ADDITIONAL MATHEMATICS
FORM 5
MODULE 7

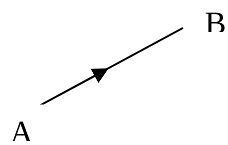
VECTORS

VECTORS

7.0 CONCEPTUAL MAP

VECTORS
A vector is a quantity that has a magnitude and a direction

Notation of Vectors



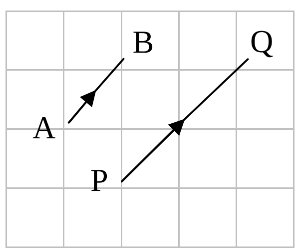
$\overrightarrow{AB} = \underset{\approx}{a}$

$\overrightarrow{BA} = -\underset{\sim}{a}$

Magnitude $\left| \overrightarrow{AB} \right|$ or $\left| -\underset{\sim}{a}\right.$

Multiplication of vector By a scalar

$|ka| = k|\underset{\sim}{a}|$



$\overrightarrow{AB} = \underset{\sim}{a}$; $\overrightarrow{PQ} = 2\overrightarrow{AB}$ ate

$= 2\underset{\sim}{a}$

Two vectors are parallel if one of the vectors is the scalar multiple of the other vector

$\overrightarrow{AB} = \frac{1}{2}\overrightarrow{PQ}$ hence $\overrightarrow{AB} \,//\, \overrightarrow{PQ}$

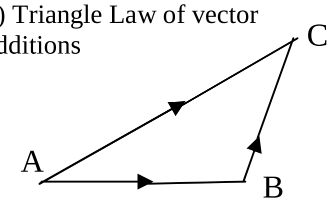
Addition and subtraction of vectors

Parallel vectors
Example

(a) $2\overrightarrow{AB} + 3\overrightarrow{AB} = 5$

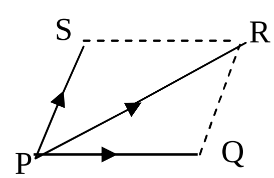
(b) $4\underset{\sim}{a} + 2\underset{\sim}{a} = 6\underset{\sim}{a}$

Non-parallel vectors
(a) Triangle Law of vector Additions



$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

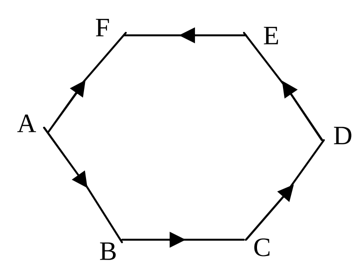
(b) Parallelogram Law of vector Additions



$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

$= \overrightarrow{PQ} + \overrightarrow{PS}$

(c) Polygon Law



$\overrightarrow{AF} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF}$

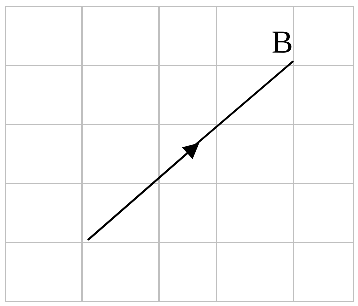
Expression of a vector as the linear combination of a few vectors

$\overrightarrow{DC} = \overrightarrow{DE} + \overrightarrow{EA} + \overrightarrow{AB} + \overrightarrow{BC}$

7.1 INTRODUCTIONS TO VECTOR

Sharpen Your Skills 1

Find the magnitude and direction of $\overrightarrow{AB}$ vector.



Distance of AB $= \sqrt{3^2 + 3^2}$

$= \sqrt{18}$

$=$ 4.24 unit

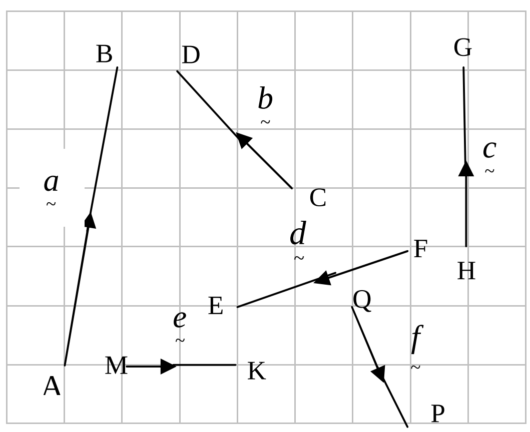
Magnitude $\left|\overrightarrow{AB}\right|$ = 4.24 unit

Direction = North-East

PRACTICE 7.1

A

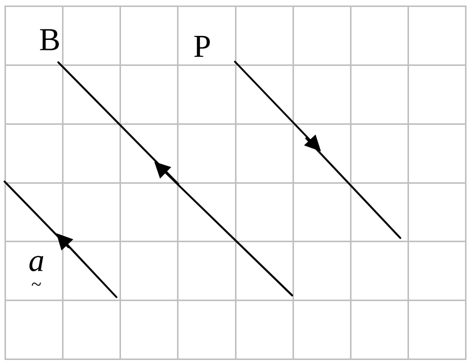
Write the notation of vector and find the magnitude and direction



