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Periodic Seasonal Reg-ARFIMA–GARCH Models for Daily Electricity Spot Prices

Siem Jan KOOPMAN, Marius OOMS, and M. Angeles CARNERO

Novel periodic extensions of dynamic long-memory regression models with autoregressive conditional heteroscedastic errors are considered for the analysis of daily electricity spot prices. The parameters of the model with mean and variance specifications are estimated simultaneously by the method of approximate maximum likelihood. The methods are implemented for time series of 1,200–4,400 daily price observations in four European power markets. Apart from persistence, heteroscedasticity, and extreme observations in prices, a novel empirical finding is the importance of day-of-the-week periodicity in the autocovariance function of electricity spot prices. In particular, the very persistent daily log prices from the Nord Pool power exchange of Norway are effectively modeled by our framework, which is also extended with explanatory variables to capture supply-and-demand effects. The daily log prices of the other three electricity markets—EEX in Germany, Powernext in France, and APX in The Netherlands—are less persistent, but periodicity is also highly significant. The dynamic behavior differs from market to market and depends primarily on the method of power generation: hydro power, power generated from fossil fuels, or nuclear power. The article improves on existing models in capturing the memory characteristics, which are important in derivative pricing and real option analysis.

KEY WORDS: Autoregressive fractionally integrated moving average model; Generalized autoregressive conditional heteroscedasticity model; Holiday effects; Long-memory process; Periodic autoregressive model; Volatility.

1. INTRODUCTION

Electricity supply has been the responsibility of public-private companies in many OECD countries until recently. It is anticipated that the private trading of electricity will further intensify in the future and eventually move toward fully privatized electricity markets. In such markets, large volumes of electricity power will be traded in the short and long term together with future contracts and options. Although similarities with financial markets exist with respect to its operations, the price formation at electricity markets is more complex, because it depends strongly on the short-term characteristics of the energy supply function. The instantaneous nature of electricity and the availability of different plant technologies lead to atypical supply functions. On the other hand, electricity demand functions typically depend on weather variables, seasons in the year, day-of-week effects, and holidays. These characteristics of electricity supply-and-demand functions determine the specific behavior of electricity prices encountered in empirical work. The dynamic behavior of prices is important for derivative pricing and real option analysis. Therefore, the empirical time series modeling of electricity prices is important for financial traders and investors.

Following the standard practice of modeling volatility in financial returns, we are interested in the conditional mean and variance of price innovations. For many efficient financial and commodity markets, log prices are assumed to behave as a random walk, and price innovations are obtained by simply taking first differences of log prices. The mean process of electricity log prices cannot be described simply by a random walk because of its specific characteristics. (See Escribano, Peña, and Villaplana 2002; Bunn and Karakatsani 2003 for reviews of the salient features of electricity prices.) The following characteristics are often considered: (a) *seasonality* in prices is due to the strong dependence of electricity demand on weather conditions as well as on social and economic activities, leading to different holiday and seasonal effects; (b) *mean-reversion* in electricity prices exists because weather is a dominant factor and influences equilibrium prices through changes in demand; (c) *jumps and spikes* can be due to the difficulty in storing large quantities of electricity so that supply and demand shocks cannot be easily smoothed out; and (d) *volatility clustering* is considered as a typical feature in financial markets where heavy trading takes place on underlying assets.

The literature on modeling and analyzing electricity prices is growing quickly; see the collection of articles by Bunn (2004), where different linear and nonlinear time series techniques are adopted in empirical work. Particular contributions of interest in the literature have been made by Lucia and Schwartz (2002) and Knittel and Roberts (2005), who argued for a mean-reversion model with deterministic seasonal mean functions and applied it to daily prices from the Nord Pool electricity power exchange and to Californian hourly electricity prices. Escribano et al. (2002) focused on volatility aspects using generalized autoregressive conditional heteroscedasticity (GARCH) models with possibly a jump-diffusion intensity parameter for daily spot prices from different electricity markets. Knittel and Roberts (2005) also included GARCH and jump processes in their model specification for hourly electricity prices.

In this article the importance of regression effects, periodicity, long memory, and volatility in electricity prices is highlighted, and a simultaneous model for these features is proposed. The parameters in this model are jointly estimated by the method of approximate maximum likelihood using daily electricity spot prices from different exchange markets. The importance of periodicity has been acknowledged by Wilkinson

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and Winsen (2002) and Hernáez et al. (2004), who pointed out that the pattern of prices varies across day types. We go further and look at periodicities for the deterministic yearly seasonal effect and for the day-of-week effects in the mean, variance, and autocovariance structures. Therefore, the parameters associated with the dynamics in the model are different for different days of the week. Haldrup and Nielsen (2004) estimated nonlinear nonperiodic long-memory models for hourly Nord Pool prices. Periodic seasonal long-memory models have not been considered in electricity prices. Here we show that seasonal long memory is an important feature in daily electricity prices. Although volatility has been considered in most empirical studies on electricity prices, we argue that volatility is a function not only of past squared price innovations, but that seasonal factors and other fixed effects in the variance equation are also important.

Equal attention is given to the modeling of the mean, variance, and autocovariance functions of the daily time series. The mean process includes deterministic effects and explanatory and intervention variables. Most coefficients in the mean are allowed to vary with day of the week (periodic). The variance process depends on day-of-week levels and yearly and half-yearly cosine waves, with deviations from these deterministic functions modeled by a GARCH process with a Student-t density. The autocovariances are determined by seasonal long-memory dynamics and lagged dependent variables with periodic coefficients. The different sets of parameters are treated simultaneously during the estimation process based on approximate maximum likelihood. The likelihood function is constructed as follows. The time series is corrected for the mean and the autocovariance features using the appropriate recursive filter for which the initial observations are treated as fixed and known. The resulting innovations are used as input for the conditional GARCH likelihood function for a t-distribution. The empirical study focuses on Nord Pool daily electricity spot prices between January 4, 1993 and April 10, 2005—that is more than 12 years of data (640 weeks)—equaling 4,480 daily observations. To show the robustness of the new modeling framework for daily electricity spot prices, we extend the model with explanatory variables capturing significant and interpretable demand supply-and-demand effects in the Nord Pool market. We also present and discuss the results for three emerging electricity markets in Europe.

The article is organized as follows. Section 2 describes the markets and datasets and provides the motivation for our modeling approach. Section 3 discusses the specification of the model and develops a simultaneous approach for estimation and inference regarding the parameters of the model. Section 4 reviews the empirical results for the daily prices of the Nord Pool market with and without nondeterministic explanatory variables. Section 5 shows that the same modeling approach can be successfully applied to emerging electricity markets in mainland Europe. Section 6 concludes.

2. EUROPEAN ELECTRICITY MARKETS AND DAILY SPOT PRICES

2.1 Some Facts About Electricity Markets

First, we examine the time series of daily spot electricity prices from the Nord Pool exchange market in Norway. Then we analyze price data from three other emerging European electricity markets: European Energy Exchange (EEX) in Germany, Powernext in France, and the Amsterdam Power Exchange (APX) in The Netherlands. These markets have started in different years, and thus the four daily time series are of different length. The oldest market is Nord Pool, started in 1991 for the trading of hydroelectric power generated in Norway. Sweden joined in 1996, Finland joined in 1998 and Denmark joined in 1999. In this article we consider only the Norwegian electricity prices. Most of this electricity is generated in hydroelectric power stations, and thus supply depends heavily on weather conditions. The average production capability of Norway's hydro power plants is about 113 Terawatt hours (1 TWh $= 10^9$ KWh) per year. However, this production depends on precipitation levels. The EEX market is the largest electricity market in mainland Europe, with 60 TWh traded in 2004. Powernext in France started in November 2001, and the volume traded reached 14.1 TWh in 2004, 3% of France's electricity consumption. The spot market APX has been operational since May 1999, and in 2004 a total of 13.4 TWh was traded on this market. All four markets operate as "day-ahead" markets that concentrate on daily trade for electricity delivered on the next day. Daily series are constructed as the average of 24 price series for the different hours of the day. The resulting prices are referred to as spot prices.

