Métodos Numéricos - MAT 1105

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para todos los estudiantes de la Facultad Nacional de Ingeniería

Sea el sistema

$$
\begin{aligned}
2 \cdot x_1 - x_2 + 4 \cdot x_3 + x_4 - x_5 &= 7 \\
-x_1 + 3 \cdot x_2 - 2 \cdot x_3 - x_4 + 2 \cdot x_5 &= 1 \\
5 \cdot x_1 + x_2 + 3 \cdot x_3 - 4 \cdot x_4 + x_5 &= 33 \\
3 \cdot x_1 - 2 \cdot x_2 - 2 \cdot x_3 - 2 \cdot x_4 + 3 \cdot x_5 &= 24 \\
-4 \cdot x_1 - x_2 - 5 \cdot x_3 + 3 \cdot x_4 - 4 \cdot x_5 &= -49
\end{aligned}
$$

Reescribiendo

$$
\begin{aligned}
(2) \cdot x_1 + (-1) \cdot x_2 + (4) \cdot x_3 + (1) \cdot x_4 + (-1) \cdot x_5 &= 7 \\
(-1) \cdot x_1 + (3) \cdot x_2 + (-2) \cdot x_3 + (-1) \cdot x_4 + (2) \cdot x_5 &= 1 \\
(5) \cdot x_1 + (1) \cdot x_2 + (3) \cdot x_3 + (-4) \cdot x_4 + (1) \cdot x_5 &= 33 \\
(3) \cdot x_1 + (-2) \cdot x_2 + (-2) \cdot x_3 + (-2) \cdot x_4 + (3) \cdot x_5 &= 24 \\
(-4) \cdot x_1 + (-1) \cdot x_2 + (-5) \cdot x_3 + (3) \cdot x_4 + (-4) \cdot x_5 &= -49
\end{aligned}
$$

Expresando en forma matricial

$$
\begin{pmatrix}
2 & -1 & 4 & 1 & -1 \\
-1 & 3 & -2 & -1 & 2 \\
5 & 1 & 3 & -4 & 1 \\
3 & -2 & -2 & -2 & 3 \\
-4 & -1 & -5 & 3 & -4
\end{pmatrix}
\times
\begin{pmatrix}
x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5
\end{pmatrix}
=
\begin{pmatrix}
7 \\ 1 \\ 33 \\ 24 \\ -49
\end{pmatrix}
$$

Usando la matriz aumentada con coeficientes y terminos independientes

$$
\left[\begin{array}{ccccc|c}
2 & -1 & 4 & 1 & -1 & 7 \\
-1 & 3 & -2 & -1 & 2 & 1 \\
5 & 1 & 3 & -4 & 1 & 33 \\
3 & -2 & -2 & -2 & 3 & 24 \\
-4 & -1 & -5 & 3 & -4 & -49
\end{array}\right]
$$

Intercambiando fila 1 por la fila 3

$$
\left[\begin{array}{ccccc|c}
2 & -1 & 4 & 1 & -1 & 7 \\
-1 & 3 & -2 & -1 & 2 & 1 \\
5 & 1 & 3 & -4 & 1 & 33 \\
3 & -2 & -2 & -2 & 3 & 24 \\
-4 & -1 & -5 & 3 & -4 & -49
\end{array}\right]
$$

Multiplicando la fila 1 por $(1/5)$ y sumando a la fila 2

$$
\left[\begin{array}{ccccc|c}
5 & 1 & 3 & -4 & 1 & 33 \\
-1 & 3 & -2 & -1 & 2 & 1 \\
2 & -1 & 4 & 1 & -1 & 7 \\
3 & -2 & -2 & -2 & 3 & 24 \\
-4 & -1 & -5 & 3 & -4 & -49
\end{array}\right]
\times \left(\frac{1}{5}\right)
$$

Calculos auxiliares:

$5 \times (1/5) + -1 = 0$ | $1 \times (1/5) + 3 = 16/5$ | $3 \times (1/5) + -2 = -7/5$ | $-4 \times (1/5) + -1 = -9/5$ | $1 \times (1/5) + 2 = 11/5$ | $33 \times (1/5) + 1 = 38/5$

Multiplicando la fila 1 por $(-2/5)$ y sumando a la fila 3

$$\left[\begin{array}{ccccc|c} 5 & 1 & 3 & -4 & 1 & 33 \\ 0 & 16/5 & -7/5 & -9/5 & 11/5 & 38/5 \\ 2 & -1 & 4 & 1 & -1 & 7 \\ 3 & -2 & -2 & -2 & 3 & 24 \\ -4 & -1 & -5 & 3 & -4 & -49 \end{array}\right] \times \left(-\frac{2}{5}\right)$$

