1 Details about Concentration Inequalities

1.1 Bernstein's Inequality

Assume that there are m classes at the E2E, each denoted as variable $X_j, j = 1, \ldots, m$. $X_1, X_2, \ldots, X_m$ are independent and satisfy for given some $\{(a_j, b_j), \forall j \in \{1, \ldots, m\}\}$ and constant C there are $P(X_j \in [a_j, b_j]) = 1, \forall j \in \{1, \ldots, m\}$ and $b_j - a_j = c_j \leq C$. Denote $S = X_1 + X_2 + \ldots + X_m$ as the sum of random variables X_j and $V = \sum_{i=1}^{m} Var(X_j)$ is the sum of variances. Then we have Bernstein's inequality,

$$P(S \leq E(S) + \alpha) \geq 1 - exp(-\frac{\alpha^2/2}{V + C\alpha/3}) \tag{1}$$

1.2 K-order Hoeffding's Inequality

Similar to Bernstein's inequality, we assume that $X_1, X_2, \ldots, X_m$ are independent and satisfy for given some $\{(a_j, b_j), \forall j \in \{1, \ldots, m\}\}$ there is $P(X_j \in [a_j, b_j]) = 1, \forall j \in \{1, \ldots, m\}$. Denote new variable $Y_j = X_j - a_j$, which satisfies $P(Y_j \in [0, c_j]) = 1, \forall j \in \{1, \ldots, m\}$. Denote $S = Y_1 + Y_2 + \ldots + Y_m$ and we have $\mathbb{E}(Y_j^k) = \mu_j^k$. K is the order of origin moment. Thus, we have following K-order Hoeffding's inequality:

$$P(S \leq E(S) + \alpha) \geq 1 - exp(-\frac{2\alpha^2}{\sum_{j=1}^{m} b_j^2 C_K(\frac{4\alpha b_j}{D}, b_j, \mu_j^1, \ldots, \mu_j^K)}) \tag{2}$$

where $D = \sum_{j=1}^{m} (\mu_j^2/\mu_j^1)^2$ and $C_K(\frac{4\alpha b_j}{D}, b_j, \mu_j^1, \ldots, \mu_j^K)$ is a value calculated by all the orders of origin moments of Y_j. However, the concentration inequality does not make any assumptions about the form of the distribution of the variables; hence, the lower bound provided by the inequality may be imprecise. Additionally, as the inequalities are solely based on data-driven background, they may only serve as rough estimates for the upper and lower bounds of the variables and this may introduce extra errors into the confidence evaluation.

Since only data of items are available, if we manage to calculate the ratio of the combination of data meeting the upper bound W, to the total number of data combinations, then we can know whether the solution satisfies the chance constraint. For uncertain solution $S = [s_1, s_2, \ldots, s_m]$, we count the number of $\sum_{i=1}^{m} d_{i,j_i,l} \leq W$ and check if it exceeds $P_0 L^m$. Since P_0 is usually a number very close to 1 under high confidence level requirement, it is more practical to compare the number of $\sum_{i=1}^{m} d_{s_i l} \geq W$ with $(1 - P_0)L^m$. If the number of $\sum_{i=1}^{m} d_{s_i l} \geq W$ is greater than $(1 - P_0)L^m$, then we can determine quickly that it is an infeasible solution.

2 Proof

Theorem 2.1. *The popped sum in the k^{th} iteration is exactly the k^{th} largest sum.*

Proof. Suppose the i^{th} largest sum is $S_r = d_{s_1 l_1} + d_{s_2 l_1} + \ldots + d_{s_m l_m}$.

- For any pair in the combination or queue Q that has already been output, the combinations with indices less than or equal to it must have been popped or exist in Q.

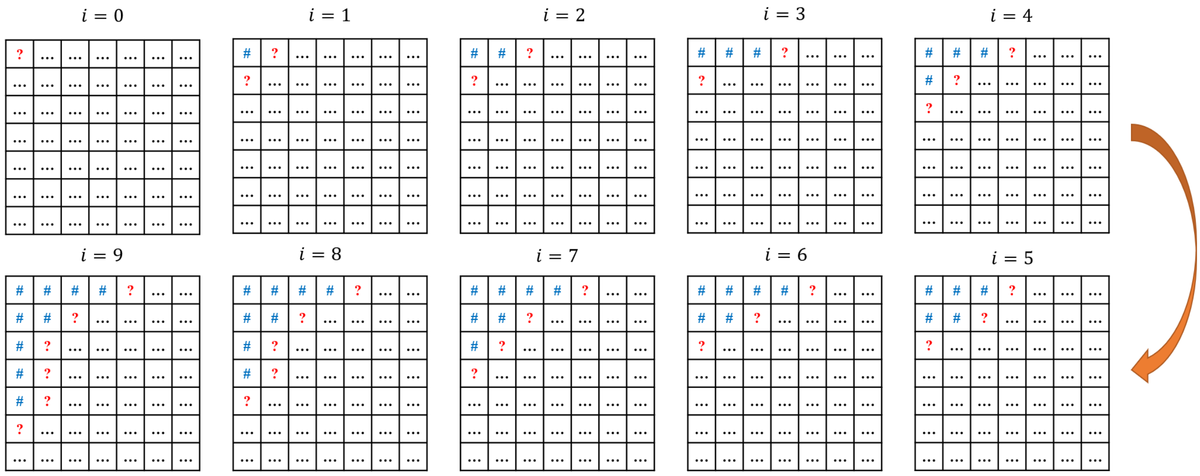

Figure 1: An example of exact evaluation when $m = 2$

- Construct a chain S, then we have the following chain, denoted as $S_0 \leq S_1 \leq, \ldots, \leq S_l$.

