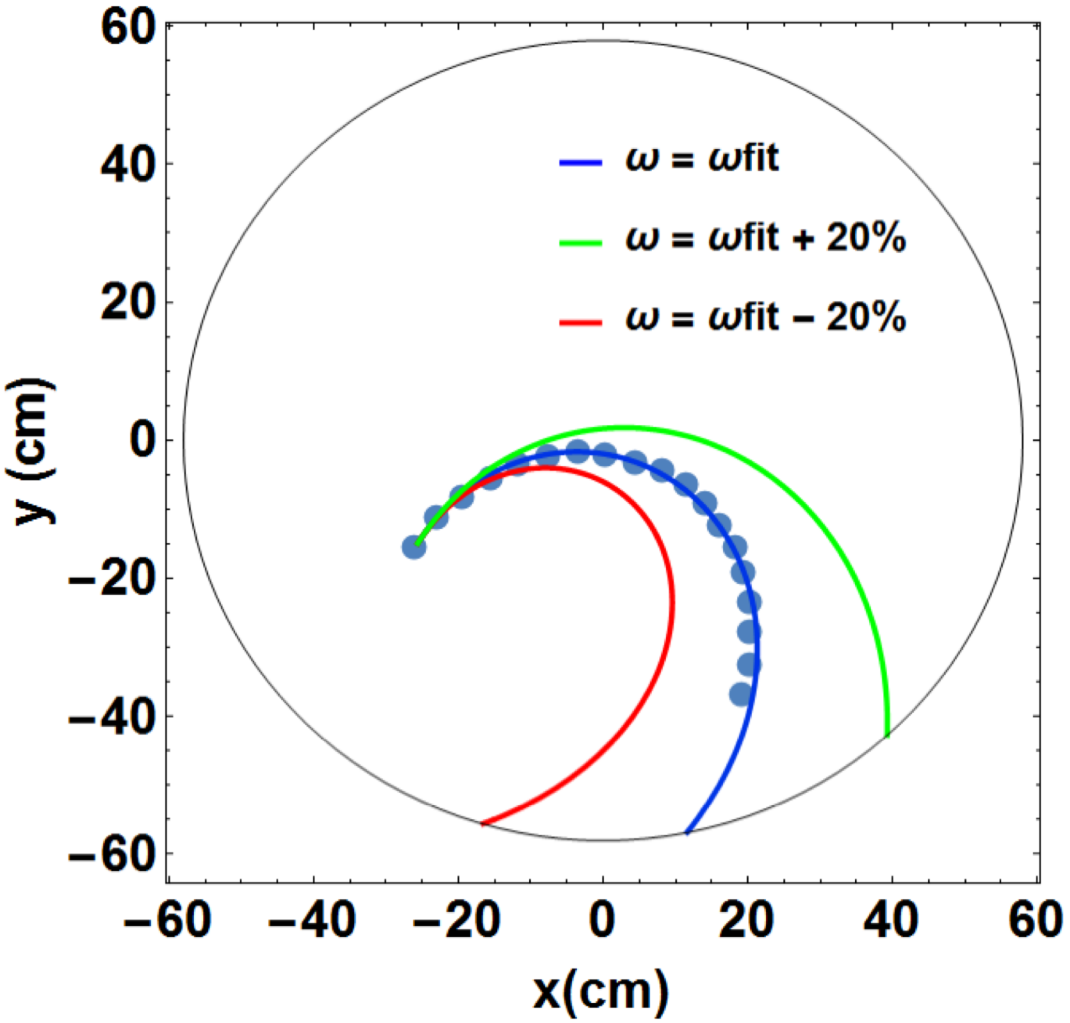
Transformation to IRF

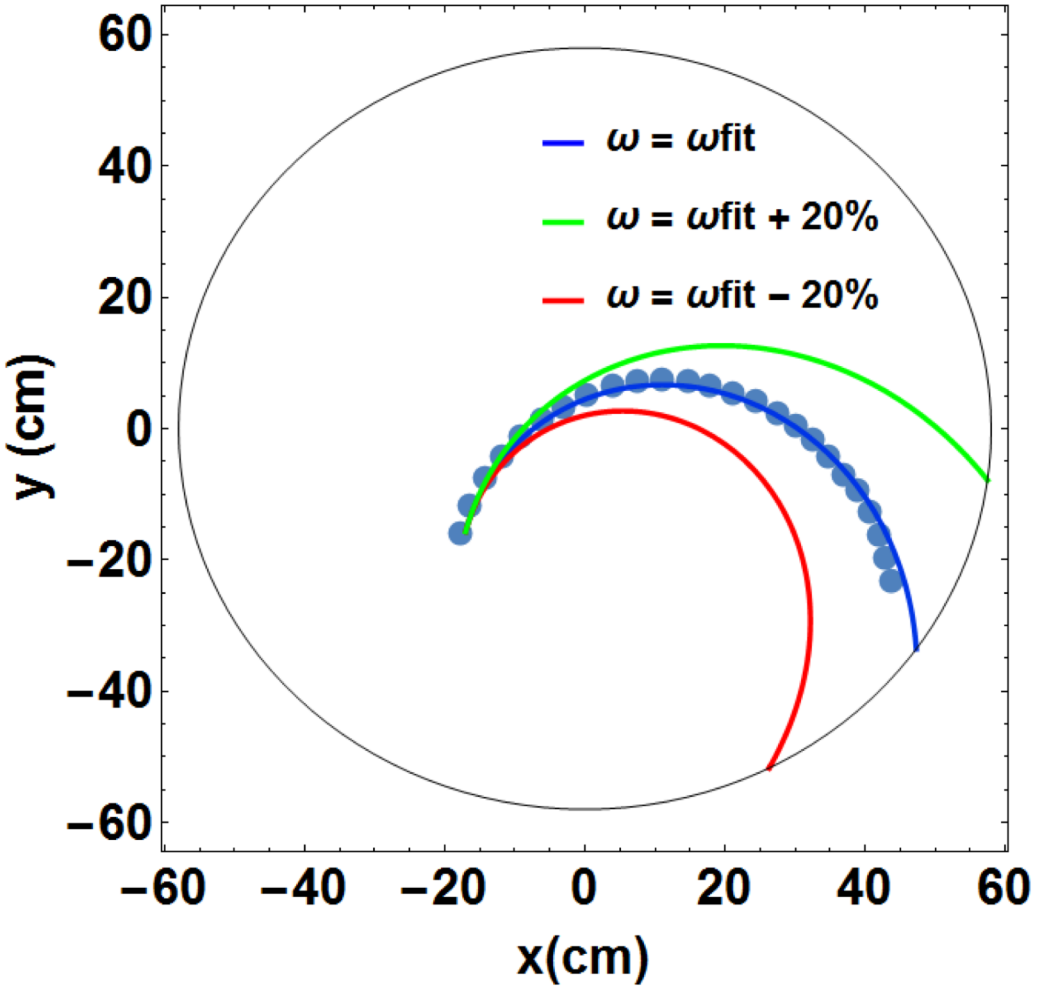
ω_{IRF}= (2.041 ± 0.045) $\frac{rad}{s}$

v_0= (88.7 ± 1.5) $\frac{cm}{s}$

From IRF Data

Results


ω = ωfit
ω = ωfit + 20%
ω = ωfit − 20%
y (cm)
x(cm)
60
40
20
0
−20
−40
−60
−60
−40
−20
0
20
40
60


ω = ωfit
ω = ωfit + 20%
ω = ωfit − 20%
y (cm)
x(cm)
60
40
20
0
−20
−40
−60
−60
−40
−20
0
20
40
60

Possible Sources of Error

- Unleveled Rotating Platform
- Vibrations in NIRF Camera
- Uneven Surface
- Parallax of Camera Lens
- Uneven Weighting of Hockey Puck

Summary

- Should see same angular velocities
- IRF velocities should match
- Theory and observed paths
- Improvements