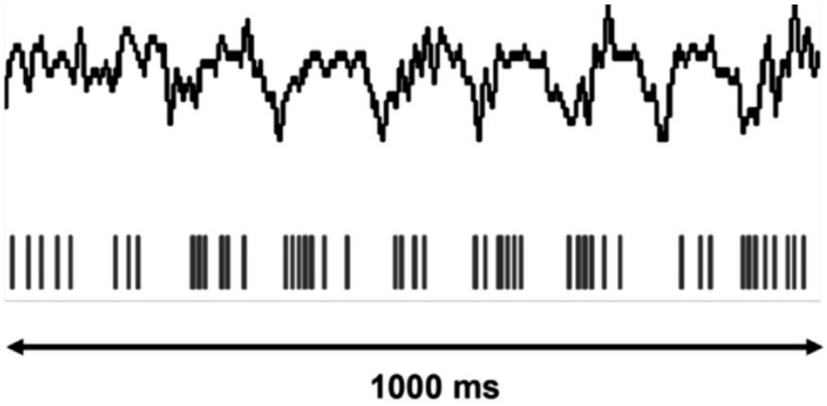
Lecture 6: LFP and Spectral Analyses

Some slides courtesy of Dr. Mark A. Kramer and Dr. Michael Fee

Two Types of Data: Spikes and Local Field Potentials

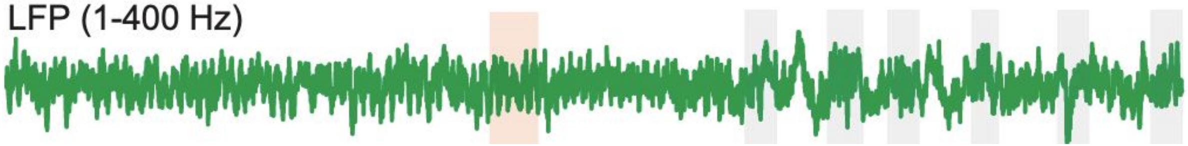
Two different types of data:

- Spikes (list of times, point process)
- Local Field Potentials (continuous)



What is the LFP?

- the **low-frequency** component of the extracellular voltage(often analyzed in ~1–300 Hz; sometimes up to ~500 Hz depending on lab conventions)
- **What it mostly reflects**: population transmembrane currents, dominated by synaptic and dendritic processes (plus return currents).
  - Often described as more “input / processing” (subthreshold synaptic drive + local dendritic integration)
  - In contrast, spikes are more “output” (suprathreshold action potentials)
- **But it’s not purely synaptic**: depending on band and brain area, LFP can include:
  - spike-related components (fast “spike bleed-through” at higher frequencies),
  - intrinsic membrane currents, dendritic spikes, and other sources.



Einevoll et al. 2013; Kajikawa and Schroeder 2011; Lindén et al. 2011; Jadhav et al. 2016

How local is the LFP? (spatial spread)

The spatial reach is not fixed; it depends strongly on:

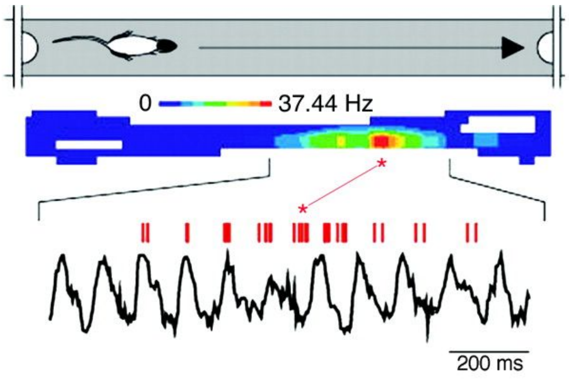
- frequency (lower frequencies spread farther),
- reference choice (common reference vs local reference),
- conductivity + geometry (layering, CSF, white matter),
- neuronal morphology and synapse distribution (aligned dendrites amplify dipoles),
- correlation structure of synaptic activity (synchronized inputs can look “large” and widespread),
- and recording configuration (electrode spacing, impedance, shank layout).

Einevoll et al. 2013; Kajikawa and Schroeder 2011; Lindén et al. 2011; Jadhav et al. 2016

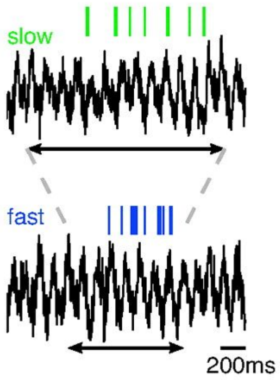
Preprocessing for LFPs

1. Inspect raw + mark bad channels/periods (head bangs, saturation, unplugged channels, huge motion pops)
2. Reference / re-reference (common average, local CAR, bipolar, etc.)
3. Artifact handling (detect & mask transient artifacts; handle stimulation artifacts separately)
4. Low-pass filter to LFP band (<500 Hz)
5. Downsample (to LFP rate, e.g., 1–2 kHz or lower as needed)
6. Bandpass for analyses (e.g., delta/theta/beta/gamma bands)

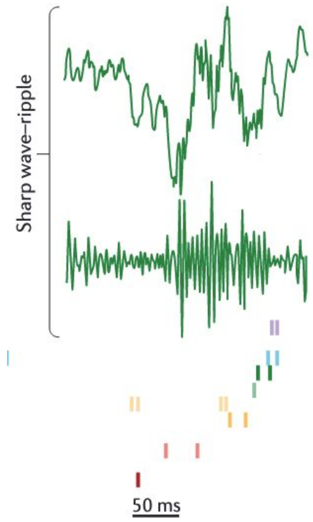
Relevance for spatial navigation



Theta Phase Precession



Faster theta
with speed



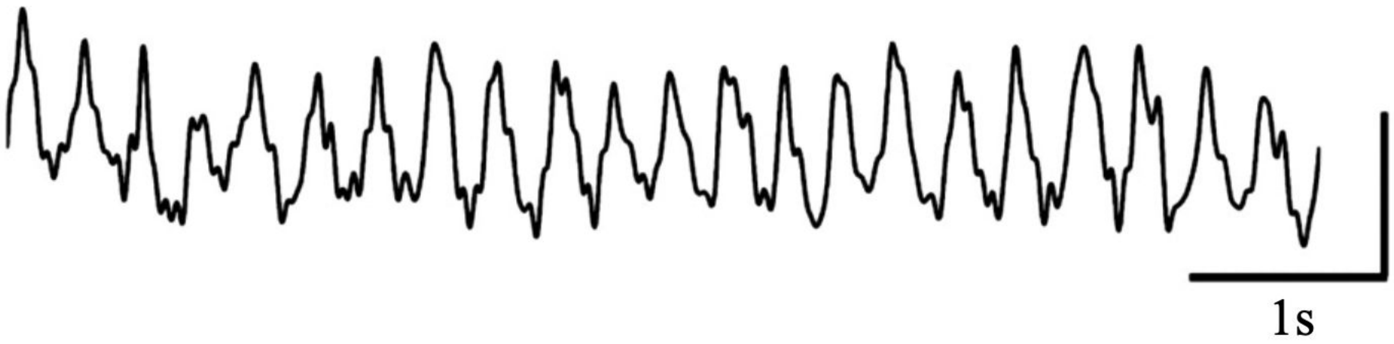
Large bursts of activity when
immobile or asleep

Buzsaki & Draguhn 2004; Geisler et al. 2007; Colgin 2016

Brain rhythms: theta

Theta: 4-8 Hz

Note: Theta frequency range different; the borders of ranges are not exact.

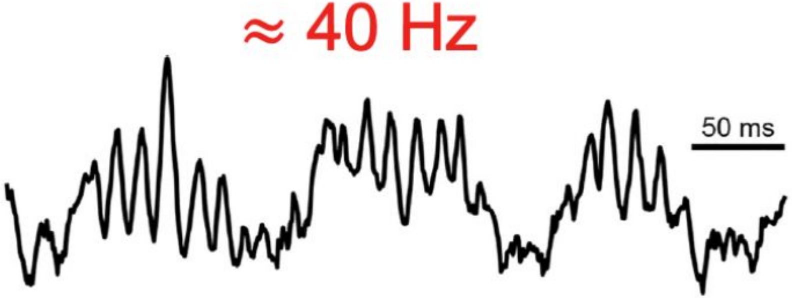


Function: Not completely understood.

In rats: learning and memory
location
motor behavior
sleep
emotional arousal
fear conditioning

Brain rhythms: gamma

Gamma: 30-50 Hz



[Fernandez-Ruiz et al., Neuron, 2023]

Function: Associated with a broad range of processes:
"binding"
attention
movement preparation
memory formation
conscious awareness

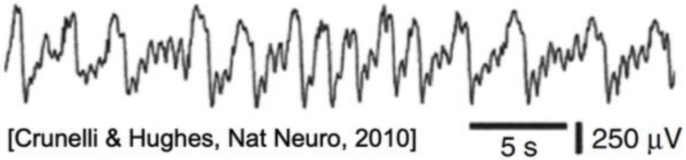
Slower

–**Delta**: 1-4 Hz

Sleep, learning, motivation

–**Slow cortical potential**: < 1 Hz

Emergence of consciousness?



[Crunelli & Hughes, Nat Neuro, 2010]

Faster

– **High gamma**: 50-120 Hz

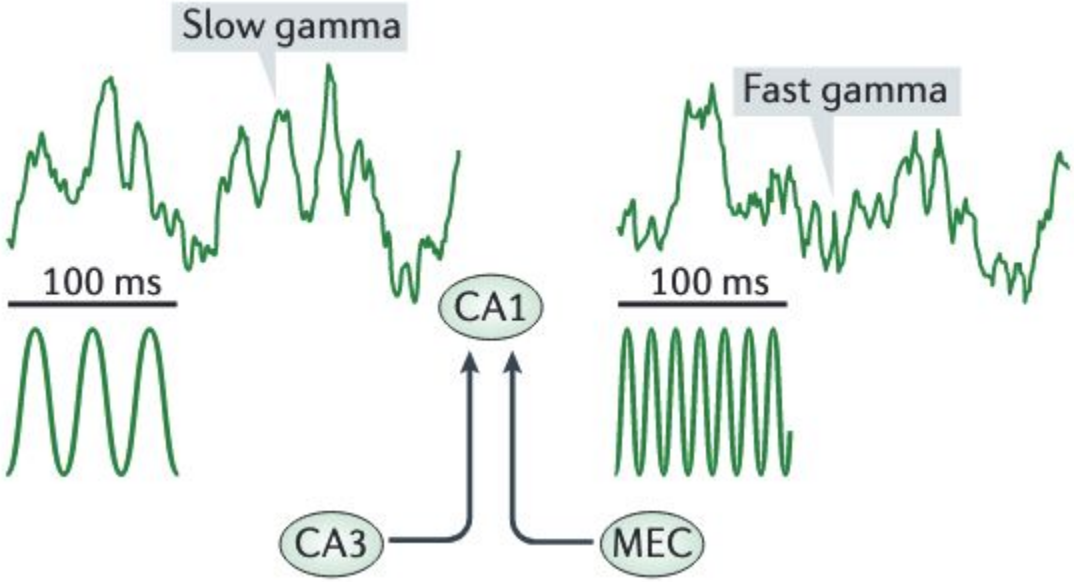
Coordination of neural activity

– **Ripples, HFO, UFO**: > 120 Hz

Replay of memories,
onset of seizures . . .

[Buzsaki, Hippocampus, 2015]

Rhythms associated with input from different brain regions



Colgin 2016

Relevance for spatial navigation

How do we extract and quantify these rhythms?

Remember: sinusoids . . .

$$V[t] = A\cos(2\pi f t)$$

Voltage as a function of time

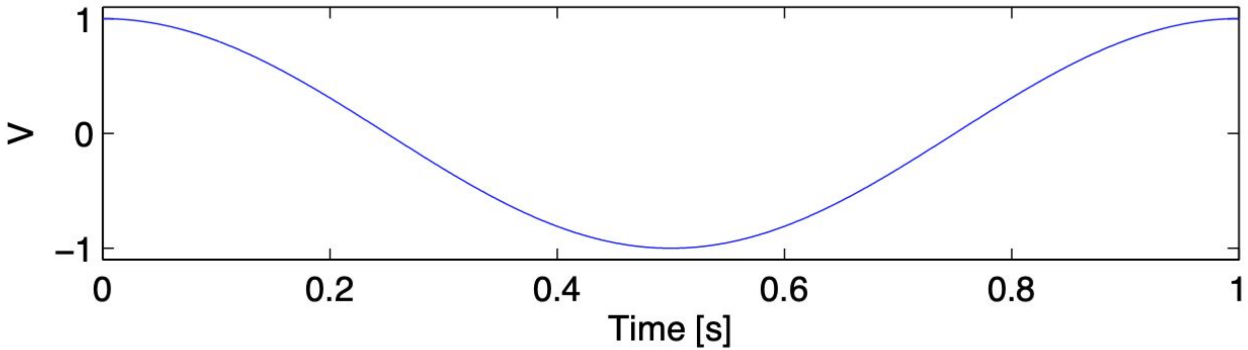
Amplitude

Frequency [Hz]

Time [s]

Ex. Consider: f = 1 Hz

Q: What does it look like?



