UNIVERSITY OF OXFORD

Saïd Business School

Bond Total Return Swaps

Theory, Pricing & Practice

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PART ONE: Theory

Bond Total Return Swaps

- Introduction to Bond TRS
- Why Trade a TRS?
- Contract Specifications
- Constant Units vs Constant Notional

PART TWO: Pricing & Practice

Case Studies

- Pricing Formulae
- Bond TRS Case Study

Quant Research Papers
https://ssrn.com/author=1728976

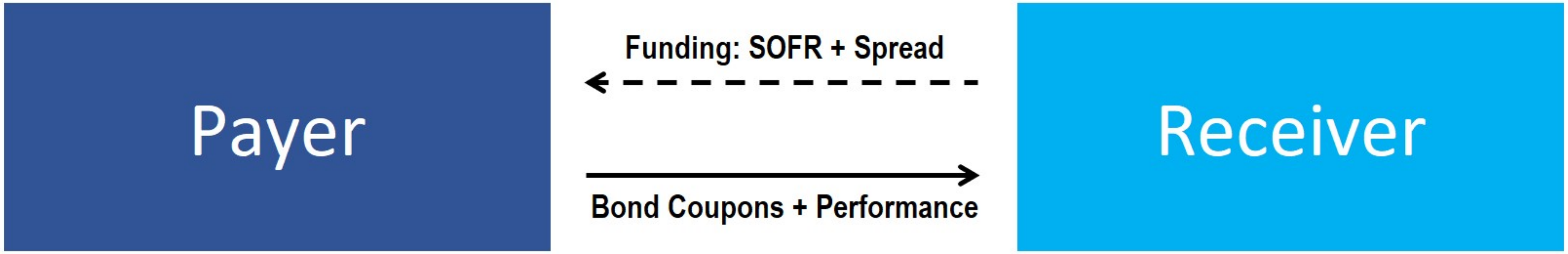
Book: Low Latency Interest Rate Markets
https://github.com/nburgessx/SwapsBook

PART ONE - THEORY

Bond TRS – Introduction to Bond TRS

What is a Bond TRS?

- The funded purchase of a Bond, where we receive the coupons and performance cash flows
- The 'payer' passes all coupons and changes in Bond price are passed to the 'receiver'



Cash Flows

- The payer holds the underlying bond, but passes on all bond income to receiver
- Bond income comes in the form of bond coupons & capital appreciation (bond performance)
- This means if bond prices rise (fall) the profit (loss) is passed to receiver
- In the event of a default the contract is unwound and the receiver pays the loss given default (LGD)

Bond TRS – Why Trade a TRS?

Pros

- **Capital Efficiency** - Much cheaper and less capital intensive than purchasing the bond directly
- **Leverage** – Capital efficiency helps increase leverage with minimal cash outlay
- **Market Access** – Direct market access not required

Cons

- **Counterparty and Default Risk** – Exposed to both counterparty and bond default risk
- **Interest Rate Risk** – Interest rate changes impact both funding costs and bond returns
- **Regulatory Risk** – Off-balance sheet and may attract increased regulatory scrutiny and associated costs.

Bond TRS – Trade Features

1. OTC, Customizable and Highly Bespoke
2. Fixed Cash Amount or Fixed Number of Bonds

   i.e. **Constant Notional** or **Constant Units**
3. Receive Bond **Coupons & Performance**
4. Pay Fixed or Float Funding
5. **Interest Rate & Credit Risk** (to Bond & Arranger)
6. Receiver Exposed to **Loss Given Default** (LGD)
7. Performance Projections Linked to Repo Rates
8. Pay Performance Regularly or At Maturity Only
9. **Regular Performance Payments Reduce PFE[1] Exposure** and XVA costs

[1] Potential Future Exposure

Bond TRS – Contract Specifications Example

Notional Specification

- 50,000 Bonds Purchased at 104.54.33%
- Cash Equivalent USD 5,227,165 (= 50,000 x 100 x 104.5433%)
- Bond Leg Specified as Number of Bonds (i.e. Units)

