Department of Mathematics
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# 10-th Exercise Sheet in „Computer Algebra“

Deadline: Thursday, 12 January 2012, 10.00 h

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**Exercise 1.** Let $f_1, f_2 \in A$ such that $f = f_1 \cdot f_2 \in I$ and $\langle f_1, f_2 \rangle = A$. Prove that $I = \langle I, f_1 \rangle \cap \langle I, f_2 \rangle$.

**Exercise 2.** Let $I$ be a radical ideal. Prove that, for every $h \notin I$, the ideal quotient $I : \langle h \rangle$ is a radical ideal.

**Exercise 3** (*Shape Lemma*)**.** Let $K$ be a field of characteristic 0, and let $I \subset K[x], x = (x_1, \ldots, x_n)$, be a zero–dimensional radical ideal. Prove that for almost all changes of coordinates $I = \langle x_1 + g_1(x_n), \ldots, x_{n-1} + g_{n-1}(x_n), g_n(x_n) \rangle$ for suitable $g_1, \ldots, g_n \in K[x_n]$.
Hint: Use Chinese Remainder Theorem and remark that the case that $I$ is a prime ideal was proven in the lecture.

**Exercise 4.** Let $f \in K[x_1, \ldots, x_n]$ and the variable $x_i$ be given. Then $f$ has a unique representation of the form $f = \sum_{j=0}^{k} f_j \cdot x_i^j$ with $f_j \in K[x_1, \ldots x_{i-1}, x_{i+1}, \ldots, x_n]$. Write a Singular procedure that returns $f_k$.