Lecture 5

Overview of Power Electronics & Intro to DC-to-AC Inverters

Objectives:

- Discuss the "voltage regulator problem", and ways to mitigate the issues.
- Discuss the basic Buck and Boost converter topologies, derive the voltage transfer characteristics, and examine the inductor & capacitor ripples.
- Examine using the Buck converter as a DC-to-AC converter and examine the H-bridge topology for generating a single-phase AC output.

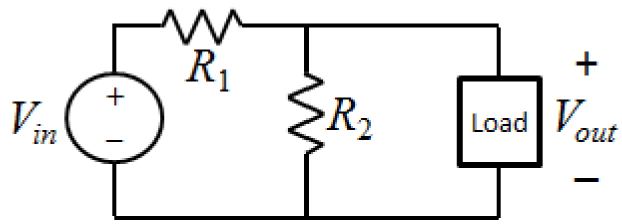
Keywords:

Linear Regulator
Switching Regulator
Buck converter
Boost converter
Switching frequency
Switching period
Duty cycle
Deadtime
Average inductor voltage
Average capacitor current
Inductor current ripple
Capacitor voltage ripple
Pulse Width Modulation (PWM)
H-bridge
Inverter

The Voltage Regulator Problem

To illustrate the most basic need for power electronics, we start with the simple voltage regulator scenario, keeping an eye on load regulation, device power losses, and efficiency.

Let's imagine that we have a 10V battery and we'd like to use it to power some microcontroller circuitry. However, the microcontroller and associated loads must run at 5V. How can we step down the battery voltage to power the MCU? The simplest solution is to use a resistor divider to bring the voltage down.



$$V_{out} = \frac{(R_2 // R_L)}{(R_2 // R_L) + R_1} V_{in}$$

This solution has two main problems:

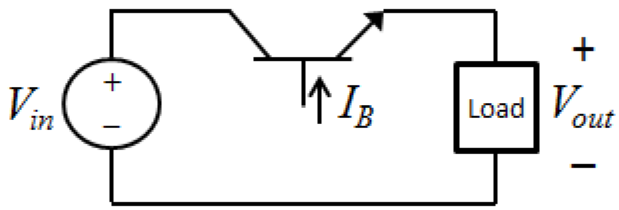
- The resistor divider circuit has poor voltage regulation. Since the load R_L is parallel to the lower resistor R_2, any change in load will alter the voltage divider ratio, changing the output voltage.
- All of the output current must flow through the top resistor R_1. This current, when multiplied by the voltage dropped across it, will lead to high losses and low efficiency.

You could decrease the value of both divider resistors (R_1 & R_2) to make it less sensitive to load changes, but the efficiency and power losses become worse. The steady-state current (sometimes called quiescent current) through the divider will become larger, and the voltage drops remain the same.

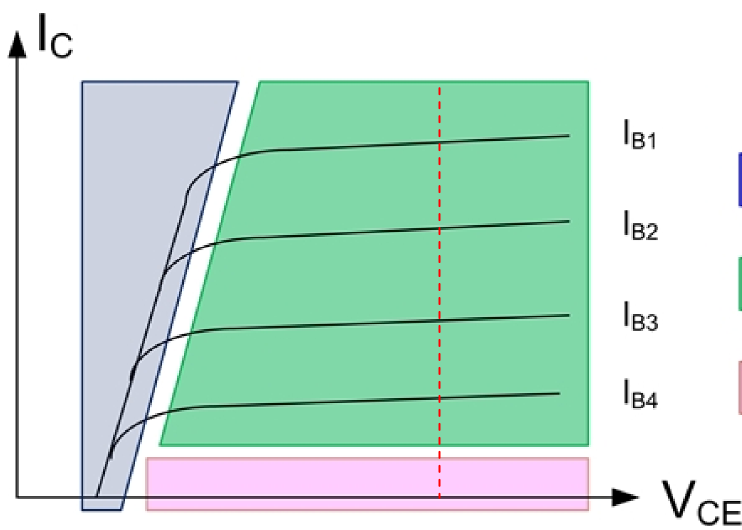
These two issues counteract each other – a better solution is needed.

Using a Transistor in the Active Range

To fix the voltage regulation problem, we could attempt to use a transistor which would operate in the active (amplifier) region.



The voltage drop could be controlled (by modulating the base / gate) to always give the correct output voltage, regardless of the demanded output load current. This approach is commonly called a linear regulator – they're used in low-power voltage regulation applications.

