Artificial Intelligence Problems **(2017–2018)**
Third year – Grado en Ingeniería Informática,
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Genetic Algorithms

1. Assume that n employees have to carry out n tasks, and we know the exact time, q_{ij}, that the i-th employee (e_i)will spend in accomplishing the j-th task (T_j). The problem is to assign tasks to employees in a one-to-one manner in such a way that all tasks are completed in the minimum total working time.

   Below is a table with an example for $n = 4$.

		Tasks			
		T_1	T_2	T_3	T_4
Employee	e_1	12	43	15	7
	e_2	9	10	6	4
	e_3	5	13	29	2
	e_4	4	11	17	9

   Two possible assignations are:

   - $e_1 \to T_2$, $e_2 \to T_3$, $e_3 \to T_1$, $e_4 \to T_4$, with total working time equal to: $43 + 6 + 5 + 9 = 63$.
   - $e_1 \to T_4$, $e_2 \to T_3$, $e_3 \to T_1$, $e_4 \to T_2$, with total working time equal to: $7 + 6 + 5 + 11 = 29$ (optimal).

   a) Describe this problem as an optimization problem: the set of possible candidates, and the function to optimize.

   b) Represent the problem to be solved by a genetic algorithm. That is: describe the genes, length of the chromosomes, decoding function and fitness function.

2. A group of N people from different countries are going to have dinner sitting at a circular table with N seats. Each person can speak two languages (not necessarily the same for all of them). The problem is to find a configuration in such a way that everyone is able to talk with their two neighbours.

   Describe the problem as an optimization problem and represent it to be solved by a genetic algorithm.

3. We have 10 cards numbered from 1 to 10. We wish to distribute them in two piles (P_1 y P_2), in such a way that the sum of the numbers in the cards of P_1 gets the closest to 36, and the product of the numbers in the cards of P_2 gets the closest to 360.

   Describe the problem as an optimization problem and represent it in order to be solved by a genetic algorithm.

4. Describe the necessary ingredients for a representation, in the genetic algorithms framework, of the following problem:

   Find the minimum of the function $f(x) = x^m$ in a domain of the form $[0, 2^n) \cap \mathbb{N}$, for a given pair of natural numbers n and m.

5. Consider the following simplified version of the "knapsack problem". We have n objects, such that each object i ($1 \leq i \leq n$) has a volume v_i. The problem is to select a subset of elements to be carried out in a knapsack of maximum capacity V, in such a way that the free space in the knapsack is minimized.

   Represent the problem to be solved by a genetic algorithm. That is: describe the genes, length of the chromosomes, decoding function and fitness function.

6. We are preparing meals for a trekking trip and we have n pieces of food. Each piece of food p_i has a volume v_i, a weight w_i, and provides c_i calories. You need to decide which pieces of food to bring with you in the trip, taking into account that the knapsack has a capacity V, you can only carry a maximum weight W, and you need at least C calories in your meal.

   Represent the problem to be solved by a genetic algorithm.

7. Represent the N-queens problem to be solved by a genetic algorithm.

8. Design an appropriate representation in order to find, by using a genetic algorithm, the first 5 decimal digits of number π.
   (Hint: the function $sen(\frac{x}{2})$ restricted to the interval $[3, 4]$ has a maximum at $x = \pi$.)

9. The *map coloring problem* can be stated in a general way as follows:

   > Given a map with N countries, assign a color to each country in such a way that any pair of countries having common border cannot get the same color, and no more than M different colors are used.

   How can this problem be formulated as an optimization problem? Represent the *map coloring problem* to be solved by a genetic algorithm.

10. A manager has to assign 10 repair projects, and gets proposals from 5 applicants (each applicant submits a budget proposal for each of the 10 repair projects). The assignment has the restriction that each applicant can take care of at most two projects.

    Formulate the problem of finding an assignment that minimizes the total budget in order to solve it by using a genetic algorithm with fitness-proportional selection.

11. A city hall has a 10.000 EUR budget to invest on youth initiatives. The young people from the town have submitted 3.000 proposals, but the total sum excedes the available budget. The problem is to select which proposals will get funded, in such a way that the maximum amount of money is spent (without exceeding the 10.000 EUR limit). Moreover, each person should be awarded at most two of his/her proposals.

    Describe this problem as an optimization problem and specify the elements needed in order to be solved by a genetic algorithm.

12. Let P be a population of seven chromosomes, $C_i, i = 1, \ldots, 7$ having the following values of the objective function: $F(C_1) = 5$, $F(C_2) = 3$, $F(C_3) = 7$, $F(C_4) = 1$, $F(C_5) = 9$, $F(C_6) = 2$ and $F(C_7) = 1$.

    Suppose that we want to select five chromosomes. Explain how can we do this, using different selection methods:

    - Fitness-proportional selction, by means of the *roulette wheel method*. To this aim we generate the following sequence of five random numbers between 0 and 28: $[5, 3, 16, 4, 20]$.
    - Tournement selction, with $k = 4$.
    - Elitist selection

13. A webmaster wishes to include some adds on his web, and he settles for that one big banner and two secondary smaller banners. He/she needs to choose among 5 advertisements ($A1, A2, \ldots, A5$), knowing that each add A_i will produce an income of X_i euros if included in the main banner, or Y_i euros if included in one of the smaller ones. Assume that each banner can host only one add (i.e. two adds cannot be placed in the same banner), and the same add cannot appear more than once in the web (i.e. cannot appear in more than one banner).

    a) Formulate the problem of maximizing the income in order to solve it by means of a genetic algorithm using *roulette wheel* selection.

    b) Consider a random population of 7 individuals and perform one step of the algorithm.

14. Design your own genetic algorithm, specifying in which part we hqave randomness, and the mechanisms used to priorize the best individuals and to introduce diversity.

15. In what sense we can say that a genetic algorithm tries to avoid the problem of local optima?