3-Phase AC Motor Control – EE 499 – 01

Independent Study

Allan Ray Taylor

10:15am – 12:20pm TF

Eddie Schodowski

Summer 2016

EE-499 Independent Study – Three-Phase AC Motor Control
Course Syllabus

Description
This is a hands-on Electrical Engineering elective class which focuses on permanent magnet (PM) three-phase AC electric motor theory, the development of control algorithms for such machines, and the power electronic drives used to energize such machines. A low-power PM machine and power electronics inverter will be analyzed and tested (with time permitting).

Instructor
Name: Allan Taylor, M.S.E., Lecturer
Office: 2-703V AB, ECE Dept.
Hours: Wed. 10:15AM – 12:20PM, or by appt.; walk-ins welcome
Phone: 810-762-9500 ext.5656
Email: ataylor@kettering.edu

Prerequisites
MATH-204 (Diff Equations & Laplace Transforms) ✓
PHYS-224/225 (Electricity & Magnetism + Lab) ✓
CE320 (Microcomputers 1 + Lab) (haven't taken)
EE320 (Electronics 1 + Lab) ✓

Co-Requisites (at least 2 of 4)
EE342 (Electrical Machines + Lab)
EE432 (Feedback Control Systems + Lab) ✓
EE424 (Power Electronics & Applications + Lab)
EE434 (Digital Signal Processing + Lab)

Schedule
Two 120-minute lecture sessions per week.
For Summer 2016 term: Section 01 meets Tues/Fri 10:15AM – 12:20PM in room 2-703V AB.

Textbook
There is no required textbook; the following textbooks are recommended as a reference:

[1] DeDoncker, R., Pulle, D., & Veltman, A. (2011). *Advanced Electric Drives – Analysis, Modeling, & Control.* London, New York. Springer Science & Business Media. ISBN: 978-94-007-0179-3.

[2] Sul, S. (2011). *Control of Electric Machine Drive Systems.* Hoboken, NJ: J. Wiley & Sons. ISBN: 978-0-470-59079-9.

Materials
Students will need access to a computer which has the following software packages and licenses installed: MathWorks MATLAB, Altium Designer, TI Code Composer Studio, & National Instruments LabVIEW.

Attendance
To ensure that all students are participating and to avoid issues later in the course, **attendance is mandatory**. Each student is allowed one unexcused absence. If you fail to attend class on two or more occasions during the term your grade may be negatively affected and/or you may be dropped from the class. Repeated or excessive tardiness may also have a negative impact on grades. Students should make every effort to contact the instructor as soon as possible in advance of a known absence or after an unforeseen one.

ASSESSMENT

Throughout the course, students will be introduced to a variety of theoretical topics and software packages used to develop simulation materials or design files. Weekly progress reports (8 total) are required from each student as they progress through the course. No exams will be given in this course but students will be required to complete an end-of-term project which focuses on the hardware or software development of an inverter.

Progress Reports

Weekly progress reports will be due at class-time on Tuesday of each week, and should briefly summarize the student's work through the previous week. Late reports will be accepted but will lose points per unit time.

Report Guidelines

- Reports should be 1 to 4 pages in length – any theoretical analyses, developed equations or Simulink models, circuit board design files or images, or summaries of microcontroller software or HMI LabVIEW code should be included.
- The report must have the student's name, day and time of class, and week number in the upper corner.
- All pages must include page numbers and must be stapled together in the upper left corner.
- Reports (and any images or equations within) must be computer generated – handwritten submissions are not allowed.

GRADING

All grades will be posted in Blackboard. The weighting and calculation of grades and the letter grade conversion for the course are outlined below:

Progress Reports (8x)	80%
Term Project	20%
	100%

100-95	A
94-90	A-
89-87	B+
86-83	B
82-80	B-

79-77	C+
76-73	C
72-70	C-
69-65	D+
64-60	D
59-0	F

INSTRUCTOR'S RESPONSIBILITIES

The following is a list of responsibilities that you may expect from the instructor:

- Provide lectures that reach as broad a range of learning styles as is feasible within the constraints of the amount of material that must be covered and the time available for lectures.
- Be available outside of class for explanations and answers tailored to individual students.
- Grade materials promptly to keep each student's progress on Blackboard up to date.
- Make the student aware of the learning objectives and provide assignments that both teach basic use and application of the objectives.

STUDENT'S RESPONSIBILITIES

The following describes what is expected of a student who wishes to do well in the course:

- Attend class regularly and inform the instructor in advance if you must miss a lecture.
- Spend a minimum of 2 to 3 hours externally, per hour of lecture, on course activities outside the classroom. These activities include reading the textbook and / or reviewing lecture notes, completing homework assignments, reviewing the course objectives, seeking help with the instructor, etc.
- Begin working on an assignment shortly after it is released. This will enable you to better understand the following lectures and class discussions.
- Monitor your progress in the course through Blackboard.
- **Ask the instructor** if you have questions about *anything* (lecture material, homework questions, your performance in the course, etc.). It is your responsibility to seek help from the instructor when you do not yet feel you fully understand a topic.

COURSE SCHEDULE

EE-499 Independent Study - Three Phase AC Motor Control

Summer 2016 - Last Updated 07/07/2016

Week	Date	Topics	Reading	HW
Week 1	12-Jul	Three-Phase Electric Machine Theory		
	15-Jul	Reference-Frame Math & Vector Control		
Week 2	19-Jul	Motor Model Development with Simulink		PR01 Due
	22-Jul	Motor Control Simulation with Simulink		
Week 3	26-Jul	Flux Weakening Control for High Speeds (skipped)		PR02 Due
	29-Jul	Power Electronics Overview of 3ph VSI		
Week 4	2-Aug	PWM Modulation Schemes		PR03 Due
	5-Aug	Intro to Altium Designer IDE		
Week 5	9-Aug	Ref-Dsgn Invrtr Schematics - Power Circuits		PR04 Due
	12-Aug	Ref-Dsgn Invrtr Schematics - Control Circuits		
Week 6	16-Aug	Ref-Dsgn Invrtr PCB Layout - Power Circuits		PR05 Due
	19-Aug	Ref-Dsgn Invrtr PCB Layout - Control Circuits		
Week 7	23-Aug	Texas Instruments Digital Signal Processors		PR06 Due
	26-Aug	Intro to Code Composer Studio IDE		
Week 8	30-Aug	SW PI Controllers & Digital Filters	Where digital control comes into play.	PR07 Due
	2-Sep	NO CLASS – LABOR DAY		
Week 9	6-Sep	LabVIEW Introduction & HMI Development		PR08 Due
	9-Sep	SW Serial Port Communication		
Week 10	13-Sep	Open-Loop Control Demo of PMAC Motor		
	16-Sep	Advanced Topics – Position Observers		
Week 11	20-Sep	Overflow Day - Work on Term Projects		
	23-Sep	Hands-On Demonstration of Motor Control		

STUDENTS WITH DOCUMENTED DISABILITIES

The University will make reasonable accommodations for persons with documented disabilities. Students need to register with the Wellness Center every term they are enrolled in classes. To be assured of having services when they are needed, students should contact the Wellness Center during the first week of each term. Note that it is the student's responsibility to arrange accommodations with each professor.

For more information on "Disability Services," refer to the *Student Life* section of the current Undergraduate Catalog (see page 24). This information is also noted in the Student Handbook (see page 31).
http://www.kettering.edu/academics/academic-resources/office-registrar/academic-course-catalogs (pg 24)
http://www.kettering.edu/current-students/student-life/student-life-resources (pg 31)

ETHICS IN THE UNIVERSITY AND ACADEMIC INTEGRITY

Kettering University values academic honesty and integrity. Cheating, collusion, misconduct, fabrication, and plagiarism are serious offenses. Each student has a responsibility to understand, accept, and comply with the University's standards of academic conduct as set forth in our statement, "Ethics in the University," and "Academic Integrity" as well as policies established by individual professors.

Several clarifications of the policy to note are as follows. Seeking assistance for a graded assignment from others, which includes but is not limited to another group in the class, another student, a tutor, other professors, or any online sources, could be considered cheating and is frowned upon. Allowing another group access to your work (this term or any following term) could also be considered cheating. You may get assistance in understanding course notes and book problems from any source. You may get unlimited assistance in anything from the course instructor.

For more information on "Ethics" and "Academic Integrity", refer to the *Student Life* section of the current Undergraduate Catalog (see page 19). This information is also noted in the Student Handbook (pages 42 - 44).
http://www.kettering.edu/academics/academic-resources/office-registrar/academic-course-catalogs (pg 19)
http://www.kettering.edu/current-students/student-life/student-life-resources (pg 42 - 44)

ACADEMIC ASSISTANCE

In addition to your professors, academic assistance with class work and writing is available from the Academic Success Center (ASC), located in room 3-322 of the Academic Building or contacted at (810)-762-7995 or academicsuccess@kettering.edu

Eddie
Schodowski

EE 499 - Week 1 Tuesday

7/12/2016

· Want to get in how to build one of these things

MATLAB for simulation
Altium for building / PCB Layout
TI Code Composer Studio (CCS) for software
LabVIEW for HMI

Anything we think about or talk about, just make a report about it for our own future reference.

Today we'll talk about Three-Phase E-Machine Theory.

$\boxed{\text{Note on variables}}$ (There's a method to this madness)

- V_R, I_S - constant values (like RMS or PK values)
- v_R, i_s - varies with time
- $\vec{V}_R, \vec{I}_s$ - complex numbers (phasors) (or bold face)
- $\vec{v}_R, \vec{i}_s$ - "space vectors, complex numbers that vary with time" (Have to describe w/ time AND magnitude/angle)

Units

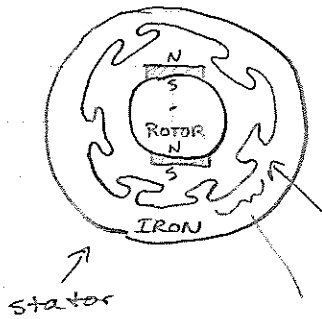
$P = V \cdot I$ (always an effort times a flow rate)
$P = F \cdot v$
$P = T \cdot \omega$

Page 2

Motor Construction

When we build e-machines, they're all similar (w/ some differences though)

- stator windings wound into slots

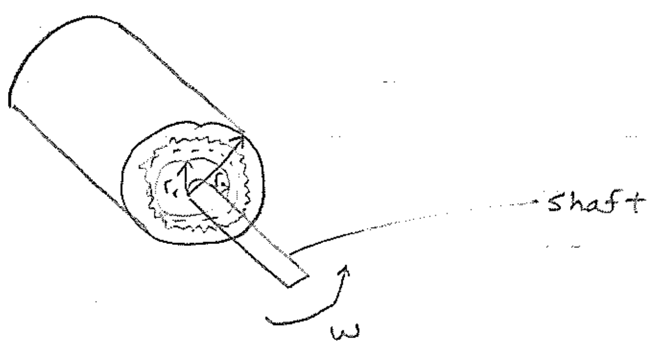


2 magnets on the rotor polarized on the same axis

stator teeth

stator slots

When you look at it end-on, the wires are either all going one way or the other where the slots are.

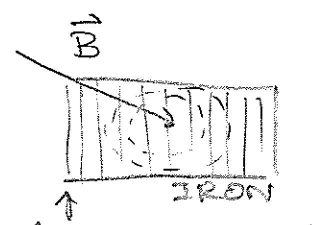


They take thin sheets of iron and insulate/laminate them and attach them together.

They do this to reduce Eddy current losses.

- Steel laminations reduces Eddy current losses

Eddy currents...



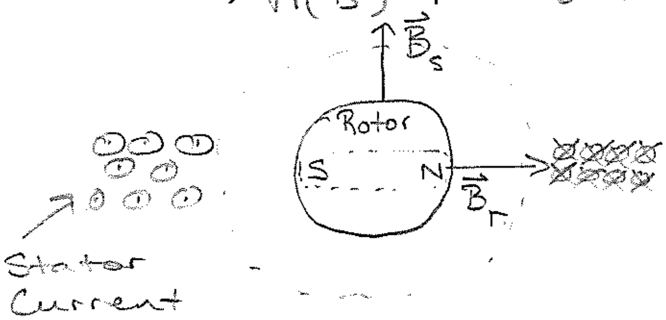
lamination path has more resistance

(?) • Voltage gets induced from a moving current (just double check)

By cutting into the laminations, they get less losses because it's very skinny and very resistive.

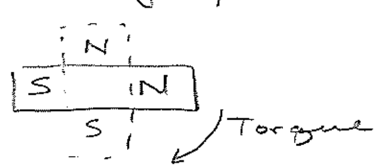
<u>Windings</u>

- Torque is made from the interaction of magnetic fields (Fields from rotor & stator)
  - ↳ PM (Permanent Magnet) Field on Rotor
  - ↳ $\vec{H}(\vec{B})$ from current in stator



(Imagine this is one winding for the stator)

Same thing as / similar:



Essentially all about repulsion / attraction.

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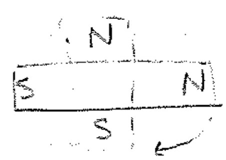
Realize that the torque depends on the angle.

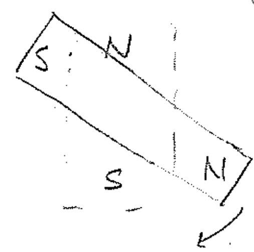
DC motor → mechanical commutator
AC motor → electrical commutator

"Max torque is when the stator field is 90° from the rotor angle."

Max torque: $\Theta = 90°$

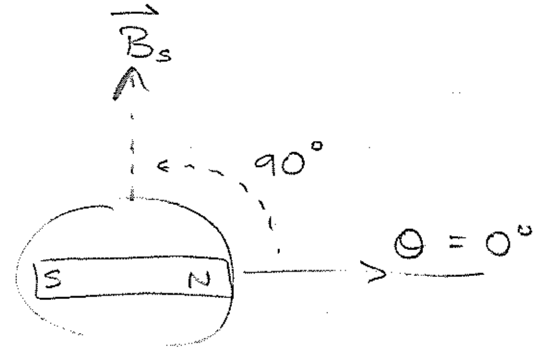


Smaller torque: $\Theta < 90°$

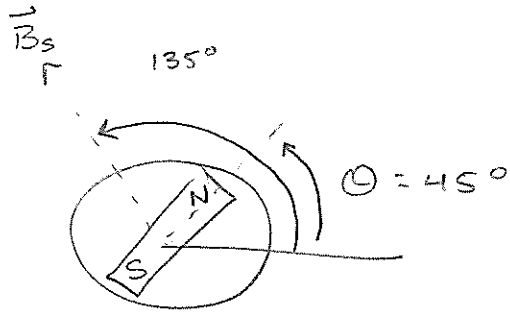


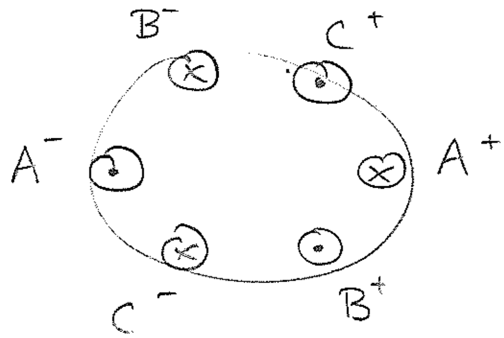
<u>Lorentz Force Equation</u>

$$\vec{F} = \vec{B} \times \vec{i}\, l$$
$$\vec{T} = \vec{F} \cdot r_r$$

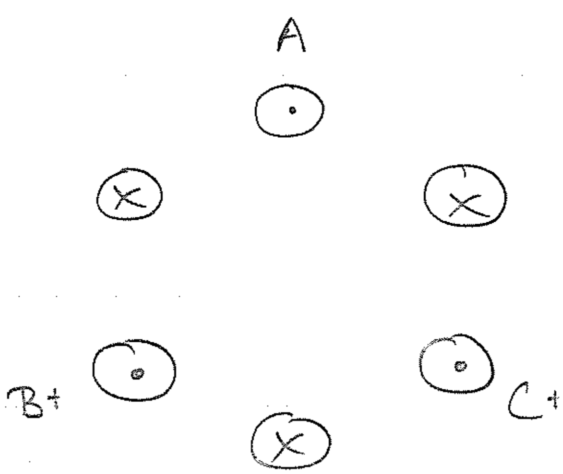


"You always want the fields of the windings to be 90° from the fields of the rotor."



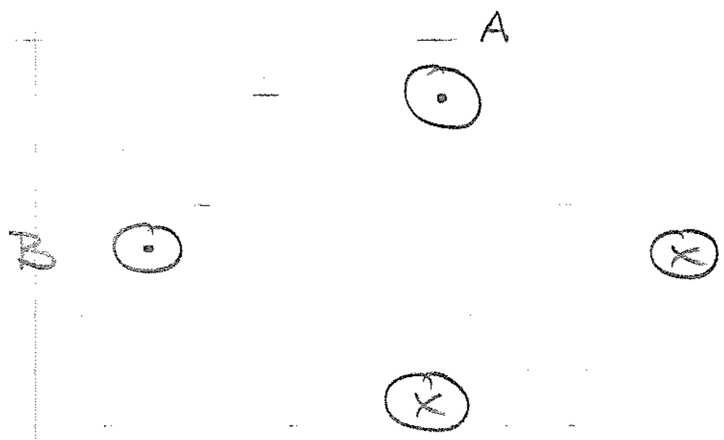


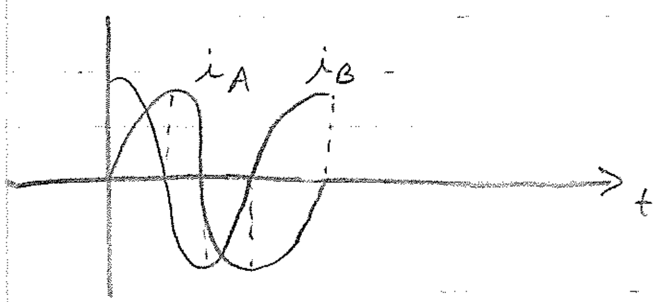
← 3 windings arranged spatially in the stator (all 120° apart)



Fun fact: you can also make a two-phase machine

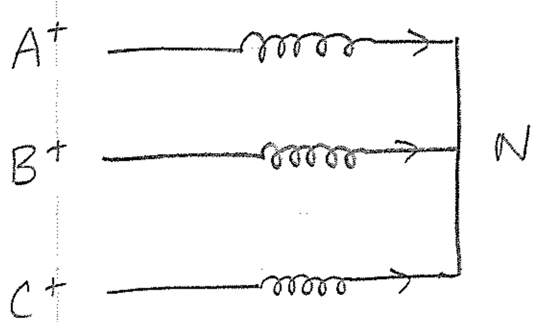
Two-phase motor





This is how the coils are positioned, but we haven't talked about how they're connected.

3-Phase:



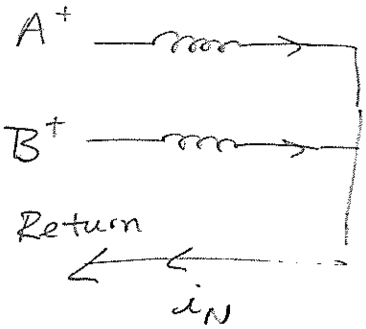
If you do KCL, these all add up to zero.
Hence, balanced three-phase sys.

$$i_A = I_m \sin(\omega t)$$
$$i_B = I_m \sin(\omega t - 120°)$$
$$i_C = I_m \sin(\omega t + 120°)$$

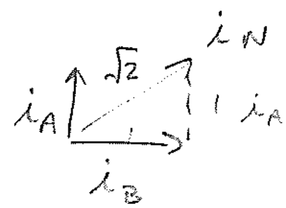
$$i_A + i_B + i_C = 0$$

Balanced 3ϕ system.

With the two-phase motor, you need a return path.



$$i_A = I_m \sin(\omega t)$$
$$i_B = I_m \cos(\omega t)$$



$$i_N = i_A + i_B = \sqrt{2}\, I_m \sin(\omega t + 45^\circ)$$

Needs you to be able to handle 40% more current in return path because it doesn't all elegantly cancel out.

They do the same thing with the field, some just use more copper.

Some more copper

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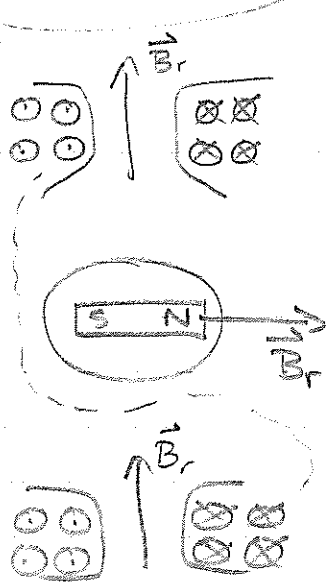
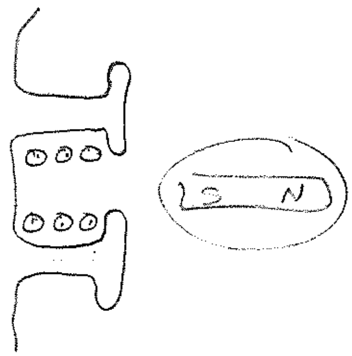
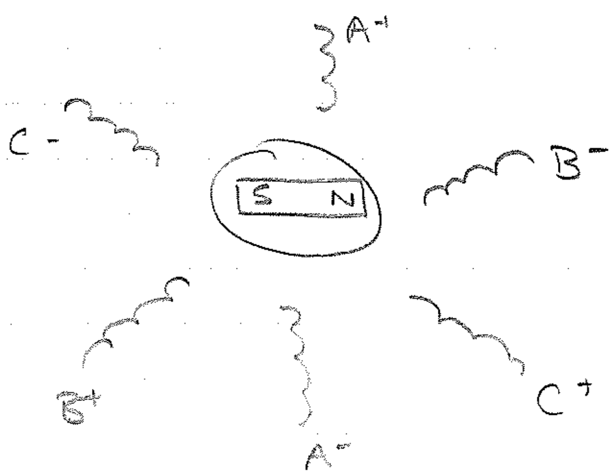
Concentrated vs. Distributed Windings





(Do later)

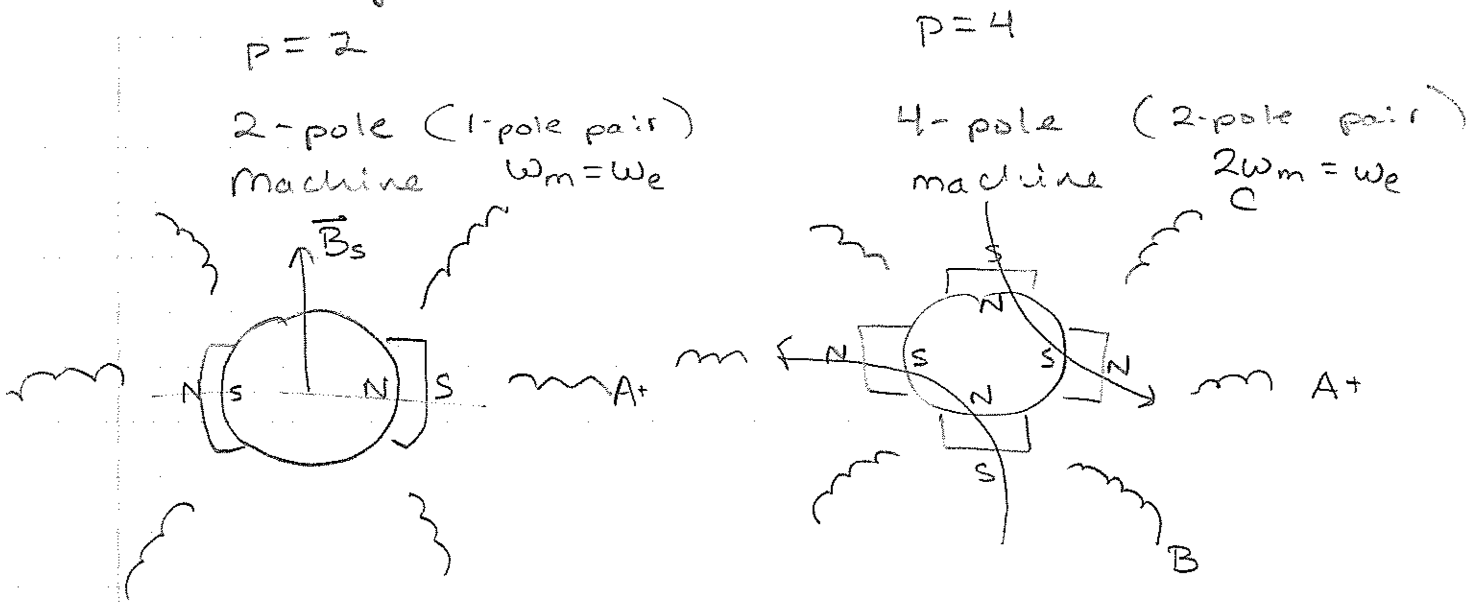
Drawn abbreviation:



(Both have same fields, just different constructions)

Pole Pairs

- A machine usually has more than 2 magnetic poles... this increases torque density.



(The more magnets you put in there, the more energy you can store for torque)

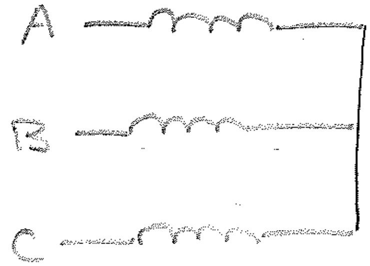
$$\frac{P}{2}\omega_m = \omega_e$$

The number of pole pairs can get pretty high. The bike in Allan's office has 46 poles.

600 rpm
$f_m = 10$ Hz
$f_e = 230$ Hz

$\boxed{\text{Page 10}}$ As an EE, how do you work with this thing?

<u>Faraday's Law</u>



Because all currents are the same (balanced) we can simplify and look at only 1-phase.