Coupon Payments

- Bond Leg Receives Coupons & Performance
- Funding Leg Pays SOFR + 100 bps
- Performance Paid at Maturity Only
- Credit Risky i.e. Cash Flows Scaled by Survival Probability
- Pays Loss Given Default (LGD) = 1 – Recovery Rate

Leg	**Bond Leg**	**Funding Leg**
PayOrReceive	Receive	Pay
Notional	50,000	5,227,165
NotionalType	Units	Cash
NotionalExchange	No Exchanges	No Exchanges
Currency	USD	USD
UnderlyingBond	UST 6.0% 15-02-2026	
CashFlowType	Coupons + Performance	
FloatIndex		SOFR O/N Index
FixedRateOrFloatSpread		100 bps
PayCoupons	On Coupon Dates	
PayPerformance	At Maturity	
Leverage	1.00	1.00
IsCreditRisky	TRUE	TRUE
CreditCurve	US Treasury	US Treasury
IncludeLossGivenDefault	TRUE	
RecoveryRate	40%	
TradedBondPrice	104.5433%	
DiscountCurve	SOFR O/N Index	SOFR O/N Index
CreditCurve	US Treasury	

Bond TRS – Constant Units

Constant Units

- **Fixed Number of Bonds** (Units)
- **Variable Cash Notional** over Time
- Every Period **Cash Notional Exchanges**
- Bond Fixings Required

Constant Units

Time	Notional	Units	Bond Fixing	Performance	Coupon	Total Payment
Past	10,000,000	80,736	123.86	108,994	155,417	264,411
Past	10,108,994	80,736	125.21	-662,038	155,417	-506,620
Past	9,446,956	80,736	117.01	235,750	155,417	391,167
Past	9,682,706	80,736	119.93	-1,984,499	155,417	-1,829,081
Past	7,698,208	80,736	95.35	-919,587	155,417	-764,169
Past	6,778,621	80,736	83.96	668,497	155,417	823,914
Past	7,447,118	80,736	92.24	-796,867	155,417	-641,450
Past	6,650,250	80,736	82.37	169,546	155,417	324,964
Current	**6,819,797**	**80,736**	**84.47**	**-74,277**	**155,417**	**81,140**
Future	6,745,519	80,736	83.55	0	155,417	0
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$$Units = \left(\frac{Cash\ Notional}{Face\ Value \times Bond\ Fixing}\right)$$

$$Units = \left(\frac{10{,}000{,}000}{100 \times 123.86\%}\right) = 80{,}736$$

$$Coupon = Units \times Face\ Value \times Rate \times Year\ Fraction$$

$$Coupon = 80{,}736 \times 100 \times 3.85\% \times 0.5$$
$$= 155{,}416.80$$

Bond TRS – Constant Notional

Constant Notional

- **Fixed Cash Notional**
- **Variable Bond Units** over Time
- No Notional Exchanges Required

Constant Notional

Time	Notional	Units	Bond Fixing	Performance	Coupon	Total Payment
Past	10,000,000	80,736	123.86	108,994	155,417	264,411
Past	10,000,000	79,866	125.21	-654,900	153,742	-501,158
Past	10,000,000	85,463	117.01	249,551	164,516	414,067
Past	10,000,000	83,382	119.93	-2,049,529	160,510	-1,889,019
Past	10,000,000	104,877	95.35	-1,194,546	201,888	-992,659
Past	10,000,000	119,104	83.96	986,184	229,276	1,215,460
Past	10,000,000	108,413	92.24	-1,070,035	208,695	-861,340
Past	10,000,000	121,403	82.37	254,947	233,702	488,649
Current	**10,000,000**	**118,385**	**84.47**	**-108,914**	**227,892**	**118,977**
Future	10,000,000	119,689	83.55	0	230,401	0
Future	10,000,000	119,689	83.55	0	230,401	0
Future	10,000,000	119,689	83.55	0	230,401	0
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$$Coupon = Units \times Face\ Value \times Rate \times Year\ Fraction$$

