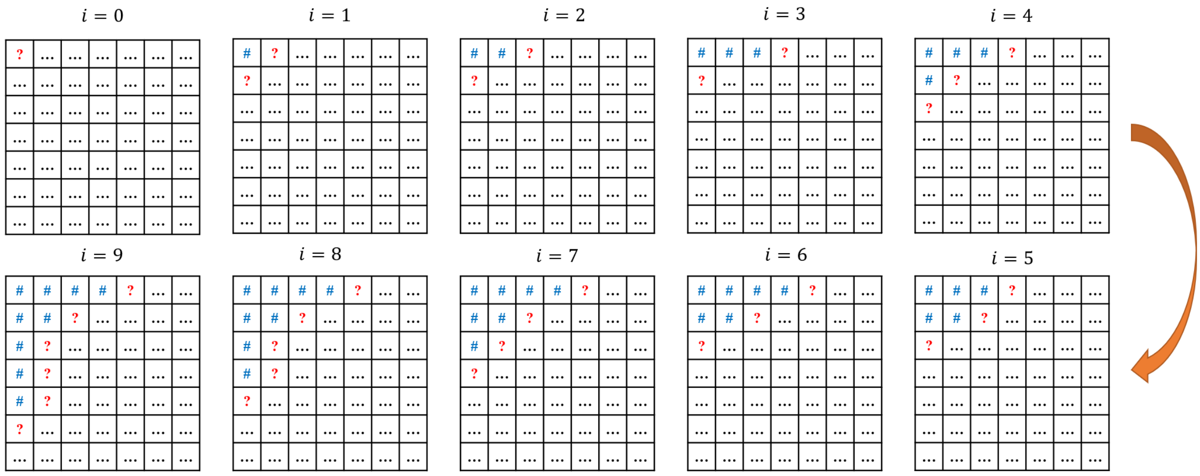

Figure 1: An example of exact evaluation when $m = 2$

1 Proof of Theorem 4.1

Proof. Suppose the k^{th} largest sum is $S_r = d_{s_1 l_1} + d_{s_2 l_1} + \ldots + d_{s_m l_m}$.

- For any pair in the combination or queue Q that has already been output, the combinations with indices less than or equal to k must have been popped or exist in Q.
- Construct a chain S, then we have the following chain, denoted as $S_0 \le S_1 \le, \ldots, \le S_l$.

$$
\begin{aligned}
& d_{s_1 0} + d_{s_2 0} + \ldots + d_{s_m 0}, \\
\ge & d_{s_1 1} + d_{s_2 0} + \ldots + d_{s_m 0}, \\
\ge & \ldots, \\
\ge & d_{s_1 l_1} + d_{s_2 0} + \ldots + d_{s_m 0} \\
\ge & \ldots, \\
\ge & d_{s_1 l_1} + d_{s_2 l_2} + \ldots + d_{s_m 0} \\
\ge & \ldots, \\
\ge & d_{s_1 l_1} + d_{s_2 l_2} + \ldots + d_{s_m l_m}
\end{aligned}
$$

- Let S_j be the first sum in chain S which has not been popped from queue Q before the j^{th} iteration. During the j^{th} iteration, when S_{j-1} is popped, S_j is added to Q.
- For the k^{th} iteration, the popped sum S_k satisfies $S_k \le S_j \le S_l = d_{s_1 l_1} + d_{s_2 l_1} + \ldots + d_{s_m l_m}$.
- By induction, since the $(k-1)^{th}$ largest sum has been popped before the k^{th} iteration, the sum popped in the k^{th} iteration is exactly the k^{th} largest sum $S_l = d_{s_1 l_1} + d_{s_2 l_1} + \ldots + d_{s_m l_m}$.

□

The proof ends. An example is shown when $m = 2$ in Fig.1.

2 A Theoretical Quantitative Analysis between Estimation Error and Sample Size

To theoretically quantify the relationship between estimation error and sample size of data, we construct a new random variable S that is the sum of the random variables that make up the

Table 1: Relationship between EE and sample size

Estimation error(%)	5	0.5	0.05	0.005
Hoeffding's bound	139	13863	1386295	138629437
Chernoff's bound [1]	278	27726	2772589	277258873
Alamo's bound [2]	1573	15723	157227	1572269

solution of CCMCKP. We also define a Bernoulli variable $Z \in \{0, 1\}$ as below:

$$Z = \begin{cases} 1, & S \leq W \\ 0, & S > W \end{cases} \tag{1}$$

Since Z is a Bernoulli variable, the expectation $\mathbb{E}[Z]$ is also the true confidence level p defined in the original chance constraint. Hoeffding's inequality shows that for independent identically distributed random variables $Z_1, Z_2, \ldots, Z_L$ taking values $[a, b]$, for any $\epsilon > 0$, the following inequality holds,

$$\mathcal{P}(|\mathbb{E}[Z] - \frac{1}{L}\sum_{l=1}^{L} Z_l| \geq \epsilon) \leq 2exp(-\frac{2L\epsilon^2}{(b-a)^2}) \tag{2}$$

We only care about whether the difference between the sample mean and the true confidence is greater than the estimation error ϵ. Let $\mathbb{E}Z = p$ and $a = 0, b = 1$, the inequality becomes

$$\mathcal{P}(p - f(Z_1, Z_2, \ldots, Z_L) \leq -\epsilon) \leq exp(-2L\epsilon^2) \tag{3}$$

where $f(Z_1, Z_2, \ldots, Z_L)$ denotes the confidence evaluation algorithm on some combination of items and p is the true confidence level. Assuming that at least 5 of the 10 solutions in the output solution set are true feasible solutions, and $f(Z_1, Z_2, \ldots, Z_L)$ is accurate, we have $1 - exp(-2L\epsilon^2) \geq 5/10$, leading to

$$L \geq \frac{ln2}{2\epsilon^2} \tag{4}$$

The inequality indicates that a large amount of samples is necessary if we do not have other information about the random variables. Practically, it is verified that the amount of data required to meet the corresponding error requirement for the true confidence is much smaller than the theoretical value obtained through Monte Carlo simulation.