$$
\begin{aligned}
&d_{s_1 0} + d_{s_2 0} + \ldots + d_{s_m 0}, \\
\geq &d_{s_1 1} + d_{s_2 0} + \ldots + d_{s_m 0}, \\
\geq &\ldots, \\
\geq &d_{s_1 l_1} + d_{s_2 0} + \ldots + d_{s_m 0} \\
\geq &\ldots, \\
\geq &d_{s_1 l_1} + d_{s_2 l_2} + \ldots + d_{s_m 0} \\
\geq &\ldots, \\
\geq &d_{s_1 l_1} + d_{s_2 l_2} + \ldots + d_{s_m l_m}
\end{aligned}
$$

- Let S_j be the first sum in chain S which was not popped from queue Q before the j^{th} iteration. During the j^{th} iteration, when S_{j-1} is popped, S_j is added to Q.
- Thus, during the k^{th} iteration, the popped sum S_i satisfies $S_i \leq S_j \leq S_l = d_{s_1 l_1} + d_{s_2 l_1} + \ldots + d_{s_m l_m}$.
- By induction, since the $(i-1)^{th}$ largest sum has been popped before the i^{th} iteration, so the sum popped in the i^{th} iteration is exactly the i^{th} largest sum0 $S_l = d_{s_1 l_1} + d_{s_2 l_1} + \ldots + d_{s_m l_m}$.

Then we finish the proof. □

A simple example is shown when $m = 2$ in Fig.1.

3 A Theoretical Quantitative Analysis between Estimation Error and Sample Size

To theoretically quantify the relationship between estimation error and sample size of data, we construct a new random variable S that is the sum of multiple random variables such that the variable itself is a random variable. Then we define a Bernoulli variable $Z \in \{0, 1\}$, i.e.,

$$
Z = \begin{cases} 1, & S \leq W \\ 0, & S > W \end{cases} \tag{3}
$$

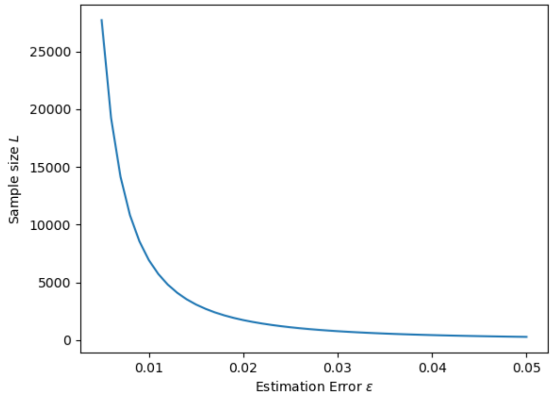

Figure 2: Sample size requirement with different error in theory

for a combination of items with L sample size, where p is the true confidence level, i.e., the expectation $\mathbb{E}Z$ of the variable Z. Hoeffding's inequality shows that for independent identically distributed random variables $Z_1, Z_2, \ldots, Z_L$ taking values $[a, b]$, for any $\epsilon > 0$, the following inequality holds,

$$Pr(|\mathbb{E}[Z] - \frac{1}{L}\sum_{l=1}^{L} Z_l| \geq \epsilon) \leq 2exp(-\frac{2L\epsilon^2}{(b-a)^2}) \tag{4}$$

For our problem, we are concerned with the true value is lower than the estimate and greater than the error, i.e., where ϵ is the estimation error corresponding to the confidence level. For the Bernoulli distribution, we have $\mathbb{E}Z = p$ and $a = 0, b = 1$. Thus, the inequality becomes

$$Pr(p - f(Z_1, Z_2, \ldots, Z_L) \leq -\epsilon) \leq exp(-2L\epsilon^2) \tag{5}$$

where $f(Z_1, Z_2, \ldots, Z_L)$ denotes the confidence evaluation algorithm on some combination of items. Assuming that at least 5 of the 10 solutions in the output solution set are true feasible solutions, and $f(Z_1, Z_2, \ldots, Z_L)$ is accurate, then we have $1 - exp(-2L\epsilon^2) \geq 5/10$. Finally, it leads to

$$L \geq \frac{ln2}{2\epsilon^2} \tag{6}$$

As this bound is derived from Hoeffding's inequality, Chernoff bound is proposed in [1] and a more tight one is proposed in [2]. However, all of these inequalities indicate that a large amount of samples is necessary. The curve of the variation of L with ϵ is shown in Figure 2.
The minimum sample sizes required for different estimation error are listed in Table 1. Overall, Hoeffding provides the tightest bounds, especially suitable for larger errors. However, as error requirements become smaller, the bounds provided by Alamo are more advantageous. Practically, it is verified that the amount of data required to meet the corresponding error requirement for the true confidence will be much smaller than the theoretical value obtained.

Table 1: Relationship between EE and sample size

Estimation error(%)	5	0.5	0.05	0.005
Hoeffding's bound	139	13863	1386295	138629437
Chernoff's bound [1]	278	27726	2772589	277258873
Alamo's bound [2]	1573	15723	157227	1572269

4 Deterministic method based on Gaussian Assumption and Mix Integer Linear Programming for DDCCMCKP

4.1 Risk-averse knapsack problem method

Yang and Chakraborty [3] proposed an algorithm for the chance-constrained knapsack problem (CCKP) assuming known Gaussian distributions of the random variables (mean and variance are known). CCKP was then transformed into an iterative Risk-averse Knapsack Problem (RA-KP), which could be solved via dynamic programming. The implement on DDCCMCKP of this work is easy. We first calculated the sample mean and standard deviation of the item weights, $\mu_{ij} = \mathbb{E}(w_{ij}), \sigma_{ij}^2 = Var(w_{ij}), i \in \{1, ..., m\}, j \in N_i$, and then plug them into the algorithm of Yang's,

$$\min \quad \sum_{i=1}^{m} \sum_{j \in N_i} c_{ij} x_{ij} \tag{7a}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} \sum_{j \in N_i} \mu_{ij} x_{ij} + \Phi^{-1}(P_0) \sqrt{\sum_{i=1}^{m} \sum_{j \in N_i} \sigma_{ij}^2 x_{ij}} \leq W \tag{7b}$$

$$\sum_{j \in N_i} x_{ij} = 1, \forall i \in \mathcal{M} \tag{7c}$$

$$x_{ij} \in \{0, 1\}, \forall i \in \mathcal{M}, j \in N_i \tag{7d}$$

where Φ represents the CDF of a standard normal distribution with zero mean and unit variance.