Easy because everything is just shifted by 120°. (360°/3 = 120°)

a single phase voltage

flux that links coil A

$$V_a(t) = R_s\, i_A(t) + \frac{d\lambda_A(t)}{dt}$$

stator windings

(Changing magnetic fields produce a voltage)

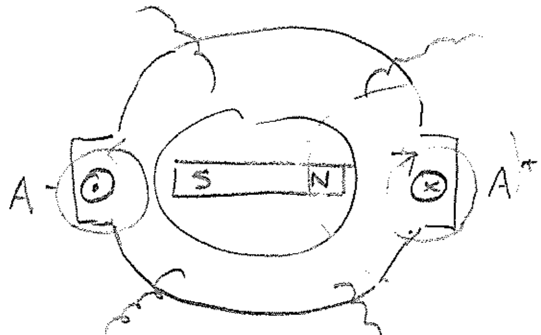
fictitious current which represents i_B, i_C, λ_{pm}

flux from permanent magnet

$$\lambda_A(t) = L_s\, i_A(t) + M\, i_m(t)$$

self inductance of coil A

coupling inductance to coil A



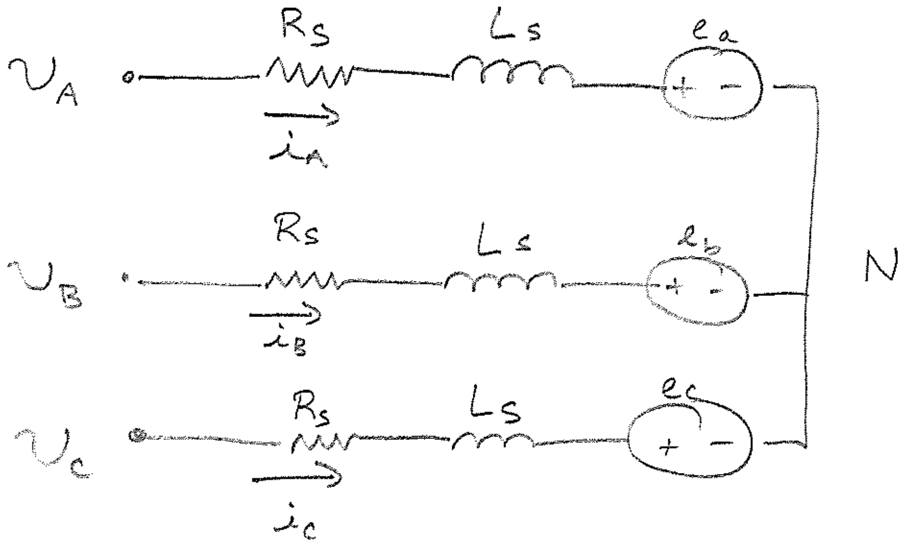
~~The other part of the flux that interacts with the permanent magnet~~

Essentially, he was talking about how all of this breaks down as far as induced voltages go.

$$v_A(t) = R_s i_A(t) + L_s \frac{d i_A(t)}{dt} + \frac{d \lambda_m(t)}{dt}$$

$$= R_s i_A(t) + L_s \frac{d i_A(t)}{dt} + e_a(t)$$

Back EMF term



All voltages and currents are all sinusoidal and phase-shifted by 120°.

$$\omega = \frac{d\theta}{dt}$$

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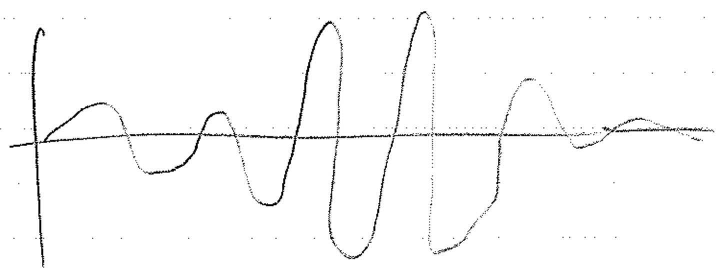
$$e_a(t) = \omega K \cos(\theta)$$

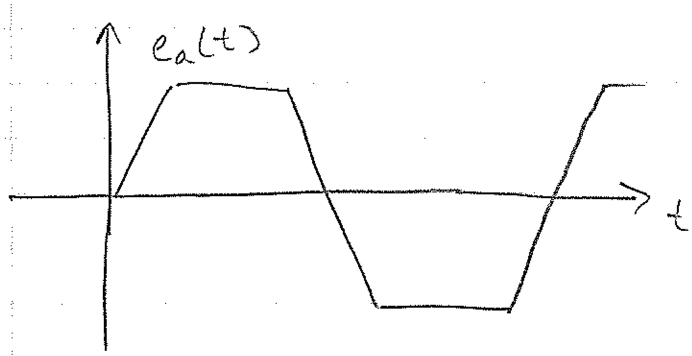
$$e_a(t) = \frac{d\lambda_m}{dt} \quad \leftarrow k\sin(\theta)$$

Permanent
Magnet
Synchronous
Machine
(PMSM) or PMSG

Because $\omega = \frac{d\theta}{dt}$, depending on how fast you spin the motor, the back EMF will generate a small or large voltage.



Brushless DC machine (Still an AC machine)
(BLDC)
Back EMF is trapezoidal!
Not sinusoidal!

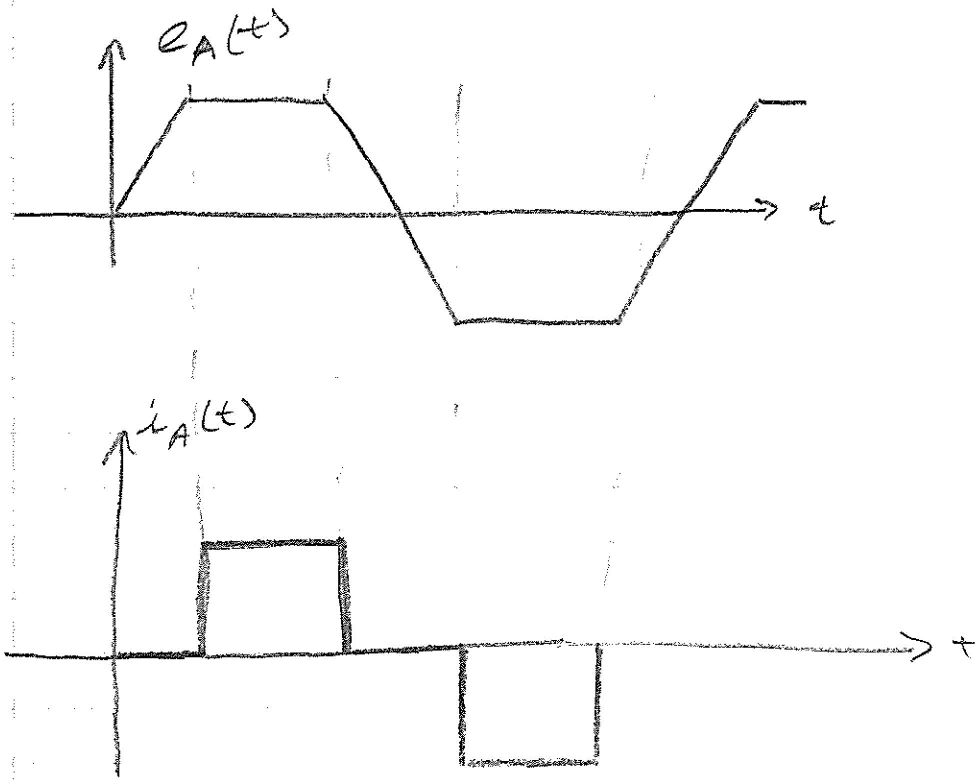


What's the impetus for a trapezoidal waveform?

Note: Ask Questions on Quora to include in the progress reports!!!

In a sinusoidal machine, we made the assumption that the current is sinusoidal and in phase with the voltage.

Let's make the current look like a square wave.



PI controller, done,
Simple.

For small, cheaper systems, they'll use a BLDC.

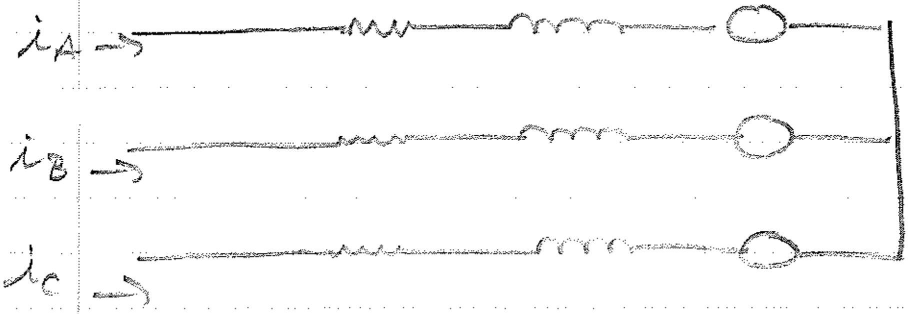
Eddie
Schodowski

EE 499 - Week 1 Friday

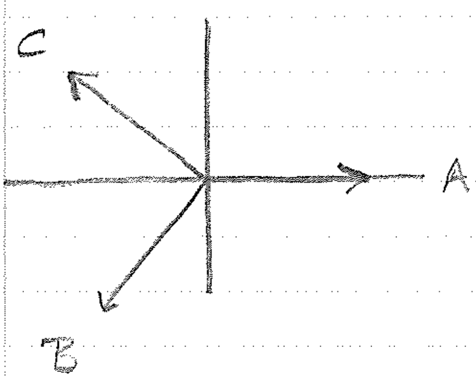
7/15/2016

Coordinate Transforms

- Three currents are not linearly independent



$i_A + i_B + i_C = 0$



The net $\vec{B}$ field can be described w/ cartesian coordinates.

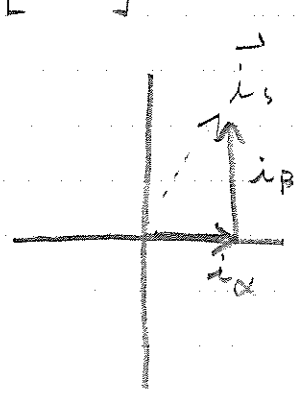
(3 variables → 2)

Clarke Transform

Very important

$$\begin{array}{c} x \\ y \end{array} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}$$

Describe rotating net field in x,y components



"The Gamma phase is zero if all currents balanced"

Page 2

$$\begin{array}{c} x \\ y \\ \text{mean} \end{array}\begin{bmatrix} i_\alpha \\ i_\beta \\ i_\gamma \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}$$

indicates imbalance in system

(ignoring for the meantime)

If i_α and i_β are my x & y components, I can represent them together as one space vector!

$\vec{i_s} = i_\alpha + j\, i_\beta$ (really, a combination of multiple currents)

- Phasors describe <u>ONE</u> sinusoidal signal, usually at steady-state and const. freq.
- Space vectors describe SEVERAL sinusoidal terms, and vary w/ time.

<u>Phasor Example</u>

$i_A(t) = I_m \cos(\omega t + \theta_i)$ A

$\vec{I_A} = I_m \angle \theta_i$ A

<u>Euler's Identity</u>

$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$

$e^{j\theta_i} = \cos(\theta_i) + j\sin(\theta_i)$

$\mathrm{Re}\{I_m e^{j(\omega t + \theta_i)}\}$

$I_m\, \mathrm{Re}\{e^{j\omega t} \cdot e^{j\theta_i}\}$

$I_m e^{j\theta_i} \times \mathrm{Re}\{e^{j\omega t}\}$

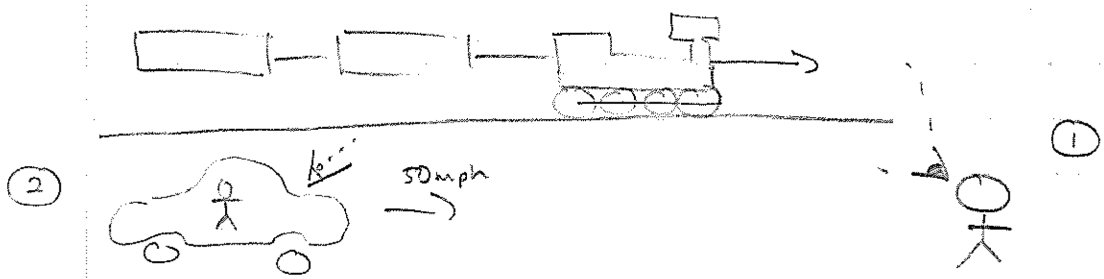
Clarke Transform Using Space Vectors

$$\vec{i}_s = (i_\alpha + j i_\beta) = \frac{2}{3}\left(i_A e^{j0} + i_B e^{j2\pi/3} + i_C e^{j4\pi/3}\right)$$

(basically this gives us the Clarke Transform)

Reference Frames

- Imagine a train moving at 50 mph



(1) Scenario 1, you are stationary
"I see 50 mph"

(2) Scenario 2, you are in a car moving at 50mph in the same direction.
"The train is stationary"

We will apply this to the flux of rotor.

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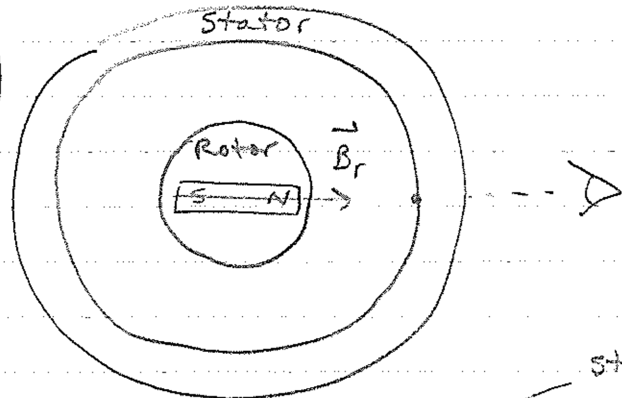
Rotor Flux

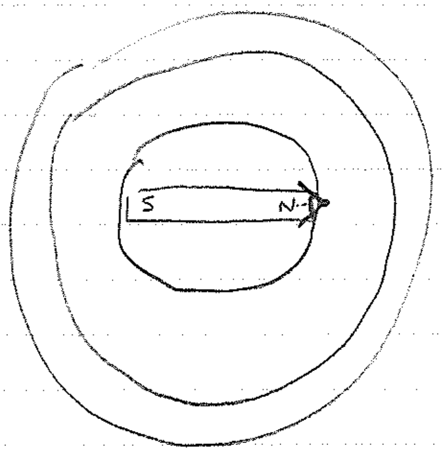
Stator Frame of Reference



stator frame of ref

$$\lambda_{pm}^{s}(t) = \Lambda_{pm}\cos(\omega t) \quad \text{Wb}$$

Rotor Frame of Reference



Remember, $\lambda_{pm}(t)$ is the flux linkage due to PM.

$$\lambda_{pm}^{r}(t) = \Lambda_{pm}$$

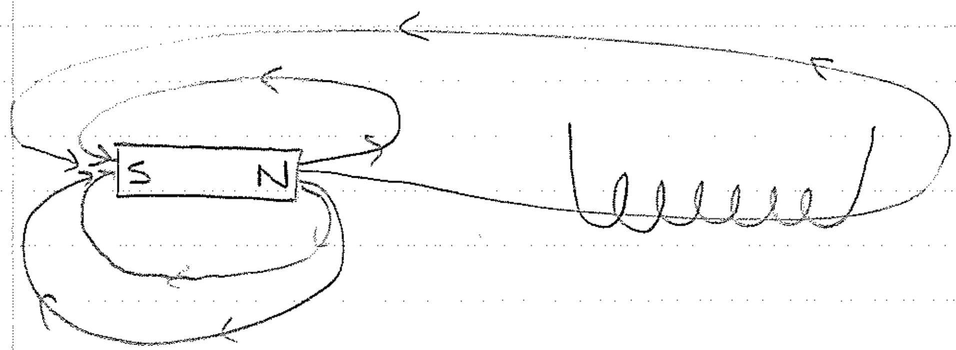
generated by PM or electromagnet or inductor... $\longrightarrow$ Φ flux

flux linkage λ Λ

Flux Linkage

$$\lambda = k \frac{NB}{\mu_0} \cos\Theta$$

λ Λ



$\vec{H}$ $\vec{B}$

Flux linkage is the flux from the PM going through the coil.

Why do we want to do this?
2 Main Reasons.

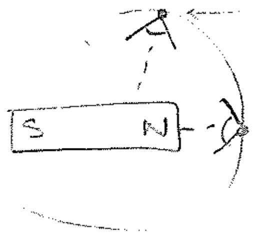
1. All AC components become DC in the rotor frame of reference! We can use simple PI control
2. When we analyze the electromagnetics, the equation becomes very simple... in rotor frame.

$$T = \frac{3}{2} \frac{P}{2} i_q \Lambda_{pm}$$

← magnet Flux or "strength"

↑ quadrature-Axis current

\# of poles



Also aligned to rotor, but not the rotor flux.

Different from rotor position

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How do we do this? (physically)

- magnets in a PM machine are FIXED to the rotor ... Thus, if we know rotor position, we know <u>rotor flux position</u>

(This is not true in an induction machine)

(*) - We can use an angular position sensor! (*)

We can put the α and β numbers as a complex number to make a space vector.

<u>Park Transformation</u>

- we can move from stator frame to <u>rotor frame</u> using another transform matrix

direct axis

quadrature axis

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

These i_d and i_q currents are DC currents.

same i_q in the Torque equation earlier.

$$\vec{i_s}^{\,r} = \vec{i_s}^{\,s} \cdot e^{-j\Theta_r} \quad \text{where} \quad \vec{i_s}^{\,r} = (i_d + j i_q)$$

$$\text{and} \quad \vec{i_s}^{\,s} = (i_\alpha + j i_\beta)$$

actual Θ of rotor

$$e^{-j\theta} = \cos(-\theta) + j\sin(-\theta)$$

$$= \cos(\theta) - j\sin(\theta)$$

$$\vec{i}_s^{\,s} \cdot e^{-j\theta_r} = (i_\alpha + j i_\beta)(\cos\theta - j\sin\theta)$$

$$= i_\alpha \cos\theta - j\, i_\alpha \sin\theta + j\, i_\beta \cos\theta + i_\beta \sin\theta$$

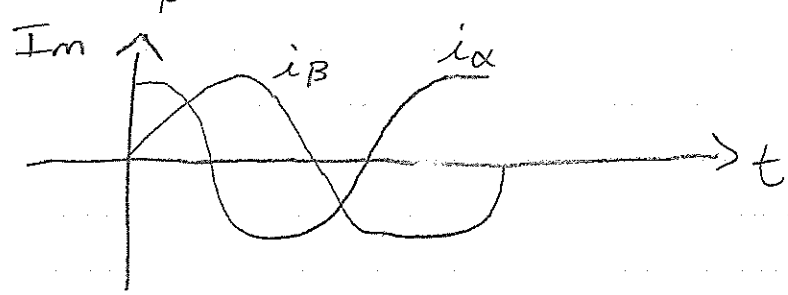
$$= \underbrace{(i_\alpha \cos\theta + i_\beta \sin\theta)}_{i_d} + j\underbrace{(i_\beta \cos\theta - i_\alpha \sin\theta)}_{i_q}$$

i_d and i_q can change with time, but at a steady state, they'll be flat DC values.

Proof that i_d, i_q are constant:

Let $i_\alpha = I_m \cos(\omega t)$

$i_\beta = I_m \sin(\omega t)$



Page 8

$$i_d = I_m \cos(\omega t)\cos(\theta) + I_m \sin(\omega t)\sin(\theta)$$

$$= I_m \frac{1}{2}\left[\cos(\omega t - \theta) + \cos(\omega t + \theta)\right]$$

$$+ I_m \frac{1}{2}\left[\cos(\omega t - \theta) - \cos(\omega t + \theta)\right]$$

$$= I_m \frac{1}{2}\left[1 + \cos(2\omega t) + 1 - \cos(2\omega t)\right]$$

$$i_d = I_m \checkmark \qquad \cos(0) = 1$$

(Remember, $\omega t = \theta$)

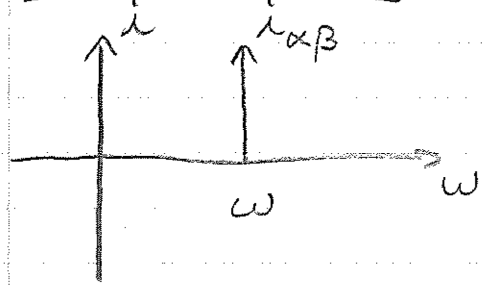
So, i_d is just a number.

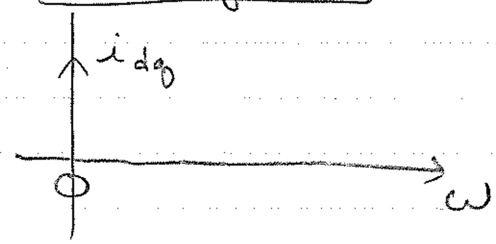
$i_q = 0$

3 eqn's $\rightarrow$ 2 eqn's $\rightarrow$ 1 space vector

Q: Why is the $2/3$ important in the Clarke Transformation? **Page 9**

$\boxed{\text{Stationary Frame}}$ $\boxed{\text{Rotating Frame}}$





Basically, the Clarke Transformation shifts it to $\omega = 0$ to make things easier for us.

$\boxed{\text{Machine Equations in Rotor Frame}}$ (1)

(1) Caveat x this regarding of α, β

$$\vec{V}_s^s = R_s \vec{i}_s^s + L_s \frac{d\vec{i}_s^s}{dt} + \frac{d\vec{\lambda}_{pm}^s}{dt} \quad \text{(Stator frame)}$$

(Rotor frame)

$$\vec{V}_s^s e^{-j\theta} = \left[R_s \vec{i}_s^s + \underbrace{L_s \frac{d\vec{i}_s^s}{dt}} + \frac{d\vec{\lambda}_{pm}^s}{dt} \right] e^{-j\theta}$$

$$\vec{V}_s^r = R_s \vec{i}_s^r + L_s \left[\frac{d}{dt}\left\{ \vec{i}_s^s e^{-j\theta} \right\} + j\omega \vec{i}_s^s e^{-j\theta} \right]$$

$$+ \left[\frac{d}{dt}\left\{ \vec{\lambda}_{pm}^s e^{-j\theta} \right\} + j\omega \vec{\lambda}_{pm}^s e^{-j\theta} \right]$$

$$\vec{V}_s^r = R_s \vec{i}_s^r + L_s \frac{d\vec{i}_s^r}{dt} + j\omega L_s \vec{i}_s^r + \cancel{\frac{d\vec{\lambda}_{pm}^r}{dt}} + j\omega \vec{\lambda}_{pm}^r$$

(Note: currents w/ $\frac{d}{dt}$'s don't easily transform, another term pops out)

From: $j\omega L_s i_q$ on the page before

$$j\vec{i}_s^{\,r} = j(i_d + j i_q) = j i_d - i_q$$

$\boxed{\text{Page 10}}$ (Space vector notation so 2 EQns here)

Real
$$v_d = R_s i_d + L_s \frac{di_d}{dt} - \omega L_s i_q + 0$$

Imaginary
$$v_q = R_s i_q + L_s \frac{di_q}{dt} + \omega L_s i_d + \omega \Lambda_{pm}$$

everything referring to stator here

Burn these into your mind!
These describe the model for the machine.
We'll simulate and then control it!

Resistive Drop

Inductor Voltage

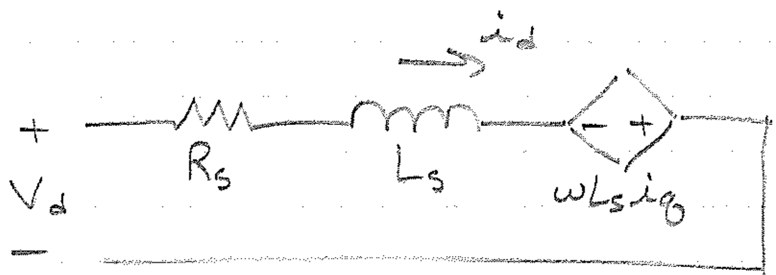
Back EMF term

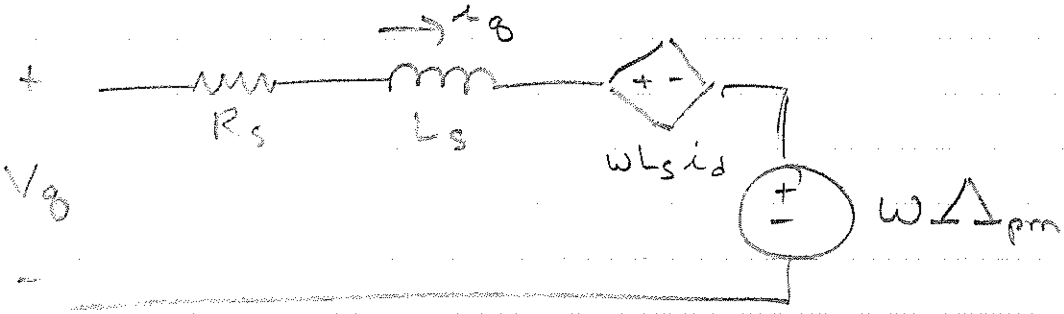
Somewhat fictitious term, change in current w/ respect to time, so view it as acceleration of sorts.

$$V = \frac{d\Phi}{dt}$$

$$\Phi = \Phi_m \cos(\omega t)$$

Rotor Frame = dq frame





Use superposition ... turn V_q & $\omega \Lambda_{pm}$ OFF

Find $\dfrac{i_d(s)}{V_d(s)} = H(s)$

(Equation you get when turning off sources)

$$i_q(s) = -\frac{\omega L_s i_d(s)}{sL_s + R_s}, \quad i_d(s) = \frac{V_d + \omega L_s i_q}{sL_s + R_s}$$

$$i_d(s) = \frac{V_d + \omega L_s\left(\frac{-\omega L_s i_d(s)}{sL_s + R_s}\right)}{sL_s + R_s}$$

(move $i_d(s)$'s over to same side)

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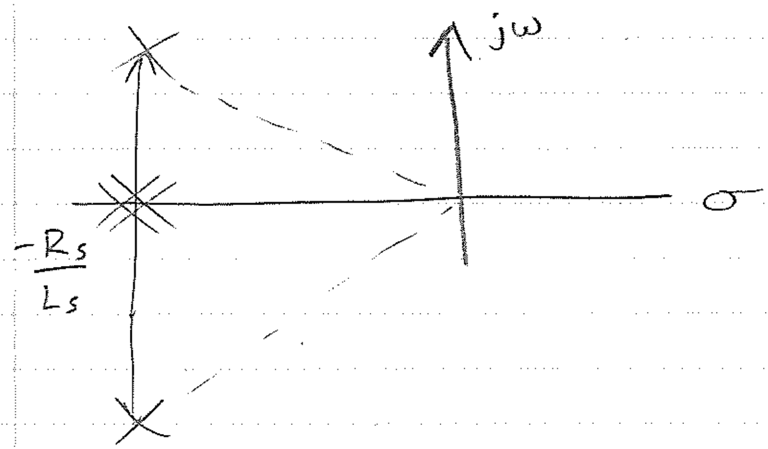
$$i_d(s)\left[1+\frac{\omega^2 L_s^2}{(sL_s+R_s)^2}\right]=\frac{V_d}{sL_s+R_s}$$

$$\frac{i_d}{V_d}=\frac{sL_s+R_s}{s^2L_s^2+2sL_sR_s+R_s^2+\omega^2L_s^2}$$

$$D(s)=0 \rightarrow s=\frac{-R_s}{L_s}\pm j\omega$$

poles change depending on the speeds

Easier to control at low speeds.



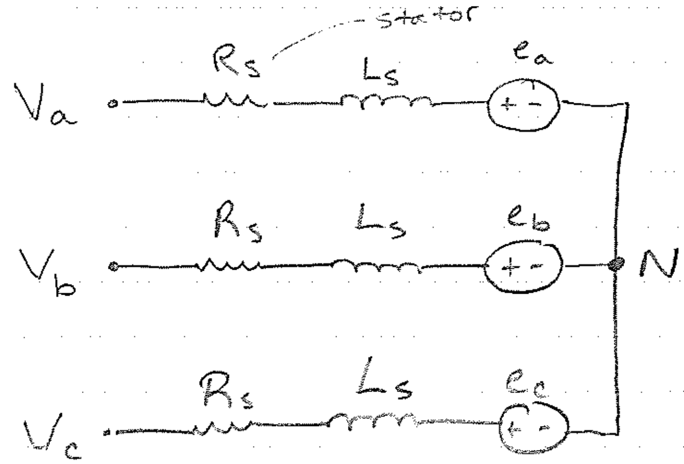
Need to learn control theory.

Eddie Schodowski

EE 499 - Week 2 Tuesday

7/19/2016

Page 1



$$V_a = R_s i_a + L_s \frac{di_a}{dt} + e_a(t)$$

where $e_a(t) = \dfrac{d\lambda_{pma}(t)}{dt}$

$$\lambda_{pma}(t) = \Lambda_{pm} \cos(\omega t)$$

$$= \Lambda_{pm} \cos(\Theta)$$

$$e_a(t) = \omega \Lambda_{pm} \sin(\Theta)$$

$\omega t = \Theta$

$i_a + i_b + i_c = 0$

Clarke Transform

$$\begin{array}{c} x \\ y \end{array} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = T_c \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \xrightarrow{\text{Park Transform}} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = T_p \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

Page 2

$$V_d = R_s i_d + L_s \frac{di_d}{dt} - \omega_e L_s i_q$$

$$V_q = R_s i_q + L_s \frac{di_q}{dt} + \omega_e L_s i_d + \omega \Lambda_{pm}$$

Relative Permeability of Iron:

$\mu_{r,Fe} >> 1$

Relative Permeability of Air:

$\mu_{r,Air} \approx 1$

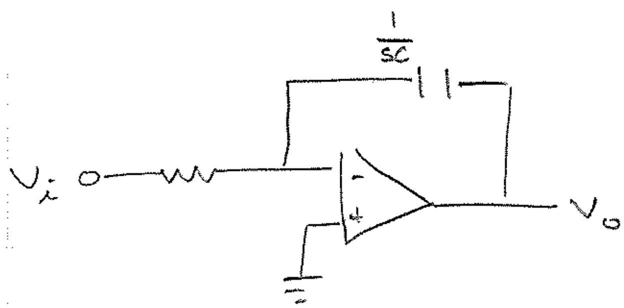
$\mu_{r,pm} \approx 1$

There are surface permanent magnets and interior permanent magnets.

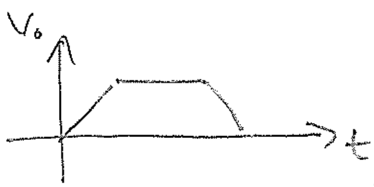
Why is it in Simulink?