Notation	Magnitude	Direction
$\overrightarrow{AB}$	5.099 unit	North-East

7.2 MULTIPLICATION OF VECTOR BY SCALAR

Sharpen Your Skills 2



1. Express $\overrightarrow{AB}$ and $\overrightarrow{PQ}$ as a scalar product of $\underset{\sim}{a}$

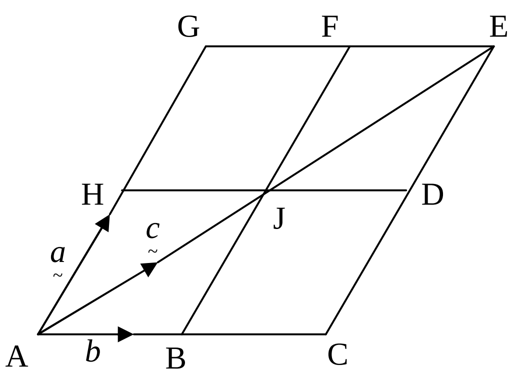
Solution

$\overrightarrow{AB} = 2\,\underset{\sim}{a}$

$\overrightarrow{PQ} = -1\frac{1}{2}\,\underset{\sim}{a}$

PRACTICE 7.2.1 Q

1. In the figure, ACEG is a parallelogram. It is given that $\overrightarrow{AH} = \underset{\sim}{a}$, $\overrightarrow{AB} = \underset{\sim}{b}$ and A $\overrightarrow{AJ} = \underset{\sim}{c}$. Points B, D, F, H are the midpoints of AC, CE, EG and AG respectively. Find each of the following vectors in terms of $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$



(a) $\overrightarrow{AE}$ =

(b) $\overrightarrow{AC}$ =

(c) $\overrightarrow{AG}$ =

(d) $\overrightarrow{FB}$ =

(e) $\overrightarrow{HD}$ =

Sharpen Your Skills 3

Given $\overrightarrow{AB} = \underset{\sim}{u}$ and $\overrightarrow{BC} = 4\underset{\sim}{u}$, shows that A, B and C are collinear

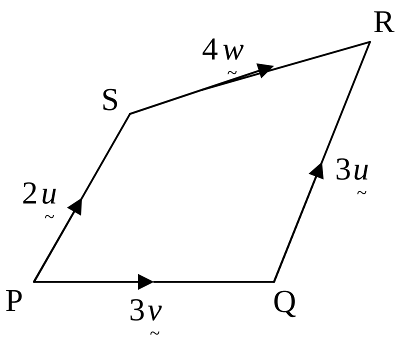
$\overrightarrow{AB} = \underset{\sim}{u}$, $\overrightarrow{BC} = 4\underset{\sim}{u}$

$\overrightarrow{BC} = 4\,\overrightarrow{AB}$ $\quad\therefore\ \overrightarrow{AB} \parallel \overrightarrow{BC}$

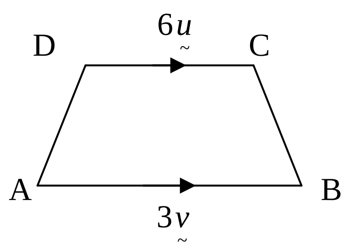
Point B is common point. $\therefore$ A, B and C are collinear

PRACTICE 7.2.2

1.



Find the parallel vector of each of the following



2. Figure below shows a trapezium ABCD in which $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are parallel. Given that $\overrightarrow{DC} = 6\underset{\sim}{u}$, $\left|\overrightarrow{AB}\right| = 4$ cm and $\left|\overrightarrow{DC}\right| = 3$ cm, express $\overrightarrow{AB}$ in terms of $\underset{\sim}{u}$.

3. Given that $\overrightarrow{AB}$ $\overrightarrow{AB} = 4\underset{\sim}{x}$ and $\overrightarrow{BC} = 6\underset{\sim}{x}$. Shows that the point A, B and C are collinear.

4. Given that $(2h - 3)\underset{\sim}{a} = (k + 5)\underset{\sim}{b}$, find the value of h and k if $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are non-parallel and non-zero vectors.

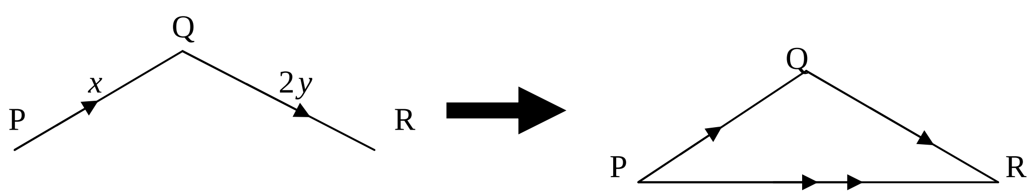
7.3 ADDITIONS AND SUBTRATIONS VECTORS

Sharpen Your Skills 4

1. $\underset{\sim}{a} + 2\underset{\sim}{a} + 2\underset{\sim}{a}$ Find the sum of the vectors

Sum of vectors $= 10\underset{\sim}{a}$

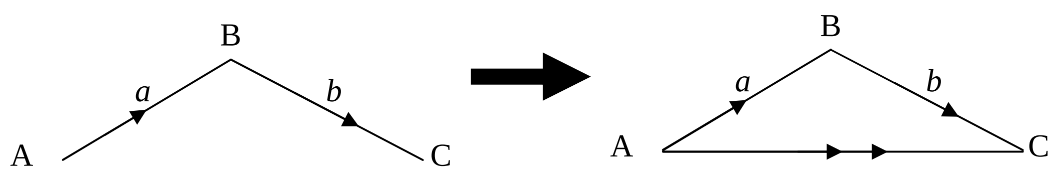
2.