2.2 Time Series Descriptives of Nord Pool Electricity Spot Prices

We consider spot prices from the Nord Pool electricity market for January 4, 1993–April 10, 2005. Figure 1 plots the daily spot prices, denoted by Y_t and computed as the average of the 24 hourly prices, together with the daily first differences of $y_t = \log Y_t$. The spot prices vary over the years and are subject to yearly cycles, weekly patterns, persistent level changes, and spikes. The first differences of log prices (returns) show clear patterns of volatility clustering. It is tempting to conclude from these graphs that electricity spot prices exhibit the typical features of daily prices from other financial markets. However,

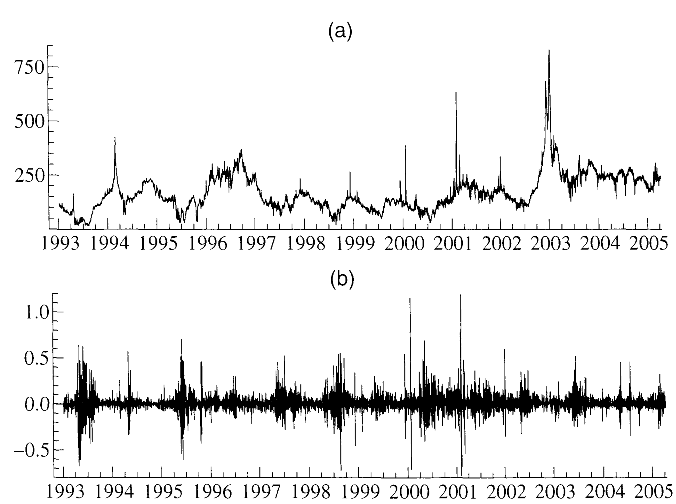

Figure 1. Daily Spot Prices for Nord Pool. (a) Prices in Norwegian kroners (NOK) per MWh, Y_t. (b) Daily returns: changes in log-transformed prices Δy_t. Sample: January 4, 1993–April 10, 2005. 1 euro is approximately 8 NOK.

Table 1. Descriptive Statistics Log Prices y_t and Covariates for European Power Markets

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Nord Pool: Δy_t							
Mean	.1124	.0105	−.0018	−.0082	−.0176	−.0667	−.0273
SD	.127	.095	.079	.067	.073	.100	.057
$r(1)$	−.26	−.22	.17	−.06	−.15	−.03	.15
$r(2)$	−.54	−.08	−.14	.04	.03	.13	.14
$r(7)$	.26	.05	−.08	−.03	.09	.46	.18
$r(21)$	.24	.01	−.03	.06	.02	.26	.05
EEX: y_t							
Mean	3.362	3.428	3.412	3.390	3.328	3.084	2.821
SD	.355	.374	.330	.381	.296	.271	.294
$r(1)$	.64	.62	.78	.74	.77	.73	.87
$r(2)$	.55	.42	.60	.61	.73	.62	.67
$r(7)$	.37	.23	.44	.29	.45	.46	.71
$r(21)$	.26	.14	.25	.21	.31	.48	.62
Powernext: y_t							
Mean	3.308	3.402	3.390	3.358	3.309	3.066	2.806
SD	.411	.333	.359	.343	.314	.305	.382
$r(1)$	.60	.67	.88	.76	.85	.84	.80
$r(2)$	.65	.58	.64	.72	.70	.71	.69
$r(7)$	.32	.41	.43	.44	.48	.58	.55
$r(21)$	.27	.34	.31	.34	.38	.44	.46
APX: y_t							
Mean	3.572	3.607	3.583	3.573	3.483	3.219	2.932
SD	.506	.504	.481	.438	.368	.293	.325
$r(1)$	.46	.79	.84	.72	.74	.41	.58
$r(2)$	.39	.33	.74	.64	.62	.41	.34
$r(7)$	.28	.26	.21	.33	.39	.48	.37
$r(21)$	.09	.17	.17	.28	.22	.38	.33
Nord Pool consumption: $x_{17,t}$							
Mean	12.6714	12.6756	12.6718	12.6771	12.6636	12.6046	12.5891
SD	.19890	.19391	.19372	.19669	.19895	.20269	.20683

NOTE: Mean refers to sample means. SD, sample standard deviations. $r(\tau)$: Periodic autocorrelation of Δy_t, y_t, $x_{17,t}$ for a lag of τ days. Prices are for Nord Pool in NOK per MWh; other prices are in Euros per MWh. Samples: Nord Pool log price changes and Reservoir levels: 638 weeks, January 18/1993–April 10/2005, EEX: 182 weeks, October 15/2002–April 10/2005, Powernext: 174 weeks, December 10/2002–April 10/2005, APX: 222 weeks, January 08/2001–April 10/2005. Reservoir levels as a percentage of total Norwegian capacity have sample mean 62.225 and standard deviation 20.657. NordPool log power consumption in MWh (around 300,000 MWh day). Sample log power consumption: 222 weeks, February 26/2001–April 10/2005. Sources: *www.statnett.no*, *www.eex.de*, *www.powernext.fr*, and *www.apx.nl*.

closer inspection reveals clear evidence that the dynamic properties of electricity spot prices are more intricate.

Table 1 presents summary statistics for the first differences of log prices for all data points associated with a particular day of the week. Note that for Monday, the first difference is with respect to the Sunday spot price, Tuesday's first difference is with respect to the Monday price, and so on. The reported periodic autocorrelations are computed as described by McLeod (1994); for example, the second column shows $r_{\mathrm{Mon}}(21) = \mathrm{corr}(\Delta p_t, \Delta p_{t-21}) = .24$. The large day-to-day differences in such autocorrelations motivate a periodic time series modeling approach. The persistence of the autocorrelations at the seasonal lags 7, 14, and so on is pronounced and clearly periodic and must be modeled explicitly; for example, for Mondays, the first seasonal correlations are .26, .21, .24, and .25, and for Saturdays, we have .46, .31, .26, and .25. The seasonal autocorrelations for the remaining days are not significant from lag 14 onward. The inclusion of long autoregressive polynomials in the model may capture these dynamics. A parsimonious alternative is to model the persistence by a seasonal fractional integration process. These findings of periodicity pertain to both the deterministic part and the dynamic part of the price process, as we illustrate in the following sections. Due to space considerations, other statistics are not presented here, but the autocorrelations remain periodic when nonstationarities due to other day-of-the-week effects and yearly weather cycles have been removed from the data by regression or by seasonal differencing. This is also evident from model estimates presented herein.

2.3 Explanatory Variables for Nord Pool Prices

Although univariate time series modeling of electricity prices is important in its own right, it is interesting to extend the analysis using publicly available data on the determinants of power demand and supply. The two most relevant and closely watched variables for the hydropower market of Nord Pool are daily data on Norwegian power consumption and weekly measurements of the overall water reservoir levels in Norway. Figure 2(a) shows a time series plot of the water reservoir levels as a percentage of total Norwegian capacity for 1993–2005. Figure 2(b) presents daily aggregate power consumption data, which are only available for 2001–2005. Both series are dominated by yearly cycles. In addition, the reservoir levels seem to exhibit long memory, and the power consumption clearly shows a varying weekly pattern. These features might explain some of the dynamic characteristics of electricity prices in a meaningful way. We expect a negative effect on prices of (unexpected) positive shocks in water levels and a positive effect of (unexpected) positive shocks in consumption. In this article we do not attempt to model power consumption and water reservoir levels.

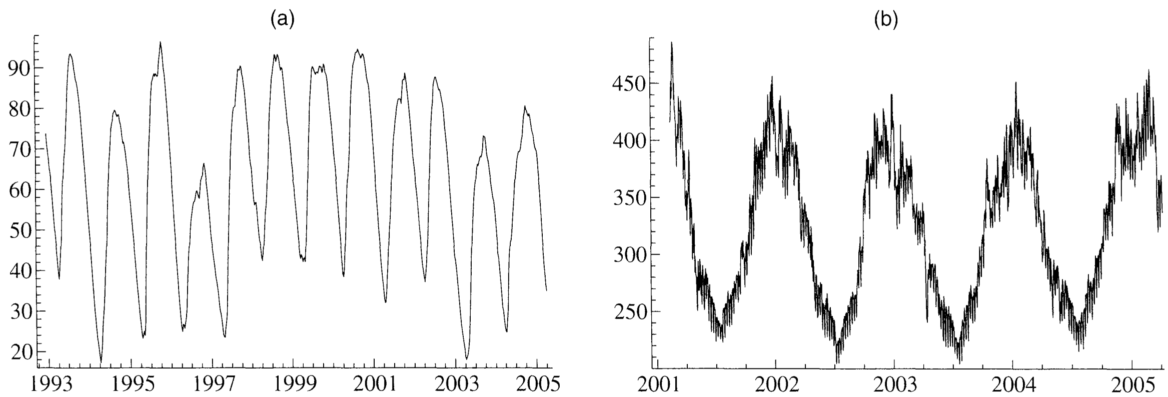

Figure 2. Weekly Water Reservoir Levels and Daily Consumption for Nord Pool. (a) Reservoir levels as a percentage of total Norwegian capacity. Sample: January 4, 1993–April 10, 2005. (b) Daily Norwegian power consumption in GWh/day. Sample: February 26, 2001–April 10, 2005. Source: www.statnett.no.

