Review of results

March 30, 2023

1 Introduction

In 1983, Pendry derived a bound on the flow of information, which reads

$$\dot{I}^2 \leq \frac{\pi}{3\hbar \ln^2 2}\dot{E}. \tag{1}$$

This result is derived using classical statistical mechanics [1]. However, this classical framework neglects the effects of genuine quantum mechanical phenomena, such as quantum correlations. We show that this inequality is violated in a model system with quantum correlations, and provide a bound on information flow, which corrects for contributions from correlations.

2 The model

Consider a chain of qubits in the Heisenberg-XY model. Restricting their interaction to their nearest neighbors allows us to draw a comparison to Pendry's original derivation with fermions using Jordan-Wigner transform, mapping a chain of spin-1/2 qubits to a chain of spinless fermions. The information flow is then the time derivative of

In particular, we have the Hamiltonian of this system as

$$H = -\sum_i \sigma_i^z + \sum_j \frac{J_j}{2}(\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y) \tag{2}$$

with

$$J_k = \frac{\lambda}{2}\sqrt{j \cdot (N-j)}.$$

This ensures a predictable periodic behaviour of the chain. This can be made apparent by considering the local magnetization $\langle \sigma_i^z \rangle$ of the qubits. Figure 1 shows the time evolution of the quibts' magnetization.[1]

[1] We choose λ to be 1. It was shown in [2] that at time $t = \pi/\lambda$, perfect state transfer is achieved

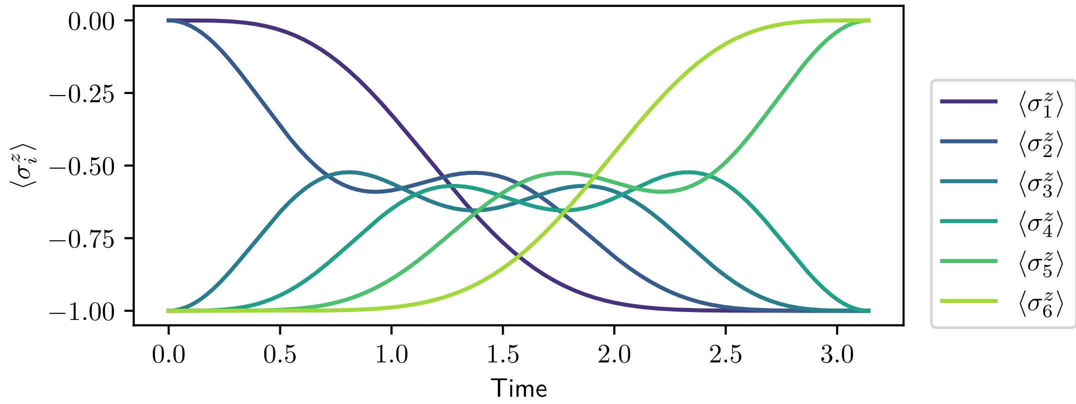

Figure 2: Time Evolution of the individual magnetizations of the qubits $\langle \sigma_i^z \rangle$, where the interaction is governed by Eq. (2). Qubits $i = 1$ and $i = 2$ are initially prepared in a thermal state $\rho_{1,2} = \exp\left(-\beta_{1,2}\sigma_{1,2}^z\right)/Z$ and qubits $i \neq 1, 2$ are prepared in the ground state.

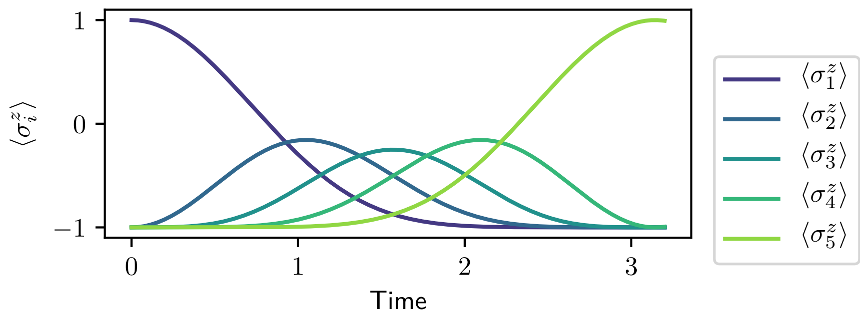

Figure 1: Time evolution of the individual magnetizations $\langle \sigma_i^z \rangle$ for the system with Hamiltonian defined as in Eq. (2). Qubit $i = 1$ is prepared in the excited state at time $t = 0$. Qubits $i > 1$ are prepared in the ground state at time $t = 0$.

