FINA3340:

Risk Management Project (I)

Managing the Market Risk Exposure with Hang Seng Index Futures

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Executive Summary

The following report will examine how we can use Hang Seng Index Futures to hedge our HK $100 million stock portfolio against the potential market risk of the position during the next three-month period - September 7, 2018 to December 7, 2018. The report will first introduce the design of our hedging strategy and then use both “Backtesting” method and scenario analysis to evaluate the effectiveness of our risk elimination strategy. In the end, it would also cover the partial hedge strategy.

PART 1 Hedging Scheme

We have to first admit there are four considerations in our practical implementation of hedging:

1. Standardized future contracts can’t perfectly match our time horizon, thus, we will utilize the alternative best futures contract: the one with the contract months to be December, and it will be closed out before the maturity day;
2. The portfolio being hedged is different from the full composition of Hang Seng Index;
3. The distribution of rate of return is more stable over time, thus, we would base our design on past return data rather than the absolute change. In this way, we assume that standard deviations of past rate of return will approximately be the same as in the future;
4. As the contract has certain maturity date, the time to maturity for the contract is changing, which deviates from the perfect theoretical model. So, here we would use estimated futures price based on underlying asset price to get rid of the influence of changing time premium.

Combining all of the above, we reach at our core formula to get the optimal hedging unit of the futures contract:

$$N^* = [\beta_{S,I} \times exp\{(r - q) \times (t_2 - t_1)\}] \times \frac{S_1}{F_1} \times \frac{N_A}{Q_F} \qquad (*)$$

where $\beta_{S,I}$ is the correlation coefficient between unhedged portfolio return and Hang Seng Index return, t_2 is the closing out date, t1 is the hedge implementation date, $S_1 \times N_A$ is the portfolio value at t_1, F_1 is futures price of Hang Seng Index futures contract at t_1, Q_F is the futures contract multiplier, r and q are risk-free interest rate and dividend yield of Hang Seng Index in continuously compounding form respectively.

Below, the report would find out step by step the elements in this equation to get the optimal units.

(a) $\boldsymbol{\beta_{S,I}}$

Practically, for better risk hedging, our portfolio’s risk exposure should be constant. To maintain this constant beta, the weighting of each individual stock in the portfolio would be constant during the next three months. That is, the number of shares may vary every day due to the fluctuation of stock prices. We can get the weighting of each individual stock's dollar value at hedge implementation time (t_1) from APPENDIX (I): MUTUAL FUND PORTFOLIO.

7-Sep-18				
Stock Name	Market Price	Number of Shares	Market Value	Weight
CATHAY PACIFIC AIRWAYS	11.4	100000	1,140,000	1.14%
CHINA MOBILE	75.65	60000	4,539,000	4.54%
CHINA RESOURCES BEER HOLDINGS	33.1	750000	24,825,000	24.82%
CHEUNG KONG INFR.HDG.	59.75	20000	1,195,000	1.19%
CLP HOLDINGS	91.2	20000	1,824,000	1.82%
HANG SENG BANK	203.2	20000	4,064,000	4.06%
HONG KONG AND CHINA GAS	15.76	100000	1,576,000	1.58%
HSBC HOLDINGS	67.2	360000	24,192,000	24.19%
HYSAN DEVELOPMENT	39.25	200000	7,850,000	7.85%
LENOVO GROUP	5.33	200000	1,066,000	1.07%
LI & FUNG	2.07	1080000	2,235,600	2.24%
MTR	39.85	50000	1,992,500	1.99%
SUN HUNG KAI PROPERTIES	111.8	200000	22,360,000	22.36%
TELEVISION BROADCASTS	22.95	50000	1,147,500	1.15%
		Total	100,006,600	100.00%

Exhibit 1. Portfolio Weighting Overview

Combining the daily realized return of each individual stock, the time-series of historical return of our portfolio could be attained. In this process, two formulas are used:

$$R_{i,t} = (P_{i,t} - P_{i,t-1})/P_{i,t-1} \times 100\%$$

where $R_{i,t}$ is the daily realized return of individual stock (i^{th}) at trading day (t) and $P_{i,t}$ is the price of individual stock (i^{th}) at trading day (t).

$$R_{S,t} = \sum_{i=1}^{N} (\omega_i \times R_{i,t})$$

where $R_{S,t}$ is the daily realized return of our portfolio at trading day (t).