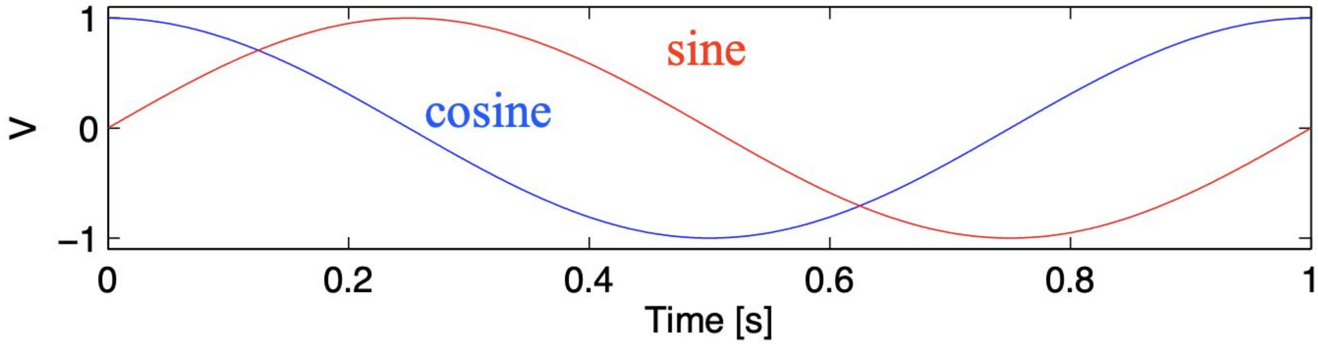
Remember: sinusoids . . .

In addition to cosine, there's also sine:

$$V[t] = B\sin(2\pi f t)$$

Voltage as a function of time — Amplitude — Frequency [Hz] — Time [s]

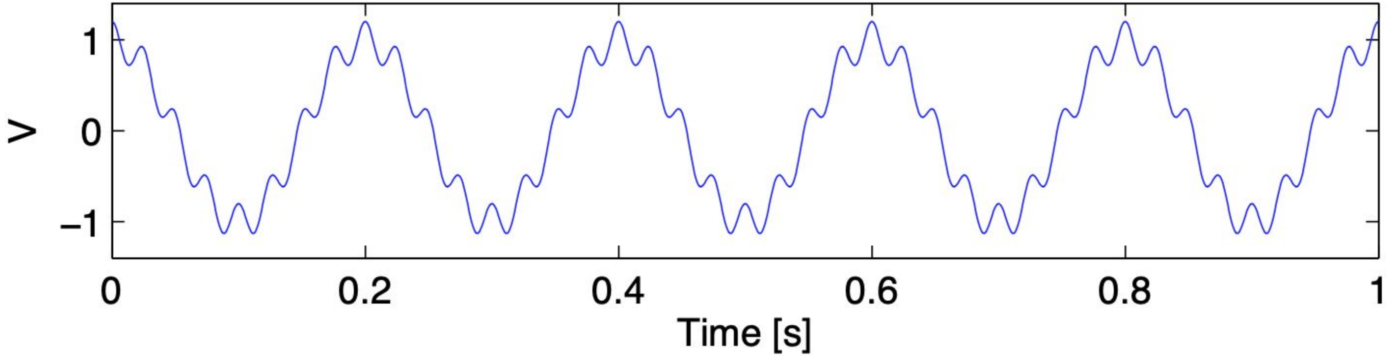
Ex. Consider: f = 1 Hz



Q: What's the difference?

Example: rhythmic signal

Consider the signal below:



Q: What are the rhythms?

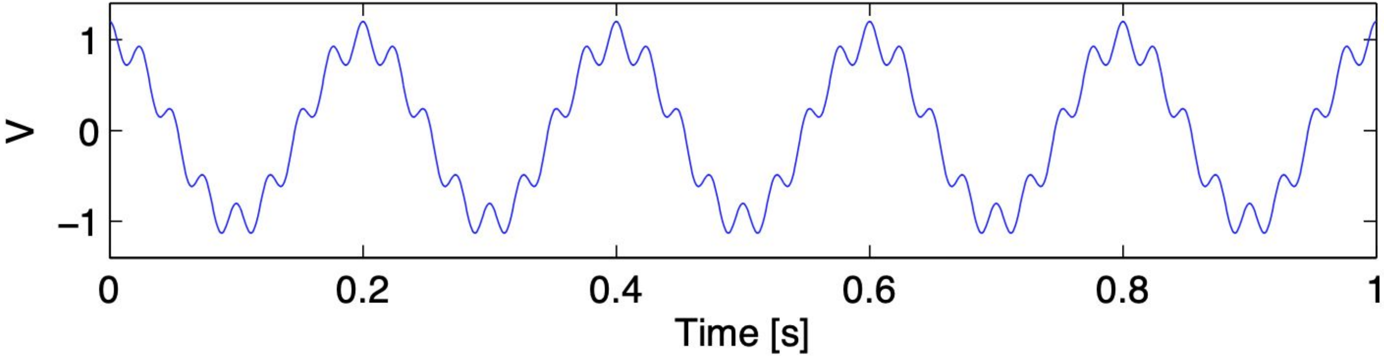
A: Apply visual inspection . . .   Slow and fast

5 Hz   40 Hz

Q: What has larger amplitude?

Example: rhythmic signal

So, we can represent this signal . . .



. . . as the sum of two sinusoids:

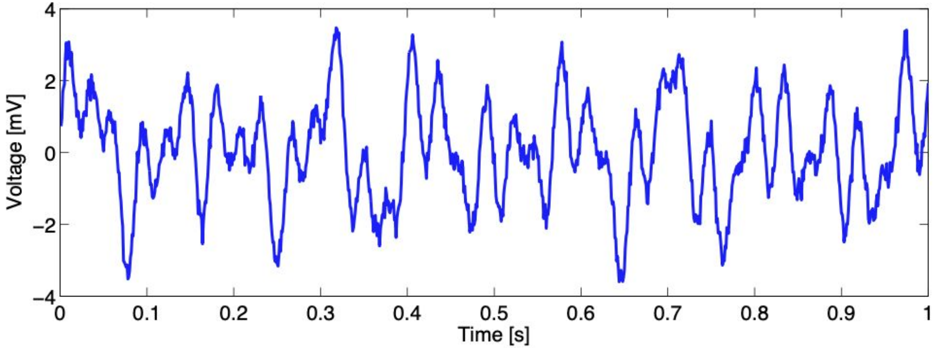
A slow, large amplitude sinusoid + a fast, small amplitude sinusoid

$$V[t] = \underset{1.0}{A_1} \cos(2\pi \underset{5\text{ Hz}}{f_1} t) + \underset{0.1}{A_2} \cos(2\pi \underset{40\text{ Hz}}{f_2} t)$$

We get a <u>simpler</u> representation of the signal. That's the idea of the spectrum.

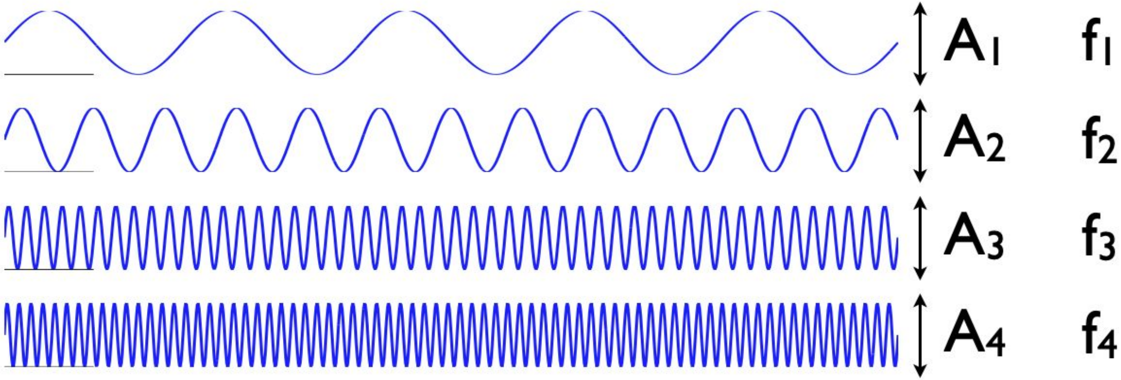
Idea: Spectrum

Consider:

V = 

- Decompose signal into oscillations at different frequencies.

$V =$ ... $+$ ... $+$ ... $+$ ... $+ \ldots$



Represent V as a sum of sinusoids (e.g., part 7 Hz, part 10 Hz, . . .)

Idea: Spectrum

So, in equations:

$$V[t] = \underbrace{A_1}_{\text{amplitude}} \cos(2\pi \underbrace{f_1}_{\text{frequency}} t) + B_1 \sin(2\pi f_1 t) + \underbrace{A_2}_{\text{amplitude}} \cos(2\pi \underbrace{f_2}_{\text{frequency}} t) + B_2 \sin(2\pi f_2 t) + \ldots$$

Or, more generally:

$$V[t] = \sum_j A_j \cos(2\pi f_j t) + B_j \sin(2\pi f_j t)$$

sum over many frequencies

amplitude

oscillation at frequency f_j

Note: A_j and B_j can be zero.
(some rhythms make no contribution to V[t])

Note: A_j and B_j are large when f_j is a good match to the data.

Why do we need sine and cosine?

<u>Note</u>: Think of sine & cosine as accounting for **phase**.

$$\underbrace{C_j}_{\text{amplitude}} \cos(2\pi \underbrace{f_j}_{\text{freq}} t + \underbrace{\phi_j}_{\text{phase}})$$

<u>Aside</u>: $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

$$= \underbrace{\boxed{C_j} \cos(2\pi f_j t) \boxed{\cos(\phi_j)}}_{A_j} - \underbrace{\boxed{C_j} \sin(2\pi f_j t) \boxed{\sin(\phi_j)}}_{B_j}$$

$$= \boxed{A_j} \cos(2\pi f_j t) + \boxed{B_j} \sin(2\pi f_j t)$$

Decompose V[t] into sine/cosine or amplitude/phase.

Idea: Spectrum

Q: How do we find A_j and B_j ?

$$V[t] = \sum_j A_j \cos(2\pi f_j t) + B_j \sin(2\pi f_j t)$$

A: Consider A_j and use **orthogonality of sinusoids**.

$$\int_0^T \cos(2\pi f_j t)\ \cos(2\pi f_k t)\, dt = \begin{cases} 0 & \text{if} \quad f_j \neq f_k \\ \text{T/2} & \text{if} \quad f_j = f_k \end{cases}$$

frequencies

Choose T so f_j and f_k complete an integer number of cycles.

integrate over time

Idea: Spectrum

Q: How do we find A_j and B_j ?

$$V[t] = \sum_j A_j \cos(2\pi f_j t) + B_j \sin(2\pi f_j t)$$

A: Consider A_j and use **orthogonality of sinusoids**.

$$\int_0^T \cos(2\pi f_j t)\ \sin(2\pi f_k t)\, dt = \ 0 \qquad \text{for all} \quad f_j\,, f_k$$

sine

Return to our original equation

$$V[t] = \sum_j A_j \cos(2\pi f_j t) + B_j \sin(2\pi f_j t)$$

<u>Task</u>: find A_j, B_j

Pick frequency f_k , multiply both sides by $\cos(2\pi f_k t)$, and integrate over time …

$$\int_0^T V[t]\cos(2\pi f_k t)dt = \int_0^T \sum_j A_j \cos(2\pi f_j t)\cos(2\pi f_k t)dt$$

$$+ \int_0^T \sum_j B_j \sin(2\pi f_j t)\cos(2\pi f_k t)dt$$

Consider each integral …

Idea: Spectrum

by orthogonality

$$\boxed{\int_0^T} \sum_j A_j \boxed{\cos(2\pi f_j t)\cos(2\pi f_k t)dt} \quad = 0 \text{ if } f_j \neq f_k \text{, or T/2 if } f_j = f_k$$

$$\boxed{\int_0^T} \sum_j B_j \boxed{\sin(2\pi f_j t)\cos(2\pi f_k t)dt} \quad = 0$$

$A_k\, T/2$ if $j = k$, 0 otherwise

So $$\int_0^T V[t]\cos(2\pi f_k t)dt = \int_0^T \sum_j A_j \cos(2\pi f_j t)\cos(2\pi f_k t)dt$$

$$+ \underbrace{\cancel{\int_0^T \sum_j B_j \sin(2\pi f_j t)\cos(2\pi f_k t)dt}}_{0}$$

$$= A_k\, T/2$$

Idea: Spectrum

$$\int_0^T V[t]\cos(2\pi f_k t)dt = A_k T/2$$

Solve for A_k

$$A_k = \frac{2}{T}\int_0^T V[t]\cos(2\pi f_k t)dt$$

We've solved for amplitude A_k

Depends on observed data V[t] multiplied by cosine we choose (f_k)

Idea: Spectrum

Return to our original equation

$$V[t] = \sum_j A_j \cos(2\pi f_j t) + B_j \sin(2\pi f_j t)$$

Q: How do we find A_j and B_j ?

A: Consider A_j and use **orthogonality of sinusoids**.

