Mathematical Modeling and Simulation of a Zombie Epidemic

Eddie Shim[1], Hengyi Wu[2], and Melody Duan[3]

I. Introduction

Everywhere we see zombies: from video games and comic books to literature and performance art. Recently, there has even been a pop culture phenomenon of introducing classical literature to the zombie genre, producing works such as *Pride and Prejudice and Zombies* [1]. That the zombies are pervasive in mainstream media cannot be disputed. However, descriptions and characteristics of zombies seem to vary widely from one account to another. Certain zombies are described as fast moving and ferociously intelligent in their attacks, while others creep around slowly, mindlessly following the scent of humans. There is lack of a clear articulation of what constitutes a zombie, besides the quality of undead.

The zombie modeled in this particular simulation is one most famously propagated by the cult-classic *Night of the Living Dead*: a slow-moving, re-animated corpse who feeds on the flesh of the living[2]. Although there exist other 'types' of zombies, we have chosen not to consider them in this simulation; while we should try to be as broad as possible when establishing mathematical models, it would be difficult to capture many different, possibly contradicting, components simultaneously. By setting into concrete the characteristics of the model zombie, we are able to establish some guidelines for the mathematical model.

Lastly, while the scenarios considered here are unlikely to become realistic, this model may have certain applications in the real world. One example includes analyzing the spread of political views, religion, or any such beliefs.

II. Basic Model

For the basic model, we consider three classes:

1. S = number of susceptible humans
2. I = number of infected humans
3. Z = number of zombies
4. D = number of dead zombies

All three of the above variables are functions of time.

The parameters of the model are as follows:

α = infectivity of the disease

δ = zombification rate for an infected person

κ = death rate for the zombie population

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All parameters are given as probabilities per unit time. Susceptibles can become infected through transmission via an encounter with a zombie (parameter α). Humans in the infected class who are affected by the virus shortly expire and resurrect to become a zombie (parameter δ). Only humans can become infected through contact with zombies, and only zombies have a craving for human flesh so we do not consider any other life forms in the model. New zombies can only come from one source:

1. Infected who have deceased

In addition, we assume no birth rate in this epidemic model, as the event occurs over a short span of time. However, we assume humans will actively seek to defeat the zombies. This can be done by removing the head or destroying the brain of the zombie (parameter κ). We also assume that zombies do not attack other zombies.

Thus, the basic model is given by

$$\frac{dS}{dt} = -\alpha\left(\frac{I+Z}{N}\right)S \tag{1}$$

$$\frac{dI}{dt} = +\alpha\left(\frac{I+Z}{N}\right)S - \delta I \tag{2}$$

$$\frac{dZ}{dt} = +\delta I - \kappa\left(\frac{I+S}{N}\right)Z \tag{3}$$

$$N = S + I + Z + D \tag{4}$$

At this point, we will create a fourth class simply to keep count of the dead zombies (D)

$$\frac{dD}{dt} = +\kappa\left(\frac{I+S}{N}\right)Z \tag{5}$$

The SIZD model is illustrated in Figure 1.

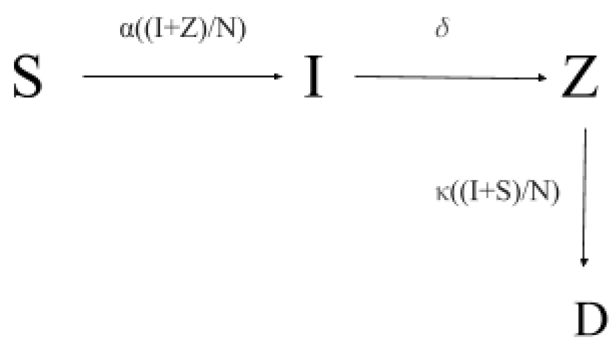


FIG. 1. The SIZD model flowchart

This model is a slightly varied and more complicated version of the SIR model usually used to characterize infectious, epidemic diseases [3].

III. Computer Program of Basic Model

```
%Project 2
%deterministic model of a zombie epidemic
%Eddie (Gyu Myung) Shim, Hengyi Wu, Melody J Duan

clear
```

```
a=.1;                   %rate of infection
delta=.5;               %rate of infected dying to zombie state
kappa = .8;       %rate of death for zombies
S=95;                   %initial susceptible
I=5;                    %initial infected
Z=0;                    %initial zombies
D=0;                    %initial dead
N=S+I+Z+D;              %total population

T=50;
dt=1;
clockmax=ceil(T/dt);

s=zeros(1,clockmax);    %array to save values of s
i=s;
z=s;
d=s;
t=0:dt:(T-dt);
for clock=1:clockmax
      S2I=dt*a*((I+Z)/N)*S;   %susceptible to infection per dt
      I2Z=dt*delta*I;         %infected to zombie per dt
      Z2D=dt*kappa*((I+S)/N)*Z;     %zombie to death per dt

      S=S-S2I;
      I=I+S2I-I2Z;
      Z=Z+I2Z-Z2D;


      D=D+Z2D;

      if (Z<0)
      negativeZ = Z;
      Z= Z-negativeZ;
      D= D+negativeZ;

      end

      s(clock)=S;
      i(clock)=I;
      z(clock)=Z;
      d(clock)=D;
end

figure(1)
plot(t,s,t,i,t,z,t,d,'linewidth',2)
%ylim([0 1100]);
legend('(S):susceptible population','(I):infected population','(Z):zombie
population', '(D):dead population')
```

