

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/325787310>

Speeded Up Detection of Squared Fiducial Markers

Article in *Image and Vision Computing* · June 2018

DOI: 10.1016/j.imavis.2018.05.004

CITATIONS
217

READS
18,574

3 authors:



Francisco J. Romero-Ramirez

University of Cordoba (Spain)

9 PUBLICATIONS 281 CITATIONS

[SEE PROFILE](#)



Rafael Muñoz-Salinas

University of Cordoba (Spain)

89 PUBLICATIONS 2,710 CITATIONS

[SEE PROFILE](#)



Rafael Medina-Carnicer

University of Cordoba (Spain)

89 PUBLICATIONS 1,673 CITATIONS

[SEE PROFILE](#)

Some of the authors of this publication are also working on these related projects:



Computer Science Doctor (PhD) [View project](#)



Cost-effective Techniques for Patient Positioning in Percutaneous Radiotherapy Using Optical Imaging Systems [View project](#)

Speeded Up Detection of Squared Fiducial Markers

Francisco J. Romero-Ramirez¹, Rafael Muñoz-Salinas^{1,2,*}, Rafael Medina-Carnicer^{1,2}

Abstract

Squared planar markers have become a popular method for pose estimation in applications such as autonomous robots, unmanned vehicles or virtual trainers. The markers allow estimating the position of a monocular camera with minimal cost, high robustness, and speed. One only needs to create markers with a regular printer, place them in the desired environment so as to cover the working area, and then registering their location from a set of images.

Nevertheless, marker detection is a time-consuming process, especially as the image dimensions grows. Modern cameras are able to acquire high resolutions images, but fiducial marker systems are not adapted in terms of computing speed. This paper proposes a multi-scale strategy for speeding up marker detection in video sequences by wisely selecting the most appropriate scale for detection, identification and corner estimation. The experiments conducted show that the proposed approach outperforms the state-of-the-art methods without sacrificing accuracy or robustness. Our method is up to 40 times faster than the state-of-the-art method, achieving over 1000 fps in 4K images without any parallelization.

Keywords: Fiducial Markers, Marker Mapping, SLAM.

¹ 1. Introduction

Pose estimation is a common task for many applications such as autonomous robots [1, 2, 3], unmanned vehicles [4, 5, 6, 7, 8] and virtual assistants [9, 10, 11, 12], among other.

Cameras are cheap sensors that can be effectively used for this task. In the ideal case, natural features such as keypoints or texture [13, 14, 15, 16] are employed to create a map of the environment. Although some of the traditional problems of previous methods for this task have been solved in the last few years, other problems remain. For instance, they are subject to filter stability issues or significant computational requirements.

In any case, artificial landmarks are a popular approach for camera pose estimation. Square fiducial markers, comprised by an external squared black border and an internal identification code, are especially attractive because the camera pose can be estimated from the four corners of a single marker [17, 18, 19, 20]. The recent work of [21] is

a step forward the use of this type of markers in large-scale problems. One only need to print the set of markers with a regular printer, place them in the area under which the camera must move, and take a set of pictures of the markers. The pictures are then analyzed and the three-dimensional marker locations automatically obtained. Afterward, a single image spotting a marker is enough to estimate the camera pose.

Despite the recent advances, marker detection can be a time-consuming process. Considering that the systems requiring localization have in many cases limited resources, such as mobile phones or aerial vehicles, the computational effort of localization should be kept to a minimum. The computing time employed in marker detection is a function of the image size employed: the larger the images, the slower the process. On the other hand, high-resolution images are preferable since markers can be detected, even if far from the camera, with high accuracy. The continuous reduction in the cost of the cameras, along with the increase of their resolution, makes necessary to develop methods able to reliably detect the markers in high-resolution images.

The main contribution of this paper is a novel method for detecting square fiducial markers in video sequences. The proposed method relies on the idea that markers can be detected in smaller versions of the image, and employs a multi-scale approach to speed up computation while maintaining the precision and accuracy. In addition, the system is able to dynamically adapt its parameters in order

*Corresponding author

Email addresses: fj.romero@uco.es (Francisco J. Romero-Ramirez), in1musar@uco.es (Rafael Muñoz-Salinas), rmedina@uco.es (Rafael Medina-Carnicer)

¹Departamento de Informática y Análisis Numérico, Edificio Einstein. Campus de Rabanales, Universidad de Córdoba, 14071, Córdoba, Spain, Tlfn:(+34)957212289

²Instituto Maimónides de Investigación en Biomedicina (IMIBIC). Avenida Menéndez Pidal s/n, 14004, Córdoba, Spain, Tlfn:(+34)957213861

49 to achieve maximum performance in the analyzed video
50 sequence. Our approach has been extensively tested and
51 compared with the state-of-the-art methods for marker de-
52 tection. The results show that our method is more than an
53 order of magnitude faster than state-of-the-art approaches
54 without compromising robustness or accuracy, and with-
55 out requiring any type of parallelism.

56 The remainder of this paper is structured as follows.
57 Section 2 explains the works most related to ours. Sec-
58 tion 3 details our proposal for speeding up the detection
59 of markers. Finally, Section 4 gives a exhaustive analysis
60 of the proposed method and Section 5 draws some conclu-
61 sions.

62 2. Related works

63 Fiducials marker systems are commonly used for camera
64 localization and tracking when robustness, precision, and
65 speed are required. In the simplest case, points are used
66 as fiducial markers, such as LEDs, retroreflective spheres
67 or planar dots [22, 23]. However, their main drawback is
68 the need of a method to solve the assignment problem, i.e.,
69 assigning a unique and consistent identifier to each element
70 over time. In order to ease the problem, a common solution
71 consists in adding an identifying code into each marker.
72 Examples of this are planar circular markers [24, 25], 2D-
73 barcodes [26, 27] and even some authors have proposed
74 markers designed using evolutionary algorithms [28].

75 Amongst all proposed approaches, those based on
76 squared planar markers have gained popularity. These
77 markers consist of an external black border and an inter-
78 nal code (most often binary) that uniquely identifies each
79 marker (see Fig 1). Their main advantage is that the pose
80 of the camera can be estimated from a single marker.

81 ARTToolKit [29] is one of the pioneer proposals. They
82 employed markers with a custom pattern that is identified
83 by template matching. This identification method, how-
84 ever, is prone to error and not very robust to illumination
85 changes. In addition, the method’s sensitivity degrades
86 as the number of markers increases. As a consequence,
87 other authors improved that work by using binary BCH
88 codes [30] (which allows a more robust error detection) and
89 named it ARTToolKit+ [31]. The project was halted and
90 followed by the Studierstube Tracker project [32], which is
91 privative. Similar to the ARTToolKit+ project is the dis-
92 continued project ARTag [33].

93 BinARyID [34] is one of the first systems that proposed
94 a method for generating customizable marker codes. In-
95 stead of using a predefined set of codes, they proposed
96 a method for generating the desired number of codes for
97 each particular application. However, they do not consider

98 the possibility of error detection and correction. AprilT-
99 ags [18], however, proposed methods for error detection
100 and correction, but their approach was not suitable for a
101 large number of markers.

102 The work ArUco [17] is probably the most popular sys-
103 tem for marker detection nowadays. It adapts to non-
104 uniform illumination, and is very robust, being able to
105 do error detection and correction of the binary codes im-
106 plemented. In addition, the authors proposed a method
107 to obtain optimal binary codes (in terms of intermarker-
108 distance) using Mixed Integer Linear Programming [35].
109 Chilitags [36] is a variation of ArUco that employs a sim-
110 pler method for decoding the marker binary codes. As we
111 show in the experimental section, the method has a bad
112 behavior in high-resolution images.

113 The recent work [21] is a step towards the applicabil-
114 ity of such methods to large areas, proposing a method
115 for estimating the three-dimensional location of a set of
116 markers freely placed in the environment (Fig 1). Given
117 a set of images taken with a regular camera (such as a
118 mobile phone), the method automatically estimates their
119 location. This is an important step that allows extending
120 the robust localization of fiducial markers to very large
121 areas.

122 Although all fiducial marker systems aim maximum
123 speed in their design, few specific solutions have been pro-
124 posed to speed up the detection process. The work of
125 Johnston *et. al.* [37] is an interesting example in which
126 the authors propose a method to speed up computation by
127 parallelizing the image segmentation process. Neverthe-
128 less, both speed and computing power is a crucial aspect,
129 especially if the localization system needs to be embedded
130 in devices with limited resources.