$$\vec{PR} = \vec{PQ} + \vec{QR}$$

$$= \underset{\sim}{x} + \underset{\sim}{y}$$

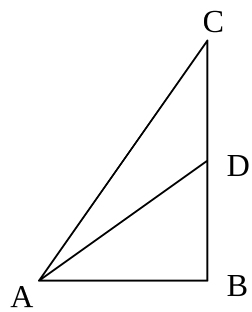
3.



$$\vec{AC} = \vec{AB} + \vec{BC}$$
$$= \underset{\sim}{a} + (-\underset{\sim}{b})$$
$$= \underset{\sim}{a} - \underset{\sim}{b}$$

PRACTICE 7.3

1.



In the above figure, point D is on BC,

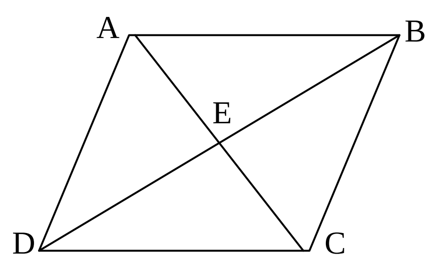
(a) Write

(i) $\vec{DA}$ in term of $\vec{CA}$ and $\vec{DC}$

(ii) $\vec{AD}$ in term of $\vec{BD}$ and $\vec{AB}$

(b) Find the resultant vector for $\vec{CA} + \vec{BC}$

2.



In the figure, ABCD is a parallelogram which diagonals AC and BD intersect each other at point E. Find the resultant vector for each of the following.

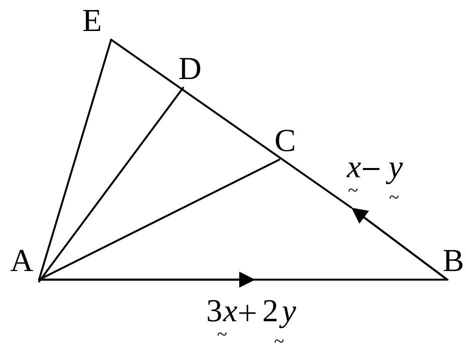
(a) $\vec{AB} + \vec{BD}$

(b) $\vec{CE} + \vec{ED}$

(c) $\vec{CA} + \vec{BC}$

(d) $\vec{EB} + \vec{DE}$

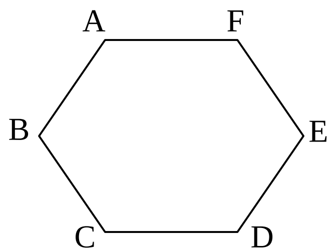
3.



Based on the above figure, BCDE is a straight line with $\vec{BC} = \vec{CD} = \vec{DE}$. Given that $\vec{AB} = 3\underset{\sim}{x} + 2\underset{\sim}{y}$ and $\vec{BC} = \underset{\sim}{x} - \underset{\sim}{y}$. Express each of the following vectors in terms of $\underset{\sim}{a}$ *and / or* $\underset{\sim}{b}$

(a) $\vec{ED}$ (a) $\vec{AC}$ (a) $\vec{DA}$

4.



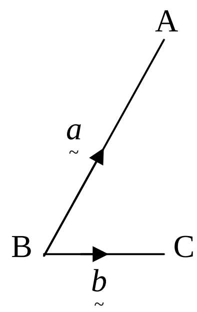
In the above figure, ABCDEF is a regular hexogen. Find the resultant vector for each of the following

(a) $\vec{AB} + \vec{BC} + \vec{CD}$

(b) $\vec{AC} + \vec{DE} + \vec{CD}$

(c) $\vec{AB} + \vec{CD} + \vec{EF} + \vec{BC} + \vec{DE} + \vec{FA}$

5.



In the above figure, $\overrightarrow{BA} = \underset{\sim}{a}$ and $\overrightarrow{BC} = \underset{\sim}{b}$. If Q is a point above $\overrightarrow{BC}$ so that $\overrightarrow{BQ} : \overrightarrow{QC} = 1:3$, express $\overrightarrow{QA}$ in terms of $\underset{\sim}{a}$ *and* $\underset{\sim}{b}$.