4.2 Mix Integer Linear Programming

Ji and Lejeune [4] studied the distribution-robust chance-constrained planning (DRCCP) optimization problem with data-driven Wasserstein fuzzy sets and proposed an algorithm and reconstruction framework suitable for all types of distribution-robust chance-constrained optimization problems, which was then applied to multidimensional knapsack problems. When the decision variables are binary, the proposed LP (who becomes MILP) formulations are equivalent reformulations of the CC. To fit the CC into [4]'s modelling and solution framework, we first formulate $\text{Prob}(\sum_{i=1}^{m} \sum_{j \in N_i} w_{ij} x_{ij} \leq W) \geq P_0$ as:

$$\text{Prob}(\sum_{i=1}^{m} \sum_{j \in N_i} w_{ij} x_{ij} > W) \leq 1 - P_0 \tag{8}$$

Define $b = -W, \epsilon = 1 - P_0, \tilde{w}_{ij} = -w_{ij}, \forall i \in \mathcal{M}, j \in N_i$. Let $\bar{x} \in \{0, 1\}^{NM}$ as the vectorization of $x \in \{0, 1\}^{N \times M}$, $\xi \in \mathbb{R}^{NM}$ is the vectorization of $\hat{w} \in \mathbb{R}^{N \times M}$. Finally, we have the standard form of 8 as [4]:

$$\text{Prob}(\xi^T \bar{x} < b) \leq \epsilon \tag{9}$$

Then we can utilize Theorem 10 in [4] to reformulate the above inequation 8, and get the MILP formulation of DDCCMCKP:

$$(\bar{x}, \lambda, \eta, s, \phi) \in \mathcal{Z}_{LP}^{MCKP} = \begin{cases} \lambda\theta + \frac{1}{L}\sum_{l=1}^{L} s_l \leq \epsilon, & \\ \lambda \geq 0, & \\ s_l \geq 0, \eta_l \geq 0, & \forall l \in \mathcal{L} \\ 1 + \eta_l b^{'} - \sum_{q\in\mathcal{Q}} \xi_{ql}^0 \phi_{ql} \leq s_l, & \forall l \in \mathcal{L} \\ \phi_{ql} \leq \lambda, & \forall q \in \mathcal{Q}, l \in \mathcal{L} \\ \phi_{ql} \leq \eta_l, & \forall q \in \mathcal{Q}, l \in \mathcal{L} \\ \phi_{ql} \leq U_{\eta_l}\bar{x}_q, & \forall q \in \mathcal{Q}, l \in \mathcal{L} \\ \phi_{ql} \geq 0, & \forall q \in \mathcal{Q}, l \in \mathcal{L} \\ \phi_{ql} \geq \eta_l + U_{\eta_l}\bar{x}_q - U_{\eta_l}, & \forall q \in \mathcal{Q}, l \in \mathcal{L} \\ \sum_{i=Nm-N+1}^{Nm} \bar{x}_i = 1, & \forall m \in \mathcal{M} \\ \bar{x}_q \in \{0,1\}, & \forall q \in \mathcal{Q} \end{cases} \tag{10}$$

where $b^{'} = b - \delta$, δ is a user-defined infinite small positive number. $\mathcal{Q}$ is the set of all $N \times M$ items, $\mathcal{M}$ is the set of M classes. Note that $\sum_{i=Nm-N+1}^{Nm} \bar{x}_i = 1, \forall m \in \mathcal{M}$ is multiple-choice constraint in MCKP. U_η is the upper bound of η and can be determined by:

$$U_{\eta_l} = \begin{cases} \mathcal{K}^{'*}, & if \exists \bar{x} : \bar{x}^T \xi_l^0 - b > 0 \\ 0, & otherwise \end{cases} \tag{11}$$

where $\mathcal{K}^{'*}$ is the optimal objective value of the linear programming problem:

$$\begin{aligned} \mathcal{K}^{'*} = & \min \bar{x}^T \xi_l^0 - b \\ & \text{s.t.} b - \bar{x}^T \xi_l^0 \leq 0 - \tau^{'} \\ & \bar{x} \in \bar{\mathcal{X}} \end{aligned} \tag{12}$$

where $\tau^{'}$ a user-defined small non-negative number.
Overall, we first solve linear programming 12 to get $U_{\eta_l}, \forall l \in \mathcal{L}$ then solve MILP $\mathcal{Z}_{LP}^{MCKP}$ 10 to find the optimal solution of DDCCMCKP.

5 Complete experimental results

In this section, we show the complete experimental results of DDALS on LAB and APP.

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[4] R. Ji and M. A. Lejeune, "Data-driven distributionally robust chance-constrained optimization with wasserstein metric," *Journal of Global Optimization*, vol. 79, no. 4, pp. 779–811, 2021.

Table 2: Performance on benchmarks of LAB. The highest average real confidence level and the lowest cost are highlighted through bolding and being underlined.

