COSI 127b
Introduction to Database Systems

Lecture 12: Normalization (2)

Midterm

Will Do Quick Review during Monday's Class

Summary: Covers all topics up through PS 4

No late submissions (solutions posted Mon after class)

Review: Good DB Design

Three Approaches:

1. Ad hoc:
   - use Entity-Relationship Model to model data requirements
   - translate ER design into relational schema

   Issue: How to tell if design is "good"?

2. Theoretical:
   - construct universal relations (e.g., `Borrower-All`)
   - decompose above using known functional dependencies

   Issue: Time-Consuming and Complex

3. Practical:
   - use ER Model to produce 1st cut DB design
   - use FDs to refine and verify

Review: Functional Dependencies

Previously:

- *What "$\mathtt{A_1, \ldots, A_n} \rightarrow \mathtt{B}$" means*
- *When sets of FDs are equivalent ($\mathtt{F} \equiv \mathtt{G}$)*
  - *if $F^+ = G^+$ (FD set closures)*
  - *algorithms: Attribute Closures or Armstrong's Axioms*

Today:

- *Minimal FD Sets (F_c = "Canonical Cover" of F)*
- *Canonical Cover Algorithm*

Coming Up:

- *DB Design using FDs and FD Algorithms*

Review: Functional Dependencies

In General:

$A_1, \ldots, A_n \rightarrow B$

Informally:

If 2 tuples agree on their values for $A_1, \ldots, A_n$,
then they will also agree on their values for B

Formally:

$$\boxed{\forall t, u\ (t[A_1] = u[A_1] \wedge \ldots \wedge t[A_n] = u[A_n]) \Rightarrow t[B] = u[B])}$$

Review: FD Closures (F^+)

Idea: Some FDs are implied by others

Borrower-All

lno	cname	cstreet	ccity	bname	amt	bcity	assets
L-17	Jones	Main	Harrison	Downtown	1000	Brooklyn	9M
L-23	Smith	North	Rye	Redwood	2000	Palo Alto	2.1M
L-15	Hayes	Main	Harrison	Perry	1500	Horseneck	1.7M
L-17	Jackson	Senator	Brooklyn	Downtown	1000	Brooklyn	9M
L-93	Curry	Walnut	Stanford	Mianus	500	Horseneck	0.4M
L-11	Smith	North	Rye	R.H.	900	Horseneck	8M
L-16	Adams	Spring	Pittsfield	Perry	1300	Horseneck	1.7M

lno → bname

+

Borrower-All

lno	cname	cstreet	ccity	bname	amt	bcity	assets
L-17	Jones	Main	Harrison	Downtown	1000	Brooklyn	9M
L-23	Smith	North	Rye	Redwood	2000	Palo Alto	2.1M
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L-16	Adams	Spring	Pittsfield	Perry	1300	Horseneck	1.7M

bname → bcity

implies

Borrower-All

lno	cname	cstreet	ccity	bname	amt	bcity	assets
L-17	Jones	Main	Harrison	Downtown	1000	Brooklyn	9M
L-23	Smith	North	Rye	Redwood	2000	Palo Alto	2.1M
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L-16	Adams	Spring	Pittsfield	Perry	1300	Horseneck	1.7M

lno → bcity

Review: FD Closures (F^+)

Idea: Some FDs are implied by others

$$\{\texttt{lno} \rightarrow \texttt{bname},\ \texttt{bname} \rightarrow \texttt{bcity}\}$$

$$\equiv$$

$$\{\texttt{lno} \rightarrow \texttt{bname},\ \texttt{bname} \rightarrow \texttt{bcity},\ \texttt{lno} \rightarrow \texttt{bcity}\}$$

$$\equiv$$

$$\{\texttt{lno} \rightarrow \texttt{bname bcity},\ \texttt{bname} \rightarrow \texttt{bcity},\ \texttt{lno} \rightarrow \texttt{bcity}\}$$

$$\equiv$$

$$\{\texttt{lno} \rightarrow \texttt{bname bcity},\ \texttt{bname} \rightarrow \texttt{bcity},\ \texttt{lno} \rightarrow \texttt{lno}\}$$

Review: FD Closures (F^+)

Given FD sets over R, F and G, how to decide if $F \equiv G$?

- Idea: Compare sets of FDs that F, G imply (closures)

$F \equiv G$ *if and only if* $F^+ = G^+$

Two ways to determine F^+

- Attribute Closures
- Armstrong's Axioms

Review: FD Closures (F^+)

Algorithm 1: Using Attribute Closures

```
ALGORITHM FD-Closure (F: {FDs})
-- using Att-Closure
BEGIN
  Result ← {}
  Atts ← <all attributes appearing in FDs in F>
  FOREACH Z ⊆ Atts DO
   Result ← Result ∪ {Z → Att-Closure(Z,F)}
  RETURN Result
END
```

```
ALGORITHM Att-Closure (Z: {Attributes}, F: {FDs})
BEGIN
  Result ← Z
  REPEAT UNTIL STABLE
   FOR EACH functional dependency in F, X → Y DO
     IF X ⊆ Result THEN Result ← Result ∪ Y
  RETURN Result
END
```

Review: FD Closures (F^+)

Algorithm 2: Using Armstrong's Axioms

```
ALGORITHM FD-Closure (F: {FDs})
-- using Armstrong's Axioms
BEGIN
  Result ← F
  REPEAT UNTIL STABLE
    IF for any of Armstrong's Axioms (if A then B),
          A matches part of Result THEN
             Result ← Result ∪ B
  RETURN Result
END
```

Review: FD Closures (F^+)

Algorithm 2: Using Armstrong's Axioms

1. *Reflexivity*
   - if $Y \subseteq X$ then $\mathbf{X} \rightarrow \mathbf{Y}$
2. *Augmentation*
   - if $X \rightarrow Y$ then $\mathbf{WX} \rightarrow \mathbf{WY}$
3. *Transitivity*
   - if $X \rightarrow Y$ and $Y \rightarrow Z$ then $\mathbf{X} \rightarrow \mathbf{Z}$
4. *Union*
   - if $X \rightarrow Y$ and $X \rightarrow Z$ then $\mathbf{X} \rightarrow \mathbf{YZ}$
5. *Decomposition*
   - if $X \rightarrow YZ$ then $\mathbf{X} \rightarrow \mathbf{Y}$ and $\mathbf{X} \rightarrow \mathbf{Z}$
6. *Pseudotransitivity*
   - if $X \rightarrow Y$ and $WY \rightarrow Z$ then $\mathbf{WX} \rightarrow \mathbf{Z}$

Today

One more algorithm over FD sets:

- Canonical Cover (F_C): a "minimal" version of FD set, F
- F_C the "minimal" version of F?
  1. equivalent to F ($F_C^+ = F^+$)
  2. "smaller" than other FD sets equivalent to F:
     a) fewer FDs:
        $\{A \rightarrow B,\ B \rightarrow C\} < \{A \rightarrow B,\ B \rightarrow C,\ A \rightarrow C\}$
     b) fewer attributes in FDs:
        $\{A \rightarrow B,\ B \rightarrow C\} < \{A \rightarrow BC,\ B \rightarrow C\}$

Canonical Cover (F_c)

Idea: Some FDs are implied by others

$\{\texttt{lno} \rightarrow \texttt{bname},\ \texttt{bname} \rightarrow \texttt{bcity}\}$

$\equiv$

$\{\texttt{lno} \rightarrow \texttt{bname},\ \texttt{bname} \rightarrow \texttt{bcity},\ \texttt{lno} \rightarrow \texttt{bcity}\}$

$\equiv$

$\{\texttt{lno} \rightarrow \texttt{bname bcity},\ \texttt{bname} \rightarrow \texttt{bcity}\}$

$\equiv$

$\{\texttt{lno} \rightarrow \texttt{bname bcity},\ \texttt{bname} \rightarrow \texttt{bcity},\ \texttt{lno} \rightarrow \texttt{lno}\}$

Which is the Canonical Cover?

Canonical Cover (F_C)

One Use for Canonical Cover:

Given:

$F = \{A \rightarrow BC,\ B \rightarrow CE,\ A \rightarrow E,\ AD \rightarrow H,\ D \rightarrow B\}$

then:

$F_c = \{A \rightarrow BH,\ B \rightarrow CE,\ D \rightarrow B\}$

Observe:

- F requires 5 global integrity constraints to enforce
- F_c requires 3 global integrity constraints to enforce

Another Use: Normalization Algorithms (Later)

Canonical Cover Algorithm

Example 1:

input: $F = \{A \rightarrow BC,\ B \rightarrow CE,\ A \rightarrow E,\ AC \rightarrow H,\ D \rightarrow B\}$

output: $F_c = \{A \rightarrow BH,\ B \rightarrow CE,\ D \rightarrow B\}$

- $F_c \equiv F$
- *no* G *that is equivalent to* F *is "smaller" than* F_c

Example 2:

input: $F = \{A \rightarrow BC,\ B \rightarrow C,\ A \rightarrow B,\ AB \rightarrow C,\ AC \rightarrow D\}$

output: $F_c = \{A \rightarrow BD,\ B \rightarrow C\}$

Canonical Cover Algorithm

Basic Algorithm (will refine later)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  …

END
```

Canonical Cover Algorithm

Basic Algorithm (will refine later)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)
```

e.g.: **A** → **B**, **A** → **C** *becomes* **A** → **BC**

B → **CD**, **B** → **DA** *becomes* **B** → **ACD**

Canonical Cover Algorithm

Basic Algorithm (will refine later)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD’s in F
     (Armstrong’s Axioms)

  2. Remove extraneous attributes from each FD in F

  …

END
```

Canonical Cover Algorithm

Basic Algorithm (will refine later)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F
```

e.g.: A → BC *becomes* A → C *if* B *is extraneous in* ***RHS***

or A → B *if* C *is extraneous in* ***RHS***

AB → C *becomes* A → C *if* B *is extraneous* **in** ***LHS***

or B → C *if* A *is extraneous* **in** ***RHS***

Canonical Cover Algorithm

Basic Algorithm (will refine later)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  Is B extraneous in A → BC?


    b) LHS:  Is B extraneous in AB → C?