IV. Model With Control Method

The basic model often produced results that were pessimistic for the prospects of humanity. In an effort to contain the zombie epidemic, we decided to introduce a vaccine into the model. Our vaccine would not provide treatment for the zombie individual to return to human form, but only to remove humans from the susceptible class to a new recovered class; thus, our vaccine provides immunity.The most natural implementation of a vaccine is to allow a fixed number of susceptible people to be vaccinated each day. We now must consider five classes for the vaccine model

1. S = numbers of susceptible humans
2. I = number of infected humans
3. Z = number of zombies
4. D = number of dead zombies
5. R = number of recovered humans, immune to the infection

With a new parameter

$\beta =$ number of people vaccinated per unit time

Thus the equations for this model are given by the following:

$$\frac{dS}{dt} = -\alpha(\frac{I+Z}{N})S - \beta \quad (6)$$

$$\frac{dI}{dt} = +\alpha(\frac{I+Z}{N})S - \delta I \quad (7)$$

$$\frac{dZ}{dt} = +\delta I - \kappa(\frac{I+S}{N})Z \quad (8)$$

$$\frac{dD}{dt} = +\kappa(\frac{I+S}{N})Z \quad (9)$$

$$\frac{dR}{dt} = +\beta \quad (10)$$

$$N = S + I + Z + D + R \quad (11)$$

The model is illustrated in figure 2.

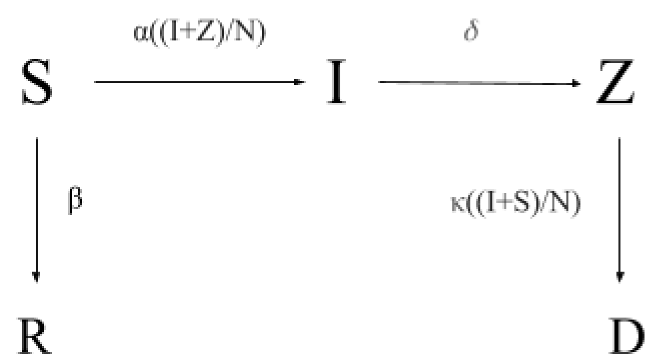


FIG. 2. SIZD model with vaccine

V. Computer Program of Model with Vaccine

```
%Project 2
```

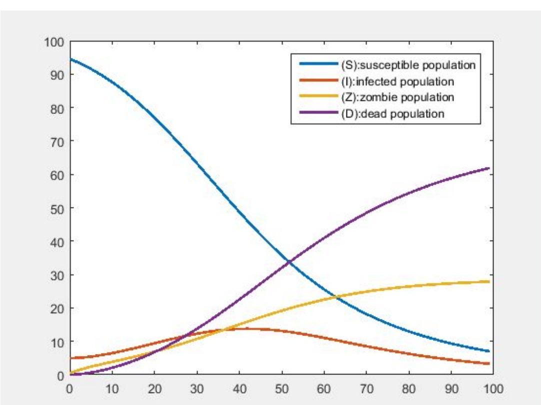
```
%deterministic model of a zombie epidemic
%Eddie (Gyu Myung) Shim, Hengyi Wu, Melody J Duan

clear

a=.3;                   %rate of infection
delta=.5;               %rate of infected succumbing to zombie state
kappa = .6;             %rate of death for zombies
beta=1;                 %rate of vaccination
S=95;                   %initial susceptible
I=5;                    %initial infected
Z=0;                    %initial zombies
D=0;                    %initial dead
R=0;
N=S+I+Z+D+R;            %total population


T=100;
dt=1;
clockmax=ceil(T/dt);

s=zeros(1,clockmax);    %array to save values of s
i=s;
z=s;
d=s;
r=s;
t=0:dt:(T-dt);
for clock=1:clockmax
      S2I=dt*a*((I+Z)/N)*S;   %susceptible to infection per dt
      I2Z=dt*delta*I;         %infected to zombie per dt
      Z2D=dt*kappa*((I+S)/N)*Z; %zombie to death per dt
      S2R=dt*beta;

      S=S-S2I-S2R;
      I=I+S2I-I2Z;
      Z=Z+I2Z-Z2D;
      D=D+Z2D;
    R=R+S2R;

    if (S<0)
      R=R+S2R+S;
        S=0;
        beta=0;

      end

      if (Z<0)
      negativeZ = Z;
```

```
        Z= 0;
        D= D+negativeZ;

        end

        s(clock)=S;
        i(clock)=I;
        z(clock)=Z;
        d(clock)=D;
      r(clock)=R;
end

figure(1)
plot(t,s,t,i,t,z,t,d,t,r,'linewidth',2)
%ylim([0 1100]);
legend('(S):susceptible population','(I):infected population','(Z):zombie
population', '(D):dead population','(R):recovered population')
```

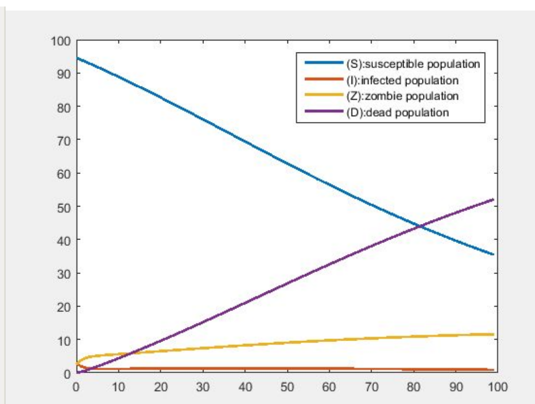
VI. Results and Discussion

We are mainly interested in the effects of three variables -- alpha, delta, and kappa. These represent the rate of infection, rate of infected to zombie, and rate of death (killing) of zombies respectively. We hypothesized there would be a few distinct cases in the model, dependent on how we adjusted these three variables. Therefore, to systematically search for these distinct cases, we decided to observe the three variables in three different states (high, medium, low rates) and mapped out 27 cases.