Harmonic Regression

Similarly

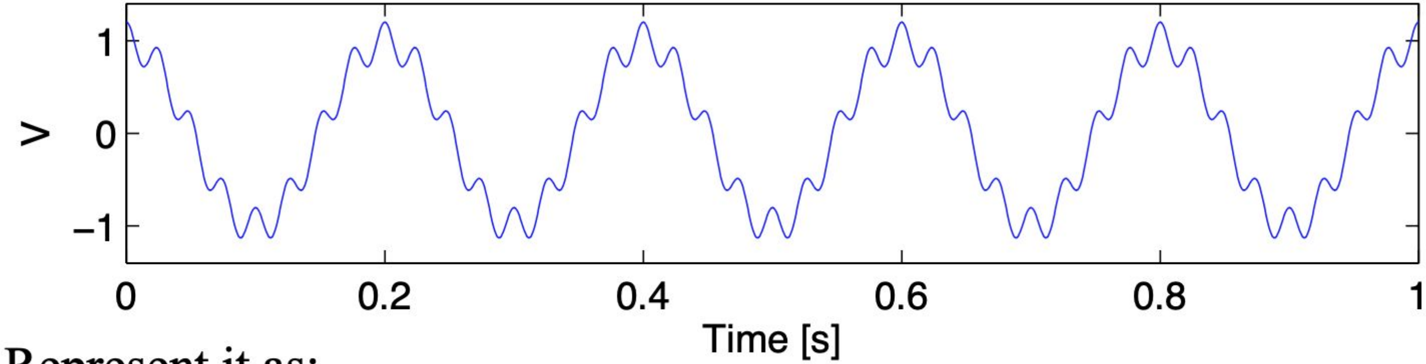
$$A_k = \frac{2}{T}\int_0^T V[t]\cos(2\pi f_k t)dt \qquad B_k = \frac{2}{T}\int_0^T V[t]\sin(2\pi f_k t)dt$$

Big idea: We can decompose V[t] into a sum of sin/cos functions, and we have a strategy to find the amplitudes A_j, B_j

Example: rhythmic signal

Q: So what?

A: Represent V[t] in a simpler way … remember:



Represent it as:

$$V[t] = A_1 \cos(2\pi f_1 t) + A_2 \cos(2\pi f_2 t)$$

1.0 5 Hz 0.1 40 Hz

To represent V[t] we need 4 numbers:

Amplitudes = {1, 0.1}

Frequencies = {5, 40} Hz

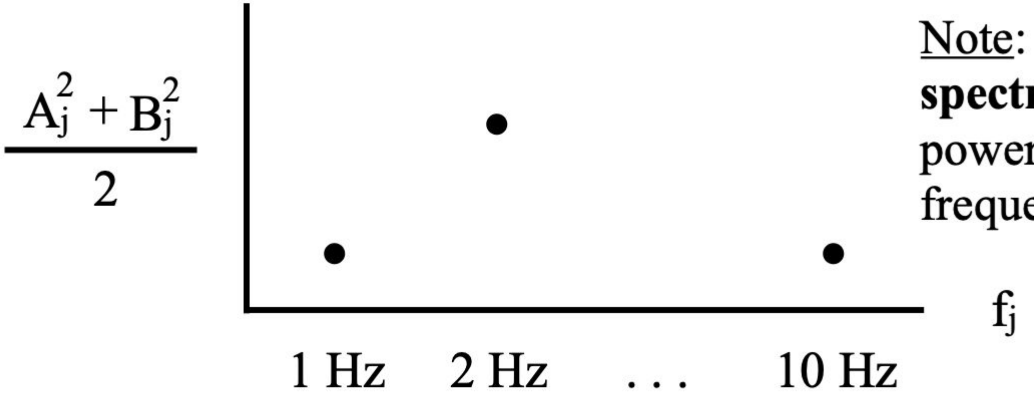
These 4 numbers completely summarize the data.

Plot: Spectrum

We can represent these amplitudes and frequencies graphically:

Plot: $\dfrac{A_j^2 + B_j^2}{2}$ versus f_j for each j

<u>Note</u>: The summed amplitudes squared.
Called the "power"

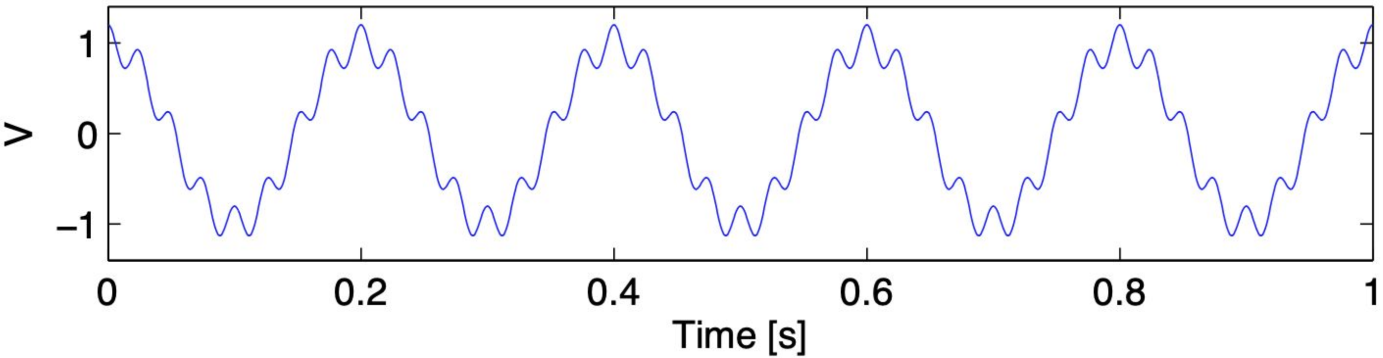


<u>Note</u>: This is called the **spectrum**; a plot of power versus frequency

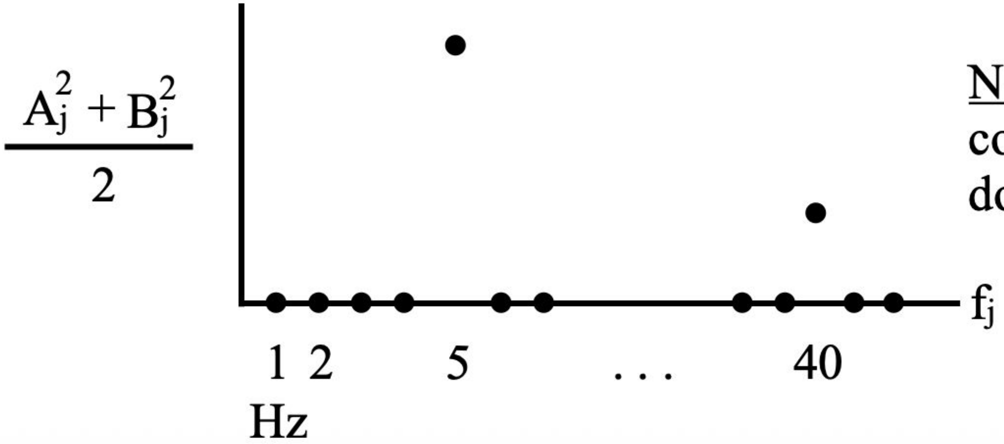
The peaks represent the <u>dominant rhythms</u> in the signal.

Example: rhythmic signal

Ex:



Plot the spectrum:

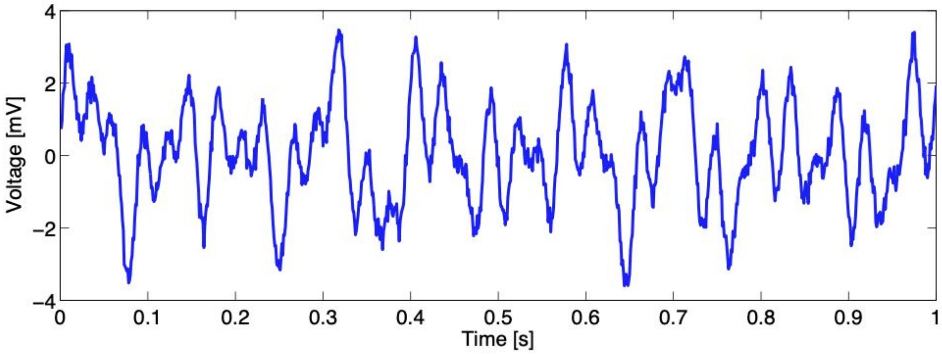


<u>Note</u>: The peaks correspond to the dominant rhythms

Example: rhythmic signal

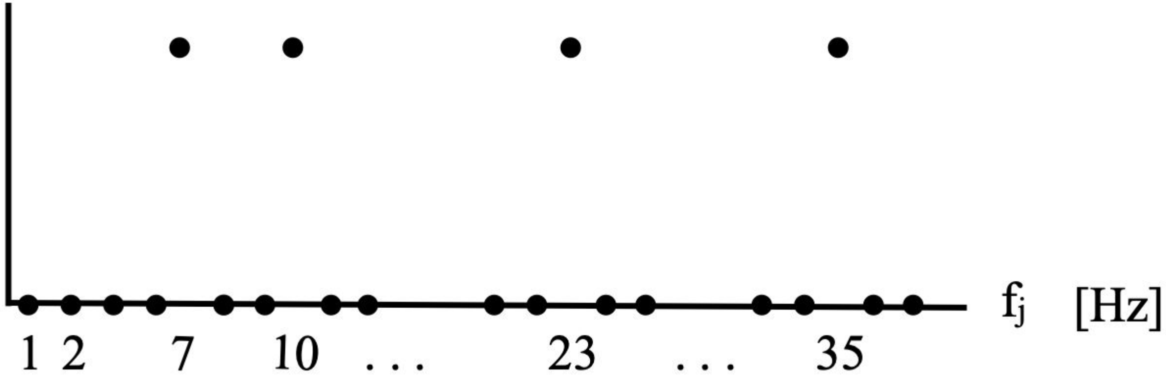
Ex.

$V =$



<u>Note</u>: It's complicated   Plot the spectrum:

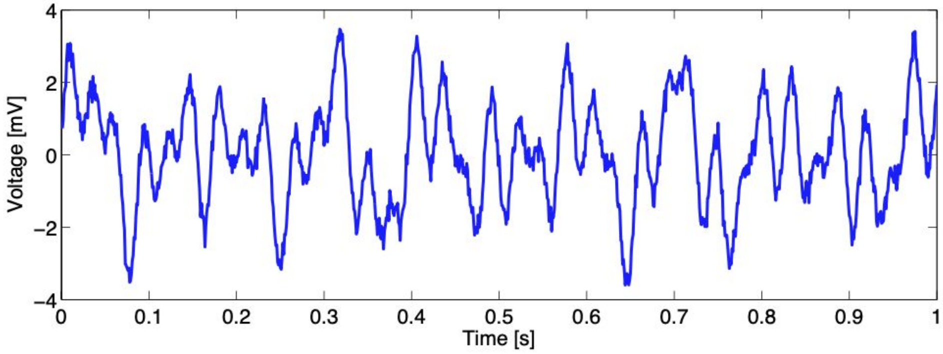
$$\frac{A_j^2 + B_j^2}{2}$$



Q: What's happening here?

Example: rhythmic signal

So, by computing the power spectrum, we find the complicated signal:

V = 

We find it's the sum of 4 sinusoids at frequencies:

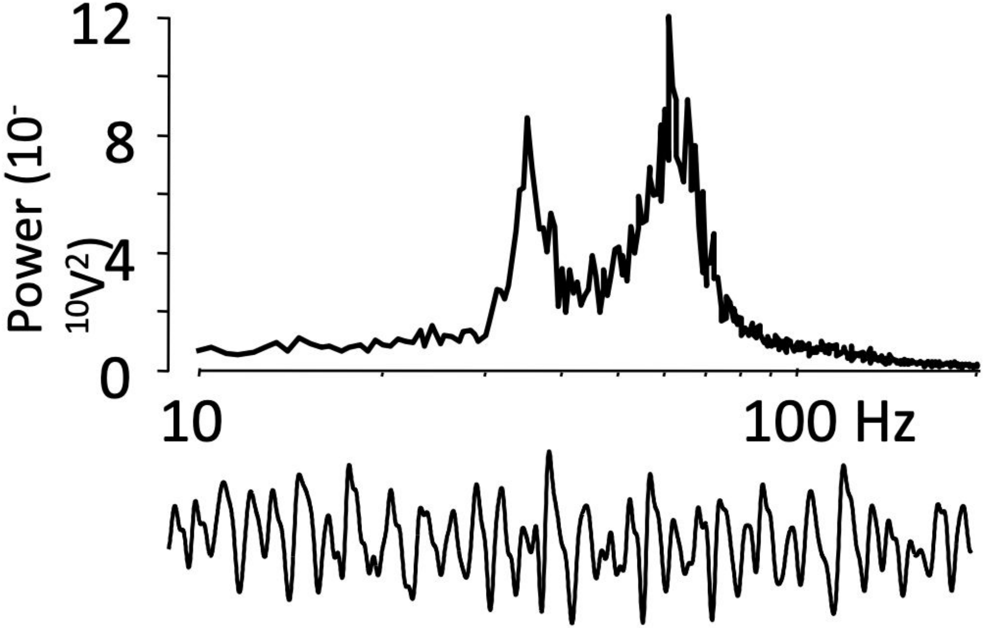
7 Hz, 10 Hz, 23 Hz, and 35 Hz

A much simpler representation of brain activity.

Example: Real world

Q: What does the power spectrum of real-world brain signals look like?