This perfect transfer works for an arbitrary initial state and size of the system. Consider Fig. 2, where we scaled up the system to include $N = 6$ qubits and have the initial state of qubit 1 and 2 as a thermal state. We can verify if ineq. (1) holds. Figure 3 shows the information and energy flow over time. Here, information flow is given by

$$\dot{I} = -\partial_t \operatorname{Tr}[\rho \ln \rho]. \tag{3}$$

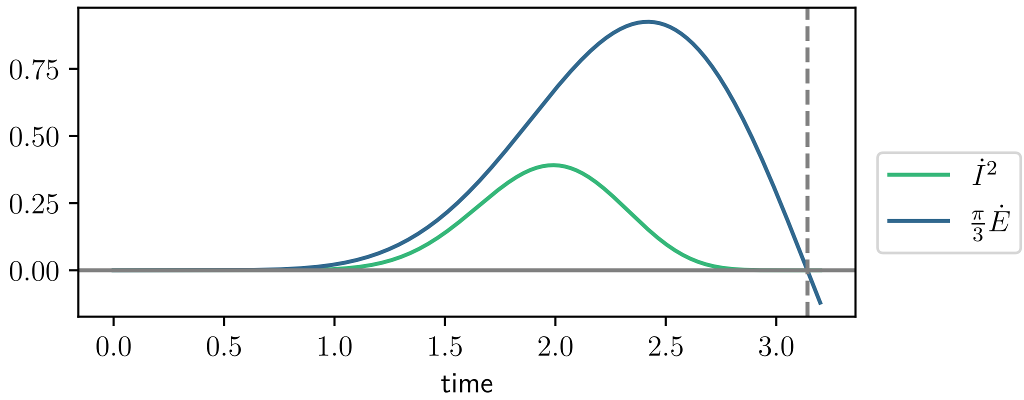

Figure 3: Information and energy flow of the last qubit in the chain. Here, the chain propagates a thermal state from one end to the other.

3 Quantum correlations

In a work by Micadei et al., it was shown that certain quantum correlations *invert* the flow of energy [3]. Heat flows, initially, not from hot to cold, but from cold to hot. The two-qubit state is then described by a product state with an added correlation term of the form

$$\chi = -i\alpha(\sigma_k^+ \sigma_{k+1}^- - \sigma_k^- \sigma_{k+1}^+), \tag{4}$$

with

$$|\alpha_{\max}| = \frac{1}{4\cosh\beta_1 \cosh\beta_2}. \tag{5}$$

Figure 4 shows the chain with initial correlations at positions 1 and 2 of the chain.

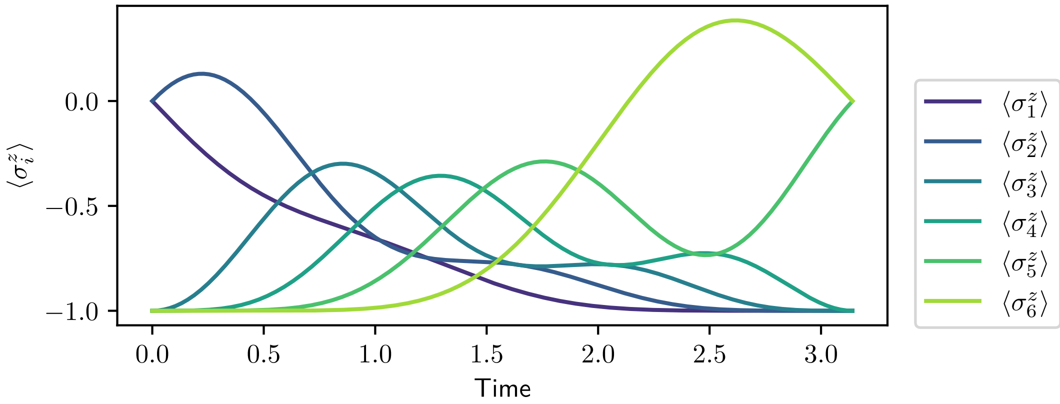

Figure 4: Time Evolution of the individual magnetizations of the qubits $\langle\sigma_i^z\rangle$, where the interaction is governed by Eq. (2). Qubits $i = 1$ and $i = 2$ are prepared in an initially correlated state $\rho_{12} = \rho_1 \otimes \rho_2 + \chi$ and qubits $i \neq 1, 2$ are prepared in the ground state. Locally, Qubit 1 and 2 are in a thermal state each, with identical inverse temperature $\beta = 1/300$.