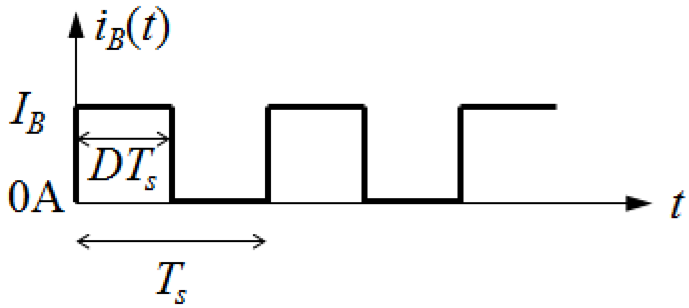


The problem with this approach is that it still does not fix the efficiency problem. What we've essentially done here is to make a voltage divider between the transistor, and the load itself.

The transistor's effective resistance is changing (through modulating the base / gate) as the load resistance (the load current) changes – all while keeping the output voltage constant (5V in our previous scenario). We're moving between I_B curves in the image above, while maintaining a constant V_{CE} (red line).

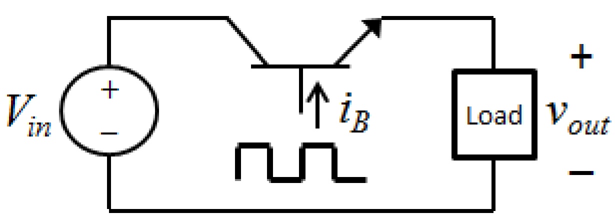
The Switching Regulator

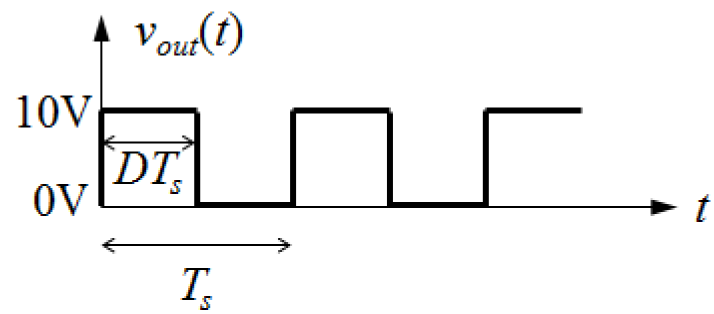
Rather than continuously operating the transistor, we will periodically switch it ON and OFF. Thus, in our example, the base current would look like a square wave versus time, with values of I_B and 0A. The output voltage would also be a square-wave, with values of 10V and 0V when the transistor is ON or OFF, respectively.



$$T_s = \text{switching period}$$
$$f_s = 1/T_s = \text{switching frequency}$$
$$D = \text{duty cycle}, 0 \le D \le 1$$

If we set the ratio between ON time and OFF time to be 1:1, or in other words, set the ON time to be 50% of the total switching period T_s, the average output voltage over time will be equal to 5V (assuming a 10V input). The percentage of ON-time with respect to the total period is referred to as the <u>duty cycle</u>, and is denoted using variable "*D*". The duty cycle (or duty ratio) is a percentage, thus its value is always between 0 and 1.





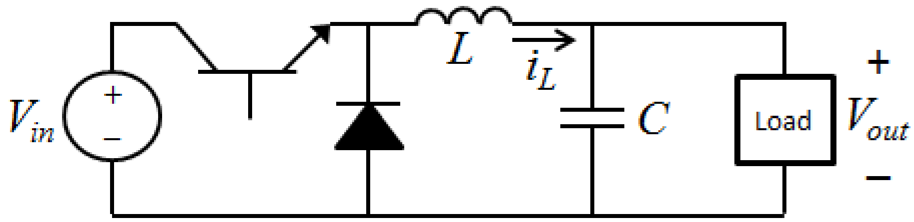
One key difference here is that when the transistor is ON, we will provide a strong base / gate drive to force the transistor to operate in the linear (saturation) region. This will guarantee that the output voltage is close to the supply value (10V) for the ON-time duration and will minimize the transistor conduction losses.

With this approach, we have solved the two previous issues (load regulation and efficiency), but we've introduced a third problem – our output voltage is not a constant DC voltage value like we originally wanted.

Switching Converter Analysis

A pulsed voltage is OK for resistive loads, but if the load were a microcontroller, we would destroy it after applying the first 10V pulse.

To get around this issue, we place an LC filter in between the switch and output. This effectively smoothens the square-wave voltage to a constant value. A diode must be placed to ground to allow a freewheeling current path for the inductor current while the transistor is OFF. This new topology is called a Buck converter.