END
```

Removing Extraneous Attributes

Extraneous Attributes in Right-Hand Side (RHS):

"Is **B** extraneous in A → **B**C?" *means* …

*can we replace "*A → **B**C*" with "*A → C*" in* F *without changing* F^+*?*

Extraneous Attributes in Left-Hand Side (LHS):

"*Is* **B** *extraneous in* A**B** → C?" *means* …

*can we replace "*A**B** → C*" with "*A → C*" in* F *without changing* F^+*?*

Extraneous Attributes in RHS

Is B extraneous in $A \rightarrow BC$?

A Simple (but Expensive) Test:

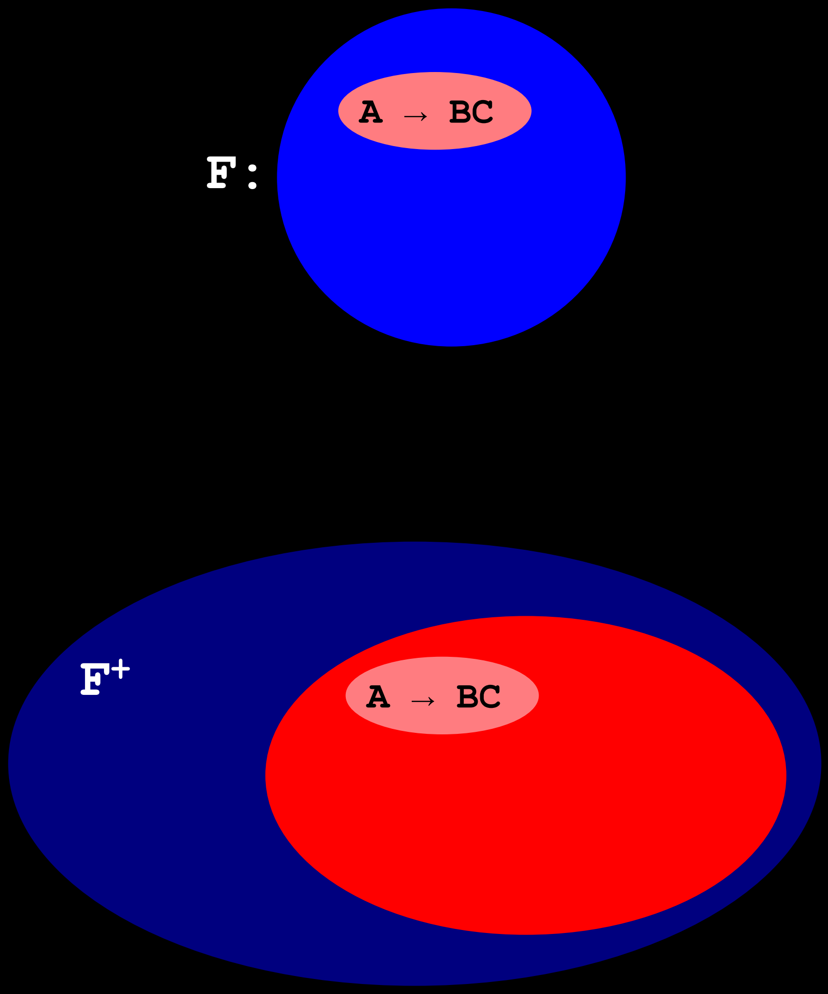
1. *Replace "$A \rightarrow BC$" with "$A \rightarrow C$" in F to construct F_2*
   - $F_2 = F - \{A \rightarrow BC\} \cup \{A \rightarrow C\}$

2. *Test: Is $F^+ = F_2^+$?*
   - *Method:* `FD-Closure (F) = FD-Closure (F`$_2$`)?`
   - *Cost: $$$$*

Is there a less expensive way to compare F^+ and F_2^+ here?

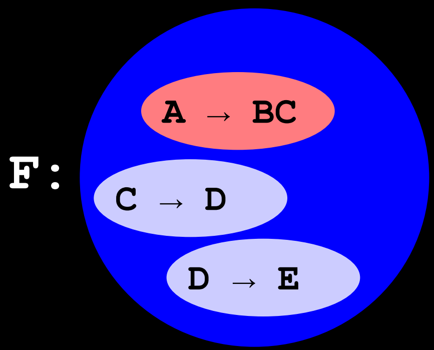
Extraneous Attributes in RHS

Is $F^+ = F_2{}^+$?



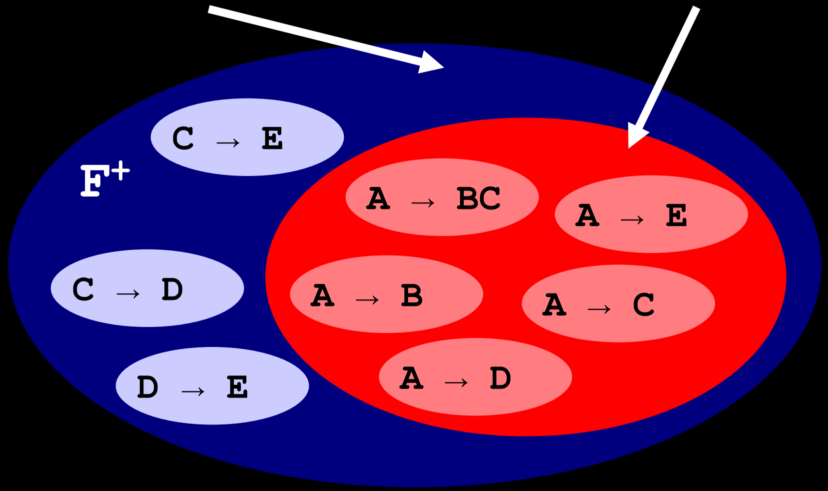
Extraneous Attributes in RHS

Is $F^+ = F_2^+$?



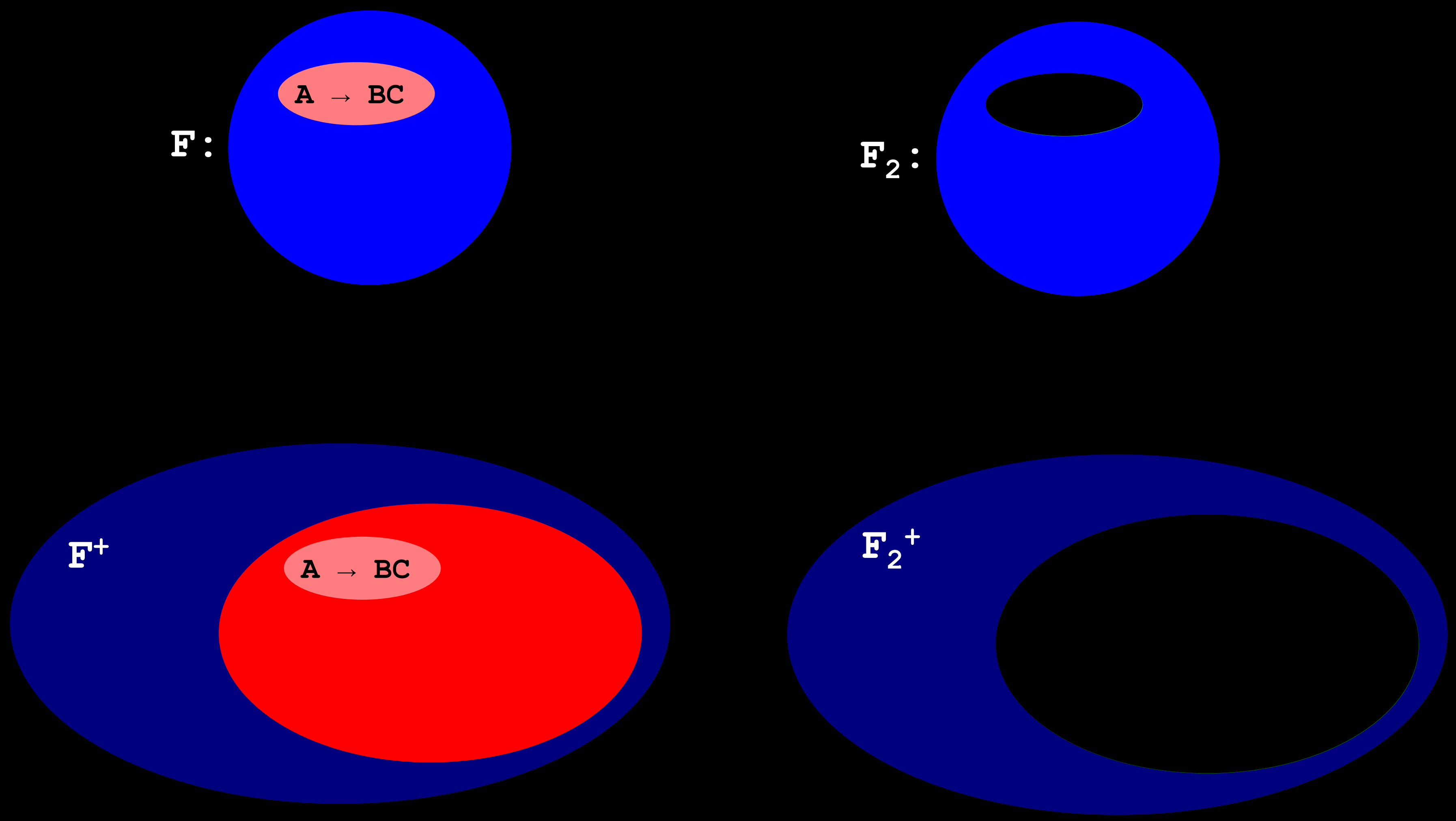
Given F = {A → BC, C → D, D → E}

(F − (A → BC))$^+$ F$^+$ − (F − (A → BC))$^+$



Extraneous Attributes in RHS

Is $F^+ = F_2^+$?



Extraneous Attributes in RHS

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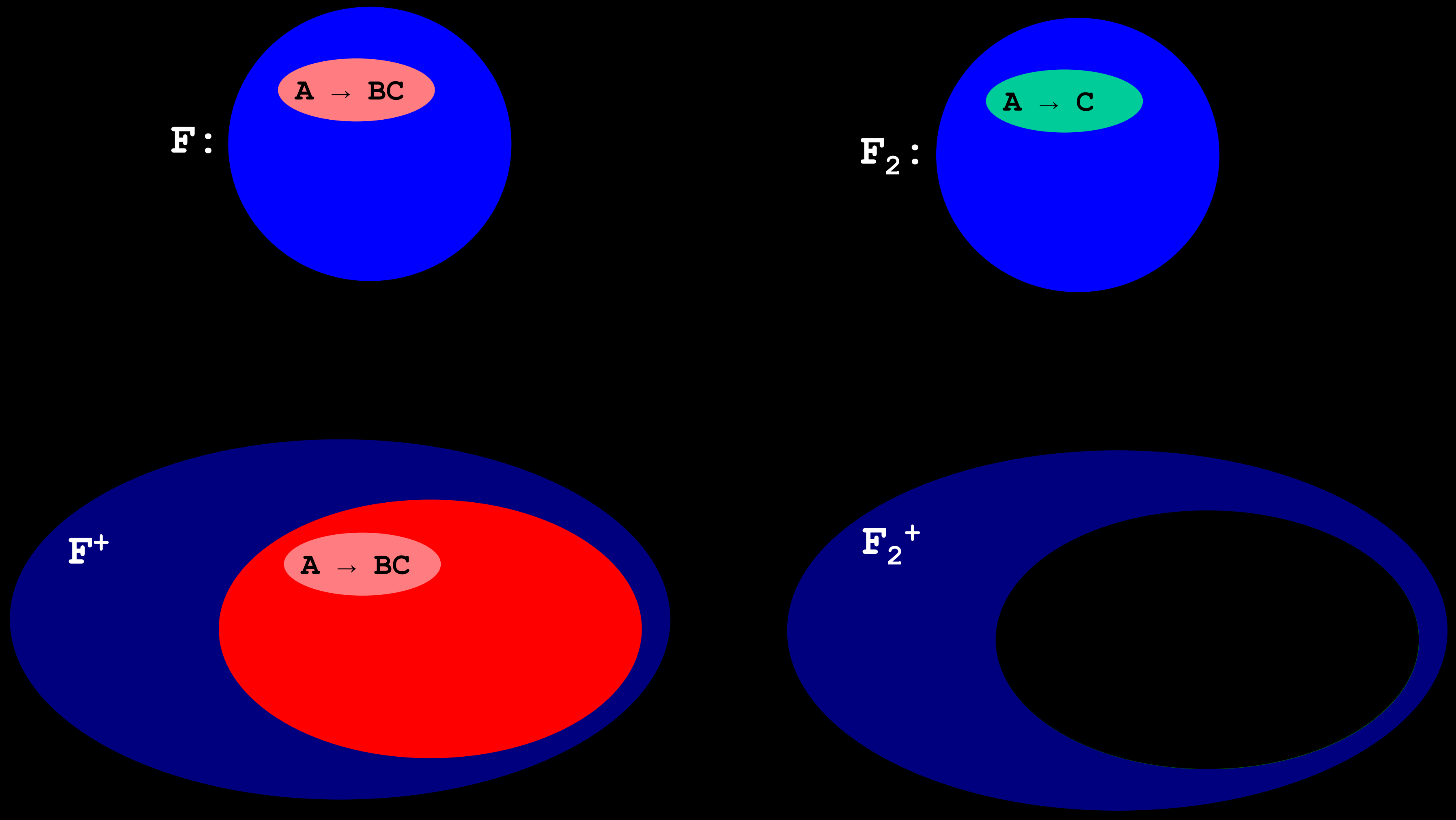

Extraneous Attributes in RHS

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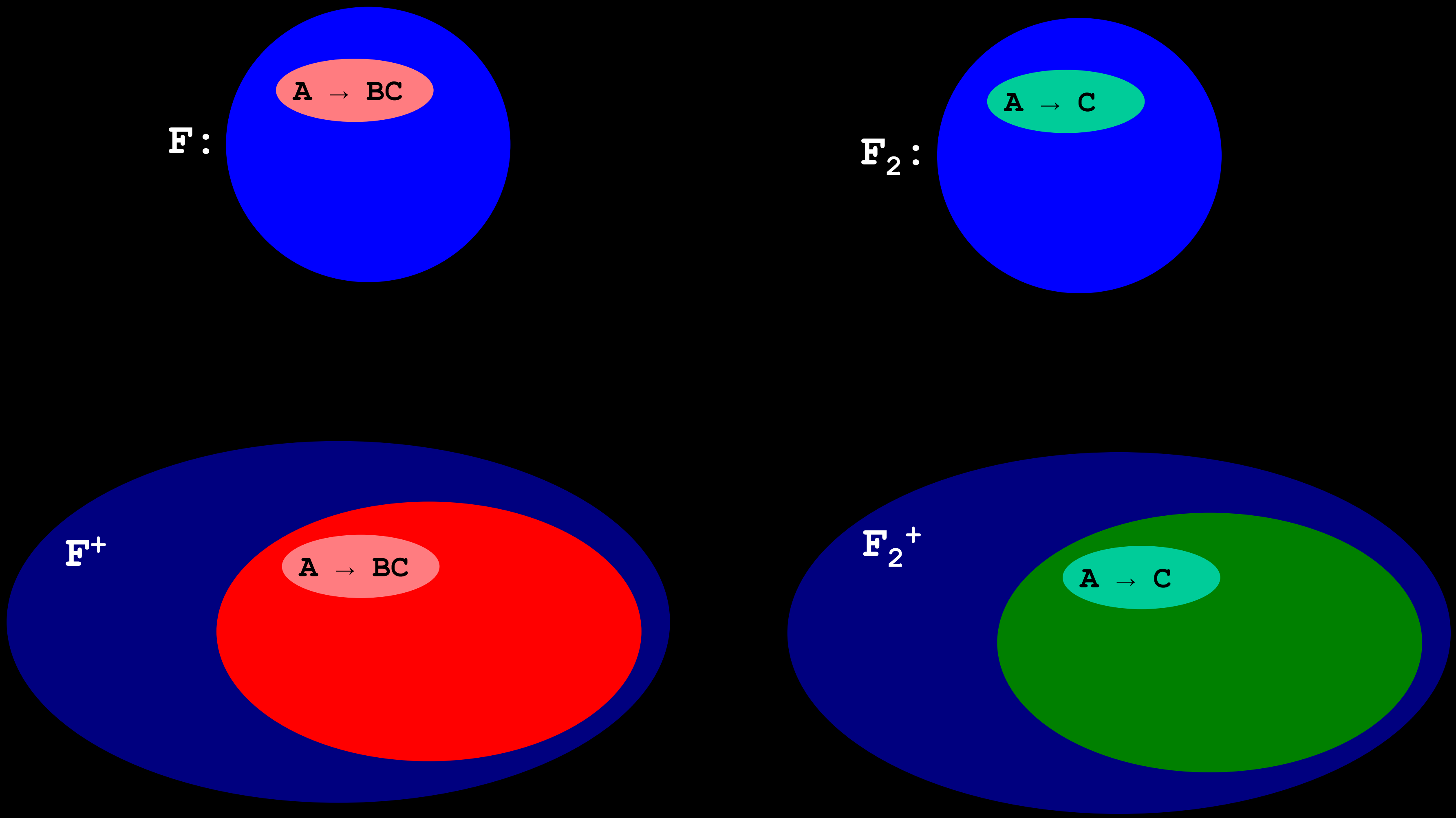

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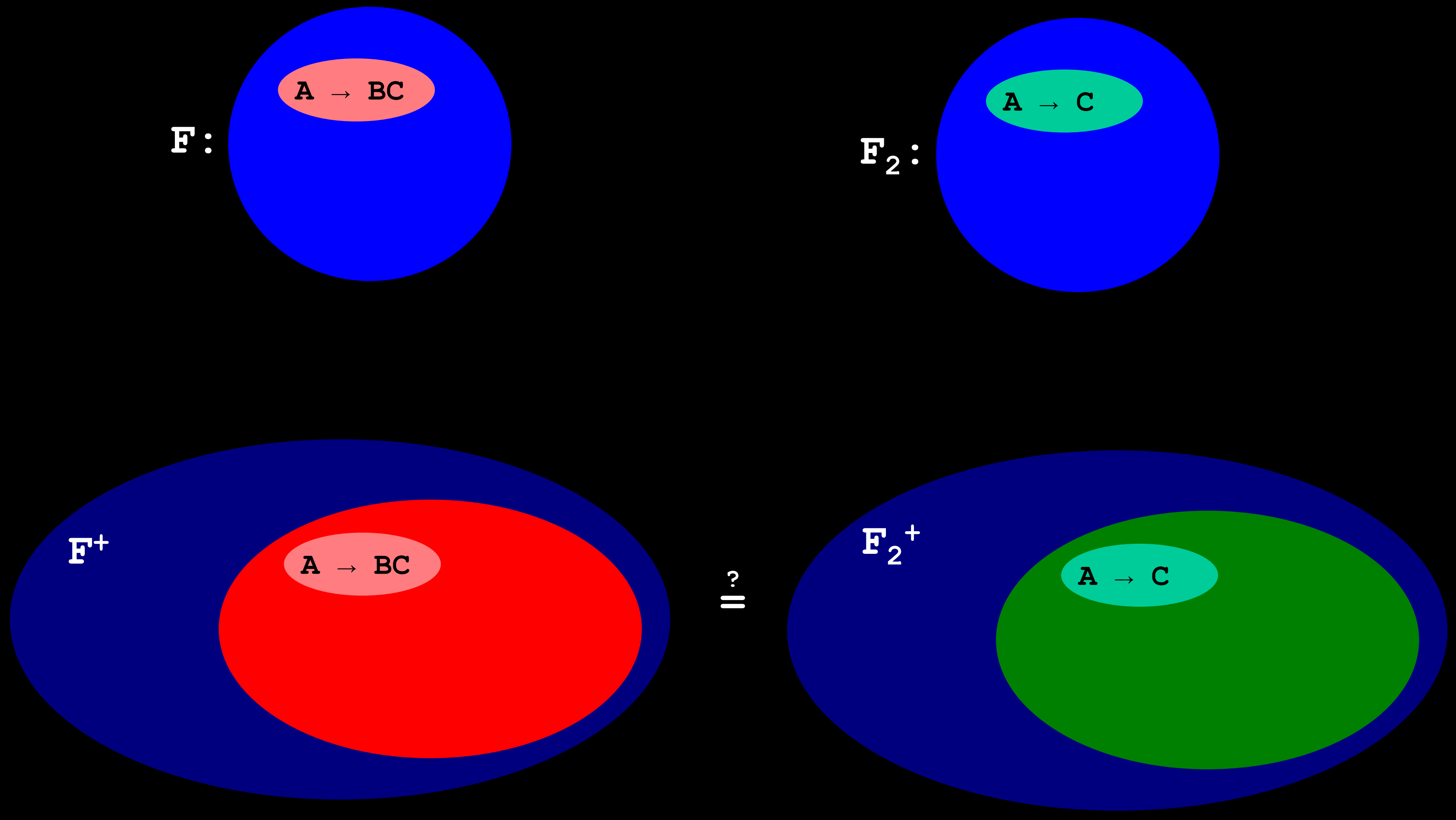

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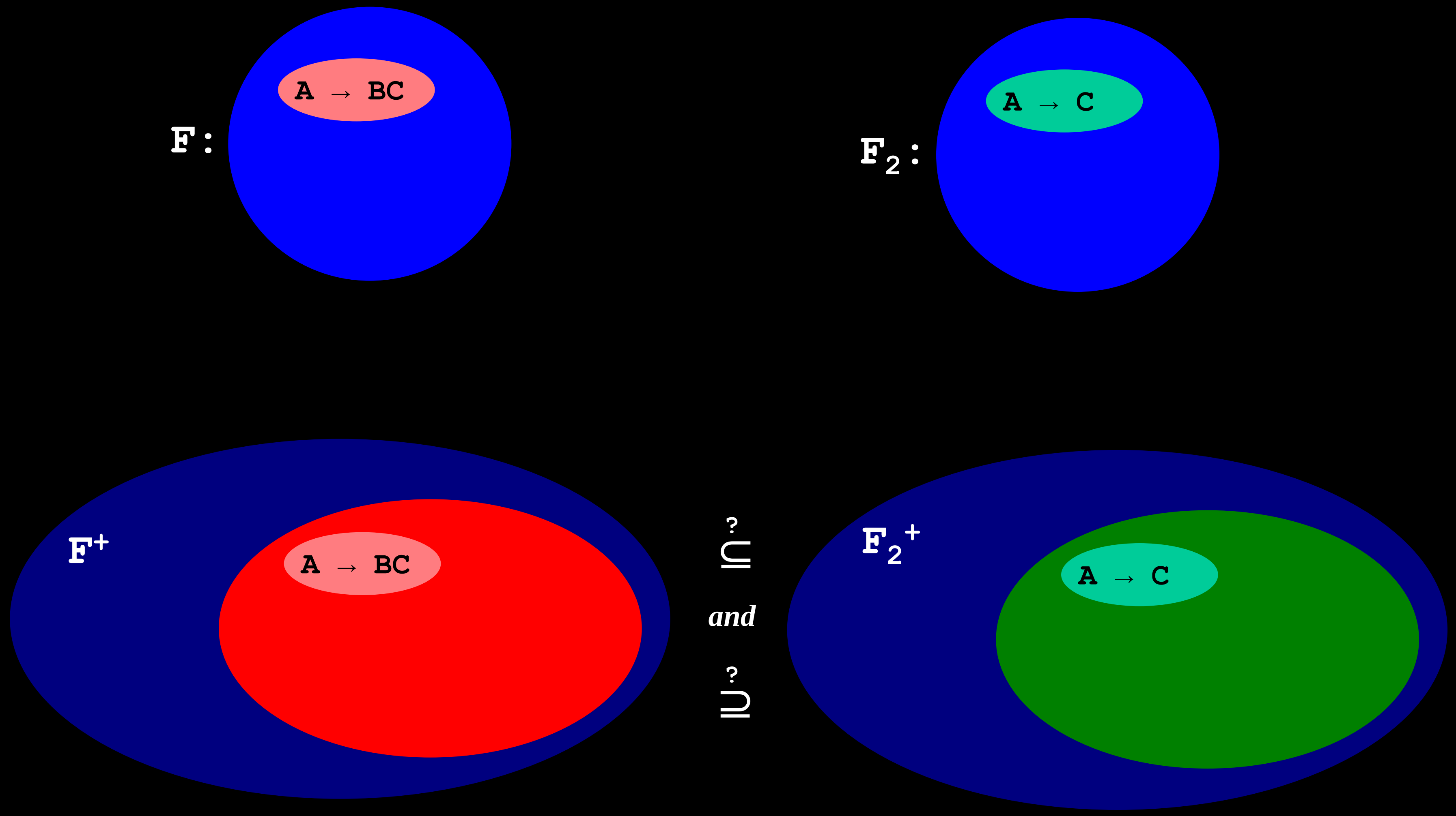

Extraneous Attributes in RHS

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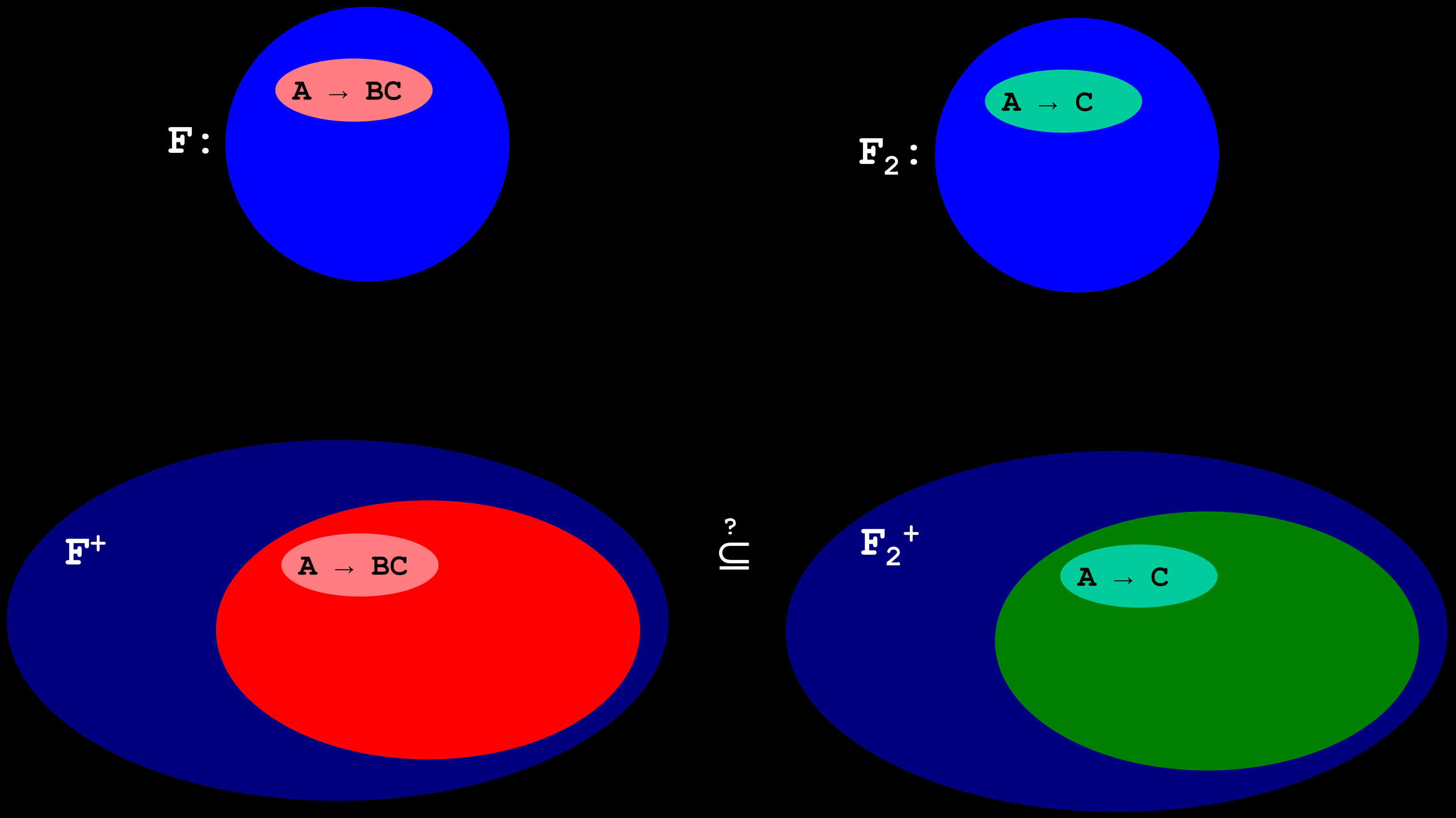

Extraneous Attributes in RHS

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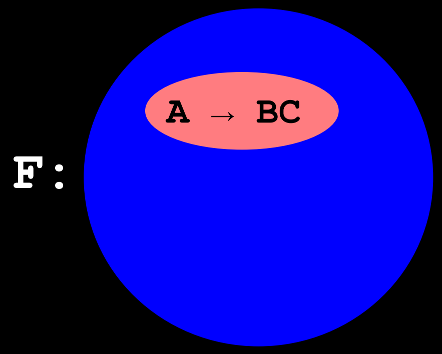
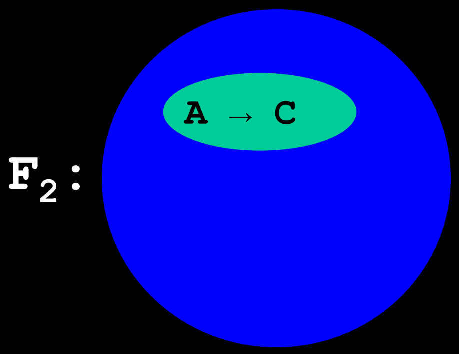
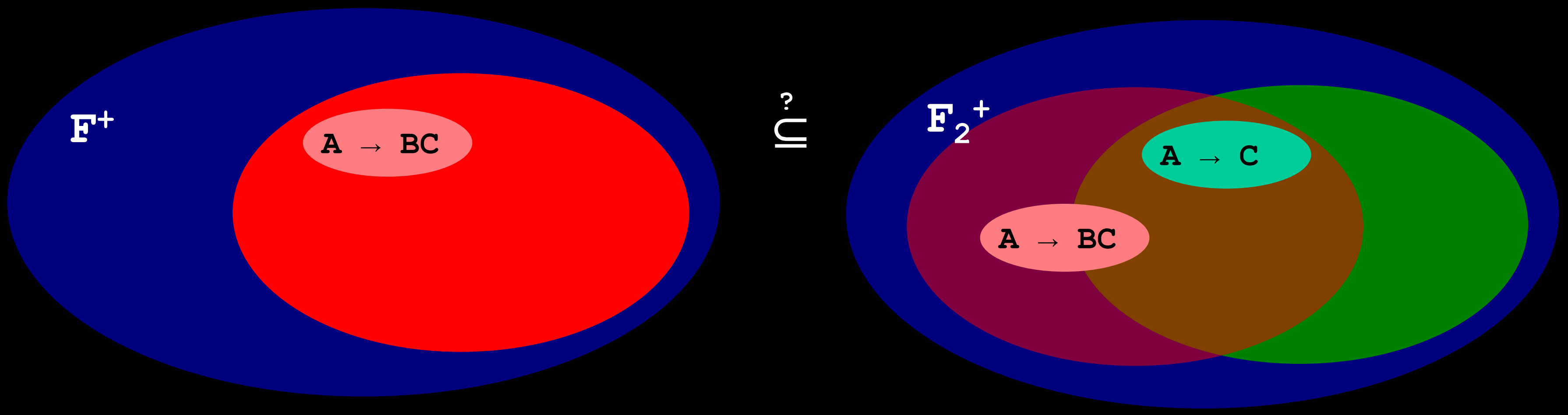


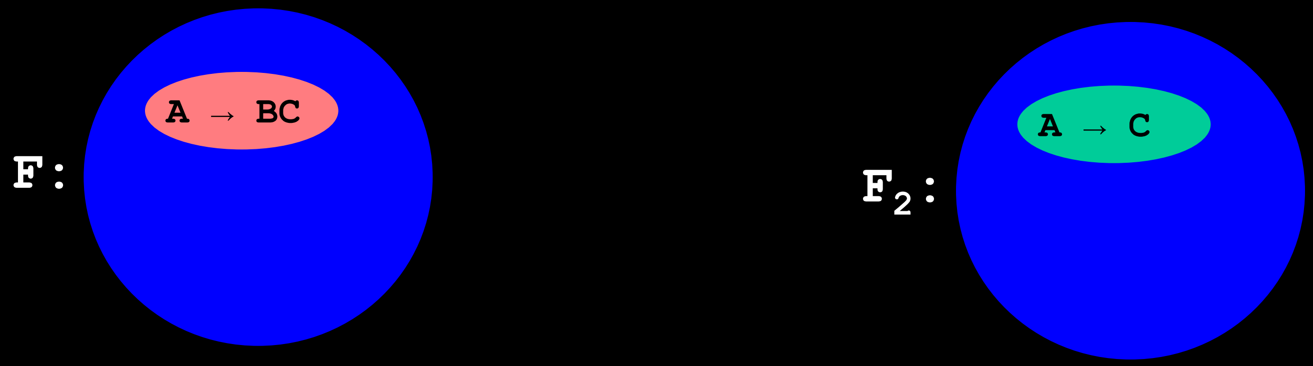


a) Is $(A \rightarrow BC) \in F_2^+$?

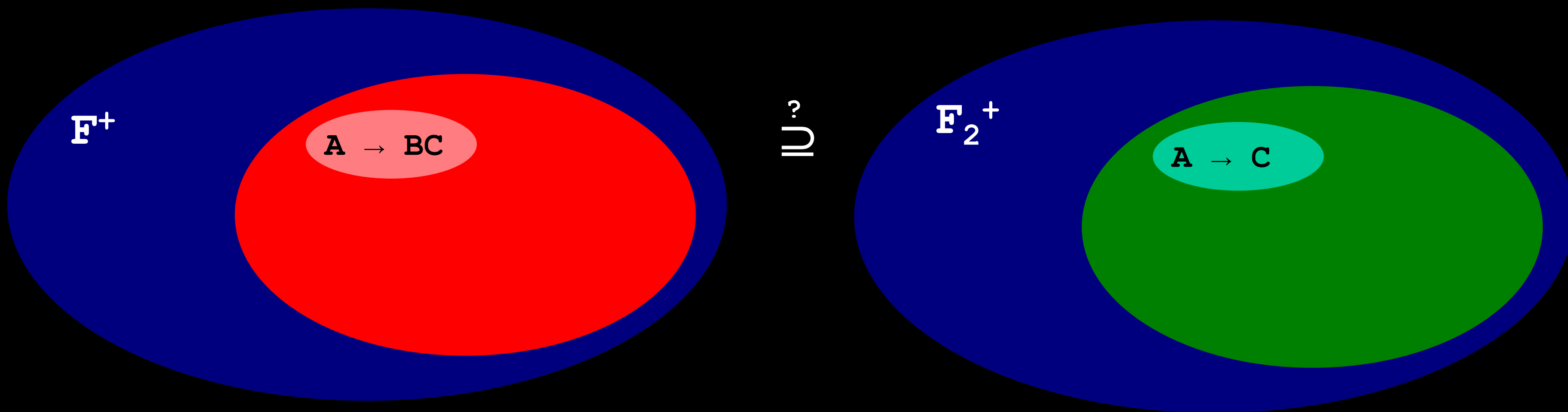


Extraneous Attributes in RHS

Is $F^+ = F_2^+$?

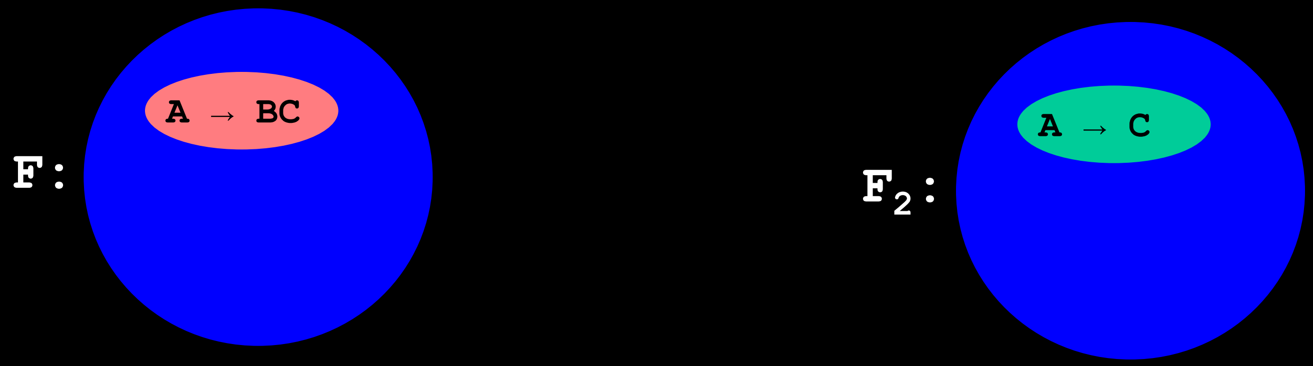


a) Is (**A** → **BC**) $\in F_2^+$?



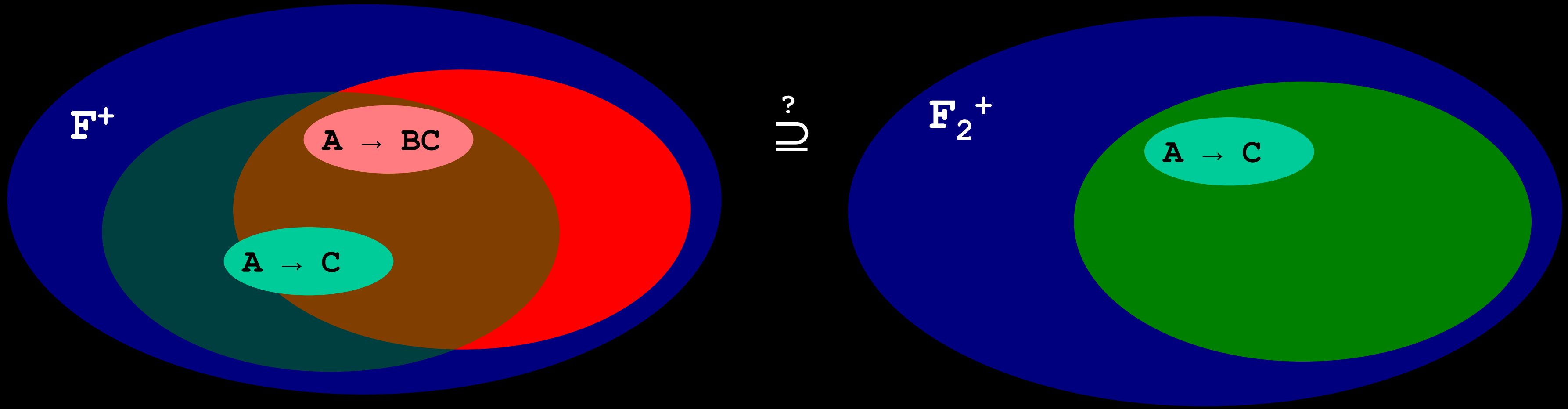
Extraneous Attributes in RHS

Is $F^+ = F_2^+$?



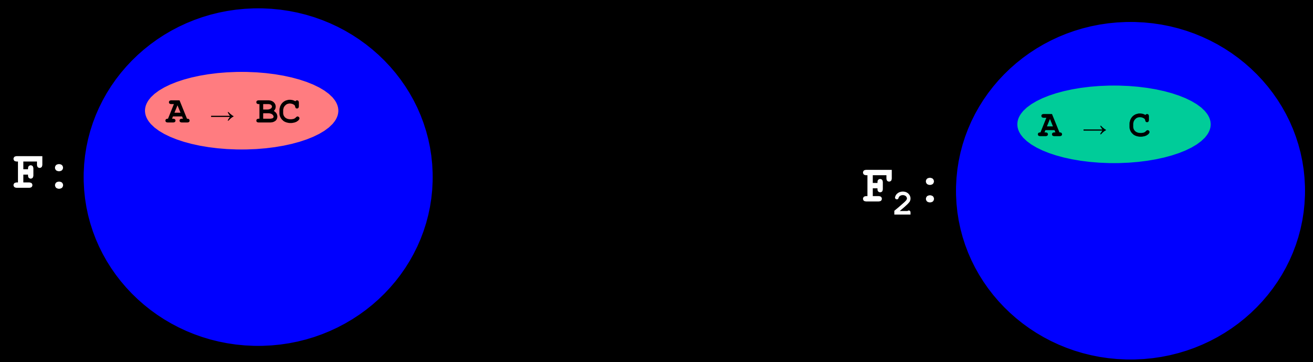