7.4 SPM QUESTION

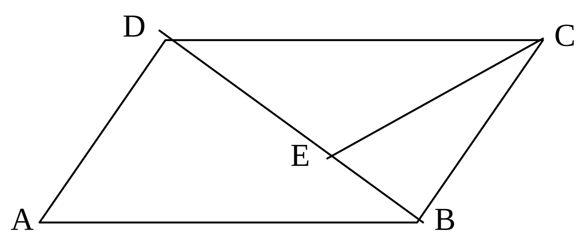
1. SPM 2003

P	=	2a + 3b
q	=	4a – b
r =	ha + (h – k)b, where h and k are constants	

Use the above information to find the values of h and k when r = 3p – 2q

[3 marks]

2. Diagram shows a parallelogram ABCD with BED as a straight line.



Given that $\vec{AB} = 6\underset{\sim}{p}, \vec{AD} = 4\underset{\sim}{p}$ and DE = 2EB, express in terms of p and q

(a) $\vec{BD}$

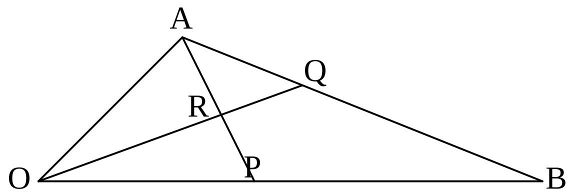
(b) $\vec{EC}$

[4 marks]

3. **SPM 2004**

Diagram below shows triangles OAB. The straight line AP intersects the straight line OQ at R. It is given that $OP = \frac{1}{3}OB$,

$AQ = \frac{1}{4}AB, \overrightarrow{OP} = 6\underset{\sim}{x}\ and\ \overrightarrow{OP} = 2\underset{\sim}{y}$



(a) Express in terms of $\underset{\sim}{x}$ and/or $\underset{\sim}{y}$:

(i) $\overrightarrow{AP}$,

(ii) $\overrightarrow{OQ}$ [4 marks]

(b) (i) Given that $\overrightarrow{AR} = h\overrightarrow{AP}$, state $\overrightarrow{AR}$ in terms of h, $\underset{\sim}{x}$ and $\underset{\sim}{y}$

(ii) Given that $\overrightarrow{RQ} = k\overrightarrow{OQ}$, state $\overrightarrow{RQ}$ in terms of k, $\underset{\sim}{x}$ and $\underset{\sim}{y}$

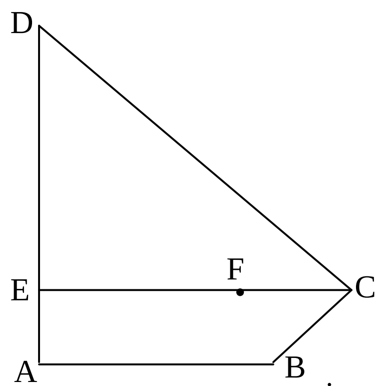
[2 marks]

(c) Using $\overrightarrow{AR}$ and $\overrightarrow{RQ}$ from (b), find the value of h and of k

[4 marks]

4. **SPM 2005**

In diagram, ABCD is a quadrilateral. AED and EFC are straight lines.



It is given that $\overrightarrow{AB} = 20\underset{\sim}{x}$,

$\overrightarrow{AE} = 8\underset{\sim}{Y}$, $\overrightarrow{DC} = 25\underset{\sim}{x} - 24\underset{\sim}{y}$, $AE = \frac{1}{4}AD$ and $EF = \frac{3}{5}EC$

(a) Express in terms of $\underset{\sim}{x}$ and/or $\underset{\sim}{y}$:

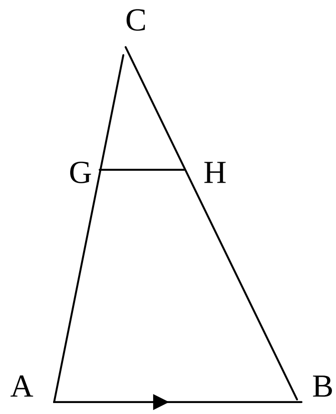
(i) $\overrightarrow{BD}$,

(ii) $\overrightarrow{EC}$ [3 marks]

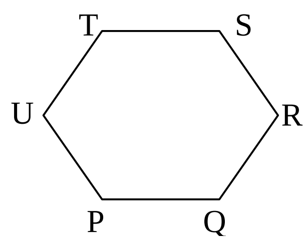
(b) Show that the points B, F and D are collinear [2 marks]

(c) If $|\underset{\sim}{x}| = 2$ *and* $|\underset{\sim}{y}| = 3$, find $|\overrightarrow{BD}|$ [2 marks]

ASSESSMENT



1. In the above figure, GH : AB = 3 : 10 and GH is parallel to $\overrightarrow{AB}$. If AB= $10\underset{\sim}{a}$, find $\overrightarrow{GH}$ in terms of $\underset{\sim}{a}$. [2 Marks]

2. 

In the above figure PQRSTU is a regular hexagon. Express $\overrightarrow{PQ} + \overrightarrow{PT} - \overrightarrow{RS}$ as a single vector.

3. Given $\overrightarrow{AB} = (k + 1)\,\underset{\sim}{a}$ and $\overrightarrow{BC} = 2\underset{\sim}{b}$. If A, B and C are collinear, $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BC}\right|$ and $\underset{\sim}{b} = 3\underset{\sim}{a}$. Find the value of k. [3 Marks]

4. Given that OABC is a rectangle where OA = 6 cm and OC = 5cm. If $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$, find [3 Marks]

(a) AC in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$

(b) $\left|\underset{\sim}{a} + \underset{\sim}{b}\right|$

5. In ΔOPQ, $\overrightarrow{OP} = \underset{\sim}{p}$ and $\overrightarrow{OQ} = \underset{\sim}{q}$. T is a point on PQ where PT : TQ=2 : 1. Given that M is the midpoint of OT, express $\overrightarrow{PM}$ in terms of $\underset{\sim}{p}$ and $\underset{\sim}{q}$. [3 Marks]