Benchmark	small $T_{\max}$					large $T_{\max}$				
	C	**ET**	**ECL**	**RCL**	**FSR**	**C**	**ET**	**ECL**	**RCL**	**FSR**
ss1-Greedy	22.2±0.0	2.0±0.0	0±0.0	99.6±0.0	1	15.1±0.0	3.0±0.0	99.9±0.0	99.9±0.0	1
ss1-GA	18.7±0.1	354±4.2	99.8±0.3	99.6±0.8	0.8	**10.8±0.0**	253.3±4.1	99.9±0.1	99.0±1.2	0.6
ss1-EDA	21.5±1.4	354.5±2.9	-	99.5±0.5	0.1	13.0±3.4	234.7±2.8	99.8±0.3	99.9±0.1	1
ss1-DDMLS(O)	**18.6±0.0**	351.7±5.8	99.9±0.0	99.9±0.0	1	**10.8±0.0**	231±13.7	99.9±0.0	99.9±0.0	1
ss1-DDMLS(V1)	**18.6±0.0**	351.7±5.8	99.9±0.0	99.9±0.0	1	**10.8±0.0**	231±13.7	99.9±0.0	99.9±0.0	1
ss1-DDMLS(V2)	**18.6±0.0**	351.7±5.8	99.9±0.0	99.9±0.0	1	**10.8±0.0**	231±13.7	99.9±0.0	99.9±0.0	1
ss1-DDMLS(V3)	20.7±2.0	360	99.6±0.3	99.2±0.9	0.3	13.0±1.7	218	99.8±0.3	99.1±1.0	0.7
ss2-Greedy	**19.4±0.0**	2.0±0.0	99.9±0.0	99.9±0.0	1	16.5±0.0	4.0±0.0	99.9±0.0	99.9±0.0	1
ss2-GA	**19.4±0.0**	384.5±2.6	99.9±0.0	99.9±0.0	1	11.1±1.4	303.6±3.4	99.7±0.2	99.7±0.4	1
ss2-EDA	19.9±0.4	385.9±2.7	99.9±0.0	99.9±0.1	1	14.8±2.2	304.5±2.6	99.7±0.2	99.7±0.4	1
ss2-DDMLS(O)	**19.4±0.0**	378.8±8.2	99.9±0.0	99.9±0.0	1	**10.4±0.0**	298.9±15.5	99.8±0.0	99.9±0.0	1
ss2-DDMLS(V1)	**19.4±0.0**	378.8±8.2	99.9±0.0	99.9±0.0	1	**10.4±0.0**	298.9±15.5	99.8±0.0	99.9±0.0	1
ss2-DDMLS(V2)	**19.4±0.0**	378.8±8.2	99.9±0.0	99.9±0.0	1	12.3±2.0	298.9±15.5	99.9±0.1	99.9±0.0	1
ss2-DDMLS(V3)	21.8±2.4	377	99.9±0.0	99.9±0	1	13.8±1.6	319	99.8±0.3	99.9±0.1	1
ss3-Greedy	24.4±0.0	2.0±0.0	100.0±0.0	99.9±0.0	1	21.8±0.0	3.0±0.0	100±0	99.9±0.0	1
ss3-GA	22.6±1.5	303.4±2.8	99.9±0.0	99.7±0.2	1	12.9±0.7	262.4±1.6	99.9±0.0	99.9±0.1	1
ss3-EDA	24.1±2.3	304.1±2.3	99.9±0.1	99.6±0.6	0.9	14.0±2.0	264.5±2.9	99.9±0.0	99.9±0.1	1
ss3-DDALS(O)	**21.4±0.0**	291.9±6.2	99.9±0.0	99.5±0.0	1	**12.6±0.0**	254.4±4.7	100.0±0.0	99.9±0.0	1
ss3-DDALS(V1)	**21.4±0.0**	291.9±6.2	99.9±0.0	99.5±0.0	1	**12.6±0.0**	254.4±4.7	100.0±0.0	99.9±0.0	1
ss3-DDALS(V2)	**21.4±0.0**	291.9±6.2	99.9±0.0	99.5±0.0	1	**12.6±0.0**	254.4±4.7	100.0±0.0	99.9±0.0	1
ss3-DDALS(V3)	23.9±1.7	279	99.8±0.2	99.1±0.8	0.8	15.7±1.9	260	99.9±0.3	99.6±0.9	0.9
ss4-Greedy	30.1±0.0	2.0±0.0	99.9±0.0	99.9±0.0	1	27.5±0.0	3.0±0.0	99.7±0.0	99.4±0.0	1
ss4-GA	23.8±5.5	768.9±8.5	99.9±0.3	99.7±0.4	1	17.2±3.0	562.6±5.7	99.7±0.3	99.6±0.3	1
ss4-EDA	**17.4±0.0**	763.4±2.5	99.9±0.0	99.9±0.0	1	13.0±1.0	553.4±2.7	99.8±0.2	98.9±0.4	1
ss4-DDALS(O)	**17.4±0.0**	759.9±25.8	99.9±0.0	99.9±0.0	1	**11.2±0.0**	525.3±39.1	99.2±0.0	99.0±0.0	1
ss4-DDALS(V1)	**17.4±0.0**	759.9±25.8	99.9±0.0	99.9±0.0	1	12.1±0.0	525.3±39.1	99.5±0.0	98.9±0.0	1
ss4-DDALS(V2)	**17.4±0.0**	759.9±25.8	99.9±0.0	99.9±0.0	1	14.0±1.2	525.3±39.1	99.9±0.2	99.0±0.3	1
ss4-DDALS(V3)	24.6±4.0	735	99.8±0.2	99.5±0.5	1	13.8±1.3	561	99.8±0.3	99.5±0.5	1
ls1-Greedy	58.8±0.0	2.0±0.0	99.8±0.0	99.8±0.0	1	56.0±0.0	3.0±0.0	99.6±0.0	99.5±0.0	1
ls1-GA	50.6±0.9	3007.2±4.1	99.1±0.1	99.2±0.1	1	41.0±4.7	1412.6±8.3	99.3±0.3	99.2±0.4	1
ls1-EDA	54.7±2.1	3003.1±2.2	99.2±0.1	99.3±0.1	1	37.7±0	1404.6±2.3	99.3±0.0	99.3±0.0	1
ls1-DDMLS(O)	**49.8±0.1**	2928.8±71.5	99.3±0.0	99.2±0.0	1	**32.0±0**	1371.3±12.4	99.2±0.0	99.1±0.0	1
ls1-DDMLS(V1)	52.2±0.3	2928.8±71.5	99.6±0.1	99.6±0.1	1	34.6±0.9	1371.3±12.4	99.6±0.0	99.6±0.1	1
ls1-DDMLS(V2)	49.9±0.3	2928.8±71.5	99.3±0.0	99.3±0.1	1	33.1±1.1	1371.3±12.4	99.4±0.2	99.3±0.0	1
ls1-DDMLS(V3)	51.2±0.9	2919	99.3±0.2	99.3±0.3	1	35.1±1.5	1383	99.3±0.2	99.2±0.3	1
