1.

a) $V = (x,y,z) = 5x^2 - 3xy + xyz$, Point: $(3,4,5)$, vector: $(1,1,1)$

$$\frac{\langle 1,1,1\rangle}{\sqrt{3}} = \langle 1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3} \rangle = \langle \sqrt{3}/3, \sqrt{3}/3, \sqrt{3}/3 \rangle$$

$$D_{\vec{u}} V(x,y,z) = \frac{\partial V}{\partial x}(u_1) + \frac{\partial V}{\partial y}(u_2) + \frac{\partial V}{\partial z}(u_3)$$

$$\frac{\partial V}{\partial x} = 10x - 3y + yz, \quad \frac{\partial V}{\partial y} = 0 - 3x + xz, \quad \frac{\partial V}{\partial z} = 0 - 0 + xy$$

$$(\sqrt{3}/3)(10x - 3y + yz) + (\sqrt{3}/3)(-3x + xz) + (\sqrt{3}/3)(xy)$$

$$(\sqrt{3}/3)[(10(3) - 3(4) + (4)(5)) + (-3(3) + (3)(5)) + ((3)(4))]$$

$$= \sqrt{3}/3\,[(30 - 12 + 20) + (-9 + 15 + 12)]$$

$$= 38 + 18 = 56(\sqrt{3}/3) = \boxed{32.3316}$$

b)

$$\frac{\partial V}{\partial x} = 10x - 3y + yz, \quad \frac{\partial V}{\partial y} = 0 - 3x + xz, \quad \frac{\partial V}{\partial z} = 0 - 0 + xy$$

$$\langle [10(3) - 3(4) + (4)(5)], [-3(3) + (3)(5)], [(3)(4)] \rangle$$

$$= \boxed{\langle 38, 6, 12 \rangle}$$

c) $\| \langle 38, 6, 12 \rangle \|$

$$= \sqrt{(38)^2 + (6)^2 + (12)^2}$$

$$= \sqrt{1444 + 36 + 144}$$

$$= \boxed{\sqrt{1624}}$$

6.

a) $f(x,y,z) = xy^3 - xyz + 7x^2$, surface: $f(x,y,z) = e^{(xyz-6)} + 23$, Point: $(2,1,3)$

$$xy^3 - xyz + 7x^2 = 0$$

$$\vec{\nabla}(xy^3 - xyz + 7x^2)$$

$$\vec{n} = \vec{\nabla}(xy^3 - xyz + 7x^2) = (y^3 - yz + 14x,\ 3y^2x - xz,\ -xy)$$

$$\vec{n} = (1^3 - (1)(3) + 14(2),\ 3(1)^2(2) - (2)(3),\ -(2)(1))$$

$$= (26, 0, -2)$$

$$\boxed{26(x-2) + 0(y-1) - 2(z-3) = 0}$$

$$e^{(xyz-6)} + 23 = 0$$

$$\vec{\nabla}(e^{(xyz-6)} + 23)$$

$$\vec{n} = \vec{\nabla}(e^{(xyz-6)} + 23) = (e^{(xyz-6)}yz,\ e^{(xyz-6)}xz,\ e^{(xyz-6)}xy)$$

$$\vec{n} = (e^{((2)(1)(3)-6)}(1)(3),\ e^{((2)(1)(3)-6)}(2)(3),\ e^{((2)(1)(3)-6)}(2)(1))$$

$$= (3, 6, 2)$$

$$\boxed{3(x-2) + 6(y-1) + 2(z-3) = 0}$$

c) $\mathbb{R}^2 \to \mathbb{R}$ $f(x,y) = x^2y + \cos(x^2y - y)$ at $(-1,1)$

$$x^2y + \cos(x^2y - y) = z$$

$$x^2y + \cos(x^2y - y) - z = 0$$

$$\vec{n} = \vec{\nabla}(x^2y + \cos(x^2y - y) - z) = (2xy - \sin(x^2y - y)\,2xy,\ x^2 - \sin(x^2y - y)(x^2 - 1),\ -1)$$

$$\vec{n} = (2(-1)(1) - \sin((-1)^2(1) - 1)\,2(-1)(1),\ (-1)^2 - \sin((-1)^2(1) - 1)((-1)^2 - 1),\ -1)$$

$$= (-2, 1, -1)$$

$$(-1)^2(1) + \cos((-1)^2(1) - 1) = z$$

$$1 + \cos(0) = z$$

$$2 = z$$

$$\boxed{-2(x+1) + (1)(y-1) - 1(z-2) = 0}$$

2. (x_0, y_0, z_0) , surface: $2x^2+y^2+3z^2=1$

$\frac{\partial f}{\partial x} = 4x$, $\frac{\partial f}{\partial y} = 2y$, $\frac{\partial f}{\partial z} = 6z$