By fitting a simple regression between our portfolio and Hang Seng Index portfolio, $\beta_{S,I}$ can be estimated as slope of the regression, which reflects the sensitivity of the realized return on our portfolio with respect to the realized return on the Hang Seng Index portfolio. The period of historical data we choose is the past one year realized return data, from Sep-8-2017 to Sep-8-2018. Thus, we get $\beta_{S,I} \approx 0.6439$.

(b) r & q

To be utilized in our core formula, the interest rate and dividend yield are in continuously compounding form. The next step is to convert annually compounding form of these two into continuously compounding form. Hence, we have:

$$(1 + 1.15\%/4)^4 = e^r$$

$$1 + 3\% = e^q$$

where 1.15% is the 3-month HKD risk-free interest rate on September 7, 2018 and 3% is the dividend yield for Hang Seng Index portfolio. Thus, $r \approx 1.1484\%$ and $q \approx 2.9559\%$.

(c) Theoretical Hang Seng Index futures price on September 7th
Using formula

$$F_t = I_t \times exp\{(r - q) \times (T - t)\}$$

where F_t is the futures price, I_t is the level of Hang Seng Index at time t, respectively, and T is the maturity day of the contract (We will discuss this calculation method in detail in part 4 of this report), we find the theoretical $\mathrm{F} = 26825.61$ on September 7th, which is 44.4 points lower than the settlement price on September 7th. This suggests that the futures contract is overpriced and we may earn some extra benefit from shorting the contract.

(d) The remaining Parameter
For simplicity, $t_2 - t_1$is considered as 0.25 year, and the unhedged portfolio value $S_1 \times N_A$is viewed as 100 million Hong Kong dollars value. From Hang Seng Index Futures Prices as at September 7, 2018, the settlement price for third delivery month (December 2018) is $F_1 = 26870$. According to APPENDIX (IV): CONTRACTUAL SPECIFICATION FOR HANG SENG INDEX FUTURES IN HKFE, $Q_F = \$50$.

(e) Recap the above
Plugging in all the values into the core formula, we reach at $N^* \approx 47.7122$. Moreover, N^*must be an integer, so we take $N^* = 48$. That is, by taking 48 units of short position in Hang Seng Index Futures contract, market risk in our portfolio can be almost fully hedged, which is our best hedging scheme design up to now. The specific Hang Seng Index Futures contract, which matches our investment horizon the best, would be the one with the contract months of December. This contract's delivery month is December 2018 and would have its maturity date, which is also the last trading day, of December 28th.

PART 2 Total Risk Exposure with/without Hedging Scheme

From the time-series data of historical realized returns, daily standard deviation can be easily estimated. The period of historical data we choose is the past one year realized return data, and we use the formula for sample (not population) standard deviation. Thus, by calculation, we find $\sigma_{1d} \approx 0.0088$and $\sigma_{3d} \approx 0.0105$.

Since the number of trading days from September 8, 2018 to December 28, 2018 is 62, the standard deviation of three-month returns can be estimated using the formula

$$\sigma_1 = \sigma_{1d} \times \sqrt{62} \approx 0.0694$$
$$\sigma_3 = \sigma_{3d} \times \sqrt{62} \approx 0.0823$$

The correlation coefficient between the realized returns on our portfolio and the realized returns of Hang Seng Index portfolio ($\rho_{S,I}$) can be estimated using the correlation function in EXCEL. Thus, by calculation, we find $\rho_{S,I} \approx 0.7637$.