ls2-Greedy	66.6±0.0	2.0±0.0	99.7±0.0	99.7±0.0	1	66.6±0.0	2.0±0.0	99.9±0.0	100.0±0.0	1
ls2-GA	53.1±2.9	9623.1±21.8	99.2±0.1	99.2±0.2	1	43.3±3.1	3317.4± 17.2	99.6±0.3	99.5±0.3	1
ls2-EDA	53.5±3.1	9604.4±2.4	99.1±0.0	99.2±0.0	1	38.7±2.7	3304.6± 2.5	99.5±0.3	99.5±0.2	1
ls2-DDMLS(O)	**47.9±0.0**	9548.9±46.4	99.0±0.0	99.0±0.0	1	**27.2±0.0**	3305.6±342.6	99.3±0.0	99.3±0.0	1
ls2-DDMLS(V1)	51.3±0.0	9548.9±46.4	99.4±0.0	99.5±0.0	1	27.9±0.4	3305.6±342.6	99.6±0.0	99.6±0.0	1
ls2-DDMLS(V2)	49.1±0.0	9548.9±46.4	99.2±0.0	99.2±0.0	1	28.0±0.5	3305.6±342.6	99.6±0.1	99.6±0.1	1
ls2-DDMLS(V3)	51.2±2.4	9518	99.2±0.1	99.2±0.1	1	29.2±1	3197	99.4±0.2	99.4±0.3	1
ls3-Greedy	101.8±0.0	3.0±0.0	99.9±0.0	99.9±0.0	1	101.8±0.0	4.0±0.0	99.8±0.0	99.9±0.0	1
ls3-GA	85.7±5.6	7213.2±11.3	99.1±0.1	98.8±0.1	1	71.3±4.0	5209.5±7.2	99.4±0.3	99.3±0.3	1
ls3-EDA	82.5±0.4	7202.6±2.8	99.1±0.2	98.8±0.2	1	73.4±3.3	5203.7±2.8	99.7±0.3	99.6±0.4	1
ls3-DDMLS(O)	**74.9±0.2**	7233.5±58.3	99.1±0.0	98.7±0.0	1	**63.4±0.9**	5217.5±274.1	99.3±0.2	99.1±0.2	1
ls3-DDMLS(V1)	79.6±1.3	7233.5±58.3	99.6±0.1	99.4±0.1	1	65.2±2.1	5217.5±274.1	99.6±0.1	99.5±0.2	1
ls3-DDMLS(V2)	75.9±0.7	7233.5±58.3	99.4±0.1	99.1±0.1	1	65.0±1.9	5217.5±274.1	99.7±0.2	99.6±0.2	1
ls3-DDMLS(V3)	77.3±1.5	7175	99.3±0.1	99.0±0.2	1	65.2±0.8	5109	99.4±0.3	99.3±0.4	1
ls4-Greedy	169.7±0.0	5±0.0	99.8±0.0	99.8±0.0	1	162.5±0.0	6.0±0.0	99.9±0.0	99.9±0.0	1
ls4-GA	123.1±8.8	11307.2±8.0	99.1±0.1	99.1±0.2	1	104.1±5.1	8409.1±5.6	99.1±0.2	99.2±0.2	1
ls4-EDA	126.2±2.3	11304.8±2.1	99.2±0.2	99.4±0.2	1	114.8±4.5	8403.4±2.8	99.6±0.2	99.6±0.2	1
ls4-DDALS(O)	**106.6±0.8**	11289.9±92.0	99.1±0.1	99.2±0.1	1	**92.8±0.7**	8376.6±378.8	99.1±0.1	99.2±0.1	1
ls4-DDALS(V1)	109.5±0.4	11289.9±92.0	99.7±0.1	99.7±0.1	1	97.9±2.2	8376.6±378.8	99.6±0.1	99.7±0.1	1
ls4-DDALS(V2)	108.9±1.3	11289.9±92.0	99.6±0.2	99.7±0.2	1	93.8±1.4	8376.6±378.8	99.3±0.2	99.4±0.2	1
ls4-DDALS(V3)	108.7±1.1	11171	99.2±0.2	99.3±0.2	1	96.5±1.7	8177	99.1±0.1	99.2±0.1	1
ls5-Greedy	213.8±0.0	3.0±0.0	99.9±0.0	99.8±0.0	1	205.5±0.0	4.0±0.0	99.9±0.0	99.9±0.0	1
ls5-GA	192.6±6.7	29013.4±7.6	99.0±0.0	98.8±0.0	1	170.2±7.7	18313±10.3	99.1±0.2	99.0±0.3	0.9
ls5-EDA	195.2±2.2	29004.2±3.1	99.1±0.1	99.0±0.2	1	165.9±1.9	18305.5±2.8	99.3±0.2	99.3±0.2	1
ls5-DDMLS(O)	**178.1±0.8**	28840.1±289.5	99.0±0.0	98.9±0.0	1	**144.5±1.0**	18276.1±395.4	99.0±0.0	99.0±0.1	1
ls5-DDMLS(V1)	184.5±1.4	28840.1±289.5	99.3±0.0	99.2±0.0	1	155.8±3.3	18276.1±395.4	99.6±0.1	99.5±0.1	1
ls5-DDMLS(V2)	179.6±1.9	28840.1±289.5	99.2±0.0	99.0±0.0	1	147.3±3.4	18276.1±395.4	99.2±0.2	99.1±0.2	1
ls5-DDMLS(V3)	182.9±1.6	28624	99.1±0.0	99.0±0.0	1	147.4±4.0	17573	99.1±0.2	99.1±0.2	1
ls6-Greedy	256.3±0.0	4.0±0.0	99.4±0.0	99.4±0.0	1	250.0±0.0	7.0±0.0	99.8±0.0	99.7±0.0	1
ls6-GA	233.5±14.0	34609.3±6.0	99.0±0.0	98.9±0.0	1	187.2±9.3	20211.9±8.3	99.0±0.0	99.0±0.2	0.9
ls6-EDA	214.1±2.8	34604.9±2.8	99.3±0.2	99.2±0.2	1	181.7±1.8	20205.1±2.7	99.4±0.2	99.3±0.2	1
ls6-DDMLS(O)	**198.5± 0.5**	34570.7±619.3	99.0±0.0	98.9±0.0	1	**162.2±1.1**	20178.8±1031.6	99.1±0.1	99.0±0.1	1
ls6-DDMLS(V1)	206.8± 2.3	34570.7±619.3	99.5±0.0	99.4±0.1	1	169.6±1.4	20178.8±1031.6	99.5±0.0	99.5±0.1	1
ls6-DDMLS(V2)	201.5± 2.4	34570.7±619.3	99.3±0.2	99.2±0.2	1	163.1±1.5	20178.8±1031.6	99.3±0.1	99.3±0.1	1
ls6-DDMLS(V3)	202.6± 2.0	33681	99.1±0.1	99.1±0.1	1	165.0±2.6	19387	99.2±0.2	99.1±0.1	1

