

שאלה 2.a

1. Two given generators / lazy lists , $l1$ and $l2$, are equivalent if and only if
For every $i \in N$:

(*) Generators - $take(l1, i) = take(l2, i)$

(**) Lazy lists – $(eq? (take\ l1\ i) (take\ l2\ i)) \rightarrow \#t$

2. We will prove that `evenSquares1` and `evenSquares2` generators equivalent by induction on n , the number of elements to generate (the 2nd parameter of `take`).

Base case: let $n=0$. So `take` will return an empty list on every generator, particularly
On `evenSquares1`, `evenSquares2`.

Means, - $take(evenSquares1, 0) = take(evenSquares2, 0)$

assumption: Assume, for $n = k$, that (*) holds for every $i, i \leq k$; that is, that

$take(evenSquares1, i) = take(evenSquares2, i)$

Proof: let $n=k+1$. According to our assumption, the lists that we get from performing `take` in the lists are equal until their k element. k^{th} value is supposed to be an even number & a square number. Let's call the next number that holds both conditions m .

EvenSquare1 - `mapGen` is a generator of all square numbers, starting from k^{th} value (we already returned k).

`FilterGen` will return a generator of all square numbers that are even, starting from k^{th} value.

When we perform `take` for the $k+1$ time on `filterGen`, it will return the next number on its list that holds both conditions (even & square) , means m .

EvenSquare2 - `FilterGen` will return a generator of all even numbers, starting from k^{th} value.

`MapGen` will return a generator of all even numbers that are square numbers, starting from k^{th} value.

When we perform `take` for the $k+1$ time on `mapGen`, it will return the next number on its list that holds both conditions (even & square) , means m ; that is, the $k+1$ element is equal in both results, and -

$take(evenSquares1, k + 1) = take(evenSquares2, k + 1)$

3. We will prove that fibs1 and fibs2 lazy lists equivalent by induction on n, the number of elements to generate (the 2nd parameter of take).

Base case: let $n=0$. So take will return an empty list on every lazy list, particularly On fibs1, fibs2.

Means, - $(eq? (take\ fibs1\ 0) (take\ fibs2\ 0)) \rightarrow \#t$

assumption: Assume, for $n = k$, that $(**)$ holds for every $i, i \leq k$; that is, that

$(eq? (take\ fibs1\ i) (take\ fibs2\ i)) \rightarrow \#t$

Proof: let $n=k+1$. According to our assumption, the lists that we get from performing take in the lists are equal until their k element. k^{th} value is supposed to be next number on Fibonacci sequence. Let's call it m.

every time we call take function, it gets the next element of the lzl (head lzl) and performs the lambda of the lzl. So according to the assumption, the k-1 and k elements are valid.

fibs1 – the lambda of the list is a recursive call, that creates the next value by adding the k-1 and k elements, so it is the next number on Fibonacci sequence – m.

fibs2 – it is built like - $(cons\ a\ (cons\ \ \ b\ (cons\ c\ \dots)))\dots$

the lambda of the list is a call to lzl-add, which receives 2 lazylists and creates the next cons of the lzl, by adding the head of the 2 lists. We call lzl-add with the k and k-1 elements, so we get the next number on Fibonacci sequence – m; that is, the k+1 element is equal in both results, and -

$(eq? (take\ fibs1\ [k + 1]) (take\ fibs2\ [k + 1])) \rightarrow \#t$

שאלה 3.a.2

נוכיח את הטענה באינדוקציה על n גודל הרשימה הראשונה.

בסיס: $n=0$ –

$$a-e[(append\$ '() l2 c)] \rightarrow^* a-e[(c '() l2)] = a-e[(c (append '() l2))]$$

הנחה: עבור $n=k \in \mathbb{N}$ הטענה מתקיימת לכל i , $k \geq i$. כלומר, אם גודל $l1$ הוא i , אז מתקיים:

$$(append\$ l1 l2 c) = (c (append l1 l2))$$

צעד: יהי $k \in \mathbb{N}$, $n=k+1$, כלומר, אם גודל $l1$ הוא $n=k+1$ אז:

$$a-e[(append\$ l1 l2 c)] \rightarrow^*$$

נכנס למקרה ה-else, ולכן יתבצע:

$$a-e[(append\$ (cdr l1) l2 (lambda(res-acc) (c (cons (car l1) res-acc))))]$$

\rightarrow^*

נשים לב שהרשימה $(cdr l1)$ שנשלחת ל- $append\$$ היא בגודל k , ולכן מהנחת האינדוקציה, נקבל ש-