ANSWERS

PRACTICE 7.1

notation	magnitude	direction
$\overrightarrow{CD}$	2.828	North west
$\overrightarrow{HG}$	3	North
$\overrightarrow{FE}$	3.162	South-west
$\overrightarrow{MK}$	2	East
$\overrightarrow{QP}$	2.236	South-west

PRACTICE 7.2.1

1. (a) $2\underset{\sim}{c}$

   (b) $2\underset{\sim}{b}$

   (c) $2\underset{\sim}{a}$

   (d) $-2\underset{\sim}{a}$

   (e) $2\underset{\sim}{b}$

PRACTICE 7.2.2

1. $\overrightarrow{PS}$ □ $\overrightarrow{QR}$

2. $8\underset{\sim}{u}$

3. h = $\frac{3}{2}$, k= -5

PRACTICE 7.3

1. (a) (i) $\overrightarrow{DC} + \overrightarrow{CA}$

   (ii) $\overrightarrow{AB} + \overrightarrow{DC}$

   (b) $\overrightarrow{BA}$

2. (a) $\overrightarrow{AD}$

   (b) $\overrightarrow{CD}$

   (c) $\overrightarrow{BA}$

   (d) $\overrightarrow{DB}$

3. (a) $\underset{\sim}{y} - \underset{\sim}{x}$

   (b) $4\underset{\sim}{x} + \underset{\sim}{y}$

   (c) $-5\underset{\sim}{x}$

4. (a) $\overrightarrow{AD}$

   (b) $\overrightarrow{AE}$

   (c) $\underset{\sim}{0}$

5. $-\frac{1}{4}\underset{\sim}{b} + \underset{\sim}{a}$

PAST YEARS QUESTION

1. SPM 2004

(a) (i) $\vec{AP} = -2\underset{\sim}{y} + 6\underset{\sim}{x}$

(ii) $\vec{OQ} = \frac{3}{2}\underset{\sim}{y} + \frac{9}{2}\underset{\sim}{x}$

(b) (i) $\vec{AR} = h(6\underset{\sim}{x} - 2\underset{\sim}{y})$

(ii) $\vec{RQ} = k(\frac{9}{2}\underset{\sim}{x} + \frac{3}{2}\underset{\sim}{y})$

(c) $k = \frac{1}{3}$, $h = \frac{1}{2}$

2. h = -2 , h = -13

3. (a) $-6\underset{\sim}{p} + 4\underset{\sim}{q}$

(b) $2\underset{\sim}{p} + \frac{8}{3}\underset{\sim}{q}$

2. SPM 2005

(a) (i) $\vec{BD} = -20\underset{\sim}{x} + 32\underset{\sim}{y}$

(ii) $\vec{EC} = 25\underset{\sim}{x}$

(c) $\left|\vec{AD}\right| = 104$

ASSESSMENT

1. $3\underset{\sim}{a}$

2. $\vec{PR}$

3. k = 5

4. (a) $\underset{\sim}{b} - 2\underset{\sim}{a}$

(b) 13 units

5. $\frac{1}{3}\underset{\sim}{q} + \frac{5}{6}\underset{\sim}{p}$

ADDITIONAL MATHEMATICS
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MODULE 8

VECTORS

VECTORS

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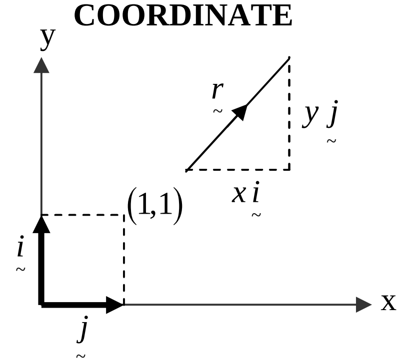
8.0 CONCEPTUAL MAP

VECTOR

VECTOR IN THE CARTESIAN COORDINATE



$$\underset{\sim}{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ \underset{\sim}{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

A vector $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} = \begin{pmatrix} x \\ y \end{pmatrix}$

The Magnitude

$$|\underset{\sim}{r}| = \sqrt{x^2 + y^2}$$

The Unit Vector

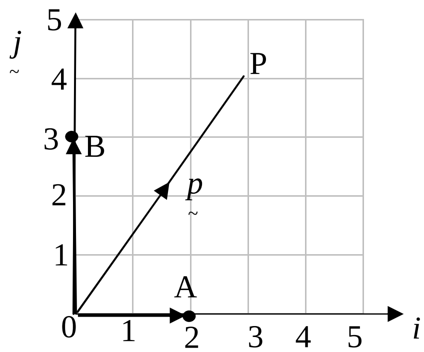
$$\hat{\underset{\sim}{r}} = \frac{\underset{\sim}{r}}{|\underset{\sim}{r}|} = \frac{1}{\sqrt{x^2 + y^2}}(x\underset{\sim}{i} + y\underset{\sim}{j})$$

For any two points P and Q, the position vector of Q relative to P is defined by $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$, where O is the origin.