Ex. From a slice of rat cortex:

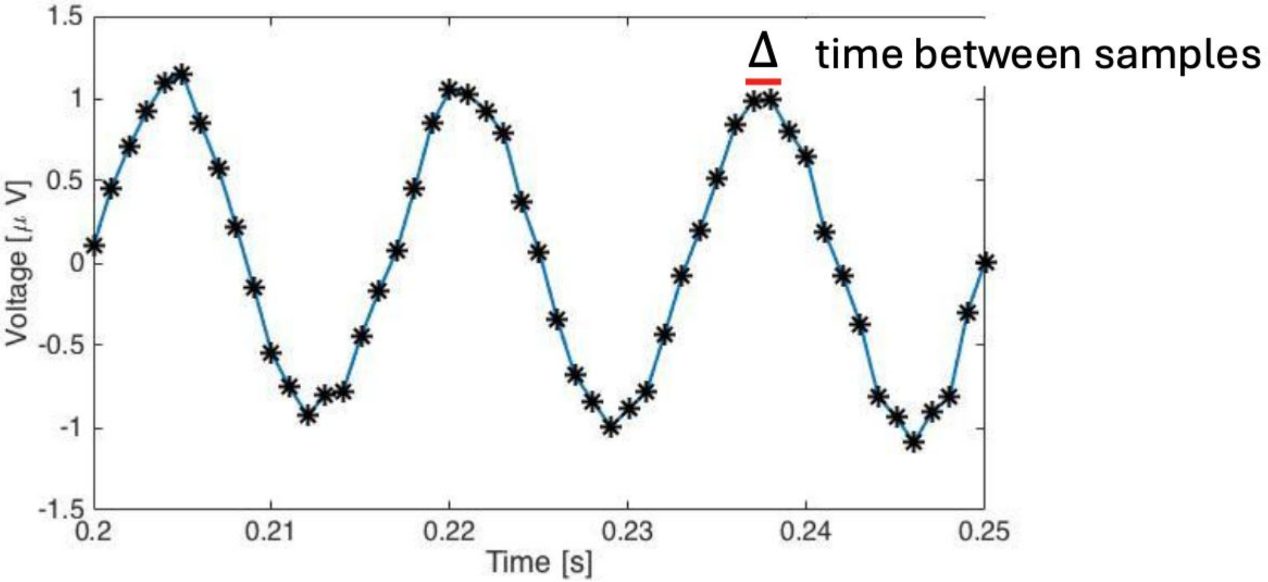


[Ainsworth et al, Neuron, 2012]

Q: What rhythms are dominant?

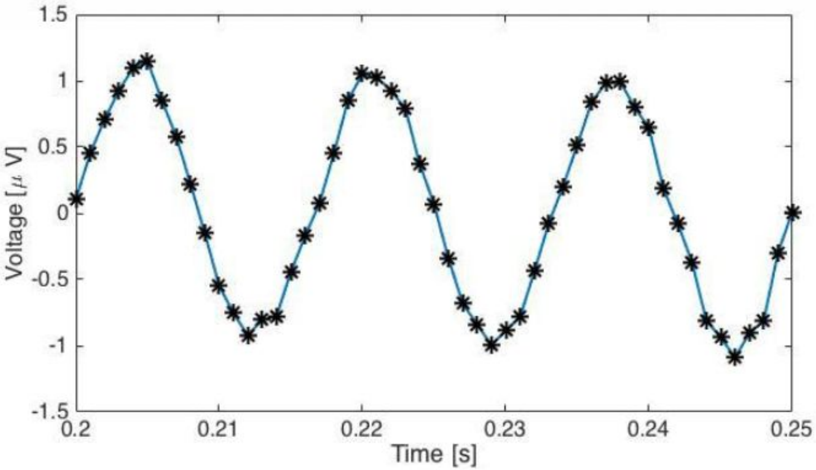
Although the signal is continuous, we only sample in discrete time

Consider a small snippet of data



Notation: x_n = Data at index n

Data is <u>not</u> continuous



<u>Notation</u>

x (EEG, LFP, MEG, ...)

x_n = Data at index n

t_n = Time at index n, $t_n = \Delta n$ where Δ = sampling interval

f_j = Frequency at index j, $f_j = j/T$ where T = total time of observation

Discrete time Fourier transform

Previously $$V[t] = \sum_j A_j \cos(2\pi f_j t) + B_j \sin(2\pi f_j t)$$

$$A_k = \frac{2}{T}\int_0^T V[t]\cos(2\pi f_k t)dt$$ and $$B_k = \frac{2}{T}\int_0^T V[t]\sin(2\pi f_k t)dt$$

Compare data V[t] to cosine at frequency f_k , does it match?

Now replace A_k, B_k:

$$\boxed{X_j = \sum_{n=1}^{N} x_n \exp(-2\pi i f_j t_n)}$$????

<u>Fourier transform</u> of the data *x*.

Discrete time Fourier transform

Define the parts

$$i^2 = -1$$

$$X_j = \sum_{n=1}^{N} x_n \boxed{\exp(-2\pi i f_j t_n)}$$????

FT of x
index j

sum over
all time n

data x
index n

frequency
index j

$f_j = j/T$

time
index n

$t_n = \Delta n$

Discrete time Fourier transform

Euler's formula
$e^{i\theta} = \cos(\theta) + i\sin(\theta)$

Fourier transform intuition:

Feynman: "*the most remarkable formula in mathematics*"

Data as a function of time index n

Data as a function of frequency index j

$$X_j = \sum_{n=1}^{N} \boxed{x_n} \boxed{\exp(-2\pi i f_j t_n)}$$

Sinusoids at frequency f_j

Euler's formula:

$$\exp(-2\pi\, i f_j\, t_n) = \cos(-2\pi\, f_j\, t_n) + i \sin(-2\pi\, f_j\, t_n).$$

So, at each time (index n) multiply data x_n by sinusoids at frequency f_j
Then sum up over all time.

Discrete time Fourier transform

Fourier transform intuition:

Data as a function of time index n

Data as a function of frequency index j

$$X_j = \sum_{n=1}^{N} \boxed{x_n} \boxed{\exp(-2\pi i f_j t_n)}.$$

Sinusoids at frequency f_j

Idea: compare our data x_n to sinusoids at frequency f_j and see how well they "match".

Good match: X_j = big Bad match: X_j = small

X_j reveals the frequencies f_j that match our data.

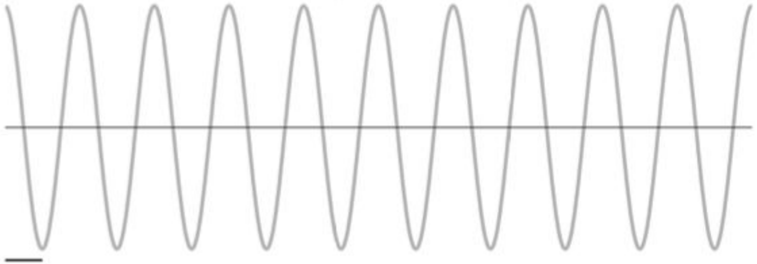
Spectrum: idea

Fourier transform <u>intuition</u>: **"Compare" data to sinusoids at different frequencies**

Match: X_j at frequency f_j is <u>large</u>

Mismatch: X_j at frequency f_j is <u>small</u>

<u>Example:</u>



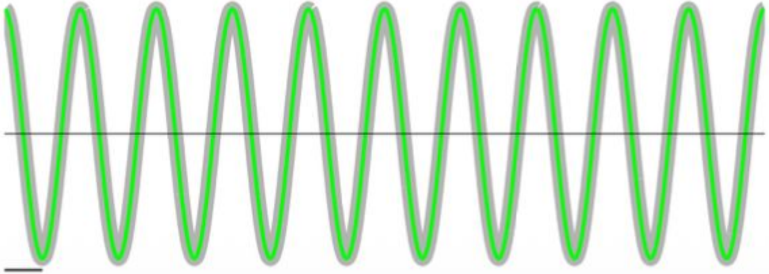
"Data"
10 Hz cosine

4 Hz

Multiply (+,-,+,-,...) & add

... <u>small</u> value

4 Hz does not match data



10 Hz

Multiple (+,+,+,+,...) & add

... <u>large</u> value

10 Hz matches data

Discrete time Fourier transform

Sound familiar? $$X_j = \sum_{n=1}^{N} x_n \exp(-2\pi i f_j t_n)$$ Fourier transform of the data *x*.

replace with Euler's formula

$$\cos(-2\pi f_j t_n) + i \sin(-2\pi f_j t_n).$$

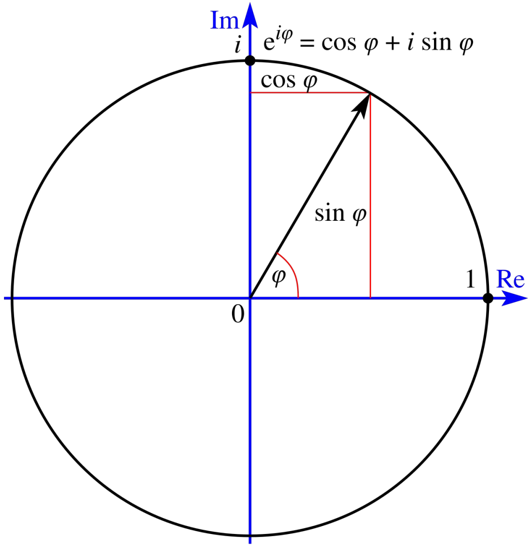
$$X_j = \boxed{\left(\sum_{n=1}^{N} x_n \cos(-2\pi f_j t_n)\right)} + i\boxed{\left(\sum_{n=1}^{N} x_n \sin(-2\pi f_j t_n)\right)}$$

Looks like $$\boxed{A_k} = \frac{2}{T}\int_0^T V[t]\cos(2\pi f_k t)dt \quad \boxed{B_k} = \frac{2}{T}\int_0^T V[t]\sin(2\pi f_k t)dt$$

Same idea: compare data to sinusoids and see how well they match

Why do we need sine and cosine?

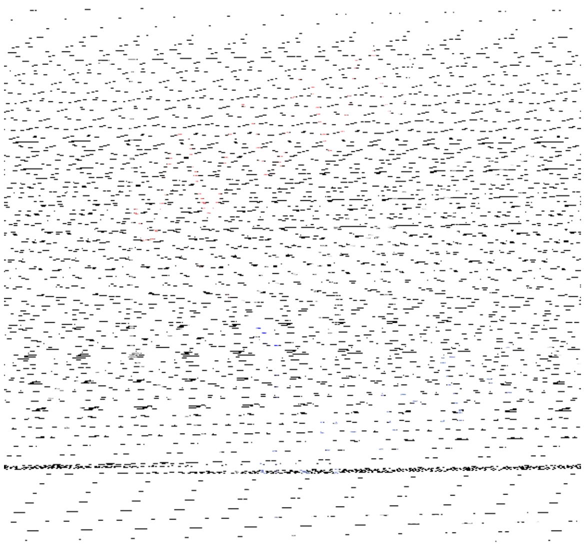
We can’t properly represent a full oscillation if we only use cosine or sine.



With only sine (or cosine), you lose phase—so you can’t represent an arbitrary oscillation

Phasors (pew pew)

This is sometimes known as a complex phasor



Harmonic regression vs. Fourier transform

Harmonic regression:

- parametric: choose frequencies to look at

Fourier transform

- non-parametric: look over a range of set frequencies

Spectrum: definition

(change sign of i everywhere)

The power of data x at frequency index j

$$S_{xx,j} = \frac{2\Delta^2}{T} X_j X_j^*$$

Previously $\frac{A_j^2 + B_j^2}{2}$

Same idea!

Constant that depends on sampling interval, duration of recording

FT of data

Complex conjugate of FT of data

Power at frequency f_j indicates how well sinusoids at f_j "match" our data.

Good match → High power at frequency f_j

Frequencies

f_j = Frequency at index j, $f_j = j/T$ where T = total time of observation

$$f_j = \left\{ 0, \frac{1}{T}, \frac{2}{T}, \ldots, \frac{1}{2\Delta} - \frac{1}{T}, \boxed{\frac{1}{2\Delta}}, -\left(\frac{1}{2\Delta} - \frac{1}{T}\right), -\left(\frac{1}{2\Delta} - \frac{2}{T}\right), \ldots, \frac{-2}{T}, \frac{-1}{T} \right\}$$

<u>Two important quantities</u>

Largest frequency: **Nyquist frequency** $f_{NQ} = \dfrac{1}{2\Delta} = \dfrac{f_0}{2}$ half the sampling frequency

$f_0 = \dfrac{1}{\Delta}$ sampling frequency

Frequency bin spacing $df = \dfrac{1}{T}$ reciprocal of total recording duration

Frequencies

Visualize f_j as a **vector**

index	0	1	2	...	$N/2-1$	$\frac{N}{2}$	$\frac{N}{2}+1$		N-2	N-1
freq	0	df	$2df$	...	$f_{NQ}-df$	f_{NQ}	$-(f_{NQ}-df)$	...	$-2df$	$-df$

Step forward in intervals of frequency resolution *df*

... until reaching maximum frequency f_{NQ}

then, frequencies are <u>negative</u>, starting with the largest magnitude.

decrease in magnitude until -*df*

Frequencies

Note: length(t_n) = N
length(f_j) = N

time and frequency vectors have the same length N

- If we record N data points, then we have N frequencies to examine.

Note: Frequencies f_j include negative values.

Consider $cos(2\pi f \mathrm{t})$ $\xrightarrow{\text{Euler's formula}}$ $\dfrac{e^{2\pi i f \mathrm{t}} + e^{-2\pi i f \mathrm{t}}}{2}$

a real signal

f > 0 f < 0

We need positive & negative frequencies to represent a real signal.

Frequencies

<u>Note</u>: Frequencies f_j include <u>negative values</u>.

Important fact: when data x_n is <u>real</u> (no imaginary component), then negative frequency spectrum is <u>redundant</u>.

$$S_{xx,j} \text{ at } f_j = S_{xx,j} \text{ at } -f_j$$

Frequencies

Q: Is x_n real?

