

Fourth Assignment in “Introduction to Numerical Analysis”, Fall Sem. 2016

Linear Algebra Part

Question 1.

Show that:

$$\|x - y\| \geq \left| \|x\| - \|y\| \right|; \quad x, y \in \mathbb{R}^n$$

Question 2.

Show that for all p-norm:

$$\|AB\| \leq \|A\| \|B\|; \quad A, B \in \mathbb{R}^n \times \mathbb{R}^n$$

Question 3.

For the matrix A given by $\begin{pmatrix} 9.7 & 6.6 \\ 4.1 & 2.8 \end{pmatrix}$ estimate the $\text{cond}(A)$. Use the 2-norm.

Question 4.

Solve the linear system:
$$\begin{cases} 4x_1 - x_2 + x_3 = 7 \\ 4x_1 - 8x_2 + x_3 = -21 \\ -2x_1 + x_2 + 5x_3 = 15 \end{cases}$$

using:

- **Jacobi**
- **Gauss Seidel**

Starting in both from the initial guess: $\underline{x}^0 = (1, 2, 2)^T$

Calculate 10 iterations. It is recommended to solve this question using MATLAB. If you use MATLAB, submit your code and output.

Question 5.

The values $x_1 = x_2 = 1.000$ are the solutions to:
$$\begin{cases} 1.133x_1 + 5.281x_2 = 6.414 \\ 24.14x_1 - 1.210x_2 = 22.93 \end{cases}$$

- Use four-digit arithmetic (with rounding) and **Gaussian Elimination** without pivoting to find a computed approximate solution to the system.
- Same as above, but use partial pivoting.

Question 6.

Use the power method with 9 iterations to locate an eigenvalue and eigenvector for the matrix (written in Matlab notation): $[[5,-1,7]; [-1,-1,1]; [7,1,5]]$.

If you decide to solve this manually, check with MATLAB and submit the code you wrote. Else, just submit the MATLAB code and output.

Good luck.