Table 3: Performance on benchmarks of APP. Due to the introduction of retransmission mechanisms, there is a significant difference in the results between LAB and APP.

Benchmark	**small $T_{\max}$**					**large $T_{\max}$**				
	C	**ET**	**ECL**	**RCL**	**FSR**	**C**	**ET**	**ECL**	**RCL**	**FSR**
ss1-Greedy	9.2±0.0	4.0±0.0	99.3±0.0	96.7±0	0	9.0±0.0	60.0±0.0	99.6±0.0	98.3±0.0	0
ss1-GA	10.5±2.3	337.4±5.2	99.4±0.3	96.9±0.7	0	8.6±2.1	303.7±2.7	99.5±0.1	97.2±0.0	0
ss1-EDA	**9.0±0.0**	332.5±2.3	99.3±0.0	96.5±0.0	0	8.0±0.9	304.2±2.4	99.6±0.0	97.7±0.6	0
ss1-DDMLS(O)	**9.0±0.0**	311.4±10.2	99.2±0.0	96.5±0.0	0	**7.3±0.0**	285.5±14.8	99.6±0.0	97.2±0.0	0
ss1-DDMLS(V1)	14.0±0.0	311.4±10.2	99.9±0.0	97.2±0.0	0	**7.3±0.0**	285.5±14.8	99.6±0.0	97.2±0.0	0
ss1-DDMLS(V2)	13.5±1.5	311.4±10.2	99.8±0.2	97.1±0.2	0	**7.3±0.0**	285.5±14.8	99.6±0.0	97.2±0.0	0
ss1-DDMLS(V3)	14.8±3.4	321	99.7±0.3	98.0±1.0	**0.5**	11.1±2.4	300	99.5±0.0	97.7±0.6	**0.1**
ss2-Greedy	6.4±0.0	3±0.0	99.7±0.0	98.6±0.0	1	6.4±0.0	3.0±0.0	99.8±0.0	99.0±0.0	1
ss2-GA	5.3±0.0	301.8±2.2	99.2±0.0	97.3±0.4	0	5.3±0.0	272.6±2.8	99.5±0.1	98.1±0.2	0
ss2-EDA	5.4±0.2	303.1±3.0	99.3±0.2	97.4±0.7	0.1	5.4±0.2	274.9±3.0	99.6±0.2	98.3±0.4	0.3
ss2-DDALS(O)	**5.2±0.0**	307±7.6	99.2±0.0	97.3±0.0	0	**5.2±0.0**	291±10.6	99.6±0.0	98.1±0.0	0
ss2-DDALS(V1)	5.5±0.1	307±7.6	99.8±0.0	97.5±0.3	0	**5.2±0.0**	291±10.1	99.6±0.0	98.1±0.0	0
ss2-DDALS(V2)	5.5±0.1	307±7.6	99.8±0.0	97.5±0.3	0	5.4±0.1	291±10.1	99.9±0.1	98.2±0.1	0
ss2-DDALS(V3)	5.4±0.1	295	99.4±0.3	97.7±0.4	0	5.3±0.1	272.6±2.8	99.5±0.1	98.1±0.2	0
ss3-Greedy	18.3±0.0	6.0±0.0	99.9±0.0	96.3±0	0	13.8±0.0	8.0±0.0	99.6±0.0	97.6±0.0	0
ss3-GA	17.5±0.3	634.6±4.0	99.3±0.3	95.3±0.5	0	12.1±1.0	503.6±2.5	99.2±0.2	94.0±1.6	0
ss3-EDA	**17.1±0.3**	634.5±3.5	99.1±0.2	93.9±1.1	0	11.2±0.2	503.7±3.1	99.5±0.2	94.7±0.8	0
ss3-DDMLS(O)	**17.1±0.2**	675.9±36.2	99.1±0.1	94.2±0.7	0	**11.1±0.1**	499.7±21.7	99.5±0.2	94.6±0.7	0
ss3-DDMLS(V1)	18.1±0.2	675.9±36.2	99.6±0.1	96.7±0.6	0	11.5±1.0	499.7± 1.7	99.6±0.0	95.4±1.1	0
ss3-DDMLS(V2)	18.1±0.3	675.9±36.2	99.8±0.2	96.7±0.7	0	11.3±0.8	499.7±21.7	99.6±0.2	94.9±1.0	0
ss3-DDMLS(V3)	19.0±0.8	632	99.5±0.3	96.3±0.5	0	12.4±0.8	499	99.5±0.2	95.5±0.1	0
ss4-Greedy	34.9±0.0	3.0±0.0	100.0±0.0	100.0±0.0	1	27.2±0.0	4.0±0.0	100.0±0.0	99.0±0.0	1
