Department of Mathematics
Wintersemester 2011/2012

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4th Exercise Sheet in „Computer Algebra“

Deadline: Thursday, 17 November 2011, 10.00 h

Exercise 1. Let $R = \mathbb{Q}[x,y,z]/\langle x^2+y^2+z^2\rangle$, $M = R^3/\langle (x,xy,xz)\rangle$, and let $N = R^2/\langle (1,y)\rangle$. Moreover, let $\varphi = \varphi_A : M \to N$ be the R-module homomorphism given by the matrix

$$A = \begin{pmatrix} x^2+1 & y & z \\ yz & 1 & -y \end{pmatrix}.$$

(a) Compute $\mathrm{Ker}(\varphi)$.

(b) Test whether $(x^2, y^2) \in \mathrm{Im}(\varphi)$, or not.

(c) Compute $\mathrm{Im}(\varphi) \cap \{ f \in N \mid f \equiv (h,0) \mod \langle (1,y)\rangle \text{ for some } h \in R \}$.

(d) Compute $\mathrm{Ann}_R\big(\mathrm{Im}(\varphi)\big)$.

Exercise 2. Let R be a Noetherian ring and $M = \langle f_1,\dots,f_k\rangle = \langle g_1,\dots,g_s\rangle \subset R^r$. Prove that $\mathrm{syz}(f_1,\dots,f_k) \oplus R^s \cong \mathrm{syz}(g_1,\dots,g_s) \oplus R^k$.

Exercise 3. Let R be a local Noetherian ring, let M be a finitely generated R-module, and let $\{f_1,\dots,f_k\}$, $\{g_1,\dots,g_k\}$ be two minimal sets of generators. Prove that $\mathrm{syz}(f_1,\dots,f_k) \cong \mathrm{syz}(g_1,\dots,g_k)$, and conclude that the i–th syzygy module $\mathrm{syz}_i(M)$ is well–defined up to isomorphism.

Exercise 4. Change your Singular procedure `standard` computing a standard basis algorithm in such a way that

(1) the pair-set P does not contain duplicates and is sorted in ascending order with respect to $\mathrm{lcm}(\mathrm{LM}(f_1), \mathrm{LM}(f_2))$, $(f_1,f_2) \in P$, and

(2) it takes an optional parameter such that if this optional parameter is the string „minimal“, the procedure returns a minimal standard basis and if this optional parameter is missing, the procedure just returns some standard basis as before.

Hint: If you define the head of the procedure `standard` as `proc standard(ideal G, list #)`, then `#` is an optional parameter of type `list` and with `size(#)==0` you can test whether it is there or not, while with `#[1]` you can access its entry if it is there.