Métodos Numéricos - MAT 1105

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Oruro - 2020

para todos los estudiantes de la Facultad Nacional de Ingeniería

Sea el sistema

$$\begin{aligned}
4 \cdot x_1 + 2 \cdot x_2 + 5 \cdot x_3 + 3 \cdot x_4 + 5 \cdot x_5 &= 36 \\
5 \cdot x_1 + 2 \cdot x_2 + 5 \cdot x_3 + 4 \cdot x_4 + 5 \cdot x_5 &= 42 \\
5 \cdot x_1 + 2 \cdot x_2 + 6 \cdot x_3 + 4 \cdot x_4 + 5 \cdot x_5 &= 43 \\
7 \cdot x_1 + 3 \cdot x_2 + 7 \cdot x_3 + 4 \cdot x_4 + 7 \cdot x_5 &= 53 \\
5 \cdot x_1 + 2 \cdot x_2 + 5 \cdot x_3 + 3 \cdot x_4 + 5 \cdot x_5 &= 38
\end{aligned}$$

Reescribiendo

$$\begin{aligned}
(4) \cdot x_1 + (2) \cdot x_2 + (5) \cdot x_3 + (3) \cdot x_4 + (5) \cdot x_5 &= 36 \\
(5) \cdot x_1 + (2) \cdot x_2 + (5) \cdot x_3 + (4) \cdot x_4 + (5) \cdot x_5 &= 42 \\
(5) \cdot x_1 + (2) \cdot x_2 + (6) \cdot x_3 + (4) \cdot x_4 + (5) \cdot x_5 &= 43 \\
(7) \cdot x_1 + (3) \cdot x_2 + (7) \cdot x_3 + (4) \cdot x_4 + (7) \cdot x_5 &= 53 \\
(5) \cdot x_1 + (2) \cdot x_2 + (5) \cdot x_3 + (3) \cdot x_4 + (5) \cdot x_5 &= 38
\end{aligned}$$

Expresando en forma matricial

$$\begin{pmatrix} 4 & 2 & 5 & 3 & 5 \\ 5 & 2 & 5 & 4 & 5 \\ 5 & 2 & 6 & 4 & 5 \\ 7 & 3 & 7 & 4 & 7 \\ 5 & 2 & 5 & 3 & 5 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 36 \\ 42 \\ 43 \\ 53 \\ 38 \end{pmatrix}$$

Usando la matriz aumentada con coeficientes y terminos independientes

$$\left[\begin{array}{ccccc|ccccc|c} 4 & 2 & 5 & 3 & 5 & 1 & 0 & 0 & 0 & 0 & 36 \\ 5 & 2 & 5 & 4 & 5 & 0 & 1 & 0 & 0 & 0 & 42 \\ 5 & 2 & 6 & 4 & 5 & 0 & 0 & 1 & 0 & 0 & 43 \\ 7 & 3 & 7 & 4 & 7 & 0 & 0 & 0 & 1 & 0 & 53 \\ 5 & 2 & 5 & 3 & 5 & 0 & 0 & 0 & 0 & 1 & 38 \end{array}\right]$$

Intercambiando fila 1 por la fila 4

$$\left[\begin{array}{ccccc|ccccc|c} 4 & 2 & 5 & 3 & 5 & 1 & 0 & 0 & 0 & 0 & 36 \\ 5 & 2 & 5 & 4 & 5 & 0 & 1 & 0 & 0 & 0 & 42 \\ 5 & 2 & 6 & 4 & 5 & 0 & 0 & 1 & 0 & 0 & 43 \\ 7 & 3 & 7 & 4 & 7 & 0 & 0 & 0 & 1 & 0 & 53 \\ 5 & 2 & 5 & 3 & 5 & 0 & 0 & 0 & 0 & 1 & 38 \end{array}\right]$$

Dividiendo la fila 1 por (7)

$$\left[\begin{array}{ccccc|ccccc|c} 7 & 3 & 7 & 4 & 7 & 0 & 0 & 0 & 1 & 0 & 53 \\ 5 & 2 & 5 & 4 & 5 & 0 & 1 & 0 & 0 & 0 & 42 \\ 5 & 2 & 6 & 4 & 5 & 0 & 0 & 1 & 0 & 0 & 43 \\ 4 & 2 & 5 & 3 & 5 & 1 & 0 & 0 & 0 & 0 & 36 \\ 5 & 2 & 5 & 3 & 5 & 0 & 0 & 0 & 0 & 1 & 38 \end{array}\right] \; / \, (7)$$

Calculos auxiliares:

$(7)\,/\,(7) = 1$ | $(3)\,/\,(7) = {}^{3}/_{7}$ | $(7)\,/\,(7) = 1$ | $(4)\,/\,(7) = {}^{4}/_{7}$ | $(7)\,/\,(7) = 1$ | $(0)\,/\,(7) = 0$ | $(0)\,/\,(7) = 0$ | $(0)\,/\,(7) = 0$ | $(1)\,/\,(7) = {}^{1}/_{7}$ | $(0)\,/\,(7) = 0$ | $(53)\,/\,(7) = {}^{53}/_{7}$

Multiplicando la fila 1 por (-5) y sumando a la fila 2

$$\left[\begin{array}{ccccc|ccccc|c} 1 & {}^{3}/_{7} & 1 & {}^{4}/_{7} & 1 & 0 & 0 & 0 & {}^{1}/_{7} & 0 & {}^{53}/_{7} \\ 5 & 2 & 5 & 4 & 5 & 0 & 1 & 0 & 0 & 0 & 42 \\ 5 & 2 & 6 & 4 & 5 & 0 & 0 & 1 & 0 & 0 & 43 \\ 4 & 2 & 5 & 3 & 5 & 1 & 0 & 0 & 0 & 0 & 36 \\ 5 & 2 & 5 & 3 & 5 & 0 & 0 & 0 & 0 & 1 & 38 \end{array}\right] \times (-5)$$

Calculos auxiliares:

$1 \times (-5) + 5 = 0$ | ${}^{3}/_{7} \times (-5) + 2 = -{}^{1}/_{7}$ | $1 \times (-5) + 5 = 0$ | ${}^{4}/_{7} \times (-5) + 4 = {}^{8}/_{7}$ | $1 \times (-5) + 5 = 0$ | $0 \times (-5) + 0 = 0$ | $0 \times (-5) + 1 = 1$ | $0 \times (-5) + 0 = 0$ | ${}^{1}/_{7} \times (-5) + 0 = -{}^{5}/_{7}$ | $0 \times (-5) + 0 = 0$ | ${}^{53}/_{7} \times (-5) + 42 = {}^{29}/_{7}$

Multiplicando la fila 1 por (-5) y sumando a la fila 3

$$\left[\begin{array}{ccccc|ccccc|c} 1 & {}^{3}/_{7} & 1 & {}^{4}/_{7} & 1 & 0 & 0 & 0 & {}^{1}/_{7} & 0 & {}^{53}/_{7} \\ 0 & -{}^{1}/_{7} & 0 & {}^{8}/_{7} & 0 & 0 & 1 & 0 & -{}^{5}/_{7} & 0 & {}^{29}/_{7} \\ 5 & 2 & 6 & 4 & 5 & 0 & 0 & 1 & 0 & 0 & 43 \\ 4 & 2 & 5 & 3 & 5 & 1 & 0 & 0 & 0 & 0 & 36 \\ 5 & 2 & 5 & 3 & 5 & 0 & 0 & 0 & 0 & 1 & 38 \end{array}\right] \times (-5)$$

Calculos auxiliares:

$1 \times (-5) + 5 = 0$ | ${}^{3}/_{7} \times (-5) + 2 = -{}^{1}/_{7}$ | $1 \times (-5) + 6 = 1$ | ${}^{4}/_{7} \times (-5) + 4 = {}^{8}/_{7}$ | $1 \times (-5) + 5 = 0$ | $0 \times (-5) + 0 = 0$ | $0 \times (-5) + 0 = 0$ | $0 \times (-5) + 1 = 1$ | ${}^{1}/_{7} \times (-5) + 0 = -{}^{5}/_{7}$ | $0 \times (-5) + 0 = 0$ | ${}^{53}/_{7} \times (-5) + 43 = {}^{36}/_{7}$

Multiplicando la fila 1 por (-4) y sumando a la fila 4

$$\left[\begin{array}{ccccc|ccccc|c} 1 & {}^{3}/_{7} & 1 & {}^{4}/_{7} & 1 & 0 & 0 & 0 & {}^{1}/_{7} & 0 & {}^{53}/_{7} \\ 0 & -{}^{1}/_{7} & 0 & {}^{8}/_{7} & 0 & 0 & 1 & 0 & -{}^{5}/_{7} & 0 & {}^{29}/_{7} \\ 0 & -{}^{1}/_{7} & 1 & {}^{8}/_{7} & 0 & 0 & 0 & 1 & -{}^{5}/_{7} & 0 & {}^{36}/_{7} \\ 4 & 2 & 5 & 3 & 5 & 1 & 0 & 0 & 0 & 0 & 36 \\ 5 & 2 & 5 & 3 & 5 & 0 & 0 & 0 & 0 & 1 & 38 \end{array}\right] \times (-4)$$

