1 Detailed Modification of GDSO and DRO for CCMCKP

1.1 GDSO

Yang and Chakraborty [1] propose an algorithm for the chance-constrained knapsack problem (CCKP) assuming known Gaussian distributions of the random variables (the values of mean and variance are known). In their work, CCKP is transformed into an iterative Risk-averse Knapsack Problem (RAKP), which could be solved via dynamic programming. The implementation of their work for CCMCKP is direct. We first calculate the sample mean and standard deviation of the item weights, i.e., $\mu_{ij} = \mathbb{E}(w_{ij}), \sigma_{ij}^2 = Var(w_{ij}), i \in \{1, ..., m\}, j \in N_i$, and then put them into the algorithm.

$$\begin{aligned}
\min \quad & \sum_{i=1}^{m} \sum_{j \in N_i} c_{ij} x_{ij} && \text{(1a)} \\
\text{s.t.} \quad & \sum_{i=1}^{m} \sum_{j \in N_i} \mu_{ij} x_{ij} + \Phi^{-1}(P_0) \sqrt{\sum_{i=1}^{m} \sum_{j \in N_i} \sigma_{ij}^2 x_{ij}} \leq W && \text{(1b)} \\
& \sum_{j \in N_i} x_{ij} = 1, \forall i \in \mathcal{M} && \text{(1c)} \\
& x_{ij} \in \{0, 1\}, \forall i \in \mathcal{M}, j \in N_i && \text{(1d)}
\end{aligned}$$

where Φ represents the CDF of a standard normal distribution with zero mean and unit variance. The above optimization problem can also be transformed into so called Risk-averse Multiple-Choice Knapsack Problem (RAMCKP).

$$\begin{aligned}
\min \quad & \sum_{i=1}^{m} \sum_{j \in N_i} c_{ij} x_{ij} && \text{(2a)} \\
\text{s.t.} \quad & \sum_{i=1}^{m} \sum_{j \in N_i} (\mu_{ij} + \sigma_{ij}^2) x_{ij} \leq W' && \text{(2b)} \\
& \sum_{j \in N_i} x_{ij} = 1, \forall i \in \mathcal{M} && \text{(2c)} \\
& x_{ij} \in \{0, 1\}, \forall i \in \mathcal{M}, j \in N_i && \text{(2d)}
\end{aligned}$$

where λ is called risk-aver parameter and W' is the adapted W in the RAMCKP. The above optimization problem can be solved by dynamic programming.

1.2 DRO

Ji and Lejeune [2] studied the distributionally robust chance-constrained programming optimization problem with data-driven Wasserstein ambiguity sets. They propose an algorithm and reconstruction framework suitable for all types of distributionally robust chance-constrained optimization problems, which is then applied to multidimensional knapsack problems. When the decision variables are binary, the proposed MILP formulations are equivalent reformulations of chance constraints. To fit the chance constraints into [2]'s modelling and solution framework, we formulate $\text{Prob}(\sum_{i=1}^{m} \sum_{j \in N_i} w_{ij} x_{ij} \leq W) \geq P_0$ as:

$$\text{Prob}(\sum_{i=1}^{m} \sum_{j \in N_i} w_{ij} x_{ij} > W) \leq 1 - P_0 \tag{3}$$

Define $b = -W, \epsilon = 1 - P_0, \tilde{w}_{ij} = -w_{ij}, \forall i \in \mathcal{M}, j \in N_i$. Let $\bar{x} \in \{0,1\}^{NM}$ as the vectorization of $x \in \{0,1\}^{N\times M}$, $\xi \in \mathbb{R}^{NM}$ is the vectorization of $\hat{w} \in \mathbb{R}^{N\times M}$. Finally, we have the standard form of 3 as [2]:

$$\text{Prob}(\xi^T \bar{x} < b) \leq \epsilon \tag{4}$$

Then we can utilize Theorem 10 in [2] to reformulate the above inequation 3, and get the MILP formulation of CCMCKP:

$$(\bar{x}, \lambda, \eta, s, \phi) \in \mathcal{Z}_{LP}^{MCKP} = \begin{cases} \lambda\theta + \frac{1}{L}\sum_{l=1}^{L} s_l \leq \epsilon, & \\ \lambda \geq 0, & \\ s_l \geq 0, \eta_l \geq 0, & \forall l \in \mathcal{L} \\ 1 + \eta_l b^{'} - \sum_{q\in\mathcal{Q}} \xi_{ql}^0 \phi_{ql} \leq s_l, & \forall l \in \mathcal{L} \\ \phi_{ql} \leq \lambda, & \forall q \in \mathcal{Q}, l \in \mathcal{L} \\ \phi_{ql} \leq \eta_l, & \forall q \in \mathcal{Q}, l \in \mathcal{L} \\ \phi_{ql} \leq U_{\eta_l}\bar{x}_q, & \forall q \in \mathcal{Q}, l \in \mathcal{L} \\ \phi_{ql} \geq 0, & \forall q \in \mathcal{Q}, l \in \mathcal{L} \\ \phi_{ql} \geq \eta_l + U_{\eta_l}\bar{x}_q - U_{\eta_l}, & \forall q \in \mathcal{Q}, l \in \mathcal{L} \\ \sum_{i=Nm-N+1}^{Nm} \bar{x}_i = 1, & \forall m \in \mathcal{M} \\ \bar{x}_q \in \{0,1\}, & \forall q \in \mathcal{Q} \end{cases} \tag{5}$$

where $b^{'} = b - \delta$, δ is a user-defined infinite small positive number. $\mathcal{Q}$ is the set of all $N \times M$ items, $\mathcal{M}$ is the set of M classes. Note that $\sum_{i=Nm-N+1}^{Nm} \bar{x}_i = 1, \forall m \in \mathcal{M}$ is multiple-choice constraint in MCKP. U_η is the upper bound of η and can be determined by:

$$U_{\eta_l} = \begin{cases} \mathcal{K}^{'*}, & if \exists \bar{x} : \bar{x}^T \xi_l^0 - b > 0 \\ 0, & otherwise \end{cases} \tag{6}$$

where $\mathcal{K}^{'*}$ is the optimal objective value of the linear programming problem:

$$\begin{aligned} \mathcal{K}^{'*} &= \min \bar{x}^T \xi_l^0 - b \\ &\text{s.t.} b - \bar{x}^T \xi_l^0 \leq 0 - \tau^{'} \\ &\quad \bar{x} \in \bar{\mathcal{X}} \end{aligned} \tag{7}$$

where $\tau^{'}$ a user-defined small non-negative number.
In summary, we first solve linear programming 7 and obtain $U_{\eta_l}, \forall l \in \mathcal{L}$. Then we solve MILP $\mathcal{Z}_{LP}^{MCKP}$ 5 to find the optimal solution of CCMCKP.