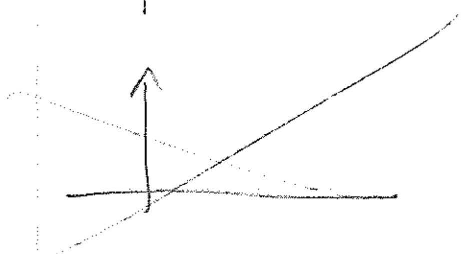
Q: What is an integrator?

$\boxed{\text{Page 3}}$



$$\frac{V_o}{V_i} = -\frac{\frac{1}{sC}}{R} = \frac{-1}{s} \cdot \frac{1}{\cancel{RC}} \; 1$$





Whatever.

Page 4

Relearn integrating factor.

$$\frac{dx}{dt} = 1 - 0.1x$$

$$\frac{dx}{dt} + \frac{1}{10}x = 1 \qquad e^{\int \frac{1}{10} dt} = e^{\frac{1}{10}t}$$

(Multiplied EQn by integrating factor)

$$\left[e^{\frac{1}{10}t}\frac{dx}{dt} + \frac{1}{10}e^{\frac{1}{10}t}x\right] = e^{1/10\, t}$$

$$\frac{d}{dt}\left[e^{(1/10)t}x\right] = e^{\frac{1}{10}t}$$

$$e^{\frac{1}{10}t}x = 10e^{\frac{1}{10}t}$$

$$\boxed{x = 10e^{-\frac{1}{10}t}} \Rightarrow 10\left[1 - e^{-\frac{1}{10}t}\right]$$

When he made the simple model on Simulink, how did he know the sol'n to the DE checked out and that

Look up Sample Theory / Nyquist Theory (Sample twice the frequency)

(Get a mouse for Simulink)

Model Development:

How do you know where to start?
Get the derivative over to one side.
Use the integrator to numerically cheat and determine what the solution to the DE looks like.

$$\frac{di_d}{dt} = \frac{1}{L_s}\left[V_d - R_s i_d + \omega_e L_s i_q\right]$$

$$\frac{di_q}{dt} = \frac{1}{L_s}\left[V_q - R_s i_q - \omega_e L_s i_d - \omega \Lambda_{pm}\right]$$

Gain blocks are for multiplying constants (yeah, gain is constant - overthought it for a second)

I should play around with Simulink.

In the real world, most systems are nonlinear.

"Linear systems are a subset of nonlinear systems."

Nonlinear Systems by Slotine
"Applied Nonlinear Control" by Slotine

Simplify non-linear systems to linear systems, and check if its close enough.

Intro → Theoretical Analysis → Experimental Confirmation

Page 6

Usually, job at the physicist that the model works out. That's what conference papers are for. Check if they're close enough to the real world.

$$\boxed{T_{em} = \frac{3}{2}\frac{P}{2} i_q \Lambda_{pm}}$$

$$F_{motor} = \underbrace{K_m vel}_{\text{friction}} + m\frac{dvel}{dt} + \underbrace{F_{load}}_{\text{(wind or whatever)}}$$

What is load?

$$T_{em} = B_m \omega_m + J_m \frac{d\omega_m}{dt} + T_L$$

Nm

$\frac{Nm}{rad/sec}$

Nm

$$\frac{d\omega}{dt} = \frac{1}{J_m}\left[T_{em} - B_m\omega_m - T_L\right]$$

He built Simulink models around this stuff and then made them subsystems and then connect them to build out the entire system.

Q: What is the electrical speed? ω_e

Where $\frac{P}{2}\omega_m = \omega_e$

Variable-step solvers → a form of numerical method.

Allan Taylor typically does fixed-step, and you basically are doing a form of integration (rectangles, over/under, trapezoids).

But, to be aware, it's possible that the type of numerical method could make your system model unstable and this stuff can affect your poles and zeros!

You need to declare your variables.

2 ways to run PM machines

(1) Machine wants to lock onto the coils. The PM's want to follow that rotating electrical field.
(If you don't advance it, then it just stays there @ the stable position)

Locking Speeds → Everything is in the d-axis.
Why does it get that way?

If a power plant goes down, it's a huge deal to get it all phase/speed aligned.

Page 8

What is an inverter and a motor drive?
What are the best books that Allan Taylor has read on EE?

Look up synchronous machines.

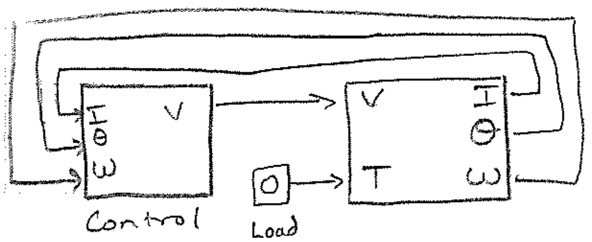
VVVF Control (Variable Voltage, Variable Frequency Control)

Eddie Schodowski

EE 499 - Week 2 Friday

7/22/2016

"Motor Control Simulation with Simulink"



Common buzzword: Field-Oriented Control (FOC)

or

Vector Control

Want to align w/ Rotor Flux Field (Θ)

Equation for Torque is purely determined by i_q and constants.

Torque Control (I)

- speed control
- Decoupling the DQ axes (simplifies control)

} ω

Objective: Regulate the current

$$V_d = R_s i_d + L_s \frac{di_d}{dt} - \omega_e L_s i_q$$

$$V_q = R_s i_q + L_s \frac{di_q}{dt} - \omega_e L_s i_d + \omega_e \Lambda_{pm}$$

$T_{em} = \frac{3}{2} \frac{P}{2} i_q \lambda_{pm}$

Page 2

Let's look at controlling i_d and i_q

$i_d^* = 0\ A \rightarrow$ it does not produce torque

ref. value

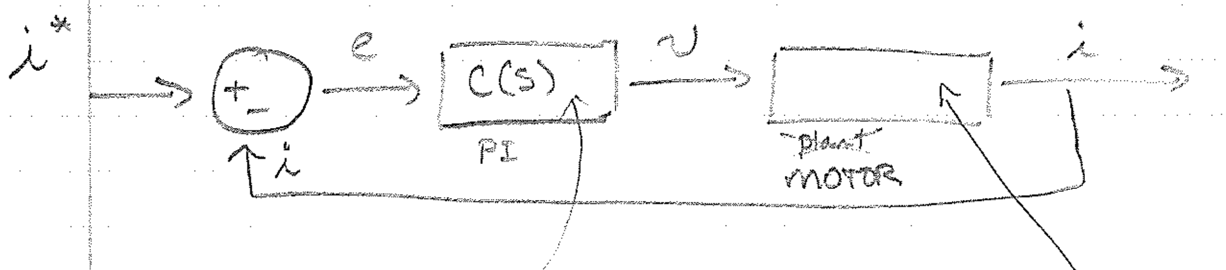
(*) $i_q^* = ??$ depends on the torque I want. (*)

$i_q^* > 0$, I accelerate the machine in positive direction or decelerate in negative direction.

$i_q^* < 0$, I decelerate in positive (regen) or accelerate negative.

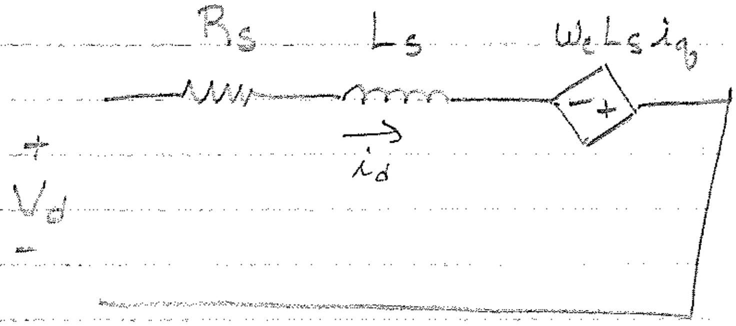
"To control the currents, I'm going to use PI controllers."

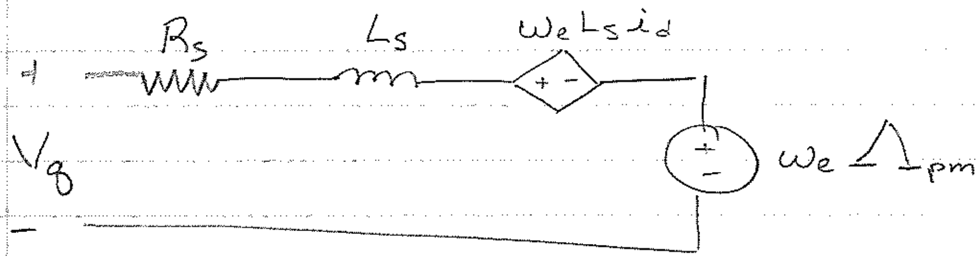
Use PI controllers to regulate current!
↳ we need one for each axis



$C(s) = K_p + \frac{K_i}{s}$

$\frac{1}{sL_s + R_s}$





$$C(s) = K_p + \frac{K_i}{s}$$

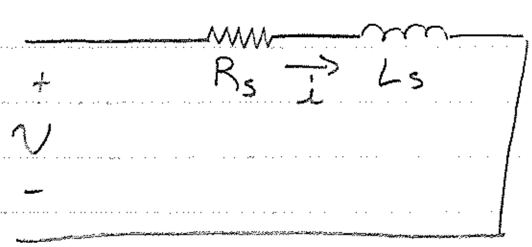
$$\text{poles} = -\frac{R_s}{L_s} \pm j\omega$$

It's <u>not</u> linear time invariant.

$$\text{zeros} = -\frac{R_s}{L_s}$$

When you drop this to $\omega = 0$, the sources drop off.

Let's analyze at zero speed!



$$v = R_s i + s L_s i$$

$$\frac{i}{v} = \frac{1}{sL_s + R_s}$$

"When I apply a voltage to the motor terminals, this is how the current responds."

Page 4

Sometimes when you're doing a PI controller, you put a saturation block on there.

Real voltage controller has finite amount it can generate.

Integrator windup

We want to prevent integrating up to really large (or small) #'s

Simulink → right click → create mask
You're going to want to put a bunch of parameters in.
This makes debugging go much faster.

Proportional gain, k_p
Integral gain, k_i
Initial condition, i_0
Saturation Limits, lims

$$TF_{CL}(s) = \frac{G}{1+GH}$$

$$H = 1$$

$$G = \left(K_p + \frac{K_I}{s}\right)\left(\frac{1}{sL_s + R_s}\right)$$

$$= \left(\frac{sK_p + k_i}{s}\right)\left(\frac{1}{sL_s + R_s}\right)$$

$$= \frac{\frac{sK_p + K_i}{s} \cdot \frac{1}{sL_s + R_s}}{1 + \frac{sK_p + K_i}{s} \cdot \frac{1}{sL_s + R_s}}$$

$$= \frac{sK_p + K_i}{(\quad S \quad)(sL_s + R_s) + (sK_p + K_i)}$$

$$= \frac{sK_p + K_i}{s^2 L_s + s(R_s + K_p) + K_i}$$

$$= \frac{\frac{K_i}{L_s}\left(s\frac{K_p}{K_i} + 1\right)}{s^2 + s\left(\frac{R_s + K_p}{L_s}\right) + \frac{K_i}{L_s}}$$

We want current controllers to be the fastest in our system.

We want current response to be fast!
Let's choose T_{sttl} to be $\approx$ 5 msec

~~Controller~~
DSP runs at 10 KHz
(100 μsec) = $T_{sampling\text{-}time}$

Settling Time:

$$T_{sttl} = \frac{3.9}{\zeta \omega_n} = 0.005\,s$$

$$D(s) = \left(s^2 + s2\zeta\omega_n + \omega_n^2\right)$$

(We're trying to find one of our PI controller coefficients)

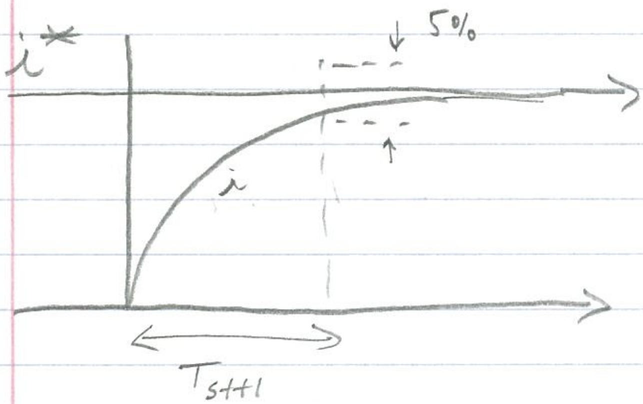
Page 6

$$0.005 = \frac{3.9}{\frac{R_s + K_p}{2L_s}}$$

Assume $R_s = 25\,m\Omega$
$L_s = 100\,\mu H$

$$K_p = \frac{3.9(2L_s)}{T_{st+1}} - R_s \approx 0.131 \text{ for } K_p$$

round down to $0.1 = K_p$
Want to reduce noise from current sensors
Be cautious about ratcheting this up too high.



For K_i let's keep the poles real!
No overshoot!

Closed-loop Poles :

$$s = -\frac{R_s + K_p}{2L_s} \pm \sqrt{\left(\frac{R_s + K_p}{2L_s}\right)^2 - \frac{K_i}{L_s}}$$

If you set the radical equal to zero, you can find the boundary conditions.

$$\left(\frac{R_s + K_p}{2L_s}\right)^2 - \frac{K_i}{L_s} = 0$$

$$\frac{(R_s + K_p)^2}{4L_s} = K_i$$

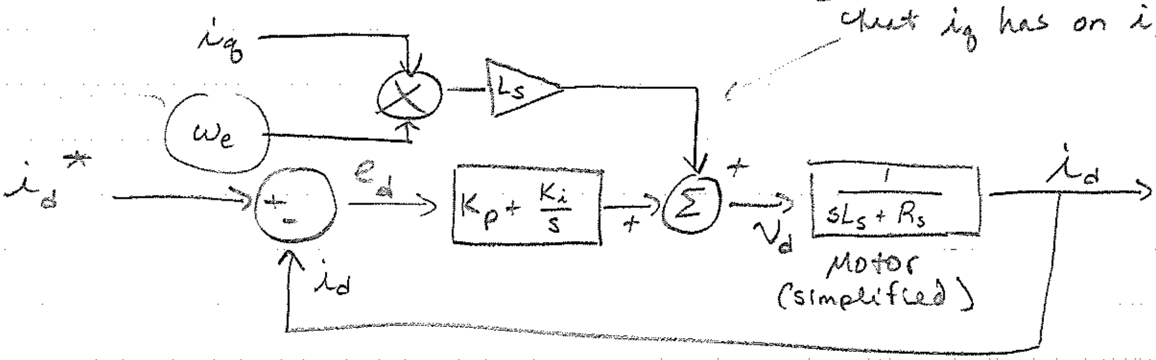
$K_i \approx 39$

Let's pick $\boxed{K_i = 20}$

Page 8

We made the assumption that our motor is time-invariant, but it really is time varying. We have to handle that and use "feed-forward" terms.

cancels out the effects that i_q has on i_d.



That's the important part.
Previously we assumed zero speed.

$$V_{q} = (PI + FF)$$

Feed Forward
(you know that you're going to experience discrepancies due to the time varying terms, so you just account for it in the model)

ω_m^* is the reference / what you're feeding in

Q: How do PI controllers take input in one unit and output different units?

Eddie
Schodowski

EE 499 - Week 3 Monday

7/26/2016

Page 1

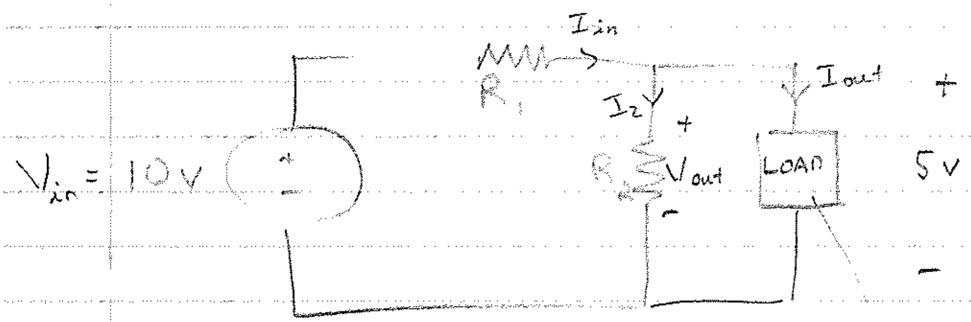
Now that we know which signals to generate, how do we generate the voltage signals?

Voltage Regulator Problem

- We have a battery (10V) and I want to power 5V load (μCU) — microcontroller unit

How do we step down voltages?

Voltage Division



$$R_L = \frac{V_{out}}{I_{out}} = \frac{5V}{\dots}$$

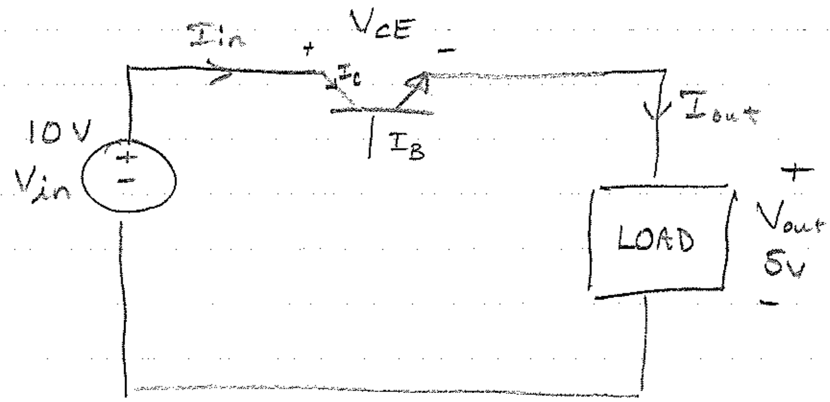
$$V_{out} = \frac{(R_2 // R_L)}{(R_2 // R_L) + R_1} \cdot V_{in}$$

Two Issues:

- Voltage regulation depends on R_L, load resistance
  - (won't always have $V_o = 5V$)
- Power Loss on R_1 and R_2

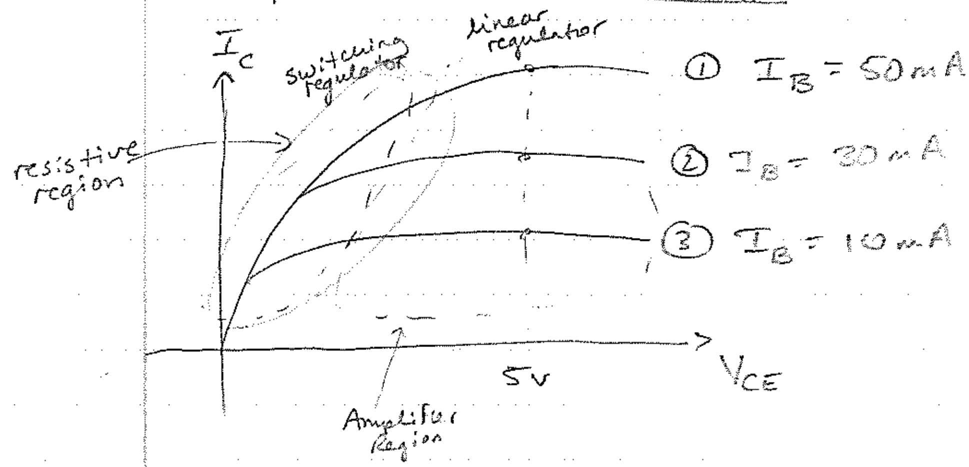
Using a transistor in Amplifier Region

Page 2



Still have lots of power loss on transistor.

← **Linear Regulator**
(Makes transistor act like a variable resistor)



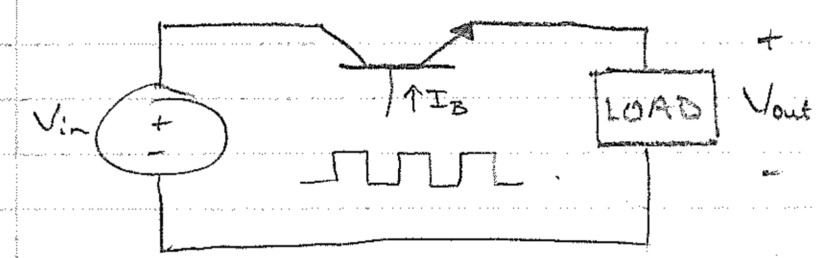
Really a 3D graph. These are the curves when you set the following values of I_B.

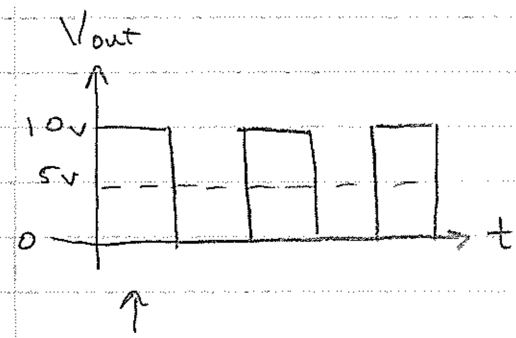
Transistor acts as a "controlled resistor"
Voltage division with load!

This fixes the first issue but doesn't really fix the second one (power loss).

Slope is $\frac{A}{V}$ which is $\left(\frac{V}{A}\right)^{-1}$, viewed as $\left(\frac{V}{I}\right)^{-1} = R^{-1} = G$, conductance.

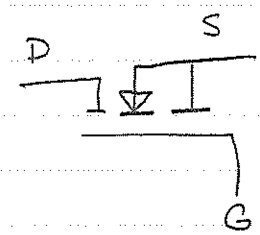
We can operate the transistor in the resistive region but we must pulse the base/gate.



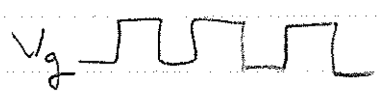


- This signal is generated by another circuit w/ a clock. Electronics I/II

This is okay for resistor loads, but would fry a μCU!



MOSFETs are current controllers
BJTs are voltage controllers



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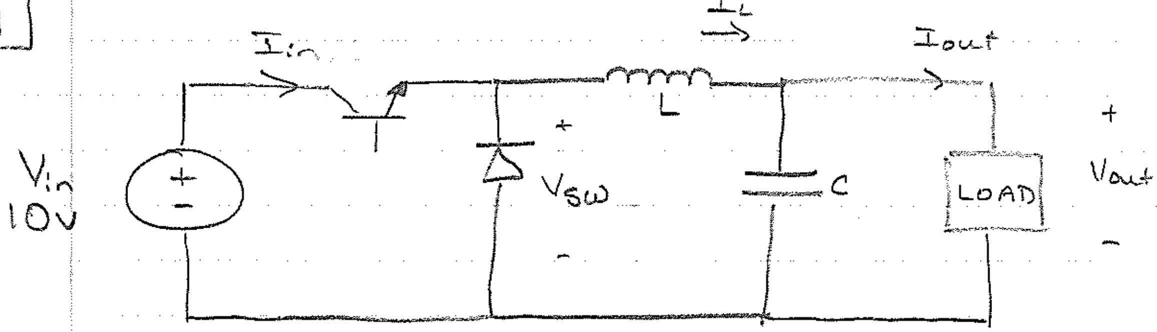
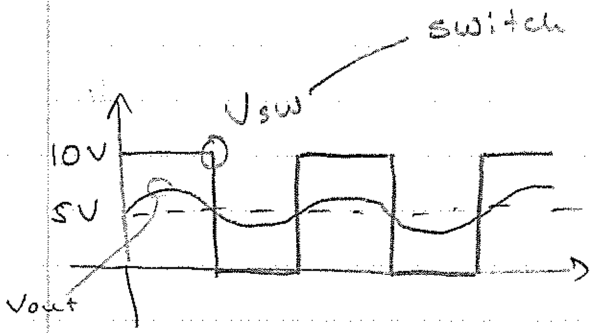
Buck Converter

- Switching Regulator (Allows us to maintain 5V even when switched (?!))

(modulation does typically come from a MOSFET, however)



- We can add an LC Filter to smoothen the pulsed voltage
- We must add the diode to provide a current path for the inductor when the switch is OFF

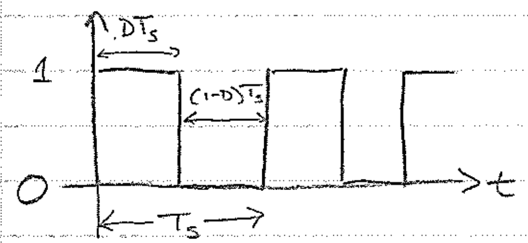


← Regulated at 5v, REGARDLESS OF THE LOAD. The beauty of voltage regulators!

Linear Regulator operates in Amplifier Region. more power loss
Switching Regulator operates in Resistive Region. Way less power loss.

Assumptions for Power Converter Analysis

1. Circuit is at steady state
2. Inductor current is continuous
   L is large, ripple current is small
3. Capacitor voltage is continuous
   C is large, voltage ripple is small
4. Switching period is T_s
   Switch is ON for $D \cdot T_s$ seconds
   OFF for $(1-D)T_s$ seconds
5. Components are ideal, No resistive loss,



$D = 0.5$

"D" is the duty cycle...
- It is the ratio of ON time to the period T_s
- It is a number from 0 to 1

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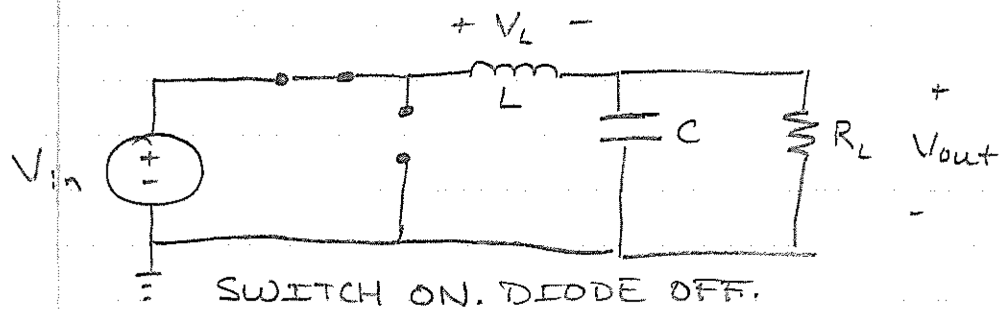
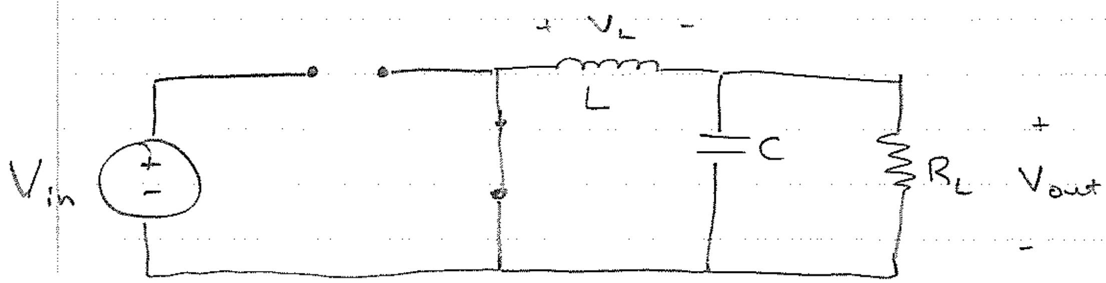
Let's analyze the inductor voltage,



SWITCH ON. DIODE OFF.

$$V_{L,ON} = V_{in} - V_{out}$$



SWITCH OFF. DIODE ON.

$$V_{L,OFF} = 0 - V_{out}$$

Steady state implies that the change in average inductor current is zero.