3 Detailed Adaptation of GDSO and DRO for CCMCKP

3.1 GDSO

Yang and Chakraborty [3] assumed known Gaussian distributions of the random variables in their work. They transformed the chance-constrained knapsack problem into several iterative risk-averse knapsack problems, which could be solved by dynamic programming method. We adapt their method, referred to as the Gaussian-distribution stochastic optimization (GDSO), to address CCMCKP. A statistical fitting approach is first applied to sample data to obtain the means and standard deviations of the Gaussian distributions, i.e., $\mu_{ij} = \mathbb{E}(w_{ij}), \sigma_{ij}^2 = Var(w_{ij}), i \in \{1, ..., m\}, j \in N_i$. The CCMCKP is formulated as below:

$$\min \quad \sum_{i=1}^{m}\sum_{j\in N_i} c_{ij}x_{ij} \tag{5a}$$

$$\text{s.t.} \quad \sum_{i=1}^{m}\sum_{j\in N_i} \mu_{ij}x_{ij} + \Phi^{-1}(P_0)\sqrt{\sum_{i=1}^{m}\sum_{j\in N_i}\sigma_{ij}^2 x_{ij}} \le W \tag{5b}$$

$$\sum_{j\in N_i} x_{ij} = 1, \forall i \in \mathcal{M} \tag{5c}$$

$$x_{ij} \in \{0,1\}, \forall i \in \mathcal{M}, j \in N_i \tag{5d}$$

where Φ represents the CDF of a standard normal distribution with zero mean and unit variance. The above optimization problem can be transformed into risk-averse multiple-choice knapsack problem as below:

$$\min \quad \sum_{i=1}^{m}\sum_{j\in N_i} c_{ij}x_{ij} \tag{6a}$$

$$\text{s.t.} \quad \sum_{i=1}^{m}\sum_{j\in N_i} (\mu_{ij} + \lambda\sigma_{ij}^2)x_{ij} \le W^{'} \tag{6b}$$

$$\sum_{j\in N_i} x_{ij} = 1, \forall i \in \mathcal{M} \tag{6c}$$

$$x_{ij} \in \{0,1\}, \forall i \in \mathcal{M}, j \in N_i \tag{6d}$$

where λ is risk-averse parameter and $W^{'}$ is the adapted W in the original CCMCKP.

3.2 DRO

Ji and Lejeune [4] studied distributionally robust multidimensional knapsack problem with Wasserstein ambiguity sets. To fit the chance constraint into Ji's framework, we reformulate the chance constraint as:

$$\mathcal{P}(\sum_{i=1}^{m}\sum_{j\in N_i} w_{ij}x_{ij} > W) \le 1 - P_0 \tag{7}$$

Then we define $b = -W, \epsilon = 1 - P_0, \tilde{w}_{ij} = -w_{ij}, \forall i \in \mathcal{M}, j \in N_i$. $\bar{x} \in \{0,1\}^{NM}$ is the vectorization of $x \in \{0,1\}^{N\times M}$, and $\xi \in \mathbb{R}^{NM}$ is the vectorization of $\hat{w} \in \mathbb{R}^{N\times M}$. Finally, we have the standard form as [4]:

$$\mathcal{P}(\xi^T\bar{x} < b) \le \epsilon \tag{8}$$

We utilize Theorem 10 in [4] to reformulate the inequation (7), and get the MILP formulation of

CCMCKP:

$$(\bar{x},\lambda,\eta,s,\phi)\in\mathcal{Z}_{LP}^{MCKP}=\begin{cases}\lambda\theta+\frac{1}{L}\sum_{l=1}^{L}s_l\le\epsilon, & \\ \lambda\ge 0, & \\ s_l\ge 0,\eta_l\ge 0, & \forall l\in\mathcal{L}\\ 1+\eta_l b^{'}-\sum_{q\in\mathcal{Q}}\xi_{ql}^{0}\phi_{ql}\le s_l, & \forall l\in\mathcal{L}\\ \phi_{ql}\le\lambda, & \forall q\in\mathcal{Q},l\in\mathcal{L}\\ \phi_{ql}\le\eta_l, & \forall q\in\mathcal{Q},l\in\mathcal{L}\\ \phi_{ql}\le U_{\eta_l}\bar{x}_q, & \forall q\in\mathcal{Q},l\in\mathcal{L}\\ \phi_{ql}\ge 0, & \forall q\in\mathcal{Q},l\in\mathcal{L}\\ \phi_{ql}\ge\eta_l+U_{\eta_l}\bar{x}_q-U_{\eta_l}, & \forall q\in\mathcal{Q},l\in\mathcal{L}\\ \sum_{i=Nm-N+1}^{Nm}\bar{x}_i=1, & \forall m\in\mathcal{M}\\ \bar{x}_q\in\{0,1\}, & \forall q\in\mathcal{Q}\end{cases}\tag{9}$$

where $b^{'}=b-\delta$, δ is a user-defined infinite small positive number. $\mathcal{Q}$ is the set of all $N\times M$ items, and $\mathcal{M}$ is the set of M classes. Note that $\sum_{i=Nm-N+1}^{Nm}\bar{x}_i=1,\forall m\in\mathcal{M}$ is the multiple-choice constraint in the CCMCKP. U_η is the upper bound of η and can be determined by:

$$U_{\eta_l}=\begin{cases}\mathcal{K}^{'*}, & if\quad \exists\bar{x}:\bar{x}^T\xi_l^0-b>0\\ 0, & otherwise\end{cases}\tag{10}$$

where $\mathcal{K}^{'*}$ is the optimal objective value of the linear programming problem:

$$\begin{aligned}\mathcal{K}^{'*}=&\min\bar{x}^T\xi_l^0-b\\ \text{s.t.}\quad & b-\bar{x}^T\xi_l^0\le 0-\tau^{'}\\ &\bar{x}\in\bar{\mathcal{X}}\end{aligned}\tag{11}$$

where $\tau^{'}$ is a user-defined small non-negative number. Based on the reformulation of CCMCKP, we first solve (11) and obtain $U_{\eta_l},\forall l\in\mathcal{L}$. The the MILP $\mathcal{Z}_{LP}^{MCKP}$ is solved with Gurobi solver. The optimal solution of MILP is also the optimal solution of CCMCKP.

