



Métodos Numéricos - MAT 1105

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para todos los estudiantes de la Facultad Nacional de Ingeniería

Sea el sistema

$$\begin{aligned} -3 \cdot x_2 + x_3 &= 7 \\ 2 \cdot x_1 + x_2 + 3 \cdot x_3 &= 15 \\ x_1 + x_3 &= 6 \end{aligned}$$

Reescribiendo

$$\begin{aligned} (0) \cdot x_1 + (-3) \cdot x_2 + (1) \cdot x_3 &= 7 \\ (2) \cdot x_1 + (1) \cdot x_2 + (3) \cdot x_3 &= 15 \\ (1) \cdot x_1 + (0) \cdot x_2 + (1) \cdot x_3 &= 6 \end{aligned}$$

Expresando en forma matricial

$$\begin{pmatrix} 0 & -3 & 1 \\ 2 & 1 & 3 \\ 1 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7 \\ 15 \\ 6 \end{pmatrix}$$

Usando la matriz aumentada con coeficientes y terminos independientes

$$\left[\begin{array}{ccc|ccc|c} 0 & -3 & 1 & 1 & 0 & 0 & 7 \\ 2 & 1 & 3 & 0 & 1 & 0 & 15 \\ 1 & 0 & 1 & 0 & 0 & 1 & 6 \end{array} \right]$$

Intercambiando fila 1 por la fila 2

$$\left[\begin{array}{ccc|ccc|c} 2 & 1 & 3 & 0 & 1 & 0 & 15 \\ 0 & -3 & 1 & 1 & 0 & 0 & 7 \\ 1 & 0 & 1 & 0 & 0 & 1 & 6 \end{array} \right]$$

Dividiendo la fila 1 por (2)

$$\left[\begin{array}{ccc|ccc|c} 1 & 1/2 & 3/2 & 0 & 1/2 & 0 & 15/2 \\ 0 & -3 & 1 & 1 & 0 & 0 & 7 \\ 1 & 0 & 1 & 0 & 0 & 1 & 6 \end{array} \right] / (2)$$

Calculos auxiliares:

$$\boxed{(2)/(2) = 1} \quad \boxed{(1)/(2) = 1/2} \quad \boxed{(3)/(2) = 3/2} \quad \boxed{(0)/(2) = 0} \quad \boxed{(1)/(2) = 1/2} \quad \boxed{(0)/(2) = 0} \quad \boxed{(15)/(2) = 15/2}$$

Multiplicando la fila 1 por (-1) y sumando a la fila 3

$$\left[\begin{array}{ccc|ccc|c} 1 & 1/2 & 3/2 & 0 & 1/2 & 0 & 15/2 \\ 0 & -3 & 1 & 1 & 0 & 0 & 7 \\ 1 & 0 & 1 & 0 & 0 & 1 & 6 \end{array} \right] \times (-1)$$

Calculos auxiliares:

$$\boxed{1 \times (-1) + 1 = 0} \quad \boxed{1/2 \times (-1) + 0 = -1/2} \quad \boxed{3/2 \times (-1) + 1 = -1/2} \quad \boxed{0 \times (-1) + 0 = 0} \quad \boxed{1/2 \times (-1) + 0 = -1/2}$$

$$\boxed{0 \times (-1) + 1 = 1} \quad \boxed{15/2 \times (-1) + 6 = -3/2}$$

Dividiendo la fila 2 por (-3)

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 3/2 & 0 & 1/2 & 0 & 15/2 \\ 0 & -3 & 1 & 1 & 0 & 0 & 7 \\ 0 & -1/2 & -1/2 & 0 & -1/2 & 1 & -3/2 \end{array} \right] / (-3)$$

Cálculos auxiliares:

$$(-3) / (-3) = 1 \quad (1) / (-3) = -1/3 \quad (1) / (-3) = -1/3 \quad (0) / (-3) = 0 \quad (0) / (-3) = 0 \quad (7) / (-3) = -7/3$$

Multiplicando la fila 2 por $(-1/2)$ y sumando a la fila 1

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 3/2 & 0 & 1/2 & 0 & 15/2 \\ 0 & 1 & -1/3 & -1/3 & 0 & 0 & -7/3 \\ 0 & -1/2 & -1/2 & 0 & -1/2 & 1 & -3/2 \end{array} \right] \times \left(-\frac{1}{2} \right)$$

Cálculos auxiliares:

$$1 \times (-1/2) + 1/2 = 0 \quad -1/3 \times (-1/2) + 3/2 = 5/3 \quad -1/3 \times (-1/2) + 0 = 1/6 \quad 0 \times (-1/2) + 1/2 = 1/2$$

$$0 \times (-1/2) + 0 = 0 \quad -7/3 \times (-1/2) + 15/2 = 26/3$$

Multiplicando la fila 2 por $(1/2)$ y sumando a la fila 3

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 5/3 & 1/6 & 1/2 & 0 & 26/3 \\ 0 & 1 & -1/3 & -1/3 & 0 & 0 & -7/3 \\ 0 & -1/2 & -1/2 & 0 & -1/2 & 1 & -3/2 \end{array} \right] \times \left(\frac{1}{2} \right)$$