2.4 Time Series Descriptives of Other Electricity Spot Prices

Time series of the logarithms of spot prices (in Euros/MWh) from three European emerging electricity markets are presented in Figure 3. The time series from EEX, Powernext, and APX have different lengths and are shorter than the time series of Nord Pool. The dynamic properties of the three time series seem different from the behavior of Nord Pool. This is most likely due to the type of electricity traded on the different markets. Most of the electricity traded on the Nord Pool market is produced by hydro power generation and thus depends on long-run weather conditions. In the APX and EEX, most of the electricity traded is thermal (from the burning of coal and gas), whereas the Powernext market trades electricity produced mainly by nuclear power plants. These new markets are also strongly linked to each other by high-voltage power lines. This leads to clear comovements in prices unrelated to the price swings in the Nord Pool market. We do not model the three series simultaneously, however.

Table 1 also contains periodic descriptive statistics for the EEX, Powernext, and APX markets. Because these prices are less persistent than the Nord Pool prices, we present descriptive statistics for log prices rather than for returns. Table 1 shows that the mean and variance of these daily electricity prices also depend on the day of the week, as does the autocorrelation structure. There are several similarities across the markets. For example, the mean is larger on Tuesdays and smaller on Sundays, whereas the correlation between Wednesdays and Tuesdays (.78, .88, and .84 for EEX, Powernext, and APX) is higher than the correlation between Mondays and preceding Sundays (.64, .60, and .46), as would be expected. Turning to periodic

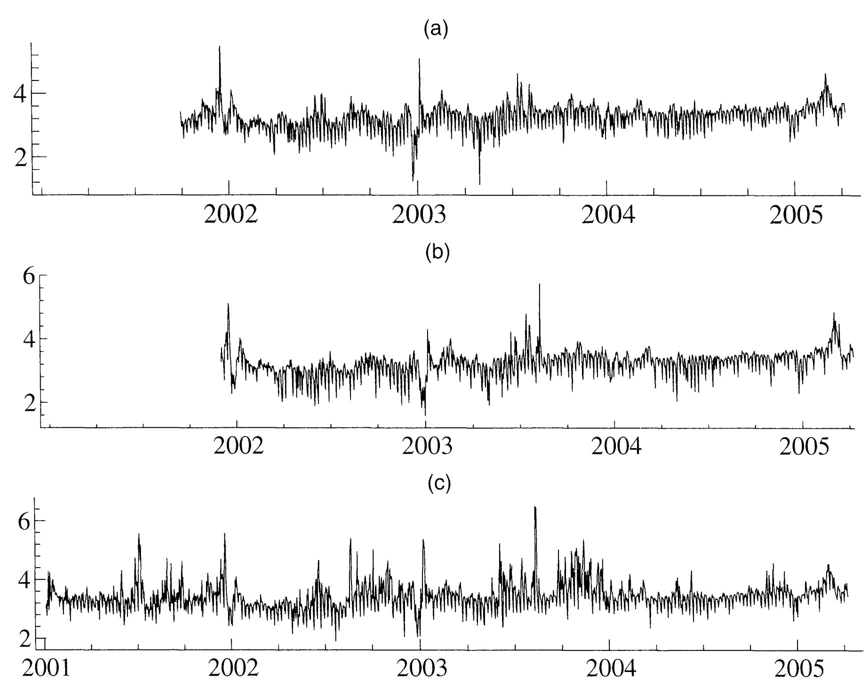

Figure 3. Log Daily Spot Prices for Three New European Electricity Markets. Prices in Euros/MWh. Samples: EEX (a): October 1, 2001–April 10, 2005; Powernext (b): December 3, 2001–April 10, 2005, APX: January 1, 2001–April 10, 2005. Sources (c): www.eex.de, www.powernext.fr, and www.apx.nl.

long-memory characteristics, the 3-week autocorrelations for Mondays (.26, .27, and .09) are considerably lower than those for Saturdays (.48, .44, and .38).

3. THE PERIODIC SEASONAL REG–ARFIMA–GARCH MODEL

3.1 Model Specification

Consider a time series of logged electricity prices y_t for $t = 1, \dots, T$ and with s periods or seasons. Period j is a modulus function of the time index t, that is, $j = j(t) = 1 + t \bmod s$. In the empirical section, we analyze daily log-transformed electricity spot prices, where the seasonal variation is due mainly to weekly patterns and thus the seasonal length is $s = 7$. Seasonal variations due to monthly and quarterly patterns can be captured by specific covariates in the model. We first describe our model before relating it to the existing literature.

An effective model for capturing the salient features of electricity price series as discussed in the previous section is the regression model with seasonal periodic autoregressive fractionally integrated moving average (ARFIMA) disturbances, that is,

$$\Phi_j(L^s)(1 - L^s)^{D_j}(y_t - \mu_t) = \Theta_j(L^s)\varepsilon_t,$$
$$\varepsilon_t | \mathcal{F}_{t-1} \sim \mathrm{NID}(0, \sigma_t^2), \quad t = 1, \dots, T, \tag{1}$$

with $j = j(t)$ and where $\mu_t = E(y_t | \mathcal{F}_{t-1})$ and $\sigma_t^2 = \mathrm{var}(y_t | \mathcal{F}_{t-1})$ are the conditional mean and variance functions for an appropriate filtration $\mathcal{F}_t$. The periodic polynomials $\Phi_j(L^s)$ and $\Theta_j(L^s)$ are in the lag operator L defined by $L^k y_t = y_{t-k}$. These polynomials do not play a role in the empirical analysis of this article, and thus we do not consider these lag polynomials further and have $\Phi_j(L^s) = \Theta_j(L^s) = 1$. The scalar periodic coefficient D_j determines the order of seasonal fractional integration for which the stationarity and invertibility conditions apply, that is, $|D_j| < .5$ for period $j = 1, \dots, s$. Using a binomial expansion, we formally define

$$(1 - L^s)^{D_j} = \sum_{i=1}^{\infty} \frac{\Gamma(i - D_j)}{\Gamma(-D_j)\Gamma(i+1)} L^{is}, \tag{2}$$

where $\Gamma(\cdot)$ is the gamma function. It follows from (2) that fractional integration implies an infinite-order lag polynomial.

The conditional mean function is given by

$$\mu_t = \phi_{1j} y_{t-1} + \cdots + \phi_{pj} y_{t-p} + \sum_{k=1}^{K} (\delta_{k0j} x_{kt} + \delta_{k1j} x_{k,t-1} + \cdots + \delta_{krj} x_{k,t-r}) \tag{3}$$

for $t = \max(p, r) + 1, \dots, T$, where ϕ_{ij} for $i = 1, \dots, p$ and δ_{kij} for $k = 1, \dots, K$ and $i = 0, 1, \dots, r$ are periodic regression coefficients for periods $j = 1, \dots, s$ when y_t or x_{kt} are observed in period $j = j(t)$. The set of coefficients ϕ_{ij} implies a periodic autoregressive polynomial for y_t that is assumed to be causal. In practice, we take $p < s$ and $r < s$, so that seasonal lags do not play a role in the conditional mean equation; they play a role only in the ARFIMA specification. The covariates x_{kt} can be assumed to be either deterministic or weakly exogenous, for example, $E(x_{k,t-i}\varepsilon_t) = 0$ for $k = 1, \dots, K$ and $i = 0, 1, \dots, r$. The mean function μ_t is referred to as conditional because it is properly defined only when past values of y_t and concurrent and past x_{kt}'s are treated as known. Therefore, μ_t is properly defined only for $t = \max(p, r) + 1, \dots, T$.

