

## Do Now

Use the limit definition of derivative to find the derivative of  $f(x) = x^2$ . Check your answer using the power rule.

With extra time, evaluate the following:

$$\frac{d}{dx}(7x^{-6} - 5\sqrt{x})$$

$$\frac{d}{dz}\left(\frac{z^{\frac{3}{2}} + 2}{z}\right)$$

$$\frac{d}{dt}\left(\frac{x + 2x^{\frac{3}{2}}}{\sqrt{x}}\right)$$

AIMS: - find the function and point given the limit definition of derivative  
- interpret the meaning of the derivative

**Unit 3.5 - Derivative Meaning and More Limit Definition**

What are the following questions asking you to find?

1.)  $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$       2.)  $\lim_{h \rightarrow 0} \frac{3(x+h)^2 - 4(x+h) - 3 - (3x^2 - 4x - 3)}{h}$

Answer each question if possible. If not, explain what the question is asking you to find, stating the specific function.

3.)  $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$       4.)  $\lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$       5.)  $\lim_{h \rightarrow 0} \frac{\ln(2x+2h) - \ln 2x}{h}$

6.)  $\lim_{h \rightarrow 0} \frac{2^{x+h} - 2^x}{h}$       7.)  $\lim_{h \rightarrow 0} \frac{(x+h)^{10} - x^{10}}{h}$

Answer each question if possible. If not, explain what the question is asking you to find, stating the specific function and x-location.

$$1.) \lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h}$$

$$2.) \lim_{h \rightarrow 0} \frac{\sqrt[4]{16+h} - 2}{h}$$

$$3.) \lim_{h \rightarrow 0} \frac{(1+h)^{10} - 1}{h}$$

$$4.) \lim_{h \rightarrow 0} \frac{3\left(\frac{1}{2} + h\right)^5 - 3\left(\frac{1}{2}\right)^5}{h}$$

$$5.) \lim_{h \rightarrow 0} \frac{\sin(\pi + h)}{h}$$

$$6.) \lim_{h \rightarrow 0} \frac{\frac{1}{(1+h)^2} - 1}{h}$$

**III. Interpretation of Derivative**

1.) Let  $P(t)$  be the population of the United States at time  $t$ . The table gives the approximate values of this function by providing midyear population estimates from 1992 to 2000. Estimate the value of  $P'(1996)$ , including units in your answer

$t$ (year)	1992	1994	1996	1998	2000
$P(t)$ (population)	10,036	10,109	10,152	10,175	10,186

2.) What would the units of  $P''(1996)$  be?  $P'''(1996)$ ?

2.) The position of a car at time  $t$  in hours is given by  $P = f(t)$  miles

a.) What is the meaning of the derivative  $f'(t)$ ? What are its units?

b.) What does it mean to say that  $f'(3) = 50$

c.) What does it mean to say that  $f'(6) = -80$

d.) What would the units of  $f'(x)$  be?  $f''(x)$ ?

3.) A manufacturer produces bolts of a fabric with a fixed width. The cost of producing  $x$  yards of this fabric is  $C = f(x)$  dollars

a.) What is the meaning of the derivative  $f'(x)$ ? What are its units?

b.) What does it mean to say  $f'(1000) = 9$ ?

c.) What does it mean to say  $f'(200) = -10$ ?

d.) What would the units of  $f'(x)$  be?  $f''(x)$ ?

**AP Question:**

t (minutes)	0	4	9	15	20
W(t) (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time  $t$  is modeled by a strictly increasing, twice-differentiable function  $W$ , where  $W(t)$  is measured in degrees Fahrenheit and  $t$  is measured in minutes. At time  $t=0$ , the temperature of the water is 55. The water is heated for 30 minutes, beginning at time  $t=0$ . Values of  $W(t)$  at selected times  $t$  for the first 20 minutes are given in the table above.

- a.) Use the data in the table to estimate  $W'(12)$ . Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

**AP Question:**

Let  $f(x) = 4x^3 - 3x - 1$ . An equation of the line tangent to  $y = f(x)$  at  $x = 2$  is

- a.)  $y = 25x - 5$
- b.)  $y = 45x + 65$
- c.)  $y = 45x - 65$
- d.)  $y = 65 - 45x$
- e.)  $y = 65x - 45$



AIMS: - find the function and point given the limit definition of derivative  
- interpret the meaning of the derivative

## TTL

1.) The number of bacteria after  $t$  hours in a controlled laboratory experiment is  $n = f(t)$

a.) What is the meaning of the derivative  $f'(t)$ ? What are its units?