$$Coupon = 80{,}736 \times 100 \times 3.85\% \ \times 0.5$$
$$= 155{,}416.80$$

PART TWO – PRICING & PRACTICE

Bond TRS – Pricing Formula

Bond TRS has three main components,

- Underlying Bond Coupons
- Bond Performance
- Funding Component

The Present Value of a TRS is the Sum of Discounted Cash Flows from these components.

However, in the event of bond default the TRS is unwound and the receiver of the bond coupons must pay the Loss Given Default (LGD) to the bond holder i.e. the payer. The LGD must be included in the PV calculation.

Bond TRS – Pricing

Bond TRS PV = ϕ (PV(Bond Coupon) + PV(Performance) – PV(Funding Cash Flows))

where **ϕ** = { +1 when receiving and -1 when paying bond coupons }

Bond TRS – Pricing the Bond Coupons

Coupon PV

- For simplicity let's define the quantity or **Bond Notional (N_B)** as Units x Face Value
- This gives **Coupon** = Bond Notional x Rate x Year Fraction
- Leading to **PV(Coupon)** = Bond Notional x Rate x Year Fraction x P(Survival) x Disc Factor
- Let's assume we have n Bond coupons

$$PV(\text{Bond Coupons}) = \sum_{i=1}^{n} N_B \,.\, r \,.\, \tau_i .\, \underbrace{Q(t_0, t_i)}_{P(Survive)} \,.\, \underbrace{P(t_0, t_i)}_{Disc\ Factor}$$

Bond TRS – Pricing the Bond Performance

Bond Performance PV

- **Bond Performance has the largest impact on the TRS PV**
- The performance captures the Change in Bond Price using Bond Fixings (or resets)
- Change in Bond Price = Price at End of Coupon Period – Bond Price at Start of Coupon Period
- Leading to PV(Performance) = Bond Notional x Change in Price x P(Survival) x Disc Factor

$$PV(\text{Performance}) = \sum_{i=1}^{n} N_B.\left(B(t_{i-1}) - B(t_i)\right).\underbrace{Q(t_0, t_i)}_{P(Survive)}.\underbrace{P(t_0, t_i)}_{Disc\ Factor}$$

When Paying Performance on Coupon Dates

$$PV(\text{Performance}) = N_B.\big(B(t_0) - B(T)\big).\underbrace{Q(t_0, T)}_{P(Survive)}.\underbrace{P(t_0, T)}_{Disc\ Factor}$$

When Paying Performance at Maturity, T

Bond TRS – Estimating or Projecting Future Bond Prices

Projecting Future Bond Prices

- **Future Bond Prices can be implied from Repo Rates**
- The repo rate acts like a **Bond Growth Factor** as if we can place a bond on deposit and earn bond interest
- Repo rates work on **Dirty Bond Prices**

Calculation Using Repo Rates

- The simple projected bond price is calculated as below,

  $B_T = B_t(1 + r\,\tau)$

- We should however deduct repo interest from coupons (c) paid during the projection period,

  $B_T = B_t(1 + r\,\tau_1 - r\,c\,\tau_2)$

- where B_t is the Dirty Bond Price at time t, r the repo rate, c the coupon amount paid, t_1 denotes the time from t to T and t_2 the time from the coupon payment to time T

