Nonlocal optimal probabilistic cloning of qubit states via twin photons

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Non-local optimal probabilistic cloning of qubit states via twin-photons

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Here we propose a feasible experimental setup for implementing optimal probabilistic cloning of two arbitrary nonorthogonal qubit states. This proposal is based on linear optics together with a pair of entangled twin-photons generated via spontaneous parametric down conversion. The states to be cloned are encoded in the polarization degree of freedom of one of the photons while two perfect clones are generated and encoded in the polarization and the spatial degree of freedom of the resting photon. Therefore, the clones are produced non-locally. The proposed implementation requires the application of recovery unitary operations onto the photon hosting the clones which are conditional on measurements performed on the other photon in a process akin to quantum teleportation.

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I. INTRODUCTION

The encoding of information in the states of quantum systems has been subject of a constant research effort in the last decades. Processes such as quantum teleportation [1], dense coding [2], and quantum error correction [3] exhibit properties unlike anything known from classical systems. On the other hand, classical processes such as the copying of information acquire unexpected features when examined from the perspective of the laws of quantum mechanics. This latter process, the copying of quantum encoded information, has proven to be forbidden by the linearity of quantum mechanics [4–7].

This no-cloning theorem does not preclude, however, the possibility of deterministically generating approximate copies of unknown states [8–12] or the probabilistic generation of perfect copies of a set of known states [13]. A probabilistic quantum cloning machine (PQCM) generates perfect copies of an arbitrary state, selected at random from a finite set of known nonorthogonal linearly independent states [14], with a non-unitary probability of success. Analytical solutions are known for the cloning of two arbitrary states and for the cloning of states with particular symmetries [15, 16]. Recently, the optimal probabilistic cloning machine has been experimentally implemented for first time [17] in the case of two clones of qubit states. This demonstration is carried out employing nuclear magnetic resonance techniques.

Here, we propose an all-optical experimental setup for implementing the PQCM in the case of two clones of qubit states. This setup is based on a single pair of entangled twin-photons and linear optics. Initially, two photons in a maximally entangled polarization state are created via spontaneous parametric down conversion. The polarization of one of the photons encodes the states to be cloned, which are selected randomly from a set of two nonorthogonal polarization states. This photon undergoes a sequence of unitary transformations and measurements which generates a positive-operator valued measure (POVM) with six elements. Two of them are associated to the failure of the cloning attempt. The subset of four remaining elements of the POVM leads to the conclusive generation of two perfect clones in the resting photon. This encodes one of the clones in its polarization degree of freedom while the other clone is encoded in its spatial degree of freedom. The generation of the clones requires the application of unitary transformations conditionally on the outcomes of the POVM. The total success probability achieved by our proposal equals the optimal one. Thus, the proposed experimental setup allows for implementing the process of optimal probabilistic cloning. Furthermore, since the clones are encoded in a photon different than the one encoding the states to be cloned, the experimental setup allows for the demonstration of a non-local, or remote, optimal probabilistic cloning. Since the setup uses two Hilbert spaces for each photon and one maximally entangled state it is not equivalent to a clone and teleport strategy, or its inverse.

II. PROBABILISTIC CLONING

The probabilistic cloning machine is constructed as a unitary-reduction process where a unitary transformation U is concatenated to a projective measurement. The unitary transformation acts jointly onto the original system o, which encodes the state to be cloned, the copy system c, which encodes the copy, and an ancillary system a. The action of the unitary transformation U is defined as

$$U|\psi_\pm\rangle_o|\Sigma\rangle_{ca} = \sqrt{p}|\psi_\pm\rangle_o|\psi_\pm\rangle_c|0\rangle_a + \sqrt{1-p}|\Phi\rangle_{oc}|1\rangle_a, \tag{1}$$

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where $|\psi_+\rangle_o$ and $|\psi_-\rangle_o$ are two nonorthogonal states encoded in the original system and $|\Sigma\rangle_{ca}$ is a blanck initial bipartite state of the systems copy and ancilla. After the application of the unitary transformation a measurement of the observable $O = \lambda_0|0\rangle_a\langle 0| + \lambda_1|1\rangle_a\langle 1|$ projects the ancilla system onto the state $|0\rangle_a$ with probability p. In this case original and copy systems are described by exactly the same state and the cloning process succeeds. Otherwise, with probability $1-p$, these systems are left in the state $|\Phi\rangle_{oc}$ independently of the initial state of the original system. In this case the cloning attempt fails. The optimal cloning probability p is given by [14]

$$p = \frac{1}{1+|\langle\psi_+|\psi_-\rangle|}, \tag{2}$$

where we have assumed that the states to be cloned are generated with the same a priory probability.