In order to estimate the risk exposure and its effectiveness when using the hedging scheme, we are able to obtain the following:

Total fluctuation of our asset value without hedge:

$$(N_A \times S_1)^2 \times {\sigma_1}^2 \approx 4.8147 \times 10^{13}$$

Total fluctuation of our hedged portfolio value without hedge:

$$(N_A \times S_1)^2 \times {\sigma_1}^2 \times (1 - {\rho_{S,I}}^2) \approx 2.0069 \times 10^{13}$$

Total fluctuation of market asset value eliminated by our hedging scheme:

$$(N_A \times S_1)^2 \times {\sigma_1}^2 \times {\rho_{S,I}}^2 \approx 2.8078 \times 10^{13}$$

The corresponding systematic risk proportion in our portfolio is able to be hedged while some specific risk is not. Hence, the hedging effectiveness, which is the proportion of total market risk which can be eliminated among the total risk of our portfolio, is equal to ${\rho_{S,I}}^2$, that is, around 58.32%. This indicates that by our hedging scheme, the total risk of our originally unhedged portfolio can be eliminated by approximately 58.32%. Theoretically, the proportion is over 50%, so the scheme is effective to a moderate degree, even if not perfect hedge. But in real life, transaction costs, changing standard deviation and correlation, and many other factors should also be taken into consideration, so it may not be that optimistic than we expect.

PART 3 Backtesting

Besides the theoretical examination of the hedging effectiveness, a backtesting method is adopted to evaluate the performance of the scheme using real-life data. Here, we assume the weighting of each individual stock in our portfolio remain constant as the weighting shown in Exhibit.1. Thereafter we set up a 3-month hedging scheme on March 7, 2018. Using the same method as stated in part 1, we find:

$$\beta_{S,I} \approx 0.5879$$

$$r \approx 0.5996\%$$
$$F_1 = 29940$$

Here we pick the 3-month risk-free rate spotted on March-7-2018 to calculate the continuously compounding form. Other parameter values such as $S_1 \times N_A$and Q_F are the same as part 1. Plugging these results into the formula $(*)$ and round N^* to be an integer, we find $N^* = 39.00$. Thus, to fully hedge the market risk in our portfolio, we should short 39 units of Hang Seng Index Futures contract, with a delivery month of June 2018.

Since the realized returns of the portfolio and the data of Hang Seng Index Futures price $(F_2 = 31535)$ on June 7, 2018 are available now, the performance of our hedging scheme can be evaluated.

P/L from Hang Seng Index Futures position:

$$-(F_2 - F_1) \times N^* \times Q_F \approx -3110250$$

P/L from unhedged portfolio:

$$RR \times S_1 \times N_A \approx 6896375.4$$

where RR is the rate of return of unhedged portfolio over March 7, 2018 to June 7, 2018.

Thus, total P/L of the hedged portfolio is the sum of the two components above, which is approximately 3786125.4. In percentage form, the total return is 3.79% of the initial investment (unhedged portfolio value).

In this backtesting, the P/L of the unhedged portfolio is actually higher than the P/L of the hedged portfolio, but it does not mean that our hedging scheme is poor, since over this period (i.e. March 7, 2018 to June 7, 2018), the performance of Hang Seng Index fluctuated moderately and did not experience a sharp decrease. On March 7, 2018, the Hang Seng Index Futures price for the third delivery month (June 2018) is 29940, but the theoretical price for this Futures is approximately 29981, calculated by formula

$$F_t = I_t \times exp\{(r - q) \times (T - t)\}.$$

Note that this is 41 points higher than actual price, meaning that the futures contract was actually underpriced on March 7. This may be because people were over pessimistic about the market and expected Hang Seng Index to drop. Since we would take a short position in the futures, the underpricing may bring us some extra loss. Nevertheless, on June 7, 2018, the Hang Seng Index was actually higher, so that $F_2$was much higher than F_1, which further resulted in the loss in our short position in futures.

PART 4 Scenario Analysis

Apart from the backtesting method to evaluate the effectiveness of our hedging scheme using the past data, we then conduct a scenario analysis to evaluate the performance of the scheme based on different hypothetical cases of future market performance.

To estimate the realized return on Hang Seng Index portfolio, we add together the rate of change of Hang Seng Index and the dividend yield of Hang Seng Index portfolio. The reason why the dividend yield is also included is that theoretically, it is also a part of the return that the holder can get.

Realized return on Hang Seng Index portfolio:

$$R_{M,i} = \frac{(I_{t2,i} - 26973.47)}{26973.47} + \frac{3\%}{4}$$

where $R_{M,i}$ is the adjusted return of Hang Seng Index in case i and $I_{t2,i}$is the hypothetical Hang Seng Index level in case i, which may take the value of 10000, 15000, 20000, 25000, 30000, and 35000.