Cálculos auxiliares:

$$1 \times (1/2) + -1/2 = 0 \quad -1/3 \times (1/2) + -1/2 = -2/3 \quad -1/3 \times (1/2) + 0 = -1/6 \quad 0 \times (1/2) + -1/2 = -1/2$$

$$0 \times (1/2) + 1 = 1 \quad -7/3 \times (1/2) + -3/2 = -8/3$$

Dividiendo la fila 3 por $(-2/3)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 5/3 & 1/6 & 1/2 & 0 & 26/3 \\ 0 & 1 & -1/3 & -1/3 & 0 & 0 & -7/3 \\ 0 & 0 & -2/3 & -1/6 & -1/2 & 1 & -8/3 \end{array} \right] / \left(-\frac{2}{3} \right)$$

Cálculos auxiliares:

$$(-2/3) / (-2/3) = 1 \quad (-1/6) / (-2/3) = 1/4 \quad (-1/2) / (-2/3) = 3/4 \quad (1) / (-2/3) = -3/2 \quad (-8/3) / (-2/3) = 4$$

Multiplicando la fila 3 por $(-5/3)$ y sumando a la fila 1

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 5/3 & 1/6 & 1/2 & 0 & 26/3 \\ 0 & 1 & -1/3 & -1/3 & 0 & 0 & -7/3 \\ 0 & 0 & 1 & 1/4 & 3/4 & -3/2 & 4 \end{array} \right] \times \left(-\frac{5}{3} \right)$$

Cálculos auxiliares:

$$1 \times (-5/3) + 5/3 = 0 \quad 1/4 \times (-5/3) + 1/6 = -1/4 \quad 3/4 \times (-5/3) + 1/2 = -3/4 \quad -3/2 \times (-5/3) + 0 = 5/2$$

$$4 \times (-5/3) + 26/3 = 2$$

Multiplicando la fila 3 por $(1/3)$ y sumando a la fila 2

$$\left[\begin{array}{ccc|ccc|c} 1 & 0 & 0 & -1/4 & -3/4 & 5/2 & 2 \\ 0 & 1 & -1/3 & -1/3 & 0 & 0 & -7/3 \\ 0 & 0 & 1 & 1/4 & 3/4 & -3/2 & 4 \end{array} \right] \times \left(\frac{1}{3} \right)$$

Calculos auxiliares:

$$\boxed{1 \times (1/3) + -1/3 = 0} \quad \boxed{1/4 \times (1/3) + -1/3 = -1/4} \quad \boxed{3/4 \times (1/3) + 0 = 1/4} \quad \boxed{-3/2 \times (1/3) + 0 = -1/2}$$

$$\boxed{4 \times (1/3) + -7/3 = -1}$$

$$\left[\begin{array}{ccc|ccc|c} 1 & 0 & 0 & -1/4 & -3/4 & 5/2 & 2 \\ 0 & 1 & 0 & -1/4 & 1/4 & -1/2 & -1 \\ 0 & 0 & 1 & 1/4 & 3/4 & -3/2 & 4 \end{array} \right]$$

De la fila 1 podemos ver que:

$$\left[\begin{array}{ccc|ccc|c} 1 & 0 & 0 & -1/4 & -3/4 & 5/2 & 2 \\ 0 & 1 & 0 & -1/4 & 1/4 & -1/2 & -1 \\ 0 & 0 & 1 & 1/4 & 3/4 & -3/2 & 4 \end{array} \right]$$

$$\boxed{x_1 = 2}$$

De la fila 2 podemos ver que:

$$\left[\begin{array}{ccc|ccc|c} 1 & 0 & 0 & -1/4 & -3/4 & 5/2 & 2 \\ 0 & 1 & 0 & -1/4 & 1/4 & -1/2 & -1 \\ 0 & 0 & 1 & 1/4 & 3/4 & -3/2 & 4 \end{array} \right]$$

$$\boxed{x_2 = -1}$$

De la fila 3 podemos ver que:

$$\left[\begin{array}{ccc|ccc|c} 1 & 0 & 0 & -1/4 & -3/4 & 5/2 & 2 \\ 0 & 1 & 0 & -1/4 & 1/4 & -1/2 & -1 \\ 0 & 0 & 1 & 1/4 & 3/4 & -3/2 & 4 \end{array} \right]$$

$$\boxed{x_3 = 4}$$

Finalmente las soluciones al sistema de ecuaciones son:

$$\begin{cases} x_1 = 2 \\ x_2 = -1 \\ x_3 = 4 \end{cases}$$

La matriz Inversa es:

$$\begin{pmatrix} -1/4 & -3/4 & 5/2 \\ -1/4 & 1/4 & -1/2 \\ 1/4 & 3/4 & -3/2 \end{pmatrix}$$

Verificamos que la matriz inversa sea correcta
Expresando en forma matricial

$$\begin{pmatrix} 0 & -3 & 1 \\ 2 & 1 & 3 \\ 1 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} -1/4 & -3/4 & 5/2 \\ -1/4 & 1/4 & -1/2 \\ 1/4 & 3/4 & -3/2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Validando soluciones
El sistema original es:

$$(0) \cdot x_1 + (-3) \cdot x_2 + (1) \cdot x_3 = 7$$

$$(2) \cdot x_1 + (1) \cdot x_2 + (3) \cdot x_3 = 15$$

$$(1) \cdot x_1 + (0) \cdot x_2 + (1) \cdot x_3 = 6$$

Reemplazando

$$(0) \cdot 2 + (-3) \cdot -1 + (1) \cdot 4 = 7$$

$$(2) \cdot 2 + (1) \cdot -1 + (3) \cdot 4 = 15$$

$$(1) \cdot 2 + (0) \cdot -1 + (1) \cdot 4 = 6$$

$$7 = 7$$

$$15 = 15$$

$$6 = 6$$