FINA3340:

Risk Management Project (II)

Managing the Market Value of Equity for Bank

Group members

Name	UID
Cai Yitong	3035235185
Yang Zhibo	3035232884
Ke Daqi	3035234600

Executive Summary

The following report will examine the interest rate risk exposure taken by ABC Bank and specifically the investment of Treasury debts. To investigate and control the interest rate risk, we will mainly utilize the Macaulay Duration and convexity and introduce scenario analysis to test the influence of potential future interest rate change. In the end, under certain constraints on duration GAP, we suggest investing in T-Debt C with interpretation and limitation of the choice.

Part I Interest Rate Risk of Net Worth

a. Calculation of Macaulay Duration

First we calculate the weight distribution of ABC Bank's Assets and Liabilities.

For Assets, $w_i = \frac{Market\ Value\ of\ Asset_i}{Total\ Market\ Value\ of\ Assets}$. For liabilities, $w_i = \frac{Market\ Value\ of\ Liability_i}{Total\ Market\ Value\ of\ Liabilities}$.

The weights will be used to calculated Macaulay Durations for the bank's assets and liabilities separately. To calculate the overall duration for Assets or Liabilities, we use formula:

$$D_{Macaulay}^{Portfolio} = \sum_{i=1}^{n} w_i \times D_{Macaulay}^{Item\ (i)}$$

Then we have: $D_{Macaulay}^{Asset} = 4.00$, $D_{Macaulay}^{Liabilities} = 2.066$. In other words, the overall Macaulay Duration for the bank's assets is 4 years and the overall Macaulay Duration for the bank's liabilities is 2.066 years.

b. Calculation and Analysis of Duration Gap

As our focus would be the interest rate risk taken by the net worth of the bank, we would look at Duration Gap to reflect the Macaulay Duration of equity due to the difficulty to directly get this. Due to the equation Assets = Liabilities + Equity, we have

$$DUR_{Macaulay}^{GAP} = D_{Macaulay}^{Asset} - \frac{L}{A} \times D_{Macaulay}^{Liabilities} = \frac{E}{A} \times D_{Macaulay}^{Equity}$$

where L, A and E stand for total market value of liabilities (HK$ 530 billion), assets (HK$ 600 billion) and equity (HK$ 70 billion), respectively. We arrive at $DUR_{Macaulay}^{GAP} = 2.175$, and further, $D_{Macaulay}^{Equity} = 18.6429$. Note that the bank's net worth is just the market value of Stockholders' Equity, we have the Macaulay Duration for the bank's net worth is 18.6429 years.

From the above calculation, the bank's Duration Gap is positive. Due to the ability of Duration to reflect the interest rate risk, which is the percentage change of portfolio value with respect to the interest rate change, we have the equation:

$$\frac{\Delta NetWorth}{NetWorth} = -DUR_{Macaulay}^{GAP} \times \frac{\Delta y}{1 + \left(\frac{y}{2}\right)} \times \frac{A}{NetWorth}$$

Thus, positive duration gap implies that: when interest rate increases, $\Delta y > 0$, as Duration Gap, Asset value and net worth for Bank ABC are all positive, $\frac{\Delta NetWorth}{NetWorth} < 0$, that is, the bank's net worth will decrease due to an increase in market interest rate; Conversely, when interest rate decreases, $\Delta y < 0$,and hence, $\frac{\Delta NetWorth}{NetWorth} > 0$, the bank's net worth will increase responding to a decrease in the market interest rate.

c. Scenario Analysis

Based on the assumption that each hypothetical interest rate change Δy is applied to ALL interest rates (discount rates) of all balance sheet items, we have the following for each item in assets or liabilities separately: $\Delta A_i/A_i \approx {D_{Mac}}^{Ai} * \Delta y/(1+y_i/2)$ $,\Delta L_i/L_i \approx {D_{Mac}}^{Ai} * \Delta y/(1+y_i/2)$
Hence, we can find the change in the market value of equity by $\Delta E = \Delta A - \Delta L$. For instance, when $\Delta y = 1\%$, we proceed the following:

Interest rate change			**1%**		
% Change of Commercial Loan		-2.3697%	% Change of Savings & Time Deposit		-1.4837%
% Change of Mortgage Loan		-11.7359%	% Change of Certificates of Deposit		-3.4347%
% Change of US Treasury Notes		-4.4401%			
Balance Sheet (in HK$ Billion)					
Asset	Market Value	Net Change	**Liabilities and Equity**	Market Value	Net Change
Cash	200.0000	0.0000	Savings & Time Deposit	374.3620	-5.6380
Commercial Loan	58.5782	-1.4218	Certificates of Deposit	144.8479	-5.1521
Mortgage Loan	105.9169	-14.0831	**Total Liabilities**	519.2099	-10.7901
US Treasury Notes	172.0079	-7.9921			
Building and Equip	40.0000	0.0000	**Equity**	57.2931	-12.7069
Total	576.5030	-23.4970	**Total**	576.5030	-23.4970

Exhibit 1. Example of Calculation Method (Scenario: $\Delta y = 1\%$)

Applying the same method continuously to calculate the change under other hypothetical scenarios, we summarize the following table and plot them in a graph.

Interest rate change	**-1%**	**0%**	**1%**	**2%**	**3%**	**4%**	**5%**
ΔMV of Total Assets	23.4970	0.0000	-23.4970	-46.9941	-70.4911	-93.9881	-117.4852
ΔMV of Total Liabilities	10.7901	0.0000	-10.7901	-21.5802	-32.3703	-43.1604	-53.9505
ΔMV of Equity	12.7069	0.0000	-12.7069	-25.4139	-38.1208	-50.8278	-63.5347

Exhibit 2. Variations of Market Values for Assets, Liabilities and Equity (in HK$ Billion)

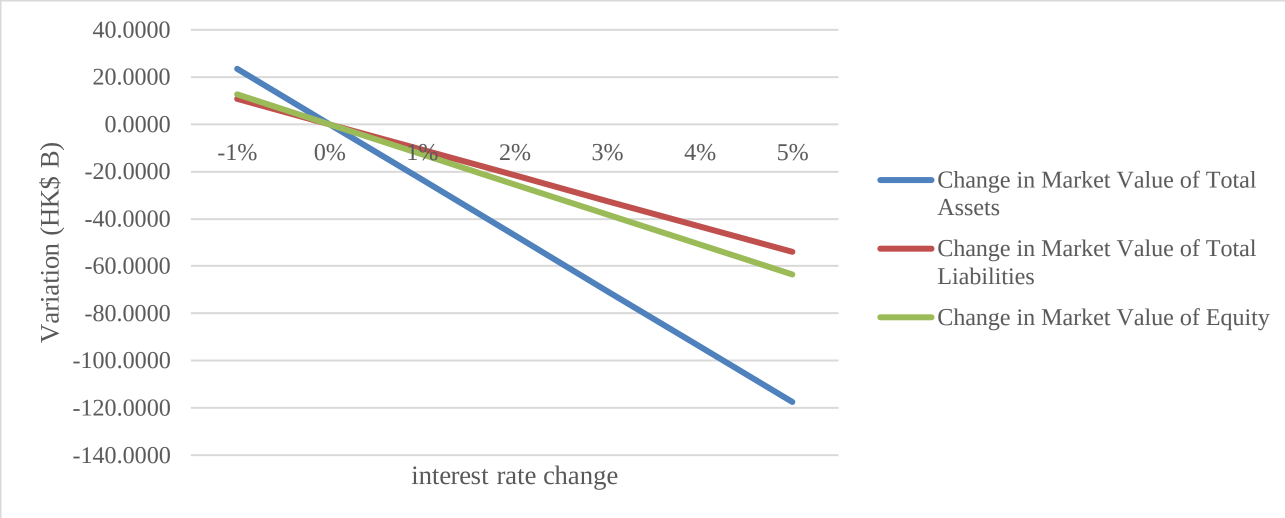


Exhibit 3. Visualization of the Variations of MV for Assets, Liabilities and Equity