III. PROPOSAL FOR PROBABILISTIC CLONING VIA TWIN PHOTONS

In the case of photons the implementation of a PQCM requires at least of two photons. The implementation of the machine needs three different Hilbert spaces and a projective measurement on one of them. It is possible in principle to consider several degrees of freedom of a single photon. However, the projective measurement on the ancilla's Hilbert space destroys the photon together with the clones encodes in it. For this reason we consider two photons. Furthermore, according Eq. (1) the machine also requires a unitary transformation which creates entanglement among the involved Hilbert spaces. Therefore, our starting point is the maximally entangled polarization state of photons 1 and 2 given by

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}}(|h\rangle_1|h\rangle_2 + |v\rangle_1|v\rangle_2), \tag{3}$$

where h and v indicate horizontal and vertical lineal polarization. This state is typically produced by spontaneous parametric down conversion of type II. The use of a maximally entangled state as a resource is already considered in the theoretical description of the PQCM [13]. A combination of two polarizing beam splitters (PBS_1 and PBS_2) and a half-wave plate (HWP_1) transforms the previous state into the state

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}|h\rangle_1(|0\rangle_1|h\rangle_2|0\rangle_2 + |1\rangle_1|v\rangle_2|1\rangle_2), \tag{4}$$

where $|0\rangle_i$ and $|1\rangle_i$ are two distinguishable paths for photons $i=1,2$ and the polarization of photon 1 is now factorized from the other degrees of freedom. This degree of freedom encodes the nonorthogonal states $|\psi_\pm\rangle_1$ to be cloned. These are generated by a second set of two half-wave plates HWP_s as $|\psi_\pm\rangle_1 = a|h\rangle_1 \pm b|v\rangle_1$ where the coefficients $a=\cos(\theta/2)$ and $b=\sin(\theta/2)$ are controlled by the tilting angle of HWP_s. A third half-wave plate HWP_3 placed on path 1 of photon 2 transforms vertical polarization into horizontal polarization. This leads to the state

$$\begin{aligned}|\Psi_2\rangle =& \tfrac{1}{\sqrt{2}}(a|h\rangle_1|1\rangle_1|h\rangle_2|1\rangle_2 \pm b|v\rangle_1|0\rangle_1|h\rangle_2|0\rangle_2 \\ &+ a|h\rangle_1|0\rangle_1|h\rangle_2|0\rangle_2 \pm b|v\rangle_1|1\rangle_1|h\rangle_2|1\rangle_2).\end{aligned} \tag{5}$$

Photons propagating on path 0 are deviated to paths $\tilde{0}$ and 0 by polarizing beam splitter PBS_3. Photons propagating on path 1 are deviated to paths $\tilde{1}$ and 1 by polarizing beam splitter PBS_4. Considering half-wave plates HWP_4 and HWP_5 on paths 0 and $\tilde{1}$ respectively, which transform vertical into horizontal polarization, we obtain the state

$$\begin{aligned}|\Psi_3\rangle =& \tfrac{1}{\sqrt{2}}(a|h\rangle_1|1\rangle_1|h\rangle_2|1\rangle_2 \pm b|h\rangle_1|0\rangle_1|h\rangle_2|0\rangle_2 \\ &+ a|h\rangle_1|\tilde{0}\rangle_1|h\rangle_2|0\rangle_2 \pm b|h\rangle_1|\tilde{1}\rangle_1|h\rangle_2|1\rangle_2).\end{aligned} \tag{6}$$

This set of transformations concludes the preparation of the input state of the PQCM. With the exception of half-wave plates HWP_S, these transformations do not depend on the coefficients defining the states to be cloned. As we shall see, this preparation stage is crucial in achieving the optimal success probability of the PQCM.

The next stage implements the PQCM. This is composed of a set of half-wave plates, beam splitters and polarizing beam splitters. Here, the information about the states to be cloned enters as the angle θ defining coefficients $\alpha, \beta, \tilde{\alpha}$ and $\tilde{\beta}$ in polarization superpositions created by means of half-wave plates. These coefficients are given by the relations

$$\begin{aligned}\alpha &= \frac{1}{\sqrt{1+\tan^4(\theta/2)}}, \quad \beta = \frac{\tan^2(\theta/2)}{\sqrt{1+\tan^4(\theta/2)}}, \\ \tilde{\alpha} &= \sqrt{\frac{1+\tan^4(\theta/2)}{2}}, \quad \tilde{\beta} = \sqrt{\frac{1-\tan^4(\theta/2)}{2}}.\end{aligned} \tag{7}$$