Mathematical Definition of the Average →

$$\overline{V_L} = 0V = \left[\int_0^{DT_s} V_{L,ON}\,dt + \int_{DT_s}^{T_s} V_{L,OFF}\,dt\right]\frac{1}{T_s}$$

(sub in)

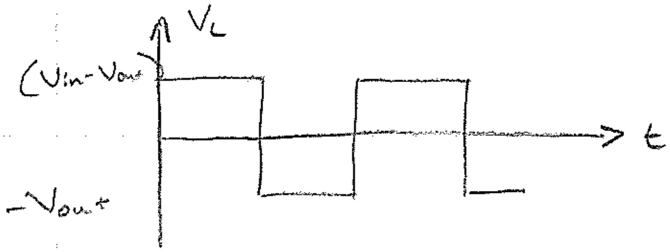
$$0 = \left[(V_{in} - V_{out})DT_s + (-V_{out})(T_s - DT_s)\right]\frac{1}{T_s}$$

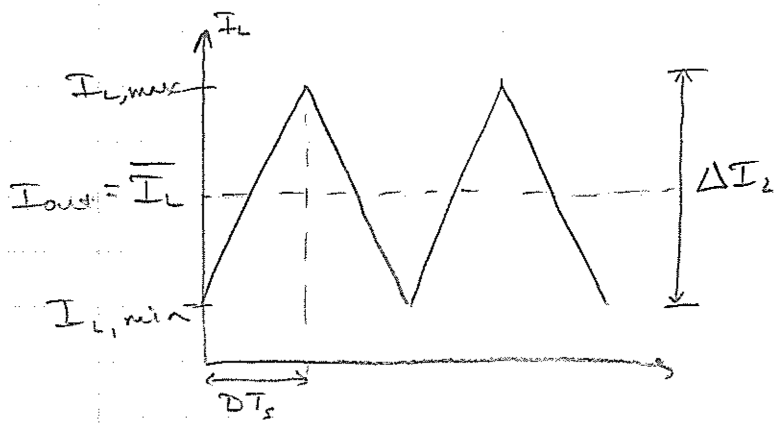
$$0 = V_{in}D - \cancel{V_{out}D} - V_{out}(1 - \cancel{D})$$

$$\boxed{V_{out} = D \cdot V_{in}}$$

Essentially, PWM.

If you can control the pulse width of your switch, you can control your output voltage.





(Diagram of Δ wave in the notes)

$$\Delta i_L = \frac{1}{L}\int_0^{DT_s} V_{L,ON}\, dt$$

$$\Delta i_L = \frac{1}{L}(V_{in} - V_{out}) DT_s$$

$$\Delta i_L = \frac{V_{in} - V_{out}}{L f_s} \rightarrow \Delta i_L = \frac{(V_{in} - DV_{in})D}{L f_s} = \boxed{\frac{V_{in}(D - D^2)}{L f_s}}$$

switching frequency

Similar to $y = x - x^2$

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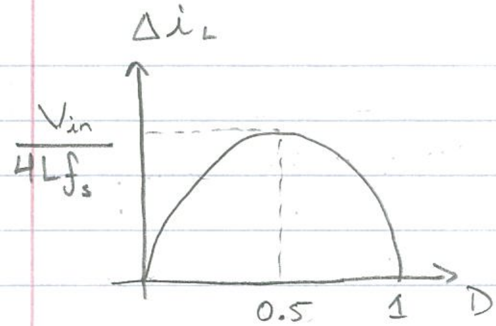
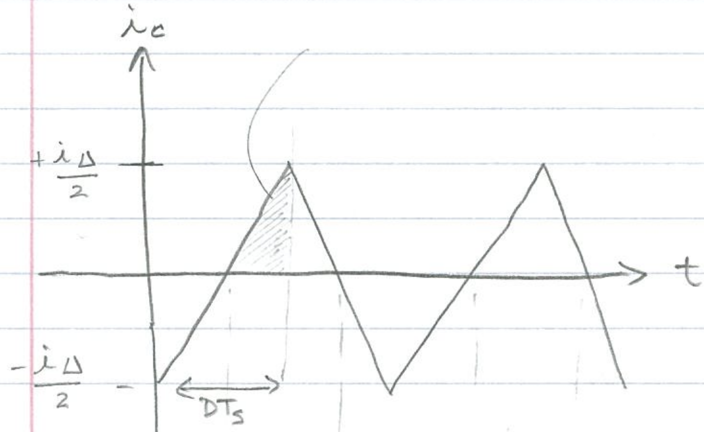
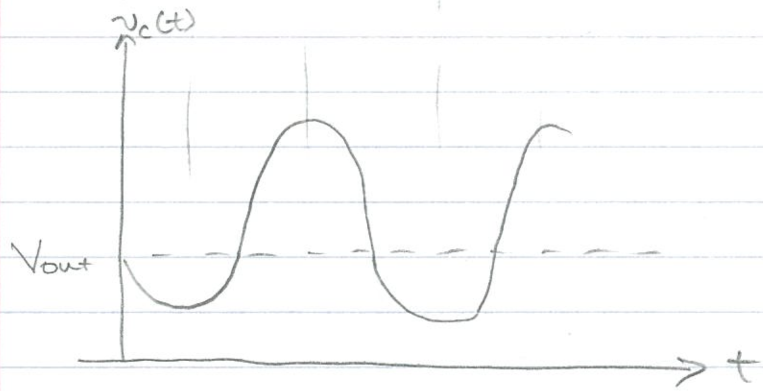


$i_L = I_{out} + i_c$





Recall,

$v_c = \frac{1}{c}\int i_o \, dt$

$$\Delta v_c = \frac{2}{c}\int_0^{\frac{T_s}{4}} \left(\frac{\Delta i_L}{2}\cdot\frac{4}{T_s}\right) t \, dt$$

$$\Delta v_c = \frac{\cancel{2}}{c}\cdot\frac{\Delta i_L}{\cancel{2}}\frac{4}{T_s}\frac{1}{\cancel{2}}t^2 \Big|_0^{\frac{T_s}{4}}$$

$$= \frac{2\Delta i_L}{CT_s}\left[\frac{T_s^2}{16} - 0\right]$$

$$= \frac{T_s}{8C}\left(\frac{V_{out}(1-D)}{Lf_s}\right)$$ (cont.)

(cont.)

$$\boxed{\Delta V_c = \frac{V_{out}(1-D)}{8LCf_s^2}} \quad \boxed{\Delta i_L = \frac{V_{in}(D-D^2)}{Lf_s}}$$

V_{out} V_{in}

(Recall, $T_s = \frac{1}{f_s}$).

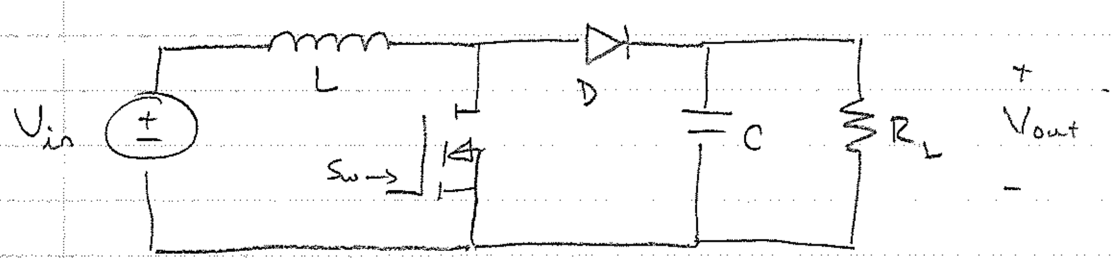
(Can basically treat like a design problem then)

Voltage Ripple of 1% $\longrightarrow 5 * 0.01 = 0.05$

$V_{out} = 5v$, $V_{in} = 10V$

$$V_{ripple} = \frac{\Delta V_c}{V_{out}} = \frac{(1-D)}{8LCf_s^2}$$

$$= \frac{0.05}{5} = 1\% = \frac{(1-0.5)}{8LCf_s^2}$$



Boost Converter

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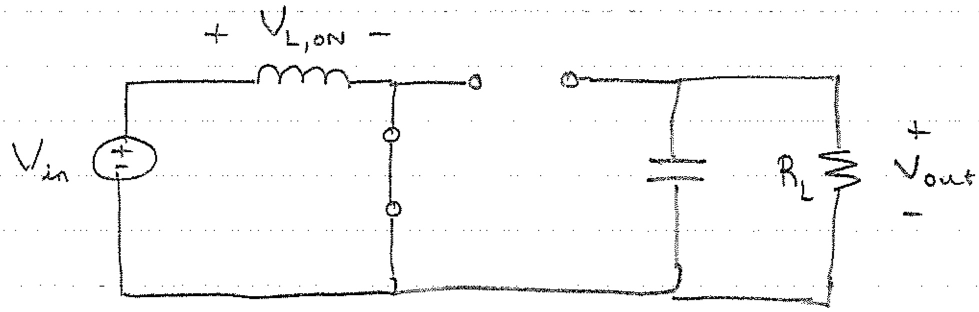
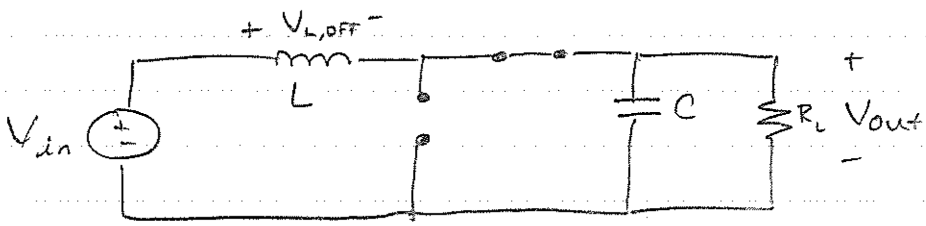


$$V_{L,ON} = V_{IN}$$



$$0V = \frac{1}{T_s}\left[\int_0^{DT_s} V_{in}\,dt + \int_{DT_s}^{T_s} (V_{in} - V_{out})\,dt\right]$$

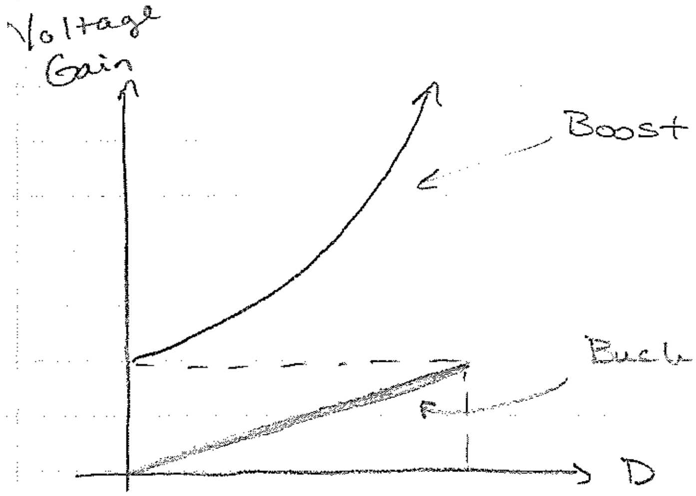
$$0 = V_{in}D + (V_{in} - V_{out})(1-D)$$

$$= \cancel{V_{in}D} + (V_{in} - V_{out}) - \cancel{D(V_{in}} - V_{out})$$

$$V_{out}(1-D) = V_{in}$$

$$\boxed{V_{out} = \frac{1}{1-D} V_{in}}$$ ← Boost Converter

Buck Gain $\rightarrow$ 0 & 1 between b/c goes down
Boost Gain $\rightarrow$ >1 b/c goes up



$$\frac{V_{out}}{V_{in}} = D \quad \text{(Buck)}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{1-D} \quad \text{(Boost)}$$

Rule of Thumb is to not take your Boost Converter beyond 4 to 5 voltage gain.

$$\Delta i_L = \frac{D V_{in}}{L f_s}$$

$$\Delta V_c = \frac{V_{out} D}{R_L C f_s}$$

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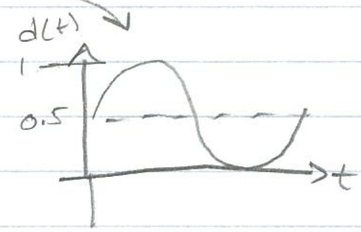
DC/AC Conversion

- $M_i \Rightarrow$ modulation index (magnitude of the AC component)

- Use a Buck & let duty cycle vary sinusoidally

$$d(t) = \left(M_i \sin(\omega t) + 1\right)\frac{1}{2}$$

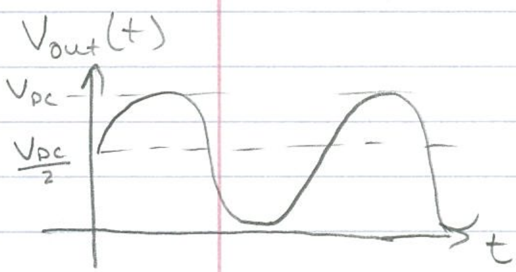


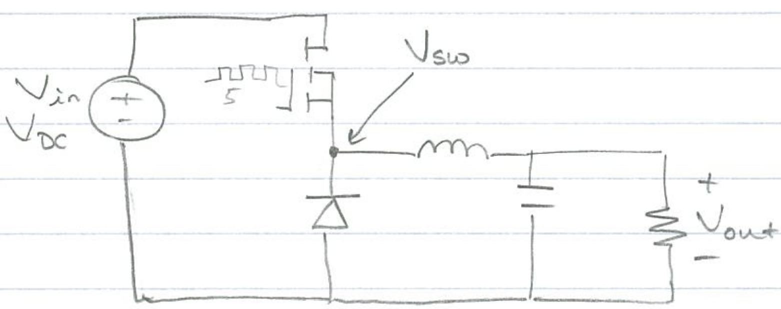
$$V_{out}(t) = d(t)\, V_{in}$$

$$V_{out}(t) = \left(1 + M_i \sin(\omega t)\right)\frac{V_{in}}{2}$$

→ If the magnitude of the AC component is zero, then the M_i is also zero.

$$0 \leq M_i \leq 1$$



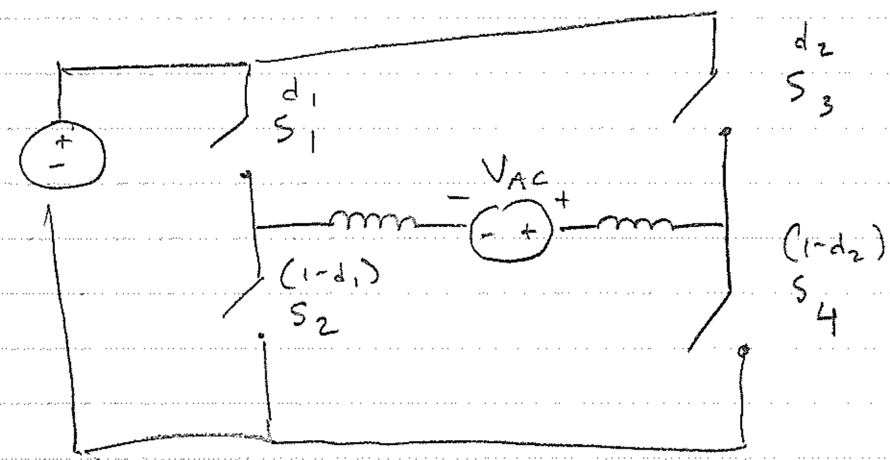


"Your output voltage has a DC bias on it."

The duty cycle varying sinusoidally allows you to essentially turn DC → AC. (nuances to this)

H-Bridge:

We can use another Buck converter to make a differential AC output.



(We could replace our diode with a transistor)
(previous page)

Control S_1 & S_2 and S_3 & S_4 to be opposite, you cancel out the $\frac{1}{2}V_{DC}$ bias.

$$d_1(t) = (M_i \sin(\omega t) + 1)\frac{1}{2}$$

$$V_{o1}(t) = d_1(t) V_{DC}$$

$$d_2(t) = (-M_i \sin(\omega t) + 1)\frac{1}{2}$$

$$V_{o2}(t) = d_2(t) V_{DC}$$

$$V_{AC} = V_{o1} - V_{o2}$$

$$\boxed{V_{AC} = M_i \sin(\omega t) V_{DC}}$$

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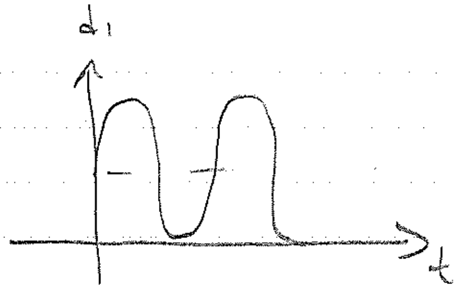
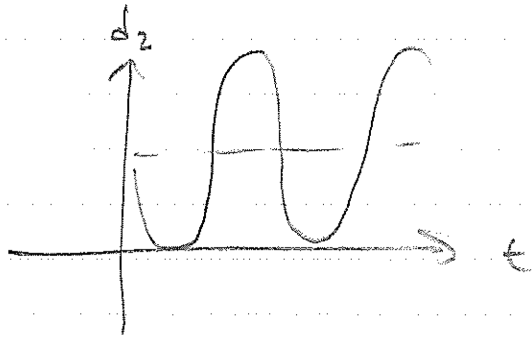
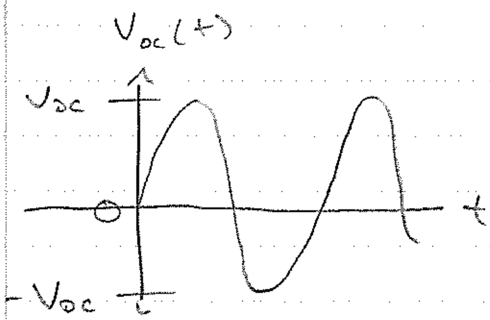




With this H-Bridge, you can generate twice the amount of voltage magnitude. Tradeoff is twice as many switching components (and gate signals).



This is essentially how we're generating the AC voltages.

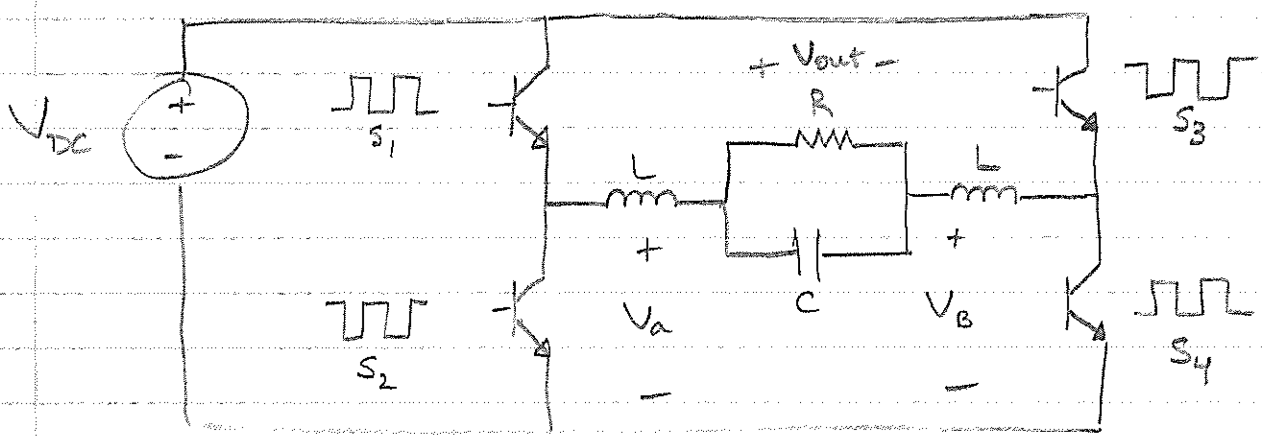
→ Also, DC offset goes away, so now centered at 0.

Eddie
Sokolowski

EE 499 - Week 3 Friday

7/29/2014

We left off talking about an H-Bridge circuit.



Replace lower ~~switch~~ diode w/ switch and give complementary gate signal.

$$V_a(t) = \frac{V_{DC}}{2}\left(1 + M_i \sin(\omega t)\right)$$

$$V_b(t) = \frac{V_{DC}}{2}\left(1 - M_i \sin(\omega t)\right)$$

$$V_{out}(t) = V_a(t) - V_b(t)$$

$$= \frac{V_{DC}}{2}\left[1 + M_i \sin(\omega t) - 1 + M_i \sin(\omega t)\right]$$

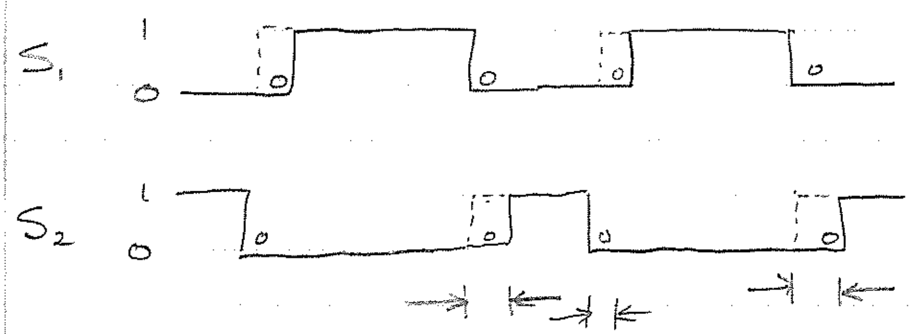
$$V_{out}(t) = V_{DC} M_i \sin(\omega t)$$

DC/AC converter → inverter (different from —▷o—)

Note: Talk to Kaiser after class

Page 2

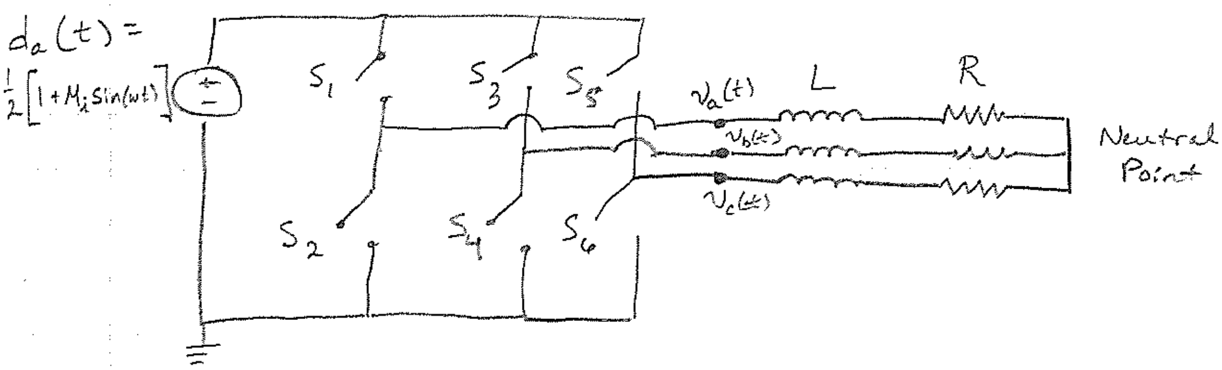
We need to insert time delays in the control signals of a complementary pair of switches.



This is called "deadtime" / "deadband"

We must allow time for the previously ON switch to shut down before turning on the next one.

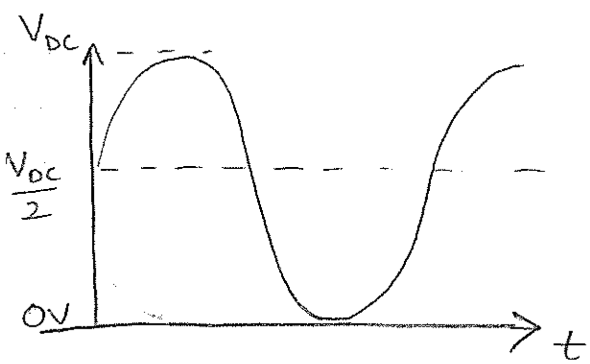
The Three-Phase Inverter



$$V_a(t) = \frac{V_{DC}}{2}\left[1 + M_i \sin(\omega t)\right]$$

$$V_b(t) = \frac{V_{DC}}{2}\left[1 + M_i \sin\left(\omega t - \frac{2\pi}{3}\right)\right]$$

$$V_c(t) = \frac{V_{DC}}{2}\left[1 + M_i \sin\left(\omega t + \frac{2\pi}{3}\right)\right]$$



(Basically graph that I can't draw of three phase)

If we apply KVL, we can find the neutral voltage is at $\frac{V_{DC}}{2}$ for a balanced system.

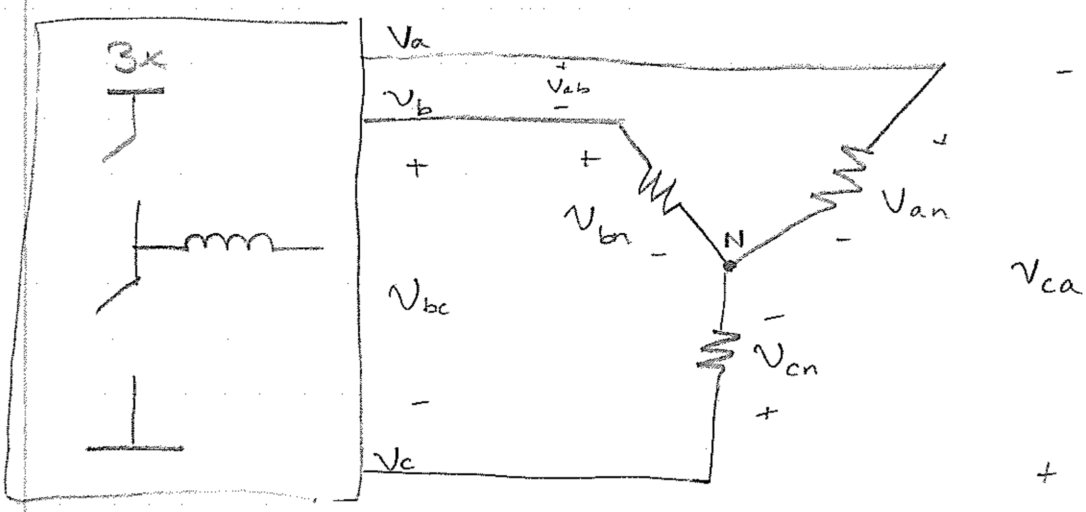
Page 4

Line to Neutral

$$V_{an}(t) = \frac{V_{DC}}{2}\left[M_i \sin(\omega t)\right]$$

$$V_{bn}(t) = \frac{V_{DC}}{2}\left[M_i \sin\left(\omega t - \frac{2\pi}{3}\right)\right]$$

$$V_{cn}(t) = \frac{V_{DC}}{2}\left[M_i \sin\left(\omega t + \frac{2\pi}{3}\right)\right]$$

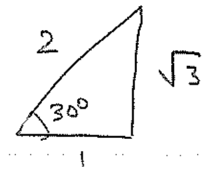


- Line-to-GND → V_a (has the $\frac{1}{2}V_{DC}$ in it)
- Line-to-Neutral → V_{an}
- Line-to-Line → $V_{ab} = V_a - V_b$

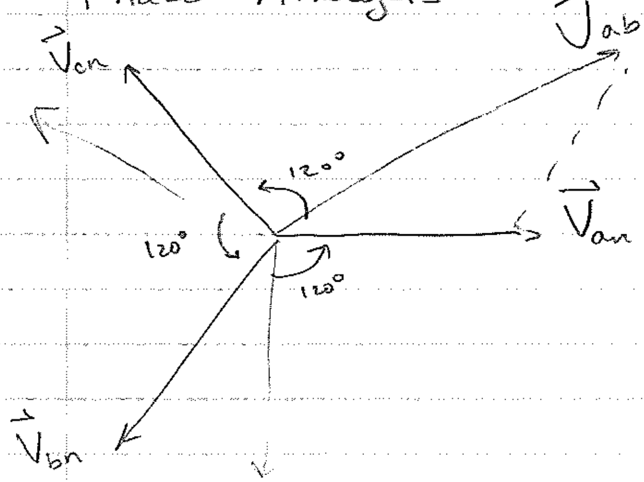
also called phase voltage

$$(1+j0)-\left(-\tfrac{1}{2}+j\tfrac{\sqrt{3}}{2}\right)$$

$$1\angle 0^\circ - 1\angle -120^\circ = \sqrt{3}\angle 30^\circ$$



Phasor Analysis



Magnitude difference b/w line-line and line-neutral is $\sqrt{3}$.