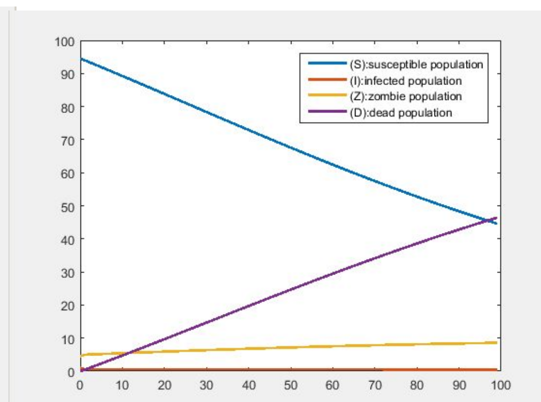
alpha = 0.1, delta = 0.1, kappa = 0.1



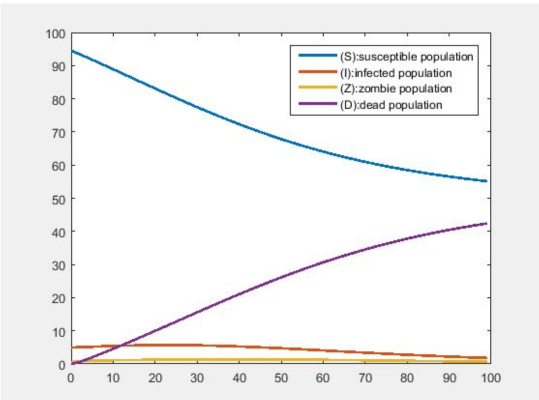
alpha = 0.1, delta = 0.5, kappa = 0.1



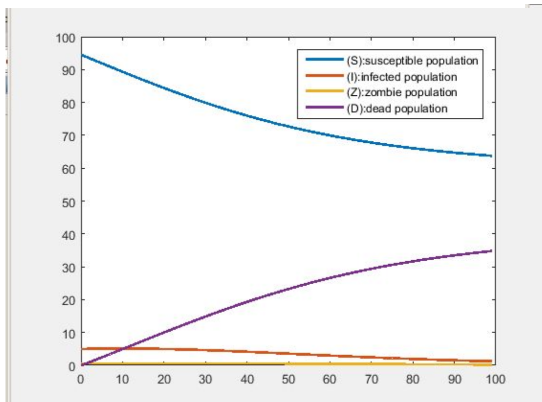
alpha = 0.1, delta = 0.9, kappa = 0.1



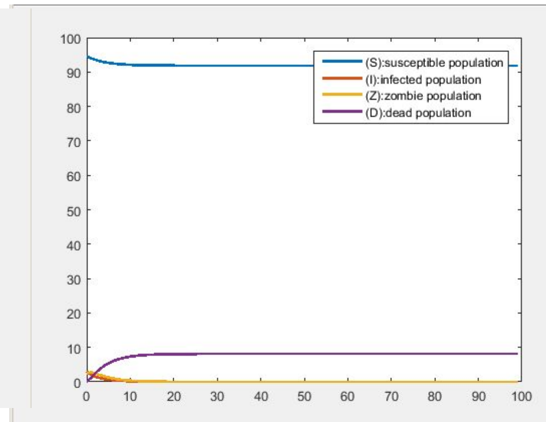
alpha = 0.1, delta = 0.1, kappa = 0.5



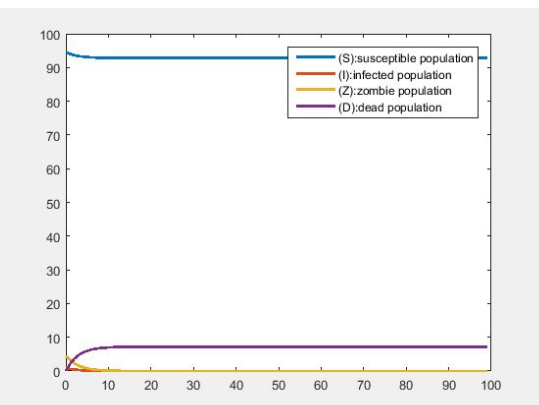
alpha = 0.1, delta = 0.1, kappa = 0.9



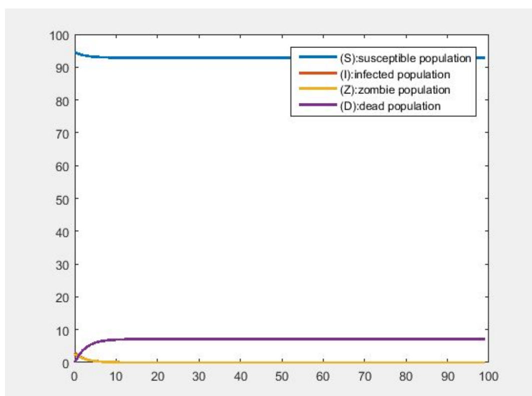
alpha = 0.1, delta = 0.5, kappa = 0.5



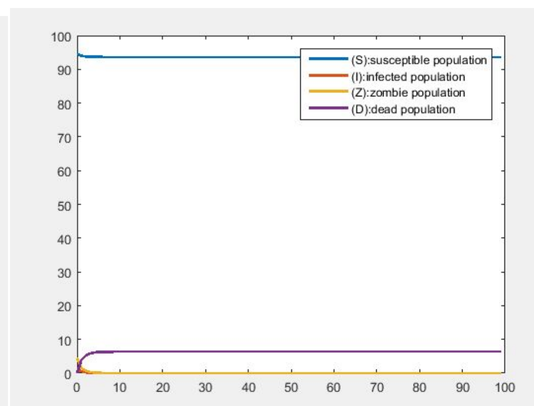
alpha = 0.1, delta = 0.9, kappa = 0.5



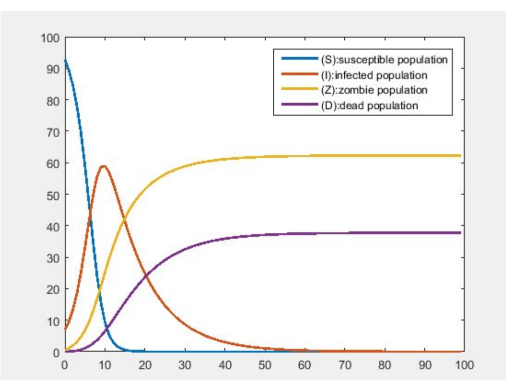
alpha = 0.1, delta = 0.5, kappa = 0.9



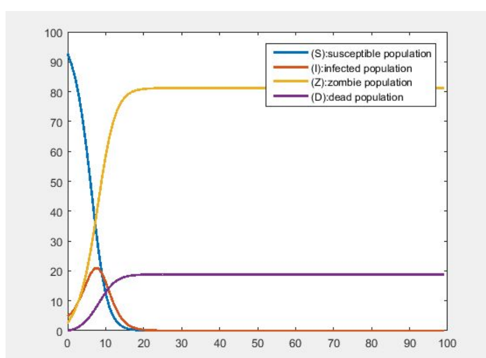
alpha = 0.1, delta = 0.9, kappa = 0.9



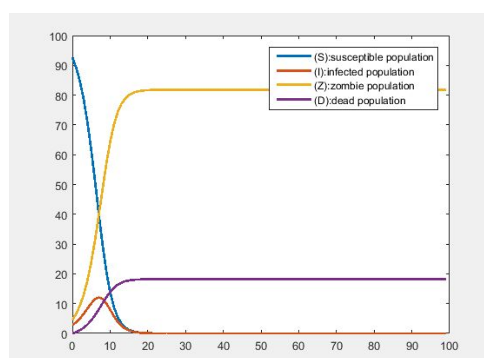
alpha = 0.5, delta = 0.1, kappa = 0.1



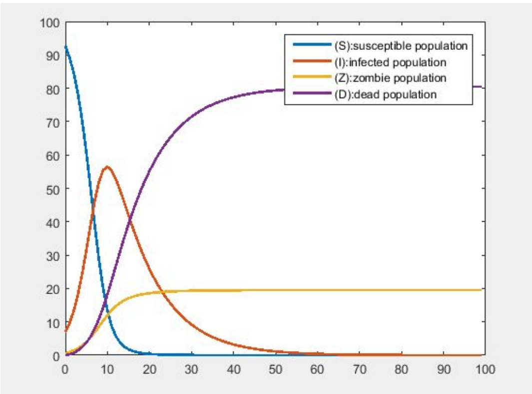