A: Yes (in neuroscience)

Only inspect $f_j > 0$

$$f_j = \left\{0, \frac{1}{T}, \frac{2}{T}, \cdots, \frac{1}{2\Delta} - \frac{1}{T}, \frac{1}{2\Delta}, \boxed{-\left(\frac{1}{2\Delta} - \frac{1}{T}\right), -\left(\frac{1}{2\Delta} - \frac{2}{T}\right), \ldots, \frac{-2}{T}, \frac{-1}{T}}\right\}$$

Ignore negative frequencies (redundant)

Spectrum: df

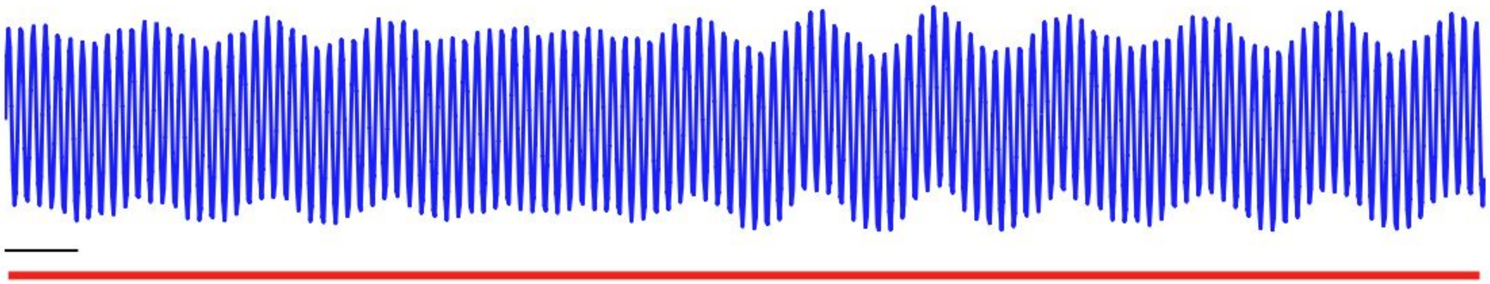
- What is df?

$$\boxed{df = \frac{1}{T}}$$

frequency **bin spacing**

where T = Total duration of recordings.

Ex.



T = 2 s so df = 0.5 Hz

Q: How do we improve frequency **bin spacing**

A: Increase T or record for longer time.

Not the same as freq resolution!

Spectrum: F_{NQ}

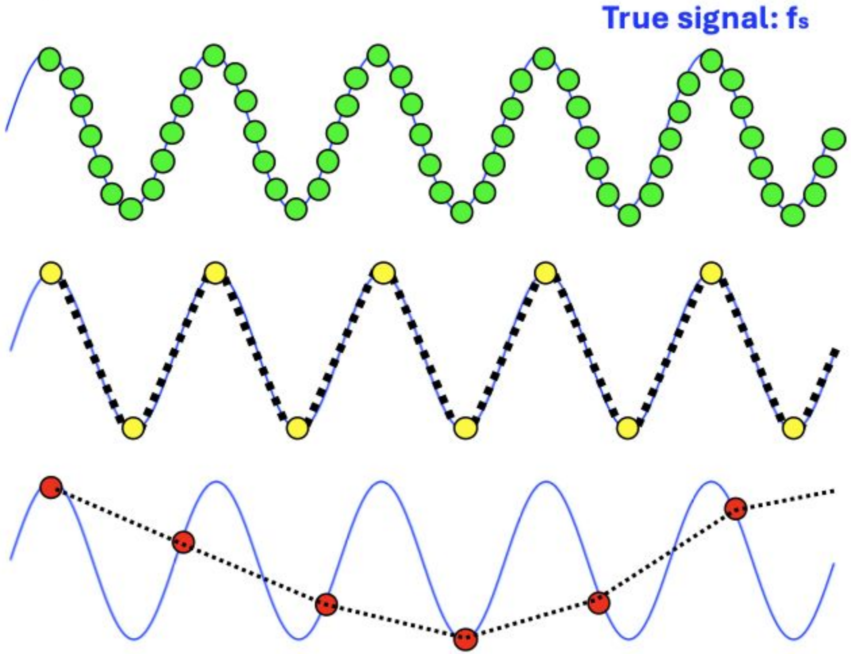
- What is F_{NQ}?

$$\boxed{F_{NQ} = f_0/2}$$

Nyquist frequency

where f_0 = sampling frequency.

The **highest** frequency we can observe.



Sample:

$f_0 >> 2\ f_s$

Accurate reconstruction

$f_0 = 2\ \boxed{f_s}$

Max freq we can observe at this sample rate!

2 samples/cycle

Enough to reconstruct signal, but just barely.

$f_0 < 2\ f_s$

High frequency (in data) mapped to low frequency (**aliased**).

All hope lost! Indistinguishable from true low frequency signals.

Spectrum: $df, \ F_{NQ}$

<u>Summary</u>

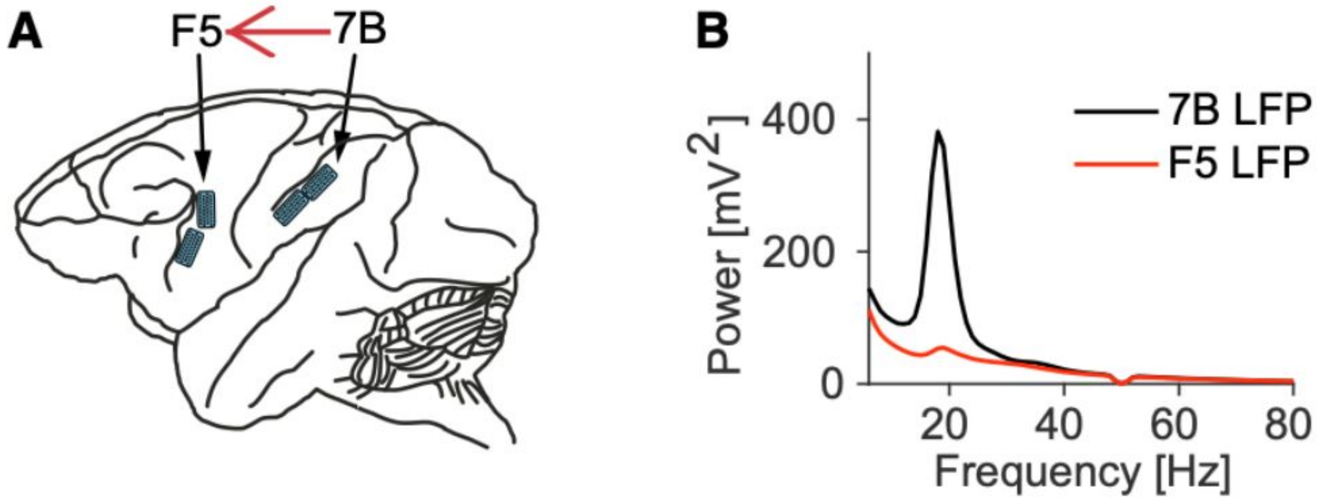
Frequency bin spacing $$df = \frac{1}{T}$$ ← Duration of recording

Nyquist frequency $$f_{\mathrm{NQ}} = \frac{f_0}{2}$$ ← Sampling frequency

For finer frequency **bin spacing** record more data.

To observe higher frequencies: increase sampling rate.

Example



[Schneider et al, Neuron, 2021]

Q. What rhythms dominate?

Quiz

Require a 2 Hz frequency **bin spacing** and to observe rhythms up to 500 Hz. What is the experiment / recording setup?

$$df =$$

$$f_{\mathrm{NQ}} =$$

Spectrum: things to know

- Subtract the mean / detrend
- Units
- Scale
- Tapers
- Frequency resolution
- Spectrogram

Subtract the mean to remove the DC component (0 Hz)

- The mean is a constant offset in voltage.
- A constant offset isn't an oscillation, but it appears in the spectrum at 0 Hz (as a big component).
- If you keep it, the spectrum can look dominated by "low-frequency power," making real rhythms harder to see.
  - Leaving it in can (1) dominate the scale and (2) spill into nearby low frequencies because we analyze finite chunks of data.
- When we say "oscillations," we usually mean fluctuations around a baseline
- Sometimes the baseline isn't constant - so you can detrend in more sophisticated ways (removing the ERP)

Spectrum: units

Q. What are the units of the spectrum?

$$S_{xx,j} = \frac{2\Delta^2}{T} X_j X_j^*$$

$$\frac{[s]^2}{[s]} \quad [V][V]$$

$$[s] \quad [V]^2$$

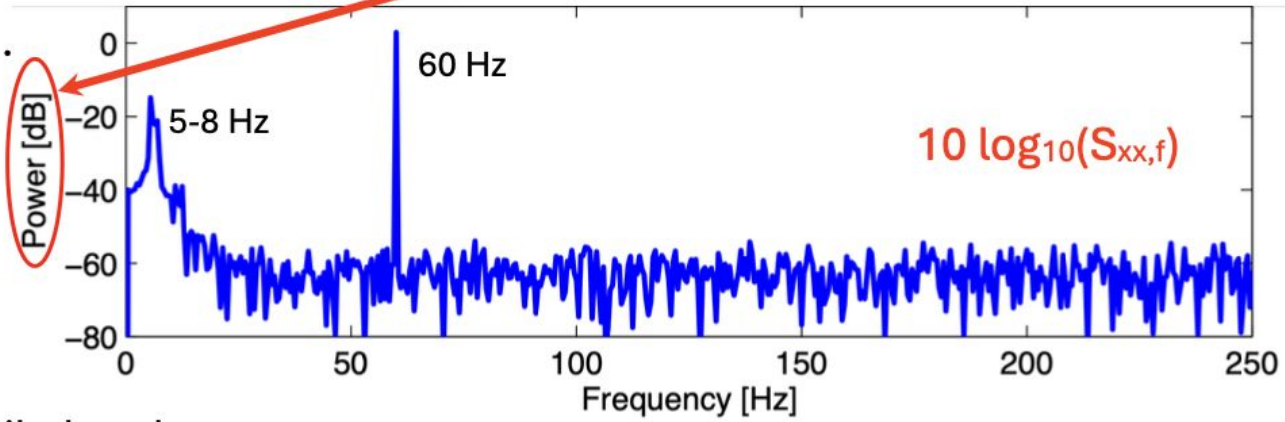
$$\boxed{\frac{[V]^2}{[Hz]}}$$

"volts squared per Hertz"

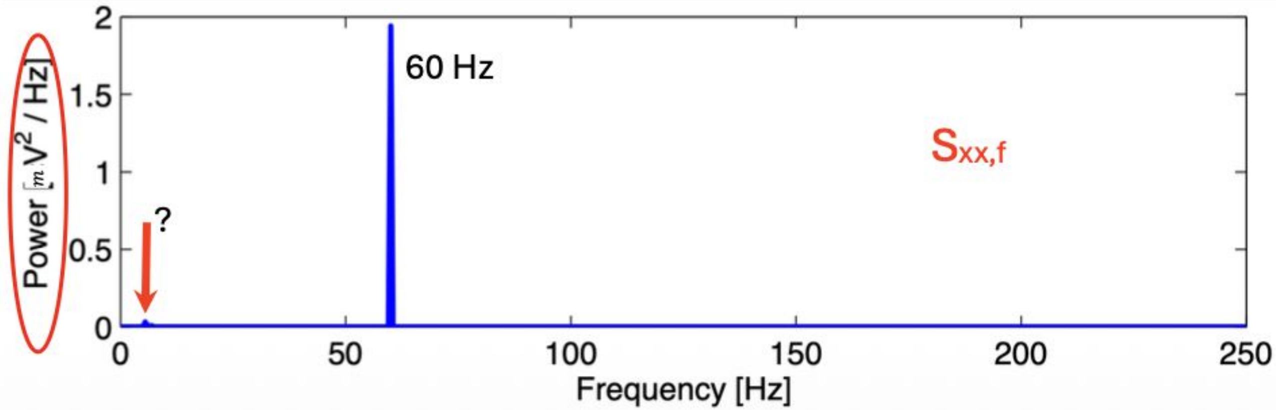
Spectrum: scale

Emphasize lower amplitude signals

A note on scale .



Without the decibel scale . . .



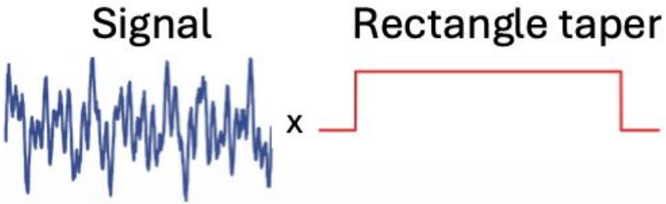
Spectrum: tapers

Doing nothing, we make an implicit taper choice . . .

. . . Data goes on forever . . .

Fourier transforms implicitly assume that the signal has infinite duration

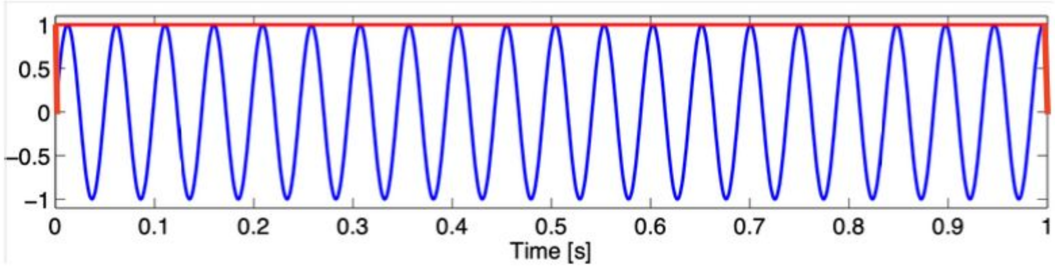
What we’re observing:

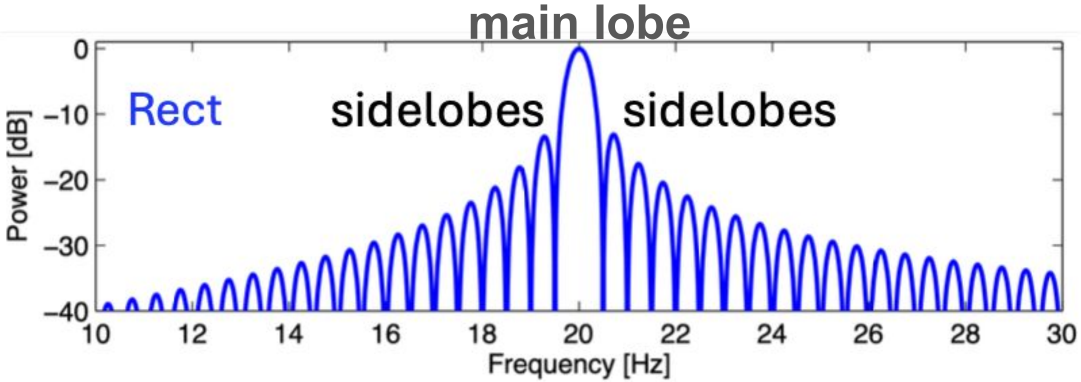


Spectrum: tapers

The rectangle taper impacts the power spectrum.