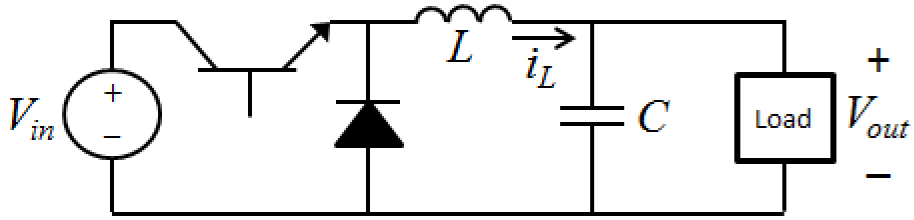


Next we'll analyze the characteristics of this circuit, but to do so, we'll make some assumptions. The basic assumptions for many power electronics converter analyses are listed below:

1. The circuit is operating at steady-state.
2. The inductor current is continuous, the inductance is large enough so that the current ripple is negligible, and the average inductor voltage is zero.
3. The capacitor voltage is continuous, the capacitance is large enough so that the voltage ripple is negligible, and the average capacitor current is zero.
4. The switching period is T_s: the switch is closed (ON) for time $D \cdot T_s$ and open (OFF) for time (1-D)$\cdot T_s$. The diode operates under the opposite conditions.
5. The components are ideal (no resistive losses or $I \cdot R$ voltage drops).

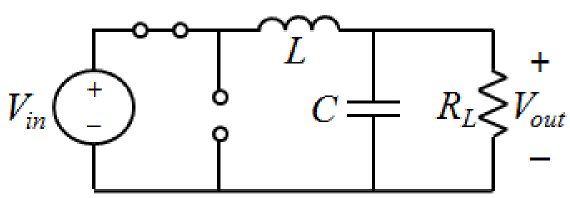
The Buck Converter

The Buck converter is a commonly used power-electronics-circuit which can step down the input voltage to a lower DC value. The inductance, capacitance, and switching frequency are selected to minimize the ripple due to switching.



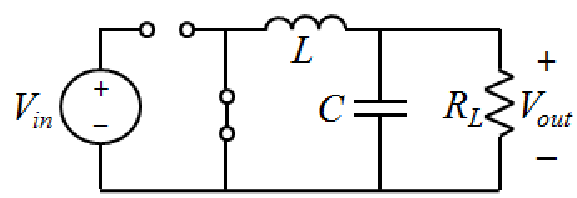
If we analyze the Buck converter topology during its two switching states, we can derive the voltage transfer characteristic between input and output. For the Buck converter analysis, let's compute the time-averaged voltage applied to the series inductor by examining the voltage over the two time-intervals.

Switch ON



$$V_{L,on} = (V_{in} - V_{out})$$

Switch OFF



$$V_{L,off} = (0\mathrm{V} - V_{out})$$

We can compute the time-averaged inductor voltage below, which should be equal to zero at steady-state (assumption #2):

$$\bar{v}_L = 0\mathrm{V} = \frac{1}{T_s}\left[\int_0^{DT_s} V_{L,on}\,dt + \int_{DT_s}^{T_s} V_{L,off}\,dt\right]$$

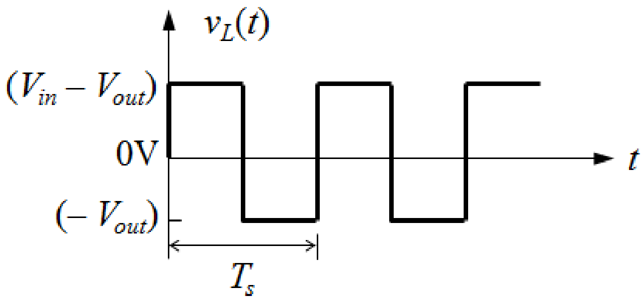
$$0\mathrm{V} = \frac{1}{T_s}\left[(V_{in} - V_{out})DT_s - V_{out}(1-D)T_s\right]$$

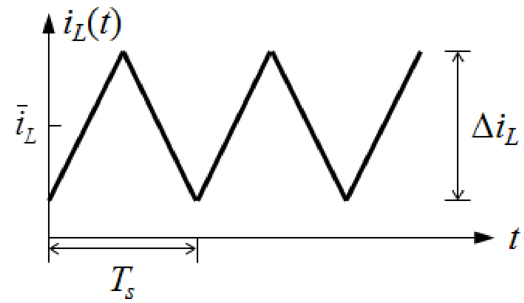
$$0\mathrm{V} = V_{in}D - V_{out}D - V_{out} + V_{out}D$$

$$\boxed{V_{out} = DV_{in}}$$

Buck Converter Ripple Analysis

The voltage across the inductor in the Buck converter scenario is a square wave voltage. Thus the current will be a triangular wave.