2 Proof

Theorem 2.1. *The popped sum in the k^{th} iteration is exactly the k^{th} largest sum.*

Proof. Suppose the i^{th} largest sum is $S_r = d_{s_1 l_1} + d_{s_2 l_1} + \ldots + d_{s_m l_m}$.

- For any pair in the combination or queue Q that has already been output, the combinations with indices less than or equal to it must have been popped or exist in Q.
- Construct a chain S, then we have the following chain, denoted as $S_0 \leq S_1 \leq, \ldots, \leq S_l$.

$$\begin{aligned} &d_{s_1 0} + d_{s_2 0} + \ldots + d_{s_m 0}, \\ \geq &d_{s_1 1} + d_{s_2 0} + \ldots + d_{s_m 0}, \\ \geq &\ldots, \\ \geq &d_{s_1 l_1} + d_{s_2 0} + \ldots + d_{s_m 0} \\ \geq &\ldots, \\ \geq &d_{s_1 l_1} + d_{s_2 l_2} + \ldots + d_{s_m 0} \\ \geq &\ldots, \\ \geq &d_{s_1 l_1} + d_{s_2 l_2} + \ldots + d_{s_m l_m} \end{aligned}$$

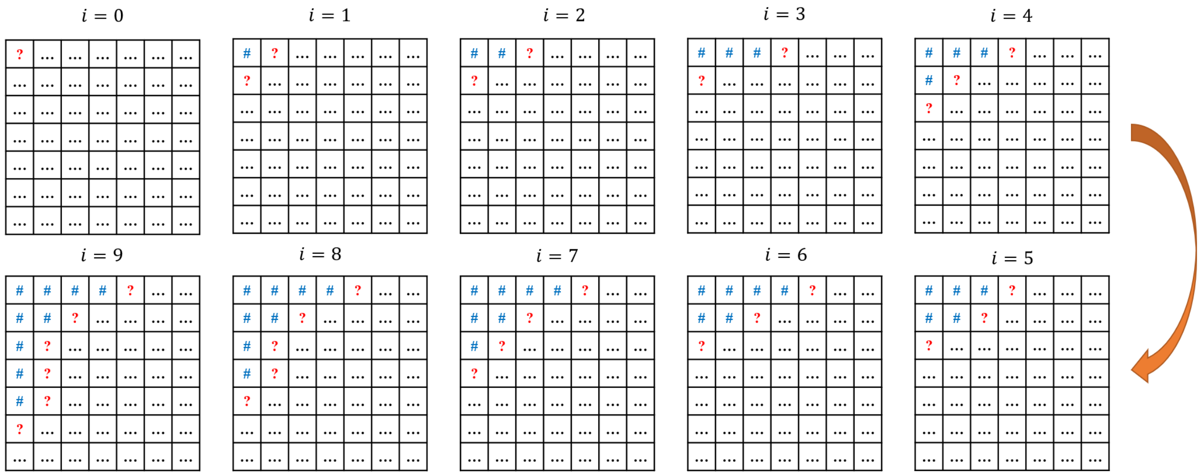

Figure 1: An example of exact evaluation when $m = 2$

- Let S_j be the first sum in chain S which was not popped from queue Q before the j^{th} iteration. During the j^{th} iteration, when S_{j-1} is popped, S_j is added to Q.
- Thus, during the k^{th} iteration, the popped sum S_i satisfies $S_i \leq S_j \leq S_l = d_{s_1 l_1} + d_{s_2 l_1} + \ldots + d_{s_m l_m}$.
- By induction, since the $(i-1)^{th}$ largest sum has been popped before the i^{th} iteration, so the sum popped in the i^{th} iteration is exactly the i^{th} largest sum0 $S_l = d_{s_1 l_1} + d_{s_2 l_1} + \ldots + d_{s_m l_m}$.

Then we finish the proof. $\square$

A simple example is shown when $m = 2$ in Fig.1.

3 A Theoretical Quantitative Analysis between Estimation Error and Sample Size

To theoretically quantify the relationship between estimation error and sample size of data, we construct a new random variable S that is the sum of multiple random variables such that the variable itself is a random variable. Then we define a Bernoulli variable $Z \in \{0, 1\}$, i.e.,

$$Z = \begin{cases} 1, & S \leq W \\ 0, & S > W \end{cases} \tag{8}$$

for a combination of items with L sample size, where p is the true confidence level, i.e., the expectation $\mathbb{E}Z$ of the variable Z. Hoeffding's inequality shows that for independent identically distributed random variables $Z_1, Z_2, \ldots, Z_L$ taking values $[a, b]$, for any $\epsilon > 0$, the following inequality holds,

$$Pr(|\mathbb{E}[Z] - \frac{1}{L}\sum_{l=1}^{L} Z_l| \geq \epsilon) \leq 2exp(-\frac{2L\epsilon^2}{(b-a)^2}) \tag{9}$$

For our problem, we are concerned with the true value is lower than the estimate and greater than the error, i.e., where ϵ is the estimation error corresponding to the confidence level. For the Bernoulli distribution, we have $\mathbb{E}Z = p$ and $a = 0, b = 1$. Thus, the inequality becomes

$$Pr(p - f(Z_1, Z_2, \ldots, Z_L) \leq -\epsilon) \leq exp(-2L\epsilon^2) \tag{10}$$