Calculos auxiliares:

$1 \times (-4) + 4 = 0$ | ${}^{3}/_{7} \times (-4) + 2 = {}^{2}/_{7}$ | $1 \times (-4) + 5 = 1$ | ${}^{4}/_{7} \times (-4) + 3 = {}^{5}/_{7}$ | $1 \times (-4) + 5 = 1$ | $0 \times (-4) + 1 = 1$ | $0 \times (-4) + 0 = 0$ | $0 \times (-4) + 0 = 0$ | ${}^{1}/_{7} \times (-4) + 0 = -{}^{4}/_{7}$ | $0 \times (-4) + 0 = 0$ | ${}^{53}/_{7} \times (-4) + 36 = {}^{40}/_{7}$

Multiplicando la fila 1 por (-5) y sumando a la fila 5

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 3/7 & 1 & 4/7 & 1 & 0 & 0 & 0 & 1/7 & 0 & 53/7 \\ 0 & -1/7 & 0 & 8/7 & 0 & 0 & 1 & 0 & -5/7 & 0 & 29/7 \\ 0 & -1/7 & 1 & 8/7 & 0 & 0 & 0 & 1 & -5/7 & 0 & 36/7 \\ 0 & 2/7 & 1 & 5/7 & 1 & 1 & 0 & 0 & -4/7 & 0 & 40/7 \\ 5 & 2 & 5 & 3 & 5 & 0 & 0 & 0 & 0 & 1 & 38 \end{array}\right] \times(-5)$$

Calculos auxiliares:

$1 \times (-5) + 5 = 0$ | $3/7 \times (-5) + 2 = -1/7$ | $1 \times (-5) + 5 = 0$ | $4/7 \times (-5) + 3 = 1/7$ | $1 \times (-5) + 5 = 0$

$0 \times (-5) + 0 = 0$ | $0 \times (-5) + 0 = 0$ | $0 \times (-5) + 0 = 0$ | $1/7 \times (-5) + 0 = -5/7$ | $0 \times (-5) + 1 = 1$

$53/7 \times (-5) + 38 = 1/7$

Intercambiando fila 2 por la fila 4

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 3/7 & 1 & 4/7 & 1 & 0 & 0 & 0 & 1/7 & 0 & 53/7 \\ 0 & -1/7 & 0 & 8/7 & 0 & 0 & 1 & 0 & -5/7 & 0 & 29/7 \\ 0 & -1/7 & 1 & 8/7 & 0 & 0 & 0 & 1 & -5/7 & 0 & 36/7 \\ 0 & 2/7 & 1 & 5/7 & 1 & 1 & 0 & 0 & -4/7 & 0 & 40/7 \\ 0 & -1/7 & 0 & 1/7 & 0 & 0 & 0 & 0 & -5/7 & 1 & 1/7 \end{array}\right]$$

Dividiendo la fila 2 por $(2/7)$

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 3/7 & 1 & 4/7 & 1 & 0 & 0 & 0 & 1/7 & 0 & 53/7 \\ 0 & 2/7 & 1 & 5/7 & 1 & 1 & 0 & 0 & -4/7 & 0 & 40/7 \\ 0 & -1/7 & 1 & 8/7 & 0 & 0 & 0 & 1 & -5/7 & 0 & 36/7 \\ 0 & -1/7 & 0 & 8/7 & 0 & 0 & 1 & 0 & -5/7 & 0 & 29/7 \\ 0 & -1/7 & 0 & 1/7 & 0 & 0 & 0 & 0 & -5/7 & 1 & 1/7 \end{array}\right] / \left(\frac{2}{7}\right)$$

Calculos auxiliares:

$(2/7) \,/\, (2/7) = 1$ | $(1) \,/\, (2/7) = 7/2$ | $(5/7) \,/\, (2/7) = 5/2$ | $(1) \,/\, (2/7) = 7/2$ | $(1) \,/\, (2/7) = 7/2$ | $(0) \,/\, (2/7) = 0$

$(0) \,/\, (2/7) = 0$ | $(-4/7) \,/\, (2/7) = -2$ | $(0) \,/\, (2/7) = 0$ | $(40/7) \,/\, (2/7) = 20$