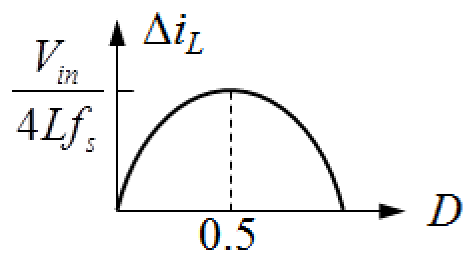




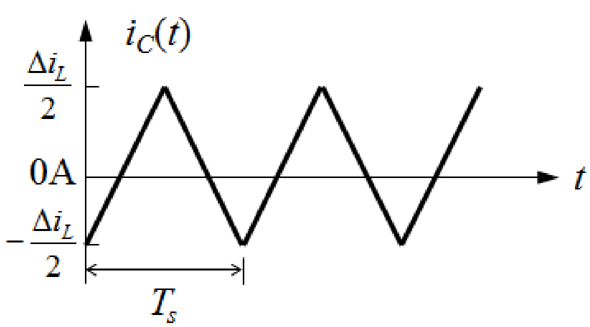
If we examine the inductor voltage during the ON-time and substitute our voltage transfer expression, we can obtain an expression for the ripple current on the inductor as a function of the duty ratio – max ripple occurs when *D* = 50%.

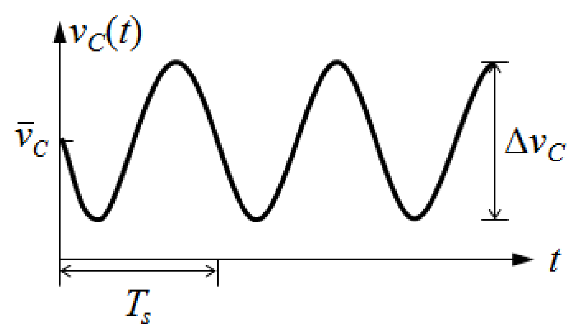
$$\Delta i_L = \frac{1}{L}\int_0^{DT_s} V_{L,on}\,dt = \frac{1}{L}(V_{in} - V_{out})DT_s$$

$$\Delta i_L = \frac{(V_{in} - DV_{in})D}{L\ f_s} = \frac{(D - D^2)V_{in}}{L\ f_s}$$



Using KCL, the difference in load current and inductor current will be the capacitor current. If the voltage ripple is small, the average inductor current will be equal to the load current. Using the same method as above, we can find Δv_C.



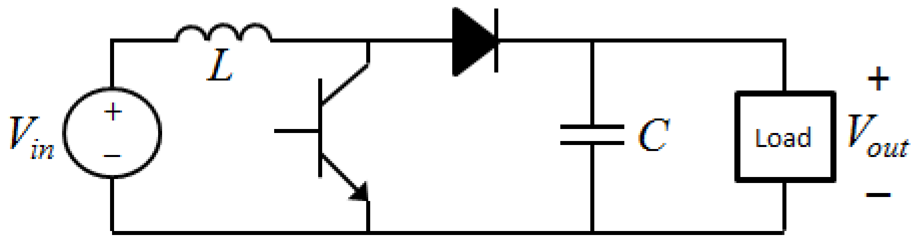


The capacitor current rises from zero linearly to half Δi_L in a quarter of a period.

$$\Delta v_C = \frac{2}{C}\int_0^{T_s/4}\left(\frac{\Delta i_L}{2}\frac{4}{T_s}t\right)dt = \frac{V_{out}(1-D)}{8LCf_s^{\,2}}, \quad \text{where} \quad \Delta i_L = \frac{V_{out}(1-D)}{L\ f_s}$$

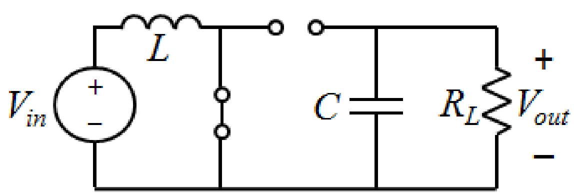
The Boost Converter

The Boost converter has the opposite characteristic to the Buck converter. Rather than stepping down the voltage, a Boost converter's output voltage is higher than the input voltage. The switch, diode, and inductor are swapped.