Calculos auxiliares:

$5 \times (-2/5) + 2 = 0$ | $1 \times (-2/5) + -1 = -7/5$ | $3 \times (-2/5) + 4 = 14/5$ | $-4 \times (-2/5) + 1 = 13/5$ | $1 \times (-2/5) + -1 = -7/5$ | $33 \times (-2/5) + 7 = -31/5$

Multiplicando la fila 1 por $(-3/5)$ y sumando a la fila 4

$$\left[\begin{array}{ccccc|c} 5 & 1 & 3 & -4 & 1 & 33 \\ 0 & 16/5 & -7/5 & -9/5 & 11/5 & 38/5 \\ 0 & -7/5 & 14/5 & 13/5 & -7/5 & -31/5 \\ 3 & -2 & -2 & -2 & 3 & 24 \\ -4 & -1 & -5 & 3 & -4 & -49 \end{array}\right] \times \left(-\frac{3}{5}\right)$$

Calculos auxiliares:

$5 \times (-3/5) + 3 = 0$ | $1 \times (-3/5) + -2 = -13/5$ | $3 \times (-3/5) + -2 = -19/5$ | $-4 \times (-3/5) + -2 = 2/5$ | $1 \times (-3/5) + 3 = 12/5$ | $33 \times (-3/5) + 24 = 21/5$

Multiplicando la fila 1 por $(4/5)$ y sumando a la fila 5

$$\left[\begin{array}{ccccc|c} 5 & 1 & 3 & -4 & 1 & 33 \\ 0 & 16/5 & -7/5 & -9/5 & 11/5 & 38/5 \\ 0 & -7/5 & 14/5 & 13/5 & -7/5 & -31/5 \\ 0 & -13/5 & -19/5 & 2/5 & 12/5 & 21/5 \\ -4 & -1 & -5 & 3 & -4 & -49 \end{array}\right] \times \left(\frac{4}{5}\right)$$

Calculos auxiliares:

$5 \times (4/5) + -4 = 0$ | $1 \times (4/5) + -1 = -1/5$ | $3 \times (4/5) + -5 = -13/5$ | $-4 \times (4/5) + 3 = -1/5$ | $1 \times (4/5) + -4 = -16/5$ | $33 \times (4/5) + -49 = -113/5$

Multiplicando la fila 2 por $(7/16)$ y sumando a la fila 3

$$\left[\begin{array}{ccccc|c} 5 & 1 & 3 & -4 & 1 & 33 \\ 0 & 16/5 & -7/5 & -9/5 & 11/5 & 38/5 \\ 0 & -7/5 & 14/5 & 13/5 & -7/5 & -31/5 \\ 0 & -13/5 & -19/5 & 2/5 & 12/5 & 21/5 \\ 0 & -1/5 & -13/5 & -1/5 & -16/5 & -113/5 \end{array}\right] \times \left(\frac{7}{16}\right)$$

Calculos auxiliares:

$16/5 \times (7/16) + -7/5 = 0$ | $-7/5 \times (7/16) + 14/5 = 35/16$ | $-9/5 \times (7/16) + 13/5 = 29/16$ | $11/5 \times (7/16) + -7/5 = -7/16$ | $38/5 \times (7/16) + -31/5 = -23/8$

Multiplicando la fila 2 por $(13/16)$ y sumando a la fila 4

$$\left[\begin{array}{ccccc|c} 5 & 1 & 3 & -4 & 1 & 33 \\ 0 & 16/5 & -7/5 & -9/5 & 11/5 & 38/5 \\ 0 & 0 & 35/16 & 29/16 & -7/16 & -23/8 \\ 0 & -13/5 & -19/5 & 2/5 & 12/5 & 21/5 \\ 0 & -1/5 & -13/5 & -1/5 & -16/5 & -113/5 \end{array}\right] \times \left(\frac{13}{16}\right)$$

Calculos auxiliares:

$16/5 \times (13/16) + -13/5 = 0$ | $-7/5 \times (13/16) + -19/5 = -79/16$ | $-9/5 \times (13/16) + 2/5 = -17/16$ | $11/5 \times (13/16) + 12/5 = 67/16$ | $38/5 \times (13/16) + 21/5 = 83/8$