4 Full experimental results

Full results on the problem instances of LAB and APP in Section VI-C are presented in Table 2 and Table 3, respectively.

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Table 2: Full results of performance on the problem instances of LAB.

Benchmark	small $T_{\max}$						large $T_{\max}$					
	C	ET	ECL	RCL	FSR	Time	C	ET	ECL	RCL	FSR	Time
ss1-Greedy	22.2±0.0	2.0±0.0	0±0.0	99.6±0.0	**1**	**0.1±0.0**	**10.8±0.0**	4.0±0.0	99.9±0.0	99.9±0.0	**1**	**0.1±0.0**
ss1-GA	18.7±0.1	356.2±4.9	99.8±0.3	99.7±0.7	0.87	7.1±0.5	10.8±0.1	233.7±3.8	99.9±0.1	99.5±0.9	0.87	3.1±0.3
ss1-EDA	21.2±1.6	354.3±2.8	29.9±45.7	99.5±0.5	0.93	8.8±3.6	12.8±2.4	234.6±2.8	99.8±0.3	99.7±0.6	0.9	2.4±1.2
ss1-DDALS(O)	**18.6±0.0**	349.5±7.6	99.9±0.0	99.9±0.0	**1**	2.2±0.1	**10.8±0.0**	234.6±17.2	99.9±0.0	99.9±0.0	**1**	1.7±0.3
ss1-DDALS(V1)	**18.6±0.0**	349.5±7.6	99.9±0.0	99.9±0.0	**1**	2.2±0.1	**10.8±0.0**	234.6±17.2	99.9±0.0	99.9±0.0	**1**	1.7±0.3
ss1-DDALS(V2)	**18.6±0.0**	349.5±7.6	99.9±0.0	99.9±0.0	**1**	2.2±0.1	**10.8±0.0**	234.6±17.2	99.9±0.0	99.9±0.0	**1**	1.7±0.3
ss1-DDALS(V3)	20.7±2.0	345	74.7±0.4	99.3±0.8	0.75	2.3	13.0±1.7	228	99.8±0.3	99.1±1.0	0.7	1.7
ss2-Greedy	**19.4±0.0**	2.0±0.0	99.9±0.0	99.9±0.0	**1**	**3.9±0.1**	13.8±0.0	4.0±0.0	99.5±0.0	99.0±0.0	**1**	**7.3±0.2**
ss2-GA	**19.4±0.0**	385.6±3.9	99.9±0.0	99.9±0.0	**1**	555.9±133.9	11.4±1.6	308.5±6.0	99.8±0.1	99.8±0.3	**1**	1311.7±217.9
ss2-EDA	19.8±0.4	384.7±2.8	99.9±0.0	99.9±0.0	**1**	144.3±29.2	15.5±2.3	305.0±3.1	99.7±0.2	99.7±0.3	**1**	582.4±476.3
ss2-DDALS(O)	**19.4±0.0**	387.2±5.2	99.9±0.0	99.9±0.0	**1**	1241.9±73.9	**10.4±0.0**	302.0±10.7	99.8±0.0	99.9±0.0	**1**	555.1±80.6
ss2-DDALS(V1)	**19.4±0.0**	387.2±5.2	99.9±0.0	99.9±0.0	**1**	1241.9±73.9	**10.4±0.0**	302.0±10.7	99.8±0.0	99.9±0.0	**1**	555.1±80.6
ss2-DDALS(V2)	**19.4±0.0**	387.2±5.2	99.9±0.0	99.9±0.0	**1**	1241.9±73.9	12.1±2.0	302.0±10.7	99.9±0.1	99.9±0.0	**1**	555.1±80.6
ss2-DDALS(V3)	21.8±2.4	392	99.9±0.0	99.9±0	**1**	1228.1	14.3±2.2	296	99.8±0.2	99.8±0.3	**1**	432.5
ss3-Greedy	**21.4±0.0**	3.0±0.0	99.9±0.0	99.5±0.0	**1**	**0.2±0.0**	17.4±0.0	4.0±0.0	100.0±0.0	100.0±0.0	**1**	**0.4±0.0**
ss3-GA	22.4±1.4	306.0±3.7	99.9±0.0	99.7±0.2	**1**	31.7±1.9	14.2±2.4	263.5±2.9	99.7±0.4	99.2±1.3	0.73	37.9±1.1
ss3-EDA	24.8±1.9	305.3±3.0	99.9±0.1	99.8±0.5	0.93	44.1±3.0	13.7±1.3	264.5±2.8	99.9±0.0	99.9±0.1	**1**	42.8±3.4
ss3-DDALS(O)	**21.4±0.0**	288.5±9.3	99.9±0.0	99.5±0.0	**1**	9.6±0.5	**12.6±0.0**	254.4±4.7	100.0±0.0	99.9±0.0	**1**	14.7±0.9
ss3-DDALS(V1)	**21.4±0.0**	288.5±9.3	99.9±0.0	99.5±0.0	**1**	9.6±0.5	**12.6±0.0**	254.4±4.7	100.0±0.0	99.9±0.0	**1**	14.7±0.9
ss3-DDALS(V2)	**21.4±0.0**	288.5±9.3	99.9±0.0	99.5±0.0	**1**	9.6±0.5	**12.6±0.0**	254.4±4.7	100.0±0.0	99.9±0.0	**1**	14.7±0.9
ss3-DDALS(V3)	23.9±1.7	284	99.8±0.2	99.1±0.8	0.75	9.1	15.5±1.7	264	99.9±0.3	99.5±0.9	0.9	14.9
ss4-Greedy	30.1±0.0	2.0±0.0	99.9±0.0	99.9±0.0	**1**	**0.1±0.0**	14.9±0.0	5.0±0.0	99.9±0.0	99.9±0.0	**1**	**0.3±0.0**