Power in a 3ϕ system

$$P = 3V_{\phi RMS}\, I_{\phi RMS}\ pf \qquad (\text{where } pf = \cos(\theta_v-\theta_i))$$

$$= \left[V_{an,RMS} * I_{aRMS} + V_{bn,RMS} * I_{bRMS} + \cdots\right]$$

$$P = \sqrt{3}\ V_{ll} * I_{\phi RMS}$$

Page 6

Sinusoidal PWM

Max AC Voltage for SPWM

$M_i = 1$

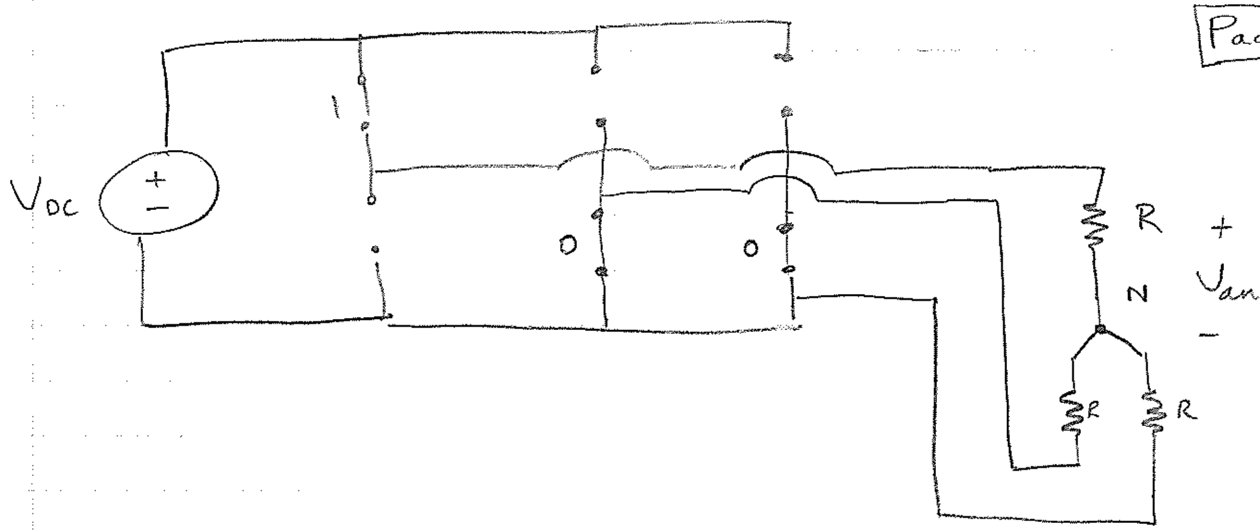
$|V_{an}| = \frac{V_{DC}}{2}$ } Maximum we got as sine PWM.

Let's look at voltages as space vectors in complex plane...

Let's examine the switching state "001" $= V_1$

We have 8 possible switching states

$S_{5,6}$	$S_{3,4}$	$S_{1,2}$	
0	0	0	V_0
0	0	1	V_1
0	1	0	V_2
0	1	1	V_3
1	0	0	V_4
1	0	1	V_5
1	1	0	V_6
1	1	1	V_7



IF resistors are balanced...
the phase A voltage will be

$$V_{an} = \frac{R}{R + \frac{1}{2}R} V_{DC} = \frac{2}{3} V_{DC}$$

parallel combination

(Break)

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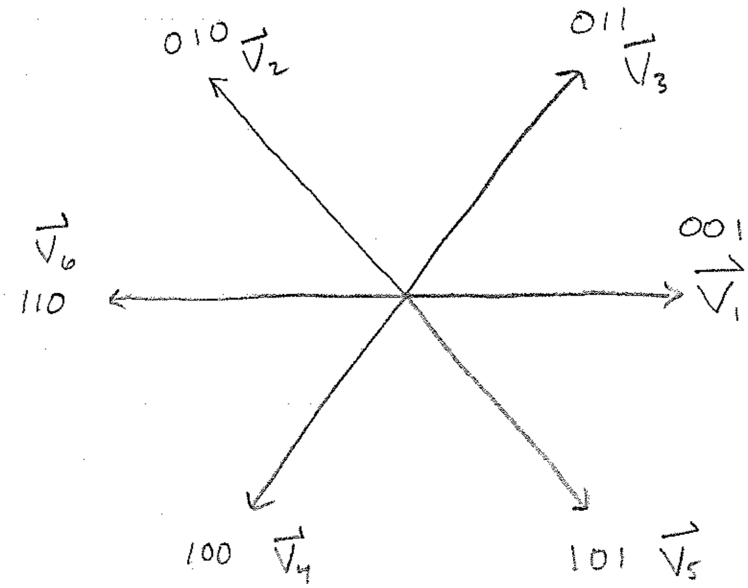
Space Vector Voltage Diagram

$$|\vec{V}_1| = \frac{2}{3}V_{DC}$$

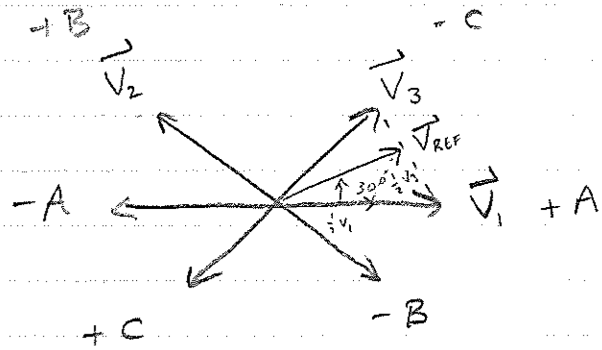
$$\angle V_1 = 0^\circ$$



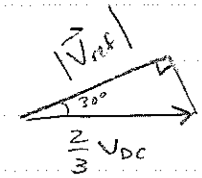
The idea with grey code is that it's a different way to count so you only flip one bit on each transition.

Binary	Grey Code	Grey Code
00	00	000
01	01	001
10	11	011
11	10	010
		110
		111 ?
		101
		100

Magnitude of $\frac{2}{3} V_{DC}$
(angles are multiples of $60°$)



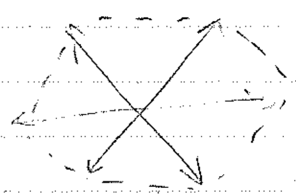
$$\vec{V}_{REF} = \frac{1}{2}\vec{V}_1 + \frac{1}{2}\vec{V}_3$$



$$|\vec{V}_{REF}| = \cos(30)\,\frac{2}{3}V_{DC}$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{2}{3} V_{DC}$$

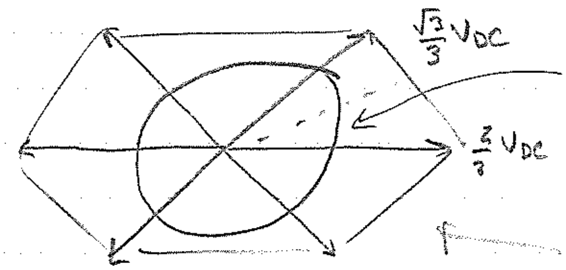
$$= \frac{\sqrt{3}}{3} V_{DC}$$



The biggest 3ϕ voltage we can generate is limited by the hexagon.

"If you increase modulation index, you get overmodulation."

Page 10



Playing with the switching, we can get out of the SPWM circle

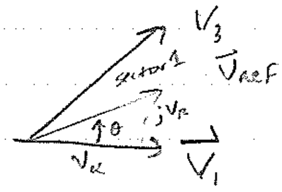
We can't just increase $M_i > 1$

this will cause distortion

$$\frac{|SV\ PWM|}{|S\ PWM|} = \frac{\frac{\sqrt{3}}{3}V_{DC}}{\frac{1}{2}V_{DC}} = \frac{\sqrt{3}\cdot 2}{3} = \frac{2}{\sqrt{3}} \cong 1,155$$

SV PWM can make 15% larger voltages than SPWM!

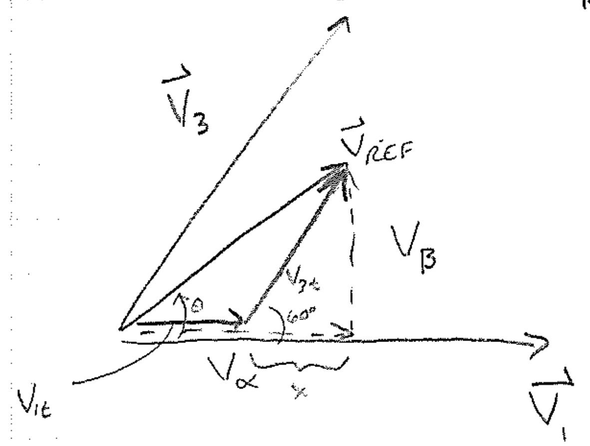
How to calculate SVPWM duty ratios?



$\vec{V}_{REF}$ is the voltage I want from PI controllers.

$$\vec{V}_{REF} = V_\alpha + jV_\beta$$

We want to construct $\vec{V}_{REF}$ from $\vec{V}_1$ and $\vec{V}_3$



Basically want to find magnitudes of $\vec{V}_1$ and $\vec{V}_3$ that will end up generating $\vec{V}_{REF}$

$$\sin(60°) = \frac{V_\beta}{V_{3t}}$$

$$\boxed{V_{3t} = \frac{V_\beta}{\sqrt{3}/2}}$$

$$\tan(60°) = \frac{V_\beta}{x}$$

$$x = \frac{V_\beta}{\sqrt{3}}$$

$$V_\alpha = V_{1t} + x$$

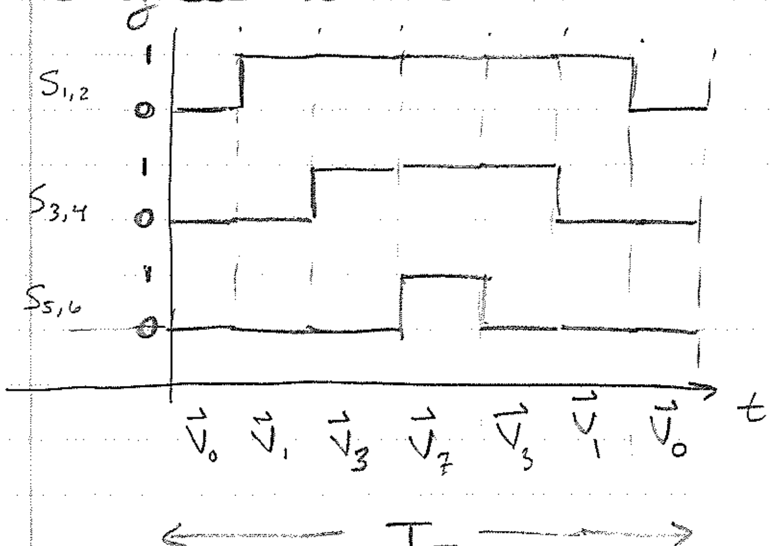
$$\boxed{V_{1t} = V_\alpha - \frac{V_\beta}{\sqrt{3}}}$$

Page 12

$$T_1 = \frac{V_1 t}{\frac{2}{3} V_{DC}} \qquad T_2 = \frac{V_2 t}{2/3\ V_{DC}}$$

$$= \frac{3V_\alpha - \sqrt{3} V_\beta}{2 V_{DC}} \qquad = \frac{\sqrt{3}\, V_\beta}{V_{DC}}$$

Grey code is useful b/c it minimizes switching loss.



$$D_a = \frac{T_1 + T_2 + T_{0/7}}{T_{sw}}$$

Half the time @ "111"

$$T_{0/7} = \frac{1}{2}(1 - T_1 - T_2)$$

$$D_b = \frac{T_2 + T_{0/7}}{T_{sw}}$$

$$D_c = \frac{T_{0/7}}{T_{sw}}$$

Basically, want to center align these pulses.

Eddie Schodowski

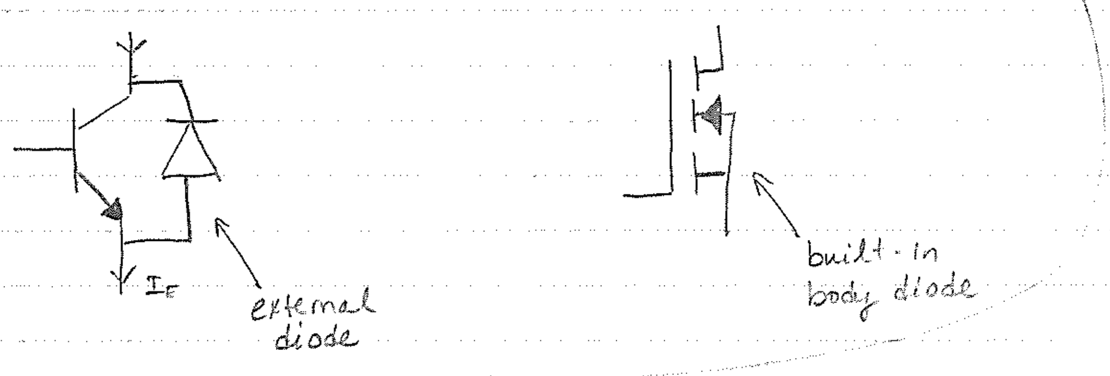
EE 499 - Week 4 Tuesday

8/2/2016

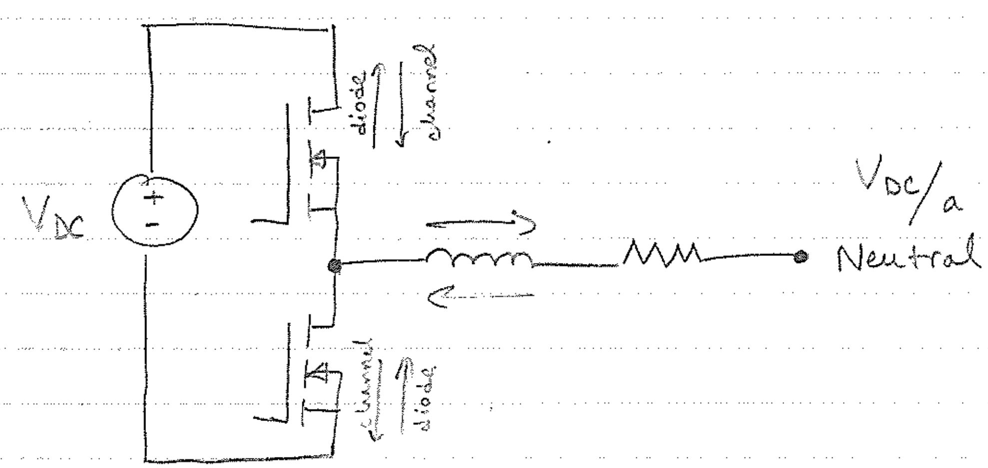
Page 1

(1) Power Losses
(2) Component Selection

Losses on the inverter came in 3 parts
(1) Conduction Loss → Joule heating due to I^2R
(2) Switching Loss → We PWM the transistors; they cannot turn ON/OFF the voltage/current instantaneously
(3) Reverse Recovery Loss → We need anti-parallel diodes



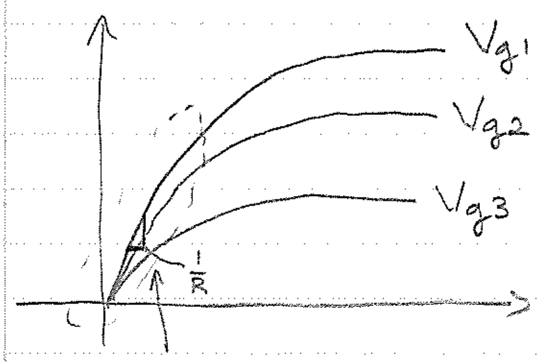
- on each switch... to transition a diode from ON-to-OFF, we must sweep out carriers from the depletion region.

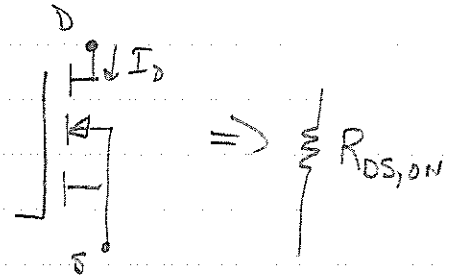


Page 2

Conduction Loss

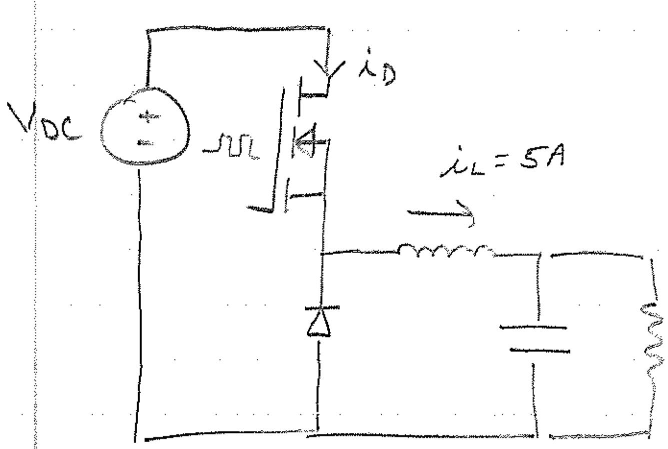
- When the MOSFET is ON, we can model it as simply a resistance

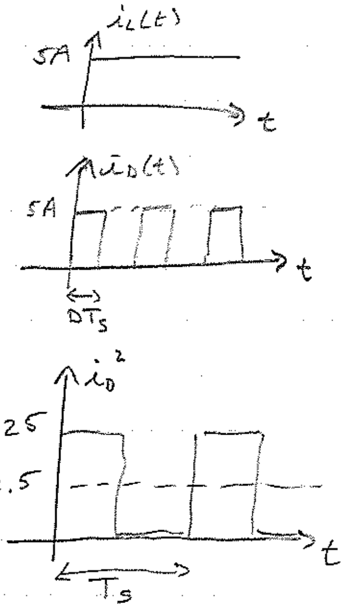




$$P_{con} = I_{D,RMS}^2 \quad R_{DS,ON}$$

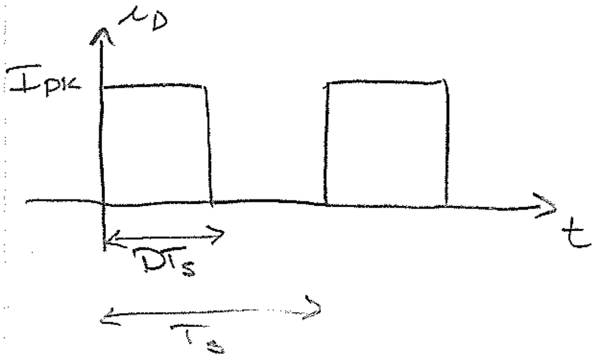
What's the RMS value of a square wave? (Assume $i_L \approx$ constant)





let $D = 0.5$

$$I_{D,RMS} = \sqrt{12.5}\ A \approx 3.53\ A$$



$$I_{D\,rms} = \sqrt{\frac{1}{T_s}\int_0^{T_s} i_D^{\,2}\, dt}$$

$$= \sqrt{\frac{1}{T_s}\left(I_{pk}^{\,2}\, t\right)\Big|_0^{DT_s}}$$

$$= \sqrt{\frac{1}{\cancel{T_s}}\left(I_{pk}^{\,2}\, D\cancel{T_s}\right)}$$

$$= \boxed{I_{pk}\sqrt{D}}$$

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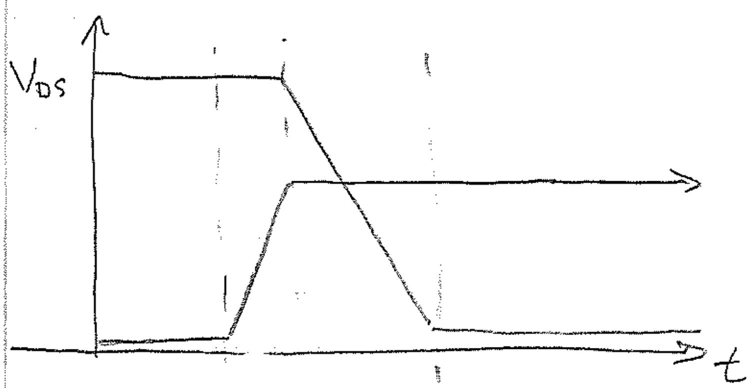
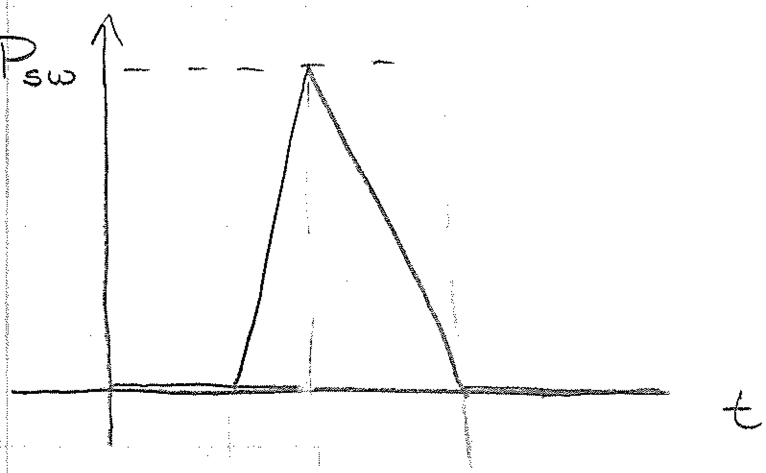
Switching Loss

OFF to ON Transition



$V_{DC} * I_L = P_{SW}$



$E = \int P\,dt$

$E_{ON} = \frac{1}{2} V_{DC} * I_L t_{ON}$

$E_{OFF} = \frac{1}{2} V_{DC} * I_L t_{OFF}$

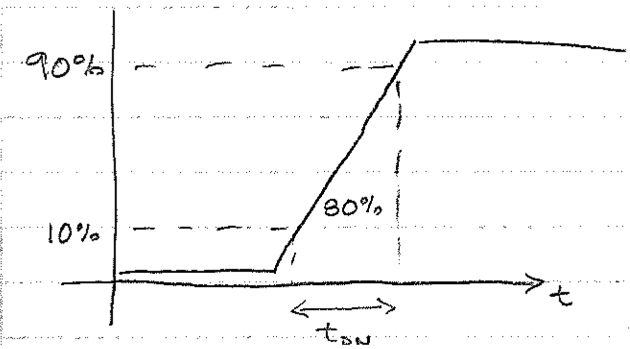
$E_{SW} = \frac{1}{2} V_{DC} * I_L (t_{ON} + t_{OFF}$

What if we're PWMing at $f_{SW} = 10\text{ kHz}$?

$P_{SW} = E_{SW} * f_{SW}$

$P_{SW} = \frac{1}{2} V_{DC} I_L (t_{ON} + t_{OFF}) f_{SW}$

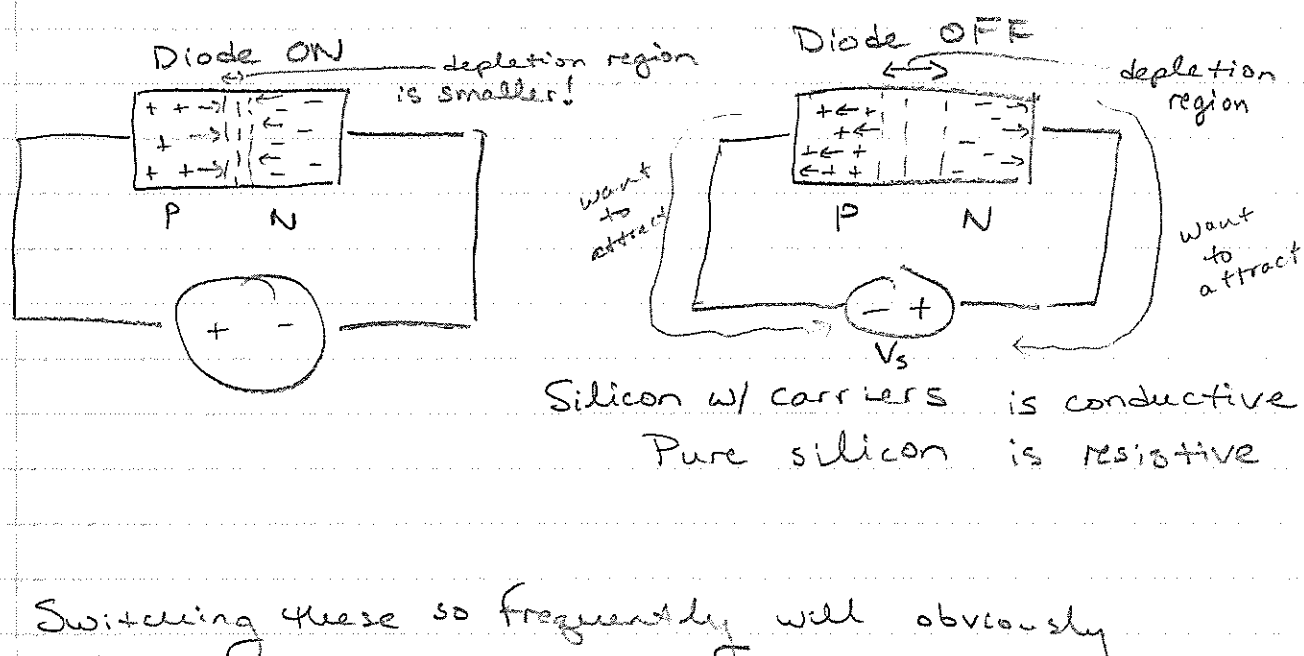
Most spec sheets list t_{ON} and t_{OFF} as the 10% to 90% time. We must account for this.



$$\frac{1}{0.8} = \frac{1}{4/5} = \frac{5}{4} = 1.25$$

$$P_{SW} = \frac{1}{2} V_{DC} I_L (t_{ON} + t_{OFF}) f_{SW} * 1.25$$

Reverse Recovery Loss



Silicon w/ carriers is conductive
Pure silicon is resistive

Switching these so frequently will obviously have implications.

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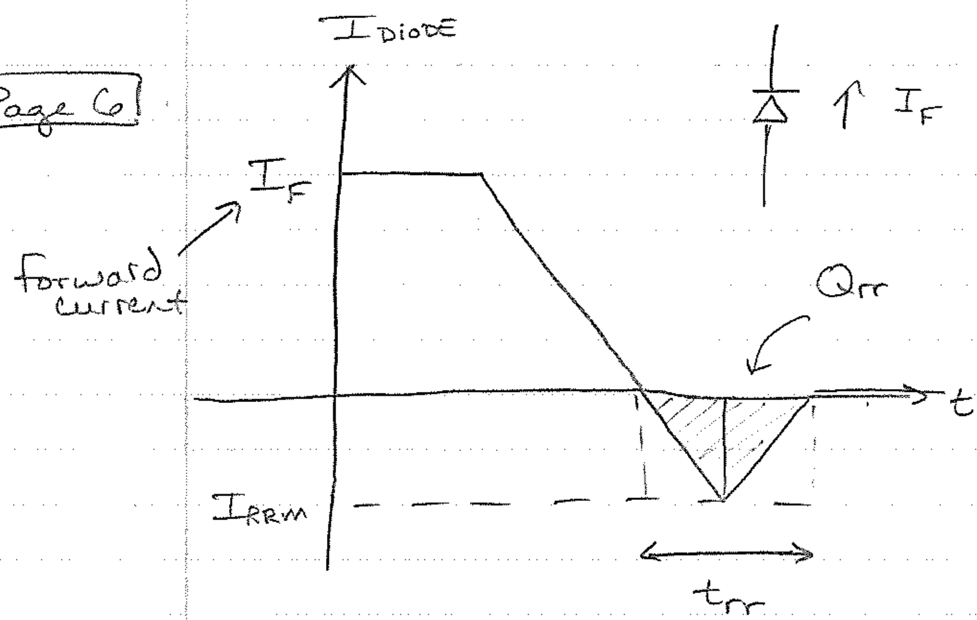


$Q_{rr} = \frac{1}{2} I_{rrm} t_{rr}$

$\text{Volts} = \frac{\text{Joule}}{\text{Coulomb}}$

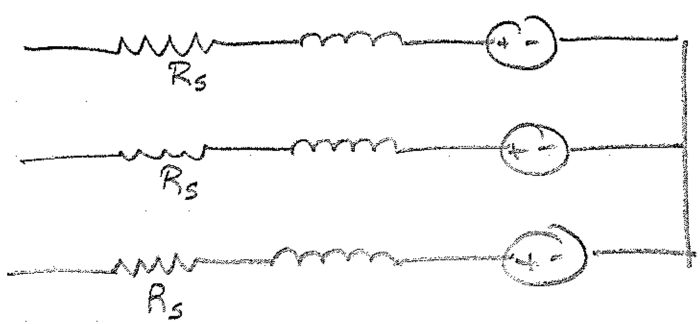
$E_{rr} = V_{DC} * Q_{rr}$

$P_{rr} = V_{DC} * Q_{rr} * f_{sw}$

(break)

- A good design will balance the loss between P_{con} and P_{sw}
- Try to keep P_{rr} as small as possible

Other losses to consider
→ machine losses

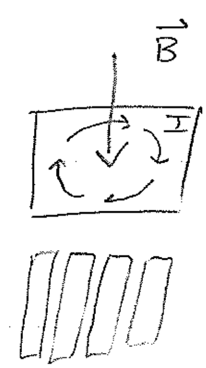


$$P_{cu} = 3 I_{AC}^2 R_s$$

→ Machine Iron Losses

P_{fe} = ??? (Can look this up, very complex and probably even books written on this stuff.)

