

Homework 3: Convex Sets

Instructions

Please submit a .qmd file along with a rendered pdf to the Brightspace page for this assignment. You may use whatever language you like within your qmd file, I recommend python, julia, or R.

Problem 1 (cvx-book 2.12):

Which of the following sets are convex? For each case give the reason(s) why or why not

- A slab, i.e., a set of the form $\{x \in \mathbb{R}^n \mid \alpha \leq \mathbf{a}^T \mathbf{x} \leq \beta\}$.
- A rectangle, i.e., a set of the form $\{x \in \mathbb{R}^n \mid \alpha_i \leq x_i \leq \beta_i, i = 1, \dots, n\}$. A rectangle is sometimes called a hyperrectangle when $n > 2$.
- A wedge, i.e., $\{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{a}_1^T \mathbf{x} \leq \mathbf{b}_1, \mathbf{a}_2^T \mathbf{x} \leq \mathbf{b}_2\}$
- The set of points closer to a given point than a given set, i.e., $\{\mathbf{x} \mid \|\mathbf{x} - \mathbf{x}_0\|^2 \leq \|\mathbf{x} - \mathbf{y}\|^2 \text{ for all } y \in S\}$ where $S \subset \mathbb{R}^n$.
- The set of points closer to one set than another, i.e., $\{\mathbf{x} \mid \text{dist}(\mathbf{x}, S) \leq \text{dist}(\mathbf{x}, T)\}$, where S, T are subsets of \mathbb{R}^N , and $\text{dist}(x, S) = \inf\{\|\mathbf{x} - \mathbf{z}\|^2 \mid \mathbf{z} \in S\}$.

Problem 2 (cvx-book 2.15):

Some sets of probability distributions. Let x be a real-valued random variable with probability distribution

$$\mathbf{prob}(x = a_i) = p_i, i = 1, \dots, n$$

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where: $a_1 < a_2 < \dots < a_n$. Of course $\mathbf{p} \in \mathbb{R}^n$ lies in the standard probability simplex $P\{\mathbf{p} \mid 1^T \mathbf{p} = 1, \mathbf{p} \succeq 0\}$. Which of the following conditions are convex in \mathbf{p} ? (That is, for which of the following conditions is the set of $\mathbf{p} \in P$ that satisfy the condition convex?) For each case give the reason(s) why or why not.