Bond TRS – Pricing the Funding Leg

Funding Leg PV

- The client borrow funds to purchase the underlying bond
- In exchange for receiving bond coupons and performance we pay funding costs
- The funding leg has a **Cash Notional (N_C)**, not to be confused with the bond notional (N_B)
- Let's assume we are funding our position in USD using SOFR as the floating index plus a spread, denoted (F+s)
- Then the **Funding Cost** or **Float Coupon** = Cash Notional x (SOFR + Spread) x Year Fraction
- Leading to **PV(Funding Leg)** = Cash Notional x (SOFR + Spread) x Year Fraction x P(Survival) x Disc Factor
- Let's assume we have m floating coupons

$$PV(\text{Float Coupons}) = \sum_{j=1}^{m} N_C \,.\, (F_j + s)\,.\,\tau_j .\, \underbrace{Q(t_0, t_j)}_{P(Survive)} \,.\, \underbrace{P(t_0, t_j)}_{Disc\ Factor}$$

Bond TRS – TRS Pricing I

Putting Everything Together …

Bond TRS PV = ϕ (PV(Bond Coupon) + PV(Performance) – PV(Funding Cash Flows))

where **ϕ** = { +1 when receiving and -1 when paying bond coupons }

$$PV(\mathrm{TRS}) = \varphi\left(\underbrace{\sum_{i=1}^{n} N_B\, r\, \tau_i\, Q(t_0,t_i)\, P(t_0,t_i)}_{Bond\ Coupons} + \underbrace{\sum_{i=1}^{n} N_B\big(B(t_{i-1}) - B(t_i)\big)\, Q(t_0,t_i)\, P(t_0,t_i)}_{Bond\ Performance} - \underbrace{\sum_{i=1}^{m} N_C\,(F_j + s)\,\tau_j\, Q(t_0,t_j)\, P(t_0,t_j)}_{Funding\ Cost}\right)$$

Bond TRS – TRS Pricing II

Definition: Risky Discount Factor

- A cash flow that can default is said to be **Credit Risky**
- To compute the PV of a credit risky cash flow we must discount the cash flow **AND** include the **Probability of Survival** i.e. the probability of no default so we can receive the cash flow.
- The **Risky Discount Factor** does just this and is defined as $\tilde{P}(t_0, t_i) = Q(t_0, t_i)\, P(t_0, t_i)$

Using Risky Discount Factor Notation gives,

$$PV(\mathrm{TRS}) = \varphi \left(\underbrace{\sum_{i=1}^{n} N_B\, r\, \tau_i\, \tilde{P}(t_0, t_i)}_{Bond\ Coupons} + \underbrace{\sum_{i=1}^{n} N_B \big(B(t_{i-1}) - B(t_i)\big)\, \tilde{P}(t_0, t_i)}_{Bond\ Performance} - \underbrace{\sum_{i=1}^{n} N_C\, (F_i + s)\, \tau_i\, \tilde{P}(t_0, t_i)}_{Funding\ Cost} \right)$$

Bond TRS – TRS Pricing III

What about the Loss Given Default?

- If the bond defaults the TRS is unwound and the **Loss Given Default** is paid by the bond coupon receiver
- We must incorporate the LGD defined as $N_B(1\text{-}RR)$, where RR is the recovery rate of the bond
- In developed markets the recovery rate is usually assumed to be 40% for senior subordinated bonds
- The probability of default during a coupon period is given by $P(\text{Default}) = Q(t_0, t_{i-1}) - Q(t_0, t_i)$, which is the probability of surviving to the start of the coupon period minus that of surviving to the end of the coupon period.

$$PV(\text{TRS}) = \varphi \left(\begin{array}{c} \underbrace{\sum_{i=1}^{n} N_B \, r \, \tau_i \, \tilde{P}(t_0, t_i)}_{Bond\ Coupons} + \underbrace{\sum_{i=1}^{n} N_B \big(B(t_{i-1}) - B(t_i)\big) \, \tilde{P}(t_0, t_i)}_{Bond\ Performance} - \underbrace{\sum_{i=1}^{n} N_C \, (F_i + s) \, \tau_i \, \tilde{P}(t_0, t_i)}_{Funding\ Cost} \\ - \underbrace{\sum_{i=1}^{n} N_B \, (1 - RR) \, \big(Q(t_0, t_{i-1}) - Q(t_0, t_i)\big) \, P(t_0, t_i)}_{Loss\ Given\ Default} \end{array} \right)$$

Bond TRS – The Breakeven Funding Spread (Par Spread)

What is the Breakeven Funding Spread or Par Spread?