The conditional time-varying variance process for $\sigma_t^2 = \mathrm{var}(y_t | \mathcal{F}_{t-1})$ is specified by the GARCH model with regression effects and scaled by seasonal factors, that is,

$$\sigma_t^2 = \exp(\lambda_j) h_t,$$
$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \sum_{k=1}^{K^*} \gamma_k z_{kt}, \tag{4}$$
$$t = \max(p, r) + 2, \dots, T,$$

with $j = j(t)$ and unknown coefficients α_0, α_1, β_1, and $\gamma_1, \dots, \gamma_{K^*}$. The seasonal factors $\lambda_1, \dots, \lambda_s$ are also unknown coefficients, but not all can be identified, and thus we restrict $\lambda_1 = 0$. Higher-order lags for h_t and ε_t^2 in (4) can be considered as well, but these do not play a role in our empirical analysis. The covariates z_{kt} are assumed to be deterministic for $k = 1, \dots, K^*$ and typically consist of time functions and dummy variables. The autoregressive process for h_t is initialized by its unconditional mean, which is replaced by the corresponding sample mean in the estimation process, discussed later. We assume and impose that $0 \le \alpha_1 + \beta_1 \le 1$.

Initially, the distribution of ε_t in (1) is assumed to be Gaussian. Given the fact that time series of electricity spot prices are usually fat-tailed, we consider the Student-t distribution for the disturbances ε_t, that is

$$\varepsilon_t | \mathcal{F}_{t-1} \sim t_\nu(0, \sigma_t^2), \qquad t = 1, \dots, T, \tag{5}$$

with degrees of freedom ν, mean 0, and time-varying variance σ_t^2. The disturbances $\varepsilon_1, \dots, \varepsilon_T$ are assumed to be serially uncorrelated. In the case of t-disturbances for model (1) with μ_t and σ_t^2 given by (3) and (4), respectively, the time index t takes the values $t = \max(p, r) + 2, \dots, T$.

The periodic seasonal regression ARFIMA model with seasonal heteroscedasticity and GARCH disturbances (the periodic seasonal Reg-ARFIMA–GARCH model) has a conditional mean equation with periodic coefficients. This suggests that each day of the week can be described by a different model. If all coefficients are periodic, including the ones of the conditional variance function, then we can isolate the periods from one another and estimate separate time-invariant models for the different periods (see Tiao and Grupe 1980). But we do not pursue this approach, because our focus is on more subtle periodic formulations in which only certain parameters are periodic. Before discussing estimation details, we briefly describe the main origins of our model in the existing literature.

Our model combines ideas from different strands of the statistical, geophysical and econometric literature. Periodic autoregressive models were first applied by Hannan (1955) and Jones and Brelsford (1967). Gladysev (1961) first analyzed the periodic correlation function and multivariate representation, whereas Tiao and Grupe (1980) discussed the consequences for traditional autoregressive moving average (ARMA) modeling if the underlying process really follows a periodic ARMA model. Estimation methods and algorithms for periodic ARMA models have been developed by Pagano (1978), Vecchia (1985), and Li and Hui (1988), among others. McLeod (1994) discussed the empirical identification of periodic autoregressive models. Further developments on likelihood evaluation and asymptotic theory for different estimators of periodic ARMA models have been discussed by Lund and Basawa (2000) and Basawa and Lund (2001).

The fractional differencing model introduced by Adenstedt (1974) has become a standard model for long-memory behavior. The generalization toward the ARFIMA model (1) with $s = 1$, $\sigma_t^2 = \sigma^2$ and no periodic coefficients was introduced by Granger and Joyeux (1980) and Hosking (1981). Statistical properties and inference for ARFIMA and other long-memory processes were discussed extensively by Beran (1994b), Baillie (1996), and, more recently, Robinson (2003). Carlin, Dempster, and Jonas (1985) provided an early analysis of ARFIMA models with seasonal fractional integration parameter D. As a final step, Ooms and Franses (2001) let the seasonal fractional D be periodic.

A novelty in this article is the introduction of a GARCH process for the variance of a periodic seasonal Reg-ARFIMA model. The GARCH model was developed by Engle (1982) and Bollerslev (1986). The statistical properties of GARCH processes are well established (see, e.g., Bollerslev, Engle, and Nelson 1994). Furthermore, Bollerslev and Ghysels (1996) introduced a periodic version of the GARCH model, that is slightly different than ours. The inclusion of regression effects in the variance specification of a nonseasonal and nonperiodic AR–GARCH model in the context of modeling electricity prices was considered by Byström (2005).

The present article extends ARFIMA–GARCH-t models with seasonal and periodic features. Baillie, Chung, and Tieslau (1996) first applied nonperiodic ARFIMA–GARCH models to price indexes. Ling and Li (1997) derived conditions for asymptotic normality of the approximate (Gaussian) maximum likelihood (ML) estimator in the ARFIMA–GARCH model.

3.2 Maximum Likelihood Estimation

The exact Gaussian log-likelihood function of the standard ARFIMA model (1) with $\boldsymbol{\mu}_t = \boldsymbol{\mu}$, $\sigma_t^2 = \sigma^2$, and $s = 1$ is given by

$$\log L(\mathbf{y}; \boldsymbol{\psi}) = -\frac{T}{2}\log 2\pi\sigma^2 - \frac{1}{2}\log|\mathbf{V}_\mathbf{y}| - \frac{1}{2\sigma^2}(\mathbf{y}-\boldsymbol{\mu})'\mathbf{V}_\mathbf{y}^{-1}(\mathbf{y}-\boldsymbol{\mu}), \quad (6)$$

where the parameter vector $\boldsymbol{\psi}$ collects all unknown coefficients of the model (1) and

$$\mathbf{y} = (y_1, \ldots, y_T)' \qquad \text{and} \qquad \sigma^2\mathbf{V}_\mathbf{y} = \mathrm{var}(\mathbf{y}),$$

with the variances and autocovariances in $\mathbf{V}_\mathbf{y}$ for an ARFIMA process computed by efficient methods such as those developed by Sowell (1992) and Doornik and Ooms (2003), who also discussed efficient methods for computing $\log L(\mathbf{y}; \boldsymbol{\psi})$ for the ARFIMA model using Durbin–Levinson methods for the necessary Choleski decomposition of $\mathbf{V}_\mathbf{y}$. The generalization toward an ARFIMA model with $s > 1$, periodic coefficients, and seasonal lags can in principle be implemented for evaluating the log-likelihood function. However, the computation of $\mathbf{V}_\mathbf{y}$ is intricate and not practical for large T, because no analytical expressions for $\mathbf{V}_\mathbf{y}$ exist in the case of a periodic seasonal ARFIMA model.

The time-varying conditional mean function (3) for the model (1) does not lead to further complexities if x_{kt} for $k = 1, \ldots, K$ is completely deterministic. In the case where any x_{kt} is correlated with ε_t, the likelihood approach requires a multivariate approach to appropriately deal with the autocovariance structure in $\mathbf{V}_\mathbf{y}$. This is beyond the scope of this article, however. Furthermore, when the time-varying conditional variance σ_t^2 is modeled as the GARCH process (4), other complexities for a full-information ML approach arise. Therefore, we adopt the approximate likelihood approach of Beran (1994a), which effectively amounts to removing the log-determinant term $\log|\mathbf{V}_\mathbf{y}|$ from the log-likelihood function (6) and truncating the infinite autoregressive representation (2). The resulting estimator belongs to a wider class of M estimators for which the estimator can be shown to converge almost surely to its true value at the rate of $\sqrt{T}$ (see Beran 1994a for more details). In the empirical study of the next section, the sample size is sufficiently large so that we can rely on these asymptotic results.

