.

GREAT ZIMBABWE UNIVERSITY

HCS 123

GARY MAGADZIRE SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

BSc HONOURS IN COMPUTER SCIENCE: PART 1 SEMESTER 1

EXAMINATION:Assignment I

HCS123: Discrete Mathematics

DATE:12 July 2021

Time : OF YOUR CHOICE hours

Candidates may attempt **ALL** questions : 80 MARKS

A1. (a) Find the unit vector in the same direction as the vector $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$. [4]

(b) Determine whether or not the point $(22, 9, -1)$ lies on the straight line that passes through the point $(2, 1, 3)$ and is parallel to the vector $5\mathbf{i} + 2\mathbf{j} - 1\mathbf{k}$. [3]

(c) Find b if $3\mathbf{i} + b\mathbf{j} + 2\mathbf{k}$ and $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ are orthogonal. [3]

(d) Let $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ and $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$. Find the vector component of $\mathbf{b}$ onto $\mathbf{a}$. [4]

(e) Calculate the cosine of the angle θ between the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = -\mathbf{i} + 2\mathbf{j} + \mathbf{k}$. [3]

(f) Let π be a plane which contains the point $(1, 2, 1)$ and suppose the vector $3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ is normal to the plane. Find the vector equation and the Cartesian equation of the plane π. [4]

A2. (a) As two planes fly by each other, their flight paths are given by straight lines

$$L_1 : x = 1 - t_1,\;\; y = -2 - 3t_1,\;\; z = 4 + t_1.$$

$$L_2 : x = 2 - 2t_2,\;\; y = -4 + 3t_2,\;\; z = 1 + 4t_2.$$

Show that the lines are skew, and therefore that there is no danger of the planes colliding. [10]

(b) The straight line l passes through the point $(2, -1, -3)$ and is parallel to the vector $1\mathbf{i} + 1\mathbf{j} - 2\mathbf{k}$ and the straight line m that passes through the point $(1, 2, 1)$ and is parallel to the vector $1\mathbf{i} - 1\mathbf{j} - 3\mathbf{k}$. Determine whether or not the lines intersect and if they intersect find the points of intersection. [8]

(c) Find the vector equation of a straight line passing through the point $(2, -1, 3)$ and parallel to the vector $5\mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$. [4]

(d) Give an example of a relation R on $A = \{a, b, c\}$ such that,

(i) R is both symmetric and antisymmetric. [2]

(ii) R is neither symmetric nor antisymmetric. [2]

(iii) R is transitive but $R \cup R^{-1}$ is not transitive(R). [3]

A3. (a) Suppose C is a collection of relations S on a set A, and let T be the intersection of the relations S in C, that is, $T = \bigcap(S \in C)$. Prove that

(i) If every S is symmetric, then T is symmetric. [5]

(ii) If every S is transitive, then T is transitive. [5]

A4. (a) Consider the following five relations on the set $G = \{a, b, c\}$
$R = \{(a, a); (a, b); (a, c); (c, c)\}$, $\emptyset$ = empty relation,
$S = \{(a, a); (a, b); (b, a); (b, b); (c, c)\}$, $G \times G$ = universal relation,
$T = \{(a, a); (a, b); (b, b); (b, c)\}$.
Determine whether or not each of the above relations on G is

(i) reflexive [3]

(ii) symmetric [3]

(iii) transitive [3]

(iv) antisymmetric. [3]

(b) Consider the $\mathbb{Z}$ of integers and an integer $m > 1$. We say that x is congruent to y modulo m, written as $x \equiv y(\text{mod m})$ if $x - y$ is divisible by m. Show that this defines an equivalence relation on $\mathbb{Z}$. [8]

END OF QUESTION PAPER