131 Our work can be seen as an improvement of the ArUco
132 system, that according to our experience, is one of the most
133 reliable fiducial marker systems nowadays (see Sec 4 for
134 further details). We propose a novel method for marker de-
135 tection and identification that allows to speed up the com-
136 puting time in video sequences by wisely exploiting tempo-
137 ral information and applying multi-scale approach. In
138 contrast to previous works, no parallelization is required in
139 our method, thus making it especially attractive for mobile
140 devices with limited computational resources.

141 3. Speeded up marker detection

142 This section provides a detailed explanation of the
143 method proposed for speeding up the detection of squared
144 planar markers. First, Sect. 3.1 provides an overview of
145 the pipeline employed in the previous work, ArUco [17],
146 for marker detection and identification, highlighting the
147 parts of the process susceptible to be accelerated. Then,

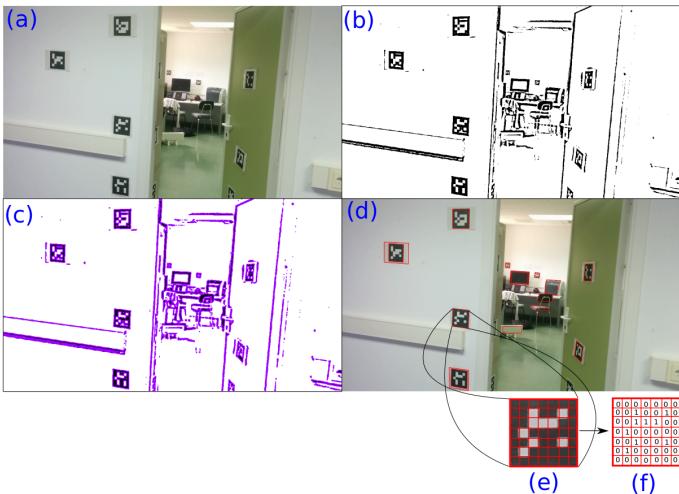


Figure 1: **Detection and identification pipeline of ArUco.** (a) Original image. (b) Image thresholded using an adaptive method. (c) Contours extracted. (d) Filtered contours that approximate to four-corner polygons. (e) Canonical image computed for one of the squared contours detected. (f) Binarization after applying Otsu’s method.

148 Sect. 3.2 explains the proposed method to speed up the
149 process.

150 3.1. Marker detection and identification in ArUco

151 The main steps for marker detection and identification
152 proposed in ArUco [17] are depicted in Figure 1. Given the
153 input image I (Figure 1a), the following steps are taken:

- 154 • Image segmentation (Figure 1b). Since the designed
155 markers have an external black border surrounded by
156 a white space, the borders can be found by segmen-
157 tation. In their approach, a local adaptive method is
158 employed: the mean intensity value m of each pixel
159 is computed using a window size w_t . The pixel is set
160 to zero if its intensity is greater than $m - c$, where c
161 is a constant value. This method is robust and ob-
162 tains good results for a wide range of values of its
163 parameters w_t and c .
- 164 • Contour extraction and filtering (Figures 1(c,d)). The
165 contour following algorithm of Suzuki and Abe [38]
166 is employed to obtain the set of contours from the
167 thresholded image. Since most of the contours ex-
168 tracted correspond to irrelevant background ele-
169 ments, a filtering step is required. First, contours too small
170 are discarded. Second, the remaining contours are
171 approximated to its most similar polygon using the
172 Douglas and Peucker algorithm [39]. Those that do
173 not approximate well to a four-corner convex polygon
174 are discarded from further processing.
- 175 • Marker code extraction (Figures 1(e,f)). The next
176 step consists in analyzing the inner region of the re-

177 maining contours to determine which of them are valid
178 markers. To do so, perspective projection is first re-
179 moved by computing the homography matrix, and the
180 resulting canonical image (Fig. 1e) is thresholded us-
181 ing the Otsu’s method [40]. The binarized image
182 (Fig. 1f) is divided into a regular grid and each ele-
183 ment is assigned a binary value according to the ma-
184 jority of the pixels in the cell. For each marker candi-
185 date, it is necessary to determine whether it belongs
186 to the set of valid markers or if it is a background el-
187 ement. Four possible identifiers are obtained for each
188 candidate, corresponding to the four possible rota-
189 tions of the canonical image. If any of the identifiers
190 belong to the set of valid markers, then it is accepted.

- 191 • Subpixel corner refinement. The last step consists in
192 estimating the location of the corners with subpixel
193 accuracy. To do so, the method employs a linear
194 regression of the marker’s contour pixels. In other
195 words, it estimates the lines of the marker sides em-
196 ploying all the contour pixels and computes the in-
197 tersections. This method, however, is not reliable for
198 uncalibrated cameras with small focal lenses (such as
199 fisheye cameras) since they usually exhibit high dis-
200 tortion.

201 When analyzing the computing times of this pipeline,
202 it can be observed that the Image segmentation and the
203 Marker code extraction steps are consuming most of the
204 computing time. The time employed in the image segmen-
205 tation step is proportional to the image size, that also in-
206 fluences the length of the contours extracted and thus the
207 computing time employed in the Contour extraction and
208 filtering step. The extraction of the canonical image (in
209 the Marker code extraction step) involves two operations.
210 First, computing the homography matrix, which is cheap.
211 But then, the inner region of each contour must be warped
212 to create the canonical image. This step requires access to
213 the image pixels of the contour region performing an inter-
214 polation in order to obtain the canonical image. The main
215 problem is that the time required to obtain the canonical
216 image depends on the size of the observed contour. The
217 larger a contour in the original image, the more time it is
218 required to obtain the canonical image. Moreover, since
219 most of the contours obtained do not belong to markers,
220 the system may employ a large amount of time computing
221 canonical images that will be later rejected.

222 A simpler approach to solving that problem would be to
223 directly sample a few sets of pixels from the inner region
224 of the marker. This is the method employed in ChiliTags.
225 However, as it will be shown in the experimental section,
226 it is prone to many false negatives.

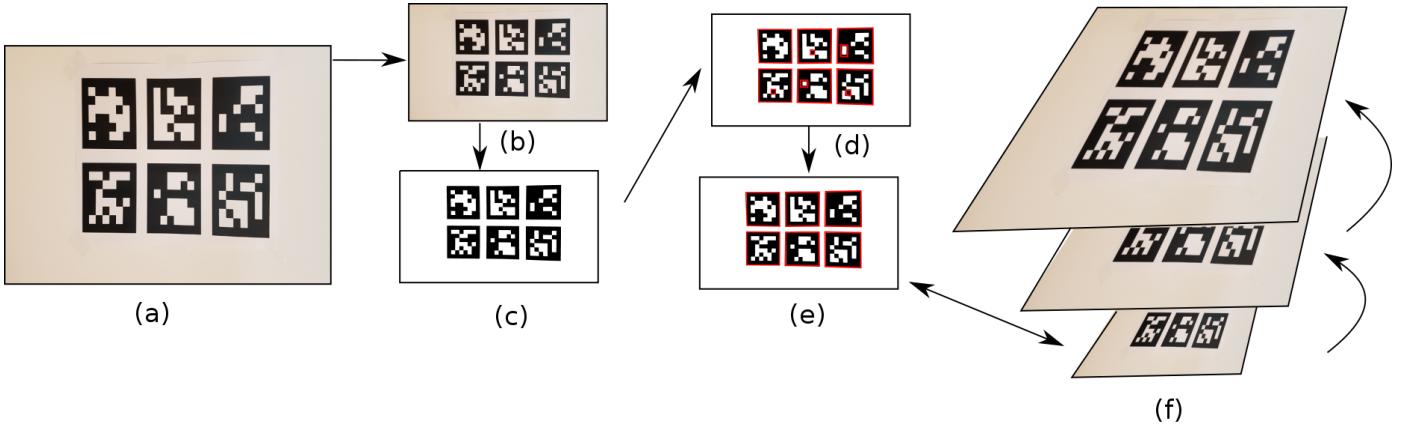


Figure 2: **Process pipeline** Main steps for fast detection and identification of squared planar markers.(a) Original input image. (b) Resized image for marker search. (c) Thresholed image. (d) Rectangles found (pink). (e) Markers detected with its corresponding identification. The image pyramid is used to speed up homography computation. (f) The corners obtained in (e) are upsampled to find their location in the original image with subpixel precision.

227 3.2. Proposed method

228 The key ideas of our proposal in order to speed up the
 229 computation are explained below. First, while the adaptive
 230 thresholding method employed in ArUco is robust to
 231 many illumination conditions without altering its param-
 232 eters, it is a time-consuming process that requires a con-
 233 volution. By taking advantage of temporal information,
 234 the adaptive thresholding method is replaced by a global
 235 thresholding approach.