8.1 VECTOR IN THE CARTESIAN COORDINATES

1. State the following vector in terms in $\underset{\sim}{i}$ and $\underset{\sim}{j}$ and also in Cartesian coordinates

<table>
<tr>
<td>Example



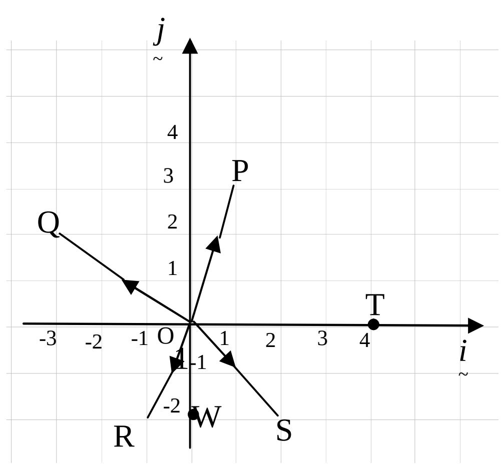
</td>
<td>Solutions

$\overrightarrow{OA} = 2\underset{\sim}{i} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$\overrightarrow{OB} = 3\underset{\sim}{j} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

$\overrightarrow{OP} = \underset{\sim}{p} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$

$= \begin{pmatrix} 3 \\ 4 \end{pmatrix}$</td>
</tr>
<tr>
<td><u>Exercise</u>



</td>
<td>Solutions
<table>
<tr><td>(a) $\overrightarrow{OP} =$</td><td>(b) $\overrightarrow{OQ} =$</td></tr>
<tr><td>(c) $\overrightarrow{OR} =$</td><td>(d) $\overrightarrow{OS} =$</td></tr>
<tr><td>(e) $\overrightarrow{OT} =$</td><td>(f) $\overrightarrow{OW} =$</td></tr>
</table>
</td>
</tr>
</table>

2. Find the magnitude for each of the vectors

Example $3\underset{\sim}{i} - 2\underset{\sim}{j} = \sqrt{3^2 + 2^2}$ $= \sqrt{13}$ unit	(a) $2\underset{\sim}{i} + 5\underset{\sim}{j} =$
(b) $5\underset{\sim}{i} - 12\underset{\sim}{j} =$	(c) $-\underset{\sim}{i} - j =$

3. Find the magnitude and unit vector for each of the following

Example $\underset{\sim}{r} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$ Solution : Magnitude, $\left\|\underset{\sim}{r}\right\| = \sqrt{3^2 + 4^2}$ $= 5$ unit vector, $\underset{\sim}{\hat{r}}, = \frac{1}{5}(4\underset{\sim}{i} + 3\underset{\sim}{j})$	(a) $\underset{\sim}{r} = 2\underset{\sim}{i} - 6\underset{\sim}{j}$
(b) $\underset{\sim}{a} = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$	(c) $\underset{\sim}{h} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

4. Given that $\underset{\sim}{a} = 3\underset{\sim}{i} - 2\underset{\sim}{j}$, $\underset{\sim}{b} = 2\underset{\sim}{i} + 5\underset{\sim}{j}$ and $\underset{\sim}{c} = -4\underset{\sim}{i} + 3\underset{\sim}{j}$. Simplify of the following

(a) $\underset{\sim}{a} + \underset{\sim}{b} - \underset{\sim}{c}$	(b) $\underset{\sim}{a} + 2\underset{\sim}{b} - 3\underset{\sim}{c}$

5. Given that $\underset{\sim}{a} = -2\underset{\sim}{i} + 3\underset{\sim}{j}$, $\underset{\sim}{b} = \underset{\sim}{i} - 4\underset{\sim}{j}$ and $\underset{\sim}{c} = 2\underset{\sim}{i} + 5\underset{\sim}{j}$. Determine the unit vector in the same direction with vector $2\underset{\sim}{a} - 3\underset{\sim}{b} + \underset{\sim}{c}$.

6. If $\overrightarrow{OA} = -\underset{\sim}{i} + 3\underset{\sim}{j}$, $\overrightarrow{OB} = -2\underset{\sim}{j}$ and $\overrightarrow{BC} = 4\underset{\sim}{i} - 3\underset{\sim}{j}$. Find