3.3 Approximate Maximum Likelihood Estimation and Inference

In the case of model (1) with $\boldsymbol{\mu}_t$ given by (3) and $\sigma_t^2 = \sigma^2$, the estimation of $\boldsymbol{\psi}$ is carried out as follows. For realizations $y_1, \ldots, y_T$, with a given $\boldsymbol{\psi}$ and by truncating the infinite autoregressive polynomial (2), we compute $\varepsilon_t = \varepsilon_t(\boldsymbol{\psi})$ in a standard way as implied by model (1) for $t = \max(p, r) + 1, \ldots, T$. Because the truncated autoregressive polynomial is long, the earlier disturbances ε_t for $t = \max(p, r) + 1, \max(p, r) + 2, \ldots$ are based on polynomials of varying and lower dimensions. Finite-sample modifications, like those of Haslett and Raftery (1989), are not implemented because T is large; however, the periodic nature of the coefficients is taken into account. The estimation of $\boldsymbol{\psi}$ is then based on

$$\widehat{\boldsymbol{\psi}} = \arg\min_{\boldsymbol{\psi}} S(\boldsymbol{\psi}), \qquad S(\boldsymbol{\psi}) = \sum_{t=\max(p,r)+1}^{T} \varepsilon_t^2, \quad (7)$$

with $\varepsilon_t = \varepsilon_t(\boldsymbol{\psi})$. This is an M estimator discussed by Beran (1994a).

In the case that σ_t^2 is modeled by the GARCH specification (4), the disturbances $\varepsilon_t(\boldsymbol{\psi})$ are obtained in the same way and taken as input for the GARCH likelihood function given by

$$\ell(\boldsymbol{\psi}) = -\frac{1}{2}\sum_{t=\max(p,r)+1}^{T}(\log 2\pi + \log\sigma_t^2 + \sigma_t^{-2}\varepsilon_t^2), \quad (8)$$

where $\sigma_t^2 = \sigma_t^2(\boldsymbol{\psi})$ is given by (4) for $t = \max(p, r) + 1, \ldots, T$. Because the process of h_t in (4) is defined by a recursion, their values for $t = \max(p, r) + 2, \ldots, T$ can be computed conditionally on the initial value $h_{\max(p,r)+1}$ set equal to the estimated sample variance of $\varepsilon_t = \varepsilon_t(\boldsymbol{\psi})$ for $t = \max(p, r) + 1, \ldots, T$, as is common in the literature. Asymptotically, the choice of initialization is negligible (see Francq and Zakoïan 2004). It should be noted that σ_t^2 is computed recursively and also requires the input of $\varepsilon_t = \varepsilon_t(\boldsymbol{\psi})$. In this case, estimation of $\boldsymbol{\psi}$ is based on $\widehat{\boldsymbol{\psi}} = \arg\max_{\boldsymbol{\psi}} \ell(\boldsymbol{\psi})$.

Finally, we consider model (1) with disturbances ε_t modeled by the t-distribution with variance σ_t^2 and number of degrees of freedom ν for $t = \max(p, r) + 1, \dots, T$. The GARCH likelihood function for t-disturbances is given by

$$\ell^*(\boldsymbol{\psi}) = \{T - \max(p, r)\} \log c(\nu) - \frac{1}{2} \sum_{t=\max p, r+1}^{T} \Big[\log d_t(\nu) + (\nu + 1) \log\{1 + d_t(\nu)^{-1} \varepsilon_t^2\}\Big], \quad (9)$$

where

$$c(\nu) = \frac{\Gamma(\frac{\nu}{2} + \frac{1}{2})}{\Gamma(\frac{\nu}{2})}, \qquad d_t(\nu) = (\nu - 2)\sigma_t^2, \qquad \nu > 2,$$

with $j = j(t)$ for $t = \max(p, r) + 1, \dots, T$. The shape coefficient ν is also part of the parameter vector $\boldsymbol{\psi}$ and can be estimated.

Actual ML estimation of $\boldsymbol{\psi}$ involves maximization of $\ell(\boldsymbol{\psi})$ or $\ell^*(\boldsymbol{\psi})$ with respect to $\boldsymbol{\psi}$ using a numerical optimization method, such as the quasi-Newton method (see Fletcher 1987). We used the MaxBFGS() routine in Ox (see Doornik 1999). Starting values for this optimization can be obtained from the M estimates for ϕ_{ij}, δ_{ij} and D_j based on (7) and on quasi-ML (QML) estimates for the GARCH parameter based on (8). Standard errors of the estimates and Wald test statistics are obtained from numerical second-order derivatives of (9).

We found interesting differences between the inefficient (Gaussian) M estimates and efficient ML estimates, even though our samples are large. The strong persistence in the volatility, measured as $\widehat{\alpha}_1 + \widehat{\beta}_1$ being close to unity, has a profound influence on the estimation of the autoregressive parameters. Consequently, the efficiency of estimates and tests increases. Boswijk and Klaassen (2004) discussed the empirical relevance of this efficiency gain for AR–GARCH-t models. Under the assumption that $E|\varepsilon_t|^4$ exists, Jensen and Rahbek (2004) showed that the asymptotic behavior of the QML estimator of the GARCH(1, 1) parameters is continuous around $\alpha_1 + \beta_1 = 1$. The estimator is also asymptotically normal if $\alpha_1 + \beta_1 > 1$. Francq and Zakoïan (2004) derived asymptotic normality of QML estimators of stable ARMA–GARCH(p, q) models under weak conditions. In our case, the innovations of electricity prices are fat-tailed, and thus we cannot use the QML estimator, and the inference is directly based on the Student-t likelihood. This approach is also applicable in stable GARCH models when $E|\varepsilon_t|^4$ does not exist, and also is more efficient (see Berkes and Horvath 2004, ex. 2.4).

4. EMPIRICAL RESULTS FOR NORD POOL

In this section we present empirical results for the daily Nord Pool data, for which data characteristics are summarized in Section 2.2. It is hinted that a seasonal periodic heteroscedastic long-memory model may be adequate to capture the dynamics in the conditional mean of the series. The Nord Pool series is sufficiently long, making a parametric long-memory analysis using the approximate ML method feasible.

4.1 Time Series Model for Nord Pool Prices

The time series model (1)–(5) using deterministic functions of time for the x_{kt} and z_{kt} variables is estimated using the method described in Sections 3.2 and 3.3 with $p = 3$. A yearly cycle is part of the x_{kt}'s as in Lucia and Schwartz (2002). This cycle captures the smooth seasonal swings in the supply-and-demand functions of electricity. The prices are subject to significant holiday effects in demand that lead to low returns on holidays and high returns thereafter. The AR parameters ϕ_{ij} for holidays differ from those for normal weekend days. The degrees of freedom are not sufficient to allow estimation of ϕ_{ij} separately for each type of holiday. Instead, dummy variables for each type of holiday are included and their effects on prices measured, contemporaneously and for a maximum of p lags. For a holiday occurring on the same day of the week each year (e.g., Ascension Day), we have $p + 1$ parameters in the model to measure its effect. For other types of holidays, we have $(p + 1) \times s$ parameters in the model, because these holiday effects may depend on the day of the week. Finally, the conditional variance is explained by both yearly and half-yearly cycles in z_{kt} following Byström (2005).

The estimation results are presented in Table 2. To economize on the estimation output, we omit standard errors. Instead, we present Wald test statistics the nullity and for the nonperiodicity of sets of parameters. The seasonal integration parameters D_j are largest for Monday and Saturday, as expected from the autocorrelations presented in Table 1. These estimated parameters are significant, clearly periodic and smaller than .5. The AR parameters ϕ_{ij} are also clearly periodic. The third-order lag is particularly important for Monday. For Thursday and Friday, the AR polynomial of the model reduces to the difference operator, as is usual in models for returns in stock markets. The periodic AR polynomial of the model is stable, however. The largest inverse root of the characteristic polynomial equals .95 (see Boswijk and Franses 1996 for unit root tests in periodic AR models).

The yearly cycle and the holiday effects measured by δ_{kij} are significant. For example, the electricity price is approximately 18% lower on Ascension Day than on a normal Thursday; this effect is based on 12 Ascension Day observations. The periodic effect of a holiday with a fixed calendar date (e.g., May 1) is more difficult to measure because it varies with the day of the week, and its estimate is sometimes based on only one or two observations. As far as the volatility equation is concerned, significant periodicity is found for the log-variance parameter λ_j. Monday and Saturday are more volatile than other days. Furthermore, the chi-squared tests show that significant yearly and half-yearly cycles in the volatility are detected in our analysis. The estimates of the GARCH parameters α_1 and β_1 are on the boundary of the admissable parameter space. As a result, we have high persistence in the conditional variance, a typical finding in many financial applications. The t distribution of the errors is fat-tailed with $\widehat{\nu} = 3.98$ estimated degrees of freedom.

