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# 1 Tabelas

tipo	bits	min...max	precisao
char	8	0..127	2
signed char	8	-128..127	2
unsigned char	8	0..255	2
short	16	-32.768 .. 32.767	4
unsigned short	16	0 .. 65.535	4
int	32	-2x10**9 .. 2 x 10**9	9
unsigned int	32	0 .. 4x10**9	9
int64_t	64	-9 x 10**18 .. 9 x 10**18	18
uint64_t	64	0 .. 18 x 10**18	19

Tabela 1: Limites de representação de dados

0! = 1
1! = 1
2! = 2
3! = 6
4! = 24
5! = 120
6! = 720
7! = 5.040
8! = 40.320
9! = 362.880
10! = 3.628.800
11! = 39.916.800
12! = 479.001.600 [limite do (unsigned) int]
13! = 6.227.020.800
14! = 87.178.291.200
15! = 1.307.674.368.000
16! = 20.922.789.888.000
17! = 355.687.428.096.000
18! = 6.402.373.705.728.000
19! = 121.645.100.408.832.000
20! = 2.432.902.008.176.640.000 [limite do (u)int64.t]

Tabela 2: Fatorial

Tipo	%
char	c
int	d
float	e, E, f, g, G
int (octal)	o
int (hexa)	x, X
uint	u
char*	s

Tabela 3: scanf() - %[\*][width][modifiers]type

modifiers	tipo
h	short int (d, i, n), or unsigned short int (o, u, x)
l	long int (d, i, n), or unsigned long int (o, u, x), or double (e, f, g)
ll	long long int (d, i, n), or unsigned long long int (o, u, x)
L	long double (e, f, g)

Tabela 4: scanf() %[\*][width][modifiers]type

função	descrição
atof	Convert string to double
atoi	Convert string to integer
atol	Convert string to long integer
strtod	Convert string to double
strtol	Convert string to long integer
strtoul	Convert string to unsigned long integer

Tabela 5: stdlib

função	descrição
cos	Compute cosine
sin	Compute sine
tan	Compute tangent
acos	Compute arc cosine
asin	Compute arc sine
atan	Compute arc tangent
atan2	Compute arc tangent with two parameters
cosh	Compute hyperbolic cosine
sinh	Compute hyperbolic sine
tanh	Compute hyperbolic tangent
exp	Compute exponential function
frexp	Get significand and exponent
ldexp	Generate number from significand and exponent
log	Compute natural logarithm
log10	Compute common logarithm
modf	Break into fractional and integral parts
pow	Raise to power
sqrt	Compute square root
ceil	Round up value
fabs	Compute absolute value
floor	Round down value
fmod	Compute remainder of division

Tabela 6: math (angulos em radianos)

## 2 Codigos

### 2.1 Exemplos

Código 1: Modelo

```

1 #include <stdio.h>
2 #include <stdlib.h>
3 #include <string.h>
4 #include <math.h>
5
6 #include <inttypes.h>
7 #include <ctype.h>
8 #include <limits.h>
9
10 #include <algorithm>
11 #include <utility>
12 #include <iostream>
13
14 #include <map>
15 #include <set>
16 #include <vector>
17 #include <list>
18 #include <queue>
19 #include <sstream>
20
21 using namespace std;
22
23 #define abs(a) ((a) > 0 ? (a) : -(a))
24
25 int main()
26 {
27     int n;
28
29     cin >> n;
30
31     for (int i = 0; i < n; i++)
32     {
33
34     }
35
36     while (cin >> n)
37     {
38
39     }
40     return 0;
41 }

```

Código 2: comparcao de ponto flutuante

```

1 /**
2  * -1 se x < y
3  * 0 se x = y
4  * 1 se x > y
5  */

```

```

6 const double EPS = 1e-10;
7 #define _inline(f...) f() __attribute__((always_inline)); f
8 _inline(int cmp)(double x, double y = 0, double tol = EPS)
9 {
10     return (x <= y + tol) ? (x + tol < y) ? -1 : 0 : 1;
11 }

```

Código 3: .vimrc para a configuração do vim

```

1 set ai noet ts=4 sw=4 bs=2
2 set cindent

```

Código 4: função que acelara o cin. Não deve ser usada com printf

```

1 std::cout.sync_with_stdio(false);

```

Código 5: printf

```

1 /* printf example */
2 #include <stdio.h>
3
4 int main()
5 {
6     printf ("Characters: %c %c \n", 'a', 65);
7     printf ("Decimals: %d %ld\n", 1977, 650000L);
8     printf ("Preceding with blanks: %10d \n", 1977);
9     printf ("Preceding with zeros: %010d \n", 1977);
10    printf ("Some different radixes: %d %x %o %#x %#o \n", 100, 100, 100,
11            100, 100);
12    printf ("floats: %4.2f %+0e %E %4.2f\n", 3.1416, 3.1416, 3.1416,
13            3.1);
14    printf ("Width trick: %*d \n", 5, 10);
15    printf ("%s \n", "A string");
16    return 0;
17 }
18 /* %[flags (-, +, etc)][width][.precision][length (h,l,L)]specifier
19 Characters: a A
20 Decimals: 1977 650000
21 Preceding with blanks:      1977
22 Preceding with zeros: 0000001977
23 Some different radixes: 100 64 144 0x64 0144
24 floats: 3.14 +3e+000 3.141600E+000 3.10
25 Width trick:      10
26 A string
27 */

```

Código 6: exemplo de map

```

1 #include <iostream>
2 #include <map>
3 using namespace std;
4
5 int main ()

```

```

6 {
7     map<char,int> mymap;
8     map<char,int>::iterator it;
9     pair<map<char,int>::iterator,bool> ret;
10
11     // first insert function version (single parameter):
12     mymap.insert ( pair<char,int>('a',100) );
13     mymap.insert ( pair<char,int>('z',200) );
14
15     ret=mymap.insert (pair<char,int>('z',500) );
16     if (ret.second==false)
17     {
18         cout << "element 'z' already existed";
19         cout << " with a value of " << ret.first->second << endl;
20     }
21
22     // third insert function version (range insertion):
23     map<char,int> anothermap;
24     anothermap.insert(mymap.begin(),mymap.find('c'));
25
26     // showing contents:
27     cout << "mymap contains:\n";
28     for ( it=mymap.begin(); it != mymap.end(); it++ )
29         cout << (*it).first << " => " << (*it).second << endl;
30
31     map<char,string> mymap;
32     mymap['a']="an element";
33     if (mymap.count('a') > 0)
34         cout << mymap['a'] << " is an element of mymap.\n";
35
36     while (!mymap.empty())
37     {
38         cout << mymap.begin()->first << " => ";
39         cout << mymap.begin()->second << endl;
40         map<char, int>::iterator erasedelement = mymap.erase(mymap.begin());
41         ;
42     }
43     return 0;
44 }

```

Código 7: exemplo de set e multiset

```

1 #include <iostream>
2 #include <set>
3 using namespace std;
4
5 int main ()
6 {
7     multiset<int> mymultiset;
8     multiset<int>::iterator it;
9
10    // set some initial values:
11    for (int i=1; i<=5; i++) mymultiset.insert(i*10);    // 10 20 30 40 50
12
13    cout << "size: " << (int) mymultiset.size() << endl;

```

```

14  cout << "count: " << (int) mymultiset.count(10) << endl;
15
16  it=mymultiset.find(20);
17  mymultiset.erase (it);
18
19  if (! mymultiset.empty)
20      mymultiset.erase (mymultiset.find(40));
21
22  for (it=mymultiset.begin(); it!=mymultiset.end(); it++)
23      cout << " " << *it;
24
25  int myints[]={19,72,4,36,20,20};
26  multiset<int> first  (myints,myints+3);    // 4,19,72
27  multiset<int> second (myints+3,myints+6);  // 20,20,36
28
29  first.swap(second); // troca conteudo. o primeiro fica [20,20,36] e o
                       // segundo [4,19,72]
30
31  return 0;
32 }