The first part of the PQCM acts onto photon 2. This photon is firstly transformed by two half-wave plates. The first one, HWP_6, is located on propagation path 0 and generates a equally weighted superposition of horizontal and vertical polarization. The second one, HWP_7, is located at propagation path 1 and creates a polarization superposition of the form $\alpha|h\rangle_2 + \beta|v\rangle_2$. Therefater, a CNOT transformation between polarization and path is carried out via a polarizing beam splitter PBS_5. This generates the state

$$|\Psi_4\rangle = \frac{1}{\sqrt{2}}(|\phi\rangle + |\tilde{\phi}\rangle), \tag{8}$$

with

$$\begin{aligned}|\phi\rangle =& a\ |h\rangle_1|1\rangle_1(\alpha|h\rangle_2|0\rangle_2 + \beta|v\rangle_2|1\rangle_2) \\ &\pm b\frac{1}{\sqrt{2}}|h\rangle_1|0\rangle_1(|h\rangle_2|1\rangle_2 + |v\rangle_2|0\rangle_2),\end{aligned} \tag{9}$$

and

$$\begin{aligned}|\tilde{\phi}\rangle =& a\ |h\rangle_1|\tilde{0}\rangle_1\frac{1}{\sqrt{2}}(|h\rangle_2|1\rangle_2 + |v\rangle_2|0\rangle_2) \\ &\pm b|h\rangle_1|\tilde{1}\rangle_1(\alpha|h\rangle_2|0\rangle_2 + \beta|v\rangle_2|1\rangle_2).\end{aligned} \tag{10}$$

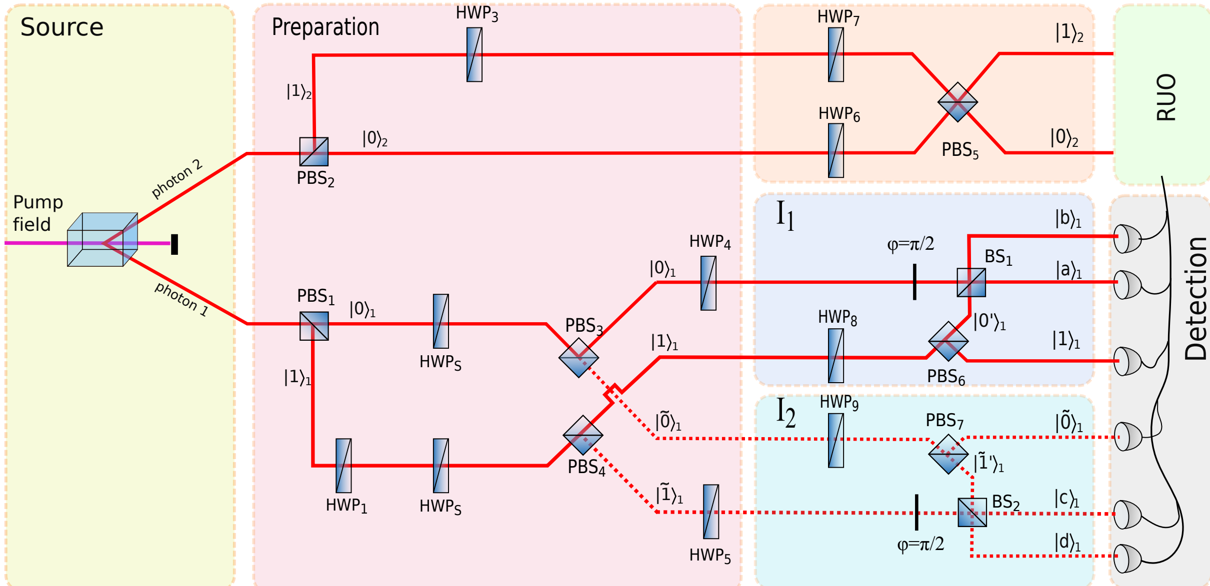

FIG. 1. Experimental setup for implementing non-local optimal probabilistic cloning. Two photons are generated in a maximally entangled polarization state. Photon 1 encodes the states to be cloned and photon 2 encodes the clones. Photon 1 is processed with the help of the lossy interferometers I_1 and I_2 and thereafter photo-detected. Conditional on the results of the photo-detection process unitary operations are applied onto photon 2 leading the generation of the clones.

