Do Now

Evaluate the following:

$\frac{d}{dx}(kx^n) = k \cdot n\, x^{n-1}$

$$\frac{d}{dx}\left(7x^{-6} - 5\sqrt{x}\right)$$

$=$

$$\frac{d}{dz}\left(\frac{z^{\frac{3}{2}}+2}{z}\right)$$

$(z^{\frac{3}{2}}+2)z^{-1}$

$\frac{d}{dx}(z^{\frac{1}{2}}+2z^{-1})$

$\boxed{\frac{1}{2}z^{-\frac{1}{2}} - 2z^{-2}}$

$$\frac{d}{dt}\left(\frac{x+2x^{\frac{3}{2}}}{\sqrt{x}}\right)$$

$\frac{d}{dx}\left(\frac{x}{x^{1/2}} + \frac{2x^{3/2}}{x^{1/2}}\right)$

$\frac{d}{dx}(x^{\frac{1}{2}} + 2x^1)$

$\boxed{\frac{1}{2}x^{-1/2} + 2}$

AIMS: - find the function and point given the limit definition of derivative
- interpret the meaning of the derivative

Unit 3.5 - Derivative Meaning and More Limit Definition

I. Beyond the First Derivative

$f(x) = 3x^3 + 2x^2 + x + 1 = y$

$f'(x) = 9x^2 + 4x + 1 = \frac{dy}{dx}$

$f''(x) = 18x + 4 = \frac{d^2y}{dx^2}$

$f'''(x) = 18 = \frac{d^3y}{dx^3}$

$f''''(x) = 0 = \frac{d^4y}{dx^4}$

Find the 3rd derivative of the following functions:

$y = 3x^8 + 2x + 1$ $\qquad f(x) = -5\sqrt{x} + \frac{1}{x}$ $\qquad (x^{2.3} + 5x^{-2} - 100x + 4)$

II. Limit Definition Questions

What are the following questions asking you to find?

$$\boxed{\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}}$$

1.) $\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$

$\frac{d}{dx}(\sin x)$

$\boxed{\cos x}$

2.) $\lim_{h\to 0}\frac{3(x+h)^2-4(x+h)-3-(3x^2-4x-3)}{h}$

$\frac{d}{dx}(3x^2-4x-3)=\boxed{6x-4}$

$f(x)=3x^2-4x-3$

$f(x+h)=3(x+h)^2-4(x+h)-3$

Determine the function you are being asked to find the derivative of:

3.) $\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$

$\frac{d}{dx}(\sqrt{x})$

$\frac{1}{2}x^{-1/2}$

4.) $\lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

$\frac{d}{dx}\left(\frac{1}{x^2}\right)=\frac{d}{dx}(x^{-2})$

$=\boxed{-2x^{-3}}$

5.) $\lim_{h\to 0}\frac{\ln(2x+2h)-\ln 2x}{h}$

$\frac{d}{dx}(\ln 2x)$

6.) $\lim_{h\to 0}\frac{2^{x+h}-2^x}{h}$

$\frac{d}{dx}(2^x)$

7.) $\lim_{h\to 0}\frac{(x+h)^{10}-x^{10}}{h}$

$\frac{d}{dx}(x^{10})=10x^9$

Each of the following limits represents the derivative of some function f at some number a. Find f and a for each case:

$\sqrt{4+h}-\sqrt{4}$

1.) $\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}$

$f(x)=\sqrt{x}$

$f(x+h)=\sqrt{x+h}$

$\frac{d}{dx}(\sqrt{x})$

$x=4$

2.) $\lim_{h\to 0}\frac{\sqrt[4]{16+h}-2}{h}$

3.) $\lim_{h\to 0}\frac{(1+h)^{10}-1}{h}$

$f(x)=x^{10}$

$f(x+h)=(x+h)^{10}$

at $x=1$

$\frac{d}{dx}(x^{10})=10x^9$

$=10$

4.) $\lim_{h\to 0}\frac{3\left(\frac{1}{2}+h\right)^5-3\left(\frac{1}{2}\right)^5}{h}$

$f(x)=3(x)^5$

$x=\frac{1}{2}$

$\frac{d}{dx}(3x^5)=15x^4$

5.) $\lim_{h\to 0}\frac{\sin(\pi+h)}{h}=$

$f(x)=\sin x$

$f(x+h)=\sin(x+h)$

$x=\pi$

$=\cos\pi=-1$

6.) $\lim_{h\to 0}\frac{\frac{1}{(1+h)^2}-1}{h}$

$f(x)=\frac{1}{x^2}$

$f(x+h)=\frac{1}{(x+h)^2}$

$x=1$

III. Interpretation of Derivative

1.) Let P(t) be the population of the United States at time t. The table gives the approximate values of this function by providing midyear population estimates from 1992 to 2000. Estimate the value of P'(1996)

t (year)	1992	1994	1996	1998	2000
P(t) (population)	10,036	10,109	10,152	10,175	10,186

b.) Interpret the meaning of P'(1996) $= 20 \frac{\text{people}}{\text{year}}$

At $t=1996$, the pop. is increasing by 20 people/year

2.) The position of a car at time t in hours is given by $P = f(t)\, miles$

a.) What is the meaning of the derivative $f'(t)$? What are its units?