ss4-GA	23.1±4.4	773.5±8.0	99.9±0.2	99.7±0.4	**1**	65.6±2.5	16.4±3.0	559.3±8.8	99.8±0.3	99.4±0.6	0.9	60.9±3.1
ss4-EDA	**17.4±0.0**	764.5±3.0	99.9±0.0	99.9±0.0	**1**	98.8±1.7	13.4±1.0	554.3±2.6	99.8±0.2	99.1±0.41	0.93	83.5±3.3
ss4-DDALS(O)	**17.4±0.0**	757.9±31.9	99.9±0.0	99.9±0.0	**1**	21.4±1.0	**11.2±0.0**	538.8±31.9	99.2±0.0	99.0±0.0	**1**	23.6±1.1
ss4-DDALS(V1)	**17.4±0.0**	757.9±31.9	99.9±0.0	99.9±0.0	**1**	21.4±1.0	12.3±0.7	538.8±31.9	99.6±0.1	98.9±0.2	**1**	23.6±1.1
ss4-DDALS(V2)	**17.4±0.0**	757.9±31.9	99.9±0.0	99.9±0.0	**1**	21.4±1.0	14.1±0.9	538.8±31.9	99.9±0.2	99.0±0.1	**1**	23.6±1.1
ss4-DDALS(V3)	25.3±4.1	822	99.8±0.2	99.6±0.5	**1**	22.9	14.3±1.5	514	99.6±0.3	99.2±0.7	0.9	24.1
ls1-Greedy	58.5±0.0	2.0±0.0	99.8±0.0	99.8±0.0	**1**	**0.3±0.0**	45.2±0.0	4.0±0.0	99.7±0.0	99.4±0.0	**1**	**0.6±0.1**
ls1-GA	50.8±1.0	3009.2±8.5	99.2±0.2	99.2±0.2	**1**	776.1±30.2	40.7±4.2	1411.0±9.0	99.5±0.3	99.3±0.4	0.9	458.3±9.4
ls1-EDA	54.3±1.7	3004.9±2.5	99.2±0.1	99.3±0.1	**1**	678.1±7.6	36.9±1.9	1403.6±2.7	99.3±0.1	99.3±0.0	**1**	417.2±8.1
ls1-DDALS(O)	**49.7±0.1**	2924.1±50.1	99.3±0.1	99.2±0.0	**1**	476.7±14.6	**32.0±0.0**	1387.8±24.5	99.2±0.0	99.1±0.0	**1**	240.0±6.2
ls1-DDALS(V1)	52.2±0.4	2924.1±50.1	99.6±0.1	99.5±0.1	**1**	476.7±14.6	34.4±0.7	1387.8±24.5	99.6±0.0	99.5±0.1	**1**	240.0±6.2
ls1-DDALS(V2)	50.0±0.4	2924.1±50.1	99.3±0.0	99.3±0.1	**1**	476.7±14.6	34.1±1.7	1387.8±24.5	99.5±0.2	99.5±0.2	**1**	240.0±6.2
ls1-DDALS(V3)	51.9±1.1	2889	99.3±0.2	99.3±0.2	**1**	475.2	34.7±1.3	1357	99.4±0.2	99.3±0.2	**1**	233.2
ls2-Greedy	66.6±0.0	2.0±0.0	99.7±0.0	99.7±0.0	**1**	**0.3±0.0**	66.6±0.0	2.0±0.0	100.0±0.0	100.0±0.0	**1**	**0.3±0.0**
ls2-GA	53.0±3.2	9626.6±20.4	99.1±0.1	99.1±0.2	**1**	2106.4±68.7	44.4±6.8	3315.4±14.1	99.4±0.3	99.4±0.3	**1**	806.4±32.0
ls2-EDA	50.9±3.1	9604.7±2.9	99.0±0.0	99.2±0.0	**1**	1955.5±17.2	38.9±2.8	3304.4±2.9	99.5±0.2	99.6±0.2	**1**	902.6±23.7
ls2-DDALS(O)	**47.9±0.0**	9628.4±89.2	99.0±0.0	99.0±0.0	**1**	815.0±16.3	**27.2±0.0**	3191.7±71.5	99.3±0.0	99.3±0.0	**1**	373.4±11.6
ls2-DDALS(V1)	53.3±2.3	9628.4±89.2	99.4±0.0	99.5±0.0	**1**	815.0±16.3	28.0±0.7	3191.7±71.5	99.6±0.0	99.6±0.1	**1**	373.4±11.6
ls2-DDALS(V2)	49.7±0.9	9628.4±89.2	99.3±0.1	99.3±0.1	**1**	815.0±16.3	28.4±0.9	3191.7±71.5	99.6±0.2	99.7±0.2	**1**	373.4±11.6
ls2-DDALS(V3)	50.8±1.7	9682	99.2±0.2	99.2±0.2	**1**	816.1	29.2±1	3072	99.4±0.2	99.4±0.3	**1**	356.9
ls3-Greedy	100.9±0.0	3.0±0.0	99.9±0.0	99.8±0.0	**1**	**1.0±0.0**	101.2±0.0	5.0±0.0	99.83±0.0	98.5±0.0	0	**1.7±0.0**
ls3-GA	83.4±4.6	7209.0±7.3	99.1±0.1	98.8±0.2	**1**	52257.1±127.9	73.9±5.8	5210.3±8.4	99.2±0.3	99.2±0.3	**1**	3499.6±73.8
ls3-EDA	82.2±1.3	7204.9±2.4	99.1±0.2	98.9±0.2	**1**	3924.2±137.9	74.8±3.4	5204.5±2.4	99.7±0.2	99.6±0.3	**1**	3132.0±82.4
ls3-DDALS(O)	**74.9±0.2**	7276.1±68.6	99.1±0.0	98.7±0.0	**1**	2534.7±29.4	**63.4±0.9**	5182.9±197.4	99.4±0.2	99.3±0.2	**1**	1785.0±73.6
ls3-DDALS(V1)	79.6±1.5	7276.1±68.6	99.6±0.1	99.4±0.1	**1**	2534.7±29.4	64.4±1.9	5182.9±197.4	99.6±0.1	99.5±0.2	**1**	1785.0±73.6