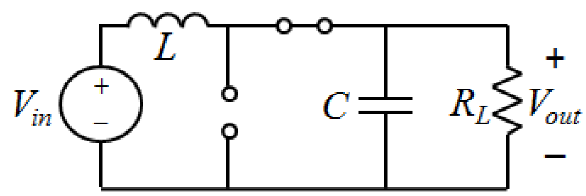
For the Boost converter analysis, again we compute the time-averaged voltage applied to the series inductor over the two time intervals.

Switch ON



$$V_{L,on} = \left(V_{in} - 0\text{V}\right)$$

Switch OFF



$$V_{L,off} = \left(V_{in} - V_{out}\right)$$

We can compute the time-averaged inductor voltage below, which should be equal to zero at steady-state:

$$\bar{v}_L = 0\text{V} = \frac{1}{T_s}\left[\int_0^{DT_s} V_{L,on}\,dt + \int_{DT_s}^{T_s} V_{L,off}\,dt\right]$$

$$0\text{V} = \frac{1}{T_s}\left[V_{in}DT_s + \left(V_{in} - V_{out}\right)\left(1-D\right)T_s\right]$$

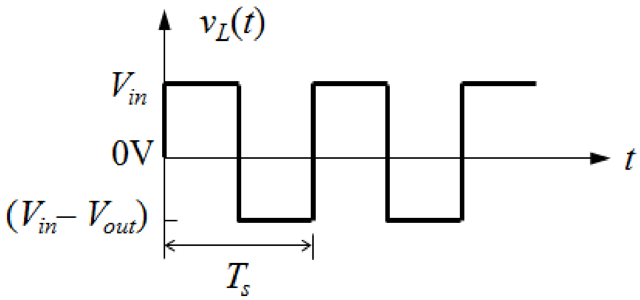
$$0\text{V} = V_{in}D + V_{in} - V_{out} - V_{in}D + V_{out}D$$

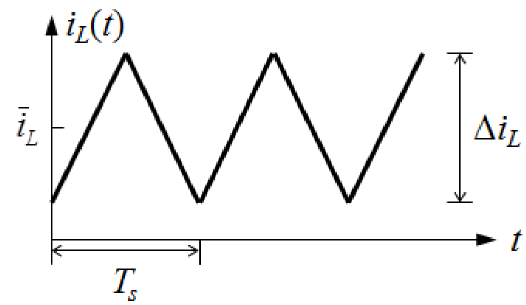
$$0\text{V} = V_{in} - V_{out}\left(1-D\right)$$

$$\boxed{V_{out} = \frac{1}{\left(1-D\right)}V_{in}}$$

Boost Converter Ripple Analysis

The voltage across the inductor in a Boost converter is also a square-wave voltage. Thus, the inductor current ripple will be triangular.

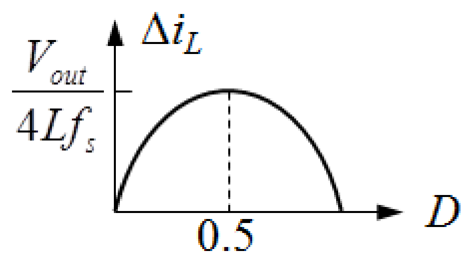




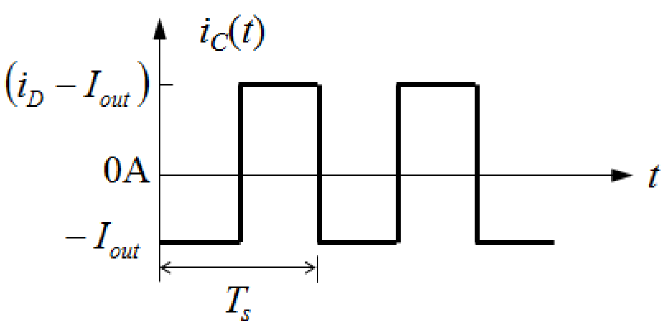
If we examine the inductor voltage during the ON-time and substitute our voltage transfer expression, we can obtain an expression for the ripple current on the inductor as a function of the duty ratio – max ripple occurs when D = 50%.

