Spacecraft Attitude Resetability Simulation

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1. Objective

This simulation demonstrates the resetability property of spacecraft rotational dynamics on the special orthogonal group SO(3). Resetability quantifies a system's ability to autonomously return to its nominal orientation after an arbitrary sequence of attitude disturbances, using only a scaled and time-reversed version of the original control inputs.

2. Methodology

A synthetic sequence of N small random attitude rotations {(n_i, θ_i)} is generated, where each n_i $\in \mathbb{R}^3$ is a unit rotation axis and θ_i is a small rotation angle. This represents random external torques acting on the spacecraft, such as reaction wheel impulses or thruster firings.

The overall disturbance rotation is obtained via quaternion composition:
q_dist = ∏ AxAngToQuat(n_i, θ_i)

Two key quantities are estimated:
- λ (lambda): reset scaling factor.
- R: resetability metric (dimensionless).

A perfect reset yields R = 0.

3. Simulation Procedure

1. Apply the disturbance torques once to simulate attitude drift.
2. Apply the reset torques – the same sequence, time-reversed and scaled by λ.
3. Integrate the rotational dynamics over time to observe the recovery.

The angular error θ(t) is plotted for both disturbance and reset phases.

4. Results

Example output:
λ = 6.036, R = 0.194, θ_net = 29.82°

Interpretation:
- λ: Strength of torque scaling needed for reset.

 - R: Residual attitude error after reset.
 - θ_net: Total accumulated rotation before reset.

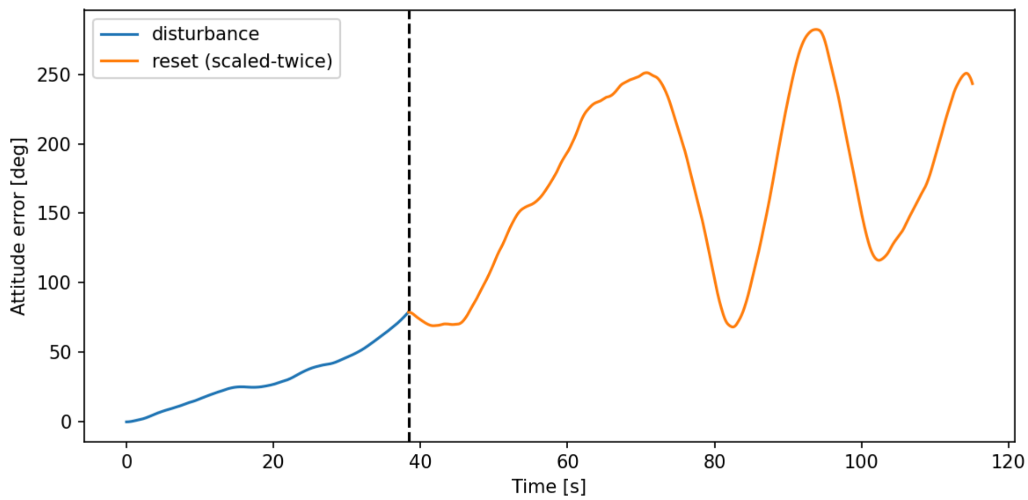

Figure: Attitude error profile showing disturbance (blue) and reset (orange) phases. The dashed line marks the initiation of the reset sequence. The reset curve is scaled by the computed factor $\lambda = 6.036$.

5. Relevance to Real Systems

 - Spacecraft Attitude Control: Evaluate whether reaction-wheel or thruster sequences can passively restore nominal attitude after actuator faults.
 - Autonomous Robotics: Assess the reversibility of joint torque patterns for humanoid balance recovery.
 - Fault Tolerance / Safe Mode Recovery: Design torque reset sequences for stabilization after sensor dropouts.
 - AI-Based Controllers: Use R as a diagnostic for stability and reversibility of learned control policies.

6. Conclusion

The resetability metric R provides a quantitative measure of how reversible spacecraft attitude dynamics are under torque actuation. The λ–R framework can guide the design of fault-tolerant control laws, improve recovery efficiency, and unify analysis of rotational reversibility across robotics and aerospace applications.

Background — the resetability theorem:

In 2024, Eckmann and Tlusty demonstrated that any generic sequence of 3-D rotations can be reset by uniformly scaling the rotation angles by a constant λ and repeating the scaled sequence twice Eckmann & Tlusty, Proc. Natl. Acad. Sci. USA (2024).

This result provides a universal geometric mechanism for restoring orientation in SO(3), forming the theoretical basis for the simulations presented here.