- This is the funding spread that gives a **TRS price of zero** or Par i.e. both trade legs have equal value
- To calculate the par spread we rearrange the equation for PV(TRS) for the funding cost "s" parameter
- We often rename the parameter "p" to indicate it is the par spread

The par spread is computed as,

$$p = \left(\frac{PV(TRS\ with\ s = 0)}{N_C\, \tau_i\, \tilde{P}(t_0, t_i)} \right)$$

The term in the denominator is called the Risky Annuity,

$$Risky\ Annuity(Float\ Leg) = N_C\, \tau_i\, \tilde{P}(t_0, t_i)$$

Giving,

$$s = \left(\frac{PV(TRS\ with\ s = 0)}{Risky\ Annuity\ Float\ Leg} \right)$$

Appendix – Approximate Bond TRS Price I

Bond TRS Approximate Pricing Methodology

We can compute an approximate Bond TRS PV as follows,

Steps:

1. **Bond Coupons** – Compute the PV of Bond Coupons within the TRS start and end dates
2. **Bond Performance** – Compute the Performance using Bond Prices as follows,
    i. **Current Bond Price** - Price the Bond
    ii. **Forward Bond Price** - Price the Bond again but with all coupons up to the TRS Maturity Date Removed
3. **Funding Leg** – Compute as Normal
4. **Loss Given Default** – Trivial to include, however for an approximate TRS PV we could perhaps ignore this for investment grade bonds i.e. assume Prob(Default) ≈ 0.

Appendix – Approximate Bond TRS Price II

What does the approximation look like when we net everything together?

1. Full Bond Cash Flows to Bond Maturity
2. Funding Leg has a Negative Notional Exchange Equal to the Current Bond Price Paid at TRS Maturity
3. Funding Leg to Maturity of TRS

Illustration – Bond Leg Coupons and Performance

Bond
Bond coupons plus final notional exchange

Bond Price 108.11
Fwd Price 106.73

Time	Coupon	DF	PV
1	5	0.9615	4.81
2	5	0.9246	4.62
3	5	0.8890	4.44
4	5	0.8548	4.27
5	5	0.8219	4.11
6	5	0.7903	3.95
7	5	0.7599	3.80
8	5	0.7307	3.65
9	5	0.7026	3.51
10	105	0.6756	70.93

TRS Coupons
Bond coupons are paid to the client
Only include coupons during TRS lifetime of TRS

Total PV 9.43

Time	Coupon	DF	PV
1	5	0.9615	4.81
2	5	0.9246	4.62
3			
4			
5			
6			
7			
8			
9			
10			

TRS Performance
Future bond price less initial price paid at maturity
i.e. Future bond cashflows minus initial bond PV

Total PV -1.27

Time	Coupon	DF	PV
1		0.9615	0.00
2	-108.11	0.9246	-99.95
3	5	0.8890	4.44
4	5	0.8548	4.27
5	5	0.8219	4.11
6	5	0.7903	3.95
7	5	0.7599	3.80
8	5	0.7307	3.65
9	5	0.7026	3.51
10	105	0.6756	70.93

TRS
Bond coupons plus performance

Total PV 8.16

Time	Coupon	DF	PV
1	5	0.9615	4.81
2	-103.11	0.9246	-95.33
3	5	0.8890	4.44
4	5	0.8548	4.27
5	5	0.8219	4.11
6	5	0.7903	3.95
7	5	0.7599	3.80
8	5	0.7307	3.65
9	5	0.7026	3.51
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Have questions or want further info?

Contact

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