ls3-DDALS(V2)	76.6±1.2	7276.1±68.6	99.4±0.1	99.2±0.1	**1**	2534.7±29.4	64.8±1.7	5182.9±197.4	99.7±0.2	99.7±0.2	**1**	1785.0±73.6
ls3-DDALS(V3)	77.1±1.3	7290	99.3±0.1	99.0±0.2	**1**	2520.8	63.8±0.5	5114	99.3±0.2	99.1±0.3	**1**	1750.2
ls4-Greedy	167.5±0.0	7.0±0.0	99.2±0.0	99.1±0.0	**1**	**3.5±0.0**	167.6±0.0	8.0±0.0	99.9±0.0	99.9±0.0	**1**	**3.7±0.2**
ls4-GA	124.1±8.7	11309.8±7.4	99.1±0.2	99.1±0.3	0.97	12310.4±282.8	107.5±6.8	8411.4±9.8	99.1±0.1	99.0±0.3	0.93	8437.8±157.5
ls4-EDA	123.6±3.6	11305.1±3.4	99.3±0.2	99.4±0.2	**1**	9498.8±243.3	116.6±3.2	8403.9±3.1	99.6±0.2	99.7±0.2	**1**	7970.2±162.7
ls4-DDALS(O)	**107.1±1.0**	11360.9±206.6	99.1±0.1	99.2±0.1	**1**	5537.5±95.9	**92.6±1.1**	8483.8±335.8	99.1±0.1	99.0±0.3	0.93	4113.9±147.9
ls4-DDALS(V1)	109.5±0.5	11360.9±206.6	99.7±0.1	99.7±0.1	**1**	5537.5±95.9	98.8±2.1	8483.8±335.8	99.6±0.1	99.6±0.1	**1**	4113.9±147.9
ls4-DDALS(V2)	108.9±1.1	11360.9±206.6	99.6±0.2	99.6±0.2	**1**	5537.5±95.9	94.4±2.0	8483.8±335.8	99.4±0.2	99.3±0.3	**1**	4113.9±147.9
ls4-DDALS(V3)	110.8±3.5	11576	99.3±0.2	99.4±0.2	**1**	5651.3	95.1±2.5	7964	99.1±0.1	99.3±0.1	**1**	3883.5
ls5-Greedy	226.7±0.0	2.0±0.0	99.9±0.0	99.9±0.0	**1**	**1.0±0.1**	216.5±0.0	5.0±0.0	99.8±0.0	99.8±0.0	**1**	**2.9±0.2**
ls5-GA	191.6±6.3	29012.8±9.4	99.0±0.0	98.9±0.1	**1**	44167.0±536.9	165.6±9.6	18311.6±8.5	99.1±0.1	98.9±0.2	0.97	24188.0±369.9
ls5-EDA	195.8±3.1	29004.5±2.8	99.2±0.1	99.0±0.2	**1**	30259.7±544.7	168.5±4.0	18303.9±3.3	99.4±0.2	99.4±0.2	**1**	22596.0±330.0
ls5-DDALS(O)	**178.2±0.9**	28935.6±446.0	99.0±0.0	98.9±0.0	**1**	18837.0±279.3	**145.3±1.5**	18509.9±825.5	99.0±0.0	99.0±0.0	**1**	11986.4±506.2
ls5-DDALS(V1)	184.5±2.2	28935.6±446.0	99.3±0.0	99.2±0.0	**1**	18837.0±279.3	155.9±3.6	18509.9±825.5	99.5±0.1	99.5±0.1	**1**	11986.4±506.2
ls5-DDALS(V2)	180.9±2.0	28935.6±446.0	99.2±0.1	99.1±0.1	**1**	18837.0±279.3	148.4±3.2	18509.9±825.5	99.3±0.2	99.3±0.2	**1**	11986.4±506.2
ls5-DDALS(V3)	180.0±1.8	28519	99.0±0.0	98.9±0.0	**1**	18654.3	145.6±1.0	17804	99.1±0.1	99.1±0.1	**1**	11506.5
ls6-Greedy	266.2±0.0	2.0±0.0	99.9±0.0	99.9±0.0	**1**	**1.7±0.0**	260.9±0.0	4.0±0.0	99.8±0.0	99.8±0.0	**1**	**3.2±0.0**
ls6-GA	226.1±10.9	34609.0±8.2	99.0±0.1	98.9±0.1	**1**	62450.7±680.7	192.7±13.8	20213.2±10.3	99.0±0.1	99.0±0.1	**1**	38172.5±307.5
ls6-EDA	215.5±1.1	34604.7±2.7	99.3±0.1	99.2±0.2	**1**	43323.9±115.5	181.9±3.3	20204.8±2.9	99.4±0.2	99.4±0.2	**1**	31784.8±192.8
ls6-DDALS(O)	**198.9± 0.8**	34546.2±496.2	99.0±0.0	98.9±0.1	**1**	28114.3±402.1	**161.7±1.3**	20060.4±974.6	99.1±0.1	99.0±0.1	1	15872.1±537.1
ls6-DDALS(V1)	205.3±1.7	34546.2±496.2	99.5±0.1	99.4±0.1	**1**	28114.3±402.1	169.3±2.3	20060.4±974.6	99.5±0.1	99.5±0.1	**1**	15872.1±537.1
ls6-DDALS(V2)	201.5±1.7	34546.2±496.2	99.3±0.1	99.2±0.1	**1**	28114.3±402.1	163.4±2.3	20060.4±974.6	99.4±0.1	99.3±0.1	**1**	15872.1±537.1
ls6-DDALS(V3)	203.1±2.1	34251	99.1±0.1	99.1±0.2	**1**	27936.2	164.0±1.6	18641	99.2±0.2	99.1±0.2	**1**	15077.8