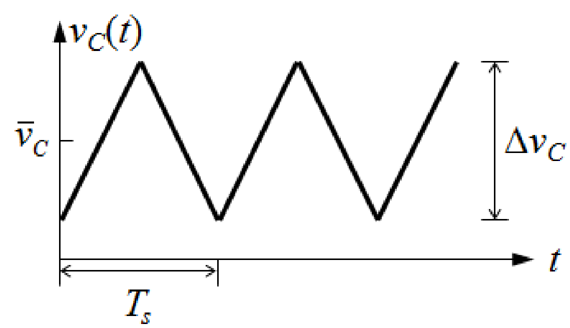
$$\Delta i_L = \frac{1}{L}\int_0^{DT_s} V_{L,on}\,dt = \frac{1}{L}(V_{in})DT_s$$

$$\Delta i_L = \frac{DV_{in}}{L\ f_s} = \frac{V_{out}\left(D - D^2\right)}{Lf_s}$$



Using KCL, the difference in load current and diode current will be the capacitor current. Assuming the ripples to be small, we can approximate the diode and load currents to be constant. Using the same method as above, we can find Δv_C.





The capacitor current during the ON-state is V_{out} divided by load resistance R_L.

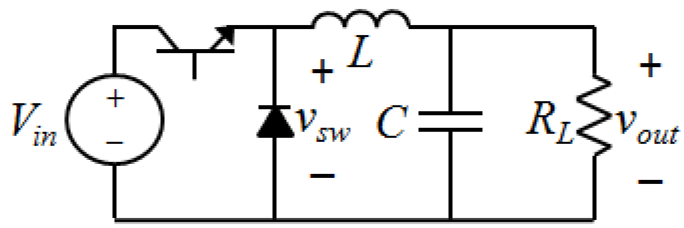
$$\Delta v_o = \frac{1}{C}\int_0^{DT_s}\left(\frac{V_{out}}{R_L}\right)dt = \frac{V_{out}D}{R_L C\ f_s}$$

Buck Converter as a DC/AC Converter

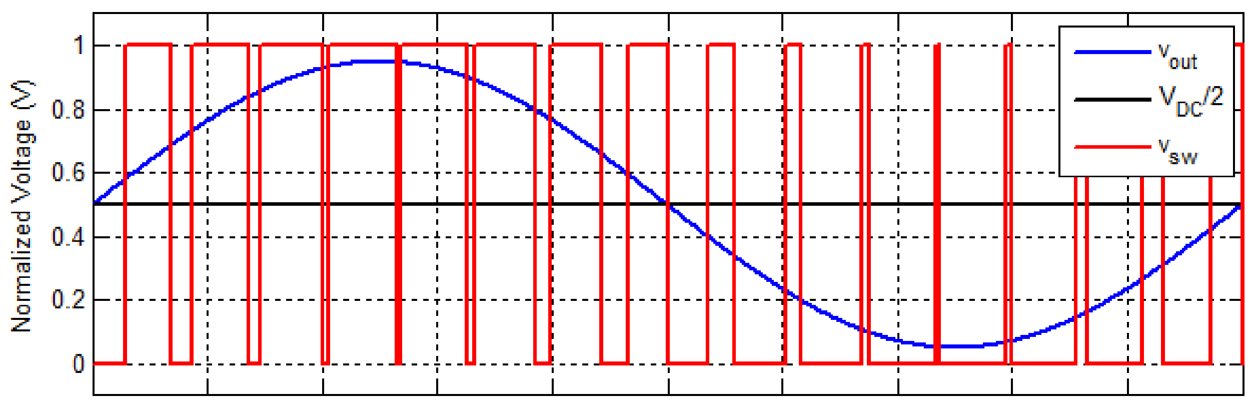
If the output voltage of a Buck converter is simply the input voltage multiplied by the duty ratio, what if we control the duty ratio to be sinusoidal over time? The average voltage output of the Buck converter will also be sinusoidal! We could call this output voltage a pulse-width modulated (PWM) sine wave.

$$d(t) = \tfrac{1}{2}\left[\sin(\omega t) + 1\right] \qquad\qquad v_{out}(t) = d(t)V_{in}$$

To help visualize this, let's plot the switching node voltage, labeled v_{sw}, and the output voltage together. The circuit diagram and plot are shown below.



Notice that the pulse widths get wider towards the peak of the sine wave, and get narrower during the trough of the sine wave.



