

10-601 Machine Learning

HW4, Section 2. Inference – Solutions

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2

2.1

$$\begin{aligned}
 P(O = t, W = t, H = t, S = f, F = f) &= P(O = t)P(H = t|O = t, W = t)P(W = t)P(S = f|W = t)P(F = f|S = f) \\
 &= 0.5 * 0.9 * 0.4 * (1 - 0.6) * (1 - 0.1) \\
 &= 0.0648
 \end{aligned}$$

2.2

$$\begin{aligned}
 P(O = t, W = t, H = t, F = f) &= \sum_{s_i \in S} P(O = t, W = t, H = t, S = s_i, F = f) \\
 &= P(O = t)P(H = t|O = t, W = t)P(W = t)P(S = f|W = t)P(F = f|S = f) + \\
 &\quad P(O = t)P(H = t|O = t, W = t)P(W = t)P(S = t|W = t)P(F = f|S = t) \\
 &= (0.0648) + (0.5 * 0.9 * 0.4 * 0.6 * 0.6) \\
 &= 0.1296
 \end{aligned}$$

2.3

$$\begin{aligned}
 P(H = f|O = f, W = t, S = t, F = t) &= \frac{P(H = f, O = f, W = t, S = t, F = t)}{P(O = f, W = t, S = t, F = t)} \\
 &= \frac{P(H = f, O = f, W = t, S = t, F = t)}{\sum_{h_i \in H} P(H = h_i, O = f, W = t, S = t, F = t)} \\
 &= \frac{0.5 * 0.5 * 0.4 * 0.6 * 0.4}{(0.5 * 0.5 * 0.4 * 0.6 * 0.4) + (0.5 * 0.5 * 0.4 * 0.6 * 0.4)} \\
 &= 0.5
 \end{aligned}$$

2.4

$$\begin{aligned}
 P(H = t|S = t, F = t) &= \frac{P(H=t, S=t, F=t)}{P(S=t, F=t)} \\
 &= \frac{\sum_{o_i \in O} \sum_{w_j \in W} P(H=t, S=t, F=t, O=o_i, W=w_j)}{\sum_{h_k \in H} \sum_{o_i \in O} \sum_{w_j \in W} P(H=h_k, S=t, F=t, O=o_i, W=w_j)} \\
 &= \frac{P(F=t|S=t) \sum_{w_j \in W} P(W=w_j) P(S=t|W=w_j) \sum_{o_i \in O} P(O=o_i) P(H=t|O=o_i, W=w_j)}{P(F=t|S=t) \sum_{w_j \in W} P(W=w_j) P(S=t|W=w_j) \sum_{o_i \in O} P(O=o_i) \sum_{h_k \in H} P(H=h_k|O=o_i, W=w_j)} \\
 &= \frac{0.4 * (0.4 * 0.6 * (0.5 * 0.9 + 0.5 * 0.5)) + 0.6 * 0.2 * (0.5 * 0.7 + 0.5 * 0.2)}{0.4 * (0.4 * 0.6 * 1 + 0.6 * 0.2 * 1)} \\
 &= 0.6167
 \end{aligned}$$

2.5

$$\begin{aligned}
P(F = t|H = t) &= \frac{P(F=t, H=t)}{P(H=t)} \\
&= \frac{\sum_{o_i \in O} P(O=o_i) \sum_{w_j \in W} P(W=w_j) P(H=t|O=o_i, W=w_j) \sum_{s_k \in S} P(S=s_k|W=w_j) P(F=t|S=s_k)}{\sum_{o_i \in O} P(O=o_i) \sum_{w_j \in W} P(W=w_j) P(H=t|O=o_i, W=w_j) \sum_{s_k \in S} P(S=s_k|W=w_j) \sum_{f_n \in F} P(F=f_n|S=s_k)} \\
&= \frac{0.5*(0.4*0.9*(0.6*0.4+0.4*0.1))+0.6*0.7*(0.2*0.4+0.8*0.1))+0.5*(0.4*0.5*(0.6*0.4+0.4*0.1))+0.6*0.2*(0.2*0.4+0.8*0.1))}{0.5*(0.4*0.9*1+0.6*0.7*1)+0.5*(0.4*0.5*1+0.6*0.2*1)} \\
&= 0.2211
\end{aligned}$$