236 Second, instead of using the original input image, a
 237 smaller version is employed. This is based on the fact
 238 that, in most cases, the useful markers for camera pose
 239 estimation must have a minimum size. Imagine an image
 240 of dimensions 1920×1080 pixels, in which a marker is de-
 241 tected as a small square with a side length of 10 pixels.
 242 Indeed, the estimation of the camera pose is not reliable
 243 at such small resolution. Thus, one might want to set a
 244 minimum length to the markers employed for camera pose
 245 estimation. For instance, let say that we only use markers
 246 with a minimum side length of $\tau_i = 100$ pixels, i.e., with a
 247 total area of 10.000 pixels. Another situation in which we
 248 can set a limit to the length of markers is when processing
 249 video sequences. It is clear that the length of a marker
 250 must be similar to its length in the previous frame.

251 Now, let us also think about the size of the canonical
 252 images employed (Figure 1e). The smaller the image, the
 253 faster the detection process but the poorer the image qual-
 254 ity. Our experience, however, indicates that very reliable
 255 detection of the binary code can be obtained from very
 256 small canonical images, such 32×32 pixels. In other words,
 257 all the rectangles detected in the image, no matter their
 258 side length, are reduced to canonical images of side length
 259 $\tau_c = 32$ pixels, for the purpose of identification.

260 Our idea, then, is to employ a reduced version of the
 261 input image, using the scale factor $\frac{\tau_c}{\tau_i}$, so as to speed up
 262 the segmentation step. In the reduced image, the smallest
 263 allowed markers, with a side length of 100 pixels in the
 264 original image, appears as rectangles with a side length of
 265 32 pixels. As a consequence, there will be no loss of quality
 266 when they are converted into the canonical image.

267 This idea has one drawback: the location of the corners
 268 extracted in the low resolution image is not as good esti-
 269 mations as the ones that can be obtained in the original
 270 image. Thus, the pose estimated with them will have a
 271 higher error. To solve that problem, a corner upsampling
 272 step is included, in which the precision of the corners is re-
 273 fined up to subpixel accuracy in the original input image
 274 by employing an image pyramid.

275 Finally, it must be considered that the generation of
 276 the canonical image is a very time-consuming operation
 277 (even if the process is done in the reduced image) that
 278 proportional to the contour length. We propose a method
 279 to perform the extraction of the canonical images in almost
 280 constant time (independently of the contour length) by
 281 wisely employing the image pyramid.

282 Below, there is a detailed explanation of the main steps
 283 of the proposed method, using Figure 2 to ease the expla-
 284 nation.

- 285 1. *Image Resize:* Given the input image I (Fig 2a), the
 286 first step consists in obtaining a resized version I^r
 287 (Fig 2b) that will be employed for segmentation. As
 288 previously pointed out, the size of the reduced image
 289 is calculated as:

$$I_w^r = \left\lfloor \frac{\tau_c}{\tau_i} I_w \right\rfloor; I_h^r = \left\lfloor \frac{\tau_c}{\tau_i} I_h \right\rfloor, \quad (1)$$

290 where the subscripts w and h denotes width and height

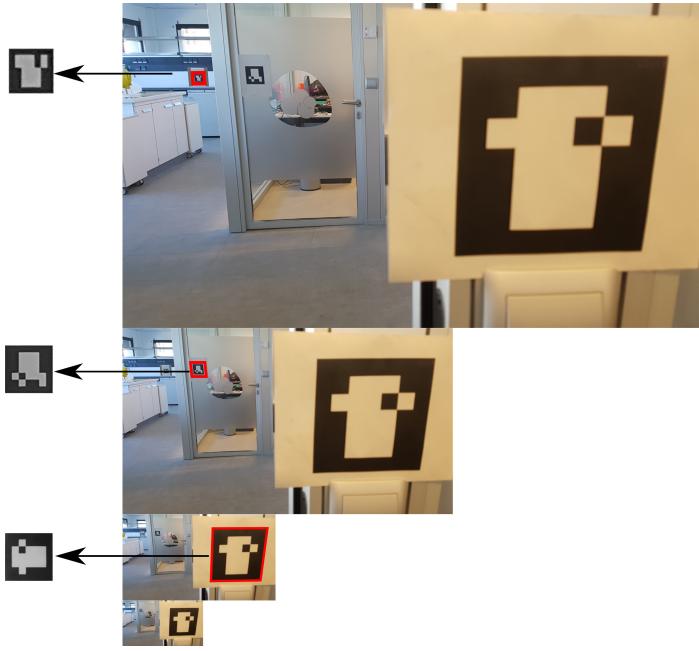


Figure 3: **Pyramidal Warping.** Scene showing tree marker at different resolutions. The left column shows the canonical images warped from the pyramid of images. Larger markers are warped from smaller images. For each marker, the image of the pyramid that minimizes the warping time while preserving the resolution is selected.

respectively. In order to decouple the desired minimum marker size from the input image dimensions, we define τ_i as:

$$\tau_i = \tau_c + \max(I_w, I_h)\tau_i \mid \tau_i \in [0, 1], \quad (2)$$

where the normalized parameter τ_i indicates the minimum marker size as a value in the range $[0, 1]$. When $\tau_i = 0$, the reduced image will be the same size as the original image. As τ_i tends to one, the image I^r becomes smaller, and consequently, the computational time required for the following step is reduced. The impact of this parameter in the final speed up is measured in the experimental section.

2. *Image Segmentation:* As already indicated, a global threshold method is employed using the following policy. If no markers were detected in the previous frame, a random threshold search is performed. The random process is repeated up to three times using the range of threshold values [10, 240]. For each tested threshold value, the whole pipeline explained below is performed. If after a number of attempts, no marker is found, it is assumed that no markers are visible in the frame. If at least one marker is detected, a histogram is created using the pixel values of all detected markers. Then, Otsu's algorithm [40] is employed to select the optimal threshold for the next frame. The calcu-

lated threshold is applied to I^r in order to obtain I^t (Fig 2c). As we show experimentally, the proposed method can adapt to smooth and abrupt illumination changes.

3. *Contour Extraction and Filtering:* First, contours are extracted from the image I^t using Suzuki and Abe algorithm [38], then small contours are removed. Since the extracted contours will rarely be squared (due to perspective projection), their perimeter is employed for rejection purposes: those with a perimeter smaller than $P(\tau_c) = 4 \times \tau_c$ pixels are rejected. For the remaining contours, a polygonal approximation is performed using Douglas and Peucker algorithm [39], and those that do not approximate to a convex polygon of four corners are also rejected. Finally, the remaining contours are the candidates to be markers (Fig 2d).

4. *Image Pyramid Creation:* An image pyramid

$$\mathcal{I} = (I^0, \dots, I^n)$$

with a set of resized versions of I , is created. I^0 denotes the original image and the subsequent images I^i are created by subsampling I^{i-1} by a factor of two. The number n of images in the pyramid is such that the smallest image dimensions is close to $\tau_c \times \tau_c$, i.e.,

$$n = \operatorname{argmin}_{v \mid I^v \in \mathcal{I}} |(I_w^v I_h^v) - \tau_c^2|. \quad (3)$$

5. *Marker Code Extraction:* In this step the canonical images of the remaining contours must be extracted and then binarized. Our method uses the pyramid of images \mathcal{I} previously computed to ensure that the process is performed in constant time, independently of the input image and contour sizes. The key principle is selecting, for each contour, the image from the pyramid in which the contour length is most similar to the canonical image length $P(\tau_c)$. In this manner, warping is faster.

