Assignment 3
Due no later than June 25 at 23:00

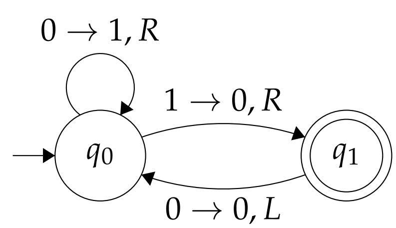
For full credit it is enough to accumulate 10 points.

Please give succinct, unambiguous, and well-phrased answers to the problems on the assignment. These qualities will be taken into account in the assessment. Each problem needs to be written on a single page and the whole assignment needs to be submitted on Gradescope.

A bonus of **1 point** will be given if the assignment is typeset in LaTeX. You may draw the automatons outside of latex (paint, take picture of handwritten), and insert picture files, but the rest must be typeset in LaTeX to receive full bonus.

Exercise 1 (2 point). Language of Turing Machine.

Let M be the following Turing Machine over the alphabet $\Sigma = \{0, 1\}$.



Let $L = \overline{L(M)}$ be the complement of the language accepted by M. Give a succinct expression for the constraints $\langle$something$\rangle$ so that $L = \{w \in \Sigma^* \mid \text{something}\}$. Give a brief justification.

Exercise 2 (2 points) Language classification.

a) Show that $L_a = \{\langle M, x_1, x_2\rangle \mid M \text{ is a TM and } M \text{ halts on } x_1 \text{ or } x_2\}$ is in RE.

b) Show that $L_b = \{\langle M, x\rangle \mid M \text{ is a TM and } M \text{ halts on } x \text{ in exactly } 2^{|x|} \text{ steps}\}$ is in REC.

Exercise 3 (3 points). Using Turing Machines for GCD computation.

The Euclidean algorithm computes the greatest common divisor of two nonnegative integers:

```
int gcd(m,n)
{
  if (n==0) return m
  else return gcd(n, m mod n)
}
```

Give a Turing machine with input alphabet $\{1, \#\}$ that on input $1^m\#1^n$ halts with $1^{\gcd(m,n)}$ written on its tape. Your description should be at the level of the description from class for the Turing machine that accepts the set $\{ww \mid w \in \{a,b\}^*\}$. In particular, do not give a state diagram or a list of transitions.

Exercise 4 (3 points). Reverse of encoding

Consider the problem

Can a given TM ever accept the reverse of its own encoding?

Give a succinct description of the language that encapsulates the problem

$$\textsc{ReverseAccept} = \{ \text{ something} \mid \text{something else } \}.$$

and prove the language is undecidable using reduction.