a) Is $(A \rightarrow BC) \in F_2^+$?

b) Is $(A \rightarrow C) \in F^+$?



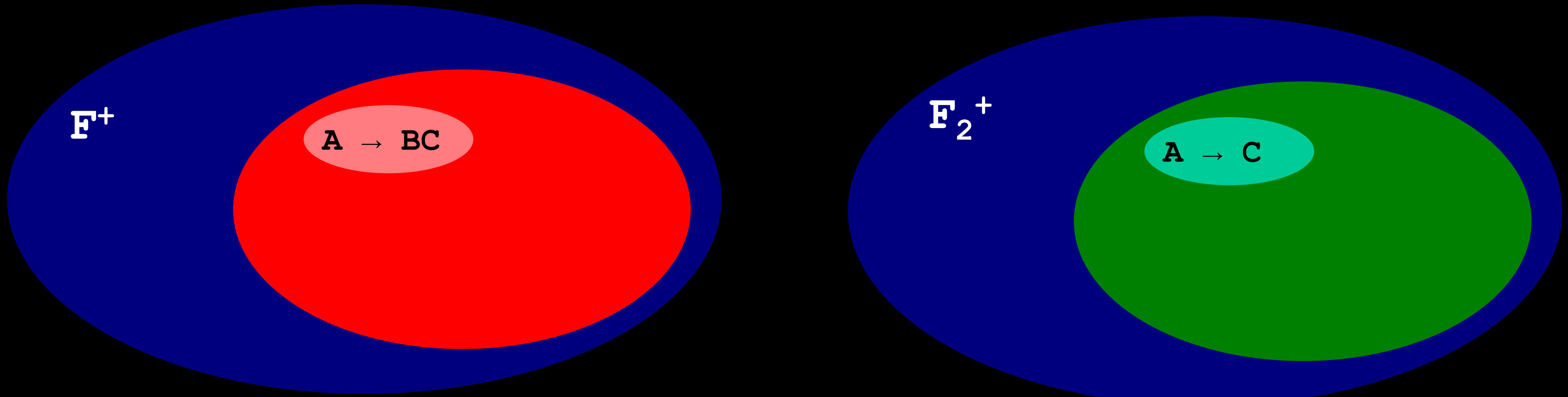
Extraneous Attributes in RHS

Is $F^+ = F_2^+$?



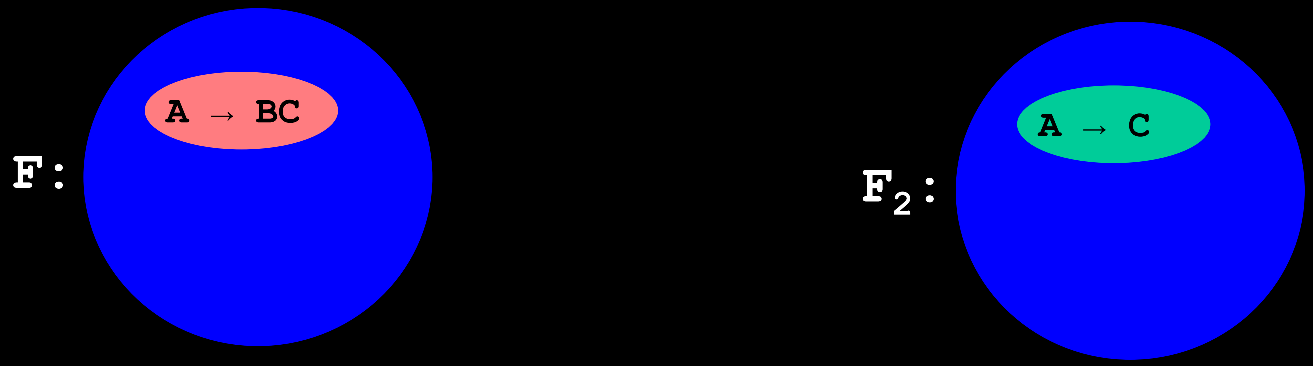
a) Is $(A \rightarrow BC) \in F_2^+$?

b) Is $(A \rightarrow C) \in F^+$?



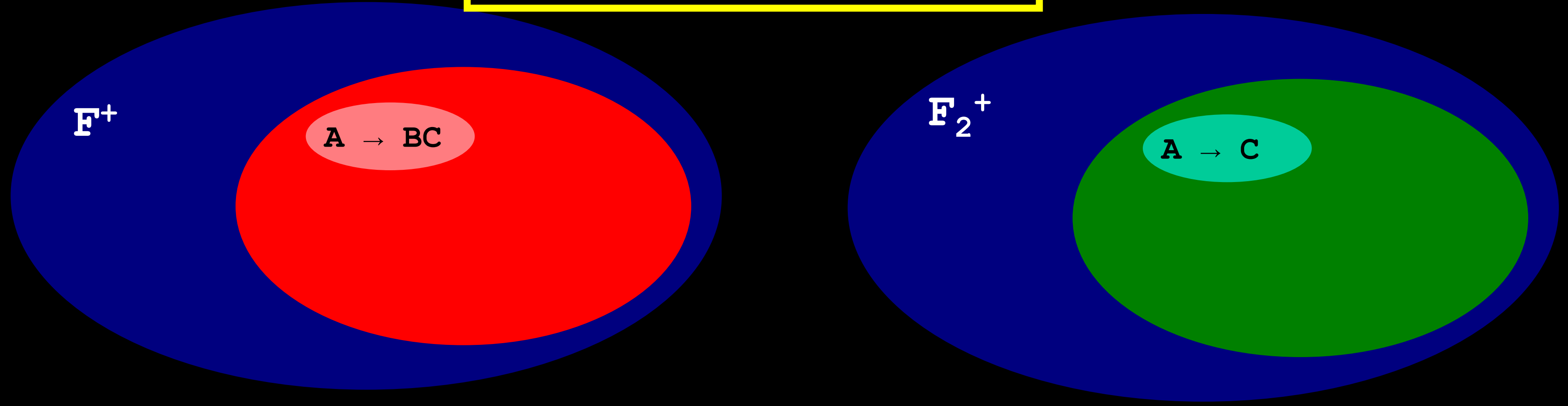
Extraneous Attributes in RHS

Is $F^+ = F_2^+$?



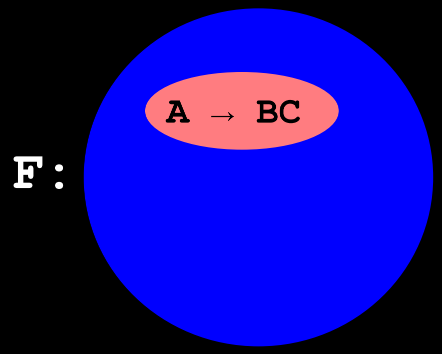
a) Is ($A \rightarrow BC$) $\in F_2^+$?

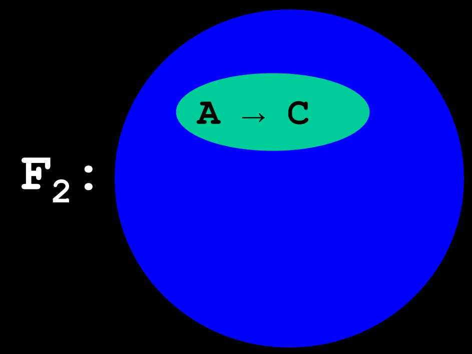
$\boxed{\text{b) Is } (A \rightarrow C) \in F^+ \text{ ?}}$



Extraneous Attributes in RHS

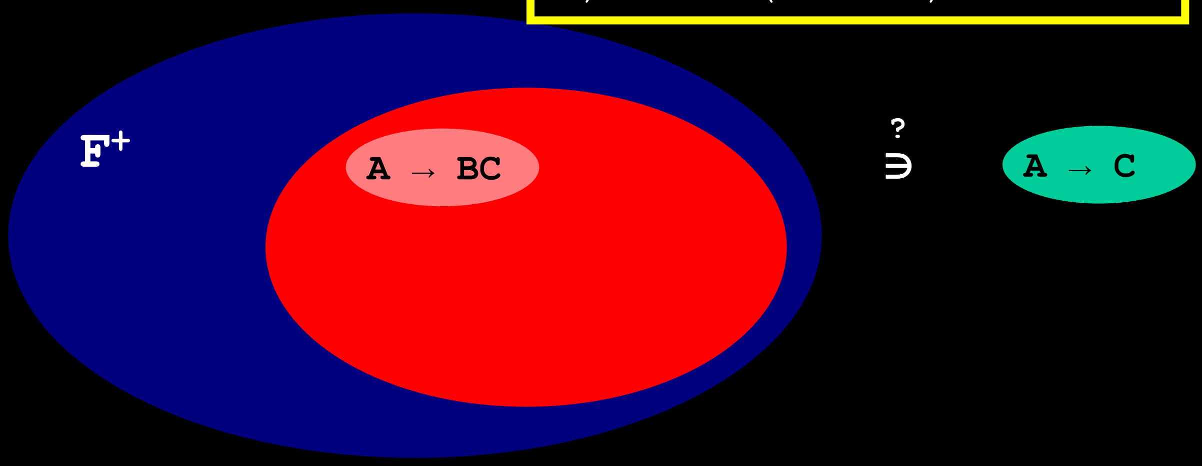
Is $F^+ = F_2^+$?





a) Is $(A \to BC) \in F_2^+$?

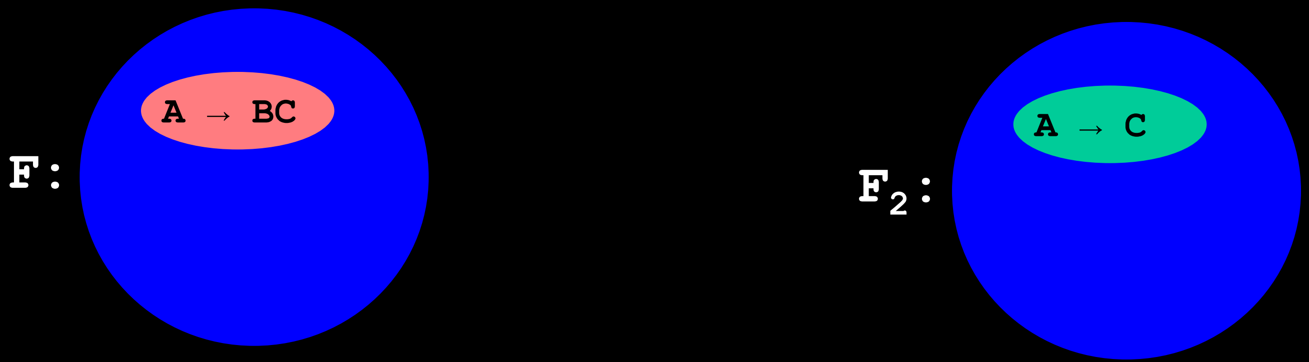
$\boxed{\text{b) Is } (A \to C) \in F^+ \text{ ?}}$



$\boxed{\textit{Key Point: } (A \to BC) \textit{ same as } (A \to B) + (A \to C)\ !}$

Extraneous Attributes in RHS

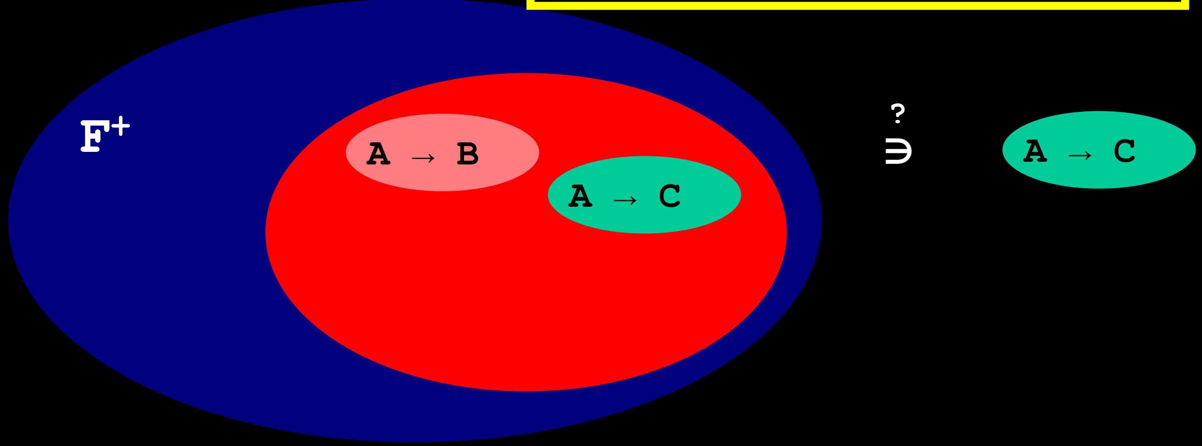
Is $F^+ = F_2^+$?



a) Is ($A \rightarrow BC$) $\in F_2^+$?

$\boxed{\text{b) Is } (A \rightarrow C) \in F^+ \text{ ?}}$

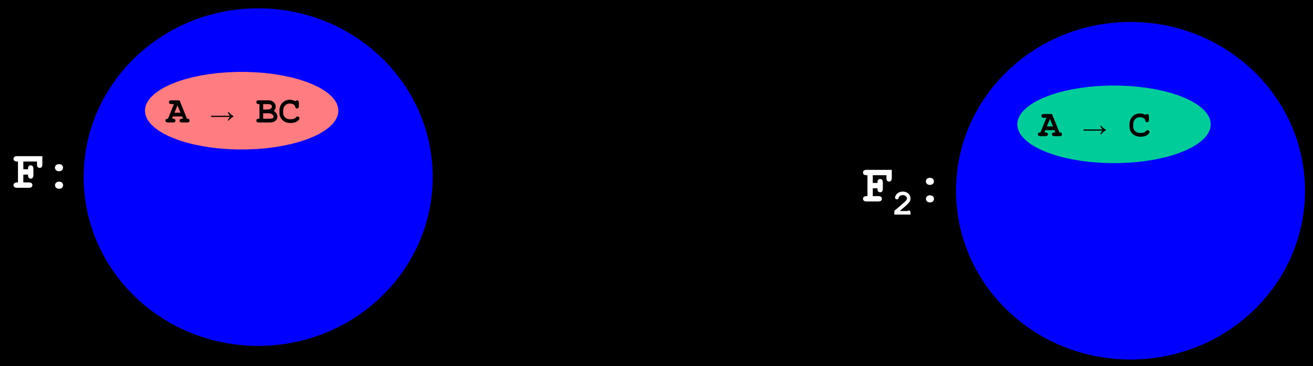
Always true!!



$\boxed{\textit{Key Point: } (A \rightarrow BC) \textit{ same as } (A \rightarrow B) + (A \rightarrow C) \text{ !}}$

Extraneous Attributes in RHS

Is $F^+ = F_2^+$?



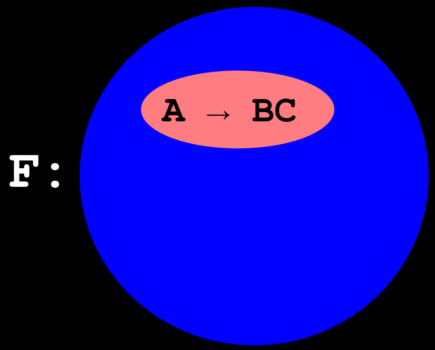
a) Is $(A \rightarrow BC) \in F_2^+$?

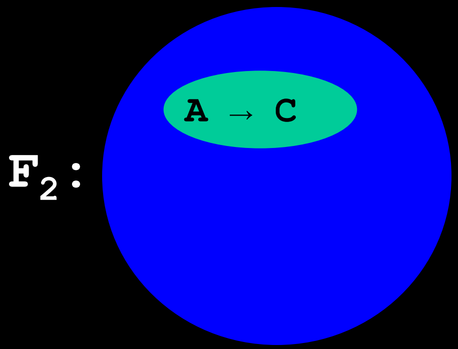
$\boxed{\text{b) Is } (A \rightarrow C) \in F^+ \text{ ?}}$

Always true!!

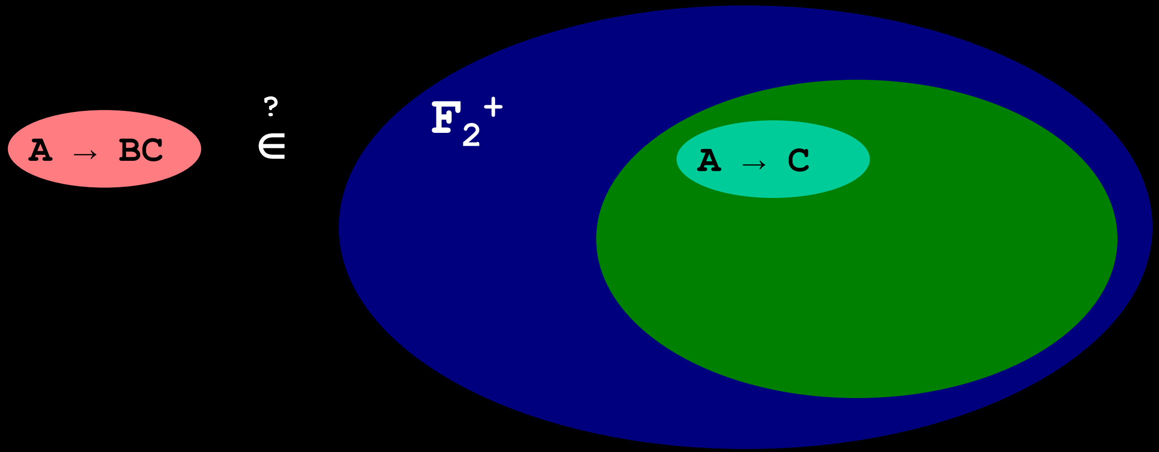
Extraneous Attributes in RHS

Is $F^+ = F_2^+$?





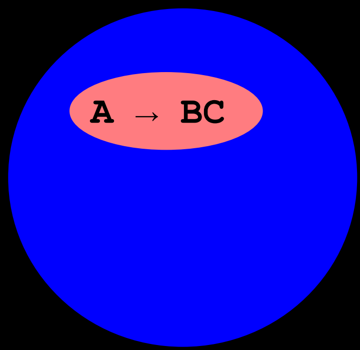
Is $(A \rightarrow BC) \in F_2^+$?



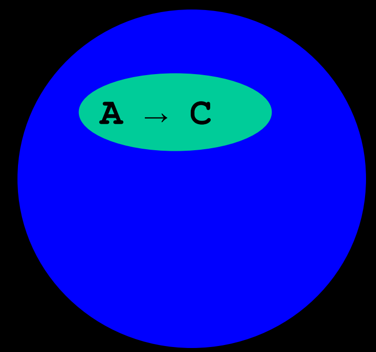
Extraneous Attributes in RHS

Is $F^+ = F_2^+$?

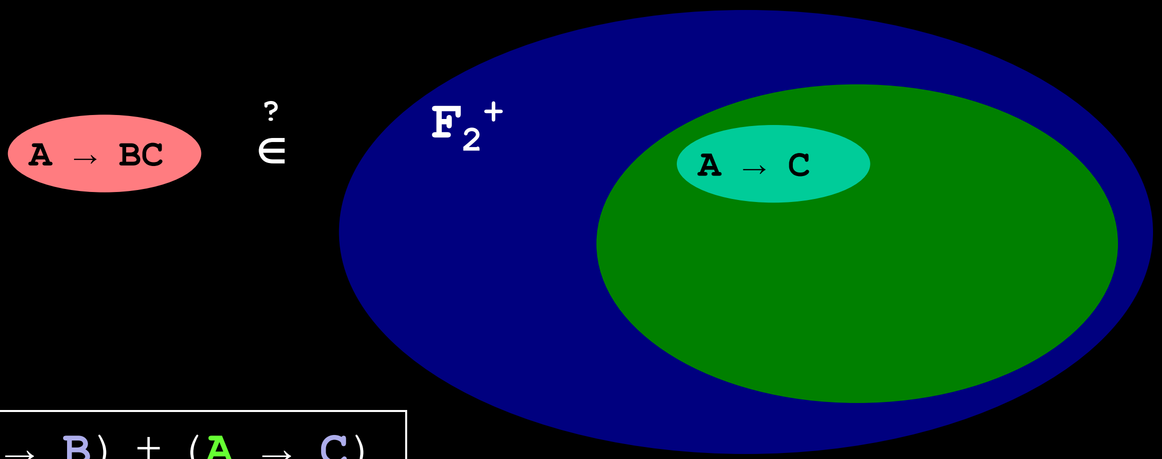
F:



$\mathbf{F_2}$**:**



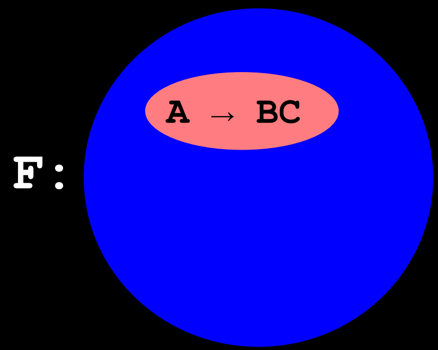
Is (**A** → **BC**) $\in F_2^+$?

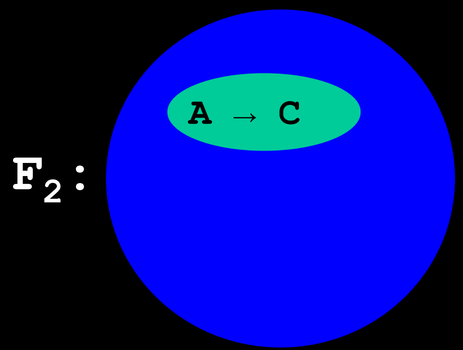


Note: (**A** → **BC**) *same as* (**A** → **B**) + (**A** → **C**)

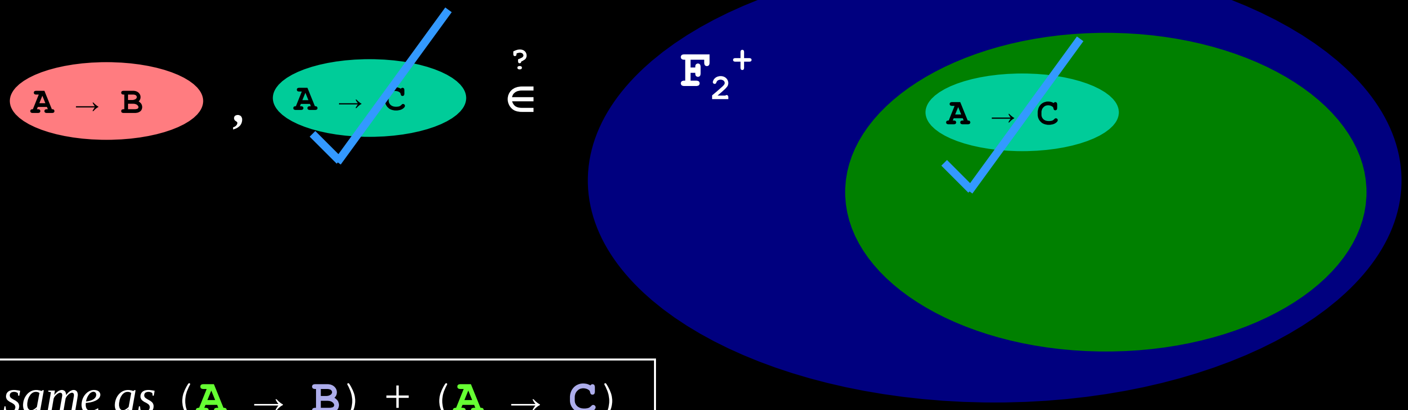
Extraneous Attributes in RHS

Is $F^+ = F_2^+$?





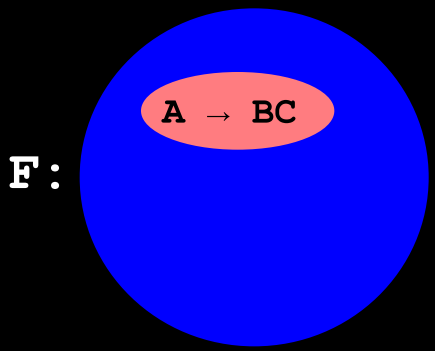
Is $(A \rightarrow BC) \in F_2^+$?

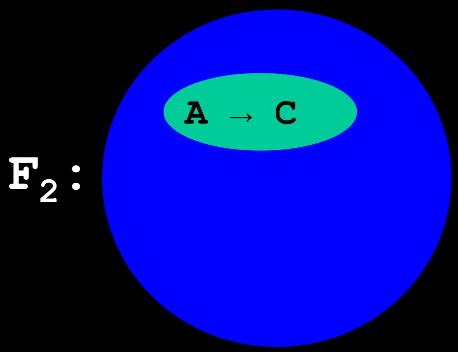


Note: $(A \rightarrow BC)$ *same as* $(A \rightarrow B) + (A \rightarrow C)$

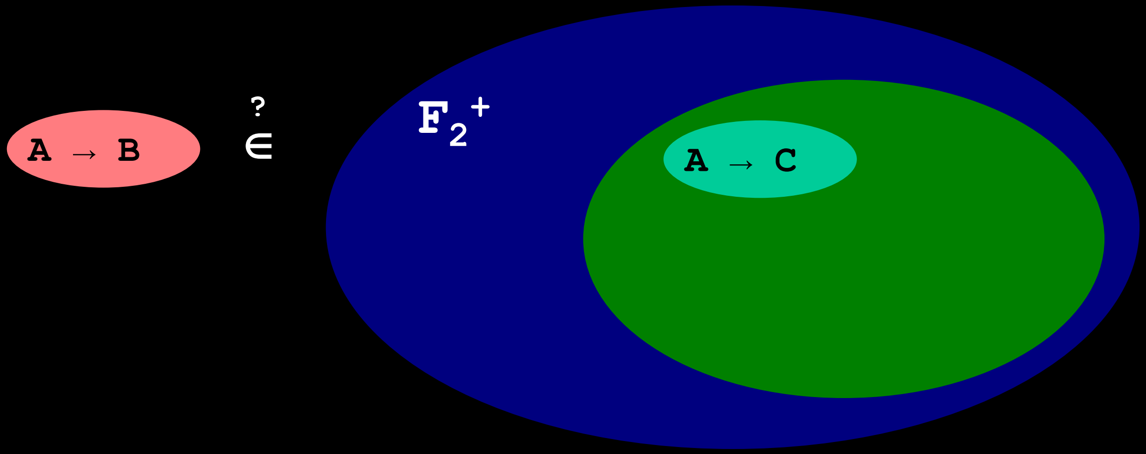
Extraneous Attributes in RHS

Is $F^+ = F_2^+$?



