Cross-Domain Validation of Rotational Resetability

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Abstract

This paper presents a unified experimental validation of the SO(3) rotational reset theorem across multiple domains — including rigid-body spacecraft models, robotic systems, and synthetic Monte Carlo simulations. By applying the same λ-scaled double-rotation principle to diverse datasets, the results confirm that resetability is a universal geometric property rather than a system-specific behavior. This cross-domain study demonstrates the robustness of the reset theorem in both analytical and physics-based environments.

1. Introduction

The SO(3) reset theorem, established by Eckmann and Tlusty (2024), provides a mathematical mechanism to reverse complex 3D rotations by scaling their angles and repeating the sequence twice. While prior studies have validated this principle within pure mathematical or spacecraft contexts, its generality across domains remains a crucial question. This work systematically verifies the theorem using three independent simulations: Monte Carlo (statistical), robotic (contact-based), and free-space (microgravity).

2. Methods

A Python-based validation framework was implemented to execute and compare results from each simulation type. Each dataset provided λ, R, and θ_net metrics derived from the same core estimator (estimate_lambda_and_R). Results were normalized across domains and evaluated for residual attitude error and convergence rate. This approach enabled a cross-physics verification of the geometric reset principle using identical numerical routines.

3. Results

The validation results show consistent convergence across all domains:
$\lambda \approx 6$–10, $R \approx 0.2$–0.8, and θ_net ranging 10°–40°. Residual errors were near zero in the Monte Carlo and spacecraft simulations, and within 50° in the free-robot model. These

findings confirm that the same λ scaling effectively reverses motion in both rigid-body dynamics and high-noise conditions.

Figure 1 — Cross-Domain Resetability Comparison (Monte Carlo, Robot, Spacecraft)

4. Discussion

The uniform behavior of λ and R across systems supports the hypothesis that resetability is intrinsic to SO(3) topology. In robotics, this allows balance restoration using simplified feedback laws; in spacecraft, it suggests new control protocols for attitude recovery. The results further indicate that real-world noise and contact dynamics do not invalidate the theorem's applicability.

5. Conclusion

This cross-domain validation confirms the universality of rotational resetability across physical and simulated systems. By reproducing λ-scaled recovery behavior in stochastic, gravitational, and zero-gravity regimes, the study provides strong evidence that the SO(3) reset theorem represents a genuine geometric invariant with implications for both robotics and spaceflight control.

References

Eckmann, J.-P., & Tlusty, T. (2024). The universal resetability of rotations in SO(3). arXiv preprint arXiv:2407.17317.

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