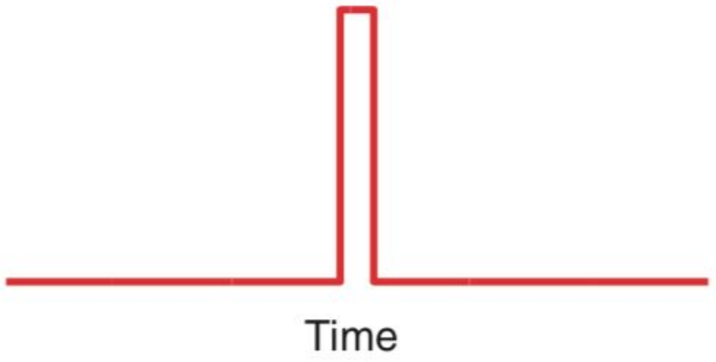
Pure sinusoid at 20 Hz

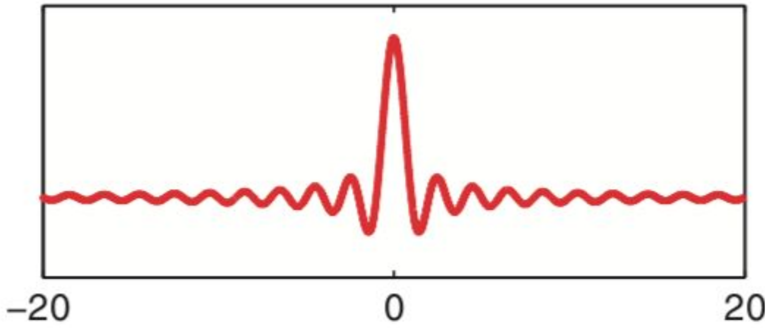




Sharp peak is “smeared out” . . .

Fourier transform of a rectangular taper is a sinc function

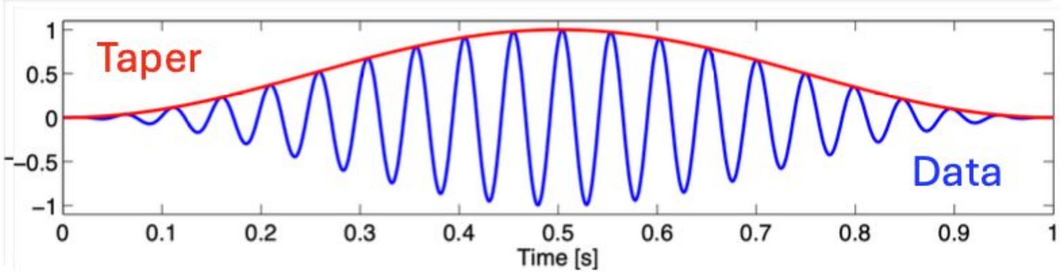




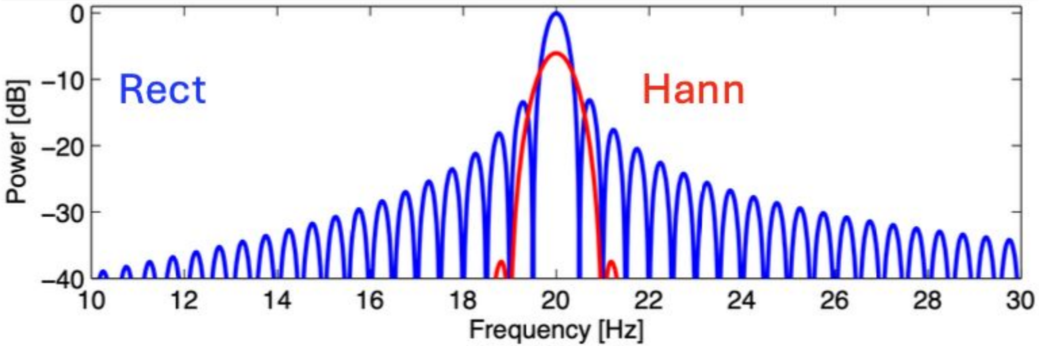
When we take the Fourier transform of a signal, we implicitly convolve the transformed signal with the sinc function

Spectrum: tapers

Idea: smooth the sharp edges of rectangle taper.



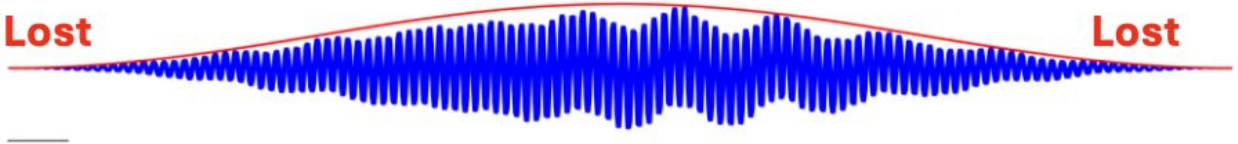
Compute spectrum of tapered data.

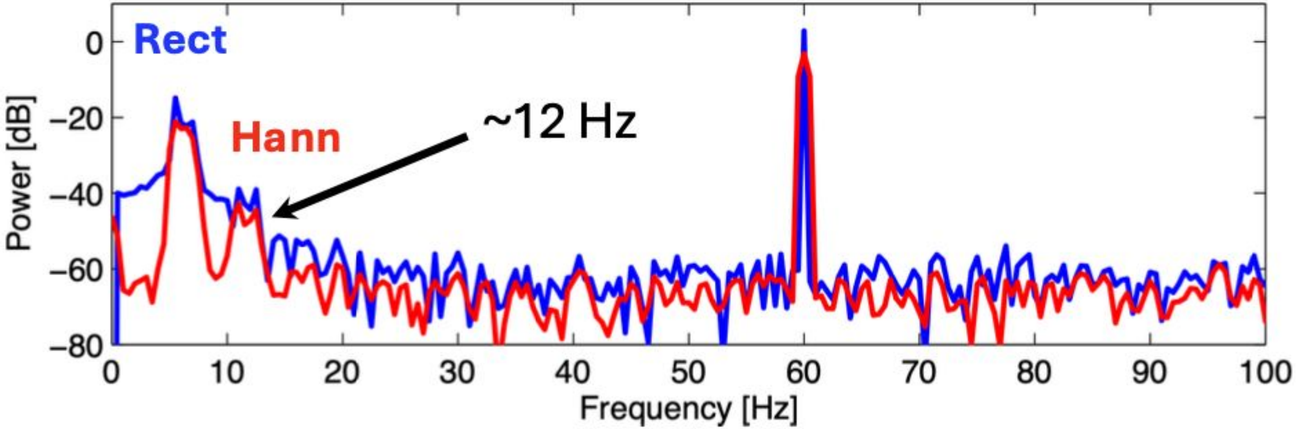


Taper reduces the sidelobes.

Spectrum: tapers

Ex:





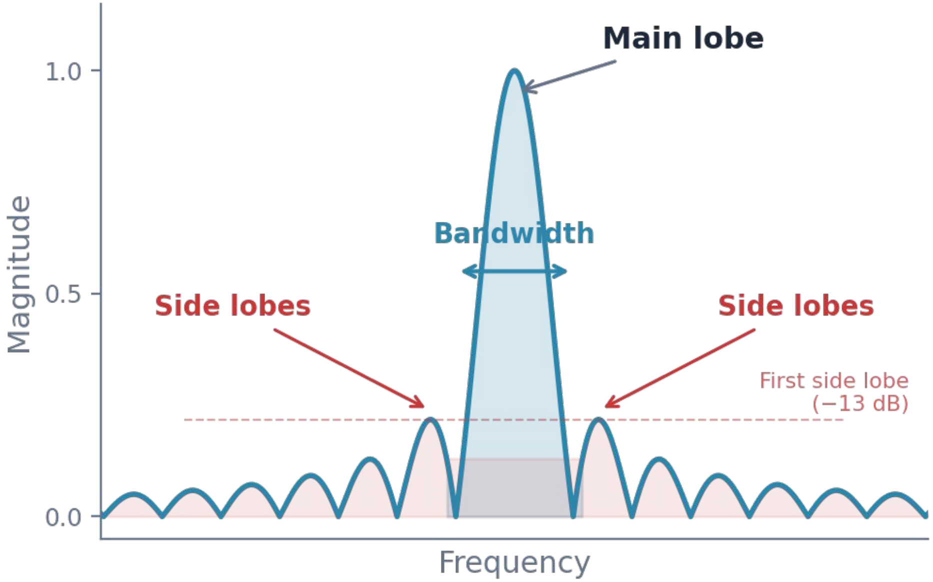
Good: Reduced sidelobes reveals a new peak.

Bad: Broader peaks & lose data at edges.

"*More lives have been lost looking at the [rectangular tapered spectrum] than by any other action involving time series.*" [Tukey 1980]

Choosing Windows / Tapers

The spectral kernel and its properties



Different kernels vary in:

Main lobe width

Frequency resolution — how precisely you localize peaks

First side lobe height

Worst-case leakage into adjacent frequency bins

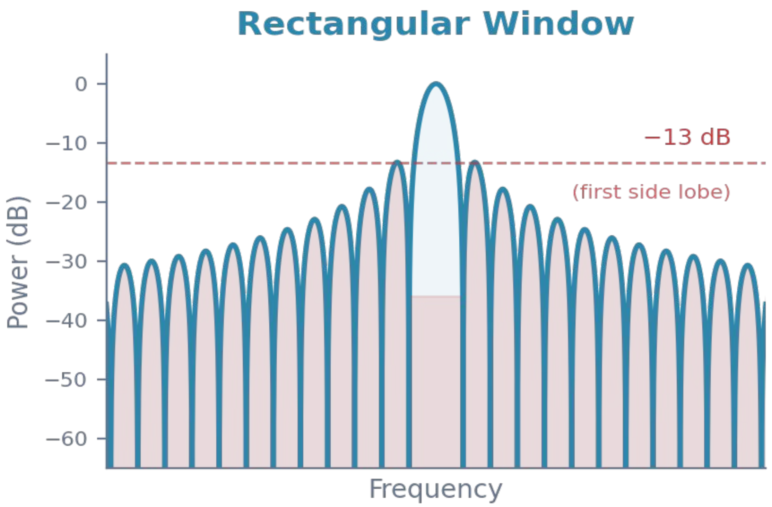
Side lobe fall-off rate

How quickly leakage decays at distant frequencies

The Fundamental Trade-off

Narrow main lobe ↔ Low side lobes — you can't have both

Narrow Main Lobe

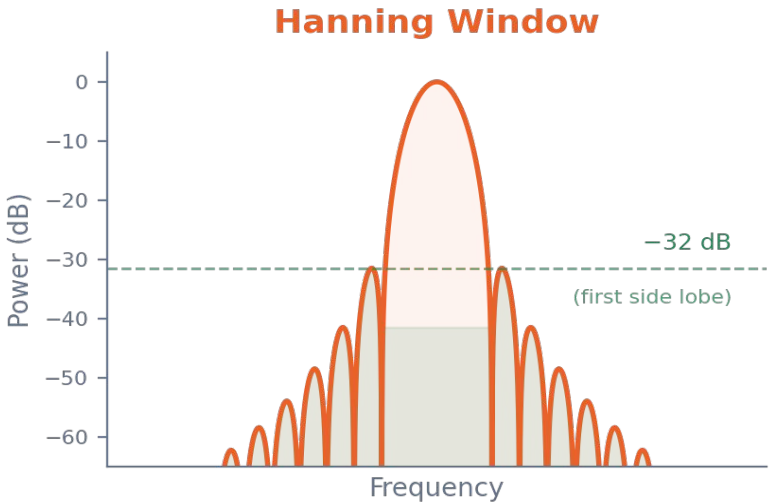


✓ Good frequency resolution

✗ Large side lobes ⟶ spectral leakage

Narrow-band bias

Wide Main Lobe



✗ Poor frequency resolution (bias)

✓ Small side lobes ⟶ less leakage

Wide-band bias

Which Source of Bias is Worse?