Note that this sinusoidal voltage has a DC offset of ½ of the DC input, which we'll now call V_{DC}. Thus, the maximum amplitude of the AC signal (ignoring the offset) would also be ½ V_{DC}; the RMS value would be ~70% of the peak value.

$$v_{out}(t) = \frac{V_{DC}}{2}\left[1 + \sin(\omega t)\right] \qquad\qquad v_{AC}(t) = \frac{V_{DC}}{2}\sin(\omega t)$$

Modulation Index

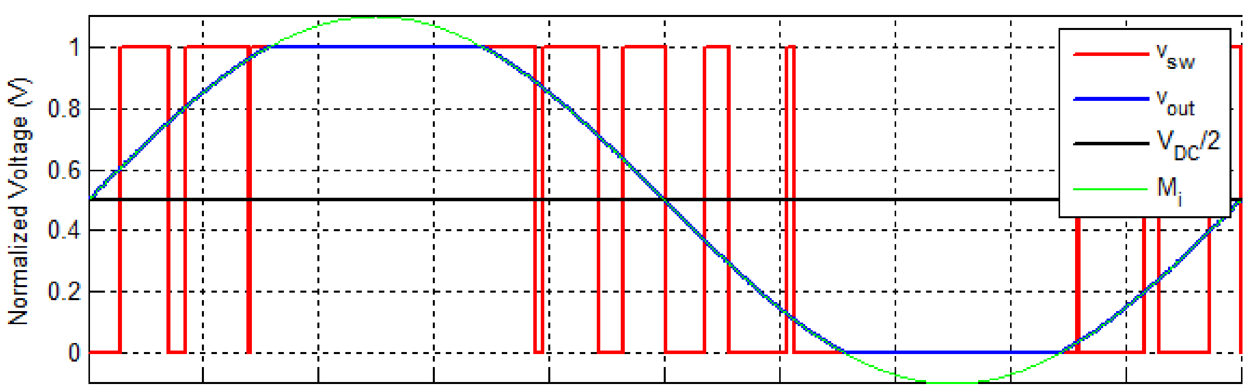
We can also control the magnitude of the AC component by changing the amplitude of the sinusoidal component in the duty cycle. Here, the variable M_i is called the modulation index, and is also a value from 0 to 1.

$$v_{out}(t) = \frac{V_{DC}}{2}\left[1 + M_i \sin(\omega t)\right], \quad \text{where} \quad 0 \le M_i \le 1$$

The modulation index can also be described as the ratio of AC voltage magnitude (the peak value) to half of the DC voltage magnitude, as shown below:

$$M_i = \frac{V_{AC,pk}}{V_{DC}/2}$$

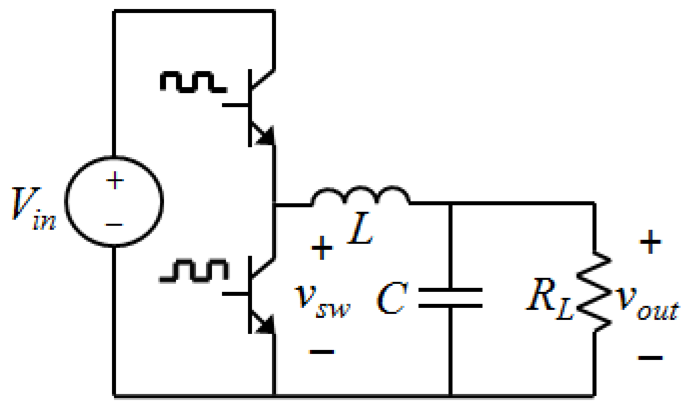
It's possible in some cases for the modulation index to be greater than 1, but the resulting voltage will not be a pure sine wave. The peaks and troughs will be clipped to $+V_{DC}$ and 0V, respectively, causing distortion.



If we examine the Fourier series of the over-modulated voltage, it will contain harmonics (as expected) but the fundamental component will be larger than half of the DC voltage. This can be desirable if, for example, the harmonics of the voltage can be safely ignored. In this case, over-modulation allows us to generate an even bigger fundamental voltage.