The last rows of Table 2 and Figure 4 present diagnostics for the standardized residuals $\widehat{\eta}_t = \widehat{\varepsilon}_t / \widehat{\sigma}_t$ that are normalized by the transformation $\widehat{\eta}_t^* = F_G^{-1}[F_t(\widehat{\eta}_t)]$ for $t = 1, \dots, n$, where $F_G(\cdot)$ and $F_t(\cdot)$ are the cumulative density functions of the standard normal and t distributions. Because the standard diagnostic statistics and graphs are designed for residuals that are assumed normal, and because the model disturbances are assumed to come from a t density, this transformation for the residuals is justified. The Ljung–Box $Q(\cdot)$ test statistics for serial correlation in the normalized scaled residuals do not exhibit significant serial correlation, whereas evidence of erratic behavior in $\widehat{\eta}_t^*$ is limited and evidence of nonnormal behavior of $\widehat{\eta}_t^*$ is not

Table 2. ML Estimates of Daily Log Prices for Nord Pool January 18, 1993–April 10, 2005, Deterministic x_t

Periodic parameters	j	*Mon, 1*	*Tue, 2*	*Wed, 3*	*Thu, 4*	*Fri, 5*	*Sat, 6*	*Sun, 7*	m	χ^2_m $\theta = 0$	m	χ^2_m $\theta = \iota\theta_0$
	D_j	.146	.005	−.012	.033	.071	.119	.078	7	90.3	6	39.6
	$\phi_{1,j}$	.682	1.011	1.150	1.097	1.035	1.181	1.097	7	4,765	6	19.7
	$\phi_{2,j}$	−.097	.043	−.178	−.080	.028	.048	.016	7	19.7	6	16.0
	$\phi_{3,j}$	.381	−.067	.022	−.016	−.064	−.214	−.116	7	92.4	6	92.3
Constant	$\delta_{1,0,j}$	.202	.065	.030	−.010	−.006	−.102	.008	7	42.6	6	39.5
CosYear	$\delta_{2,0,j}$	−.175	−.010	−.042	.002	.026	.115	.083	7	50.0	6	49.6
SinYear	$\delta_{3,0,j}$	−.014	−.074	−.066	−.027	−.035	−.059	−.038	7	44.4	6	6.8
Maundy	$\delta_{4,0,4}$				−.025				1	14.7		
Good Fri	$\delta_{5,i,5}$					−.033	.062	−.000	3	57.4		
Easter	$\delta_{6,i,1}$	−.067	.130	−.013	.002				4	173.5		
Ascension	$\delta_{7,i,4}$				−.178	.156	.011	−.008	4	188.5		
Pentecost	$\delta_{8,\ldots,1}$	−.112	.145	−.010	.012				4	107.8		
Dec 24	$\delta_{9,0,j}$	−.169	−.050	−.096	−.079	−.016	−.031	−.004	7	82.3		
Dec 25	$\delta_{10,0,j}$	−.135	−.040	.015	.006	−.010	.054	−.032	7	26.7		
Dec 26	$\delta_{11,0,j}$	−.005	.033	.024	−.014	.039	.064	.011	28	138.4		
	$\delta_{11,1,j-1}$	−.003	.026	.111	.095	.033	.094	.010				
	$\delta_{11,2,j-2}$	.038	.006	−.009	−.035	−.012	.007	−.014				
	$\delta_{11,3,j-3}$	.051	−.014	.001	.019	.051	.021	−.055				
Jan 1	$\delta_{12,0,j}$	−.116	−.016	−.053	.012	.033	.042	.001	28	1502		
	$\delta_{12,1,j-1}$	.110	.165	.608	.125	.055	.058	−.002				
	$\delta_{12,2,j-2}$	.062	.010	−.056	−.404	.047	.014	−.011				
	$\delta_{12,3,j-3}$	−.005	−.026	.043	.004	−.019	−.015	−.026				
May 1	$\delta_{13,0,j}$	−.166	−.144	−.102	−.049	−.171	−.124	−.166	28	424.8		
	$\delta_{13,1,j-1}$	.312	.134	.193	.078	.130	.222	.099				
	$\delta_{13,2,j-2}$	.086	.030	−.014	−.038	.038	.043	−.105				
	$\delta_{13,3,j-3}$	.192	.016	.096	−.159	.004	−.030	−.030				
May 17	$\delta_{14,0,j}$	−.043	.019	−.197	−.013	−.071	−.124	−.217	28	425.7		
	$\delta_{14,1,j-1}$	.101	.055	.362	.224	−.023	.061	−.064				
	$\delta_{14,2,j-2}$	.016	.072	−.071	.005	−.050	−.118	−.008				
	$\delta_{14,3,j-3}$	.025	−.016	.071	−.048	−.011	−.173	−.049				
log var	λ_j	0	−.371	−.611	−.742	−.671	−.018	−.730	6	131.7		

Nonperiodic parameters	*CosY* γ_1	*SinY* γ_2	*CosHY* γ_3	*SinHY* γ_4	$10^3\alpha_0$	α_1	β_1	ν		
	−.240	.089	.145	.0079	.504	.406	.594	3.98		
χ^2_1	8.00	4.47	6.35	.00	15.5		154			
LL:	7,429.04	#pars:	205	T:	4,466		AIC/T:	−3.2351	ρ_1:	.951
AC: Q(28):	37.4	Q(91):	86.3	in var:	Q^*(28):	55.3	Q^*(91):	151.5		

NOTE: Model (1)–(5), Section 3.1. $\delta_j(L)'x_t$: Periodic effects in conditional mean. Deterministic regressors x_t: Constant term, Yearly cycle $\times 10^{-1}$ and 11 holidays with nonoverlapping daily lags. Maundy, ..., Pentecost, $\delta_{k,i,j}$: coefficients for lags $i = 0, 1, 2$ or $i = 0, 1, 2, 3$ of day-of-the-week j of regressor k. Dec 24, ..., May 17, $\delta_{k,i,j-i}$: coefficients for lag i of day-of-the-week $j - i$, where $j - i$ follows modulo 7 arithmetic. λ_j: periodicity parameters conditional variances. $\gamma' z_t$: Nonperiodic effects conditional variance. Deterministic regressors z_t: Yearly cycle $\times 10^{-3}$ and Half-yearly cycle $\times 10^{-3}$. m: degrees of freedom of asymptotic χ^2_m Wald test statistics, $\theta = 0$: test for nullity of parameter set. $\theta = \iota\theta_0$: test for equality of parameters in corresponding row. Asymptotic critical values at 95% and 99% for $m = 7$: 14.1 and 18.5, for $m = 6$: 12.6 and 16.8. α_1, β_1: GARCH parameters. ν: shape parameter t-distribution. LL: approximate log-likelihood. T: number of observations. AIC: ($-2 \times$ LL + 2#pars). #pars: number of parameters. ρ_1: largest inverse root of characteristic polynomial of periodic AR part. AC (AutoCorrelation): $Q(\cdot)$: Ljung–Box statistics on normalized residuals. in var: $Q^*(\cdot)$: idem for squared normalized residuals.

apparent. The high number of 0's in the empirical histogram in Figure 4(c) stems from holiday dummy variables that occur once and put associated residuals to 0. Strong evidence of serial correlation in $\hat{\eta}_t^{*2}$ is not present, especially in the short run.

4.2 Explanatory Variables for Nord Pool Prices

Prediction errors of the pure time series model can be taken as stemming from changes in the electricity supply-and-demand functions. To verify this proposition, we extend the analysis for Nord Pool prices by adding explanatory variables to the time series model. First, we consider weekly data on Monday's water reservoir levels, in both demeaned levels and demeaned weekly differences, as a proxy for supply effects. Levels and first differences are less correlated than levels, and lagged levels and associated test statistics are easier to interpret. Furthermore, the coefficients of first differences measure short-run effects, whereas those of levels capture long-run effects (see Johnston and Dinardo 1997, chap. 8). The time series model of Table 2 was extended with these supply variables and fitted to the same sample. The estimation results show that, ceteris paribus, a positive change in the water levels has a significant negative effect on electricity prices, except on Mondays, when the measurements for the new week are not yet publicly available. The parameter estimates of the pure time series model are not much affected by the introduction of the water levels. The noticeable exception is the effect of the deterministic yearly cycle in the conditional mean of prices, which is largely replaced by the effect of the yearly cycle in changing water levels.