Let us consider a detected contour $\vartheta \in I^r$, and denote by $P(\vartheta)^j$ its perimeter in the image $I^j \in \mathcal{I}$. Then, the best image $I^h \in \mathcal{I}$ for homography computation is selected as:

$$I^h \mid h = \operatorname{argmin}_{j \in \{0, 1, \dots, n\}} |P(\vartheta)^j - P(\tau_c)|. \quad (4)$$

The pyramidal warping method employed can be better understood in Fig. 3, which shows a scene with three markers at different distances. The left images represent the canonical images obtained while the right images show the pyramid of images. In our method, the canonical image of the smallest marker is extracted from the largest image in the pyramid (top

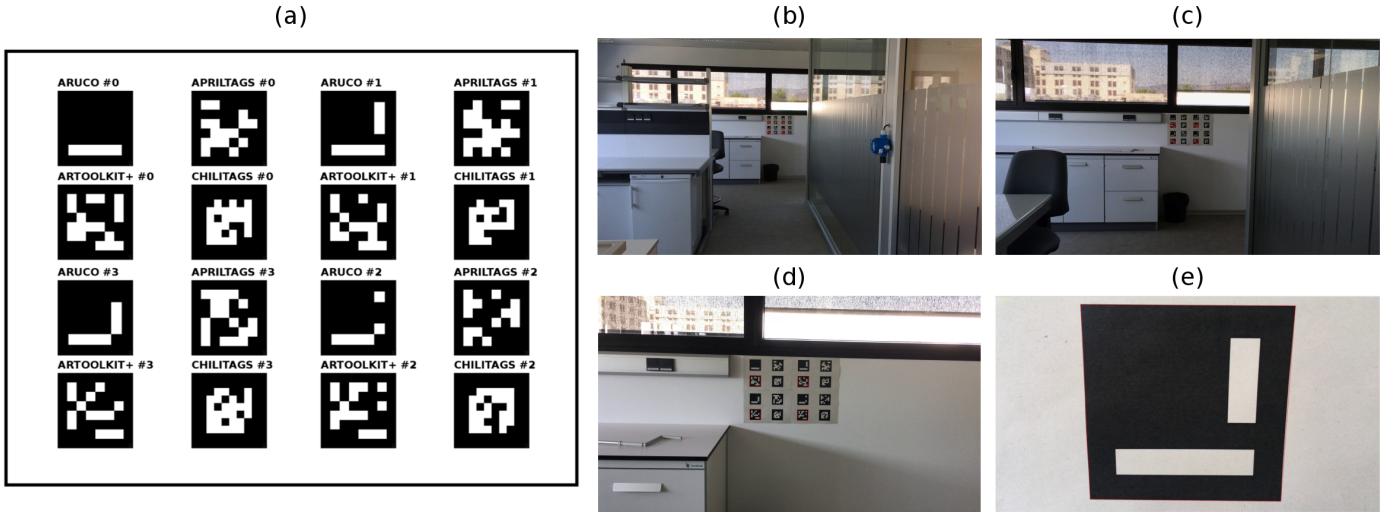


Figure 4: **Test sequences.** (a) The set of 16 markers employed for evaluation. There are four markers from each method tested: ArUco, AprilTags, ArToolKit+ and ChiliTags. (b-e) Images from the video sequences used for testing. The markers are seen as small as in (b), and as big as in (e), where the marker represents the 40% of the total image area.

row of Fig 3). As the length of the marker increases, smaller images of the pyramid are employed to obtain the canonical view. This guarantees that the canonical image is obtained in almost constant time using the minimum possible computation.

Finally, for each canonical image, the Otsu's method [40] for binarization is employed, and the inner code analyzed to determine whether it is a valid marker or not. This is a very cheap operation.

6. Corner Upsampling: So far, markers have been detected in the image I^r . However, it is required to precisely localize their corners in the original image I . As previously indicated, the precision of the estimated camera pose is directly influenced by the precision in the corner localization. Since the difference in size between the images I and I^r can be very large, a direct upsampling can lead to errors. Instead, we proceed in incremental steps looking for the corners in larger versions of the image I^r until the image I is reached.

For the corner upsampling task, the image $I^i \in \mathcal{I}$ of the pyramid with the most similar size to I^r is selected in the first place, i.e.,

$$I^i = \underset{I^v \in \mathcal{I}}{\operatorname{argmin}} |(I_w^v I_h^v) - (I_w^r I_h^r)|. \quad (5)$$

Then, the position of each contour corner in the image I^i is computed by simply upsampling the corner locations. This is, however, an approximate estimation that does not precisely indicate the corner position in the image I^i . Thus, a corner refinement process is done in the vicinity of each corner so as to find its best

location in the selected image I^i . For that purpose, the method implemented in the OpenCV library [41] has been employed. Once the search is done in I^i for all corners, the operation is repeated for the image I^{i-1} , until I^0 is reached. In contrast to the ArUco approach, this one is not affected by lens distortions.

7. Estimation of τ_i : The parameter τ_i has a direct influence in the computation time. The higher it is, the faster the computation. A naive approach consists in setting a fixed value for this parameter. However, when processing video sequences, the parameter can be automatically adjusted at the end of each frame. In the first image of the sequence, the parameter τ_i is set to zero. Thus, markers of any size are detected. Then, for the next frame, τ_i is set to a value slightly smaller than the size of the smallest marker detected in the previous frame. In this way, markers could be detected even if the camera moves away from them. Therefore, the parameter τ_i can be dynamically updated as:

$$\tau_i = (1 - \tau_s)P(\vartheta^s)/4 \quad (6)$$

where ϑ^s is the marker with the smallest perimeter found in the image, and τ_s is a factor in the range $(0, 1]$ that accounts for the camera motion speed. For instance, when $\tau_s = 0.1$, it means that in the next frame, τ_i is such that markers 10% smaller than the smallest marker in the current image will be sought. If no markers are detected in a frame, τ_i is set to zero so that in the next frame markers of any size can be detected.

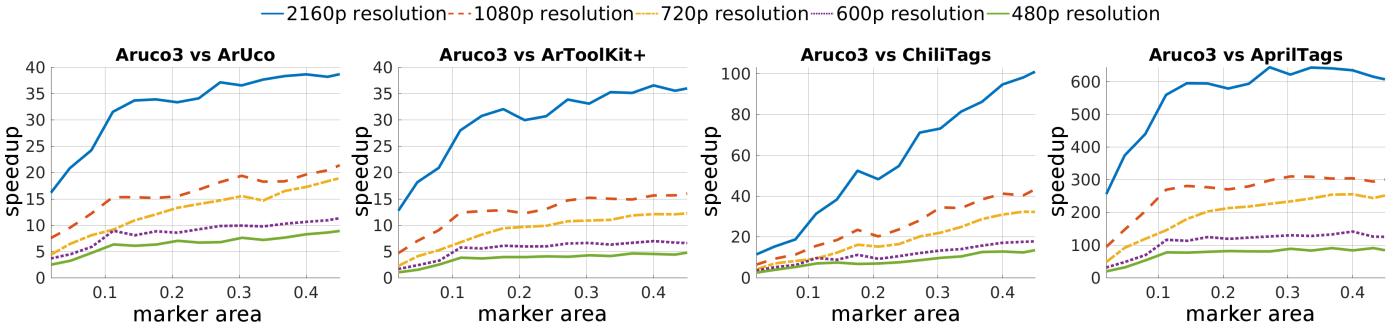


Figure 5: SpeedUp of ArUco3 compared to ArUco, ArToolKit+, ChiliTags and AprilTags for resolutions: 4K (3840×2160), 1080p (1920×1080), 720p (1280×720), 600p (800×600) and 480p (640×480). The horizontal axis represents the percentage of area occupied by the markers in each frame, and the vertical axis one indicates how many times ArUco3 is faster.

401 As can be observed, the proposed pipeline includes a
 402 number of differences with respect to the original ArUco
 403 pipeline that allows increasing significantly the processing
 404 speed as we show next.

405 4. Experiments and results

406 This section shows the results obtained to validate the
 407 methodology proposed for the detection of fiducial mark-
 408 ers.

409 First, in Sect 4.1, the computing times of our proposal
 410 are compared to the best alternatives found in the liter-
 411 ature: AprilTags [18], ChiliTags [36], ArToolKit+ [31],
 412 as well as ArUco [17] which is included in the OpenCV
 413 library³. Then, Sect. 4.2 analyses and compares the sensi-
 414 tivity of the proposed method with the above-mentioned
 415 methods. The main goal is to demonstrate that our ap-
 416 proach is able to reliably detect the markers with a very
 417 high true positive ratio, under a wide range of marker reso-
 418 lutions, while keeping the false positive rate to zero. After-
 419 ward, Sect. 4.3 studies the impact of the different system
 420 parameters on the speed and sensitivity, while Sect. 4.4
 421 evaluates the precision in the estimation of the corners.
 422 Finally, Sect. 4.5 shows the performance of the proposed
 423 method in a realistic video sequence with occlusions, illu-
 424 mination, and scale changes.

425 To carry out the first three experiments, several videos
 426 have been recorded in our laboratory. Figure 4(b-e) shows
 427 some images of the video sequences employed. For these
 428 tests, a panel with a total of 16 markers was printed (Fig-
 429 ure 4a), four from each one of the fiducial markers em-
 430 ployed. The sequences were recorded at different distances
 431 at a frame rate of 30 fps using an Honor 5 mobile phone at
 432 4K resolution. The videos employed are publicly available
 433 ⁴ for evaluation purposes.