```

Código 8: exemplo de list

```

1  #include <iostream>
2  #include <list>
3  using namespace std;
4
5  int main ()
6  {
7      list<int> mylist (2,100);           // two ints with a value of 100
8      mylist.push_front (200);
9      mylist.push_back (300);
10
11     it = mylist.begin();
12     mylist.insert (it,10);
13     mylist.insert (it,2,20); // two ints with a value of 20
14
15     mylist.reverse(); // Reverses the order of the elements in the list.
16
17     cout << "mylist contains:";
18     for (list<int>::iterator it=mylist.begin(); it!=mylist.end(); ++it)
19         cout << " " << *it;
20
21     cout << "Popping out the elements in mylist:";
22     while (!mylist.empty())
23     {
24         cout << " " << mylist.front();
25         mylist.pop_front();
26     }
27
28     while (!mylist.empty())
29     {
30         cout << " " << mylist.back();
31         mylist.pop_back();
32     }
33

```

```

34  cout << mylist.size() << endl;
35
36  return 0;
37 }

```

Código 9: exemplo de queue

```

1  #include <iostream>
2  #include <queue>
3  using namespace std;
4
5  int main ()
6  {
7      queue<int> myqueue;
8      int sum (0);
9
10     for (int i=1;i<=10;i++) myqueue.push(i);
11
12     myqueue.back() -= myqueue.front();
13
14     cout << "size: " << (int) myqueue.size() << endl;
15
16     while (!myqueue.empty())
17     {
18         sum += myqueue.front();
19         myqueue.pop();
20     }
21
22     cout << "total: " << sum << endl;
23
24     return 0;
25 }

```

Código 10: exemplo de priority queue

```

1  #include <iostream>
2  #include <queue>
3  using namespace std;
4
5  int main ()
6  {
7      priority_queue<int> mypq;
8
9      mypq.push(30);
10     mypq.push(100);
11     mypq.push(25);
12     mypq.push(40);
13
14     cout << "size: " << (int) mypq.size() << endl;
15
16     cout << "Popping out elements...";
17     while (!mypq.empty())
18     {
19         cout << " " << mypq.top();
20         mypq.pop();

```

```

21 }
22 cout << endl;
23
24 return 0;
25 }

```

Código 11: exemplo de stack

```

1 #include <iostream>
2 #include <stack>
3 using namespace std;
4
5 int main ()
6 {
7     stack<int> mystack;
8     int sum = 0;
9
10    mystack.push(10);
11    mystack.push(20);
12
13    mystack.top() -= 5;
14
15    while (!mystack.empty())
16    {
17        sum += mystack.top();
18        mystack.pop();
19    }
20
21    cout << "size: " << (int) mystack.size() << endl;
22
23    return 0;
24 }

```

Código 12: exemplo de vector

```

1 #include <iostream>
2 #include <vector>
3 using namespace std;
4
5 int main ()
6 {
7     vector<int> myvector (3,100); // (100 100 100)
8     vector<int>::iterator it;
9
10    myvector.reserve(100);
11
12    for (int i=0; i<myvector.size(); i++)
13        myvector.at(i)=i; // = myvector[i] = i
14
15    it = myvector.begin();
16    it = myvector.insert ( it , 200 );
17    myvector.insert (it,2,300);
18
19    vector<int> anothervector (2,400);
20    int myarray [] = { 501,502,503 };

```

```

21 vector<int> initializer (myarray/*PointerInicio*/, myarray+sizeof(
    myarray)/sizeof(int)/*PointerFim*/);
22 myvector.insert (it+2,anothervector.begin(),anothervector.end());
23 myvector.insert (myvector.begin(), myarray, myarray+3);
24
25 cout << "myvector contains:";
26 for (it=myvector.begin(); it<myvector.end(); it++)
27     cout << " " << *it;
28 cout << endl;
29
30 // erase the 6th element
31 myvector.erase (myvector.begin()+5);
32 int sum;
33 while (!myvector.empty())
34 {
35     sum += myvector.back();
36     myvector.pop_back();
37 }
38
39 return 0;
40 }

```

Código 13: exemplo de string

```

1 #include <iostream>
2 #include <string>
3 using namespace std;
4
5 int main ()
6 {
7     string str ("There are two needles in this haystack with needles.");
8     string str2 ("needle");
9     size_t found;
10
11    // different member versions of find in the same order as above:
12    found=str.find(str2);
13    if (found!=string::npos)
14        cout << "first 'needle' found at: " << int(found) << endl;
15
16    found=str.find("needles are small",found+1,6);
17    if (found!=string::npos)
18        cout << "second 'needle' found at: " << int(found) << endl;
19
20    found=str.find("haystack");
21    if (found!=string::npos)
22        cout << "'haystack' also found at: " << int(found) << endl;
23
24    found=str.find('.');
25    if (found!=string::npos)
26        cout << "Period found at: " << int(found) << endl;
27
28    // let's replace the first needle:
29    str.replace(str.find(str2),str2.length(),"preposition");
30    cout << str << endl;
31
32    string str="We think in generalities, but we live in details.";

```

```

33          // quoting Alfred N. Whitehead
34  string str2, str3;
35  size_t pos;
36
37  str2 = str.substr (12,12); // "generalities"
38
39  pos = str.find("live");    // position of "live" in str
40  str3 = str.substr (pos);   // get from "live" to the end
41
42  cout << str2 << ' ' << str3 << endl;
43
44
45  return 0;
46 }
47 /*
48 first 'needle' found at: 14
49 second 'needle' found at: 44
50 'haystack' also found at: 30
51 Period found at: 51
52 There are two prepositions in this haystack with needles.
53 generalities live in details.
54 */

```

---

Código 14: exemplo de stringstream

---

```

1 #include <string>
2 #include <sstream>
3 #include <iostream>
4
5
6 using namespace std;
7
8
9 int main() {
10  string linha="Olah mundo";
11  stringstream separador(linha); // Tokenizador
12  string word;
13  separador >> word; // word=Olah
14  char mu[4];
15  separador.readsome(mu, 3); // Le " mu" (inclui espaco)
16  cout << separador.tellg() << endl; // 7 (posicao de leitura)
17  cout << separador.tellp() << endl; // 0, posicao de escrita
18
19  separador.seekp(separador.str().size());
20  separador << " cruel"; // separador = "Olah mundo cruel"
21
22  separador.seekp(5);
23  separador << "doido"; // separador = "Olah doido cruel" (sobrescrito)
24  return 0;
25 }

```

---

Código 15: exemplo de ordenação

---

```

1 #include <iostream>
2 #include <algorithm>

```

```

3 #include <vector>
4 using namespace std;
5
6 bool myfunction (int i,int j) { return (i<j); }
7
8 struct myclass {
9  bool operator() (int i,int j) { return (i<j);}
10 } myobject;
11
12 int compare (const void * a, const void * b)
13 {
14  return ( *(int*)a - *(int*)b );
15 }
16
17
18 int main () {
19  int myints[] = {32,71,12,45,26,80,53,33};
20  vector<int> myvector (myints, myints+8); // 32 71 12 45
21                                           26 80 53 33
22  // using default comparison (operator <):
23  sort (myvector.begin(), myvector.begin()+4); // (12 32 45 71)
24                                           26 80 53 33
25  // using function as comp
26  sort (myvector.begin()+4, myvector.end(), myfunction); // 12 32 45
27                                           71(26 33 53 80)
28  // using object as comp
29  sort (myvector.begin(), myvector.end(), myobject); // (12 26 32 33
30                                           45 53 71 80)
31
32  // if stable is need
33  stable_sort (myvector.begin(), myvector.end(), myfunction);
34
35  // Rearranges the elements in the range [first,last), in such a way
36  // that the subrange [first,middle)
37  // contains the smallest elements of the entire range sorted in
38  // ascending order, and the subrange
39  // [middle,end) contains the remaining elements without any specific
40  // order.
41  partial_sort (myvector.begin(), myvector.begin()+3, myvector.end());
42
43  qsort (myints, 8, sizeof(int), compare);
44
45  return 0;
46 }

```

---

Código 16: pesquisa binária

---

```

1 int compareMyType (const void * a, const void * b)
2 {
3  if ( *(MyType*)a > *(MyType*)b ) return 1;
4  if ( *(MyType*)a == *(MyType*)b ) return 0;
5  if ( *(MyType*)a < *(MyType*)b ) return -1;
6 }
7
8 int key = 40;

```

```
9 item = (int*) bsearch (&key, values, n, sizeof (int), compareMyType);
```

Código 17: Arredondamento e output em outras bases

```
1 #include <iostream>
2 #include <iomanip> // setprecision()
3 using namespace std;
4
5 int main () {
6     double a = 3.1415926534;
7     double b = 2006.0;
8     double c = 1.0e-10;
9
10    // setprecision(1) => 1 casa decimal apos a virgula
11    cout << fixed << setprecision(1) << 9.09090901 << endl;
12    cout << fixed << setprecision(2) << 9.09090901 << endl;
13    cout << fixed << setprecision(3) << 9.09090901 << endl;
14    cout << fixed << setprecision(2) << 9.1 << endl;
15
16    // anula o efeito de setprecision
17    cout.unsetf(ios::floatfield);
18
19    // 5 digitos no maximo
20    cout.precision(5);
21
22    cout << a << '\t' << b << '\t' << c << endl;
23    cout << fixed << a << '\t' << b << '\t' << c << endl;
24    cout << scientific << a << '\t' << b << '\t' << c << endl;
25
26    // Sets the basefield format flag for the str stream to dec, hex or
27    // oct.
28    int n = 70;
29    cout << dec << n << endl;
30    cout << hex << n << endl;
31    cout << oct << n << endl;
32
33    return 0;
34 }
35 /* output
36 9.1
37 9.09
38 9.10
39 3.1416 2006 1e-10
40 3.14159 2006.00000 0.00000
41 3.14159e+00 2.00600e+03 1.00000e-10
42 70
43 46
44 106
45 */
```

Código 18: máximo divisor comum e mínimo múltiplo comum

```
1 int gcd(int x, int y)
2 {
3     return y ? gcd(y, x % y) : abs(x);
4 }
5 uint64_t lcm(int x, int y)
6 {
7     if (x && y) return abs(x) / gcd(x, y) * uint64_t(abs(y));
8     else return uint64_t(abs(x | y));
9 }
```

Código 19: decide se um número é primo

```
1 bool isPrime(int n)
2 {
3     if (n < 0) return isPrime(-n);
4     if (n == 1) return true;
5     if (n < 5 || n % 2 == 0 || n % 3 == 0) return (n == 2 || n == 3);
6
7     int maxP = sqrt(n) + 2;
8     for (int p = 5; p < maxP; p += 6)
9     {
10        if (n % p == 0 || n % (p+2) == 0) return false;
11    }
12    return true;
13 }
```

Código 20: Retorna a fatoração em números primos de abs(n).

```
1 typedef map<int, int> prime_map;
2 void squeeze(prime_map& M, int& n, int p)
3 {
4     for (; n % p == 0; n /= p) M[p]++;
5 }
6 void factor(int n, prime_map& M)
7 {
8     if (n < 0) { factor(-n, M); return; }
9     if (n < 2) return;
10
11    squeeze(M, n, 2);
12    squeeze(M, n, 3);
13
14    int maxP = sqrt(n) + 2;
15    for (int p = 5; p < maxP; p += 6)
16    {
17        squeeze(M, n, p);
18        squeeze(M, n, p+2);
19    }
20    if (n > 1) M[n]++;
21 }
```

## 2.2 Teoria dos números

Código 21: Calcula Valor de  $a^b \bmod n$  de forma rápida.



```

1 int mpow(int a, int b, int n = 10)
2 {
3     if(b == 0)
4         return 1;
5     else {
6         long long res = mpow(a, b/2, n);
7         res = (res*res) % n;
8         if(b%2 == 1)
9             res = (res*a) % n;
10        return (int) res;
11    }
12 }

```

Código 22: Calcula  $(a*b)\%c$  de forma rápida.

```

1 long long mulmod(long long a, long long b, long long c)
2 {
3     long long x = 0;
4     long long y = a % c;
5     while(b > 0)
6     {
7         if(b & 1ll) x = (x + y) % c;
8         y = (y << 1) % c;
9         b >>= 1;
10    }
11    return x % c;
12 }

```

Código 23: Computa  $x$  tal que  $a*x = b \pmod{c}$ . Quando a equação não tem solução, retorna algum valor arbitrário errado, mas basta conferir o resultado.

```

1 long long axbmodc(long long a, long long b, long long c)
2 {
3     return a ? (axbmodc(c % a, (a - b % a) % a, a) * c + b) / a : 0;
4 }

```

Código 24: **Baby-step Giant-step algorithm** Calcula o menor valor de  $e$  para  $b^e = n \pmod{p}$ . Retorna -1 se eh impossivel

```

1 #define inv_mult( a, n ) axbmodc(a, 1, n)
2
3 long long discreteLlogarithm( long long b, long long n, long long p )
4 {
5     if ( n == 1 ) return 0;
6
7     map < long long, int > table;
8     long long m = sqrt(p) + 1, pot = 1, pot2 = 1;
9
10    for (int j = 0; j < m; j++)
11    {
12        if ( pot == n ) return j;
13        table[( n * inv_mult( pot, p ) ) % p] = j;
14        pot = ( pot * b ) % p;

```

```

15    }
16
17    for (int i = 0; i < m; i++)
18    {
19        if ( table.find( pot2 ) != table.end() ) return i * m + table[pot2
20        ];
21        pot2 = ( pot * pot2 ) % p;
22    }
23    return -1;
24 }

```

## 2.3 Estruturas de dados

Código 25: Números de precisão harbitrária.

```

1 const int DIG = 4;
2 const int BASE = 10000; // BASE**3 < 2**51
3 const int TAM = 1000;
4
5 struct BigInt
6 {
7     int num[TAM], numDigits;
8     BigInt(int x = 0): numDigits(1)
9     {
10         memset(num, 0, sizeof(num));
11         num[numDigits++] = x; fixInvariant();
12     }
13     BigInt(char *s): numDigits(1)
14     {
15         memset(num, 0, sizeof(num));
16         int sign = 1;
17
18         while (*s && !isdigit(*s))
19         {
20             if (*s++ == '-') sign *= -1;
21         }
22
23         char *t = strdup(s), *p = t + strlen(t);
24
25         while (p > t)
26         {
27             *p = 0; p = max(t, p - DIG);
28             sscanf(p, "%d", &num[numDigits]);
29             num[numDigits++] *= sign;
30         }
31
32         free(t);
33         fixInvariant();
34     }
35
36     BigInt& fixInvariant(int m = 0)
37     {
38         numDigits = max(m, numDigits);

```

```

39     int sign = 0;
40
41     for (int i = 1, carry = 0; i <= numDigits || carry && (numDigits =
42         i); i++)
43     {
44         num[i] += carry;
45         carry = num[i] / BASE;
46         num[i] %= BASE;
47         if (num[i]) sign = (num[i] > 0) ? 1 : -1;
48     }
49
50     for (int i = 1; i < numDigits; i++)
51     {
52         if (num[i] * sign < 0)
53         {
54             num[i] += sign * BASE;
55             num[i+1] -= sign;
56         }
57     }
58
59     while (numDigits && !num[numDigits]) numDigits--;
60     return *this;
61
62     //Comparacao
63     int cmp(const BigInt& x = 0) const
64     {
65         int i = max(numDigits, x.numDigits), t = 0;
66         while (1)
67         {
68             if ((t = ::cmp(num[i], x.num[i])) || i-- == 0) return t;
69         }
70     }
71
72     bool operator <(const BigInt& x) const { return cmp(x) < 0; }
73     bool operator >(const BigInt& x) const { return cmp(x) > 0; }
74     bool operator <=(const BigInt& x) const { return cmp(x) <= 0; }
75     bool operator >=(const BigInt& x) const { return cmp(x) >= 0; }
76     bool operator ==(const BigInt& x) const { return cmp(x) == 0; }
77     bool operator !=(const BigInt& x) const { return cmp(x) != 0; }
78
79     //operacoes fundamentais
80     BigInt& operator +=(const BigInt& x)
81     {
82         for (int i = 1; i <= x.numDigits; i++) num[i] += x.num[i];
83         return fixInvariant(x.numDigits);
84     }
85     BigInt& operator -=(const BigInt& x)
86     {
87         for (int i = 1; i <= x.numDigits; i++) num[i] -= x.num[i];
88         return fixInvariant(x.numDigits);
89     }
90
91     void multiAndAcumWithShift(const BigInt& x, int m, int b)
92     { // *this += (x * m) << b;

```

```

93         for (int i = 1, carry = 0; (i <= x.numDigits || carry) && (
94             numDigits = i + b); i++)
95         {
96             num[i+b] += x.num[i] * m + carry;
97             carry = num[i+b] / BASE;
98             num[i+b] %= BASE;
99         }
100     }
101     BigInt operator *(const BigInt& x) const
102     {
103         BigInt r;
104         for (int i = 1; i <= numDigits; i++) r.multiAndAcumWithShift(x,
105             num[i], i-1);
106         return r;
107     }
108     BigInt div(const BigInt& x)
109     {
110         if (x == 0) return 0;
111
112         BigInt q;
113         q.numDigits = max(numDigits - x.numDigits + 1, 0);
114         int d = x.num[x.numDigits] * BASE + x.num[x.numDigits-1];
115
116         for (int i = q.numDigits; i > 0; i--)
117         {
118             int j = x.numDigits + i - 1;
119             q.num[i] = int((num[j] * double(BASE) + num[j-1]) / d);
120             multiAndAcumWithShift(x, -q.num[i], i-1);
121             if (i == 1 || j == 1) break;
122             num[j-1] += BASE * num[j];
123             num[j] = 0;
124         }
125
126         fixInvariant(x.numDigits);
127         return q.fixInvariant();
128     }
129
130     BigInt& operator *=(const BigInt& x) { return *this = (*this) * x; }
131     BigInt operator +(const BigInt& x) { return BigInt(*this) += x; }
132     BigInt operator -(const BigInt& x) { return BigInt(*this) -= x; }
133     BigInt operator -( ) { BigInt r = 0; return r -= *this; }
134     BigInt& operator /=(const BigInt& x) { return *this = div(x); }
135     BigInt& operator %=(const BigInt& x) { div(x); return *this; }
136     BigInt operator /(const BigInt& x) { return BigInt(*this).div(x); }
137     BigInt operator %(const BigInt& x) { return BigInt(*this) %= x; }
138
139     // I/O
140     operator string() const
141     {
142         ostringstream s; s << num[numDigits];
143         for (int i = numDigits - 1; i > 0; i--)
144         {
145             s.width(DIG);
146             s.fill('0');

```

```

147         s << abs(num[i]);
148     }
149
150     return s.str();
151 }
152
153 friend ostream& operator <<(ostream& o, const BigInt& x)
154 {
155     return o << (string) x;
156 }
157
158 friend istream& operator >>(istream& in, BigInt& x)
159 {
160     string num;
161     in >> num;
162     x = BigInt((char*) num.c_str());
163     return in;
164 }
165
166 // potencia e raiz
167 BigInt pow(int x)
168 {
169     if (x < 0) return (*this == 1 || *this == -1) ? pow(-x) : 0;
170     BigInt r = 1;
171     for (int i = 0; i < x; i++) r *= *this;
172     return r;
173 }
174
175 BigInt root(int x)
176 {
177     if (cmp() == 0 || cmp() < 0 && x % 2 == 0) return 0;
178     if (*this == 1 || x == 1) return *this;
179     if (cmp() < 0) return -(*this).root(x);
180     BigInt a = 1, d = *this;
181     while (d != 1)
182     {
183         BigInt b = a + (d /= 2);
184         if (cmp(b.pow(x)) >= 0) { d += 1; a = b; }
185     }
186
187     return a;
188 }
189 };

```

## 2.4 Programação Dinâmica

Código 26: **Sub Set Sum**: Verifica se há um subconjunto dos elementos do vetor cuja soma seja igual a soma pedida.

```

1 //soma maxima dos elementos do vetor
2 #define MAXSUM 10000
3 int n;
4 int vet[TAM];
5 bool m[MAXSUM];

```

```

6
7 //M->soma maxima dos elementos do vetor c->soma procurada
8 bool subSetSum(int M, int c)
9 {
10     for (int i = 0; i <= M; i++) m[i] = false;
11     m[0] = true;
12
13     for(int i = 0; i < n; i++)
14     {
15         for(int j = M; j >= vet[i]; j--)
16         {
17             m[j] |= m[j - vet[i]];
18         }
19     }
20
21     return m[c];
22 }

```

---

### Código 27: **Lis: longest increasing (decreasing) subsequence** $O(n^2)$

---

```

1 #define TAM 10000
2 int c[TAM];
3 int A[TAM];
4 int H[TAM];
5
6 void ssctf(int n)
7 {
8     for (int m = 1; m <= n; m++)
9     {
10         c[m] = H[m];
11         for (int i = m - 1; i > 0; i--)
12         {
13             if (A[i] < A[m] && c[i] + H[m] > c[m])
14             {
15                 c[m] = c[i] + H[m];
16             }
17         }
18     }
19 }
20
21 void ssdtf(int n)
22 {
23     for (int m = 1; m <= n; m++)
24     {
25         c[m] = H[m];
26         for (int i = m - 1; i > 0; i--)
27         {
28             if (A[i] > A[m] && c[i] + H[m] > c[m])
29             {
30                 c[m] = c[i] + H[m];
31             }
32         }
33     }
34 }
35
36 int lis1d(int n, bool inc = true)

```

```

37 {
38     if (inc) ssctf(n);
39     else ssdtf(n);
40
41     int max = 0;
42
43     for (int i = 1; i <= n; i++)
44         if (max < c[i])
45             max = c[i];
46
47     return max;
48 }

```

---

Código 28: Lis: longest increasing subsequence  $O(n \log n)$

---

```

1 // Longest Increasing Subsequence - LIS  $O(n \log n)$ 
2 #define fori(i, n) for ( int i = 0; i < (n); ++i )
3 void lis( const vector< int > & v, vector< int > & asw )
4 {
5     vector<int> pd(v.size(), 0), pd_index(v.size()), pred(v.size());
6     int maxi = 0, x, j, ind;
7
8     fori(i, v.size())
9     {
10         x = v[i];
11         j = lower_bound( pd.begin(), pd.begin() + maxi, x ) - pd.begin();
12         //j = upper_bound( pd.begin(), pd.begin() + maxi, x ) - pd.begin()
13         ; para lds
14         pd[j] = x;
15         pd_index[j] = i;
16         if( j == maxi ) { maxi++; ind = i; }
17         pred[i] = j ? pd_index[j-1] : -1;
18     }
19     // return maxi; se a sequencia nao precisa ser refeita
20
21     int pos = maxi-1, k = v[ind];
22     asw.resize( maxi );
23
24     while ( pos >= 0 )
25     {
26         asw[pos--] = k;
27         ind = pred[ind];
28         k = v[ind];
29     }

```

## 2.5 Grafos

---

Código 29: Verifica se o grafo é aciclico.

---

```

1 #define TAM 100
2 #define BRANCO 0
3 #define CINZA 1

```

```

4 #define PRETO 2
5 bool grafo[TAM][TAM];
6 int pass[TAM];
7
8 bool dfs( int v )
9 {
10     pass[v] = CINZA;
11
12     for ( int i = 0; i < TAM; i++)
13     {
14         if ( grafo[v][i] )
15         {
16             if (pass[i] == CINZA) return false;
17             if (pass[i] == BRANCO && !dfs(i)) return false;
18         }
19     }
20
21     pass[v] = PRETO;
22     return true;
23 }
24
25 bool aciclico()
26 {
27     memset( pass, BRANCO, TAM*sizeof(int) );
28
29     for ( int i = 0; i < TAM; i++)
30     {
31         if (pass[i] == BRANCO)
32         {
33             if (!dfs(i)) return false;
34         }
35     }
36
37     return true;
38 }

```

---

Código 30: Dijkstra Caminho minimo 1 para todos pesos positivos.

---

```

1 #include <queue>
2
3 typedef vector<map<int, int> > AdjList;
4 typedef AdjList Grafo;
5
6 int dist[MAX_VERTICES];
7 int prev[MAX_VERTICES]; // para recuperar o caminho usando um disjoint
8                             forest set
9
10 void dijkstra(Grafo& grafo, int source)
11 {
12     for ( int i = 0; i < grafo.size(); i++)
13     {
14         dist[i] = INF;
15         prev[i] = -1;
16     }
17
18     dist[source] = 0;

```

```

18 priority_queue<pair<int, int> > heap;
19 heap.push(make_pair(0, source));
20
21 while (!heap.empty())
22 {
23     int u = heap.top().second;
24     heap.pop();
25
26     // para cada vizinho de u
27     for (map<int, int>::iterator i = grafo[u].begin(); i != grafo[u].
28         end(); i++)
29     {
30         int totalDist = dist[u] + (*i).second;
31         if (totalDist <= dist[(*i).first])
32         {
33             dist[(*i).first] = totalDist;
34             heap.push(make_pair(totalDist, (*i).first));
35             prev[(*i).first] = u;
36         }
37     }
38 }

```

Código 31: Floresta disjunta de arvores

```

1 #define SIZE 100
2
3 struct dsf
4 {
5     int element_count;
6     int parent[SIZE];
7     int rank[SIZE];
8 };
9 typedef struct dsf * disjoint_set_forest_p;
10
11 void dsf_init(disjoint_set_forest_p forest, int element_count)
12 {
13     forest->element_count = element_count;
14     memset(forest->parent, 0, element_count*sizeof(int));
15     memset(forest->rank, 0, element_count*sizeof(int));
16
17     for (int i = 0; i < element_count; ++i)
18         forest->parent[i] = i;
19 }
20
21 int dsf_find_set(disjoint_set_forest_p forest, int i)
22 {
23     if (i != forest->parent[i])
24     {
25         forest->parent[i] = dsf_find_set(forest, forest->parent[i]);
26     }
27     return forest->parent[i];
28 }
29
30 void dsf_union(disjoint_set_forest_p forest, int i, int j)
31 {

```

```

32     int x = dsf_find_set(forest, i);
33     int y = dsf_find_set(forest, j);
34
35     if (forest->rank[x] > forest->rank[y])
36     {
37         forest->parent[y] = x;
38     }
39     else
40     {
41         forest->parent[x] = y;
42         if (forest->rank[x] == forest->rank[y])
43         {
44             forest->rank[y]++;
45         }
46     }
47 }

```

Código 32: Kruskal Arvore geradora mínima kruskal

```

1 typedef vector<map<int, int> > AdjList;
2 struct Grafo
3 {
4     int edgeCnt;
5     AdjList adj;
6 };
7
8 struct edge
9 {
10     int u;
11     int v;
12     int weight;
13 };
14
15 int edge_compare(const void * e1, const void * e2)
16 {
17     struct edge * p1 = (struct edge *) e1;
18     struct edge * p2 = (struct edge *) e2;
19     int f = p1->weight - p2->weight;
20     if (f < 0)
21     {
22         return -1;
23     }
24     else if (f == 0)
25     {
26         return edge_compare1(e1, e2);
27     }
28     else
29     {
30         return 1;
31     }
32 }
33
34 struct edge * get_edge_list(Grafo& graph)
35 {
36     int edge_count = graph.edgeCnt;

```

```

37     struct edge *edges = (struct edge*) malloc(edge_count * sizeof(
38         struct edge));
39     int current_edge = 0;
40     for (int i = 0; i < graph.adj.size(); ++i)
41     {
42         for (map<int, int>::iterator j = graph.adj[i].begin(); j !=
43             graph.adj[i].end(); j++)
44         {
45             struct edge e;
46             e.u = i < (*j).first ? i : (*j).first;
47             e.v = i > (*j).first ? i : (*j).first;
48             e.weight = (*j).second;
49             edges[current_edge++] = e;
50         }
51     }
52     return edges;
53 }
54
55 void kruskal(Grafo& graph, Grafo& mst)
56 {
57     // Obtain a list of edges and sort it by weight in  $O(E \lg E)$  time
58     int edge_count = graph.edgeCnt;
59     struct edge *edges = get_edge_list(graph);
60     qsort(edges, edge_count, sizeof(struct edge), edge_compare);
61
62     disjoint_set_forest dsf;
63     dsf_init(&dsf, edge_count);
64
65     for (int i = 0; i < edge_count; ++i)
66     {
67         struct edge e = edges[i];
68         int uset = dsf_find_set(dsf, e.u);
69         int vset = dsf_find_set(dsf, e.v);
70         if (uset != vset)
71         {
72             mst.adj[e.u][e.v] = e.weight;
73             mst.edgeCnt++;
74             dsf_union(dsf, uset, vset);
75         }
76     }
77     free(edges);
78 }
79
80 }

```

Código 33: verifica se um grafo é bipartido

```

1 #define TAM 200
2
3 bool grafo[TAM][TAM];
4 int pass[TAM];
5 int n;
6
7 bool bipartido(int v, int color = 1)

```

```

8 {
9     pass[v] = color;
10    int thisColor = color;
11    bool ret = true;
12
13    color = color == 1 ? 2 : 1;
14
15    for (int i = 0; i < n; i++)
16    {
17        if (grafo[v][i])
18        {
19            if (!pass[i]) ret = dfs(i, color);
20            else if (pass[i] == thisColor) return false;
21
22            if (!ret) return false;
23        }
24    }
25
26    return ret;
27 }

```

Código 34: faz a ordenação topológica de um grafo acíclico

```

1 #define UNVISITED -1
2
3 int grafo[SIZE][SIZE];
4 int prof[SIZE];
5 int sorted[SIZE];
6 int nordem;
7
8 void dfsTopsort(int no)
9 {
10     for (int viz = 0; viz < SIZE; viz++)
11     {
12         if (grafo[no][viz])
13         {
14             if (prof[viz] == UNVISITED)
15             {
16                 prof[viz] = prof[no] + 1;
17                 dfsTopsort(viz);
18             }
19         }
20     }
21
22     sorted[nordem++] = no;
23 }
24
25 void topSort(int nvt)
26 {
27     memset(prof, UNVISITED, nvt*sizeof(int));
28     nordem = nvt - 1;
29
30     for (int i = 0; i < nvt; i++)
31     {
32         if (prof[i] == UNVISITED)
33         {

```

```

34     prof[i] = 0;
35     dfsTopsort(i);
36 }
37 }
38 }

```

---

Código 35: calcula fluxo máximo, **Ford-Fulkerson**

---

```

1 #define TAM 1000
2 #define MAX_INT 1000000
3
4 int grafo[TAM][TAM];
5 int pred[TAM];
6 int f[TAM][TAM];
7 bool visitados[TAM];
8 int fila[TAM];
9
10 bool bfs(int n, int ini, int fim)
11 {
12     int no, s = 0, e = 0;
13     fila[e++] = ini;
14
15     while (s != e)
16     {
17         no = fila[s++];
18
19         if (visitados[no] == 2) continue;
20         visitados[no] = 2;
21
22         for (int i = 0; i < n; i++)
23         {
24             if (visitados[i] < 2)
25             {
26                 if (grafo[no][i] - f[no][i] > 0)
27                 {
28                     pred[i] = no;
29                     if (i == fim) return true;
30                     if (visitados[i] == 0)
31                     {
32                         fila[e++] = i;
33                         visitados[i] = 1;
34                     }
35                 }
36             }
37         }
38     }
39
40     return false;
41 }
42
43 bool dfs(int s, int t, int size)
44 {
45     visitados[s] = true;
46     if (s == t) return true;
47
48     for (int v = 0; v < size; v++)

```

```

49     {
50         if (!visitados[v] && grafo[s][v] - f[s][v] > 0)
51         {
52             pred[v] = s;
53             if (dfs(v, t, size)) return true;
54         }
55     }
56
57     return false;
58 }
59
60 bool findPath(int s, int t, int size)
61 {
62     memset(visitados, false, sizeof(bool)*size);
63     pred[s] = s;
64     // Aqui pode ser usado tanto busca em largura quanto em profundidade.
65     // busca em largura geralmente apresenta tempos de execucao bem
66     menores.
67     return bfs(size, s, t);
68     //return dfs(s, t, size);
69 }
70
71 int maxFlow(int size, int s, int t)
72 {
73     int delta;
74
75     for (int i = 0; i < size; i++)
76     {
77         memset(f[i], 0, sizeof(int)*size);
78     }
79
80     while(1)
81     {
82         bool path = findPath(s, t, size);
83         if (!path) break;
84
85         delta = MAX_INT;
86         for (int c = t; pred[c] != c; c = pred[c])
87         {
88             delta = min(delta, grafo[pred[c]][c] - f[pred[c]][c]);
89         }
90
91         for (int c = t; pred[c] != c; c = pred[c])
92         {
93             f[pred[c]][c] += delta;
94             f[c][pred[c]] -= delta;
95         }
96     }
97
98     int soma = 0;
99
100     for (int i = 0; i < size; i++)
101     {
102         soma += f[i][t];
103     }

```

```

104     return soma;
105 }

```

Código 36: calcula fluxo máximo, algoritmo mais eficiente porém muito maior em tempo de codificação

```

1  const int VT = 100;
2  const int AR = VT * VT;
3
4  struct grafo
5  {
6      // lista de adjacencias representada na forma de vetor
7      int nvt, nar;
8      int dest[2 * AR];
9      int adj[VT][2 * VT];
10     int nadj[VT];
11
12     int cap[AR]; // capacidade do arco
13     int fluxo[AR];
14     int ent[VT];
15
16     int padj[VT], lim[VT], nivel[VT], qtd[VT];
17
18     int inv(int a) { return a ^ 0x1; }
19     int orig(int a) { return dest[inv(a)]; }
20     int capres(int a) { return cap[a] - fluxo[a]; }
21
22     void inic(int n = 0)
23     {
24         nvt = n;
25         nar = 0;
26         memset(nadj, 0, sizeof(nadj));
27     }
28
29     //////////////////////////////////////////
30     // Adiciona uma aresta ao grafo.
31     //
32     // "int u" apenas para Fluxos;
33     //
34     int aresta(int i, int j, int u = 0)
35     {
36         int ar = nar;
37         cap[nar] = u;
38         dest[nar] = j;
39         adj[i][nadj[i]] = nar++;
40         nadj[i]++;
41
42         cap[nar] = 0;
43         dest[nar] = i;
44         adj[j][nadj[j]] = nar++;
45         nadj[j]++;
46         return ar;
47     }
48
49     void revbfs(int ini, int fim)
50     {

```

```

51     int i, no, viz, ar;
52     queue<int> fila;
53
54     memset(nivel, NULO, sizeof(nivel));
55     memset(qtd, 0, sizeof(qtd));
56
57     nivel[fim] = 0;
58     fila.push(fim);
59
60     while (!fila.empty())
61     {
62         no = fila.front();
63         fila.pop();
64         qtd[nivel[no]]++;
65
66         for (i = 0; i < nadj[no]; i++)
67         {
68             ar = adj[no][i];
69             viz = dest[ar];
70
71             if (cap[ar] == 0 && nivel[viz] == NULO)
72             {
73                 nivel[viz] = nivel[no] + 1;
74                 fila.push(viz);
75             }
76         }
77     }
78
79     int admissivel(int no)
80     {
81         while (padj[no] < nadj[no])
82         {
83             int ar = adj[no][padj[no]];
84             if (nivel[no] == nivel[dest[ar]] + 1 && capres(ar) > 0) return
85                 ar;
86             padj[no]++;
87         }
88
89         padj[no] = 0;
90         return NULO;
91     }
92
93     int retrocede(int no)
94     {
95         int i, ar, viz, menor = NULO;
96
97         if (--qtd[nivel[no]] == 0) return NULO;
98
99         for (i = 0; i < nadj[no]; i++)
100         {
101             ar = adj[no][i]; viz = dest[ar];
102             if (capres(ar) <= 0) continue;
103             if (menor == NULO || nivel[viz] < nivel[menor]) menor = viz;
104         }
105

```



```

106     if (menor != NULO) nivel[no] = nivel[menor];
107     qtd[++nivel[no]]++;
108
109     return ((ent[no] == NULO) ? no : orig(ent[no]));
110 }
111
112 int avanca(int no, int ar)
113 {
114     int viz = dest[ar];
115     ent[viz] = ar;
116     lim[viz] = min(lim[no], capres(ar));
117     return viz;
118 }
119
120 int aumenta(int ini, int fim)
121 {
122     int ar, no = fim, fmax = lim[fim];
123
124     while (no != ini)
125     {
126         fluxo[ar = ent[no]] += fmax;
127         fluxo[inv(ar)] -= fmax;
128         no = orig(ar);
129     }
130
131     return fmax;
132 }
133
134 int maxflow(int ini, int fim)
135 {
136     int ar, no = ini, fmax = 0;
137
138     memset(fluxo, 0, sizeof(fluxo));
139     memset(padj, 0, sizeof(padj));
140
141     revbfs(ini, fim);
142
143     lim[ini] = INF;
144     ent[ini] = NULO;
145
146     while (nivel[ini] < nvt && no != NULO)
147     {
148         if ((ar = admissivel(no)) == NULO)
149         {
150             no = retrocede(no);
151         }
152         else if ((no = avanca(no, ar)) == fim)
153         {
154             fmax += aumenta(ini, fim);
155             no = ini;
156         }
157     }
158     return fmax;
159 }
160 };

```

## 2.6 Geometria

Código 37: ponto e poligono

```

1 struct point
2 {
3     double x, y;
4     double z; // para pontos no espaco
5     point(double x = 0, double y = 0, double z = 0): x(x), y(y), z(z) {}
6
7     point operator +(point q) { return point(x + q.x, y + q.y, z + q.z); }
8     point operator -(point q) { return point(x - q.x, y - q.y, z - q.z); }
9     point operator *(double t) { return point(x * t, y * t, z * t); }
10    point operator /(double t) { return point(x / t, y / t, z / t); }
11    double operator *(point q) { return x * q.x + y * q.y + z * q.z; }
12    point vec(point q) { return point(y * q.z - z * q.y, z * q.x - x * q.z,
13                                     x * q.y - y * q.x); }
13    double operator %(point q) { return x * q.y - y * q.x; }
14
15    int cmp(point q) const
16    {
17        if (int t = ::cmp(x, q.x)) return t;
18        else if (int t = ::cmp(y, q.y)) return t;
19        return ::cmp(z, q.z);
20    }
21
22    bool operator ==(point q) const { return cmp(q) == 0; }
23    bool operator !=(point q) const { return cmp(q) != 0; }
24    bool operator < (point q) const { return cmp(q) < 0; }
25
26    friend ostream& operator <<(ostream& o, point p) {
27        return o << "(" << p.x << ", " << p.y << ", " << p.z << ")";
28    }
29    static point pivot;
30 };
31
32 // para pontos 2D
33 double abs(point p) { return hypot(p.x, p.y); }
34 double arg(point p) { return atan2(p.y, p.x); }
35
36 point point::pivot;
37
38 typedef vector<point> polygon;
39
40 int ccw(point p, point q, point r)
41 {
42     return cmp((p - r) % (q - r));
43 }
44
45 double angle(point p, point q, point r)
46 {
47     point u = p - q, v = r - q;
48     return atan2(u % v, u * v);
49 }

```

Código 38: Decide se q está sobre o segmento fechado pr.

```

1 bool between(point p, point q, point r)
2 {
3     return ccw(p, q, r) == 0 && cmp((p - q) * (r - q)) <= 0;
4 }

```

Código 39: Decide se os segmentos fechados pq e rs têm pontos em comum.

```

1 bool seg_intersect(point p, point q, point r, point s)
2 {
3     point A = q - p;
4     point B = s - r;
5     point C = r - p;
6     point D = s - q;
7
8     int a = cmp(A % C) + 2 * cmp(A % D);
9     int b = cmp(B % C) + 2 * cmp(B % D);
10
11     if (a == 3 || a == -3 || b == 3 || b == -3) return false;
12     if (a || b || p == r || p == s || q == r || q == s) return true;
13
14     int t = (p < r) + (p < s) + (q < r) + (q < s);
15     return t != 0 && t != 4;
16 }

```

Código 40: Calcula a distância do ponto r ao segmento pq.

```

1 double seg_distance(point p, point q, point r)
2 {
3     point A = r - q;
4     point B = r - p;
5     point C = q - p;
6
7     double a = A * A, b = B * B, c = C * C;
8
9     if (cmp(b, a + c) >= 0) return sqrt(a);
10    else if (cmp(a, b + c) >= 0) return sqrt(b);
11    else return fabs(A % B) / sqrt(c);
12 }

```

Código 41: Classifica o ponto p em relação ao polígono T. Retorna 0, -1 ou 1 dependendo se p está no exterior, na fronteira ou no interior de T, respectivamente.

```

1 int in_poly(point p, polygon& T)
2 {
3     double a = 0;
4     int N = T.size();
5     for (int i = 0; i < N; i++)
6     {
7         if (between(T[i], p, T[(i+1) % N])) return -1;
8         a += angle(T[i], p, T[(i+1) % N]);
9     }
10    return cmp(a) != 0;

```

```

11 }

```

Código 42: Convex Hull.

```

1 #include <cstdio>
2 #include <algorithm>
3 #include <vector>
4 #include <string>
5 #include <cmath>
6
7 using namespace std;
8
9 #define INF 1e9
10 #define EPS 1e-9
11
12 int cmp(double a, double b = 0.0) {
13     return a+EPS < b ? -1 : a-EPS > b;
14 }
15
16 struct Point {
17     double x, y;
18     Point(double a=0.0, double b=0.0) {x=a, y=b;}
19     Point operator+(const Point &P) const {return Point(x+P.x, y+P.y);}
20     Point operator-(const Point &P) const {return Point(x-P.x, y-P.y);}
21     Point operator*(double c) const {return Point(x*c, y*c);}
22     Point operator/(double c) const {return Point(x/c, y/c);}
23     double operator!() const {return sqrt(x*x+y*y);}
24     bool operator==(const Point &p) const {return !cmp(x, p.x) && !cmp(y, p.y);}
25     bool operator<(const Point &p) const {if (cmp(x, p.x)) return cmp(x, p.x) < 0; return cmp(y, p.y) < 0;}
26     void print(string prefix = "") const {printf("%s%.31f %.31f\n", prefix.c_str(), x, y);}
27 };
28
29 typedef vector<Point> Polygon;
30
31 double cross(Point A, Point B) {
32     return A.x*B.y - B.x*A.y;
33 }
34
35 Point pmin;
36
37 bool lessThan(Point A, Point B) {
38     if (cmp(cross(A-pmin, B-pmin))) return cmp(cross(A-pmin, B-pmin)) > 0;
39     return cmp(!(pmin-A), !(pmin-B)) < 0;
40 }
41
42 int sort(Polygon &p) {
43     int imin = 0, i, j, n = p.size();
44
45     for (i=1; i < p.size(); i++) {
46         if (p[i] < p[imin]) imin = i;
47     }
48
49     swap(p[0], p[imin]);

```

```

50 pmin = p[0];
51
52 sort(p.begin()+1,p.end(),lessThan);
53
54 for (i=n-1; i > 0 && !cmp(cross(p[i]-p[0],p[i-1]-p[0])); i--);
55 if (i == 0) return 1;
56 for (j=0; j < (n-i)/2; j++) swap(p[i+j],p[n-j-1]);
57 return 0;
58 }
59
60 Polygon convex_hull(Polygon &p) { // tirar o & para nao alterar o
    poligono original
61     int hs = 2, n = p.size();
62     Polygon hull;
63
64     if (p.size() < 3) return p;
65
66     int isline = sort(p);
67     hull.push_back(p[0]), hull.push_back(p[1]);
68
69     for (int i=2; i <= n-isline; i++) {
70         while (hull.size() > 1 && cmp(cross(p[i%n]-hull[hs-1],hull[hs-2]-
            hull[hs-1])) <= 0) { // trocar o <= para < para manter pontos
                intermediarios
71             hull.pop_back(), hs--;
72         }
73         if (i < n) hull.push_back(p[i]), hs++;
74     }
75
76     return hull;
77 }
78
79 int main() {
80     Polygon p;
81
82     while (...) {
83         double x = ...;
84         double y = ...;
85         p.push_back(Point(x,y));
86     }
87
88     Polygon hull = convex_hull(p);
89
90     for (int i=0; i < hull.size(); i++) {
91         ...
92     }
93 }

```

```

3 #include <algorithm>
4
5 using namespace std;
6
7 #define fori(i,n) for(int i=0; i < (n); ++i)
8 #define forr(i,a,b) for(int i=(a); i <= (b); ++i)
9 #define ford(i,a,b) for(int i=(a); i >= (b); --i)
10 #define sz size()
11
12 const double EPS=1e-9; const int INF = 0x3f3f3f3f;
13
14 #define all(x) (x).begin(),(x).end()
15
16 int cmpD(double x, double y=0, double tol=EPS) {
17     return (x <= y+tol) ? (x+tol<y) ? -1 : 0 : 1;
18 }
19
20 struct simplex {
21     // max c * x, s.t: A * x <= b; x >= 0
22     simplex(const vector< vector< double > > &A_, const vector< double
        > &b_) : A(A_), b(b_), c(c_) {}
23     const vector< double > &c_ : A(A_), b(b_), c(c_) {}
24     vector< vector< double > > A; vector< double > b, c, sol;
25     vector< bool > N; vector< int > kt; int m, n;
26     void pivot(int k, int l, int e) {
27         int x = kt[l]; double p = A[l][e];
28         fori(i,k) A[l][i] /= p;
29         b[l] /= p; N[e] = false;
30         fori(i,m) if (i != l) {b[i] -= A[i][e]*b[l]; A[i][x] = -A[i][e]*A[
            l][x];}
31         fori(j,k) if (N[j]) {
32             c[j] -= c[e] * A[l][j];
33             fori(i,m) if (i != l) A[i][j] -= A[i][e] * A[l][j];
34         }
35         kt[l] = e; N[x] = true; c[x] = -c[e] * A[l][x];
36     }
37     vector< double > go(int k) {
38         vector< double > res;
39         while (1) {
40             int e = -1, l = -1;
41             fori(i,k) if (N[i] && cmpD(c[i]) > 0) { e = i; break; }
42             if (e == -1) break;
43             fori(i,m) if (cmpD(A[i][e]) > 0 && (l == -1 || cmpD(b[i] / A
                [i][e],
44                 b[l] / A[l][e], 1e-20) < 0)) l = i;
45             if (l == -1) return vector< double >(); // unbounded
46             pivot(k, l, e);
47         }
48         res.resize(k, 0);
49         fori(i,m) res[kt[i]] = b[i];
50         return res;
51     }
52     vector< double > solve() {
53         m = A.sz; n = A[0].sz; int k = m+n+1;
54         N = vector< bool >(k, true); vector< double > c_copy = c;
55         c.resize(n+m); kt.resize(m);

```

## 2.7 Algebra Linear

Código 43: Simplex

```

1 #include <iostream>
2 #include <vector>

```

```

56     fori(i,m) {
57         A[i].resize(k); A[i][n+i] = 1; A[i][k-1] = -1;
58         kt[i] = n+i; N[kt[i]] = false;
59     }
60     int l = min_element(all(b)) - b.begin();
61     if(cmpD(b[l]) < 0) {
62         c = vector<double>(k,0);
63         c[k-1] = -1; pivot(k, l, k-1); sol=go(k);
64         if(cmpD(sol[k-1])>0) return vector<double>(); // infeasible
65         fori(i,m) if(kt[i] == k-1) {
66             fori(j,k-1) if(N[j] && cmpD( A[i][j] ) != 0 ) {
67                 pivot( k, i, j); break;
68             }
69         }
70         c=c.copy; c.resize(k,0);
71         fori(i,m) fori(j,k) if(N[j]) c[j] -= c[kt[i]]*A[i][j];
72     }
73     sol = go(k-1);
74     if(!sol.empty()) sol.resize(n);
75     return sol;
76 }
77 };
78
79 // Como usar
80 int main() {
81     /* Exemplo: Maximize cx Subject to Ax <= b */
82     vector<vector<double>> A(9);
83     double Av[][3] = {{1,1,0}, {0,0,-1}, {-1,-1,0},
84                       {0,0,1}, {1,0,0}, {0,1,0},
85                       {0,0,1}, {1,0,1}, {0,1,0}};
86
87     for(int i=0; i < 9; i++) {
88         A[i].insert(A[i].begin(), &(Av[i][0]), &(Av[i][3])); // Sim, [3]!
89         // Ou seja, idx-final+1
90     }
91
92     vector<double> c(3, 1); // c=[1 1 1]
93     double bv[] = {2,-1,-2,1,2,1,1,2,1};
94     vector<double> b(bv, bv+sizeof(bv)/sizeof(double));
95
96     simplex sim(A,b,c);
97     vector<double> s = sim.solve();
98     if(!s.size()) cout << "Impossible\n";
99     else
100     for(int i=0; i < s.size(); i++) {
101         cout << s[i] << endl;
102     }

```

## 2.8 Casamento de strings

Código 44: String matching - Algoritmo KMP -  $O(n + m)$

1 //  $F[i]$  - size of the largest prefix of  $pattern[0..i]$  that is also a

```

2 // suffix of  $pattern[1..i]$ . Ex:  $pattern = \{a,b,a,c,a,b\}$ ,  $F =$ 
3 //  $\{0,0,1,0,1,2\}$ 
4 #define MAX_PATTERN_SIZE 10010
5 int F[MAX_PATTERN_SIZE];
6 void build_failure_function( const string & pattern )
7 {
8     int m = pattern.size();
9     F[0] = -1;
10    for (int i = 0; i < m; i++)
11    {
12        F[i+1] = F[i] + 1;
13        while ( F[i+1] > 0 && pattern[i] != pattern[ F[i+1]-1 ] )
14            F[i+1] = F[ F[i+1]-1 ] + 1;
15    }
16
17    // retorna a posicao inicial de cada ocorrencia de pattern em text
18    vector<int> KMP( const string & text, const string & pattern )
19    {
20        build_failure_function( pattern );
21        vector<int> start_positions;
22        int j = 0, m = pattern.size(), n = text.size();
23
24        for (int i = 0; i < n; i++)
25        {
26            while ( true )
27            {
28                if ( text[i] == pattern[j] )
29                {
30                    if ( ++j == m )
31                    {
32                        start_positions.push_back( i - m + 1 );
33                        j = F[j];
34                    }
35                    break;
36                }
37
38                if ( j == 0 ) break;
39                j = F[j];
40            }
41        }
42
43        return start_positions;
44    }

```

## 2.9 Outros

Código 45: josephus problem

```

1 /**
2 The Josephus problem (or Josephus permutation) is a theoretical problem
related to a certain counting-out game. There are people standing in
a circle waiting to be executed. After the first man is executed,
certain number of people are skipped and one man is executed. Then

```

```

again, people are skipped and a man is executed. The elimination
proceeds around the circle (which is becoming smaller and smaller as
the executed people are removed), until only the last man remains,
who is given freedom. The task is to choose the place in the initial
circle so that you are the last one remaining and so survive.
3  */
4
5  using namespace std;
6
7  int josephus(int n, int m)
8  {
9      int res = 0;
10     vector<int> people;
11     int loc = 0;
12
13     for (int i = 0; i < n; i++) people.push_back(i+1);
14
15     while (people.size() > 1)
16     {
17         if (loc >= people.size())
18             loc %= people.size();
19
20         people.erase(people.begin()+loc);
21         loc += (m-1);
22     }
23
24     return people[0];
25 }

```

Código 46: Gera as permutações dos elementos da string

```

1  bool nextPermutation(string& number)
2  {
3      bool isBigger = true;
4      int i, j;
5
6      for (i = number.size() - 1; i >= 0; i--)
7      {
8          if (number[i] < number[i+1]) break;
9      }
10
11     if (i != -1)
12     {
13         isBigger = false;
14
15         for (j = number.size() - 1; j >= i+1; j--)
16         {
17             if (number[j] > number[i])
18             {
19                 break;
20             }
21         }
22
23         int tmp = number[i];
24         number[i] = number[j];
25         number[j] = tmp;

```

```

26
27         j = number.size() - 1;
28         i++;
29
30         while (i < j)
31         {
32             tmp = number[i];
33             number[i] = number[j];
34             number[j] = tmp;
35             i++;
36             j--;
37         }
38     }
39
40     return isBigger;
41 }

```

Código 47: Exemplo de geração de permutações dos elementos da string, usando back-tracking

```

1  #define TAM 10
2  int cnt = 0;
3  int total; //numero de elementos de elem
4  int elem[TAM];
5  bool usados[TAM]; //ZERE ME
6
7  void enumera(int num)
8  {
9      if (num == total)
10     {
11         cnt++;
12         for (int i = 0; i < total; i++) cout << elem[i];
13         cout << endl;
14         return;
15     }
16
17     for (int i = 0; i < total; i++)
18     {
19         if (!usados[i])
20         {
21             elem[num] = i;
22             usados[i] = true;
23             enumera(num + 1);
24             usados[i] = false;
25         }
26     }
27 }

```

## 3 Biblioteca C/C++

### 3.1 I/O

Ignorando entradas na família scanf:

Código 48: Ignora os dois floats do meio. Retornará 2 no sucesso.

```
1 scanf("%f %*f %*f %d", &a, &b);
```

## 3.2 Map

Código 49: Referencias map

```
1 #include <map>
2 #include <string>
3 #include <cstdio>
4
5 using namespace std; // USE ISTO!!!
6
7 class Comparadora;
8
9 class Pessoa {
10     int idade;
11     string nome;
12     friend class Comparadora;
13 public:
14     Pessoa(string nome, int idade) {
15         this->idade = idade;
16         this->nome = nome;
17     }
18     void print() const {
19         printf("Nome: %s Idade: %d\n", nome.c_str(), idade);
20     }
21 };
22
23 class Comparadora { // Ordena crescentemente
24 public: // <- IMPORTANTE
25     bool operator() (const Pessoa &a, const Pessoa &b)
26     {
27         int idDif = a.idade-b.idade;
28         if(idDif < 0) return true;
29         else if(idDif==0) return a.nome.compare(b.nome) < 0 ? true : false
30         ;
31         else return false;
32     }
33
34 int main() {
35     Pessoa r("Rangelz", 86);
36     Pessoa r2("Rangelzao", 86);
37
38     map<Pessoa, string, Comparadora> alunos;
39     alunos[r]="UFMG";
40     alunos[r2]="PUC";
41     // Iterator
42     for(map<Pessoa, string, Comparadora>::iterator it=alunos.begin(); it
43         != alunos.end(); it++) {
44         it->first.print();
45         printf("\t%s\n\n", it->second.c_str());
46     }
```

```
46 // Find
47 if(alunos.find(Pessoa("Rangelz", 86)) != alunos.end()) { // Achou!
48     printf("Achei Rangel!\n");
49 }
50 return 0;
51 }
```