Table 3: Full results of performance on the problem instances of APP.

Benchmark	small $T_{\max}$						large $T_{\max}$					
	C	ET	ECL	RCL	FSR	Time	C	ET	ECL	RCL	FSR	Time
ss1-Greedy	14.0±0.0	4.0±0.0	99.9±0.0	97.2±0.0	0	**0.01±0.00**	7.3±0.0	5.0±0.0	99.6±0.0	97.1±0.0	0	**0.1±0.0**
ss1-GA	9.7±1.7	335.9±3.9	99.3±0.2	96.6±0.2	0	7.0±1.2	7.4±0.8	304.8±3.4	99.4±0.2	97.2±0.0	0	9.6±1.5
ss1-EDA	9.1±0.1	334.5±3.1	99.3±0.0	96.6±0.1	0	12.9±1.1	8.3±0.9	305.0±2.9	99.5±0.1	97.8±0.6	0	6.7±1.2
ss1-DDALS(O)	**9.0±0.0**	312.1±8.3	99.2±0.0	96.5±0.0	0	1.4±0.3	**7.3±0.0**	285.2±15.8	99.6±0.0	97.1±0.0	0	3.2±0.3
ss1-DDALS(V1)	14.0±0.0	312.1±8.3	99.9±0.0	97.2±0.0	0	1.4±0.3	**7.3±0.0**	285.2±15.8	99.6±0.0	97.1±0.0	0	3.2±0.3
ss1-DDALS(V2)	13.8±0.9	312.1±8.3	99.8±0.1	97.1±0.1	0	1.4±0.3	**7.3±0.0**	285.2±15.8	99.6±0.0	97.2±0.0	0	3.2±0.3
ss1-DDALS(V3)	14.1±2.7	321	99.6±0.3	97.9±1.0	**0.5**	1.5	11.1±2.4	300	99.5±0.0	97.8±0.7	**0.1**	2.7
ss2-Greedy	5.9±0.0	6.0±0.0	99.8±0.0	98.5±0.0	0	**12.6±0.4**	5.9±0.0	6.0±0.0	99.9±0.0	98.9±0.0	**1**	**8.7±0.2**
ss2-GA	5.3±0.1	303.2±2.3	99.2±0.0	97.3±0.5	0	4508.8±1198.4	5.3±0.1	272.9±2.2	99.5±0.1	98.1±0.2	0.03	3422.4±352.0
ss2-EDA	5.5±0.3	305.0±2.7	99.2±0.2	97.6±0.7	0.1	3323.2±1532.5	5.6±0.3	274.1±2.4	99.5±0.2	98.5±0.5	0.47	1185.3±1048.5
ss2-DDALS(O)	**5.2±0.0**	300.0±10.9	99.2±0.0	97.3±0.0	0	1010.5±98.7	**5.2±0.0**	297.5±7.8	99.6±0.0	98.1±0.0	0	862.2±78.7
ss2-DDALS(V1)	5.4±0.0	300.0±10.9	99.8±0.1	97.4±0.2	0	1010.5±98.7	**5.2±0.0**	297.5±7.8	99.6±0.0	98.1±0.0	0	862.2±78.7
ss2-DDALS(V2)	5.4±0.0	300.0±10.9	99.8±0.0	97.4±0.2	0	1010.5±98.7	5.4±0.1	297±7.8	99.9±0.1	98.2±0.2	0.1	862.2±78.7
ss2-DDALS(V3)	5.4±0.1	306	99.5±0.3	97.6±0.4	0	1086.0	5.5±0.1	298	99.7±0.2	98.5±0.3	0.5	858.0
ss3-Greedy	18.3±0.0	6.0±0.0	99.9±0.0	96.3±0	0	**0.5±0.0**	14.2±0.0	8.0±0.0	99.8±0.0	97.3±0.0	0	**0.6±0.0**
ss3-GA	17.7±0.7	634.9±5.2	99.4±0.4	95.0±1.8	0	179.2±2.5	12.0±1.1	503.6±3.3	99.4±0.3	95.2±1.5	0	152.9±2.2
ss3-EDA	17.6±0.2	635.1±2.8	99.9±0.2	95.9±0.1	0	116.7±3.4	13.8±1.1	503.9±2.7	99.8±0.3	96.4±0.7	0	98.2±2.8
ss3-DDALS(O)	**17.0±0.2**	690.1±31.5	99.1±0.1	93.9±0.7	0	34.5±3.1	**11.1±0.1**	505.8±14.6	99.6±0.2	94.7±0.5	0	30.5±1.2
ss3-DDALS(V1)	18.0±0.3	690.1±31.5	99.7±0.2	96.5±0.5	0	34.5±3.1	11.2±0.7	505.8±14.6	99.7±0.0	95.1±0.7	0	30.5±1.2
ss3-DDALS(V2)	18.0±0.4	690.1±31.5	99.9±0.1	96.4±0.4	0	34.5±3.1	11.3±0.8	505.8±14.6	99.6±0.2	94.9±0.8	0	30.5±1.2
ss3-DDALS(V3)	17.7±0.3	736	99.4±0.3	95.6±1.6	0	35.4	12.7±1.0	507	99.4±0.2	95.3±0.7	0	29.8
ss4-Greedy	34.9±0.0	3.0±0.0	100.0±0.0	100.0±0.0	**1**	**0.2±0.0**	28.5±0.0	4.0±0.0	100.0±0.0	99.0±0.0	**1**	**0.3±0.0**
ss4-GA	27.5±1.4	2008.9±7.9	99.8±0.2	90.3±0.5	0	251.8±24.5	20.5±2.7	1608.4±7.4	99.4±0.3	92.9±4.8	0	322.5±9.2
ss4-EDA	**26.1±0.0**	2005.7±2.3	100.0±0.0	90.1±0.0	0	333.9±11.8	18.3±2.7	1604.5±2.8	99.9±0.1	85.2±6.9	0	287.9±19.6