This dynamics has implications on ineq. (1). Figure 5 shows the analogous plot to Fig. 3.

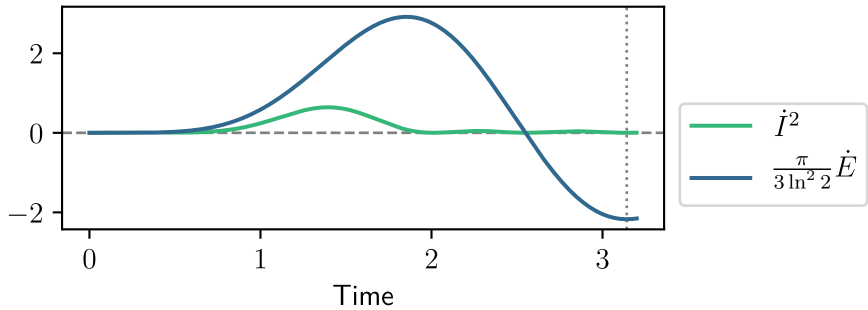

Figure 5: Energy flow and information flow of qubit N for initially correlated qubits 1 and 2. The dotted vertical line indicates $t = \pi$, which is the time of state inversion, i.e. where the dynamics reverse, dictated by the interaction.

This shows that this type of initial state leads to a reversal of heat flow and, as a consequence, to a violation of ineq. (1).

3.1 Bound on information flow

Seeing as we are able to violate ineq. (1), we are now interested in a more general bound, which also holds for systems with quantum correlations.

As preliminaries, we separate correlated and uncorrelated energy flow,

$$\dot{E} = \dot{E}_0^t + \dot{E}_\chi^t, \tag{6}$$

with subscript χ denoting contributions from correlations, and rewritie quantum relative entropy as

$$\mathcal{S}(\rho^t \,||\, \rho^0_{\text{th}}) = -\Delta S + \beta \Delta E. \tag{7}$$

Taking time derivative and isolating $\dot{S}$,

$$\dot{S}_0 = \beta \dot{E}_0^t - \dot{\mathcal{S}}_0(\rho_N^t \,||\, \rho_N^0) \tag{8}$$
$$\dot{S} = \beta \left(\dot{E}_0^t + \dot{E}_\chi^t\right) - \dot{\mathcal{S}}_\chi(\rho_N^t \,||\, \rho_N^0) \tag{9}$$

After subtracting eq. (8) from eq. (9) one has

$$\dot{S} - \dot{S}_0 = \beta \dot{E}_\chi^t - \Delta_\chi \dot{\mathcal{S}}(\rho_N^t \,||\, \rho_N^0) \tag{10}$$

Isolating $\dot{S}$ again, then squaring the result gives the exact result of

$$\dot{S}^2 = \left(\dot{S}_0 + \beta \dot{E}_\chi - \Delta_\chi \dot{\mathcal{S}}\left(\rho_N^t \,||\, \rho_N^0\right)\right)^2 \tag{11}$$

Inserting ineq. (1) into above expression we obtain

$$\dot{S}^2 \leq \frac{\pi}{3\ln^2 2}\dot{E}_0^t + 2\dot{S}_0\left(\beta \dot{E}_\chi - \Delta \dot{\mathcal{S}}\left(\rho_N^t \,||\, \rho_N^0\right)\right) + \left(\beta \dot{E}_\chi - \Delta \dot{\mathcal{S}}\left(\rho_N^t \,||\, \rho_N^0\right)\right)^2 \tag{12}$$

Figure 6 shows the comparisons of the previous and the corrected bound.

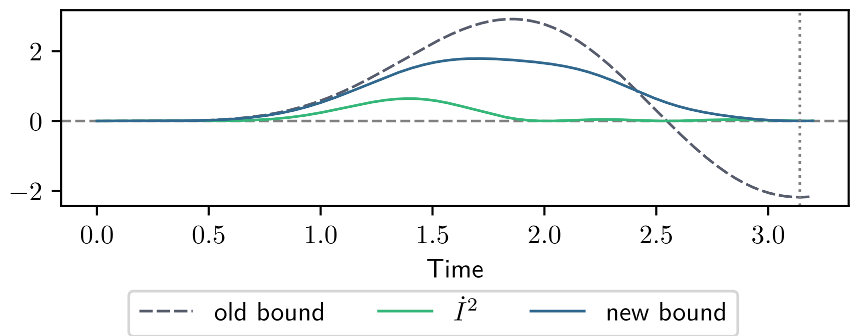

Figure 6: Bound according to ineq. (12). "Old bound" given by ineq. (1). In the case shown, qubit 1 and qubit 2 are correlated with equal temperatures $\beta = 1/300$.

