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Computerlab 2: Car Suspension System I

- Analysis of the Car Suspension System using Simulink -

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Car Suspension System

1 Simulink Model of the Car Suspension System

An automobile suspension system consists of a spring and a hydraulic shock absorber as a damping part. The differential equation of the system is:

$$m\frac{d^2y}{dt^2}+b\frac{dy}{dt}+ky=F_{ext}$$

input force
restoring force
friction force

b = friction coefficient

k = spring coefficient

$m = m_{Car}/4$ (car has 4 wheels)

This is a 2nd order LTI-system. The input of the system is the external force F_{ext}, the output of the system is the vertical deflection y of the car. The system parameters of the <u>non</u>normalized system are:

$$a_2=m;\ a_1=b;\ a_0=k;\ b_2=0;\ b_1=0;\ b_0=1$$

We want to analyse the car suspension system using a Simulink-model and a MatLab-program for a general 2nd order LTI-system.

1.1 Create the Simulink model of a 2nd order LTI-System

We rewrite the general differential equation of the 2nd order LTI-system:

$$a_2\frac{d^2y}{dt^2}+a_1\frac{dy}{dt}+a_0y=b_2\frac{d^2u}{dt^2}+b_1\frac{du}{dt}+b_ou$$

$$\Rightarrow \qquad a_2\frac{d^2y}{dt^2}=b_2\frac{d^2u}{dt^2}+b_1\frac{du}{dt}+b_ou-a_1\frac{dy}{dt}-a_0y$$

$$\Rightarrow \quad \frac{d^2y}{dt^2}=\frac{1}{a_2}\left(b_2\frac{d^2u}{dt^2}+b_1\frac{du}{dt}+b_ou-a_1\frac{dy}{dt}-a_0y\right)$$

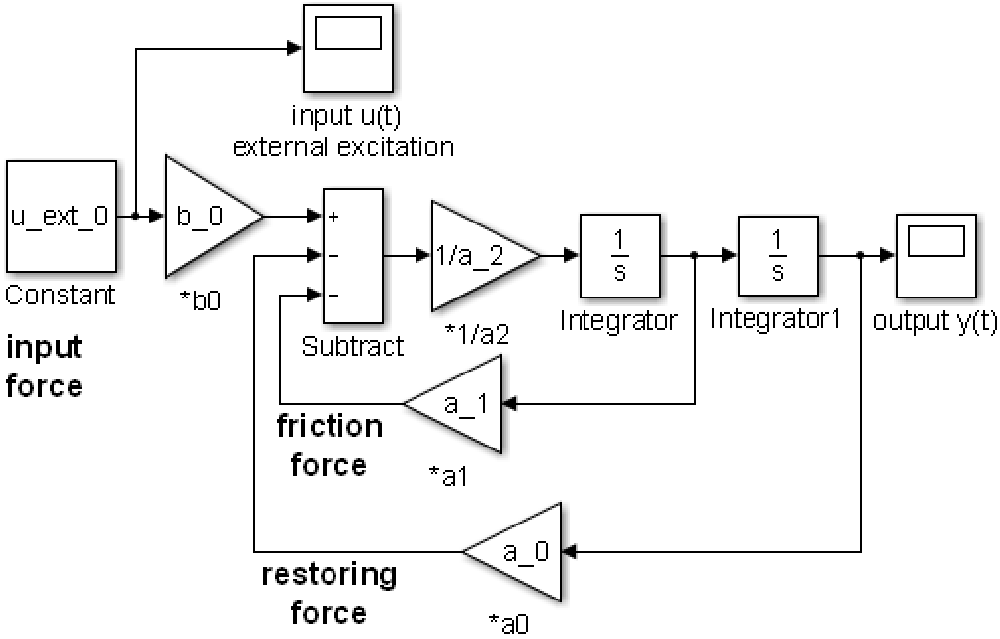
Using $y=\iint\left(\frac{d^2y}{dt^2}\right)dtdt$ we get:

$$\Rightarrow \quad \boxed{y=\frac{1}{a_2}\iint\left(b_2\frac{d^2u}{dt^2}+b_1\frac{du}{dt}+b_ou-a_1\frac{dy}{dt}-a_0y\right)dtdt}$$

In our case the system parameters b_2 and b_1 are equal to zero. So we get:

$$\boxed{y = \frac{1}{a_2} \iint \left(b_o u - a_1 \frac{dy}{dt} - a_0 y \right) dtdt}$$

From this we create the Simulink model **LTI_2_constant_input.slx**. If you have difficulties in creating the system follow the step-by-step description in the appendix.



