



# Métodos Numéricos - MAT 1105

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*para todos los estudiantes de la Facultad Nacional de Ingeniería*

Sea el sistema

$$3 \cdot x_1 + 3 \cdot x_2 + 2 \cdot x_3 = 810$$

$$x_1 + 2 \cdot x_2 + x_3 = 410$$

$$2 \cdot x_1 + x_2 + 2 \cdot x_3 = 490$$

Reescribiendo

$$(3) \cdot x_1 + (3) \cdot x_2 + (2) \cdot x_3 = 810$$

$$(1) \cdot x_1 + (2) \cdot x_2 + (1) \cdot x_3 = 410$$

$$(2) \cdot x_1 + (1) \cdot x_2 + (2) \cdot x_3 = 490$$

Expresando en forma matricial

$$\begin{pmatrix} 3 & 3 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 810 \\ 410 \\ 490 \end{pmatrix}$$

Usando la matriz aumentada con coeficientes y terminos independientes

$$\left[ \begin{array}{ccc|ccc} 3 & 3 & 2 & 1 & 0 & 0 & 810 \\ 1 & 2 & 1 & 0 & 1 & 0 & 410 \\ 2 & 1 & 2 & 0 & 0 & 1 & 490 \end{array} \right]$$

Dividiendo la fila 1 por (3)

$$\left[ \begin{array}{ccc|ccc} 3 & 3 & 2 & 1 & 0 & 0 & 810 \\ 1 & 2 & 1 & 0 & 1 & 0 & 410 \\ 2 & 1 & 2 & 0 & 0 & 1 & 490 \end{array} \right] / (3)$$

Calculos auxiliares:

$$(3) / (3) = 1 \quad (3) / (3) = 1 \quad (2) / (3) = 2/3 \quad (1) / (3) = 1/3 \quad (0) / (3) = 0 \quad (0) / (3) = 0 \quad (810) / (3) = 270$$

Multiplicando la fila 1 por  $(-1)$  y sumando a la fila 2

$$\left[ \begin{array}{ccc|ccc} 1 & 1 & 2/3 & 1/3 & 0 & 0 & 270 \\ 1 & 2 & 1 & 0 & 1 & 0 & 410 \\ 2 & 1 & 2 & 0 & 0 & 1 & 490 \end{array} \right] \times (-1)$$

Calculos auxiliares:

$$1 \times (-1) + 1 = 0 \quad 1 \times (-1) + 2 = 1 \quad 2/3 \times (-1) + 1 = 1/3 \quad 1/3 \times (-1) + 0 = -1/3 \quad 0 \times (-1) + 1 = 1$$

$$0 \times (-1) + 0 = 0 \quad 270 \times (-1) + 410 = 140$$

Multiplicando la fila 1 por  $(-2)$  y sumando a la fila 3

$$\left[ \begin{array}{ccc|ccc} 1 & 1 & 2/3 & 1/3 & 0 & 0 & 270 \\ 0 & 1 & 1/3 & -1/3 & 1 & 0 & 140 \\ 2 & 1 & 2 & 0 & 0 & 1 & 490 \end{array} \right] \times (-2)$$

Cálculos auxiliares:

$$\begin{array}{|l|} \hline 1 \times (-2) + 2 = 0 \\ \hline 1 \times (-2) + 1 = -1 \\ \hline 2/3 \times (-2) + 2 = 2/3 \\ \hline 1/3 \times (-2) + 0 = -2/3 \\ \hline 0 \times (-2) + 0 = 0 \\ \hline 0 \times (-2) + 1 = 1 \\ \hline 270 \times (-2) + 490 = -50 \\ \hline \end{array}$$

Dividiendo la fila 2 por (1)

$$\left[ \begin{array}{ccc|ccc|c} 1 & 1 & 2/3 & 1/3 & 0 & 0 & 270 \\ 0 & 1 & 1/3 & -1/3 & 1 & 0 & 140 \\ 0 & -1 & 2/3 & -2/3 & 0 & 1 & -50 \end{array} \right] / (1)$$

Cálculos auxiliares:

$$\begin{array}{|l|} \hline (1) / (1) = 1 \\ \hline (1/3) / (1) = 1/3 \\ \hline (-1/3) / (1) = -1/3 \\ \hline (1) / (1) = 1 \\ \hline (0) / (1) = 0 \\ \hline (140) / (1) = 140 \\ \hline \end{array}$$

Multiplicando la fila 2 por (-1) y sumando a la fila 1

$$\left[ \begin{array}{ccc|ccc|c} 1 & 1 & 2/3 & 1/3 & 0 & 0 & 270 \\ 0 & 1 & 1/3 & -1/3 & 1 & 0 & 140 \\ 0 & -1 & 2/3 & -2/3 & 0 & 1 & -50 \end{array} \right] \times (-1)$$

Cálculos auxiliares:

$$\begin{array}{|l|} \hline 1 \times (-1) + 1 = 0 \\ \hline 1/3 \times (-1) + 2/3 = 1/3 \\ \hline -1/3 \times (-1) + 1/3 = 2/3 \\ \hline 1 \times (-1) + 0 = -1 \\ \hline 0 \times (-1) + 0 = 0 \\ \hline 140 \times (-1) + 270 = 130 \\ \hline \end{array}$$

Multiplicando la fila 2 por (1) y sumando a la fila 3

$$\left[ \begin{array}{ccc|ccc|c} 1 & 0 & 1/3 & 2/3 & -1 & 0 & 130 \\ 0 & 1 & 1/3 & -1/3 & 1 & 0 & 140 \\ 0 & -1 & 2/3 & -2/3 & 0 & 1 & -50 \end{array} \right] \times (1)$$