alpha = 0.5, delta = 0.5, kappa = 0.1



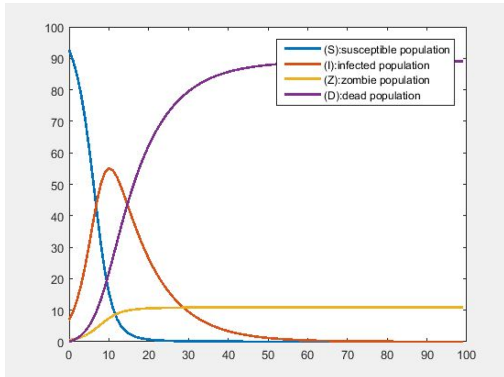
alpha = 0.5, delta = 0.9, kappa = 0.1



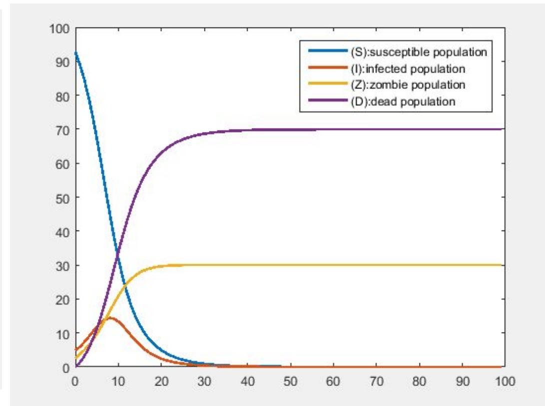
alpha = 0.5, delta = 0.1, kappa = 0.5



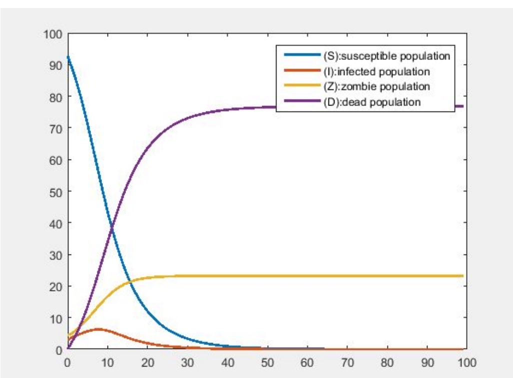
alpha = 0.5, delta = 0.1, kappa = 0.9



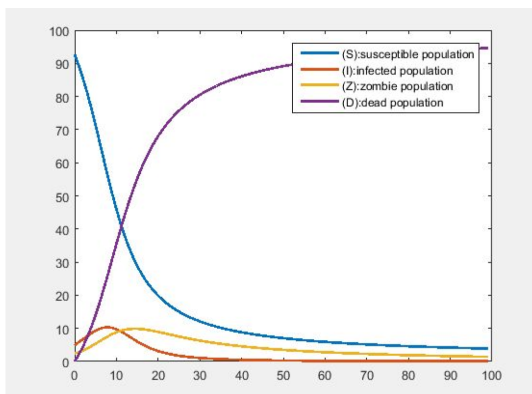
alpha = 0.5, delta = 0.5, kappa = 0.5



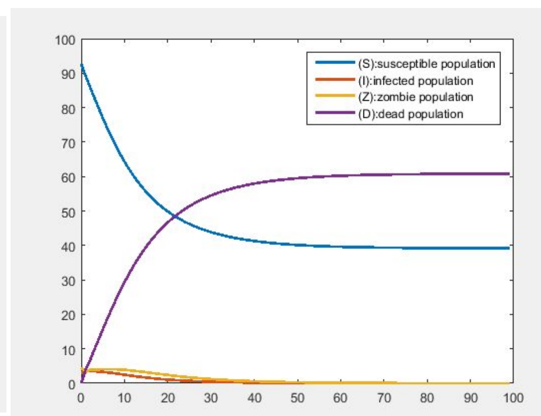
alpha = 0.5, delta = 0.9, kappa = 0.5



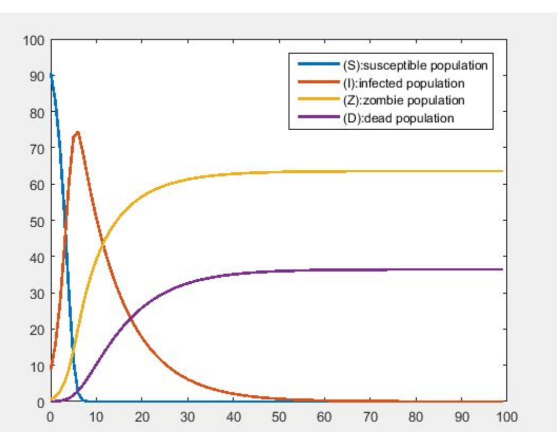
alpha = 0.5, delta = 0.5, kappa = 0.9



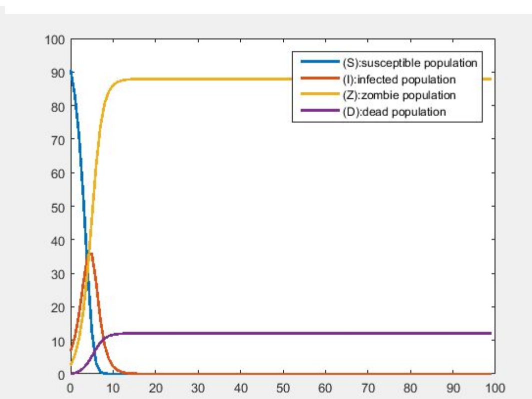
alpha = 0.5, delta = 0.9, kappa = 0.9



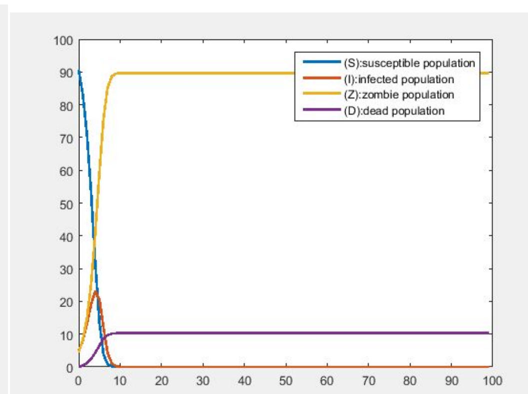
alpha = 0.9, delta = 0.1, kappa = 0.1



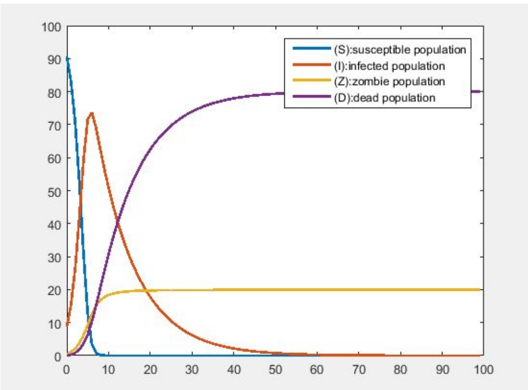
alpha = 0.9, delta = 0.5, kappa = 0.1



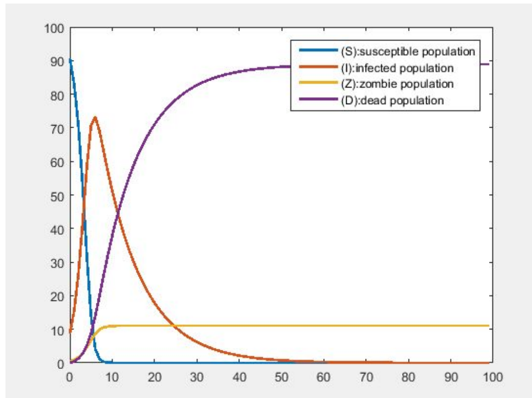
