QUANTUM HARDWARE

1. Natural & Artificial Atoms

1.1 Why [AMO]-control

→ long trapping durations → large statistics

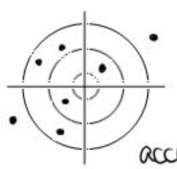
A) - short noise limit $\sim \sqrt{N}$ ↰ # of experiments

- QM projection noise $\sim \sqrt{N}$

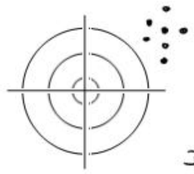
$|0\rangle + |1\rangle$ → measurement ≙ projection into $|0\rangle$ or $|1\rangle$

- Heisenberg limit $\sim N$
  (entanglement brings another $\sqrt{N}$)

B)



accurate, not precise



precise, not accurate

→ long coherence times

spin flip

T_1-time

phase

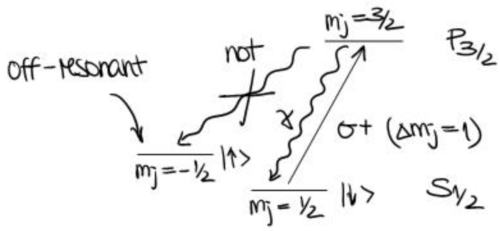
phase

$T_2^{(*)}$ time

$\Delta E \Delta t \geq \hbar$

$\hbar \Delta\omega \Delta t \geq \hbar \rightarrow \Delta\omega \geq \frac{1}{\Delta t}$ $\quad \Delta t_{T_1/T_2^*} \gg \rightarrow \Delta\omega \ll$ ↰ small error in energy

→ high detection efficiency (at low noise)



selection rules keep it a closed transition
(no $\Delta m = 2$ possible)

→ high operational fidelity

→ single qubit (Rabi flopping) 99,9999... %

Rotation

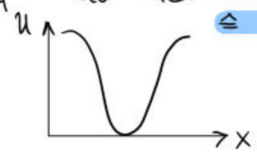
→ two qubit $\begin{matrix}\uparrow \\ \uparrow\end{matrix}$ $\begin{matrix}\uparrow \\ \uparrow + \downarrow\end{matrix}$ } ⇒ $\uparrow\uparrow + \downarrow\downarrow$

magnets try to align

99,94%

1.2 state of the art AMO - some numbers

@ trap: U_0 = 1eV ≙ 12 000 K $(k_B T)$
≙ $2{,}5 \cdot 10^{14}$ Hz $(h\nu)$ microwave $\sim 10^9$ Hz



↳ basically infinite (cor. temperature)

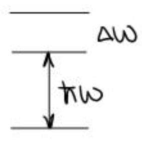
@ coherence times (see above)

T_1 → years

T_2^* → minutes - hours

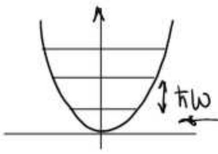
↑ ΔB (e.g. Zeeman shift) as change of ΔE

$|0\rangle - |1\rangle \longleftrightarrow |0\rangle + |1\rangle$



@ sensitivity (detection) 99,999...% (better $1-10^{-4}$)
lifetime in $P_{3/2}$ $\tau = 10^{-8}$ s
$\Gamma = \frac{1}{\tau} \approx 2\pi \cdot 10$ MHz $\triangleq$ rate of emitted photons (in $|\downarrow\rangle$)
(no photons if $|\uparrow\rangle$)

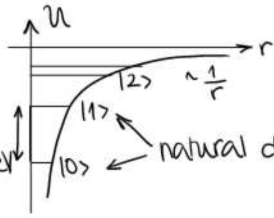
@ incoherent control



harmonic oscillator $\frac{1}{2}m\omega^2 x^2 \overset{\omega=\sqrt{D/m}}{=} \frac{1}{2}m\frac{D}{m}x^2 = \frac{1}{2}Dx^2$ — motional DOF, trapping

cooling to motional ground state

for qubit: $\bar{e}$ DOF



$E_R = -13{,}6$ eV

natural decoupled two level system

1.3 historical view (trapping)

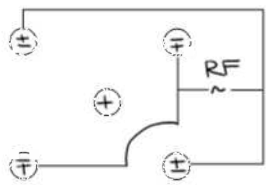
fundamental ↯ Schrödinger "never single quanta"
technical ↯ no trap for charged particles

1) $\vec{E} = -\vec{\nabla}\phi$
2) $\operatorname{div}\vec{E} = \rho/\epsilon_0$
3) $\Delta\phi = 0$

$\Big\}\ \operatorname{div}\vec{E} = -\Delta\phi = \rho/\epsilon_0 = 0$ ← no charge in center

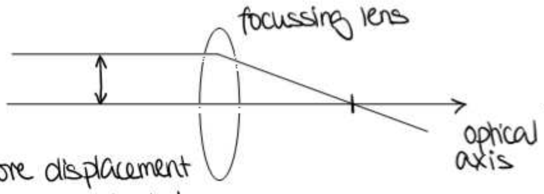
→ in 3D: $\phi(x,y,z) = \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2 + \frac{1}{2}m\omega_z^2 z^2$
$\Delta\phi = m\omega_x^2 + m\omega_y^2 + m\omega_z^2 = 0 \longrightarrow$ ↯ one of the $\omega_{xyz}^2 < 0$
↯ deconfinement → loss in 1D

$\boxed{\text{Dehmelt + Paul}}$ e.g. Paultrap

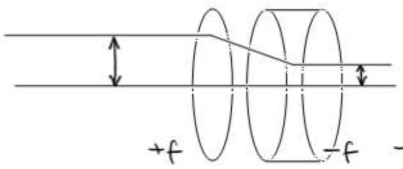


time averaged potential

<u>Analogon</u> (? why netto effect: focus + defocussing ⇏ 0 ?)



+ →



less deflection
→ netto focussing

19/04/23

Σ Di Vincenzo Criteria

1) qubit (scalable)
2) initialization → low enough temperatures to allow for that otherwise Boltzmann distribution → excited state
3) detection

} incoherent interaction

4) coherence times
5) single & two qubit operations

} coherent

2 Light – matter interaction

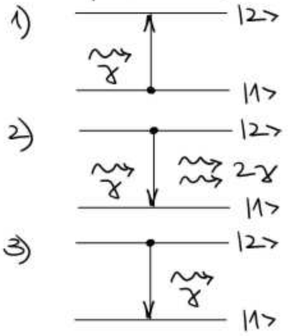
≙ em wave (MHz, GHz, THz, vis)

two-level system ≙ qubit

$\bar{e}$ quantum of charge

γ quantum of energy

Basic processes: incoherent



stimulated absorption

stimulated emission

spontaneous emission

≙ random in time → $\Gamma = \frac{1}{\tau}$

polarization

energy

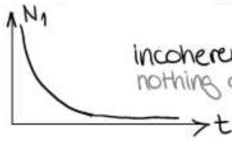
Einstein: (rate equations)

coupling strength / Einstein coefficient

$$\frac{dN_1}{dt} = -B_{12} N_1 \rho(\omega)$$

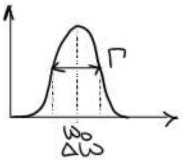
spectral density

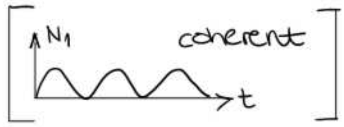
$$\boxed{N_1(t) = N_{10}\, e^{-t/\tau}}$$



incoherent

nothing comes back

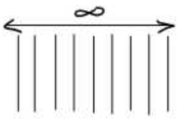




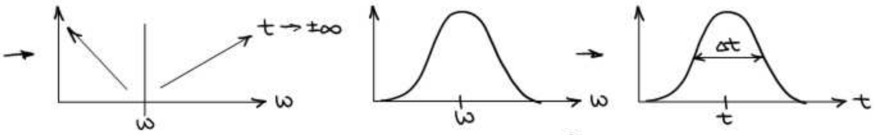
side remarks #1 spectral density $\rho(\omega)$

plane waves: $A e^{i(\omega t - kz)}$

do not exist



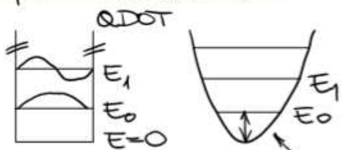
≙ infinite energy ↯



plane wave

pulse = Σ / ∫ of plane waves

#2 spontaneous emission



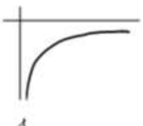
harmonic oscillator (see Transmon qubit, em-field motion)

vacuum energy (quantum fluctuations)

$\Delta x \Delta p \geq \hbar/2$

↳ okay, but is it real?

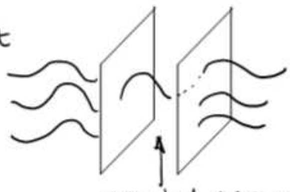
→ H-atom (Lamb-shift)



$\frac{1}{r}$ + em-fluctuations

→ energy level shift

→ Casimir effect



all λ

restricted amount of λ

→ radiation pressure

$p_{out} > p_{inside}$

↳ two hands at 1μm → 10^{-7} N (weight of water droplet)

→ spontaneous emission ≙ 'stimulated' by vacuum fluctuations

→ MEMS → Casimir force dominant (↳ vacuum fluctuations dominant)

↑ rate equations (incoherent)

↓ coherent

3. unitary evolution

3.1 Schrödinger equation

Differential equation? → Newton: $m\ddot{x} + Dx = 0$
→ Maxwell:

↳ $\partial/\partial t \leftrightarrow \partial/\partial x$ or $\partial^2/\partial x^2$

Wish list:
1) de Broglie $p = \hbar k = \hbar \frac{2\pi}{\lambda} = \frac{h}{\lambda_{dB}}$
2) linear differential equations
3) plane wave as solution
} if plane wave is solution → superposition of plane waves is also a solution

(1) $|\psi\rangle = A e^{i(kx-\omega t)}$

(2) $\partial/\partial t\, |\psi\rangle = -i\omega |\psi\rangle$ ← $\hbar\omega = E$

(3) $\partial/\partial x\, |\psi\rangle = ik|\psi\rangle = i\frac{p}{\hbar}|\psi\rangle$ de Broglie

$\boxed{\frac{\partial}{\partial x} = i\frac{p}{\hbar} \quad \hat{p} = -i\hbar\frac{\partial}{\partial x}}$

(4) $\partial^2/\partial x^2\, |\psi\rangle = -k^2|\psi\rangle = -\frac{p^2}{\hbar^2}|\psi\rangle = \frac{p^2}{2m}\left(-\frac{2m}{\hbar^2}\right)|\psi\rangle$ ← $E_{kin} = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

(5) $i\hbar \cdot (2)$:

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = \hbar\omega|\psi\rangle \overset{(4)}{=} \underbrace{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}}_{\text{Hamiltonian for free particle}}|\psi\rangle$$

← Δ for 3D

→ $$\boxed{i\hbar\frac{\partial}{\partial t}|\psi\rangle = \Big(\underbrace{-\frac{\hbar^2}{2m}\Delta}_{E_{kin}} + V(\vec{x})\Big)|\psi\rangle}$$ time dependent Schrödinger equation

Reminder #1 $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ → Heisenbergs uncertainty principle

#2 $|\psi\rangle \hat{=}$ probability amplitude

$|\psi|^2 = \psi\cdot\psi^* \hat{=}$ probability density

↳ $\int|\psi|^2 dV \hat{=}$ probability

$\boxed{\text{expectation value} \quad \langle N\rangle = \int \psi N \psi^* \, dV}$

$\boxed{\text{charge density, mass density} \leftarrow \text{integral } \int dV \to \text{charge / mass}}$

Dirac notation: $\psi\psi^* = \langle\psi|\psi\rangle$

3.2 Schrödinger for atom (two levels) & plane waves

1) $i\hbar\frac{\partial}{\partial t}|\psi\rangle = (\hat{H}_{atom} + \hat{H}_{int})|\psi\rangle$
↳ take complete basis of the undisturbed atom

$|p_2\rangle$, $|p_1\rangle$ $\hat{H}_{atom}$

2) $\hat{H}_{atom}|p_{1/2}\rangle = E_{1/2}|p_{1/2}\rangle$ ← eigenstates
↑ eigenenergies/-values

3) Ansatz for solving Schrödingers equation: $\boxed{|\psi(t)\rangle^{new} = c_1(t)\,e^{-iE_1t/\hbar}\,|p_1\rangle^{old\,1} + c_2(t)\,e^{-iE_2t/\hbar}\,|p_2\rangle^{old\,2}}$

4) multiply from left: $\langle p_1|e^{iE_1t/\hbar}$ (later $\langle p_2|e^{iE_2t/\hbar}$)

5) $\langle p_1|e^{iE_1t/\hbar}\, i\hbar\frac{\partial}{\partial t}\left[e^{-iE_1t/\hbar}c_1(t)|p_1\rangle + e^{-iE_2t/\hbar}c_2(t)|p_2\rangle\right]$ left

$= \langle p_1|e^{iE_1t/\hbar}\left(\hat{H}_a + \hat{H}_i\right)\left[e^{-iE_1t/\hbar}c_1(t)|p_1\rangle + e^{-iE_2t/\hbar}c_2(t)|p_2\rangle\right]$ right

6) $\xrightarrow{a}$ $\langle p_1|\hat{H}_a|p_2\rangle = 0$ | $\langle p_1|p_2\rangle = 0$: orthonormal

$\xrightarrow{b}$ $\langle p_1|\hat{H}_a|p_1\rangle c_1(t) = E_1 \cancel{c_1}(t)$

first part left: $i\hbar\frac{\partial}{\partial t}\left(e^{-iE_1t/\hbar}c_1(t)\right) \overset{\text{chain rule}}{=} i\hbar\underbrace{\left(-i\frac{E_1}{\cancel{\hbar}}\cancel{c_1(t)} + \dot{c}_1(t)\right)e^{-iE_1t/\hbar}}_{\text{left over}}$

7) $i\hbar \frac{\partial}{\partial t} c_1(t) \underbrace{\langle p_1|p_1\rangle}_{1} \overset{\langle p_1|p_2\rangle = 0}{=} c_1(t) \langle p_1|\hat{H}_i|p_1\rangle + c_2(t) \langle p_1|\hat{H}_i|p_2\rangle \, e^{i(E_1-E_2)t/\hbar}$ (I)

8) same for $\langle p_2| e^{+iE_2 t/\hbar}$ → $i\hbar \frac{\partial}{\partial t} c_2(t) = c_1(t) \langle p_2|\hat{H}_i|p_1\rangle e^{i(E_2-E_1)t/\hbar} + c_2(t) \langle p_2|\hat{H}_i|p_2\rangle$ (II)

$E_{1|2} = \hbar\omega_{1|2}$

Σ 9) $$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} = \begin{pmatrix} \langle p_1|\hat{H}_i|p_1\rangle & \langle p_1|\hat{H}_i|p_2\rangle \overbrace{e^{i(E_1-E_2)t/\hbar}}^{-i\omega_{21}t} \\ \langle p_2|\hat{H}_i|p_1\rangle \underbrace{e^{i(E_2-E_1)t/\hbar}}_{i\omega_{21}t} & \langle p_2|\hat{H}_i|p_2\rangle \end{pmatrix} \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix}$$

transition matrix elements

3.3 Interaction in dipole approximation

$$\boxed{\hat{H}_i = \overbrace{e\vec{r}}^{\text{dipole}} \cdot \vec{E}(\vec{R}, t) + \underbrace{\frac{\partial E}{\partial r}\Big|_{\vec{r}} + \frac{\partial^2 E}{\partial r^2} \vec{r}^{\,2} + \dots}_{\text{multipole expansion}}}$$

location of proton

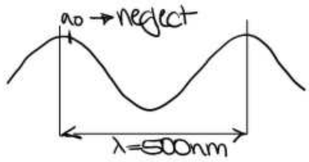
relative distance, e^- from proton

neglect all but first term?

→ sketch: $\lambda_{em} \approx 500\,nm$ Bohr radius

$e^- - p \approx \text{Å} \approx 10^{-10}\,m \cdot 0.5 = a_0$

$\frac{\lambda_{em}}{a_0} = \frac{5\cdot 10^{-7}\,m}{5\cdot 10^{-11}\,m} \approx 10^4$



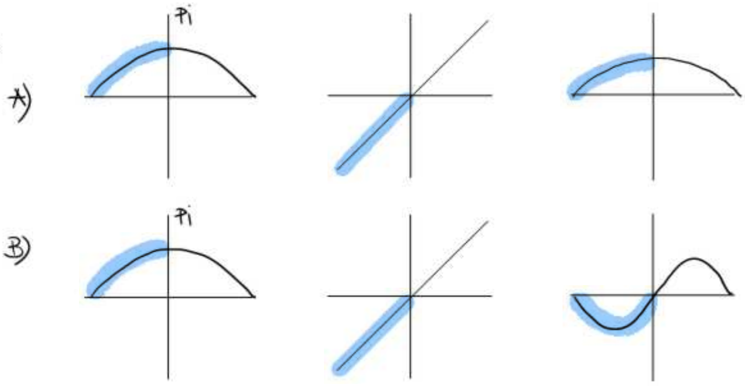
→ $$\boxed{\hat{H}_i = \underbrace{e\vec{r}}_{-\vec{d}} \, \underbrace{\vec{E}_0 \cos(\omega_L t)}_{\vec{E}(t)}}$$

sketch: Transition matrix elements

24/04/23

1 $\langle p_i|\hat{H}_i|p_i\rangle = 0$ why?

2 options



→ diagonal elements are 0

→ off-diagonal elements ≙ transition matrix elements

3.4 Coupling strength

$$\langle p_i|H_i|p_j\rangle_{i\neq j} = \underbrace{\langle p_i|e\vec{r}|p_j\rangle \cdot E_0}_{\hbar\Omega_{ij}} \cos\omega_L t$$

$$\boxed{\Omega_{12} = \Omega_{21}^{*} = \frac{eE_0 \langle p_1|\vec{r}|p_2\rangle}{\hbar}}$$ Rabi frequency ≙ figure of merit of coupling strength

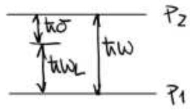
3.5 Solving schrödingers equation

(1) $$i\begin{pmatrix}\dot{c}_1(t)\\ \dot{c}_2(t)\end{pmatrix} = \begin{pmatrix}0 & \Omega_{12}e^{-i\omega_{21}t}\cos\omega_L t\\ \Omega_{12}e^{+i\omega_{21}t}\cos\omega_L t & 0\end{pmatrix}\begin{pmatrix}c_1(t)\\ c_2(t)\end{pmatrix}$$

(2) Euler's formula $\cos\omega_L t = \frac{1}{2}\left(e^{i\omega_L t}+e^{-i\omega_L t}\right)$

$$i\begin{pmatrix}\dot{c}_1(t)\\ \dot{c}_2(t)\end{pmatrix} = \begin{pmatrix}0 & \frac{\Omega_{12}}{2}e^{-i\omega_{21}t}\left(e^{i\omega_L t}+e^{-i\omega_L t}\right)\\ \frac{\Omega_{12}}{2}e^{i\omega_{21}t}\left(e^{i\omega_L t}+e^{-i\omega_L t}\right) & 0\end{pmatrix}\begin{pmatrix}c_1(t)\\ c_2(t)\end{pmatrix}$$

(3) $$i\begin{pmatrix}\dot{c}_1(t)\\ \dot{c}_2(t)\end{pmatrix} = \frac{\Omega}{2}\begin{pmatrix}0 & e^{-i(\omega_{21}+\omega_L)t}+e^{-i(\omega_{21}-\omega_L)t}\\ e^{i(\omega_{21}+\omega_L)t}+e^{i(\omega_{21}-\omega_L)t} & 0\end{pmatrix}\begin{pmatrix}c_1(t)\\ c_2(t)\end{pmatrix}$$



oscillates at $\approx 2\omega_L \approx 2\omega_{21}$ (10^{15} Hz) → averages out

Rotating wave approximation $\quad e^{\pm i\delta t} \quad \delta = (\omega_{21}-\omega_L)$ energy detuning

(4) $$i\begin{pmatrix}\dot{c}_1(t)\\ \dot{c}_2(t)\end{pmatrix} = \frac{\Omega}{2}\begin{pmatrix}0 & e^{-i\delta t}\\ e^{i\delta t} & 0\end{pmatrix}\begin{pmatrix}c_1(t)\\ c_2(t)\end{pmatrix}$$ → search for solutions (of c_1 and c_2)

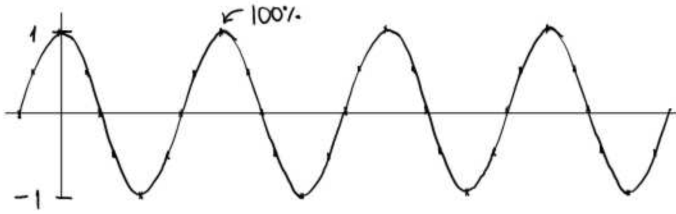
3.6 Solution on resonance

I) $\dot{c}_1(t) = -i\frac{\Omega}{2}c_2(t)$

II) $\dot{c}_2(t) = -i\frac{\Omega}{2}c_1(t)$ → take 2nd derivative $\quad \dot{\text{II}}$) $\ddot{c}_2(t) = -i\frac{\Omega}{2}\dot{c}_1(t) \overset{(I)}{=} -\frac{\Omega^2}{4}c_2(t)$

→ $c_2(t) = \sin\left(\frac{\Omega}{2}t\right)$
$\quad c_1(t) = \cos\left(\frac{\Omega}{2}t\right)$

amplitude ≠ population!

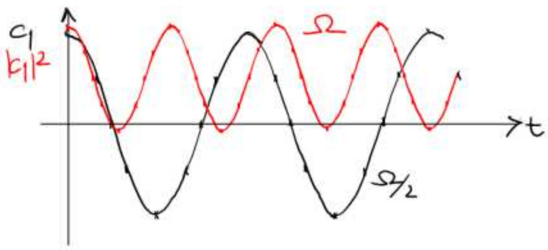


coherent oscillation
→ 100% population dependent on time
≙ qubit at work (one-qubit gates)

→ $|c_1(t)|^2 = \cos^2\left(\frac{\Omega}{2}t\right)$
$\quad |c_2(t)|^2 = \sin^2\left(\frac{\Omega}{2}t\right)$

$$\boxed{\begin{aligned}&\cos a\cdot\cos b = \tfrac{1}{2}\left(\cos(a+b)+\cos(a-b)\right) \quad \text{for } a-b=0\\ &\cos^2 a = \tfrac{1}{2}(\cos 2a + 1)\end{aligned}}$$

→ $|c_1(t)|^2 = \frac{1}{2}\left(1+\cos\Omega_{12}t\right)$
$\quad |c_2(t)|^2 = \frac{1}{2}\left(1-\cos\Omega_{12}t\right)$ } Rabi oscillation of population



Σ 1) system qubit oscillates with Ω_{12} $\quad |p_1\rangle \leftrightarrow |p_2\rangle$
2) population exchange by 100% (coherent)
3) Increasing E_0 $(\Omega \sim E_0)$ ≙ increasing $\Omega_{\text{Rabi},12}$

(typically 1µs for ion traps
10ns for solide state
compared to 10^{-15}s)

Two cases

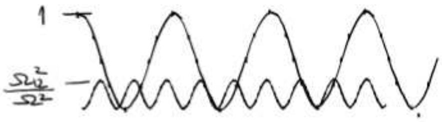
1) for $t_{int} = \frac{\pi}{\Omega_{21}}$ (≙ π-pulse)
$c_1(t=\pi/\Omega_{21}) = \cos\left(\frac{\Omega_{21}}{2}\pi/\Omega_{21}\right) = \cos\pi/2 = 0$

2) for $t_{in} = \frac{\pi}{2\Omega_{21}}$ (≙ π/2-pulse)
$c_1\left(t=\frac{\pi}{2\Omega_{21}}\right) = \cos\pi/4 = \frac{1}{\sqrt{2}}$

→ for π/2-pulse on p_1 → $|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left(|p_1\rangle + |p_2\rangle\right)$ ≙ superposition state

3.7 Dynamics of off-resonant case ($\delta \neq 0$)

$$\Omega = \sqrt{\Omega_{12}^2 + \delta^2}$$
$$|c_2(t)|^2 = \frac{\Omega_{12}^2}{\Omega_{12}^2 + \delta^2} \sin^2\left(\frac{\Omega}{2} t\right)$$



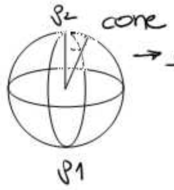
! 1) amplitude <
2) Ω-rate increases

→ $\delta = 0$

rotating state on the surface of the Bloch sphere
(! measurement = projection on z-axis)

→ $\delta \neq 0$

projection $\ll 1$



→ Ω faster

> Projection noise
> $\frac{1}{\sqrt{2}}(|\rho_1\rangle + |\rho_2\rangle) \rightarrow$ 50% in ρ_1, in ρ_2
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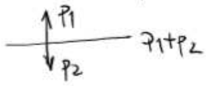
> 
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4. Density matrix formalism

4.1 Two level density matrix

$$\hat{\rho} = |\psi\rangle\langle\psi| = (c_1|\rho_1\rangle + c_2|\rho_2\rangle)(c_1^*\langle\rho_1| + c_2^*\langle\rho_2|)$$
$$= c_1c_1^*|\rho_1\rangle\langle\rho_1| + c_2c_1^*|\rho_2\rangle\langle\rho_1| + c_1c_2^*|\rho_1\rangle\langle\rho_2| + c_2c_2^*|\rho_2\rangle\langle\rho_2|$$

$$\rightarrow \hat{\rho} = \begin{pmatrix} c_1c_1^* & c_1c_2^* \\ c_2c_1^* & c_2c_2^* \end{pmatrix} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \qquad \rho_{ij} = \langle\rho_i|\rho|\rho_j\rangle = \langle\rho_i|\psi\rangle\langle\psi|\rho_j\rangle = c_i c_j^*$$

4.1.1 Examples

1) statistical mixture: $\hat{\rho}_m = p_1|\rho_1\rangle\langle\rho_1| + p_2|\rho_2\rangle\langle\rho_2| = \begin{pmatrix} p_1 & 0 \\ 0 & p_2 \end{pmatrix}$

not amplitudes $(c_2c_2^*) \hat{=}$ probabilities

2) coherent superposition state (pure) $\quad |\psi\rangle = \frac{1}{\sqrt{2}}(|\rho_1\rangle + |\rho_2\rangle) \quad c_1 = c_2 = \frac{1}{\sqrt{2}}$

$$\hat{\rho}_c = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} = 1/2 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

off-diagonal: coherence $\hat{=}$ phase information

4.1.2 Analysis of differences ($\hat{\rho}_m$ and $\hat{\rho}_c$)

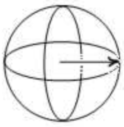
→ diagonal elements $\hat{=}$ probabilities
→ off-diagonal elements $\hat{=}$ coherences (relative phase between $|\rho_1\rangle$ and $|\rho_2\rangle$)

→ For $\hat{\rho}_c$: $\mathrm{Tr}(\hat{\rho}_c) = \mathrm{Tr}(\hat{\rho}_c^2)$ $\quad$ eg. $\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

For $\hat{\rho}_m$:
$$\begin{pmatrix} p_1 & 0 \\ 0 & p_2 \end{pmatrix}\begin{pmatrix} p_1 & 0 \\ 0 & p_2 \end{pmatrix} = \begin{pmatrix} p_1^2 & 0 \\ 0 & p_2^2 \end{pmatrix}$$
↳ $p_1 + p_2 = 1$ $\qquad$ ↳ $p_1^2 + p_2^2 \leq 1$ $\qquad$ for $p_1^2 + p_2^2 < 1$: sign for mixed state

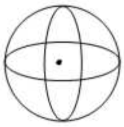
4.1.3 Analysis of difference in experiment

$|\Psi_c\rangle = \frac{1}{\sqrt{2}}(|p_1\rangle + |p_2\rangle)$
↳ pure state. live on surface of Bloch sphere



all atoms of the ensemble point into same direction

for mixed state
↳ live inside the Bloch sphere



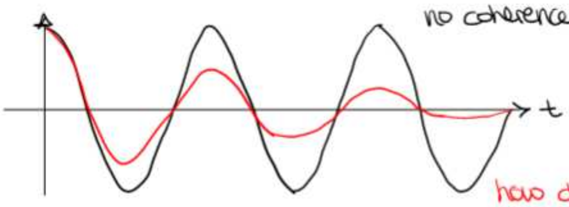
centred in Bloch sphere (no phase information)
$\hat{\rho}_m = \frac{1}{2}|p_1\rangle\langle p_1| + \frac{1}{2}|p_2\rangle\langle p_2|$

derive minimal results → Quantum Liouville equation $i\hbar\,\partial_t \hat{\rho} = [\hat{H}, \hat{\rho}]$, $\hat{H} = \hat{H}_0 + \hat{H}_{int}$
→ decoherence → Γ damping rates (decoherence)

$\sum$ coherence (Schrödinger) · exp. damping $e^{-\Gamma t}$

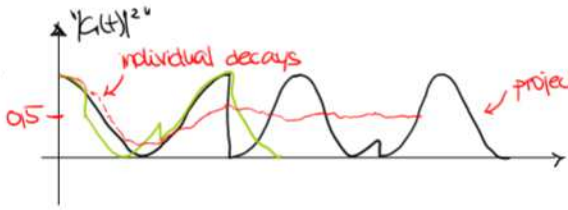
e.g. numerical result

no coherence → $\Gamma = 0$



how does single qubit evolve?