(a) $\overrightarrow{OC}$	(b) $\left|\overrightarrow{AC}\right|$

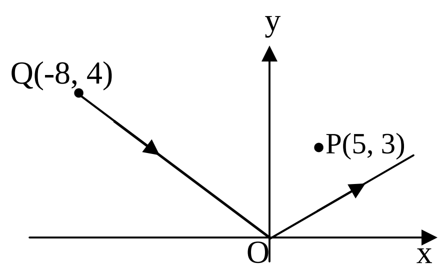
Example Given that $\underset{\sim}{a} = 6\underset{\sim}{i} + 7\underset{\sim}{j}$ and $\underset{\sim}{b} = p\underset{\sim}{i} + 2\underset{\sim}{j}$ .find the possible value (or values) of p for each of the following cases. (a) $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel. (b) $\lvert\underset{\sim}{a}\rvert = \lvert\underset{\sim}{b}\rvert$ Solutions $\underset{\sim}{a} = k\underset{\sim}{b}$ $6\underset{\sim}{i} + 7\underset{\sim}{j} = k(p\underset{\sim}{i} + 2\underset{\sim}{j})$ $6\underset{\sim}{i} + 7\underset{\sim}{j} = kp\underset{\sim}{i} + 2k\underset{\sim}{j}$ By equating the coefficients of $\underset{\sim}{i}$ 6 = kp ………(1) By equating the coefficients of $\underset{\sim}{j}$ 7 = 2k ……….(2) k = $\frac{7}{2} = 3\frac{1}{2}$ Subtitute k = $\frac{7}{2}$ into (1) 6 = $\frac{7}{2}$p p = $\frac{12}{7} = 1\frac{5}{7}$ (b) $\lvert\underset{\sim}{a}\rvert = \lvert\underset{\sim}{b}\rvert$ $\sqrt{6^2 + 7^2} = \sqrt{p^2 + 2^2}$ $\sqrt{85} = \sqrt{p^2 + 4}$ $p^2 + 4 = 85$ $p^2 = 81$ $p = \pm 9$	7. Given $\underset{\sim}{r} = 4\underset{\sim}{i} + (k+3)\underset{\sim}{j}$ and $\underset{\sim}{s} = (k+4)\underset{\sim}{i} + 14\underset{\sim}{j}$. If $\underset{\sim}{r}$ is parallel to $\underset{\sim}{s}$, find (a) the value of k (b) $\lvert\underset{\sim}{s} - 3\underset{\sim}{r}\rvert$ for the positive values of k

Example O is the origin, P and Q are two points such that $\vec{OP} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\vec{OQ} = \begin{pmatrix} k^2 - 3k \\ -6 \end{pmatrix}$. If the points O, P and Q are collinear, find the possible values of k. Solutions Since the points O, P and Q are collinear, $\vec{OP} = m\vec{OQ}$ (m is a constant) $\begin{pmatrix} 2 \\ -3 \end{pmatrix} = m\begin{pmatrix} k^2 - 3k \\ -6 \end{pmatrix}$ $\begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} m(k^2 - 3k) \\ -6m \end{pmatrix}$ $m(k^2 - 3k) = 2$ ……..(1) $-6m = -3$ …………...(2) $m = \frac{1}{2}$ From (1): $\frac{1}{2}(k^2 - 3k) = 2$ $k^2 - 3k = 4$ $k^2 - 3k - 4 = 0$ $(k - 4)(k + 1) = 0$ $k = 4$ or -1	8. Given that $\vec{PQ} = 6\underset{\sim}{i} - 8\underset{\sim}{j}$ and $\vec{PR} = 2\underset{\sim}{i} + b\underset{\sim}{j}$. If P, Q and R are collinear, find the value of b.

8.2 SPM QUESTIONS

SPM 2003/no. 12 / paper 1.

1. Diagram 2 shows two vectors, $\overrightarrow{OP}$ and $\overrightarrow{QO}$.



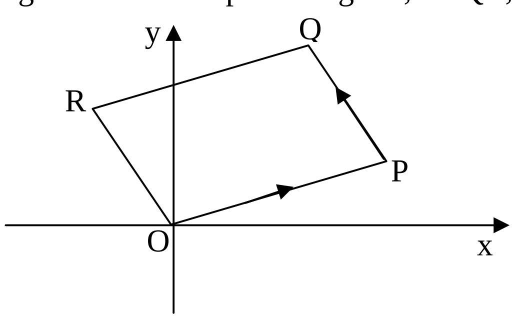
Express

(a) $\overrightarrow{OP}$ in the form $\begin{pmatrix} x \\ y \end{pmatrix}$,

(b) $\overrightarrow{OQ}$ in the form $x\mathbf{i} + y\mathbf{j}$. [2 marks]

SPM 2005 / no. 15 / Paper 1

2. Diagram 5 shows a parallelogram, OPQR, drawn on a Cartesian plane.



It is given that $\overrightarrow{OP} = 6\underset{\sim}{i} + 4\underset{\sim}{j}$ and $\overrightarrow{PQ} = -4\underset{\sim}{i} + 5\underset{\sim}{j}$. Find $\overrightarrow{PR}$. [3 marks]

SPM 2004

3. Given that O(0, 0), A(-3, 4) and B(2, 16), find in terms of the unit vectors, $\underset{\sim}{i}$ and $\underset{\sim}{j}$

   (a) $\overrightarrow{AB}$

   (b) the unit vector in the direction of $\overrightarrow{AB}$ [4 marks]

SPM 2004

4. Given that A(-2, 6), B(4, 2) and C(m, P), find the value of m and of p such that $\overrightarrow{AB} + 2\overrightarrow{BC} = 10\underset{\sim}{i} - 12\underset{\sim}{j}$. [4 marks]

SPM 2003

5. Give that $\vec{AB} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$, $\vec{OB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\vec{CD} = \begin{pmatrix} k \\ 5 \end{pmatrix}$, find

(a) the coordinates of A [2 marks]

(b) the unit vector in the direction of $\vec{OA}$ [2 marka]

(c) the value of k, if $\vec{CD}$ is parallel to $\vec{AB}$ [2 marks]

