Entropy for Data Science

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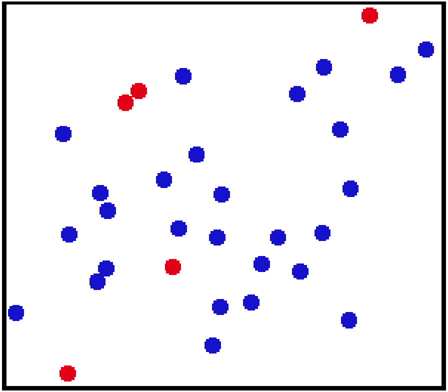
Outline

- History
  - Statistical Mechanics
  - Information Theory
- Shannon Entropy
  - Uniform Distribution
  - Normal Distribution
- Tsallis Entropy

History

Statistical Mechanics

Consider a box with N particles of a monatomic gas



How would you model this?

Statistical Mechanics - State Variables?

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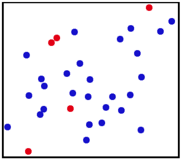
- Characterize System Behaviors

Statistical Mechanics - Ensemble Statistics



Assume each particle obeys Newton's Law

- v_0 and x_0 determines system
- Impractical for large N

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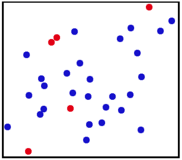

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James Maxwell's Kinetic Theory of Gases

- Consider Ensamble Statistics

$$PV = \frac{Nm\bar{v}^2}{3} \tag{2}$$

Statistical Mechanics Entropy

Average Behaviour $\rightarrow$ Macroscopic Properties

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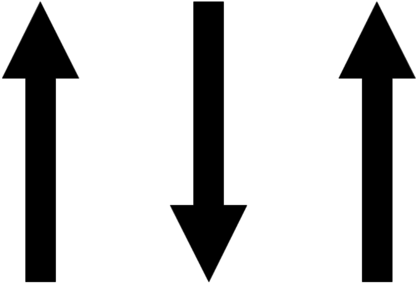
Ludwig Boltzman's statistical mechanical entropy

$$S = k_b \ln(\Omega) \tag{3}$$

Ω is the multiplicity of a given macrostate

Macrostates, Microstates, and Multiplicity

Consider non-interacting paramagnet with 3 dipoles



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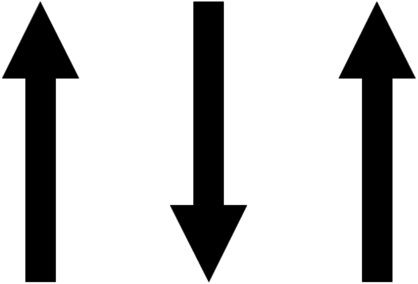

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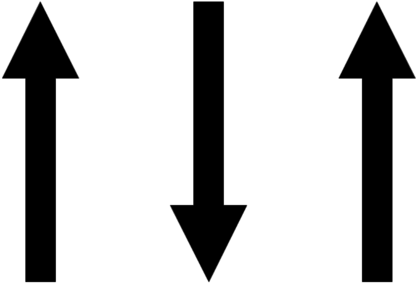

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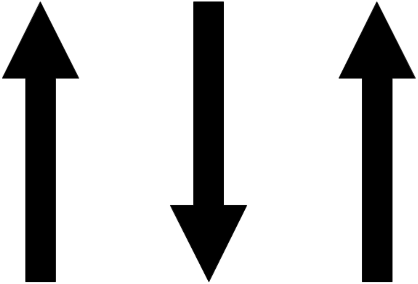
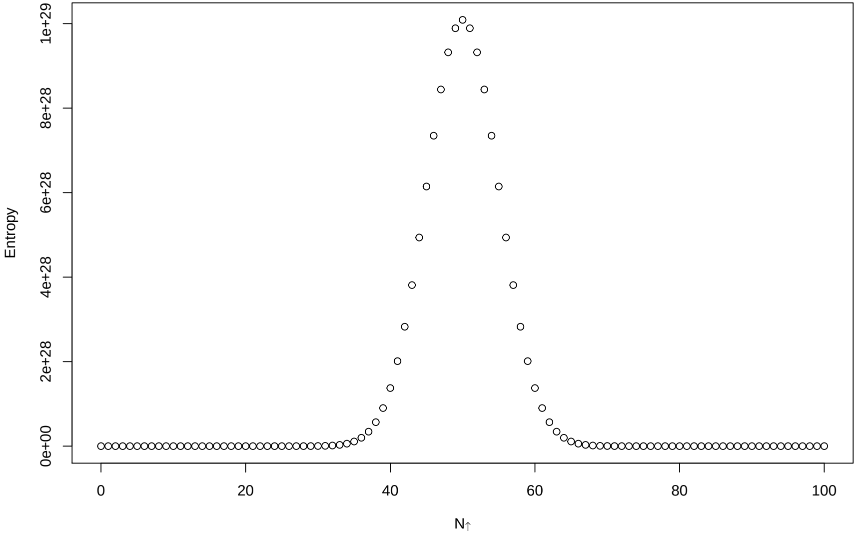


- Macrostate - 2 Up, 1 Down
- Microstate - $\uparrow, \downarrow, \uparrow$
- $\Omega = \binom{N}{N_\uparrow} = \binom{3}{2}$

Entropy of 100 Dipole Paramagnet



Interpretation

Features of paramagnet entropy

- Minimum at 0 and 100 $\rightarrow$ 1 microstate each
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Measure of “mixed-up-ness" of a physical system

- Higher entropy $\rightarrow$ more mixing (randomness)
- Lower entropy *rightarrow* less mixing