Using the linear regression model, we can find that the variation of the market value of assets, liabilities and equity fit in a perfect linear relationship with the interest rate change using the above method. Such linearity property aligns with the formula we use: $\Delta A_i/A_i \approx {D_{Mac}}^{Ai} * \Delta y/(1+y_i/2)$, $\Delta L_i/L_i \approx {D_{Mac}}^{Ai} * \Delta y/(1+y_i/2)$. However, we also need to recognize that duration method to find the price change related to interest rate change is in effect an estimation, and is only reliable within a small range of interest rate change. Combining the above, we can approximately have the following relationship

$$\Delta A(in\ HK\$B) \approx -2349.7037\Delta y \ (1)$$
$$\Delta L(in\ HK\$B) \approx -1079.0092\Delta y \ (2)$$
$$\Delta E(in\ HK\$B) \approx -1270.6945\Delta y \ (3)$$

Therefore, when the interest rate increases by 1%, the market value of assets, liabilities and equity will drop by around \$23.497B, \$10.790B and \$12.707B respectively. It is evident that the sum of coefficients in (2) and (3) is equal to the coefficient in (1), so that the sum of the variations of the liabilities and equity caused by interest rate change will be equivalent to the variation of the asset caused by the same interest rate change. The market value of assets changes most significantly with respect to certain interest rate change among these three items. And with the above linear relationship, when $\Delta E = -30B$, $\Delta y \approx 2.3609\%$, that is, a 2.3609% increase of interest rate would approximately induce the market value of equity drop from \$70B to \$40B.

Part II US Treasury Securities to Invest

a. Market Price

Given the information of these three kinds of T-Debts, we can calculate the market prices using PV function in Excel or discounted cash flow. We assume the face value is HK\$100. Since the cash payment is made at the end of each period, the 5th input in the function is 0. For instance, for T-Debt (A), the function is =-PV(yield to maturity/2, payment period = time to maturity*2, annual coupon/2, par value, type)=-PV(1.30%, 10, 2.55, 100, 0). The summary of market price is in Exhibit 4 below.

b. Macaulay Duration

The Macaulay Duration could be calculated using the DURATION function in Excel. For US Treasury notes and bonds, we choose "1" as our basis. Thus, the duration function used would be DURATION(Settlement, Maturity, Coupon, Yield, Frequency, Basis), where settlement and maturity are all dates, coupon and yield are all annually, frequency is 2 to represent semi-annual payment of coupon and semi-annual interest rate. The duration of T-Debt (A), (B) and (C) are shown in Exhibit 4 below.

T-Debt	Coupon Rate	TTM	YTM	r	N	PMT	FV	**Market Price**	**Macaulay Duration**
A	5.10%	5	2.60%	1.30%	10	2.55	100	$ 111.6508	4.5097
B	0.50%	10	2.80%	1.40%	20	0.25	100	$ 80.0601	9.7303
C	2.20%	20	3.10%	1.55%	40	1.1	100	$ 86.6599	15.9480

Exhibit 4. Market Price and Macaulay Duration for US Treasury Debt Securities

c. Convexity

Convexity is defined as $\frac{1}{P}\frac{\partial^2 P}{\partial y^2}$.We use three methods to calculate convexity values below, the first is to use Convexity formula on the half-year basis and then multiply by 4.

$$\text{Convexity in Half}-\text{Year Basis} = \frac{1}{P} \times \frac{1}{(1+y_{semi})^2} \times \sum_{i=1}^{n} [\frac{c_{semi,i}}{(1+y_{semi})^{t_i}} \times (t_{semi,i}^2 + t_{semi,i})]$$

The second method to get an approximate value is calculating modified durations for the base case and the case where the interest rate increases by 0.1%, then we can get the variation of modified duration with respect to the interest rate change, and thus,

$$\text{Convexity} \approx [(D_{Modified}{}^{Base})^2 - \Delta D_{Modified}/\Delta y]$$

The third method is to calculate the effective convexity. choosing $\underline{s} = 0.1\%$, we find $P^-$and $P^+$and thus, the

$$\text{Effective Convexity} = \frac{P_- + P_+ - 2P}{P \times \bar{S}^2}$$

The results are shown as follows:

	Market Price	D_{Macaulay}	Convexity 1	Convexity 2	Convexity 3
A	$111.6508	4.5097	23.3098	23.3094	23.3098
B	$80.0601	9.7303	98.4862	98.4883	98.4872
C	$86.6599	15.9480	292.6851	292.7987	292.6952

Exhibit 5. Convexity values of the three US Treasury Debt Securities

It can be seen that the convexity value calculated from three methods are roughly the same, however, as Method 2 and 3 are kind of estimation methods, we would mainly rely on convexity calculated from the formula.