Multiplicando la fila 2 por $(-3/7)$ y sumando a la fila 1

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 3/7 & 1 & 4/7 & 1 & 0 & 0 & 0 & 1/7 & 0 & 53/7 \\ 0 & 1 & 7/2 & 5/2 & 7/2 & 7/2 & 0 & 0 & -2 & 0 & 20 \\ 0 & -1/7 & 1 & 8/7 & 0 & 0 & 0 & 1 & -5/7 & 0 & 36/7 \\ 0 & -1/7 & 0 & 8/7 & 0 & 0 & 1 & 0 & -5/7 & 0 & 29/7 \\ 0 & -1/7 & 0 & 1/7 & 0 & 0 & 0 & 0 & -5/7 & 1 & 1/7 \end{array}\right] \times \left(-\frac{3}{7}\right)$$

Calculos auxiliares:

$1 \times (-3/7) + 3/7 = 0$ | $7/2 \times (-3/7) + 1 = -1/2$ | $5/2 \times (-3/7) + 4/7 = -1/2$ | $7/2 \times (-3/7) + 1 = -1/2$

$7/2 \times (-3/7) + 0 = -3/2$ | $0 \times (-3/7) + 0 = 0$ | $0 \times (-3/7) + 0 = 0$ | $-2 \times (-3/7) + 1/7 = 1$ | $0 \times (-3/7) + 0 = 0$

$20 \times (-3/7) + 53/7 = -1$

Multiplicando la fila 2 por $(1/7)$ y sumando a la fila 3

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & -1/2 & -1/2 & -1/2 & -3/2 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 7/2 & 5/2 & 7/2 & 7/2 & 0 & 0 & -2 & 0 & 20 \\ 0 & -1/7 & 1 & 8/7 & 0 & 0 & 0 & 1 & -5/7 & 0 & 36/7 \\ 0 & -1/7 & 0 & 8/7 & 0 & 0 & 1 & 0 & -5/7 & 0 & 29/7 \\ 0 & -1/7 & 0 & 1/7 & 0 & 0 & 0 & 0 & -5/7 & 1 & 1/7 \end{array}\right] \times \left(\frac{1}{7}\right)$$

Calculos auxiliares:

$1 \times (1/7) + -1/7 = 0$ | $7/2 \times (1/7) + 1 = 3/2$ | $5/2 \times (1/7) + 8/7 = 3/2$ | $7/2 \times (1/7) + 0 = 1/2$ | $7/2 \times (1/7) + 0 = 1/2$

$0 \times (1/7) + 0 = 0$ | $0 \times (1/7) + 1 = 1$ | $-2 \times (1/7) + -5/7 = -1$ | $0 \times (1/7) + 0 = 0$ | $20 \times (1/7) + 36/7 = 8$

Multiplicando la fila 2 por $(1/7)$ y sumando a la fila 4

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & -1/2 & -1/2 & -1/2 & -3/2 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 7/2 & 5/2 & 7/2 & 7/2 & 0 & 0 & -2 & 0 & 20 \\ 0 & 0 & 3/2 & 3/2 & 1/2 & 1/2 & 0 & 1 & -1 & 0 & 8 \\ 0 & -1/7 & 0 & 8/7 & 0 & 0 & 1 & 0 & -5/7 & 0 & 29/7 \\ 0 & -1/7 & 0 & 1/7 & 0 & 0 & 0 & 0 & -5/7 & 1 & 1/7 \end{array}\right] \times \left(\frac{1}{7}\right)$$

Calculos auxiliares:

$1 \times (1/7) + -1/7 = 0$ | $7/2 \times (1/7) + 0 = 1/2$ | $5/2 \times (1/7) + 8/7 = 3/2$ | $7/2 \times (1/7) + 0 = 1/2$ | $7/2 \times (1/7) + 0 = 1/2$

$0 \times (1/7) + 1 = 1$ | $0 \times (1/7) + 0 = 0$ | $-2 \times (1/7) + -5/7 = -1$ | $0 \times (1/7) + 0 = 0$ | $20 \times (1/7) + 29/7 = 7$

Multiplicando la fila 2 por $(1/7)$ y sumando a la fila 5

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & -1/2 & -1/2 & -1/2 & -3/2 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 7/2 & 5/2 & 7/2 & 7/2 & 0 & 0 & -2 & 0 & 20 \\ 0 & 0 & 3/2 & 3/2 & 1/2 & 1/2 & 0 & 1 & -1 & 0 & 8 \\ 0 & 0 & 1/2 & 3/2 & 1/2 & 1/2 & 1 & 0 & -1 & 0 & 7 \\ 0 & -1/7 & 0 & 1/7 & 0 & 0 & 0 & 0 & -5/7 & 1 & 1/7 \end{array}\right] \times \left(\frac{1}{7}\right)$$