ss4-GA	27.3±1.1	2010±8.4	99.8±0.3	90.4±0.5	0	21.5±2.9	1614.8±9.8	99.6±0.3	93.2±4.8	0
ss4-EDA	**26.1±0.0**	2005.7±2.3	100.0±0.0	90.1±0.0	0	**16.4±0.0**	1602.7±2.1	99.9±0.0	80.3±0.0	0
ss4-DDALS(O)	**26.1±0.0**	1988.2±30.4	100.0±0.0	90.1±0.0	0	16.7±0.9	1579.4±76.8	99.8±0.2	81.8±4.4	0
ss4-DDALS(V1)	**26.1±0.0**	1988.2±30.4	100.0±0.0	90.1±0.0	0	16.7±1.0	1579.4±76.8	99.9±0.0	81.5±3.6	0
ss4-DDALS(V2)	**26.1±0.0**	1988.2±30.4	100.0±0.0	90.1±0.0	0	18.0±1.9	1579.4±76.8	99.9±0.1	86.1±7.0	0
ss4-DDALS(V3)	29.1±2.4	2022	99.9±0.2	91.3±2.9	0	19.9±1.7	1589	99.7±0.3	92.0±5.3	0
ls1-Greedy	84.1±0.0	2.0±0.0	99.3±0.0	99.5±0.0	1	65.3±0.0	5.0±0.0	99.1±0.0	99.0±0.0	1
ls1-GA	65.0±1.9	6432.1±6.4	99.1±0.2	98.8±0.1	1	59.3±3.9	6332.7±6.7	99.0±0.0	98.8±0.2	1
ls1-EDA	68.9±0.0	6423.1±2.9	99.1±0.0	99.4±0	1	61.35±0	6323.2±3.2	99.2±0.0	99.1±0.0	1
ls1-DDMLS(O)	**61.5±0.0**	6417.8±16.3	99.2±0.0	98.5±0	1	**53.9±2.3**	6318±65.9	99.0±0.0	98.6±0.1	0.9
ls1-DDMLS(V1)	63.4±1.4	6417.8±16.3	99.6±0.0	99.1±0.1	1	61.8±0.5	6318±65.9	99.7±0.3	99.4±0.2	1
ls1-DDMLS(V2)	62.3± 0.3	6417.8±16.3	99.4±0.2	98.9±0.3	1	54.8±2.8	6318±65.9	99.1±0.1	98.8±0.1	1
ls1-DDMLS(V3)	62.9±0.8	6403	99.2±0.1	98.7±0.2	1	57.4±2.6	6340±65.9	99.1±0.0	98.7±0.1	1
ls2-Greedy	85.7±0.0	3.0±0.0	99.9±0.0	99.9±0.0	1	85.7±0.0	3.0±0.0	100.0±0.0	100.0±0.0	1
ls2-GA	76.9±2.7	22617.4±10.9	99.0±0.1	98.9±0.2	1	61.2±2.1	21327.1±20.3	99.1±0.0	99.1±0.1	1
ls2-EDA	75.9±0.0	22604.3±3.3	99.0±0.0	98.9±0.0	1	59.2±0.3	21306.5±2.5	99.2±0.0	99.2±0.1	1
ls2-DDALS(O)	**71.9±0.3**	22591.3±63.6	99.0±0.0	99.0±0.0	1	**57.2±0.0**	21233.5±51.9	99.3±0.1	99.3±0.1	1
ls2-DDALS(V1)	76.6±1.1	22591.3±63.6	99.6±0.1	99.5±0.1	1	58.5±0.8	21233.5±51.9	99.6±0.1	99.6±0.1	1
ls2-DDALS(V2)	72.9±0.6	22591.3±63.6	99.3±0.2	99.2±0.2	1	57.5±0.2	21233.5±51.9	99.4±0.1	99.5±0.1	1
ls2-DDALS(V3)	73.1±0.7	22564	99.2±0.1	99.1±0.1	1	57.9±0.4	21245	99.2±0.1	99.2±0.2	1
ls3-Greedy	158.3±0.0	3.0±0.0	99.4±0.0	99.3±0.0	1	158.3±0.0	3.0±0.0	99.6±0.0	99.9±0.0	1
ls3-GA	119.4±3.9	21008.5±7.5	99.1±0.1	98.4±0.4	0.7	122.6±5.3	21007.5±4.2	99.0±0.1	98.4±0.4	0.5
ls3-EDA	117.0±0.7	21005.3±3.0	99.4±0.0	98.9±0.0	1	115.0±2.0	21004.8±2.6	99.3±0.2	99.0±0.1	0.6
ls3-DDMLS(O)	**112.0±1.2**	20942.2±211.2	99.1±0.0	98.9±0.1	0.9	**111.4± 1.5**	20956.2±193.7	99.3±0.3	98.8±0.4	0.5
ls3-DDMLS(V1)	119.0±2.7	20942.2±211.2	99.5±0.1	98.9±0.3	0.9	114.4±4.3	20956.2±193.7	99.5±0.2	99.0±0.4	0.7