Part III Investment Decision

a. Maximum Amount of Cash to be Invested

As we would use cash to purchase T-Debts, the sum of cash and T-Debt would stay at HK$200 billion and setting the duration GAP not higher than 2.5 years as our restriction, we can find the maximum amount of cash which can be used to invest in T-note using Solver in Excel based on Duration Gap formula introduced above. Taking T-Debt A as an example, we would like to maximize MV(A) subject to *Cash + T-note A =200 and* $D^{Gap}{}_{Macaulay} = D^{Asset}{}_{Macaulay} - (L/A) * D^{Asset}{}_{Macaulay} \leq 2.5$. Since $(L/A) * D^{Asset}{}_{Macaulay} = 1.825$, which is unchanged, we can rewrite the above relation: $D^{Gap}{}_{Macaulay} \leq 4.325$. Similarly, we can find the maximum amount of cash to be invested in T-Debt B and C.

Asset	Market Value (HK Billion)	Interest Rate	Macaulay Duration
Cash	**156.7593**	-	-
Commercial Loan	60	11.00%	2.5
Mortgage Loan	120	4.50%	12
US Treasury Notes	180	2.70%	4.5
Building and Equip	40	-	-
T-note A	**43.2407**	**2.60%**	**4.5097**
Total	600		
Asset	Market Value (HK Billion)	Interest Rate	Macaulay Duration
Cash	179.9595	-	-
Commercial Loan	60	11.00%	2.5
Mortgage Loan	120	4.50%	12
US Treasury Notes	180	2.70%	4.5
Building and Equip	40	-	-
T-note B	**20.0405**	**2.80%**	**9.7303**
Total	600		
Asset	Market Value (HK Billion)	Interest Rate	Macaulay Duration
Cash	**187.7729**	-	-
Commercial Loan	60	11.00%	2.5
Mortgage Loan	120	4.50%	12
US Treasury Notes	180	2.70%	4.5
Building and Equip	40	-	-
T-note C	**12.2271**	**3.10%**	**15.9480**
Total	600		

Exhibit 6. Maximum Amount of Cash which can be Invested in T-Debt

Therefore, the maximum amount of cash that can be individually invested in T-Debt (A), (B) and (C) separately are HK$43.2407 billion, HK$20.0405 billion and HK$12.2271 billion.

b. Choose among T-note/bond

(1) Choose T-Debt C

Duration: As the boss would invest the amount of cash into one of these T-notes/bonds to keep the portfolio's duration gap to be exactly 2.5 years, and our investment will not change the structure of liabilities, we have that each of these three choices would have the same duration for assets, duration for liabilities and duration gap.

Convexity: convexity of the whole portfolio should be our concern instead of the convexity per unit of T-Debt. As cash has the convexity of zero and all the other assets and liabilities are fixed, the increase in the convexity of Assets should be the market value weighting multiply by the T-Debt convexity. And the difference of convexity contribution between the new financial position and the original position is just the convexity effect of the new T-Debt investment. The reasons are as follows:

Overall Convexity for Assets = Convexity of T-Debt × (Cash amount invested/Total Assets) + Convexity contributed from other assets and liabilities excluding T-Debt.

Total change of assets value due to convexity = Change of Assets value due to convexity of other assets + Change of Assets value due to convexity of T-debt, where T-debt stands for the potential investment in T-note/bond (A), or (B), or (C), and other assets stands for Total Assets exclude the potential T-Debt investment.