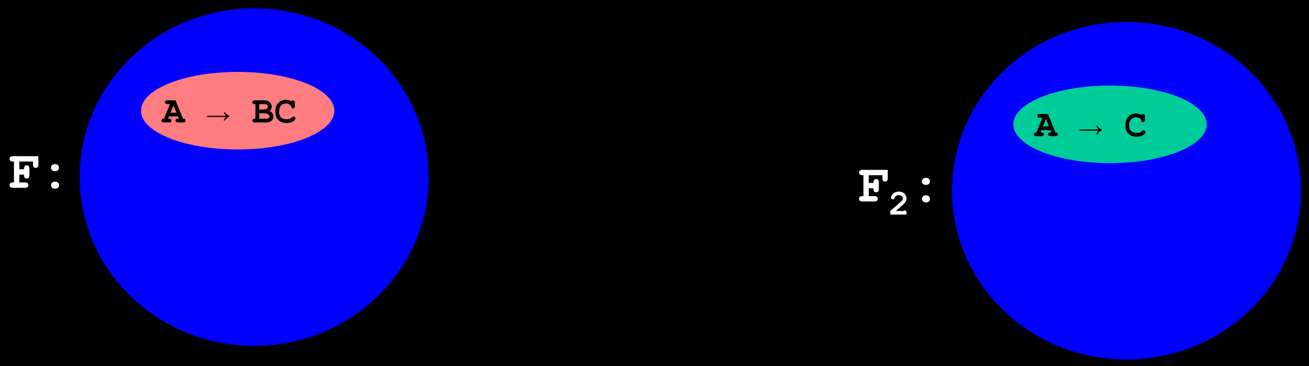


Is $(A \rightarrow BC) \in F_2^+$?

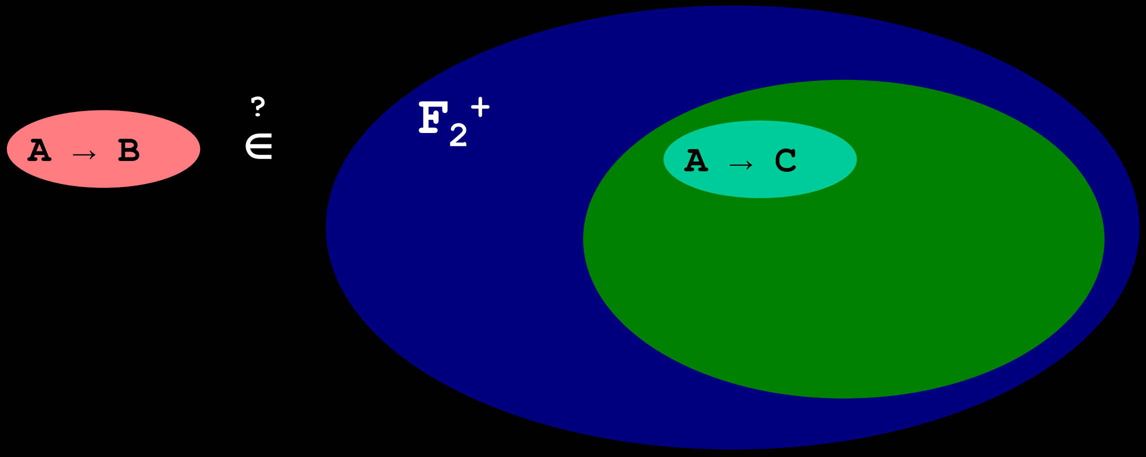


Extraneous Attributes in RHS

Is $F^+ = F_2^+$?

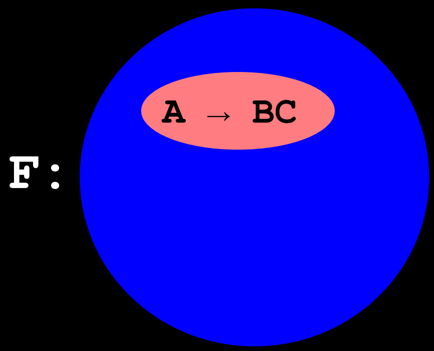


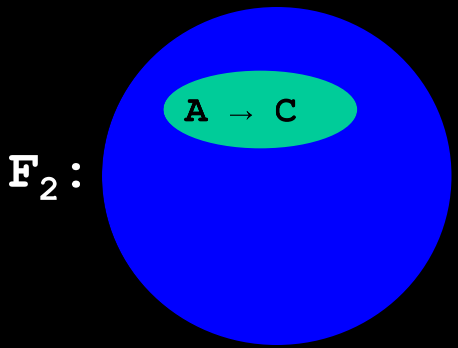
Is $(A \rightarrow B) \in F_2^+$?



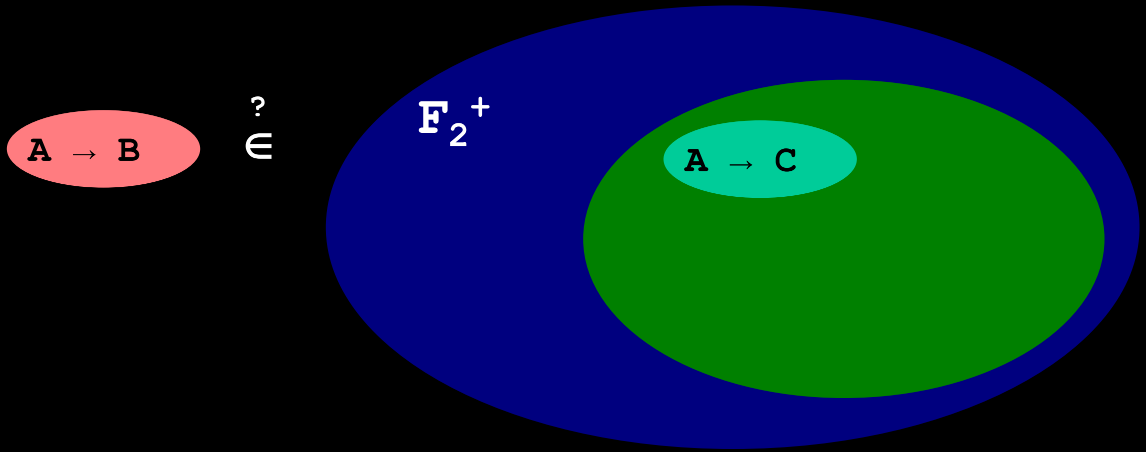
Extraneous Attributes in RHS

Is $F^+ = F_2^+$?





Is $(A \rightarrow B) \in (F - \{A \rightarrow BC\} \cup \{A \rightarrow C\})^+$?



Extraneous Attributes in RHS

Is B extraneous in $A \rightarrow BC$?

A Simple (but Expensive) Test:

1. *Replace "$A \rightarrow BC$" with "$A \rightarrow C$" in F to construct F_2*
   - $F_2 = F - \{A \rightarrow BC\} \cup \{A \rightarrow C\}$

2. *Test: Is $F^+ = F_2^+$?*
   - *Method:* `FD-Closure (F) = FD-Closure (F₂)?`
   - *Cost: $$$$*

Is there a less expensive way to compare F^+ and F_2^+ here?

Extraneous Attributes in RHS

Is B extraneous in $A \rightarrow BC$?

A Simple (but Expensive) Test:

1. *Replace "*$A \rightarrow BC$*" with "*$A \rightarrow C$*" in* F *to construct* F_2
   - $F_2 = F - \{A \rightarrow BC\} \cup \{A \rightarrow C\}$

2. *Test: Is* $F^+ = F_2^+$*?*
   - *Method:* FD-Closure (F) = FD-Closure (F_2)?
   - *Cost: $$$$*

Is there a less expensive way to compare F^+ *and* F_2^+ *here?*

A: Yes. Test:

Is $(A \rightarrow B) \in F_2^+$?

i.e., Is $(A \rightarrow B) \in (F - \{A \rightarrow BC\} \cup \{A \rightarrow C\})^+$?

Extraneous Attributes in RHS

$$F_2 = F - \{A \rightarrow BC\} \cup \{A \rightarrow C\}$$

Comparing Extraneous Attributes Tests:

1. *Test: Is* $F^+ = F_2^+$*?*
   - *Method:* $\text{FD-Closure}(F) = \text{FD-Closure}(F_2)$?
   - *Cost: $$$$*

2. *Test: Is* $(\mathbf{A} \rightarrow \mathbf{B}) \in F_2^+$*?*
   - *Method:*

Extraneous Attributes in RHS

$F_2 = F - \{A \rightarrow BC\} \cup \{A \rightarrow C\}$

Comparing Extraneous Attributes Tests:

1. *Test: Is* $F^+ = F_2^+$?
   - *Method:* FD-Closure (F) = FD-Closure (F_2)?
   - *Cost: $$$$*

2. *Test: Is* $(A \rightarrow B) \in F_2^+$?
   - *Method:* (**A** → **B**) ∈ FD-Closure (F_2)?
   - *Cost: $$*

Is there a cheaper way to do this?

A: Yes

Method: B ∈ Att-Closure ({A},F_2)?
Cost: $

Canonical Cover Algorithm

Basic Algorithm (will refine later)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  Is B extraneous in A → BC?


    b) LHS:  Is B extraneous in AB → C?

END
```

