Quiz Date: Wednesday September 20, 2023

Exercises

1. For a positive integer n, evaluate the sum

$$\binom{n}{n}5^n + \binom{n}{n-1}5^{n-1} + \binom{n}{n-2}5^{n-2} + \cdots + \binom{n}{2}5^2.$$

2. Suppose 25 golf balls are to be distributed among seven golf players. In how many ways can the balls be distributed so that first player receives only one or two balls?

3 For this problem you do not need to simplify the answers. Let

$$f = (x_1 + x_2 + x_3)^{1000}$$

and let

$$g = (x_1 + x_2 - 3x_3)^{1000}.$$

- Find the coefficient of $x_1^2x_2^3x_3^2$ in f and g?
- What is the sum of all the coefficients in f and g?
- How many terms are there in the expansion of f and g?
  Hint: The expression $(x_1 + x_2)^3$ when expanded completely has 4 terms.

4. A. How many ways can I distribute 10 identical pens and 7 identical erasers to a class of four students?
   B. How many ways can I distribute 10 identical pens and 7 identical erasers to a class of three students such that each student recevies at least 2 pens and at least two erasers?

5 We will count the number of cycles in K_6.

A. How many cycles on 6 vertices are in K_6?
B. How many cycles on 5 vertices are in K_6?
C. How many cycles on 4 vertices are in K_6?

6 Consider the word CANADA

A. In how many ways can the letters from this word be arranged?
B. For the arrangements in part (A), how many have all three A's together?

7. A carton has 11 white balls, 11 pink balls, 11 yellow balls and 11 red balls. Each set of 11 balls is numbered 0 to 10. Suppose six balls are chosen from the box.

A. How many ways can there be six of the same colour?
B. How many ways can there be four of one colour and two of another colour?
C. How many ways can there be three of one colour and three of a different colour.
D. How many ways can there be two of one colour, two of a second colour, and two of a third colour?