1	2	3
Main lobe shape	**Large first side lobe**	**Slow side lobe fall-off**
Smooths across nearby frequencies, blurring sharp peaks	Strong leakage into adjacent bands — can create phantom peaks	Distant frequencies contaminate the estimate — broadband leakage

Takeaway: Side lobe bias (2 & 3) is generally worse

We know that oscillations change in time

Theta power is present in hippocampus when the animal is running, not when immobile (mostly)

Ripple power is present in brief bursts during immobility in the hippocampus

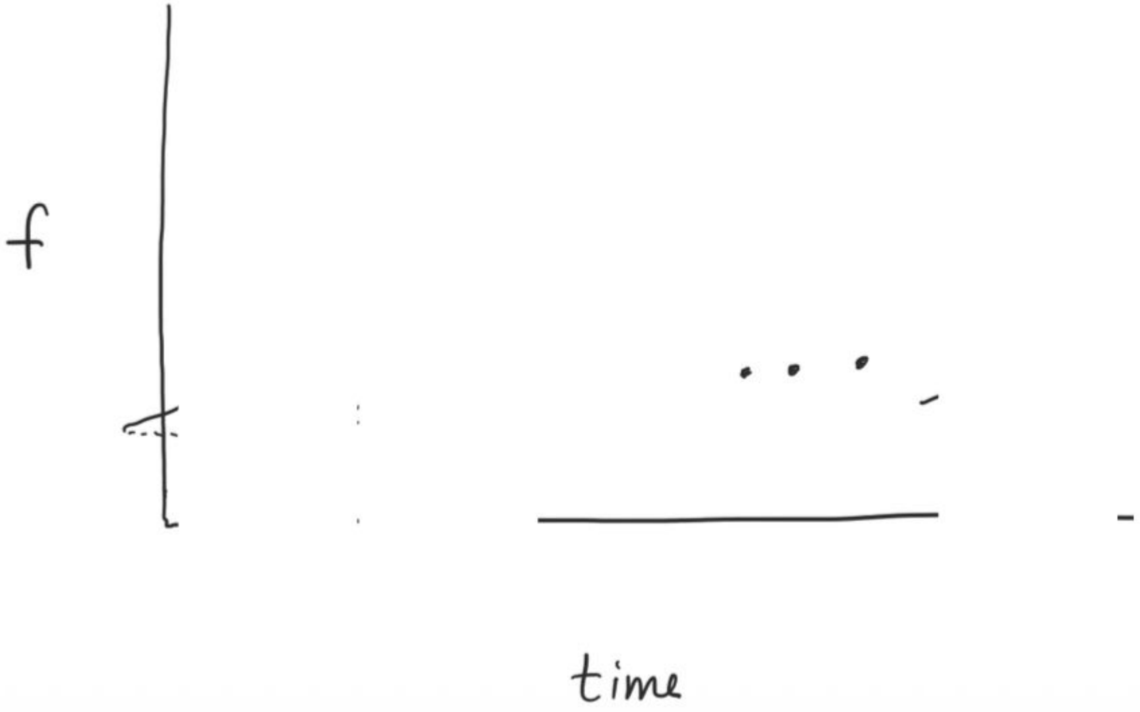
Spectrum: spectrogram

Idea: Divide data into smaller intervals, then compute the spectrum in each interval

Spectrum: spectrogram

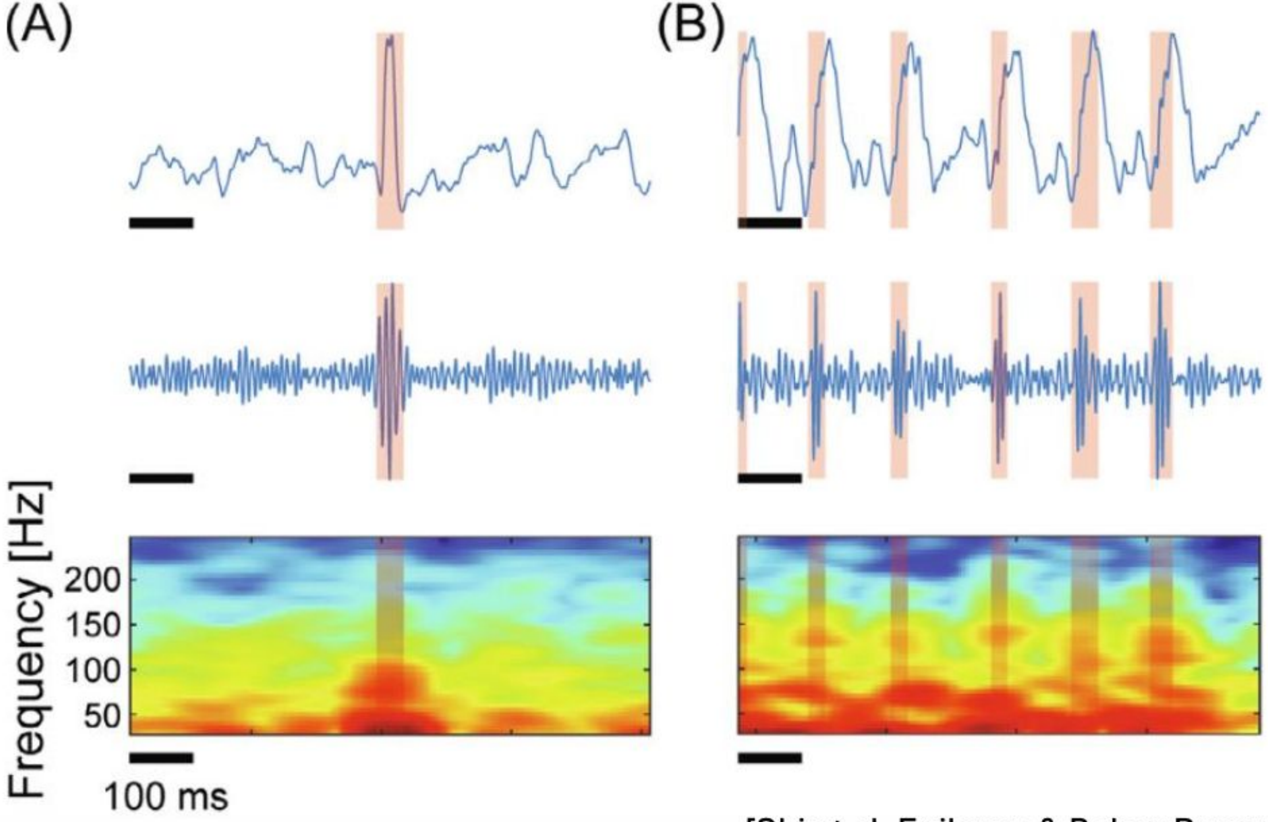
Display as

spectrogram



Usually, color indicates power

Spectrum: spectrogram

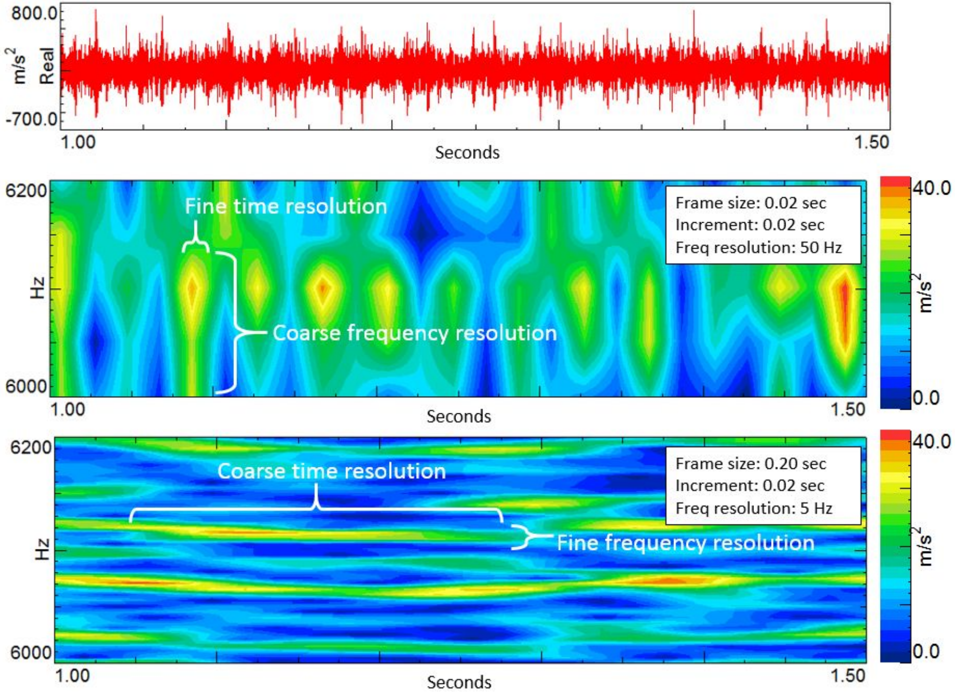


[Shi et al, Epilepsy & Behav Reports, 2022]

Time-frequency trade off

Frequency is about repeating patterns. To know whether something is 8 Hz or 10 Hz, you need to observe multiple cycles → that takes time.

Time localization needs short windows. If you only look at a short chunk, you can tell when something happened, but you lose precision about which exact frequency it was.



https://community.sw.siemens.com/s/article/wavelets-time-frequency-analysis

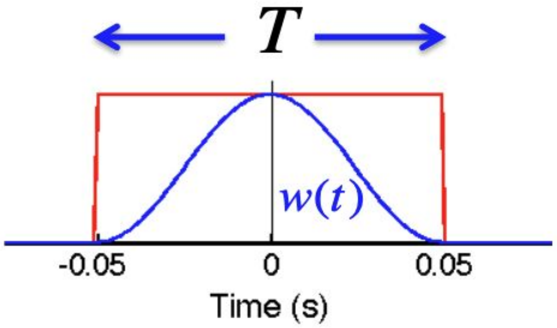
T = temporal resolution

Frequency resolution (Main-lobe width) scales like $1/T$, but the proportionality constant depends on the window/taper

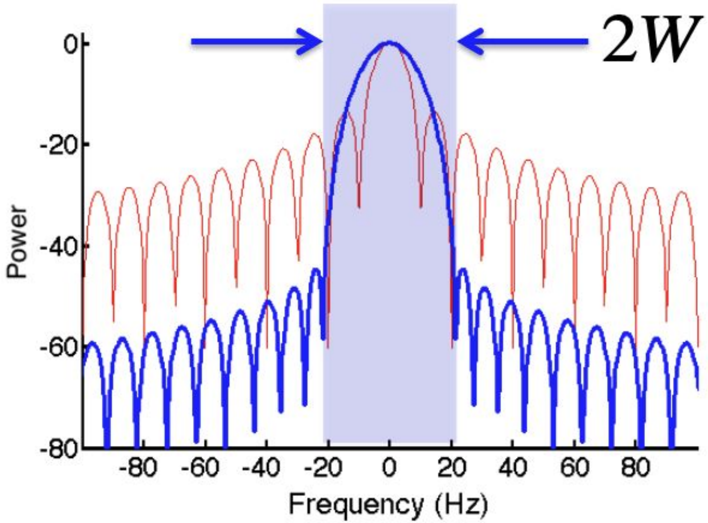
How do we find tapers that balance temporal and frequency resolution?

Finding good tapers: one strategy

We want to find a strictly time-localized function [-T/2,T/2] whose Fourier Transform is maximally localized within a finite window in the frequency domain [-W,W].



dpss_taper.m.m

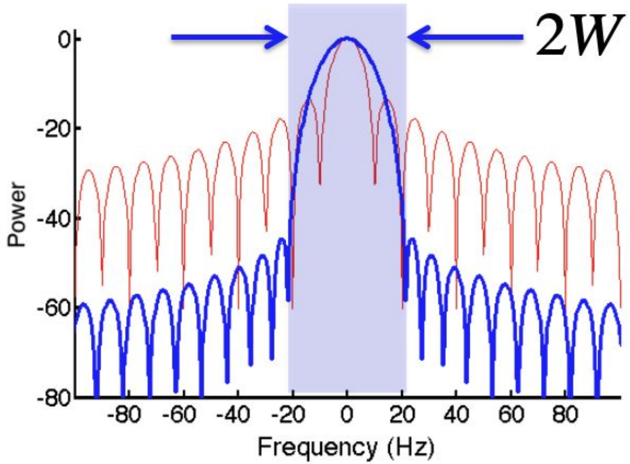


- We want to find a function of time w(t) that maximizes the spectral concentration.

$$\lambda = \frac{\int_{-W}^{W} |U(f)|^2 \, df}{\int_{-\infty}^{\infty} |U(f)|^2 \, df}$$

U(f) is the F.T. of w(t)

$$U(f) = \int_{-\infty}^{\infty} w(t) e^{-i2\pi ft} \, dt$$

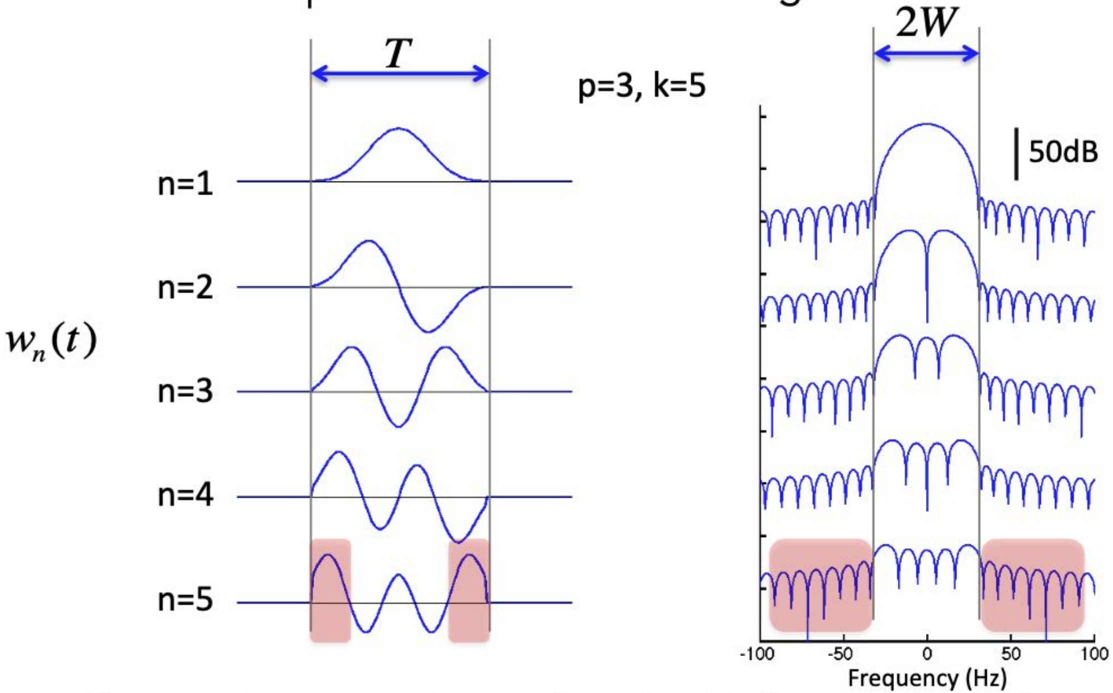


- Maximizing λ gives a set of k=2WT-1 functions called Slepian functions for which λ is very close to 1.