Calculos auxiliares:

$1 \times (1/7) + -1/7 = 0$ | $7/2 \times (1/7) + 0 = 1/2$ | $5/2 \times (1/7) + 1/7 = 1/2$ | $7/2 \times (1/7) + 0 = 1/2$ | $7/2 \times (1/7) + 0 = 1/2$

$0 \times (1/7) + 0 = 0$ | $0 \times (1/7) + 0 = 0$ | $-2 \times (1/7) + -5/7 = -1$ | $0 \times (1/7) + 1 = 1$ | $20 \times (1/7) + 1/7 = 3$

Dividiendo la fila 3 por $(3/2)$

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & -1/2 & -1/2 & -1/2 & -3/2 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 7/2 & 5/2 & 7/2 & 7/2 & 0 & 0 & -2 & 0 & 20 \\ 0 & 0 & 3/2 & 3/2 & 1/2 & 1/2 & 0 & 1 & -1 & 0 & 8 \\ 0 & 0 & 1/2 & 3/2 & 1/2 & 1/2 & 1 & 0 & -1 & 0 & 7 \\ 0 & 0 & 1/2 & 1/2 & 1/2 & 1/2 & 0 & 0 & -1 & 1 & 3 \end{array}\right] / \left(\frac{3}{2}\right)$$

Calculos auxiliares:

$(3/2) / (3/2) = 1$ | $(3/2) / (3/2) = 1$ | $(1/2) / (3/2) = 1/3$ | $(1/2) / (3/2) = 1/3$ | $(0) / (3/2) = 0$ | $(1) / (3/2) = 2/3$

$(-1) / (3/2) = -2/3$ | $(0) / (3/2) = 0$ | $(8) / (3/2) = 16/3$

Multiplicando la fila 3 por $(1/2)$ y sumando a la fila 1

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & -1/2 & -1/2 & -1/2 & -3/2 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 7/2 & 5/2 & 7/2 & 7/2 & 0 & 0 & -2 & 0 & 20 \\ 0 & 0 & 1 & 1 & 1/3 & 1/3 & 0 & 2/3 & -2/3 & 0 & 16/3 \\ 0 & 0 & 1/2 & 3/2 & 1/2 & 1/2 & 1 & 0 & -1 & 0 & 7 \\ 0 & 0 & 1/2 & 1/2 & 1/2 & 1/2 & 0 & 0 & -1 & 1 & 3 \end{array}\right] \times \left(\frac{1}{2}\right)$$

Calculos auxiliares:

$1 \times (1/2) + -1/2 = 0$ | $1 \times (1/2) + -1/2 = 0$ | $1/3 \times (1/2) + -1/2 = -1/3$ | $1/3 \times (1/2) + -3/2 = -4/3$

$0 \times (1/2) + 0 = 0$ | $2/3 \times (1/2) + 0 = 1/3$ | $-2/3 \times (1/2) + 1 = 2/3$ | $0 \times (1/2) + 0 = 0$ | $16/3 \times (1/2) + -1 = 5/3$

Multiplicando la fila 3 por $(-7/2)$ y sumando a la fila 2

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & 0 & 0 & -1/3 & -4/3 & 0 & 1/3 & 2/3 & 0 & 5/3 \\ 0 & 1 & 7/2 & 5/2 & 7/2 & 7/2 & 0 & 0 & -2 & 0 & 20 \\ 0 & 0 & 1 & 1 & 1/3 & 1/3 & 0 & 2/3 & -2/3 & 0 & 16/3 \\ 0 & 0 & 1/2 & 3/2 & 1/2 & 1/2 & 1 & 0 & -1 & 0 & 7 \\ 0 & 0 & 1/2 & 1/2 & 1/2 & 1/2 & 0 & 0 & -1 & 1 & 3 \end{array}\right] \times \left(-\frac{7}{2}\right)$$

Calculos auxiliares:

$1 \times (-7/2) + 7/2 = 0$ | $1 \times (-7/2) + 5/2 = -1$ | $1/3 \times (-7/2) + 7/2 = 7/3$ | $1/3 \times (-7/2) + 7/2 = 7/3$

$0 \times (-7/2) + 0 = 0$ | $2/3 \times (-7/2) + 0 = -7/3$ | $-2/3 \times (-7/2) + -2 = 1/3$ | $0 \times (-7/2) + 0 = 0$ | $16/3 \times (-7/2) + 20 = 4/3$