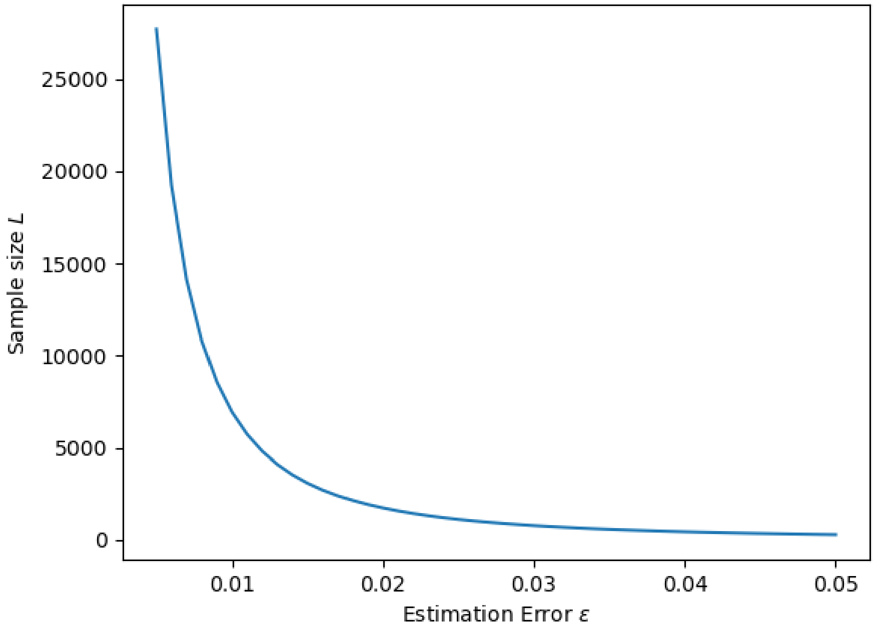

Figure 2: Sample size requirement with different error in theory

Table 1: Relationship between EE and sample size

Estimation error(%)	5	0.5	0.05	0.005
Hoeffding's bound	139	13863	1386295	138629437
Chernoff's bound [3]	278	27726	2772589	277258873
Alamo's bound [4]	1573	15723	157227	1572269

where $f(Z_1, Z_2, \ldots, Z_L)$ denotes the confidence evaluation algorithm on some combination of items. Assuming that at least 5 of the 10 solutions in the output solution set are true feasible solutions, and $f(Z_1, Z_2, \ldots, Z_L)$ is accurate, then we have $1 - exp(-2L\epsilon^2) \geq 5/10$. Finally, it leads to

$$L \geq \frac{ln2}{2\epsilon^2} \tag{11}$$

As this bound is derived from Hoeffding's inequality, Chernoff bound is proposed in [3] and a more tight one is proposed in [4]. However, all of these inequalities indicate that a large amount of samples is necessary. The curve of the variation of L with ϵ is shown in Figure 2.
The minimum sample sizes required for different estimation error are listed in Table 1.
Overall, Hoeffding provides the tightest bounds, especially suitable for larger errors. However, as error requirements become smaller, the bounds provided by Alamo are more advantageous. Practically, it is verified that the amount of data required to meet the corresponding error requirement for the true confidence will be much smaller than the theoretical value obtained.

4 Complete experimental results

In this section, we show the complete experimental results of DDALS and other baselines on LAB and APP.

Table 2: Performance on benchmarks of LAB. The highest average real confidence level and the lowest cost are highlighted through bolding and being underlined.