434 In the video, there are frames in which the markers ap-
 435 pear as small as can be observed in Figure 4b, where
 436 the area of each marker occupies only 0.5% of the image,
 437 and frames in which the marker is observed as big as in
 438 Figure 4e, where the marker occupies 40% of total im-
 439 age area. In total, the video sequences recorded sum up
 440 to 10.666 frames. The video frames have been processed
 441 at different resolutions so that the impact of the image
 442 resolution in the computing time can be analyzed. In par-
 443 ticular, the following the standard image resolutions have
 444 been employed: 4K (3840×2160), 1080p (1920×1080),
 445 720p (1280×720), 600p (800×600) and 480p (640×480).

446 All tests were performed using an Intel® Core™ i7-
 447 4700HQ 8-core processor with 8 GB RAM and Ubuntu
 448 16.04 as the operating system. However, only one execu-
 449 tion thread was employed in the tests performed.

450 It must be indicated that the code generated as part of
 451 this work has been publicly released as the version 3 of the
 452 popular ArUco library⁵. So, in the experiments section,
 453 the method proposed in this paper will be referred to as
 454 ArUco3.

455 4.1. Speedup

456 This section compares the computing times of the pro-
 457 posed method with the most commonly used alternatives
 458 AprilTags, ArToolKit+, ChiliTags, and ArUco. To do so,
 459 we compute the speedup of our approach as the ratio be-
 460 tween the computing time of an alternative (t_1) and the
 461 computing time of ArUco3 (t_2) in processing the same im-
 462 age:

$$463 \text{SpeedUp} = t_1/t_2 \quad (7)$$

464 In our method, the value $\tau_c = 32$ was employed in all the
 465 sequences, while τ_i and the segmentation threshold where
 466 automatically computed as explained in the Steps 2 and 7
 467 of the proposed method (Sect. 3.2).

³<https://opencv.org/>

⁴<https://mega.nz/#F!DnA1wIAQ!6f6owb81G0E7Sw3EfddUXQ>

⁵<http://www.uco.es/grupos/ava/node/25>

Table 1: Mean computing times (milliseconds) of the different steps of the proposed method for different resolutions.

| | Resolution | | | | |
|---|------------|-------|-------|-------|-------|
| | 480p | 600p | 720p | 1080p | 2160p |
| Step 1:Image Resize | 0.037 | 0.050 | 0.057 | 0.068 | 0.101 |
| Step 2:Image Segmentation | 0.044 | 0.048 | 0.059 | 0.084 | 0.351 |
| Step 3:Contour Extraction and Filtering | 0.219 | 0.250 | 0.301 | 0.403 | 1.109 |
| Step 4:Image Pyramid Creation | 0.037 | 0.076 | 0.096 | 0.186 | 0.476 |
| Step 5:Marker code extraction | 0.510 | 0.519 | 0.542 | 0.547 | 0.583 |
| Step 6:Corner Upsampling | 0.058 | 0.065 | 0.079 | 0.096 | 0.134 |
| Time (ms) | 0.903 | 1.009 | 1.133 | 1.384 | 2.755 |

Fig. 5 shows the speedup of our approach for different image resolutions. The horizontal axis represents the relative area occupied by the marker in the image, while the vertical axis represents the speedup. A total of 30 speed measurements were performed for each image, taking the median computing time for our evaluation. In the tests, the speedup is evaluated as a function of the observed marker area in order to better understand the behavior of our approach.

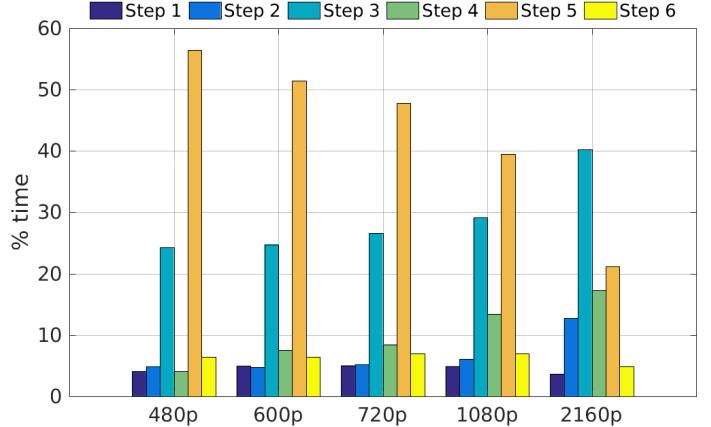
The tests conducted clearly show that the proposed method (ArUco3) is faster than the rest of the methods and that the speedup increases with the image resolution and with the observed marker area. Compared to ArUco implementation in the OpenCV library, the proposed method is significantly faster, achieving a minimum speedup of 17 in 4K resolutions, up to 40 in the best case.

In order to properly analyze the computing times of the different steps of the proposed method (Sect. 3.2), Table 1 shows a summary for different image resolutions. Likewise, Fig. 6 shows the percentage of the total time required by each step. Please notice that Step 7 (Eq. 6) has been omitted because its computing time is negligible.

As can be seen, the two most time-consuming operations are Step 3 and 5. In particular, Step 5 requires special attention, since it proves the validity of the multi-scale method proposed for marker warping. It can be observed in the table, that the amount of time employed by Step 5 is constant across all resolutions. In other words, the computing time does not increase significantly with the image resolution. Also notice how the time of Step 3 increases in 2160p. It is because this step involves operations that depend on the image dimensions, which grow quadratically. An interesting future work is to develop methods reducing the time for contour extraction and filtering in high-resolution images.

In any case, considering the average total computing time, the proposed method achieves in average more than 360 fps in 4K resolutions and more than 1000 fps in the lowest resolution, without any parallelism.

Figure 6: **Main steps ArUco3 times.** Percentage of time of the global computation required by each of the steps for resolutions: 4K, 1080p, 720p, 600p and 480p.



4.2. Sensitivity analysis

Correct detection of markers is a critical aspect that must be analyzed to verify that the proposed algorithm is able to obviate redundant information present in the scene, extracting exclusively marker information. Fig. 7 shows the True Positive Rate (TPR) of the proposed method as a function of the area occupied by the marker in the image for different image resolutions.

As can be observed, below certain marker area, the detection is not reliable. This is because the observed marker area is very small, making it difficult to distinguish the different bits of the inner binary code. Once the observed area of the marker reaches a certain limit, the proposed method achieves perfect detection in all resolutions. It must be remarked, that the False Positive Rate is zero in all cases tested. Since it is a binary problem, the True Negative Rate is one (TNR=1-FPR).

For a comparative evaluation performance between ArUco3 and the other methods, the TPR has been analyzed individually and the results are shown in Fig. 7. As can be observed, ArUco behaves exactly like ArUco3. AprilTags, however, has very poor behavior in all resolutions, especially as the marker or the image sizes increases. As we already commented in Sect. 2, AprilTags does not rely on warping the marker image but instead does a subsampling of a few pixels on the image in order to obtain the binary code. This may be one of the reasons for its poor performance. ArToolkit+ behaves reasonably well across all the image resolutions and marker areas, while Chilitags shows a somewhat unreliable behavior in all resolutions but 480p.

In conclusion, the proposed approach behaves similar to the previous version of ArUco.