Canonical Cover Algorithm

Basic Algorithm (will refine later)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  Is B extraneous in A → BC?
```

$$\text{Is } (A \rightarrow B) \in (F - \{A \rightarrow BC\} \cup \{A \rightarrow C\})^{+}?$$

```
    b) LHS:  Is B extraneous in AB → C?

END
```

Extraneous Attributes in RHS

Is B extraneous in $A \rightarrow BC$?

$$\boxed{F = \{A \rightarrow BC,\ B \rightarrow C\}}$$

Is **B** *extraneous in* **A** $\rightarrow$ **BC**?

i.e., Is $(A \rightarrow B) \in (F - \{A \rightarrow BC\} \cup \{A \rightarrow C\})^+$?

i.e., Is $(A \rightarrow B) \in \{A \rightarrow C,\ B \rightarrow C\}^+$?

A: *No:* $(A \rightarrow B) \notin \{A \rightarrow C,\ B \rightarrow C\}^+$

When asked or for partial credit, prove your answers as above

Extraneous Attributes in RHS

Is B extraneous in $A \rightarrow BC$?

$$\boxed{F = \{A \rightarrow BC, \ B \rightarrow C\}}$$

Is **C** *extraneous in* $A \rightarrow BC$?

i.e., Is $(A \rightarrow C) \in (F - \{A \rightarrow BC\} \cup \{A \rightarrow B\})^+$?

i.e., Is $(A \rightarrow C) \in \{A \rightarrow B, \ B \rightarrow C\}^+$?

A: *Yes:* $(A \rightarrow C) \in \{A \rightarrow B, \ B \rightarrow C\}^+$

When asked or for partial credit, prove your answers as above

Extraneous Attributes in LHS

Is B extraneous in $AB \rightarrow C$?

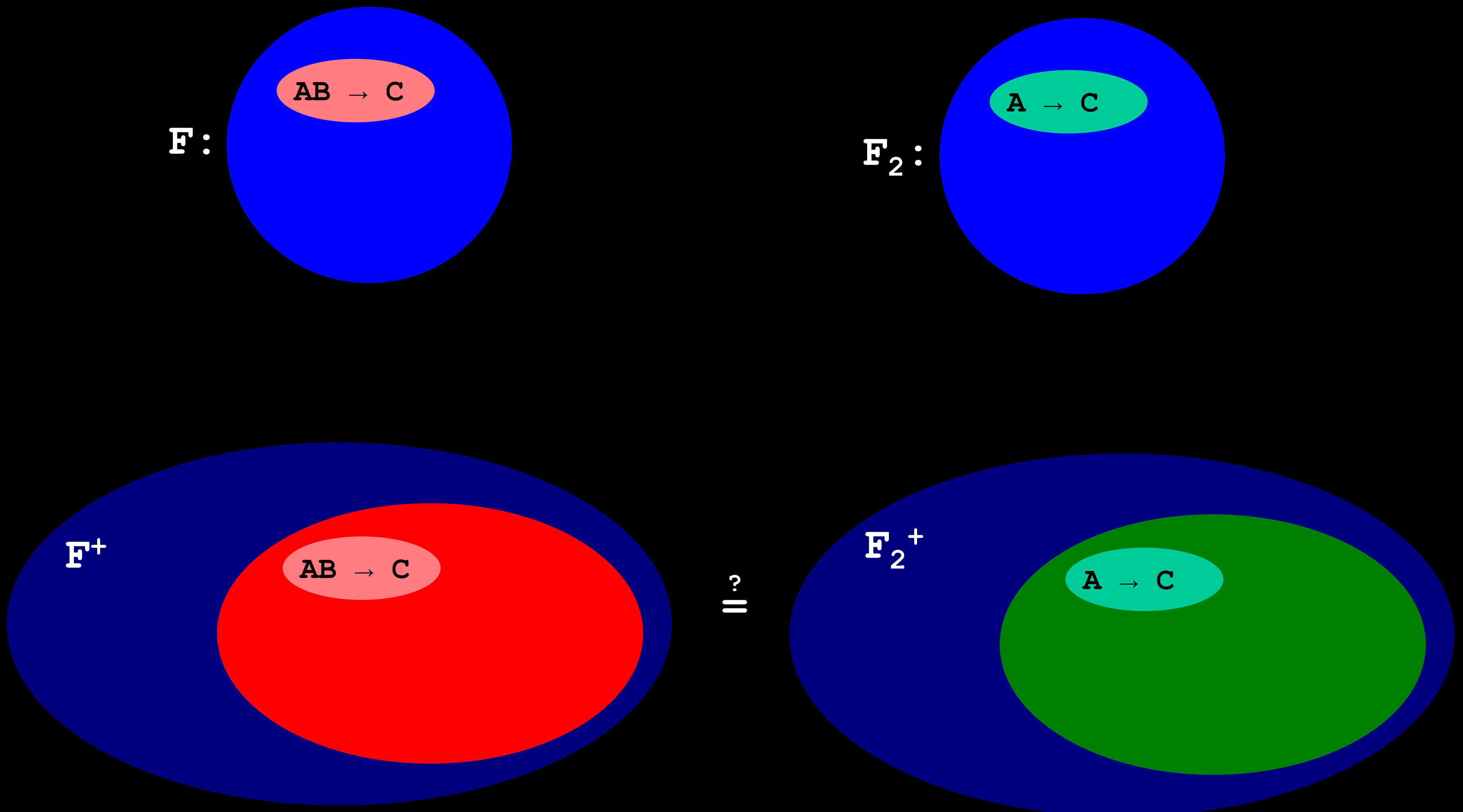
A Simple (but Expensive) Test:

1. *Replace "$AB \rightarrow C$" with "$A \rightarrow C$" in F to construct F_2:*
   - $F_2 = F - \{AB \rightarrow C\} \cup \{A \rightarrow C\}$
2. *Test: Is $F_2^+ = F^+$?* *If yes, then B is extraneous*

Is there a less expensive way to compare F^+ and F_2^+ here?

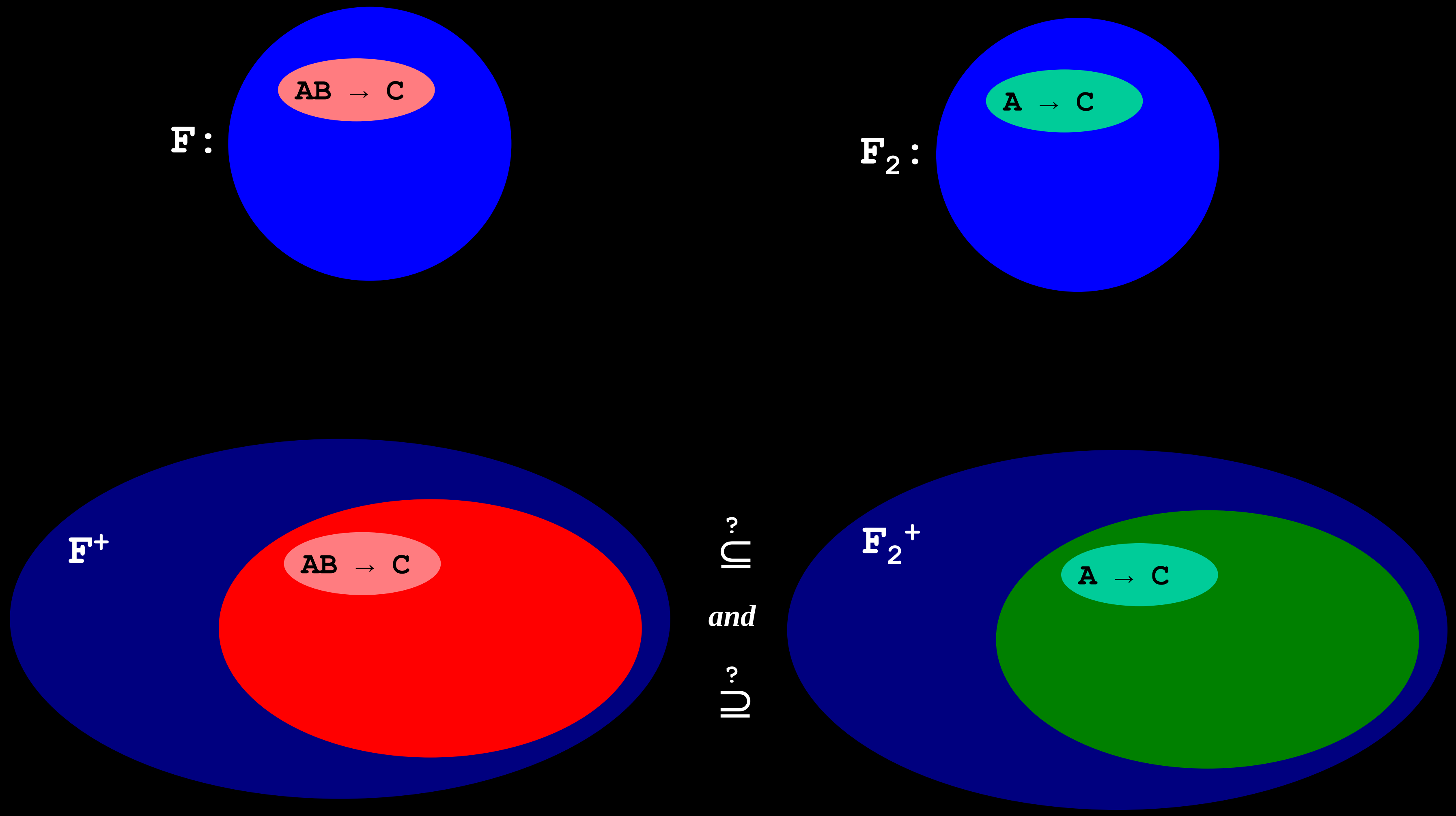
Extraneous Attributes in LHS

Is $F^+ = F_2^+$?



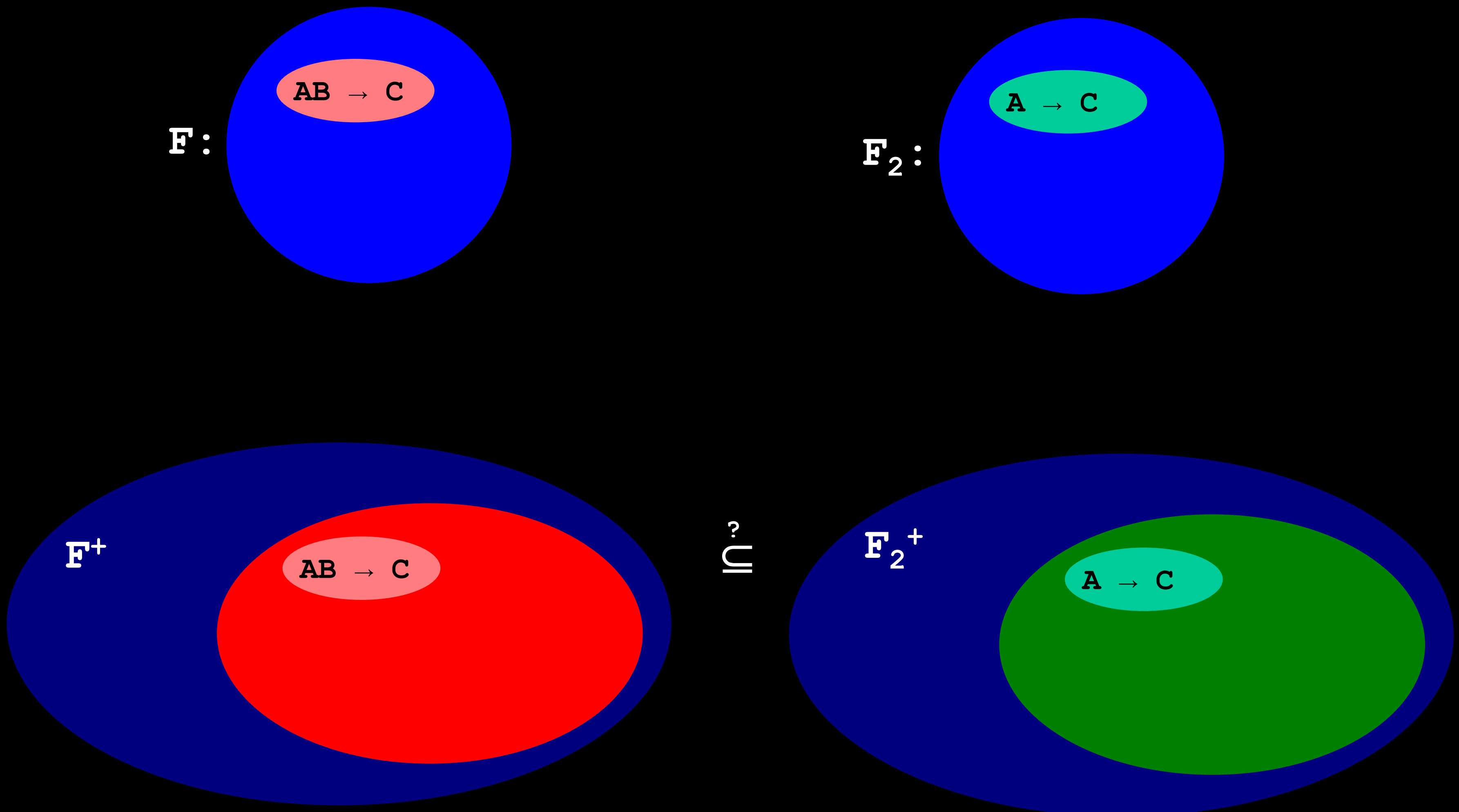
Extraneous Attributes in LHS

Is $F^+ = F_2^{\ +}$?



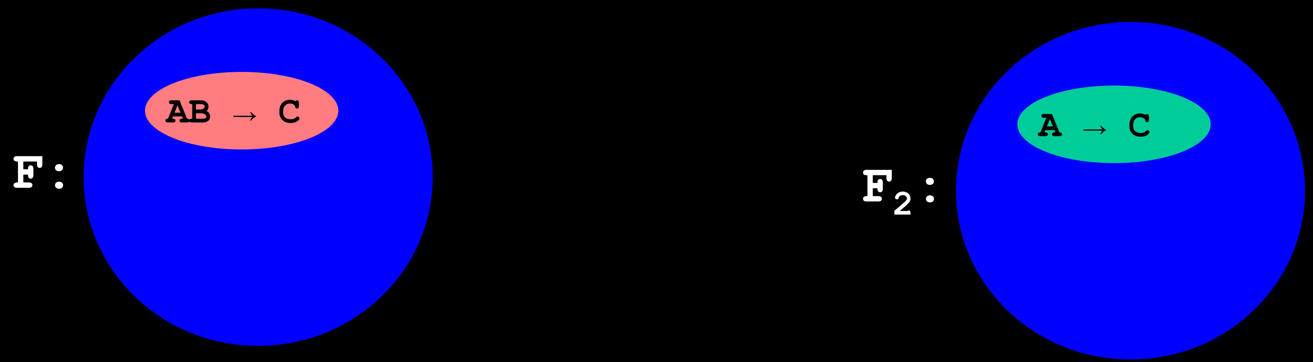
Extraneous Attributes in LHS

Is $F^+ = F_2^+$?

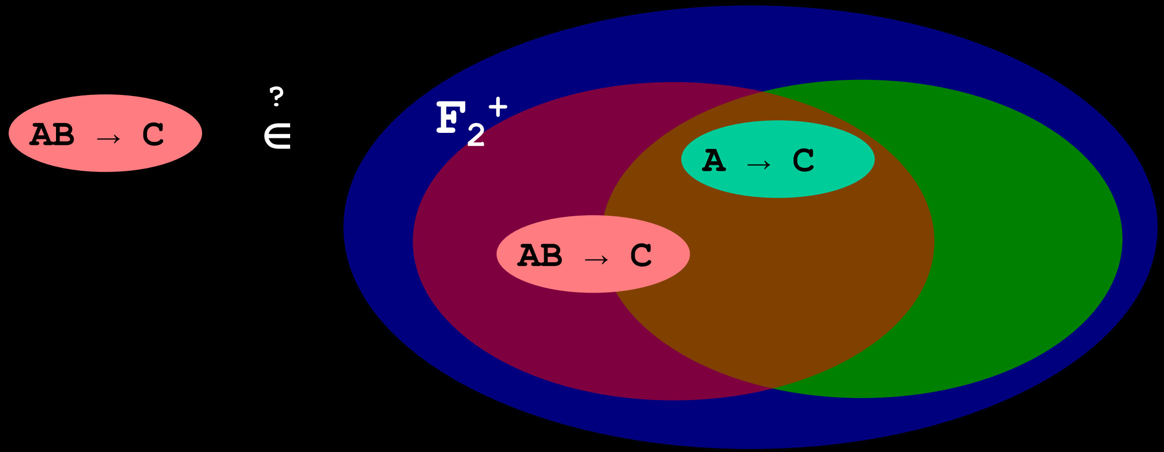


Extraneous Attributes in LHS

Is $F^+ = F_2^+$?

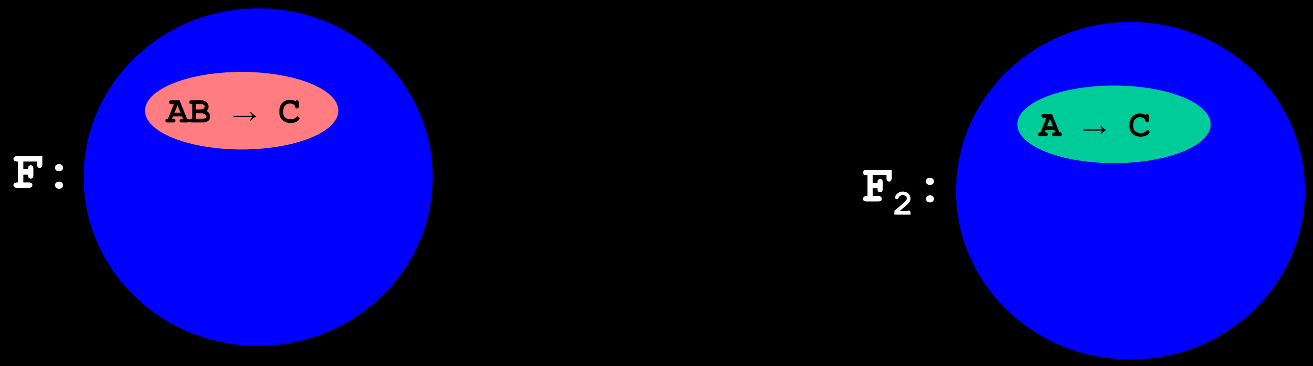


a) Is $(AB \rightarrow C) \in F_2^+$?

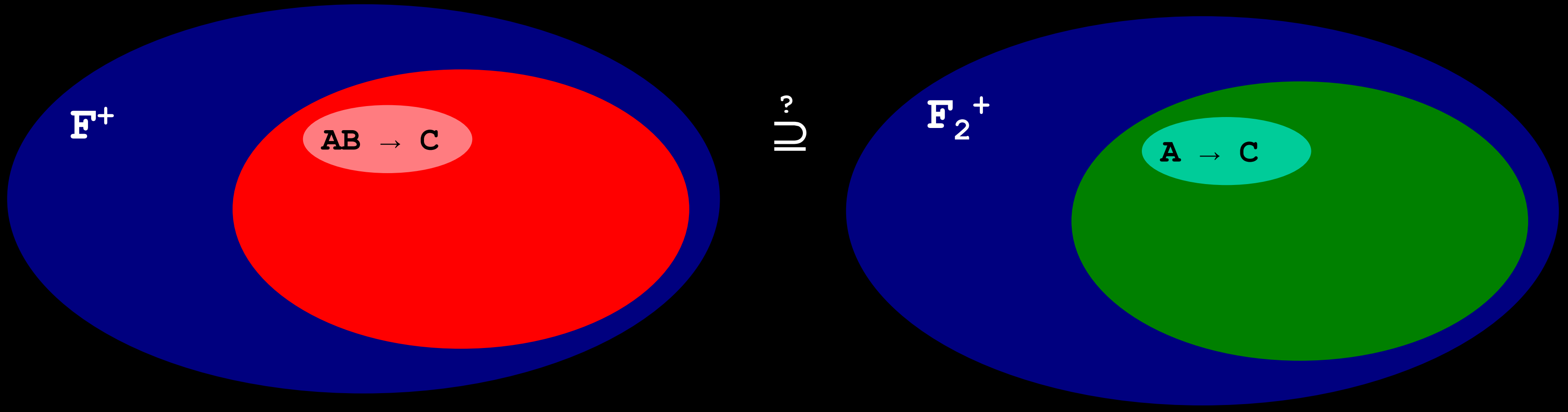


Extraneous Attributes in LHS

Is $F^+ = F_2^+$?



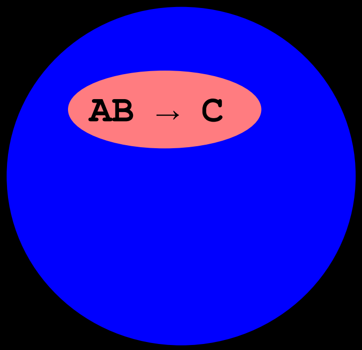
a) Is (AB → C) ∈ F_2^+ ?



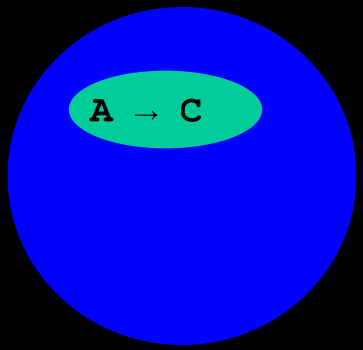
Extraneous Attributes in LHS

Is $F^+ = F_2^+$?

F:

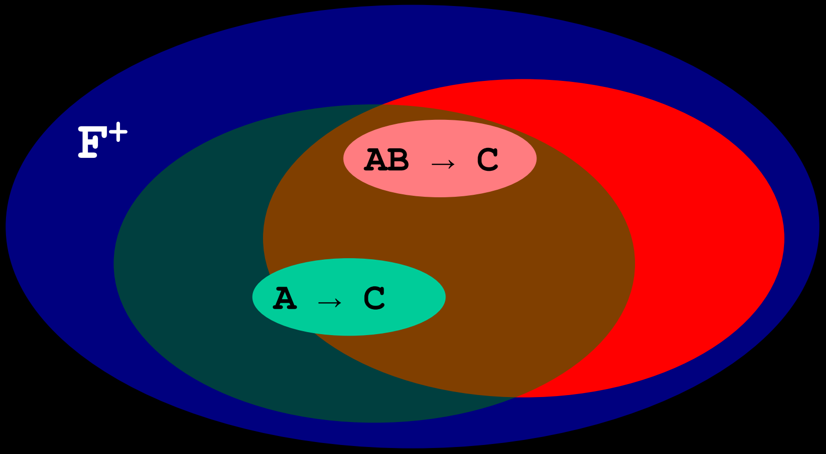


F_2:

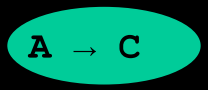


a) Is (**AB** → **C**) $\in F_2^+$?

b) Is (**A** → **C**) $\in F^+$?

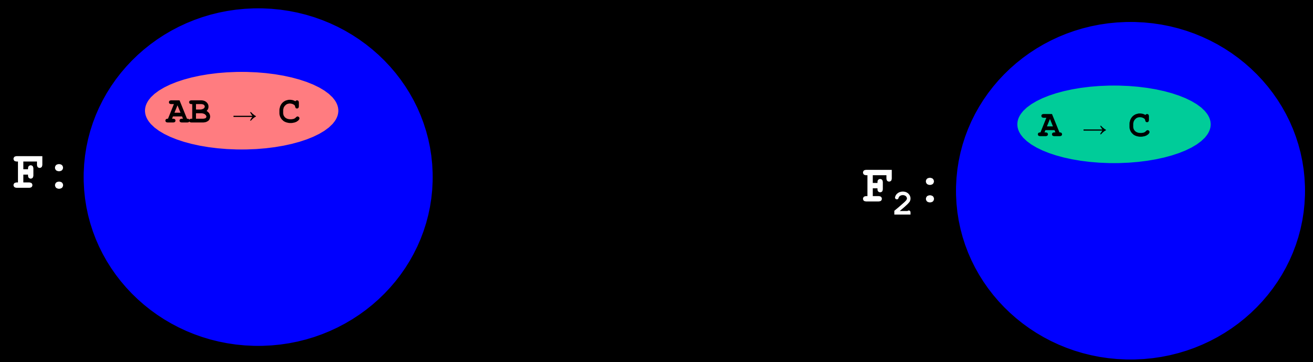


$\overset{?}{\ni}$



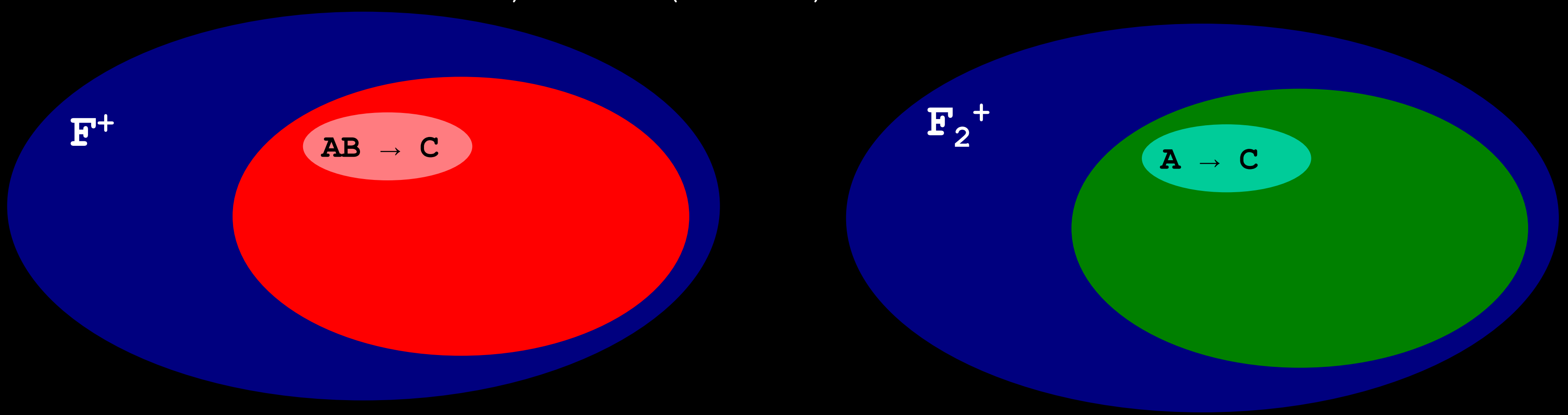
Extraneous Attributes in LHS

Is $F^+ = F_2^+$?



a) Is $(AB \rightarrow C) \in F_2^+$?

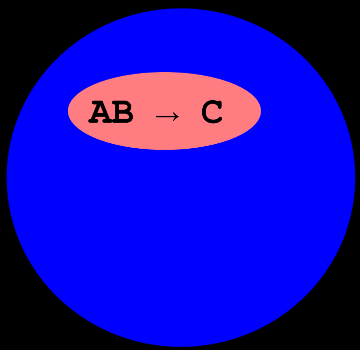
b) Is $(A \rightarrow C) \in F^+$?



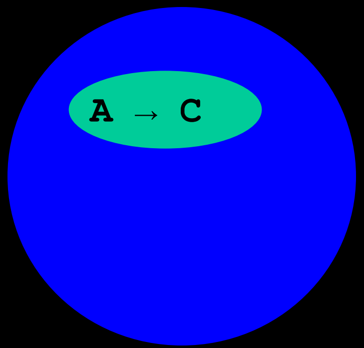
Extraneous Attributes in LHS

Is $F^+ = F_2^+$?

F:

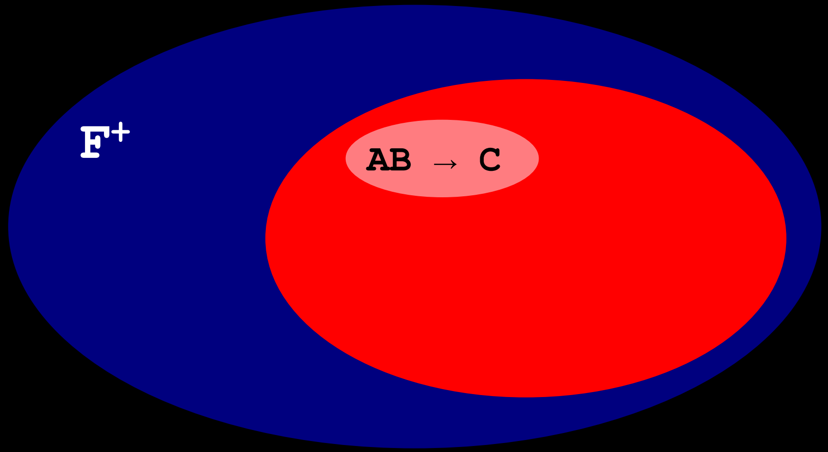


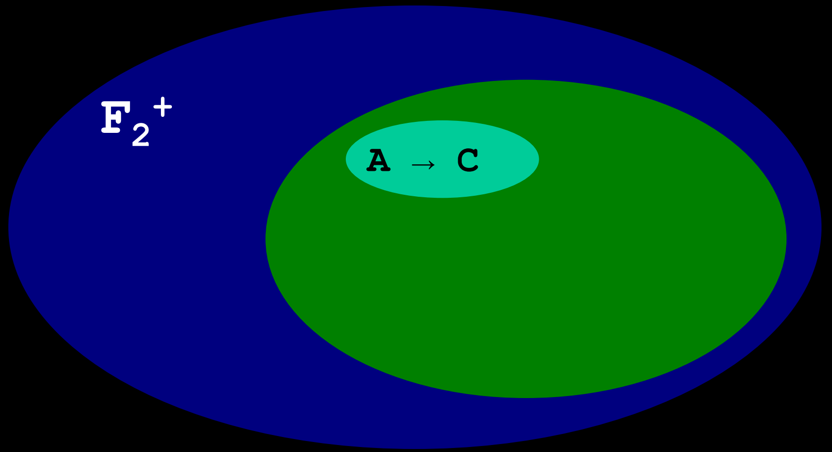
F_2:



a) Is (**AB** → **C**) $\in F_2^+$?

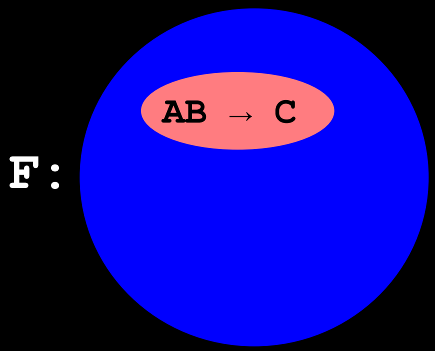
b) Is (**A** → **C**) $\in F^+$?

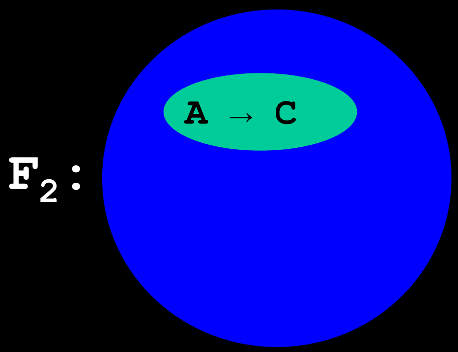




Extraneous Attributes in LHS

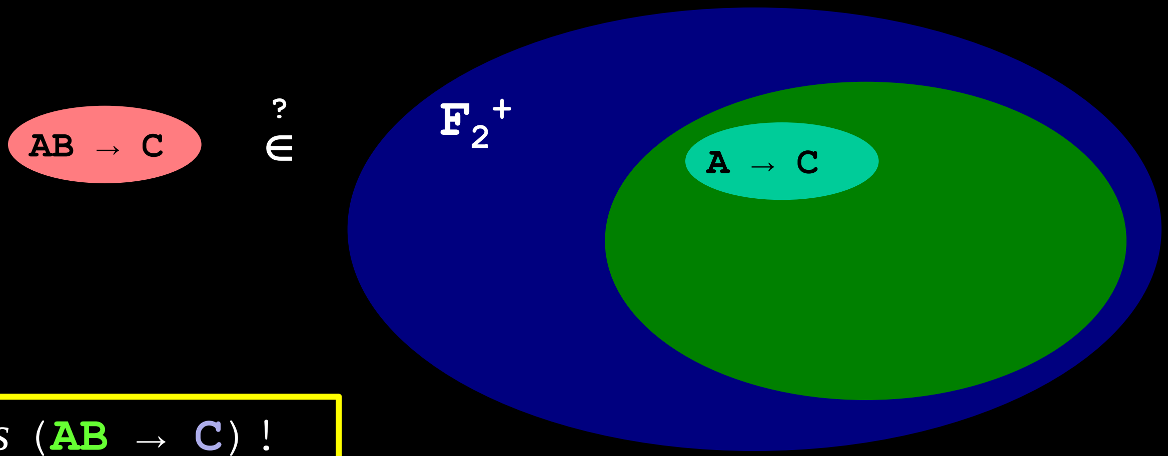
Is $F^+ = F_2^+$?





a) Is $(AB \rightarrow C) \in F_2^+$?

b) Is $(A \rightarrow C) \in F^+$?



Key point: $(A \rightarrow C)$ *implies* $(AB \rightarrow C)$!

Extraneous Attributes in LHS

Is $F^+ = F_2^+$?

"**A** → **C**" *implies* "**AB** → **C**"

<u>Proof (by Armstrong's Axioms):</u>

(1) **A** → **C** Given

(2) **AB** → **BC** Augmentation (1)

Review: Armstrong's Axioms

1. *Reflexivity*
   - if $Y \subseteq X$ then $\mathbf{X} \rightarrow \mathbf{Y}$
2. *Augmentation*
   - if $X \rightarrow Y$ then $\mathbf{WX} \rightarrow \mathbf{WY}$
3. *Transitivity*
   - if $X \rightarrow Y$ and $Y \rightarrow Z$ then $\mathbf{X} \rightarrow \mathbf{Z}$
4. *Union*
   - if $X \rightarrow Y$ and $X \rightarrow Z$ then $\mathbf{X} \rightarrow \mathbf{YZ}$
5. *Decomposition*
   - if $X \rightarrow YZ$ then $\mathbf{X} \rightarrow \mathbf{Y}$ and $\mathbf{X} \rightarrow \mathbf{Z}$
6. *Pseudotransitivity*
   - if $X \rightarrow Y$ and $WY \rightarrow Z$ then $\mathbf{WX} \rightarrow \mathbf{Z}$

Extraneous Attributes in LHS

Is $F^+ = F_2^+$?

"$\mathbf{A} \rightarrow \mathbf{C}$" *implies* "$\mathbf{AB} \rightarrow \mathbf{C}$"

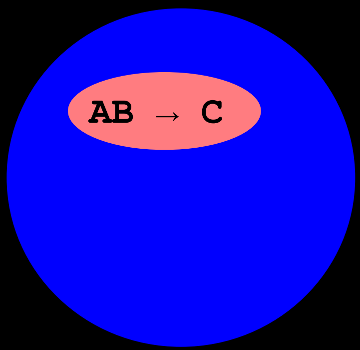
Proof (by Armstrong's Axioms):