b.) What does it mean to say that $f'(3) = 50$

c.) What does it mean to say that $f'(6) = -80$

3.) A manufacturer produces bolts of a fabric with a fixed width. The cost of producing x yards of this fabric is $C = f(x)$ *dollars*

a.) What is the meaning of the derivative $f'(x)$? What are its units?

$\frac{\text{dollar}}{\text{yard}}$

b.) What does it mean to say $f'(1000) = 9$?

At $x = 1000$, cost is increasing by 9 dollars/yard

c.) What does it mean to say $f'(200) = -10$?

AP Question:

t (minutes)	0	4	9	15	20
W(t) (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where W(t) is measured in degrees Fahrenheit and t is measured in minutes. At time t=0, the temperature of the water is 55. The water is heated for 30 minutes, beginning at time t=0. Values of W(t) at selected times t for the first 20 minutes are given in the table above.

a.) Use the data in the table to estimate W'(12). Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

AP Question:

Let $f(x) = 4x^3 - 3x - 1$. An equation of the line tangent to $y = f(x)$ at $x = 2$ is

a.) $y = 25x - 5$

b.) $y = 45x + 65$

c.) $y = 45x - 65$

d.) $y = 65 - 45x$

e.) $y = 65x - 45$

AIMS: - find the function and point given the limit definition of derivative
- interpret the meaning of the derivative

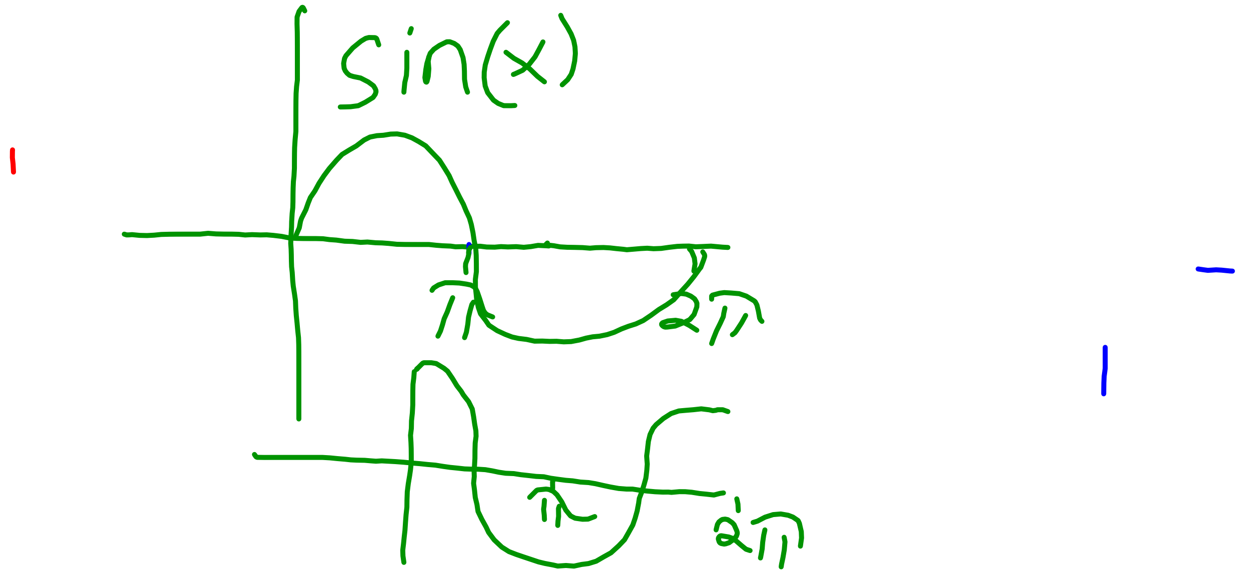
TTL

1.) The number of bacteria after t hours in a controlled laboratory experiment is $n = f(t)$

a.) What is the meaning of the derivative $f'(t)$? What are its units?

b.) What does it mean to say $f'(5) = 100$

2.) $\dfrac{d}{dx}(5x-7)^2$


sin(x)
π
2π
π
2π