1 Iterative Deepening for Functional Programming Enumeration

We present here an detailed exposition of an iterative deepening type algorithm for enumerating functional programs.

Algorithm 1 Cost bounded depth first search on typed functional programs.

Require: G a grammar of typed, weighted primitives. A weight γ associated with a new application. A requested type τ . A current substitution σ .

```
1: function CBSEARCH(G, b, \tau, \sigma)
                                                               ▶ A cost bounded depth first search.
          out = []
 2:
 3:
          c_{min} \leftarrow \infty
          for e, \eta, w \in G do
                                                                                          \triangleright \tau is the type of e
 4:
               \sigma' \leftarrow unify(\eta, \tau, \sigma)
 5:
               if null(\sigma)) then
                                                                                ▶ If types can be unified.
 6:
                     continue
 7:
               else
 8:
 9:
                     out \leftarrow cons(out, (e, \sigma', w))
                     if w > b then
10:
                          c_{min} \leftarrow min(w, c_{min})
11:
                     end if
12:
               end if
13:
          end for
14:
          if \gamma > b then
15:
               c_{min} \leftarrow min(\gamma, c_{min})
16:
               return (out, c_{min})
17:
          else
18:
               (lhss, k_{\ell}) \leftarrow DBSEARCH(G, b - \gamma, \eta \rightarrow \tau, \sigma)
19:
20:
               c_{min} \leftarrow \gamma + k_{\ell}
               for e_{\ell}, \sigma_{\ell}, w_{\ell} \in \text{lhss do}
21:
                    if \gamma + w_{\ell} \leq b then
22:
                          (\text{rhss}, k_r) \leftarrow \text{DBSEARCH}(G, b - \gamma - w_\ell, \eta, \sigma_\ell)
23:
                          if \gamma + w_{\ell} + k_r > b then
24:
                               c_{min} \leftarrow min(c_{min}, \gamma + w_{\ell} + k_r)
25:
                          end if
26:
                          for e_r, \sigma_r, w_r \in \text{rhss do}
27:
                               if \gamma + w_{\ell} + w_r \leq b then
28:
                                    out \leftarrow cons(out, (app(e_{\ell}, e_r), \sigma_r, \gamma + w_{\ell} + w_r))
29:
30:
                               end if
                          end for
31:
                    end if
32:
               end for
33:
               return (out, c_{min})
34:
35:
          end if
36: end function
```

Algorithm 2 Cost bounded depth first search on typed functional programs, version 2. This version adds a modest bounding heuristic cost. Every expansion carries with it an induced cost. We modify the previous version in the following way: every time we proceed with an application, we reduce the available bound by the minimum required to complete the application if it comes back successfully. In this first version, we just reduce the bound by the smallest primitive in the grammar.

Require: G a grammar of typed, weighted primitives. A weight γ associated with a new application. A requested type τ . A current substitution σ .

```
1: function CBSEARCH(G, b, \tau, \sigma)
                                                               ▶ A cost bounded depth first search.
          out = []
 2:
 3:
          c_{min} \leftarrow \infty
 4:
          for e, \eta, w \in G do
                                                                                          \triangleright \tau is the type of e
               \sigma' \leftarrow unify(\eta, \tau, \sigma)
 5:
               if null(\sigma)) then
                                                                                 \triangleright If types can be unified.
 6:
                     continue
 7:
               else
 8:
                     out \leftarrow cons(out, (e, \sigma', w))
 9:
10:
                     if w > b then
                          c_{min} \leftarrow min(w, c_{min})
11:
                     end if
12:
               end if
13:
          end for
14:
          if \gamma + w_{min} > b then
15:
               c_{min} \leftarrow min(\gamma + wMin, c_{min})
16:
               return (out, c_{min})
17:
          else
18:
                w_{min} \leftarrow \min\{w|(-,-,w) \in G\}
19:
                (lhss, k_{\ell}) \leftarrow DBSEARCH(G, b - \gamma - w_{min}, \eta \rightarrow \tau, \sigma)
20:
               c_{min} \leftarrow \gamma + k_{\ell}
21:
               for e_{\ell}, \sigma_{\ell}, w_{\ell} \in \text{lhss do}
22:
                     if \gamma + w_{\ell} \leq b then
23:
                          (\text{rhss}, k_r) \leftarrow \text{DBSEARCH}(G, b - \gamma - w_\ell, \eta, \sigma_\ell)
24:
                          if \gamma + w_{\ell} + k_r > b then
25:
                               c_{min} \leftarrow min(c_{min}, \gamma + w_{\ell} + k_r)
26:
                          end if
27:
                          for e_r, \sigma_r, w_r \in \text{rhss do}
28:
29:
                               if \gamma + w_{\ell} + w_{r} \leq b then
                                    out \leftarrow cons(out, (app(e_{\ell}, e_r), \sigma_r, \gamma + w_{\ell} + w_r))
30:
31:
                               end if
                          end for
32:
                     end if
33:
               end for
34:
               return (out, c_{min})
35:
          end if
36:
37: end function
```