alpha = 0.9, delta = 0.9, kappa = 0.1



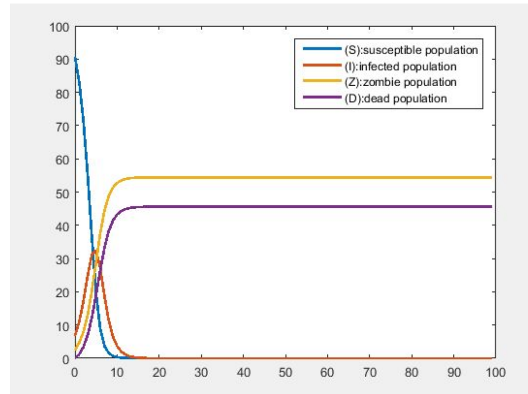
alpha = 0.9, delta = 0.1, kappa = 0.5



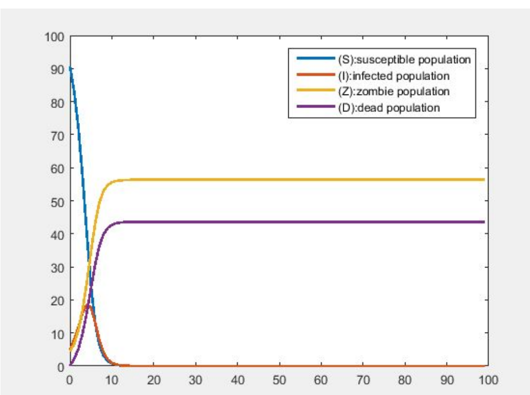
alpha = 0.9, delta = 0.1, kappa = 0.9



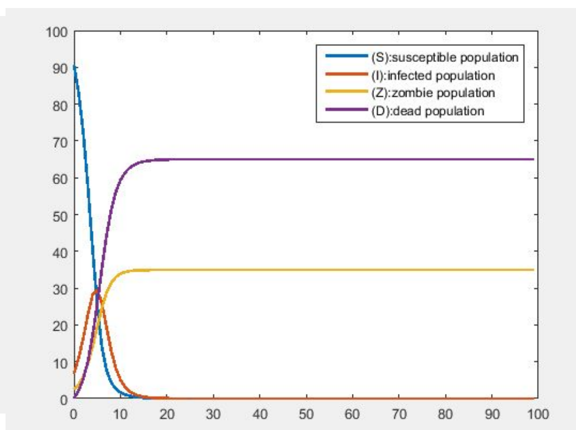
alpha = 0.9, delta = 0.5, kappa = 0.5



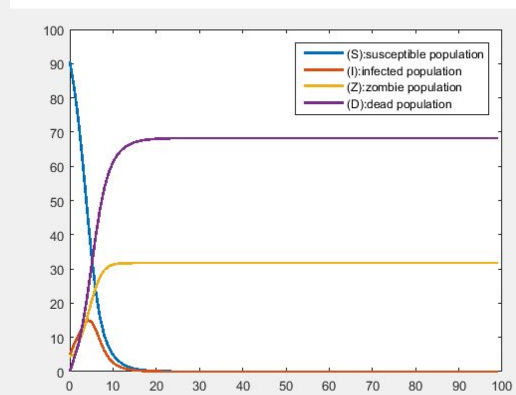
alpha = 0.9, delta = 0.9, kappa = 0.5



alpha = 0.9, delta = 0.5, kappa = 0.9



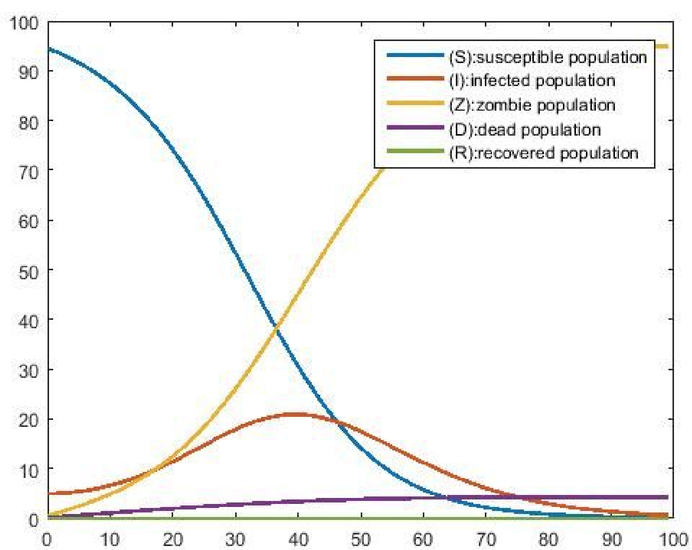
alpha = 0.9, delta = 0.9, kappa = 0.9



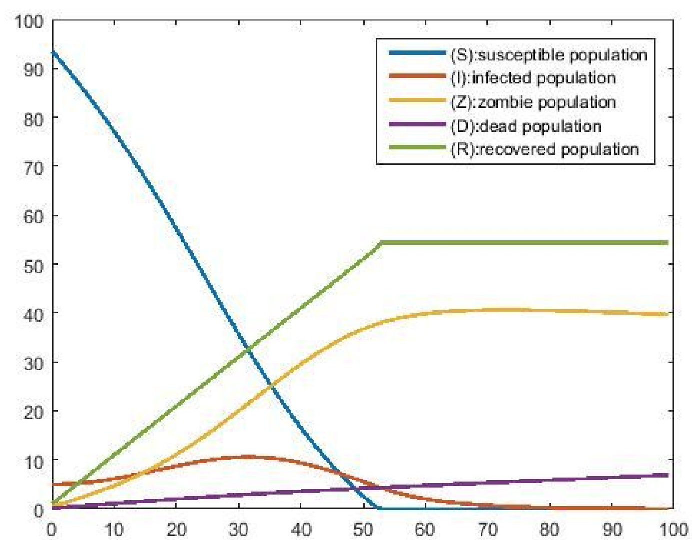
We noticed that by differing the alpha, delta, and kappa, there were distinct end results for our simulations. We noticed 6 cases in which the susceptible were prevalent, namely when alpha was low and kappa was high. This was in line with our intuition, where a low infectivity rate and high kill rate meant zombies could not thrive enough to overtake the healthy population. There were 0 cases in which infected prevaled, which makes sense because all infected people eventually become zombies, no matter how low delta is as long as it is nonzero. There were 8 cases in which zombies were prevalent, namely with mid-high alphas, mid-high deltas, and low kappas. From here we observed that alpha was the key variable; it is the bottleneck which controlled the rate of depleting the susceptible population. Even if the humans killed zombies at an extremely high rate, as long as the infectivity rate was not too small, the result would be that the zombie population would prevail. Lastly, there were 13 cases where death prevailed, which was occurred with mid-high alphas, and high kappas. This intuitively means a high infection rate and a high kill rate, leading to a high death rate.