```
(1) A → C       Given
(2) AB → BC     Augmentation (1)
(3) AB → C      Decomposition (2)
```

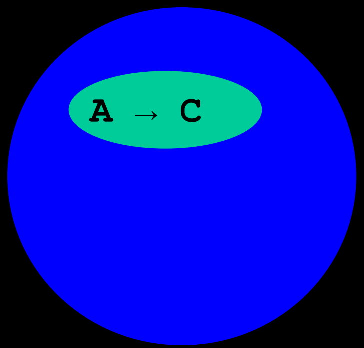
Extraneous Attributes in LHS

Is $F^+ = F_2^+$?

F:



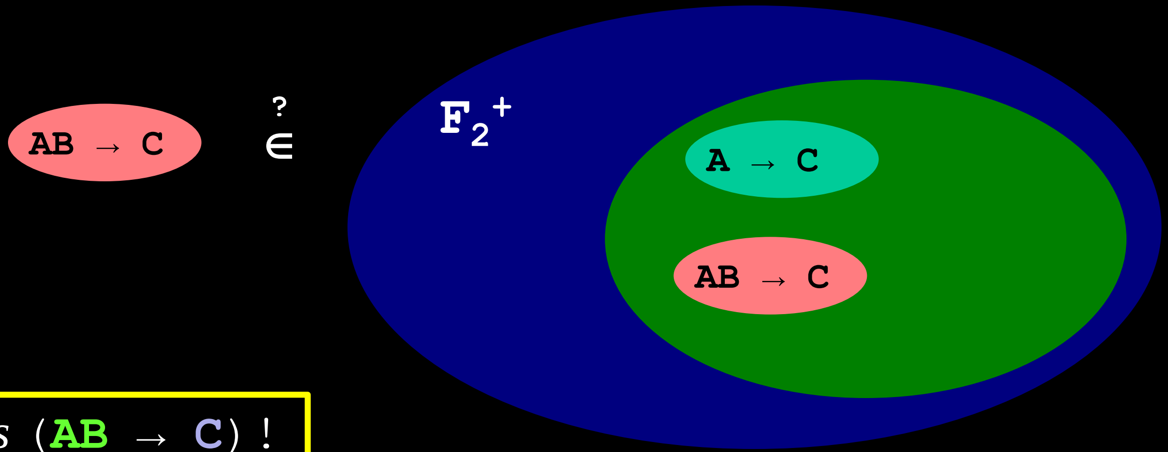
F_2:



a) Is (**AB** → **C**) $\in F_2^+$?

Always True!!

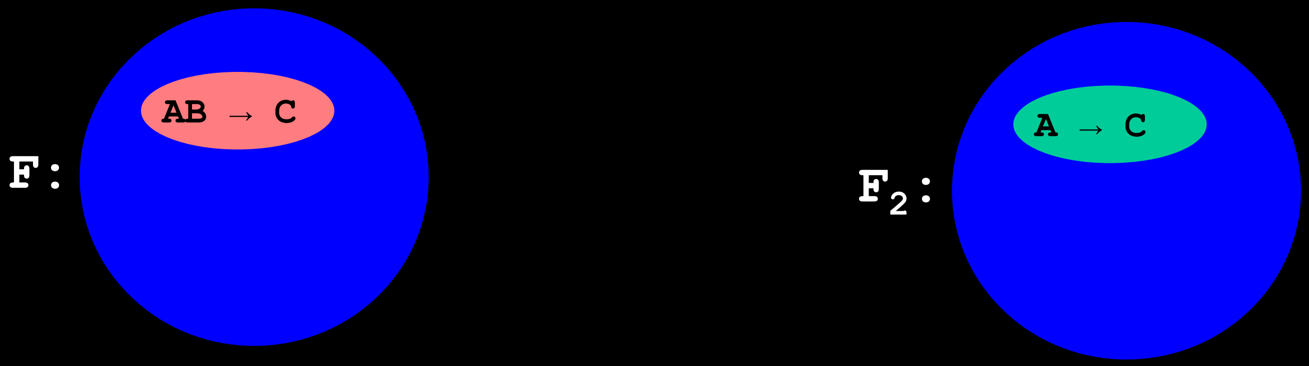
b) Is (**A** → **C**) $\in F^+$?



Key point: (**A** → **C**) *implies* (**AB** → **C**) !

Extraneous Attributes in LHS

Is $F^+ = F_2^+$?



a) Is $(AB \rightarrow C) \in F_2^+$?

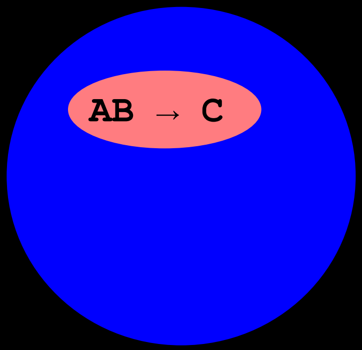
Always True!!

b) Is $(A \rightarrow C) \in F^+$?

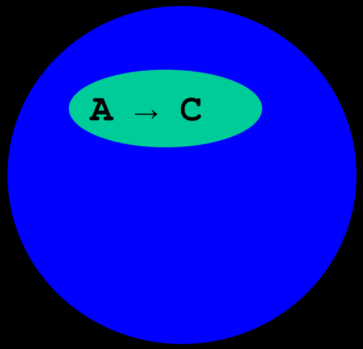
Extraneous Attributes in LHS

Is $F^+ = F_2^+$?

F:



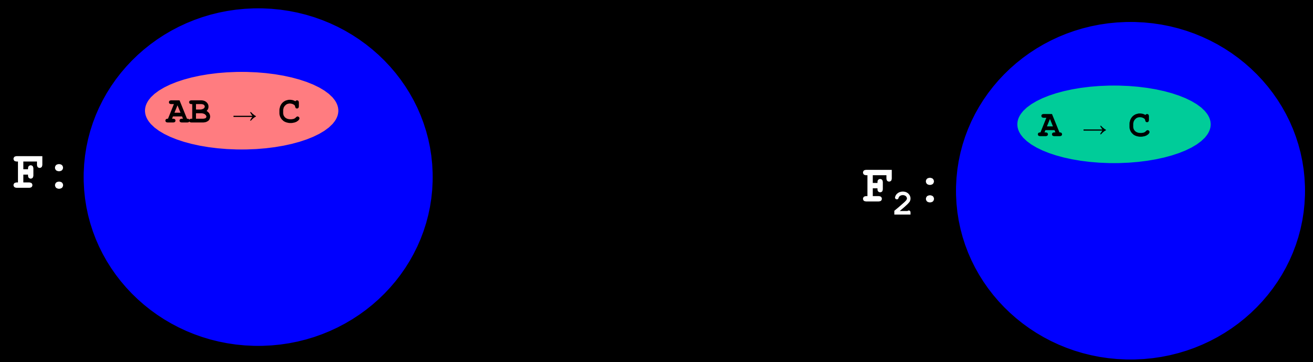
F_2:



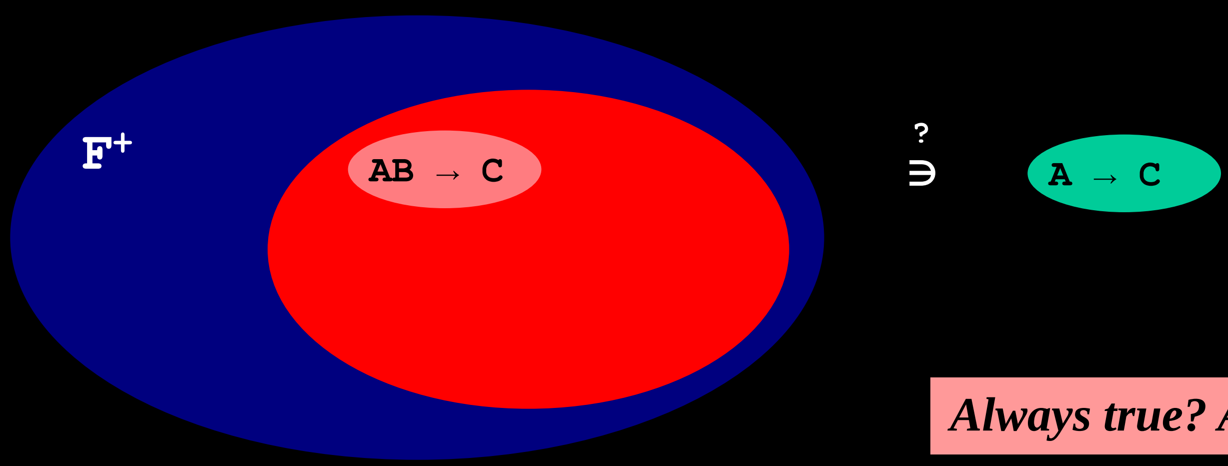
Is (**A** → **C**) ∈ F^+ ?

Extraneous Attributes in LHS

Is $F^+ = F_2^{\ +}$?



Is $(\mathbf{A} \rightarrow \mathbf{C}) \in F^+$?



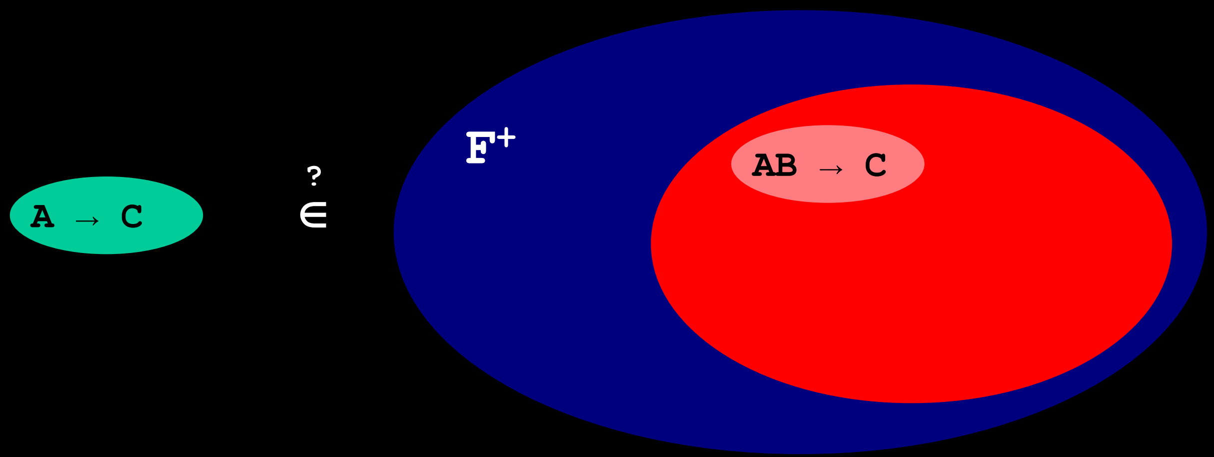
Always true? Always false?

Extraneous Attributes in LHS

Is $F^+ = F_2^+$?

Is $(A \rightarrow C) \in F^+$?

Always true? Always false?



A	C
1	3
1	3

Sometimes true …

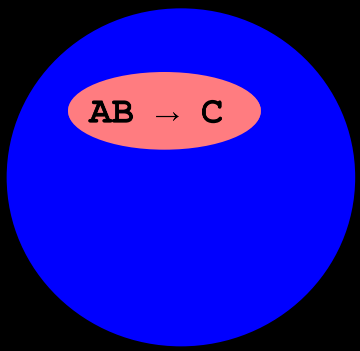
A	C
1	3
1	4

… , sometimes false

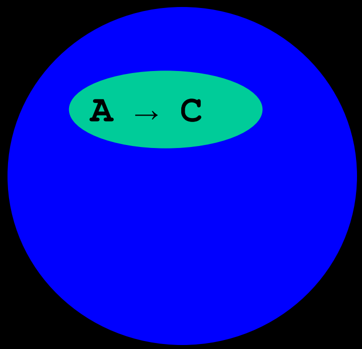
Extraneous Attributes in LHS

Is $F^+ = F_2^+$?

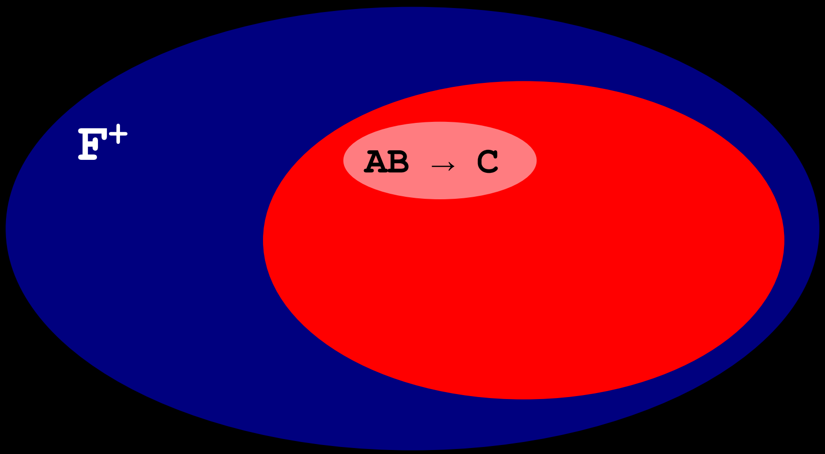
F:



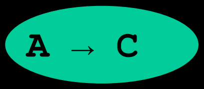
F_2:



Is $(A \to C) \in F^+$?



$\overset{?}{\ni}$



Sometimes true, sometimes false

Extraneous Attributes in LHS

"Is B extraneous in $AB \rightarrow C$?"

A Simple (but Expensive) Test:

1. *Replace "$A \rightarrow BC$" with "$A \rightarrow C$" in F to construct F_2*
   - $F_2 = F - \{A \rightarrow BC\} \cup \{A \rightarrow C\}$
2. *Test: Is $F_2^+ = F^+$?* *If yes, then B is extraneous*

Is there a less expensive way to compare F^+ and F_2^+ here?

A: Yes. Test: Is $(A \rightarrow C) \in F^+$?

Canonical Cover Algorithm

Basic Algorithm (will refine later)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  Is B extraneous in A → BC?
```