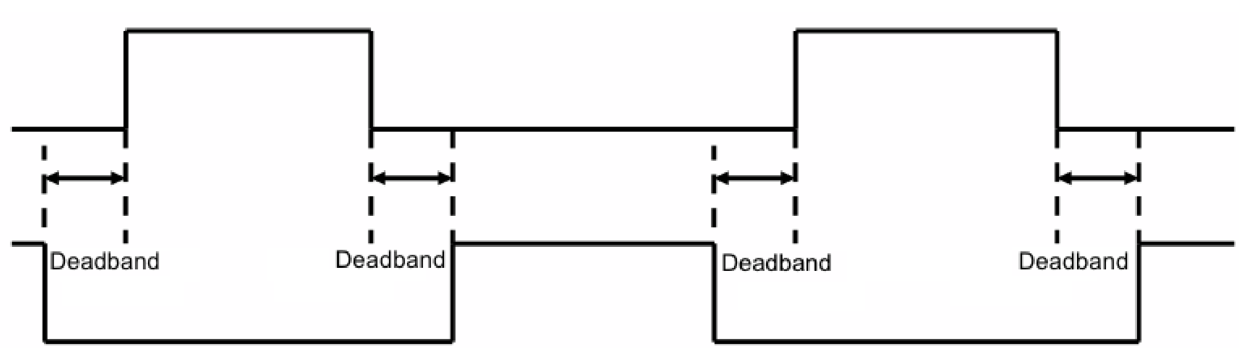
Replacing the Diode w/ a Switch

If the current must be allowed go negative (e.g. a sinusoidal AC current), we must replace the freewheeling diode with a second semiconductor switch. This will ensure the center node is always connected to ground when the top switch is OFF, regardless of the inductor current direction (with a diode, the diode will only conduct when the current flows towards v_{out}).



To drive the second switch, we can simply apply the logical inverse of the PWM signal applied to the upper switch. Thus, the two switches operate as a complementary pair. When the top switch is ON, the lower is OFF, and vice-versa.

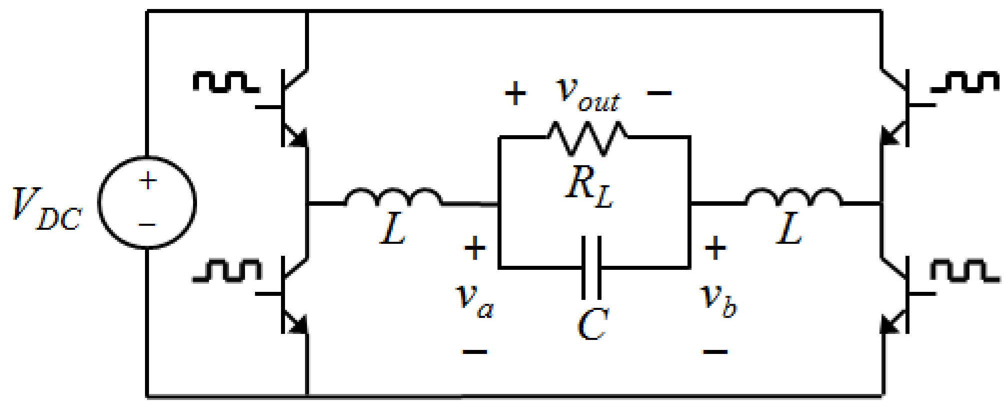
Usually, a small time-delay is inserted between the turn-OFF and turn-ON points of both switches. This delay is often called deadtime or deadband. It is a period where both gating signals are held OFF to ensure that the previously-active switch has time to fully shutdown before enabling the complementary switch.



If both switches become active at the same time, the DC input source will become short-circuited, causing a large current that will destroy the transistors!

The H-Bridge Circuit

We can combine two Buck converter circuits to make a differential AC output voltage. This circuit topology is commonly called an H-bridge circuit, due to the arrangement of switches and the load.



We can describe the average voltage at both switching nodes, v_a and v_b, as a sinusoidal function. In practice, the second Buck converter is controlled to generate a sinusoidal voltage which is 180° out of phase with the first.

$$v_a(t) = \frac{V_{DC}}{2}\left[1 + M_i \sin(\omega t)\right] \qquad v_b(t) = \frac{V_{DC}}{2}\left[1 - M_i \sin(\omega t)\right]$$

We can compute the output voltage as the difference of v_a and v_b. Notice that when connected this way, the ½ V_{DC} terms will cancel each other out, leaving only the AC component. Furthermore, the maximum AC magnitude is now doubled, since we can apply full positive V_{DC} or full negative V_{DC} to the load.

$$\begin{aligned} v_{out}(t) = v_{AC}(t) &= v_a(t) - v_b(t) \\ &= \frac{V_{DC}}{2}\left\{\left[1 + M_i \sin(\omega t)\right] - \left[1 - M_i \sin(\omega t)\right]\right\} \\ &= V_{DC}\left[M_i \sin(\omega t)\right] \end{aligned}$$

Power electronics circuits which convert a DC voltage to an AC voltage are commonly called inverter circuits. The circuit above could be considered a single-phase inverter.