Entropy for Data Science

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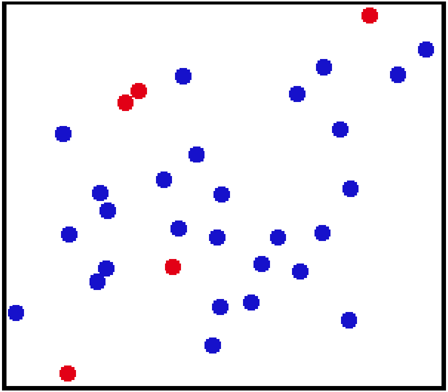
Outline

- History
  - Statistical Mechanics
  - Information Theory
- Shannon Entropy
  - Uniform Distribution
  - Normal Distribution
- Tsallis Entropy

History

Statistical Mechanics

Consider a box with N particles of a monatomic gas



How would you model this?

Statistical Mechanics - State Variables?

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$$PV = nRT \tag{1}$$

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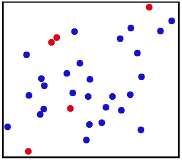
- Characterize System Behaviors

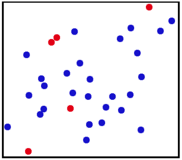
Statistical Mechanics - Ensemble Statistics



Assume each particle obeys Newton's Law

- v_0 and x_0 determines system
- Impractical for large N

Statistical Mechanics - Ensemble Statistics



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James Maxwell's Kinetic Theory of Gases

- Consider Ensamble Statistics

$$PV = \frac{Nm\bar{v}^2}{3} \tag{2}$$

Statistical Mechanics Entropy

Average Behaviour $\rightarrow$ Macroscopic Properties

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Ludwig Boltzman's statistical mechanical entropy

$$S = k_b \ln(\Omega) \tag{3}$$

Ω is the multiplicity of the macrostate

Microstates, Macrostates, and Multiplicity

Consider three coin flips

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- $\Omega = \binom{3}{2}$