ss4-DDALS(O)	**26.1±0.0**	1988.9±27.8	100.0±0.0	90.1±0.0	0	33.3±2.3	**16.6±0.8**	1582.8±54.4	99.8±0.1	81.3±3.7	0	61.3±3.9
ss4-DDALS(V1)	**26.1±0.0**	1988.9±27.8	100.0±0.0	90.1±0.0	0	33.3±2.3	16.8±1.1	1582.8±54.4	99.8±0.1	81.8±4.3	0	61.3±3.9
ss4-DDALS(V2)	**26.1±0.0**	1988.9±27.8	100.0±0.0	90.1±0.0	0	33.3±2.3	18.3±2.2	1582.8±54.4	99.9±0.1	86.9±7.3	0	61.3±3.9
ss4-DDALS(V3)	29.7±2.5	1944	99.9±0.2	92.2±3.9	0.2	28.3	20.8±2.9	1586	99.7±0.3	88.3±6.3	0	59.3
ls1-Greedy	84.1±0.0	2.0±0.0	99.3±0.0	99.5±0.0	**1**	**0.3±0.0**	84.1±0.0	2.0±0.0	99.9±0.0	99.9±0.0	**1**	**0.3±0.0**
ls1-GA	64.7±2.9	6437.8±11.7	99.1±0.1	98.7±0.1	**1**	3441.1±15.0	60.2±4.0	6332.0±11.5	99.0±0.1	98.7±21.9	0.9	3381.1±21.9
ls1-EDA	68.9±0.0	6424.7±2.5	99.0±0.0	99.4±0.0	**1**	2266.4±90.7	61.4±0.2	6324.3±2.9	99.2±0.0	99.1±0.0	**1**	2250.8±102.6
ls1-DDALS(O)	**61.5±0.0**	6418.0±19.6	99.2±0.0	98.5±0.0	**1**	955.6±9.4	**53.7±2.6**	6304.6±65.6	99.0±0.0	98.7±0.3	**1**	958.1±14.7
ls1-DDALS(V1)	63.1±1.0	6418.0±19.6	99.6±0.0	99.1±0.1	**1**	955.6±9.4	62.1±1.7	6304.6±65.6	99.7±0.2	99.5±0.3	**1**	958.1±14.7
ls1-DDALS(V2)	62.3±0.3	6418.0±19.6	99.4±0.1	98.9±0.3	**1**	955.6±9.4	55.3±3.2	6304.6±65.6	99.2±0.2	98.8±0.2	**1**	958.1±14.7
ls1-DDALS(V3)	62.8±0.7	6417	99.2±0.1	98.7±0.2	**1**	950.7	55.2±1.2	6351 ±65.9	99.1±0.1	98.8±0.1	**1**	935.6
ls2-Greedy	89.1±0.0	2.0±0.0	99.9±0.0	99.9±0.0	**1**	**0.2±0.0**	89.1±0.0	2.0±0.0	100.0±0.0	100.0±0.0	**1**	**0.3±0.0**
ls2-GA	76.7±2.8	22616.5±10.5	99.1±0.1	99.0±0.2	**1**	3925.1±155.5	60.9±2.7	21324.5±13.3	99.1±0.1	99.1±0.1	**1**	3382.2±64.9
ls2-EDA	79.3±1.1	22604.1±2.6	99.5±0.1	99.3±0.1	**1**	7174.3±44.2	59.9±0.8	21304.8±3.2	99.2±0.0	99.2±0.0	**1**	6970.8±88.9
ls2-DDALS(O)	**72.0±0.3**	22631.7±62.4	99.0±0.0	99.0±0.0	**1**	841.3±7.6	**57.2±0.0**	21265.2±73.3	99.3±0.1	99.3±0.0	**1**	776.4±6.6
ls2-DDALS(V1)	76.2±1.1	22631.7±62.4	99.5±0.1	99.5±0.1	**1**	841.3±7.6	59.0±1.2	21265.2±73.3	99.6±0.0	99.6±0.1	**1**	776.4±6.6
ls2-DDALS(V2)	73.2±0.4	22631.7±62.4	99.4±0.1	99.3±0.1	**1**	841.3±7.6	57.6±0.5	21265.2±73.3	99.4±0.1	99.5±0.1	**1**	776.4±6.6
ls2-DDALS(V3)	73.7±1.5	22536	99.2±0.1	99.0±0.2	**1**	846.2	58.3±0.6	21274	99.3±0.1	99.3±0.1	**1**	769.6
ls3-Greedy	163.3±0.0	3.0±0.0	99.9±0.0	99.4±0.0	**1**	**1.1±0.0**	163.3±0.0	3.0±0.0	99.9±0.0	99.9±0.0	**1**	**1.0±0.0**
ls3-GA	121.7±7.3	21012.1±8.6	99.0±0.0	98.6±0.4	0.8	17477.6±60.9	121.1±4.1	21011.8±7.1	99.0±0.0	98.4±0.4	0.53	17447.7±62.8
ls3-EDA	117.4±1.0	21004.5±2.5	99.4±0.0	98.9±0.0	**1**	13400.8±127.8	116.8±1.1	21004.8±2.6	99.6±0.1	99.4±0.2	0.97	13425.6±127.7
ls3-DDALS(O)	**111.9±1.0**	20999.3±184.8	99.1±0.1	98.9±0.1	**1**	5734.4±64.0	**111.7±1.6**	20910.0±266.1	99.3±0.3	98.8±0.4	0.6	5702.1±80.0
ls3-DDALS(V1)	118.1±2.0	20999.3±184.8	99.5±0.2	98.9±0.3	0.9	5734.4±64.0	113.8±4.2	20910.0±266.1	99.5±0.2	99.2±0.3	0.97	5702.1±80.0
ls3-DDALS(V2)	114.1±2.4	20999.3±184.8	99.3±0.2	98.8±0.1	**1**	5734.4±64.0	112.6±2.4	20910.0±266.1	99.5±0.3	99.0±0.4	0.8	5702.1±80.0