Let us note that it is possible to map state $|\tilde{\phi}\rangle$ onto state $|\phi\rangle$ by interchanging the states of photon 2 and the path states $\{|\tilde{0}\rangle, |\tilde{1}\rangle\}$ with states $\{|1\rangle, |0\rangle\}$. These two states $|\phi\rangle$ and $|\tilde{\phi}\rangle$ are transformed using two lossy interferometers I_1 and I_2 respectively. The arms of interferometer I_1 are formed by the propagation paths 0 and 1 of photon 1. On path 1 half-wave plate HWP_8 generates the polarization superposition $\tilde{\alpha}|h\rangle_1 + \tilde{\beta}|v\rangle_1$ and on path 0 a phase difference of $\pi/2$ is introduced. Thereafter, the polarizing beam splitter PBS_6 allows vertically polarized photons to leave the interferometer. Finally, photons propagating at paths 0 and 1 are merged at beam splitter BS_1 whose output ports are designated by b and a. After these transformations a photo-detection on path 1 projects photon 2 onto the entangled state

$$\alpha|h\rangle_2|0\rangle_2 + \beta|v\rangle_2|1\rangle_2, \tag{11}$$

which does not corresponds to a cloning process. In this case the cloning attempt fails. A photo-detection on path a projects photon 2 onto the state

$$a\tilde{\alpha}(\alpha|h\rangle_2|0\rangle_2 + \beta|v\rangle_2|1\rangle_2) \pm \frac{b}{\sqrt{2}}(|1\rangle_2|h\rangle_2 + |0\rangle_2|v\rangle_2), \tag{12}$$

which considering Eqs. (7) can be cast in the form

$$(a|h\rangle_2 \pm b|v\rangle_2)(a|0\rangle_2 \pm b|1\rangle_2). \tag{13}$$

This state clearly corresponds to a successful cloning process in which the states to be cloned are encoded in photon 1 and its two clones are encoded in photon 2. Finally, a photo-detection on path b leads to the state

$$(a|h\rangle_2 \mp b|v\rangle_2)(a|0\rangle_2 \mp b|1\rangle_2), \tag{14}$$

for photon 2. In this case the cloning attempt interchanges the clones of states $|\psi_\pm\rangle_1$. This situation can be solved by applying a recovery unitary operation (RUO) on photon 2 conditional to a photo-detection on path b. This operator corresponds to a phase shift of π on path 1 and half-wave plates HWP on paths 0 and 1 with angle $\theta = 0$. After these transformations the state of Eq. (14) is mapped onto the state of Eq. (13) representing a new successful cloning attempt. Considering state $|\phi\rangle$ only we achieve a cloning probability of $p/2$, where p is given by Eq. (2) with $|\langle\psi_-|\psi_+\rangle| = \cos(\theta)$, that is we obtain the half of the maximum probability. It is possible to achieve the maximum probability by considering the action of interferometer I_2 onto state $|\tilde{\phi}\rangle$, which is analogous to the action of I_1 onto state $|\phi\rangle$. The arms of this interferometer are formed by the propagation paths $\tilde{0}$ and $\tilde{1}$ of photon 1. In this case a photo-detection on path 0 leads to a failure of the cloning process while photo-detections on paths c and d leads to successful cloning attempts after the application of recovery unitary operations. A photo-detection on path c requires the application of a CNOT transformation between path and polarization of photon 2. This operation is carried out with the help of a polarizing beam splitter followed by half-wave plates on each path. The first half-wave plate generates the transformation

$$\frac{1}{\sqrt{2}}(|h\rangle_2 + |v\rangle_2) \rightarrow (\alpha|h\rangle_2 + \beta|v\rangle_2), \tag{15}$$

on path 0 and the second half-wave plate implements the inverse of the previous transformation on path 1. Thereafter, paths 0 and 1 are interchanged. Finally, a photo-detection on path d requires the application of the recovery operation employed in the case of a photo-detection on path c followed by the recovery operation employed in case of a photo-detection on path b. Thus, the action of both interferometers I_1 and I_2 leads to an overall success probability of the experimental setup here proposed which is given by the optimal cloning probability p of Eq. (2).

Other two strategies can be employed to transmit clones to a distant party. We can first generate the clones and then teleport them. In this case the clones are generated locally and the teleportation process requires two maximally entangled channels, one for each clone. The inverse process also leads to the transmission of clones. The states to be cloned are first teleported and then cloned. In this case the generation of the clones it is also local.

From the viewpoint of the resources employed in the proposed experimental setup, none of these two strategies can be implemented. We have made use of two photons each endowed with two Hilbert spaces corresponding to polarization and path. Initially both photons are in a maximally entangled state of polarization being the spatial degree of freedom of both photons in a separable state. In the previous strategies the cloning is local and must be carried out in a single photon. Since the PQCM requires three Hilbert spaces and each photon has only two degrees of freedom, this task can not be performed.