Multiplicando la fila 2 por $(1/16)$ y sumando a la fila 5

$$\left[\begin{array}{ccccc|c} 5 & 1 & 3 & -4 & 1 & 33 \\ 0 & 16/5 & -7/5 & -9/5 & 11/5 & 38/5 \\ 0 & 0 & 35/16 & 29/16 & -7/16 & -23/8 \\ 0 & 0 & -79/16 & -17/16 & 67/16 & 83/8 \\ 0 & -1/5 & -13/5 & -1/5 & -16/5 & -113/5 \end{array}\right] \times \left(\frac{1}{16}\right)$$

Calculos auxiliares:

$16/5 \times (1/16) + -1/5 = 0$ | $-7/5 \times (1/16) + -13/5 = -43/16$ | $-9/5 \times (1/16) + -1/5 = -5/16$ | $11/5 \times (1/16) + -16/5 = -49/16$ | $38/5 \times (1/16) + -113/5 = -177/8$

Intercambiando fila 3 por la fila 4

$$\left[\begin{array}{ccccc|c} 5 & 1 & 3 & -4 & 1 & 33 \\ 0 & 16/5 & -7/5 & -9/5 & 11/5 & 38/5 \\ 0 & 0 & 35/16 & 29/16 & -7/16 & -23/8 \\ 0 & 0 & -79/16 & -17/16 & 67/16 & 83/8 \\ 0 & 0 & -43/16 & -5/16 & -49/16 & -177/8 \end{array}\right]$$

Multiplicando la fila 3 por $(35/79)$ y sumando a la fila 4

$$\left[\begin{array}{ccccc|c} 5 & 1 & 3 & -4 & 1 & 33 \\ 0 & 16/5 & -7/5 & -9/5 & 11/5 & 38/5 \\ 0 & 0 & -79/16 & -17/16 & 67/16 & 83/8 \\ 0 & 0 & 35/16 & 29/16 & -7/16 & -23/8 \\ 0 & 0 & -43/16 & -5/16 & -49/16 & -177/8 \end{array}\right] \times \left(\frac{35}{79}\right)$$

Calculos auxiliares:

$-79/16 \times (35/79) + 35/16 = 0$ | $-17/16 \times (35/79) + 29/16 = 106/79$ | $67/16 \times (35/79) + -7/16 = 112/79$ | $83/8 \times (35/79) + -23/8 = 136/79$

Multiplicando la fila 3 por $(-43/79)$ y sumando a la fila 5

$$\left[\begin{array}{ccccc|c} 5 & 1 & 3 & -4 & 1 & 33 \\ 0 & {}^{16}/_{5} & {}^{-7}/_{5} & {}^{-9}/_{5} & {}^{11}/_{5} & {}^{38}/_{5} \\ 0 & 0 & {}^{-79}/_{16} & {}^{-17}/_{16} & {}^{67}/_{16} & {}^{83}/_{8} \\ 0 & 0 & 0 & {}^{106}/_{79} & {}^{112}/_{79} & {}^{136}/_{79} \\ 0 & 0 & {}^{-43}/_{16} & {}^{-5}/_{16} & {}^{-49}/_{16} & {}^{-177}/_{8} \end{array}\right] \times \left(-\frac{43}{79}\right)$$

Calculos auxiliares:

$\boxed{{}^{-79}/_{16} \times ({}^{-43}/_{79}) + {}^{-43}/_{16} = 0}$ $\boxed{{}^{-17}/_{16} \times ({}^{-43}/_{79}) + {}^{-5}/_{16} = {}^{21}/_{79}}$ $\boxed{{}^{67}/_{16} \times ({}^{-43}/_{79}) + {}^{-49}/_{16} = {}^{-422}/_{79}}$
$\boxed{{}^{83}/_{8} \times ({}^{-43}/_{79}) + {}^{-177}/_{8} = {}^{-2194}/_{79}}$

Multiplicando la fila 4 por $({}^{-21}/_{106})$ y sumando a la fila 5

$$\left[\begin{array}{ccccc|c} 5 & 1 & 3 & -4 & 1 & 33 \\ 0 & {}^{16}/_{5} & {}^{-7}/_{5} & {}^{-9}/_{5} & {}^{11}/_{5} & {}^{38}/_{5} \\ 0 & 0 & {}^{-79}/_{16} & {}^{-17}/_{16} & {}^{67}/_{16} & {}^{83}/_{8} \\ 0 & 0 & 0 & {}^{106}/_{79} & {}^{112}/_{79} & {}^{136}/_{79} \\ 0 & 0 & 0 & {}^{21}/_{79} & {}^{-422}/_{79} & {}^{-2194}/_{79} \end{array}\right] \times \left(-\frac{21}{106}\right)$$

