write-up

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Full Model

For $i = 1, \ldots, n$ observations, denote each observation's group of scores as $\mathbf{Y}_i$, where

$$\mathbf{Y}_i = \begin{pmatrix} Y_{i,1} \\ Y_{i,2} \\ Y_{i,3} \end{pmatrix} = \begin{pmatrix} \text{political participation score} \\ \text{political satisfaction score} \\ \text{economic satisfaction score} \end{pmatrix}$$

We will assume the likelihood of the data follows a multivariate normal distribution. The full model then becomes:

$$\mathbf{Y}_i \mid \boldsymbol{\theta}, \Sigma \sim MVN(\boldsymbol{\theta}, \Sigma).$$
$$\boldsymbol{\theta} \sim MVN(\boldsymbol{\mu}_0, \Lambda_0)$$
$$\Sigma \sim \text{inverseWishart}(\nu_o, S_o^{-1}).$$

Let $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)$ denote the mean scores for 1) political participation, 2) political satisfaction, and 3) economic satisfaction.

Let Σ denote the covariance matrix, where the $(i,j)^{\text{th}}$ component of Σ is the covariance between Y_i and~Y_j, giving component variances along the diagonal of Σ.

Hyperparameter settings

We will set the prior mean on $\boldsymbol{\theta}$ to be the average political participation, political satisfaction, and economic satisfaction scores. Specifically, let $\boldsymbol{\mu}_0 = (.5, .5, .5)^T$.

Since the true mean cannot be below 0 or above 1, we will use a prior variance that puts little probability outside this range. Thus, the prior variances on $\boldsymbol{\theta}$ can be set as $\Lambda_0 = (., ., .)^T$ so that the prior probability that $P(\theta_j \neq [0, 1]) = 0.05$. We will also take the prior correlation of 0.5, or rather $\lambda_{1,2} = ./2 = ..$

For setting prior settings for Σ, we will take S_o to be equal to $\mathbf{\Lambda}$. We will center Σ around S_o by setting $\nu_0 = p + . = ..$