After creating the model you declare the parameters of each block by clicking twice on the icon and write the name of the variable into the corresponding line:

- Gains of the amplifiers are the system parameters as denoted in the above diagram.
- For the input of the system you use the variable u_ext_0.
- For the initial conditions you use in the left integrator block the variable for the initial velocity dy_dt_0; in the right integrator block the initial deflection y_0.

Declare the general parameters for the simulink simulation by the following steps:

Select **Simulation** in the drop-down-menue of the model
Click on **Model Configuration Parameters**
Select **Solver** in the list on the left side
For **Stop time** use the variable **t_stop**
For **Max step size** use **t_sample**;
Press **OK**

2 Analysis of the Car Suspension System using Simulink

We want to use the above Simulink model to derive the response of the car suspension system from simulations:

a) Initial response,
b) Step response,
c) Transient response,
d) Frequency response.

We use a parameter program for the configuration of the model (see next section). By this way we do not have to change these values in the Simulink model directly and we make sure that we declare all parameters needed in the model.

The car suspension system shall be characterized by the following values:

- friction coefficient: b = 700 Ns/m (in case of underdamping),
- spring coefficient: k = 130000 N/m,
- mass of the car: m_{Car} = 1580 kg.

As the car has 4 wheels so we have to use $m = m_{Car} / 4$.

We can use the Simulink model for other systems that are PT2-elements just by modifying the parameter program.

2.1 Initial response

2.1.1 Create the Parameterprogram for Simulation of the Initial Response

The initial response of the system is the response due to initial conditions when no input is applied. To simulate the initial response, we use:

- no input signal: $u(t) = 0$ ($\Leftrightarrow$ no input force)
- initial conditions: $y(t = 0\text{s}) = 1$ cm (= y_0 = initial value of 2. integrator)
  $v(t = 0\text{s}) = 0$ m/s (= dy_dt_0 = initial value of 1. integrator).

These initial conditions correspond to a deflection of the car and then leaving the car to itself.

Write the short MatLab-program **CSS_para_initial.m** to define the system parameters and to assign the values:

```
% Parameter program for the simulink model
% of the car suspension system
% to study the initial response

clear           % delete all variables
close all       % close all figure windows

% initial conditions of the system
y_0 = 0.01;     % initial deflection in m (2nd integrator)
dy_dt_0 = 0;    % initial velocity in m/s (1st integrator)

% external force
u_ext_0 = 0;    % no external force for initial response

% constant coefficients of the system
k = 130000;     % constant of the spring in N/m
m_car = 1580;   % mass of the car
m = m_car/4;    % car has 4 wheels

w_0 = sqrt(k/m);  % natural frequency

% damping coefficient
b = 0;            % no damping
%b = 700;         % underdamping
%b = 2*m*w_0      % critical damping
%b = 30000;       % overdamping

% system parameters
a_0 = k;
a_1 = b;
a_2 = m;
b_0 = 1;
% !!!  b_1 and b_2 are not used in the model  !!!

% calculations
disp('natural angular frequency of the system in 1/s: ')
w_0 = sqrt(k/m)
disp('oscillation period of the undamped system in s: ')
T_0 = 2*pi/w_0
disp('decay coefficient of the system in 1/s: ')
delta = b/(2*m)
disp('coefficient for critical damping in kg/s: ')
b_crit = 2*m*delta

% Parameters for Simulink
t_stop = 4;                    % time intervall for simulation
N_sample = 1000;               % Number of samples in the simulation
t_sample = t_stop / N_sample;  % sampling time

% end of parameter program
```

This parameter program also calculates the natural frequency of the system, the oscillation period and the decay coefficient.

2.1.2 Simulations of the Initial Response of the Car Suspension System

We use the Simulink model to analyse the initial response of the car suspension system for 4 different values of damping.

a) **No damping:** Calculate the natural frequency of the system and check the result by simulation using b = 0 (no friction).