The basic building blocks of the proposed setup have been already experimentally implemented. The set of transformations acting on photon 1 have been used to implement a Bell measurement acting onto a pure polarization-path entangled state of a single photon [18], to conclusively increase or decrease the inner product between two nonorthogonal states [19] and to control the interference fringes in an optical quantum eraser [20, 21]. All this were implemented with twin-photons at 702.2 nm produced by pumping a crystal with a continuous wave (cw) Ar^+ ion laser at 351,1 nm with output power between 100-150 mW. Let us note that half-wave plates HWP_3 and HWP_7 can be merge into s single half-wave plate. The set of transformations acting on photon 2 are the lossy interferometers I_1 and I_2. Each one of these is composed of a set of transformations which are similar to the transformations acting onto photon 1 but replacing one of the mirrors used to close the interferometer by a polarizing beam splitter, which allow the losses. The input ports of interferometers I_1 and I_2 receive photons originated at two different polarizing beam splitters (PBS_3 and PBS_4 in Fig. 1). Here, half-wave plates HWP_1 and HWP_s can also be merge into a single half-wave plate. This class of arrangement has also been already used in experiments. In particular it was used to implement quantum teleportation [22] of a qubit via a quantum channel formed by two modes of the electromagnetic field populated by the vacuum and a single photon Fock state. This experiment was carried out using twin-photons at 727.6 nm generated by pumping a crystal by a cw Ar^+ ion laser at 363.8 nm with an average output power close to 100 mW. The proposal requires the application of recovery unitary operators, all these are well within the reach of current experimental techniques. Nevertheless, since quantum tomography must be used on the clones to asses the quality of the cloning process it is simpler to avoid the use of the recovery unitary operators and perform the tomography on the cloned states and compare them with the states prior to the application of the recovery unitary operators, as in the case of passive quantum teleportation [23].

An alternative physical implementation of this nonlocal probabilistic cloning can be based on the so called spatial qubits [24, 25]. In this case the source consists of a crystal producing twin-photons generated via spontaneous parametric down conversion of type I in the state $|h\rangle_1|h\rangle_2|\xi\rangle_{1,2}$, where the transversal momentum correlations of both photons is described by the entangled state $|\xi\rangle_{1,2}$. Placing a single half-wave plate on path of photon 1 it is possible to encode the states to be cloned as $|\psi_\pm\rangle_1|h\rangle_2|\xi\rangle_{1,2}$. Since the polarization degree of freedom already encodes the states to be teleported we need to resort to another degree of freedom of photon 1. For this purpose we place at the propagation path of each photon an optical diffractive element, such as for instance a double-slit array or spatial light modulator. It has been experimentally shown that this together with the manipulation of the angular spectrum of the pump field allow to control the transversal correlations of twin-photons to produce effective higher dimensional spatial qudits [24]. Immediately after the double-slit arrays, one at each propagation path, the state of photons 1 and 2 is given by

$$|\Psi_\pm^{(1)}\rangle = |\psi_\pm\rangle_1|h\rangle_2\frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2), \qquad (16)$$

where the *slit states* $|0\rangle_i$ and $|1\rangle_i$ denote the state of photon $i = 1$ or $i = 2$ after passing through slits 0 or 1 respectively at each double-slit array. We can now identify the copy system with a spatial qubit of photon 1, that is $|0\rangle_c \to |0\rangle_1$ and $|1\rangle_c \to |1\rangle_1$, and the ancilla system with a spatial qubit of photon 2 together with its polarization degree of freedom. After the double-slit array photon 1 propagates until reaching the set of polarizing beam splitters PBS_3 and PBS_4. Free space propagation and tomography of this class of states has been experimentally demonstrated within laboratory scales [25, 26] up to a fidelity of 98%. Thereafter, photons 1 and 2 follow the set of transformations indicated in Fig. 1.

IV. SUMMARY

In summary, we propose an experimental setup for implementing the optimal probabilistic quantum cloning machine. The proposal is based on the use of two photons each endowed with two Hilbert spaces. Initially both photons are in a maximally entangled polarization state.

The states to be cloned are encoded in the polarization of one of the photons while the two perfect clones are encoded in the resting photon. Here, one of the clones is encoded in polarization and the other clone is encoded in the spatial degree of freedom. Thereby, the cloning process succeeds with the optimal probability and in a non-local way. The cloning process is characterized by the use of conditional operations in a process similar to quantum teleportation.

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