TECHNISCHE UNIVERSITÄT
KAISERSLAUTERN

Department of Mathematics
Wintersemester 2011/2012

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9-th Exercise Sheet in „Computer Algebra“

Deadline: Thursday, 05 January 2012, 10.00 h

Exercise 1. Let $I \subset K[x_1, \ldots, x_n]$ be an ideal and moreover assume that $K[x_1, \ldots, x_r] \subset K[x_1, \ldots, x_n]/I$ is a Noether normalization. Prove that I is equidimensional if and only if every non–zero $f \in K[x_1, \ldots, x_r]$ is a non–zerodivisor in $K[x_1, \ldots, x_n]/I$.

Exercise 2. Use Exercise 1 to check whether $\langle x^2 + xy, xz \rangle$ is equidimensional.

Exercise 3. Let K be a field with $\mathrm{char}(K) = 0$, $\overline{K}$ its algebraic closure and $P \subset K[x_1, \ldots, x_n]$ a maximal ideal. Prove that $P\overline{K}[x_1, \ldots, x_n]$ is a radical ideal.

Exercise 4. Let $I \subset K[x_1, \ldots, x_n]$ be a zero-dimensional ideal. Write a SINGULAR procedure to compute $\sqrt{I} \subset K[x_1, \ldots, x_n]$, the radical of I.