After estimating the hypothetical realized return on Hang Seng Index portfolio, the conditional expected return on our portfolio can be estimated as well using CAPM. That is,

$$E[R_i] = R_f + \beta_{S,I}\{E[R_{M,i}] - R_f\}$$

where $\beta_{S,I} \approx 0.6439$and $R_f = 1.15\%/4$.
In CAPM, the rates are in annually compounding form, so that we do not need to convert them into continuously compounding forms here.

	Hypothetical Hang Seng Index					
	10000	15000	20000	25000	30000	35000
R_M	-62.18%	-43.64%	-25.10%	-6.57%	11.97%	30.51%
$E[R_i]$	-39.93%	-28.00%	-16.06%	-4.13%	7.81%	19.75%
P/L (unhedged)	-39,934,121.4	-27,998,013.8	-16,061,906.2	-4,125,798.7	7,810,308.9	19,746,416.5

Exhibit 2. Expected returns and P/L from unhedged portfolio under different scenarios

According to APPENDIX(IV): CONTRACTUAL SPECIFICATION FOR HANG SENG INDEX FUTURES IN HKFE, the last trading day is the business day immediately preceding the last business day of the contract month, so that it will be December 28, 2018 (*T*). We will close out the contract on December 7, 2018 (t_2). Hence, theoretical price for Hang Seng Index Futures price can be found using the formula:

$$F_{t2,i} = I_{t2,i} \times exp\{(r - q) \times (T - t_2)\}$$

where $r \approx 1.1484\%$ and $q \approx 2.9559\%$, which have been converted to continuously compounding form in part 1. Here we use the 3-month risk-free interest rate on September 7th because the risk-free interest rate on December 7th is unknown, and September 7th is the closest available day. However, in reality, this rate will probably change during the 3 months.

Since there are 21 days from December 7, 2018 to December 28, 2018, $T - t_2$can be considered as 21/365.

	Hypothetical Hang Seng Index					
	10000	15000	20000	25000	30000	35000
$F_{t2,i}$	9,989.6	14,984.4	19,979.2	24,974.0	29,968.8	34,963.6
P/L (futures)	40,512,945.8	28,525,418.7	16,537,891.6	4,550,364.5	-7,437,162.6	-19,424,689.7

Exhibit 3. Theoretical futures price and P/L from futures position under different scenarios

Hence, by adding P/L from unhedged portfolio and P/L from futures position, we can find the P/L of hedged portfolio. By adding the total P/L to the initial total portfolio value, we get the total value of the investment at the Futures closing-out date.

	Hypothetical Hang Seng Index					
	10000	15000	20000	25000	30000	35000
P/L (unhedged)	-39,934,121.4	-27,998,013.8	-16,061,906.2	-4,125,798.7	7,810,308.9	19,746,416.5
P/L (futures)	40,512,945.8	28,525,418.7	16,537,891.6	4,550,364.5	-7,437,162.6	-19,424,689.7
Total P/L	578,824.4	527,404.9	475,985.4	424,565.8	373,146.3	321,726.8
Total Value	100,578,824.4	100,527,404.9	100,475,985.4	100,424,565.8	100,373,146.3	100,321,726.8
P/L (%)	0.58%	0.53%	0.48%	0.42%	0.37%	0.32%

Exhibit 4. Overall performance under different scenarios

Therefore, under this hedging scheme, theoretically, we are always able to make moderate profits, which is about 0.4% or 0.5%, under different scenarios. Since all the market risks are eliminated (actually not because the contract number N* must be rounded to an integer, but the difference is small), the fluctuation from Hang Seng Index only will not have impact on the hedged portfolio value directly. The gain of the portfolio mainly comes from the risk-free interest.

However, there are many strong assumptions here, which tend to differ from real life. For example, the future price at the Futures closing-out date we use here is theoretical. In real life, due to the influence of macro economy and people's expectation, the future price may be different. Additionally, risk-free interest rate may fluctuate. Moreover, in our analysis, we use the expected return of unhedged the portfolio, but realized return is probably different from the expected return. And the performance of the companies can be unstable sometimes, so that firm-specific risks may also be something we need to consider. For instance, if government launches a scheme to curb the climbing housing price, then stocks related to real estate (e.g. Sun Hung Kai Properties) may be influenced.