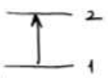
Monte Carlo of single atom
N = 1
N = 3
N = 10(0)



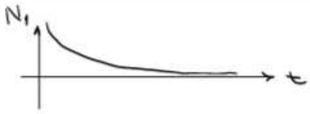
! only ensemble average allows to derive expectation values

Recap 26/04/23

(1) incoherent evolution: Einstein rate equation



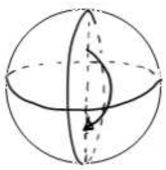
$\frac{dN_1}{dt} \propto -N_1 \rightarrow e^{-\Gamma t}$

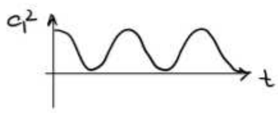


(2) coherent evolution: Schrödinger equation $|c_1(t)|^2 = \frac{1}{2}(1 + \cos(\Omega_{12} t))$
↳ Rabi rate $\Omega_{12} = \frac{e E_0 \langle 1|\hat{r}|2\rangle}{\hbar}$

faster if E_0 gets bigger

on resonance $\delta = 0$





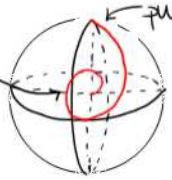
1
projection on z-axis

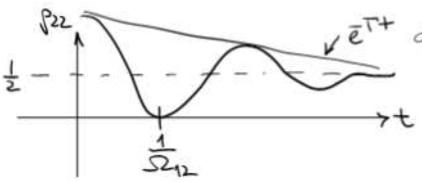
(3) (in)coherent evolution: density matrix formalism $\hat{\rho} = |\Psi\rangle\langle\Psi| = \begin{pmatrix} c_1 c_1^* & c_1 c_2^* \\ c_1^* c_2 & c_2 c_2^* \end{pmatrix}$

generic form $\hat{\rho} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$ populations
phase coherence

difference between e.g. $c_1 c_1^*$ and ρ_{11}? 1) same for pure states
2) for mixed states multiplying ρ_{ij} by $e^{-\Gamma t}$

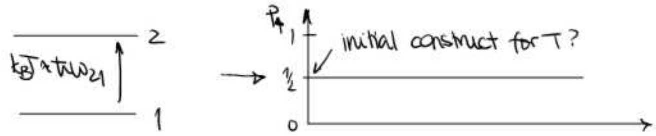
mixed state





⇒ !! you see ensemble average!

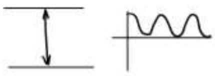
Today: Cooling



$\hookrightarrow N_2 \rightarrow$ cooling important

1) deceleration $\rightarrow$ force needed $\vec{F} = \frac{d\vec{p}}{dt}$ ← photons provide momentum kicks $p = \hbar k = \frac{2\pi}{\lambda}\hbar = \frac{h}{\lambda}$ de Broglie
2) deceleration $\neq$ cooling → you need more
   $\hookrightarrow$ 2 light forces + velocity depending detuning $\rightarrow$ Doppler cooling
   random motion in ensemble → temperature

5. Incoherent control for (laser) cooling

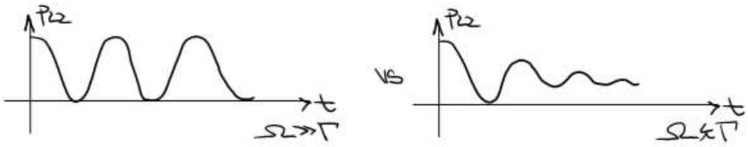


we search for steady state $\hat{=}$ we have a temperature (randomness stays @ same level)

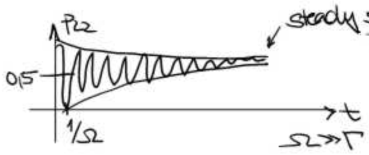
$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = 0$$

5.1 Time scales

a) $t \ll \frac{1}{\Gamma}$ ← decoherence rate



b) $t \gg \frac{1}{\Gamma}$



steady state
thermalization has occured, still have fluctuations
→ temperature

requirement $\frac{d}{dt}\rho_{22} = 0$
$\frac{d}{dt}(\rho_{22} - \rho_{11}) = 0$
still motion

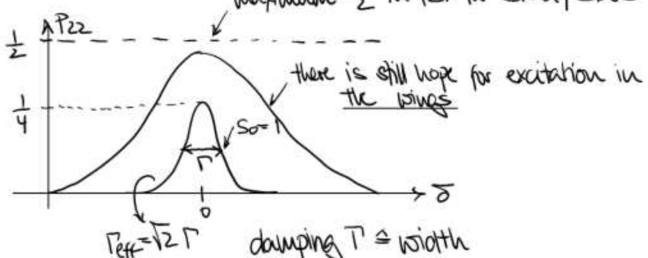
5.2 Steady-state ($\hat{=}$ end of dynamics)

step by step $\frac{d}{dt}(\rho_{22} - \rho_{11}) = 0$ with optical Bloch equations
$\rho_{22}(\Omega_{12}, \Gamma, \delta)$
$s_0 = \frac{2\Omega_{12}^2}{\Gamma^2}$ saturation parameter

$$\boxed{\rho_{22} = \frac{s_0}{1 + s_0 + 4\delta^2/\Gamma^2} \cdot \frac{1}{2}}$$

$\hookrightarrow$ photon scattering rate Γ_{scat}

light-matter interaction → saturation



damping $\Gamma \hat{=}$ width
$\Gamma_{eff} = \sqrt{1+s_0}\,\Gamma$

5.3 Light force ($v=0$)

$F = \frac{\Delta p}{\Delta t}$ $\boxed{\Gamma_{scat} = \rho_{22}\,\Gamma_{\text{spontaneous emission}} \quad [\tfrac{1}{s}]}$

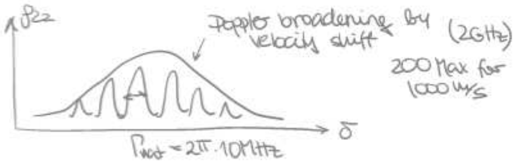
decay rate

Force: $\boxed{F_{max,scat} = \overbrace{\hbar k}^{\Delta p}\,\frac{\Gamma_{SE}}{2}}$ maximum, but close to, because $\rho_{22} \rightarrow \frac{1}{4}$ for s_0

$\Gamma_{scat,max}$

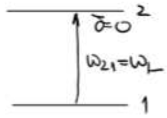
5.4 Light force $(v \neq 0) \leftarrow \nabla \neq 0 \rightarrow T \neq 0$

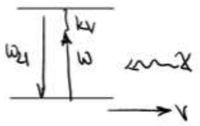
↳ Doppler shift



plane wave $\quad e^{i(kx-\omega t)} \xrightarrow[v\neq 0]{} e^{i(k(x_0+vt)-\omega t)} = e^{i(kx_0 + \underbrace{(kv-\omega)}_{\Delta\omega,\ \omega_{eff}} t)}$

→ velocity dependent frequency shift



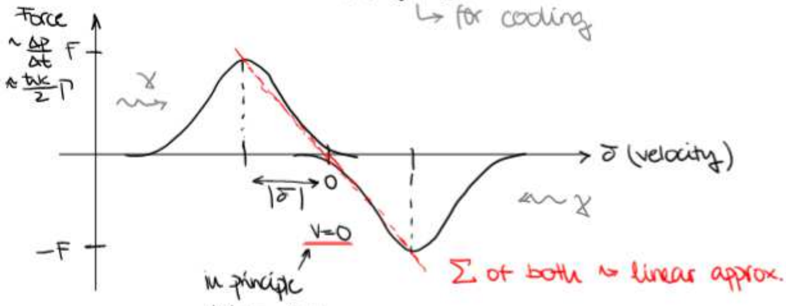


for $v \neq 0$

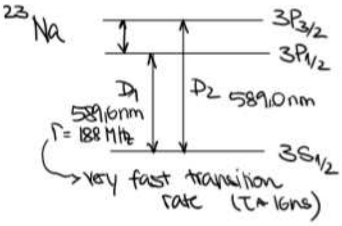
⇒ $\underline{\delta < 0}$ red detuned

→ the velocity v of the particle tunes the red-detuned laser frequency into resonance
↳ laser & atom are counter propagating

principle in 1D: two counter propagating lasers (both $\delta < 0$)
↳ for cooling



$\left.\begin{matrix} +v \rightarrow -F \\ -v \rightarrow +F \end{matrix}\right\} \Rightarrow v = 0\ ?$ → $T_{Doppler} \approx 1\,mK$ ($\approx 1\,m/s$) limit due random kicks



$F = \frac{dp}{dt} \qquad \Gamma_{scatt} = \rho_{22}\,\Gamma$
$\qquad\qquad\qquad\qquad ↳ \frac{1}{2}$

$\Delta p = \hbar k = \frac{h}{\lambda} \qquad F = ma \rightarrow a = \frac{F}{m} = \frac{\hbar k \Gamma}{2m} = \frac{\hbar c^2}{2mc^2}\,\frac{2\pi}{\lambda}\,\Gamma$

$\hbar c \approx 200\,MeV \cdot 10^{-15}\,m$

$mc^2 \approx 1\,GeV$

(↳ atomic mass unit amu)

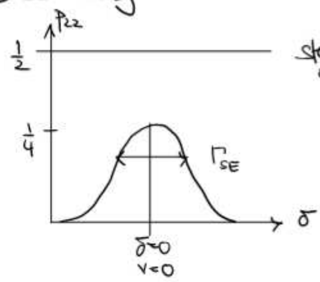
$$\rightarrow a = \frac{200\,MeV \cdot 10^{-15}\,m \cdot 3{,}00 \cdot 10^8 \frac{m}{s}\, 2\pi}{2 \cdot 23\,GeV \cdot 589{,}6 \cdot 10^{-9}\,m \cdot 16 \cdot 10^{-9}\,s} = \frac{10^8 \cdot 3\, 2\pi}{23 \cdot 6 \cdot 10}\ \frac{m}{s^2} \approx 10^6\ \frac{m}{s^2} = \underline{10^5 g}$$

$\rightarrow s = ut + \frac{a}{2}t^2 \qquad v = u + at = 0 \rightarrow t$

$\qquad\qquad\qquad\qquad u = \sqrt{\frac{k_B T}{m}}$

08/05/23

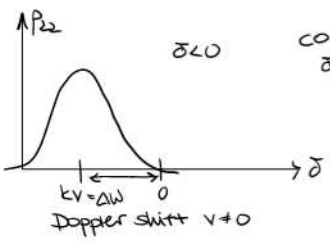
@ cooling



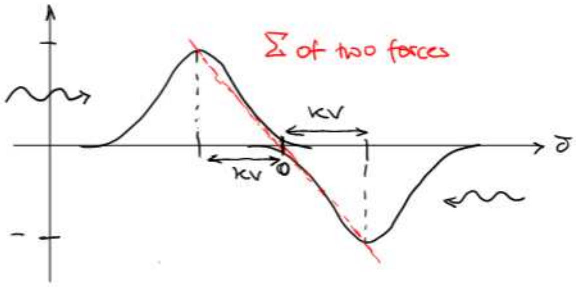
steady state
50% population max [stimulated emission vs. absorption]

force $\hat{=} \frac{\Delta p}{\Delta t} \approx \frac{\hbar k\, \Gamma_{SE}}{2}$ → $10^5 g$

→ 1000 m/s decelerated within 1m



cooling
$\delta = 0$



Why not T = 0 ?
$\langle p \rangle = 0$: emission in random direction

! $\langle p^2 \rangle \neq 0$
due to random kicks
≙ random walk in phase space

5.5 Temperature limit for T_D

$$\boxed{k_B T_{Doppler} = \frac{\hbar\Gamma}{2}}$$

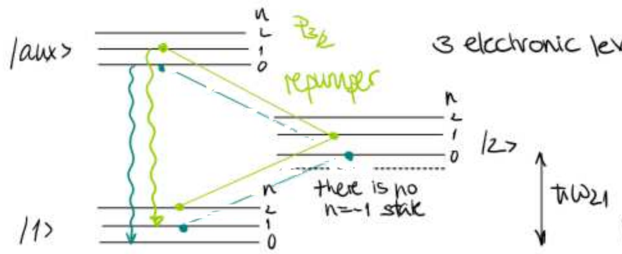
→ $\Gamma \approx 2\pi \cdot 10\,MHz$
→ $T_D \approx 0{,}1 - 1\,mK$

$10^{8} - 10^{7}$

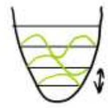
$\boxed{\text{collect: } 12000K \mathrel{\hat=} 1eV \mathrel{\hat=} 2{,}5 \cdot 10^{14}\,Hz \text{ (IR)}}$
↳ $\omega_{21}/2\pi$

5.6 Beyond the "ultimate" limit T_D

EIT or sideboard cooling: closely related to circuit QED
picture three electronic levels



3 electronic levels ⊕ motional levels in the trap



↳ hyperfine qubit $\mathrel{\hat=} 1\,GHz$ ($S_{1/2}$)
optical qubit $\mathrel{\hat=} 10^{14}\,Hz$ ($S_{1/2} - D_{5/2}$)

red $\Leftrightarrow$ $\delta < 0$ ($\delta = \omega_{exc}$)

1) RSB: $|1\rangle|n=2\rangle \rightarrow |2\rangle|n=1\rangle$
2) repumper: → SE $|2\rangle|n=1\rangle \rightarrow |aux\rangle|n=1\rangle \xrightarrow{SE} |1\rangle|n=1\rangle$
3) $|1\rangle|n=1\rangle \xrightarrow{RSB} |2\rangle|n=0\rangle$
4) repumper: $|2\rangle|n=0\rangle \rightarrow |aux\rangle|n=0\rangle \xrightarrow{SE} \underline{|1\rangle|n=0\rangle}$
motional ground state

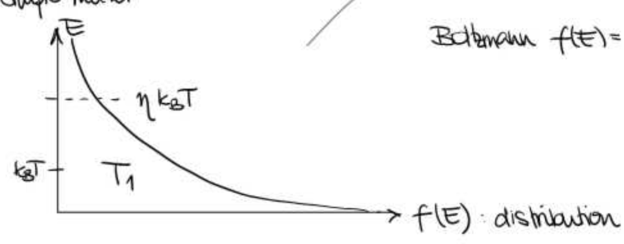
Important summary:

→ you need spontaneous emission (SE) for cooling! If not: coherent oscillations → no dump of entropy
→ cycle stops with last recoil into $|1\rangle|n=0\rangle$ [dark]
⇒ $T_{recoil} \leftrightarrow \Delta p = \hbar k$ ⇒ $T_{recoil} \approx 1\,\mu K$
→ RSB $|1\rangle|n=1\rangle \rightarrow |2\rangle|n=0\rangle$
$|1\rangle|n=0\rangle \not\rightarrow$
} conditional operation for two qubit gates

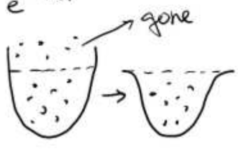
5.7 Evaporative cooling

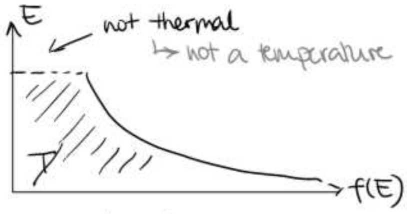
→ select fast ones ⇒ remove them
+ thermalisation
simple model

lower potential that fast atoms can escape

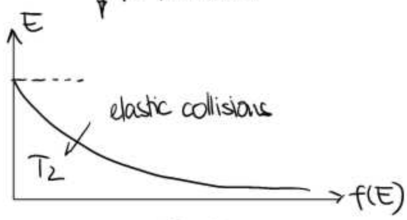


Boltzmann $f(E) = e^{-E/k_BT}$





↓ re-thermalize



$T_2 < T_1$

→ next out: $E_{out\,\#2} = -\eta k_B T_2$ (equilibrium)

efficiency? → ramping too fast: no interaction → no re-thermalisation
→ ramping too slow: other losses (e.g. inelastic "collisions"/effect)
↳ independent on amount of collisions

figure of merit: phase space density

$$\boxed{\rho := n\,\lambda_{dB}^3} \quad [1/\mathrm{cm}^3]\ [\mathrm{cm}^3]$$

$$\lambda_{dB} = \frac{h}{p} = \frac{h}{m\bar{v}} = \frac{\hbar}{\sqrt{2\pi m k_B T}} \qquad \tfrac{1}{2}m v^2 \sim k_B T$$

(dB: de Broglie)

$$\frac{\Gamma_{elastic}}{\Gamma_{inelastic}} \approx 1000 \rightarrow 10^{-11}\,\mathrm{K}$$

for $\rho \approx 1 \Rightarrow$ degenerate gases (e.g. Bose-Einstein condensate)

5.8 Dilution fridges

(1960 ~ 200mK, today ~ 10mK)

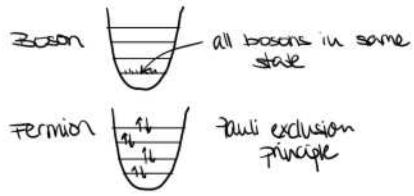
step by step

5.8.1 $^{3/4}$He

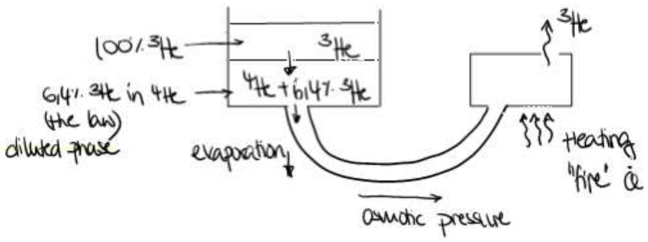
superfluid @ 2,17 K

1) evaporative cooling ^{4}He (Boson) → boils at 4,2K → lower pressure $p_c \approx 0{,}1$mbar → $T_{base} \approx 1$K
2) evaporative cooling ^{3}He (Fermion) → ($m < m_{^4He}$ → $P_{vapor} \gg$) → $T \approx 0{,}25$K
   less binding energy

(two ^{3}He fermions → one boson ⇒ superfluid)



5.8.2 ^{3}He - ^{4}He -mixtures

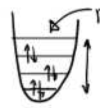


Why 6,4%?

^{3}He closer to ^{4}He (than other ^{3}He) — fermions repell each other

→ van der Waals forces $\sim \frac{1}{r^6}$ → better boundary of ^{3}He in ^{4}He

→ limit ^{3}He fermions



→ equally costly to stick with ^{3}He

≙ equal chemical potential for ^{3}He in ^{3}He vs ^{3}He in ^{4}He

5.8.3 Removal of ^{3}He

heating (see PDF) ⇒ heat ^{3}He/^{4}He upstream
  → you evaporate ^{3}He
  → refill ^{3}He out of ^{3}He rock bath (100% pure phase) → this gets cooled
⇒ more heat $\dot{Q}$ → T<? limit $\dot{Q}$: evaporation of ^{4}He
  ↳ no osmotic pressure anymore

additional challenges:
  → heat exchange < for T< $\left(\frac{1}{T^2}\right)$
  → viscosity of ^{3}He < for T< → heating by friction

$T_{\text{dilution fridge}} \approx 10$mK
  ↳ 2mK → 10^4 more ^{3}He needed ↯

see also magnetic cooling ↔ (entropy cooling) $\quad dS = \frac{\partial Q}{T}$

use dof electrons + photons/phonons } couple → complete gate set 08/05/23

qubit → cooling $\hbar\omega_{21} \gg k_B T$ $\xrightarrow{\text{atoms}}$ 1mK Doppler → 1μK (sideband cooling)
artificial atoms → 1(0)mK (solid state)

$2{,}5 \cdot 10^{14}$ Hz $\hat{=}$ 1eV $\hat{=}$ 12 000K

→ 1mK → $2{,}5 \cdot 10^{7}$ Hz $\hat{=}$ 25 MHz
(10^{-7})

compare to $\omega_{osc}/2\pi \approx$ 2MHz in traps

→ 1μK → $2{,}5 \cdot 10^{4}$ Hz $\hat{=}$ 25 kHz
(10^{-10})

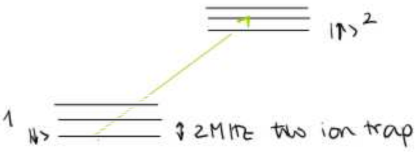
6 Coherent qubit control

↳ gates

Intro $|1\rangle|n=0\rangle \xrightarrow[\pi]{BSB} |2\rangle|n=1\rangle$

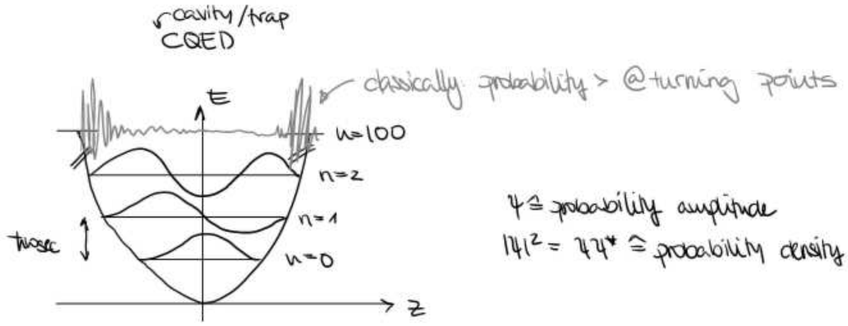
$|1\rangle|n=0\rangle \xrightarrow[\pi/2]{BSB} |1\rangle|n=0\rangle + |2\rangle|n=1\rangle$

! entangled state (everything within the quantum correlation)



6.1. Phonon (motional) DOF (Phonon in cavity)

→ no micro motion
→ no anharmonicities } harmonic trap

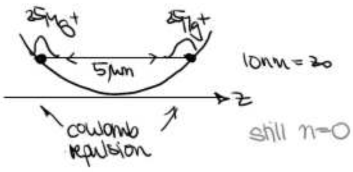


$\psi \hat{=}$ probability amplitude
$|\psi|^2 = \psi\psi^* \hat{=}$ probability density

summary
1) Eigenstates $E_n = \hbar\omega(n+\frac{1}{2})$
2) Eigenfunctions $\psi_n = \underbrace{\left(\frac{m\omega}{\pi\hbar 2^{2n}(n!)^2}\right)^{1/4}}_{\text{normalization}} \cdot \underbrace{\exp\left\{-\frac{m\omega}{\hbar}\frac{z^2}{2}\right\}}_{\text{gauss}} \underbrace{H_n\left(\frac{m\omega}{\hbar} z\right)}_{\text{hermite polynomials}}$
3) $\hat{z} = z_0(\hat{a} + \hat{a}^+)$ ($\hat{a}$: annihilation, $\hat{a}^+$: creation)
4) $\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle$
$\hat{a}^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$
5) $z_0 \hat{=}$ width of groundstate ψ_0 $\quad z_0 = \sqrt{\langle\psi_0|\hat{z}^2|\psi_0\rangle} = \sqrt{\frac{\hbar}{2m\omega}}$
groundstate n=0 → vacuum fluctuations

e.g. $\omega_{osc}/2\pi$ = 2MHz → $z_0 = \sqrt{\frac{\hbar c\, c}{2mc^2\omega}} = \sqrt{\frac{200 \cdot 10^6 eV \cdot 10^{-15} m \cdot 3 \cdot 10^8 m/s}{2 \cdot 10^9 eV \cdot 25 \cdot 2\pi \cdot 2 \cdot 10^6\, 1/s}} = \sqrt{\frac{10^{-7}}{10^9}}$ m
$m_{^{25}Mg^+}$
$= \sqrt{10^{-16}}$ m $= 10^{-8}$ m = 10nm

compare to distance between ions:



10nm = z_0

$U = \frac{1}{2}m\omega^2 z^2$

→ $\frac{d}{z_0} = \frac{5000}{10} = 500$

→ no overlap of ψ

→ phonons needed for "data exchange"/ communication

Schrödingers picture $c(t)$: pure state of motion $\psi_{motion} = \sum_{n=0}^{\infty} c_n(t)|n\rangle = \sum_{n=0}^{\infty} c_n e^{-in\omega t}|n\rangle$

- → $n <$: $P>$ for $z=0$ (classically a turning point)
- → equidistant levels: $\Delta E = \hbar\omega$
- → Heisenberg $\Delta x \Delta p \geq \hbar/2$
- → $n \gg$ (classical)

6.2 Electronic DOF (internal)

↳ qubit

Quantization field (B_z) → $|1\rangle \wedge |2\rangle$ related to σ_z, S_z

$\hat{H}_e = -\hat{\vec{\mu}}\vec{B}_z = \frac{\mu_m B \hbar}{\hbar}\hat{S}_z$

(we still have $\hat{S}_x, \hat{S}_y$) ~ we could work with $\vec{E}$-field, we chose $\vec{B}$

6.3 Interaction Hamiltonian

$\hat{H}_{interaction} = -\vec{\mu}\vec{B}_x(x,t)$ $\quad B_x \ll B_z$

↳ $B_0\cos(\omega t + \phi)$

in co-rotating frame → no B_z visible, but $\frac{1}{2}B_x$ → flipping spin if on resonance

$\hbar\Omega_{Rabi} = -\frac{\mu B_0}{2}$

linear polarized B_x → B_{left}, B_{right} $\quad$ $1/2$ of the amplitude

$$\hat{H}_{inter} = -\frac{\mu B_0}{\hbar}\hbar\hat{S}_x\cos(kz - \omega_L t + \phi) = \underbrace{-\mu B_0 \hat{S}_x \tfrac{1}{2}\left(e^{i(kz-\omega_L t+\phi)} + e^{-i(kz-\omega_L t+\phi)}\right)}_{5\ \text{steps}}$$

1) Raising & Lowering spin

$$\left.\begin{aligned}\hat{S}_+ &= \hat{S}_x + i\hat{S}_y \\ \hat{S}_- &= \hat{S}_x - i\hat{S}_y\end{aligned}\right\} \quad \hat{S}_+ + \hat{S}_- = 2\hat{S}_x \rightarrow \boxed{\hat{S}_x = \tfrac{1}{2}(\hat{S}_+ + \hat{S}_-)}$$

$$\boxed{\begin{aligned}\hat{S}_x &= \tfrac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix} \\ \hat{S}_y &= \tfrac{1}{2}\begin{pmatrix}0&-i\\i&0\end{pmatrix} \\ \rightarrow \hat{S}_+ &= \begin{pmatrix}0&0\\1&0\end{pmatrix}, \quad \underbrace{\hat{S}_+\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}}_{\text{flipping}}\end{aligned}}$$

$$\rightarrow \hat{H}_{int} = \hbar\Omega_{Rabi}\,\tfrac{1}{2}(\hat{S}_+ + \hat{S}_-)\left(e^{i(kz-\omega_L t+\phi)} + e^{-i(kz-\omega_L t+\phi)}\right)$$

2) Heisenberg picture

$\hat{H}'_{int} = U_0^{\dagger}(t)\hat{H}_{int}U_0(t)$ where $U_0(t) = e^{-i\hat{H}_0/\hbar\, t}$ with $\hat{H}_0 = \hat{H}_{motion} + \hat{H}_{electronic} = \hbar\omega_z(a^{\dagger}a + \tfrac{1}{2}) + \hbar\underset{\omega_{21}}{\omega_0} S_z$

SR $|\psi(t)\rangle = U(t)|\psi(0)\rangle$ → $\langle\psi(t)|H|\psi(t)\rangle = \langle U\psi|H|U\psi\rangle = \langle\psi|\underbrace{U^{\dagger}HU}_{\text{interaction picture}}|\psi\rangle$

→ transformation

$$\hat{a}^{\dagger} \rightarrow \hat{a}e^{-i\omega_z t} \qquad \hat{S}_+ \rightarrow \hat{S}_+ e^{i\omega_0 t}$$
$$\hat{a} \rightarrow \hat{a}^{\dagger}e^{i\omega_z t} \qquad \hat{S}_- \rightarrow \hat{S}_- e^{-i\omega_0 t}$$

$$\rightarrow H_{int} = \frac{\hbar\Omega}{2}\left(\hat{S}_+e^{i\omega_0 t} + \hat{S}_-e^{-i\omega_0 t}\right)\left(e^{i(kz-\omega_L t+\phi)} + e^{-i(kz-\omega_L t+\phi)}\right)$$

$k = 2\pi/\lambda$, $z = z_0(a + a^{\dagger})$

3) Quantization of harmonic oscillator

$$H_{int} = \frac{\hbar\Omega}{2}\left(\hat{S}_+e^{i\omega_0 t} + \hat{S}_-e^{-i\omega_0 t}\right)\left[e^{ikz_0(a^{\dagger}e^{i\omega_z t} + ae^{-i\omega_z t})} + e^{-ikz_0(a^{\dagger}e^{i\omega_z t} + ae^{-i\omega_z t})}\right]e^{-i(\omega_L t+\phi)}$$

ω_0
ω_L: Laser/magnetic field frequency
$\omega_0 \approx \omega_L$ → we want

4) rotating wave approximation

$\omega_0+\omega_L \approx 2\omega_0 \rightarrow$ oscillation fast, averages out

$\omega_L-\omega_0=\delta$ detuning small

hermitian conjugate

$$\hat{H}'_{\substack{int\\RWA}} = \frac{\hbar\Omega}{2}\hat{S}_+\left[e^{ikz_0(ae^{-i\omega_0 t}+a^+e^{i\omega_0 t})}\right]e^{-i\overbrace{(\omega_L-\omega_0)}^{\delta}t+i\phi}+hc$$

with $kz_0=\eta\ll 1$

Lamb Dicke regime

5) Lamb Dicke

$e^{\eta} \xrightarrow{\eta\ll1} 1+\eta+\cancel{\frac{1}{2}\eta^2}+\ldots$

What does $\eta=kz_0\ll1$ mean?

a) $k=\frac{2\pi}{\lambda}$, $z_0=\sqrt{\frac{\hbar}{2m\omega}}$ $\Bigg\}$ $kz_0=k\sqrt{\frac{\hbar}{2m\omega_z}}=\sqrt{\frac{\hbar k^2}{2m\omega_z}}=\sqrt{\frac{\hbar^2k^2}{2m\hbar\omega_z}}=\sqrt{\frac{E_{kin}}{E_{vibration}}}\ll1$

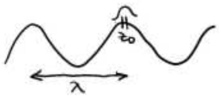
trapping frequency

λ: wavelength of Bx-field

$\hat{=}$ photon recoil energy $E_{recoil}=\frac{\hbar^2k^2}{2m}\ll$ phonon energy

$\Rightarrow$ no vibrational excitation by scattering (see sideband cooling)

b) $\eta\ll1 \Rightarrow z_0\ll\lambda$



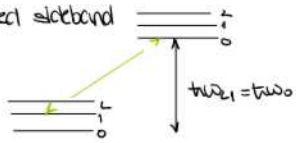
$$\Rightarrow \hat{H}'_{\substack{int\\RWA}} \approx \frac{\hbar\Omega}{2}\hat{S}_+\left[1+i\eta(ae^{-i\omega_0 t}+a^+e^{i\omega_0 t})\right]e^{-i\delta t}e^{i\phi}+hc$$

6.4 Operations (carrier & sidebands)

1) carrier: $\delta=0 \rightarrow \omega_L=\omega_0$

$$\boxed{\hat{H}'_{\substack{int\\RWA\\carrier}}=\frac{\hbar}{2}\Omega e^{i\phi}\hat{S}_+ + hc}$$

2) red sideband



$\omega_L=\omega_0-\omega_z \rightarrow \delta=-\omega_z$

$$\boxed{\hat{H}'_{\substack{int\\RWA\\RSB}}=\frac{\hbar}{2}\Omega e^{i\phi}i\eta\underbrace{\hat{S}_+\hat{a}}_{RSB}+hc}$$

coupling of qubit and motion

3) Blue sideband $\delta=\omega_z$

$$\boxed{\hat{H}'_{\substack{int\\RWA\\BSB}}=\frac{\hbar}{2}\Omega e^{i\phi}i\eta\hat{S}_+\hat{a}^+ + hc}$$

! sidebands do not flip at $\Omega_{\substack{Rabi\\carrier}}=\Omega$ but $\Omega_{sideband}=\underset{\ll1}{\eta}\Omega_{carrier}$

$\rightarrow$ η too small $\rightarrow$ T operation $\gg$
η too large $\rightarrow$ no seperated operations $\Big\}\rightarrow \eta\approx0.1$

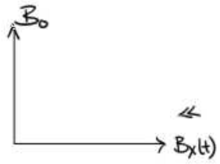
typical for ions: $\Omega_{Rabi}/2\pi \approx 1\,MHz$
$\Omega_{SB}/2\pi \approx 0.1\,MHz \rightarrow$ eg $10\mu s$ for π-pulse

10/05/23

coupling spin and motion (phonons/photons)

$|\uparrow\rangle_z, |\downarrow\rangle_z$ $\quad |n=\#\rangle$

steps

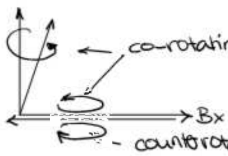


#1 $\hat{S}_x \rightarrow \frac{1}{2}(\hat{S}_++\hat{S}_-)$

#2 Schrödinger $\rightarrow$ Heisenberg
time dep. in state $\Psi(t)$ — time dep. in H $H(t)$ — Interaction picture $\hat{H}_0 \rightarrow \hat{H}_{int}$ $\llcorner B_x(t)$

#3 $\hat{S}_z$ $\llcorner \omega_0,\omega_{21}$ — $a^+, \hat{a}$ $\llcorner \omega_z$ $\rightsquigarrow \hat{H}_0$

#4 $\hat{H}_{int}$



co-rotating $\omega_0-\omega_L=\delta$

linear polarized

counterrotating $\omega_0+\omega_L\approx2\omega_0$

$\eta=\sqrt{\frac{\frac{\hbar^2k^2}{2m}}{\hbar\omega}}$ $\ll1$ $\rightarrow$ no change of $|n\rangle$ by spontaneous emission

Absorption photon: $p_x=\hbar k_x$
$E_{recoil}=\frac{p_x^2}{2m}=\frac{\hbar^2k^2}{2m}$

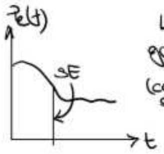
$\rightarrow$ Taylor the e^{η}

#5 Lamb-Dicke-approximation $e^{i\underset{\eta}{kz_0}(a^+e^{i\omega_z t}+ae^{-i\omega_z t})}$

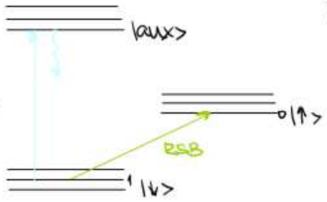
$$\rightarrow \hat{H}_{int}=\hat{S}_+\Big[\underset{carrier}{1}+ikz_0(\underset{BSB}{\hat{a}^+}e^{i\omega_z t}+\underset{RSB}{\hat{a}}e^{-i\omega_z t})\Big]e^{i\delta t}\underset{today}{e^{i\phi}}+hc$$

6.4. experimental implementation

Implementation

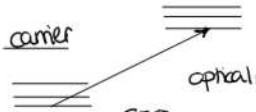


$\Gamma_{dipole} \sim \frac{1}{\tau} \gg$
↳ $\tau = ns = 10^{-9}s$
good for incoherent control (cooling, detection, state preparation)



$\Gamma \sim \frac{1}{\tau_a} \ll \infty$
narrow linewidth (~forbidden quadrupol transition)
good for coherent operations → gates

6.4.1 optical qubit



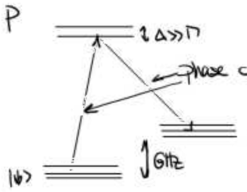
$\tau = 1s \rightarrow \Gamma = 1Hz$
D=2
optical, VIS range

Ca^+
Ba^+

$\rightarrow \langle S | \hat{r}^2 | D \rangle$

! consider the linewidth of transition

6.4.2 hyperfine qubits



phase coherent lasers → two photon stimulated Raman transition

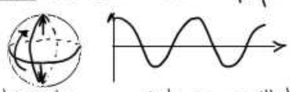
S

seperated by VIS (see above)
just hyperfine → GHz

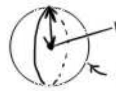
THINK AGAIN ABOUT THIS - LOOK AT IT

Sidemark

carrier on Bloch vector: flips spins



sideboard: state is oscillating through the center

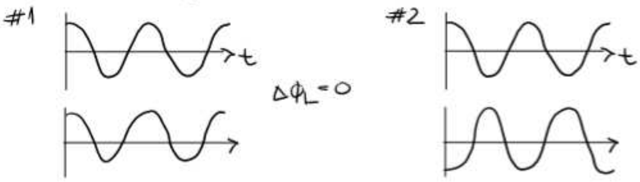


Blochsphere-picture: π_{pure} $|\downarrow\rangle|0\rangle \rightarrow |\uparrow\rangle|1\rangle$
$\pi/2$ $|\downarrow\rangle|0\rangle \rightarrow |\downarrow\rangle|0\rangle + |\uparrow\rangle|1\rangle$

7. Full single qubit

from Bloch circle to Bloch sphere

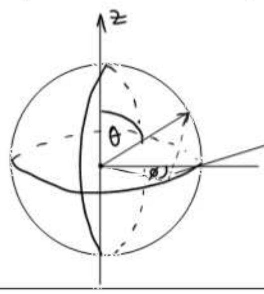
Phase of the driving source



Rotations don't commute
↳ order plays important role

7.1 single qubit on the Bloch sphere

! two-dimensional complex vector space
↳ complex numbers relate to phase differences ($e^{i\pi/2} = i$)



$|\psi\rangle = \cos(\theta/2)\,|\downarrow\rangle + e^{i\phi}\sin(\theta/2)\,|\uparrow\rangle$

phase

|↑> & |↓> orthogonal, but on z-axis in Bloch sphere
+z / -z orthonormal here
↕ both

$|\psi\rangle = \alpha|\downarrow\rangle + \beta|\uparrow\rangle$ $\quad \alpha, \beta \in \mathbb{C} \rightarrow$ 4 numbers
$|\alpha|^2 + |\beta|^2 = 1 \rightarrow$ 3 numbers left

$\alpha = r_\downarrow e^{i\phi_\downarrow}$
$\beta = r_\uparrow e^{i\phi_\uparrow}$

$\rightarrow |\psi\rangle = r_\downarrow e^{i\phi_\downarrow}|\downarrow\rangle + r_\uparrow e^{i\phi_\uparrow}|\uparrow\rangle = \underbrace{e^{i\phi_\downarrow}}_{\text{global phase}}\left(r_\downarrow|\downarrow\rangle + r_\uparrow e^{i(\phi_\uparrow - \phi_\downarrow)}|\uparrow\rangle\right)$

global phase → not measurable / not physical
phase difference

Exercise: |↑> → |↓> $\theta = 0, \theta = \pi$ / $\theta = \pi/2$: superposition state

Summary 1) $\theta(t) \hat{=}$ Rabi-rate $\Omega \cdot \Delta t$ pulse duration

see also $\boxed{c_1(t) = \sin(\frac{\Omega}{2} t)}$

2) $\phi(t) \hat{=}$ relative phase of a first pulse with respect to a second pulse
↳ defines axes of rotation

7.2 Example of Quantum Information Processing (QIP) - single qubit (theory)

$|\downarrow\rangle$ $|\uparrow\rangle$

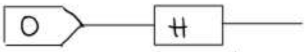
$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

7.2.1 Not-gate

qubit flip: $\alpha|0\rangle + \beta|1\rangle \rightarrow \beta|0\rangle + \alpha|1\rangle$

$\begin{pmatrix}\alpha\\\beta\end{pmatrix}$ $\quad \underbrace{\begin{pmatrix}0&1\\1&0\end{pmatrix}}_{\sigma_x}\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \begin{pmatrix}\beta\\\alpha\end{pmatrix}$ ✓

7.2.2 Create superposition (Hadamard gate)



$|0\rangle$ $\quad \frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$ $\quad \frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ ✓

$$H \cdot H \cdot |0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}2\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

↯ H·H reproduces itself
→ we need rotations (θ, ϕ)

7.2.3 Rotation (x,y,z)

$$R_z = \begin{pmatrix} e^{-i\gamma_z/2} & 0 \\ 0 & e^{i\gamma_z/2}\end{pmatrix}$$

$$R_y = \begin{pmatrix} \cos(\gamma_y/2) & -\sin(\gamma_y/2) \\ \sin(\gamma_y/2) & \cos(\gamma_y/2)\end{pmatrix}$$

$$R_x = \begin{pmatrix} \cos(\gamma_x/2) & -i\sin(\gamma_x/2) \\ -i\sin(\gamma_x/2) & \cos(\gamma_x/2)\end{pmatrix}$$

! R_z rotates states within x/y plane
$R_{x,y} \hat{=}$ Rabi rotations around x and y axis

Example

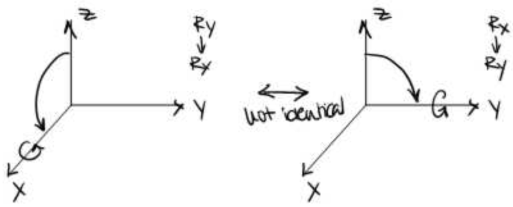
$$R_y\left(\frac{\pi/2}{2}\right)\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1&-1\\1&1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$R_y\left(\frac{\pi/2}{2}\right)R_y\left(\frac{\pi/2}{2}\right)\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1&-1\\1&1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0\\2\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$$

✓ flipped spin after $\pi/2 \cdot \pi/2$

Notes

1) matrix of any rotation can be written as product of 2x2 matrices
2) rotations do not commute



3) Pauli matrices are a complete basis $(\mathbb{1}, \sigma_1, \sigma_2, \sigma_3)$
4) Rabi flopping → $\theta(t)$
relative phase (of two pulses) → axis of rotation

8. Two qubit gate (Cirac & Zoller 1995)

15/05/23

Universal set of gate: → single qubit operation (Hadamard $\frac{\pi}{2}$)

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \rightarrow R_x(\pi)R_y(\pi/2) = iH$$

→ two qubit gate

Intro: classically (bits 0 OR 1)

CNOT ≙ conditional flip: flip the target bit, if the control bit is <u>one</u>

c t

$|00\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$

$|01\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$

$|10\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$

$|11\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$

truth table

c t	c t
0 0	0 0
0 1	0 1
1 0	1 1
1 1	1 0

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

e.g. CNOT

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Quantum case:

$|c\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|c\rangle|t\rangle \rightarrow$

$\frac{1}{\sqrt{2}}(|0\rangle_c + |1\rangle_c)|0\rangle_t \rightarrow \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

100% correlation   ! Einstein et al / Schrödinger cat / spooky interaction of a distance

best precursor

CNOT

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \leftrightarrow$$

spin flip

phase gate

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

phase flip

8.1 Cirac & Zoller gate

protocol

c   t   5µm   rf trap

... o o o o o o o o o ...   + COM centre of mass mode

#2   #1   10nm

phonons as data bus   ↳ not overlapping

$\underset{\rightarrow}{o}\ \underset{\rightarrow}{o}$ 2 MHz in phase oscillation

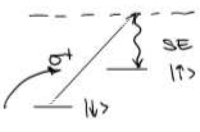
steps   qubit ≙ e-DOF (within every ion)

nochmal angucken

0) initialize qubit: motional DOF → cooling side band $T \approx 1\mu K$

$(\Delta t_{SB} \approx 1\mu s) \rightarrow \Delta t_{SB} > \frac{1}{\omega_{sec}}$

internal electronic DOF → optical pumping



1) swap e-DOF of control (qubit c #2) on the COM mode

phonon mode

2) C-Phase on ion #1 (qubit target) → conditional on COM mode

n = $|0\rangle$ / $|1\rangle$ COM

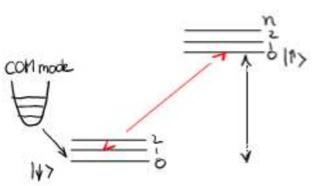
3) swap information of COM mode back on control qubit

8.2 Implementation

8.2.1 step #1:

product

$$(a|\downarrow\rangle_2 + b|\uparrow\rangle_2) \cdot \underset{n=}{|0\rangle_{COM}} = a|\downarrow 0\rangle + b|\uparrow 0\rangle$$



RSB: e.g. $|\downarrow\rangle|1\rangle \rightarrow |\uparrow\rangle|0\rangle$

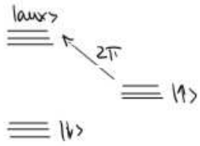
$|\uparrow\rangle|0\rangle \rightarrow |\downarrow\rangle|1\rangle$!

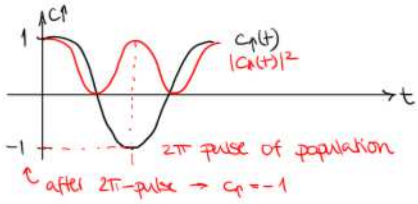
coherent operation
→ both pathways

now: RSB π-pulse on c-qubit

$$a|\downarrow_2 0\rangle + b|\uparrow_2 0\rangle \xrightarrow{\binom{RSB}{\pi}} a|\downarrow_2 0\rangle + b|\downarrow_2 1\rangle = \overset{\text{product state}}{|\downarrow_2\rangle}\underbrace{(a|0\rangle + b|1\rangle)}_{\text{superposition of c-qubit is now on the COM mode}}$$

8.2.2 operation #2





! amplitude evolved by π only
$e^{i\pi} = -1$

8.2.3 swap information back (COM → c-qubit)

→ $|0\rangle_{COM}$ → next operator

c-qubit should stay the same → as truth table shows
→ every algorithm can then be written

8.3 watch the gate

0) initialize

$$[a|\downarrow_1\rangle + b|\uparrow\rangle_1] \overset{\text{product}}{\cdot} [c|\downarrow_2\rangle + d|\uparrow\rangle_2] \overset{\text{product}}{\cdot} |n=0\rangle_{COM}$$

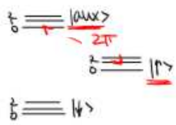
1) RSB π on ion #2 (control):

$$[ac|\downarrow_1\downarrow_2 0\rangle + ad|\downarrow_1\uparrow_2 0\rangle + bc|\uparrow_1\downarrow_2 0\rangle + bd|\uparrow_1\uparrow_2 0\rangle]$$

$$\rightarrow [ac|\downarrow_1\downarrow_2 0\rangle + ad|\downarrow_1\downarrow_2 1\rangle + bc|\uparrow_1\downarrow_2 0\rangle + bd|\uparrow_1\downarrow_2 1\rangle]$$

↓ RSB 2π on 1

2) RSB 2π on ion #1 (target):



$$[ac|\downarrow_1\downarrow_2 0\rangle + ad|\downarrow_1\downarrow_2 1\rangle + bc|\uparrow_1\downarrow_2 0\rangle - bd|\uparrow_1\downarrow_2 1\rangle]$$

3) RSB π on ion #2 (control)

$$[ac|\downarrow_1\downarrow_2 0\rangle + ad|\downarrow_1\uparrow_2 0\rangle + bc|\uparrow_1\downarrow_2 0\rangle - bd|\uparrow_1\uparrow_2 0\rangle]$$

no motion left behind

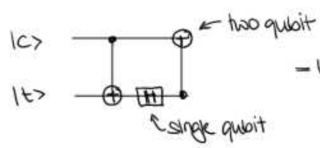
$$\hat{=} [ac|\downarrow_1\downarrow_2\rangle + ad|\downarrow_1\uparrow_2\rangle + bc|\uparrow_1\downarrow_2\rangle - bd|\uparrow_1\uparrow_2\rangle] \overset{\text{product}}{\cdot} \overset{n=0}{|0\rangle_{COM}}$$

eq. $\downarrow\downarrow \hat{=} \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$

$$\begin{pmatrix}ac\\ad\\bc\\bd\end{pmatrix} \xrightarrow{\text{gate}} \begin{pmatrix}ac\\ad\\bc\\-bd\end{pmatrix}$$

algorithm

$$\underset{\text{c-phase gate}}{\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}} \underset{\text{initial}}{\begin{pmatrix}ac\\ad\\bc\\bd\end{pmatrix}} = \underset{\text{final}}{\begin{pmatrix}ac\\ad\\bc\\-bd\end{pmatrix}}$$



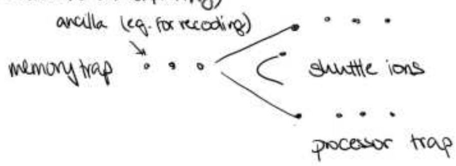
= (N-times) → arbitrary algorithm
↳ Shor's algorithm factorizing large #

8.4 Challenges

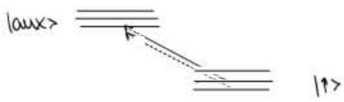
↯ 1: . . . . more ions → more modes (# 10^7)

$\updownarrow \Delta\omega$ resolution of modes $\Delta t_{gate} > \frac{1}{\Delta\omega}$

= (possible solution: multiplexing)



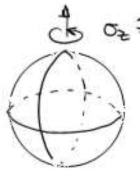
↯ 2: Cirac Zoller requires $T=0$
fidelity suffers for $T\neq 0$



~ (solved by different gate resolutions)
geometric phase gate for $T\neq 0$ (principle 100% possible)
current status: 99,94%
↳ NIST / Oxford

C-phase gate $\stackrel{?}{=}$ CNOT 17/05/23
[Cirac & Zoller]

picture #1
physics as argument



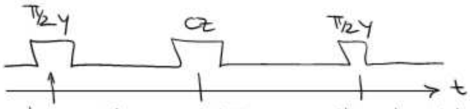
$\sigma_z \hat{=}$ rotation
phase rotation in ↑↓ basis (z)
$R_x, R_y \rightarrow$ flops on ↑↓
$\sigma_x\ \sigma_y$

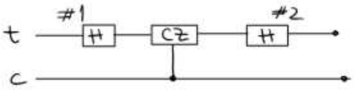
change basis
= quantization axis

$\pi/2$ - around y-axis → points along x-axis
↓
now $R_z(\sigma_z)$

$|+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$
$|-\rangle - \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$ flips between $|+\rangle, |-\rangle$

picture #2
algorithm part





H: Hadamard $\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$ target qubit (because control qubit does not change)

$$\underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}}_{\#2}\underbrace{\begin{pmatrix}1&0\\0&-1\end{pmatrix}}_{Cz}\underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}}_{\#1}\begin{pmatrix}\cancel{\alpha}\\\cancel{\beta}\\\gamma\\\delta\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}1&1\\-1&1\end{pmatrix}\begin{pmatrix}\cancel{\alpha}\\\cancel{\beta}\\\gamma\\\delta\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0&2\\2&0\end{pmatrix}\begin{pmatrix}\cancel{\alpha}\\\cancel{\beta}\\\gamma\\\delta\end{pmatrix} = \underbrace{\begin{pmatrix}0&1\\1&0\end{pmatrix}}_{CNOT}\begin{pmatrix}\cancel{\alpha}\\\cancel{\beta}\\\gamma\\\delta\end{pmatrix}$$

← time

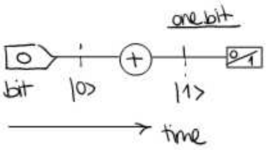
α, β: control
γ, δ: target

→ CZ: controlled phase gate ↔ CNOT

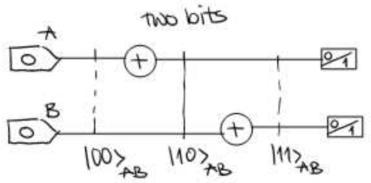
to treat control & target qubit →
$$\begin{array}{c} \text{control} \\ \begin{pmatrix}\boxed{\begin{matrix}1&0\\0&1\end{matrix}}&\begin{matrix}0&0\\0&0\end{matrix}\\\begin{matrix}0&0\\0&0\end{matrix}&H/Cz\end{pmatrix}\end{array}$$

9. Quantum Algorithms

9.1 Classical network



→ 100% $|1\rangle$



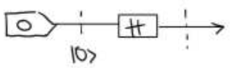
} 100% $|11\rangle$

↳ single & two bit operations (conditional) → any classical algorithm

9.2 Quantum network

9.2.1 single qubit (→ here: Hadamard)

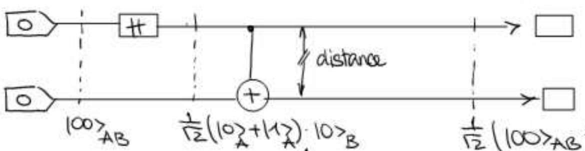
single qubit



$\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \hat{=}$ superposition state
↑
$|0\rangle$

9.2.2 two qubit (→ here CNOT)

classical measurement ?