$$a-e [((lambda(res-acc) (c (cons (car l1) res-acc))) (append (cdr l1) l2))] \rightarrow^*$$

$$a-e [(c (cons (car l1) (append (cdr l1) l2)))] =$$

$$a-e [(c (append l1 l2))]$$

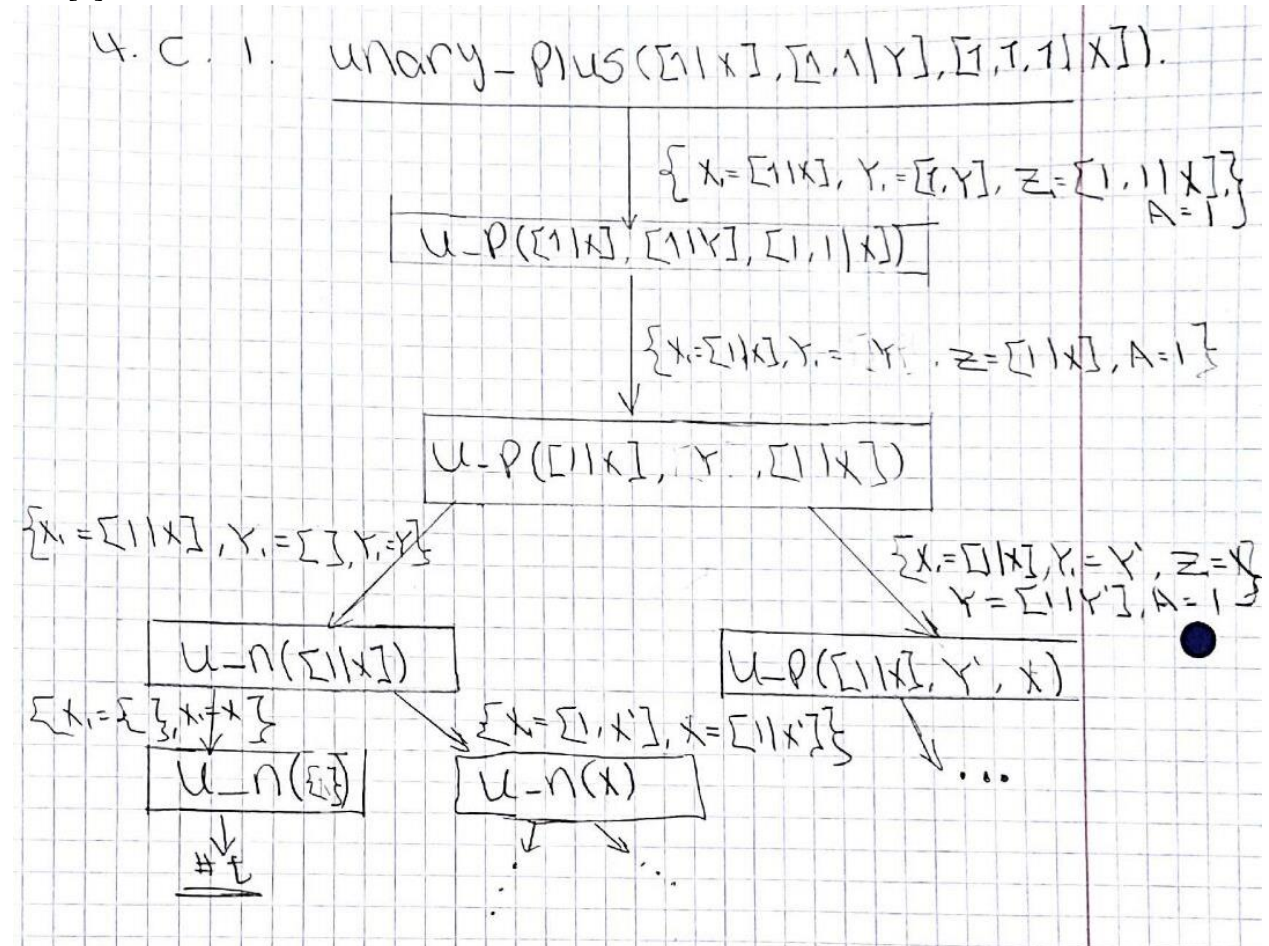
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4.b. The result of all these operations is "fail" because the number of arguments inside is unequal on both side.

The procces of the algo unify:

Open the proc p for both side, after the proc v, and in that moment, there are not the same number of arguments.

4.c. [1]



[2] Is a success, because the one of the leaf return true.

[3] The tree is infinite because the tree can calcitonin an infinite set of value that answer true.

Example: $X = Y = [1 | []], [1 | 1 | []] \dots$