

Defining Parker's Concatenation

Eddie Antonio Santos

April 18, 2017

The goal is to define an operator, $a||b$ such that the result is the concatenation of (base-10) digits of each of its operands.

1 Implementation

The implementation shifts the left-hand operand a by the number of digits in the right-hand operand b . To do this, we must determine, mathematically, how many digits are in the right-hand operand.

1.1 Digits

First, we wish to determine how many digits there are in a number. This will indicate how many times we should multiply a by ten.

$$digits(x) = \lfloor \log_{10} x \rfloor + 1$$

```
digits :: Integral a => a -> a
digits i = floor(log10 x) + 1
        where x = fromIntegral i
```

1.2 Concatenation

Now that we can get the number of digits of an arbitrary number, we can implement the operator.

To shift a base-10 number by some number of digits d to the left, multiply the number by ten to the number of digits:

$$a \times 10^d$$

Since this operation effectively fills the right-hand side with zeros, a simple addition with the right-hand operand will effectively replace the zeros with the digits of the right-side operator:

$$a \times 10^{digits(b)} + b$$

Because `||` is already a default Haskell operator, we mustn't override it! Instead, we shall invent a new infix operator. To be extra obnoxious, not only will we define a new operator in Haskell (yuck!), but it will bear the appearance of an extraordinarily kawaii cat emoticon: `=^.^=` (the cat is for “concatenation”).

```
(=^.^=) :: Integral a => a -> a -> a
a =^.^= b = a * 10 ^ (digits b) + b
```

Appendix: Main function

This computes the solution to the 10,958 problem [1].

```
main = putStrLn $ show solution
      where solution = 1 * 2 =^.^= 3 + ((4 * 5 * 6) =^.^= 7 + 8) * 9
```

Appendix: Defining \log_{10}

For strange floating point reasons, `log10 1000` is less than 3, but we'll ignore that for now. The alternative solution is to use primitive values, but that's a pain.

```
log10 = logBase 10
```

Appendix: Generalization for any base

The generalized concatenation operator, defined for any base is as follows:

```
cat :: Integral a => a -> a -> a -> a
cat base a b = a * base ^ (places b) + b
      where places d = floor (log b' / log base') + 1
              base' = fromIntegral base
              b' = fromIntegral b
```

References

- [1] Matt Parker, *A 10,958 Solution*. Numberphile, Ed: Brady Haran, 2017. Available: <https://www.youtube.com/watch?v=pasyRUj7UwM>.