Benchmark	small $T_{\max}$						large $T_{\max}$					
	C	ET	ECL	RCL	FSR	Time	C	ET	ECL	RCL	FSR	Time
ss1-Greedy	22.2±0.0	2.0±0.0	0±0.0	99.6±0.0	1	**0.1±0.0**	**10.8±0.0**	4.0±0.0	99.9±0.0	99.9±0.0	1	**0.1±0.0**
ss1-GA	18.7±0.1	356.2±4.9	99.8±0.3	99.7±0.7	0.87	7.1±0.5	10.8±0.1	233.7±3.8	99.9±0.1	99.5±0.9	0.87	3.1±0.3
ss1-EDA	21.2±1.6	354.3±2.8	29.9±45.7	99.5±0.5	0.93	8.8±3.6	12.8±2.4	234.6±2.8	99.8±0.3	99.7±0.6	0.9	2.4±1.2
ss1-DDALS(O)	**18.6±0.0**	349.5±7.6	99.9±0.0	99.9±0.0	1	2.2±0.1	**10.8±0.0**	234.6±17.2	99.9±0.0	99.9±0.0	1	1.7±0.3
ss1-DDALS(V1)	**18.6±0.0**	349.5±7.6	99.9±0.0	99.9±0.0	1	2.2±0.1	**10.8±0.0**	234.6±17.2	99.9±0.0	99.9±0.0	1	1.7±0.3
ss1-DDALS(V2)	**18.6±0.0**	349.5±7.6	99.9±0.0	99.9±0.0	1	2.2±0.1	**10.8±0.0**	234.6±17.2	99.9±0.0	99.9±0.0	1	1.7±0.3
ss1-DDALS(V3)	20.7±2.0	345	74.7±0.4	99.3±0.8	0.75	2.3	13.0±1.7	228	99.8±0.3	99.1±1.0	0.7	1.7
ss2-Greedy	**19.4±0.0**	2.0±0.0	99.9±0.0	99.9±0.0	1	**3.9±0.1**	13.8±0.0	4.0±0.0	99.5±0.0	99.0±0.0	1	**7.3±0.2**
ss2-GA	**19.4±0.0**	385.6±3.9	99.9±0.0	99.9±0.0	1	555.9±133.9	11.4±1.6	308.5±6.0	99.8±0.1	99.8±0.3	1	1311.7±217.9
ss2-EDA	19.8±0.4	384.7±2.8	99.9±0.0	99.9±0.0	1	144.3±29.2	15.5±2.3	305.0±3.1	99.7±0.2	99.7±0.3	1	582.4±476.3
ss2-DDALS(O)	**19.4±0.0**	387.2±5.2	99.9±0.0	99.9±0.0	1	1241.9±73.9	**10.4±0.0**	302.0±10.7	99.8±0.0	99.9±0.0	1	555.1±80.6
ss2-DDALS(V1)	**19.4±0.0**	387.2±5.2	99.9±0.0	99.9±0.0	1	1241.9±73.9	**10.4±0.0**	302.0±10.7	99.8±0.0	99.9±0.0	1	555.1±80.6
ss2-DDALS(V2)	**19.4±0.0**	387.2±5.2	99.9±0.0	99.9±0.0	1	1241.9±73.9	12.1±2.0	302.0±10.7	99.9±0.1	99.9±0.0	1	555.1±80.6
ss2-DDALS(V3)	21.8±2.4	392	99.9±0.0	99.9±0	1	1228.1	14.3±2.2	296	99.8±0.2	99.8±0.3	1	432.5
ss3-Greedy	**21.4±0.0**	3.0±0.0	99.9±0.0	99.5±0.0	1	**0.2±0.0**	17.4±0.0	4.0±0.0	100.0±0.0	100.0±0.0	1	**0.4±0.0**
ss3-GA	22.4±1.4	306.0±3.7	99.9±0.0	99.7±0.2	1	31.7±1.9	14.2±2.4	263.5±2.9	99.7±0.4	99.2±1.3	0.73	37.9±1.1
ss3-EDA	24.8±1.9	305.3±3.0	99.9±0.1	99.8±0.5	0.93	44.1±3.0	13.7±1.3	264.5±2.8	99.9±0.0	99.9±0.1	1	42.8±3.4
ss3-DDALS(O)	**21.4±0.0**	288.5±9.3	99.9±0.0	99.5±0.0	1	9.6±0.5	**12.6±0.0**	254.4±4.7	100.0±0.0	99.9±0.0	1	14.7±0.9
ss3-DDALS(V1)	**21.4±0.0**	288.5±9.3	99.9±0.0	99.5±0.0	1	9.6±0.5	**12.6±0.0**	254.4±4.7	100.0±0.0	99.9±0.0	1	14.7±0.9
ss3-DDALS(V2)	**21.4±0.0**	288.5±9.3	99.9±0.0	99.5±0.0	1	9.6±0.5	**12.6±0.0**	254.4±4.7	100.0±0.0	99.9±0.0	1	14.7±0.9
ss3-DDALS(V3)	23.9±1.7	284	99.8±0.2	99.1±0.8	0.75	9.1	15.5±1.7	264	99.9±0.3	99.5±0.9	0.9	14.9
ss4-Greedy	30.1±0.0	2.0±0.0	99.9±0.0	99.9±0.0	1	**0.1±0.0**	14.9±0.0	5.0±0.0	99.9±0.0	99.9±0.0	1	**0.3±0.0**
ss4-GA	23.1±4.4	773.5±8.0	99.9±0.2	99.7±0.4	1	65.6±2.5	16.4±3.0	559.3±8.8	99.8±0.3	99.4±0.6	0.9	60.9±3.1
ss4-EDA	**17.4±0.0**	764.5±3.0	99.9±0.0	99.9±0.0	1	98.8±1.7	13.4±1.0	554.3±2.6	99.8±0.2	99.1±0.41	0.93	83.5±3.3
ss4-DDALS(O)	**17.4±0.0**	757.9±31.9	99.9±0.0	99.9±0.0	1	21.4±1.0	**11.2±0.0**	538.8±31.9	99.2±0.0	99.0±0.0	1	23.6±1.1
ss4-DDALS(V1)	**17.4±0.0**	757.9±31.9	99.9±0.0	99.9±0.0	1	21.4±1.0	12.3±0.7	538.8±31.9	99.6±0.1	98.9±0.2	1	23.6±1.1
ss4-DDALS(V2)	**17.4±0.0**	757.9±31.9	99.9±0.0	99.9±0.0	1	21.4±1.0	14.1±0.9	538.8±31.9	99.9±0.2	99.0±0.1	1	23.6±1.1
ss4-DDALS(V3)	25.3±4.1	822	99.8±0.2	99.6±0.5	1	22.9	14.3±1.5	514	99.6±0.3	99.2±0.7	0.9	24.1
ls1-Greedy	58.5±0.0	2.0±0.0	99.8±0.0	99.8±0.0	1	**0.3±0.0**	45.2±0.0	4.0±0.0	99.7±0.0	99.4±0.0	1	**0.6±0.1**
ls1-GA	50.8±1.0	3009.2±8.5	99.2±0.2	99.2±0.2	1	776.1±30.2	40.7±4.2	1411.0±9.0	99.5±0.3	99.3±0.4	0.9	458.3±9.4
ls1-EDA	54.3±1.7	3004.9±2.5	99.2±0.1	99.3±0.1	1	678.1±7.6	36.9±1.9	1403.6±2.7	99.3±0.1	99.3±0.0	1	417.2±8.1
ls1-DDALS(O)	**49.7±0.1**	2924.1±50.1	99.3±0.1	99.2±0.0	1	476.7±14.6	**32.0±0.0**	1387.8±24.5	99.2±0.0	99.1±0.0	1	240.0±6.2
ls1-DDALS(V1)	52.2±0.4	2924.1±50.1	99.6±0.1	99.5±0.1	1	476.7±14.6	34.4±0.7	1387.8±24.5	99.6±0.0	99.5±0.1	1	240.0±6.2
ls1-DDALS(V2)	50.0±0.4	2924.1±50.1	99.3±0.0	99.3±0.1	1	476.7±14.6	34.1±1.7	1387.8±24.5	99.5±0.2	99.5±0.2	1	240.0±6.2
ls1-DDALS(V3)	51.9±1.1	2889	99.3±0.2	99.3±0.2	1	475.2	34.7±1.3	1357	99.4±0.2	99.3±0.2	1	233.2
ls2-Greedy	66.6±0.0	2.0±0.0	99.7±0.0	99.7±0.0	1	**0.3±0.0**	66.6±0.0	2.0±0.0	100.0±0.0	100.0±0.0	1	**0.3±0.0**