From theory we calculate:

$$\omega_o = \sqrt{\frac{k}{m}} = \sqrt{\frac{\qquad}{\qquad}} = \_\_\_\_\_\_\_\_\_\_ \ 1/\text{s}$$

The corresponding period is: $T_o = \dfrac{2\pi}{\omega_o} = \dfrac{\qquad}{\qquad} = \_\_\_\_\_\_\_\_$ s

From simulations we get:

$$10 \cdot T_{o,Simu} = \_\_\_\_\_\_\_\_ \text{s} \Rightarrow T_{o,Simu} = \_\_\_\_\_\_\_\_ \text{s} \Rightarrow \omega_{o,Simu} = \frac{2\pi}{T_{o,Simu}} = \_\_\_\_\_\_\_\_ 1/\text{s}$$

b) **Underdamping:** Calculate the decay coefficient of the underdamped system with friction coefficient b = 700 Ns/m.

$$\delta = \frac{b}{2m} = \frac{700}{\qquad} = \_\_\_\_\_\_\_\_\_\_\_\_$$

c) **Critical damping:** Vary the damping coefficient b to get the initial response of the critically damped system.

$$\delta = \omega_o \quad \Rightarrow \quad b_{crit} = 2m\delta = \_\_\_\_\_\_\_\_\_\_\_\_$$

Deflection for $t = T_0$: y = ____________

Comment: ________________________

d) **Overdamping:** Repeat the simulation for an overdamped system.
value you have chosen for b: $b_{od} = \_\_\_\_\_\_\_\_\_\_$

- **Perform the simulations for all 4 cases.**
- **Use a title and labels for each axis and modify the colors (use hidden handles after running scope.m). Use appropriate size for letters and numbers (12pt. at least).**
- **Transfer the diagrams of the initial response *y(t)* into a Word document.**
- **Save the program and model on a USB-stick as well.**

2.2 Step Response

2.2.1 Create the Parameterprogram for Simulation of the Step Response

The above simulink model can also be used for simulation of the step response of the car suspension system. We apply a constant external force with an amplitude of 500 N as an input of the system, the initial conditions shall be zero. For this we only have to modify the following lines of the parameter program **CSS_para_initial.m** :

```
% Parameter program for the simulink model
% of the car suspension system
% to study the step response of the system

% initial conditions of the system
y_0 = 0;     % initial deflection in m (2nd integrator)
dy_dt_0 = 0;    % initial velocity in m/s (1st integrator)

% external force
u_ext_0 = 500;    % external force of the step input

% Parameters for Simulink
t_stop = 10;        % time intervall for simulation
```

We save this new parameter program using the name **CSS_para_step.m.**

2.2.2 Simulations of the Step Response of the Car Suspension System

We use the Simulink model to analyse the step response of the car suspension system for the 3 different values of damping we used for the initial response in case of underdamping, critical damping and overdamping.

- Perform the simulations for all 3 cases of damping.
- Give each diagram a title, a label for each axis and modify the diagram according to your desire using the hidden handles (they show up after running scope.m).
- Transfer the diagrams of the step response *y(t)* into a Word document.

Calculate the terminal value of the deflection and compare it to the result of the simulation:

$$y_{t,\max} = \frac{F_{ext,\max}}{m\omega_o^2} = \underline{\qquad\qquad\qquad} = \underline{\qquad\qquad\qquad}$$

$$y_{t,\max,simu} = \underline{\qquad\qquad\qquad}$$

2.3 Transient Response

We want to modify the above simulink model and the parameter program for simulation of the transient response of the car suspension system. We apply a harmonic external force with the amplitude of 500 N as an input of the system. The frequency shall be 10 times the natural frequency of the system. The initial conditions shall be zero.

2.3.1 Simulink Model for Simulation of the Transient Response

Step 1: Save the Simulink model **LTI_2_constant_input.slx** using the new name **LTI_2_harmonic_input.slx**

Step 2: Replace the constant input force by a sin-input from the library:
- Select **Sources** in the library
  - select **Sine Wave** for a sinusoidal input of u(t) and drag it to the work sheet
    - Click twice on this icon
      - enter Amplitude: **u_ext_0**
      - enter Frequency: **w_ext**
      - enter Sample time: **t_sample OK**

2.3.2 Parameterprogram for Simulation of the Transient Response

We want to modify the parameter program of the step response for the simulation of the transient response. In this case we apply an harmonic external force as an input. The values of the initial conditions are zero (same values as for step response).

Step 1: Save the parameter program **CSS_para_step.m** using the new name **CSS_para_transient.m**.