$\nabla f(x,y,z) = \langle 4x, 2y, 6z \rangle$

$\langle 4x, 2y, 6z \rangle = k \langle 10, 10, 10 \rangle$

$4x = 10k$, $2y = 10k$, $6z = 10k$

$x = \frac{10k}{4}$, $y = \frac{10k}{2}$, $z = \frac{10k}{6}$

$2(\frac{5}{2}k)^2 + (5k)^2 + 3(5k/3)^2 = 1$

$2(\frac{25}{4})k^2 + (5k)^2 + 3(\frac{25k^2}{9}) = 1$

$\frac{25}{2}k^2 + 25k^2 + \frac{25k^2}{3} = 1$

$k^2(\frac{25}{2} + 25 + \frac{25}{3}) = 1$

$k^2(\frac{75 + 150 + 50}{6}) = 1$

$k^2(\frac{275}{6}) = 1$

$\sqrt{k^2} = \sqrt{6/275}$

$k = \pm \frac{\sqrt{6}}{5\sqrt{11}}$

$k \pm \frac{\sqrt{66}}{55}$

$x_0 = \frac{5}{2} \cdot \frac{\sqrt{66}}{55}$

$y_0 = 5 \cdot \frac{\sqrt{66}}{55}$

$z_0 = \frac{5}{3} \cdot \frac{\sqrt{66}}{55}$

$$\boxed{\begin{array}{c} \left(\frac{\sqrt{66}}{22}, \frac{\sqrt{66}}{11}, \frac{\sqrt{66}}{33}\right) \\ \left(-\frac{\sqrt{66}}{22}, -\frac{\sqrt{66}}{11}, -\frac{\sqrt{66}}{33}\right) \end{array}}$$

3. $f: \mathbb{R}^2 \to \mathbb{R}$

$\frac{\partial F}{\partial s} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t}$

$= (7)(2) + (6)(-1)$

$= 14 - 6 = \boxed{8}$

4.

a) $f: \mathbb{R}^2 \to \mathbb{R}$, $f_x(1,1)$ and $f_y(1,1)$

$f_x(1,1) = \frac{3.4 - 3}{1.4 - 1} = \frac{0.4}{0.4} = \boxed{1}$

$f_y(1,1) = \frac{3.8 - 3}{1.4 - 1} = \frac{0.8}{0.4} = \boxed{2}$

b) $L(1,1) = 3 + 1(1.05 - 1) + 2(0.9 - 1)$

$= 3 + 0.05 - 0.2 = \boxed{2.85}$

c) $f_{xy}(1,1) = \frac{f_x(1, 1+\Delta y) - f_x(1, 1-\Delta y)}{2\Delta y}$

$f_x(1, 1.4) = \frac{4.36 - 3.8}{1.4 - 1} = \frac{0.56}{0.4} = 1.4$

$\frac{1.4 - 1.2}{1.4 - 0.6} = 0.25 = f_{xy}(1,1)$

$f_x(1, 0.6) = \frac{2.44 - 1.96}{1 - 0.6} = \frac{0.48}{0.4} = 1.2$

$\boxed{\text{positive}}$

5.

a) $f_x = x + y$, $f_y = y$

$f_{xy} = 1$

$f_{yx} = 0$

$1 \neq 0$, so there is no differentiable function $f(x,y)$ that exists

b) $\lim_{x \to 0, y \to 0} \frac{xy}{x^2+y^2}$

$x \to 0, y = 0$: $\lim_{x \to 0} \frac{xy}{x^2+y^2} = \lim_{x \to 0} \frac{0}{x^2} = 0$

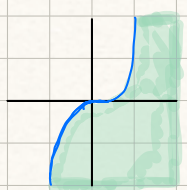
$y \to 0, x = 0$: $\lim_{y \to 0} \frac{xy}{x^2+y^2} = \lim_{y \to 0} \frac{0}{y^2} = 0$

$y = x, x \to 0$: $\lim_{x \to 0} \frac{x^2}{x^2+x^2} = \lim_{x \to 0} \frac{x^2}{2x^2} = \frac{1}{2}$

$\boxed{0 \neq 1/2 \text{, the limit DNE}}$

c) $x^3 - y \geq 0$

$y \leq x^3$



$\boxed{\text{range: } [0, \infty)}$

10.

a) right point

b) left point

c) $3x^2+y^2$