TECHNISCHE UNIVERSITÄT
KAISERSLAUTERN

Department of Mathematics
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Prof. Dr. Gerhard Pfister
Christian Eder

# 3rd Exercise Sheet in „Computer Algebra“

Deadline: Thursday, 10 November 2011, 10.00 h

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**Exercise 1.** Let $I_1, I_2$ be two ideals in $K[x]_>$ with $I_2 = \langle h_1, \ldots, h_r \rangle$, $h_i \in K[x]$. Define $h := h_1 + th_2 + t^2h_3 + \cdots + t^{r-1}h_r \in K[x,t]$. Prove that

$$I_1 : I_2 = (I_1 : h) \cap K[x]_> .$$

**Exercise 2.** Let $>$ be a global monomial ordering on $\mathrm{Mon}(x_1, \ldots, x_n)$, let $I \subset K[x]$ be an ideal, and let $G$ be a standard basis of $I$ with respect to $>$. Show that the following are equivalent:

(a) $\dim_K(K[x]/I) < \infty$,

(b) For each $i = 1, \ldots, n$ there exists an $n_i \geq 0$ such that $x_i^{n_i}$ is a leading monomial of an element of $G$.

**Exercise 3.** Implement, as SINGULAR procedure, a standard basis algorithm. Don't forget to add at least one example to your procedure.