Meanwhile, notice that convexity is a good property for investors, as the market value of the portfolio would increase more when the market interest rate decreases and decreases less when the market interest increases if it has a higher convexity and ceteris paribus, which can also be seen from the formula: $\frac{\Delta \mathrm{P}}{P} \approx -Duration_{Modified} \times \Delta y + \frac{1}{2} \times Convexity \times \Delta \mathrm{y}^2$

Then we multiply Cash invested in Debt(i) by its convexity and compare the results. The reason behind is that according to equation: Total change of Assets value due to convexity of T-debt investment = $\frac{1}{2} \times Convexity_{T-Debt} \times (\Delta y)^2 \times \text{Cash amount invested}$,

given a change of interest rate Δy, the change of Asset value due to convexity of T-debt (i) is always positive and will be larger for a larger (Convexity(i) * Cash amount invested in T-Debt (i)). The results are shown below.

Cash invested in A * Convexity (A)	Cash invested in B * Convexity (B)	Cash invested in C * Convexity (C)
1007.93	1973.73	3578.80

Exhibit 7. Comparison among Cash Invested in T-Debt(i) * Convexity (i), (in billion)

Combining the above arguments, it can be seen that even if T-Debt C has the lowest cash amount invested, it has the largest convexity benefits, while T-Debt A contributes the least. Based on the considerations on both duration and convexity, we suggest to invest in T-Debt C.

(2) Advantages and Limitations on our Choice

When facing a short investment horizon, if we expect a recent trend of interest rise, the bond price reduction would be larger for A and B compared to C. Moreover, in reality, the shift of the yield curve would not be parallel. Instead, the short-term rate would be influenced much more by the Federal Fund Rate than the long-term rate. Thus, the drop of the short-term T-Debt price would be larger because of an increase in interest rate, which is part of the advantage of our investment in T-Debt C.

The cash investment in T-Debt C would be much smaller than the other two. More cash would probably mean a better liquidity, but the difference is relatively small because Treasury debts are considered highly liquid.

T-Debt C has the longest term to maturity among these three instruments, thus is most susceptible to future uncertainties and potential risk. Even if we assume fixed exchange rate between HKD and USD and interest rate change would be applied to all interest rate with the same magnitude, in reality, these assumptions would not hold, and the debt instruments with longer maturity would be very likely to have larger volatility. Also, such longer investment term would probably introduce higher administrative cost and transaction cost.

Another factor is the potential gain/loss from the change of exchange rate. In our analysis, we assume the exchange rate between HKD and USD is fixed. However, this is not the case in reality. Therefore, if the exchange rate increases, say if it changes from HK$7.80/US$ to HK$8.00/US$, the investment in T-Debt (A) will lead to more gain because of the exchange rate change as the investment amount is the highest among the three. Other hand, if exchange rate decreases, the investment in US T-Debt (C) will lead to least loss because of the change.

In addition, interest earned under portfolio with T-Debt C is the lowest. We can calculate the estimated interest earned in half a year using:

Max. investment * YTM/2

$HKD Billion	Bond A	Bond B	Bond C
Maximum Investment	43.2407	20.0405	12.2271
Interest earned/ half year	0.5621	0.28	0.1895

Exhibit9. Interest earned (Higher opportunity cost for T-Debt C)