Step 2: Describe the purpose of the program:

```
% Parameter program for the simulink model
% of the car suspension system
% to study
%       a) the transient response of the system
%       b) the response to an harmonic input
```

Step 3: Declare the angular frequency ω_{ext} of the harmonic input to be 10 times the natural frequency of the system (amplitude of external force = 500 N as for step response):

```
% external force
u_ext_0 = 500;          % amplitude of harmonic input
w_ext = w_0*10           % external angular frequency in 1/s
```

Step 4: Add the following lines to calculate the frequency and the period of the harmonic input:

```
f_ext = w_ext/(2*pi)    % external frequency in Hz
T_ext = 1/f_ext         % period of the harmonic input
```

Step 5: Change the simulation parameters of the model:

```
N_periods = 300;              % number of periods for simulation
t_stop = N_periods*T_ext;     % time for simulation
N_sample = 100;               % number of samples/period
t_sample = T_ext/N_sample;    % sampling time (100 samples/period)
```

Run this program and write down the values written in the MatLab command window for

- the external angular frequency: ω_{ext} = ________________ 1/s
- the external frequency: f_{ext} = ________________ Hz
- the period of the harmonic input: T_{ext} = ________________ s

2.3.3 Simulations of Transient Response of the Car Suspension System

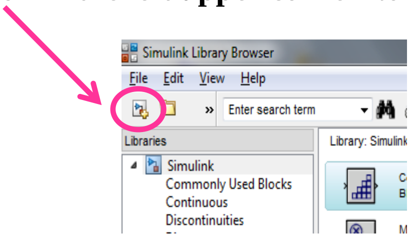
Study the transient response of the underdamped system and the critically damped system:

- Perform the simulation for the 2 different values of the friction coefficient you have deduced when studying the initial response
- Give each diagram a title, a label for each axis and modify the diagram according to your desire using the hidden handles (they show up after running **scope.m**).
- Mark the part of the transient response and the part of the steady state response in your diagrams (Use **Insert**, then select Line (Arrow, Text,…)).
- Transfer the 2 diagrams of the transient response *y(t)* into a Word document.

3 Appendix: Step-by-step description for the Simulink model

To start Simulink enter in the Matlab command window: >> **simulink**
The Simulink Library Browser will open.

Click twice on the **icon in the left upper corner** to open a sheet for creating the new model:



We start with the first part of the integrals:

1. Select **Simulink** in the left list of the library browser.

2. Select **Sources**
   select **Constant** for a constant value of the external force
   and drag it to the work sheet
   Click twice on this icon and **enter u_ext_0** as constant value;

3. **Math Operations**
   select **Sum**
   Click twice on the icon and **insert '+--'; OK**

4. **Math Operations**
   select **Gain**
   Click twice on the icon and **insert 1/a_2** as gain; **OK**

5. **Continuous**
   select **Integrator**
   Click twice on the icon and **insert dy_dt_0** as initial condition; **OK**

6. **Continuous**
   select **Integrator** again
   Click twice on the icon and **insert y_0** as initial condition; **OK**

7. **Sinks**
   select **Scope** that shall show the output y(t)
   Click on the text "scope" underneath the icon and **rename** it: **y(t)**

8. **Connect all elements** by selecting icon 1, while pressing Crtl (or Strg) select icon 2 and so on.

9. **Sinks**
   select an additional **Scope** that will show the external input u(t)
   Click twice on the icon and **rename** it by clicking on scope: **u(t)**
   Connect this scope to the input of the system

Now we want to design the friction force:

10. **Math Operations**
    select **Gain**
    click twice on the icon and **insert a_1** as gain; **OK**

11. To flip the icon we click once on the gain-icon
    select **Rotate&Flip** in the drop-down-menue
    select **Flip block**

12. Connect the output of the **first** integrator to the input of the new gain-icon.

13. Connect the output of the gain-icon to the **first** minus-input of the sum-icon.

Now we want to design the force of the spring:

14. **Math Operations**
    select **Gain**
    click twice on the icon and **insert a_0** as gain; **OK**

15. To flip the icon we click once on the gain-icon
    select **Rotate&Flip** in the drop-down-menue
    select **Flip block**

16. Connect the output of the **second** integrator to the input of the new gain-icon.

17. Connect the output of the gain-icon to the **second** minus-input of the sum-icon.

Now we want to start the simulation:

18. Run the parameter file from the **matlab command window**:
    >> **CSS_para_initial**

19. Return to the Simulink model.

20. Select **Simulation** in the drop-down-menue
    Click on **Model Configuration parameters**
    Select **Solver** in the list on the left side
    Set the Stop time to **t_stop**
    Set the Max step size to **t_sample**; **OK**

21. Click on **Start** to start the simulation.

22. After the simulation has stopped, click twice on the scope-icons in the model to open the scope-windows and click on  to autoscale the diagram.

23. Run the program **scope.m** to show the hidden handles.

24. Use nice colors. Add labels and title. Make sure that the size of the letters and numbers is not less than 12pt and that they are bold. Use a thicker line (like 2 pt) for the displayed signal.