8.3 ASSESSMENT TEST

1. Find the magnitude and unit vector of $\underset{\sim}{a} = 6\underset{\sim}{i} + 8\underset{\sim}{j}$ [2 marks]

2. Find the unit vector in the direction of $\overrightarrow{OP} = \begin{pmatrix} -12 \\ 5 \end{pmatrix}$ [2 marks]

3. Given that $\underset{\sim}{u} = 3\underset{\sim}{i} + m\underset{\sim}{j}$ and $\underset{\sim}{v} = k\underset{\sim}{i} - 3\underset{\sim}{j}$, find the value of m and of k if $\underset{\sim}{u} + m\underset{\sim}{v} = 11\underset{\sim}{i} - 4\underset{\sim}{j}$ [3 marks]

4. Three points A, B and C are such that $\overrightarrow{OA} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} k \\ 2 \end{pmatrix}$, where O is the origin. Find the value of k if the points A, B and C are collinear [4 marks]

5. It is given that $\underset{\sim}{a} = 2\underset{\sim}{i} - 2\underset{\sim}{j}, \underset{\sim}{b} = \underset{\sim}{i} + 2\underset{\sim}{j}$, P(1, -3) and Q(5, 2). If $\overrightarrow{PQ} = h\underset{\sim}{a} + k\underset{\sim}{b}$, where h and k are constants, find

(a) the value of h and k [4 marks]

(b) the unit vector in the direction of $\overrightarrow{PQ}$. [2 marks]

6. PQRS is a parallelogram. M is the midpoint of QR. Given that $\overrightarrow{PQ} = 3\underset{\sim}{i} + \underset{\sim}{j}$ and $\overrightarrow{PM} = 4\underset{\sim}{i} + 2\underset{\sim}{j}$, find

(a) (i) $\overrightarrow{QR}$

(ii) $\overrightarrow{PR}$ [4 marks]

(b) the length of PS [2 marks]

7 .Given that $\underset{\sim}{u} = \begin{pmatrix} 7 \\ 9 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} 11 \\ k-3 \end{pmatrix}$, find the possible value(s) of k for each of the following cases.

(a) $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are parallel [3 marks]

(b) $|\underset{\sim}{u}| = |\underset{\sim}{v}|$ [3 marks]

ANSWERS

Exercises

1. (a) $\overrightarrow{OP} = \underset{\sim}{i} + 3\underset{\sim}{j}$

   (b) $\overrightarrow{OQ} = -3\underset{\sim}{i} + 2\underset{\sim}{j}$

   (c) $\overrightarrow{OR} = -\underset{\sim}{i} - 2\underset{\sim}{j}$

   (d) $\overrightarrow{OS} = 2\underset{\sim}{i} - 2\underset{\sim}{j}$

   (e) $\overrightarrow{OT} = 4\underset{\sim}{i}$

   (f) $\overrightarrow{OW} = -2\underset{\sim}{j}$

2. (a) $\sqrt{13}$ units

   (b) 13 units

   (c) $\sqrt{2}$ units

3. (a) $\sqrt{40}$ units , $\dfrac{1}{\sqrt{40}}(2\underset{\sim}{i} - 6\underset{\sim}{j})$

   (b) $\sqrt{45}$ units, $\dfrac{1}{\sqrt{45}}\begin{pmatrix} -6 \\ 3 \end{pmatrix}$

   (c) $\sqrt{5}$ units, $\dfrac{1}{\sqrt{5}}\begin{pmatrix} -1 \\ -2 \end{pmatrix}$

4. (a) $9\underset{\sim}{i} + 6\underset{\sim}{j}$

   (b) $19\underset{\sim}{i} + 17\underset{\sim}{j}$

5. $\dfrac{1}{\sqrt{554}}(-5\underset{\sim}{i} + 23\underset{\sim}{j})$

6. (a) $4\underset{\sim}{i} - 5\underset{\sim}{j}$

   (b) $\sqrt{89}$

Assesment

1. $\dfrac{3}{5}\underset{\sim}{i} + \dfrac{4}{5}\underset{\sim}{j}$

2. $\begin{pmatrix} -\dfrac{12}{13} \\ \dfrac{5}{13} \end{pmatrix}$

3 m =2, k = 4

4 k = 7

5 (a) k =3, h = $\dfrac{1}{2}$

   (b) $\dfrac{1}{\sqrt{41}}(4\underset{\sim}{i} + 5\underset{\sim}{j})$

6 (a) (i) $2\underset{\sim}{i} + 2\underset{\sim}{j}$

   (ii) $5\underset{\sim}{i} + 3\underset{\sim}{j}$

   (b) 2.828 units

7 k =$17\dfrac{1}{7}$

   k = 0 or 6

7. (a) k = -11, 4
(b) $\sqrt{6}$ units

8. $b = -\frac{8}{3}$

SPM QUESTIONS

1. (a) $\vec{OP} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$

(b) $\vec{OQ} = -8\underset{\sim}{i} + 4\underset{\sim}{j}$

2. $-10\underset{\sim}{i} + \underset{\sim}{j}$

3. (a) $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$

(b) $\frac{1}{13}\begin{pmatrix} 5 \\ 12 \end{pmatrix}$

4. m = 6, p = -2

5. (a) $A(-3, 4)$

(b) $\frac{1}{5}\begin{pmatrix} -3 \\ -4 \end{pmatrix}$

(c) $k = \frac{25}{7}$