Second, we consider levels, daily differences, and lagged daily differences of demeaned log power consumption, which we take as a proxy for electricity demand. Periodic sample means of this variable are reported in Table 1. Because power consumption data are available only for a shorter period, a separate analysis is carried out for the model with both water supply and power consumption as explanatory variables. The estimation results are presented in Table 3. The effect of water level

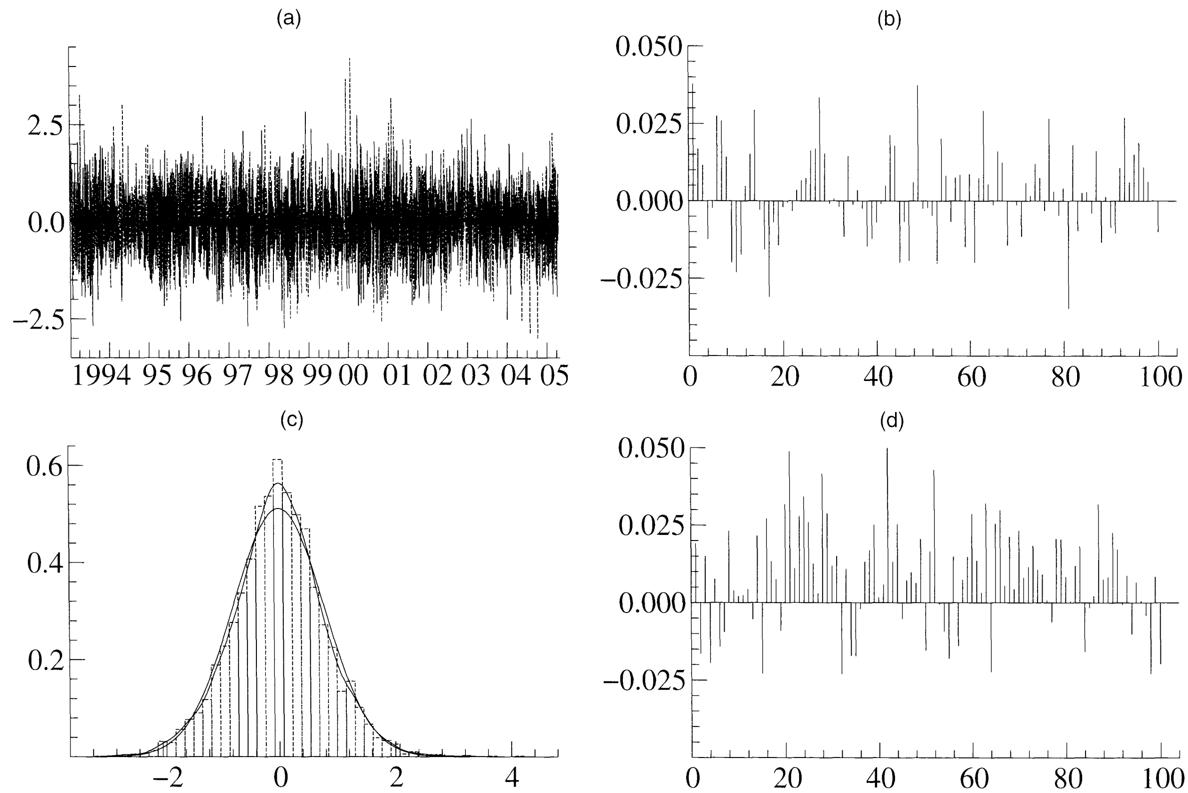

Figure 4. Diagnostics for Nord Pool Model With Deterministic x_t. Model estimates are presented in Table 2. (a): Daily normalized scaled residuals against time. (b) Autocorrelation function normalized scaled residuals against lag in days. (c) Histogram and nonparametric density estimate of normalized scaled residuals. The thick line is a reference for normal distribution. (d) Autocorrelation function squared normalized scaled residuals against lag in days. Sample: January 18, 1993–April 10, 2005.

Table 3. Effect Reservoir Levels and Power Consumption for Nord Pool, February 26, 2001–April 10, 2005

Periodic parameters	*j*	*Mon, 1*	*Tue, 2*	*Wed, 3*	*Thu, 4*	*Fri, 5*	*Sat, 6*	*Sun, 7*	*m*	χ^2_m $\theta = 0$	*m*	χ^2_m $\theta = \iota\theta_0$
	D_j	.125	.053	−.124	.006	−.118	.062	.052	7	25.9	6	25.4
	$\phi_{1,j}$	.460	.892	.995	.983	.790	.966	1.056	7	1,153	6	52.2
	$\phi_{2,j}$	.094	.076	.018	.138	−.003	.149	−.142	7	6.79	6	5.41
	$\phi_{3,j}$	.398	.003	−.014	−.136	.198	−.097	.112	7	37.1	6	36.3
Constant	$\delta_{1,0,j}$	.295	.158	.000	.072	.056	−.131	−.162	7	19.3	6	17.5
CosYear	$\delta_{2,0,j}$	−.016	−.014	−.016	.005	−.034	−.010	−.027	7	44.4	6	15.0
SinYear	$\delta_{3,j}$	−.012	−.013	−.003	−.016	−.014	−.013	−.004	7	13.0	6	3.0
...	...											
Δ_7Water	$\delta_{15,\ldots,j}$	.365	−.292	−.391	−.302	−.549	−.628	−.478	7	60.6	6	15.7
Water	$\delta_{16,\ldots,j}$	−.057	−.050	.012	−.072	−.009	−.003	.038	7	9.3	6	7.8
ΔConsu	$\delta_{17,0,j}$	.423	.468	.395	.324	.425	.372	.240	28	221.1		
	$\delta_{17,1,j-1}$	.231	.172	.045	.221	−.049	.058	−.007				
	$\delta_{17,2,j-2}$	.333	.175	−.051	.025	.082	.086	.202				
	$\delta_{17,3,j-3}$	.100	.043	−.094	−.055	−.021	−.229	−.068				
Consu	$\delta_{18,0,j}$	.071	.035	.028	−.046	.083	.025	.069	7	19.7		
log var	λ_j	0	−.410	−.737	−.654	−.715	−.054	−.406	6	36.3		
Nonperiodic parameters												
	CosY	*SinY*	*CosHY*	*SinHY*								
	γ_1	γ_2	γ_3	γ_4	$10^3\alpha_0$	α_1	β_1	ν				
	−.274	−.015	.136	.035	.403	.501	.499	3.216				
χ^2_1	2.38	.07	2.0	.2	3.57		20.1					
LL:	3,060.38	#pars:	200	*T*:	1,505		AIC/*T*:	−3.8012				
AC: *Q*(28):	39.1	*Q*(91):	119.2	in var:	*Q**(28):	40.3	*Q**(91):	93.9	ρ_1:	.923		

NOTE: Also see the note for Table 2. Coefficients $\delta_{4,i,j}, \ldots, \delta_{14,i,j}$ of holiday effects are not reported. Water: Water levels in Nord Pool area as a fraction of total capacity minus 0.6245, the sample mean over 1993–2004 as reported on Mondays for the Nord Pool area. Source: *www.statnett.no*. Δ_7Water: change with respect to previous week. Number of coefficients for the "fixed date" holidays is lower than in Table 2 due to the smaller sample. Consu: Log daily Norwegian Power consumption in MWh, corrected for day-of-the-week means over the period 2001–2005, reported in Table 1. ΔConsu: daily change in log power consumption.

increases on prices remains significantly negative for all days of the week except Monday. Positive changes in log consumption have very significant positive effects on electricity prices. Long-run effects of water levels and consumption are not significant in this basic model, which does not allow for feedbacks from prices to consumption. The fractional integration and AR parameters remain jointly significant and periodic.

These empirical results for the Nord Pool case show that the Reg-ARFIMA–GARCH model successfully describes the conditional mean and variance of the price process. The estimation results are easy to interpret and make economic sense. However, a more extensive analysis should take into account the facts that aggregate supply-and-demand functions are nonlinear and vary with hour of day. However, we should also note that historical data on aggregate supply curves for the Nord Pool are not publicly available.