ls2-GA	53.0±3.2	9626.6±20.4	99.1±0.1	99.1±0.2	1	2106.4±68.7	44.4±6.8	3315.4±14.1	99.4±0.3	99.4±0.3	1	806.4±32.0
ls2-EDA	50.9±3.1	9604.7±2.9	99.0±0.0	99.2±0.0	1	1955.5±17.2	38.9±2.8	3304.4±2.9	99.5±0.2	99.6±0.2	1	902.6±23.7
ls2-DDALS(O)	**47.9±0.0**	9628.4±89.2	99.0±0.0	99.0±0.0	1	815.0±16.3	**27.2±0.0**	3191.7±71.5	99.3±0.0	99.3±0.0	1	373.4±11.6
ls2-DDALS(V1)	53.3±2.3	9628.4±89.2	99.4±0.0	99.5±0.0	1	815.0±16.3	28.0±0.7	3191.7±71.5	99.6±0.0	99.6±0.1	1	373.4±11.6
ls2-DDALS(V2)	49.7±0.9	9628.4±89.2	99.3±0.1	99.3±0.1	1	815.0±16.3	28.4±0.9	3191.7±71.5	99.6±0.2	99.7±0.2	1	373.4±11.6
ls2-DDALS(V3)	50.8±1.7	9682	99.2±0.2	99.2±0.2	1	816.1	29.2±1	3072	99.4±0.2	99.4±0.3	1	356.9
ls3-Greedy	100.9±0.0	3.0±0.0	99.9±0.0	99.8±0.0	1	**1.0±0.0**	101.2±0.0	5.0±0.0	99.83±0.0	98.5±0.0	0	**1.7±0.0**
ls3-GA	83.4±4.6	7209.0±7.3	99.1±0.1	98.8±0.2	1	52257.1±127.9	73.9±5.8	5210.3±8.4	99.2±0.3	99.2±0.3	1	3499.6±73.8
ls3-EDA	82.2±1.3	7204.9±2.4	99.1±0.2	98.9±0.2	1	3924.2±137.9	74.8±3.4	5204.5±2.4	99.7±0.2	99.6±0.3	1	3132.0±82.4
ls3-DDALS(O)	**74.9±0.2**	7276.1±68.6	99.1±0.0	98.7±0.0	1	2534.7±29.4	**63.4±0.9**	5182.9±197.4	99.4±0.2	99.3±0.2	1	1785.0±73.6
ls3-DDALS(V1)	79.6±1.5	7276.1±68.6	99.6±0.1	99.4±0.1	1	2534.7±29.4	64.4±1.9	5182.9±197.4	99.6±0.1	99.5±0.2	1	1785.0±73.6
ls3-DDALS(V2)	76.6±1.2	7276.1±68.6	99.4±0.1	99.2±0.1	1	2534.7±29.4	64.8±1.7	5182.9±197.4	99.7±0.2	99.7±0.2	1	1785.0±73.6
ls3-DDALS(V3)	77.1±1.3	7290	99.3±0.1	99.0±0.2	1	2520.8	63.8±0.5	5114	99.3±0.2	99.1±0.3	1	1750.2
ls4-Greedy	167.5±0.0	7.0±0.0	99.2±0.0	99.1±0.0	1	**3.5±0.0**	167.6±0.0	8.0±0.0	99.9±0.0	99.9±0.0	1	**3.7±0.2**
ls4-GA	124.1±8.7	11309.8±7.4	99.1±0.2	99.1±0.3	0.97	12310.4±282.8	107.5±6.8	8411.4±9.8	99.1±0.1	99.0±0.3	0.93	8437.8±157.5
ls4-EDA	123.6±3.6	11305.1±3.4	99.3±0.2	99.4±0.2	1	9498.8±243.3	116.6±3.2	8403.9±3.1	99.6±0.2	99.7±0.2	1	7970.2±162.7
ls4-DDALS(O)	**107.1±1.0**	11360.9±206.6	99.1±0.1	99.2±0.1	1	5537.5±95.9	**92.6±1.1**	8483.8±335.8	99.1±0.1	99.0±0.3	0.93	4113.9±147.9
ls4-DDALS(V1)	109.5±0.5	11360.9±206.6	99.7±0.1	99.7±0.1	1	5537.5±95.9	98.8±2.1	8483.8±335.8	99.6±0.1	99.6±0.1	1	4113.9±147.9
ls4-DDALS(V2)	108.9±1.1	11360.9±206.6	99.6±0.2	99.6±0.2	1	5537.5±95.9	94.4±2.0	8483.8±335.8	99.4±0.2	99.3±0.3	1	4113.9±147.9
ls4-DDALS(V3)	110.8±3.5	11576	99.3±0.2	99.4±0.2	1	5651.3	95.1±2.5	7964	99.1±0.1	99.3±0.1	1	3883.5
ls5-Greedy	226.7±0.0	2.0±0.0	99.9±0.0	99.9±0.0	1	**1.0±0.1**	216.5±0.0	5.0±0.0	99.8±0.0	99.8±0.0	1	**2.9±0.2**
ls5-GA	191.6±6.3	29012.8±9.4	99.0±0.0	98.9±0.1	1	44167.0±536.9	165.6±9.6	18311.6±8.5	99.1±0.1	98.9±0.2	0.97	24188.0±369.9
ls5-EDA	195.8±3.1	29004.5±2.8	99.2±0.1	99.0±0.2	1	30259.7±544.7	168.5±4.0	18303.9±3.3	99.4±0.2	99.4±0.2	1	22596.0±330.0
ls5-DDALS(O)	**178.2±0.9**	28935.6±446.0	99.0±0.0	98.9±0.0	1	18837.0±279.3	**145.3±1.5**	18509.9±825.5	99.0±0.0	99.0±0.0	1	11986.4±506.2
ls5-DDALS(V1)	184.5±2.2	28935.6±446.0	99.3±0.0	99.2±0.0	1	18837.0±279.3	155.9±3.6	18509.9±825.5	99.5±0.1	99.5±0.1	1	11986.4±506.2
ls5-DDALS(V2)	180.9±2.0	28935.6±446.0	99.2±0.1	99.1±0.1	1	18837.0±279.3	148.4±3.2	18509.9±825.5	99.3±0.2	99.3±0.2	1	11986.4±506.2
ls5-DDALS(V3)	180.0±1.8	28519	99.0±0.0	98.9±0.0	1	18654.3	145.6±1.0	17804	99.1±0.1	99.1±0.1	1	11506.5
ls6-Greedy	266.2±0.0	2.0±0.0	99.9±0.0	99.9±0.0	1	**1.7±0.0**	260.9±0.0	4.0±0.0	99.8±0.0	99.8±0.0	1	**3.2±0.0**
ls6-GA	226.1±10.9	34609.0±8.2	99.0±0.1	98.9±0.1	1	62450.7±680.7	192.7±13.8	20213.2±10.3	99.0±0.1	99.0±0.1	1	38172.5±307.5
ls6-EDA	215.5±1.1	34604.7±2.7	99.3±0.1	99.2±0.2	1	43323.9±115.5	181.9±3.3	20204.8±2.9	99.4±0.2	99.4±0.2	1	31784.8±192.8
ls6-DDALS(O)	**198.9± 0.8**	34546.2±496.2	99.0±0.0	98.9±0.1	1	28114.3±402.1	**161.7±1.3**	20060.4±974.6	99.1±0.1	99.0±0.1	1	15872.1±537.1
ls6-DDALS(V1)	205.3±1.7	34546.2±496.2	99.5±0.1	99.4±0.1	1	28114.3±402.1	169.3±2.3	20060.4±974.6	99.5±0.1	99.5±0.1	1	15872.1±537.1
ls6-DDALS(V2)	201.5±1.7	34546.2±496.2	99.3±0.1	99.2±0.1	1	28114.3±402.1	163.4±2.3	20060.4±974.6	99.4±0.1	99.3±0.1	1	15872.1±537.1
ls6-DDALS(V3)	203.1±2.1	34251	99.1±0.1	99.1±0.2	1	27936.2	164.0±1.6	18641	99.2±0.2	99.1±0.2	1	15077.8