PART 5 Partial Hedge

(a) Hedging Scheme

While the above hedging scheme eliminates all the market risks, we find that it also eliminates the potential upside of the portfolio. Therefore, we think a partial hedging scheme that only hedge against part of the market risk could be implemented. Now we are going to set up a hedging scheme to reduce the market beta from 0.6439 to 0.5. Here Hang Seng Index Futures contract with delivery month of December 2018 is still chosen.

Then by using formula

$$N^* = [(\beta_{S,I} - {\beta_{S,I}}^*) \times exp\{(r-q) \times (t_2 - t_1)\}] \times \frac{S_1}{F_1} \times \frac{N_A}{Q_F}$$

where $\beta_{S,I} = 0.6439$ and ${\beta_{S,I}}^* = 0.5$, other parameters the same as in part 1, we get the number of futures contracts (N) to be shorted, which is 11, rounded up from 10.664.

Therefore, by shorting 11 units of Hang Seng Index Futures contracts with a delivery month of December 2018, together with the original unhedged portfolio, our "partial" hedging scheme has a market beta of 0.5.

(b) Scenario Analysis

A similar scenario analysis is carried out for this "partial" hedging scheme as well to investigate the performance of the portfolio under different realizations of market performance. We analyze the theoretical return of the hedged portfolio when Hang Seng Index goes to 10000, 15000, 20000, 25000, 30000, and 35000, respectively. Following the steps we have already stated in part 4, we get $R_{M,i}$, $E[R_i]$, $F_{t2,i}$, P/L of unhedged portfolio and futures position, and then the total P/L. Finally, we calculate the total value of the investment and total P/L in percentage of the initial investment, which is the total value of the original unhedged portfolio on September 7th. Results are shown in Exhibit 5 below.

	Hypothetical Hang Seng Index					
	10000	15000	20000	25000	30000	35000
R_M	-62.18%	-43.64%	-25.10%	-6.57%	11.97%	30.51%
$E[R_i]$	-39.93%	-28.00%	-16.06%	-4.13%	7.81%	19.75%
$F_{t2,i}$	9,989.6	14,984.4	19,979.2	24,974.0	29,968.8	34,963.6
P/L (futures *)	9,284,216.7	6,537,075.1	3,789,933.5	1,042,791.9	-1,704,349.8	-4,451,491.4
P/L (unhedged)	-39,934,121.4	-27,998,013.8	-16,061,906.2	-4,125,798.7	7,810,308.9	19,746,416.5
Total P/L*	-30,649,904.6	-21,460,938.7	-12,271,972.7	-3,083,006.8	6,105,959.1	15,294,925.1
Total Value	69,350,095.4	78,539,061.3	87,728,027.3	96,916,993.2	106,105,959.1	115,294,925.1
P/L (%)	-30.65%	-21.46%	-12.27%	-3.08%	6.11%	15.29%

Exhibit 5. Scenario Analysis under partial hedging scheme

Notice that if Hang Seng Index drops to 10000, the total loss of the portfolio is 30.65%. And if Hang Seng Index increases to 25000, the total gain would be 15.3%. Compared to the fully hedging scheme, the potential loss is much higher than the fully hedging scheme, while the potential gain is also much higher. Profit/Loss fluctuates more significantly under the partial hedging scheme, since the latter one is exposed to a certain degree of market risks. When Hang Seng Index fluctuates, our portfolio will fluctuate to some extent accordingly.

Conclusion

In this project, we first establish a full hedge by shorting 48 units of Hang Seng Index Futures contract with a delivery month of December 2018 in order to make the hedged portfolio have a β of 0. We then explore the effectiveness and performance of the scheme by calculating the theoretical hedging effectiveness, which is the proportion of total risk eliminated, backtesting on March 7th, and scenario analysis. Finally, we set up a "partial" hedging scheme to protect the potential gain from the investment, and we find that a short position in 11 units of Hang Seng Index Futures contract with a delivery month of December 2018 will make the β = 0.5.