5. EMPIRICAL RESULTS FOR OTHER EUROPEAN MARKETS

To investigate the robustness of the periodic seasonal Reg-ARFIMA–GARCH model, we repeat the analysis for daily electricity spot prices from three younger mainland European markets: EEX in Germany, Powernext in France, and APX in the Netherlands. The countries have six holidays in common. We take 12 French, 10 German, and 8 Dutch holidays into account. Table 4 reports selected parameter estimates for the models of these three new markets. The estimates of the holiday effects are not reported but are significant despite the fact that they are based on a relatively small number of observations. For APX and Powernext, extra intervention dummies are introduced for the week of July 11–17, 2003, during which a lack of cooling water in rivers threatened nuclear power production and prices were more than $e^{2.1} = 8.2$ times higher than was expected on July 11.

The estimated dynamic parameters and disturbance variances vary from day to day in all three markets. For example, Mondays produce particularly low AR(1) coefficients. The periodic patterns of the dynamic parameters vary significantly from market to market. Monday's AR(3) coefficients for Powernext deviate from those for EEX and APX. Sunday's AR(3) coefficients for APX differ from those for EEX and Powernext. Powernext and APX show long-memory behavior for Saturdays, in concordance with the autocorrelations reported in Table 1, whereas the EEX estimates indicate fractional integration for Sundays. Apparently, slowly evolving changes in the weekly seasonal patterns occurred in these markets in the first years of their existence. The autoregressive parts of models for these new market models show stronger mean reversion than in the Nord Pool.

Table 4. ML Estimates for Daily Log Prices for EEX, Powernext, and APX

Periodic parameters	*j*	*Mon, 1*	*Tue, 2*	*Wed, 3*	*Thu, 4*	*Fri, 5*	*Sat, 6*	*Sun, 7*	*m*	χ^2_m $\theta = 0$	*m*	χ^2_m $\theta = \iota\theta_0$
EEX												
	D_j	.078	−.047	−.055	−.015	.178	.101	.372	7	57.1	6	43.0
	$\phi_{1,j}$	.389	.860	.633	.752	.652	.625	.729	7	431	6	19.7
	$\phi_{2,j}$	−.092	−.086	.259	.024	.263	.049	−.032	7	18.9	6	16.7
	$\phi_{3,j}$	.511	.053	.016	.149	−.061	.078	.127	7	101	6	61.0
Const	$\delta_{1,0,j}$	.830	.596	.334	.271	.457	.598	.265	7	68.8	6	10.3
...	...											
log var	λ_j	0	.242	.093	−.056	−.076	.462	−.274	6	21.2		
Pnext												
	D_j	−.003	.205	−.089	−.105	.087	.358	−.255	7	54.8	6	52.7
	$\phi_{1,j}$	.109	.519	.720	.769	.784	.693	1.059	7	453	6	114
	$\phi_{2,j}$	.578	−.049	.241	.168	.036	.008	.492	7	56.8	6	41.6
	$\phi_{3,j}$	.151	.187	−.045	.019	.097	−.032	−.436	7	22.2	6	22.1
Const	$\delta_{1,0,j}$	.733	1.200	.263	.137	.245	.931	−.608	7	96.4	6	71.6
...	...											
03/7/11	$\delta_{16,0,j}$	2.154	−.964	−.482	−.306	.020	.385	.725	7	338	6	256
log var	λ_j	0	−.340	−.847	−.613	−.786	−.650	.518	6	70.5		
APX												
	D_j	.078	.121	.044	.090	.045	.354	.052	7	90.0	6	41.0
	$\phi_{1,j}$	.350	.520	.689	.692	.541	.286	.785	7	438	6	36.4
	$\phi_{2,j}$	.181	.036	.134	.038	.127	.026	.075	7	13.0	6	3.6
	$\phi_{3,j}$	.318	.262	.011	.078	.113	−.057	−.014	7	34.3	6	26.3
Const	$\delta_{1,0,j}$	.693	.678	.573	.657	.721	2.312	.259	7	237	6	76.0
...	...											
03/7/11	$\delta_{12,0,j}$	2.735	.738	.924	−1.988	−1.005	.171	−.021	7	408	6	403
log var	λ_j	0	−.713	−1.127	−1.009	−.832	−1.047	−.486	6	36.6		

	Nonperiodic parameters						*Nonperiodic diagnostics*					
	$10^3\alpha_0$	α_1	β_1	ν	*LL*	*#par*	*Q(28)*	*Q(91)*	*T*	ρ_1	*Q*(28)*	*Q*(91)*
EEX	8.26	.332	.409	3.516	753.3	142	65.0	132.8	1,274	.468	38.5	132.4
Pnext	1.083	.149	.851	3.257	806.2	190	52.6	120.0	1,218	.364	23.2	87.7
APX	16.68	.422	.578	3.065	335.8	137	49.0	111.8	1,554	.196	39.5	140.5

NOTE: See also notes for Table 2, Coefficients $\delta_{2,0,j},\ldots,\delta_{3,0,j}$ for periodic yearly cycle in mean and $\gamma_1,\ldots,\gamma_4$ for nonperiodic cycle in variance not reported. Const: Constant term. Pnext: Powernext. Other unreported coefficients: $\delta_{4,i,j},\ldots,\delta_{13,i,j}$ for EEX (10 holidays), $\delta_{4,i,j},\ldots,\delta_{15,i,j}$ for Powernext (12 holidays), $\delta_{4,i,j},\ldots,\delta_{11,i,j}$ for APX (8 holidays). Common holidays for Germany (EEX), France (Powernext), and the Netherlands (APX): Easter, Ascencion Day, Pentecost, Dec 25, Jan 1. Other holidays EEX: Good Friday, May 1, October 3, December 24, December 26. Other holidays Powernext: May 1, May 8, July 14, August 15, November 1, November 11, December 24. Other holidays APX: Good Friday, April 30, December 26. Powernext and APX: 03/7/11: dummy for week 11–17 July 2003 for extremely low water levels in rivers.

The largest inverse roots of the characteristic polynomial of the autoregressive component are .47 for EEX, .36 for Powernext, and .20 for APX. Finally, the volatility persistence as measured by $\alpha_1 + \beta_1$ is lower than unity for the EEX, but the volatilities for Powernext and APX are persistent.

The last three rows of Table 4 present the Box–Ljung $Q(\cdot)$ statistics for the normalized residuals in levels and squares, $\widehat{\eta}_t^*$ and $\widehat{\eta}_t^{*2}$. The residual diagnostics are satisfactory but not perfect. Long-run residual autocorrelations apparently remain present for EEX and Powernext. We further report that the normalized APX residuals are skewed to the left, and long-run autocorrelations in the squared residuals are apparent. The basic model specification can probably be improved by taking market-specific features into account.

6. CONCLUSIONS

This article presents an empirical analysis of daily spot prices for four European electricity markets using periodic seasonal Reg-ARFIMA–GARCH models to explain the dynamics in the conditional mean and variance of log prices. Day-of-the-week periodic autocovariances for short-run dynamics are modeled by lagged dependent variables, and those for long-run dynamics are modeled by seasonal ARFIMA models. Regressors capture yearly cycles, holiday effects, and possible interventions in mean and variance. The GARCH t component takes volatility clustering and extreme observations into account. The model parameters are estimated simultaneously by approximate ML methods. Given the persistent changes in volatility, simultaneous estimation of mean and variance parameters is preferred over two-step methods. Residual diagnostics show a good model fit. The resulting time series models allow for dynamic point forecasting and stochastic simulation. The Nord Pool market trades hydro power, and it is shown that a significant part of the short-term price movement can be explained by weekly water reservoir levels and daily electricity consumption. The inclusion of these explanatory variables in the model does not significantly change the estimated periodic heteroscedastic seasonal autocovariance structure in Nord Pool prices. The basic modeling framework is successful for Nord Pool prices, whereas it can be somewhat improved for prices from other European markets.

Suggestions for future extensions are more flexible distributions for the error term, smoothly time-varying (periodic) parameters, and a more extensive specification of the conditional variance equation. More parsimonious periodic autoregressive components can be estimated and tested. The model can also be used for prices at a particular hour of the day. Finally, the strong interrelationships between prices and consumption may lead to multivariate modeling approaches. The empirical findings in this article may have important consequences for the modeling and forecasting of mean and variance functions of spot prices for electricity and associated contingent assets.

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