… also called discrete prolate spheroid sequence (dpss)

Slepian tapers are orthogonal

- The set of dpss functions is also orthogonal.



- Because they are orthogonal, each will give an independent estimate of the spectrum!

Multitaper Spectral Estimation

- Select a time window width T (temporal resolution).
- Select a time-bandwidth product p=WT (i.e. set the frequency resolution).
- Compute the set of set of dpss tapers using T and p=WT
- Estimate the spectrum using each of the k= 2*p-1 tapers

$$\hat{S}_n(f) = \left| \sum_{t=1}^{N} w_n(t) y(t) e^{-i2\pi f t} \right|^2$$

- Average the estimates to get the spectrum!

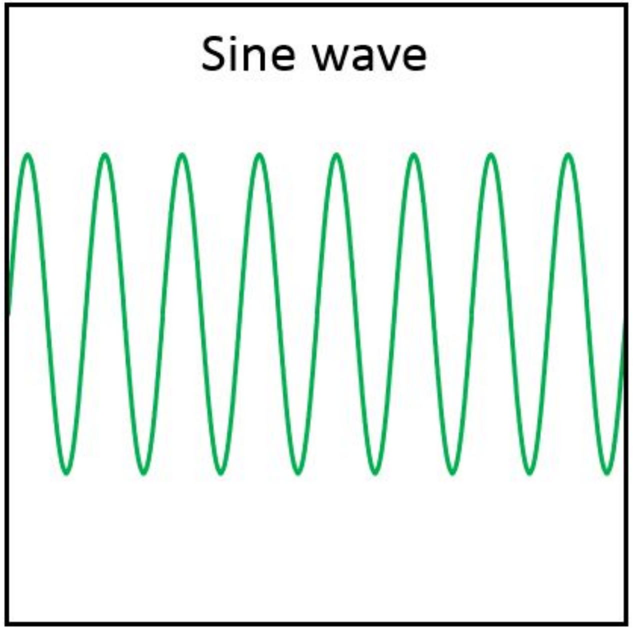
$$S(f) = \frac{1}{k} \sum_{n=1}^{k} \hat{S}_n(f)$$

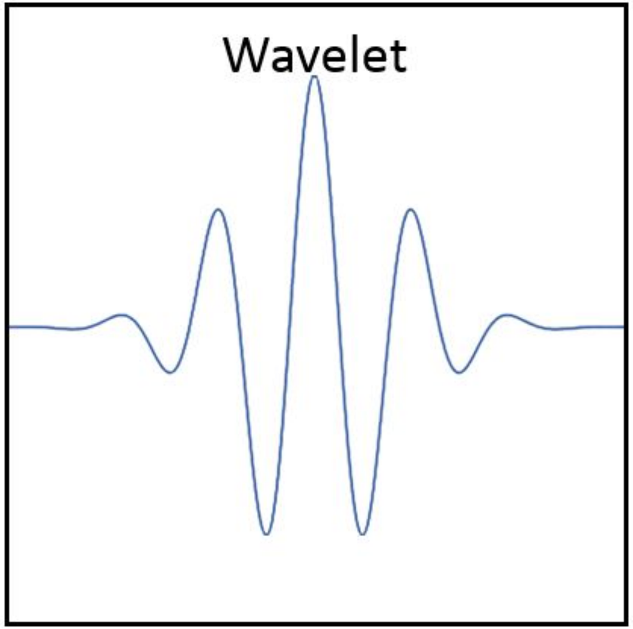
- You get multiple spectral estimates from the same piece of data.

Multitaper Spectral Estimation Advantages

- Much lower variance (more stable spectra)
- Less spectral leakage (supresses sidelobe influence)
- Can get uncertainty estimates

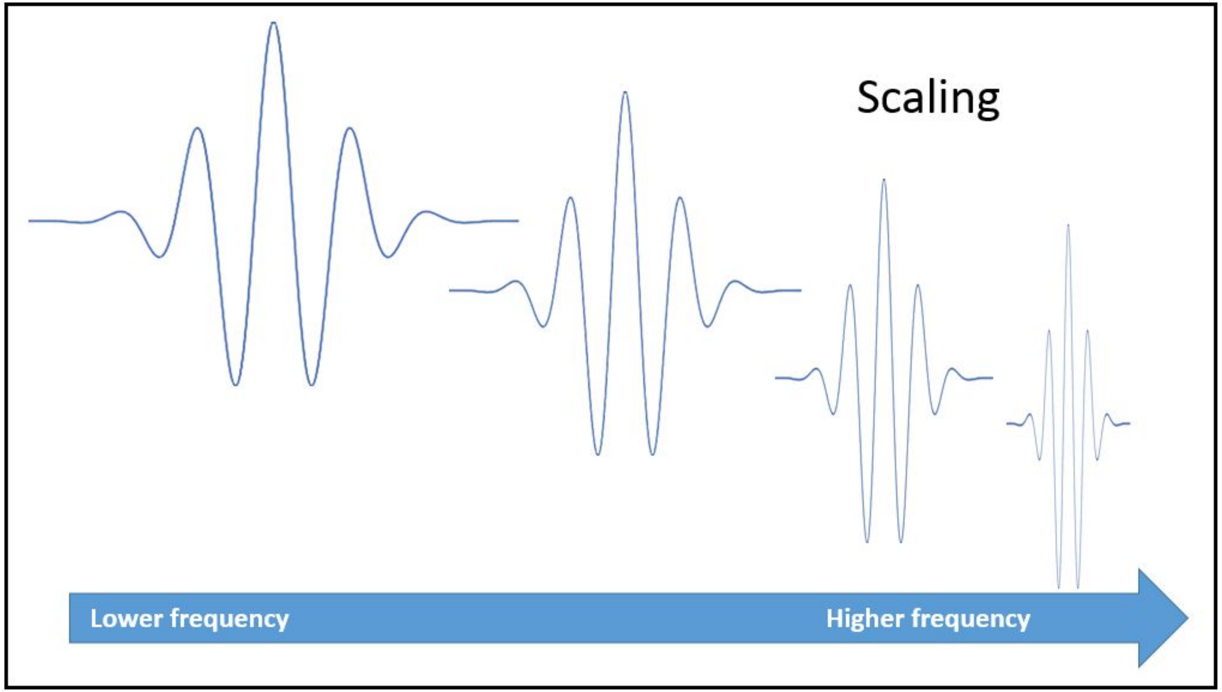
Wavelet Transform





https://community.sw.siemens.com/s/article/wavelets-time-frequency-analysis

The wavelets have short windows at high frequencies, long windows at low frequencies



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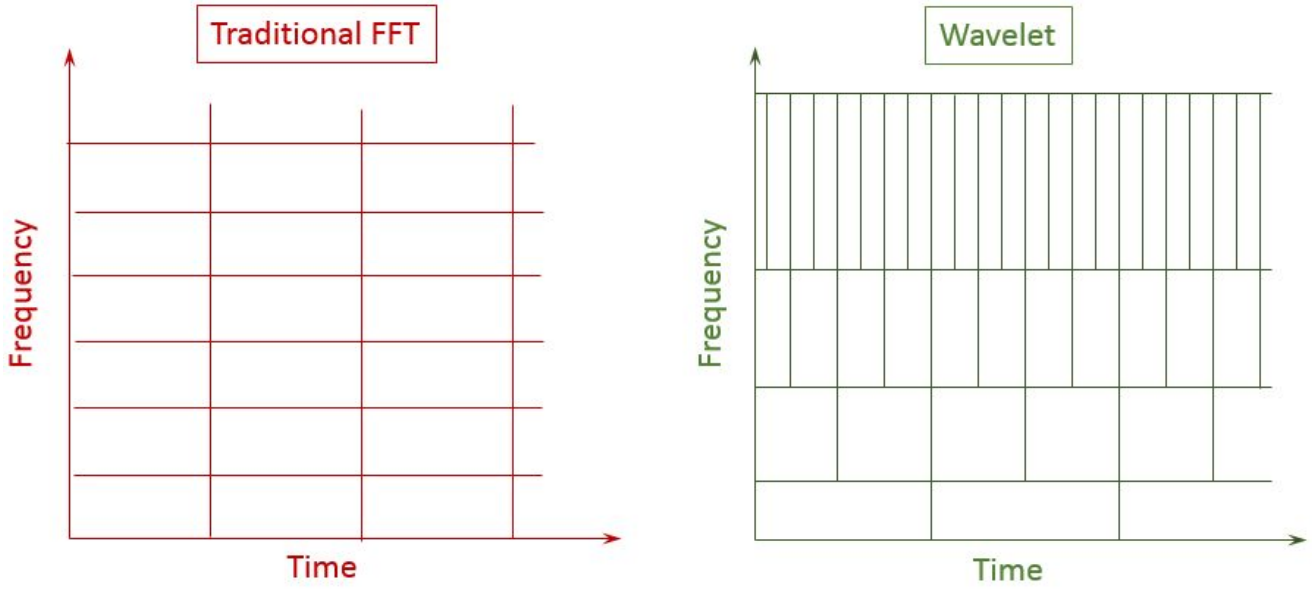

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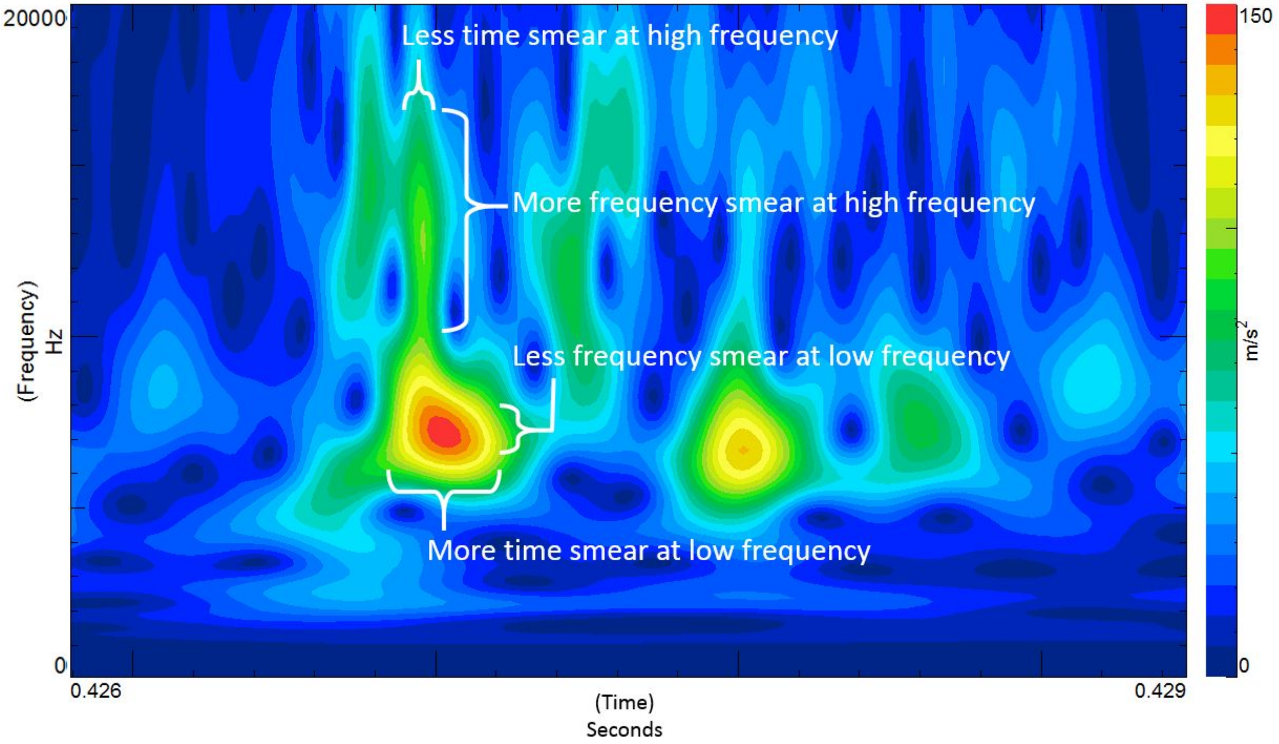

Comparison

Multitaper spectrogram

- fixed window
- minimize spectral leakage
- lower variance
- precise specification of time and frequency resoltuion

Wavelets

- adaptive windows, good visualization across frequencies
- leakage / side-lobe behavior depends on the wavelet
- still subject to time-frequency tradeoff
- linear relationship between bandwidth and frequency

Spectrum: summary