Multiplicando la fila 3 por $(-1/2)$ y sumando a la fila 4

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & 0 & 0 & -1/3 & -4/3 & 0 & 1/3 & 2/3 & 0 & 5/3 \\ 0 & 1 & 0 & -1 & 7/3 & 7/3 & 0 & -7/3 & 1/3 & 0 & 4/3 \\ 0 & 0 & 1 & 1 & 1/3 & 1/3 & 0 & 2/3 & -2/3 & 0 & 16/3 \\ 0 & 0 & 1/2 & 3/2 & 1/2 & 1/2 & 1 & 0 & -1 & 0 & 7 \\ 0 & 0 & 1/2 & 1/2 & 1/2 & 1/2 & 0 & 0 & -1 & 1 & 3 \end{array}\right] \times \left(-\frac{1}{2}\right)$$

Calculos auxiliares:

$1 \times (-1/2) + 1/2 = 0$ | $1 \times (-1/2) + 3/2 = 1$ | $1/3 \times (-1/2) + 1/2 = 1/3$ | $1/3 \times (-1/2) + 1/2 = 1/3$

$0 \times (-1/2) + 1 = 1$ | $2/3 \times (-1/2) + 0 = -1/3$ | $-2/3 \times (-1/2) + -1 = -2/3$ | $0 \times (-1/2) + 0 = 0$

$16/3 \times (-1/2) + 7 = 13/3$

Multiplicando la fila 3 por $(-1/2)$ y sumando a la fila 5

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & 0 & 0 & -1/3 & -4/3 & 0 & 1/3 & 2/3 & 0 & 5/3 \\ 0 & 1 & 0 & -1 & 7/3 & 7/3 & 0 & -7/3 & 1/3 & 0 & 4/3 \\ 0 & 0 & 1 & 1 & 1/3 & 1/3 & 0 & 2/3 & -2/3 & 0 & 16/3 \\ 0 & 0 & 0 & 1 & 1/3 & 1/3 & 1 & -1/3 & -2/3 & 0 & 13/3 \\ 0 & 0 & 1/2 & 1/2 & 1/2 & 1/2 & 0 & 0 & -1 & 1 & 3 \end{array}\right] \times \left(-\frac{1}{2}\right)$$

Calculos auxiliares:

$1 \times (-1/2) + 1/2 = 0$ | $1 \times (-1/2) + 1/2 = 0$ | $1/3 \times (-1/2) + 1/2 = 1/3$ | $1/3 \times (-1/2) + 1/2 = 1/3$

$0 \times (-1/2) + 0 = 0$ | $2/3 \times (-1/2) + 0 = -1/3$ | $-2/3 \times (-1/2) + -1 = -2/3$ | $0 \times (-1/2) + 1 = 1$

$16/3 \times (-1/2) + 3 = 1/3$

Dividiendo la fila 4 por (1)

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & 0 & 0 & -1/3 & -4/3 & 0 & 1/3 & 2/3 & 0 & 5/3 \\ 0 & 1 & 0 & -1 & 7/3 & 7/3 & 0 & -7/3 & 1/3 & 0 & 4/3 \\ 0 & 0 & 1 & 1 & 1/3 & 1/3 & 0 & 2/3 & -2/3 & 0 & 16/3 \\ 0 & 0 & 0 & 1 & 1/3 & 1/3 & 1 & -1/3 & -2/3 & 0 & 13/3 \\ 0 & 0 & 0 & 0 & 1/3 & 1/3 & 0 & -1/3 & -2/3 & 1 & 1/3 \end{array}\right] \; / \, (1)$$

Calculos auxiliares:

$(1) \,/\, (1) = 1$ | $(1/3) \,/\, (1) = 1/3$ | $(1/3) \,/\, (1) = 1/3$ | $(1) \,/\, (1) = 1$ | $(-1/3) \,/\, (1) = -1/3$ | $(-2/3) \,/\, (1) = -2/3$ | $(0) \,/\, (1) = 0$ | $(13/3) \,/\, (1) = 13/3$

Multiplicando la fila 4 por (1) y sumando a la fila 2

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & 0 & 0 & -1/3 & -4/3 & 0 & 1/3 & 2/3 & 0 & 5/3 \\ 0 & 1 & 0 & -1 & 7/3 & 7/3 & 0 & -7/3 & 1/3 & 0 & 4/3 \\ 0 & 0 & 1 & 1 & 1/3 & 1/3 & 0 & 2/3 & -2/3 & 0 & 16/3 \\ 0 & 0 & 0 & 1 & 1/3 & 1/3 & 1 & -1/3 & -2/3 & 0 & 13/3 \\ 0 & 0 & 0 & 0 & 1/3 & 1/3 & 0 & -1/3 & -2/3 & 1 & 1/3 \end{array}\right] \; \times (1)$$

