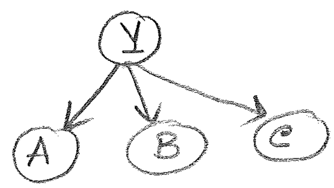
3 Bayes Nets Structure Learning and Parameter Estimation [Purna: 25 points]

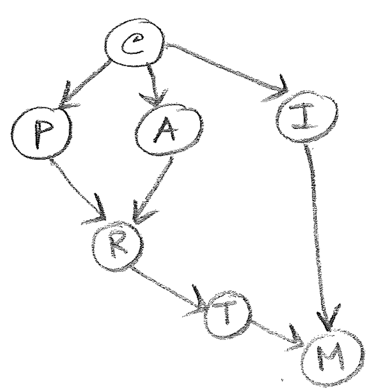
3.1 Short Questions

First, we will have some warm-up exercises.

1. Consider the Naive Bayes classifier with features A, B and C. Let the class variable be Y. How would you represent the conditional independence assumption using a Bayes net? It will have four nodes, A, B, C and Y.



2. Let P denote the event that Purna is sick. Let A denote the event that Andy is sick. Let C denote the event that its cold in Pittsburgh. Let I denote the event that the roads are icy. Let R denote the event that recitation time was changed. Let T be the event that Tom sent out an email about the late recitation. Finally, let M be the event that you missed the recitation. Now, Purna or Andy can get sick if its cold. The recitation time can change if Purna or Andy gets sick. You can miss the recitation is you didn't read Tom's email or if the roads were icy. And, last but not the least, the roads will be icy if its cold in Pittsburgh. Draw the Bayes net which reflects the dependencies in this question.



3.2 Structure Learning

This question asks you to infer an appropriate Bayes Net from the data, including both the network structure and the Conditional Probability Tables (CPTs). Consider the following training datasets. For each dataset give the network structure and CPTs that you feel best account for the dependencies in the data, while trying to keep the total number of edges in the Bayes Net small. In each case, your Bayes Net will have three nodes, called A, B and C. Some or all of these questions have multiple good answers. Please supply only one answer to each question.

Hint: Each dataset has multiple copies of a datapoint. A good first step is to write down the dataset with unique datapoints, and their respective counts. From there, try to find the conditional independence assumptions you want to express with your network. Recall that if B and C are conditionally independent given A, then for any given value of A, you should see that B and C are independent in the datasets.

1.

A	B	C
1	0	0
1	0	1
1	1	0
1	1	1
1	0	0
1	0	1
1	1	0
1	1	1
0	0	0
0	0	1
0	1	0
0	1	1
0	0	0
0	0	1
0	1	0
0	1	1

A B

C

$P(A=1) = 1/2$

$P(B=1) = 1/2$

$P(C=1) = 1/2$

A, B and C are independent of each other.

2.

A	B	C
1	0	1
1	0	1
1	1	1
0	1	1
1	0	1
0	0	0
1	0	1
0	1	1
1	1	1
0	0	0
0	1	1
1	1	1
0	1	1
0	0	0
1	1	1
0	0	0

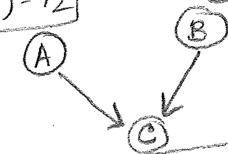
We have

A	B	C	count
1	0	1	4
0	1	1	4
1	1	1	4
0	0	0	4

$C = A \vee B$

$\Downarrow$

$\boxed{P(A=1) = 1/2}$ $\boxed{P(B=1) = 1/2}$

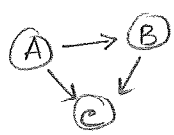


$P(C=1 \mid A=1, B=1) = 1$
$P(C=1 \mid A=0, B=1) = 1$
$P(C=1 \mid A=1, B=0) = 1$
$P(C=1 \mid A=0, B=0) = 0$

3.

A	B	C
1	0	1
1	1	0
1	0	1
1	1	0
1	1	1
0	1	1
1	1	1
0	1	1

A, B and C are dependent. There are no conditional independence assumptions.

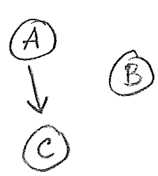


$P(A=1) = 3/4$ $P(B=1 \mid A=1) = 2/3$
$P(B=1 \mid A=0) = 1$

$P(C=1 \mid A=1, B=1) = 1/2$
$P(C=1 \mid A=1, B=0) = 1$
$P(C=1 \mid A=0, B=1) = 1$
$P(C=1 \mid A=0, B=0) =$ Not enough data

4.

A	B	C
1	1	1
1	0	1
1	0	1
0	0	0
0	0	0
0	1	0



$P(A=1) = 1/2$
$P(C=1 \mid A=1) = 1$
$P(C=1 \mid A=0) = 0$

$P(B=1) = 1/3$

A & C are identical.

$P(A=1, C=1 \mid B=1) = P(A=1, C=1) = 1/2$
$P(A=1, B=1 \mid C=1) = P(A=1 \mid C=1)\, P(B=1) = 1 \times 1/3 = 1/3$