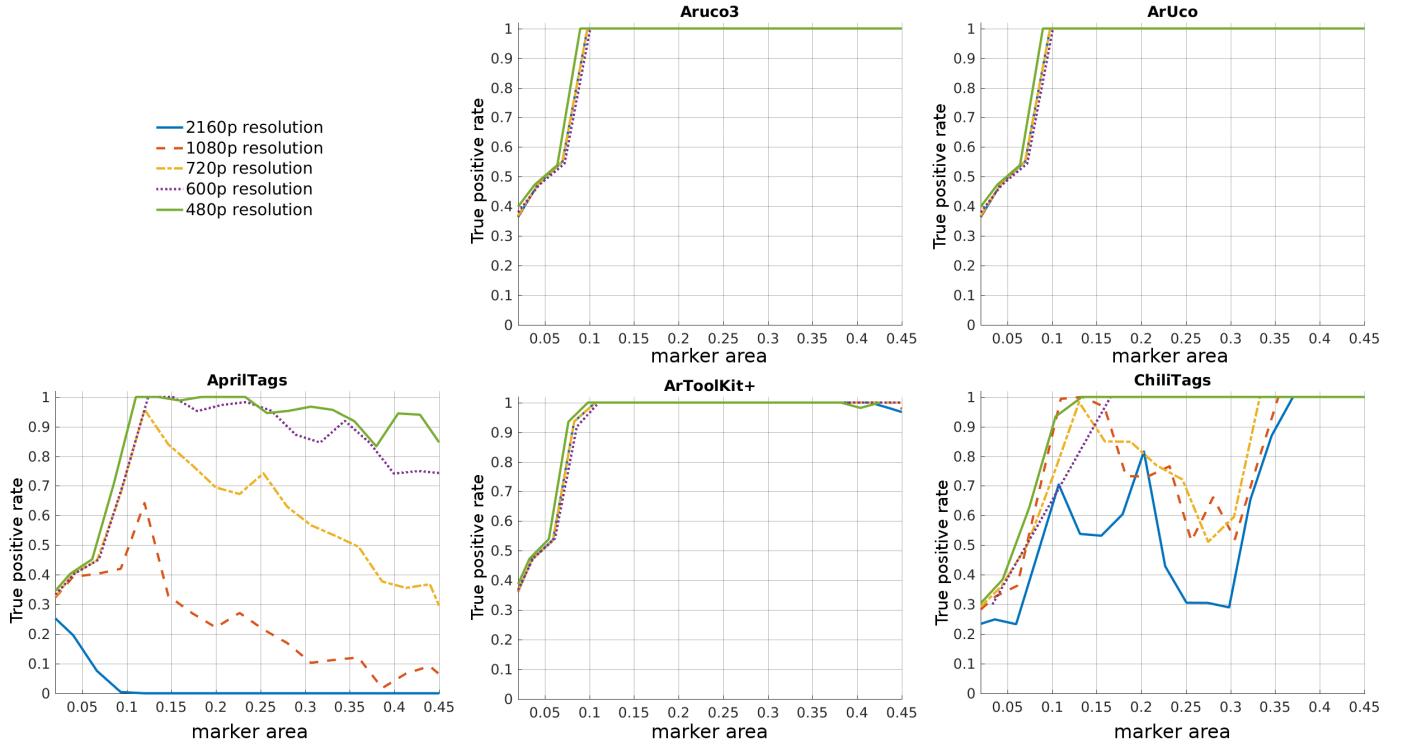


Figure 7: **True Positive Ratio.** Mean true positive ratio (TPR) for ArUco3, Chilitags, ArUco, ArToolKit+ and AprilTags for resolutions: 4K, 1080p, 720p, 600p and 480p), as function of the observed area for the set of markers.

532 4.3. Analysis of parameters

533 The computing time and robustness of the proposed
 534 method depend mainly on two parameters, namely τ_i
 535 which indicates the minimum size of the markers detected,
 536 and τ_c , the size of the canonical image.

537 The parameter τ_i has an influence on the computing
 538 time, since it determines the size of the resized image I'
 539 (Eq. 1). We have analyzed the speed as a function of
 540 this parameter and the results are shown in Fig. 8. The
 541 figure represents the horizontal axis the value τ_i , and in the
 542 vertical axis, the average speed (measured as frames per
 543 second) in the sequences analyzed, independently of the
 544 observed marker area. A different line has been depicted
 545 for each image resolution. In this case, we have set fixed
 546 the parameter $\tau_c = 32$.

547 It can be observed that the curves follow a similar pattern
 548 in the five cases analyzed. In general, the maximum
 549 increase in speed is obtained in the range of values
 550 $\tau_i = (0, 0.2)$. Beyond that point, the improvement be-
 551 comes marginal. To better understand the impact of this
 552 parameter, Table 2 shows the reduction of the input im-
 553 age size I for different values of τ_i . For instance, when
 554 $\tau_i = 0.02$, the resized image I' is 48% smaller than the
 555 original input image I (see Eq. 1). Beyond $\tau_i = 0.2$, the
 556 resized image is so small that it has not a big impact in the
 557 speedup because there are other steps with a fixed com-

puting time such as the Step 5 (Marker Code Extraction).

558 Table 2: Image size reduction for different values of τ_i .

| τ_i | 0.01 | 0.015 | 0.02 | 0.1 | 0.2 |
|----------------|------|-------|------|-----|-----|
| Size reduction | 0% | 31% | 48% | 82% | 90% |

559 In any case, it must be noticed that the proposed
 560 method is able to achieve 1000 fps in 4K resolutions when
 561 detecting markers larger than 10% ($\tau_i = 0.1$) of the image
 562 area, and the same limit of 1000 fps is achieved for 1080p
 563 resolutions for $\tau_i = 0.05$.

564 With regards to the parameter τ_c , it indirectly influences
 565 the speed since it determines the size of the resized images
 566 (Eq 1). The smaller it is, the smaller the resized image I' .
 567 Nevertheless, this parameter also has an influence on the
 568 correct detection of the markers. The parameter indicates
 569 the size of the canonical images used to identify the
 570 binary code of markers. If the canonical image is very small,
 571 pixels are mixed up, and identification is not robust. Con-
 572sequently, the goal is to determine the minimum value of
 573 τ_c that achieves the best TPR. Fig. 9 shows the TPR ob-
 574tained for different configurations of the parameter τ_c . As
 575 can be seen, for low values of the parameter τ_c (between
 576 8 and 32) the system shows problems in the detection of
 577 markers. However, for $\tau_c \geq 32$ there is no improvement in
 578 the TPR. Thus, we conclude that the value $\tau_c = 32$ is the

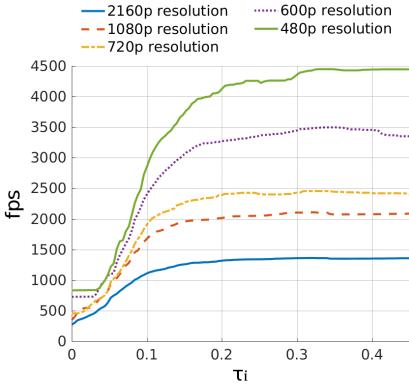


Figure 8: **Parameter τ_i .** Speed of method as a function of the parameter τ_i for the different resolutions tested.

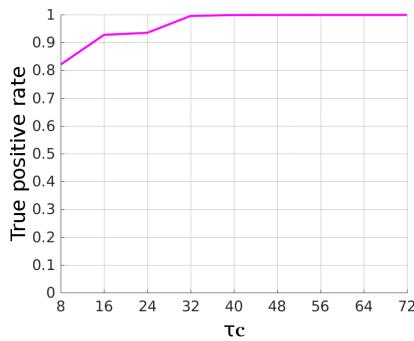


Figure 9: **Parameter τ_c .** True positive rate obtained by different configurations of parameter τ_c

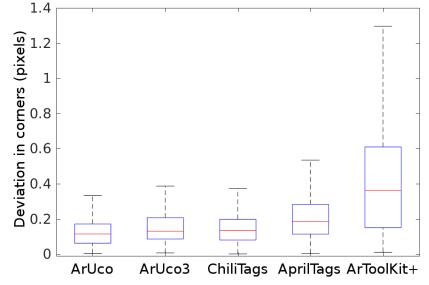


Figure 10: **Vertex jitter** measured for the different marker systems.

best choice.

4.4. Precision of corner detection

An important aspect to consider in the detection of the markers is *vertex jitter*, which refers to the noise in the estimation of the corners' location. These errors are problematic because they propagate to the estimation of the camera pose. In our method, a corner upsampling step (Step 6 in Sect. 3.2) is proposed to refine the corners' estimations from the reduced image I^r to the original image I . This section analyzes the proposed method comparing the results with the other marker systems.

In order to perform the experiments, the camera has been placed at a fixed position recording the set of markers already presented in Fig. 4a. Since the camera is not moving, the average location estimated for each corner can be considered to be the correct one (i.e., a Gaussian error distribution is assumed). Then, the standard deviation is an error measure for the localization of the corners. The process has been repeated a total of six times at varying distances and the results obtained are shown in Fig. 10 as box plots. In Table 3, the average error of each method has been indicated.

Table 3: Vertex jitter analysis: Standard deviations of the different methods in estimating the marker corners.

| Method | ArUco | ArUco3 | Chilitags | AprilTags | ArToolKit+ |
|---------------------|-------|--------|-----------|-----------|------------|
| Average error (pix) | 0.140 | 0.161 | 0.174 | 0.225 | 0.432 |

As can be observed, the ArUco system obtains the best results, followed by our proposal ArUco3. However, it can be seen that the difference between both methods is of only 0.02 pixels, which is very small to consider it relevant. Chilitags shows a similar behavior than ArUco and ArUco3, but AprilTags and ArToolKit+ exhibit worse performance.

4.5. Video sequence analysis

This section aims at showing the behavior of the proposed system in a realistic scenario. For that purpose, four markers have been placed in an environment with irregular lighting and a video sequence has been recorded using a 4K mobile phone camera. Figure 11(a-e) show the frames 1, 665, 1300, 1700 and 2100 of the video sequence. At the start of the sequence, the camera is around five meters away from the markers. The camera approaches the markers and then moves away again. As can be seen, around frame 650 (Figure 11b), the user occludes the markers temporarily.