$$\text{Is } (A \rightarrow B) \in (F - \{A \rightarrow BC\} \cup \{A \rightarrow C\})^{+}?$$

```
    b) LHS:  Is B extraneous in AB → C?

END
```

Canonical Cover Algorithm

Basic Algorithm (will refine later)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  Is B extraneous in A → BC?
```

Is $(A \rightarrow B) \in (F - \{A \rightarrow BC\} \cup \{A \rightarrow C\})^+$?

```
    b) LHS:  Is B extraneous in AB → C?
```

Is $(A \rightarrow C) \in F^+$?

```
END
```

Canonical Cover Algorithm

The Algorithm

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  Is B extraneous in A → BC?

         Is (A → B) ∈ (F – {A → BC} ∪ {A → C})+?

    b) LHS:  Is B extraneous in AB → C?

         Is (A → C) ∈ F+ ?
END
```

Extraneous Attributes in LHS

"Is B extraneous in A**B** $\rightarrow$ C?"

$\boxed{F = \{AB \rightarrow C,\ B \rightarrow A\}}$

Is **B** *extraneous in* **AB** $\rightarrow$ **C**?

i.e., Is $(A \rightarrow C) \in F^+$?

A: *No:* $(A \rightarrow C) \notin F^+$

When asked or for partial credit, prove your answers as above

Extraneous Attributes in LHS

"Is B extraneous in A**B** $\rightarrow$ C?"

$$\boxed{F = \{AB \rightarrow C,\ B \rightarrow A\}}$$

*Is **A** extraneous in **AB** $\rightarrow$ **C**?*

i.e., Is $(B \rightarrow C) \in F^+$?

A: *Yes:* $(B \rightarrow C) \in F^+$

When asked or for partial credit, prove your answers as above

Canonical Cover Cheat Sheet

Canonical Cover Algorithm

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  Is B extraneous in A → BC?
             Is (A → B) ∈ (F – {A → BC} ∪ {A → C})+?

    b) LHS:  Is B extraneous in AB → C?
             Is (A → C) ∈ F+ ?

END
```

Canonical Cover Cheat Sheet

Determining if: $\mathbf{A} \rightarrow \mathbf{B} \in \mathbf{X}^+$

1. Calculate $\mathbf{A}^+$ = **Att-Closure ({A}, X)**
   (i.e., $\mathbf{A}^+$ wrt **X**)

2. Determine if $\mathbf{B} \in \mathbf{A}^+$

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_c:

F = {A → BC, B → CE, A → E}

A: F_c = Canonical-Cover (F)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
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  2. Remove extraneous attributes from each FD in F

    a) RHS:  B extraneous in A → BC?
             Is (A → B) ∈ (F − {A → BC} ∪ {A → C})⁺)

    b) LHS:  B extraneous in AB → C?
             Is (A → C) ∈ F⁺
END
```

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_c:

F = {A → BC, B → CE, A → E}

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          Is (A → B) ∈ (F − {A → BC} ∪ {A → C})⁺)

    b) LHS:  B extraneous in AB → C?
          Is (A → C) ∈ F⁺
END
```

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_c:

F = {A → BCE, B → CE}

A: F_c = Canonical-Cover (F)

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ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  B extraneous in A → BC?
             Is (A → B) ∈ (F – {A → BC} ∪ {A → C})+)

    b) LHS:  B extraneous in AB → C?
             Is (A → C) ∈ F+
END
```

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_c:

F = {A → BCE, B → CE}

A: F_c = Canonical-Cover (F)

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ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
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  2. Remove extraneous attributes from each FD in F

    a) RHS:  B extraneous in A → BC?
          Is (A → B) ∈ (F − {A → BC} ∪ {A → C})⁺)

    b) LHS:  B extraneous in AB → C?
          Is (A → C) ∈ F⁺
END
```

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_C:

$$F = \{A \rightarrow BCE,\ B \rightarrow CE\}$$

1. **B** *extraneous in* **A** → **BCE**?

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_c:

$F = \{A \rightarrow BCE,\ B \rightarrow CE\}$

1. B *extraneous in* $A \rightarrow BCE$? *No:* $(A \rightarrow B) \notin \{A \rightarrow CE,\ B \rightarrow CE\}^+$
   Proof: $A^+ = \{A,C,E\}$ and $B \notin A^+$
2. C *extraneous in* $A \rightarrow BCE$?

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_c:

$$F = \{A \rightarrow BCE,\ B \rightarrow CE\}$$

1. B *extraneous in* $A \rightarrow BCE$? *No:* $(A \rightarrow B) \notin \{A \rightarrow CE,\ B \rightarrow CE\}^+$
   Proof: $A^+ = \{A,C,E\}$ and $B \notin A^+$
2. C *extraneous in* $A \rightarrow BCE$? *Yes:* $(A \rightarrow C) \in \{A \rightarrow BE,\ B \rightarrow CE\}^+$
   Proof: $A^+ = \{A,B,E,C\}$ and $C \in A^+$

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_c:

$$F = \{A \rightarrow BCE,\ B \rightarrow CE\}$$

1. B *extraneous in* $A \rightarrow BCE$? *No:* $(A \rightarrow B) \notin \{A \rightarrow CE,\ B \rightarrow CE\}^+$
   Proof: $A^+ = \{A,C,E\}$ and $B \notin A^+$
2. C *extraneous in* $A \rightarrow BCE$? *Yes:* $(A \rightarrow C) \in \{A \rightarrow BE,\ B \rightarrow CE\}^+$
   Proof: $A^+ = \{A,B,E,C\}$ and $C \in A^+$

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_C:

$$F = \{A \rightarrow BE,\ B \rightarrow CE\}$$

1. B *extraneous in* $A \rightarrow BCE$? *No:* $(A \rightarrow B) \notin \{A \rightarrow CE,\ B \rightarrow CE\}^+$
   Proof: $A^+ = \{A,C,E\}$ and $B \notin A^+$
2. C *extraneous in* $A \rightarrow BCE$? *Yes:* $(A \rightarrow C) \in \{A \rightarrow BE,\ B \rightarrow CE\}^+$
   Proof: $A^+ = \{A,B,E,C\}$ and $C \in A^+$
3. E *extraneous in* $A \rightarrow BE$?

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_C:

$$F = \{A \rightarrow BE,\ B \rightarrow CE\}$$

1. B *extraneous in* $A \rightarrow BCE$? *No:* $(A \rightarrow B) \notin \{A \rightarrow CE,\ B \rightarrow CE\}^+$
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2. C *extraneous in* $A \rightarrow BCE$? *Yes:* $(A \rightarrow C) \in \{A \rightarrow BE,\ B \rightarrow CE\}^+$
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   Proof: $A^+ = \{A,B,C,E\}$ and $E \in A^+$

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_C:

$$F = \{A \rightarrow B,\ B \rightarrow CE\}$$

1. B *extraneous in* A → BCE? *No:* $(A \rightarrow B) \notin \{A \rightarrow CE,\ B \rightarrow CE\}^+$
   Proof: $A^+ = \{A,C,E\}$ and $B \notin A^+$
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   Proof: $A^+ = \{A,B,C,E\}$ and $E \in A^+$

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_C:

F = {A → B, B → CE}

1. B *extraneous in* A → BCE? *No:* (A → B) ∉ {A → CE, B → CE}$^+$
   Proof: A^+ = {A,C,E} and B ∉ A^+
2. C *extraneous in* A → BCE? *Yes:* (A → C) ∈ {A → BE, B → CE}$^+$
   Proof: A^+ = {A,B,E,C} and C ∈ A^+
3. E *extraneous in* A → BE? *Yes:* (A → E) ∈ {A → B, B → CE}$^+$
   Proof: A^+ = {A,B,C,E} and E ∈ A^+
4. C *extraneous in* B → CE?

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_C:

F = {A → B, B → CE}

1. B *extraneous in* A → BCE? *No:* (A → B) $\notin$ {A → CE, B → CE}$^+$
   Proof: A^+ = {A,C,E} and B $\notin$ A^+
2. C *extraneous in* A → BCE? *Yes:* (A → C) $\in$ {A → BE, B → CE}$^+$
   Proof: A^+ = {A,B,E,C} and C $\in$ A^+
3. E *extraneous in* A → BE? *Yes:* (A → E) $\in$ {A → B, B → CE}$^+$
   Proof: A^+ = {A,B,C,E} and E $\in$ A^+
4. C *extraneous in* B → CE? *No:* (B → C) $\notin$ {A → B, B → E}$^+$
   Proof: B^+ = {B,E} and C $\notin$ B^+
5. E *extraneous in* B → CE?

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_C:

$$F = \{A \rightarrow B,\ B \rightarrow CE\}$$

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   Proof: $A^+ = \{A,C,E\}$ and $B \notin A^+$
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   Proof: $B^+ = \{B,C\}$ and $E \notin B^+$

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_c:

F = {A → B, B → CE}

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  B extraneous in A → BC?
         Is (A → B) ∈ (F - {A → BC} ∪ {A → C})+)

    b) LHS:  B extraneous in AB → C?
         Is (A → C) ∈ F+
END
```

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_C:

F = {A → B, B → CE}

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Canonical Cover Algorithm Example

Determine the canonical cover of F, F_C:

$$F = \{A \rightarrow B,\ B \rightarrow CE\}$$

1. B *extraneous in* A → BCE? *No:* $(A \rightarrow B) \notin \{A \rightarrow CE,\ B \rightarrow CE\}^+$
   Proof: $A^+ = \{A,C,E\}$ and $B \notin A^+$
2. C *extraneous in* A → BCE? *Yes:* $(A \rightarrow C) \in \{A \rightarrow BE,\ B \rightarrow CE\}^+$
   Proof: $A^+ = \{A,B,E,C\}$ and $C \in A^+$
3. E *extraneous in* A → BE? *Yes:* $(A \rightarrow E) \in \{A \rightarrow B,\ B \rightarrow CE\}^+$
   Proof: $A^+ = \{A,B,C,E\}$ and $E \in A^+$
4. C *extraneous in* B → CE? *No:* $(B \rightarrow C) \notin \{A \rightarrow B,\ B \rightarrow E\}^+$
   Proof: $B^+ = \{B,E\}$ and $C \notin B^+$
5. E *extraneous in* B → CE? *No:* $(B \rightarrow E) \notin \{A \rightarrow B,\ B \rightarrow C\}^+$
   Proof: $B^+ = \{B,C\}$ and $E \notin B^+$

Canonical Cover Algorithm Example

Determine the canonical cover of F, F_c:

$F = \{A \rightarrow BC,\ B \rightarrow CE,\ A \rightarrow E\}$

A: $F_c = \{A \rightarrow B,\ B \rightarrow CE\}$

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_c:

F = {A → BC, B → CE, A → E, AC → H, D → B}

A: F_c = Canonical-Cover (F)

```
ALGORITHM Canonical-Cover (F: {FDs})
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             Is (A → C) ∈ F⁺
END
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Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_c:

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END
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Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_c:

F = {A → BCE, B → CE, AC → H, D → B}

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Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_c:

$F = \{A \rightarrow BCE,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}$

A: $F_c = \text{Canonical-Cover}\ (F)$

```
ALGORITHM Canonical-Cover (F: {FDs})
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          Is (A → B) ∈ (F – {A → BC} ∪ {A → C})⁺)

    b) LHS:  B extraneous in AB → C?
          Is (A → C) ∈ F⁺
END
```

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_C:

F = {A → BCE, B → CE, AC → H, D → B}

1. **B** *extraneous in* **A → BCE***?*

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_C:

$F = \{A \rightarrow BCE,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}$

1. B *extraneous in* A → BCE? *No:* $(A \rightarrow B) \notin \{A \rightarrow CE,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $A^+ = \{A, C, E, H\}$ and $B \notin A^+$

2. C *extraneous in* A → BCE?

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_C:

$$F = \{A \rightarrow BCE,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}$$

1. B *extraneous in* $A \rightarrow BCE$? *No:* $(A \rightarrow B) \notin \{A \rightarrow CE,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}^+$
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Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_C:

F = {A → BE, B → CE, AC → H, D → B}

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Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_c:

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   Proof: $A^+ = \{A,B,E,C,H\}$ and $C \in A^+$
3. E *extraneous in* $A \rightarrow BE$?

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_c:

$F = \{A \rightarrow BE,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}$

1. B *extraneous in* A → BCE? *No:* $(A \rightarrow B) \notin \{A \rightarrow CE,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $A^+ = \{A,C,E,H\}$ and $B \notin A^+$
2. C *extraneous in* A → BCE? *Yes:* $(A \rightarrow C) \in \{A \rightarrow BE,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $A^+ = \{A,B,E,C,H\}$ and $C \in A^+$
3. E *extraneous in* A → BE? *Yes:* $(A \rightarrow E) \in \{A \rightarrow B,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $A^+ = \{A,B,C,E,H\}$ and $E \in A^+$

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_C:

F = {A → B, B → CE, AC → H, D → B}

1. **B** *extraneous in* **A → BCE?** *No:* **(A → B) ∉ {A → CE, B → CE, AC → H, D → B}$^+$**
   Proof: A^+ = {A,C,E,H} and B ∉ A^+
2. **C** *extraneous in* **A → BCE?** *Yes:* **(A → C) ∈ {A → BE, B → CE, AC → H, D → B}$^+$**
   Proof: A^+ = {A,B,E,C,H} and C ∈ A^+
3. **E** *extraneous in* **A → BE?** *Yes:* **(A → E) ∈ {A → B, B → CE, AC → H, D → B}$^+$**
   Proof: A^+ = {A,B,C,E,H} and E ∈ A^+

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_c:

$F = \{A \rightarrow B,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}$

1. B *extraneous in* A → BCE? *No:* $(A \rightarrow B) \notin \{A \rightarrow CE,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $A^+ = \{A,C,E,H\}$ and $B \notin A^+$
2. C *extraneous in* A → BCE? *Yes:* $(A \rightarrow C) \in \{A \rightarrow BE,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $A^+ = \{A,B,E,C,H\}$ and $C \in A^+$
3. E *extraneous in* A → BE? *Yes:* $(A \rightarrow E) \in \{A \rightarrow B,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $A^+ = \{A,B,C,E,H\}$ and $E \in A^+$
4. C *extraneous in* B → CE?

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_C:

$F = \{A \rightarrow B,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}$

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2. C *extraneous in* A → BCE? *Yes:* $(A \rightarrow C) \in \{A \rightarrow BE,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $A^+ = \{A,B,E,C,H\}$ and $C \in A^+$
3. E *extraneous in* A → BE? *Yes:* $(A \rightarrow E) \in \{A \rightarrow B,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $A^+ = \{A,B,C,E,H\}$ and $E \in A^+$
4. C *extraneous in* B → CE? *No:* $(B \rightarrow C) \notin \{A \rightarrow B,\ B \rightarrow E,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $B^+ = \{B,E\}$ and $C \notin B^+$
5. E *extraneous in* B → CE?

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_C:

F = {A → B, B → CE, AC → H, D → B}

1. B *extraneous in* A → BCE? *No:* (A → B) ∉ $\{A \to CE, B \to CE, AC \to H, D \to B\}^+$
   Proof: $A^+ = \{A,C,E,H\}$ and $B \notin A^+$
2. C *extraneous in* A → BCE? *Yes:* (A → C) ∈ $\{A \to BE, B \to CE, AC \to H, D \to B\}^+$
   Proof: $A^+ = \{A,B,E,C,H\}$ and $C \in A^+$
3. E *extraneous in* A → BE? *Yes:* (A → E) ∈ $\{A \to B, B \to CE, AC \to H, D \to B\}^+$
   Proof: $A^+ = \{A,B,C,E,H\}$ and $E \in A^+$
4. C *extraneous in* B → CE? *No:* (B → C) ∉ $\{A \to B, B \to E, AC \to H, D \to B\}^+$
   Proof: $B^+ = \{B,E\}$ and $C \notin B^+$
5. E *extraneous in* B → CE? *No:* (B → E) ∉ $\{A \to B, B \to C, AC \to H, D \to B\}^+$
   Proof: $B^+ = \{B,C\}$ and $E \notin B^+$
6. A *extraneous in* AC → H?

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_C:

F = {A → B, B → CE, AC → H, D → B}

1. B *extraneous in* A → BCE? *No:* (A → B) ∉ {A → CE, B → CE, AC → H, D → B}$^+$
   Proof: A^+ = {A,C,E,H} and B ∉ A^+
2. C *extraneous in* A → BCE? *Yes:* (A → C) ∈ {A → BE, B → CE, AC → H, D → B}$^+$
   Proof: A^+ = {A,B,E,C,H} and C ∈ A^+
3. E *extraneous in* A → BE? *Yes:* (A → E) ∈ {A → B, B → CE, AC → H, D → B}$^+$
   Proof: A^+ = {A,B,C,E,H} and E ∈ A^+
4. C *extraneous in* B → CE? *No:* (B → C) ∉ {A → B, B → E, AC → H, D → B}$^+$
   Proof: B^+ = {B,E} and C ∉ B^+
5. E *extraneous in* B → CE? *No:* (B → E) ∉ {A → B, B → C, AC → H, D → B}$^+$
   Proof: B^+ = {B,C} and E ∉ B^+
6. A *extraneous in* AC → H? *No:* (C → H) ∉ F^+
   Proof: C^+ = {C} and H ∉ C^+
7. C *extraneous in* AC → H?

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_c:

$$F = \{A \rightarrow B,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}$$

1. B *extraneous in* $A \rightarrow BCE$? *No:* $(A \rightarrow B) \notin \{A \rightarrow CE,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $A^+ = \{A,C,E,H\}$ and $B \notin A^+$
2. C *extraneous in* $A \rightarrow BCE$? *Yes:* $(A \rightarrow C) \in \{A \rightarrow BE,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $A^+ = \{A,B,E,C,H\}$ and $C \in A^+$
3. E *extraneous in* $A \rightarrow BE$? *Yes:* $(A \rightarrow E) \in \{A \rightarrow B,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $A^+ = \{A,B,C,E,H\}$ and $E \in A^+$
4. C *extraneous in* $B \rightarrow CE$? *No:* $(B \rightarrow C) \notin \{A \rightarrow B,\ B \rightarrow E,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $B^+ = \{B,E\}$ and $C \notin B^+$
5. E *extraneous in* $B \rightarrow CE$? *No:* $(B \rightarrow E) \notin \{A \rightarrow B,\ B \rightarrow C,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $B^+ = \{B,C\}$ and $E \notin B^+$
6. A *extraneous in* $AC \rightarrow H$? *No:* $(C \rightarrow H) \notin F^+$
   Proof: $C^+ = \{C\}$ and $H \notin C^+$
7. C *extraneous in* $AC \rightarrow H$? *Yes:* $(A \rightarrow H) \in F^+$
   Proof: $A^+ = \{A,B,C,E,H\}$ and $H \in A^+$

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_C:

$F = \{A \rightarrow B,\ B \rightarrow CE,\ A \rightarrow H,\ D \rightarrow B\}$

1. B *extraneous in* A $\rightarrow$ BCE? *No:* $(A \rightarrow B) \notin \{A \rightarrow CE,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $A^+ = \{A,C,E,H\}$ and $B \notin A^+$
2. C *extraneous in* A $\rightarrow$ BCE? *Yes:* $(A \rightarrow C) \in \{A \rightarrow BE,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $A^+ = \{A,B,E,C,H\}$ and $C \in A^+$
3. E *extraneous in* A $\rightarrow$ BE? *Yes:* $(A \rightarrow E) \in \{A \rightarrow B,\ B \rightarrow CE,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $A^+ = \{A,B,C,E,H\}$ and $E \in A^+$
4. C *extraneous in* B $\rightarrow$ CE? *No:* $(B \rightarrow C) \notin \{A \rightarrow B,\ B \rightarrow E,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $B^+ = \{B,E\}$ and $C \notin B^+$
5. E *extraneous in* B $\rightarrow$ CE? *No:* $(B \rightarrow E) \notin \{A \rightarrow B,\ B \rightarrow C,\ AC \rightarrow H,\ D \rightarrow B\}^+$
   Proof: $B^+ = \{B,C\}$ and $E \notin B^+$
6. A *extraneous in* AC $\rightarrow$ H? *No:* $(C \rightarrow H) \notin F^+$
   Proof: $C^+ = \{C\}$ and $H \notin C^+$
7. C *extraneous in* AC $\rightarrow$ H? *Yes:* $(A \rightarrow H) \in F^+$
   Proof: $A^+ = \{A,B,C,E,H\}$ and $H \in A^+$

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_c:

F = {A → B, B → CE, A → H, D → B}

A: F_c = Canonical-Cover (F)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  B extraneous in A → BC?
             Is (A → B) ∈ (F – {A → BC} ∪ {A → C})⁺)

    b) LHS:  B extraneous in AB → C?
             Is (A → C) ∈ F⁺
END
```

