Department of Mathematics
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Prof. Dr. Gerhard Pfister
Christian Eder

# 8-th Exercise Sheet in „Computer Algebra“

Deadline: Thursday, 15 December 2011, 10.00 h

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**Exercise 1.** Let $w = (w_1, \ldots, w_n) \in \mathbb{Z}^n$, $w_i \neq 0$ for all $i$, and let $I \subset K[x_1, \ldots, x_n]$ be an ideal. Moreover, let $I^h \subset K[x_1, \ldots, x_n, t]$ be the ideal generated by the weighted homogenizations of the elements of $I$ with respect to $t$. Prove the following statements:

(a) $I^h$ is primary (prime) if and only if $I$ is primary (prime).

(b) Let $I = Q_1 \cap \ldots \cap Q_r$ be an irredundant primary decomposition, then $I^h = Q_1^h \cap \ldots \cap Q_r^h$ is an irredundant primary decomposition, too.

Hint: For (b), prove that $(I_1 \cap I_2)^h = I_1^h \cap I_2^h$.

**Exercise 2.** Let $\operatorname{Ass}(\langle 0 \rangle) = \{P_1, \ldots, P_s\}$. Prove that $\bigcup_{i=1}^s P_i$ is the set of zerodivisors of $A$.

**Exercise 3.** Let $K$ be a field of characteristic 0, $\overline{K}$ the algebraic closure of $K$ and $I \subset K[x]$ an ideal. Prove the following statements:

(a) $I\overline{K}[x] \cap K[x] = I$.

(b) If $f \in K[x]$ is squarefree, then $f \in \overline{K}[x]$ is squarefree.

**Exercise 4.** Write a Singular procedure to compute the ecart of a given polynomial and use this to implement a normal form algorithm for non-global orderings.