Calculos auxiliares:

$1 \times (1) + -1 = 0$ | $1/3 \times (1) + 7/3 = 8/3$ | $1/3 \times (1) + 7/3 = 8/3$ | $1 \times (1) + 0 = 1$ | $-1/3 \times (1) + -7/3 = -8/3$ | $-2/3 \times (1) + 1/3 = -1/3$ | $0 \times (1) + 0 = 0$ | $13/3 \times (1) + 4/3 = 17/3$

Multiplicando la fila 4 por (-1) y sumando a la fila 3

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & 0 & 0 & -1/3 & -4/3 & 0 & 1/3 & 2/3 & 0 & 5/3 \\ 0 & 1 & 0 & 0 & 8/3 & 8/3 & 1 & -8/3 & -1/3 & 0 & 17/3 \\ 0 & 0 & 1 & 1 & 1/3 & 1/3 & 0 & 2/3 & -2/3 & 0 & 16/3 \\ 0 & 0 & 0 & 1 & 1/3 & 1/3 & 1 & -1/3 & -2/3 & 0 & 13/3 \\ 0 & 0 & 0 & 0 & 1/3 & 1/3 & 0 & -1/3 & -2/3 & 1 & 1/3 \end{array}\right] \; \times (-1)$$

Calculos auxiliares:

$1 \times (-1) + 1 = 0$ | $1/3 \times (-1) + 1/3 = 0$ | $1/3 \times (-1) + 1/3 = 0$ | $1 \times (-1) + 0 = -1$ | $-1/3 \times (-1) + 2/3 = 1$ | $-2/3 \times (-1) + -2/3 = 0$ | $0 \times (-1) + 0 = 0$ | $13/3 \times (-1) + 16/3 = 1$

Dividiendo la fila 5 por $(1/3)$

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & 0 & 0 & -1/3 & -4/3 & 0 & 1/3 & 2/3 & 0 & 5/3 \\ 0 & 1 & 0 & 0 & 8/3 & 8/3 & 1 & -8/3 & -1/3 & 0 & 17/3 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1/3 & 1/3 & 1 & -1/3 & -2/3 & 0 & 13/3 \\ 0 & 0 & 0 & 0 & 1/3 & 1/3 & 0 & -1/3 & -2/3 & 1 & 1/3 \end{array}\right] \; / \left(\frac{1}{3}\right)$$

Calculos auxiliares:

$(1/3) \,/\, (1/3) = 1$ | $(1/3) \,/\, (1/3) = 1$ | $(0) \,/\, (1/3) = 0$ | $(-1/3) \,/\, (1/3) = -1$ | $(-2/3) \,/\, (1/3) = -2$ | $(1) \,/\, (1/3) = 3$ | $(1/3) \,/\, (1/3) = 1$

Multiplicando la fila 5 por $(1/3)$ y sumando a la fila 1

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & 0 & 0 & -1/3 & -4/3 & 0 & 1/3 & 2/3 & 0 & 5/3 \\ 0 & 1 & 0 & 0 & 8/3 & 8/3 & 1 & -8/3 & -1/3 & 0 & 17/3 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1/3 & 1/3 & 1 & -1/3 & -2/3 & 0 & 13/3 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & -2 & 3 & 1 \end{array}\right] \times \left(\frac{1}{3}\right)$$

Calculos auxiliares:

$1 \times (1/3) + -1/3 = 0$ | $1 \times (1/3) + -4/3 = -1$ | $0 \times (1/3) + 0 = 0$ | $-1 \times (1/3) + 1/3 = 0$ | $-2 \times (1/3) + 2/3 = 0$ | $3 \times (1/3) + 0 = 1$ | $1 \times (1/3) + 5/3 = 2$

Multiplicando la fila 5 por $(-8/3)$ y sumando a la fila 2

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 8/3 & 8/3 & 1 & -8/3 & -1/3 & 0 & 17/3 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1/3 & 1/3 & 1 & -1/3 & -2/3 & 0 & 13/3 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & -2 & 3 & 1 \end{array}\right] \times \left(-\frac{8}{3}\right)$$

Calculos auxiliares:

$1 \times (-8/3) + 8/3 = 0$ | $1 \times (-8/3) + 8/3 = 0$ | $0 \times (-8/3) + 1 = 1$ | $-1 \times (-8/3) + -8/3 = 0$ | $-2 \times (-8/3) + -1/3 = 5$ | $3 \times (-8/3) + 0 = -8$ | $1 \times (-8/3) + 17/3 = 3$