Figure 11f shows the values of the parameter τ_i automatically calculated along the sequence and Figure 11g the processing speed. As can be observed, the system is able to automatically adapt the value of τ_i according to the observed marker area, thus adapting the computing speed of the system. The maximum speed is obtained around the frame 1300 when the camera is closest to the markers.

It can also be observed that around frame 650 when the user occludes the markers with his hand, the system is unable to detect any marker. Thus, the system searches for the full resolution image ($\tau_i = 0$) and the speed decreases. However, when the markers are observed again, the system recovers its speed.

Finally, Figure 11h shows the threshold values employed for segmentation in each frame. As can be seen, the system adapts to the illumination changes. Along the sequence, the system does not produce any false negative nor positives.

5. Conclusions and future work

This paper has proposed a novel approach for detecting fiducial markers aimed at maximizing speed while preserving accuracy and robustness. The proposed method

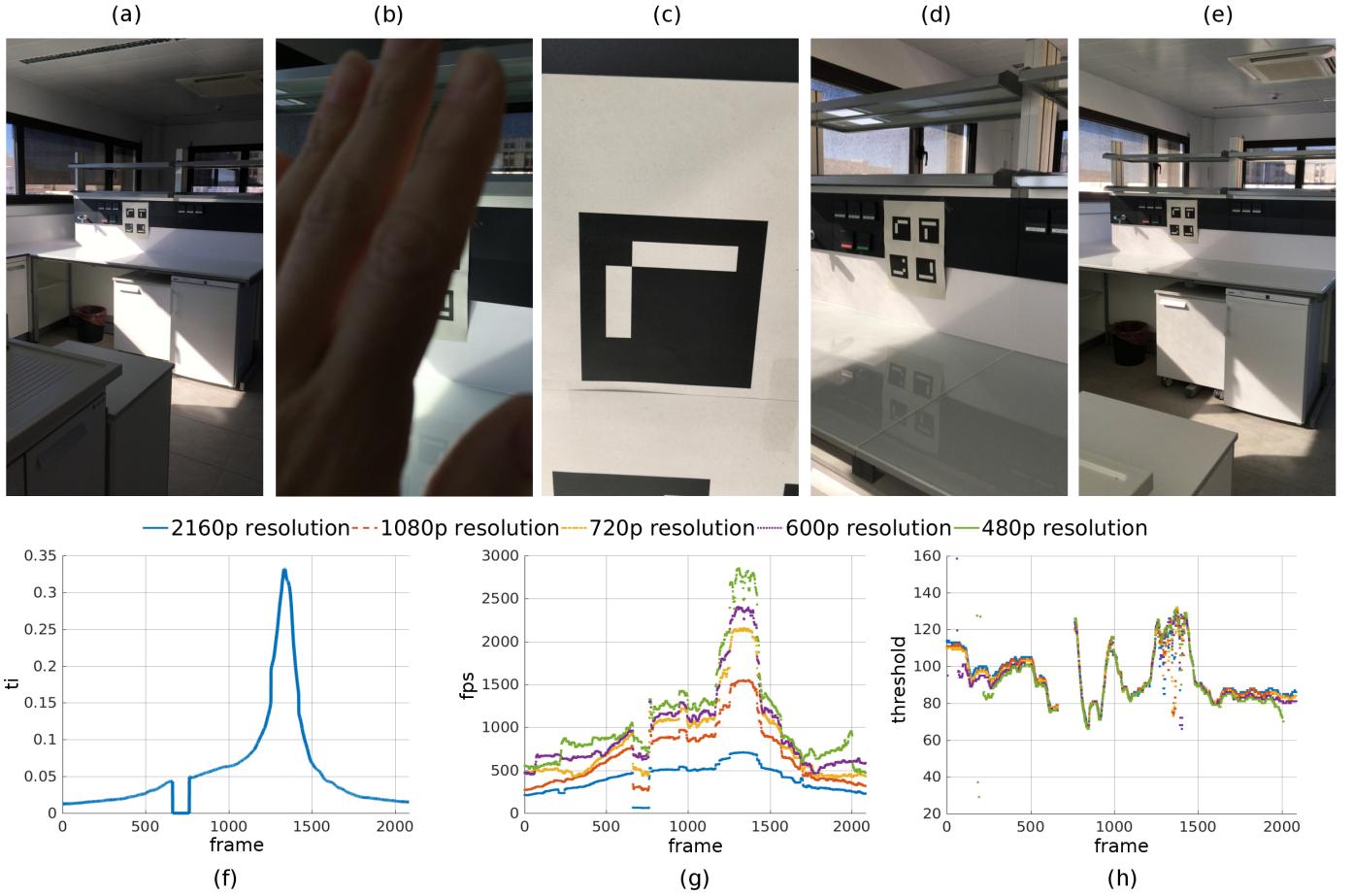


Figure 11: **Video Sequence** in a realistic scenario. (a-e) Frames of the video sequence. The camera approaches the marker and then moves away. The user occludes the camera temporarily. (f) Evolution of the parameter τ_i automatically computed. (g) Speed of the proposed method in each frame of the sequence. (h) Thresholds automatically computed for each frame. The system adapts to illumination changes.

642 is specially designed to take advantage of the increasing
 643 camera resolutions available nowadays. Instead of detecting
 644 markers in the original image, a smaller version of the
 645 image is employed, in which the detection can be done
 646 at higher speed. By wisely employing a multi-scale image
 647 representation, the proposed method is able to find the
 648 position of the marker corners with subpixel accuracy in the
 649 original image. The size of the processed image, as well
 650 as the threshold employed for segmentation, are dynam-
 651 ically adapted in each frame considering the information
 652 of the previous one. As a consequence, the system speed
 653 dynamically adapts in order to achieve the maximum per-
 654 formance.

655 As shown experimentally, the proposed method outper-
 656 forms the state-of-the-art systems in terms of computing
 657 speed, without compromising the sensitivity or the pre-
 658 cision. Our method is between 17 and 40 times faster than
 659 the ArUco approach implemented in the OpenCV library.
 660 When compared to other approaches such as Chilitags,
 661 AprilTags, and ArToolKit+, our method achieves even

higher speedups.

We consider as possible future works to investigate the
 662 use of the proposed method in fish-eye cameras. The idea
 663 is to compare the method with the rectified images if there
 664 is analyze the method's performance in presence of high
 665 distortion. Also, we as well as to characterize the perfor-
 666 mance when multiple fiducial markers with significantly
 667 different scales are present in the same image.

668 Our system, which is publicly available as open source
 669 code⁶, is a cost-effective tool for fast and precise self-
 670 localization in applications such as robotics, unman-
 671 nered vehicles or augmented reality applications.

Acknowledgments

672 This project has been funded under projects TIN2016-
 673 75279-P and IFI16/00033 (ISCIII) of Spain Ministry of
 674 Economy, Industry and Competitiveness, and FEDER.