From the simulation results of the basic model, it is clear that an outbreak of zombie infection is likely to be disastrous for the human species. Without a sufficient control method, almost all of the simulations resulted in eradication of mankind. Thus, we simulated a model in which a vaccine was introduced to give humanity a better chance at survival. Naturally implementing a vaccine will allow a greater percentage of the population to survive, but the more interesting question is how varying the three parameters (alpha, delta, and kappa) will affect the effectiveness of the vaccine.

No Vaccine

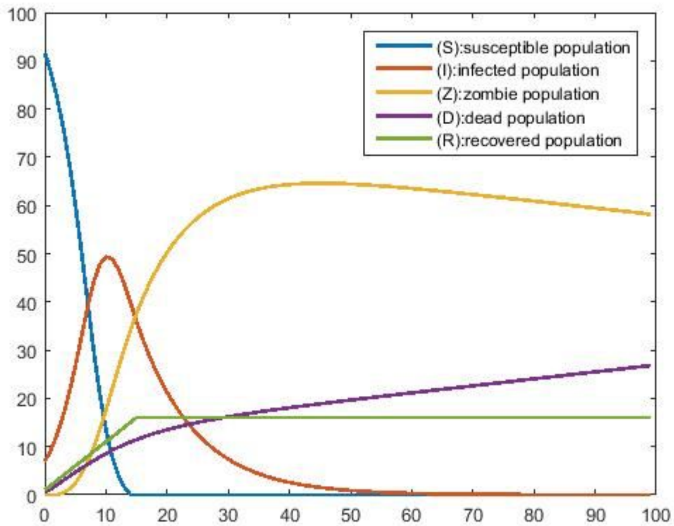


Vaccine: β=1

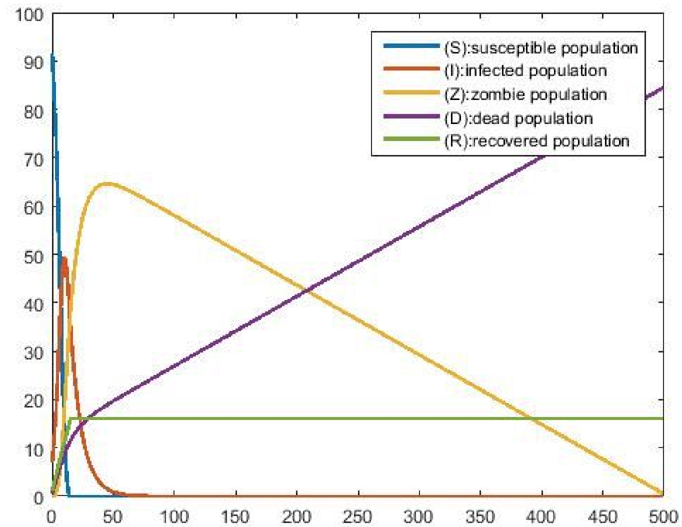


We first notice that it is guaranteed that a certain portion the population will survive since there are immune to the zombie virus. In particular, before the introduction of a vaccine, there were instances in which the human population was completely wiped out and the only categories that remained were the zombie population and the dead population. With a vaccine, there will definitively be a human population that survives the zombie onslaught. Note that the susceptible population always decreases to zero, but this does not mean that the human population is always becoming extinct. Instead the human population that remains alive all belong to the recovered population. We see that only vaccinating one person per day changes the situation dramatically and allows almost 60 people to survive.

