Defining Parker's Concatenation

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The goal is to define an operator, $a \blacksquare b$ such that the result is the concatenation of (base-10) digits of each of its operands.

1 Implementation

The implementation shifts the left-hand operand a by the number of digits in the right-hand operand b. To do this, we must determine, mathematically, how many digits are in the right-hand operand.

1.1 Digits

First, we wish to determine how many digits there are in a number. This will indicate how many times we should multiply a by ten.

$$digits(x) = \lfloor \log_{10} x \rfloor + 1$$

```
digits :: Integral a ⇒ a → a
digits i = floor(log10 x) + 1
    where x = fromIntegral i
```

1.2 Concatenation

Now that we can get the number of digits of an arbitrary number, we can implement the operator.

To shift a base-10 number by some number of digits d to the left, multiply the number by ten to the number of digits:

$$a \times 10^{d}$$

Since this operation effectively fills the right-hand side with zeros, a simple addition with the right-hand operand will effectively replace the zeros with the digits of the right-side operator:

$$a \times 10^{digits(b)} + b$$

Because `||` is already a default Haskell operator, we mustn't override it! Instead, we shall invent a new infix operator: ■. In true `#ParkerSquare` spirit, I could not get it to typeset quite perfectly in LaTeX, however, the following code compiles fine in GHC. I gave it a go.

```
(■) :: Integral a ⇒ a → a → a
a ■ b = a * 10 ^ (digits b) + b
```

2 Main

This computes the solution to the 10,958 problem [1].

```
main = putStrLn $ show solution
    where solution = 1 * 2 ■ 3 + ((4 * 5 * 6) ■ 7 + 8) * 9
```

Appendix: Generalization for any base

The generalized concatenation operator, defined for any base is as follows:

```
cat :: Integral a ⇒ a → a → a → a
cat base a b = a * base ^ (places b) + b
    where places d = floor(log b' / log base') + 1
          base' = fromIntegral base
          b' = fromIntegral b
```

Appendix: Defining $\log_{10}$

For strange floating point reasons, `log10 1000` is less than 3, but we'll ignore that for now. The alternative solution is to use primitive values, but that's a pain.

```
log10 = logBase 10
```

References

[1] Matt Parker, *A 10,958 Solution*. Numberphile, Ed: Brady Haran, 2017. Available: `https://www.youtube.com/watch?v=pasyRUj7UwM`.