"When you build a motor, you build it out of very thin sheets of metal."

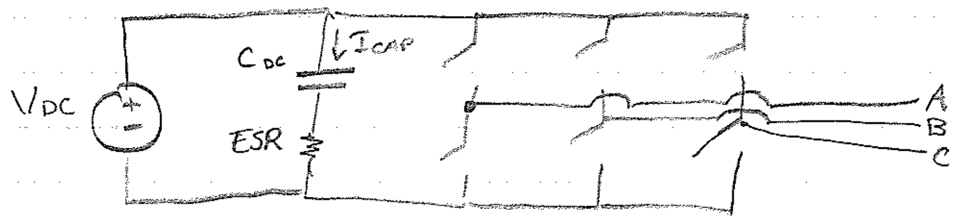


(Higher resistance.

Page 8

A well-designed machine
the P_{cu} and P_{fe} are same

→ Capacitor Loss



Capacitors have stray resistance
Equivalent Series Resistance (ESR)

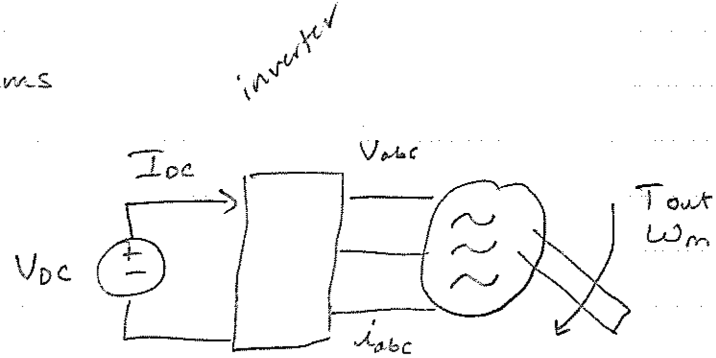
Scooter Design

Motor Specs

$V_{DC} = 48\,V$ ← (not always even clear what voltage this is)
$P_{mech} = 2000\,W$
$3\,\phi$ motor

$P_{in} = V_{DC} * I_{DC}$
$P_{AC} = 3V_{\phi rms} * I_{\phi rms}$

$P_{MECH} = T_{out} * \omega_m$



$$\eta_{inv} = \frac{P_{AC}}{P_{DC}} = \frac{P_{AC}}{P_{AC} + P_{con} + P_{sw} + P_{rr} + P_{cap}}$$

$$\eta_{mot} = \frac{P_{MECH}}{P_{AC}} = \frac{P_{MECH}}{P_{MECH} + P_{Cu} + P_{Fe} + P_{friction}}$$

$\eta_{inv} \approx 95\%$

$\eta_{mot} \approx 80\text{-}90\%$

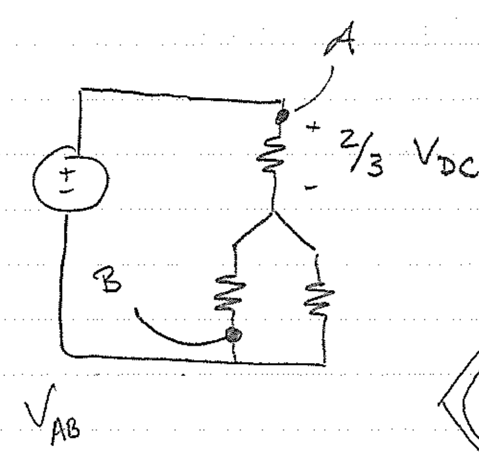
"1st thing I want to do is assume everything is at maximum power

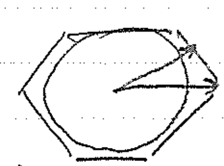
$V_{DC} = 48V$

$V_{LL\,PK} = V_{DC}$

$V_{LL\,rms} = \dfrac{V_{DC}}{\sqrt{2}}$

$$\boxed{V_{\phi\,rms} = \frac{V_{DC}}{\sqrt{2}\sqrt{3}}}$$





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"I'm trying to figure out the current."

$P_{AC} = 3 * V_\phi * I_\phi$ $\qquad$ $P_{AC} = \sqrt{3}\, V_{LL}\, I_\phi$

$P_{MECH} = 2000\ W$

$\eta_{mot} = 90\%$

$P_{AC} = \frac{2000}{0.9} \approx 2{,}200\ W$

$P_{AC} = 3\, V_\phi\, I_\phi$

$I_\phi = \frac{P_{AC}}{3\, V_\phi} = \frac{2200}{3 \times 19.6V} = \boxed{37.4\ A_{(RMS)}}$

$i_a(t) = \sqrt{2}\ 37.4 \sin(\omega t)\ A$

$P_{AC} = 2200\ W$

$\eta_{inv} = 95\%$

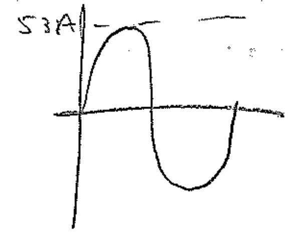
$P_{DC} = \frac{2200}{0.95} = 2{,}300\ W$

$P_{DC} = V_{DC} * I_{DC}$

$I_{DC} = \frac{P_{DC}}{V_{DC}} = \frac{2300}{48} = \boxed{47.9\ A}$

DigiKey

$i_a(t) \approx 53\,A_{peak}$!
MOSFET definitely needs to handle that.



Allow 100% tolerance on current $\rightarrow$ 100 A switch!

Allow 50% tolerance on voltage $\rightarrow$ 75 V

"Usually we only use N-channel devices for power electronics. The reason is because of the difference in carriers crossing over."

Only N-channel for power electronics $\longrightarrow$ Majority/minority carriers are different

$\mu_e > \mu_h$

N-type P-type

mobility $\Rightarrow R_{DS\ ON}$

N-channel usually has lower resistance so less loss.

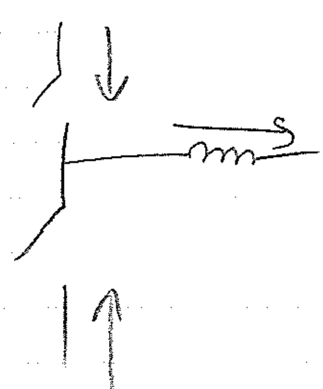
Q: How much money does DigiKey make?

$I_{D\,RMS} = I_L \sqrt{D}$

$P_{con,\,2x} = I_L^2 * R_{DS,ON}$

one phase leg

$P_{con} = 3 * I_{\phi rms} * R_{DS,ON}$



"At worst temperature, how much current can I push?"

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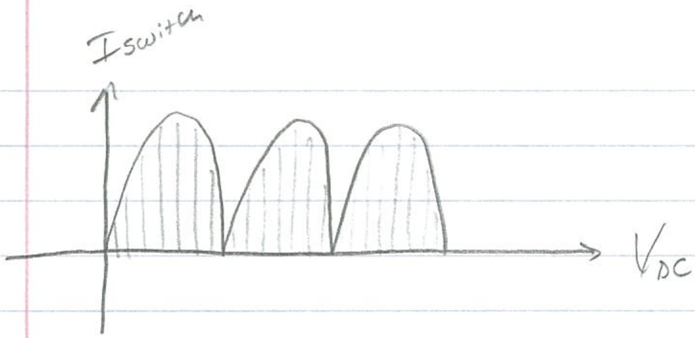
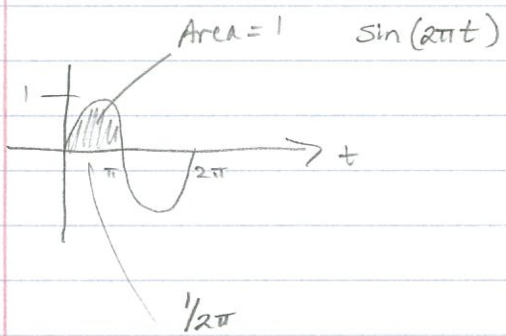
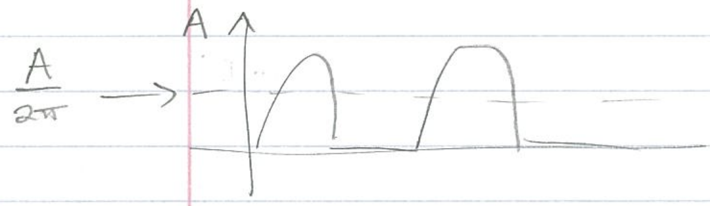
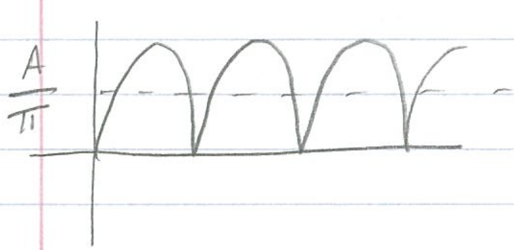








We'll just quickly look at batteries.

Hobbyking.com

$P_{mech} = 2000\ \text{W}$ (just made up)

$V_{el} = 30\ \text{mph}$

$V_{DC,nom} = 44.4\ \text{V} = (3.7 \times 12)$

$Q_{Batt} = 16\ \text{Ah}$

$E_{BATT} = 44.4\ \text{V} \cdot 16\ \text{Ah} = \boxed{710\ \text{Wh}}$

$t_{run} = \dfrac{710\ \text{Wh}}{2300\ \text{W}} = 0.3\ \text{h}$

$X_{run} = 40\ \text{mph} \times 0.3\ \text{h} = \boxed{12\ \text{mi}}$

Eddie
Schodowski

EE 499 - Week 4 Friday

8/5/2016
Page 1

Altium - Schematic Layout
- Tons of little things that make this software really easy to use.
- Just take a look at the user manual
- Can link the parts to what's in stock w/ DigiKey!
    - Generates an Excel Sheet you can upload to DigiKey
- What does VCC stand for?
- Net Class Property
- For some reason, we don't want this transistor close to high voltage
- Designator U1

3D Content Central → create 3D footprint of parts!

Can export a BOM from Altium to Digikey

Tools → Parameter Management

Copper Weighting
  Spec for PCB Manufacturing
    $1 oz\ Cu / in^2$

"Keep-out layer"

$1\ mil = 1/1000^{th}$ inch

GND plane on top and bottom layer
  GND "islands"

Eddie
Schodowski

EE 499 - Week 5 Tuesday

8/9/2016

Page 1

Walking through all the parts on this system
Also will go through good practices for power and stuff

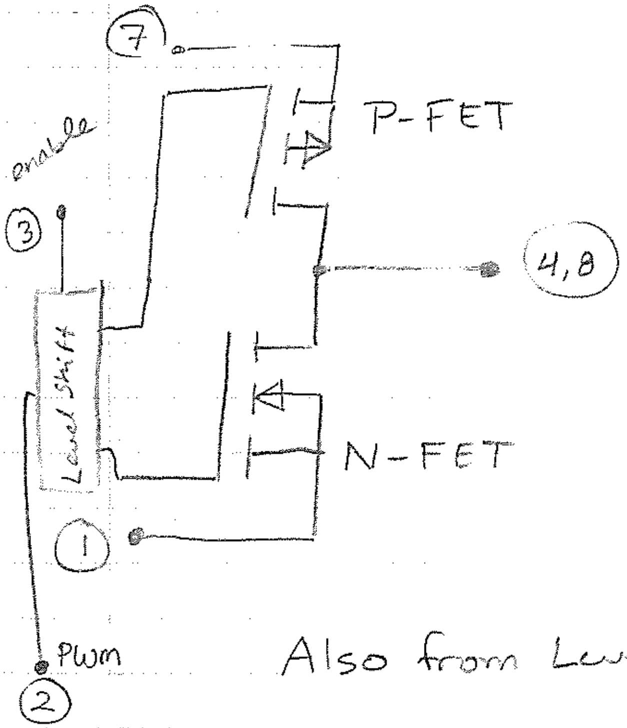
Overview

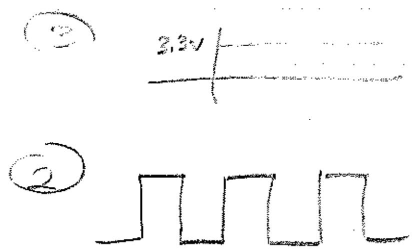
On Board:

sensing
power supplies
inverter circuits

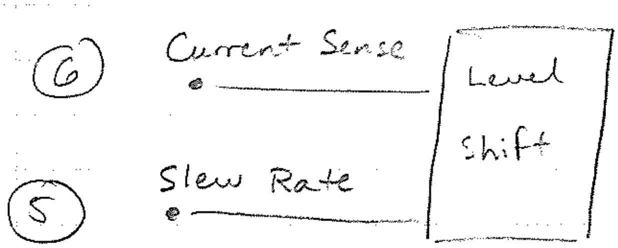
} 3 main parts on the board

"The main reason why he was able to make a credit card sized board is because of the package which is basically (?) a very small H-Bridge."





Also from Level Shift... (ran out of room)



Slew Rate controls how fast gate signals go up and down.

Page 2

DrEN → Driver Enable

They are all enabled with a common signal input.

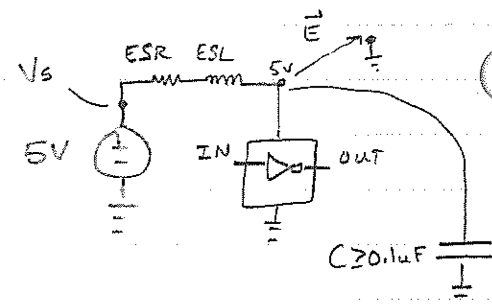
$V_R = I_D R_{sns}$

$1\,m\Omega \leq R_{sns} \leq 1\,\Omega$

for 10's of Amps, $R_{sns} \approx 50\,m\Omega$

The voltage V_R will only be nonzero when the P-FET is on.

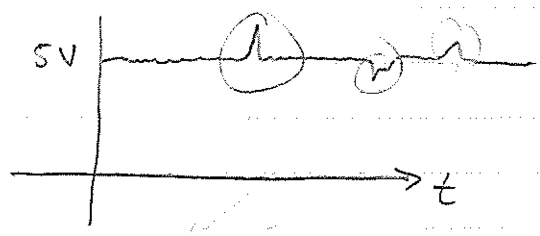
<u>Bypass Capacitors</u>



The reason we use these:

There's an equivalent series resistance, and an equivalent series inductance and also a parasitic capacitance to GND

So, we put this extra bypass capacitor next to the power pin.

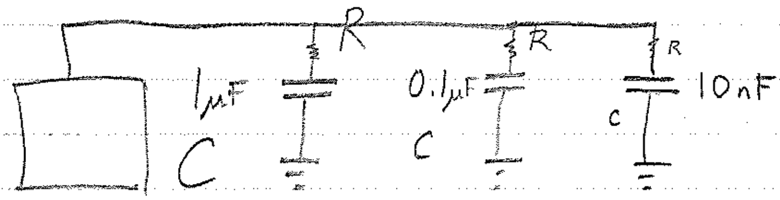


These spikes can screw up a logic chip.

By definition, capacitors want to maintain voltage.

$i = C\frac{dV}{dt}$

If i is small and C is small, the ripples are big.
If i is small and C is big, the ripples are small.



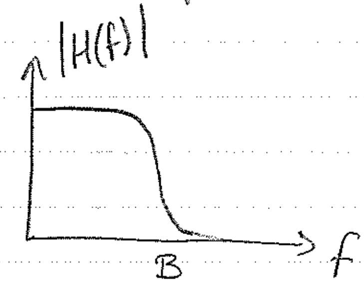
It's common for a really critical component to do that, but maybe not for an IC.
μController more common.

Cannot solder to Al!
(It's a chemistry problem)

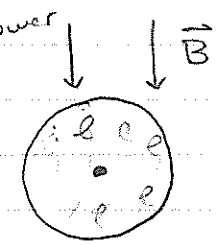
Current Sense Methods

- ~~Sense resistor~~ $V_{SNS} = I_{SNS} R_{SNS}$
  - ↳ High bandwidth (V changes at same rate as I)
  - ↳ Cheap and relatively simple
  - ↳ Common mode voltage is an issue on switching circuits
  - ↳ Added power losses → limits maximum sensing range
  - ↳ control circuit power is low
- Hall Effect Sensor
  - ↳ A current carrying conductor inside a $\vec{B}$ will cause current crowding
  - ↳ lower BW
  - ↳ Higher cost & complexity
  - ↳ consumes more control circuit power

bandwidth
"the upper limit of frequency information"



Lorentz Force
$\vec{F} = \vec{B} \times \vec{I} l$



Potentially you're wasting some copper space

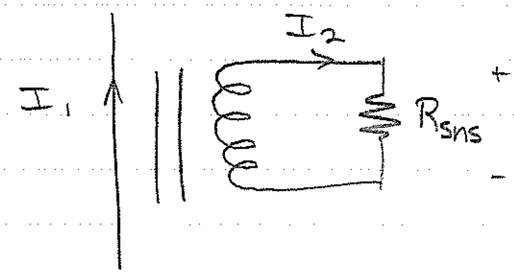
Page 4

Pros to Hall Effect Sensor:

- No level shifting issues (common mode voltage)
- Power losses in conducting path

Magnetically Coupled Current Sense

↳ Basically a transformer with the secondary-side shorted with a sense resistor



$V_{sns} = I_2 R_{sns}$

$V_{sns} = (N I_1) R_{sns}$

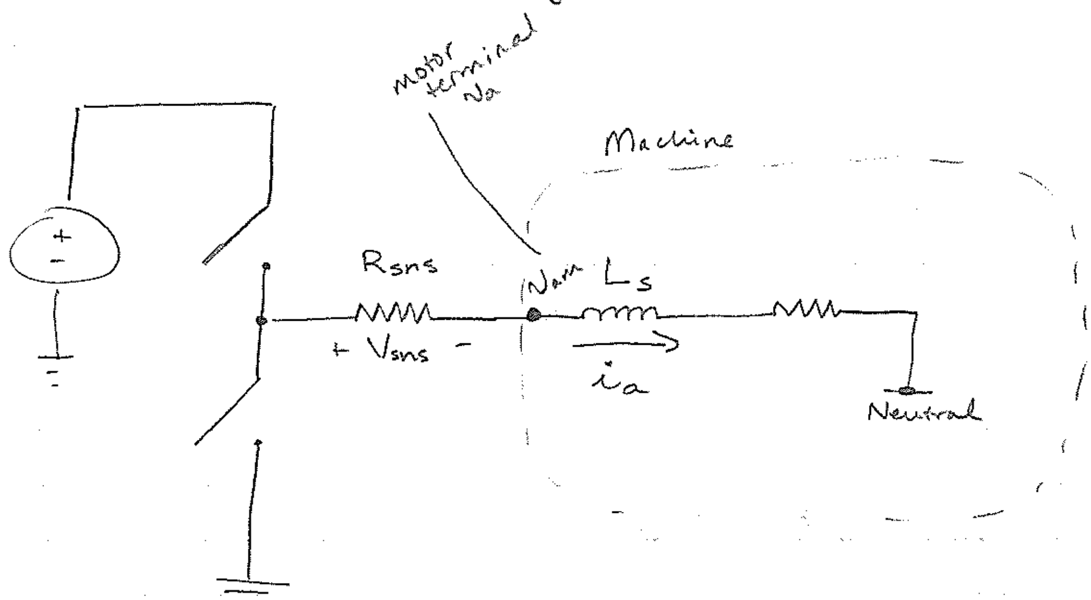
Cannot measure DC current!

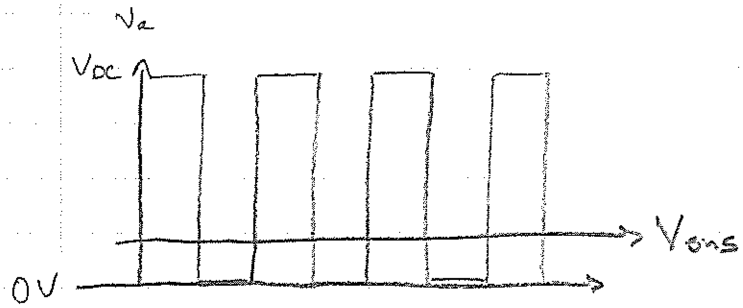
High Bandwidth

Rogowski Coil

<BREAK>

What is common-mode voltage?





Assume i_a is DC

$V_{sns} = I_a R_{sns}$

$V_{sns} = V_a - V_{am}$

V_{am} should be roughly the same as V_a, but a bit different due to the tiny little offset from V_{sns}.

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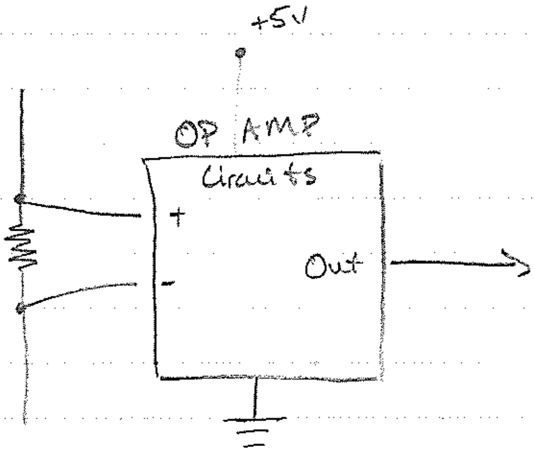


$V_{diff} = V_+ - V_-$

$V_{cm} = \frac{1}{2}(V_+ + V_-)$

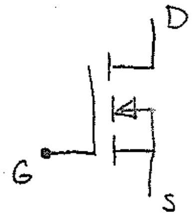
If this is a 24v system, this could mess things up with transistors because 24v would be greater than the collector voltage of 5v.

Common mode is basically the average of the two input pins.

Caps and a zener diode to choke any overvoltage

Voltage sense circuitry has a voltage divider and an op-amp

MOSFET Gate Drive



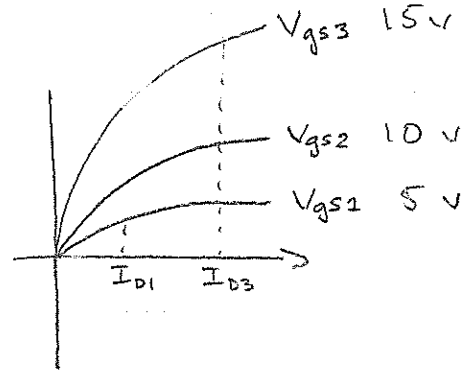
N-FET

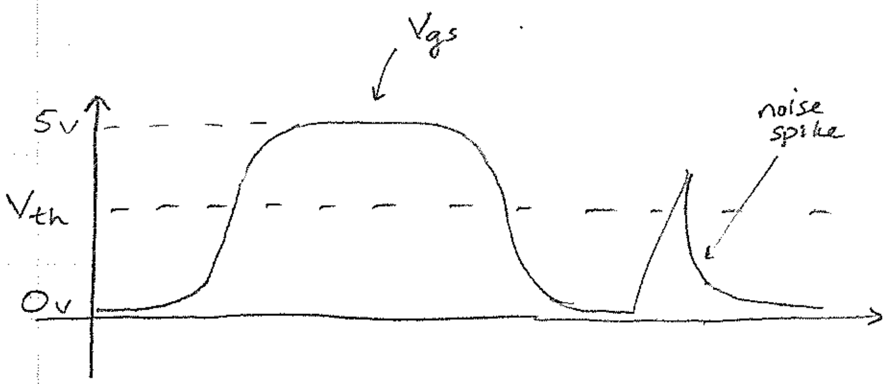
N-channel $V_{th} > 0$ → (~2V to 4V)

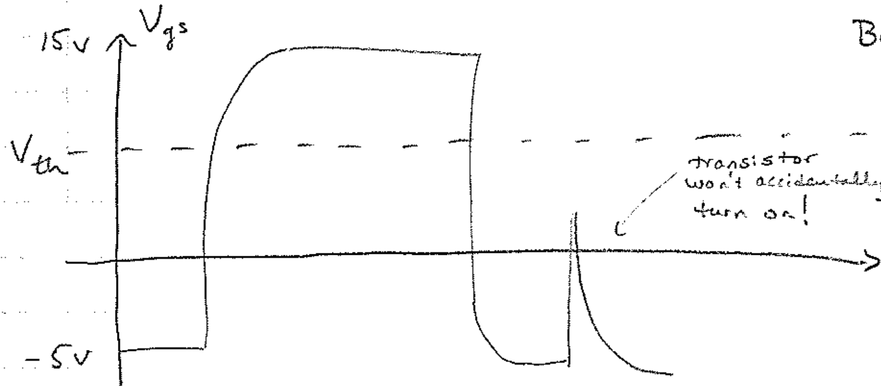
to turn on N-FET, set $V_{gs} = 5V$ OR $V_{gs} = 15V$

We want a bigger V_{gs} because it gives us a lower resistance.
The higher V_{gs} lines are actually a bit sharper.
Helps prevent it from going into the amplifier region.

We use 0V or -5V to turn OFF







Better!