? we get correlation $|00\rangle$ or $|11\rangle$

product state

no product state ≙ maximally entangled

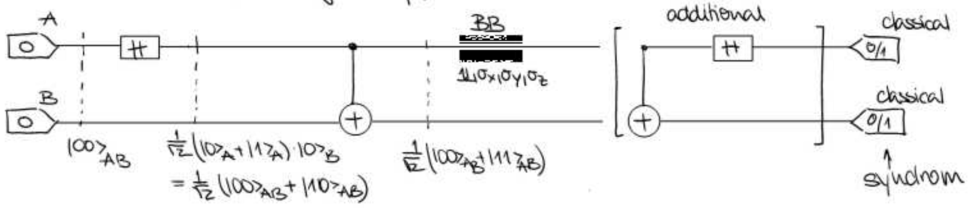
initialization & measurement only non coherent operations

9.3 Examples of algorithms

9.3.1 Black Box Problem (close to Deutsch-Jose-algorithm)

Black Box does one out of four operations ($\mathbb{1}, \sigma_x, \sigma_y, \sigma_z$) on qubit A
(you have a 2^{nd} qubit B, not in contact with the Black Box (BB))

? can we learn out of a single run, what the BB does?



<u>case #1</u> $[\mathbb{1}]$

$$\frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB}) - \overset{\mathbb{1}}{(BB)} - \boxed{CNOT} \rightarrow \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |10\rangle_{AB}) - \boxed{H} \rightarrow \overset{100\%}{\boxed{|0\rangle_A|0\rangle_B}} \quad \text{syndrom \#1}$$

computation:

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}2\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} \rightarrow \text{qubit A turns into } |0\rangle_A$$

<u>case #2</u> $[\sigma_z]_A$

product state
→ no entanglement

$$\frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB}) - \overset{\sigma_z}{(BB)} - \frac{1}{\sqrt{2}}(|00\rangle_{AB} - |11\rangle_{AB}) - \boxed{CNOT} - \frac{1}{\sqrt{2}}(|00\rangle_{AB} - |10\rangle_{AB}) = \frac{1}{\sqrt{2}}(|0\rangle_A - |1\rangle_A)\cdot|0\rangle_B - \boxed{H}$$

$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ syndrom #2

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0\\2\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix} = |1\rangle$$

$\rightarrow \boxed{|1\rangle_A|0\rangle_B}$ 100%.

<u>case #3</u> $[\sigma_x]_A$

$$\frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB}) - \overset{\sigma_x}{(BB)} - \overset{\text{flipping A}}{\frac{1}{\sqrt{2}}(|10\rangle_{AB} + |01\rangle_{AB})} - \boxed{CNOT} - \frac{1}{\sqrt{2}}(|11\rangle_{AB} + |01\rangle_{AB}) = \frac{1}{\sqrt{2}}(|1\rangle_A + |0\rangle_A)|1\rangle_B - \boxed{H}$$

$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ syndrom #3

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}2\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} = |0\rangle$$

$\rightarrow \boxed{|0\rangle_A|1\rangle_B}$ 100%.

<u>case #4</u> $[\sigma_y]_A$

$\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \rightarrow \boxed{i\,|1\rangle_A|1\rangle_B}$ syndrom #4 100%.

<u>Summary</u> 4 different syndroms for 4 different operations of the BB

$$\left.\begin{array}{l}|00\rangle \rightarrow \mathbb{1}\\ |10\rangle \rightarrow \sigma_z\\ |01\rangle \rightarrow \sigma_x\\ (i)|11\rangle \rightarrow \sigma_y\end{array}\right\} \text{ by single run you know what BB does}$$

! BB acts on qubit A only?
↯ not true: entangled state
→ operation acts on both qubits
$2^2 = 4$

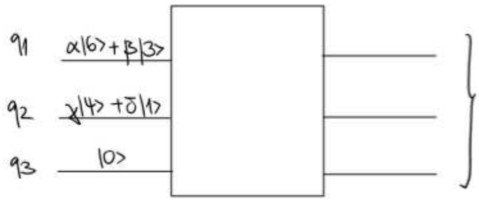
→ both qubits run through the BB!
! single shot experiment → no statistics needed (BB can do different operation next time)

9.3.2 Quantum speed up

#1 Factorization → Shor's algorithm

in a nutshell

Quantum adder

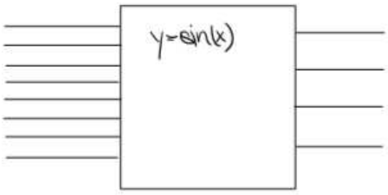


output

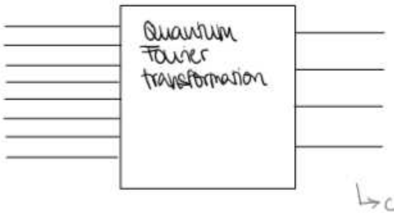
$\alpha\beta|639\rangle + \alpha\gamma|64\,10\rangle + \alpha\delta|617\rangle + \beta\gamma|347\rangle + \beta\delta|314\rangle + \gamma\delta|415\rangle$

↯ only one output

what does Shor?



superposition of all outcomes



frequency of sin(x)

→ single measurement
→ periodicity of the sin(x)

↳ code cracking

exponential speed up
for example on factorization

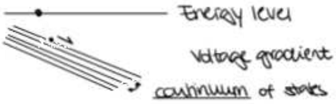
→ play with QC and learn
→ exponential Quantum simulation
→ quantum meterology (better clocks)
→ quantum sensory
↳ quantum cryptography

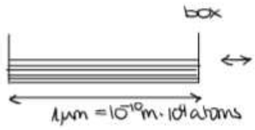
10. Solide state – semi conductors

22/05/23

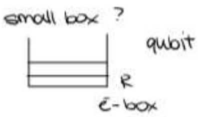
10.1 in a nutshell

⊖ electron → discrete charge
→ discrete energy levels





↔



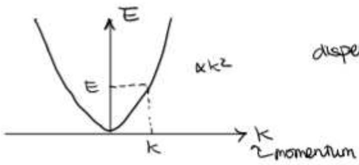
10.1.1 free electron 1D (10^3 → 3dim)

$V=0$ $\quad i\hbar \frac{\partial\psi}{\partial t} = \hat{H}\psi$

$\hat{=} \frac{\hbar^2 k^2}{2m} + \cancel{V(x)}^{\,0}$

$\hat{=} p^2/2m$

$\rightarrow \boxed{E_{kin} = \frac{\hbar^2 k^2}{2m}} \propto k^2$

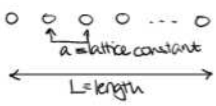


dispersion relation

10.1.2 Schrodinger (V=0)

↳ plane wave

$\psi = e^{-i(kx-\omega t)}$ in a lattice:



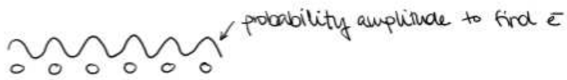
? if $\lambda_{\bar{e}}$ matches multiples of lattice constant ?

! each atom reflects tiny amount of ψ ($\sim 10^{-4}$) → 10^4 atoms → back reflection!

10.1.3 Back reflection of ψ

↳ plane waves

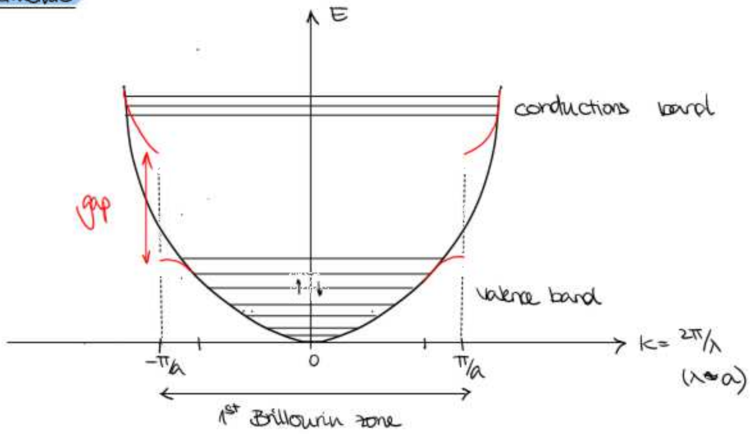
$\psi = e^{-i(kx-\omega t)} \pm e^{i(kx-\omega t)}$) $\propto \cos(kx$



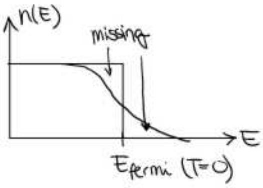
↳ $\cos^2(kx)$ $\hat{=}$ standing wave

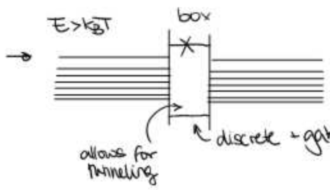
more accurate: Bloch equations ($V \neq 0$ but periodically modulated → Bragg reflexion)

10.1.4 band structure



discrete states in valence & conduction bands ($\lambda \gg a$)
+ Pauli principle → filling states up to Fermi level (for $T=0$, E_{max})
→ origin of distinction
- metal
- isolator — semiconductor



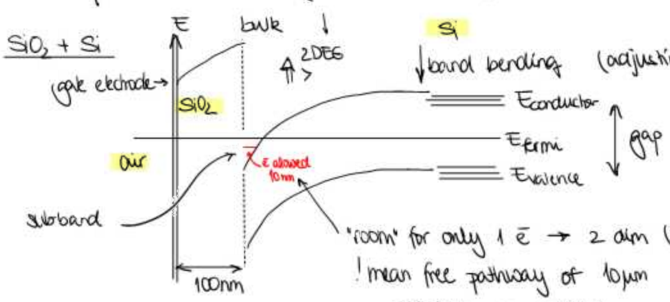


discrete - gates (voltage to electrodes) → allows to shift energy levels

10.2 2D Band structure (2DEG: 2 dimensional electron gas)

(C) MOS
↑ metal oxide semiconductor
complementary
↳ p.n donor
acceptor

1) $SiO_2 + Si$



(adjusting the gap at the interface)

$\rho \approx 10^{12}\ e/cm^2 \approx 10nm$ λ_{Fermi}

'room' for only 1 $\bar{e}$ → 2 dim (2DEG) electrons accumulate on surfaces
! mean free pathway of 10μm
→ lattices are matched
→ no dopands

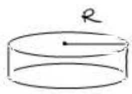
2) $\underbrace{AlGaAs}_{\text{insulator}} + \underbrace{GaAs}_{\text{semiconductor}}$ (↙ 30%)

10.2.1 e-traps

→ discrete # electrons (one)
→ discrete energy levels (subbands)
} artificial atoms

⊕ engineer (tune) it
⊖ tune ↯ (not identical)
⊖ dilution fridge ↯

parameters

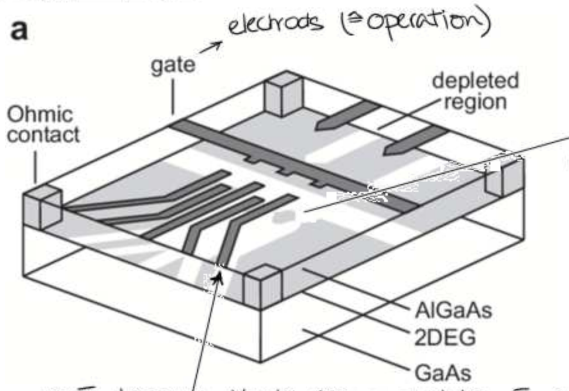


$C = 8\varepsilon\varepsilon_0 R$
$V(R) = \frac{Q}{4\pi\varepsilon_0 R}$ energy stored in capacitor

1) charging energy $E_C = \frac{e^2}{2C}$ eg. $R = 1nm$ → $E_C \approx 0{,}3eV$
typical $R = 100nm$ → $E_C \approx 0{,}003eV$ (30K)
($T \approx 290K = 25mK$)

2) level spacing ΔE $R = 1nm$ → $E_R = 10eV$
$R = 100nm$ → $E_R = 0{,}001eV$

10.2.2 Quantum transistor



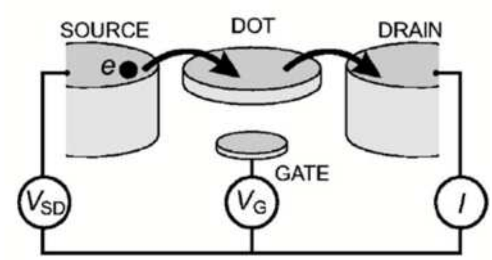
electrods (≙ operation)

quantum dot (single $\bar{e}$ trapped & isolated)

no $\bar{e}$ because $V_{gate} < 0$ → pushing $\bar{e}$ away

RMP + Physics today (→ ILIAS)

Transistor



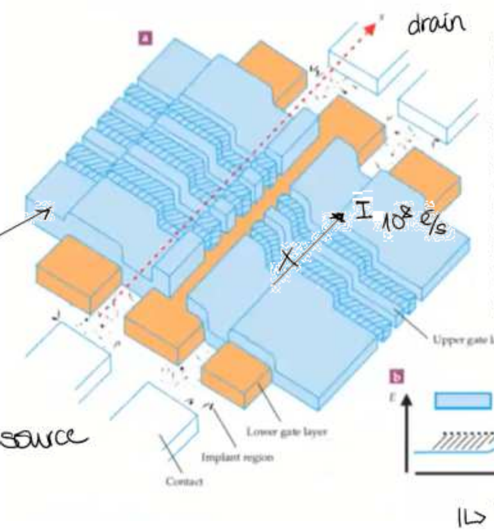


FIGURE 1. A SCHEMATIC OF TWO TUNNEL-COUPLED QUANTUM DOTS. (a) Two patterned metal layers separated by dielectrics (not shown) define the quantum dots. Voltages applied to the lower-layer gates (orange) delimit the channel, which runs parallel to the x-axis. Voltages applied to the upper-layer gates (blue) shape the potential landscape along the channel as shown in the panel b cross section. (b) An electron is confined in each of two local potential minima, the two quantum dots. Tunnel barriers separate the dots from each other and from electron reservoirs. The reservoirs, whose highest occupied electron level is at the Fermi energy E_F, are connected to contacts via implant regions.

I 10^8 $\bar{e}$/s

gate to control $\bar{e}$

$|L\rangle$ $|R\rangle$

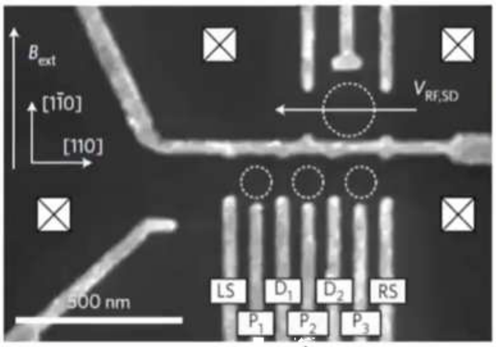
if only one "dot" is populated → qubit / quantum dot

× Quantum dot: measuring population either $|L\rangle$ or $|R\rangle$

→ 1% change of current $I \approx 1nA = 10^{-9} \cdot 10^{19} \cdot 10^{-2} = 10^{8}$ $\bar{e}$/s !

($1n = 10^{-9}$, 10^{19} atoms)

10.2.3 electronic control 1.0



→ assymetry of measurement dot

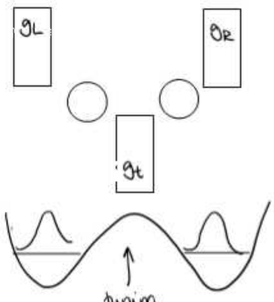
→ single site (dots) resolution

INTEL

$V_{P_{1,2,3}}$ to control $\bar{e}$-charge on dots

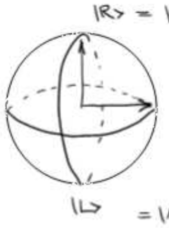
0 dimensional

10.2.4 charged qubit



tuning

tunneling barrier → controlling coupling of $|L\rangle$ and $|R\rangle$



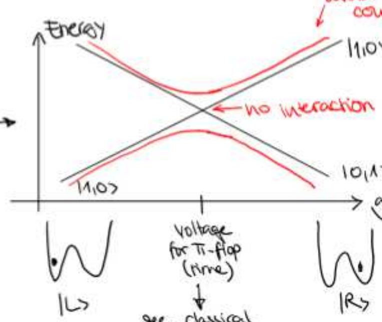
$|R\rangle = |0\ 1\rangle$

$|R\rangle + |L\rangle$

$|L\rangle = |1\ 0\rangle$

"level repulsion / avoided crossing"
coupled harmonic oscillator
→ perturbation theory

→



avoid crossing by coupling

↑ this allows for Rabi flopping

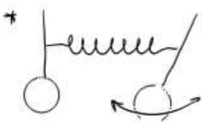
no interaction

voltage for π-flop (time)

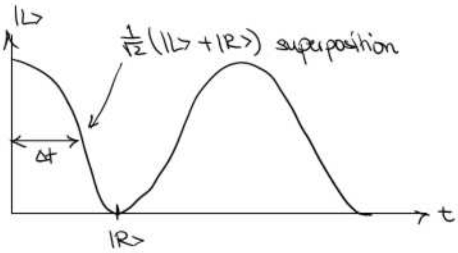
↓ see classical coupled pendula*

$|L\rangle, |R\rangle$ ground states

*



exchange of oscillation



duration ≙ π-time of gate pulse

Δt for π/2 pulse ≙ single qubit operation on the quantum dot charge qubit
on 2DEG 0-dim. quantum dot

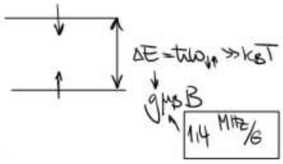
10.3. Spin (Q-Dot) qubit

24/05/23

Intro: Requirements by DeVincenzo
↳ see D.Loss PRA 1998

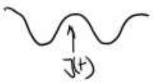
1) qubit ⇝ we had charge qubit: $|L\rangle + |R\rangle$ (two states) noise on charge
→ spin (more robust): $|\uparrow\rangle + |\downarrow\rangle$ charge noise shifts $\bar{e}$/spin, but doesn't flip it

2) initialization: T ≤: dilution fridge ≈ 10mK
B_0: (Zeeman splitting, spanning the qubit)
≈ $\boxed{1T = 10^4 G}$
$\Delta\omega \approx 1.4 \frac{MHz}{G} \cdot 10^4 G = 1.4 \cdot 10^4$ MHz ≈ limit for electronics
('limitation if λ ⇝ length of components → wire ⇔ antenna)



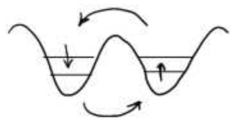
3) read out: ↯ spin has to be translated into charge → current ⊕
but during detection period ≙ charge qubit ↯

4) coherent control: single & two-qubit gate → single qubit ⇒ MW source → Rabi flopping of spin
→ two qubit gates: exchange interaction (see coupled pendula)
$\hat{H}_J = \sum J_{ij}(t)\,\sigma_i\sigma_j$



nutshell eg.

Pauli exclusion principle
⇒ no exchange

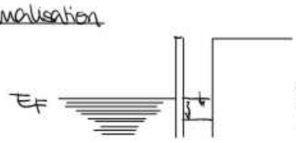


exchange interaction, controlled by J(t)

5) coherences $T_{coherence} \gg T_{gate}$
↯ GaAs → Si $\overset{①}{\to}$ ^{28}Si (pure)
↖ no nuclear spin
$\overset{②}{\to}$ ^{31}P
↖ $\bar{e}$-trap

10.3.1 Initialization

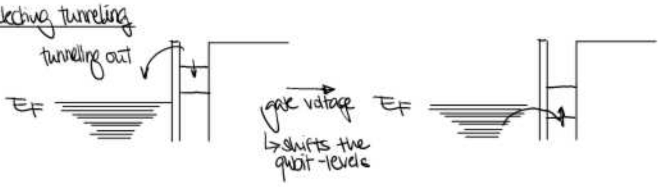
I Thermalisation



waiting for the decay via phonons in SC

T_1 should be >
⇒ long time for operation

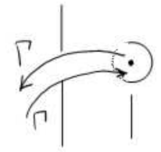
II spin-selecting tunneling



filling by tunneling (on short timescales)

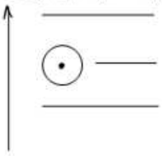
2 Möglichkeiten I langsam, II schneller
↳ „historisch"

10.3.2 Readout (charge)



dependent on $\bar{e}$ on DOT
→ change in curve

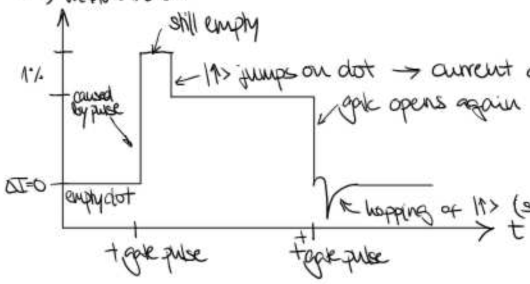
$I(t)$ drawn $\hat{=}$ 1nA $= 10^{10}\, \bar{e}/s$
$10^{9}\, \bar{e}/s$



→ $\Delta I \approx 1\% \cdot I$ → $10^{8}\, \bar{e}/s$

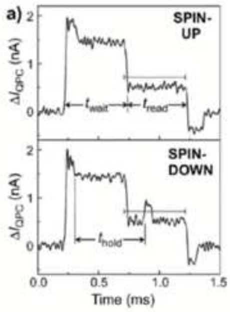
→ gate-pulse induced tunneling

$I(t)$ measured dot

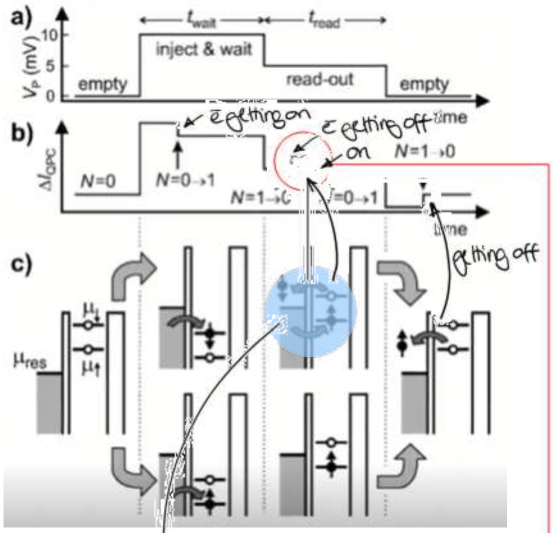


μ_{res}: chemical reservoir → think about as Fermi potential

only single electrons allowed due to Coulomb repulsion



a) gatespannung vs time
b) change in current vs time



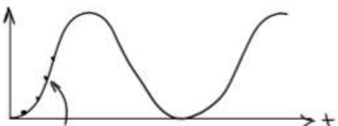
$\bar{e}$ hops of from |↓>,
followed by |↑> tunneling in
→ distinguish |↑> no hump
|↓> hump

additional peak: loosing spin |↓>
↳ |↑> spin can be populated

10.3.3 Single spin (qubit) control

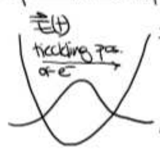
1) originally B_{AC} — $I(t)$
⊗ ∧ ⊙ ← time dependent (oscillating $B_x(t)$)
↳ co-rotating / counter rotating $B_{x/2}$
B_z quantization
→ corotating frame (processes with spin around B_z)
→ B_z vanishes for spin
⊕ B_x corotates (static for spin) → flip spin

$\hat{=}$ Rabi flopping



$B_x(t)$ stopp
→ |↑> + |↓> see ions

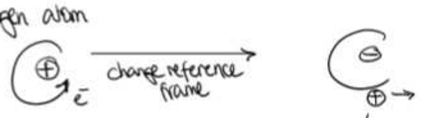
2) Spin-orbit coupling



$\bar{e}$ wavefunction

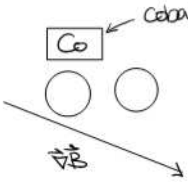
→ $B_{effective}(t)$ arises
↓
flips spin

hydrogen atom



proton $\hat{=}$ current → $\vec{L}\vec{S}$
→ B by proton
(angular momentum $\vec{L}$, $\vec{L}$ → B>)

3) (E)DSR: magnetic field gradient



$\vec{E}(t)$ on gate electron → $\vec{B}(t)$ (moving in quadrant)
↳ Rabi-flopping

Side remark

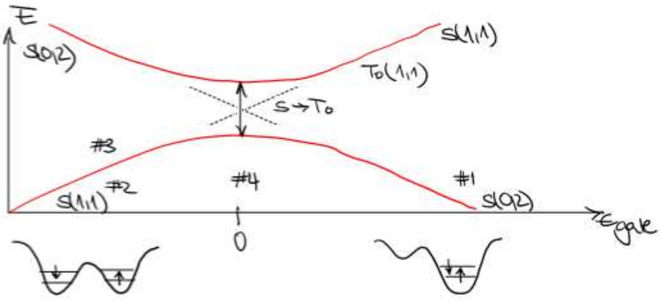
Holes might be advantageous to $\bar{e}$ (in the valence band) → stronger spin-orbit coupling
$\hat{=}$ missing $\bar{e}$ — p-orbital vs conductance band ($\bar{e}$-filled s-orbital)
→ advantageous for stronger spin-orbit coupling

10.3.4 Two-qubit gate

(! single qubit operation!)