References

[1] F. Yang and N. Chakraborty, "Algorithm for optimal chance constrained knapsack problem with applications to multi-robot teaming," in *2018 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2018, pp. 1043–1049.

[2] R. Ji and M. A. Lejeune, "Data-driven distributionally robust chance-constrained optimization with wasserstein metric," *Journal of Global Optimization*, vol. 79, no. 4, pp. 779–811, 2021.

Table 3: Performance on benchmarks of APP. Due to the introduction of retransmission mechanisms, there is a significant difference in the results between LAB and APP.

Benchmark	small $T_{\max}$						large $T_{\max}$					
	C	ET	ECL	RCL	FSR	Time	C	ET	ECL	RCL	FSR	Time
ss1-Greedy	14.0±0.0	4.0±0.0	99.9±0.0	97.2±0.0	0	**0.01±0.00**	7.3±0.0	5.0±0.0	99.6±0.0	97.1±0.0	0	**0.1±0.0**
ss1-GA	9.7±1.7	335.9±3.9	99.3±0.2	96.6±0.2	0	7.0±1.2	7.4±0.8	304.8±3.4	99.4±0.2	97.2±0.0	0	9.6±1.5
ss1-EDA	9.1±0.1	334.5±3.1	99.3±0.0	96.6±0.1	0	12.9±1.1	8.3±0.9	305.0±2.9	99.5±0.1	97.8±0.6	0	6.7±1.2
ss1-DDALS(O)	**9.0±0.0**	312.1±8.3	99.2±0.0	96.5±0.0	0	1.4±0.3	**7.3±0.0**	285.2±15.8	99.6±0.0	97.1±0.0	0	3.2±0.3
ss1-DDALS(V1)	14.0±0.0	312.1±8.3	99.9±0.0	97.2±0.0	0	1.4±0.3	**7.3±0.0**	285.2±15.8	99.6±0.0	97.1±0.0	0	3.2±0.3
ss1-DDALS(V2)	13.8±0.9	312.1±8.3	99.8±0.1	97.1±0.1	0	1.4±0.3	**7.3±0.0**	285.2±15.8	99.6±0.0	97.2±0.0	0	3.2±0.3
ss1-DDALS(V3)	14.1±2.7	321	99.6±0.3	97.9±1.0	**0.5**	1.5	11.1±2.4	300	99.5±0.0	97.8±0.7	**0.1**	2.7
ss2-Greedy	5.9±0.0	6.0±0.0	99.8±0.0	98.5±0.0	0	**12.6±0.4**	5.9±0.0	6.0±0.0	99.9±0.0	98.9±0.0	**1**	**8.7±0.2**
ss2-GA	5.3±0.1	303.2±2.3	99.2±0.0	97.3±0.5	0	4508.8±1198.4	5.3±0.1	272.9±2.2	99.5±0.1	98.1±0.2	0.03	3422.4±352.0
ss2-EDA	5.5±0.3	305.0±2.7	99.2±0.2	97.6±0.7	0.1	3323.2±1532.5	5.6±0.3	274.1±2.4	99.5±0.2	98.5±0.5	0.47	1185.3±1048.5
ss2-DDALS(O)	**5.2±0.0**	300.0±10.9	99.2±0.0	97.3±0.0	0	1010.5±98.7	**5.2±0.0**	297.5±7.8	99.6±0.0	98.1±0.0	0	862.2±78.7
ss2-DDALS(V1)	5.4±0.0	300.0±10.9	99.8±0.1	97.4±0.2	0	1010.5±98.7	**5.2±0.0**	297.5±7.8	99.6±0.0	98.1±0.0	0	862.2±78.7
ss2-DDALS(V2)	5.4±0.0	300.0±10.9	99.8±0.0	97.4±0.2	0	1010.5±98.7	5.4±0.1	297±7.8	99.9±0.1	98.2±0.2	0.1	862.2±78.7
ss2-DDALS(V3)	5.4±0.1	306	99.5±0.3	97.6±0.4	0	1086.0	5.5±0.1	298	99.7±0.2	98.5±0.3	0.5	858.0
ss3-Greedy	18.3±0.0	6.0±0.0	99.9±0.0	96.3±0	0	**0.5±0.0**	14.2±0.0	8.0±0.0	99.8±0.0	97.3±0.0	0	**0.6±0.0**
ss3-GA	17.7±0.7	634.9±5.2	99.4±0.4	95.0±1.8	0	179.2±2.5	12.0±1.1	503.6±3.3	99.4±0.3	95.2±1.5	0	152.9±2.2
ss3-EDA	17.6±0.2	635.1±2.8	99.9±0.2	95.9±0.1	0	116.7±3.4	13.8±1.1	503.9±2.7	99.8±0.3	96.4±0.7	0	98.2±2.8
ss3-DDALS(O)	**17.0±0.2**	690.1±31.5	99.1±0.1	93.9±0.7	0	34.5±3.1	**11.1±0.1**	505.8±14.6	99.6±0.2	94.7±0.5	0	30.5±1.2
ss3-DDALS(V1)	18.0±0.3	690.1±31.5	99.7±0.2	96.5±0.5	0	34.5±3.1	11.2±0.7	505.8±14.6	99.7±0.0	95.1±0.7	0	30.5±1.2
ss3-DDALS(V2)	18.0±0.4	690.1±31.5	99.9±0.1	96.4±0.4	0	34.5±3.1	11.3±0.8	505.8±14.6	99.6±0.2	94.9±0.8	0	30.5±1.2
ss3-DDALS(V3)	17.7±0.3	736	99.4±0.3	95.6±1.6	0	35.4	12.7±1.0	507	99.4±0.2	95.3±0.7	0	29.8
ss4-Greedy	34.9±0.0	3.0±0.0	100.0±0.0	100.0±0.0	**1**	**0.2±0.0**	28.5±0.0	4.0±0.0	100.0±0.0	99.0±0.0	**1**	**0.3±0.0**
ss4-GA	27.5±1.4	2008.9±7.9	99.8±0.2	90.3±0.5	0	251.8±24.5	20.5±2.7	1608.4±7.4	99.4±0.3	92.9±4.8	0	322.5±9.2