b.) What does it mean to say  $f'(5) = 100$

2.)  $\frac{d}{dx}(5x - 7)^2$

## Do Now

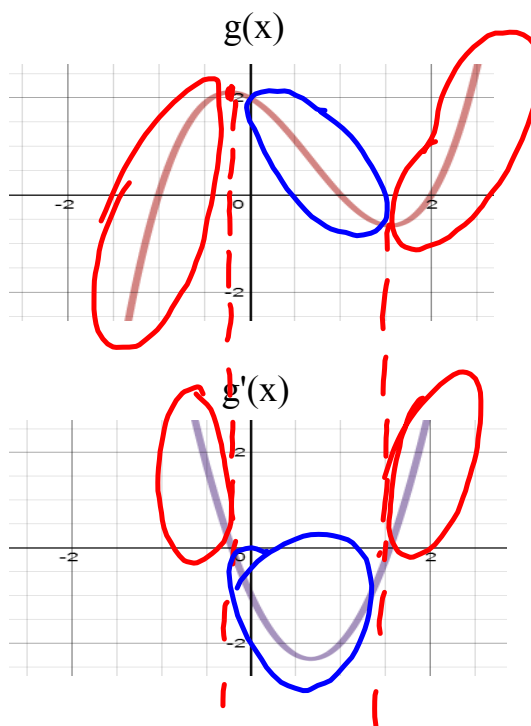
Determine the slope of  $g(x)$  at the given  $x$  values $x = -1, -0.25, 1, 1.5, 2$ 

$x$	-1	-0.25	1	1
slope	2	0	-2	

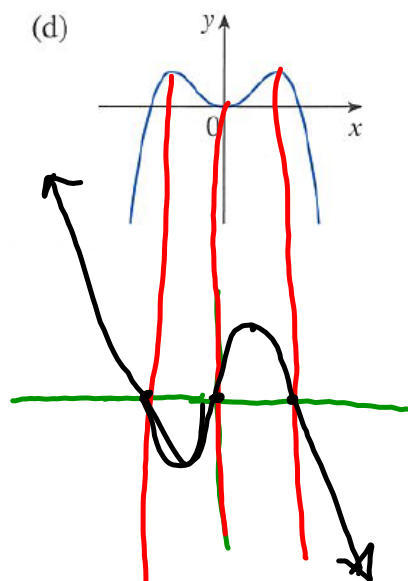
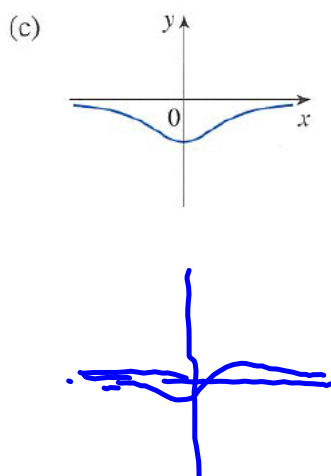
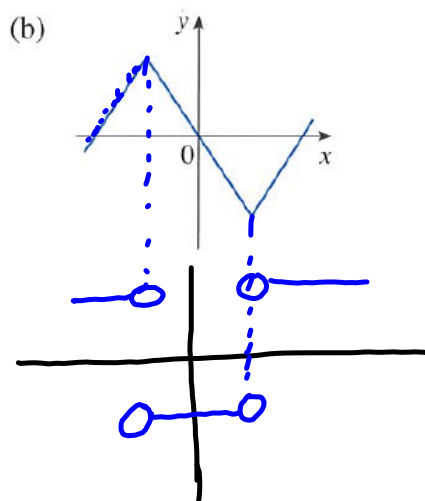
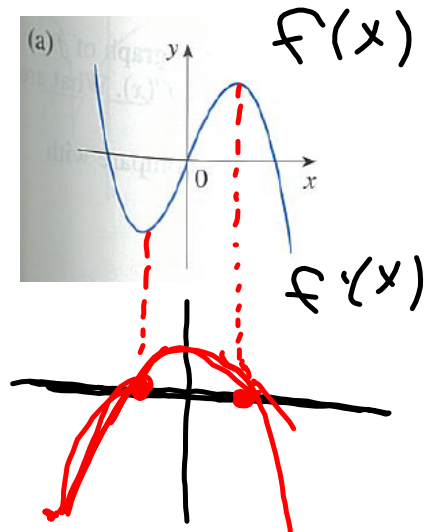
Determine the values of  $f'(x)$  at the given  $x$  values $x = -1, -0.25, 1, 1.5, 2$ 

