Chord Voicing Feedback: Signal Comparison

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1 Introduction

The chord voicing feedback data processing and analytics comprises of two main stages: filtering and signal comparison. This document will focus on a method I developed to do the signal comparison. To be clear, the user will play a chord (input a signal) and that chord will be compared with a target chord (predetermined amplitudes of frequencies) in the frequency domain.

2 Method

Consider a piano chord that comprises of some frequencies with magnitude denoted as $|H_{s1}(\omega)|$ and a predetermined chord with magnitude $|H_{s2}(\omega)|$. Now consider a discrete parametric curve described by these parameters as functions of ω

$$\vec{C} = \begin{bmatrix} \omega \\ |H_{s1}(\omega)| \\ |H_{s2}(\omega)| \end{bmatrix}.$$

For each interval of ω, we can treat the set of vectors $\{\vec{C_iC_j}\}_{i=0,j=1}^{N}$ where C_i and C_j represent the vectors corresponding to the frequencies of the chord and N is the number of vectors that the curve is made up of. We want to compare the chords so that the user input will match the predetermined. So, variation is denoted by non-constant vectors in the curve. In the continuous case, we would like to force all vector derivatives to zero as that means there is no variation between the signals. To be able to provide valuable feedback there needs to be a spectrum of how close the user is to the threshold signal. It is natural to use the inner product as a method to do this. So, for each vector $\vec{C_iC_j}$ we would like its inner product with $\langle \vec{C_iC_j}_1, \vec{C_iC_j}_2, \cdots, \vec{C_iC_j}_{m-1}, 0\rangle$ to equal one. If, the inner product is one or very close to one, the user has played the chord correctly. The general formula will be

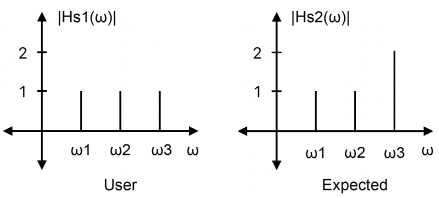
$$\left\{ \frac{1}{|[\overrightarrow{C_iC_j}]_{m\times 1}|} [\overrightarrow{C_iC_j}]_{m\times 1} \cdot \begin{bmatrix} \overrightarrow{C_iC_j}_1 \\ \overrightarrow{C_iC_j}_2 \\ \vdots \\ \overrightarrow{C_iC_j}_{m-1} \\ 0 \end{bmatrix} \right\}_{i=0,j=1}^{N} = V$$

where V is the set of inner products and $-\delta \leq v \leq \delta$ where δ is the tolerance parameter for each $v \in V$. This will determined later based on sample tests. This can be further simplified to

$$\left\{ \frac{1}{|[\overrightarrow{C_iC_j}]_{m\times 1}|} \sum_{k=1}^{m-1} ([\overrightarrow{C_iC_j}]_k)^2 \right\}_{i=0,j=1}^{N} = V$$

3 Example

For clarity, I will demonstrate with an example. Below are graphs of frequency domain representations of a triad played by the user and the expected triad.



Here we have three frequencies which means we will have two vectors of consideration. We have vectors $C_0 = \langle 1, 1, 1\rangle$, $C_2 = \langle 2, 1, 1\rangle$, and $C_2 = \langle 3, 2, 1\rangle$ which correspond to my original definition $C = \langle \omega, |H_{s1}(\omega)|, |H_{s2}(\omega)|\rangle$. So, $\overrightarrow{C_0C_1} = \langle 2, 1, 1\rangle - \langle 1, 1, 1\rangle = \langle 1, 0, 0\rangle$, $\overrightarrow{C_1C_2} = \langle 3, 2, 1\rangle - \langle 2, 1, 1\rangle = \langle 1, 0, 1\rangle$. Now that we have our C_iC_j, using my formula we get the set of inner products V.

$$\left\{ \frac{1}{|[\overrightarrow{C_iC_j}]_{m\times 1}|} \sum_{k=1}^{m-1} ([\overrightarrow{C_iC_j}]_k)^2 \right\}_{i=0,j=1}^{N} = \left\{ 1, \frac{1}{\sqrt{2}} \right\}$$

So, it can be concluded that the user needs to increase the amplitude of ω_3.