Calculos auxiliares:

$\boxed{{}^{106}/_{79} \times ({}^{-21}/_{106}) + {}^{21}/_{79} = 0}$ $\boxed{{}^{112}/_{79} \times ({}^{-21}/_{106}) + {}^{-422}/_{79} = {}^{-298}/_{53}}$ $\boxed{{}^{136}/_{79} \times ({}^{-21}/_{106}) + {}^{-2194}/_{79} = {}^{-1490}/_{53}}$

$$\left[\begin{array}{ccccc|c} 5 & 1 & 3 & -4 & 1 & 33 \\ 0 & {}^{16}/_{5} & {}^{-7}/_{5} & {}^{-9}/_{5} & {}^{11}/_{5} & {}^{38}/_{5} \\ 0 & 0 & {}^{-79}/_{16} & {}^{-17}/_{16} & {}^{67}/_{16} & {}^{83}/_{8} \\ 0 & 0 & 0 & {}^{106}/_{79} & {}^{112}/_{79} & {}^{136}/_{79} \\ 0 & 0 & 0 & 0 & {}^{-298}/_{53} & {}^{-1490}/_{53} \end{array}\right]$$

De la fila 5 podemos ver que:

$$\left[\begin{array}{ccccc|c} 5 & 1 & 3 & -4 & 1 & 33 \\ 0 & {}^{16}/_{5} & {}^{-7}/_{5} & {}^{-9}/_{5} & {}^{11}/_{5} & {}^{38}/_{5} \\ 0 & 0 & {}^{-79}/_{16} & {}^{-17}/_{16} & {}^{67}/_{16} & {}^{83}/_{8} \\ 0 & 0 & 0 & {}^{106}/_{79} & {}^{112}/_{79} & {}^{136}/_{79} \\ 0 & 0 & 0 & 0 & {}^{-298}/_{53} & {}^{-1490}/_{53} \end{array}\right]$$

$$({}^{-298}/_{53}) \cdot x_5 = {}^{-1490}/_{53}$$

$$x_5 = \frac{{}^{-1490}/_{53}}{{}^{-298}/_{53}}$$

$$\boxed{x_5 = 5}$$

De la fila 4 podemos ver que:

$$\left[\begin{array}{ccccc|c} 5 & 1 & 3 & -4 & 1 & 33 \\ 0 & {}^{16}/_{5} & {}^{-7}/_{5} & {}^{-9}/_{5} & {}^{11}/_{5} & {}^{38}/_{5} \\ 0 & 0 & {}^{-79}/_{16} & {}^{-17}/_{16} & {}^{67}/_{16} & {}^{83}/_{8} \\ 0 & 0 & 0 & {}^{106}/_{79} & {}^{112}/_{79} & {}^{136}/_{79} \\ 0 & 0 & 0 & 0 & {}^{-298}/_{53} & {}^{-1490}/_{53} \end{array}\right]$$

$$(106/79) \cdot x_4 + (112/79) \cdot x_5 = 136/79$$

$$x_4 = \frac{136/79 - (112/79) \cdot x_5}{106/79}$$

$$x_4 = \frac{136/79 - (112/79) \cdot (-1490/53)}{106/79}$$

$$x_4 = (-424/79)/(106/79)$$

$$\boxed{x_4 = -4}$$

De la fila 3 podemos ver que:

$$\left[\begin{array}{ccccc|c} 5 & 1 & 3 & -4 & 1 & 33 \\ 0 & 16/5 & -7/5 & -9/5 & 11/5 & 38/5 \\ 0 & 0 & -79/16 & -17/16 & 67/16 & 83/8 \\ 0 & 0 & 0 & 106/79 & 112/79 & 136/79 \\ 0 & 0 & 0 & 0 & -298/53 & -1490/53 \end{array}\right]$$

$$(-79/16) \cdot x_3 + (-17/16) \cdot x_4 + (67/16) \cdot x_5 = 83/8$$

$$x_3 = \frac{83/8 - (-17/16) \cdot x_4 - (67/16) \cdot x_5}{-79/16}$$

$$x_3 = \frac{83/8 - (-17/16) \cdot (136/79) - (67/16) \cdot (-1490/53)}{-79/16}$$

$$x_3 = (-237/16)/(-79/16)$$

$$\boxed{x_3 = 3}$$

De la fila 2 podemos ver que:

$$\left[\begin{array}{ccccc|c} 5 & 1 & 3 & -4 & 1 & 33 \\ 0 & 16/5 & -7/5 & -9/5 & 11/5 & 38/5 \\ 0 & 0 & -79/16 & -17/16 & 67/16 & 83/8 \\ 0 & 0 & 0 & 106/79 & 112/79 & 136/79 \\ 0 & 0 & 0 & 0 & -298/53 & -1490/53 \end{array}\right]$$

$$(16/5) \cdot x_2 + (-7/5) \cdot x_3 + (-9/5) \cdot x_4 + (11/5) \cdot x_5 = 38/5$$

$$x_2 = \frac{38/5 - (-7/5) \cdot x_3 - (-9/5) \cdot x_4 - (11/5) \cdot x_5}{16/5}$$

$$x_2 = \frac{38/5 - (-7/5) \cdot (83/8) - (-9/5) \cdot (136/79) - (11/5) \cdot (-1490/53)}{16/5}$$

$$x_2 = (-32/5)/(16/5)$$

$$\boxed{x_2 = -2}$$

De la fila 1 podemos ver que:

$$\left[\begin{array}{ccccc|c} 5 & 1 & 3 & -4 & 1 & 33 \\ 0 & {}^{16}/_{5} & -{}^{7}/_{5} & -{}^{9}/_{5} & {}^{11}/_{5} & {}^{38}/_{5} \\ 0 & 0 & -{}^{79}/_{16} & -{}^{17}/_{16} & {}^{67}/_{16} & {}^{83}/_{8} \\ 0 & 0 & 0 & {}^{106}/_{79} & {}^{112}/_{79} & {}^{136}/_{79} \\ 0 & 0 & 0 & 0 & -{}^{298}/_{53} & -{}^{1490}/_{53} \end{array}\right]$$

$$(5)\cdot x_1 + (1)\cdot x_2 + (3)\cdot x_3 + (-4)\cdot x_4 + (1)\cdot x_5 = 33$$

$$x_1 = \frac{33 - (1)\cdot x_2 - (3)\cdot x_3 - (-4)\cdot x_4 - (1)\cdot x_5}{5}$$

$$x_1 = \frac{33 - (1)\cdot ({}^{38}/_{5}) - (3)\cdot ({}^{83}/_{8}) - (-4)\cdot ({}^{136}/_{79}) - (1)\cdot (-{}^{1490}/_{53})}{5}$$

$$x_1 = {}^{(5)}/_{(5)}$$

$$\boxed{x_1 = 1}$$

Finalmente las soluciones al sistema de ecuaciones son:

$$\begin{cases} x_1 = 1 \\ x_2 = -2 \\ x_3 = 3 \\ x_4 = -4 \\ x_5 = 5 \end{cases}$$

Validando soluciones
El sistema original es:

$$(2)\cdot x_1 + (-1)\cdot x_2 + (4)\cdot x_3 + (1)\cdot x_4 + (-1)\cdot x_5 = 7$$
$$(-1)\cdot x_1 + (3)\cdot x_2 + (-2)\cdot x_3 + (-1)\cdot x_4 + (2)\cdot x_5 = 1$$
$$(5)\cdot x_1 + (1)\cdot x_2 + (3)\cdot x_3 + (-4)\cdot x_4 + (1)\cdot x_5 = 33$$
$$(3)\cdot x_1 + (-2)\cdot x_2 + (-2)\cdot x_3 + (-2)\cdot x_4 + (3)\cdot x_5 = 24$$
$$(-4)\cdot x_1 + (-1)\cdot x_2 + (-5)\cdot x_3 + (3)\cdot x_4 + (-4)\cdot x_5 = -49$$

Reemplazando

$$(2)\cdot 1 + (-1)\cdot -2 + (4)\cdot 3 + (1)\cdot -4 + (-1)\cdot 5 = 7$$
$$(-1)\cdot 1 + (3)\cdot -2 + (-2)\cdot 3 + (-1)\cdot -4 + (2)\cdot 5 = 1$$
$$(5)\cdot 1 + (1)\cdot -2 + (3)\cdot 3 + (-4)\cdot -4 + (1)\cdot 5 = 33$$
$$(3)\cdot 1 + (-2)\cdot -2 + (-2)\cdot 3 + (-2)\cdot -4 + (3)\cdot 5 = 24$$
$$(-4)\cdot 1 + (-1)\cdot -2 + (-5)\cdot 3 + (3)\cdot -4 + (-4)\cdot 5 = -49$$
$$7 = 7$$
$$1 = 1$$
$$33 = 33$$
$$24 = 24$$
$$-49 = -49$$