$S(1,1) \leftrightarrow S(0,2)$ ← here, two $\bar{e}$ allowed on dot (! Pauli)
LR LR

↑ & ↓ → singlet



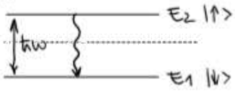
protocol:

#1 sweep S(0,2) ground state with gate
#2 → S(1,1)
#3 → degenerate with T₀(1,1)
#4 → getting to $\epsilon = 0$ → Rabi flopping $S(1,1) \Leftrightarrow T_0(1,1)$
→ after $\pi/2$ → superposition state

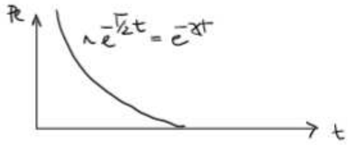
Cavities and Photons

05/06/23

TLA



$|\psi\rangle = \alpha|\downarrow\rangle + \beta|\uparrow\rangle$
$H = \frac{\omega}{2}\sigma_z$



$\sim e^{-\Gamma/2 t} = e^{-\gamma t}$

$\Gamma = A_{12}$

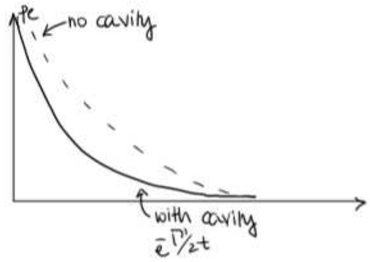
$H_{em} = \sum_k \omega_k (\hat{a}_k^\dagger \hat{a}_k + \frac{1}{2})$
mode

$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$
$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$

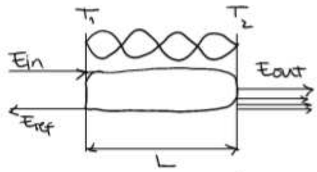
$H_{int} = \sum_k g_k (\hat{a}^\dagger \sigma^- + \hat{a}\sigma^+)$

$|\psi\rangle = c_e(t)|e,0\rangle + \sum_k c_k(t)|g,1_k\rangle$

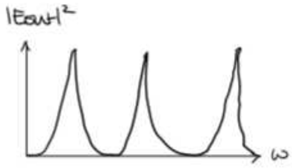
$|c_e(t)|^2 = e^{-\Gamma/2 t}$
$g \propto |\mu_{12}|^2 = |\langle\downarrow|q\hat{r}|\uparrow\rangle|^2$



$\Gamma' > \Gamma$ g depends on the environment!

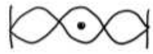


$L = n\frac{\lambda}{2}$



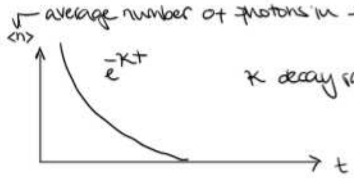
cavity standing wave transition frequency

$\omega_c \approx \omega_e$



$\Gamma' \sim \frac{1}{V}$ ← cavity volume

average number of photons in the cavity



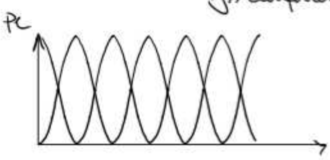
κ decay rate + line width of transmission

free space modes
$g \gg \sum_k g_k$

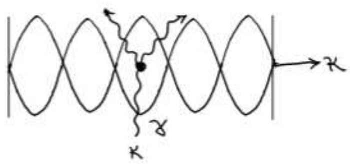
coupling constant
$g \propto \frac{1}{\sqrt{V}}$

$H = \frac{\omega}{2}\sigma_z + \omega_c \hat{a}^\dagger \hat{a} + g(\hat{a}^\dagger \sigma^- + \hat{a}\sigma^+)$ Jaynes-Cummings Hamiltonian

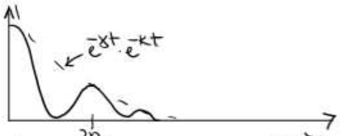
sum lost because
g ≫ compared to all other (free space) modes



just an atom between two mirrors which emits one photon
↳ most likely photon with ω_c



still have some leakage



$g \propto \sqrt{n}\, g$

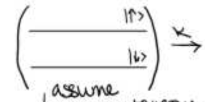
flopping because of coupling

$g > \kappa, \gamma$ strong
$g < \kappa, \gamma$ weak

trapped ions: $^{40}Ca^+$

	$g \approx 1$ MHz	22 MHz
	$\kappa \approx 250$ kHz	1 MHz
	$\gamma \approx 20$ MHz	20 MHz
	weak coupling regime	strong coupling regime

$|\psi\rangle_{atom} = \alpha|\downarrow\rangle + \beta|\uparrow\rangle$ $\quad |\psi\rangle_{atom+cavity} = \alpha|\downarrow,0\rangle + \beta|\uparrow,0\rangle$ initial state



$|\psi\rangle_{atom,cavity,fiber} = \alpha|\downarrow,0,0\rangle + \beta|\uparrow,0,0\rangle \xrightarrow{t\to\infty} |\downarrow\rangle_A |0\rangle_C (\alpha|0\rangle_F + \beta|1\rangle_F)$ → explicitely not entangled

↳ infinitely long

assume
↳ no spontaneous emission / decay → no decay in free modes

→ couple cavity to another cavity via fiber

state transfer

$|\psi\rangle = \alpha|\downarrow\rangle + \beta|\uparrow\rangle \to |\psi_{em}\rangle \to |\psi\rangle = \alpha|\downarrow\rangle + \beta|\uparrow\rangle$

atom in cavity: - couples to cavity with cavity mode / cavity coupling, g
- emits photons via spontaneous emission

→ $g \gg \gamma$: measure photon → was in excited state → fidelity 100% only if $\gamma = 0$

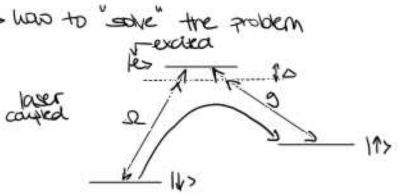
→ $g \approx \gamma$: not possible anymore ↯

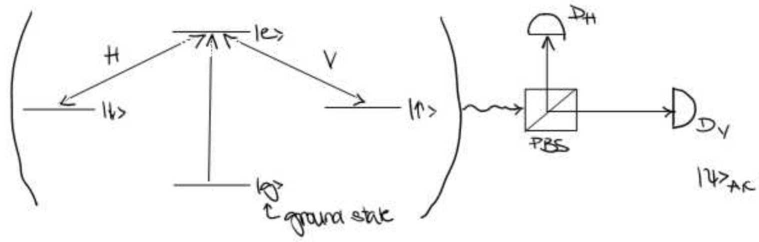
↳ spontaneous emission destroys this principle

$|e,0\rangle \xrightarrow{g} |g,1\rangle \xrightarrow{\kappa} |g,0\rangle$

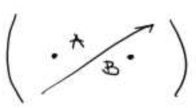
$|e,0\rangle \xrightarrow{\gamma} |g,0\rangle$

→ how to "solve" the problem





$|\psi\rangle_{AC} = \alpha|H\downarrow\rangle + \beta|V\uparrow\rangle$ (Heralded entanglement)

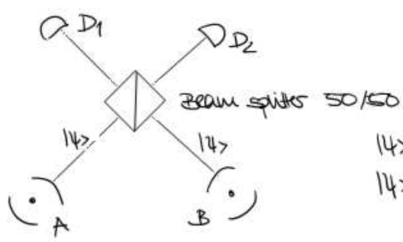


$|\psi\rangle_{AB} = |H\downarrow H\downarrow\rangle + |H\downarrow V\uparrow\rangle + |V\uparrow H\downarrow\rangle + |V\uparrow V\uparrow\rangle$

$\to |\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle$

both detectors click

two atoms coupled to the same cavity → loose information which atom has emitted which photon



Hong-Ou-Mandel

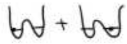
$|\psi\rangle = |H\downarrow\rangle + |V\uparrow\rangle$

$|\psi\rangle_{AB} = |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$

+ slides (2nd lecture)

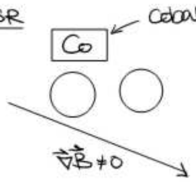
10.3.5 different flavours of spin qubits in quantum dots

0) charge qubit: $|L\rangle + |R\rangle$;

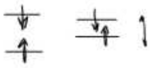


1) spin qubit: → less prone to charge noise $|\uparrow\rangle + |\downarrow\rangle$

! EDSR

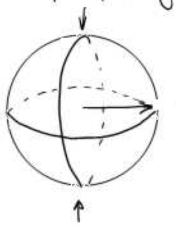


→ site dependent energy shift



different shift of qubit levels

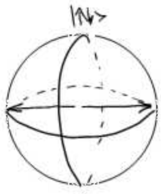
2) two spins forming one qubit



→ different basis $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, $[|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle]$

new qubit

→ $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ singulett



$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ triplet T_0

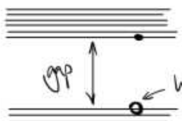
$T_+ = |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle$

$T_- = |\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle$

(two spin basis for single qubit operation → see above)

3) alternative: exploit holes (not electrons) as spin carriers

s-orbitals



conduction band

hole (with spin & effective mass)

valence band

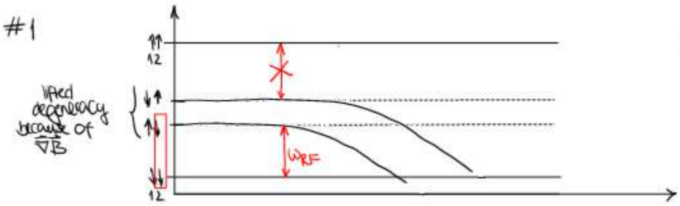
p-orbitals

↳ stronger spin-orbit-coupling (also more prone to errors!)

advantages:

10.3.6 Two qubit gates (eg. CZ)

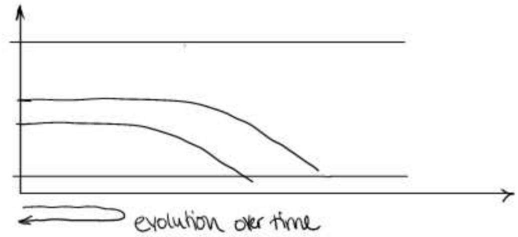
#1



conditional rotation: → if qubit #2 is $|\downarrow\rangle$ → qubit #1 gets flipped!

electrical gate voltage [coupling] (can barrier height)

#2



$$e^{i\phi} \nearrow e^{i\phi_{\uparrow\downarrow}} = e^{iE_{\uparrow\downarrow}/\hbar\, t} \quad \searrow e^{i\phi_{\downarrow\uparrow}} = e^{iE_{\downarrow\uparrow}/\hbar\, t}$$

⟵ evolution over time

regular duration time → π-phase gate

$$\boxed{\Delta\phi = \frac{(E_{\uparrow\downarrow} - E_{\downarrow\uparrow})}{\hbar} t}$$

→ record gate operation fidelity for QDOTS ≈ 99.5%.

10.3.7 Decoherence

0) charge noise ↯ only first order robust (compared to charge-qubit)
↯ second order → LS-coupling B(t) random → spin flop errors

1) ↯ Ga, As have nuclear spin

↖ 10^6 atoms for one $\bar{e}$

for full polarization ↑↑↑↑ → $A = \sum_i A_i \approx 5{,}5T$

B_z not high enough for polarization of (nuclear) spin

magnetic moment $\mu \sim \frac{1}{m}$

$m_e \to \vec{\mu}_B \approx 2000\, \vec{\mu}_{BH}$ ↖hydrogen

$B_{random} \approx \frac{5{,}5T}{\sqrt{10^6}} \approx 5{,}5mT$ → larger source of decoherence

↖ Ga atoms

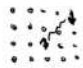
10.3.8 Fight decoherence

#1 GaAs (no stable isotopes with I=0) → $T_2^* \sim 10ns$ (~ order of magnitude of gate operations ↯)

2→ Si ($^{28}Si \to I=0$

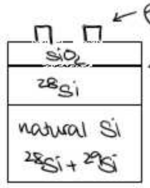
5% ^{29}Si $I = \frac{1}{2}$ → some residual fluctuations)

→ $T_2^* \approx 100 \cdot 10ns = 1\mu s$



exchange ⇒ fluctuations → $B_{eff}(t)$

3→ ^{28}Si (I=0)



→ $T_2^* \approx 100$-$250\mu s$

[^{28}Si-gates 4 orders of magnitude compared to GaAs]
↳ INTEL

11. Phosphorus atoms in ^{28}Si
↳ spin qubit ← nuclear

general: III IV V

C

Si $\boxed{P}$ → donor of $\bar{e}$

acceptors ↗ Ga Ge As

⋮

I=0

no isotopes with I=0

differences

1) Band gap <

2) spin-orbit coupling > → $T_{coherence}$ < but $t_{operation}$ <

3) m_{eff} of $\bar{e}$ < $E_{kin} = \frac{p^2}{2m}$

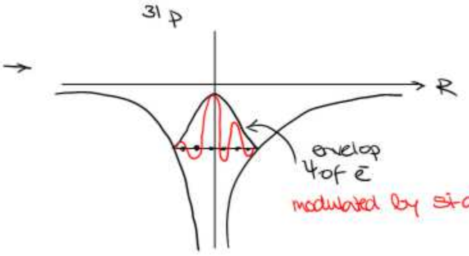
→ ⟨ψ⟩ $\hat{=}$ ⇒ larger spread

⇒ less effort on gate electronic possibility

→ now doping ^{28}Si with ^{31}P ↖donor

differences to hydrogen

→ Bohr radius: $\epsilon_{Si} \approx 11\, \epsilon_0$, $m_{eff} \approx 0{,}145\, m_e$ } $\frac{11}{0{,}145} a_0 \approx 20Å$ → $\bar{e}$ spans over many ^{28}Si



modulated by Si-atoms → ↯ impurities in ^{28}Si (e.g. ^{29}Si)
$\bar{e}$ is favorably positioned at nuclei

11.1 Nuclear spin qubit

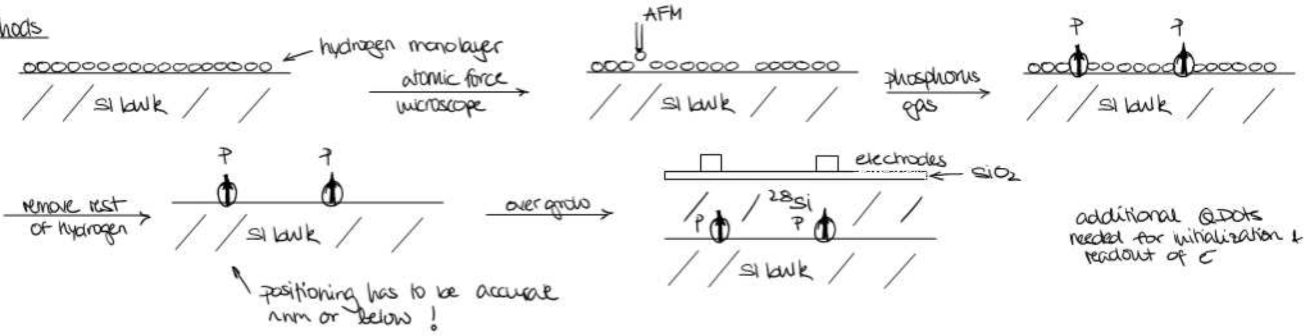
14/06/23

Idea [Kane: Nature 1998, V393, p133]: $Si + {}^{31}P_{I=1/2}$ I) nuclear spin: ⇑ = |1⟩, ⇓ = |0⟩ II) electronic spin ↑ ↓

↖ difference to GaAs: here ↓ is ground state fabrication of ^{31}P in ^{28}Si: 14/06/23

methods

#1



additional QDots needed for initialization & readout of e^-

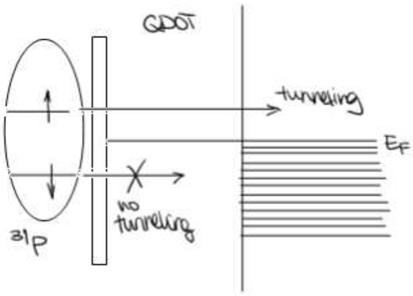
#2 ion implementation
($^{31}P^+$ ~ 14keV) → into Si

? focusing
positioning
damage to rest of lattice

11.2 Readout (1.0) + initialization

reminder: as before $B = 1{,}5T$

(inverse preparation) initialization

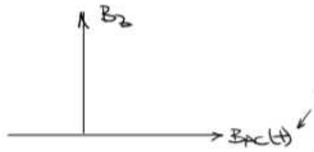


→ measuring presence of $\bar{e}$ by current + spin of e^- @ gate voltage applied

11.3 Gates (so far above only $\bar{e}$)

11.3.1 single nuclear qubit operations

→ $B_{AC}(t)$ additionally to quantization field ($B_z \sim 1{,}5T$)

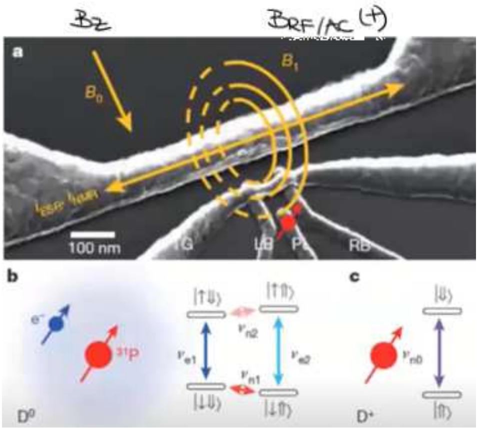


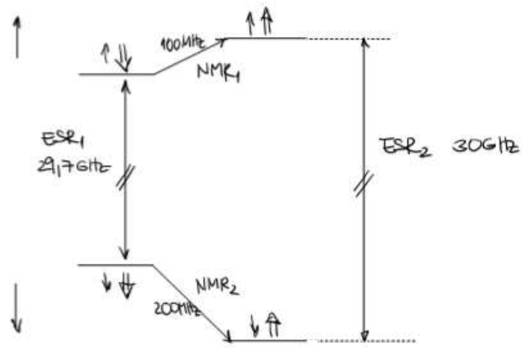
frequency? → precession/resonance frequency
linear polarization σ^+, σ^-

magnetic moment $\vec{\mu} = I\vec{A}$ $\mu \sim \frac{e}{2m}\hbar$
$m_e \sim \frac{1}{2000} m_{nuc}$ ^{31}P

→ NMR ^ ν_{NMR} ^ 100MHz ⇑
ESR ^ ν_{ESR} ^ 30GHz ↑

B_z $B_{RF/AC}(t)$





→ via NMR: Rabi flopping of ^{31}P nuclear spin (qubit)
≙ single qubit operation

here: electron is needed for spin control & detection

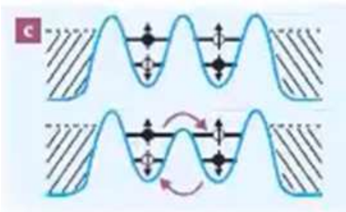
→ protocol: 1) readout of $\bar{e}$ → check for ΔI in QDOT SET
2) nuclear spin?
→ prepare $\bar{e}$ in $|\downarrow\rangle$ (i.e. $E < E_F$ → no tunneling)
→ ESR_2 π-pulse → if ⇓ → no flip (off-resonant) → no current
$\bar{e}\downarrow$

→ if ⇑ → π-flip (resonant) → current
$\bar{e}$ ↑

summary single qubit gate via NMR
nuclear qubit detection via ESR ($\bar{e}$-spin + QDOT SET)
↳ ↓ charge noise

11.3.2 two qubit gates

→ what we had in QDOTs



↑ ↓

exchange interaction of the $\bar{e}$ spin ↑↓ and ↓↑ (no effect on ↑↑ and ↓↓)

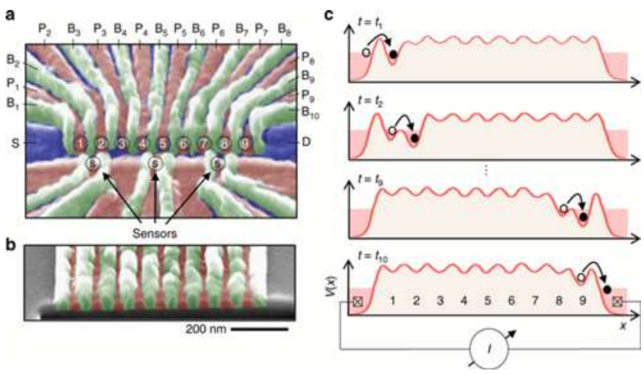
→ now two ^{31}P atoms

→ again $\bar{e}$ spin exchange interaction + nuclear spin follows
need $\bar{e}$ for – initialization
– readout
– interactions
} prone to charge again

currently (2019) Australia ↓ 5% detection error ~ <1%
Simmons ↓ $F_{2qubit\ gates}$ ~ 0,9 ≪ 99,99%

11.4 State of the art (scaling)

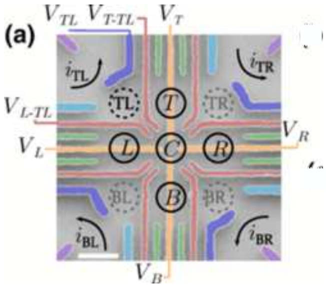
spin qubit QDOTs



→ 8/9 dots in linear chain

lack of connectivity, no long distance

→ 3×3 2D array



⊕ nearest neighbour interaction only
→ lack of

11.5 Summary

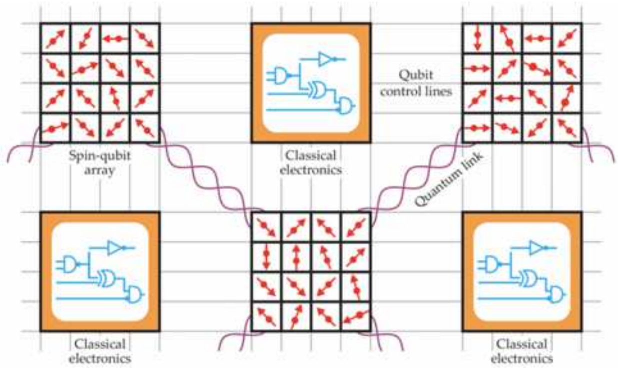
→ spin → Singulett-Triplett (QDOTs)
→ nuclear spin ^{31}P

→ initialization – loading (shift E_F / E)
– prepare (tunneling + Rabi flopping)
– readout

→ gates → Rabi ($B_{AC}(t)$) (NMR) ESR
↘ exchange interaction (directly $\bar{e}$ ↓ → mediating interaction to nuclear spin)

→ coherence ^{31}P ~ 90% / 95% 2 qubits
95% / 99% readout
→ ^{28}Si + Dot → single qubit 99,9%
two qubits 99,5%

11.6 Vision / challenges



addressing challenges
→ connectivity ↯
1) shuttling $\bar{e}$ (preserve spin) demonstrated in GaAs
2) coupling via Resonators CQED (see ions)
ion $\xrightarrow{\text{phonon}}$ ion
spin $\xrightarrow{\text{photon}}$ spin

→ Readout ↯ $\bar{e}$ needed as charge (not spin only)
→ Temperature all of these "10^9" qubits in one dilution fridge ⇝ wiring
⇝ classical electrodes
↕
⇝ operations in quantum on ps (classical electronics on ns)
⇝ positive ≈ 1,5K a lot of quantum beauty

→ cross talk (gate voltages, $\vec{E}$, B) between "10^9" dots
→ charge noise → correlated / → uncorrelated } recalibration of control fields required (dead time)

→ reliable fabrication (^{31}P at a nm distance) man-made (true future feature) ↔ not all alike
↳ ^{28}Si ($I=0$) qubit is in degeneracy (there are several ground states)

12 superconducting qubits

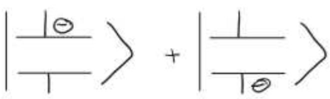
21/06/23

→ google, IBM, D-wave, (Intel)

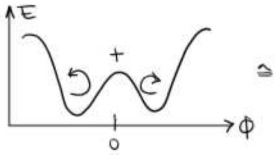
Refs Physics today 2005
RMP 2021

Intro

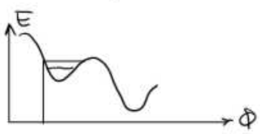
? qubit



≙ charge qubit



≙ flux qubit



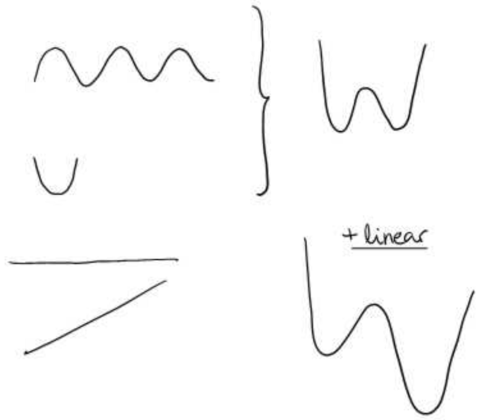
≙ phase qubit

examples (later more detailed)