3.2 Dependence of the dynamics on quantum discord

The quantum-ness of the correlations can be quantified through *geometric discord*. We investigate the dependence of certain quantities on discord by varying α in Eq. (4). Figure 7 shows the reduced maximum information flow.

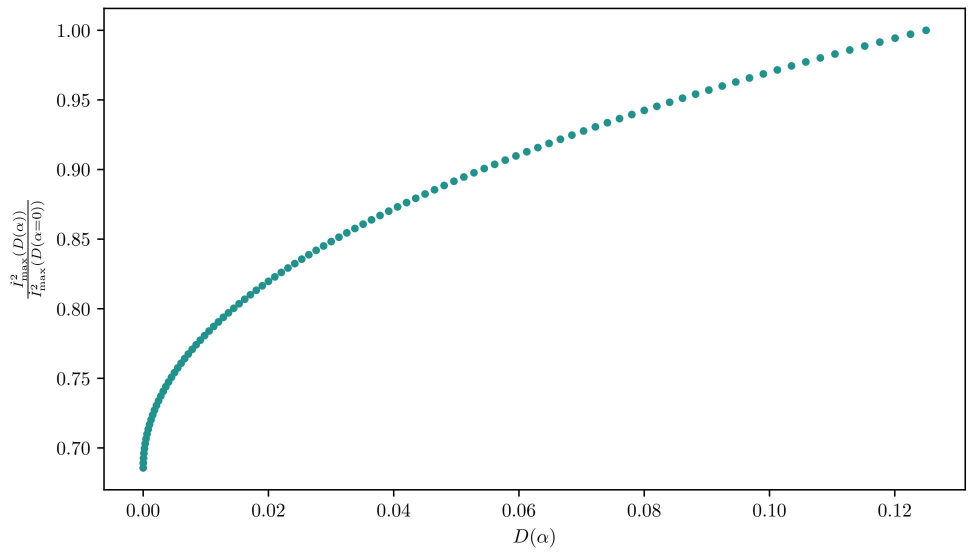

Figure 7: Reduced maximum information flow as a function of discord. The values for information flow are divided by the maximum, $\dot{I}^2_{\max}[D(\alpha = \alpha_{\max})]$, such that the range is from 0 to 1.

We also investigate the fidelity of the first and last qubits, either as pairs of or as separate qubits. Figure 8 shows the fidelity as a function of time for two different values of the geometric discord.

