TECHNISCHE UNIVERSITÄT
KAISERSLAUTERN

Department of Mathematics
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7-th Exercise Sheet in „Computer Algebra“

Deadline: Thursday, 08 December 2011, 10.00 h

Exercise 1. Let K be a field, and let $P \subset K[x_1, \ldots, x_n]$ be a prime ideal. Moreover, let $u \subset x = \{x_1, \ldots, x_n\}$ be a maximal independent set for P. Prove that $K[x]_P = K(u)[x \setminus u]_{PK(u)[x \setminus u]}$.

Exercise 2. Let K be a field, and let $P \subset K[x_1, \ldots, x_n]$ be a prime ideal. Moreover, let $u \subset x = \{x_1, \ldots, x_n\}$ be a maximal independent set for P. Prove that $K(u)[x \setminus u]/PK(u)[x \setminus u] = Q(K[x_1, \ldots, x_n]/P)$.

Exercise 3. Let A be a normal integral domain, K its quotient field, and $L \supset K$ be a field extension. Let $x \in L$ be integral over A and $f \in K[t]$ be the minimal polynomial of x. Then $f \in A[t]$.

Exercise 4. Add the Chain Criterion to your standard basis algorithm. Think of possible optimizations combining it with the Product Criterion. Moreover, implement a counter for the number of critical pairs discarded by the criteria.