ls3-DDALS(V3)	116.1±1.7	20667	99.2±0.2	98.8±0.1	**1**	5700.9	114.4±1.4	21000	99.1±0.1	98.6±0.2	0.7	5663.9
ls4-Greedy	238.1±0.0	3.0±0.0	99.4±0.0	99.0±0.0	**1**	**1.5±0.0**	193.7±0.0	15.0±0.0	99.2±0.0	98.8±0.0	**1**	**7.0±0.2**
ls4-GA	206.6±4.7	48018.6±13.4	99.0±0.0	98.6±0.3	0.73	46617.9±648.4	182.7±10.7	45015.2±11.7	99.0±0.0	98.5±0.2	0.6	42937.2±431.6
ls4-EDA	204.3±1.9	48004.5±3.5	99.0±0.0	99.0±0.0	**1**	45488.7±924.8	163.6±1.2	45003.5±2.4	99.1±0.1	98.8±0.1	**1**	42955.7±613.1
ls4-DDALS(O)	**195.1±0.5**	47999.9±83.0	99.0±0.0	98.9±0.0	**1**	16357.5±137.6	**161.5±2.1**	44440.5±344.6	99.0±0.0	98.5±0.3	0.4	15160.3±127.4
ls4-DDALS(V1)	209.2±2.8	47999.9±83.0	99.1±0.0	98.6±0.3	0.6	16357.5±137.6	169.4±4.2	44440.5±344.6	99.3±0.2	98.9±0.4	0.8	15160.3±127.4
ls4-DDALS(V2)	197.4±1.7	47999.9±83.0	99.1±0.0	99.0±0.1	**1**	16357.5±137.6	162.5±2.4	44440.5±344.6	99.1±0.1	98.6±0.2	0.7	15160.3±127.4
ls4-DDALS(V3)	196.8±0.8	48046	99.0±0.0	98.9±0.1	**1**	16280.8	163.5±3.5	44648	99.0±0.0	98.3±0.1	0.1	15162.6
ls5-Greedy	282.7±0.0	4.0±0.0	99.6±0.0	99.4±0.0	**1**	**2.8±0.2**	267.2±0.0	6.0±0.0	99.0±0.0	98.8±0.0	**1**	**3.9±0.0**
ls5-GA	238.6±8.3	81011.4±10.1	99.0±0.0	98.6±0.1	0.87	104473.1±1301.8	216.9±8.3	79012.9±8.8	99.0±0.0	98.6±0.2	0.77	101543.5±950.4
ls5-EDA	220.3±0.6	81004.3±2.8	99.1±0.0	98.7±0.0	**1**	103916.1±1686.4	200.6±1.8	79004.3±2.5	99.1±0.1	98.7±0.2	**1**	100901.5±1166.1
ls5-DDALS(O)	**217.4±1.2**	81186.6±246.3	99.0±0.0	98.6±0.0	**1**	36706.0±267.7	**199.6±2.2**	78766.1±799.8	99.0±0.0	98.6±0.1	0.8	35543.0±403.1
ls5-DDALS(V1)	225.3±3.0	81186.6±246.3	99.2±0.1	98.8±0.1	**1**	36706.0±267.7	209.4±4.9	78766.1±799.8	99.3±0.1	98.9±0.2	0.97	35543.0±403.1
ls5-DDALS(V2)	219.0±2.6	81186.6±246.3	99.1±0.1	98.7±0.1	**1**	36706.0±267.7	201.9±3.4	78766.1±799.8	99.1±0.1	98.7±0.2	0.9	35543.0±403.1
ls5-DDALS(V3)	220.9±2.3	81306	99.0±0.0	98.7±0.0	**1**	36440.4	203.6±1.1	77365	99.1±0.1	98.7±0.2	0.9	34775.3
ls6-Greedy	399.3±0.0	3.0±0.0	99.8±0.0	99.4±0.0	**1**	**1.9±0.1**	381.4±0.0	12.0±0.0	99.8±0.0	99.4±0.0	**1**	**6.9±0.1**
ls6-GA	357.0±13.2	125313.9±8.9	99.0±0.0	98.2±0.1	0	196363.3±427.9	323.2±9.8	121514.7±9.1	99.0±0.0	98.2±0.3	0.13	193984.6±4074.8
ls6-EDA	338.1±1.4	125305.6±2.9	99.0±0.0	98.6±0.0	**1**	196994.6±539.1	300.7±1.6	121504.8±2.9	99.1±0.1	98.7±0.1	**1**	193948.7±1566.8
ls6-DDALS(O)	**323.9±0.6**	125056.0±178.1	99.0±0.0	98.1±0.0	0	70604.7±449.7	**297.9±4.5**	121616.1±1093.7	99.0±0.0	98.3±0.1	0	68409.7±727.0
ls6-DDALS(V1)	332.7±2.9	125056.0±178.1	99.1±0.0	98.2±0.0	0	70604.7±449.7	**297.9±4.5**	121616.1±1093.7	99.0±0.0	98.3±0.1	0	68409.7±727.0
ls6-DDALS(V2)	325.3±2.2	125056.0±178.1	99.0±0.1	98.2±0.1	0	70604.7±449.7	199.4±5.2	121616.1±1093.7	99.1±0.1	98.3±0.2	0.1	68409.7±727.0
ls6-DDALS(V3)	326.4±2.0	124926	99.0±0.0	98.2±0.0	0	70190.2	305.0±3.9	121537	99.01±0.0	98.1±0.1	0	68162.4