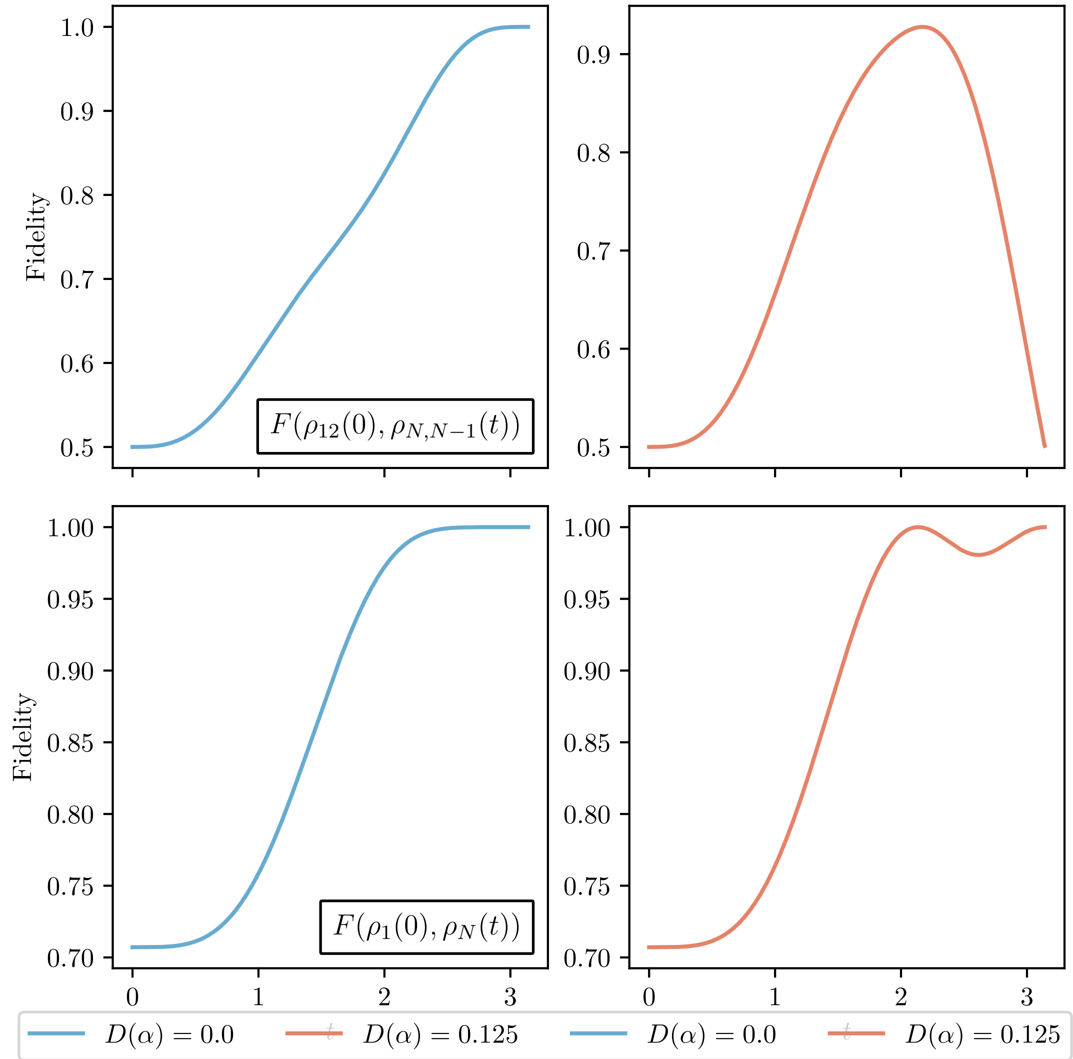

Figure 8: Fidelity of the last qubit with the initial state of the first qubit as a function of time for minimal (maximal) discord (top/bottom left). On the right: Fidelity of the last two qubits with the initial state of the first two as a function of time for minimal (maximal) discord

Figure 8 shows that when considering the single qubits, we have a state transfer, which occurs at an earlier point than $t = \pi$. This *arrival time* can also be plotted as a function of geometric discord. Figure 9 shows this relation.

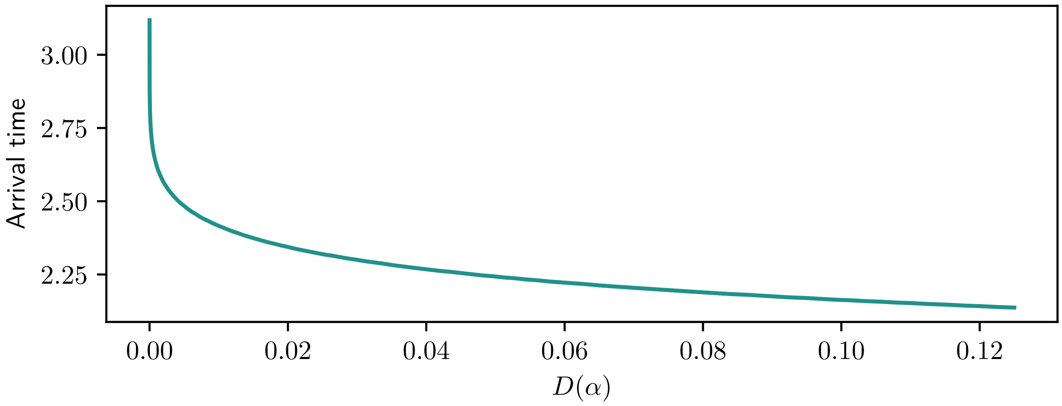

Figure 9: Arrival time (first local maximum in fidelity) as a function of geometric discord.

References

[1] J B Pendry. “Quantum limits to the flow of information and entropy”. In: *Journal of Physics A: Mathematical and General* 16.10 (July 1983), pp. 2161–2171.

[2] Matthias Christandl et al. “Perfect State Transfer in Quantum Spin Networks”. In: *Physical Review Letters* 92.18 (May 2004).

[3] Kaonan Micadei et al. “Reversing the direction of heat flow using quantum correlations”. In: *Nature Communications* 10 (June 2019), p. 2456.