Multiplicando la fila 5 por $(-1/3)$ y sumando a la fila 4

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 5 & -8 & 3 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1/3 & 1/3 & 1 & -1/3 & -2/3 & 0 & 13/3 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & -2 & 3 & 1 \end{array}\right] \times \left(-\frac{1}{3}\right)$$

Calculos auxiliares:

$1 \times (-1/3) + 1/3 = 0$ | $1 \times (-1/3) + 1/3 = 0$ | $0 \times (-1/3) + 1 = 1$ | $-1 \times (-1/3) + -1/3 = 0$ | $-2 \times (-1/3) + -2/3 = 0$ | $3 \times (-1/3) + 0 = -1$ | $1 \times (-1/3) + 13/3 = 4$

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 5 & -8 & 3 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & -2 & 3 & 1 \end{array}\right]$$

De la fila 1 podemos ver que:

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 5 & -8 & 3 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & -2 & 3 & 1 \end{array}\right]$$

$$\boxed{x_1 = 2}$$

De la fila 2 podemos ver que:

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 5 & -8 & 3 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & -2 & 3 & 1 \end{array}\right]$$

$$\boxed{x_2 = 3}$$

De la fila 3 podemos ver que:

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 5 & -8 & 3 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & -2 & 3 & 1 \end{array}\right]$$

$$\boxed{x_3 = 1}$$

De la fila 4 podemos ver que:

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 5 & -8 & 3 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & -2 & 3 & 1 \end{array}\right]$$

$$\boxed{x_4 = 4}$$

De la fila 5 podemos ver que:

$$\left[\begin{array}{ccccc|ccccc|c} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 5 & -8 & 3 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & -2 & 3 & 1 \end{array}\right]$$

$$\boxed{x_5 = 1}$$

Finalmente las soluciones al sistema de ecuaciones son:

$$\begin{cases} x_1 = 2 \\ x_2 = 3 \\ x_3 = 1 \\ x_4 = 4 \\ x_5 = 1 \end{cases}$$

La matriz Inversa es:

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 5 & -8 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & -1 & -2 & 3 \end{pmatrix}$$

Verificamos que la matriz inversa sea correcta
Expresando en forma matricial

$$\begin{pmatrix} 4 & 2 & 5 & 3 & 5 \\ 5 & 2 & 5 & 4 & 5 \\ 5 & 2 & 6 & 4 & 5 \\ 7 & 3 & 7 & 4 & 7 \\ 5 & 2 & 5 & 3 & 5 \end{pmatrix} \times \begin{pmatrix} -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 5 & -8 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & -1 & -2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Validando soluciones
El sistema original es:

$$(4) \cdot x_1 + (2) \cdot x_2 + (5) \cdot x_3 + (3) \cdot x_4 + (5) \cdot x_5 = 36$$
$$(5) \cdot x_1 + (2) \cdot x_2 + (5) \cdot x_3 + (4) \cdot x_4 + (5) \cdot x_5 = 42$$
$$(5) \cdot x_1 + (2) \cdot x_2 + (6) \cdot x_3 + (4) \cdot x_4 + (5) \cdot x_5 = 43$$
$$(7) \cdot x_1 + (3) \cdot x_2 + (7) \cdot x_3 + (4) \cdot x_4 + (7) \cdot x_5 = 53$$
$$(5) \cdot x_1 + (2) \cdot x_2 + (5) \cdot x_3 + (3) \cdot x_4 + (5) \cdot x_5 = 38$$

Reemplazando

$$(4) \cdot 2 + (2) \cdot 3 + (5) \cdot 1 + (3) \cdot 4 + (5) \cdot 1 = 36$$
$$(5) \cdot 2 + (2) \cdot 3 + (5) \cdot 1 + (4) \cdot 4 + (5) \cdot 1 = 42$$
$$(5) \cdot 2 + (2) \cdot 3 + (6) \cdot 1 + (4) \cdot 4 + (5) \cdot 1 = 43$$
$$(7) \cdot 2 + (3) \cdot 3 + (7) \cdot 1 + (4) \cdot 4 + (7) \cdot 1 = 53$$
$$(5) \cdot 2 + (2) \cdot 3 + (5) \cdot 1 + (3) \cdot 4 + (5) \cdot 1 = 38$$
$$36 = 36$$
$$42 = 42$$
$$43 = 43$$
$$53 = 53$$
$$38 = 38$$