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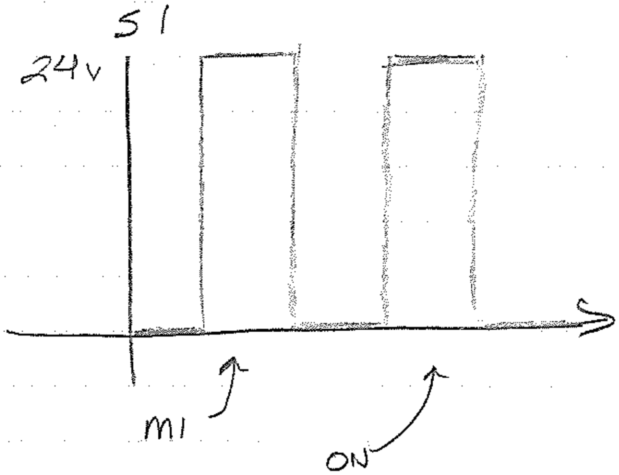
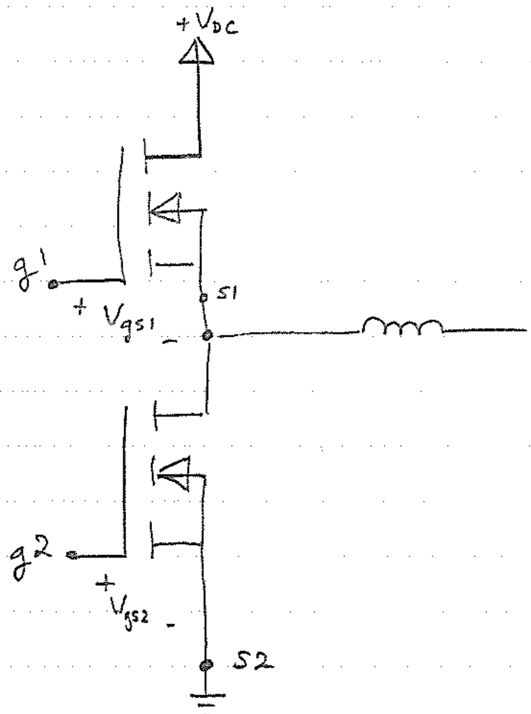
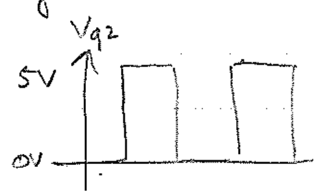
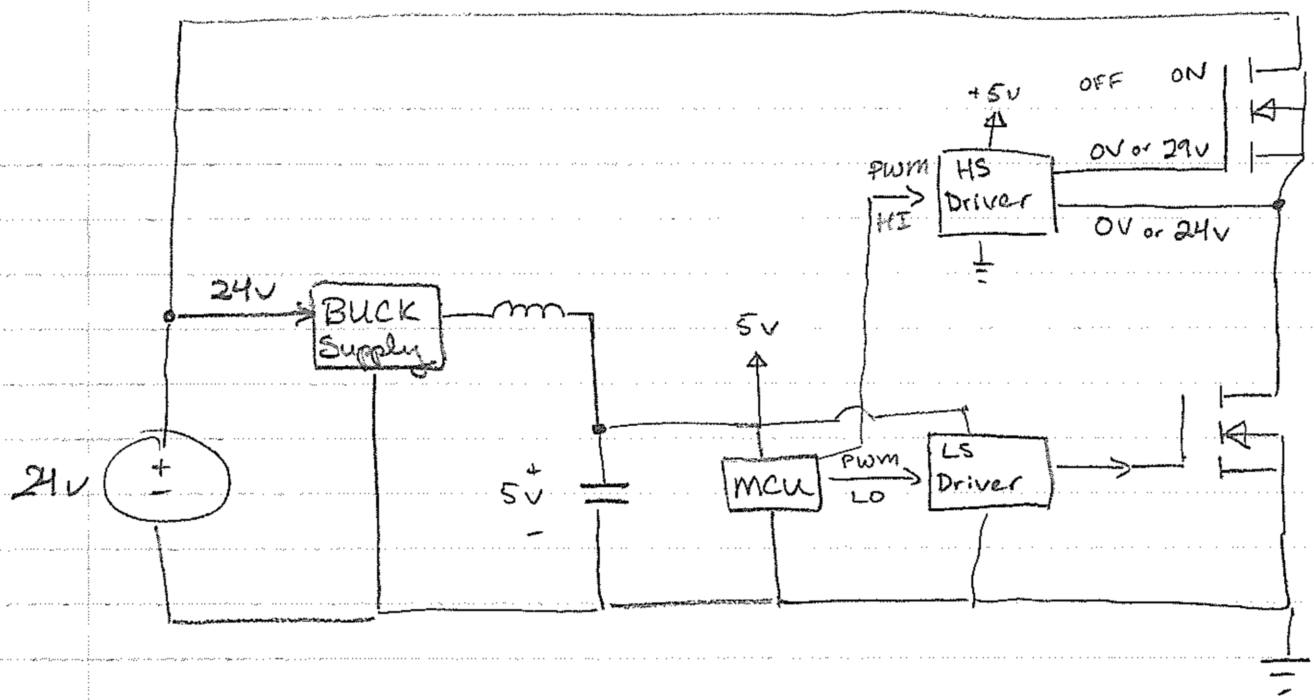




S2 is always ground (0 V)
g2 will be from 0V to 5V

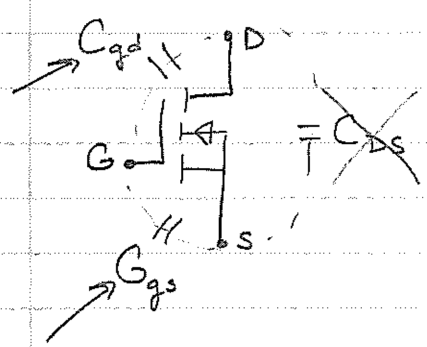




Miller Effect

A source of spikes.

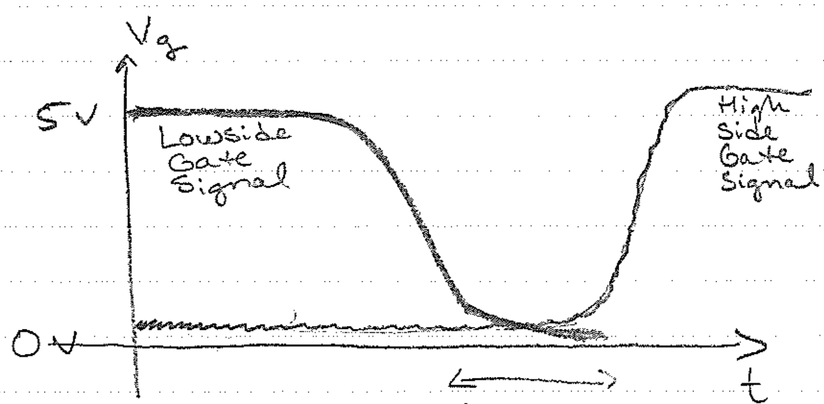
Happens when you have MOSFETS stacked in series.



Lower Switch

$V_s = 0v$

turn off event



not noise
just delineating
different signals
in my notes

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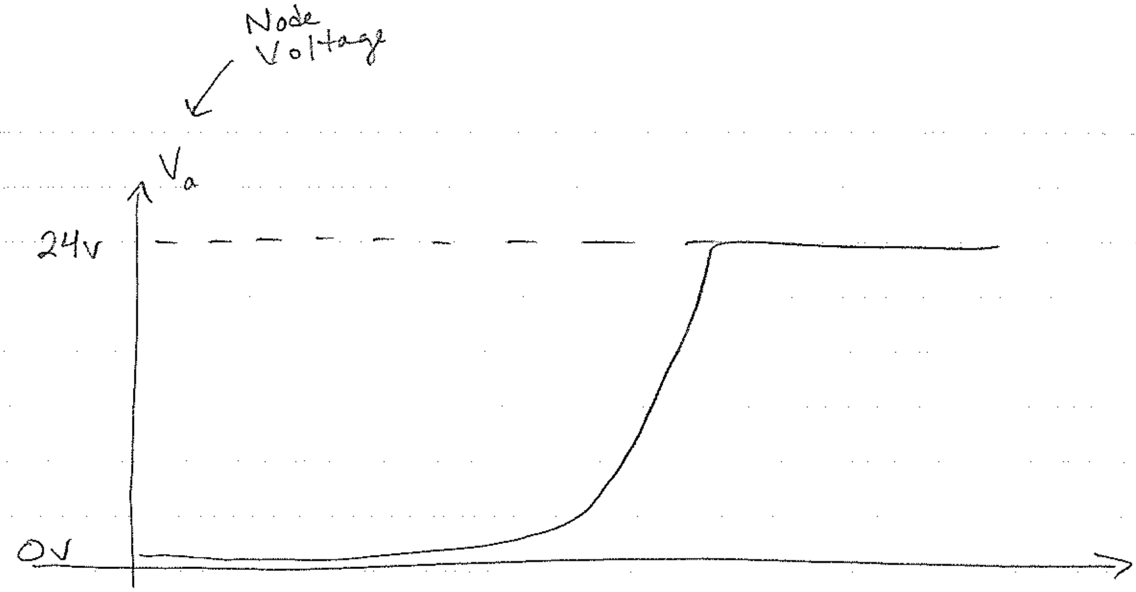
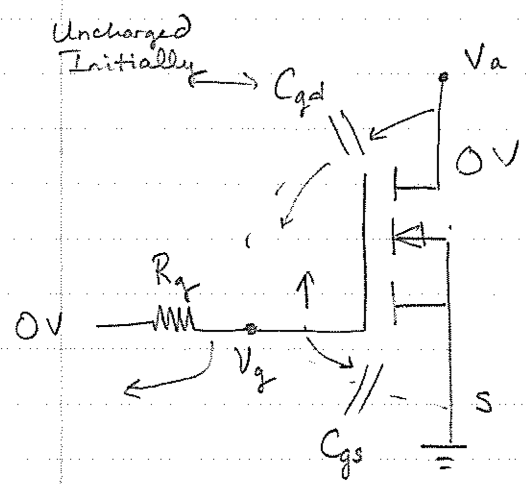


"When the upper switch is on, this V_a is going to fly up to 24v"

$V_a \uparrow$ 24v



C_{gd} ← Miller Capacitance

Eddie Schodowski

EE 499 - Week 5 Friday

8/12/2016

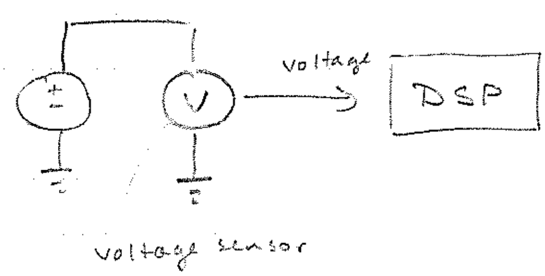
Page 1

http:// 4pcb.com
Student Program

Good to have bottom layer all a GND plane

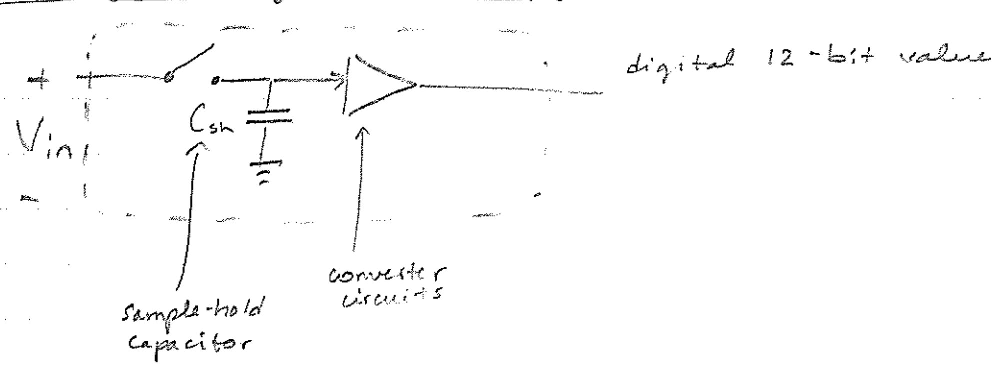
(oops, forgot to mention HKN on my Robots application

RC Filters

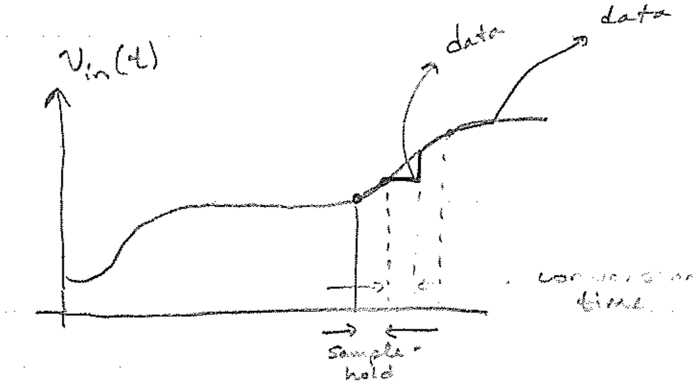


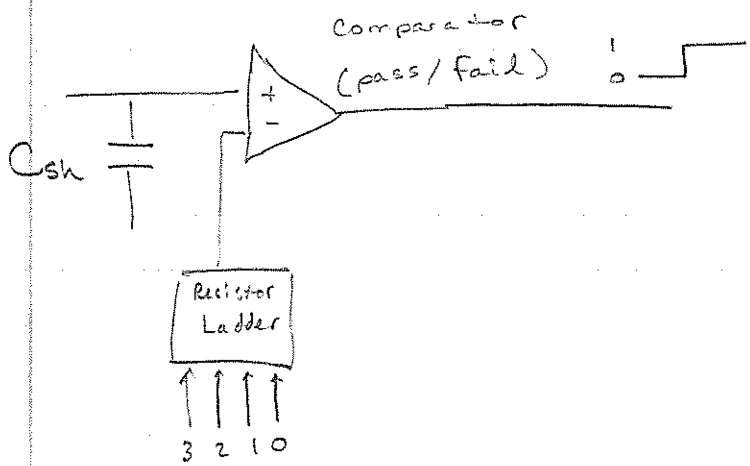
$V_{DC} = 24.37$ V

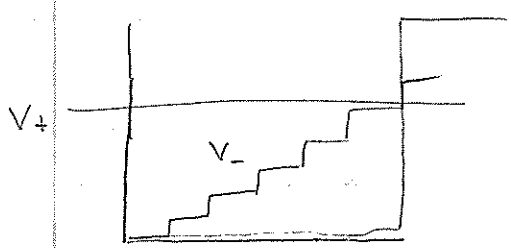
Analog to Digital Converter (A2D)



Page 2

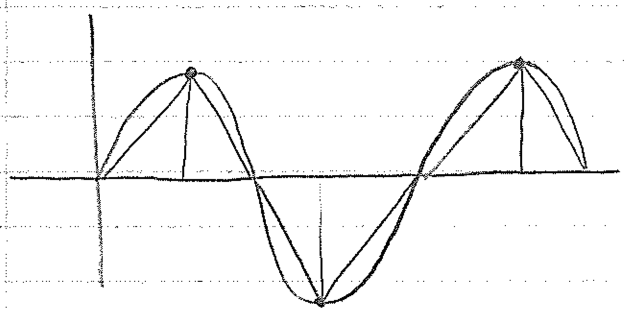






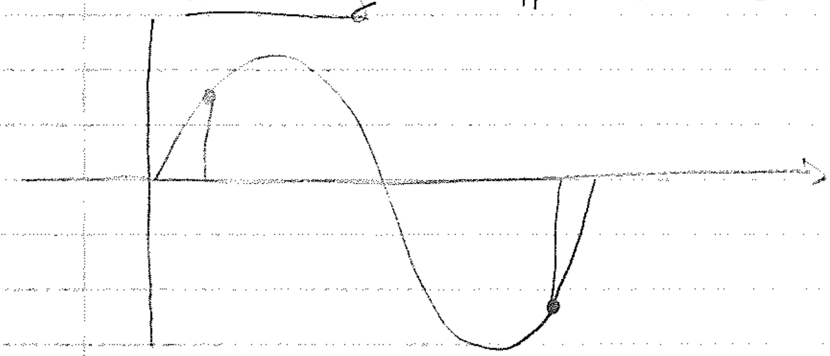
"Why do have an entire corner of my schematic dedicated to RC filters?"

Nyquist - Shannon Sampling Theorem

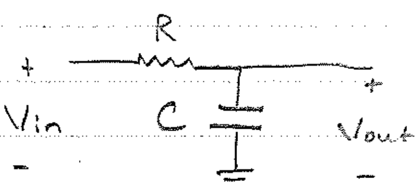


Must sample at 2x the frequency to prevent loss of information

Aliasing → Happens when we sample < 2x the frequency



With aliasing, high frequencies look like low frequencies when you reconstruct them.



$$H(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{sRC + 1}$$

Let $s = j\omega$

$$H(j\omega) = \frac{1}{j\omega RC + 1}$$

That's why you have $\sqrt{2}$ at your break freq.

$$\omega_p = \frac{1}{RC} \qquad f_p = \frac{1}{2\pi RC}$$

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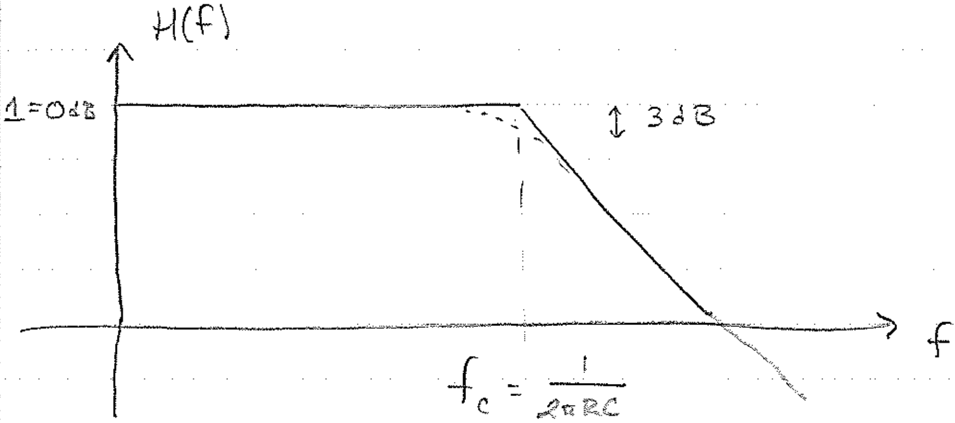
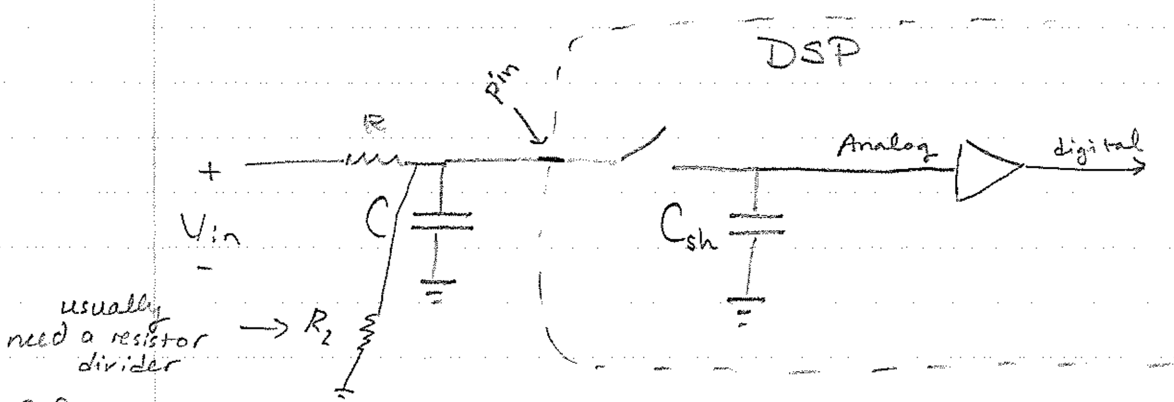


$f_{sample} = 10$ KHz
$f_c = 5$ KHz

pretty heavy here

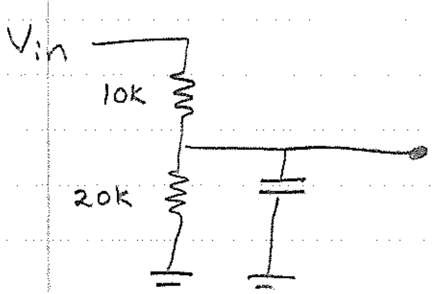
$$5000 = \frac{1}{2\pi R(10nF)}$$

$$C >> C_{sh}$$



$$R = \frac{R_1 R_2}{R_1 + R_2}$$

$$R = \frac{1}{2\pi(5000)(10nF)} = \frac{1}{10\pi(1\times10^{-5})} = \frac{1\times10^{4}}{\pi}$$



$$V_{out} = \frac{20}{20+30} V_{in}$$

typically used by microsoft

ICSP → In-Circuit Serial Programming

JTAG → (?) Boundary scan

→ clock
→ Din (data in)
→ Dout
→ Mode

There are reasons behind getting an external "onboard EEPROM"

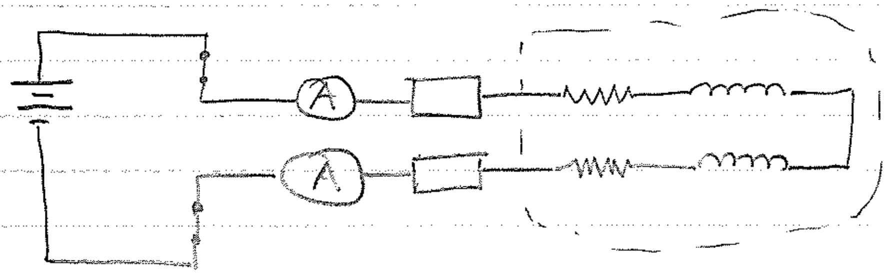
Most microcontrollers are not 5v tolerant.
Basically to conserve energy and clock transistors faster.

What if you want to go Digital to Analog?
Use PWM switch (?)
- Uses an RC circuit and can output a signal to a scope

Eddie
Sokodowski

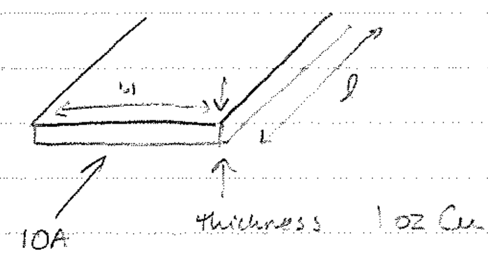
EE 499 - Week 6 Tuesday

8/16/2011



PCB Layout:

(1) Electrically
(2) Thermally
(3) Electromagnetically → (make high current traces as short as possible, minimize coupling near inductors)
(4) Mechanically

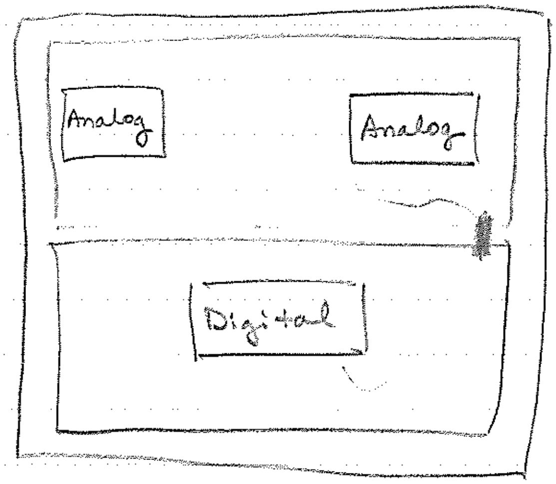


$R = \frac{\rho l}{A} = ?$ Ohms   (ρ → resistivity of copper)

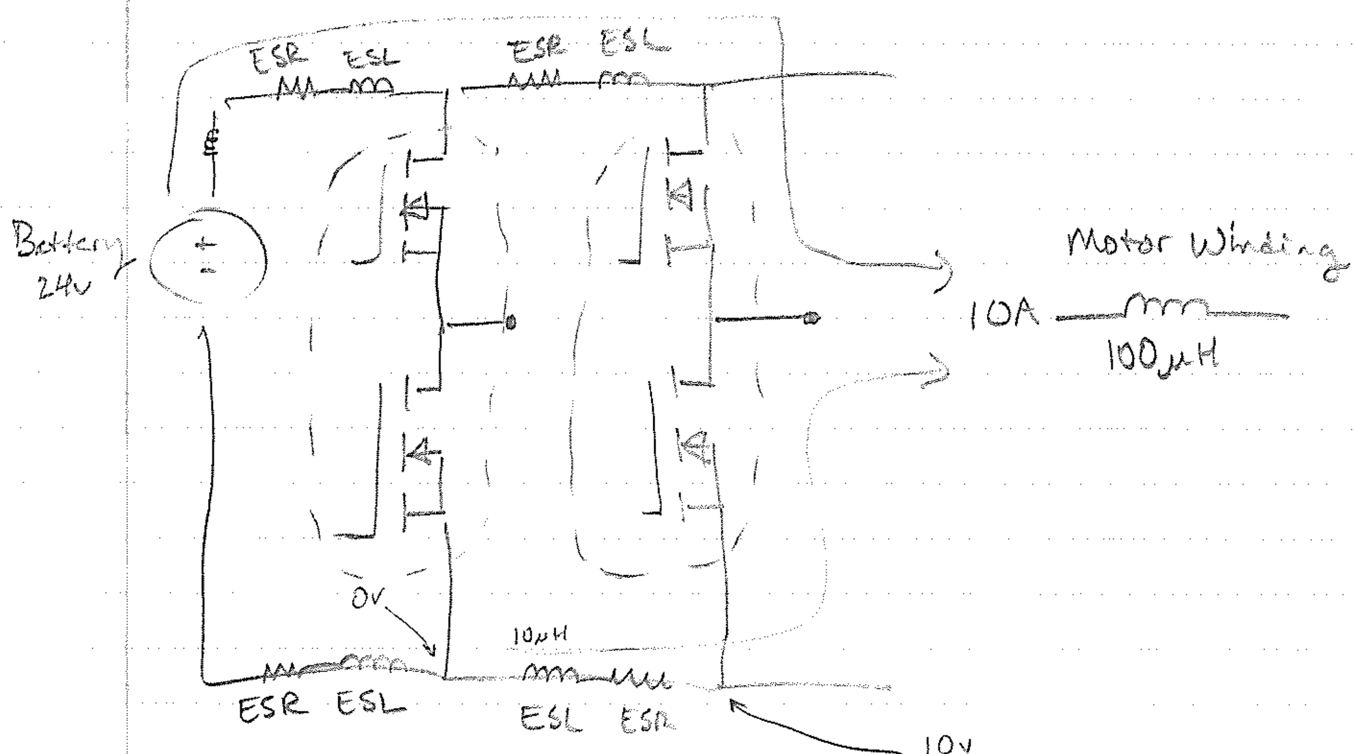
35 μm

$P = I^2 R$ Watts

Page 2

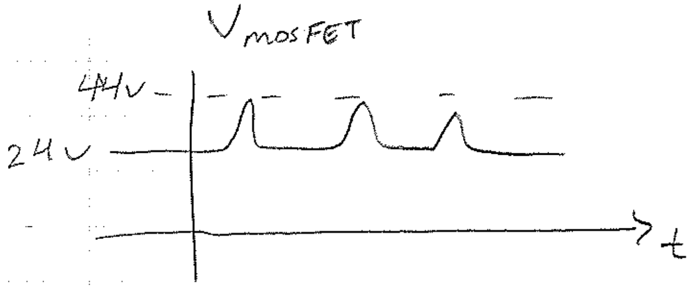


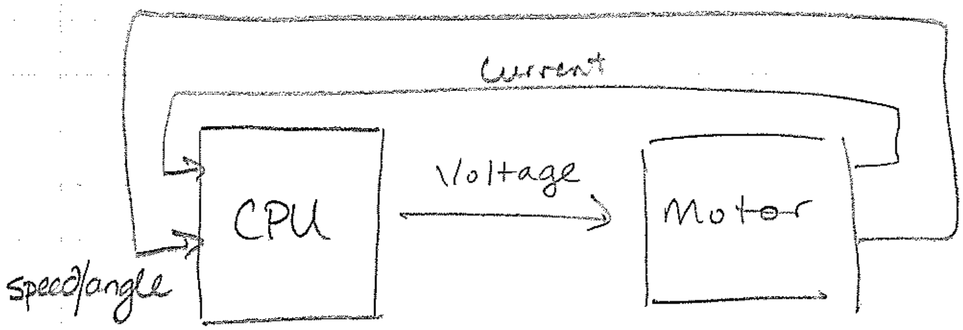
← ground connection

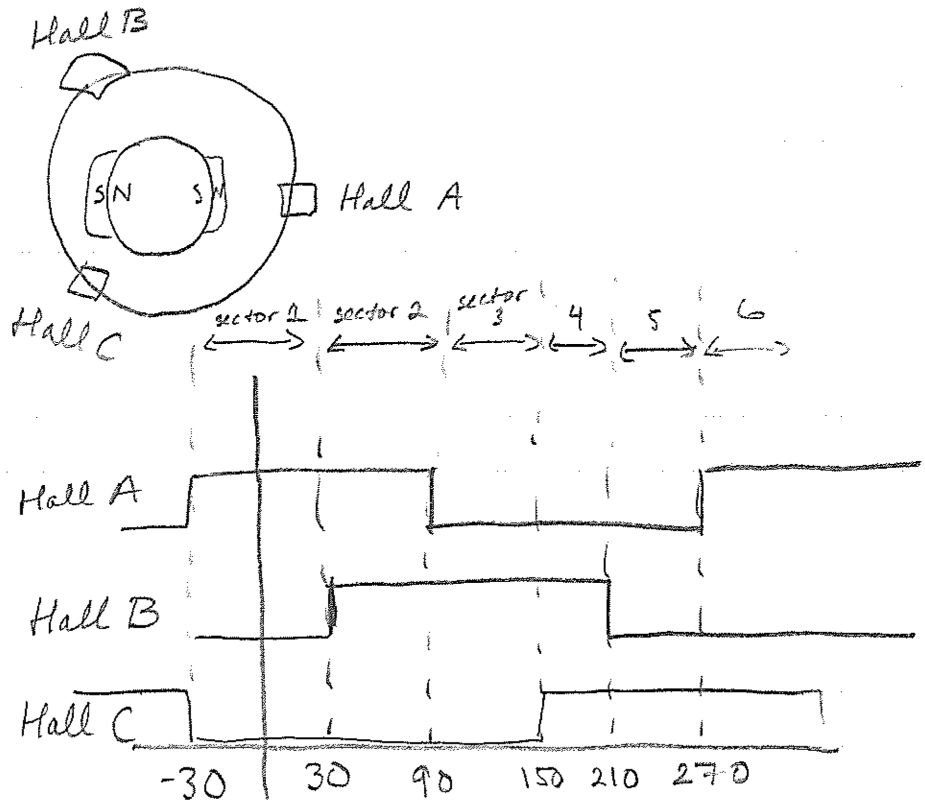


$V_{L1} = L_1 \frac{di_L}{dt}$ big! $\approx 1V \sim 10V$

$V_{L2} = L_2 \cancel{\frac{di_L}{dt}} \approx 1V \sim 10V$







Each sector is 60°; sensor resolution is $\pm 30^\circ$

The problem is that this is all it tells you.
You know the range of the angle but not the exact angle.

