DATA 609 - Homework 3: Convex Sets

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Instructions

Please submit a .qmd file along with a rendered pdf to the Brightspace page for this assignment. You may use whatever language you like within your qmd file, I recommend python, julia, or R.

Problem 1 (cvx-book 2.12):

Which of the following sets are convex? For each case give the reason(s) why or why not

a. A slab, i.e., a set of the form $\{x \in \mathbb{R}^n \mid \alpha \leq \mathbf{a}^T\mathbf{x} \leq \beta\}$.
b. A rectangle, i.e., a set of the form $\{x \in \mathbb{R}^n \mid \alpha_i \leq x_i \leq \beta_i, i = 1, \cdots, n\}$. A rectangle is sometimes called a hyperrectangle when n > 2.
c. A wedge, i.e., $\{\mathbf{x} \in \mathbf{R}^n \mid \mathbf{a_1}^T\mathbf{x} \leq \mathbf{b}_1, \mathbf{a}_2^T \leq \mathbf{b}_2\}$
d. The set of points closer to a given point than a given set, i.e., $\{\mathbf{x} \mid \|\mathbf{x} - \mathbf{x}_0\|^2 \leq \|\mathbf{x} - \mathbf{y}\|^2$ for all $y \in S\}$ where $S \subset \mathbb{R}^n$.
e. The set of points closer to one set than another, i.e., $\{\mathbf{x} \mid dist(\mathbf{x}, S) \leq dist(\mathbf{x}, T)\}$, where S, T are subsets of $\mathbf{R}^N$, and $dist(x, S) = \inf\{\|\mathbf{x} - \mathbf{z}\|^2 | \mathbf{z} \in S\}$.

Problem 1 Solution

a. The slab is a convex set because it is an intersection of two halfspaces.
b. The rectangle is a convex set because the set contains a finite intersection of halfspaces.
c. The wedge set is a convex set because like part a, it is an intersection of two halfspaces.
d. The set of points is a convex set because the set can be expressed as an intersection of halfspaces where $S = \{x \mid \|x - x_0\|_2 \leq \|x - y\|_2\}$ for fixed y
e. The set of points is not a convex set because for $x | dist(x, S) \leq dist(x, T)$, both subset S and T are not convex sets when $x \in R | x \leq -1/2$ or $x \geq 1/2$