Cálculos auxiliares:

$$\begin{array}{|l|} \hline 1 \times (1) + -1 = 0 \\ \hline 1/3 \times (1) + 2/3 = 1 \\ \hline -1/3 \times (1) + -2/3 = -1 \\ \hline 1 \times (1) + 0 = 1 \\ \hline 0 \times (1) + 1 = 1 \\ \hline 140 \times (1) + -50 = 90 \\ \hline \end{array}$$

Dividiendo la fila 3 por (1)

$$\left[ \begin{array}{ccc|ccc|c} 1 & 0 & 1/3 & 2/3 & -1 & 0 & 130 \\ 0 & 1 & 1/3 & -1/3 & 1 & 0 & 140 \\ 0 & 0 & 1 & -1 & 1 & 1 & 90 \end{array} \right] / (1)$$

Cálculos auxiliares:

$$\begin{array}{|l|} \hline (1) / (1) = 1 \\ \hline (-1) / (1) = -1 \\ \hline (1) / (1) = 1 \\ \hline (1) / (1) = 1 \\ \hline (90) / (1) = 90 \\ \hline \end{array}$$

Multiplicando la fila 3 por (-1/3) y sumando a la fila 1

$$\left[ \begin{array}{ccc|ccc|c} 1 & 0 & 1/3 & 2/3 & -1 & 0 & 130 \\ 0 & 1 & 1/3 & -1/3 & 1 & 0 & 140 \\ 0 & 0 & 1 & -1 & 1 & 1 & 90 \end{array} \right] \times \left( -\frac{1}{3} \right)$$

Cálculos auxiliares:

$$\begin{array}{|l|} \hline 1 \times (-1/3) + 1/3 = 0 \\ \hline -1 \times (-1/3) + 2/3 = 1 \\ \hline 1 \times (-1/3) + -1 = -4/3 \\ \hline 1 \times (-1/3) + 0 = -1/3 \\ \hline 90 \times (-1/3) + 130 = 100 \\ \hline \end{array}$$

Multiplicando la fila 3 por (-1/3) y sumando a la fila 2

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -4/3 & -1/3 & 100 \\ 0 & 1 & 1/3 & -1/3 & 1 & 0 & 140 \\ 0 & 0 & 1 & -1 & 1 & 1 & 90 \end{array} \right] \times \left( -\frac{1}{3} \right)$$

Calculos auxiliares:

$$\boxed{1 \times (-1/3) + 1/3 = 0} \quad \boxed{-1 \times (-1/3) + -1/3 = 0} \quad \boxed{1 \times (-1/3) + 1 = 2/3} \quad \boxed{1 \times (-1/3) + 0 = -1/3}$$

$$\boxed{90 \times (-1/3) + 140 = 110}$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -4/3 & -1/3 & 100 \\ 0 & 1 & 0 & 0 & 2/3 & -1/3 & 110 \\ 0 & 0 & 1 & -1 & 1 & 1 & 90 \end{array} \right]$$

De la fila 1 podemos ver que:

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -4/3 & -1/3 & 100 \\ 0 & 1 & 0 & 0 & 2/3 & -1/3 & 110 \\ 0 & 0 & 1 & -1 & 1 & 1 & 90 \end{array} \right]$$

$$\boxed{x_1 = 100}$$

De la fila 2 podemos ver que:

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -4/3 & -1/3 & 100 \\ 0 & 1 & 0 & 0 & 2/3 & -1/3 & 110 \\ 0 & 0 & 1 & -1 & 1 & 1 & 90 \end{array} \right]$$

$$\boxed{x_2 = 110}$$

De la fila 3 podemos ver que:

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -4/3 & -1/3 & 100 \\ 0 & 1 & 0 & 0 & 2/3 & -1/3 & 110 \\ 0 & 0 & 1 & -1 & 1 & 1 & 90 \end{array} \right]$$

$$\boxed{x_3 = 90}$$

Finalmente las soluciones al sistema de ecuaciones son:

$$\begin{cases} x_1 = 100 \\ x_2 = 110 \\ x_3 = 90 \end{cases}$$

La matriz Inversa es:

$$\begin{pmatrix} 1 & -4/3 & -1/3 \\ 0 & 2/3 & -1/3 \\ -1 & 1 & 1 \end{pmatrix}$$

Verificamos que la matriz inversa sea correcta  
Expresando en forma matricial

$$\begin{pmatrix} 3 & 3 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix} \times \begin{pmatrix} 1 & -4/3 & -1/3 \\ 0 & 2/3 & -1/3 \\ -1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Validando soluciones  
El sistema original es:

$$(3) \cdot x_1 + (3) \cdot x_2 + (2) \cdot x_3 = 810$$

$$(1) \cdot x_1 + (2) \cdot x_2 + (1) \cdot x_3 = 410$$

$$(2) \cdot x_1 + (1) \cdot x_2 + (2) \cdot x_3 = 490$$

Reemplazando

$$(3) \cdot 100 + (3) \cdot 110 + (2) \cdot 90 = 810$$

$$(1) \cdot 100 + (2) \cdot 110 + (1) \cdot 90 = 410$$

$$(2) \cdot 100 + (1) \cdot 110 + (2) \cdot 90 = 490$$

$$810 = 810$$

$$410 = 410$$

$$490 = 490$$