$x$	-1	-0.25	1	1
$f'(x)$	~	0	-2	

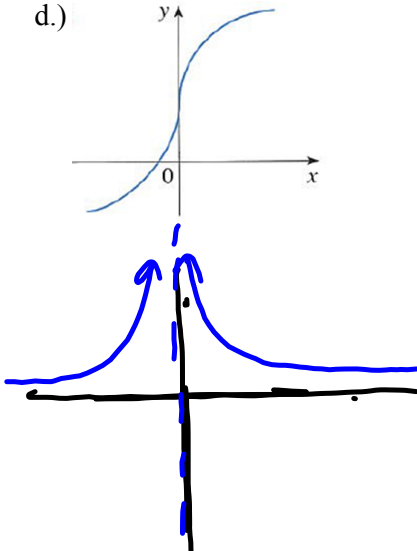
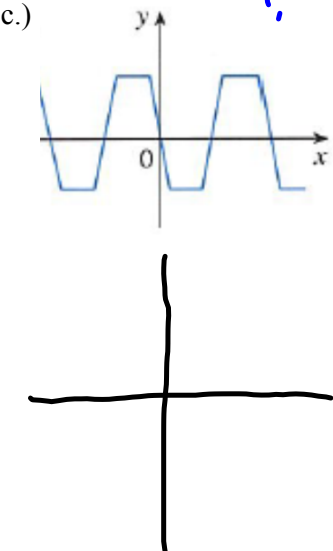
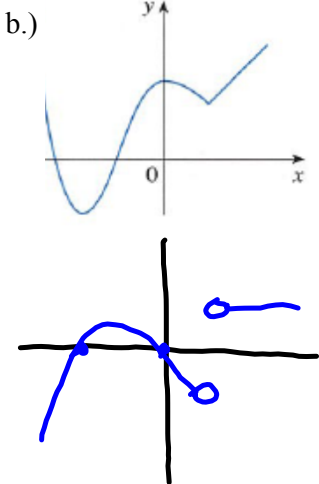
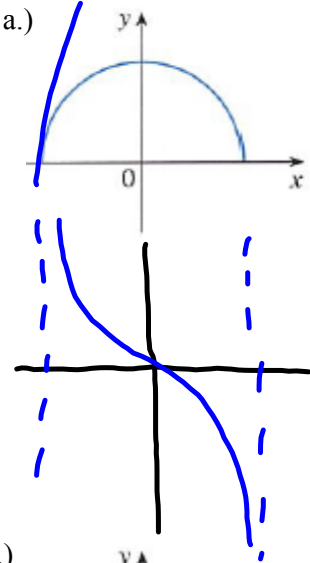
What do you notice?

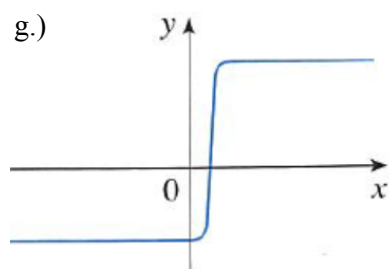
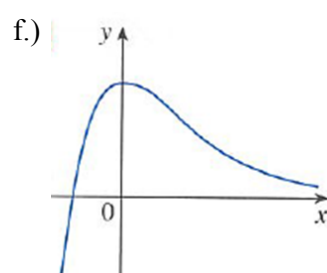
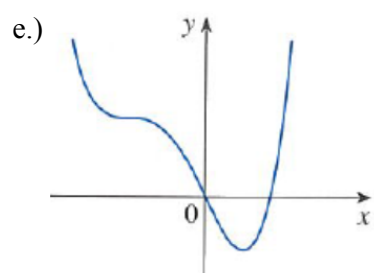


Draw the derivative graph of the following functions:



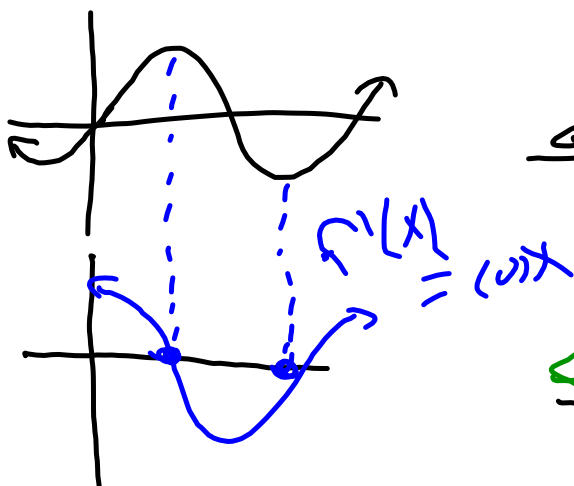
Draw the graph of the derivative for the following functions:



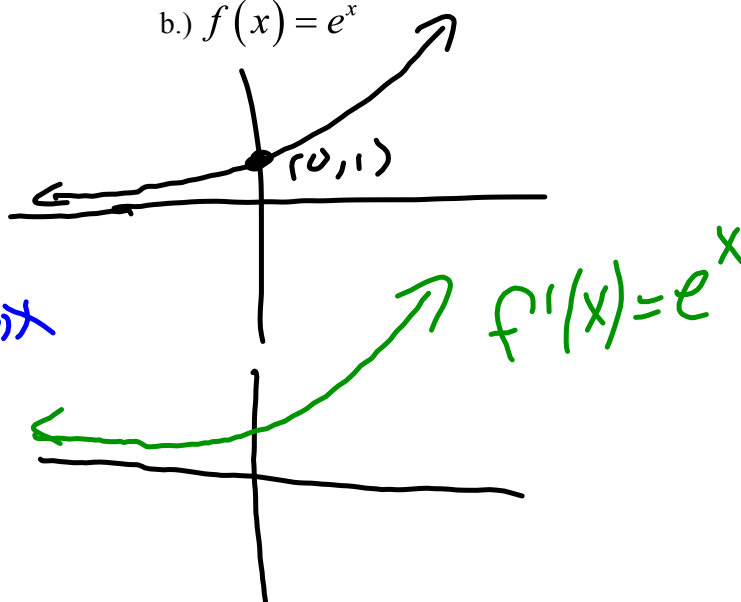


Make a sketch of the graph of  $f$  and below it sketch the graph of  $f'$ . Make a guess for the formula of  $f'(x)$  from its graph (Hint, they are all functions you know)

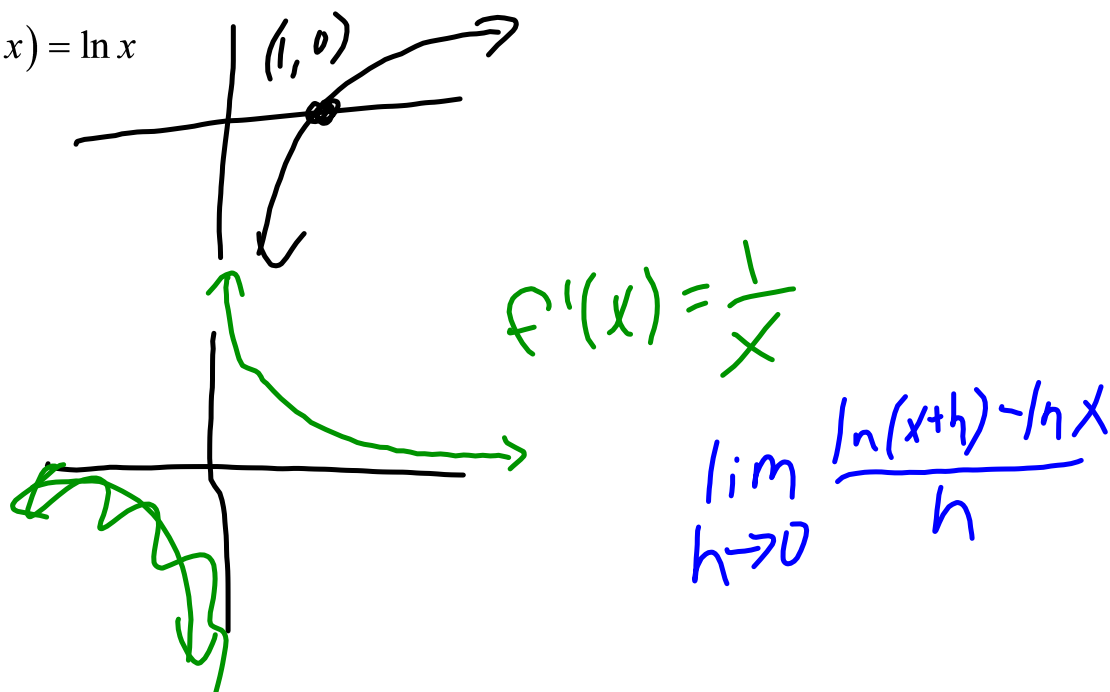
a.)  $f(x) = \sin x$



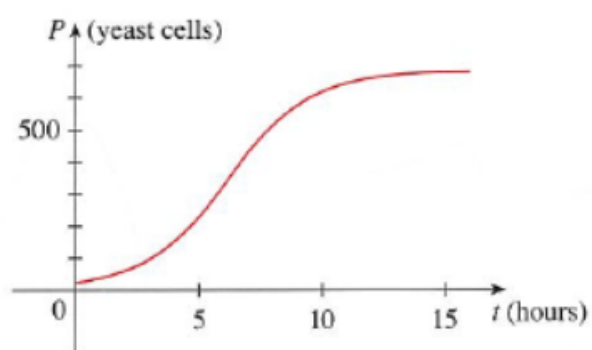
b.)  $f(x) = e^x$



c.)  $f(x) = \ln x$



Below is the graph of the population function  $P(t)$  for yeast cells in a laboratory culture. Graph the derivative  $P'(t)$ . What does the graph of  $P'(t)$  tell us about the yeast population?



AIMS: - draw the derivative graph given any function

## TTL

Draw the graph of the derivative of the function below:

