Sumário

Lista de Tabelas

Algoritmos

Complexidade	**Tamanho máximo da entrada**
1	α
$log(n)$	10^9
n	10^6
n * $log(n)$	10^4
n^2	10^3
n^2* $log(n)$	10^3
n^3	10^2
2^n	20
$n!$	10

Tabela 1: Complexidade máxima da solução em fução do tamanho da entrada

1 Tabelas

tipo	bits	min...max	precisao
char	8	0..127	2
signed char	8	-128..127	2
unsigned char	8	0..255	2
short	16	-32.768 .. 32.767	4
unsigned short	16	0 .. 65.535	4
int	32	-2x10**9 .. 2 x 10**9	9
unsigned int	32	0 .. 4x10**9	9
int64_t	64	-9 x 10**18 .. 9 x 10**18	18
uint64_t	64	0 .. 18 x 10**18	19

Tabela 2: Limites de representação de dados

0! = 1
1! = 1
2! = 2
3! = 6
4! = 24
5! = 120
6! = 720
7! = 5.040
8! = 40.320
9! = 362.880
10! = 3.628.800
11! = 39.916.800
12! = 479.001.600 [limite do (unsigned) int]
13! = 6.227.020.800
14! = 87.178.291.200
15! = 1.307.674.368.000
16! = 20.922.789.888.000
17! = 355.687.428.096.000
18! = 6.402.373.705.728.000
19! = 121.645.100.408.832.000
20! = 2.432.902.008.176.640.000 [limite do (u)int64_t]

Tabela 3: Fatorial

2 Codigos

2.1 Exemplos

Código 1: Modelo

```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <string.h>
4  #include <math.h>
5
6  #include <inttypes.h>
7  #include <ctype.h>
8  #include <limits.h>
9
10 #include <algorithm>
11 #include <utility>
12 #include <iostream>
13
14 #include <map>
15 #include <set>
16 #include <vector>
17 #include <list>
18 #include <queue>
19 #include <sstream>
```

Tipo	%
char	c
int	d
float	e, E, f, g, G
int (octal)	o
int (hexa)	x, X
uint	u
char*	s

Tabela 4: scanf() - %[*][width][modifiers]type

modifiers	tipo
h	short int (d, i, n), or unsigned short int (o, u, x)
l	long int (d, i, n), or unsigned long int (o, u, x), or double (e, f, g)
ll	long long int (d, i, n), or unsigned long long int (o, u, x)
L	long double (e, f, g)

Tabela 5: scanf() %[*][width][modifiers]type

função	descrição
atof	Convert string to double
atoi	Convert string to integer
atol	Convert string to long integer
strtod	Convert string to double
strtol	Convert string to long integer
strtoul	Convert string to unsigned long integer

Tabela 6: stdlib

função	descrição
cos	Compute cosine
sin	Compute sine
tan	Compute tangent
acos	Compute arc cosine
asin	Compute arc sine
atan	Compute arc tangent
atan2	Compute arc tangent with two parameters
cosh	Compute hyperbolic cosine
sinh	Compute hyperbolic sine
tanh	Compute hyperbolic tangent
exp	Compute exponential function
frexp	Get significand and exponent
ldexp	Generate number from significand and exponent
log	Compute natural logarithm
log10	Compute common logarithm
modf	Break into fractional and integral parts
pow	Raise to power
sqrt	Compute square root
ceil	Round up value
fabs	Compute absolute value
floor	Round down value
fmod	Compute remainder of division

Tabela 7: math (angulos em radianos)

```
20
21 using namespace std;
22
23 #define abs(a) ((a) > 0 ? (a) : -(a))
24
25 int main()
26 {
27     int n;
28
29     cin >> n;
30
31     for (int i = 0; i < n; i++)
32     {
33
34     }
35
36     while (cin >> n)
37     {
38
39     }
40     return 0;
41 }
```

Código 2: comparcao de ponto flutuante

```
1 /**
2  * -1 se x < y
3  * 0 se x = y
4  * 1 se x > y
5  */
6 const double EPS = 1e-10;
7 #define _inline(f...) f() __attribute__((always_inline)); f
8 _inline(int cmp)(double x, double y = 0, double tol = EPS)
9 {
10     return (x <= y + tol) ? (x + tol < y) ? -1 : 0 : 1;
11 }
```

Código 3: .vimrc para a configuração do vim

```
1 set ai noet ts=4 sw=4 bs=2
2 set cindent
```

Código 4: função que acelara o cin. Não deve ser usada com printf

```
1 std::cout.sync_with_stdio(false);
```

Código 5: printf

```
1 /* printf example */
2 #include <stdio.h>
3
4 int main()
5 {
6     printf ("Characters: %c %c \n", 'a', 65);
7     printf ("Decimals: %d %ld\n", 1977, 650000L);
8     printf ("Preceding with blanks: %10d \n", 1977);
9     printf ("Preceding with zeros: %010d \n", 1977);
10    printf ("Some different radixes: %d %x %o %#x %#o \n", 100, 100, 100,
          100, 100);
11    printf ("floats: %4.2f %+.0e %E %4.2f\n", 3.1416, 3.1416, 3.1416,
          3.1);
12    printf ("Width trick: %*d \n", 5, 10);
13    printf ("%s \n", "A string");
14    return 0;
15 }
16 /* %[flags (-, +, etc)][width][.precision][length (h,l,L)]specifier
17 Characters: a A
18 Decimals: 1977 650000
19 Preceding with blanks:       1977
20 Preceding with zeros: 0000001977
21 Some different radixes: 100 64 144 0x64 0144
22 floats: 3.14 +3e+000 3.141600E+000 3.10
23 Width trick:    10
24 A string
25 */
```

Código 6: exemplo de map

```
1 #include <iostream>
2 #include <map>
3 using namespace std;
4
5 int main ()
6 {
7   map<char,int> mymap;
8   map<char,int>::iterator it;
9   pair<map<char,int>::iterator,bool> ret;
10
11  // first insert function version (single parameter):
12  mymap.insert ( pair<char,int>('a',100) );
13  mymap.insert ( pair<char,int>('z',200) );
14
15  ret=mymap.insert (pair<char,int>('z',500) );
16  if (ret.second==false)
17  {
18    cout << "element 'z' already existed";
19    cout << " with a value of " << ret.first->second << endl;
20  }
21
22  // third insert function version (range insertion):
23  map<char,int> anothermap;
24  anothermap.insert(mymap.begin(),mymap.find('c'));
25
26  // showing contents:
27  cout << "mymap contains:\n";
28  for ( it=mymap.begin() ; it != mymap.end(); it++ )
29    cout << (*it).first << " => " << (*it).second << endl;
30
31  map<char,string> mymap;
```

```
  mymap['a']="an element";
  if (mymap.count('a') > 0)
      cout << mymap['a'] << " is an element of mymap.\n";

  while (!mymap.empty())
  {
     cout << mymap.begin()->first << " => ";
     cout << mymap.begin()->second << endl;
     map<char, int>::iterator erasedelement = mymap.erase(mymap.begin())
         ;
  }

  return 0;
}
```

Código 7: exemplo de set e multset

```
#include <iostream>
#include <set>
using namespace std;

int main ()
{
  multiset<int> mymultiset;
  multiset<int>::iterator it;

  // set some initial values:
  for (int i=1; i<=5; i++) mymultiset.insert(i*10);   // 10 20 30 40 50

  cout << "size: " << (int) mymultiset.size() << endl;
  cout << "count: " << (int) mymultiset.count(10) << endl;

  it=mymultiset.find(20);
  mymultiset.erase (it);

  if (! mymultiset.empty)
   mymultiset.erase (mymultiset.find(40));

  for (it=mymultiset.begin(); it!=mymultiset.end(); it++)
    cout << " " << *it;

  int myints[]={19,72,4,36,20,20};
  multiset<int> first (myints,myints+3);      // 4,19,72
  multiset<int> second (myints+3,myints+6);  // 20,20,36

  first.swap(second); // troca conteudo. o primeiro fica [20,20,36] e o
      segundo [4,19,72]

  return 0;
}
```

Código 8: exemplo de list

```
#include <iostream>
#include <list>
using namespace std;

int main ()
{
  list<int> mylist (2,100);          // two ints with a value of 100
  mylist.push_front (200);
  mylist.push_back (300);

  it = mylist.begin();
  mylist.insert (it,10);
  mylist.insert (it,2,20); // two ints with a value of 20

  mylist.reverse(); // Reverses the order of the elements in the list.

  cout << "mylist contains:";
  for (list<int>::iterator it=mylist.begin(); it!=mylist.end(); ++it)
    cout << " " << *it;

  cout << "Popping out the elements in mylist:";
  while (!mylist.empty())
  {
    cout << " " << mylist.front();
    mylist.pop_front();
  }

  while (!mylist.empty())
  {
    cout << " " << mylist.back();
    mylist.pop_back();
  }

  cout << mylist.size() << endl;

  return 0;
}
```

Código 9: exemplo de queue

```
#include <iostream>
#include <queue>
using namespace std;

int main ()
{
  queue<int> myqueue;
  int sum (0);

  for (int i=1;i<=10;i++) myqueue.push(i);

  myqueue.back() -= myqueue.front();

  cout << "size: " << (int) myqueue.size() << endl;

  while (!myqueue.empty())
  {
     sum += myqueue.front();
```

```
19        myqueue.pop();
20    }
21
22    cout << "total: " << sum << endl;
23
24    return 0;
25 }
```

Código 10: exemplo de priority queue

```
1  #include <iostream>
2  #include <queue>
3  using namespace std;
4
5  int main ()
6  {
7    priority_queue<int> mypq;
8
9    mypq.push(30);
10   mypq.push(100);
11   mypq.push(25);
12   mypq.push(40);
13
14   cout << "size: " << (int) mypq.size() << endl;
15
16   cout << "Popping out elements...";
17   while (!mypq.empty())
18   {
19      cout << " " << mypq.top();
20      mypq.pop();
21   }
22   cout << endl;
23
24   return 0;
25 }
```

Código 11: exemplo de stack

```
1  #include <iostream>
2  #include <stack>
3  using namespace std;
4
5  int main ()
6  {
7    stack<int> mystack;
8    int sum = 0;
9
10   mystack.push(10);
11   mystack.push(20);
12
13   mystack.top() -= 5;
14
15   while (!mystack.empty())
16   {
17      sum += mystack.top();
18      mystack.pop();
19   }
20
21   cout << "size: " << (int) mystack.size() << endl;
22
23   return 0;
24 }
```

Código 12: exemplo de vector

```
1  #include <iostream>
2  #include <vector>
3  using namespace std;
4
5  int main ()
6  {
7    vector<int> myvector (3,100); // (100 100 100)
8    vector<int>::iterator it;
9
10   myvector.reserve(100);
11
12   for (int i=0; i<myvector.size(); i++)
13     myvector.at(i)=i; // = myvector[i] = i
14
15   it = myvector.begin();
16   it = myvector.insert ( it , 200 );
17   myvector.insert (it,2,300);
18
19   vector<int> anothervector (2,400);
20   int myarray [] = { 501,502,503 };
21   vector<int> initializer(myarray/*PointerInicio*/, myarray+sizeof(
         myarray)/sizeof(int)/*PointerFim*/);
22   myvector.insert (it+2,anothervector.begin(),anothervector.end());
23   myvector.insert (myvector.begin(), myarray, myarray+3);
24
25   cout << "myvector contains:";
26   for (it=myvector.begin(); it<myvector.end(); it++)
27     cout << " " << *it;
28   cout << endl;
29
30   // erase the 6th element
31   myvector.erase (myvector.begin()+5);
32   int sum;
33   while (!myvector.empty())
34   {
35      sum += myvector.back();
36      myvector.pop_back();
37   }
38
39   return 0;
40 }
```

Código 13: exemplo de string

```
1  #include <iostream>
```

```cpp
#include <string>
using namespace std;

int main ()
{
  string str ("There are two needles in this haystack with needles.");
  string str2 ("needle");
  size_t found;

  // different member versions of find in the same order as above:
  found=str.find(str2);
  if (found!=string::npos)
    cout << "first 'needle' found at: " << int(found) << endl;

  found=str.find("needles are small",found+1,6);
  if (found!=string::npos)
    cout << "second 'needle' found at: " << int(found) << endl;

  found=str.find("haystack");
  if (found!=string::npos)
    cout << "'haystack' also found at: " << int(found) << endl;

  found=str.find('.');
  if (found!=string::npos)
    cout << "Period found at: " << int(found) << endl;

  // let's replace the first needle:
  str.replace(str.find(str2),str2.length(),"preposition");
  cout << str << endl;

  string str="We think in generalities, but we live in details.";
                             // quoting Alfred N. Whitehead
  string str2, str3;
  size_t pos;

  str2 = str.substr (12,12); // "generalities"

  pos = str.find("live");     // position of "live" in str
  str3 = str.substr (pos);   // get from "live" to the end

  cout << str2 << ' ' << str3 << endl;


  return 0;
}
/*
first 'needle' found at: 14
second 'needle' found at: 44
'haystack' also found at: 30
Period found at: 51
There are two prepositions in this haystack with needles.
generalities live in details.
*/
```

Código 14: exemplo de stringstream

```cpp
#include <string>
#include <sstream>
#include <iostream>


using namespace std;


int main() {
    string linha="Olah mundo";
    stringstream separador(linha); // Tokenizador
    string word;
    separador >> word; // word=Olah
    char mu[4];
    separador.readsome(mu, 3); // Le " mu" (inclui espaco)
    cout << separador.tellg() << endl; // 7 (posicao de leitura)
    cout << separador.tellp() << endl; // 0, posicao de escrita

    separador.seekp(separador.str().size());
    separador << " cruel"; // separador = "Olah mundo cruel"

    separador.seekp(5);
    separador << "doido"; // separador = "Olah doido cruel" (sobrescrito)
    return 0;
}
```

Código 15: exemplo de ordenação

```cpp
#include <iostream>
#include <algorithm>
#include <vector>
using namespace std;

bool myfunction (int i,int j) { return (i<j); }

struct myclass {
  bool operator() (int i,int j) { return (i<j);}
} myobject;

int compare (const void * a, const void * b)
{
  return ( *(int*)a - *(int*)b );
}


int main () {
  int myints[] = {32,71,12,45,26,80,53,33};
  vector<int> myvector (myints, myints+8);               // 32 71 12 45
      26 80 53 33

  // using default comparison (operator <):
  sort (myvector.begin(), myvector.begin()+4);           //(12 32 45 71)
      26 80 53 33
  // using function as comp
  sort (myvector.begin()+4, myvector.end(), myfunction); // 12 32 45
      71(26 33 53 80)
```

```
  // using object as comp
  sort (myvector.begin(), myvector.end(), myobject);      //(12 26 32 33
      45 53 71 80)

  // if stable is need
  stable_sort (myvector.begin(), myvector.end(), myfunction);

  // Rearranges the elements in the range [first,last), in such a way
      that the subrange [first,middle)
  // contains the smallest elements of the entire range sorted in
      ascending order, and the subrange
  // [middle,end) contains the remaining elements without any specific
      order.
  partial_sort (myvector.begin(), myvector.begin()+3, myvector.end());

  qsort (myints, 8, sizeof(int), compare);

  return 0;
}
```

Código 16: pesquisa binária

```
int compareMyType (const void * a, const void * b)
{
  if ( *(MyType*)a >  *(MyType*)b ) return 1;
  if ( *(MyType*)a == *(MyType*)b ) return 0;
  if ( *(MyType*)a <  *(MyType*)b ) return -1;
}

int key = 40;
item = (int*) bsearch (&key, values, n, sizeof (int), compareMyType);
```

Código 17: Arredondamento e output em outras bases

```
#include <iostream>
#include <iomanip> // setprecision()
using namespace std;

int main () {
  double a = 3.1415926534;
  double b = 2006.0;
  double c = 1.0e-10;

  // setprecision(1) => 1 casa decimal apos a virgula
  cout << fixed << setprecision(1) << 9.09090901 << endl;
  cout << fixed << setprecision(2) << 9.09090901 << endl;
  cout << fixed << setprecision(3) << 9.09090901 << endl;
  cout << fixed << setprecision(2) << 9.1 << endl;

  // anula o efeito de setprecision
  cout.unsetf(ios::floatfield);

  // 5 digitos no maximo
  cout.precision(5);

  cout << a << '\t' << b << '\t' << c << endl;
  cout << fixed << a << '\t' << b << '\t' << c << endl;
  cout << scientific << a << '\t' << b << '\t' << c << endl;

  // Sets the basefield format flag for the str stream to dec, hex or
      oct.
  int n =70;
  cout << dec << n << endl;
  cout << hex << n << endl;
  cout << oct << n << endl;

  return 0;
}
/* output
9.1
9.09
9.091
9.10
3.1416   2006     1e-10
3.14159  2006.00000       0.00000
3.14159e+00      2.00600e+03      1.00000e-10
70
46
106
*/
```

2.2 Teoria dos números

Código 18: máximo divisor comum e mínimo multiplo comum

```
int gcd(int x, int y)
{
   return y ? gcd(y, x % y) : abs(x);
}
uint64_t lcm(int x, int y)
{
   if (x && y) return abs(x) / gcd(x, y) * uint64_t(abs(y));
   else return uint64_t(abs(x | y));
}
```

Código 19: decide se um número é primo

```
bool isPrime(int n)
{
   if (n < 0) return isPrime(-n);
   if (n == 1) return true;
   if (n < 5 || n % 2 == 0 || n % 3 == 0) return (n == 2 || n == 3);

   int maxP = sqrt(n) + 2;
   for (int p = 5; p < maxP; p += 6)
   {
      if (n % p == 0 || n % (p+2) == 0) return false;
   }
```

```
    return true;
}
```

Código 20: Retorna a fatoração em números primos de abs(n).

```
typedef map<int, int> prime_map;
void squeeze(prime_map& M, int& n, int p)
{
    for (; n % p == 0; n /= p) M[p]++;
}
void factor(int n, prime_map& M)
{
    if (n < 0) { factor(-n, M); return; }
    if (n < 2) return;

    squeeze(M, n, 2);
    squeeze(M, n, 3);

    int maxP = sqrt(n) + 2;
    for (int p = 5; p < maxP; p += 6)
    {
        squeeze(M, n, p);
        squeeze(M, n, p+2);
    }
    if (n > 1) M[n]++;
}
```

Código 21: Calcula Valor de $a^b mod$ n de forma rápida.

```
int mpow(int a, int b, int n = 10)
{
  if(b == 0)
    return 1;
  else {
    long long res = mpow(a, b/2, n);
    res = (res*res) % n;
    if(b%2 == 1)
      res = (res*a) % n;
    return (int) res;
  }
}
```

Código 22: Calcula (a*b)%c de forma rápida.

```
long long mulmod(long long a, long long b, long long c)
{
    long long x = 0;
    long long y = a % c;
    while(b > 0)
    {
        if(b & 1ll) x = (x + y) % c;
        y = (y << 1) % c;
        b >>= 1;
    }
    return x % c;
}
```

Código 23: Computa x tal que a*x = b (mod c). Quando a equação não tem solução, retorna algum valor arbitrário errado, mas basta conferir o resultado.

```
long long axbmodc(long long a, long long b, long long c)
{
    return a ? (axbmodc(c % a, (a - b % a) % a, a) * c + b) / a : 0;
}
```

Código 24: **Baby-step Giant-step algorithm** Calcula o menor valor de e para $b^e = n$ mod p. Retorna -1 se eh impossivel

```
#define inv_mult( a, n ) axbmodc(a, 1, n)

long long discreteLlogarithm( long long b, long long n, long long p )
{
    if ( n == 1 ) return 0;

    map < long long, int > table;
    long long m = sqrt(p) + 1, pot = 1, pot2 = 1;

    for (int j = 0; j < m; j++)
    {
        if ( pot == n ) return j;
        table[( n * inv_mult( pot, p ) ) % p] = j;
        pot = ( pot * b ) % p;
    }

    for (int i = 0; i < m; i++)
    {
        if ( table.find( pot2 ) != table.end() ) return i * m + table[pot2
            ];
        pot2 = ( pot * pot2 ) % p;
    }

    return -1;
}
```

2.3 Estruturas de dados

Código 25: Números de precisão harbitrária.

```
const int DIG = 4;
const int BASE = 10000; // BASE**3 < 2**51
const int TAM = 1000;

struct BigInt
{
    int num[TAM], numDigits;
    BigInt(int x = 0): numDigits(1)
```

```
9     {
10       memset(num, 0, sizeof(num));
11       num[numDigits++] = x; fixInvariant();
12    }
13    BigInt(char *s): numDigits(1)
14    {
15       memset(num, 0, sizeof(num));
16       int sign = 1;
17
18       while (*s && !isdigit(*s))
19       {
20          if (*s++ == '-') sign *= -1;
21       }
22
23       char *t = strdup(s), *p = t + strlen(t);
24
25       while (p > t)
26       {
27          *p = 0; p = max(t, p - DIG);
28          sscanf(p, "%d", &num[numDigits]);
29          num[numDigits++] *= sign;
30       }
31
32       free(t);
33       fixInvariant();
34    }
35
36    BigInt& fixInvariant(int m = 0)
37    {
38       numDigits = max(m, numDigits);
39       int sign = 0;
40
41       for (int i = 1, carry = 0; i <= numDigits || carry && (numDigits =
              i); i++)
42       {
43          num[i] += carry;
44          carry = num[i] / BASE;
45          num[i] %= BASE;
46          if (num[i]) sign = (num[i] > 0) ? 1 : -1;
47       }
48
49       for (int i = 1; i < numDigits; i++)
50       {
51          if (num[i] * sign < 0)
52          {
53             num[i] += sign * BASE;
54             num[i+1] -= sign;
55          }
56       }
57
58       while (numDigits && !num[numDigits]) numDigits--;
59       return *this;
60    }
61
62    //Comparacao
63    int cmp(const BigInt& x = 0) const
64    {
65       int i = max(numDigits, x.numDigits), t = 0;
66       while (1)
67       {
68          if ((t = ::cmp(num[i], x.num[i])) || i-- == 0) return t;
69       }
70    }
71
72    bool operator <(const BigInt& x) const { return cmp(x) < 0; }
73    bool operator >(const BigInt& x) const { return cmp(x) > 0; }
74    bool operator <=(const BigInt& x) const { return cmp(x) <= 0; }
75    bool operator >=(const BigInt& x) const { return cmp(x) >= 0; }
76    bool operator ==(const BigInt& x) const { return cmp(x) == 0; }
77    bool operator !=(const BigInt& x) const { return cmp(x) != 0; }
78
79    //operacoes fundamentais
80    BigInt& operator +=(const BigInt& x)
81    {
82       for (int i = 1; i <= x.numDigits; i++) num[i] += x.num[i];
83       return fixInvariant(x.numDigits);
84    }
85    BigInt& operator -=(const BigInt& x)
86    {
87       for (int i = 1; i <= x.numDigits; i++) num[i] -= x.num[i];
88       return fixInvariant(x.numDigits);
89    }
90
91    void multiAndAcumWithShift(const BigInt& x, int m, int b)
92    { // *this += (x * m) << b;
93       for (int i = 1, carry = 0; (i <= x.numDigits || carry) && (
              numDigits = i + b); i++)
94       {
95          num[i+b] += x.num[i] * m + carry;
96          carry = num[i+b] / BASE;
97          num[i+b] %= BASE;
98       }
99    }
100
101   BigInt operator *(const BigInt& x) const
102   {
103      BigInt r;
104      for (int i = 1; i <= numDigits; i++) r.multiAndAcumWithShift(x,
             num[i], i-1);
105      return r;
106   }
107
108   BigInt div(const BigInt& x)
109   {
110      if (x == 0) return 0;
111
112      BigInt q;
113      q.numDigits = max(numDigits - x.numDigits + 1, 0);
114      int d = x.num[x.numDigits] * BASE + x.num[x.numDigits-1];
115
116      for (int i = q.numDigits; i > 0; i--)
117      {
```

```
            int j = x.numDigits + i - 1;
            q.num[i] = int((num[j] * double(BASE) + num[j-1]) / d);
            multiAndAcumWithShift(x, -q.num[i], i-1);
            if (i == 1 || j == 1) break;
            num[j-1] += BASE * num[j];
            num[j] = 0;
        }

        fixInvariant(x.numDigits);
        return q.fixInvariant();
    }

    BigInt& operator *=(const BigInt& x) { return *this = (*this) * x; }
    BigInt operator +(const BigInt& x) { return BigInt(*this) += x; }
    BigInt operator -(const BigInt& x) { return BigInt(*this) -= x; }
    BigInt operator -() { BigInt r = 0; return r -= *this; }
    BigInt& operator /=(const BigInt& x) { return *this = div(x); }
    BigInt& operator %=(const BigInt& x) { div(x); return *this; }
    BigInt operator /(const BigInt& x) { return BigInt(*this).div(x); }
    BigInt operator %(const BigInt& x) { return BigInt(*this) %= x; }

    // I/O
    operator string() const
    {
        ostringstream s; s << num[numDigits];
        for (int i = numDigits - 1; i > 0; i--)
        {
            s.width(DIG);
            s.fill('0');
            s << abs(num[i]);
        }

        return s.str();
    }

    friend ostream& operator <<(ostream& o, const BigInt& x)
    {
        return o << (string) x;
    }

    friend istream& operator >>(istream& in, BigInt& x)
    {
        string num;
        in >> num;
        x = BigInt((char*) num.c_str());
        return in;
    }

    // potencia e raiz
    BigInt pow(int x)
    {
        if (x < 0) return (*this == 1 || *this == -1) ? pow(-x) : 0;
        BigInt r = 1;
        for (int i = 0; i < x; i++) r *= *this;
        return r;
    }

    BigInt root(int x)
    {
        if (cmp() == 0 || cmp() < 0 && x % 2 == 0) return 0;
        if (*this == 1 || x == 1) return *this;
        if (cmp() < 0) return -(-*this).root(x);
        BigInt a = 1, d = *this;
        while (d != 1)
        {
            BigInt b = a + (d /= 2);
            if (cmp(b.pow(x)) >= 0) { d += 1; a = b; }
        }

        return a;
    }
};
```

2.4 Programação Dinâmica

Código 26: **Sub Set Sum**: Verifica se há um sobconjunto dos elementos do vetor cuja soma seja igual a soma pedida.

```
//soma maxima dos elementos do vetor
#define MAX_SUM 10000
int n;
int vet[TAM];
bool m[MAX_SUM];

//M->soma maxima dos elementos do vetor c->soma procurada
bool subSetSum(int M, int c)
{
    for (int i = 0; i <= M; i++) m[i] = false;
    m[0] = true;

    for(int i = 0; i < n; i++)
    {
        for(int j = M; j >= vet[i]; j--)
        {
            m[j] |= m[j - vet[i]];
        }
    }

    return m[c];
}
```

Código 27: **Lis: longest increasing (decreasing) subsequence** O(n^2)

```
#define TAM 10000
int c[TAM];
int A[TAM];
int H[TAM];

void ssctf(int n)
```

```
{
    for (int m = 1; m <= n; m++)
    {
        c[m] = H[m];
        for (int i = m -1; i > 0; i--)
        {
            if (A[i] < A[m] && c[i] + H[m] > c[m])
            {
                c[m] = c[i] + H[m];
            }
        }
    }
}

void ssdtf(int n)
{
    for (int m = 1; m <= n; m++)
    {
        c[m] = H[m];
        for (int i = m -1; i > 0; i--)
        {
            if (A[i] > A[m] && c[i] + H[m] > c[m])
            {
                c[m] = c[i] + H[m];
            }
        }
    }
}

int lis1d(int n, bool inc = true)
{
    if (inc) ssctf(n);
    else ssdtf(n);

    int max = 0;

    for (int i = 1; i <= n; i++)
        if (max < c[i])
            max = c[i];

    return max;
}
```

Código 28: **Lis: longest increasing subsequence** O(n*logn)

```
// Longest Increasing Subsequence - LIS O(n log n)
#define fori(i, n) for ( int i = 0; i < (n); ++i )
void lis( const vector< int > & v, vector< int > & asw )
{
    vector<int> pd(v.size(),0), pd_index(v.size()), pred(v.size());
    int maxi = 0, x, j, ind;

    fori(i,v.size())
    {
        x = v[i];
        j = lower_bound( pd.begin(), pd.begin() + maxi, x ) - pd.begin();
        //j = upper_bound( pd.begin(), pd.begin() + maxi, x ) - pd.begin()
            ; para lds
        pd[j] = x;
        pd_index[j] = i;
        if( j == maxi ) { maxi++; ind = i; }
        pred[i] = j ? pd_index[j-1] : -1;
    }
    // return maxi; se a sequencia nao precisa ser refeita

    int pos = maxi-1, k = v[ind];
    asw.resize( maxi );

    while ( pos >= 0 )
    {
        asw[pos--] = k;
        ind = pred[ind];
        k = v[ind];
    }
}
```

Código 29: Problema da Mochila O(n*W)

```
#include <stdio.h>

#define MAXWEIGHT 100

int n = 3; /* The number of objects */
int c[10] = {8, 6, 4}; /* c[i] is the *COST* of the ith object; i.e.
    what
                    YOU PAY to take the object */
int v[10] = {16, 10, 7}; /* v[i] is the *VALUE* of the ith object; i.e.
                     what YOU GET for taking the object */
int W = 10; /* The maximum weight you can take */

void fill_sack() {
    int a[MAXWEIGHT]; /* a[i] holds the maximum value that can be
        obtained
                    using at most i weight */
    int last_added[MAXWEIGHT]; /* I use this to calculate which object
        were
                            added */
    int i, j;
    int aux;

    for (i = 0; i <= W; ++i) {
        a[i] = 0;
        last_added[i] = -1;
    }

    a[0] = 0;
    for (i = 1; i <= W; ++i)
        for (j = 0; j < n; ++j)
            if ((c[j] <= i) && (a[i] < a[i - c[j]] + v[j])) {
                a[i] = a[i - c[j]] + v[j];
                last_added[i] = j;
            }
```

```
32
33      for (i = 0; i <= W; ++i)
34          if (last_added[i] != -1)
35              printf("Weight %d; Benefit: %d; To reach this weight I added
                    object %d (%d$ %dKg) to weight %d.\n", i, a[i], last_added[i
                    ] + 1, v[last_added[i]], c[last_added[i]], i - c[last_added[
                    i]]);
36          else
37              printf("Weight %d; Benefit: 0; Can't reach this exact weight.\n
                    ", i);
38
39      printf("---\n");
40
41      aux = W;
42      while ((aux > 0) && (last_added[aux] != -1)) {
43          printf("Added object %d (%d$ %dKg). Space left: %d\n", last_added[
                aux] + 1, v[last_added[aux]], c[last_added[aux]], aux - c[
                last_added[aux]]);
44          aux -= c[last_added[aux]];
45      }
46
47      printf("Total value added: %d$\n", a[W]);
48  }
49
50  int main(int argc, char *argv[]) {
51      fill_sack();
52
53      return 0;
54  }
```

2.5 Grafos

Código 30: Verifica se o grafo é aciclico.

```
1  #define TAM 100
2  #define BRANCO 0
3  #define CINZA 1
4  #define PRETO 2
5  bool grafo[TAM][TAM];
6  int pass[TAM];
7
8  bool dfs(int v)
9  {
10     pass[v] = CINZA;
11
12     for (int i = 0; i < TAM; i++)
13     {
14         if (grafo[v][i])
15         {
16             if (pass[i] == CINZA) return false;
17             if (pass[i] == BRANCO && !dfs(i)) return false;
18         }
19     }
20     pass[v] = PRETO;
21     return true;
22 }
23
24 bool aciclico()
25 {
26     memset(pass, BRANCO, TAM*sizeof(int));
27
28     for (int i = 0; i < TAM; i++)
29     {
30         if (pass[i] == BRANCO)
31         {
32             if (!dfs(i)) return false;
33         }
34     }
35
36     return true;
37 }
38
```

Código 31: **Dijkstra** Caminho minimo 1 para todos pesos positivos.

```
1  #include <queue>
2
3  typedef vector<map<int, int> > AdjList;
4  typedef AdjList Grafo;
5
6  int dist[MAX_VERTICES];
7  int prev[MAX_VERTICES]; // para recuperar o caminho usando um dijoint
       forest set
8
9  void dijkstra(Grafo& grafo, int source)
10 {
11     for (int i = 0; i < grafo.size(); i++)
12     {
13         dist[i] = INF;
14         prev[i] = -1;
15     }
16
17     dist[source] = 0;
18     priority_queue<pair<int, int> > heap;
19     heap.push(make_pair(0, source));
20
21     while (!heap.empty())
22     {
23         int u = heap.top().second;
24         heap.pop();
25
26         // para cada vizinho de u
27         for (map<int,int>::iterator i = grafo[u].begin(); i != grafo[u].
               end(); i++)
28         {
29             int totalDist = dist[u] + (*i).second;
30             if (totalDist <= dist[(*i).first])
31             {
32                 dist[(*i).first] = totalDist;
33                 heap.push(make_pair(totalDist, (*i).first));
```

```
34              prev[(*i).first] = u;
35          }
36      }
37  }
38 }
```

Código 32: Floresta dijunta de arvores

```
1 #define SIZE 100
2
3 struct dsf
4 {
5     int element_count;
6     int parent[SIZE];
7     int rank[SIZE];
8 };
9 typedef struct dsf * disjoint_set_forest_p;
10 typedef struct dsf disjoint_set_forest;
11
12 void dsf_init(disjoint_set_forest_p forest, int element_count)
13 {
14    forest->element_count = element_count;
15    memset(forest->parent, 0, element_count*sizeof(int));
16    memset(forest->rank, 0, element_count*sizeof(int));
17
18    for (int i = 0; i < element_count; ++i)
19       forest->parent[i] = i;
20 }
21
22 int dsf_find_set(disjoint_set_forest_p forest, int i)
23 {
24     if (i != forest->parent[i])
25     {
26         forest->parent[i] = dsf_find_set(forest, forest->parent[i]);
27     }
28     return forest->parent[i];
29 }
30
31 void dsf_union(disjoint_set_forest_p forest, int i, int j)
32 {
33     int x = dsf_find_set(forest, i);
34     int y = dsf_find_set(forest, j);
35
36     if (forest->rank[x] > forest->rank[y])
37     {
38         forest->parent[y] = x;
39     }
40     else
41     {
42         forest->parent[x] = y;
43         if (forest->rank[x] == forest->rank[y])
44         {
45             forest->rank[y]++;
46         }
47     }
48 }
```

Código 33: **Kruskal** Arvore geradora mínima kruskal

```
1 typedef vector<map<int, int> > AdjList;
2 struct Grafo
3 {
4    int edgeCnt;
5    AdjList adj;
6 };
7
8 struct edge
9 {
10    int u;
11    int v;
12    int weight;
13 };
14
15 int edge_compare(const void * e1, const void * e2)
16 {
17    struct edge * p1 = (struct edge *) e1;
18    struct edge * p2 = (struct edge *) e2;
19    int f = p1->weight - p2->weight;
20    if (f < 0)
21    {
22       return -1;
23    }
24    else  if (f == 0)
25    {
26        return edge_compare1(e1, e2);
27    }
28    else
29    {
30        return 1;
31    }
32 }
33
34 struct edge * get_edge_list(Grafo& graph)
35 {
36    int edge_count = graph.edgeCnt;
37    struct edge *edges = (struct edge*) malloc(edge_count * sizeof(
          struct edge));
38
39    int current_edge = 0;
40
41    for (int i = 0; i < graph.adj.size(); ++i)
42    {
43        for (map<int, int>::iterator j = graph.adj[i].begin(); j !=
              graph.adj[i].end(); j++)
44        {
45            struct edge e;
46            e.u = i < (*j).first ? i : (*j).first;
47            e.v = i > (*j).first ? i : (*j).first;
48            e.weight = (*j).second;
49            edges[current_edge++] = e;
50        }
51    }
```

```
52
53      return edges;
54  }
55
56  void kruskal(Grafo& graph, Grafo& mst)
57  {
58      // Obtain a list of edges and sort it by weight in O(E lg E) time
59      int edge_count = graph.edgeCnt;
60      struct edge *edges = get_edge_list(graph);
61      qsort(edges, edge_count, sizeof(struct edge), edge_compare);
62
63      disjoint_set_forest dsf;
64     dsf_init(&dsf, edge_count);
65
66      for (int i = 0; i < edge_count; ++i)
67      {
68          struct edge e = edges[i];
69          int uset = dsf_find_set(&dsf, e.u);
70          int vset = dsf_find_set(&dsf, e.v);
71          if (uset != vset)
72          {
73              mst.adj[e.u][e.v] = e.weight;
74           mst.edgeCnt++;
75              dsf_union(&dsf, uset, vset);
76          }
77      }
78
79      free(edges);
80  }
```

Código 34: verifica se um grafo é bipartido

```
1  #define TAM 200
2
3  bool grafo[TAM][TAM];
4  int pass[TAM];
5  int n;
6
7  bool bipartido(int v, int color = 1)
8  {
9     pass[v] = color;
10    int thisColor = color;
11    bool ret = true;
12
13    color = color == 1 ? 2 : 1;
14
15    for (int i = 0; i < n; i++)
16    {
17       if (grafo[v][i])
18       {
19          if (!pass[i]) ret = bipartido(i, color);
20          else if (pass[i] == thisColor) return false;
21
22          if (!ret) return false;
23       }
24    }
25
26    return ret;
27 }
```

Código 35: faz a ordenação topológica de um grafo acíclico

```
1  #define UNVISITED -1
2
3  int grafo[SIZE][SIZE];
4  int prof[SIZE];
5  int sorted[SIZE];
6  int nordem;
7
8  void dfsTopsort(int no)
9  {
10    for (int viz = 0; viz < SIZE; viz++)
11    {
12       if (grafo[no][viz])
13       {
14          if (prof[viz] == UNVISITED)
15          {
16             prof[viz] = prof[no] + 1;
17             dfsTopsort(viz);
18          }
19       }
20    }
21
22    sorted[nordem--] = no;
23 }
24
25 void topSort(int nvt)
26 {
27    memset(prof, UNVISITED, nvt*sizeof(int));
28    nordem = nvt - 1;
29
30    for (int i = 0; i < nvt; i++)
31    {
32       if (prof[i] == UNVISITED)
33       {
34          prof[i] = 0;
35          dfsTopsort(i);
36       }
37    }
38 }
```

Código 36: outro metodo para fazer a ordenação topológica de um grafo acíclico

```
1  bool topologicalSort(vector< vector< int > > &g, vector< int > &r)
2  {
3     vector< int > deg(g.size());
4     FOREACH(node, g){
5        FOREACH(ngb, *node){
6           deg[*ngb]++;
7        }
8     }
```

```
    priority_queue< int, vector< int >, greater< int > > q;
    FORN(i, 0, g.size())
        if(0 == deg[i]) q.push(i);

    while(not q.empty()){
        int node = q.top();
        q.pop();
        r.push_back(node);
        FOREACH(ngb, g[node]){
            if(--deg[*ngb] == 0) q.push(*ngb);
        }
    }

    return r.size() == g.size();
}
```

Código 37: calcula fluxo máximo, **Ford-Fulkerson**

```
#define TAM 1000
#define MAX_INT 1000000

int grafo[TAM][TAM];
int pred[TAM];
int f[TAM][TAM];
bool visitados[TAM];
int fila[TAM];

bool bfs(int n, int ini, int fim)
{
    int no, s = 0, e = 0;
    fila[e++] = ini;

    while (s != e)
    {
        no = fila[s++];

        if (visitados[no] == 2) continue;
        visitados[no] = 2;

        for (int i = 0; i < n; i++)
        {
            if (visitados[i] < 2)
            {
                if(grafo[no][i] - f[no][i] > 0)
                {
                    pred[i] = no;
                    if (i == fim) return true;
                    if(visitados[i] == 0)
                    {
                        fila[e++] = i;
                        visitados[i] = 1;
                    }
                }
            }
        }
    }

    return false;
}

bool dfs(int s, int t, int size)
{
    visitados[s] = true;
    if(s == t) return true;

    for(int v = 0; v < size; v++)
    {
        if(!visitados[v] && grafo[s][v] - f[s][v] > 0)
        {
            pred[v] = s;
            if(dfs(v, t, size)) return true;
        }
    }

    return false;
}

bool findPath(int s, int t, int size)
{
    memset(visitados, false, sizeof(bool)*size);
    pred[s] = s;
    // Aqui pode ser usado tanto busca em largura quanto em profundidade.
    // busca em largura geralmente apresenta tempos de execucao bem
        menores.
    return bfs(size, s, t);
    //return dfs(s, t, size);
}

int maxFlow(int size, int s, int t)
{
    int delta;

    for(int i = 0; i < size; i++)
    {
        memset(f[i], 0, sizeof(int)*size);
    }

    while(1)
    {
        bool path = findPath(s, t, size);
        if (!path) break;

        delta = MAX_INT;
        for(int c = t; pred[c] != c; c = pred[c])
        {
            delta = min(delta, grafo[pred[c]][c] - f[pred[c]][c]);
        }

        for(int c = t; pred[c] != c; c = pred[c])
        {
            f[pred[c]][c] += delta;
```

```
            f[c][pred[c]] -= delta;
        }
    }

    int soma = 0;

    for(int i = 0; i < size; i++)
    {
        soma += f[i][t];
    }

    return soma;
}
```

Código 38: calcula fluxo máximo, algoritmo mais eficiente porém muito maior em tempo de codificação

```
const int VT = 100;
const int AR = VT * VT;

struct grafo
{
    // lista de adjacencias representada na forma de vetor
    int nvt, nar;
    int dest[2 * AR];
    int adj[VT][2 * VT];
    int nadj[VT];

    int cap[AR]; // capacidade do arco
    int fluxo[AR];
    int ent[VT];

    int padj[VT], lim[VT], nivel[VT], qtd[VT];

    int inv(int a) { return a ^ 0x1; }
    int orig(int a) { return dest[inv(a)]; }
    int capres(int a) { return cap[a] - fluxo[a]; }

    void inic(int n = 0)
    {
        nvt = n;
        nar = 0;
        memset(nadj, 0, sizeof(nadj));
    }

    ////////////////////////////////////////////////////
    // Adiciona uma aresta ao grafo.
    //
    // "int u" apenas para Fluxos;
    //
    int aresta(int i, int j, int u = 0)
    {
        int ar = nar;
        cap[nar] = u;
        dest[nar] = j;
        adj[i][nadj[i]] = nar++;
        nadj[i]++;

        cap[nar] = 0;
        dest[nar] = i;
        adj[j][nadj[j]] = nar++;
        nadj[j]++;
        return ar;
    }

    void revbfs(int ini, int fim)
    {
        int i, no, viz, ar;
        queue<int> fila;

        memset(nivel, NULO, sizeof(nivel));
        memset(qtd, 0, sizeof(qtd));

        nivel[fim] = 0;
        fila.push(fim);

        while (!fila.empty())
        {
            no = fila.front();
            fila.pop();
            qtd[nivel[no]]++;

            for (i = 0; i < nadj[no]; i++)
            {
                ar = adj[no][i];
                viz = dest[ar];

                if (cap[ar] == 0 && nivel[viz] == NULO)
                {
                    nivel[viz] = nivel[no] + 1;
                    fila.push(viz);
                }
            }
        }
    }

    int admissivel(int no)
    {
        while (padj[no] < nadj[no])
        {
            int ar = adj[no][padj[no]];
            if (nivel[no] == nivel[dest[ar]] + 1 && capres(ar) > 0) return
                ar;
            padj[no]++;
        }

        padj[no] = 0;
        return NULO;
    }

    int retrocede(int no)
    {
```

```
95      int i, ar, viz, menor = NULO;
96
97      if (--qtd[nivel[no]] == 0) return NULO;
98
99      for (i = 0; i < nadj[no]; i++)
100     {
101        ar = adj[no][i]; viz = dest[ar];
102        if (capres(ar) <= 0) continue;
103        if (menor == NULO || nivel[viz] < nivel[menor]) menor = viz;
104     }
105
106     if (menor != NULO) nivel[no] = nivel[menor];
107     qtd[++nivel[no]]++;
108
109     return ((ent[no] == NULO) ? no : orig(ent[no]));
110  }
111
112  int avanca(int no, int ar)
113  {
114     int viz = dest[ar];
115     ent[viz] = ar;
116     lim[viz] = min(lim[no], capres(ar));
117     return viz;
118  }
119
120  int aumenta(int ini, int fim)
121  {
122     int ar, no = fim, fmax = lim[fim];
123
124     while (no != ini)
125     {
126        fluxo[ar = ent[no]] += fmax;
127        fluxo[inv(ar)] -= fmax;
128        no = orig(ar);
129     }
130
131     return fmax;
132  }
133
134  int maxflow(int ini, int fim)
135  {
136     int ar, no = ini, fmax = 0;
137
138     memset(fluxo, 0, sizeof(fluxo));
139     memset(padj, 0, sizeof(padj));
140
141     revbfs(ini, fim);
142
143     lim[ini] = INF;
144     ent[ini] = NULO;
145
146     while (nivel[ini] < nvt && no != NULO)
147     {
148        if ((ar = admissivel(no)) == NULO)
149        {
150           no = retrocede(no);
151        }
152        else if ((no = avanca(no, ar)) == fim)
153        {
154           fmax += aumenta(ini, fim);
155           no = ini;
156        }
157     }
158     return fmax;
159  }
160 };
```

2.6 Geometria

Código 39: ponto e poligono

```
1  struct point
2  {
3     double x, y;
4     double z; // para pontos no espaco
5     point(double x = 0, double y = 0, double z = 0): x(x), y(y), z(z) {}
6
7     point operator +(point q) { return point(x + q.x, y + q.y, z + q.z);
          }
8     point operator -(point q) { return point(x - q.x, y - q.y, z - q.z);
          }
9     point operator *(double t) { return point(x * t, y * t, z * t); }
10    point operator /(double t) { return point(x / t, y / t, z / t); }
11    double operator *(point q) { return x * q.x + y * q.y + z * q.z; }
12    point vec(point q) { return point(y * q.z - z * q.y, z * q.x - x * q.
          z, x * q.y - y * q.x); }
13    double operator %(point q) { return x * q.y - y * q.x; }
14
15    int cmp(point q) const
16    {
17       if (int t = ::cmp(x, q.x)) return t;
18       else if (int t = ::cmp(y, q.y)) return t;
19       return ::cmp(z, q.z);
20    }
21
22    bool operator ==(point q) const { return cmp(q) == 0; }
23    bool operator !=(point q) const { return cmp(q) != 0; }
24    bool operator < (point q) const { return cmp(q) < 0; }
25
26    friend ostream& operator <<(ostream& o, point p) {
27      return o << "(" << p.x << ", " << p.y << ", " << p.z << ")";
28    }
29    static point pivot;
30 };
31
32 // para pontos 2D
33 double abs(point p) { return hypot(p.x, p.y); }
34 double arg(point p) { return atan2(p.y, p.x); }
35
36 point point::pivot;
```

```
37
38 typedef vector<point> polygon;
39
40 int ccw(point p, point q, point r)
41 {
42     return cmp((p - r) % (q - r));
43 }
44
45 double angle(point p, point q, point r)
46 {
47     point u = p - q, v = r - q;
48     return atan2(u % v, u * v);
49 }
```

Código 40: Decide se q está sobre o segmento fechado pr.

```
1 bool between(point p, point q, point r)
2 {
3     return ccw(p, q, r) == 0 && cmp((p - q) * (r - q)) <= 0;
4 }
```

Código 41: Decide se os segmentos fechados pq e rs têm pontos em comum.

```
1 bool seg_intersect(point p, point q, point r, point s)
2 {
3     point A = q - p;
4     point B = s - r;
5     point C = r - p;
6     point D = s - q;
7
8     int a = cmp(A % C) + 2 * cmp(A % D);
9     int b = cmp(B % C) + 2 * cmp(B % D);
10
11    if (a == 3 || a == -3 || b == 3 || b == -3) return false;
12    if (a || b || p == r || p == s || q == r || q == s) return true;
13
14    int t = (p < r) + (p < s) + (q < r) + (q < s);
15    return t != 0 && t != 4;
16 }
```

Código 42: Calcula a distância do ponto r ao segmento pq.

```
1 double seg_distance(point p, point q, point r)
2 {
3     point A = r - q;
4     point B = r - p;
5     point C = q - p;
6
7     double a = A * A, b = B * B, c = C * C;
8
9     if (cmp(b, a + c) >= 0) return sqrt(a);
10    else if (cmp(a, b + c) >= 0) return sqrt(b);
11    else return fabs(A % B) / sqrt(c);
12 }
```

Código 43: Classifica o ponto p em relação ao polígono T. Retorna 0, -1 ou 1 dependendo se p está no exterior, na fronteira ou no interior de T, respectivamente.

```
1 int in_poly(point p, polygon& T)
2 {
3     double a = 0;
4     int N = T.size();
5     for (int i = 0; i < N; i++)
6     {
7         if (between(T[i], p, T[(i+1) % N])) return -1;
8         a += angle(T[i], p, T[(i+1) % N]);
9     }
10    return cmp(a) != 0;
11 }
```

Código 44: Convex Hull **graham scan**.

```
1 #define INF 1e9
2 #define EPS 1e-9
3
4 int cmp(double a, double b = 0.0) {
5     return a+EPS < b ? -1 : a-EPS > b;
6 }
7
8 struct Point {
9     double x, y;
10    Point(double a=0.0,double b=0.0){x=a,y=b;}
11    Point operator+(const Point &P) const {return Point(x+P.x,y+P.y);}
12    Point operator-(const Point &P) const {return Point(x-P.x,y-P.y);}
13    Point operator*(double c) const {return Point(x*c,y*c);}
14    Point operator/(double c) const {return Point(x/c,y/c);}
15    double operator!() const {return sqrt(x*x+y*y);}
16    bool operator==(const Point &p) const {return !cmp(x,p.x) && !cmp(y,p
          .y);}
17    bool operator<(const Point &p) const {if (cmp(x,p.x)) return cmp(x,p.
          x) < 0; return cmp(y,p.y) < 0;}
18    void print(string prefix = "") const {printf("%s%.3lf %.3lf\n",prefix
          .c_str(),x,y);}
19 };
20
21 typedef vector<Point> Polygon;
22
23 double cross(Point A, Point B) {
24     return A.x*B.y - B.x*A.y;
25 }
26
27 Point pmin;
28
29 bool lessThan(Point A, Point B) {
30     if (cmp(cross(A-pmin,B-pmin))) return cmp(cross(A-pmin,B-pmin)) > 0;
31     return cmp(!(pmin-A),!(pmin-B)) < 0;
32 }
33
34 int sort(Polygon &p) {
35     int imin = 0, i, j, n = p.size();
36
```

```
37    for (i=1; i < p.size(); i++) {
38       if (p[i] <  p[imin]) imin = i;
39    }
40
41    swap(p[0],p[imin]);
42    pmin = p[0];
43
44    sort(p.begin()+1,p.end(),lessThan);
45
46    for (i=n-1; i > 0 && !cmp(cross(p[i]-p[0],p[i-1]-p[0])); i--);
47    if (i == 0) return 1;
48    for (j=0; j < (n-i)/2; j++) swap(p[i+j],p[n-j-1]);
49    return 0;
50 }
51
52 Polygon convex_hull(Polygon &p) { // tirar o & para nao alterar o
       poligono original
53    int hs = 2, n = p.size();
54    Polygon hull;
55
56    if (p.size() < 3) return p;
57
58    int isline = sort(p);
59    hull.push_back(p[0]), hull.push_back(p[1]);
60
61    for (int i=2; i <= n-isline; i++) {
62       while (hull.size() > 1 && cmp(cross(p[i%n]-hull[hs-1],hull[hs-2]-
             hull[hs-1])) <= 0) { // trocar o <= para < para manter pontos
             intermediarios
63          hull.pop_back(), hs--;
64       }
65       if (i < n) hull.push_back(p[i]), hs++;
66    }
67
68    return hull;
69 }
70
71 int main() {
72    Polygon p;
73
74    while (...) {
75       double x = ...;
76       double y = ...;
77       p.push_back(Point(x,y));
78    }
79
80    Polygon hull = convex_hull(p);
81
82    for (int i=0; i < hull.size(); i++) {
83       ...
84    }
85    return 0;
86 }
```

2.7 Algebra Linear

Código 45: Simplex

```
1  #include <iostream>
2  #include <vector>
3  #include <algorithm>
4
5  using namespace std;
6
7  #define fori(i,n) for(int i=0; i < (n); ++i)
8  #define forr(i,a,b) for(int i=(a); i <= (b); ++i)
9  #define ford(i,a,b) for(int i=(a); i >= (b); --i)
10 #define sz size()
11
12 const double EPS=1e-9; const int INF = 0x3f3f3f3f;
13
14 #define all(x) (x).begin(),(x).end()
15
16 int cmpD(double x, double y=0, double tol=EPS) {
17    return (x <= y+tol) ? (x+tol<y) ? -1 : 0 : 1;
18 }
19
20 struct simplex {
21    // max c * x, s.t: A * x <= b; x >= 0
22    simplex( const vector< vector< double > > & A_, const vector< double
          > & b_,
23          const vector< double > & c_ ) : A( A_ ), b( b_ ), c( c_ ) {}
24    vector< vector< double > > A; vector< double > b, c, sol;
25    vector< bool > N; vector< int > kt; int m, n;
26    void pivot( int k, int l, int e ) {
27       int x = kt[l]; double p = A[l][e];
28       fori(i,k) A[l][i] /= p;
29       b[l] /= p; N[e] = false;
30       fori(i,m) if (i != l) {b[i] -= A[i][e]*b[l]; A[i][x] = -A[i][e]*A[
             l][x];}
31       fori(j,k) if ( N[j] ) {
32          c[j] -= c[e] * A[l][j];
33          fori(i,m) if ( i != l ) A[i][j] -= A[i][e] * A[l][j];
34       }
35       kt[l] = e; N[x] = true; c[x] = -c[e] * A[l][x];
36    }
37    vector< double > go( int k ) {
38       vector< double > res;
39       while ( 1 ) {
40          int e = -1, l = -1;
41          fori(i,k) if ( N[i] && cmpD( c[i] ) > 0 ) { e = i; break; }
42          if ( e == -1 ) break;
43          fori(i,m) if ( cmpD(A[i][e]) > 0 && ( l == -1 || cmpD( b[i] / A
                [i][e],
44                      b[l] / A[l][e], 1e-20 ) < 0 ) ) l = i;
45          if ( l == -1 ) return vector< double >(); // unbounded
46          pivot( k, l, e );
47       }
48       res.resize( k, 0 );
49       fori(i,m) res[kt[i]] = b[i];
```

```
50        return res;
51    }
52    vector< double > solve() {
53       m = A.sz; n = A[0].sz; int k = m+n+1;
54       N = vector< bool >( k, true ); vector< double > c_copy = c;
55       c.resize(n+m); kt.resize(m);
56       fori(i,m) {
57          A[i].resize(k); A[i][n+i] = 1; A[i][k-1] = -1;
58          kt[i] = n+i; N[kt[i]] = false;
59       }
60       int l = min_element(all(b)) - b.begin();
61       if(cmpD(b[l] ) < 0 ) {
62          c = vector<double>(k,0);
63          c[k-1] = -1; pivot(k, l, k-1); sol=go(k);
64          if(cmpD(sol[k-1])>0) return vector<double>(); // infeasible
65          fori(i,m) if(kt[i] == k-1) {
66             fori(j,k-1) if(N[j] && cmpD( A[i][j] ) != 0 ) {
67                pivot( k, i, j); break;
68             }
69          }
70          c=c_copy; c.resize(k,0);
71          fori(i,m) fori(j,k) if(N[j]) c[j] -= c[kt[i]]*A[i][j];
72       }
73       sol = go(k-1);
74       if(!sol.empty()) sol.resize(n);
75       return sol;
76    }
77 };
78
79 // Como usar
80 int main() {
81    /* Exemplo: Maximize cx Subject to Ax <= b */
82    vector<vector<double> > A(9);
83    double Av[][3] = {{1,1,0}, {0,0,-1}, {-1,-1,0},
84                      {0,0,1}, {1,0,0}, {0,1,0},
85                      {0,0,1}, {1,0,1}, {0,1,0}};
86
87    for(int i=0; i < 9; i++) {
88       A[i].insert(A[i].begin(), &(Av[i][0]), &(Av[i][3])); // Sim, [3]!
             Ou seja, idx-final+1
89    }
90
91    vector<double > c(3, 1); // c=[1 1 1]
92    double bv[] = {2,-1,-2,1,2,1,1,2,1};
93    vector<double > b(bv, bv+sizeof(bv)/sizeof(double));
94
95    simplex sim(A,b,c);
96    vector<double> s = sim.solve();
97    if(!s.size()) cout << "Impossivel\n";
98    else
99    for(int i=0; i < s.size(); i++) {
100      cout << s[i] << endl;
101   }
102 }
```

2.8 Casamento de strings

Código 46: String matching - Algoritmo **KMP** - O(n + m)

```
1  // F[i] - size of the largest prefix of pattern[0..i] that is also a
2  // suffix of pattern[1..i]. Ex: pattern = {a,b,a,c,a,b}, F =
       {0,0,1,0,1,2}
3  #define MAX_PATTERN_SIZE 10010
4  int F[MAX_PATTERN_SIZE];
5  void build_failure_function( const string & pattern )
6  {
7     int m = pattern.size();
8     F[0] = -1;
9     for (int i = 0; i < m; i++)
10    {
11       F[i+1] = F[i] + 1;
12       while ( F[i+1] > 0 && pattern[i] != pattern[ F[i+1]-1 ] )
13          F[i+1] = F[ F[i+1]-1 ] + 1;
14    }
15 }
16
17 // retorna a posicao inicial de cada ocorrencia de pattern em text
18 vector<int> KMP( const string & text, const string & pattern )
19 {
20    build_failure_function( pattern );
21    vector<int> start_positions;
22    int j = 0, m = pattern.size(), n = text.size();
23
24    for (int i = 0; i < n; i++)
25    {
26       while ( true )
27       {
28          if ( text[i] == pattern[j] )
29          {
30             if ( ++j == m )
31             {
32                start_positions.push_back( i - m + 1 );
33                j = F[j];
34             }
35             break;
36          }
37
38          if ( j == 0 ) break;
39          j = F[j];
40       }
41    }
42
43    return start_positions;
44 }
```

2.9 Outros

Código 47: Encontra o elemento mais comum no vetor.

```
int findMajority(int vec[], int n)
{
   int cnt = 0;
   int maior;
   for (int i = 0; i < n; i++)
   {
      if (cnt == 0) {maior = v; cnt = 1;}
      else if (v == maior) cnt++;
      else cnt--;
   }

   return maior;
}
```

Código 48: josephus problem

```
/**
The Josephus problem (or Josephus permutation) is a theoretical problem related to a certain counting-out game. There are people standing in a circle waiting to be executed. After the first man is executed, certain number of people are skipped and one man is executed. Then again, people are skipped and a man is executed. The elimination proceeds around the circle (which is becoming smaller and smaller as the executed people are removed), until only the last man remains, who is given freedom. The task is to choose the place in the initial circle so that you are the last one remaining and so survive.
 */

using namespace std;

int josephus(int n, int m)
{
   int res = 0;
   vector <int> people;
   int loc = 0;

   for (int i = 0; i < n; i++) people.push_back(i+1);

   while (people.size() > 1)
   {
      if (loc >= people.size())
         loc %= people.size();

      people.erase(people.begin()+loc);
      loc += (m-1);
   }

   return people[0];
}
```

Código 49: Gera as permutações dos elementos da string

```
bool nextPermutation(string& number)
{
       bool isBigger = true;
       int i, j;

       for (i = number.size() - 1; i >= 0; i--)
       {
               if (number[i] < number[i+1]) break;
       }

       if (i != -1)
       {
               isBigger = false;

               for (j = number.size() - 1; j >= i+1; j--)
               {
                       if (number[j] > number[i])
                       {
                               break;
                       }
               }

               int tmp = number[i];
               number[i] = number[j];
               number[j] = tmp;

               j = number.size() -1;
               i++;

               while (i < j)
               {
                       tmp = number[i];
                       number[i] = number[j];
                       number[j] = tmp;
                       i++;
                       j--;
               }
       }

       return isBigger;
}
```

Código 50: Exemplo de geração de permutações dos elementos da string, usando backtracking

```
#define TAM 10
int cnt = 0;
int total; //numero de elementos de elem
int elem[TAM];
bool usados[TAM]; //ZERE ME

void enumera(int num)
{
   if (num == total)
   {
      cnt++;
      for (int i = 0; i < total; i++) cout << elem[i];
      cout << endl;
      return;
```

```
    }

    for (int i = 0; i < total; i++)
    {
        if (!usados[i])
        {
            elem[num] = i;
            usados[i] = true;
            enumera (num + 1);
            usados[i] = false;
        }
    }
}
```

3 Biblioteca C/C++

3.1 I/O

Ignorando entradas na família scanf:

Código 51: Ignora os dois floats do meio. Retornará 2 no sucesso.

```
scanf("%f %*f %*f %d", &a, &b);
```

3.2 Map

Código 52: Referencias map

```
#include <map>
#include <string>
#include <cstdio>

using namespace std; // USE ISTO!!!

class Comparadora;

class Pessoa {
    int idade;
    string nome;
    friend class Comparadora;
public:
    Pessoa(string nome, int idade) {
        this->idade = idade;
        this->nome = nome;
    }
    void print() const {
        printf("Nome: %s Idade: %d\n", nome.c_str(), idade);
    }
};

class Comparadora { // Ordena crescentemente
public: // <- IMPORTANTE
    bool operator() (const Pessoa &a, const Pessoa &b)
    {
        int idDif = a.idade-b.idade;
        if(idDif < 0) return true;
        else if(idDif==0) return a.nome.compare(b.nome) < 0 ? true : false
            ;
        else return false;
    }
};

int main() {
    Pessoa r("Rangelz", 86);
    Pessoa r2("Rangelzao", 86);

    map<Pessoa, string, Comparadora> alunos;
    alunos[r]="UFMG";
    alunos[r2]="PUC";
    // Iterator
    for(map<Pessoa, string, Comparadora>::iterator it=alunos.begin(); it
        != alunos.end(); it++) {
        it->first.print();
        printf("\t%s\n\n", it->second.c_str());
    }
    // Find
    if(alunos.find(Pessoa("Rangelz", 86)) != alunos.end()) { // Achou!
        printf("Achei Rangel!\n");
    }
    return 0;
}
```