1) Josephson Junction   $\hat{H}_J \sim \cos\phi$

2) Inductance L  $\hat{H}_L \sim \phi^2$

3) Bias current $\sim \phi$



↳ IL/AS Bildchen

12.1 Basics

12.1.1 Historic view

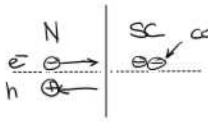
$>100a$

Hg @ 4,2 K → Resistance dropped to 0

it took 50a to get some theory BCS (Bardeen, Cooper, Schriefer)

12.1.2 Characteristics (Typ I)

- Meissner effect (field within superconductor → 0) different from normal conductors (even @ R=0)
  → B-field gets expelled, even if it was "in" for $T > T_c$
- Proximity effect:
  B-field penetrates into superconductor (SC) by 50–100nm + SC-state penetrates into normal conductors & insulators



↳ coherent conversion of quasi-particles into cooper pairs

Andrejeco reflection

we look at $\underbrace{SC-I-SC}_{\text{Josephson Junction}}$

Phase transition:

$\bar{e}$ are fermions

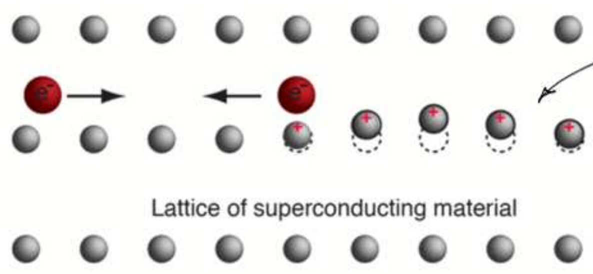
still SC in a phase transition ≙ Bose-Einstein Conductant

→ all $\bar{e}$ get into identical state by pairing → forming a Boson ($\bar{e}\bar{e}$)

Parameters that control SC-phase: T_c, I_c, B_c all connected via energy barrier (SC-gap: Δ)

→ for $B \neq 0$ → $T_c <$

? Why do $\bar{e}$ form pairs



deformation of the lattice

→ intermitted attraction between two $\bar{e}$ (cooper pair)

+ opposite momenta $\vec{k}$ and $-\vec{k}$

I) ground state: $\Psi = \sqrt{n_{co}}\, e^{-i\phi}$

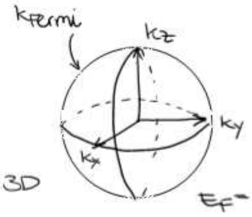
↳ density of cooper pairs

! macroscopic phase coherence

! Ψ ≙ order parameter of phase transition (e.g. zero above T_c)

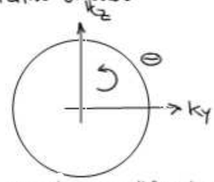
II) Cooper pairs $\bar{e}\bar{e}$ form Boson with opponent $\vec{k}$
+ spin singulett $\uparrow\downarrow + \downarrow\uparrow$
+ SC gap (protecting the CP)

1) free electron



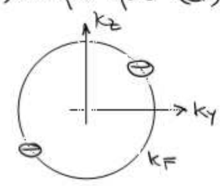
$E_F = \frac{(\hbar k_F)^2}{2m}$

2) Fermi liquid



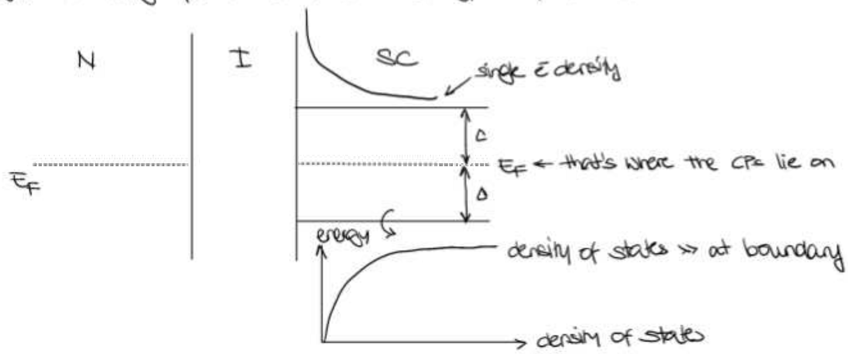
$\hat{=}$ excitation of "free" electron gas

3) Cooper pair (CP)



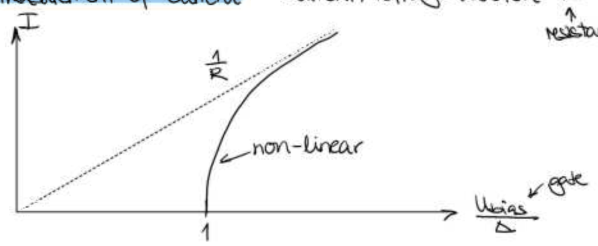
opposite momenta: two $\bar{e}$ form a pair
(not permanently the identical $\bar{e}$)

III) superconducting gap (Δ)
gap? breaking up a CP ($2\bar{e}$) costs energy → protects the SC-state

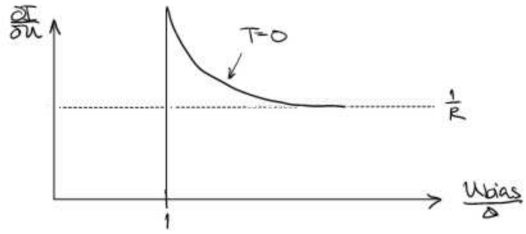


+ apply gate voltage → shift E_F

IV) measurement of current   current/voltage relation $R = U/I \rightarrow \frac{1}{R} = I/U$
(R: resistance; $\frac{1}{R}$: conductivity)



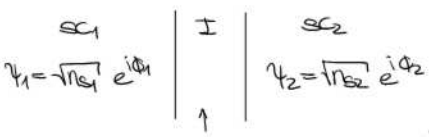
conductivity ~ density of states



→ direct access to density of states of single $\bar{e}$

12.1.3 Josephson Junction

Overlap of two SC-Ψ

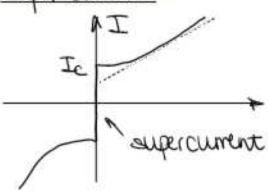


tunneling (proximity effect!)

overlap of Ψ_1 & Ψ_2 (see evanescent waves)

phase drop of JJ $\boxed{\Delta\Phi = \Phi_2 - \Phi_1}$

super current



voltage drop across JJ

supercurrent → no drop of voltage → for $I_S > I_c$ → breaking up of CPs

I) law of JJ
$\Delta\Phi \,\hat{=}\,$ phase $\hat{=}$ flux ($B\cdot A$)
$\boxed{I_S = I_c \sin(\Delta\Phi)}$

for $I_s = I_c \rightarrow \boxed{\Delta\phi = \pi/2}$

two laws of JJ

II) $\boxed{2eU = \hbar \frac{\partial\phi}{\partial t}}$ AC voltage drop + cooper pair = $\hbar\omega$ → $\boxed{\frac{2e}{\hbar} U = \frac{\partial\phi}{\partial t}}$

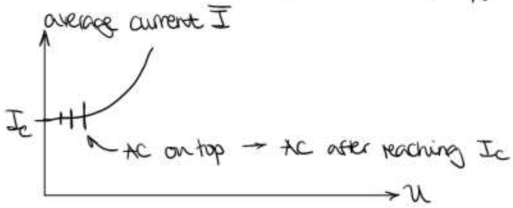
AC of CPs across the JJ

used for description of the volt

$\frac{\hbar}{2e} \hat{=} \phi_0 \hat{=}$ flux quantum

revised I) $\boxed{I_s(t) = I_c \sin(\phi(t))}$

phase $\hat{=}$ flux $\quad \phi = \frac{\Phi}{\Phi_0}$



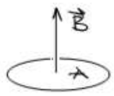
12.1.4 Josephson energy

oscillating current ⟷ energy $\hat{=}$ energy stored in JJ

$$\rightarrow \text{energy} = \int \text{power}\, dt = \int U I_s\, dt = \int \frac{\hbar}{2e}\frac{\partial\phi}{\partial t} I_c \sin(\phi(t))\, dt = \frac{\hbar I_c}{2e}\int \sin(\phi(t))\, d\phi = -\underbrace{\frac{\hbar I_c}{2e}}_{E_J = \hbar\omega_J} \cos\phi$$

ω_J: CP current, $I_c/2e$

where $\frac{\hbar}{2e} = \Phi_0$ quantum of flux



with $\phi = 2\pi \frac{\Phi}{\Phi_0}$ ← flux

phase; ← quantum of flux

$U = -L\frac{\partial I}{\partial t} \qquad \int U dt = \Phi = \int -L dI \qquad$ energy in coils $\rightarrow \frac{1}{2}LI^2 \rightarrow \frac{\Phi^2}{2L}$

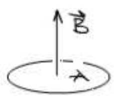
capacitor $\rightarrow \frac{1}{2}CU^2 \rightarrow \frac{Q^2}{2C}$

26/06/23

SN I) $I_s = I_c \sin(\Delta\phi)$

↳ $2\pi \frac{\Phi}{\Phi_0}$ flux

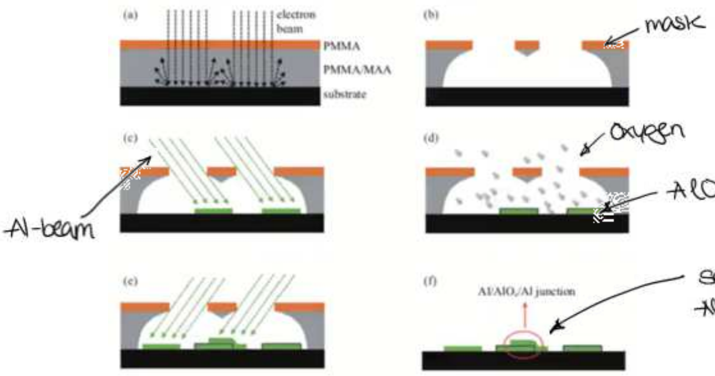
$\left.\begin{matrix} I \\ B \\ \phi \\ \Phi \end{matrix}\right\}$ ~ "identical"



$BA = \Phi$

II) $2eU = \underbrace{\hbar \frac{\partial\phi}{\partial t}}_{\hbar\omega_J}$ ← oscillating SC in JJ

12.1.5 Fabrication



↱ 50–100nm (coopers survive due to proximity effect)

SC | I | SC

Al | AlO$_x$ | Al $\quad \hat{=}$ Josephson junction

same: capacitor

loops/coils

wires

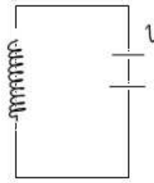
12.2 Superconducting electronics

capacitors — $C = \frac{Q}{U} \rightarrow \hat{Q} = CU$

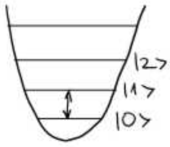
inductors — $L = \frac{\Phi}{I} \rightarrow \hat{\Phi}_{flux} = LI$

resistors — $R = \frac{U}{I} \rightarrow I = \frac{1}{R} U$

wires



$\rightarrow \omega_{res} = \frac{1}{\sqrt{LC}}$ harmonic oscillator

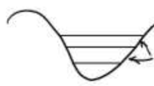


same energy shifts
→ can't talk to just 2-levels → talk to all
→ no qubit

qubit ?
↯ $\omega_{10} = \omega_{12} = \dots$
→ Rabi-drive drives all transitions
→ coherent state ↯ no two-level system

→ solution JJ: no linear dependence → no harmonic oscillator

First: let's stick to the non-qubit (quantized LC-circuit)
later → induce JJ for non-linear deformation



$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L} \qquad \left[\hat{=} \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2\right]$$

$\frac{1}{2}CU^2$, $\frac{1}{2}LI^2$

$U = \frac{Q}{C}$, $I = \frac{\Phi}{L}$

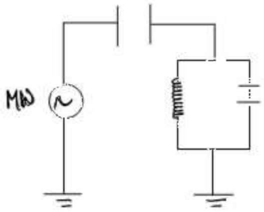
$\omega_{01} = \frac{1}{\sqrt{LC}} \approx 10\,\text{GHz}$

$10^{15}\,\text{Hz} \approx 50000\,\text{K}$ → $10\,\text{GHz} \approx 500\,\text{mK}$

→ dilution fridge (10mK)

⊕ microwave sources (single qubit operations)

→

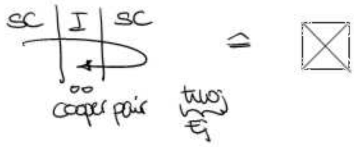


for cooper pairs (T<10K)

12.2.1 Josephson Junction (non linear element)

1) $I_S = I_c \sin(\Delta\phi)$ $\quad \Delta\phi = 2\pi \frac{\Phi_L}{\Phi_0}$

we use flux

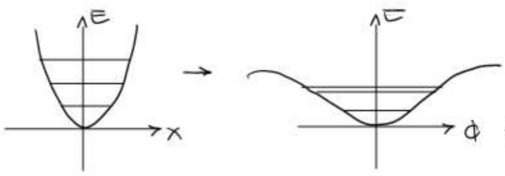


2) $L_{jj} = \frac{\partial\Phi}{\partial I}$ (remember $L = \frac{\Phi}{I}$)

take 1) $\left(\frac{\partial I}{\partial \Phi}\right)^{-1} = \left[\partial/\partial\Phi \left(I_c \sin\left(\frac{2\pi\Phi}{\Phi_0}\right)\right)\right]^{-1} = \left[I_c \cos\left(\frac{2\pi\Phi}{\Phi_0}\right) \cdot \frac{2\pi}{\Phi_0}\right]^{-1}$

$$\rightarrow \boxed{L_{JJ} = \frac{\Phi_0}{2\pi I_c} \frac{1}{\cos\left(\frac{2\pi\Phi}{\Phi_0}\right)}}$$ non-linear

12.2.2 Hamiltonian with JJ (non-linear, still classical)



flux (current, phase, B-field)

Analogue to quantization of the harmonic oscillator:

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L} \Leftrightarrow \frac{\hat{p}^2}{2m} + \frac{\hat{x}^2}{2m}$$

where $Q \Leftrightarrow p$, $\Phi \Leftrightarrow x$, $C \Leftrightarrow m$

$\frac{1}{2}\frac{C}{LC}\hat{\Phi}^2 \rightarrow \frac{1}{2}C\omega^2\hat{\Phi}^2$

$\leftrightarrow \frac{1}{2}m\omega^2\hat{x}^2$

12.2.3 Creation and Annihilation operators

$\hat{H} \to \sqrt{V} \to$ solution $\hbar\omega(a^\dagger a + \frac{1}{2})$

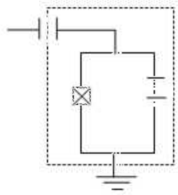
$$\hat{Q} = \sqrt{\frac{\hbar}{2Z_0}}(a^\dagger - a) \qquad Z_0 = \sqrt{\frac{L}{C}} \text{ characteristic impedance}$$
$$\hat{\Phi} = \sqrt{\frac{\hbar Z_0}{2}}(a^\dagger + a)$$

$\to \hat{H} = \hbar\omega_1(a^\dagger a + \frac{1}{2})$

still harmonic oscillator, no qubit

12.3 Charge qubit (→ include non-linearity ☒) see 12.1.4

I $\hat{H}_{qubit} = \underbrace{\frac{\hat{Q}^2}{2C}}_{E_{kin}} - \underbrace{E_J\cos\left(\frac{2\pi\Phi}{\Phi_0}\right)}_{\hbar\omega_J}$



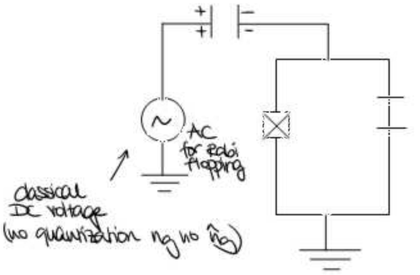
II $\hat{Q}^2 \to (2e)^2\hat{n}^2$

density operator of CP

I in II: $\hat{H}_{\substack{charge\\qubit}} = \frac{(2e)^2\hat{n}^2}{2C} - E_J\cos\left(\frac{2\pi\Phi}{\Phi_0}\right) = 4E_C\hat{n}^2 - E_J\cos\left(\frac{2\pi\Phi}{\Phi_0}\right) \qquad E_C = \frac{e^2}{2C}$

with $\left.\begin{matrix}\Delta\hat{n}^2 < \\ \Delta\phi^2 >\end{matrix}\right\}$ commutation relation

coherent control of qubit



additional charge from voltage control

$\to \boxed{\hat{n} = \hat{n}_j \pm n_g}$

$\to$ two operators $\left.\begin{matrix}\hat{\Phi} \\ \#CP\ \hat{n}_j\end{matrix}\right\}$ same commutation relation $[\hat{p}, \hat{x}] = -i\hbar$, $[\hat{n}, \hat{\Phi}] = -i\hbar$

we search for robust regime of E_J, E_C and n_g

superconducting energy gap

Bose-Einstein condensator

di Vincenzo criteria → what do I need for a qubit to be a qubit

28/06/23

Eigenvalues and Eigenstates of the charge qubit

problem: Find E_k, $|\psi_k\rangle$

$$\boxed{\hat{H} = 4E_C\hat{n}^2 - E_J\cos\Phi}$$

$\hbar\omega_J \leftarrow$ oscillation of CP

$\frac{Q^2}{2C} \to (2e)^2$ $\qquad \int P\,dt = \int U I_S\,dt = \int U I_c \sin\varphi\,dt$

(1) $\hat{\Phi}|\psi_k\rangle = \Phi_k|\psi_k\rangle$ ($\leftarrow \hat{x}$) in phase space flux

(2) $\hat{n} \to -i\hbar\,\partial/\partial\varphi$ ($\hat{p}$)

$\to$ DGL $\boxed{\left[4E_C\left(\underbrace{-i\hbar\frac{\partial}{\partial\varphi}}_{\hat{n}_j} - n_g\right)^2 - E_J\cos\Phi\right]|\psi_k\rangle = E_k|\psi_k\rangle}$

n_g: classical charge density due to voltage source

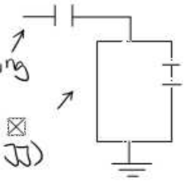
→ optimize system for robustness against Δn_g → $E_J, E_C \to \frac{E_J}{E_C}$

12.4 Transmon qubit

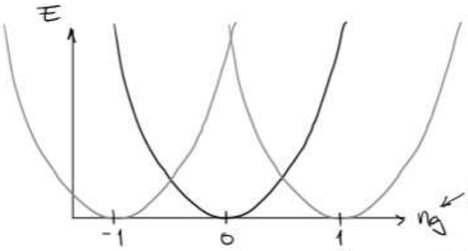
exact solution via Matthiews equation → tuning $\frac{E_J}{E_C}$

12.4.1 energy spectra (n_g, $\frac{E_j}{E_c}$)

(1) $E_j = 0$

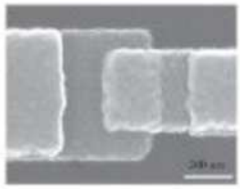


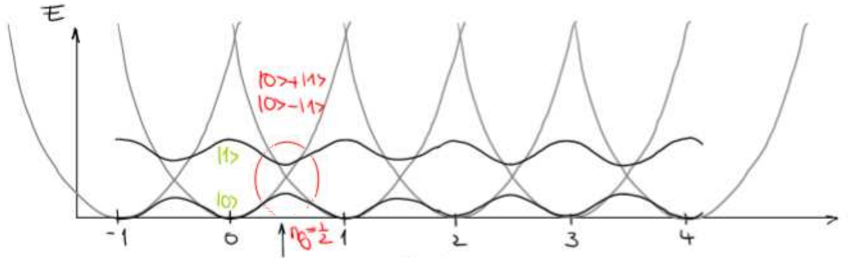
$\rightarrow H \propto n_g^2$



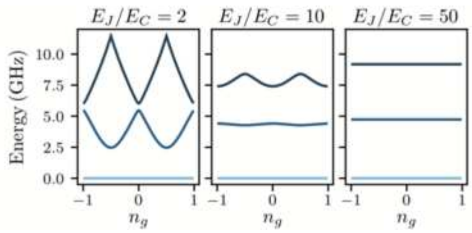
(2) add it again $E_j \ll \rightarrow \frac{E_j}{E_c} \ll 1 \rightarrow E_j$ ≙ tunneling of CP
≙ oscillating CP between n=0 and n=1
≙ coupling, when energies become degenerate
→ anti-crossing (avoided crossing)

what happens to the spectra?



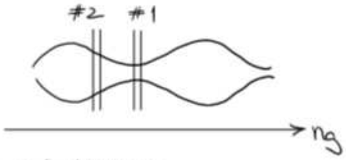


opening gaps ~ $E_j = \hbar\omega_j$
→ new basis



aim: less prone to noise $\frac{\partial n_g}{\partial V}$!
↖ noise

reduce impact of charge noise
→ search for best n_g

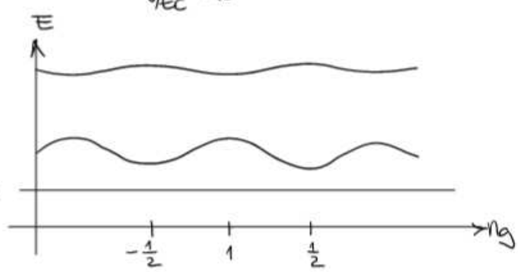


at #2 small ∂V → large change of qubit frequency
→ $T_2 \ll$ because clock frequency changes (jitter)

same

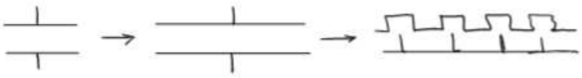
at #1 small ∂V → smaller change of qubit frequency
→ $T_2 >$

$E_j/E_c \approx 10$



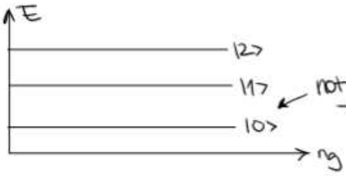
small impact of ∂V :)

theory $E_c < \rightarrow C > \rightarrow E_c \propto \frac{1}{C} < \rightarrow \frac{E_j}{E_c} >$



← to increase $\frac{\text{surface}}{\text{size}}$

(4) $\frac{E_j}{E_c} \approx 50$



← not # of CP
→ many CP

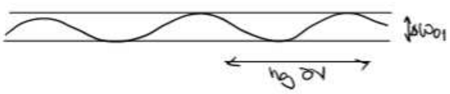
! flat $\frac{\Delta n_g}{\partial V}$ robust against charge noise

but ↯ harmonic again $E_j \gg E_c$
→ $\omega_{01} = \omega_{12}$ not a qubit anymore

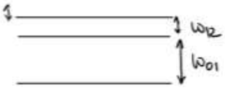
12.4.2 Transmon error budget

trade off (1) $\frac{E_j}{E_c} \gg$ as possible → flat spectra (robust against ∂V)
(2) sufficiently different ω_{01} and ω_{12} (not harmonic)

@ (1) band width

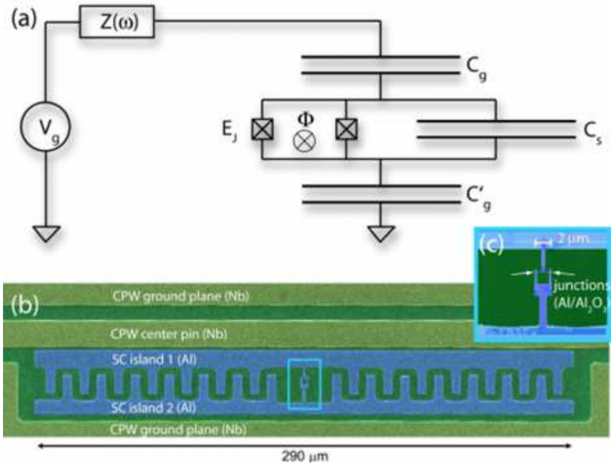


@ (2) $\omega_{01} - \omega_{12} = \delta\omega \ll$ we lose the qubit



⊕ Fourier limit on pulse duration
for $T_\pi \approx 1\mu s \rightarrow \frac{1}{T_\pi} \approx 1 MHz = \Delta\omega$
pulse duration for π-pulse

if pulse too long → decoherence (to gain resolution)
if pulse too short → lack of resolution of ω_{01}
→ optimum ≈ at $E_J/E_C \approx 50$ (minimum in decoherence while allowing for resolving the qubit)



← all feasible with device above (just more symmetric)

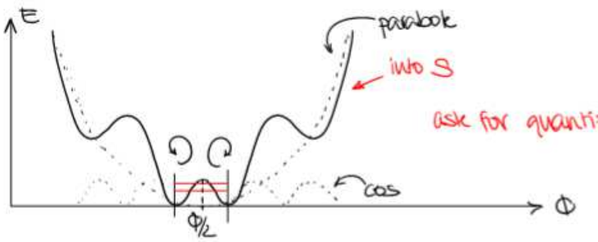
real numbers $E_J \cdot \hbar \cdot 20\,\text{GHz}$
$E_C \approx \hbar \cdot 400\,\text{MHz}$
→ errors below 1%

12.5 Different flavours

intro: charge qubit (1CP)
Transmon ~100CP flattened charge qubit

12.5.1 Flux qubit

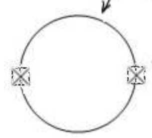
Adding inductance L → $\frac{\hat{\Phi}^2}{2L} = \hat{H}_{\text{induction}}$ → parabolic + JJ (cos)



my favourite qubit might be $|\circlearrowleft\rangle$ or $|\circlearrowright\rangle$
for $\Phi_0/2$ (allows for coupling)

$|0\rangle = |\circlearrowleft\rangle + |\circlearrowright\rangle$
$|1\rangle = |\circlearrowleft\rangle - |\circlearrowright\rangle$ flux qubit

adding L?