ss4-EDA	**26.1±0.0**	2005.7±2.3	100.0±0.0	90.1±0.0	0	333.9±11.8	18.3±2.7	1604.5±2.8	99.9±0.1	85.2±6.9	0	287.9±19.6
ss4-DDALS(O)	**26.1±0.0**	1988.9±27.8	100.0±0.0	90.1±0.0	0	33.3±2.3	**16.6±0.8**	1582.8±54.4	99.8±0.1	81.3±3.7	0	61.3±3.9
ss4-DDALS(V1)	**26.1±0.0**	1988.9±27.8	100.0±0.0	90.1±0.0	0	33.3±2.3	16.8±1.1	1582.8±54.4	99.8±0.1	81.8±4.3	0	61.3±3.9
ss4-DDALS(V2)	**26.1±0.0**	1988.9±27.8	100.0±0.0	90.1±0.0	0	33.3±2.3	18.3±2.2	1582.8±54.4	99.9±0.1	86.9±7.3	0	61.3±3.9
ss4-DDALS(V3)	29.7±2.5	1944	99.9±0.2	92.2±3.9	0.2	28.3	20.8±2.9	1586	99.7±0.3	88.3±6.3	0	59.3
ls1-Greedy	84.1±0.0	2.0±0.0	99.3±0.0	99.5±0.0	**1**	**0.3±0.0**	84.1±0.0	2.0±0.0	99.9±0.0	99.9±0.0	**1**	**0.3±0.0**
ls1-GA	64.7±2.9	6437.8±11.7	99.1±0.1	98.7±0.1	**1**	3441.1±15.0	60.2±4.0	6332.0±11.5	99.0±0.1	98.7±21.9	0.9	3381.1±21.9
ls1-EDA	68.9±0.0	6424.7±2.5	99.0±0.0	99.4±0.0	**1**	2266.4±90.7	61.4±0.2	6324.3±2.9	99.2±0.0	99.1±0.0	**1**	2250.8±102.6
ls1-DDALS(O)	**61.5±0.0**	6418.0±19.6	99.2±0.0	98.5±0.0	**1**	955.6±9.4	**53.7±2.6**	6304.6±65.6	99.0±0.0	98.7±0.3	**1**	958.1±14.7
ls1-DDALS(V1)	63.1±1.0	6418.0±19.6	99.6±0.0	99.1±0.1	**1**	955.6±9.4	62.1±1.7	6304.6±65.6	99.7±0.2	99.5±0.3	**1**	958.1±14.7
ls1-DDALS(V2)	62.3±0.3	6418.0±19.6	99.4±0.1	98.9±0.3	**1**	955.6±9.4	55.3±3.2	6304.6±65.6	99.2±0.2	98.8±0.2	**1**	958.1±14.7
ls1-DDALS(V3)	62.8±0.7	6417	99.2±0.1	98.7±0.2	**1**	950.7	55.2±1.2	6351 ±65.9	99.1±0.1	98.8±0.1	**1**	935.6
ls2-Greedy	89.1±0.0	2.0±0.0	99.9±0.0	99.9±0.0	**1**	**0.2±0.0**	89.1±0.0	2.0±0.0	100.0±0.0	100.0±0.0	**1**	**0.3±0.0**
ls2-GA	76.7±2.8	22616.5±10.5	99.1±0.1	99.0±0.2	**1**	3925.1±155.5	60.9±2.7	21324.5±13.3	99.1±0.1	99.1±0.1	**1**	3382.2±64.9
ls2-EDA	79.3±1.1	22604.1±2.6	99.5±0.1	99.3±0.1	**1**	7174.3±44.2	59.9±0.8	21304.8±3.2	99.2±0.0	99.2±0.0	**1**	6970.8±88.9
ls2-DDALS(O)	**72.0±0.3**	22631.7±62.4	99.0±0.0	99.0±0.0	**1**	841.3±7.6	**57.2±0.0**	21265.2±73.3	99.3±0.1	99.3±0.0	**1**	776.4±6.6
ls2-DDALS(V1)	76.2±1.1	22631.7±62.4	99.5±0.1	99.5±0.1	**1**	841.3±7.6	59.0±1.2	21265.2±73.3	99.6±0.0	99.6±0.1	**1**	776.4±6.6
ls2-DDALS(V2)	73.2±0.4	22631.7±62.4	99.4±0.1	99.3±0.1	**1**	841.3±7.6	57.6±0.5	21265.2±73.3	99.4±0.1	99.5±0.1	**1**	776.4±6.6
ls2-DDALS(V3)	73.7±1.5	22536	99.2±0.1	99.0±0.2	**1**	846.2	58.3±0.6	21274	99.3±0.1	99.3±0.1	**1**	769.6
ls3-Greedy	163.3±0.0	3.0±0.0	99.9±0.0	99.4±0.0	**1**	**1.1±0.0**	163.3±0.0	3.0±0.0	99.9±0.0	99.9±0.0	**1**	**1.0±0.0**
ls3-GA	121.7±7.3	21012.1±8.6	99.0±0.0	98.6±0.4	0.8	17477.6±60.9	121.1±4.1	21011.8±7.1	99.0±0.0	98.4±0.4	0.53	17447.7±62.8
ls3-EDA	117.4±1.0	21004.5±2.5	99.4±0.0	98.9±0.0	**1**	13400.8±127.8	116.8±1.1	21004.8±2.6	99.6±0.1	99.4±0.2	0.97	13425.6±127.7
ls3-DDALS(O)	**111.9±1.0**	20999.3±184.8	99.1±0.1	98.9±0.1	**1**	5734.4±64.0	**111.7±1.6**	20910.0±266.1	99.3±0.3	98.8±0.4	0.6	5702.1±80.0
ls3-DDALS(V1)	118.1±2.0	20999.3±184.8	99.5±0.2	98.9±0.3	0.9	5734.4±64.0	113.8±4.2	20910.0±266.1	99.5±0.2	99.2±0.3	0.97	5702.1±80.0
ls3-DDALS(V2)	114.1±2.4	20999.3±184.8	99.3±0.2	98.8±0.1	**1**	5734.4±64.0	112.6±2.4	20910.0±266.1	99.5±0.3	99.0±0.4	0.8	5702.1±80.0
ls3-DDALS(V3)	116.1±1.7	20667	99.2±0.2	98.8±0.1	**1**	5700.9	114.4±1.4	21000	99.1±0.1	98.6±0.2	0.7	5663.9