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_c:

F = {A → BH, B → CE, D → B}

A: F_c = Canonical-Cover (F)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  B extraneous in A → BC?
          Is (A → B) ∈ (F – {A → BC} ∪ {A → C})⁺)

    b) LHS:  B extraneous in AB → C?
          Is (A → C) ∈ F⁺
END
```

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_c:

F = {A → BH, B → CE, D → B}

A: F_c = Canonical-Cover (F)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  B extraneous in A → BC?
         Is (A → B) ∈ (F – {A → BC} ∪ {A → C})+)

    b) LHS:  B extraneous in AB → C?
         Is (A → C) ∈ F+
END
```

Canonical Cover Algorithm Exercise

Determine the canonical cover of F*,* F_c*:*

F = {A → BH, B → CE, D → B}

8. **B** *extraneous in* **A → BH***?*

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_C:

$$F = \{A \rightarrow BH,\ B \rightarrow CE,\ D \rightarrow B\}$$

8. B *extraneous in* $A \rightarrow BH$? *No:* $(A \rightarrow B) \notin \{A \rightarrow H,\ B \rightarrow CE,\ D \rightarrow B\}^+$
   Proof: $A^+ = \{A, H\}$ and $B \notin A^+$

9. H *extraneous in* $A \rightarrow BH$?

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_C:

F = {A → BH, B → CE, D → B}

8. **B** *extraneous in* **A → BH?** *No:* **(A → B) ∉ {A → H, B → CE, D → B}$^+$**
   Proof: A^+ = {A,H} and B ∉ A^+

9. **H** *extraneous in* **A → BH?** *No:* **(A → H) ∉ {A → B, B → CE, D → B}$^+$**
   Proof: A^+ = {A,B,C,E} and H ∉ A^+

10. **C** *extraneous in* **B → CE?**

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_c:

F = {A → BH, B → CE, D → B}

8. B *extraneous in* A → BH? *No:* (A → B) ∉ {A → H, B → CE, D → B}$^+$
   Proof: A^+ = {A,H} and B ∉ A^+

9. H *extraneous in* A → BH? *No:* (A → H) ∉ {A → B, B → CE, D → B}$^+$
   Proof: A^+ = {A,B,C,E} and H ∉ A^+

10. C *extraneous in* B → CE? *No:* (B → C) ∉ {A → BH, B → E, D → B}$^+$
    Proof: B^+ = {B,E} and C ∉ B^+

11. E *extraneous in* B → CE?

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_C:

F = {A → BH, B → CE, D → B}

8. B *extraneous in* A → BH? *No:* (A → B) ∉ $\{A \to H, B \to CE, D \to B\}^+$
   Proof: $A^+ = \{A,H\}$ and $B \notin A^+$

9. H *extraneous in* A → BH? *No:* (A → H) ∉ $\{A \to B, B \to CE, D \to B\}^+$
   Proof: $A^+ = \{A,B,C,E\}$ and $H \notin A^+$

10. C *extraneous in* B → CE? *No:* (B → C) ∉ $\{A \to BH, B \to E, D \to B\}^+$
    Proof: $B^+ = \{B,E\}$ and $C \notin B^+$

11. E *extraneous in* B → CE? *No:* (B → E) ∉ $\{A \to BH, B \to C, D \to B\}^+$
    Proof: $B^+ = \{B,C\}$ and $E \notin B^+$

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_c:

$F = \{A \rightarrow BH,\ B \rightarrow CE,\ D \rightarrow B\}$

A: $F_c = \text{Canonical-Cover (F)}$

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  B extraneous in A → BC?
          Is (A → B) ∈ (F − {A → BC} ∪ {A → C})⁺)

    b) LHS:  B extraneous in AB → C?
          Is (A → C) ∈ F⁺
END
```

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_c:

F = {A → BH, B → CE, D → B}

A: F_c = Canonical-Cover (F)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  B extraneous in A → BC?
             Is (A → B) ∈ (F – {A → BC} ∪ {A → C})+)

    b) LHS:  B extraneous in AB → C?
             Is (A → C) ∈ F+
END
```

Canonical Cover Algorithm Exercise

Determine the canonical cover of F*,* F_C:

$F = \{A \to BH,\ B \to CE,\ D \to B\}$

8. B *extraneous in* A $\to$ BH? *No:* $(A \to B) \notin \{A \to H,\ B \to CE,\ D \to B\}^+$
   Proof: $A^+ = \{A,H\}$ and $B \notin A^+$

9. H *extraneous in* A $\to$ BH? *No:* $(A \to H) \notin \{A \to B,\ B \to CE,\ D \to B\}^+$
   Proof: $A^+ = \{A,B,C,E\}$ and $H \notin A^+$

10. C *extraneous in* B $\to$ CE? *No:* $(B \to C) \notin \{A \to BH,\ B \to E,\ D \to B\}^+$
    Proof: $B^+ = \{B,E\}$ and $C \notin B^+$

11. E *extraneous in* B $\to$ CE? *No:* $(B \to E) \notin \{A \to BH,\ B \to C,\ D \to B\}^+$
    Proof: $B^+ = \{B,C\}$ and $E \notin B^+$

Canonical Cover Algorithm Exercise

Determine the canonical cover of F, F_c:

$F = \{A \rightarrow BC,\ B \rightarrow CE,\ A \rightarrow E,\ AC \rightarrow H,\ D \rightarrow B\}$

A: $F_c = \{A \rightarrow BH,\ B \rightarrow CE,\ D \rightarrow B\}$

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

$$F = \{A \rightarrow BC,\ B \rightarrow CA,\ C \rightarrow AB\}$$

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

$F = \{A \rightarrow BC,\ B \rightarrow CA,\ C \rightarrow AB\}$

A: F_c = Canonical-Cover (F)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  B extraneous in A → BC?
         Is (A → B) ∈ (F - {A → BC} ∪ {A → C})+)

    b) LHS:  B extraneous in AB → C?
         Is (A → C) ∈ F+
END
```

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

F = {A → BC, B → CA, C → AB}

A: F_c = Canonical-Cover (F)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  B extraneous in A → BC?
          Is (A → B) ∈ (F − {A → BC} ∪ {A → C})+)

    b) LHS:  B extraneous in AB → C?
          Is (A → C) ∈ F+
END
```

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

F = {A → BC, B → CA, C → AB}

A: F_c = Canonical-Cover (F)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
   1. Where possible, apply UNION rule to FD's in F
      (Armstrong's Axioms)

   2. Remove extraneous attributes from each FD in F

     a) RHS:  B extraneous in A → BC?
              Is (A → B) ∈ (F – {A → BC} ∪ {A → C})+)

     b) LHS:  B extraneous in AB → C?
              Is (A → C) ∈ F+
END
```

Canonical Cover Algorithm Example

Determine 2 different canonical covers of `F`:

$$F = \{A \rightarrow BC,\ B \rightarrow CA,\ C \rightarrow AB\}$$

1. **B** *extraneous in* **A** $\rightarrow$ **BC**?

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

$$F = \{A \rightarrow BC,\ B \rightarrow CA,\ C \rightarrow AB\}$$

1. B *extraneous in* $A \rightarrow BC$*?* *Yes:* $(A \rightarrow B) \in \{A \rightarrow C,\ B \rightarrow CA,\ C \rightarrow AB\}^+$
   Proof: $A^+ = \{A,C,B\}$ and $B \in A^+$

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

$$F = \{A \rightarrow C,\ B \rightarrow CA,\ C \rightarrow AB\}$$

1. B *extraneous in* $A \rightarrow BC$? *Yes:* $(A \rightarrow B) \in \{A \rightarrow C,\ B \rightarrow CA,\ C \rightarrow AB\}^+$
   Proof: $A^+ = \{A, C, B\}$ and $B \in A^+$

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

$$F = \{A \rightarrow C,\ B \rightarrow CA,\ C \rightarrow AB\}$$

1. B *extraneous in* $A \rightarrow BC$? *Yes:* $(A \rightarrow B) \in \{A \rightarrow C,\ B \rightarrow CA,\ C \rightarrow AB\}^+$
   Proof: $A^+ = \{A, C, B\}$ and $B \in A^+$
2. C *extraneous in* $B \rightarrow CA$?

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

$F = \{A \rightarrow C,\ B \rightarrow CA,\ C \rightarrow AB\}$

1. B *extraneous in* A → BC? *Yes:* $(A \rightarrow B) \in \{A \rightarrow C,\ B \rightarrow CA,\ C \rightarrow AB\}^+$
   Proof: $A^+ = \{A, C, B\}$ and $B \in A^+$
2. C *extraneous in* B → CA? *Yes:* $(B \rightarrow C) \in \{A \rightarrow C,\ B \rightarrow A,\ C \rightarrow AB\}^+$
   Proof: $B^+ = \{B, A, C\}$ and $C \in B^+$

Canonical Cover Algorithm Example

Determine 2 different canonical covers of `F`*:*

$$F = \{A \rightarrow C,\ B \rightarrow A,\ C \rightarrow AB\}$$

1. `B` *extraneous in* `A → BC`? *Yes:* $(A \rightarrow B) \in \{A \rightarrow C,\ B \rightarrow CA,\ C \rightarrow AB\}^+$
   Proof: $A^+ = \{A,C,B\}$ and $B \in A^+$
2. `C` *extraneous in* `B → CA`? *Yes:* $(B \rightarrow C) \in \{A \rightarrow C,\ B \rightarrow A,\ C \rightarrow AB\}^+$
   Proof: $B^+ = \{B,A,C\}$ and $C \in B^+$

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

$$F = \{A \rightarrow C,\ B \rightarrow A,\ C \rightarrow AB\}$$

1. B *extraneous in* A → BC? *Yes:* $(A \rightarrow B) \in \{A \rightarrow C,\ B \rightarrow CA,\ C \rightarrow AB\}^+$
   Proof: $A^+ = \{A,C,B\}$ and $B \in A^+$
2. C *extraneous in* B → CA? *Yes:* $(B \rightarrow C) \in \{A \rightarrow C,\ B \rightarrow A,\ C \rightarrow AB\}^+$
   Proof: $B^+ = \{B,A,C\}$ and $C \in B^+$
3. A *extraneous in* C → AB?

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

$F = \{A \rightarrow C,\ B \rightarrow A,\ C \rightarrow AB\}$

1. B *extraneous in* $A \rightarrow BC$? *Yes:* $(A \rightarrow B) \in \{A \rightarrow C,\ B \rightarrow CA,\ C \rightarrow AB\}^+$
   Proof: $A^+ = \{A,C,B\}$ and $B \in A^+$
2. C *extraneous in* $B \rightarrow CA$? *Yes:* $(B \rightarrow C) \in \{A \rightarrow C,\ B \rightarrow A,\ C \rightarrow AB\}^+$
   Proof: $B^+ = \{B,A,C\}$ and $C \in B^+$
3. A *extraneous in* $C \rightarrow AB$? *Yes:* $(C \rightarrow A) \in \{A \rightarrow C,\ B \rightarrow A,\ C \rightarrow B\}^+$
   Proof: $C^+ = \{C,B,A\}$ and $A \in C^+$

Canonical Cover Algorithm Example

Determine 2 different canonical covers of `F`:

$F = \{A \rightarrow C,\ B \rightarrow A,\ C \rightarrow B\}$

1. B *extraneous in* A → BC? *Yes:* $(A \rightarrow B) \in \{A \rightarrow C,\ B \rightarrow CA,\ C \rightarrow AB\}^+$
   Proof: $A^+ = \{A,C,B\}$ and $B \in A^+$
2. C *extraneous in* B → CA? *Yes:* $(B \rightarrow C) \in \{A \rightarrow C,\ B \rightarrow A,\ C \rightarrow AB\}^+$
   Proof: $B^+ = \{B,A,C\}$ and $C \in B^+$
3. A *extraneous in* C → AB? *Yes:* $(C \rightarrow A) \in \{A \rightarrow C,\ B \rightarrow A,\ C \rightarrow B\}^+$
   Proof: $C^+ = \{C,B,A\}$ and $A \in C^+$

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

F = {A → C, B → A, C → B}

A: F_c = Canonical-Cover (F)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  B extraneous in A → BC?
          Is (A → B) ∈ (F − {A → BC} ∪ {A → C})⁺)

    b) LHS:  B extraneous in AB → C?
          Is (A → C) ∈ F⁺
END
```

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

F = {A → C, B → A, C → B}

A: F_c = Canonical-Cover (F)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  B extraneous in A → BC?
          Is (A → B) ∈ (F − {A → BC} ∪ {A → C})⁺)

    b) LHS:  B extraneous in AB → C?
          Is (A → C) ∈ F⁺
END
```

Canonical Cover Algorithm Example

Determine 2 different canonical covers of `F`:

$$F = \{A \rightarrow BC,\ B \rightarrow CA,\ C \rightarrow AB\}$$

A: $F_c = \{A \rightarrow C,\ B \rightarrow A,\ C \rightarrow B\}$

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

F = {A → BC, B → CA, C → AB}

A: F_c = Canonical-Cover (F)

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ALGORITHM Canonical-Cover (F: {FDs})
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 REPEAT UNTIL STABLE
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    a) RHS:  B extraneous in A → BC?
         Is (A → B) ∈ (F – {A → BC} ∪ {A → C})+)

    b) LHS:  B extraneous in AB → C?
         Is (A → C) ∈ F+
END
```

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

F = {A → BC, B → CA, C → AB}

A: F_c = Canonical-Cover (F)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  B extraneous in A → BC?
          Is (A → B) ∈ (F − {A → BC} ∪ {A → C})⁺)

    b) LHS:  B extraneous in AB → C?
          Is (A → C) ∈ F⁺
END
```

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

$F = \{A \rightarrow BC,\ B \rightarrow CA,\ C \rightarrow AB\}$

A: F_c = Canonical-Cover (F)

```
ALGORITHM Canonical-Cover (F: {FDs})
BEGIN
 REPEAT UNTIL STABLE
  1. Where possible, apply UNION rule to FD's in F
     (Armstrong's Axioms)

  2. Remove extraneous attributes from each FD in F

    a) RHS:  B extraneous in A → BC?
          Is (A → B) ∈ (F − {A → BC} ∪ {A → C})+)

    b) LHS:  B extraneous in AB → C?
          Is (A → C) ∈ F+
END
```

Canonical Cover Algorithm Example

Determine 2 different canonical covers of `F`:

$$F = \{A \rightarrow BC,\ B \rightarrow CA,\ C \rightarrow AB\}$$

1. **C** *extraneous in* **A → BC***?*

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

$$F = \{A \rightarrow BC,\ B \rightarrow CA,\ C \rightarrow AB\}$$

1. C *extraneous in* $A \rightarrow BC$? *Yes:* $(A \rightarrow C) \in \{A \rightarrow B,\ B \rightarrow CA,\ C \rightarrow AB\}^+$

   Proof: $A^+ = \{A,B,C\}$ and $C \in A^+$

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

$$F = \{A \rightarrow B,\ B \rightarrow CA,\ C \rightarrow AB\}$$

1. C *extraneous in* $A \rightarrow BC$? *Yes:* $(A \rightarrow C) \in \{A \rightarrow B,\ B \rightarrow CA,\ C \rightarrow AB\}^+$
   Proof: $A^+ = \{A,B,C\}$ and $C \in A^+$

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

$$F = \{A \rightarrow B,\ B \rightarrow CA,\ C \rightarrow AB\}$$

1. C *extraneous in* $A \rightarrow BC$? *Yes:* $(A \rightarrow C) \in \{A \rightarrow B,\ B \rightarrow CA,\ C \rightarrow AB\}^+$
   Proof: $A^+ = \{A,B,C\}$ and $C \in A^+$
2. A *extraneous in* $B \rightarrow CA$?

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

$$F = \{A \rightarrow B,\ B \rightarrow CA,\ C \rightarrow AB\}$$

1. C *extraneous in* $A \rightarrow BC$? *Yes:* $(A \rightarrow C) \in \{A \rightarrow B,\ B \rightarrow CA,\ C \rightarrow AB\}^+$
   Proof: $A^+ = \{A,B,C\}$ and $C \in A^+$
2. A *extraneous in* $B \rightarrow CA$? *Yes:* $(B \rightarrow A) \in \{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow AB\}^+$
   Proof: $B^+ = \{B,C,A\}$ and $A \in B^+$

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

$$F = \{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow AB\}$$

1. C *extraneous in* A → BC? *Yes:* $(A \rightarrow C) \in \{A \rightarrow B,\ B \rightarrow CA,\ C \rightarrow AB\}^+$
   Proof: $A^+ = \{A,B,C\}$ and $C \in A^+$
2. A *extraneous in* B → CA? *Yes:* $(B \rightarrow A) \in \{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow AB\}^+$
   Proof: $B^+ = \{B,C,A\}$ and $A \in B^+$

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

$$F = \{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow AB\}$$

1. C *extraneous in* $A \rightarrow BC$? *Yes:* $(A \rightarrow C) \in \{A \rightarrow B,\ B \rightarrow CA,\ C \rightarrow AB\}^+$
   Proof: $A^+ = \{A,B,C\}$ and $C \in A^+$
2. A *extraneous in* $B \rightarrow CA$? *Yes:* $(B \rightarrow A) \in \{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow AB\}^+$
   Proof: $B^+ = \{B,C,A\}$ and $A \in B^+$
3. B *extraneous in* $C \rightarrow AB$?

Canonical Cover Algorithm Example

Determine 2 different canonical covers of F:

$$F = \{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow AB\}$$

1. C *extraneous in* A → BC? *Yes:* $(A \rightarrow C) \in \{A \rightarrow B,\ B \rightarrow CA,\ C \rightarrow AB\}^+$
   Proof: $A^+ = \{A,B,C\}$ and $C \in A^+$
2. A *extraneous in* B → CA? *Yes:* $(B \rightarrow A) \in \{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow AB\}^+$
   Proof: $B^+ = \{B,C,A\}$ and $A \in B^+$
3. B *extraneous in* C → AB? *Yes:* $(C \rightarrow B) \in \{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow A\}^+$
   Proof: $C^+ = \{C,A,B\}$ and $B \in C^+$

Canonical Cover Algorithm Example

Determine 2 different canonical covers of `F`*:*

$F = \{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow A\}$

1. C *extraneous in* $A \rightarrow BC$? *Yes:* $(A \rightarrow C) \in \{A \rightarrow B,\ B \rightarrow CA,\ C \rightarrow AB\}^+$
   Proof: $A^+ = \{A,B,C\}$ and $C \in A^+$
2. A *extraneous in* $B \rightarrow CA$? *Yes:* $(B \rightarrow A) \in \{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow AB\}^+$
   Proof: $B^+ = \{B,C,A\}$ and $A \in B^+$
3. B *extraneous in* $C \rightarrow AB$? *Yes:* $(C \rightarrow B) \in \{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow A\}^+$
   Proof: $C^+ = \{C,A,B\}$ and $B \in C^+$

Canonical Cover Algorithm Example

Determine 2 different canonical covers of `F`:

$$F = \{A \rightarrow BC,\ B \rightarrow CA,\ C \rightarrow AB\}$$

A2: $F_c = \{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow A\}$

Canonical Cover Algorithm Example

Determine 2 different canonical covers of `F`:

$$F = \{A \rightarrow BC,\ B \rightarrow CA,\ C \rightarrow AB\}$$

A1: $F_c = \{A \rightarrow C,\ B \rightarrow A,\ C \rightarrow B\}$

A2: $F_c = \{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow A\}$