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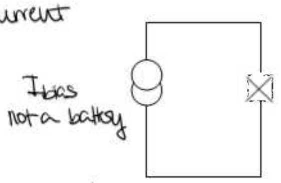
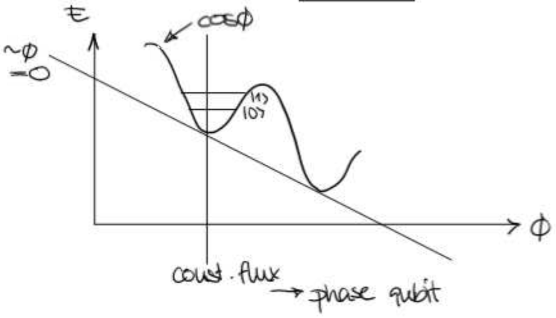
12.5.3 Phase qubit

Adding bias current



$$E = \int UI\,dt = I_{bias}\int U dt = I_{bias}\Phi$$
const. bias current

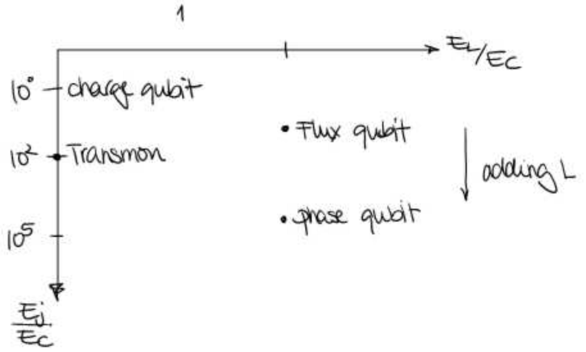
$$\left[\begin{array}{l} U = -L\dot{I} \\ \int U dt = -LI = \Phi \end{array}\right]$$



dependent on I_{bias} → tilt
→ state dependent $\Phi(t)$ → U
→ read out high efficient ≈ 99%

! phase qubit best on read out
flux qubit "best" on coherent operations
Transmon "best" on coherent operations & coupling

12.5 Different flavours

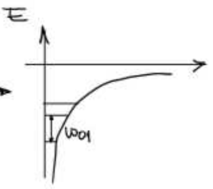


13 Cavity Quantum Electro Dynamics (CQED)

↖ photons

13.1 Intro

atomic physics ⟷ quantum optics
artificial atoms ⟷ "?" quantum electronics ⟶



Two level system ≙ qubit

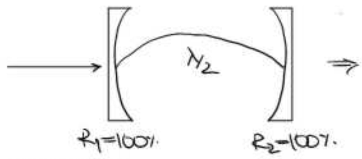
$\hat{H} = -\hat{d}E(t) \sim \Omega$

↑ dipolemoment $\hat{d}\hat{x}$

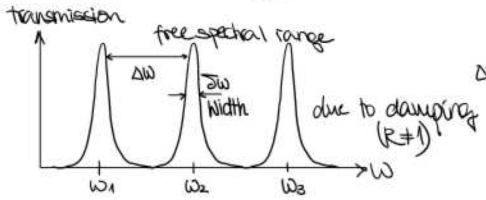
better ≙ faster (as long as $\Delta t \sim \frac{1}{\Delta\omega_{01}}$)

Serge Haroche ⤳ Rydberg atoms ⊕ $E \sim \frac{1}{\sqrt{V}}$

13.2 Basics on cavities (resonators)

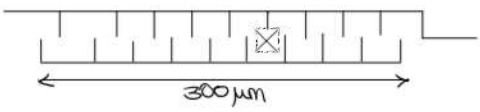


⇒ 100% transmission → constructive interferance



$\Delta\omega = \frac{c}{2L}$ ← speed of light $[\frac{m}{s}]$; L ← length of cavity $[m]$

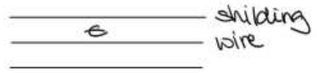
new: transmon



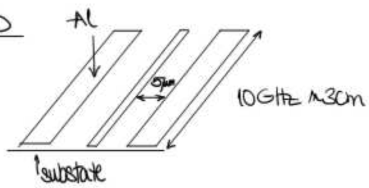
$E \sim \frac{1}{\sqrt{V}} \sim \frac{1}{\sqrt{\lambda^3}}$

→ coax cable:

cut along axis



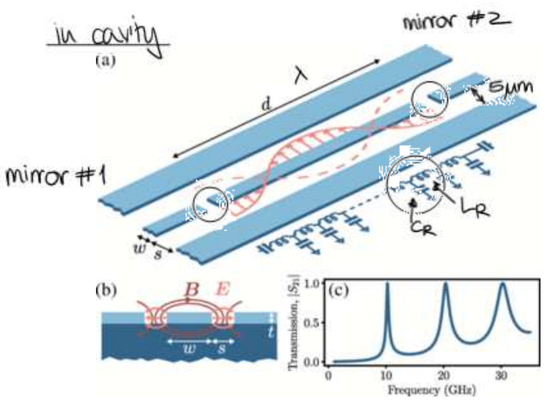
in 2D



10 GHz ∼ 3 cm

$\lambda = \frac{c}{\nu} = \frac{3 \cdot 10^8 \frac{m}{s}}{10^{10} \ 1/s} \sim 3cm$

in cavity

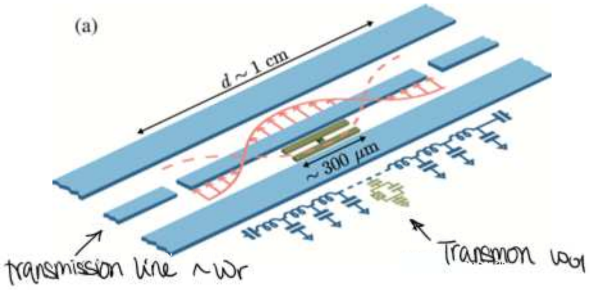


RMP

cavity $\omega_R = \frac{1}{\sqrt{L_R C_R}}$ $E \sim \frac{1}{\sqrt{V}} \sim \frac{1}{\sqrt{\lambda \cdot 10^{-3}\lambda \cdot 10^{-3}\lambda}} \sim 10^3 \sim 0.2 \frac{V}{m}$

↑ 5μm/30000μm ↑ enhancement

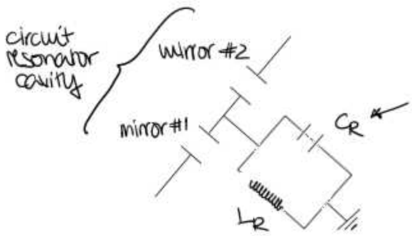
13.3 Transmon in cavity



coupling of Transmon qubit to resonator (via MW-photons)

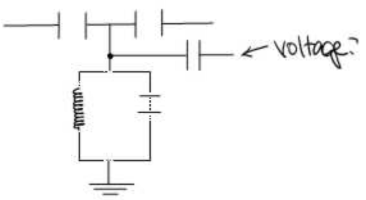
$\omega_r \to$ transmission $\to$ read-out
$\omega_{01} \to$ Rabi flopping $\to$ single qubit gate

13.3.1 Resonator circuit



$\omega_r \sim \frac{1}{\sqrt{L_R C_R}} \triangleq$ harmonic oscillator (let's pick one mode)

13.3.2 Resonator coupling



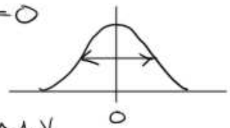
$C = \frac{Q}{V} \to \hat{V} = \frac{\hat{Q}}{C}$ ← operator

expectation value?

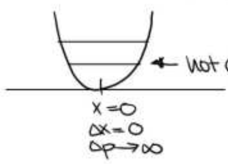
$\langle 0|p|0\rangle = \langle 0|x|0\rangle = 0$

! $\langle 0|\hat{V}|0\rangle = 0$

$\sqrt{\langle 0|\hat{V}^2|0\rangle} \neq 0$
$= \sqrt{\frac{\hbar\omega_R}{2C_R}} \sim 1\mu V$



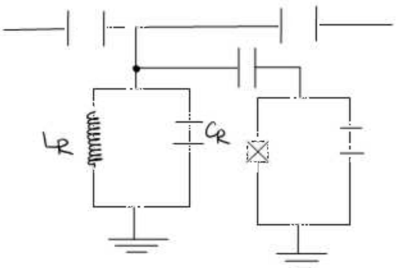
$1\mu V$ over $5\mu m$
$\to E \sim \frac{1\mu V}{5\mu m} \sim 0.2\,\frac{V}{m}$



← not @ energy = 0

harmonic oscillator (resonator)

13.4 Coupling Transmon to resonator



resonator ⊕ transmon ($\frac{E_J}{E_C} \approx 50$)
external former gate voltage by resonator

I) $\hat{H}_R = \frac{\hat{Q}_R^2}{2C_R} + \frac{\hat{\Phi}_R^2}{2L_R} + (4E_C\hat{n}^2 - E_J\cos\Phi)$

$(\hat{n}-n_g)^2$; taylor series $\Phi^2 + \Phi^4$

non-linearity → qubit (21s)

$(\hat{b}^\dagger\hat{b})^4$
$\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b}$

now:

II) $\hat{H} = \hbar\omega_r\hat{a}^\dagger\hat{a} + \hbar\omega_{01}\hat{b}^\dagger\hat{b} - E_J\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b}$

coupling? remember $V_g \to (\hat{n}-n_g)^2$
replace by $\hat{n}_r$

III) $(\hat{n}-\hat{n}_r)^2 \to \hat{n}\hat{n}_r$
$g\,(\hat{a}^\dagger-\hat{a})(\hat{b}^\dagger-\hat{b}) \to \hat{a}^\dagger\hat{b}$ and $\hat{a}\hat{b}^\dagger$ → red & blue sideband

coupling strength ; light ; matter

IV) Transmon as qubit

two levels: $\hat{b}$ and $\hat{b}^\dagger \to \hat{\sigma}_-$ and $\hat{\sigma}_+$

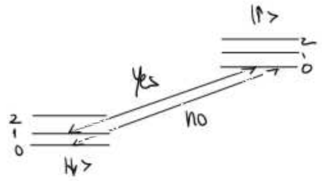
V) $H \approx \hbar\omega_r\hat{a}^\dagger\hat{a} + \frac{\hbar\omega_{01}}{2}\hat{\sigma}_z + g(\hat{a}^\dagger\hat{\sigma}_- + \hat{a}\hat{\sigma}_+)$

sideband transitions

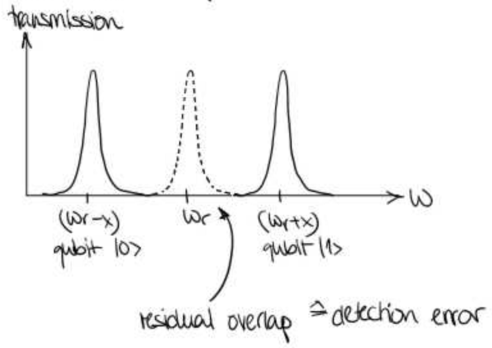
rotating wave approximation
e.g. $\hat{a}\hat{\sigma}$ cancel out

remember ions

red side band



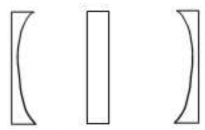
for $|\omega_{01} - \omega_r| = \Delta \ll g$



$$H = \hbar(\omega_r + \chi\hat{\sigma}^z)a^\dagger a + \hbar\frac{\omega_{01}}{2}$$
($\hat{\sigma}^z$: $\pm$, $\begin{pmatrix}1&0\\0&-1\end{pmatrix}$)

resolve spectrum → measure qubit

Analogue:



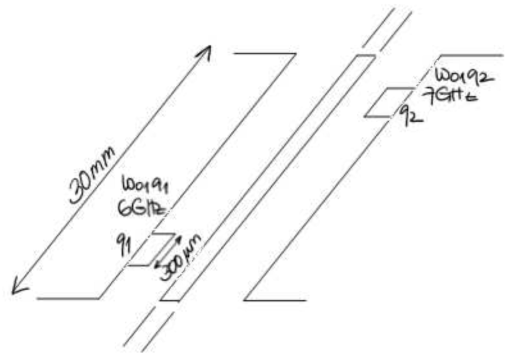
→ change effective length of cavity
→ change resonance frequency

dialectric constant $\in (0,1)$

→ if you transmit photon at $\omega_r + \chi$ → qubit was in state $|1\rangle$

13.5 DV

1) single qubit (Transmon)
2) state preperation (cooling ~10mK)
3) read out (transmission at $\omega_r + \chi$)
4) coherent operation – single qubit (Rabi flopping via ω_{01})
   – two qubit gate



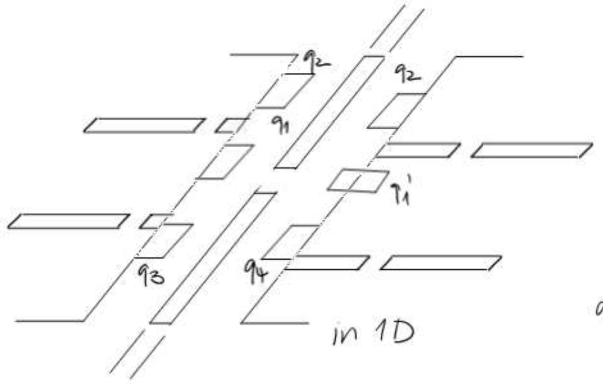
→ transfer q_1 on resonator (red side band)
→ excite q_2 in dependance of q_1 (photon in the resonator)

"the truth"

05/07/23

13.6 Scaling

Intro: Transmon + QED



not on phonons in cavity

add 3rd dimension (second layer)
↳ 2D array

Challenges:

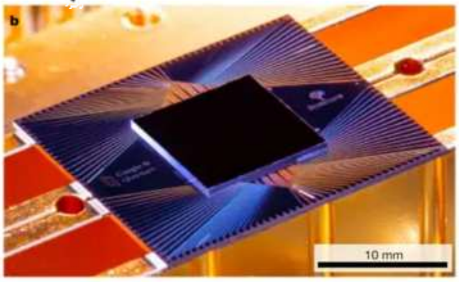


Fig. 1 | The Sycamore processor. a, Layout of processor, showing a rectangular array of 54 qubits (grey), each connected to its four nearest neighbours with couplers (blue). The inoperable qubit is outlined. **b**, Photograph of the Sycamore chip.

4 Transmons + CQED

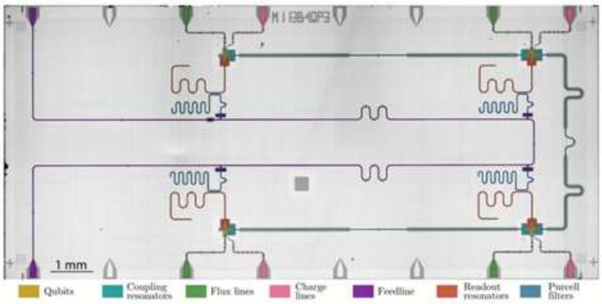


FIG. 27. False colored optical microscope image of a four-transmon device. The transmon qubits are shown in yellow, the coupling resonators are shown in cyan, the flux lines for single-qubit tuning are shown in green, the charge lines for single-qubit manipulation are shown in pink, and a common feedline for multiplexed readout is shown in purple, with transmission-line resonators for dispersive readout (red) employing Purcell filters (blue). Adapted from Andersen *et al.*, 2019.

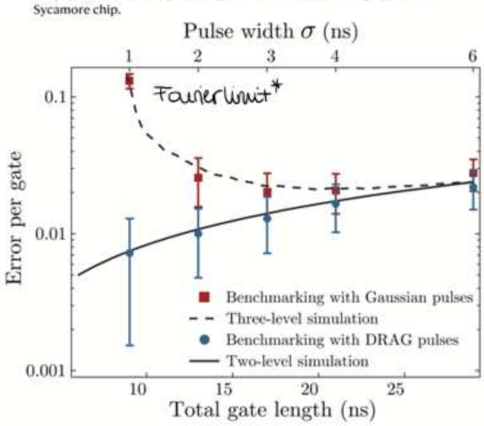


FIG. 28. Single-qubit gate errors extracted from randomized benchmarking for Gaussian and DRAG pulses as a function of total gate time and pulse width σ for Gaussian pulses. The experimental results (symbols) are compared to numerical simulations (lines) with two or three transmon levels. Adapted from Chow *et al.*, 2010.

single qubit fidelity

↳ longer gates: decoherence reduces fidelity
* error > for T_{gate} < Fourierlimit: $T_{gate} < \to \frac{1}{\Delta\omega} < \to \boxed{\Delta\omega >}$
don't resolve qubit anymore

Readout

A B S T R A C T

High-fidelity initialization, manipulation, and measurement of qubits are important in quantum computing. For the Google's Sycamore processor, the gate fidelity of single- and two-qubit logic operations has improved to>99.6%, whereas single-shot measurement fidelity remains at the level of 97%, which severely limits the application of the superconducting approach to large-scale quantum computing. The current measurement scheme relies on the dispersive interaction between the qubit and the readout resonator, which was proposed back in 2004. However, the measurement fidelity is limited by the trade-off between the state separation and relaxation time of the two-level system. Recently, an exciting phenomenon was observed experimentally, wherein the separation-decay limit could be alleviated by exploiting the cascade decay nature of the higher levels; however, the mechanism and effectiveness of this phenomenon are still unclear. Herein, we present a theoretical tool to extract different types of errors in high-level states encoding dispersive measurement. For the realistic parameters of Google's Sycamore processor, the use of state $|2\rangle$ is sufficient to suppress 92% of the decay readout error on average, where the total readout error is dominated by the background thermal excitation. We also show counter-intuitively that, the assistance of high-level states is effective in the measurement of logic 0, where there is no decay process.

→ readout very important & needed for error correction
summary: beautiful physics to be explored

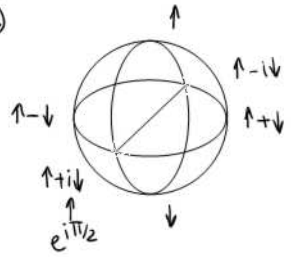
14. Quantum Cryptography

14.1 why?

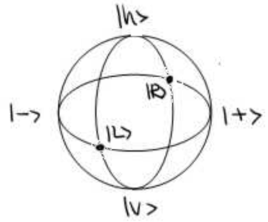
quantum information processing

1) already commercially available
2) more than QIP → qubits / photons
   → useful for many applications
3) if a quantum computer (QC) get's realized, Shor's algorithm will allow for code breaking
   → data info has to be secure for decades (50a)
   [if someone eve drops to day and stores the info → he/she will decode in 15a with a QC]

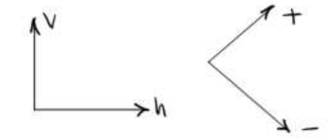
recap: e.g. $\bar{e}$ qubit (spin ½ ≙ two level system)
fermion



photon spin 1 ($\hbar$) → qubit? $m_s = \begin{smallmatrix}+1\\0\\-1\end{smallmatrix}$
boson
mass ≙ 0, v=c → one longitudinal DOF drops
$\vec{E} \perp \vec{B} \rightarrow \perp \vec{k}$



two basis:

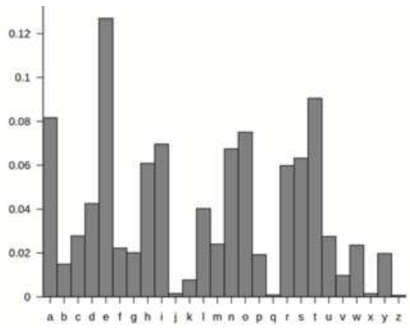


1) $|h\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $\quad |v\rangle = \begin{pmatrix}0\\1\end{pmatrix}$
2) $|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}(|h\rangle + |v\rangle)$
   $|-\rangle = \frac{1}{\sqrt{2}}(|h\rangle - |v\rangle)$
3) $|R\rangle = \frac{1}{\sqrt{2}}(|h\rangle + i|v\rangle)$
   $|L\rangle = \frac{1}{\sqrt{2}}(|h\rangle - i|v\rangle)$
   phase shift of $e^{i\pi/2}$

14.2 Crypto → historical view

see Arthur Eckart

→ sparta 400 BC (belt around cylinder)
→ ceaser 50 BC A→D (shifted by three letters XYZABC)
B→E
periodic boundary
↳ arbitrary shifts possible
→ Persian (Al Kindi) 800 AC



typical distribution of letters in english language text

→ statistics (how often do letters occur)
English (e, t, a, ... pair correlation th)
→ solution: evening out statistics by changing shifts (fast enough)
↳ see Enigma
→ solution: one time pad → random k as long as the message
key

prime numbers best example 73 → see Sheldon

1001001
64 + 8 + 1 = 73

↳ classical key distribution is based on the mathematical assumption that multiplying is easy and factoring exponentially hard

quantum key distribution: based on the laws of physics
→ Heisenberg
→ Bell inequality
↯ real devices are imperfect → Quantum Hacking

14.3 Quantum key distribution

Alice ⟷ Bob (Eve ↯)

here 1) BB84 (Bennet, Brassard 1984) Heisenberg
2) E91 (Arthur Eckert 1991) Bell

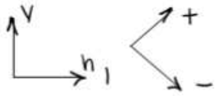
What is that key?

Alice	1) bit string	1 1 1 0 0 0 1 1 1 0 0 0
	2) key (random)	1 0 0 0 1 0 1 1 0 0 1 1
	3) X OR	0 1 1 0 1 0 0 0 1 0 1 1
Bob	4) key again	1 0 0 0 1 0 1 1 0 0 1 1
	5) X OR again	1 1 1 0 0 0 1 1 1 0 0 0

X OR operation ≙ Σ binary

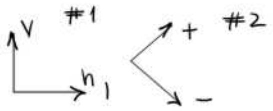
→ not random ≙ message

14.3.1 Protocoll of BB84

→ polarisation (free space) →  2 Basis ⊕ ⊗ (in fiber use phase & polarisation)

→ two channels I) Quantum channel (Eve can attack)
II) public channel (for everyone)
+ include superposition states [no-cloning theorem]
if it was possible, you could beat Heisenberg

10^6 copies → 500 000 on Δx, 500 000 on Δp } $\Delta x \Delta p \ll \rightarrow 0$

two different  basis

	0	1
⊕	↑	→
⊗	↗	↘

⚠ no single measurement can distinguish between 4 states
+ after measurement → we stay projected

14.3.2 Example BB84

1) A creates photon $|\vec{h}\rangle$ $|\vec{v}\rangle$ $|\vec{+}\rangle$ $|\vec{-}\rangle$
+ chooses basis at random
+ records which basis she used
2) B chooses random basis (⊕, ⊗) → 50% correct
50% wrong
+ records, which basis he used
3) A+B communicate on public channel about the bases they used (! not their result)
4) A+B disregard the cases of different bases
+ keep results in same basis
↳ translate result into 0 and 1

+ VL monday 10|07|23 → bis hier ausgedruckt finally → Example on BB84

10/07/23

Abit	0	1	1	0	0
Abasis	⊕	⊕	⊗	⊕	⊕
Asents	↑	→	↘	↑	↑
Bbasis	⊕	⊕	⊗	⊗	⊗
Bresult	↑	→	↘	↗	↘
?take it	0	1	1	✗	✗

} random choise

← choise of laser diode

← disregarded results

↯ random wrong

↳ wrong basis by someone

↯ random right

	0	1
⊕	↑	→
⊗	↗	↘

public share: basis they used

compare results (public channel) → check, whether errors occure even if identical basis was used

if not → secure channel

14.3.3 How to detect Eve

Abit	0	1	1
Abasis	⊕	⊕	⊗
Asent	↑	→	↘
Ebasis	+	×	+
Ebit	↑₀	↗₀	→₁ 50/50
Bbasis	⊕	⊗	⊗
Bbit	↑	↗	↗₀ 50/50
Public	V	basis off	basis V
key	0	X	0 ↯

comparison

Eve has projected intermittingly

you're allowed to copy quantum states as long as they're eigenstates (superposition → projection, no copy)

→ they choose subset of their key for the public check

wrong basis → 50% right, 50% wrong

identical basis → 100% right

→ $P_{Eve} = 1-\left(\frac{3}{4}\right)^n$ ← number of bits of the key used for check

e.g. 73

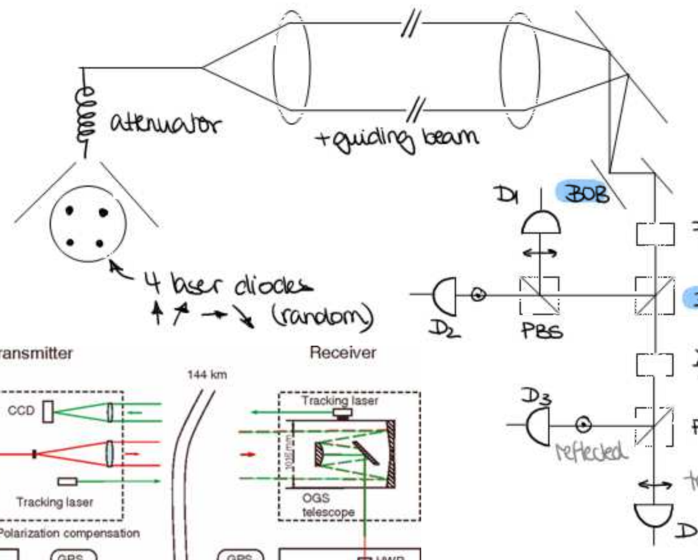
$\approx 1-10^{-9}$ "fundamentally" secure up to 10^{-9}

14.3.4 free space Quantum key distribution (QKD)

PRL 98, 010504 (2007)

RMP, Nature (Miscus)

Palma → Teneriffa (144km)



4 laser diodes ↑ ↗ → ↘ (random)

Polarisator (to compensate for changes in polarization) e.g. due to air

BS just 50/50 no PBS!

