CS312

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HW1.

Question 1

0.1 (n-f, m) $f = O(g)$, $f = \Omega(g)$, $f = \Theta(g)$

	$f(n)$	$g(n)$
(a)	$n - 100$	$n - 200$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{n-100}{n-200} = 0 \rightarrow \boxed{f = O(g)}$$

(b) $f(n) = n^{1/2}$, $g(n) = n^{2/3}$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{n^{1/2}}{n^{2/3}} = \lim_{n \to \infty} (n)^{(1/2 - 2/3)}$$

$$= \lim_{n \to \infty} n^{-1/6} = 0 \quad \boxed{f = O(g)}$$

(C) $f(n) = 100n + \log n$, $g(n) = n + (\log n)^2$

$$\lim_{n \to \infty} \frac{100n + \log n}{n + (\log n)^2} = \frac{100 + \frac{\log n}{n}}{1 + \frac{(\log n)^2}{n}}$$

$$\because \lim_{n \to \infty} \frac{\log n}{n} \rightarrow 0$$

$$\lim_{n \to \infty} \frac{(\log n)^2}{n} \rightarrow 0$$

$$\therefore \lim_{n \to \infty} \frac{100 + \log n}{1 + (\log n)^2} \rightarrow \lim_{n \to \infty} \frac{100 + 0}{1 + 0} = 100$$

$$\boxed{\therefore f = \Theta(g)}$$

(d) $f(n) = n \log n$, $g(n) = 10n \log 10n$

$$\lim_{n\to\infty} \frac{\cancel{n} \log n}{10\cancel{n} \log 10n} = \lim_{n\to\infty} \frac{\log n}{10 \underbrace{\log 10n}_{\log 10 + \log n}}$$

$$= \lim_{n\to\infty} \frac{\log n}{10(\log 10 + \log n)}$$

$$= \lim_{n\to\infty} \frac{1}{10\left(\underbrace{\frac{\log 10}{\log n}}_{\to \emptyset} + 1\right)}$$

$$= \frac{1}{10}$$

$\boxed{\therefore f = \Theta(g)}$

(e) $f(n) = \log 2n$, $g(n) = \log 3n$

$$\lim_{n\to\infty} \frac{\log 2n}{\log 3n} = \lim_{n\to\infty} \frac{\log 2 + \log n}{\log 3 + \log n}$$

$$= \lim_{n\to\infty} \frac{1}{1} = 1$$

$\boxed{\therefore f = \Theta(g)}$

(f) $f(n) = 10 \log n$, $g(n) = n \log^2 n$

$$\lim_{n\to\infty} \frac{10 \cancel{\log n}}{n \log^{\cancel{2}} n} = \lim_{n\to\infty} \frac{10}{n \log n} = 0$$

$\boxed{\therefore f = O(g)}$

(m) $f(n) = n2^n$, $g(n) = 3^n$.

$$\lim_{n\to\infty} \frac{n \cdot 2^n}{3^n} = \lim_{n\to\infty} n \cdot \frac{2^n}{3^n}$$

$$= \lim_{n\to\infty} \underset{\downarrow \atop \infty}{n} \cdot \underset{\downarrow \atop 0}{\left(\frac{2}{3}\right)^n}$$

$$= 0$$

$$\boxed{\therefore f = O(g)}$$

Question 2

0.2 Show $c > 0$, $c \in \mathbb{R}$ then $\boxed{g(n) = 1 + c + c^2 + \cdots + c^n}$

(a) $\Theta(1)$ if $c < 1$,

$$g(n) = \frac{1 - c^{n+1}}{1 - c}$$

$$\lim_{n\to\infty} g(n) = \lim_{n\to\infty} \frac{1 - c^{n+1}}{1 - c} \Rightarrow \frac{1}{1-c}$$

$$\therefore g(n) = \Theta(1)$$

(b) $\Theta(n)$ if $c = 1$,

$$g(n) = n + 1 \qquad \therefore g(n) = \Theta(n)$$

(c) $\Theta(c^n)$ if $c > 1$,

$$g(n) = \frac{1 - c^{n+1}}{1 - c}$$

$$\lim_{n\to\infty} g(n) = \lim_{n\to\infty} \frac{1 - c^{n+1}}{1 - c} \approx \frac{+c^{n+1} - 1}{c - 1} \approx \frac{c^{n+1}}{c - 1} \approx c^n$$

$$\therefore g(n) = \Theta(c^n$$

Question 3

a)
```python
def fabonacci(n):
    if n == 0 or n == 1 or n == 2:
        return 1
    return fabonacci(n - 1) + fabonacci(n - 2) * fabonacci(n - 3)
```

b)
```
function fabonacci_linear(n):
        if n == 0 or n == 1 or n == 2:
                return 1

        f = array of size (n + 1)

        f[0] = f[1] = f[2] = 1

        for i from 3 to n:
                f[i] = f[i-1] + f[i-2] * f[i-3]
        return f[n]
```