Page 4

Eddie
Sendowski

EE 499 - Week 7 Monday

8/23/2016

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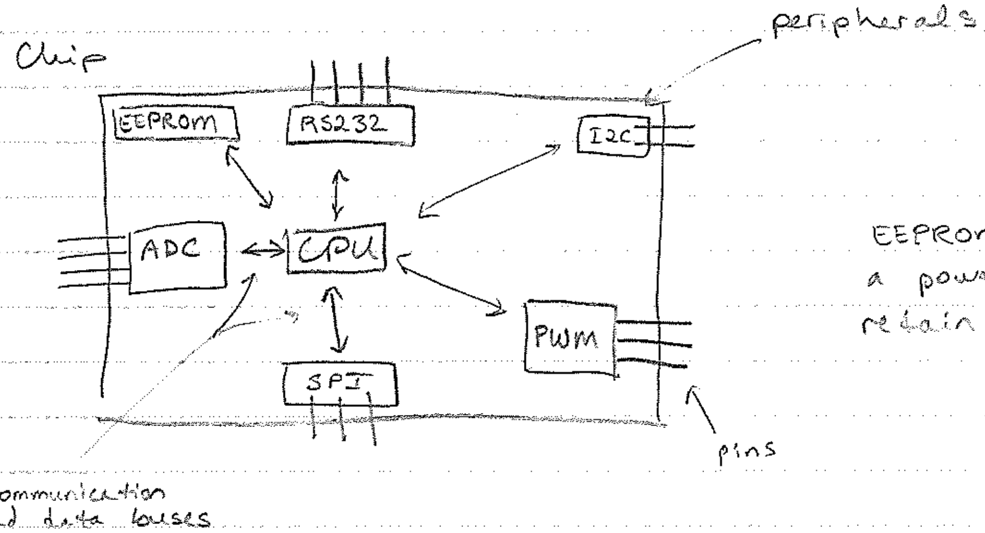
Final project

Probably will focus more on the software to learn more.



EEPROM doesn't need a power source to retain its memory.

CPU Core

- This is where the code you write is actually executed.
- Code can be stored in EEPROM or in RAM, but EEPROM is non-volatile (retains memory without power)

Peripherals

- The CPU connects to several peripherals that perform different functions...
- They can be input (ADC) or output (PWM)
- The CPU "talks" to peripherals by reading/writing special function registers (SFRs)

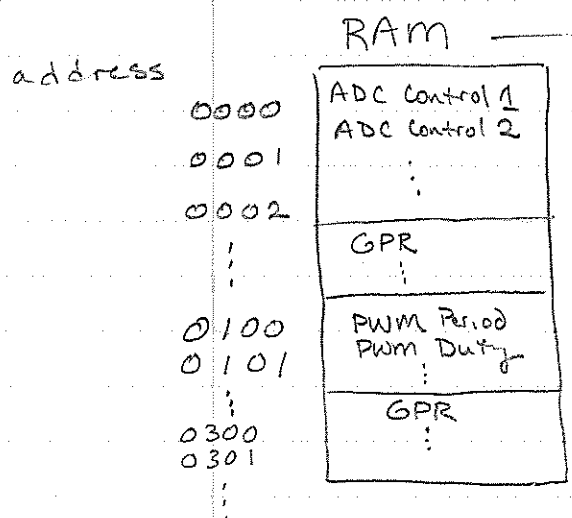
(Write Medium article about 4 month terms)

Could be the one thing that makes Kettering better

$\boxed{\text{Page 2}}$

Special Function Registers

The Random Access Memory (RAM) has two types of registers : GPRs and SFRs

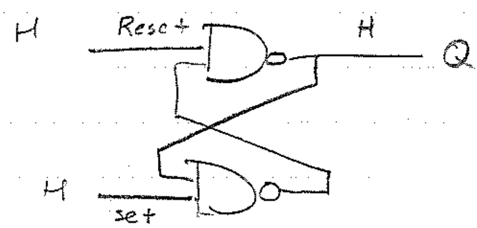


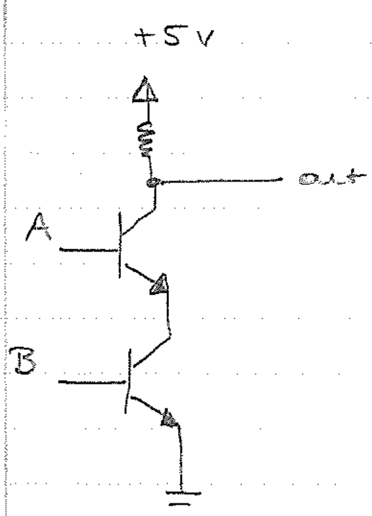
→ just a bunch of flip flops

↓

just a single bit memory cell

(Basically hardware behind memory storage)





A	B	Out
0	0	1
0	1	1
1	0	1
1	1	0

General Purpose Registers - used to store variables / data

example:

0010110(1)

↑ controls the ADC clock

To configure a peripheral, we must modify the SFR memory for that peripheral

PWM Generation inside CPU PWM Peripheral

- Fundamentally, there are 3 values we need to generate PWM in a chip:
    - Period : PRD
    - Counter : CTR
    - Compare value : CMPA or CMPB

$\Big\}$ SFR Names

these occupy a specific memory address in RAM

Page 4

- The PWM Counter counts at the CPU clock speed
  In most cases → 100 MHz

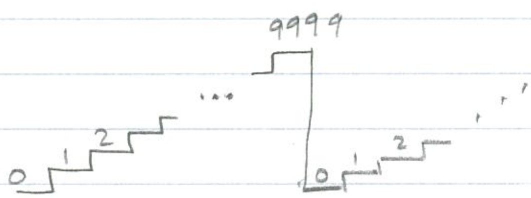
- It is a 16-bit counter
  0000 → FFFF
  $0_{10} \rightarrow 65535_{10}$

- Minimum counter frequency is

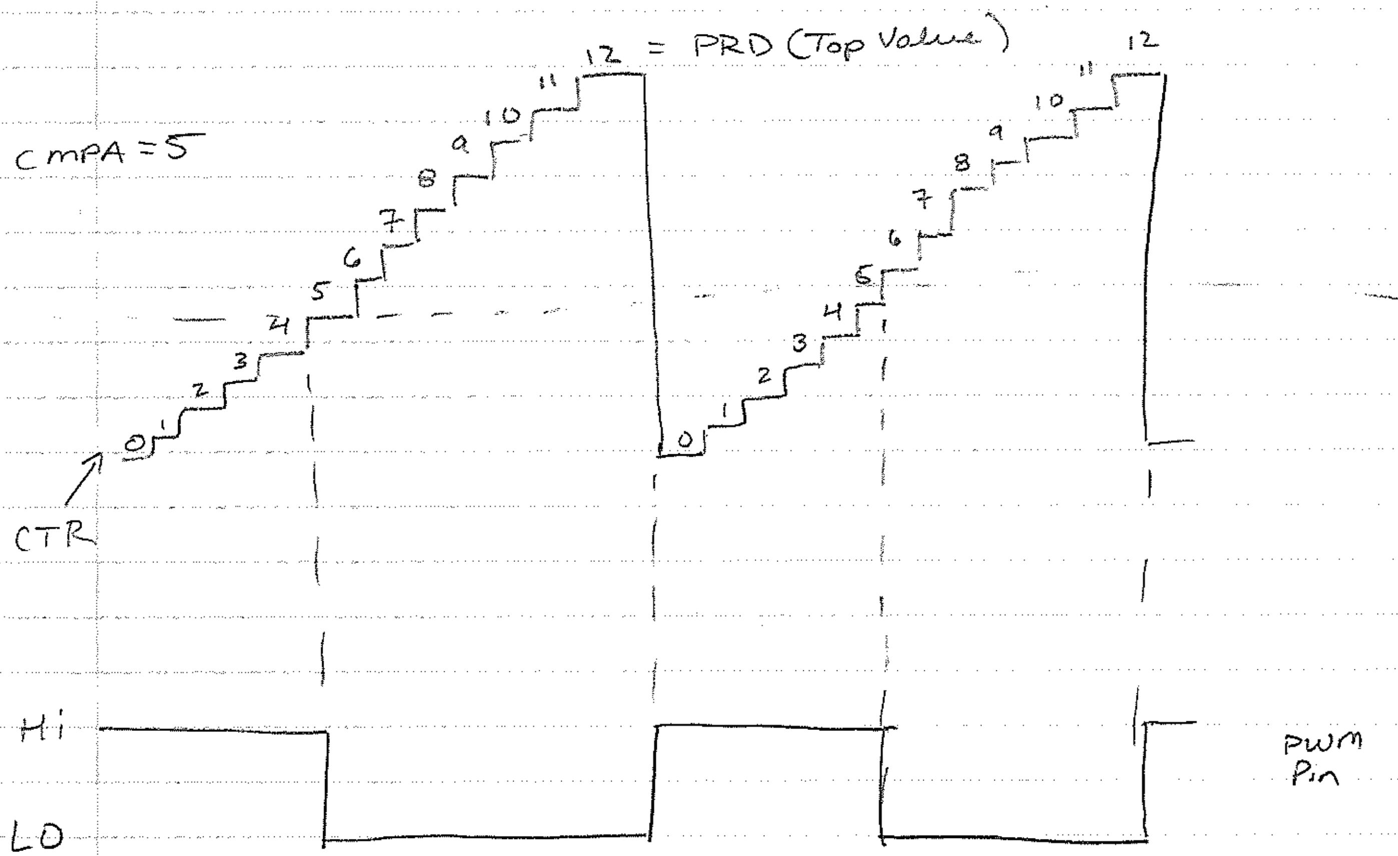
$$= \frac{\text{CPU Clock}}{2^n} = \frac{100\text{ MHz}}{65536} \approx 1.525\text{ KHz}$$

where n is the # of bits

- By default the counter period is set to maximum.
- Counter increments to FFFF, then clears
- We can change the max counter value by loading a value to the PRD register
  - Now it will count from
    0000 → PRD
    $0_{10} \rightarrow ??$
- For 100 MHz clock, we can set PRD = $\underbrace{10,000_{10}}^{\text{decimal}}$ to get a 10 KHz counter.



PWM is generated by comparing the clock value to a compare value, CMPA.



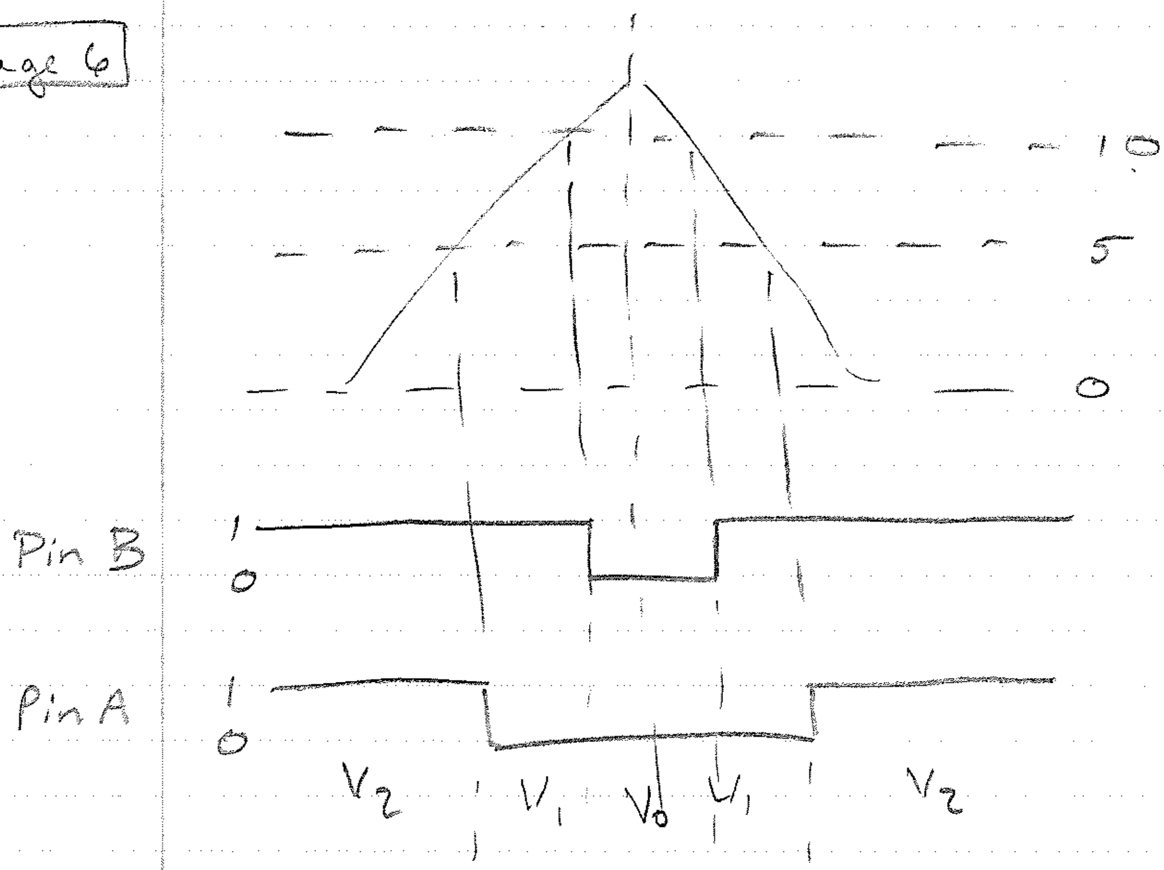
You can change the CMPA register value to adjust your duty cycle.

If CMPB = 10, you get "<u>edge-aligned PWM</u>"

In a 3 phase system, the relationship between pulses matters.

$\boxed{\text{Page 6}}$

imagine steps there



This slows down the PWM by a factor of 2.

<u>PWM Resolution</u>

- Related to the number of distinct values we have between 0_{10} and PRD.

for up-count and down-count

$$\text{PWM Resolution} = 100 * \frac{1}{PRD}$$

Example 3-bit PWM $= 12.5\% / \text{bit}$

$$PRD = 2^3 = 8$$

Q: How did long division get invented?

Eddie
Schodowski

EE 499 - Week 7 Friday

8/26/2016

Page 1

Q: Why does Sam Altman think we won't see another Bell Labs and that startups will replace it?

Header Files?
↳ Benefit of OOP in non-OOP langs like C

- Scan notes for Will
- Get AWS IoT Button ✓ Ordered

Don't always want to run the timing stuff in the main method

Oh shit I really have to do Power Semiconductors
Watch Sully's talk again
Just fucking finish thesis, don't rechange topic

Eddie Schodowski

EE 499 - Week 8 Tuesday

8/30/2016

Page 1

Will talk a bit about DSP stuff.

It's going to come into play when we try to implement the control algorithm.

Digital Control

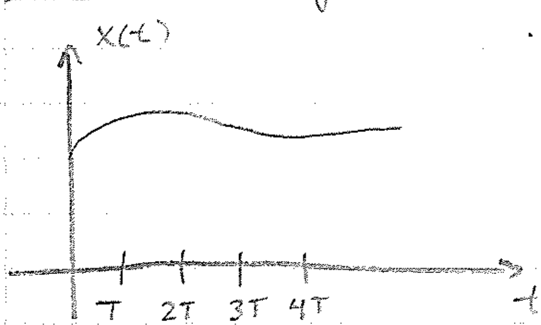
- There is no "integrate" function in C-code.
- Same with "differentiate"

"In a computer program, they don't exist, we have to approximate them"

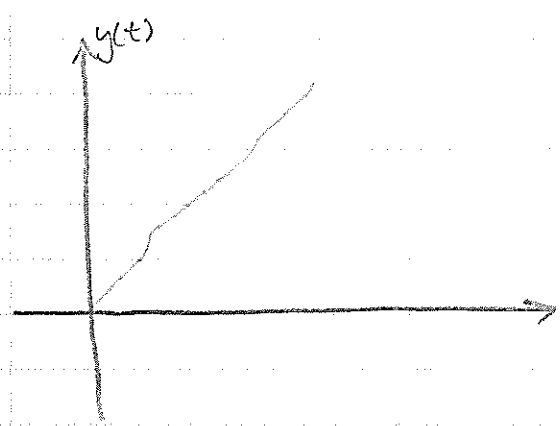
- There is a problem when we want to implement a controller or filter
    - A PI controller contains an integrator

Numerical Integration

$$y(t) = \int_0^t x(t)\,dt$$



(roughly a constant, integrate to get a slope)



T is my digital sampling period

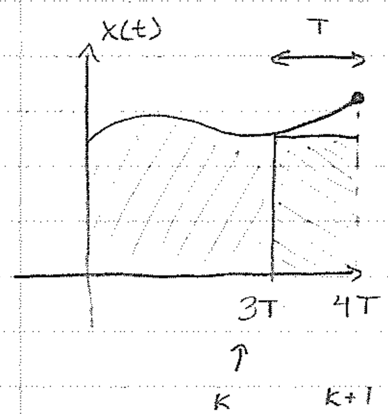
Now, integrate up to a fixed multiple of T

Page 2 $y((k+1)T) = \int_0^{(k+1)T} x(t)\,dt = \underbrace{\int_0^{kT} x(t)\,dt}_{\text{Actually equal to the answer to the previous sample.}} + \int_{kT}^{(k+1)T} x(t)\,dt$

For graph, pretend $k = 3$

$= y(kT) + \underbrace{\int_{kT}^{(k+1)T} x(t)\,dt}_{\text{Approximate!}}$

This is where numerical methods start.



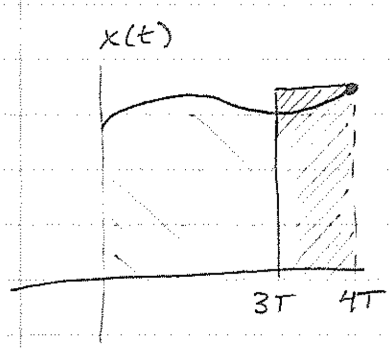
<u>Euler's Method</u>

$y((k+1)) = y(kT) + T x(kT)$

$Y(z) z = Y(z) + T X(z)$

$Y(z)(z-1) = T x(z)$

$\boxed{H(z) = \frac{T}{z-1}}$ $\boxed{s \Leftrightarrow \frac{z-1}{T}}$ ✱

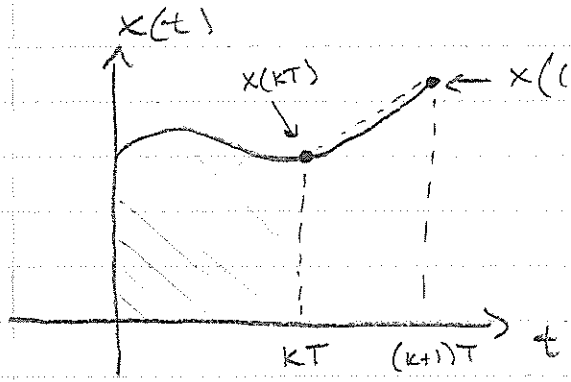


<u>Euler's Backward Method</u>

$y((k+1)T) = y(kT) + Tx((k+1)T)$

$Y(z) z = Y(z) + T X(z) z$

$\boxed{H(z) = \frac{Tz}{z-1}}$ $\boxed{s \Leftrightarrow \frac{z-1}{Tz}}$



Trapezoidal Rule
Tustin's method
Bilinear Transform

$$y((k+1)T) = y(kT) + \frac{T}{2}\left[x(kT) + x((k+1)T)\right]$$

$$Y(z)z = Y(z) + \frac{T}{2}\left[X(z) + zX(z)\right]$$

$$\boxed{H(z) = \frac{T}{2}\left(\frac{z+1}{z-1}\right)} \qquad \boxed{s \Longleftrightarrow \frac{2}{T}\left(\frac{z-1}{z+1}\right)}$$

$\int \rightarrow \frac{1}{s}$ (That's why we flip that stuff)

$$\frac{dx(t)}{dt} + \frac{5}{4}x(t) = \frac{3}{4} \longrightarrow \text{Integrating Factor}$$

$$IF = e^{\int \frac{5}{4}dt} = e^{\frac{5}{4}t}$$

$$e^{\frac{5}{4}t}\frac{dx(t)}{dt} + e^{\frac{5}{4}t}\cdot\frac{5}{4}x(t) = \frac{3}{4}e^{\frac{5}{4}t}$$

$$\int \frac{d}{dt}\left[e^{(5/4)t}x(t)\right] = \int \frac{3}{4}e^{\frac{5}{4}t}$$

constant of integration

$$e^{\frac{5}{4}t}x(t) = \frac{4}{5}\cdot\frac{3}{4}e^{\frac{5}{4}t} + C_0$$

$$x(t) = \frac{3}{5} + C_0 e^{-\frac{5}{4}t}$$

(next side)

Page 4

if $x(0) = 0$

$$0 = \frac{3}{5} + C_0 \overset{1}{e^{0}} \rightarrow C_0 = -\frac{3}{5}$$

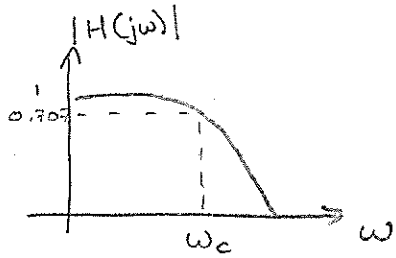
$$\boxed{x(t) = \frac{3}{5} - \frac{3}{5} e^{-\frac{5}{4}t}}$$

In some cases, you want to build a control system.

Digitizing Transfer Functions

- say we have a low pass filter

$$H(s) = \frac{\omega_c}{s + \omega_c}$$



Pretty common to use trapezoidal.
- Use bilinear transform to convert to Z-domain for digital equivalent.

Continuous: $s \rightarrow j\omega$

Discrete: $z \rightarrow e^{j\Omega T}$

$$H(z) = \frac{\omega_c}{\frac{2}{T}\left(\frac{z-1}{z+1}\right) + \omega_c}$$

$$H\left(e^{j\Omega T}\right) = \frac{\omega_c}{\frac{2}{T}\left(\frac{e^{j\Omega T} - 1}{e^{j\Omega T} + 1}\right) + \omega_c}$$

$$\left(\frac{e^{-j\Omega T/2}}{e^{-j\Omega T/2}}\right)$$

$e^{j\theta} = \cos\theta + j\sin\theta$

$e^{-j\theta} = \cos\theta - j\sin\theta$

$\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

$\sin\theta = \dfrac{e^{j\theta} - e^{j\theta}}{2j}$

Tells us how digital frequencies map to analog frequencies

$$\boxed{\omega = \frac{2}{T}\tan\left(\frac{\Omega T}{2}\right)}$$

$$H(e^{j\Omega T}) = \frac{\omega_c}{\frac{2}{T}\left(\dfrac{e^{j\Omega T/2} - e^{-j\Omega T/2}}{e^{j\Omega T/2} + e^{-j\Omega T/2}}\right) + \omega_c}$$

$$H(e^{j\Omega T}) = \frac{\omega_c}{\frac{2}{T}\left(\dfrac{2j\,\dfrac{e^{j\Omega T/2} - e^{-j\Omega T/2}}{2j}}{2\,\dfrac{e^{j\Omega T/2} + e^{-j\Omega T/2}}{2}}\right) + \omega_c}$$

$$H(e^{j\Omega T}) = \frac{\omega_c}{j\,\frac{2}{T}\left(\dfrac{\sin(\Omega T/2)}{\cos(\Omega T/2)}\right) + \omega_c}$$

$$\boxed{H(e^{j\Omega T}) = \frac{\omega_c}{j\left(\frac{2}{T}\tan(\Omega T/2)\right) + \omega_c}}$$

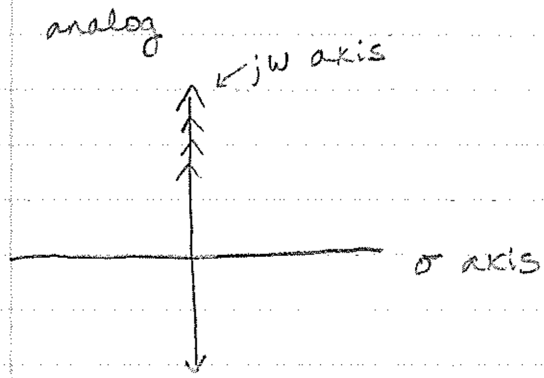
$$H(j\omega) = \frac{\omega_c}{j\omega + \omega_c}$$

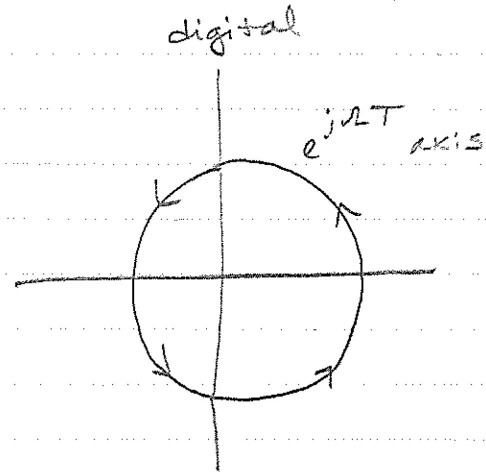
Page 6

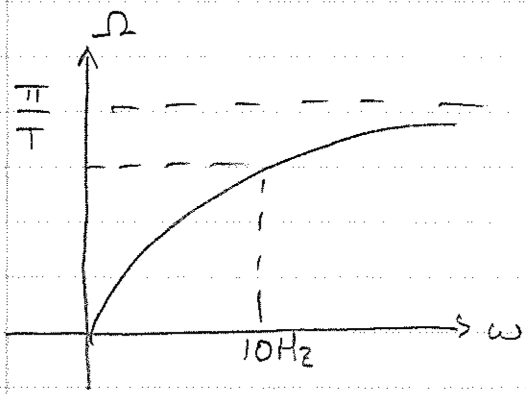
We can rearrange it to give the opposite information:

$$\boxed{\Omega = \frac{2}{T}\tan^{-1}\left(\frac{\omega T}{2}\right)}$$

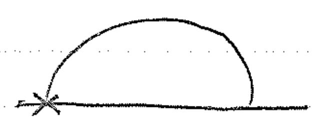
This is called the frequency prewarping equation







$$\Omega T \qquad (2\pi \underset{\uparrow}{\overset{50Hz}{50}})\ \underset{\uparrow}{\overset{f_s = 100Hz}{0.01}} = \pi$$



$$H(z) = \frac{\omega_c}{\frac{2}{T}\left(\frac{z-1}{z+1}\right) + \omega_c}$$

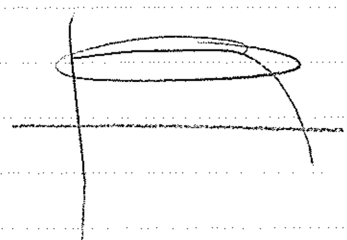
$$= \frac{T(z+1)\,\omega_c}{2(z-1) + \omega_c T(z+1)}$$

$$= \frac{Tz + T\omega_c}{z(T\omega_c+2) + (T\omega_c - 2)}$$

$$\boxed{H(z) = \frac{z\left(\frac{T}{T\omega_c+2}\right) + \left(\frac{T}{T\omega_c+2}\right)}{z + \frac{T\omega_c - 2}{T\omega_c+2}}}$$

1^{st} Order Digital LPF

"Butterworth Filter"



Shape of transfer function

$$\omega_c = \frac{2}{T}\tan(\Omega T/2)$$

$$H(z) = \frac{b_1 z + b_0}{az + a_0} \quad \text{where } a_1 = 1$$

$\boxed{\text{Page 8}}$

$$y(kT+T) = -a_0 y(kT) + b_1 x(kT+T) + b_0 x(kT)$$