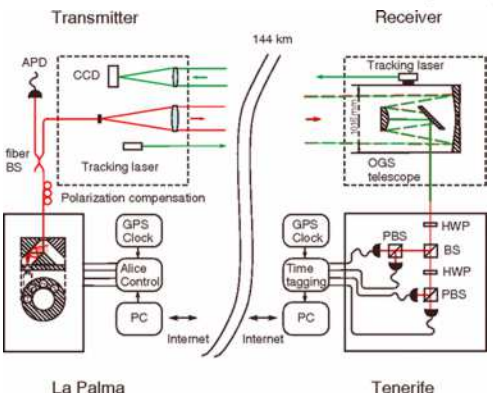
↳ polarizing beam splitter

$\frac{\lambda}{2}$ (tilt → into other basis) 22,5° (2×22,5° = 45°)

PBS

! random choise "of BOB" by his BS

≙ passive random number generator (RNG)



14.4 Hardware

14.4.1 Single photon sources

1) Attenuation down to single photon (on average single photon)

pulse possible

Eve could store one photon, waits for public communication on basis

→ 100% correct result without causing error

→ further alternate → Poissonian distribution $P_n(\mu) = \frac{\mu^n e^{-\mu}}{n!}$

mean value (μ)

\# of photon per pulse (n)

eg $\mu = 0{,}2$ (0,2 photons per pulse)

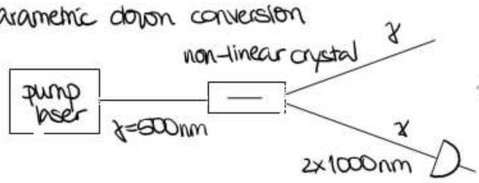
→ $P_0 = 0{,}8$
$P_1 \approx 0{,}2$
$P_2 \approx 0{,}01$

→ ? better $\mu = 0{,}02$ → $P_0 \gg$
$P_1 \approx 1\%$
$P_2 \approx 10^{-4}$

⊕ more secure
⊖ lower rate

will always be a major handicap

2) Parametric down conversion



frequency doubling stage → energy is conserved

detector: if click: other arm carries single photon

single photon source

inverted green laser pointer
(2x 1064nm → 1x 523nm)

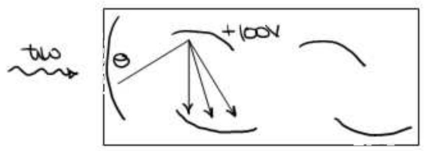
↯ non-deterministic (you know about single photon, but you can't decide when)

3) deterministic source

QM of two level system (Quantum dot, atom/ion in cavity)

14.4.2 Detection of single photons

1) Photomultipliers



⇒ $10^5 e$ but 1eV initial energy needed

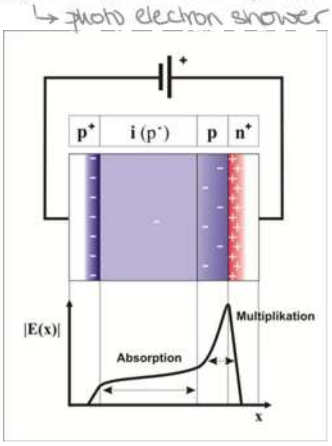
↯ $\lambda = 1550nm$ telecom ↯

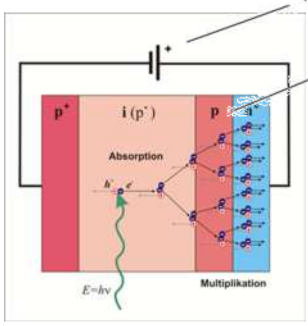
↳ E < 1eV

1eV ≙ 1,240 µm

2) Avalanche photo diode (solide state equivalent | PM)

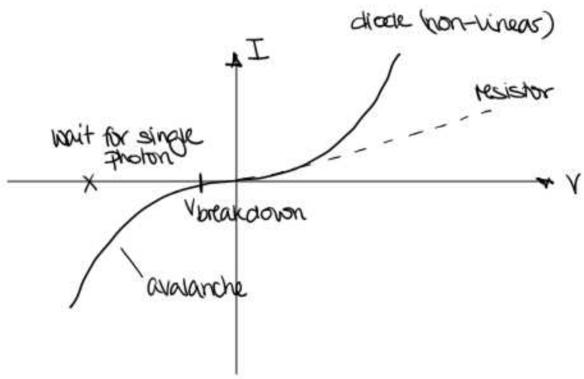
↳ photo electron shower

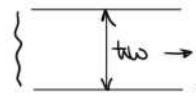




inverted diode

avalanche



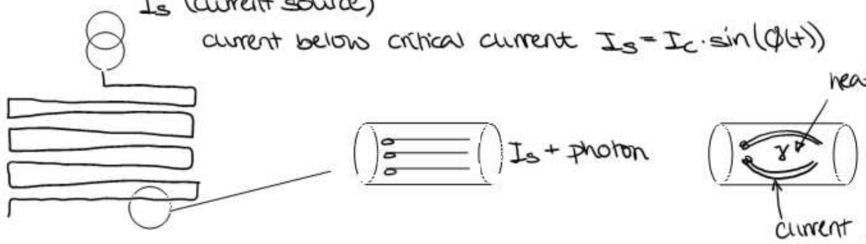
for GaAs → small band gap →  $h\nu$ → 1550nm are enough

vacuum

3) superconducting nano-wire

I_s (current source)

current below critical current $I_s = I_c \cdot \sin(\phi(t))$

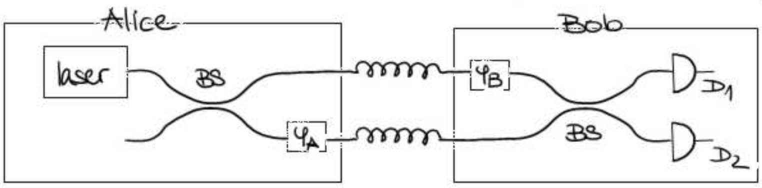


current goes up → $I \gg I_c$

→ voltage drop (observable)

14.4.3 Fibers

(change of polarization → phase)



interferometer

phase setup with extended Mach-Zehnder interferometer

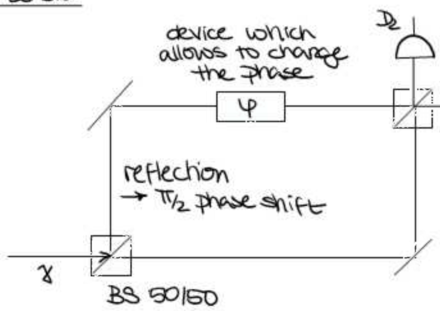
$\varphi_A = 0, \pi$ → basis ⊕ $\qquad \varphi_B = 0$ → ⊕

$\pi/2, 3\pi/2$ → basis ⊗ $\qquad \pi/2$ → ⊗

↳ same basis: count in $\begin{cases} D_1 \to 0 \\ D_2 \to 1 \end{cases}$

different bases → no correlation

#1 Mach Zender



one gets π phase shift

$2 \times \pi/2 = \pi$ → destructive interference → never clicks

both get $\pi/2$ phase shift

→ constructive interference

→ for $\varphi = 0$ D_1 will click

D_2 is off

#2 circular polarization

↻ + ↺ ≙ ↕ linear polarized light

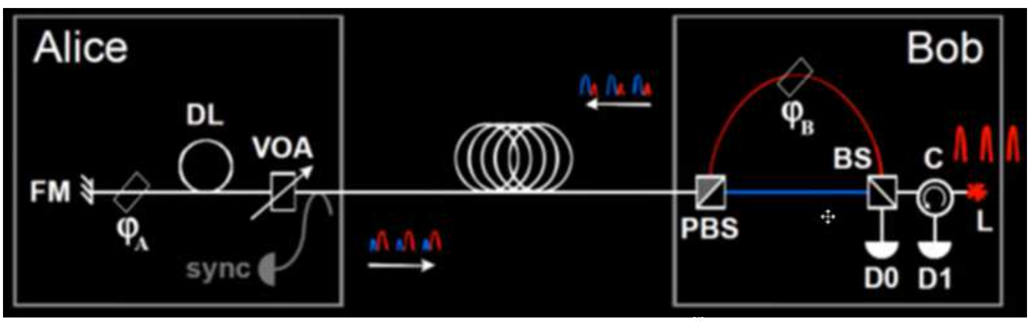
↻ + $e^{i\varphi}$ ↺ → tilted axis → chosing φ ≙ chosing basis

$\varphi_A = -45°$ or $+45°$ ≙ 0

$+135°$ or $-135°$ ≙ 1

14.5 Commercial system (IT Quantique)

see Markarov (Waterloo)



see fig. 2 of PRL

→ Bob sends & detects γ

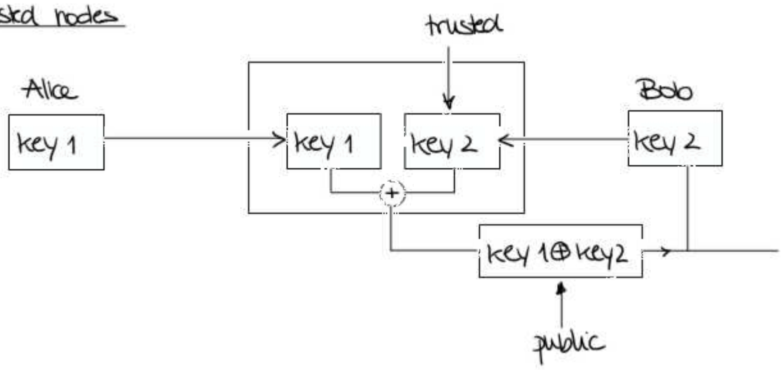
attenuated laser

↳ long & short path ~ control phase

~ in time seperated

→ limitation set by fibers $\lambda = 1550\,nm$ 300km → 50dB → 10^{-5} rate
attenuation 0,16 dB/km
↯ long distances (→ satellites)

→ trusted nodes

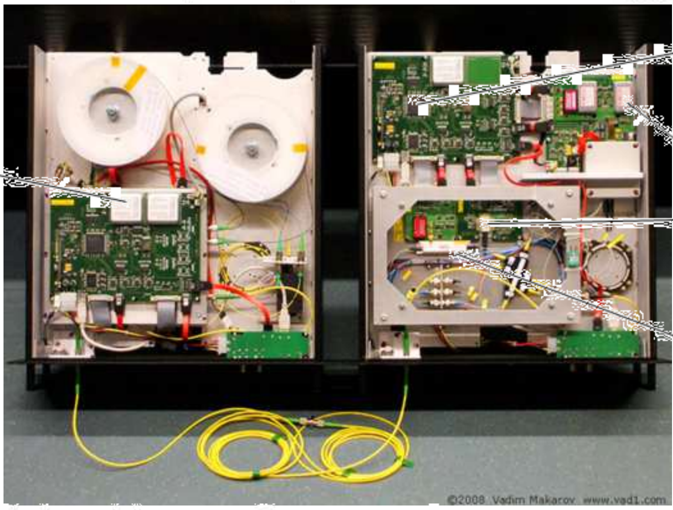


Bob wants to get K_1
$K_1 \oplus K_2 \oplus K_2 \rightarrow K_1$ again
(see XOR before)

14.6 Attacks

12/07/23

Quantique → opened by Makarov (Waterloo) Quantum Hardware



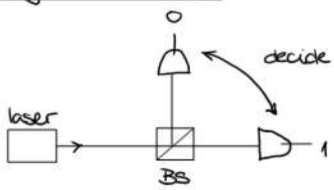
FPGA
RNG
Avalanche photodiods
laser
phase shifter

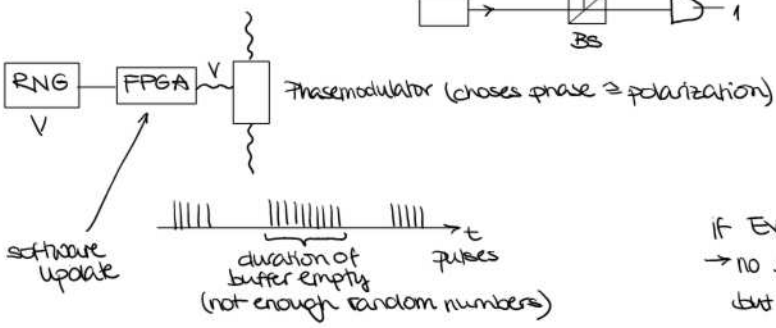
fibers: conserve phase, but polarization can differ due to temperature, tilts, ...

fundamentally secure? → check for real world

Intro: easy attacks

1. True randomness of random number generator (RNG)





if Eve would have found out (not Makarov)
→ no publication in a journal
but key gets public for Eve

2. Trust the manifucturer (deliver the device & the compounts)

compounts: · power supply
· connected to external world → communication via fibers
! if run RNG until 2024 random in real time
from then on $\underbrace{\text{stored (random) numbers}}_{\text{Eve}}$

→ expensive way out: build your chips yourself (NSA?)

14.6.1 Quantum attacs: double click

you expect ideally one count in one pulse at most

_ . _ . _ _ :

↑ double click possible

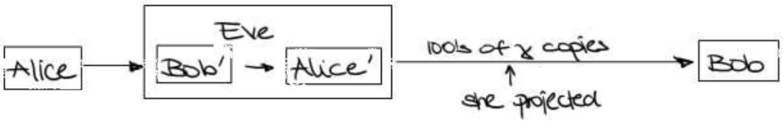
Why?

DC

→ detector provides dark cermats (out of work)

→ two photon pulses

} → disregard the mistake



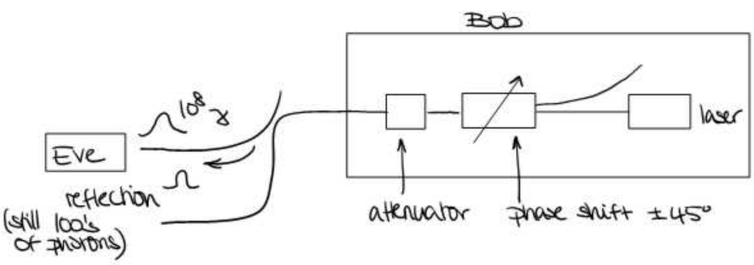
→ detects in Eve's basis → one click

→ detects not in Eve's basis → double clicks disregarded

- if Bob uses identical basis as Eve → Bob gets single clicks → keeps result
  - → if Eve chose basis of Alice (50% random) (BB84)
    - → Eve has copied the key bit
  - → if Eve chose basis different to Alice's
    - · Bob in Eve's basis (basis of A ≠ B)
      - → disregarded (BB84)
    - · Bob not in Eve's basis
      - → ⚡ double clicks → disregarded

(Eve leaves no trace other than in the # of double clicks)

14.6.2 Trojen horse



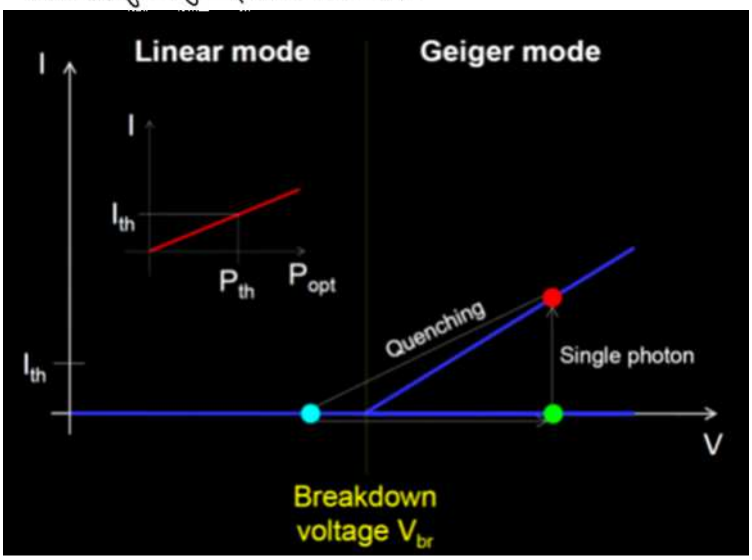
↳ Eve measures impact on her photons, detects state of phase shifts

→ knows the basis

→ can take and save copies

14.6.3 Blinding Bob

attacking single photon detectors



continious wave

↑

recepie: 1) Eve sends CW-light

2) photocurrent

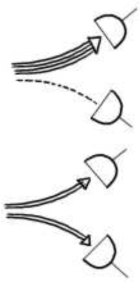
→ resistance > → voltage drop

if Eve wants Bob to get a click

→ $P_{classic}$ above $P_{threshold}$

Alice → [Eve: Bob' → Alice'] $\xrightarrow{\text{cw-laser, bright state copy}}$ Bob

\#1 Eve gets Bob linear (not Geiger anymore)
\#2 Eve sends bright state
(sufficient power to trigger the detector) → Bob uses Eve's basis
→ Bob uses different basis



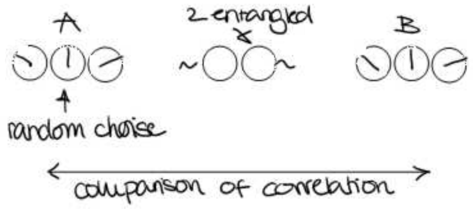
→ if Eve is right → copy
→ Bob right basis → ↯ work
→ Bob wrong basis → different from Alice → disregarded (BB84)

→ if Eve is wrong
→ Bob also wrong (compared to Alice, A+B) → disregarded (BB84)
→ Bob is right → no event (both detectors below threshold)
comparable to other loss

14.7 E91: Arthur Eckert

→ using pairs of entangled photons: Bell states $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$
↳ $|hv\rangle + |vh\rangle$ or $|+-\rangle + |-+\rangle$
↳ before detection, there is no polarization

→ A+B receive opposite results • for identical basis
→ opposite results → key
• for different bases
→ Bell inequality → violation of Bell → entanglement
→ not violated → Eve



14.8 Errors in the key

→ preperation
→ detection
→ noise
→ Eve ?
} assume all errors due to Eve

1) Error correction – classical (allowed to take copies)

1 1 1 0 0 1 (. . . .) 1
1 1 1 0 0 1 (. . . .) 1 → parity measurement $\Sigma_{\text{over all}}$ 0 ∨ 0, 1 ∨ 1 if they differ
↳ assumption: probability for single error $\ll 1$ → "lowest" P for two errors

2) Privacy amplification

simple in a nutshell: if Eve might know 50% of my bits → take two bits 0 or 1 / 0 or 1 → Eve knows one of them
sum two bits
→ 1 random number (Bob & Alice know but Eve can't)

		1
		1 ← Eve has some knowledge
0	1 0 1 1 1 0 0	0
1	1 1 0 0 0 1 1	0
1	0 1 1 1 0 0 0	1
3 bits	hash-function	0
	↳ removes information	1

7 bits

15. Quantum Computing (universal) → error correction

17/07/23

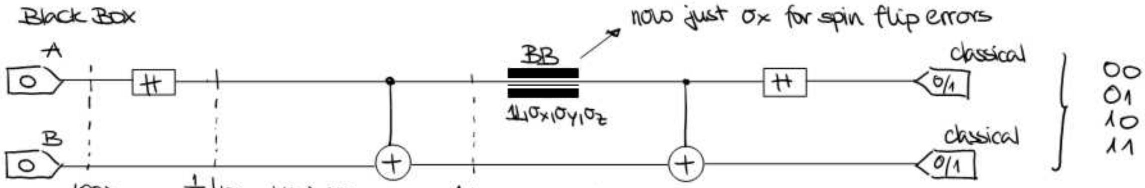
?Why ↔ currently but fidelity ≈ 10^{-4} (only)
↳ 1000 ancillae qubits per logical qubit (10^3–10^4) → 10^5–10^7 qubits needed

?Why not classical? no copies allowed (see crypto)
→ no parity check ↯ classically 10^9 $P_{1,error} \approx 1$, $P_{2,error} \approx 0$

15.1 Logical qubits/ancillae → using entanglement

→ spread information to several qubits (see black box problem)
↳ Hadamard + CNOT
↳ perform projection on ancillae only!
↳ create syndromes
↳ do corrections on logical qubit (without knowing its state)

Reminder Black Box



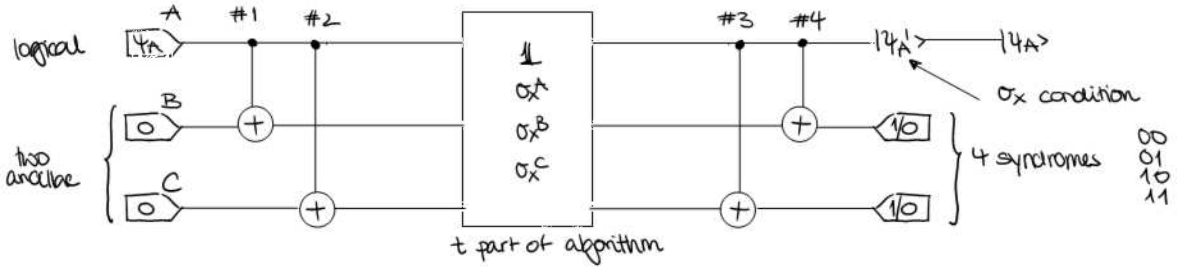
$|00\rangle_{AB}$ $\frac{1}{\sqrt{2}}(|0\rangle_A + |1\rangle_A)\cdot|0\rangle_B = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |10\rangle_{AB})$ $\frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$

projected both
→ learned out of syndrome whether $\mathbb{1}, \sigma_x, \sigma_y, \sigma_z$ occured

now: syndromes allow to project the error + correct the qubit (! without knowing the state of the qubit)

15.2 Correct for spin-flip errors (σ_x)

→ no projection of ψ_A
+ conditional action on ψ_A', if error occured to reconstruct ψ_A



protocol

0) $(\alpha|0\rangle + \beta|1\rangle)_A \cdot |0\rangle_B \cdot |0\rangle_C \xrightarrow{\binom{CNOT\#1}{AB}} \alpha|000\rangle + \beta|110\rangle \xrightarrow{\binom{CNOT\#2}{AC}} \alpha|000\rangle + \beta|111\rangle$
maximally entangled state

1) no error ≙ $\mathbb{1}$
$\alpha|000\rangle + \beta|111\rangle \xrightarrow{\binom{CNOT\#3}{AC}} \alpha|000\rangle + \beta|110\rangle \xrightarrow{\binom{CNOT\#4}{AB}} \alpha|000\rangle + \beta|100\rangle$
$\equiv (\alpha|0\rangle + \beta|1\rangle)_A \cdot |00\rangle_{BC}$
unperturbed ψ_A · product state · syndrome → $|00\rangle \hat{=} \mathbb{1}$ no error occured
→ projection of ancillae does not affect ψ_A

2) σ_x^A ≙ error on A
$\alpha|000\rangle + \beta|111\rangle \xrightarrow{\sigma_x^A} \alpha|100\rangle + \beta|011\rangle$ (spin flip) $\xrightarrow{\binom{CNOT\#3}{AC}} \alpha|101\rangle + \beta|011\rangle \xrightarrow{\binom{CNOT\#4}{AB}} \alpha|111\rangle + \beta|011\rangle$
$= \underbrace{(\alpha|1\rangle + \beta|0\rangle)_A}_{\psi_A' \text{ flipped}} \cdot \underbrace{|11\rangle_{BC}}_{\text{syndrome}}$

⚠ qubit A has been flipped, but no information about ψ_A
→ correction: σ_x^A again

3) $\sigma_x^B \hat{=}$ error on B

$\alpha|000\rangle + \beta|111\rangle \xrightarrow{\sigma_x^B} \alpha|010\rangle + \beta|101\rangle - \begin{pmatrix}\text{CNOT \#3}\\ AC\end{pmatrix} \rightarrow \alpha|010\rangle + \beta|100\rangle - \begin{pmatrix}\text{CNOT \#4}\\ AB\end{pmatrix} \rightarrow \alpha|010\rangle + \beta|110\rangle$

$\equiv \underbrace{(\alpha|0\rangle + \beta|1\rangle)_A}_{\psi_A' = \psi_A} \cdot \underbrace{|10\rangle_{BC}}_{\text{syndrome}}$

$\boxed{10}$

→ no operation needed, error occured on ancilla only

4) $\sigma_x^C \hat{=}$ error on C

$\alpha|000\rangle + \beta|111\rangle \xrightarrow{\sigma_x^C} \alpha|001\rangle + \beta|110\rangle - \begin{pmatrix}\text{CNOT \#3}\\ AC\end{pmatrix} \rightarrow \alpha|001\rangle + \beta|111\rangle - \begin{pmatrix}\text{CNOT \#4}\\ AB\end{pmatrix} \rightarrow \alpha|001\rangle + \beta|101\rangle$

$\equiv \underbrace{(\alpha|0\rangle + \beta|1\rangle)_A}_{\psi_A' = \psi_A} \cdot \underbrace{|01\rangle_{BC}}_{\text{syndrome}}$

$\boxed{01}$

→ no action needed

Σ create syndromes of ancillae

00 → nothing

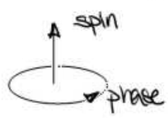
01 } only ancilla got flipped
10 }

11 → qubit got flipped → error correction needed → conditional $\sigma_x |\psi_A'\rangle = |\psi_A\rangle$

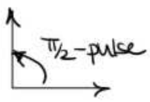
→ keeps qubit alive

15.3 Phase errors

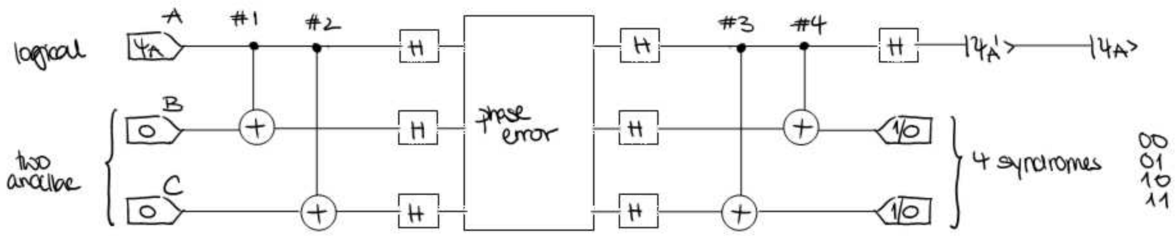
translate phase error → spin flip error



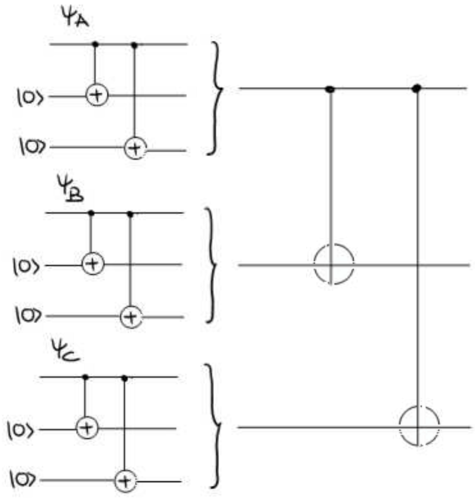
— rotation into new basis



e.g. $\frac{\pi}{2}\sigma_x$ (Hadamard gate)

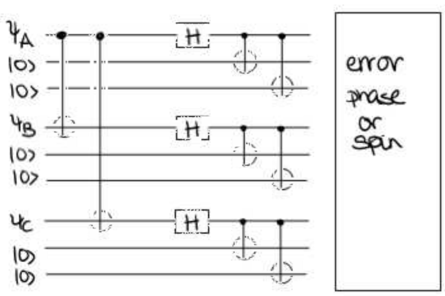


15.4 conduction of sets of qubits



additional protection via ancillae

15.5 arbitrary errors (spin, phase)



mirror action → read syndroms → conditional on ψ_A

→ Shor code (9 qubits)

Andrew sheme (7 qubits)

Laflamus (5 qubits)

↑ minimum number to correct for spin & phase error

15.6 Disclaimer

Over head scales evil with error

→ for 100% ⇝ 10^3–10^4 qubits for QC

→ for 99,99% ⇝ 100–1000 ancillae per logical qubit → 10^6 qubits over all

→ for 99% ⇝ 10^7–10^{12} qubits needed

[see Majorana qubits: google: no qubit so far]

≙ topological qubit

⊕ quantum error correction corrects all spin errors (not only complete flips)

? why: projection of ancillae projects the error!

visible error → σ_x for correction