Vaccine: β=1, T=100

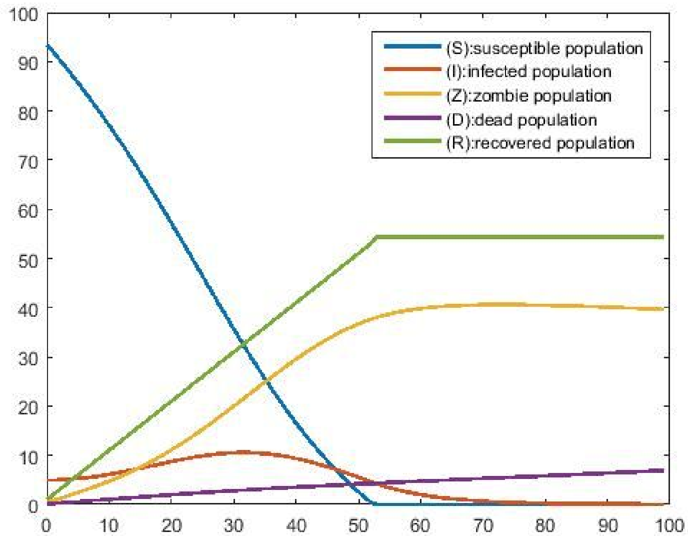


Vaccine: β=1, T=500

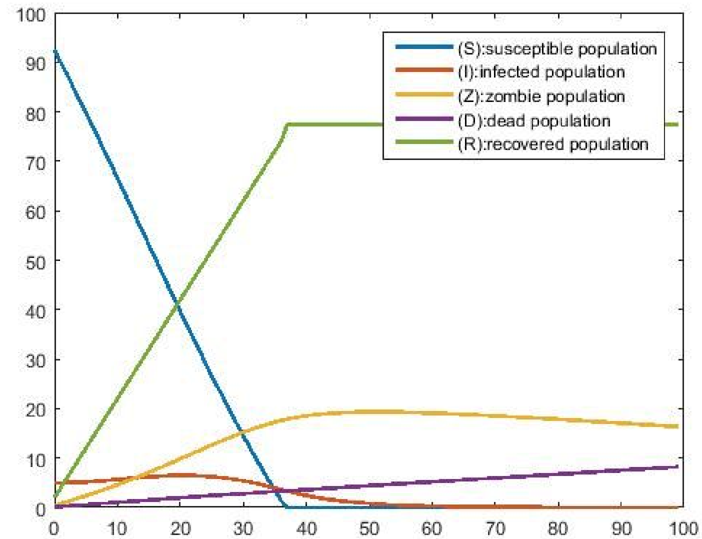


In the model without the vaccine, we generally saw the zombie population increase and then plateau when it reached a maximum. However, with the vaccine we see that by increasing the number of days to 500, the zombie population eventually dies out. Since we have a portion the population that is essentially invincible, they can fight the zombies without any risk of dying. The implication of this is that given enough time, the zombie population will be completely killed off. Thus, in this vaccine model, the only possibility is that there is a recovered population and a dead population at the end of the simulation.

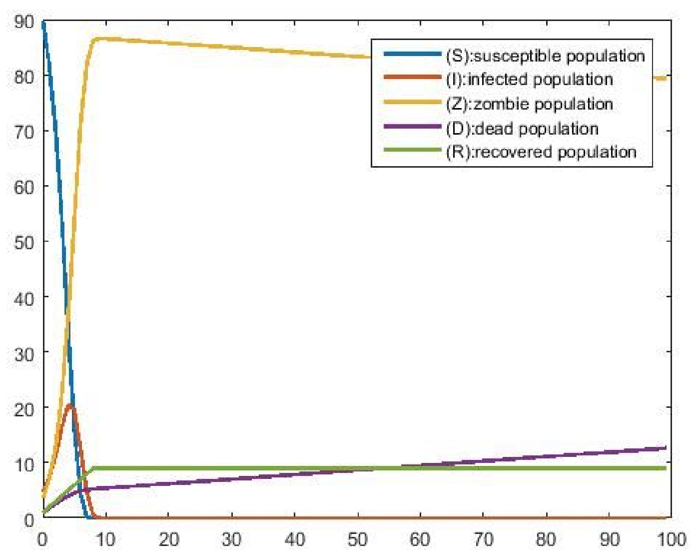
Low Infectivity Rate, β=1



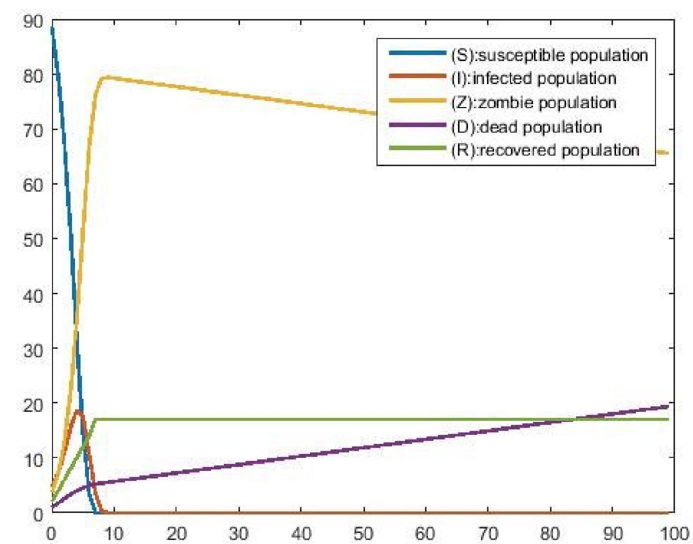
Low Infectivity Rate, β=2



High Infectivity Rate, β=1



High Infectivity Rate, β=2



Furthermore, we discovered that the "infectivity" rate (alpha) has a strong influence on the effectiveness of the vaccine. When the infectivity rate is low, there is more time to vaccinate people before they become infected. Compared to the situation in which the infectivity rate is high, large number become infected before many people can become vaccinated. We see that increasing the number of people vaccinated per day by one saves about 25 more people in the low infectivity case, but the same increase in the high infectivity case saves less than 10 more people.

VII. Further Research

We have examined here only one control method of many. Next, a quarantine or a cure may be adopted to observe results and compare differences. Perhaps in certain control methods, a situation of coexistence is possible between the species. In such a case, the zombie infection becomes endemic in a growing population.

We could slightly alter the vaccine model and still allow vaccinated people to be killed (for example in physical combat) by zombies. This seems more realistic as vaccinated people are only immune from contracting the disease, not from dying altogether. This prevents the phenomenon where the zombie population decreases to zero given enough time.

The results obtained in this simulation was from a short time-scale, where birth and death rates of the human population were ignored. For a further study, we may choose to examine the zombie infection on a longer time-scale. Although it can easily be predicted that the difficulty for human survival increases with time, it may be interesting to see if with sufficient control methods, human survival over the long term is possible.

References and Notes

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