ls4-Greedy	238.1±0.0	3.0±0.0	99.4±0.0	99.0±0.0	**1**	**1.5±0.0**	193.7±0.0	15.0±0.0	99.2±0.0	98.8±0.0	**1**	**7.0±0.2**
ls4-GA	206.6±4.7	48018.6±13.4	99.0±0.0	98.6±0.3	0.73	46617.9±648.4	182.7±10.7	45015.2±11.7	99.0±0.0	98.5±0.2	0.6	42937.2±431.6
ls4-EDA	204.3±1.9	48004.5±3.5	99.0±0.0	99.0±0.0	**1**	45488.7±924.8	163.6±1.2	45003.5±2.4	99.1±0.1	98.8±0.1	**1**	42955.7±613.1
ls4-DDALS(O)	**195.1±0.5**	47999.9±83.0	99.0±0.0	98.9±0.0	**1**	16357.5±137.6	**161.5±2.1**	44440.5±344.6	99.0±0.0	98.5±0.3	0.4	15160.3±127.4
ls4-DDALS(V1)	209.2±2.8	47999.9±83.0	99.1±0.0	98.6±0.3	0.6	16357.5±137.6	169.4±4.2	44440.5±344.6	99.3±0.2	98.9±0.4	0.8	15160.3±127.4
ls4-DDALS(V2)	197.4±1.7	47999.9±83.0	99.1±0.0	99.0±0.1	**1**	16357.5±137.6	162.5±2.4	44440.5±344.6	99.1±0.1	98.6±0.2	0.7	15160.3±127.4
ls4-DDALS(V3)	196.8±0.8	48046	99.0±0.0	98.9±0.1	**1**	16280.8	163.5±3.5	44648	99.0±0.0	98.3±0.1	0.1	15162.6
ls5-Greedy	282.7±0.0	4.0±0.0	99.6±0.0	99.4±0.0	**1**	**2.8±0.2**	267.2±0.0	6.0±0.0	99.0±0.0	98.8±0.0	**1**	**3.9±0.0**
ls5-GA	238.6±8.3	81011.4±10.1	99.0±0.0	98.6±0.1	0.87	104473.1±1301.8	216.9±8.3	79012.9±8.8	99.0±0.0	98.6±0.2	0.77	101543.5±950.4
ls5-EDA	220.3±0.6	81004.3±2.8	99.1±0.0	98.7±0.0	**1**	103916.1±1686.4	200.6±1.8	79004.3±2.5	99.1±0.1	98.7±0.2	**1**	100901.5±1166.1
ls5-DDALS(O)	**217.4±1.2**	81186.6±246.3	99.0±0.0	98.6±0.0	**1**	36706.0±267.7	**199.6±2.2**	78766.1±799.8	99.0±0.0	98.6±0.1	0.8	35543.0±403.1
ls5-DDALS(V1)	225.3±3.0	81186.6±246.3	99.2±0.1	98.8±0.1	**1**	36706.0±267.7	209.4±4.9	78766.1±799.8	99.3±0.1	98.9±0.2	0.97	35543.0±403.1
ls5-DDALS(V2)	219.0±2.6	81186.6±246.3	99.1±0.1	98.7±0.1	**1**	36706.0±267.7	201.9±3.4	78766.1±799.8	99.1±0.1	98.7±0.2	0.9	35543.0±403.1
ls5-DDALS(V3)	220.9±2.3	81306	99.0±0.0	98.7±0.0	**1**	36440.4	203.6±1.1	77365	99.1±0.1	98.7±0.2	0.9	34775.3
ls6-Greedy	399.3±0.0	3.0±0.0	99.8±0.0	99.4±0.0	**1**	**1.9±0.1**	381.4±0.0	12.0±0.0	99.8±0.0	99.4±0.0	**1**	**6.9±0.1**
ls6-GA	357.0±13.2	125313.9±8.9	99.0±0.0	98.2±0.1	0	196363.3±427.9	323.2±9.8	121514.7±9.1	99.0±0.0	98.2±0.3	0.13	193984.6±4074.8
ls6-EDA	338.1±1.4	125305.6±2.9	99.0±0.0	98.6±0.0	**1**	196994.6±539.1	300.7±1.6	121504.8±2.9	99.1±0.1	98.7±0.1	**1**	193948.7±1566.8
ls6-DDALS(O)	**323.9±0.6**	125056.0±178.1	99.0±0.0	98.1±0.0	0	70604.7±449.7	**297.9±4.5**	121616.1±1093.7	99.0±0.0	98.3±0.1	0	68409.7±727.0
ls6-DDALS(V1)	332.7±2.9	125056.0±178.1	99.1±0.0	98.2±0.0	0	70604.7±449.7	**297.9±4.5**	121616.1±1093.7	99.0±0.0	98.3±0.1	0	68409.7±727.0
ls6-DDALS(V2)	325.3±2.2	125056.0±178.1	99.0±0.1	98.2±0.1	0	70604.7±449.7	199.4±5.2	121616.1±1093.7	99.1±0.1	98.3±0.2	0.1	68409.7±727.0
ls6-DDALS(V3)	326.4±2.0	124926	99.0±0.0	98.2±0.0	0	70190.2	305.0±3.9	121537	99.01±0.0	98.1±0.1	0	68162.4

[3] R. Tempo, E.-W. Bai, and F. Dabbene, "Probabilistic robustness analysis: Explicit bounds for the minimum number of samples," in *Proceedings of 35th IEEE Conference on Decision and Control*, vol. 3. IEEE, 1996, pp. 3424–3428.

[4] T. Alamo, R. Tempo, and A. Luque, "On the sample complexity of randomized approaches to the analysis and design under uncertainty," in *Proceedings of the 2010 American Control Conference*. IEEE, 2010, pp. 4671–4676.