ls3-DDMLS(V2)	115.7±1.9	20942.2±211.2	99.4±0.1	98.8±0.1	1	111.8± 1.7	20956.2±193.7	99.4±0.3	98.9±0.4	0.6
ls3-DDMLS(V3)	115.6±1.5	20810	99.2±0.1	97.9±0.6	0.3	113.7±0.7	20819	99.0±0.0	98.3±0.2	0.1
ls4-Greedy	207.3±0.0	13.0±0.0	99.0±0.0	98.9±0.0	1	195.3±0.0	17.0±0.0	99.4±0.0	99.4±0.0	1
ls4-GA	208.5±8.3	48014.1±12.1	99.0±0.0	98.6±0.3	0.7	175.2±10.3	45014.6±10.2	99.0±0.0	98.5±0.2	0.6
ls4-EDA	205.3±2.4	48004.5±3.3	99.0±0.0	99.0±0.0	1	163.8±1.3	45005.5±2.6	99.1±0.0	98.8±0.0	1
ls4-DDALS(O)	**194.9±0.8**	47962.8±67.7	99.0±0.0	98.9±0.0	1	**163.3±3.6**	44822.1±583.5	99.0±0.0	98.4±0.1	0.3
ls4-DDALS(V1)	203.2±2.0	47962.8±67.7	99.1±0.0	99.0±0.1	1	172.8±4.5	44822.1±583.5	99.3±0.1	98.9±0.3	0.8
ls4-DDALS(V2)	196.5±1.3	47962.8±67.7	99.0±0.0	99.0±0.0	1	165.3±4.3	44822.1±583.5	99.1±0.1	98.5±0.2	0.3
ls4-DDALS(V3)	196.9±2.1	48010	99.0±0.0	98.9±0.1	1	168.2± 2.3	44174	99.0±0.0	98.9± 0.1	1
ls5-Greedy	267.2±0.0	11.0±0.0	99.1±0.0	98.6±0.0	1	264.0±0.0	12.0±0.0	99.3±0.0	98.9±0.0	1
ls5-GA	241.2±9.4	81015.9±10.3	99.0±0.0	98.6±0.1	0.7	219.9±10.5	79012.6±7.5	99.0±0.0	98.5±0.2	0.6
ls5-EDA	220.3±0.3	81002.4±2.4	99.1±0.0	98.7±0.0	1	**198.7±1.8**	79005.3±3.1	99.1±0.2	98.6±0.2	1
ls5-DDMLS(O)	**218.2±1.0**	81144.5±283.4	99.0±0.0	98.6±0.0	1	199.4±2.2	78638.8±555.0	99.0±0.1	98.6±0.1	0.9
ls5-DDMLS(V1)	228.3±2.9	81144.5±283.4	99.2±0.1	98.9±0.2	1	209.3±5.8	78638.8±555.0	99.2±0.1	98.9±0.2	0.9
ls5-DDMLS(V2)	220.3±3.0	81144.5±283.4	99.1±0.1	98.7±0.1	1	200.1±2.7	78638.8±555.0	99.1±0.1	98.7±0.2	1
ls5-DDMLS(V3)	221.5±1.5	81213	99.0±0.0	98.7±0.0	1	202.2±3.1	78784	99.1±0.1	98.9±0.2	0.9
ls6-Greedy	388.8±0.0	4.0±0.0	99.1±0.0	98.6±0.0	1	374.4±0.0	12.0±0.0	99.7±0.0	99.6±0.0	1
ls6-GA	354.5±14.3	125315.6±10.0	99.0±0.0	98.2±0.1	0	325.3±10.5	121508.9±4.8	99.0±0.0	98.2±0.2	0.1
ls6-EDA	337.8±0.9	125305.4±2.4	99.0±0.0	98.6±0.0	1	301.6±1.7	121503.7±2.6	99.1±0.1	98.7±0.1	1
ls6-DDMLS(O)	**324.2±0.9**	125232.9±226.2	99.0±0.0	98.1±0.0	0	**298.7±5.6**	121483.4±943.5	99.0±0.0	98.4±0.1	0.3
ls6-DDMLS(V1)	333.1±1.9	125232.9±226.2	99.2±0.1	98.6±0.2	0.6	311.5±7.0	121483.4±943.5	99.3±0.1	98.6±0.3	0.6
ls6-DDMLS(V2)	325.5±2.7	125232.9±226.2	99.0±0.1	98.2±0.1	0.1	301.4±5.2	121483.4±943.5	99.1±0.2	98.5±0.2	0.5
ls6-DDMLS(V3)	328.3±2.4	124923	99.0±0.0	98.3±0.1	0	304.7±3.2	121648	99.1±0.0	98.4±0.1	0.1