⁶<http://www.uco.es/grupos/ava/node/25>

678

References

- [1] R. Sim, J. J. Little, Autonomous vision-based robotic exploration and mapping using hybrid maps and particle filters, *Image and Vision Computing* 27 (1) (2009) 167 – 177, canadian Robotic Vision 2005 and 2006.
- [2] A. Pichler, S. C. Akkaladevi, M. Ikeda, M. Hofmann, M. Plasch, C. Wögerer, G. Fritz, Towards shared autonomy for robotic tasks in manufacturing, *Procedia Manufacturing* 11 (Supplement C) (2017) 72 – 82, 27th International Conference on Flexible Automation and Intelligent Manufacturing, FAIM2017, 27–30 June 2017, Modena, Italy.
- [3] R. Valencia-Garcia, R. Martinez-Béjar, A. Gasparetto, An intelligent framework for simulating robot-assisted surgical operations, *Expert Systems with Applications* 28 (3) (2005) 425 – 433.
- [4] A. Broggi, E. Dickmanns, Applications of computer vision to intelligent vehicles, *Image and Vision Computing* 18 (5) (2000) 365 – 366.
- [5] T. Patterson, S. McClean, P. Morrow, G. Parr, C. Luo, Timely autonomous identification of uav safe landing zones, *Image and Vision Computing* 32 (9) (2014) 568 – 578.
- [6] D. González, J. Pérez, V. Milanés, Parametric-based path generation for automated vehicles at roundabouts, *Expert Systems with Applications* 71 (2017) 332 – 341.
- [7] J. L. Sanchez-Lopez, J. Pestana, P. de la Puente, P. Campoy, A reliable open-source system architecture for the fast designing and prototyping of autonomous multi-uav systems: Simulation and experimentation, *Journal of Intelligent & Robotic Systems* (2015) 1–19.
- [8] M. Olivares-Mendez, S. Kannan, H. Voos, Vision based fuzzy control autonomous landing with uavs: From v-rep to real experiments, in: *Control and Automation (MED), 2015 23th Mediterranean Conference on*, 2015, pp. 14–21.
- [9] S. Pfugl, R. Vasireddy, T. Lerch, T. M. Ecker, M. Tannast, N. Boemke, K. Siebenrock, G. Zheng, Augmented marker tracking for peri-acetabular osteotomy surgery, in: *2017 39th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC)*, 2017, pp. 937–941.
- [10] J. P. Lima, R. Roberto, F. Simões, M. Almeida, L. Figueiredo, J. M. Teixeira, V. Teichrieb, Markerless tracking system for augmented reality in the automotive industry, *Expert Systems with Applications* 82 (2017) 100 – 114.
- [11] P. Chen, Z. Peng, D. Li, L. Yang, An improved augmented reality system based on andar, *Journal of Visual Communication and Image Representation* 37 (2016) 63 – 69, weakly supervised learning and its applications.
- [12] S. Khattak, B. Cowan, I. Chepurna, A. Hogue, A real-time reconstructed 3d environment augmented with virtual objects rendered with correct occlusion, in: *Games Media Entertainment (GEM)*, 2014 IEEE, 2014, pp. 1–8.
- [13] J. Engel, T. Schöps, D. Cremers, LSD-SLAM: Large-scale direct monocular SLAM, 2014.
- [14] R. Mur-Artal, J. M. M. Montiel, J. D. Tardós, Orb-slam: A versatile and accurate monocular slam system, *IEEE Transactions on Robotics* 31 (5) (2015) 1147–1163.
- [15] Cooperative pose estimation of a fleet of robots based on interactive points alignment, *Expert Systems with Applications* 45 (2016) 150 – 160.
- [16] S.-h. Zhong, Y. Liu, Q.-c. Chen, Visual orientation inhomogeneity based scale-invariant feature transform, *Expert Syst. Appl.* 42 (13) (2015) 5658–5667.
- [17] S. Garrido-Jurado, R. Muñoz Salinas, F. J. Madrid-Cuevas, M. J. Marín-Jiménez, Automatic generation and detection of highly reliable fiducial markers under occlusion, *Pattern Recognition* 47 (6) (2014) 2280–2292.
- [18] E. Olson, Apriltag: A robust and flexible visual fiducial system, in: *Robotics and Automation (ICRA), 2011 IEEE International Conference on*, 2011, pp. 3400–3407.
- [19] F. Ababsa, M. Mallem, Robust camera pose estimation using 2d fiducials tracking for real-time augmented reality systems, in: *Proceedings of the 2004 ACM SIGGRAPH International Conference on Virtual Reality Continuum and Its Applications in Industry, VRCAI '04*, 2004, pp. 431–435.
- [20] V. Mondéjar-Guerra, S. Garrido-Jurado, R. Muñoz-Salinas, M.-J. Marín-Jiménez, R. Medina-Carnicer, Robust identification of fiducial markers in challenging conditions, *Expert Systems with Applications* 93 (1) (2018) 336–345.
- [21] R. Muñoz-Salinas, M. J. Marín-Jimenez, E. Yeguas-Bolivar, R. Medina-Carnicer, Mapping and localization from planar markers, *Pattern Recognition* 73 (January 2018) 158 – 171.
- [22] K. Dorfmüller, H. Wirth, Real-time hand and head tracking for virtual environments using infrared beacons, in: *in Proceedings CAPTECH'98*. 1998, Springer, 1998, pp. 113–127.
- [23] M. Ribo, A. Pinz, A. L. Fuhrmann, A new optical tracking system for virtual and augmented reality applications, in: *In Proceedings of the IEEE Instrumentation and Measurement Technical Conference*, 2001, pp. 1932–1936.
- [24] V. A. Knyaz, R. V. Sibiryakov, The development of new coded targets for automated point identification and non-contact surface measurements, in: *3D Surface Measurements, International Archives of Photogrammetry and Remote Sensing, Vol. XXXII, part 5*, 1998, pp. 80–85.
- [25] L. Naimark, E. Foxlin, Circular data matrix fiducial system and robust image processing for a wearable vision-inertial self-tracker, in: *Proceedings of the 1st International Symposium on Mixed and Augmented Reality, ISMAR '02*, IEEE Computer Society, Washington, DC, USA, 2002, pp. 27–36.
- [26] J. Rekimoto, Y. Ayatsuka, Cybercode: designing augmented reality environments with visual tags, in: *Proceedings of DARE 2000 on Designing augmented reality environments, DARE '00*, ACM, New York, NY, USA, 2000, pp. 1–10.
- [27] M. Rohs, B. Gfeller, Using camera-equipped mobile phones for interacting with real-world objects, in: *Advances in Pervasive Computing*, 2004, pp. 265–271.
- [28] M. Kaltenbrunner, R. Bencina, reactivision: a computer-vision framework for table-based tangible interaction, in: *Proceedings of the 1st international conference on Tangible and embedded interaction, TEI '07*, ACM, New York, NY, USA, 2007, pp. 69–74.
- [29] H. Kato, M. Billinghurst, Marker tracking and hmd calibration for a video-based augmented reality conferencing system, in: *Augmented Reality, 1999. (IWAR '99) Proceedings. 2nd IEEE and ACM International Workshop on*, 1999, pp. 85–94.
- [30] S. Lin, D. J. Costello, *Error Control Coding*, Second Edition, Prentice-Hall, Inc., Upper Saddle River, NJ, USA, 2004.
- [31] D. Wagner, D. Schmalstieg, ARToolKitPlus for pose tracking on mobile devices, in: *Computer Vision Winter Workshop*, 2007, pp. 139–146.
- [32] D. Schmalstieg, A. Fuhrmann, G. Hesina, Z. Szalavári, L. M. Encarnaçāo, M. Gervautz, W. Purgathofer, The studierstube augmented reality project, *Presence: Teleoper. Virtual Environ.* 11 (1) (2002) 33–54.
- [33] M. Fiala, Designing highly reliable fiducial markers, *IEEE Transactions on Pattern Analysis and Machine Intelligence* 32 (7) (2010) 1317–1324.
- [34] D. Flohr, J. Fischer, A Lightweight ID-Based Extension for Marker Tracking Systems, in: *Eurographics Symposium on Virtual Environments (EGVE) Short Paper Proceedings*, 2007, pp. 741
742
743
744
745
746
747
748
749
750
751
752
753
754
755
756
757
758
759
760
761
762
763
764
765
766
767
768
769
770
771
772
773
774
775
776
777
778
779
780
781
782
783
784
785
786
787
788
789
790
791
792
793
794
795
796
797
798
799
800
801
802
803
804
805

- 806 59–64.
- 807 [35] S. Garrido-Jurado, R. Muñoz-Salinas, F. Madrid-Cuevas,
808 R. Medina-Carnicer, Generation of fiducial marker dictionaries
809 using mixed integer linear programming, Pattern Recognition
810 51 (2016) 481–491.
- 811 [36] Q. Bonnard, S. Lemaignan, G. Zufferey, A. Mazzei, S. Cuendet,
812 N. Li, A. Özgür, P. Dillenbourg, Chilitags 2: Robust fiducial
813 markers for augmented reality and robotics. (2013).
814 URL <http://chili.epfl.ch/software>
- 815 [37] D. Johnston, M. Fleury, A. Downton, A. Clark, Real-time po-
816 sitioning for augmented reality on a custom parallel machine,
817 Image and Vision Computing 23 (3) (2005) 271 – 286.
- 818 [38] Topological structural analysis of digitized binary images by
819 border following, Computer Vision, Graphics, and Image Pro-
820 cessing 30 (1) (1985) 32 – 46.
- 821 [39] D. H. Douglas, T. K. Peucker, Algorithms for the reduction
822 of the number of points required to represent a digitized line
823 or its caricature, Cartographica: The International Journal for
824 Geographic Information and Geovisualization 2 (10) (1973) 112
825 – 122.
- 826 [40] N. Otsu, A threshold selection method from gray-level his-
827 tograms, IEEE Transactions on Systems, Man, and Cybernetics
828 9 (1) (1979) 62–66.
- 829 [41] G. Bradski, A. Kaehler, Learning OpenCV: Computer Vision in
830 C++ with the OpenCV Library, 2nd Edition, O'Reilly Media,
831 Inc., 2013.