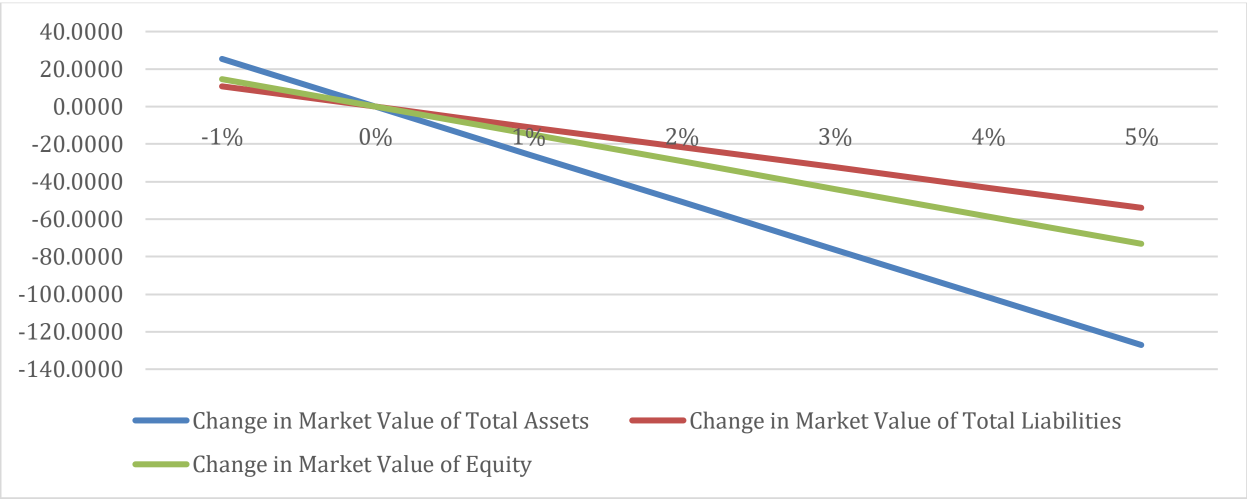
Also, the coupon received each year under portfolio with T-Debt C is much lower than the one with T-Debt A. We can calculate it by equation Total coupon = coupon per unit of T-Debt * Cash amount invested/ Current market price. Thus, in half a year, the cash flow received from T-Debt C, which is HK$155.2 million, would be lower than HK$987.6 million for T-Debt A, while remains larger than B. In this perspective, the earnings would be smaller for T-Debt C.

c. Scenario Analysis

If we use approximation based on the Macaulay durations of the assets (the same as the scenario analysis for each item in the current balance sheet), then the variations of market values for assets, liabilities and equities will be:

Summary of Scenario Analysis							
Interest rate change	-1%	0%	1%	2%	3%	4%	5%
Change in Market Value of Total Assets	25.4172	0.0000	-25.4172	-50.8345	-76.2517	-101.6690	-127.0862
Change in Market Value of Total Liabilities	10.7901	0.0000	-10.7901	-21.5802	-32.3703	-43.1604	-53.9505
Change in Market Value of Equity	14.6272	0.0000	-14.6272	-29.2543	-43.8815	-58.5086	-73.1358

Exhibit 8. Variations of market values for Assets, Liabilities and Equity (Approximation for T-Debt C)



$$\Delta A(in\ HK\$B) \approx -2541.7247\Delta y$$
$$\Delta L(in\ HK\$B) \approx -1079.0092\Delta y$$
$$\Delta E(in\ HK\$B) \approx -1462.7155\Delta y$$

Therefore, when the interest rate increases by 1%, the market value of assets, liabilities and equity will drop by around $25.725B, $10.790B and $14.627B respectively. It is also evident that the variations of the liabilities and equity caused by interest rate change will be equivalent to the variation of the asset caused by the same interest rate change.

However, duration is only estimation of the potential price change, while for T-Debt, we can use the PV function in Excel to calculate the actual price of T-Debt C instead of using approximation. Hence, in this case, the result will not be perfectly linear any more.

Summary of Scenario Analysis							
Interest rate change	-1%	0%	1%	2%	3%	4%	5%
Change in Market Value of Total Assets	25.6087	0.0000	-25.2496	-50.2045	-74.9174	-99.4310	-123.7803
Change in Market Value of Total Liabilities	10.7901	0.0000	-10.7901	-21.5802	-32.3703	-43.1604	-53.9505
Change in Market Value of Equity	14.8186	0.0000	-14.4595	-28.6244	-42.5471	-56.2706	-69.8298

Exhibit 9. Variations of market values for Assets, Liabilities and Equity

(Actual price for T-Debt C)

$$\Delta A(in\ HK\$B) \approx -2486.8659\Delta y$$
$$\Delta L(in\ HK\$B) \approx -1079.0092\Delta y$$
$$\Delta E(in\ HK\$B) \approx -1407.8567\Delta y$$

Therefore, when the interest rate increases by 1%, the market value of assets, liabilities and equity will drop by around $24.86B, $10.79B and $14.08B respectively. Also, same as the previous result, that the variations of the liabilities and equity caused by interest rate change will be equivalent to the variation of the asset caused by the same interest rate change.

From the regression line, we can find when $\Delta E = -30B$, $\Delta y \approx 2.1309$%. Hence, the market value of equity will drop from $70B to $40B when $\Delta y \approx 2.1309$%.